Title: Geometric Phases in String Theory

Date: Dec 11, 2007 11:00 AM

URL: http://pirsa.org/07120006

Abstract: I will describe the emergence of geometric (Berry) phases in supersymmetric systems.
In theories with degenerate states, non-Abelian geometric phases can arise.
I show how supersymmetry helps to ensure the existence of this phenomenon by invoking the examples of systems with (2,2) and (4,4) supersymmetry. In the former, I show how instantons contribute crucially to the form of the non-Abelian phase.
The latter system applies to D0-D4 brane dynamics in string theory, leading to a surprising re-interpretation of the Berry phase in terms of gravitational precession of a probe brane.
Towards the end will comment on some work in progress on the geometric phase in (8,8) SUSY systems.

Geometric Phases in String Theory

Julian Sonner
DAMTP and Trinity College

Perimeter Institute, Tuesday 11 Dec 2007

Goals of the Project

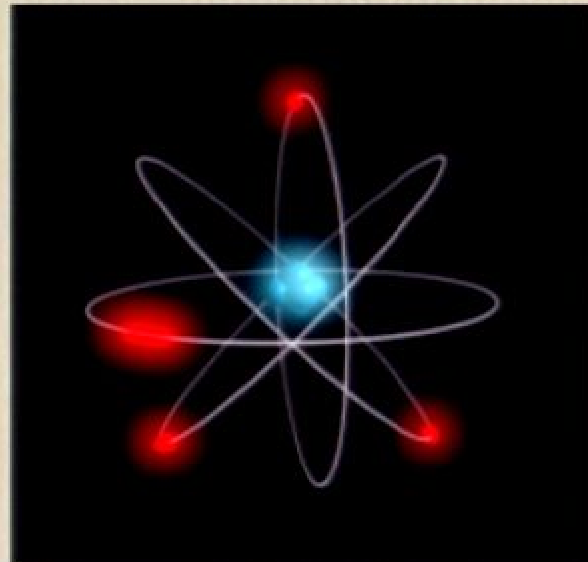
- Compute non-Abelian Berry's Phase in strongly interacting QM systems
    - "The Geometric Phase in Supersymmetric Quantum Mechanics"; arXiv 0709.0731 and Phys. Rev. D. (in press)
    - "The Geometric Phase and Gravitational precession of D-branes"; arXiv 0709.2136 and Phys. Rev. D (in press)
    - + to appear soon...
    - Work done in collaboration with Chris Pedder and David Tong
- Find applications to condensed-matter systems and/or topological quantum computation

Contents of the Talk

- Review of Geometric Phase
- Extension to non-Abelian Geometric Phase
- (2,2): briefly
- (4,4): Yang Monopole and D0-D4 system
- Overview + Outlook on (8,8) model
- Conclusions

Berry Philosophy

Parameters



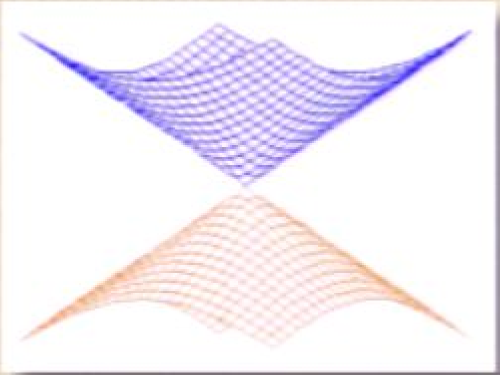
Hamiltonian

- Set system up in a particular energy eigenstate
- Change parameters slowly: Adiabatic theorem means that system clings on to eigenstate

special slide credit: Dave Tong

Review of Berry Phase I

- Canonical Example of Abelian Berry Phase:
- Spin 1/2 in external magnetic field



$$H = \vec{B} \cdot \vec{\sigma}$$

- slowly change magnetic field
- Adiabatic Theorem: Cling on to eigenstate
- Quantum Evolution gives law of parallel transport

Review of Berry Phase II

(the canonical example)

$$H_{1/2} = \vec{B} \cdot \sigma$$

- Quantization: $H_{1/2}|B_{\pm}\rangle = \pm B|B_{\pm}\rangle$
- Pick ground state and normalize to zero energy

$$\mathcal{H} = H_{1/2} - |\vec{B}|$$

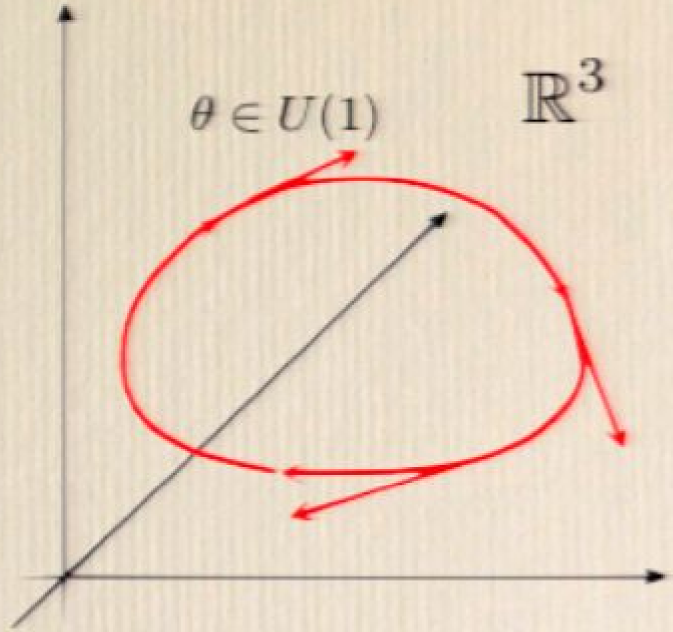
- Now ask what happens as $\vec{B}$ is varied

Review of Berry Phase III

(the general picture)

- Induce gauge connection on $\mathbb{R}^3$

$$A = i\langle B_+(t)|d|B_+(t)\rangle$$



$$|B_+(t)\rangle = \exp\left(-i\int^t E_+(t')dt'\right)\exp\left[i\int_C \langle B_+|d|B_+\rangle\right]|B_+(0)\rangle$$

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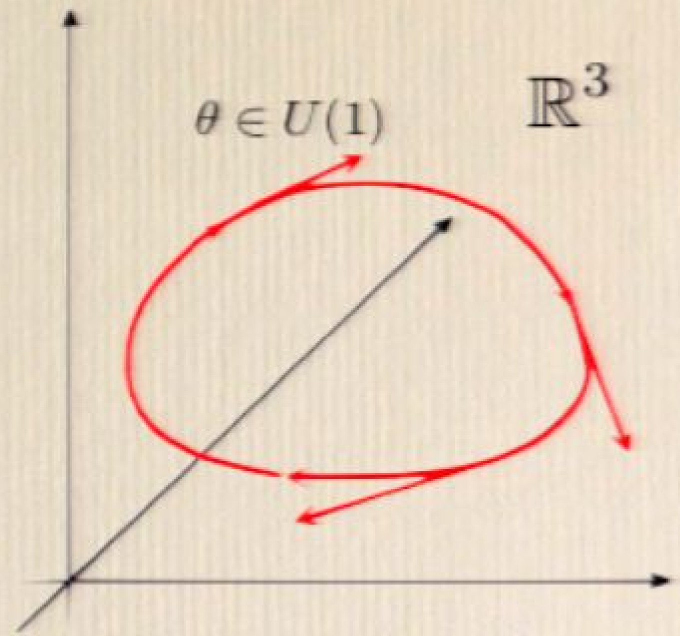

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Dynamical phase

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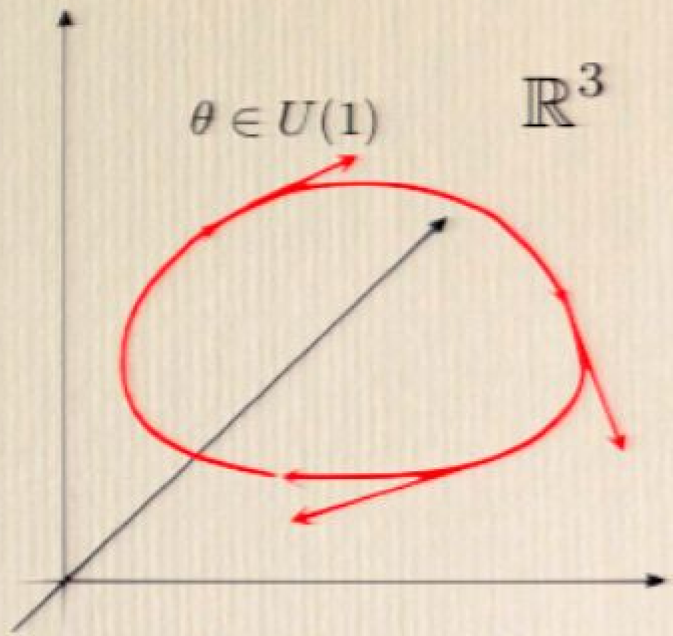

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Dynamical phase Geometric phase

Review of Berry Phase IV

(back to our example)

- Follow the ground state

$$|\Omega\rangle = \mathcal{N}|B_+\rangle$$

- Berry connection $A = i\langle\Omega|d|\Omega\rangle$ is the Dirac Monopole

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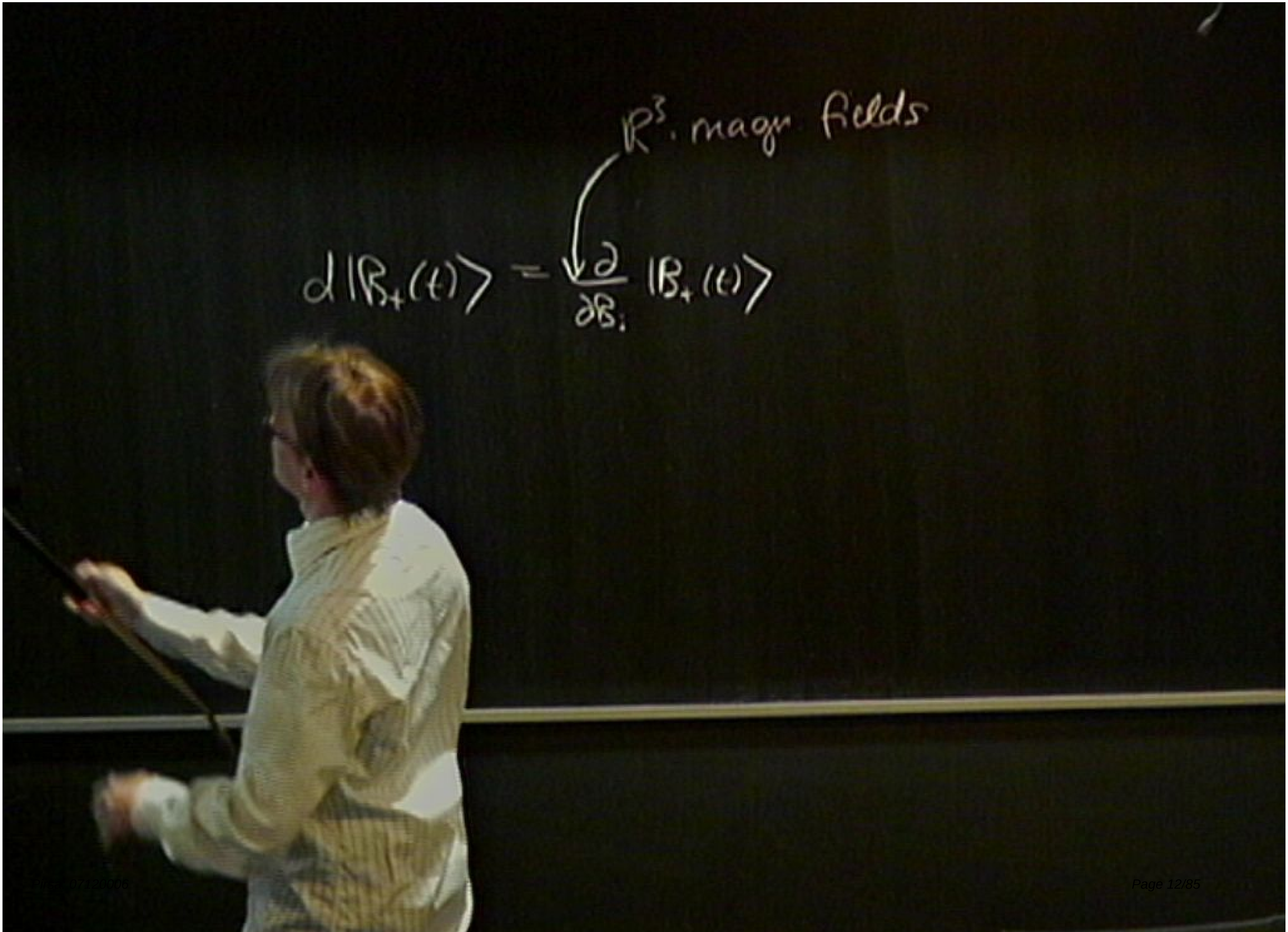

$\mathbb{R}^3$: magn. fields
$d\,|B_+(t)\rangle = \frac{\partial}{\partial B_i}|B_+(t)\rangle$

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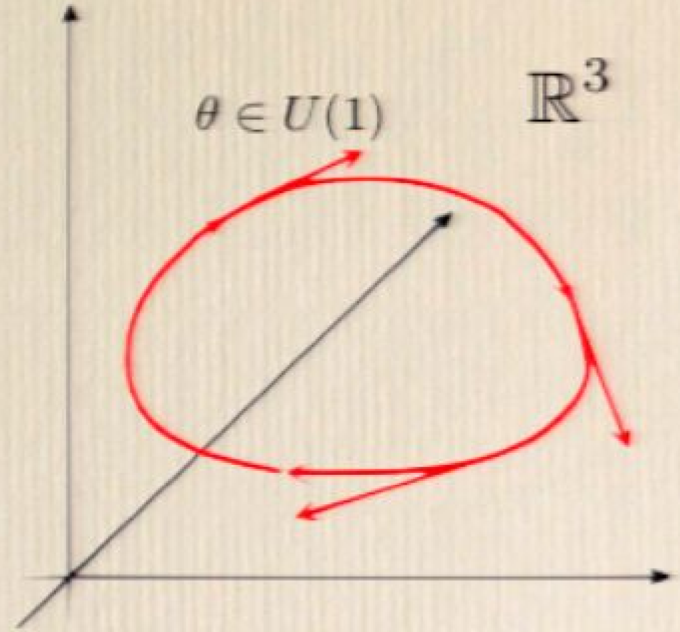

