

Title: Renormalizing large-scale structure perturbation theory: practical and fun

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URL: <http://pirsa.org/07110076>

Abstract:

Renormalizing large-scale structure perturbation theory: practical and fun (renormalizing galaxy bias)

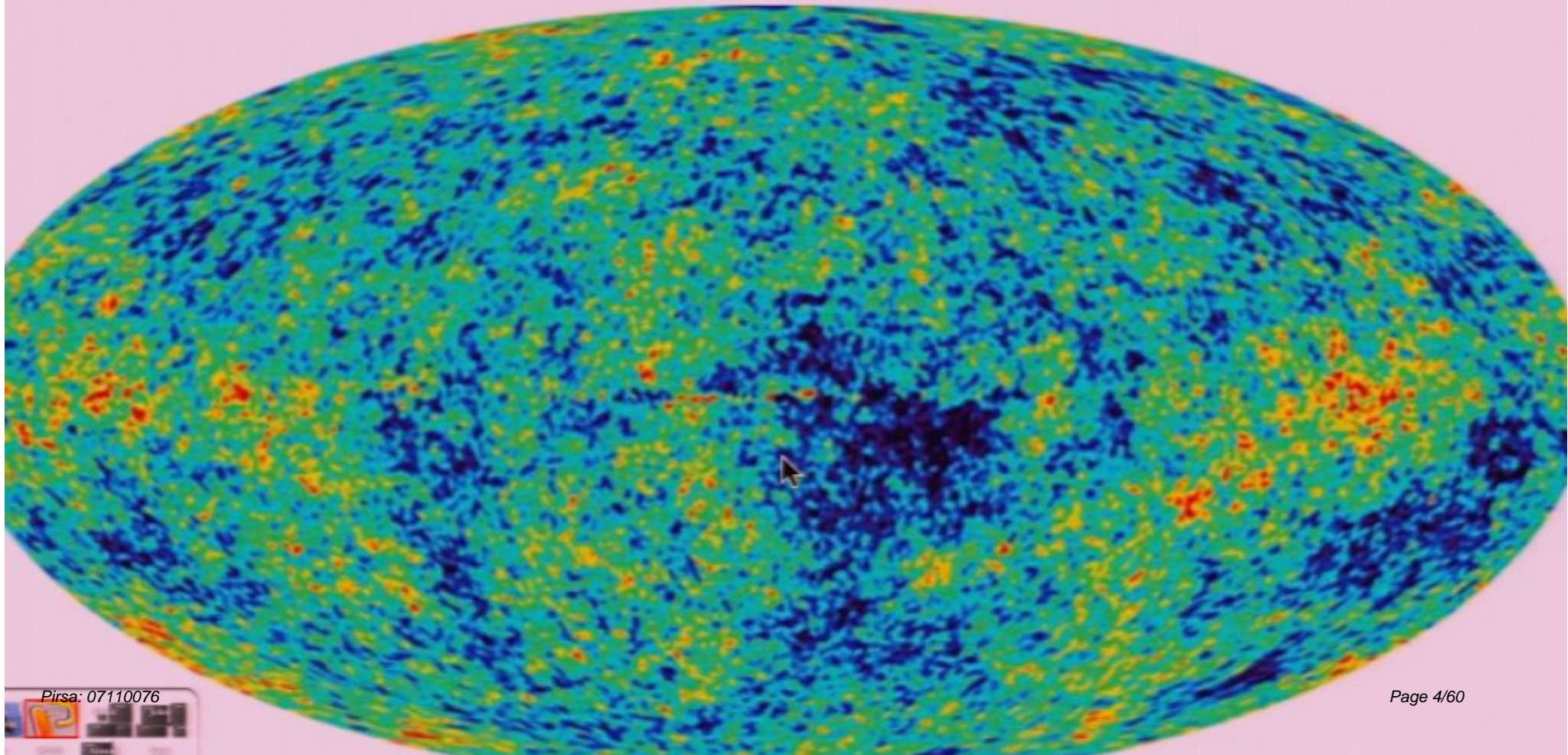
Patrick McDonald
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LSS is a growth industry

- SDSS Luminous Red Galaxies have probed ~ 1 Gpc/h
- The Universe at $z < 5$ contains ~ 700 Gpc/h!
- Lots more volume at higher z , which we might be able to see through 21cm.

WMAP CMB map (anisotropies in the 2.7 K cosmic microwave background, mapping the density fluctuations at $z \sim 1100$ - the *best* cosmological probe)



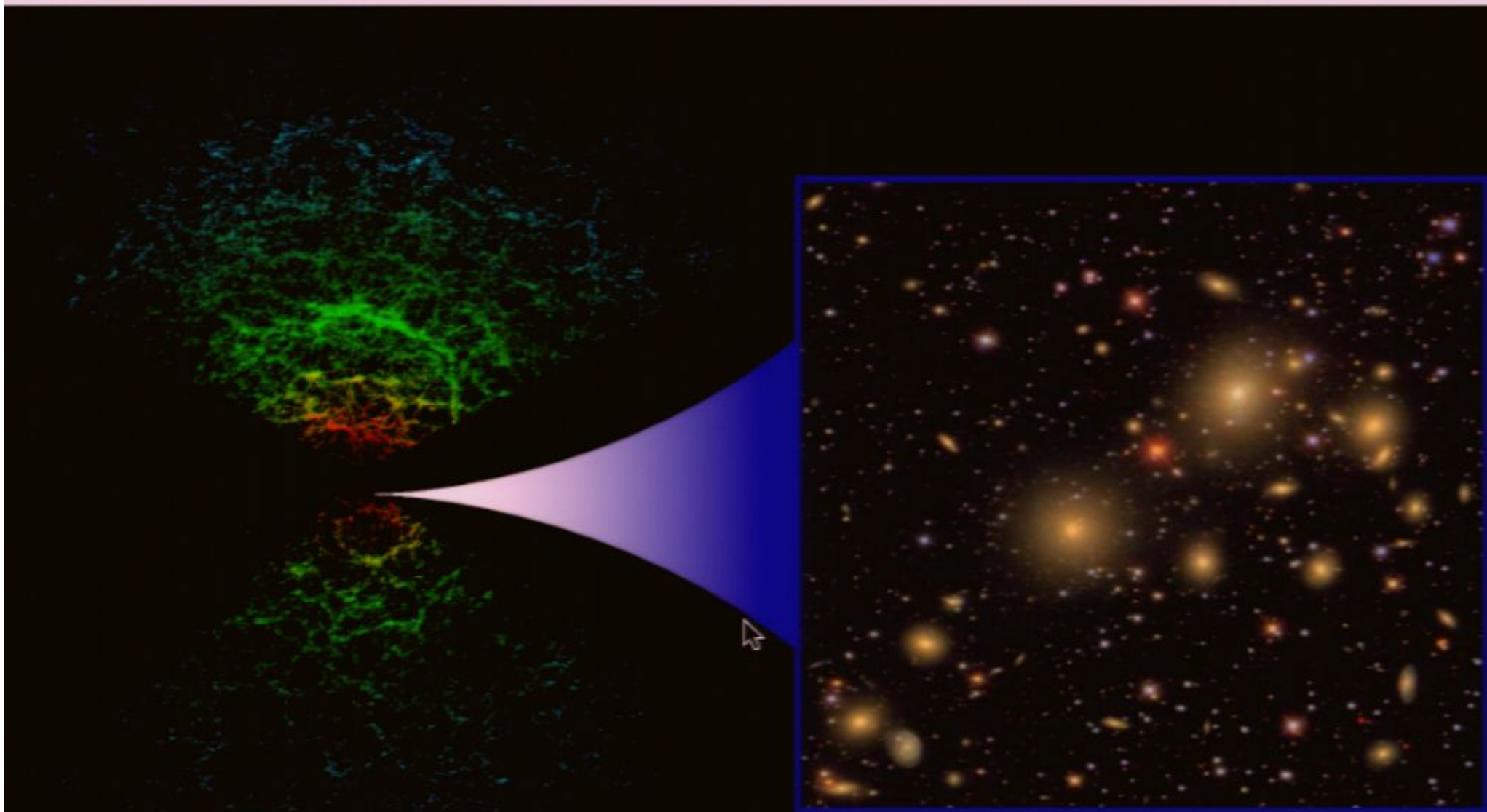
Future surveys

- SDSS III, 10000 sq. deg., LRGs to $z \sim 0.7$, BAO scale to 1% (in two z bins)
- Proposed JDEM ADEPT: $z=1-2$ with 28600 sq. deg. Covering ~ 110 cubic Gpc/h
- European SPACE mission similar
- Chang et al. 21cm $z=0-2.5$

Basic picture of LSS

- Inflation (or whatever) predicts a spectrum of small perturbations
- We measure these while they are still small using the CMB
- The fluctuations continue to grow, becoming large (non-linear) starting at small scales, galaxies form, etc.
- On large scales, fluctuations are still close to linear, and we measure them by observing galaxies (or other things) which trace mass.

