Title: The Imprints of Primordial Non-Gaussianities on Virialized Objects

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Abstract:

# The imprints of primordial Non-Gaussianities on virialized objects

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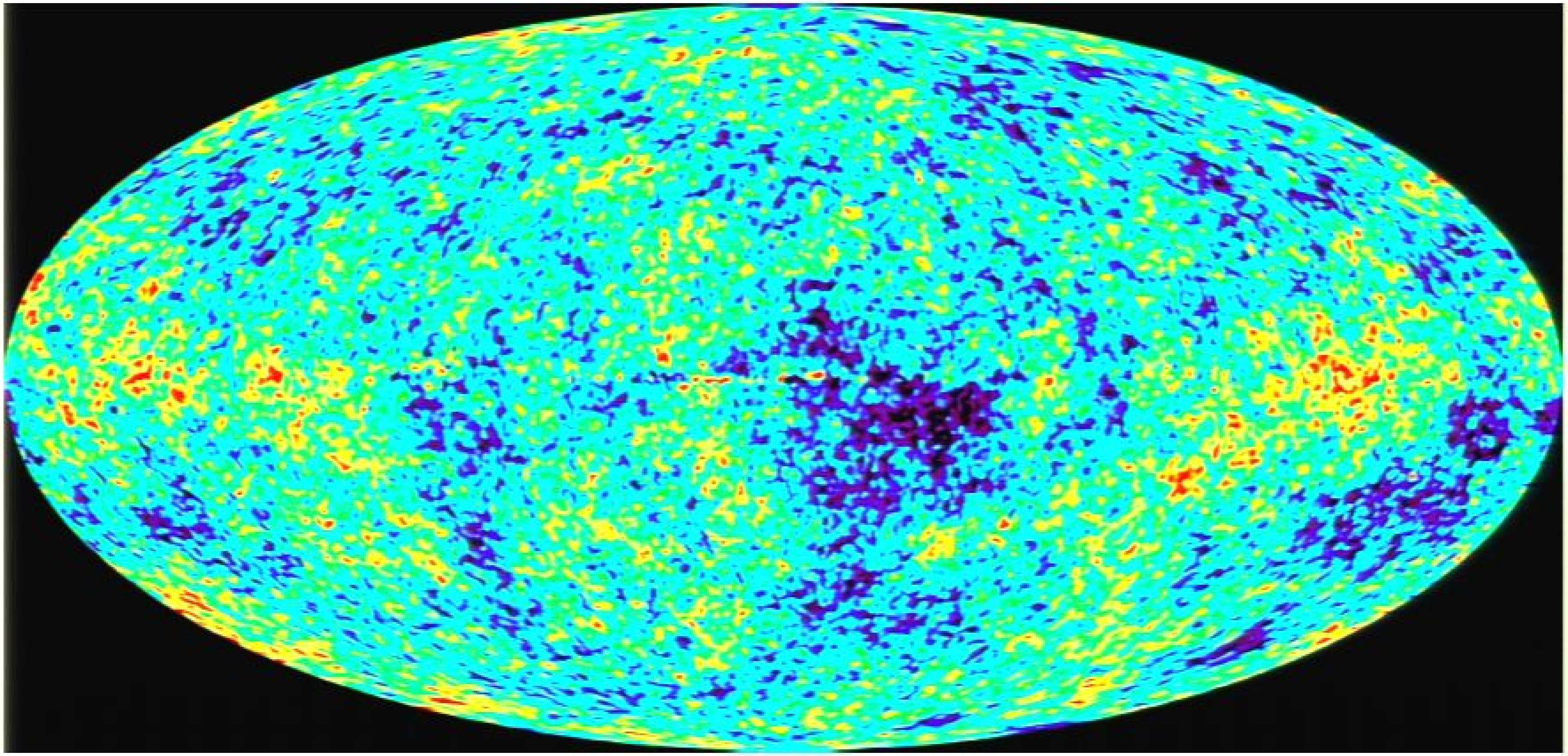
arXiv: 0710.4560

# Why is Non-Gaussianity important?



- The CMB as we observe it is Gaussian (not so obvious in fact...)
- The statistical analysis of this map supports the simplest Inflation predictions:
  - Geometry of the universe Flat at ~3% (WMAP 3 + HST)
  - Power spectrum index nearly scale invariant $N_s=0.96\pm0.016$ (WMAP3 only)
  - Limits on Non-Gaussian component to ~0.1% in power



- Gaussianity observationally supports Inflation as much as Flatness... so the level of non-