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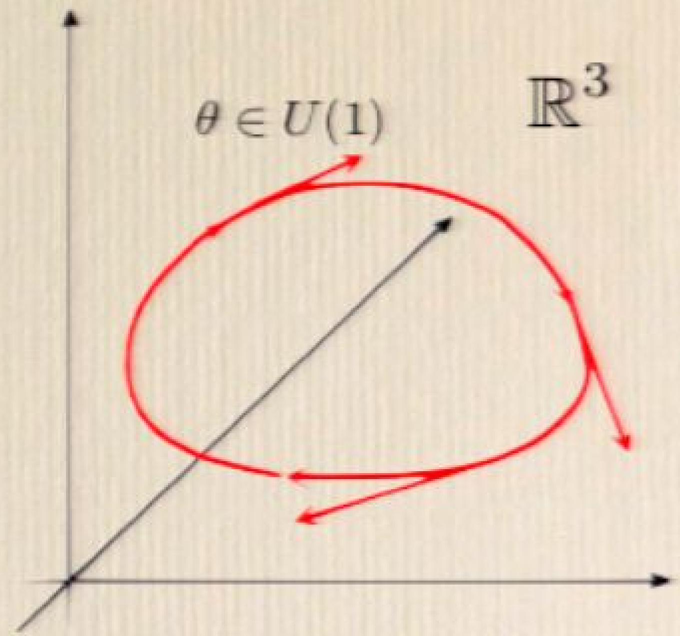

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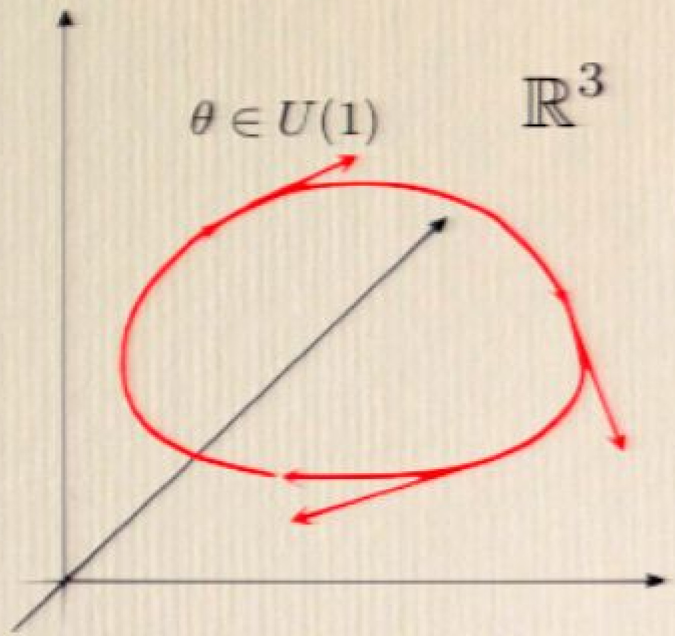

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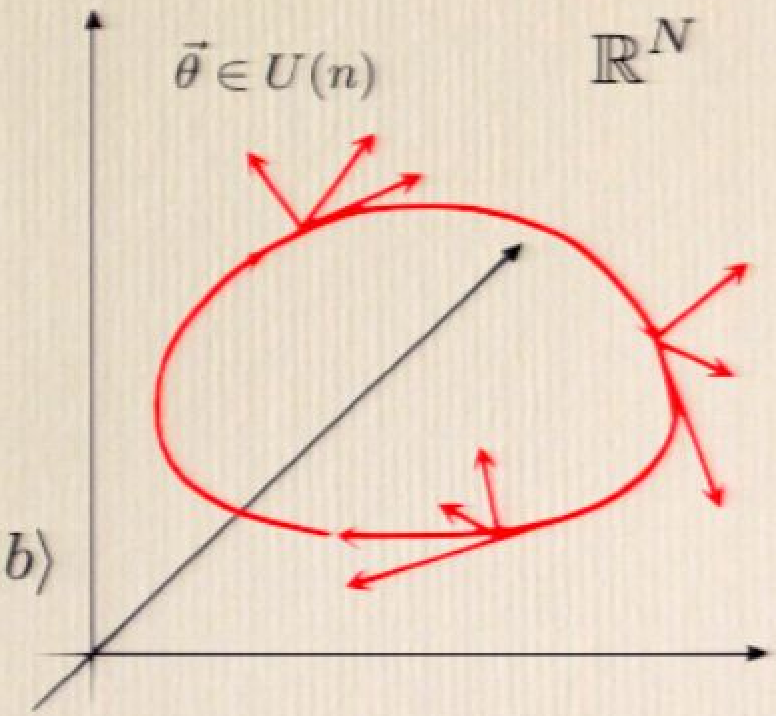

- Berry Holonomy

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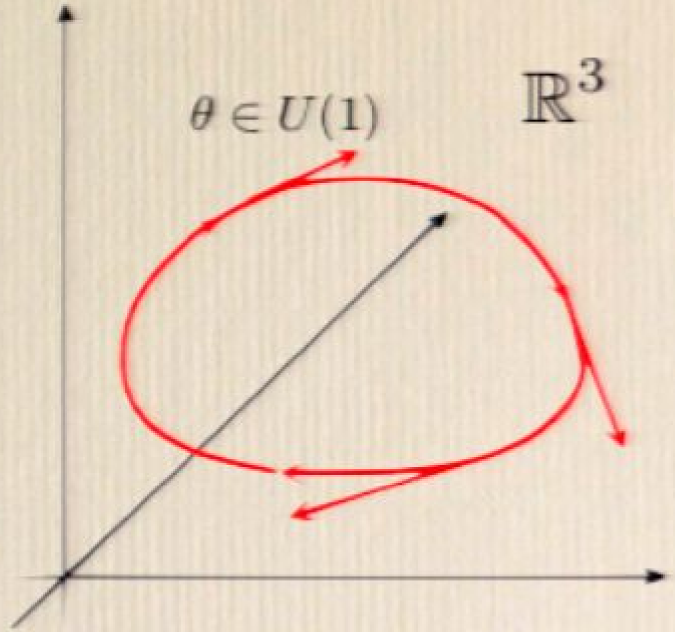

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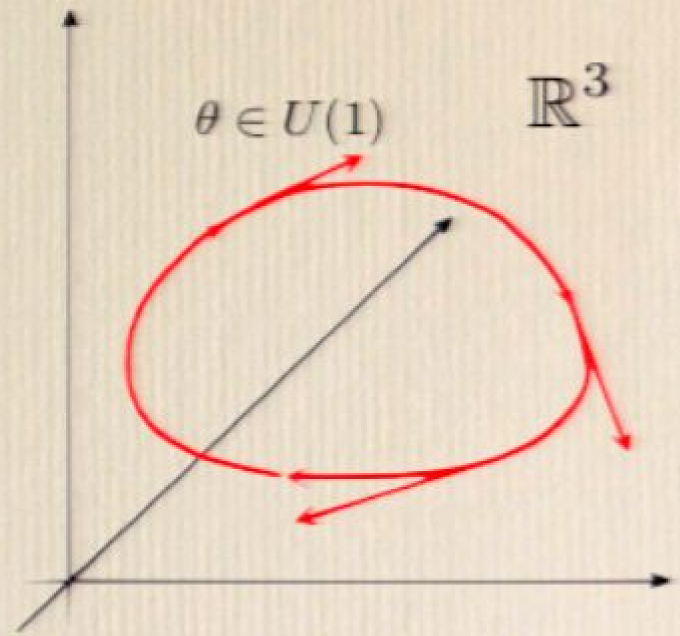

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Dynamical phase   Geo

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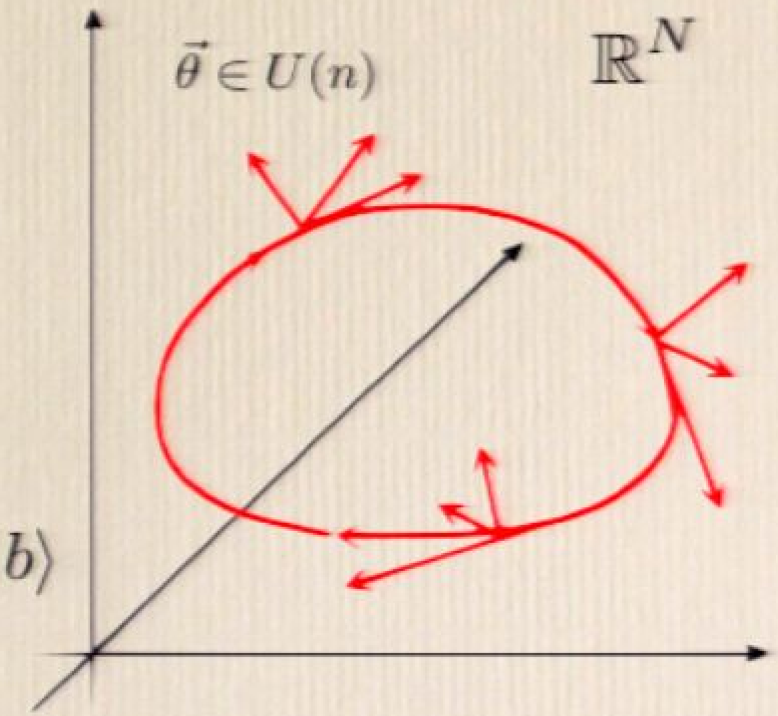

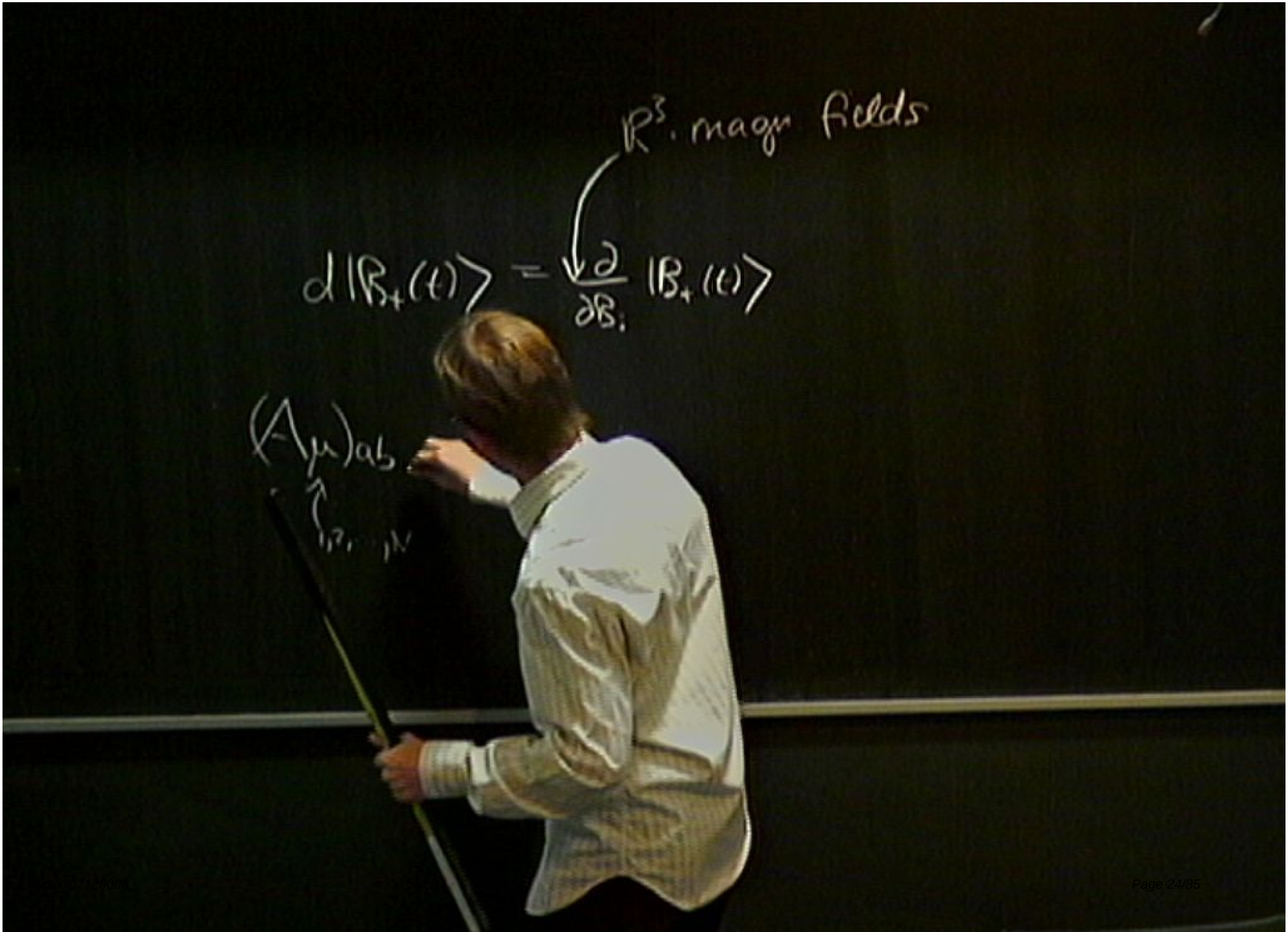
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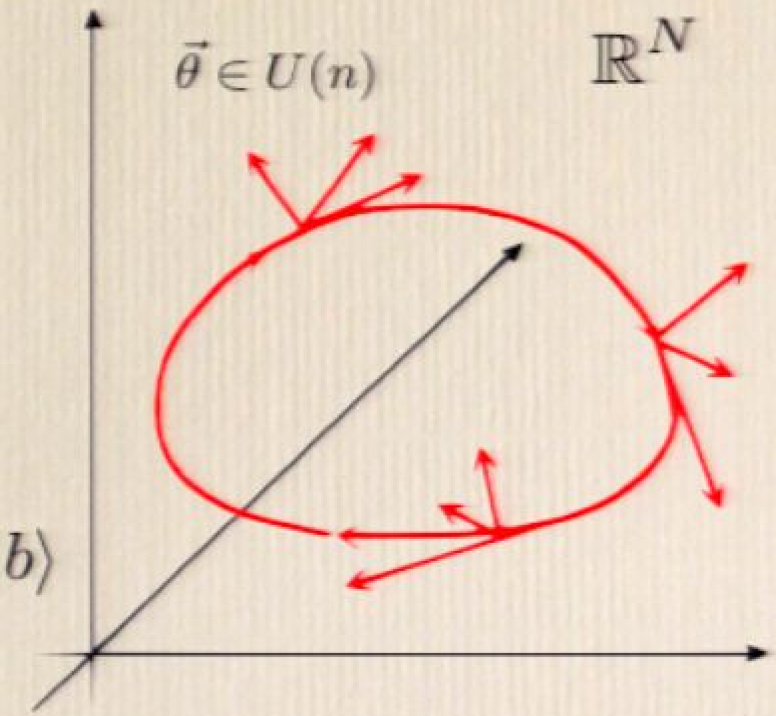

Rest of the Talk

- Demonstrate that supersymmetry and Berry phases are natural companions
- Two (very) different examples of SUSY systems and non-Abelian Berry phases
- Only have time to talk about (4,4) model in detail
- Two different mechanism to ensure degeneracy over range of parameters
- New interpretation in terms of D-brane precession