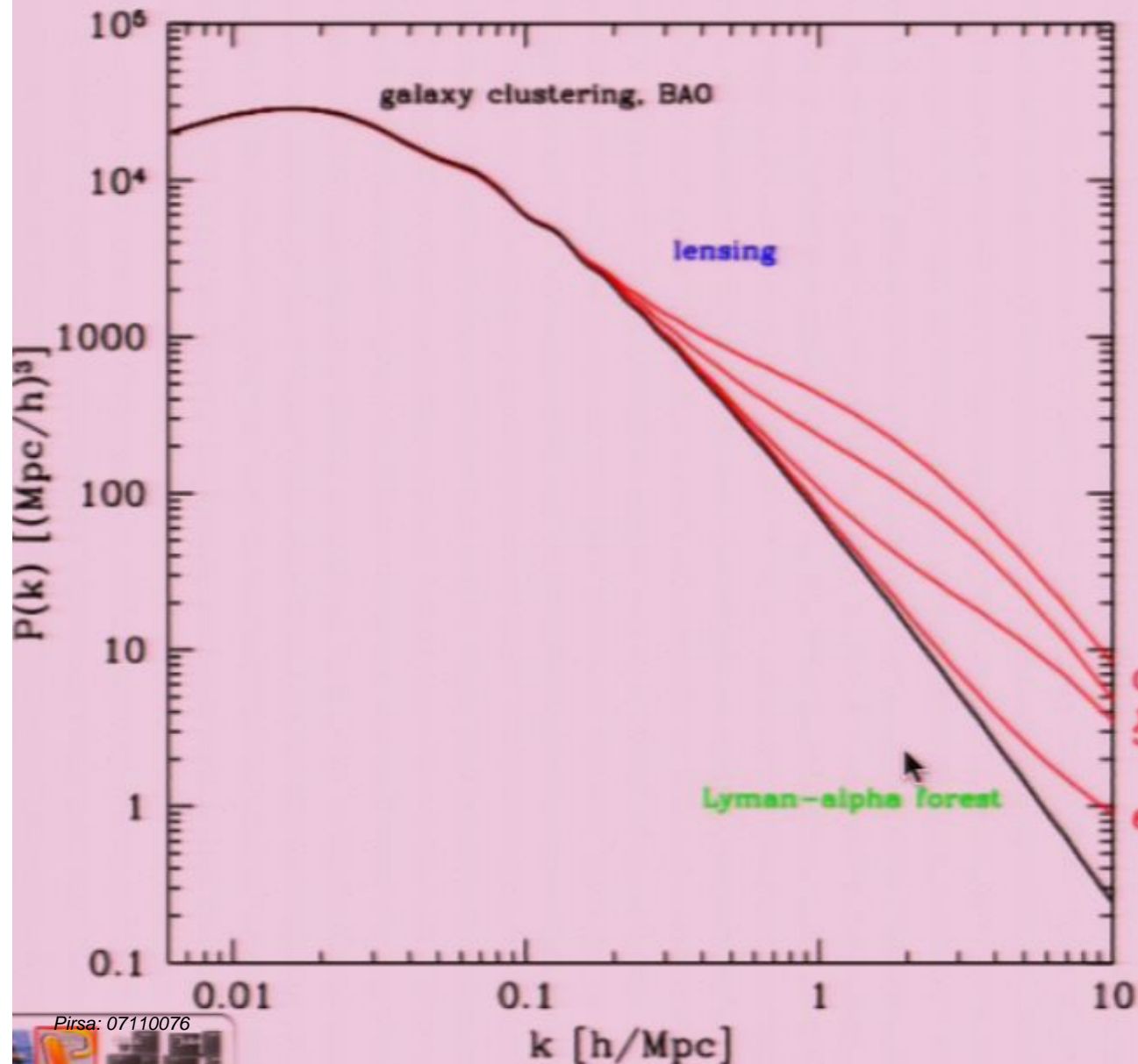
SDSS Galaxy map



Motivation for precision measurements of the galaxy (or other) power spectra.

- Dark energy through baryonic acoustic oscillations (BAO) has been a big focus lately, but older reasons haven't gone away...
- Measurement of the turnover scale gives $\Omega_m h$.
- Neutrino masses.
- Inflation through the shape of the primordial power spectrum.
- Non-Gaussianity
- Modified gravity, etc., etc.
- Generally fits together with other constraints (e.g., CMB) to break degeneracies.

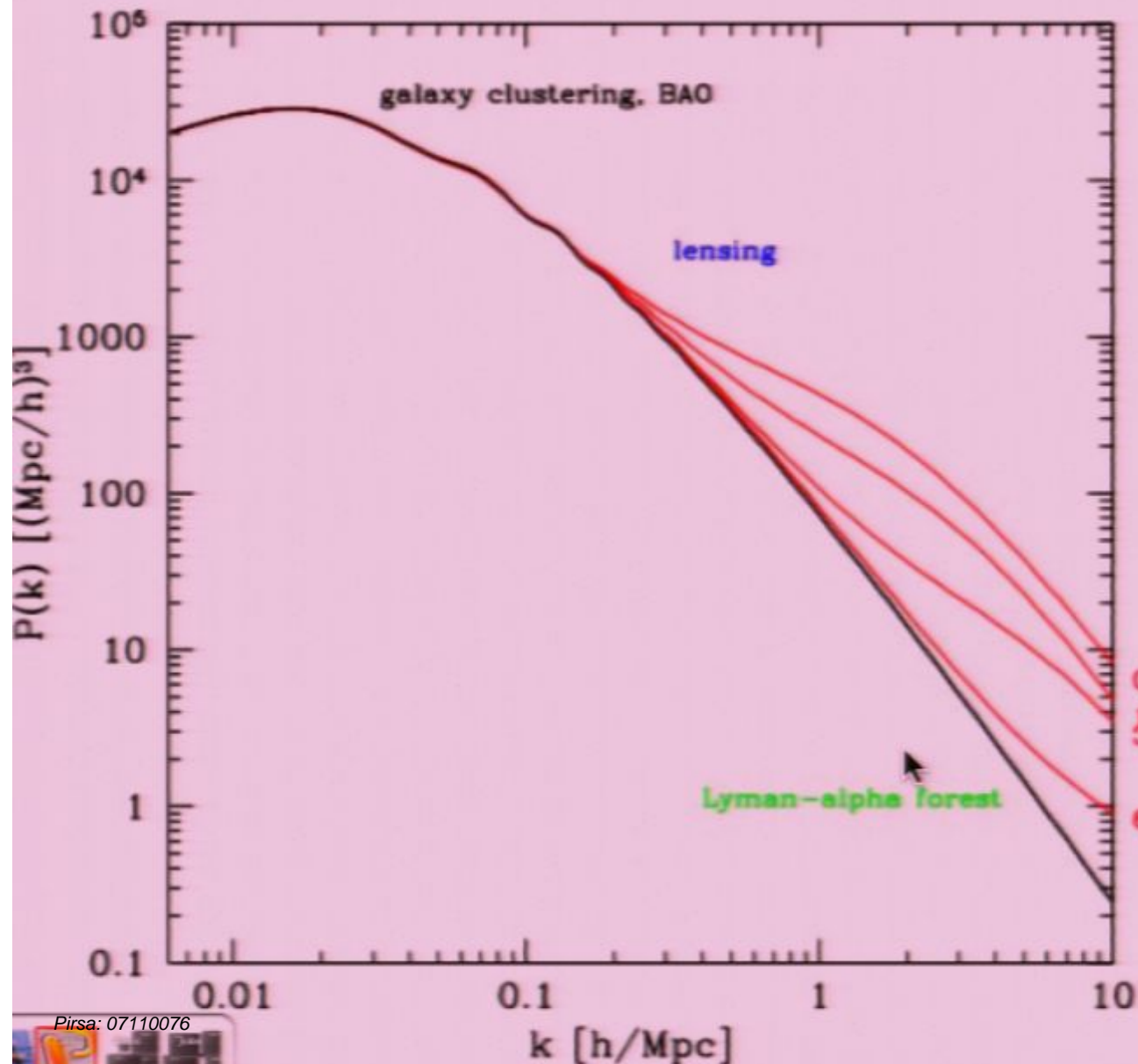
Non-linear power spectrum



Observational Motivation:

- Galaxies/BAO
- Ly α forest
- Cosmic shear
- clusters/SZ
- 21 cm(?)
- Red non-linear curves from Smith et al. simulation fits, not perfectly accurate