# Inflation, non-Gaussianity and density peaks

- Simplest inflationary models predict small but nonzero non-Gaussianity (non-linear evolution of the potential)
- For $\Phi \rightarrow \Phi + f_{NL}(\Phi^2 - \langle\Phi^2\rangle)$, expect $f_{NL} \sim O(1)$, $\Phi$ Bardeen potential (e.g. Salopek & Bond 90, Maldacena 2003)
- But other models also predict higher $f_{NL}$ ($\sim 100$) so that seeing or not-seeing primordial NG is important
- Current WMAP constraints (from T bispectrum): $f_{NL} \lesssim O(100)$ (CL 95%)
- We can expect $f_{NL} \lesssim O(10)$ from Planck
- The abundance of rare peaks can be strongly affected by this sort of skewness $\Rightarrow$ Can we constrain $f_{NL}$ using halo statistics like dN/dM or their bias?
- So far, qualitative estimates have been made using (extended) Press-Schechter, but PS mass function is off by an order of magnitude for cluster-sized halos (Verde et al. 01, Scoccimarro 04, Sefusatti et al. 06)
- Simulations are required to properly address the effect of $f_{NL}$ on dN/dM (Kang et al. 01, Grossi et al. 07)

# OUTLINE

- MASS FUNCTION:
  CONSTRUCTING A SIMPLE AND UNIVERSAL FITTING FORMULA FOR $F_{NL}$ COSMOLOGIES USING N-BODY SIMULATIONS

- LARGE SCALE HALO BIAS IN $F_{NL}$ COSMOLOGIES:
  ANALYTICS AND NUMERICAL INSIGHTS

- OBSERVATIONAL CONSEQUENCES

# GENERATING ICs FOR N-BODY SIMULATIONS WITH $F_{NL} \neq 0$

- GENERATE A REALIZATION OF $\Phi$ AS GAUSSIAN FIELD WITH A (NEARLY) SCALE INVARIANT POWER SPECTRA
- ADD THE QUADRATIC TERM IN REAL SPACE
- APPLY TRANSFER FUNCTION IN FOURIER SPACE
- USE ZELDOVICH APPROXIMATION TO GENERATE THE PARTICLE DATA
- CAVEAT: FINITE VOLUME EFFECT COULD BE TRICKY DUE TO MODE-COUPLING BUT IT SEEMS TO BE UNIMPORTANT
- WE GENERATE $512^3$ AND $1024^3$ SIMULATIONS USING THE $P^3M$ GRACOS CODE (WWW.GRACOS.ORG)

# Large Scale structures depends on $F_{NL}$

$f_{NL}$=-5000

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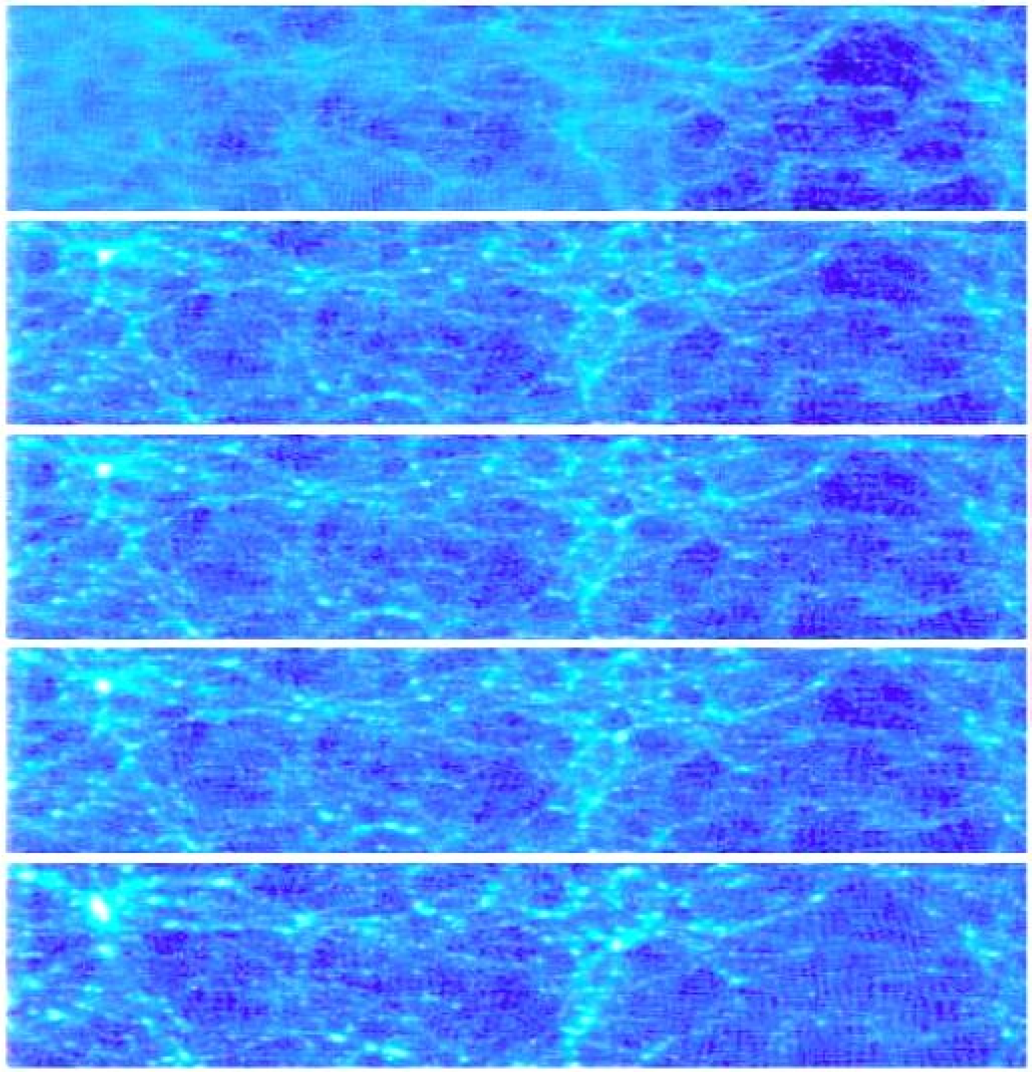