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(2,2) Geometric Phase

- (2,2) SUSY Quantum Mechanics
- Free chiral multiplet: Dirac Monopole
    - More complicated examples: non-renormalisation
- $\mathbb{CP}^1$ Sigma model with potential
    - Multiple vacua: Witten index
    - Non-Abelian Holonomy interpolates between vacua
    - Receives corrections due to BPS instantons
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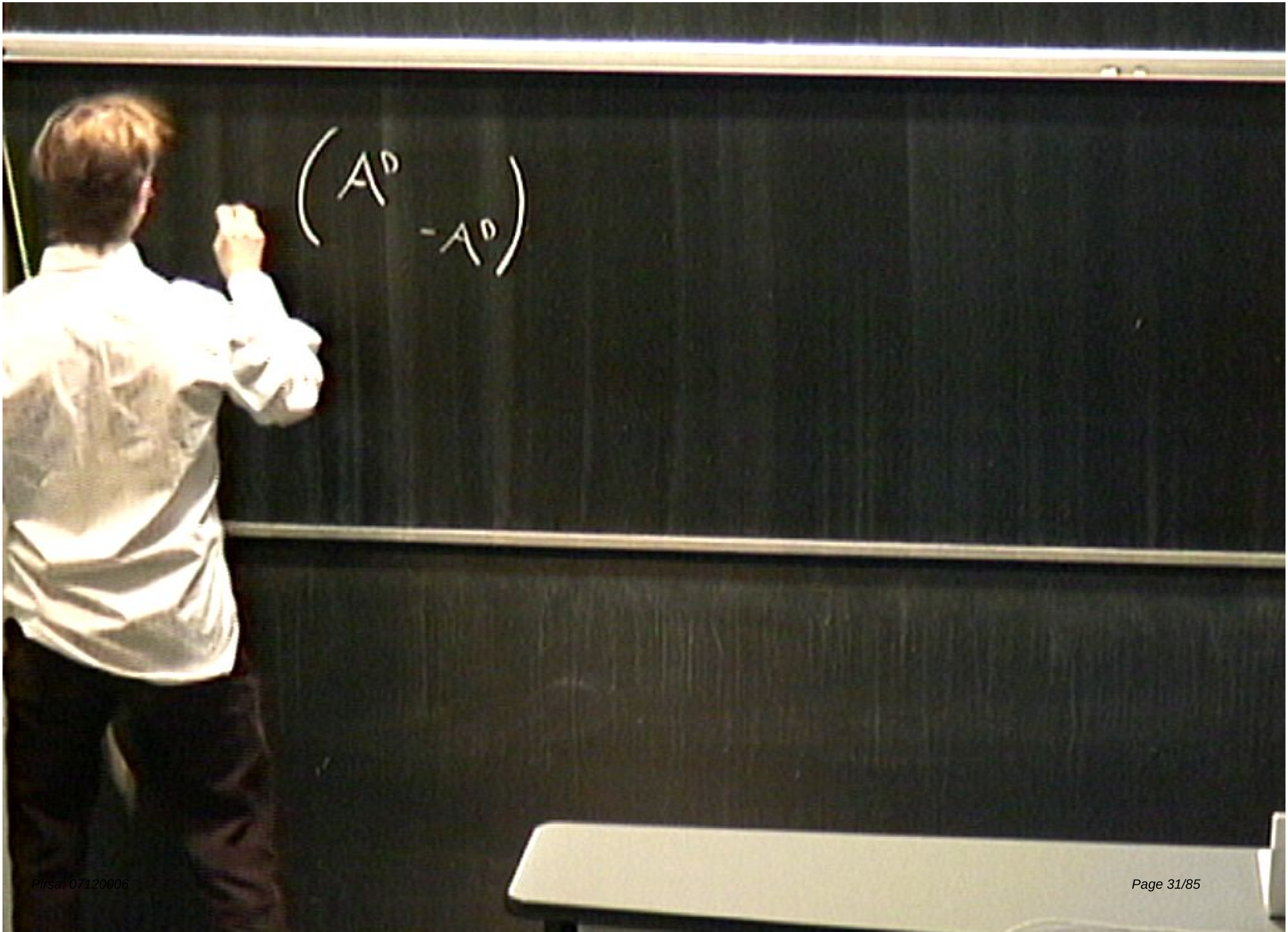
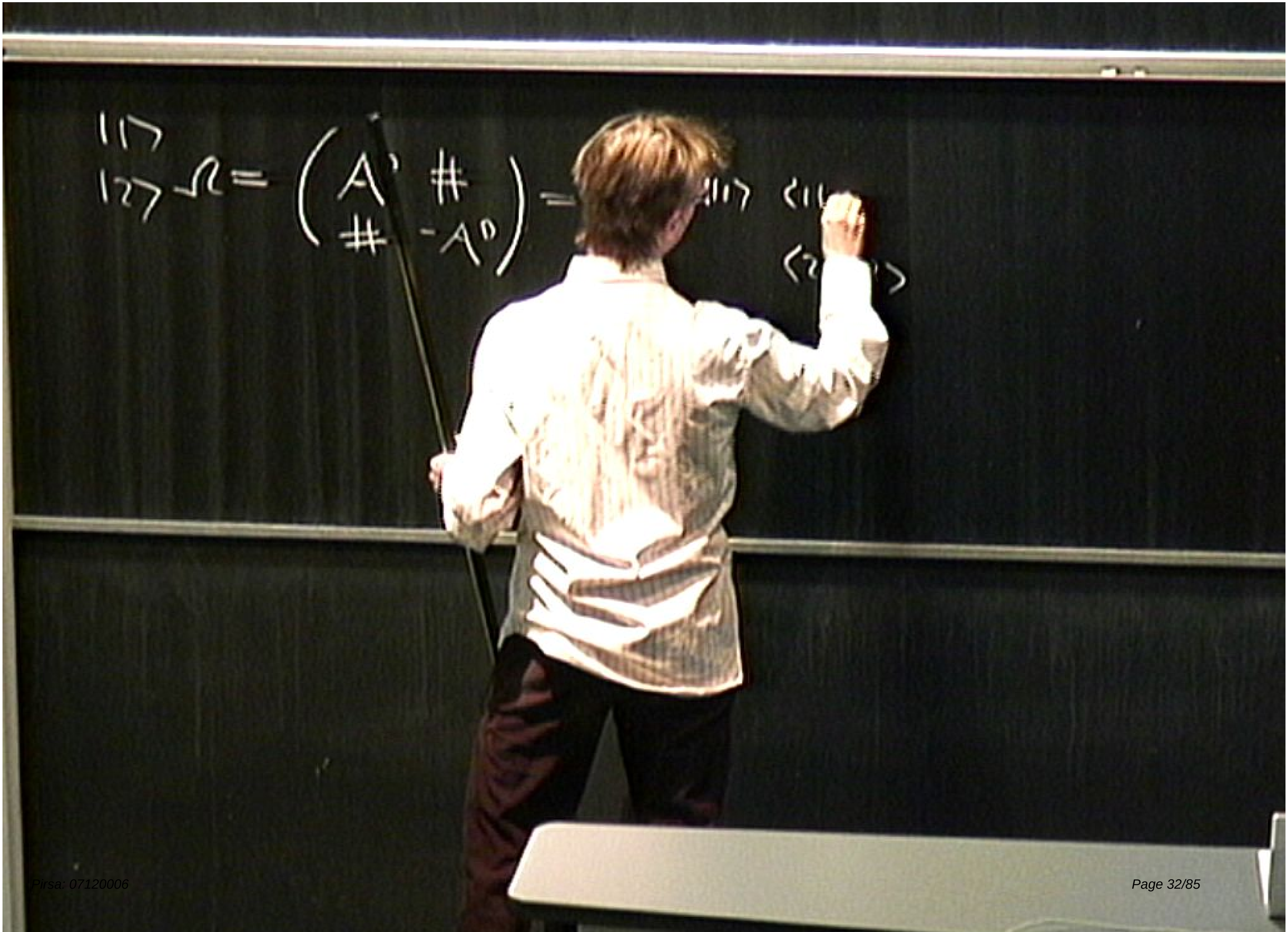
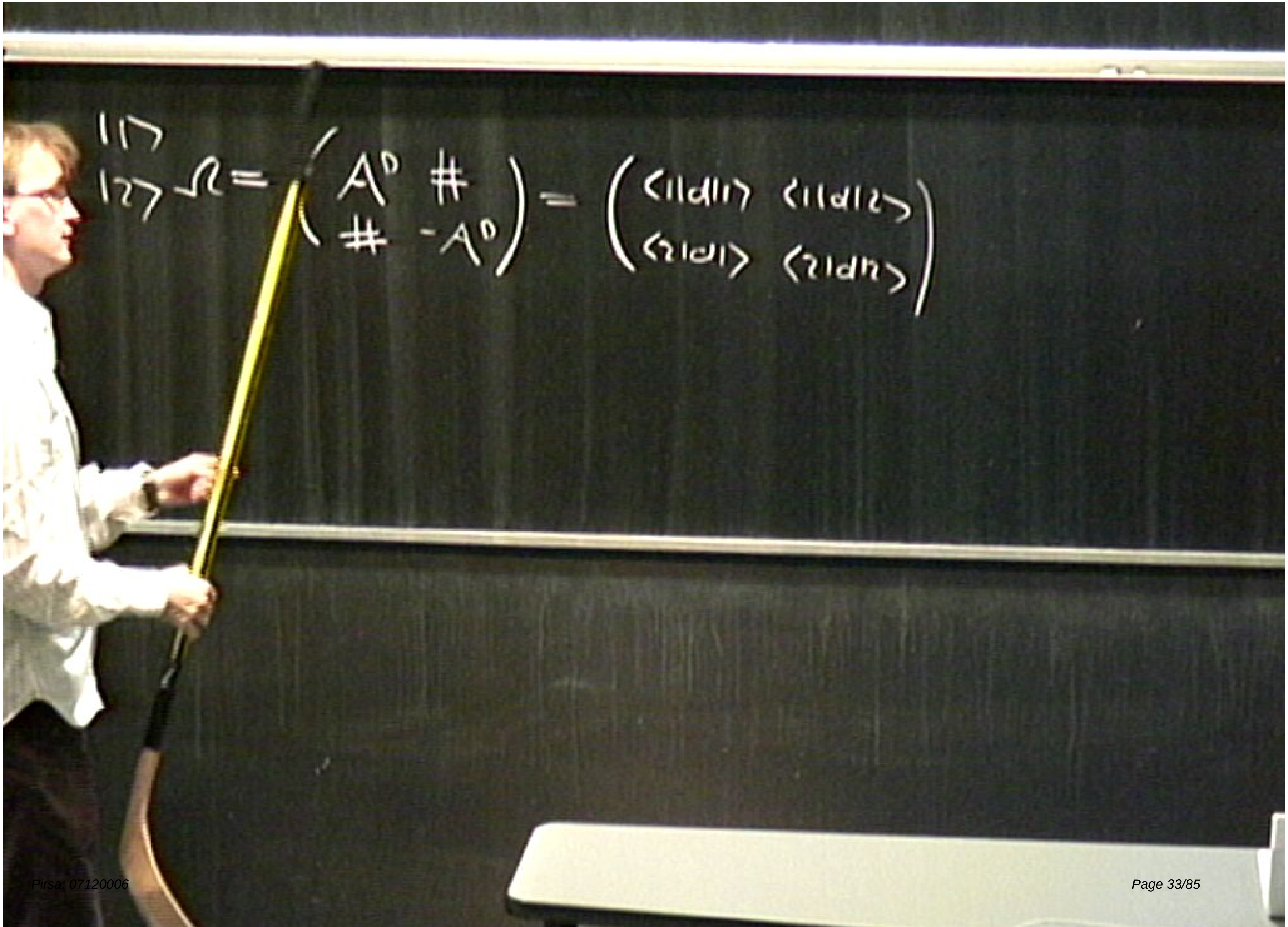

$\begin{pmatrix} A^D & \\ & -A^D \end{pmatrix}$


|1⟩
|2⟩


|1>
|2> Ω = (A⁰ # / # -A⁰) = (<1|d|1> <1|d|2> / <2|d|1> <2|d|2>)

$|1\rangle$

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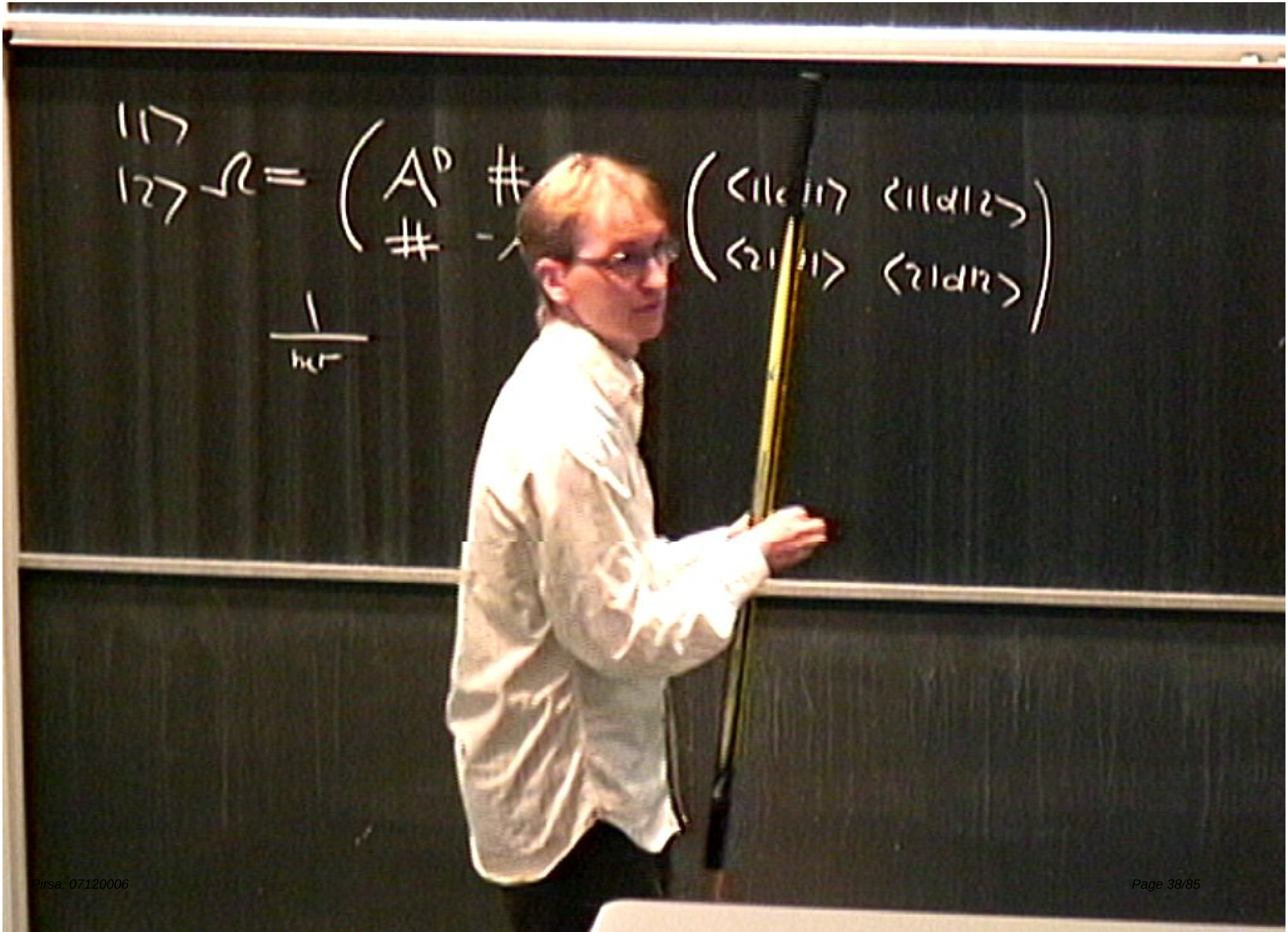