Non-linear power spectrum

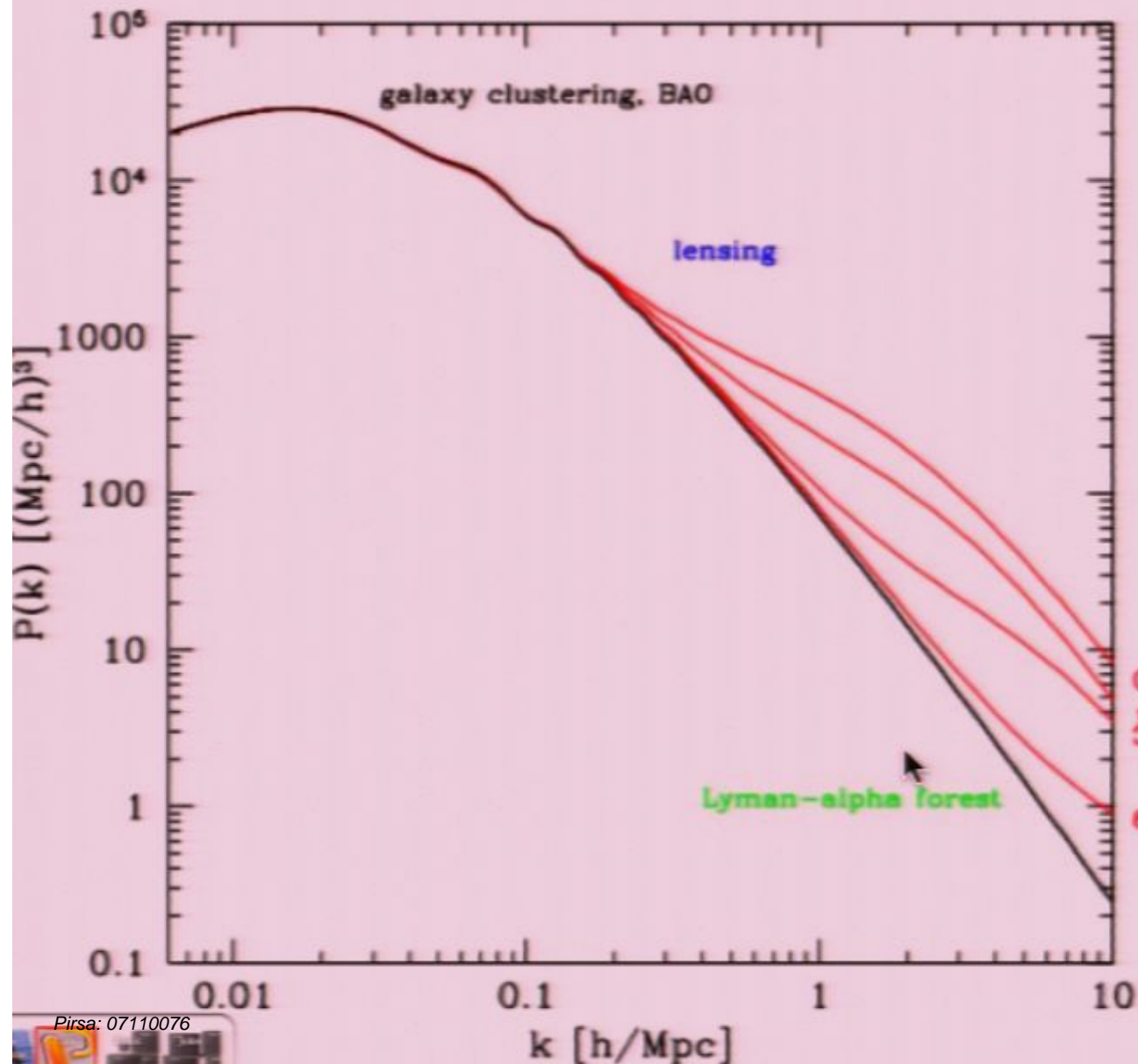


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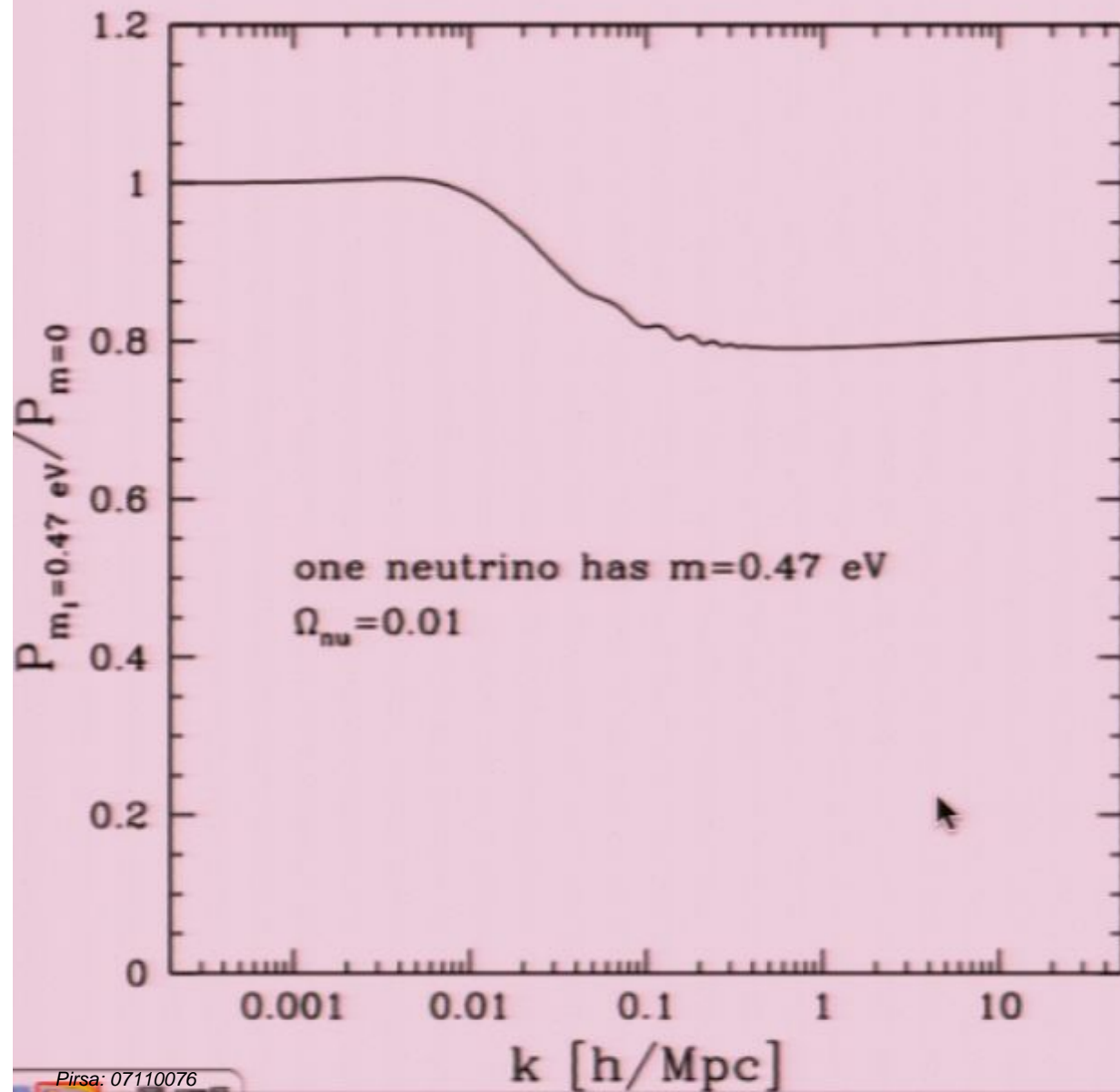
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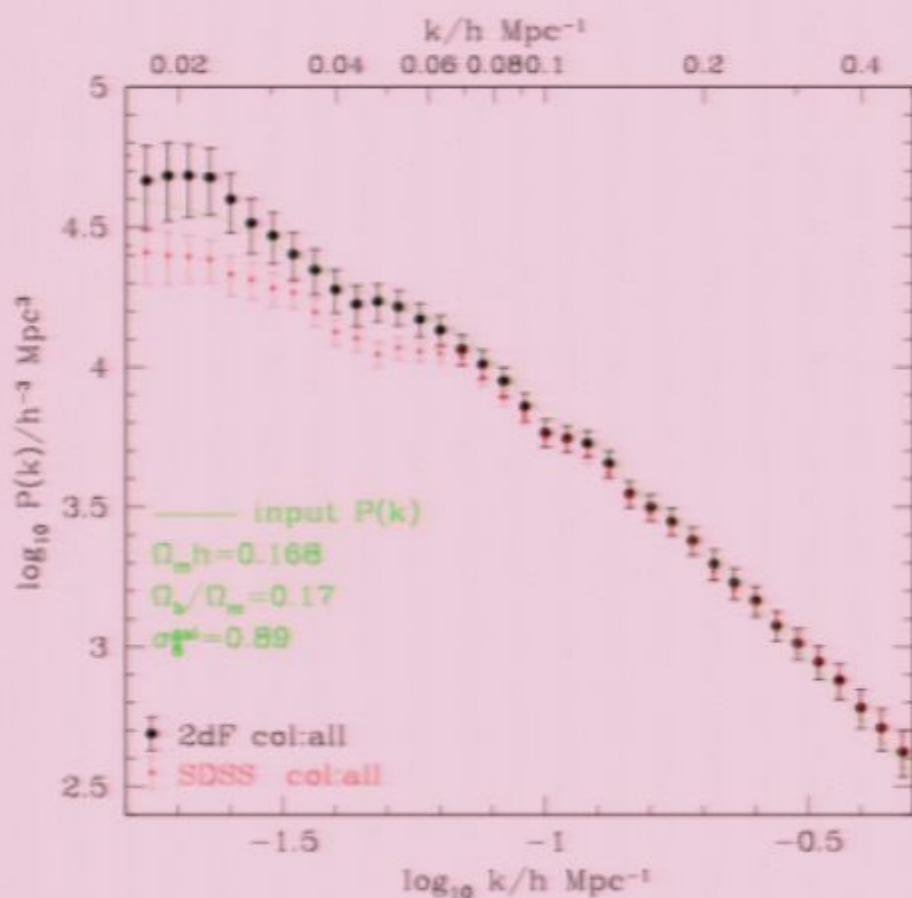


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- Effect of massive neutrinos (linear power)

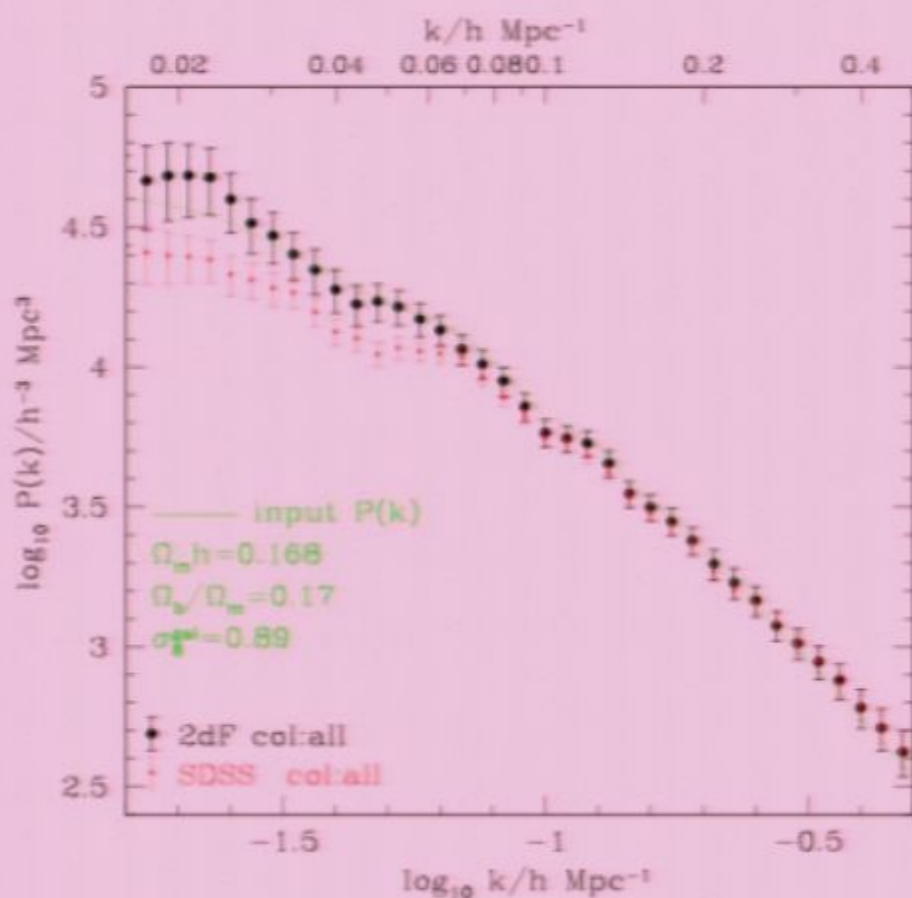
SDSS-2dF P(k) comparison



- “Bias” just means generally the differences between galaxy density and mass density.
- The difference between non-linear and linear mass density is also an issue.
- On relevant scales, linear bias (galaxy density proportional to mass density) is **not sufficient**.

Sanchez et al (2007)

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Commonly used: Q “model”

$$P_g(k) = \frac{1 + Qk^2}{1 + Ak} P_L(k)$$

Cole et al. (2005)

$A=1.7$, $Q=9.6$ (real space)

$A=1.4$, $Q=4.0$ (redshift space) from semi-analytic models.

Not clear how general, often marginalize over Q but not A

Supposed to account for both non-linearities in mass power and non-linear bias.

Not clear why this is used when many of the users themselves have better halo-based models. *Maybe* it is good enough for now.

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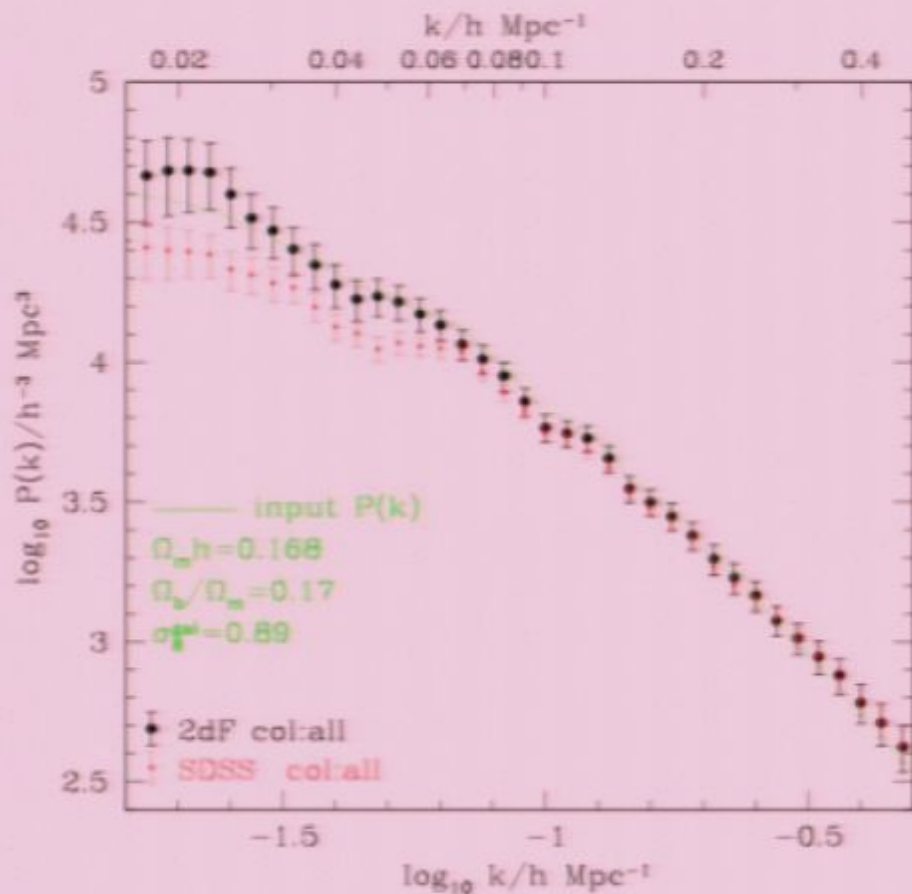
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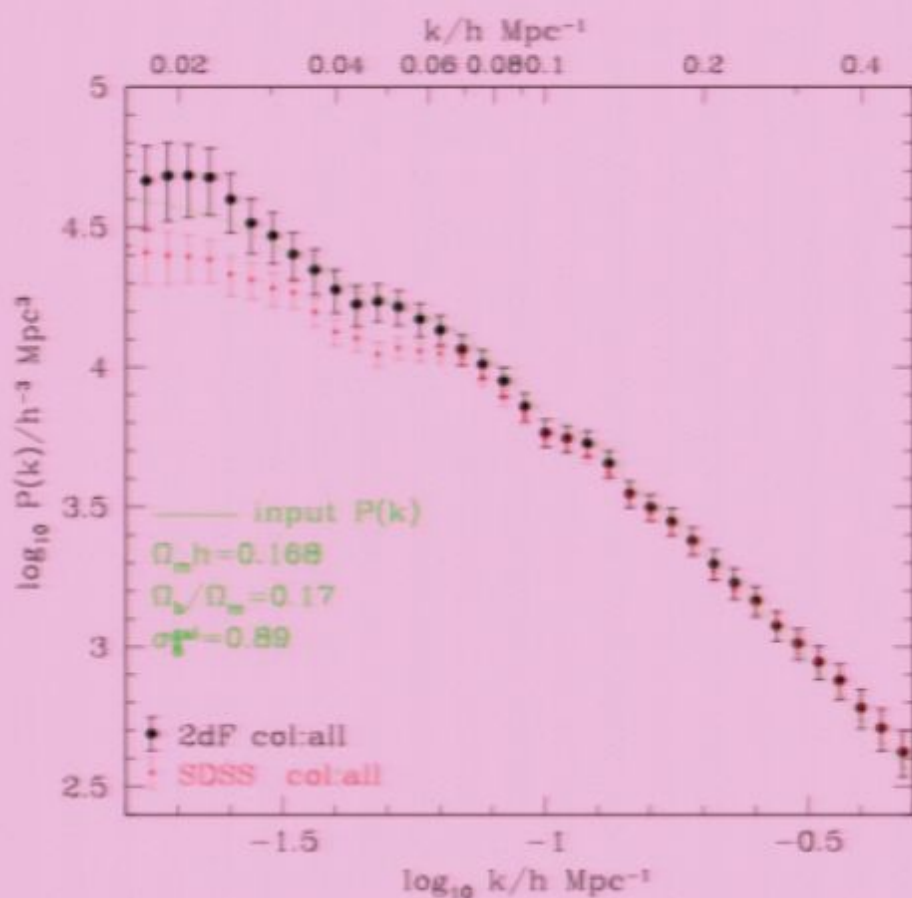
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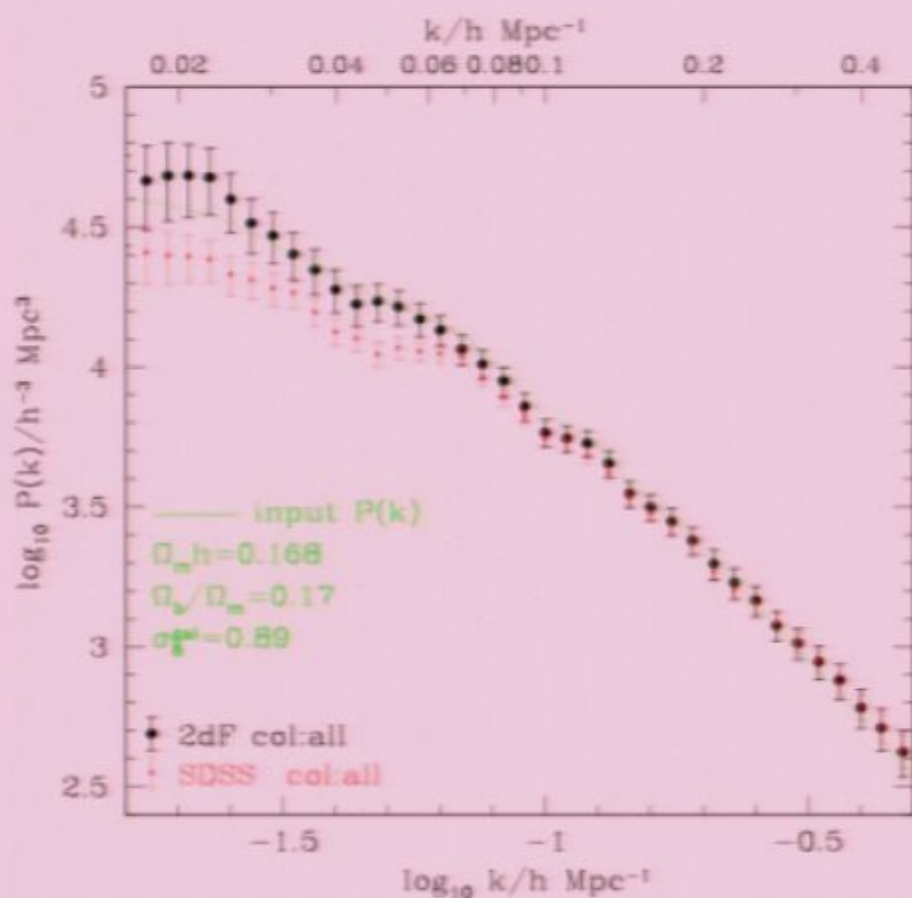
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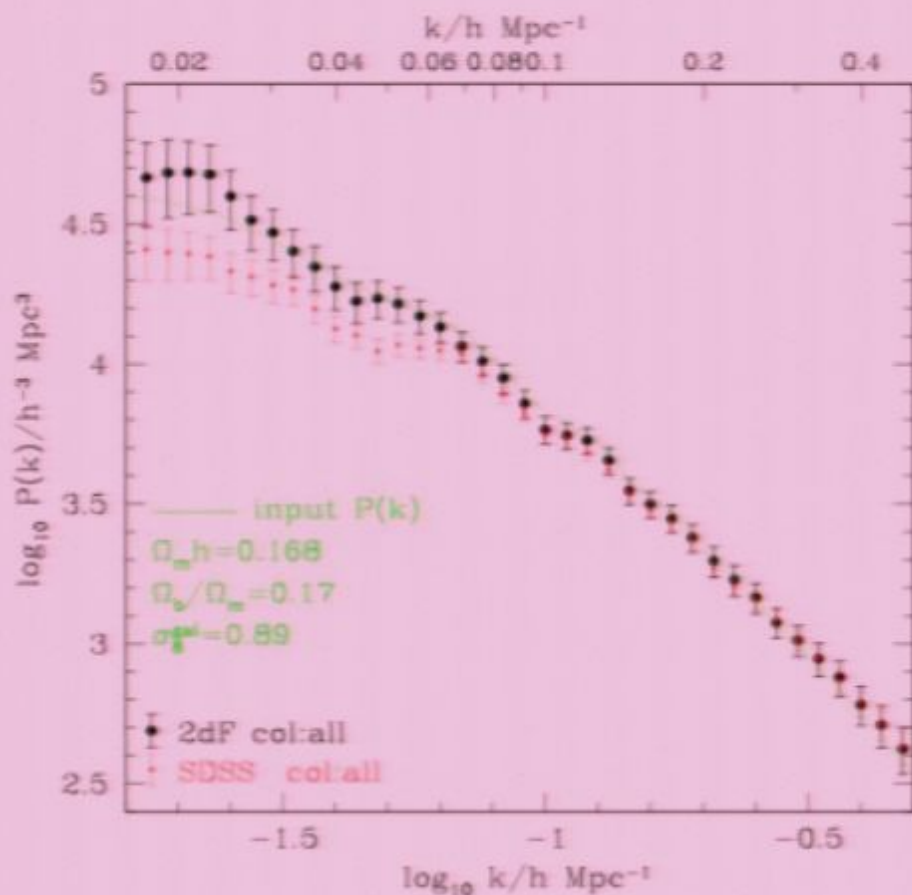
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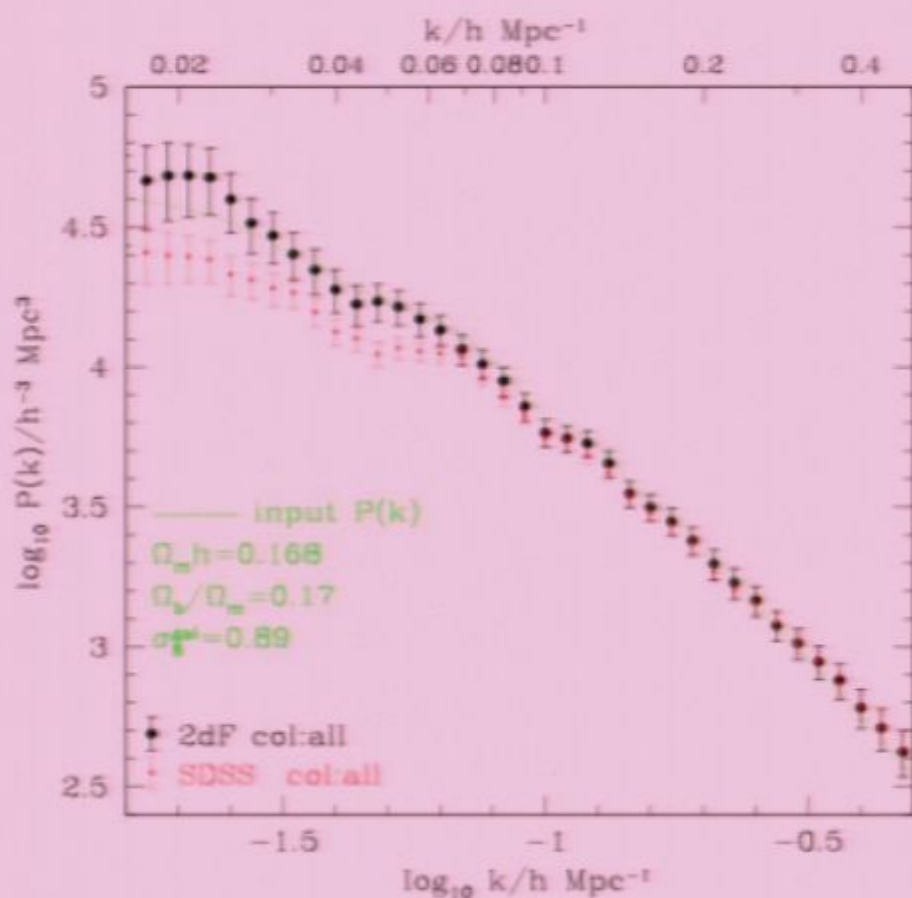
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Driving motivation:

- In order to achieve the goals of future (or even current) giant large-scale structure surveys we need precise, reliable calculations of the observable statistics, e.g., the galaxy power spectrum, which include a means to marginalize over uncertainties in the model.

Two approaches to modeling galaxy clustering:

- **Halo model** is a **bottom up** approach: take one fundamental thing that we know about individual galaxies - that they live in dark matter halos - and use this to predict large-scale clustering. There has been a lot of work on this and I'm not saying there's anything wrong with it.
- **Perturbation theory** is a **top down** approach: start with the fact that perturbations are small on very large scales, suggesting a Taylor series, and sweep small-scale details under the rug as much as possible. Less work so far.

For most of my career I would have said PT was a curiosity, not very relevant to interpreting real observations, so why am I working on it now?

- Criticism: It just doesn't work very well.
 - Response: Renormalization methods can fix that.
- Even to the extent that it does, it doesn't extend the range of scales accurately predicted very far.
 - The range where it helps is critical for BAO.
 - Higher precision data means a wider range where beyond-linear PT is both necessary and accurate (i.e., corrections can be very important, while still being small).
- The equations you're solving are incomplete (single-stream approximation).
 - It isn't clear that this is significant on relevant scales, but, to the extent that it is, we can fix it using RG methods.

Bias of tracers (McDonald 2006)

(most of the base calculations in Heavens, Matarrese, & Verde, 1998, without the renormalization interpretation)

Naïve perturbation theory: tracer density is a Taylor series in mass density perturbation (local for now):

$$\rho_g(\delta) = \rho_0 + \rho'_0 \delta + \frac{1}{2} \rho''_0 \delta^2 + \frac{1}{6} \rho'''_0 \delta^3 + \dots$$

To make sense, higher order terms should decrease in size.

Trivial warm-up: compute the mean density of galaxies:

$$\langle \rho_g \rangle = \rho_0 + \frac{1}{2} \rho''_0 \langle \delta^2 \rangle + \dots$$

2nd term is ~divergent (maybe not literally infinite, depending on the power spectrum, but certainly large)

Not a problem. Eliminate the “bare” Taylor series parameter in favor of a parameter for the *observed* mean density of galaxies.

$$\bar{\rho}_g \equiv \rho_0 + \frac{1}{2} \rho''_0 \sigma^2$$

$$\rho_g(\delta) = \bar{\rho}_g + \rho'_0 \delta + \frac{1}{2}\rho''_0 (\delta^2 - \sigma^2) + \frac{1}{6}\rho'''_0 \delta^3 + \dots$$

The mean density is a trivial example, leads to nothing new.

Now move to fluctuations:

$$\delta_g(\delta) = \frac{\rho_g(\delta) - \bar{\rho}_g}{\bar{\rho}_g} = c_1\delta + \frac{1}{2}c_2(\delta^2 - \sigma^2) + \frac{1}{6}c_3\delta^3 + \dots$$

Correlation function:

$$\xi_g(|\mathbf{x}_a - \mathbf{x}_b|) = \xi_g^{ab} = \langle \delta_g^a \delta_g^b \rangle = c_1^2 \langle \delta_a \delta_b \rangle + \frac{1}{3}c_1c_3 \langle \delta_a \delta_b^3 \rangle + \frac{1}{4}c_2^2 (\langle \delta_a^2 \delta_b^2 \rangle - \sigma^4) + c_1c_2 \langle \delta_a \delta_b^2 \rangle + \dots$$

(assuming 4th order terms Gaussian):

$$\xi_g^{ab} = c_1^2 \xi_{ab} + c_1c_3 \sigma^2 \xi_{ab} + \frac{1}{2}c_2^2 \xi_{ab}^2 + c_1c_2 \langle \delta_a \delta_b^2 \rangle + \dots$$

Going to absorb divergent part into *observable* linear bias, but not yet because another piece comes from the cubic term.