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$$\frac{1}{hcr}$$

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D0-D4 system: overview

- R-Symmetry

$$Spin(5) \times SU(2)_R \cong Sp(2) \times Sp(1)$$

- Lagrangian

$$L = L_{\mathrm{vec}} + L_{\mathrm{hyper}} + L_{\mathrm{Yuk}}$$

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D0-D4 System: Details

$$L_{\text{vector}} = \frac{1}{2g^2}\,(\dot{\vec{X}}^2 + 2i\bar{\Lambda}\dot{\Lambda})$$

$$\begin{aligned} L_{\text{hyper}} &= \sum_{i=1}^{N} |\mathcal{D}_t\phi_i|^2 + |\mathcal{D}_t\tilde{\phi}_i|^2 + i\bar{\Psi}_i\mathcal{D}_t\Psi_i - \vec{X}^2(|\phi_i|^2 + |\tilde{\phi}_i|^2) \\ &\quad -\frac{g^2}{2}(\sum_i |\phi_i|^2 - |\tilde{\phi}_i|^2)^2 - 2g^2|\sum_i \tilde{\phi}_i\phi_i|^2 \end{aligned}$$

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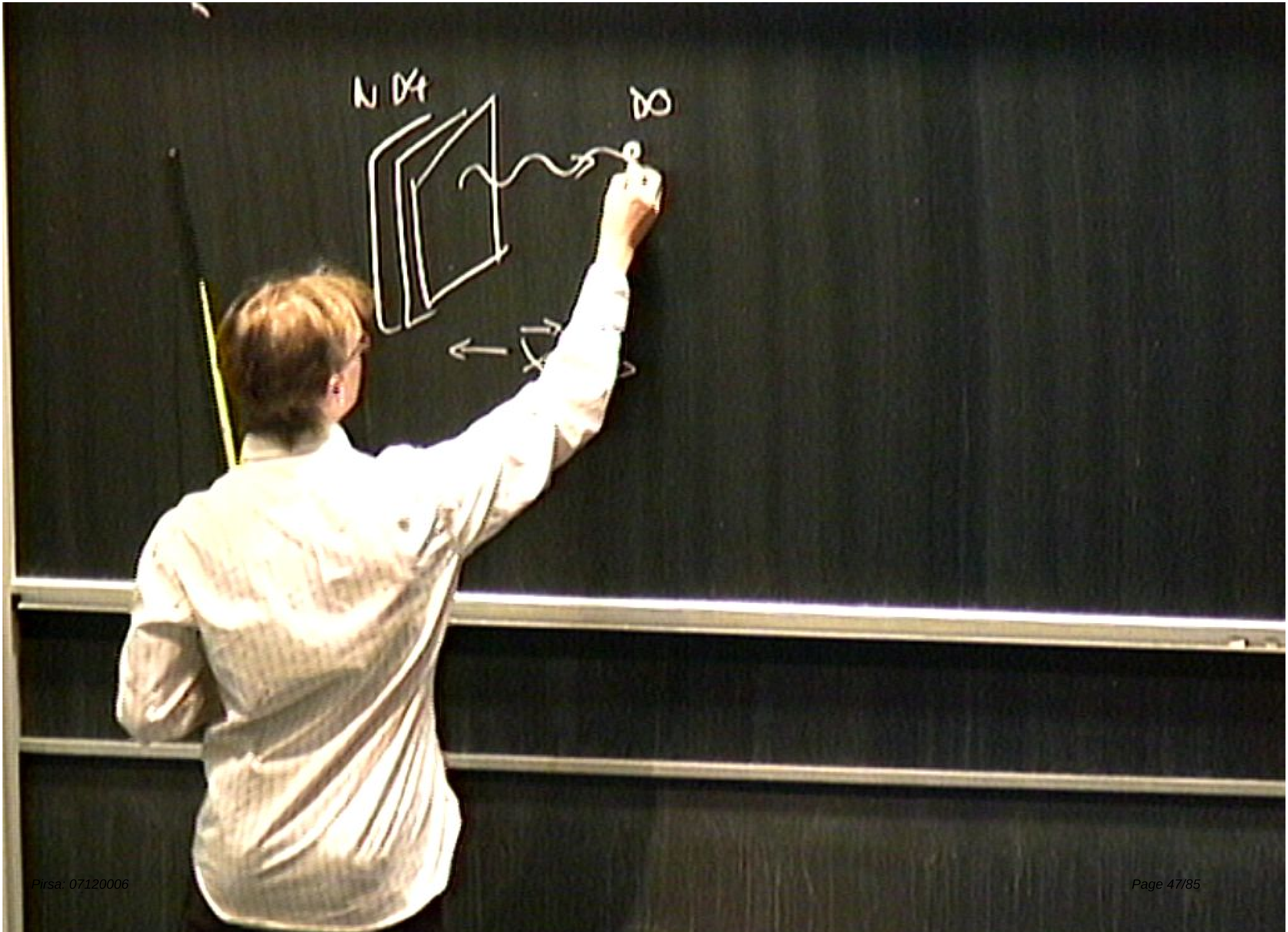
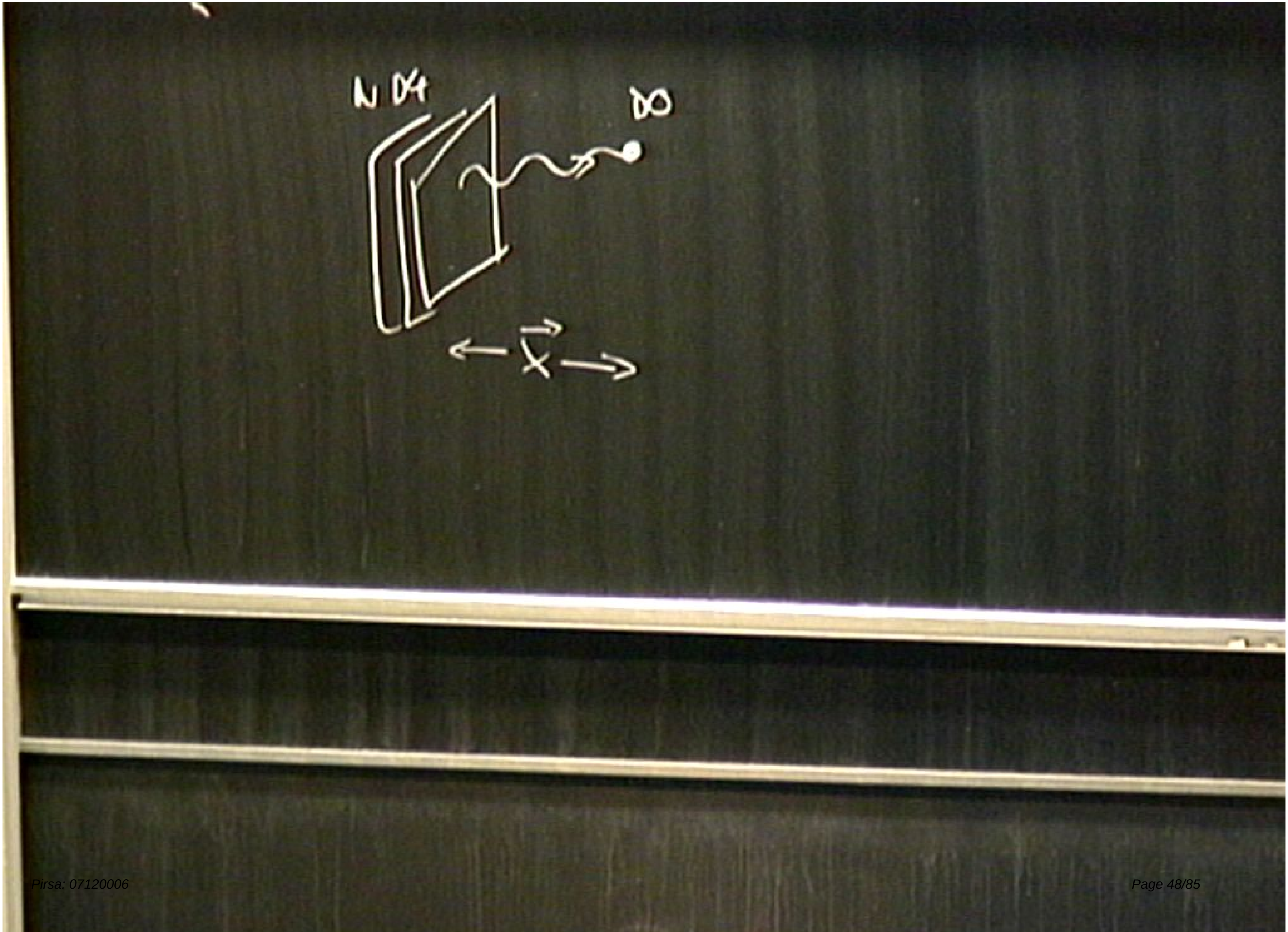

N D4
D0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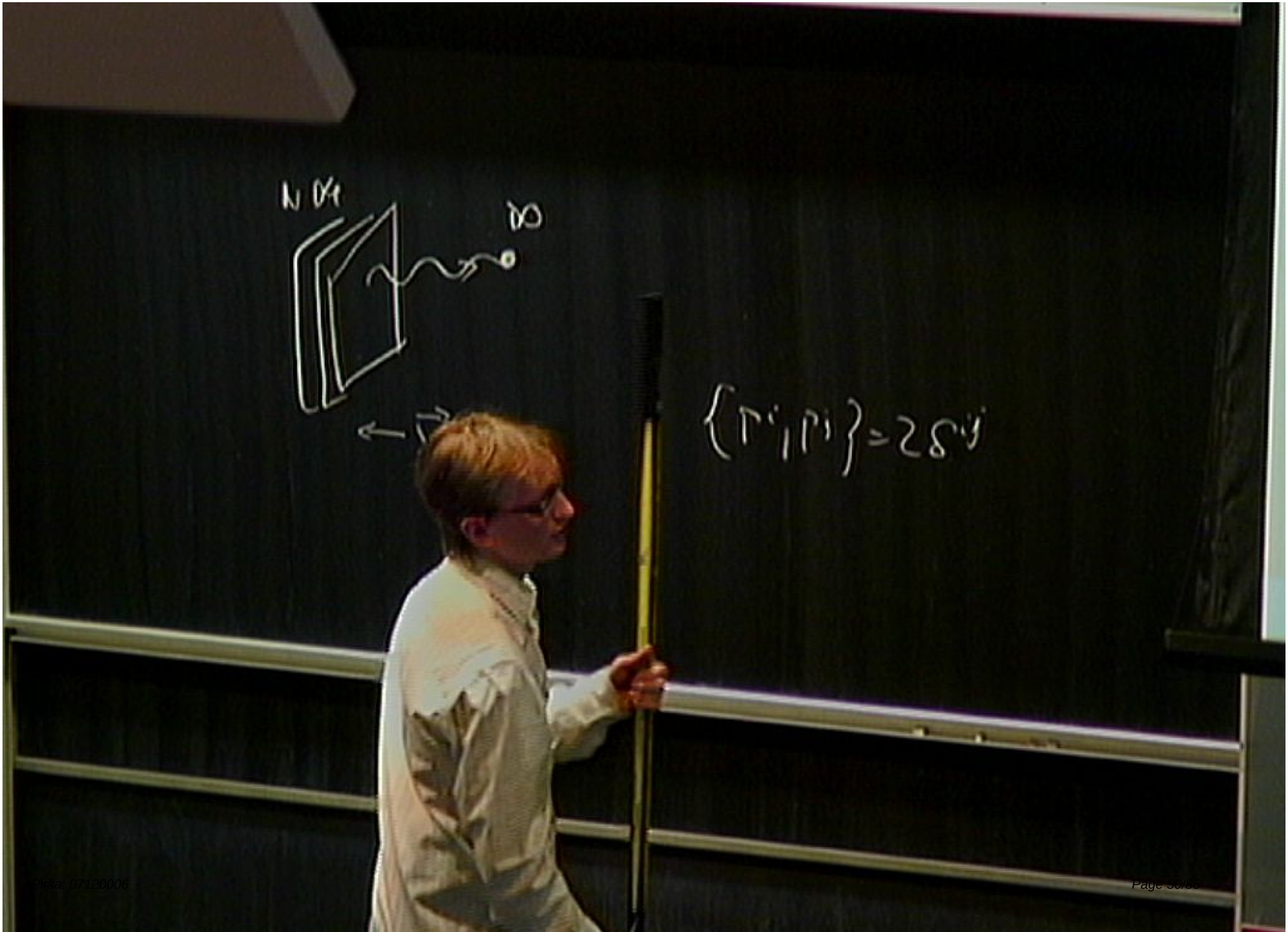
{Γi,Γj}=2δij

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Born-Oppenheimer

- Take $N = 1$ and $g^2 \to 0$

$$H = |\pi|^2 + |\tilde{\pi}|^2 + X^2 \left(|\phi|^2 + |\tilde{\phi}|^2 \right) + \bar{\Psi} \left(\vec{X} \cdot \Gamma \right) \Psi$$

State	Multiplicity	H_F Eigenvalue
$\|0\rangle$	1	0
$\bar{\Psi}_\alpha\|0\rangle$	4	$(-X)_2$, $(+X)_2$
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Table 1: The Fermionic Hilbert Space.

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$$L_{\text{hyper}} = \sum_{i=1}^{N} |\mathcal{D}_t\phi_i|^2 + |\mathcal{D}_t\tilde{\phi}_i|^2 + i\bar{\Psi}_i\mathcal{D}_t\Psi_i - \vec{X}^2(|\phi_i|^2 + |\tilde{\phi}_i|^2)$$
$$-\frac{g^2}{2}(\sum_i |\phi_i|^2 - |\tilde{\phi}_i|^2)^2 - 2g^2|\sum_i \tilde{\phi}_i\phi_i|^2$$

$$L_{\text{Yuk}} = -\bar{\Psi}(\vec{X}\cdot\vec{\Gamma})\Psi + \sqrt{2}\bar{\Psi}_\alpha(\phi\Lambda_\alpha + \tilde{\phi}^\dagger J_\alpha{}^\beta\Lambda^\star_\beta) + \text{h.c.}$$

Born-Oppenheimer

- Take $N = 1$ and $g^2 \to 0$

$$H = |\pi|^2 + |\tilde{\pi}|^2 + X^2 \left(|\phi|^2 + |\tilde{\phi}|^2 \right) + \bar{\Psi} \left(\vec{X} \cdot \Gamma \right) \Psi$$

State	Multiplicity	H_F Eigenvalue
$\|0\rangle$	1	0
$\bar{\Psi}_\alpha \|0\rangle$	4	$(-X)_2$, $(+X)_2$
$\bar{\Psi}_\alpha \bar{\Psi}_\beta \|0\rangle$	6	$-2X$, 0_4, $+2X$
$\bar{\Psi}_\alpha \bar{\Psi}_\beta \bar{\Psi}_\gamma \|0\rangle$	4	$(-X)_2$, $(+X)_2$
$\bar{\Psi}_\alpha \bar{\Psi}_\beta \bar{\Psi}_\gamma \bar{\Psi}_\delta \|0\rangle$	1	0

Table 1: The Fermionic Hilbert Space.