Standard perturbation theory for gravitational collapse:

Evolution equations:

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1 + \delta) \mathbf{v}] = 0 \quad \text{Continuity}$$

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H} \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla \phi \quad \text{Euler}$$

$$\nabla^2 \phi = 4 \pi G a^2 \bar{\rho} \delta \quad \text{Poisson}$$

Write density (and velocity) as a series of (ideally) increasingly small terms, δ_n is of order δ_1^n

$$\delta = \delta_1 + \delta_2 + \delta_3 + \dots$$

Solve evolution equations iteratively

Density in standard PT

$$\delta_{\mathbf{k}} = \delta_1(\mathbf{k}) + \int \frac{d^3\mathbf{q}}{(2\pi)^3} \delta_1(\mathbf{q}) \delta_1(\mathbf{k} - \mathbf{q}) J_S^{(2)}(\mathbf{q}, \mathbf{k} - \mathbf{q}) + \\ \int \frac{d^3\mathbf{q}_1}{(2\pi)^3} \frac{d^3\mathbf{q}_2}{(2\pi)^3} \delta_1(\mathbf{q}_1) \delta_1(\mathbf{q}_2) \delta_1(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2) J_S^{(3)}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2) + \dots$$

- Now have non-linear density field in terms of the original Gaussian fluctuations, so it is easy to evaluate statistics.

Moving to the galaxy power spectrum, and using 2nd order perturbation theory for the cubic term:

$$P_g(k) = [c_1^2 + c_1 c_3 \sigma^2 + \frac{68}{21} c_1 c_2 \sigma^2] P(k) + \frac{1}{2} c_2^2 \int \frac{d^3 \mathbf{q}}{(2\pi)^3} P_L(q) P_L(|\mathbf{k} - \mathbf{q}|) + 2 c_1 c_2 \int \frac{d^3 \mathbf{q}}{(2\pi)^3} P_L(q) P_L(|\mathbf{k} - \mathbf{q}|) J_S^{(2)}(\mathbf{q}, \mathbf{k} - \mathbf{q}) + \dots$$

The red integral has a badly behaved part. Constant as $k \rightarrow 0$ so it looks like *shot-noise*. Absorb the constant part into a free-parameter for the *observable* shot-noise power (preserve linear bias+shot noise model on large scales):

$$N = N_0 + \frac{1}{2} c_2^2 \int \frac{d^3 \mathbf{q}}{(2\pi)^3} P_L^2(q)$$

$$b_1^2 = c_1^2 + c_1 c_3 \sigma^2 + \frac{68}{21} c_1 c_2 \sigma^2$$

$$\tilde{b}_2 = \frac{c_2}{b_1}$$

Final result:

$$P_g(k) = N + b_1^2 \left[P(k) + \frac{\tilde{b}_2^2}{2} \int \frac{d^3 \mathbf{q}}{(2\pi)^3} P_L(q) [P_L(|\mathbf{k} - \mathbf{q}|) - P_L(q)] + 2\tilde{b}_2 \int \frac{d^3 \mathbf{q}}{(2\pi)^3} P_L(q) P_L(|\mathbf{k} - \mathbf{q}|) J_S^{(2)}(\mathbf{q}, \mathbf{k} - \mathbf{q}) \right]$$

Started with 4-5+ parameters:

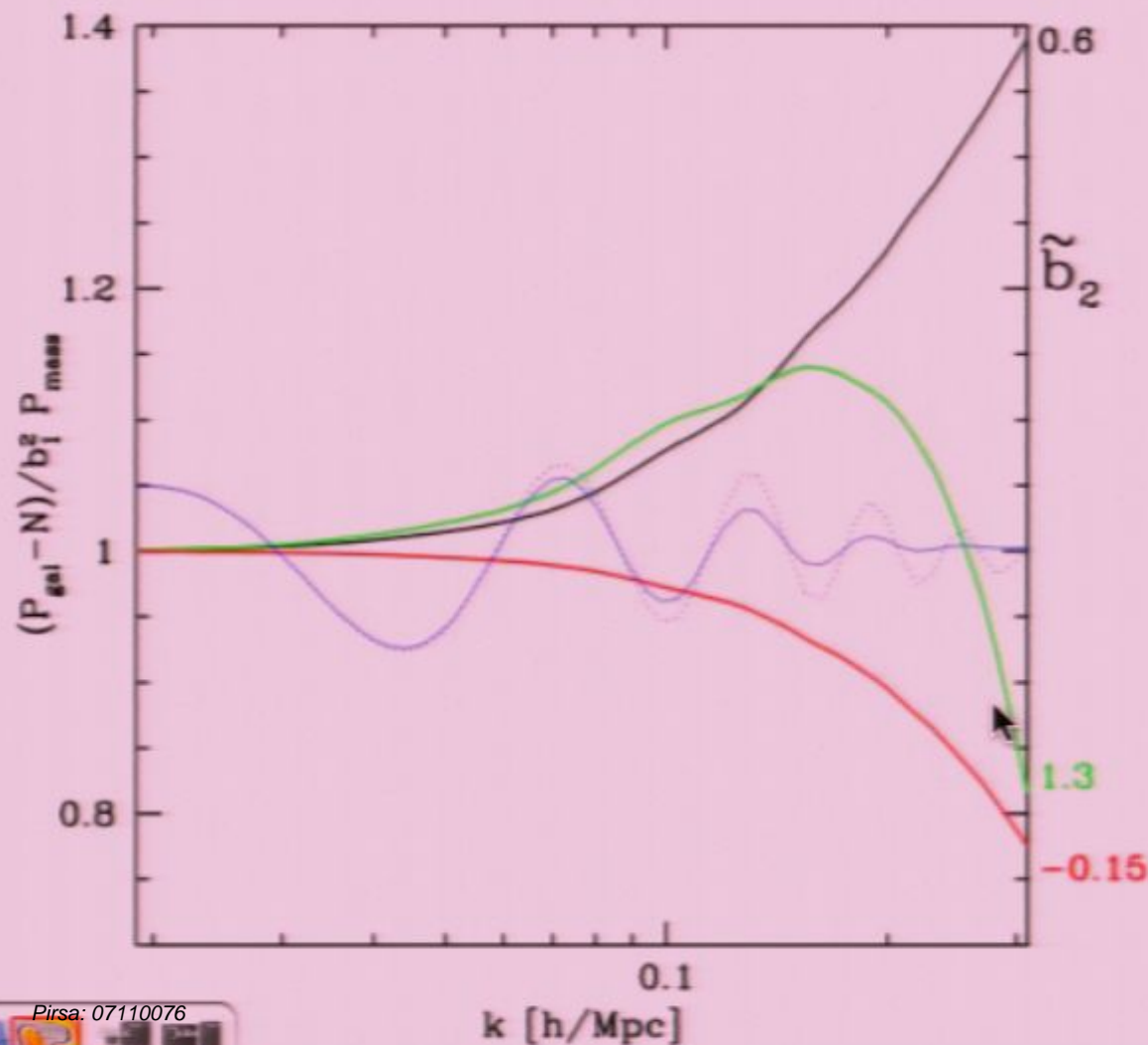
ρ_0 , ρ'_0 , ρ''_0 , ρ'''_0 , N_0 , and messy cutoffs

Now have only 4, with much more cleanly separated effects:

$\bar{\rho}_g$, b_1 , \tilde{b}_2 , and N

(reiterate: Heavens, Matarrese, & Verde, 1998 did most of this calculation already, including recognizing the generation of “effective bias” and shot-noise)

Effect of 2nd order bias in renormalized PT



Black, green, red:
fundamental 2nd order
bias effect, for labeled
values.

Blue: BAO effect, in
linear theory (dotted),
and RGPT (solid)

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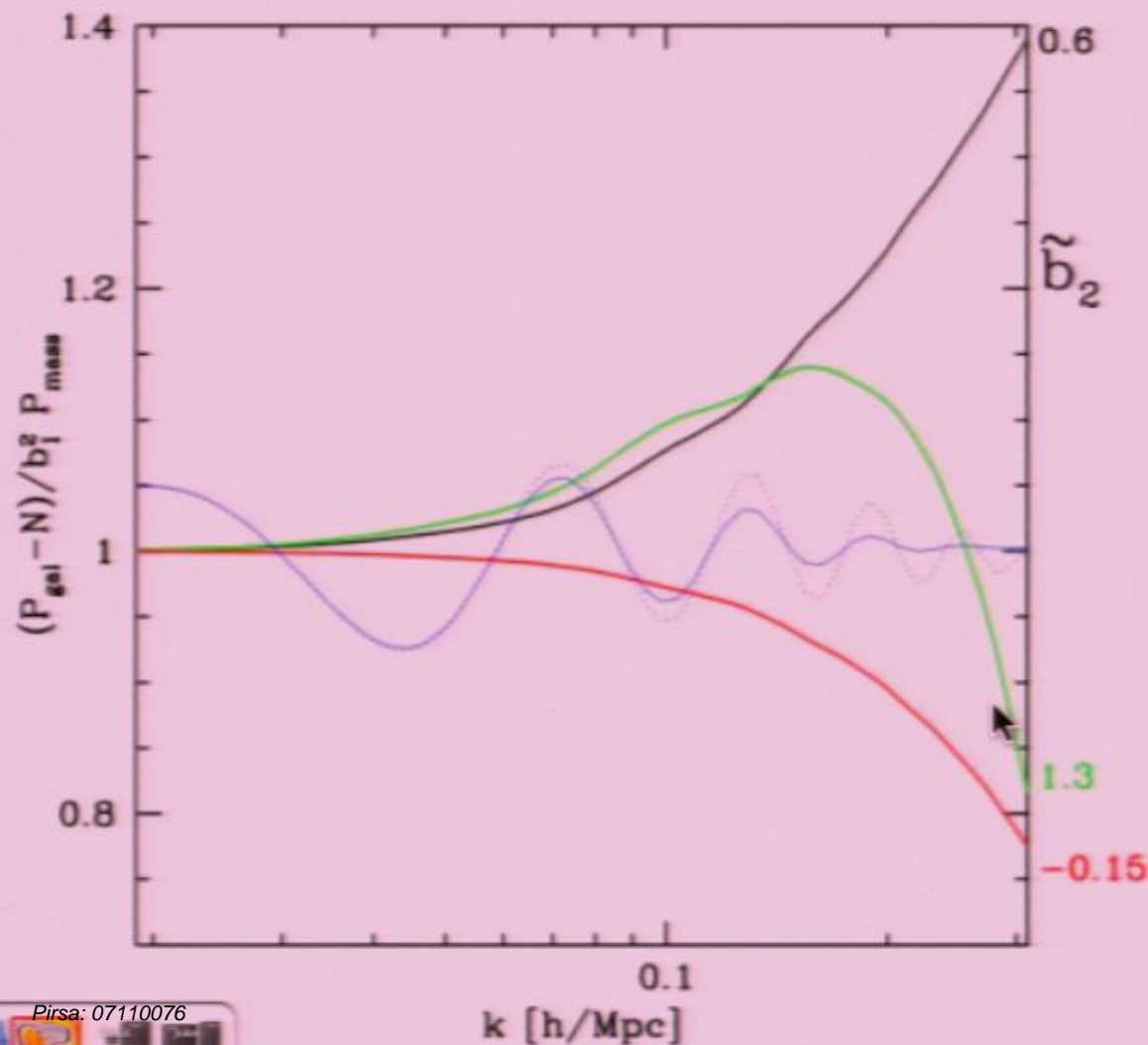
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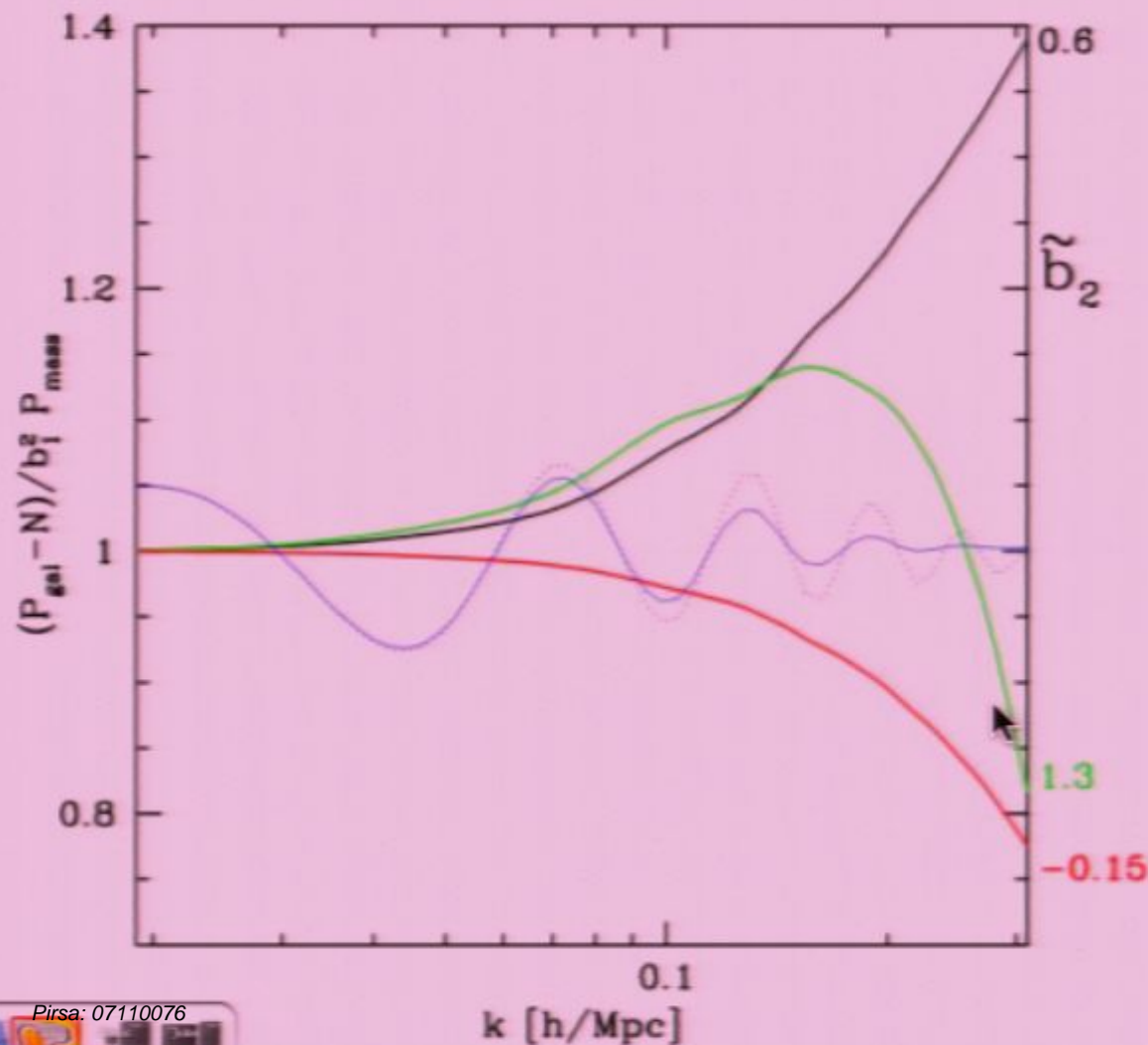
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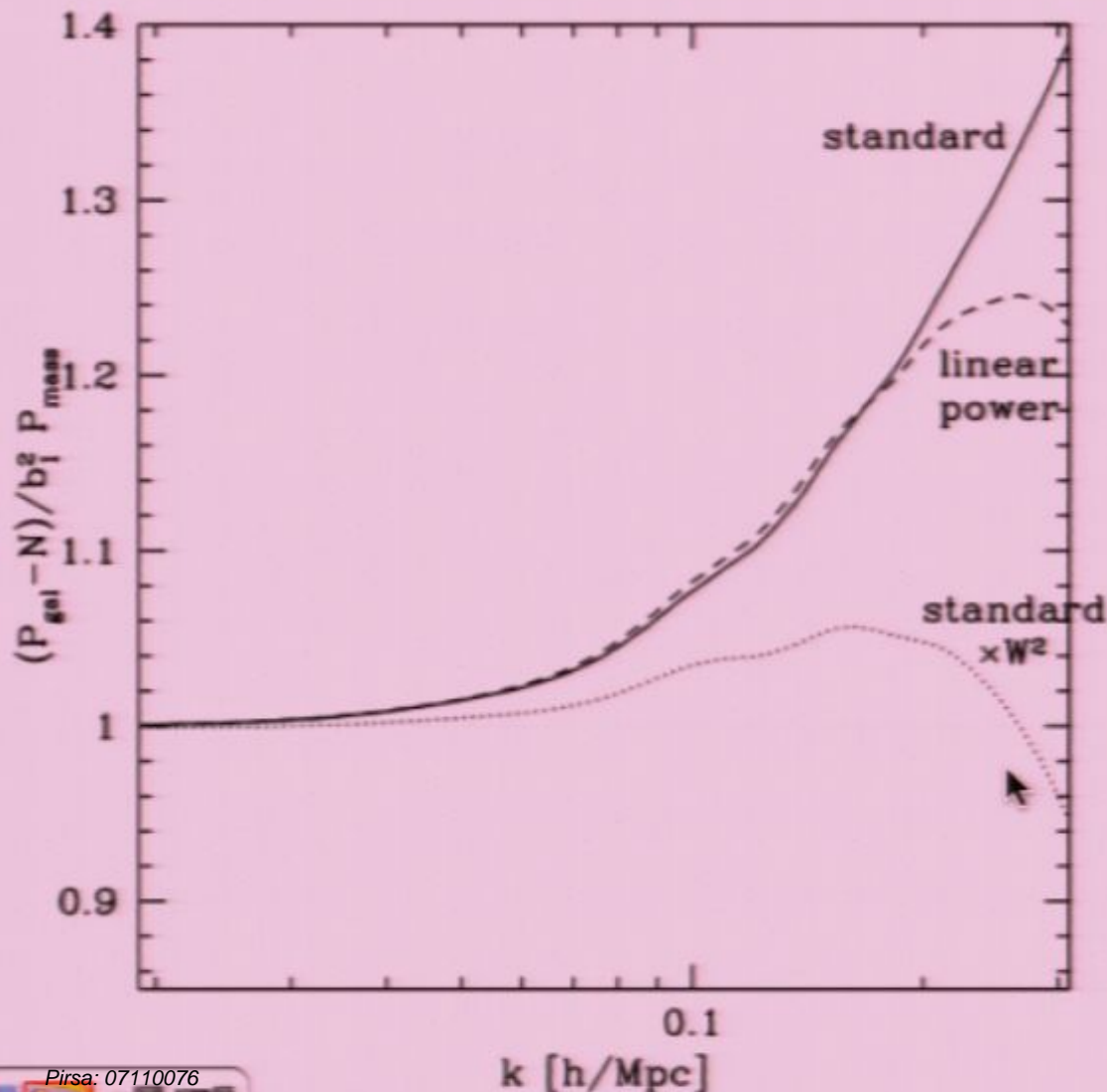
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Effect of high-k power



Standard calculation (solid) uses RG power to evaluate the bias integrals.

Dashed uses linear power.

Dotted shows the effect of 2 Mpc/h rms Gaussian smoothing (smoothing to control the Taylor series isn't a good option).

What about galaxy-mass correlation?

$$\langle \delta_g^a \delta^b \rangle = c_1 \langle \delta_a \delta_b \rangle + \frac{1}{2} c_2 \langle \delta_a^2 \delta_b \rangle + \frac{1}{6} c_3 \langle \delta_a^3 \delta_b \rangle + \dots$$

Modified bias, consistent with the previous redefinition.

No shot-noise.

$$b_1 = c_1 + \frac{1}{2} c_3 \sigma^2 + \frac{34}{21} c_2 \sigma^2$$

Same redefinitions also work for bispectrum.