QM Calculation I: Yang

- Ground state undergoes no Berry phase
- 1st excited state $\bar{\Psi}_\alpha|0\rangle$
    - pick basis $P_-\lambda_\alpha\bar{\Psi}_\alpha|0\rangle \to \{|1\rangle, |2\rangle\}$
- Berry connection

$$(A_\mu)_{ab} = \frac{-X^\nu}{2X(X-X_5)}\eta^m_{\mu\nu}\sigma^m_{ab}\,,\ A_5 = 0$$

- Yang Monopole: $S^7 \xrightarrow{S^3} S^4$
- Second Chern Number $\mathcal{C}_2 = -1$

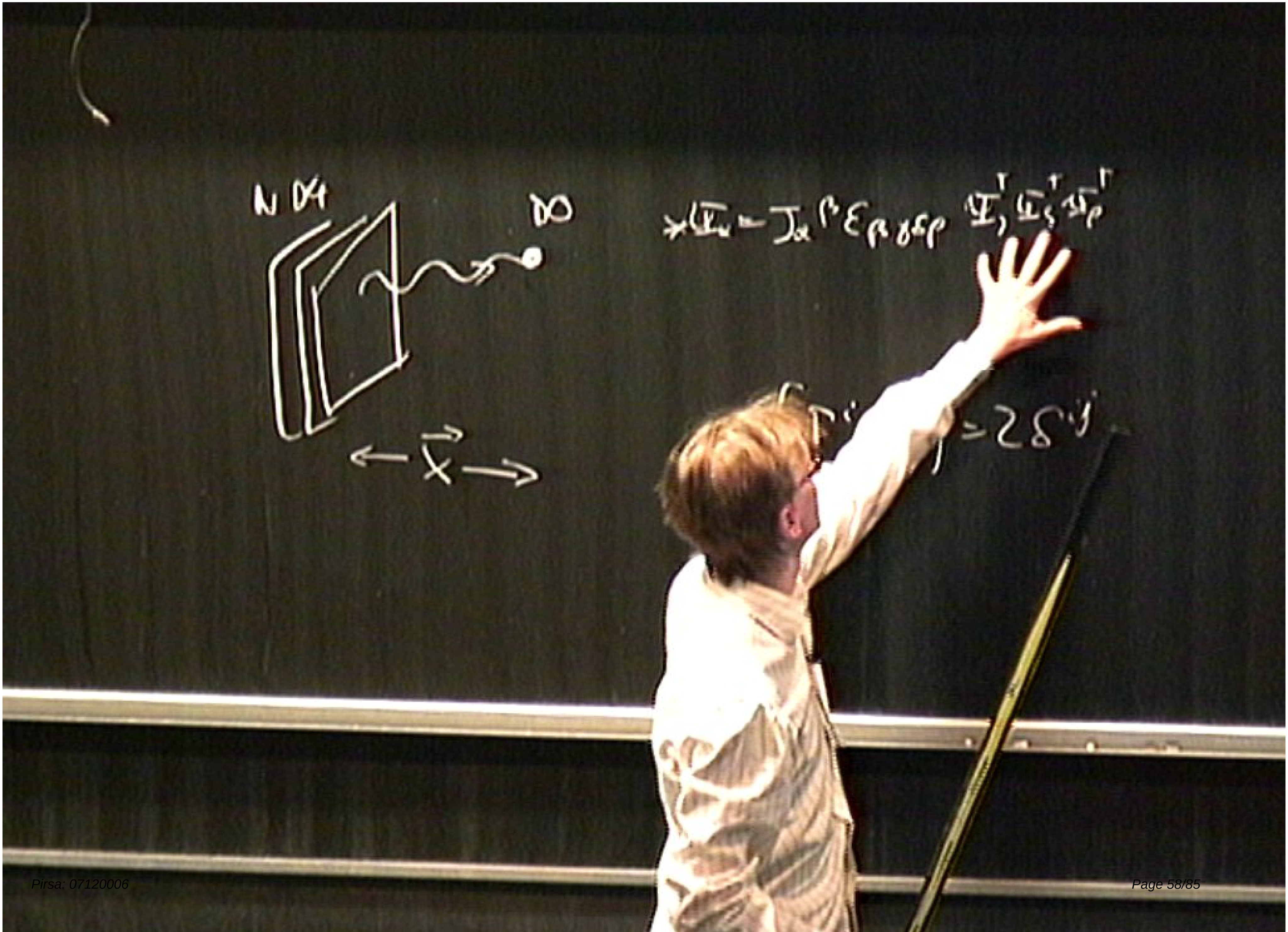
QM Calculation II: Yin

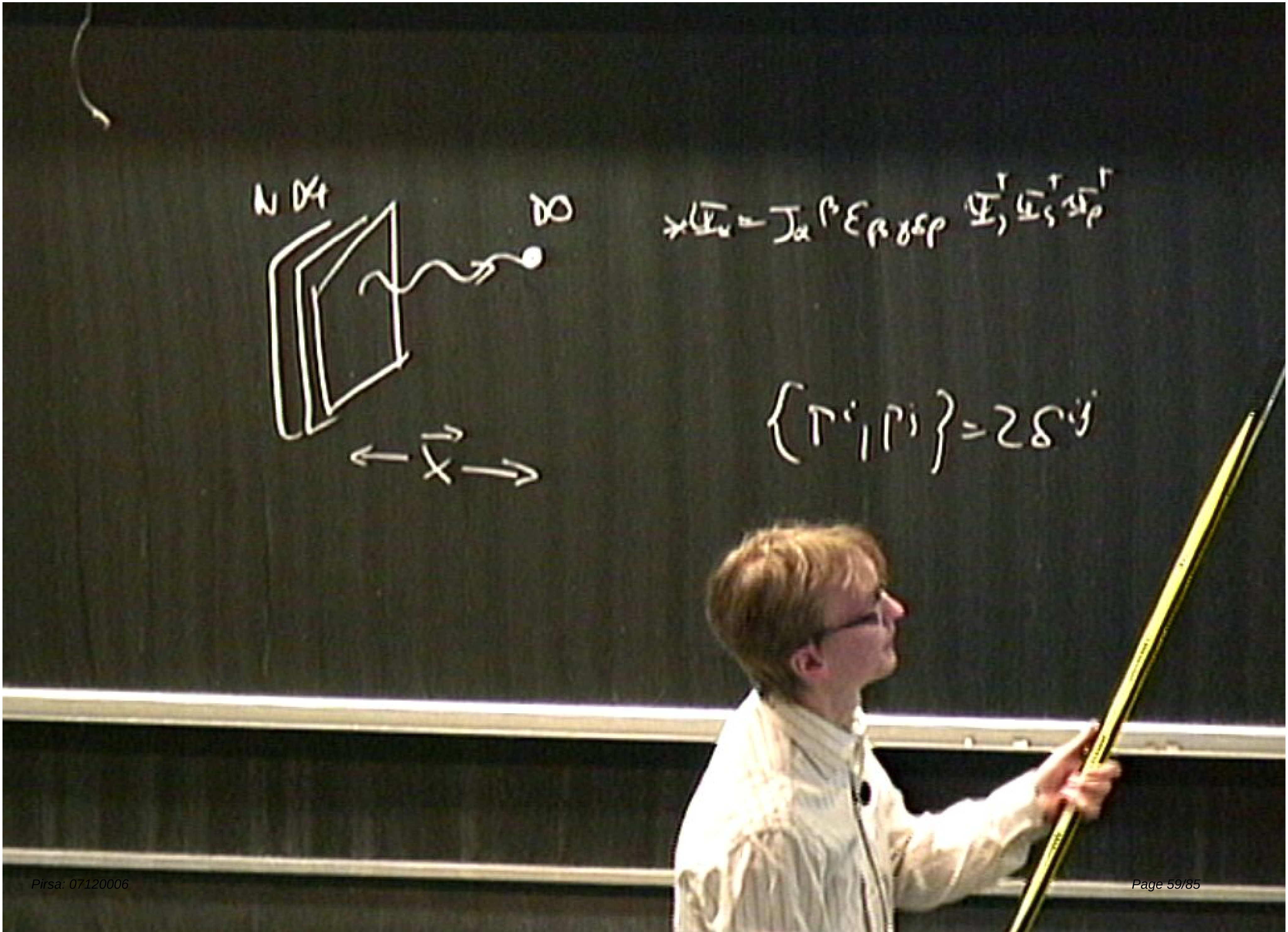
- 1st excited state $\star\bar{\Psi}_\alpha|0\rangle$
  - pick basis $\lambda_\alpha \star \bar{\Psi}_\alpha|0\rangle \rightarrow \{|3\rangle, |4\rangle\}$
- Berry connection

$$\left(\tilde{A}_\mu\right)_{ab} = \frac{-X^\nu}{2X(X - X_5)}\bar{\eta}^m_{\mu\nu}\sigma^m_{ab}\,,\ A_5 = 0$$

- Yin Monopole: Second Chern Number

$$\mathcal{C}_2 = 1$$


N D4
D0


N D4
D0
{Γi, Γj} = 2δij

QM Calculation II: Yin

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- Yin Monopole: Second Chern Number

$$\mathcal{C}_2 = 1$$

Effective Action Perspective III

(strong coupling Berry phase)

- Classical Spin precession = Geometric phase of strongly coupled quantum system
- Read off the spin connection

$$\Omega_\mu = \frac{3}{2}\frac{X^\nu}{X^2}\Gamma_{\nu\mu}$$

- Suggests new method to compute Berry phase
- Coefficient differs from weak-coupling result: Smooth interpolation or level crossings?

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- Suggests new method to compute Berry phase
- Coefficient differs from weak-coupling result: Smooth interpolation or level crossings?

What the D0 saw I

(Closed string perspective)

- Treat D0-brane as a probe

$$
\begin{aligned}
ds^2 &= \frac{1}{\sqrt{f(R)}} dx_\mu dx^\mu + \sqrt{f(R)}\, d\vec{R}\cdot d\vec{R} \\
f(R) &= 1 + \pi g_s N \frac{(\alpha')^{3/2}}{R^3}
\end{aligned}
$$

- Expand DBI action to quadratic order

What the Do saw II

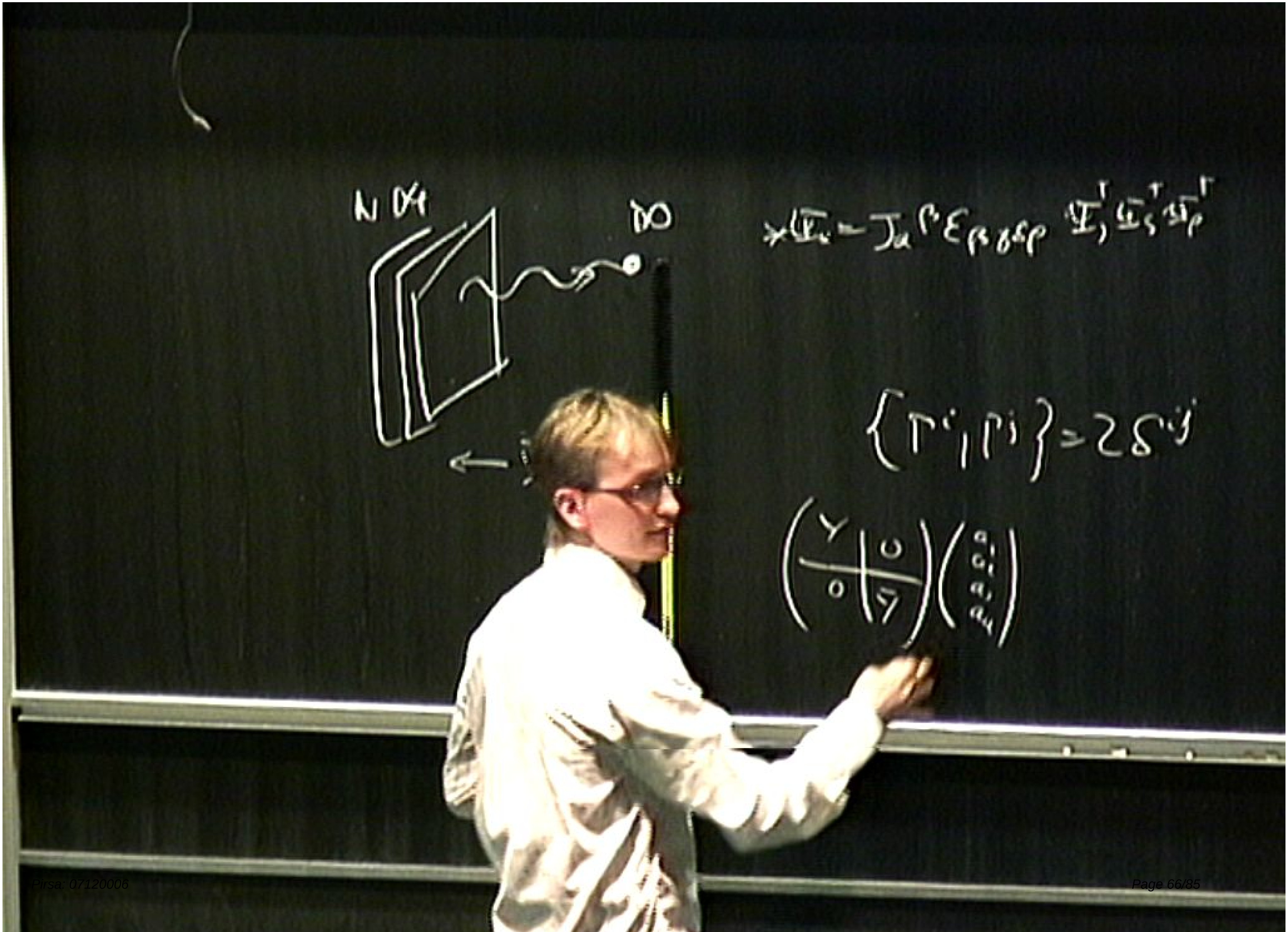
(Closed string perspective)