Can easily add cross-correlations between different types of galaxy.

Toward a completely general model...

- The only other variable in standard PT is the velocity divergence: $\theta = -\mathcal{H}^{-1} \nabla \cdot \mathbf{v}$
 - In linear theory there is no point in including it in the bias model because $\theta = \delta$ but this isn't true at higher orders.
 - Including $\theta - \delta$ in the Taylor series for bias adds one new free parameter (work with Arabindo Roy).
- But then we could start adding things like $\nabla^2 \delta$, etc., to the model, with new free parameters.
- Also, what if the galaxy-mass relation isn't perfectly local?

Non-local model (derivative expansion)

- Galaxy density depends on mass density ~everywhere.

$$\delta_g(\mathbf{x}) = f[\delta(\mathbf{x}')]]$$

- First Taylor expand in delta

$$= f[0] + \int d\mathbf{x}' K(|\mathbf{x} - \mathbf{x}'|) \delta(\mathbf{x}') + \dots$$

- Then shift the integration variable and do a spatial expansion

$$\int d\Delta\mathbf{x} K(|\Delta\mathbf{x}|) \delta(\mathbf{x} + \Delta\mathbf{x}) = \int d\Delta\mathbf{x} K(|\Delta\mathbf{x}|) \left[\delta(\mathbf{x}) + \frac{d\delta}{dx_i}(\mathbf{x}) \Delta x_i + \frac{1}{2} \frac{d^2\delta}{dx_i dx_j}(\mathbf{x}) \Delta x_i \Delta x_j + \dots \right]$$

$$= \delta(\mathbf{x}) \underbrace{\int d\Delta\mathbf{x} K(|\Delta\mathbf{x}|)}_{\equiv b} + \frac{d\delta(\mathbf{x})}{dx_i} \underbrace{\int d\Delta\mathbf{x} K(|\Delta\mathbf{x}|) \Delta x_i}_{=0} + \frac{1}{2} \frac{d^2\delta(\mathbf{x})}{dx_i dx_j} \underbrace{\int d\Delta\mathbf{x} K(|\Delta\mathbf{x}|) \Delta x_i \Delta x_j}_{= b R^2 \delta_{ij} \mathcal{O}(1)} + \dots$$

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No Signal

VGA-1

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Result is an expansion in the non-locality scale R over the scale you're observing, $1/k$

$$\delta_g(\mathbf{k}) = b \left[1 - \frac{\mathcal{O}(1)}{2} R^2 k^2 \right] \delta(\mathbf{k}) + \dots$$

$$P_g(k) = b^2 \left[1 - \tilde{b}_{k^2} R^2 k^2 \right] P(k) + \dots$$

- What looked like it might be a nightmare of new parameters turns out to be remarkably simple!
- For a reasonable $R \sim 1 \text{ Mpc}/h$, correction is $\sim 4\%$ at $k=0.2 \text{ h}/\text{Mpc}$.
- R depends on real physics of galaxy formation - should study with simulations/semi-analytic models.

Future directions

- Cosmological parameter estimation.
- Comparison to simulations.
- Finish generalizing the model.
- Redshift-space (easy if no velocity bias)
- Dynamic model?
 - Make a copy of the mass density evolution equations to describe galaxy density evolution, and add something like
$$\dot{\rho}_g = f(\rho)$$
to describe galaxy formation. This kind of this has been considered before, but now we have new tools (RG) to deal with it.

Non-local model (derivative expansion)

- Galaxy density depends on mass density ~everywhere.

$$\delta_g(\mathbf{x}) = f[\delta(\mathbf{x}')]]$$

- First Taylor expand in delta

$$= f[0] + \int d\mathbf{x}' K(|\mathbf{x} - \mathbf{x}'|) \delta(\mathbf{x}') + \dots$$

- Then shift the integration variable and do a spatial expansion

$$\int d\Delta\mathbf{x} K(|\Delta\mathbf{x}|) \delta(\mathbf{x} + \Delta\mathbf{x}) = \int d\Delta\mathbf{x} K(|\Delta\mathbf{x}|) \left[\delta(\mathbf{x}) + \frac{d\delta}{dx_i}(\mathbf{x}) \Delta x_i + \frac{1}{2} \frac{d^2\delta}{dx_i dx_j}(\mathbf{x}) \Delta x_i \Delta x_j + \dots \right]$$

$$= \delta(\mathbf{x}) \underbrace{\left[\int d\Delta\mathbf{x} K(|\Delta\mathbf{x}|) \right]}_{\equiv b} + \frac{d\delta(\mathbf{x})}{dx_i} \underbrace{\int d\Delta\mathbf{x} K(|\Delta\mathbf{x}|) \Delta x_i}_{\substack{\text{0} \\ \swarrow}} + \frac{1}{2} \frac{d^2\delta(\mathbf{x})}{dx_i dx_j} \underbrace{\left[\int d\Delta\mathbf{x} K(|\Delta\mathbf{x}|) \Delta x_i \Delta x_j \right]}_{= b R^2 \delta_{ij} \mathcal{O}(1)} + \dots$$

$$\delta_g(\mathbf{x}) = b \left[\delta(\mathbf{x}) + \frac{\mathcal{O}(1)}{2} R^2 \nabla^2 \delta(\mathbf{x}) \right] + \dots$$

Result is an expansion in the non-locality scale R over the scale you're observing, $1/k$

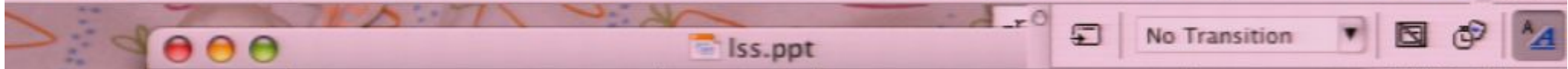
$$\delta_g(\mathbf{k}) = b \left[1 - \frac{\mathcal{O}(1)}{2} R^2 k^2 \right] \delta(\mathbf{k}) + \dots$$

$$P_g(k) = b^2 \left[1 - \tilde{b}_{k^2} R^2 k^2 \right] P(k) + \dots$$

- What looked like it might be a nightmare of new parameters turns out to be remarkably simple!
- For a reasonable $R \sim 1 \text{ Mpc}/h$, correction is $\sim 4\%$ at $k=0.2 \text{ h}/\text{Mpc}$.
- R depends on real physics of galaxy formation - should study with simulations/semi-analytic models.

Future directions

- Cosmological parameter estimation.
- Comparison to simulations.
- Finish generalizing the model.
- Redshift-space (easy if no velocity bias)
- Dynamic model?
 - Make a copy of the mass density evolution equations to describe galaxy density evolution, and add something like
$$\dot{\rho}_g = f(\rho)$$
to describe galaxy formation. This kind of this has been considered before, but now we have new tools (RG) to deal with it.



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Toward a completely general model...

- The only other variable is specified PT is the velocity dispersion: $\sigma = H^{-1} \nabla \cdot v$
- In this theory there is no policy in including it in the linear model because $\sigma \propto \delta$ has (partial) type of higher order
- In letting $\sigma \propto \delta$ to the Taylor series the linear order is the same as the previous (presumably Aristotelian) way
- But then we could start adding things like $\nabla^2 \delta$, etc., to the model with new parameters
- Also, what if the galaxy-galaxy relation isn't perfectly local?

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Non-local model (derivative expansion)

- Galaxy density depends on some density "environment", $\delta_g(x) = f(\delta(x))$
- Now Taylor expand to do this: $f(\delta) = f(0) + f'(0)\delta + \frac{f''(0)}{2}\delta^2 + \dots$
- The only free parameters available are the spatial expansion $f(\delta) = f(0) + f'(0)\delta + \frac{f''(0)}{2}\delta^2 + \dots$

$$\delta_g(x) = b \left[\delta(x) + \frac{O(1)}{2} R^2 \nabla^2 \delta(x) + \dots \right]$$

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Result is an expansion in the non-locality scale R over the scale you're observing, i.e.:

$$\delta_g(k) = b \left[1 - \frac{O(1)}{2} R^2 k^2 \right] \delta(k) + \dots$$

$$P_g(k) = b^2 \left[1 - b_2 R^2 k^2 \right] P(k) + \dots$$

- When looked like it might be a signature of new physics or primary to be a completely single
- for a somewhat Q=1 theory, correction is $\sim 40\%$ at $k=2$ h/Mpc
- if you can see physics of galaxy formation - should really help distinguish between models

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Future directions

- Cosmological parameter estimation
- Comparison to simulations
- High precision theory model
- Relativistic (any $f(R)$ or velocity field)
- Open model?

While a copy of the non-density expansion equation to show the galaxy density expansion, and add something like $\rho_g = f(\rho)$ to the other galaxy expansion. This kind of job has been considered before, but never have we written down the velocity

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Re-introducing velocity dispersion in Eulerian PT

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Digression: Renormalization group method

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|--------|----------|
| 18996 | 3 May |
| 48313 | 3 May |
| 28298 | 3 May |
| 46842 | 3 May |
| 89699 | 6 May |
| 24415 | 15 May |
| 49594 | 31 May |
| 28635 | 8 Jun 1 |
| 18930 | 8 Jun 1 |
| 17437 | 8 Jun 1 |
| 38162 | 8 Jun 1 |
| 35587 | 8 Jun 1 |
| 29922 | 8 Jun 1 |
| 25726 | 8 Jun 1 |
| 36693 | 8 Jun 1 |
| 17731 | 8 Jun 1 |
| 28921 | 8 Jun 1 |
| 36941 | 9 Jun 1 |
| 182258 | 12 Aug 1 |

Formatting Palette

▼ Presentation

Current Slide

Layout: Bulleted List

Transition: No Transition

Theme

Design template: Blank Presen

Color scheme:

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