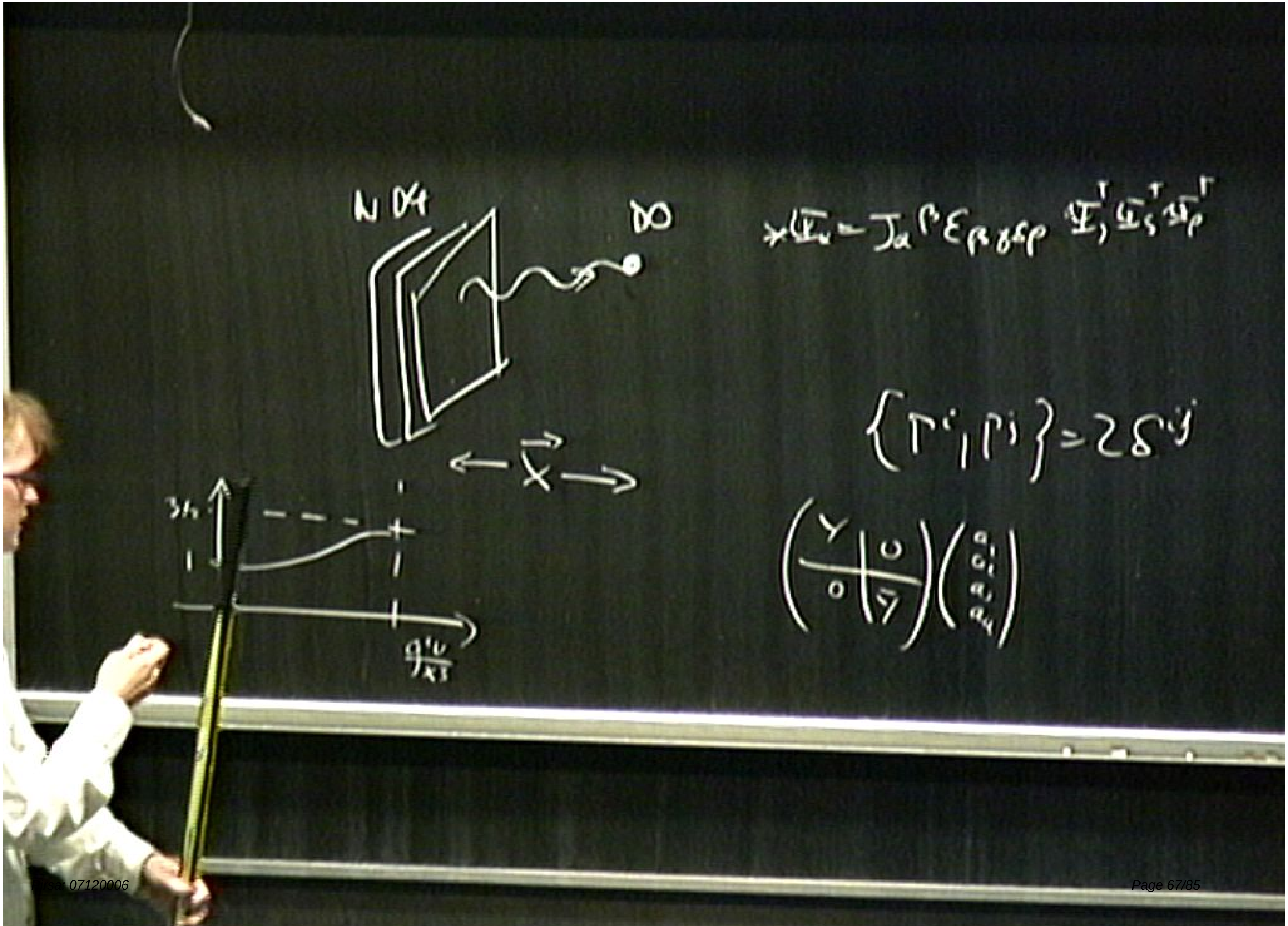
- SUSY completion (to this order)

$$L = f(X)\left(\dot{\vec{X}}^2 + i(\bar{\Lambda}\dot{\Lambda} + \dot{\bar{\Lambda}}\Lambda)\right) + i\dot{X}^\mu f_{,\nu}\bar{\Lambda}\Gamma^{\mu\nu}\Lambda$$
$$+\frac{1}{2}\left(f_{,\mu\nu} - \frac{1}{2}f_{,\mu}f_{,\nu}\right)(\bar{\Lambda}\Gamma^\mu\Lambda\bar{\Lambda}\Gamma^\nu\Lambda + \bar{\Lambda}\Gamma^\mu\bar{\Lambda}\Lambda\Gamma^\nu\Lambda)$$

- Describes spin precession of a probe particle
- Spin = R-Symmetry representation

[redacted]


N D4
D0
{Γi, Γj} = 2δij


N D4
D0
X
{Γ^i, Γ^j} = 2δ^ij

What the Do saw II

(Closed string perspective)

- SUSY completion (to this order)

$$
\begin{aligned}
L &= f(X)\left(\dot{\vec{X}}^2 + i(\bar{\Lambda}\dot{\Lambda} + \dot{\bar{\Lambda}}\Lambda)\right) + i\dot{X}^\mu f_{,\nu}\bar{\Lambda}\Gamma^{\mu\nu}\Lambda \\
&\quad + \frac{1}{2}\left(f_{,\mu\nu} - \frac{1}{2}f_{,\mu}f_{,\nu}\right)(\bar{\Lambda}\Gamma^\mu\Lambda\bar{\Lambda}\Gamma^\nu\Lambda + \bar{\Lambda}\Gamma^\mu\bar{\Lambda}\Lambda\Gamma^\nu\Lambda)
\end{aligned}
$$

- Describes spin precession of a probe particle

- Spin = R-Symmetry representation

What the D0 saw II

(Closed string perspective)

- SUSY completion (to this order)

$$L = f(X)\left(\dot{\vec{X}}^2 + i(\bar{\Lambda}\dot{\Lambda} + \dot{\bar{\Lambda}}\Lambda)\right) + i\dot{X}^\mu f_{,\nu}\bar{\Lambda}\Gamma^{\mu\nu}\Lambda$$
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- Describes spin precession of a probe particle
- Spin = R-Symmetry representation

(8,8) Berry Phase

- Consider U(2) D0-brane matrix model

- Separate branes: $SU(2) \to U(1)$
    - break down to Cartan subalgebra
    - excite a stretched string between branes
    - ask what happens as one brane moves around the other
- Berry Phase is Octonionic SO(8) connection
- Last Hopf Map

Division Algebras and SUSY

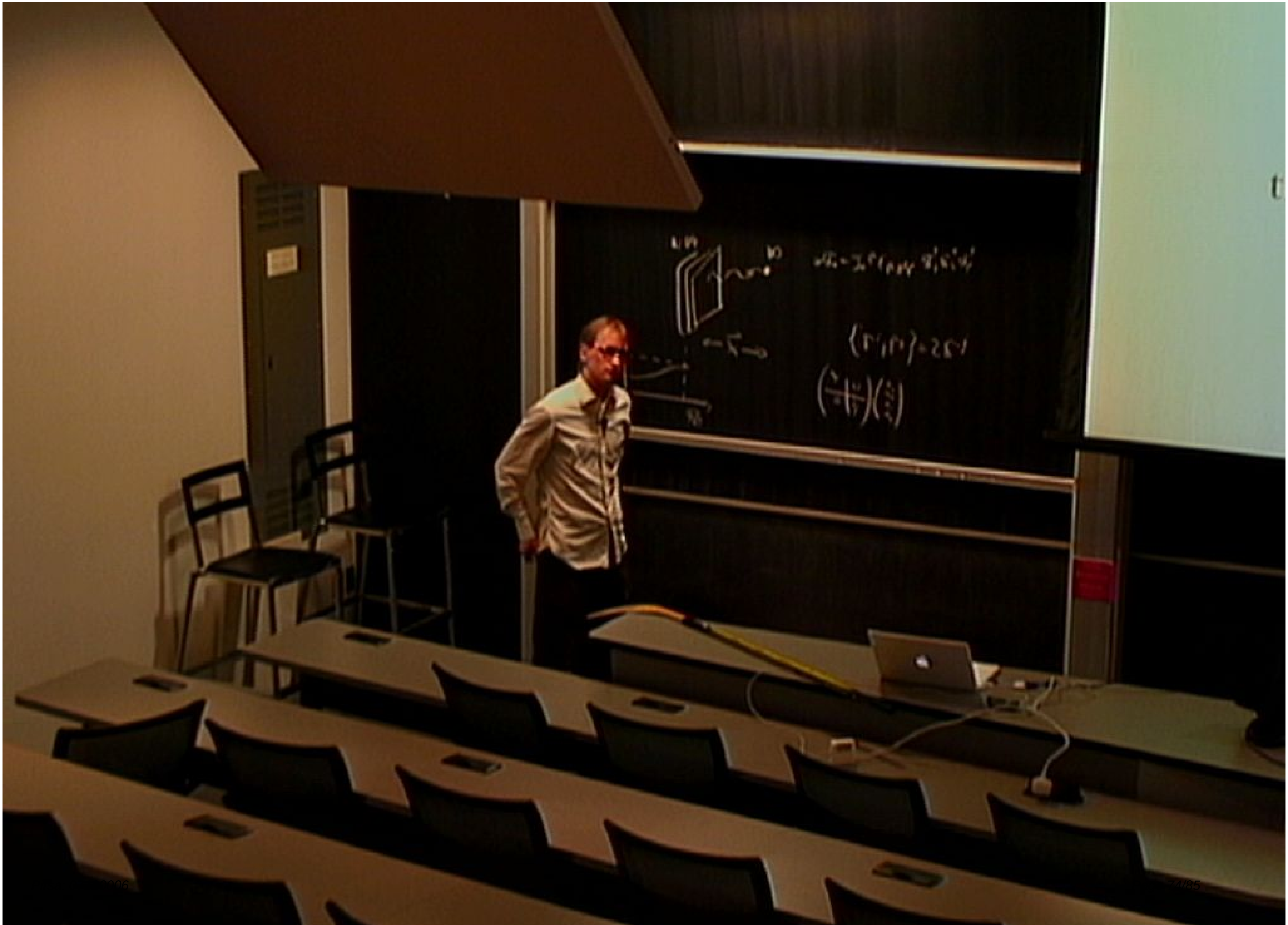
$$
\begin{array}{cccc}
S^1 & S^3 & S^7 & S^{15} \\
\big\downarrow S^0 \cong \mathbb{Z}_2 & \big\downarrow S^1 & \big\downarrow S^3 & \big\downarrow S^7 \\
\mathbb{RP}^1 & \mathbb{CP}^1 \cong S^2 & \mathbb{HP}^1 \cong S^4 & \mathbb{OP}^1 \cong S^8 \\
\big\uparrow & \big\uparrow & \big\uparrow & \big\uparrow \\
(1,1) & (2,2) & (4,4) & (8,8) \\
\pm 1 & U(1) \text{ Dirac} & SU(2) \text{ Yang} & SO(8) \text{ Oct}
\end{array}
$$

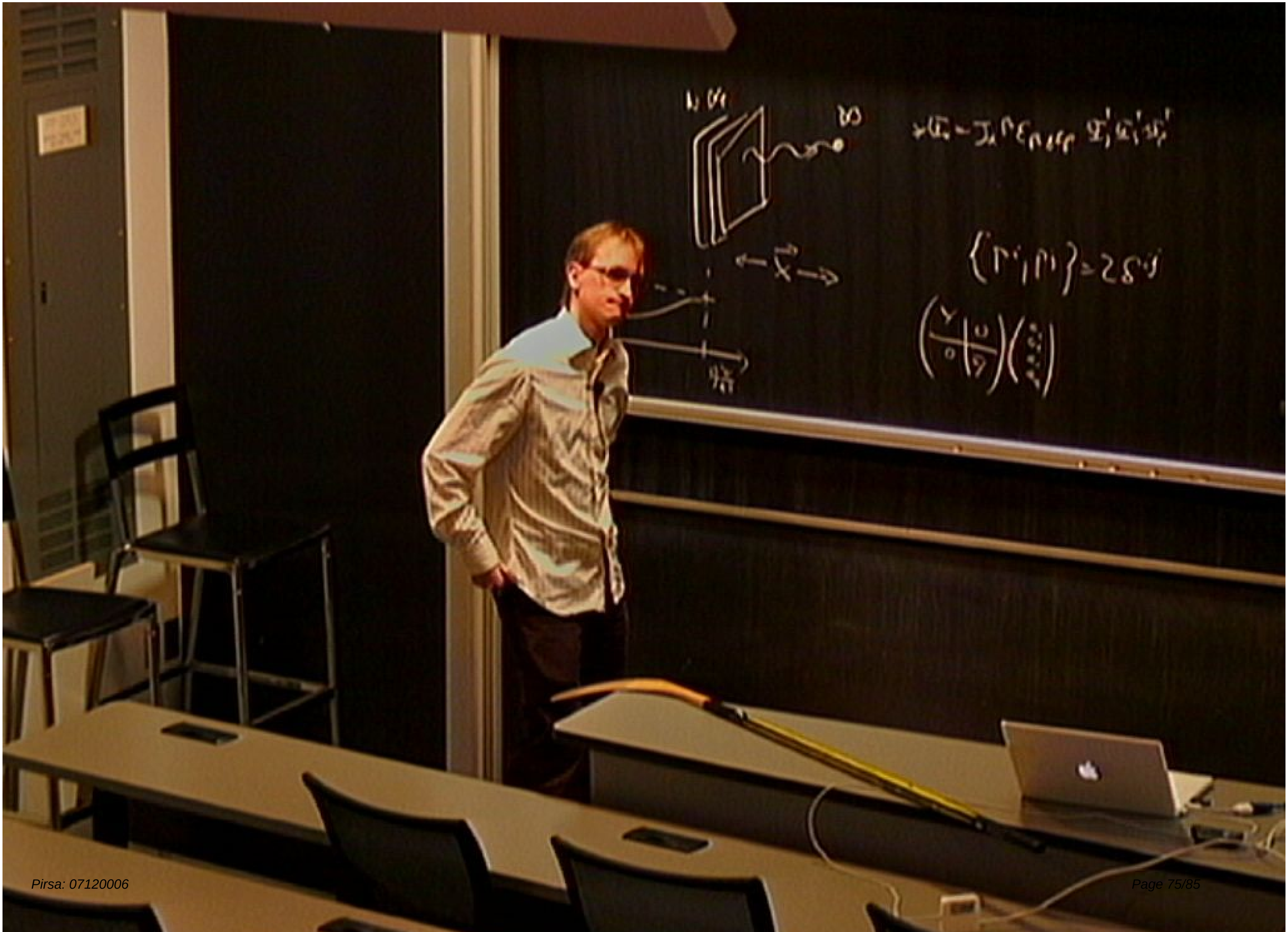
- Four Hopf Maps, four division algebras, four Berry connections

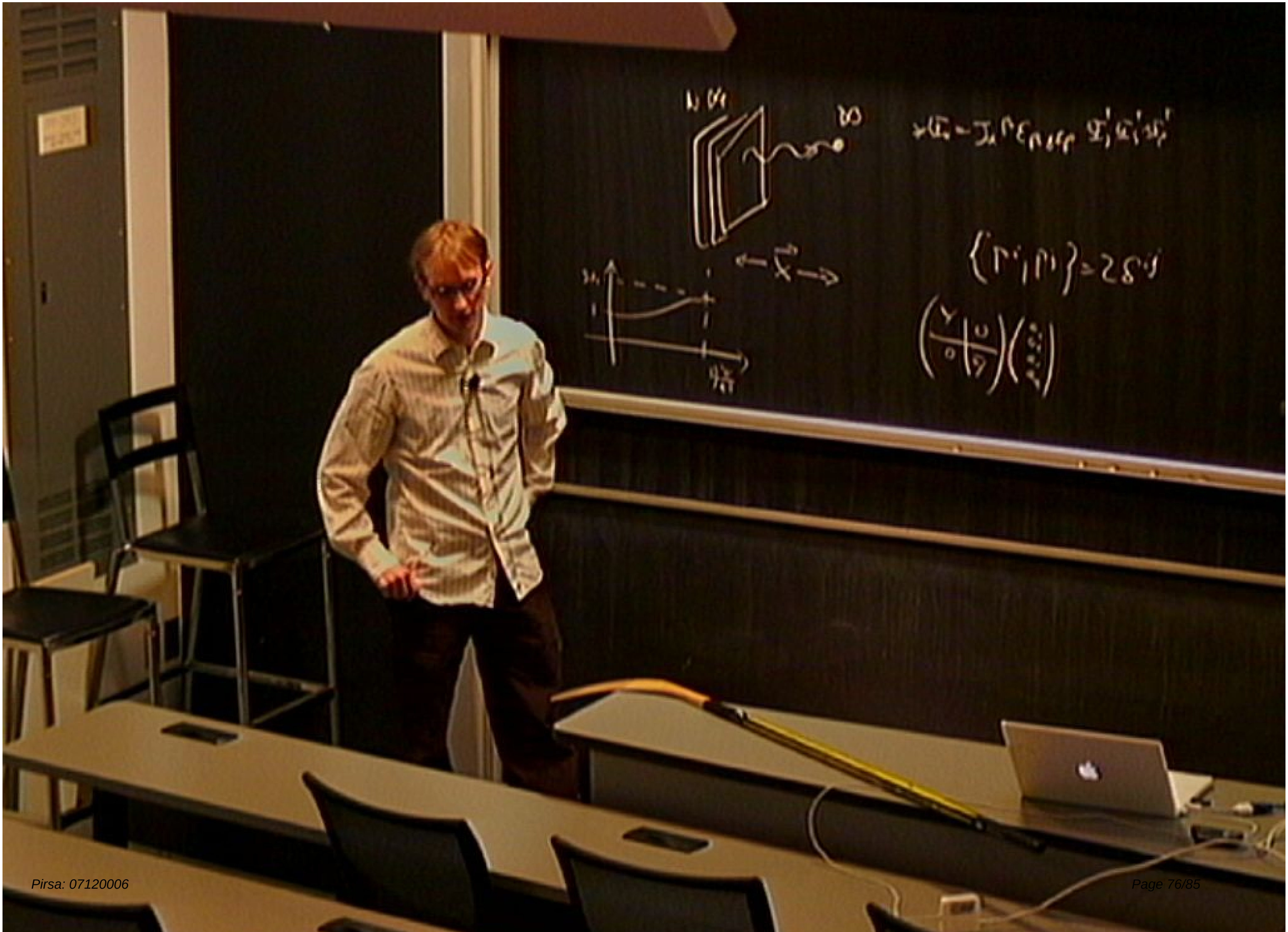
Summary

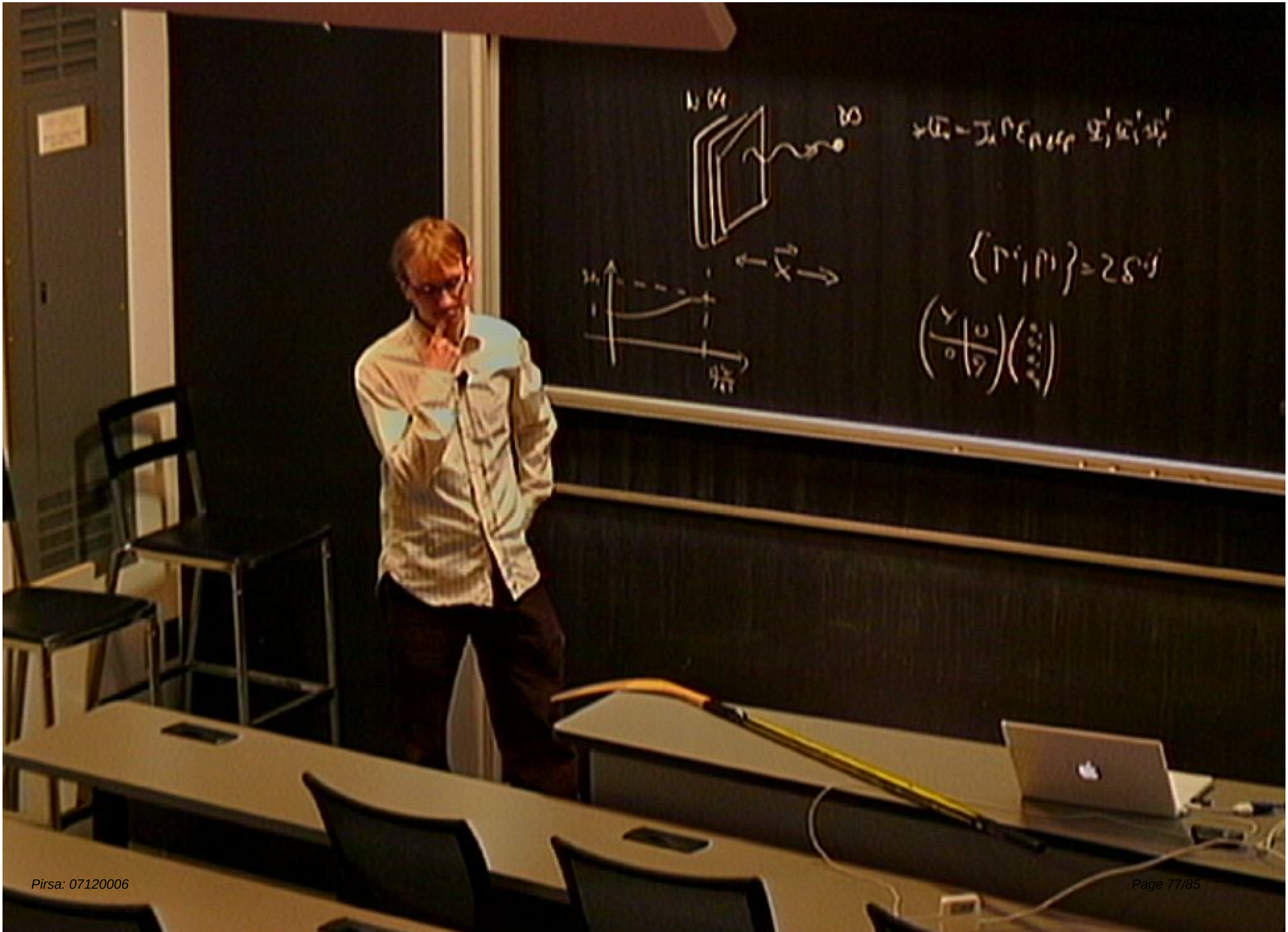
- Berry Phase is natural concept in SUSY systems
  - N=2: sign flip
  - N=4: Dirac Monopole
  - N=8: Yang Monopole
  - N=16: Octonionic Monopole
- N=4: non-Abelian phase from Witten index
- N=8: Berry phase from gravitational precession
  - Relation to d=4+1 dimensional gauge theory
  - Instantons
- Look for Applications (aka: "is it good for anything?")

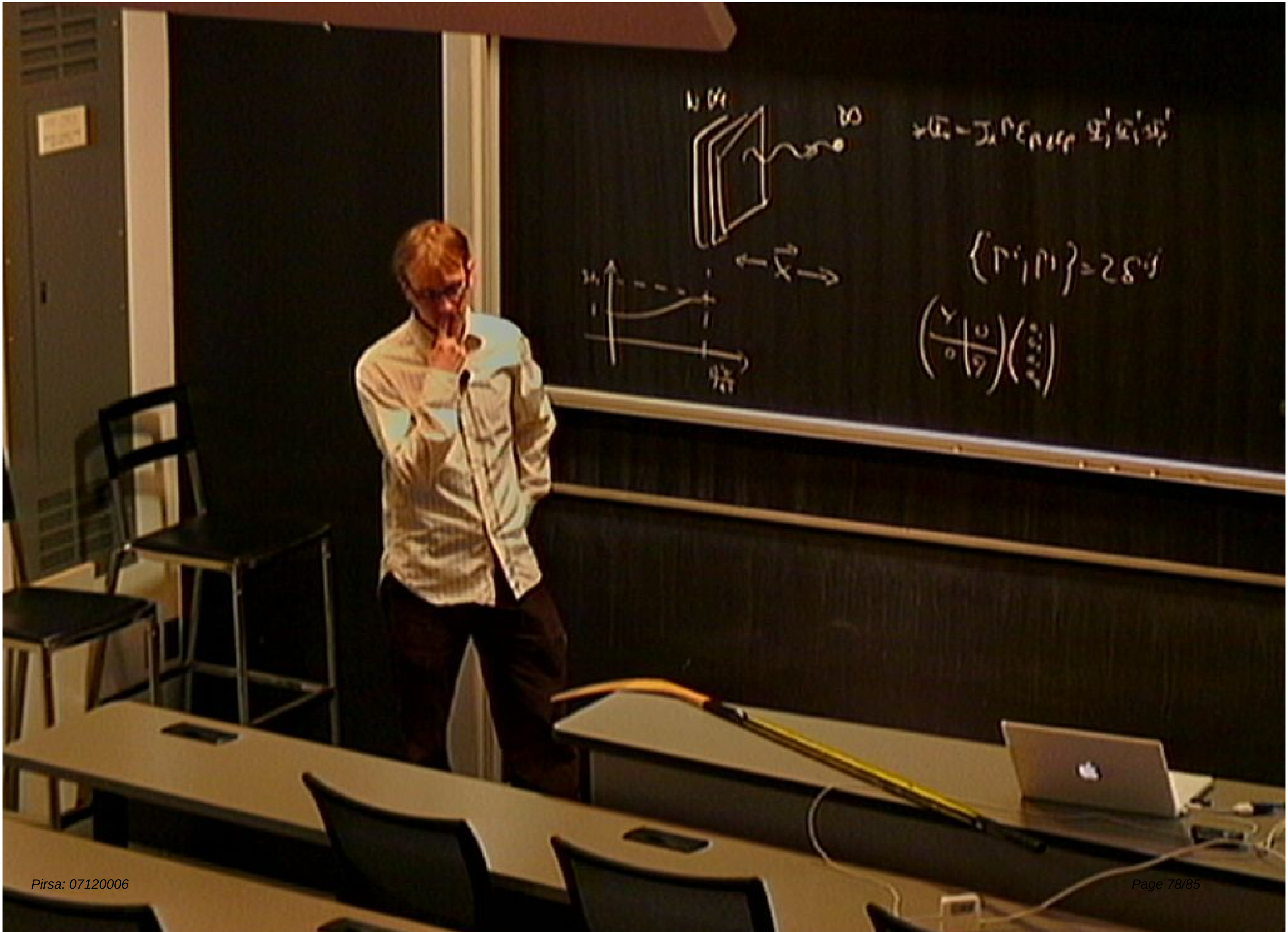
thanks for listening

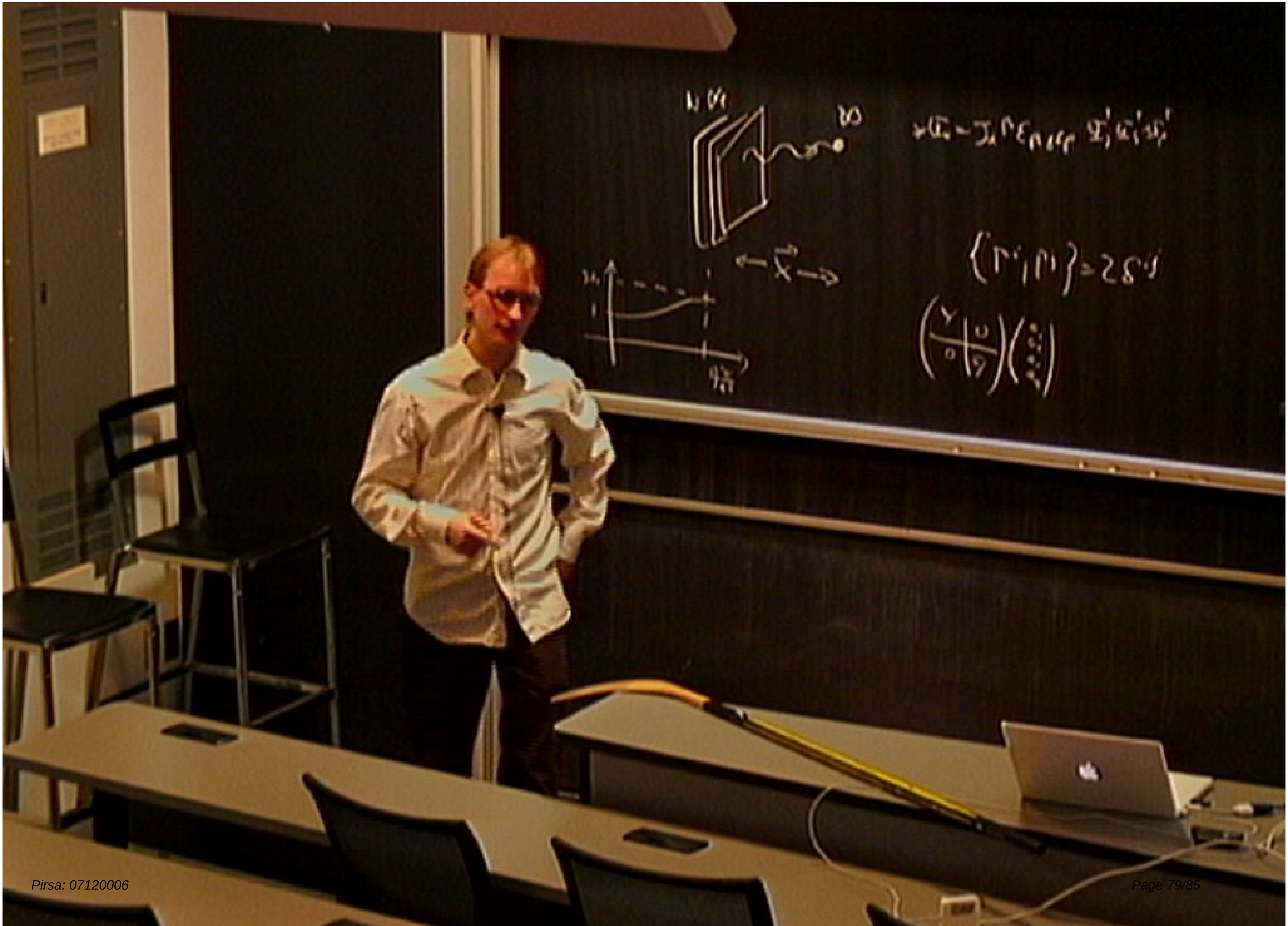


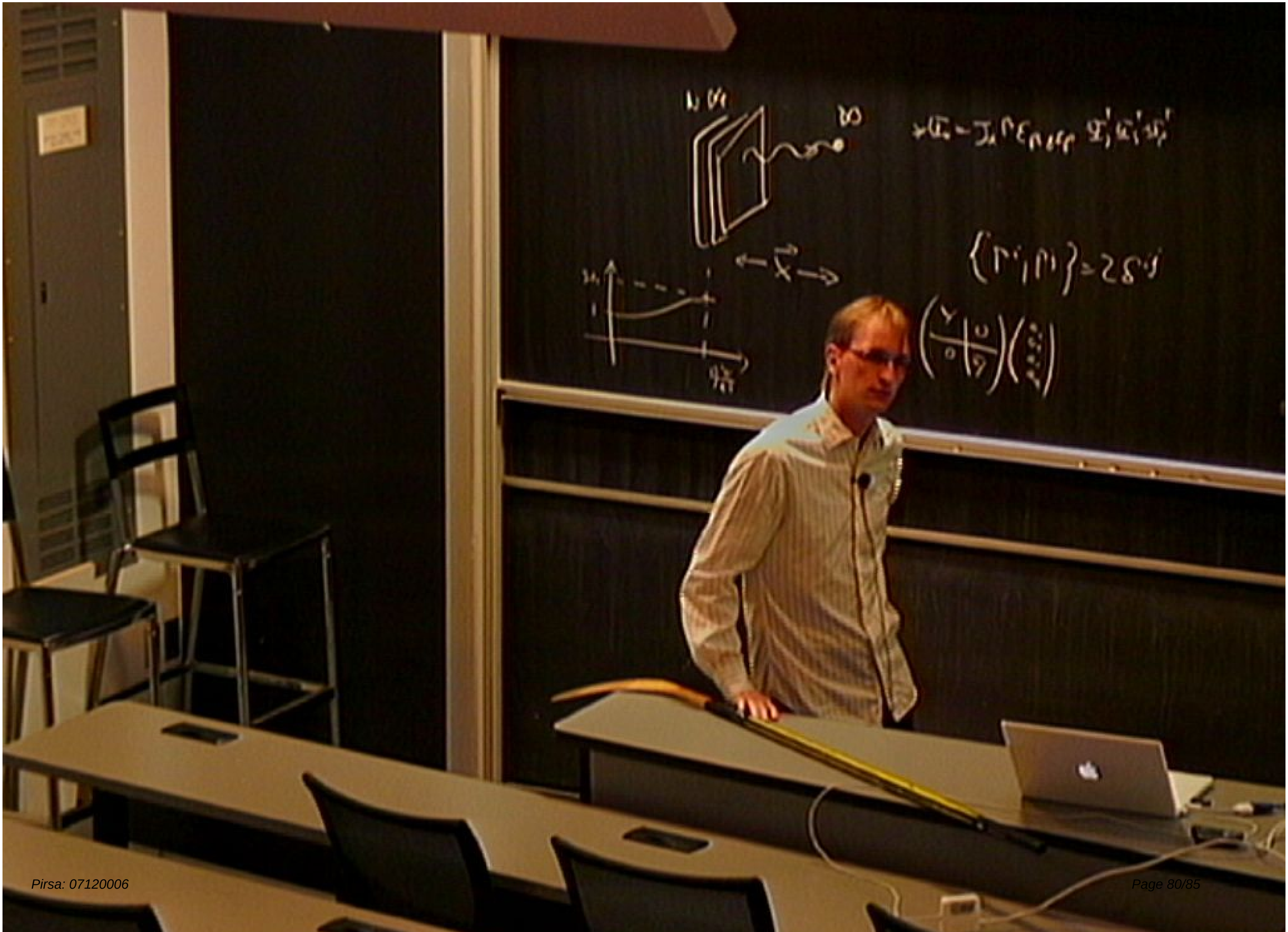


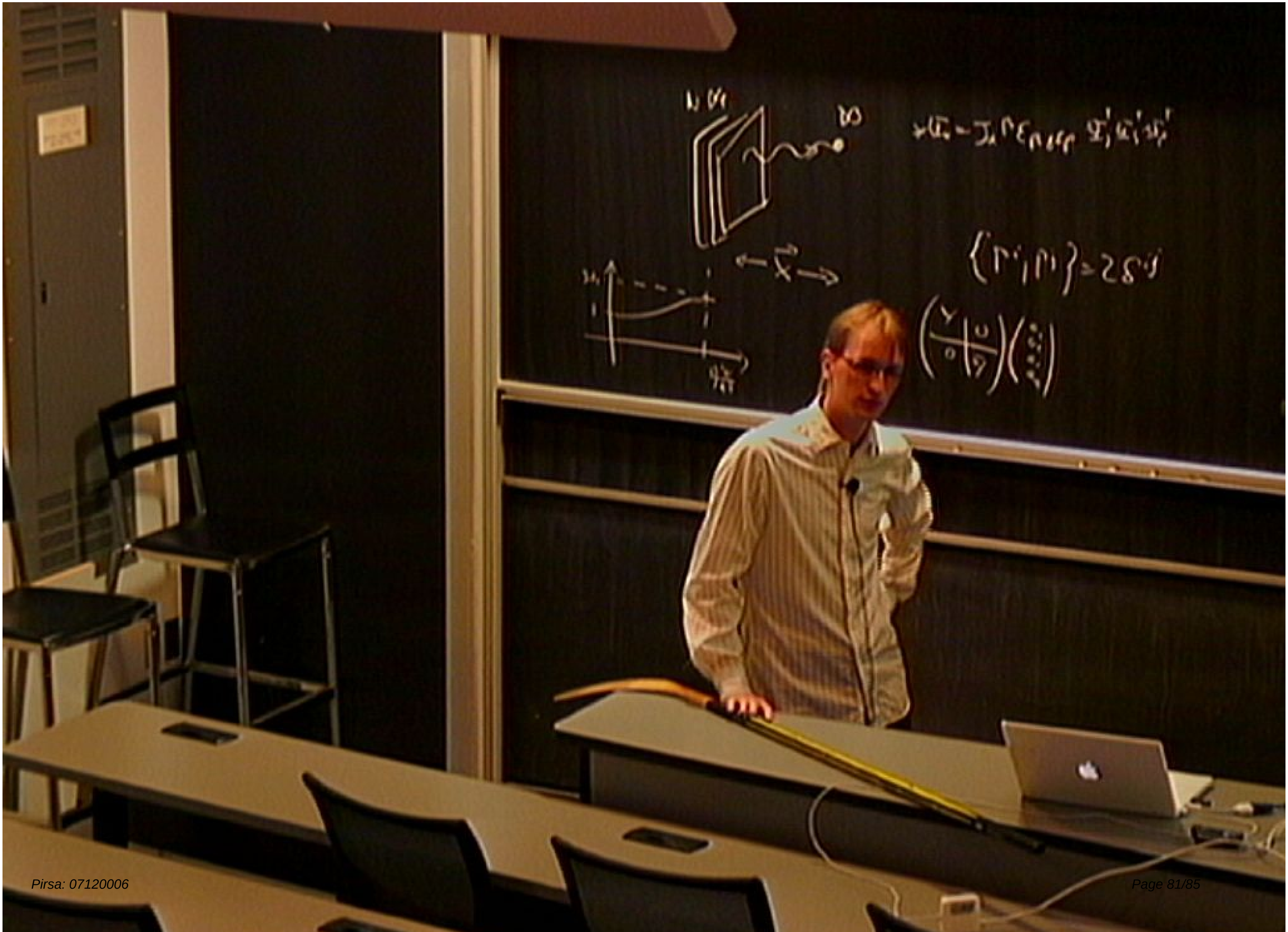


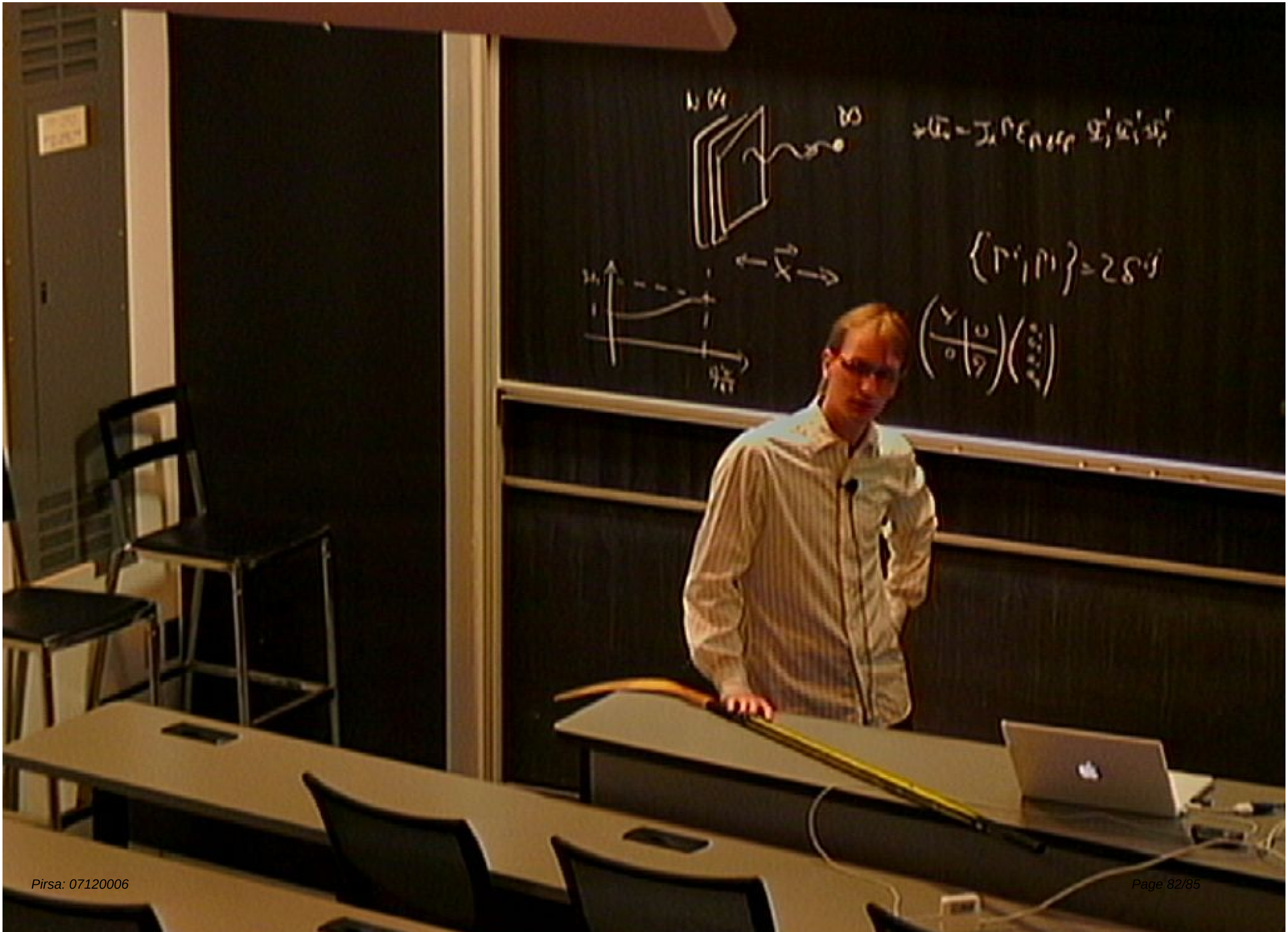


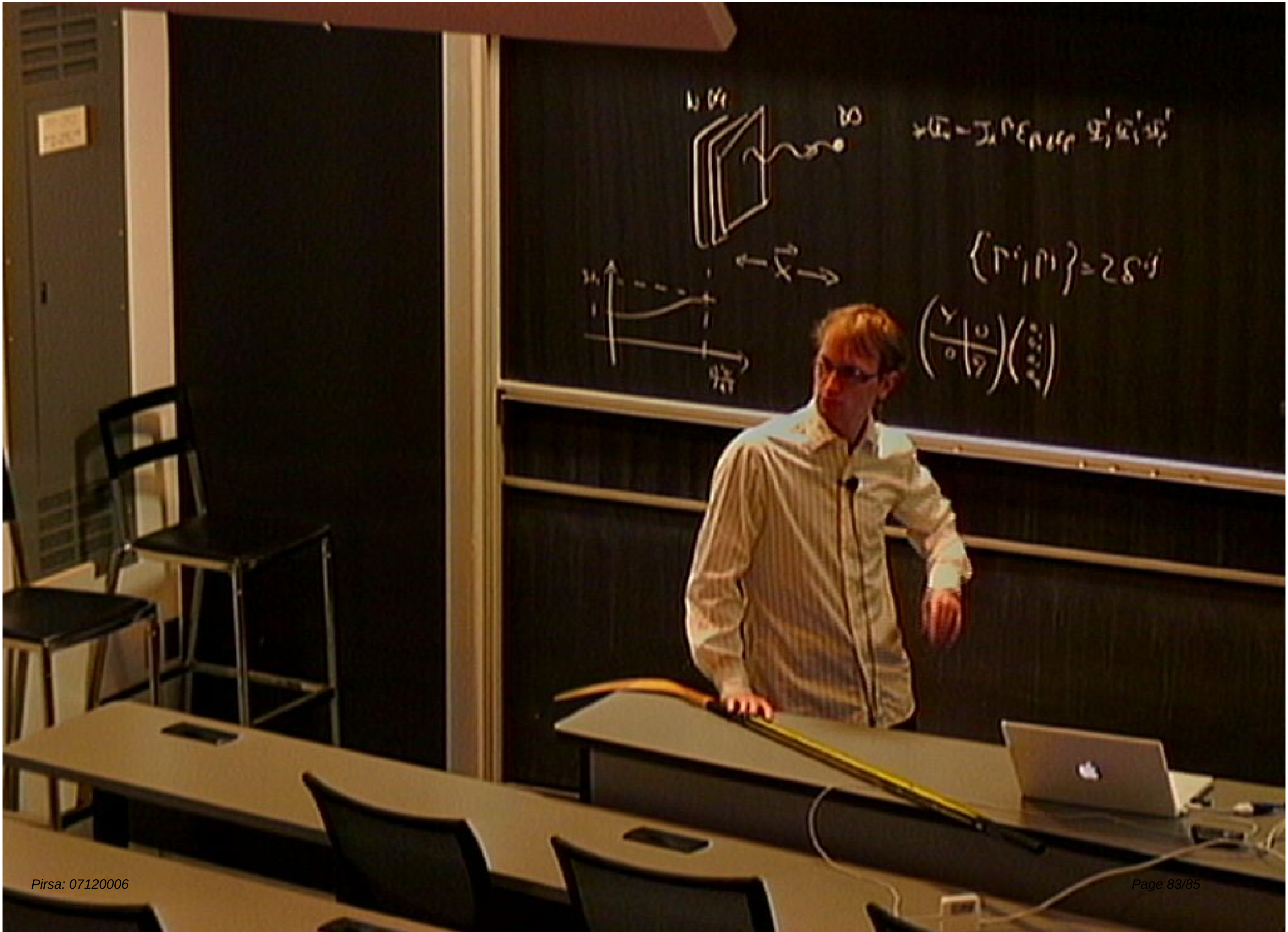












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