- Under-dense regions evolution decrease with $F_{NL}$
- Over-Dense regions evolution increase with $F_{NL}$

- Same initial Conditions, different $F_{NL}$
- Slice through a box in a simulation $N_{PART}=512^3$, L=800 Mpc/h

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- Generate a realization of $\Phi$ as Gaussian field with a (nearly) scale invariant power spectra
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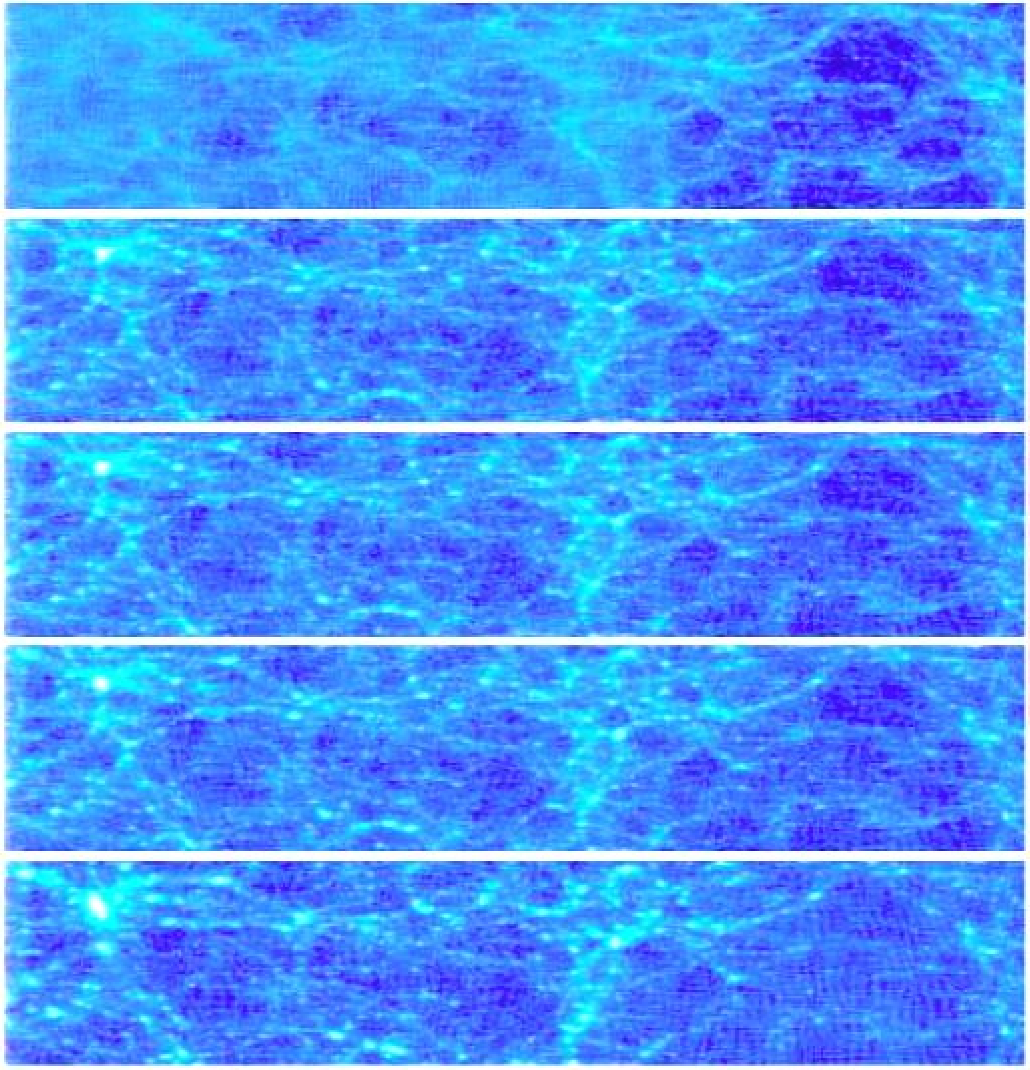

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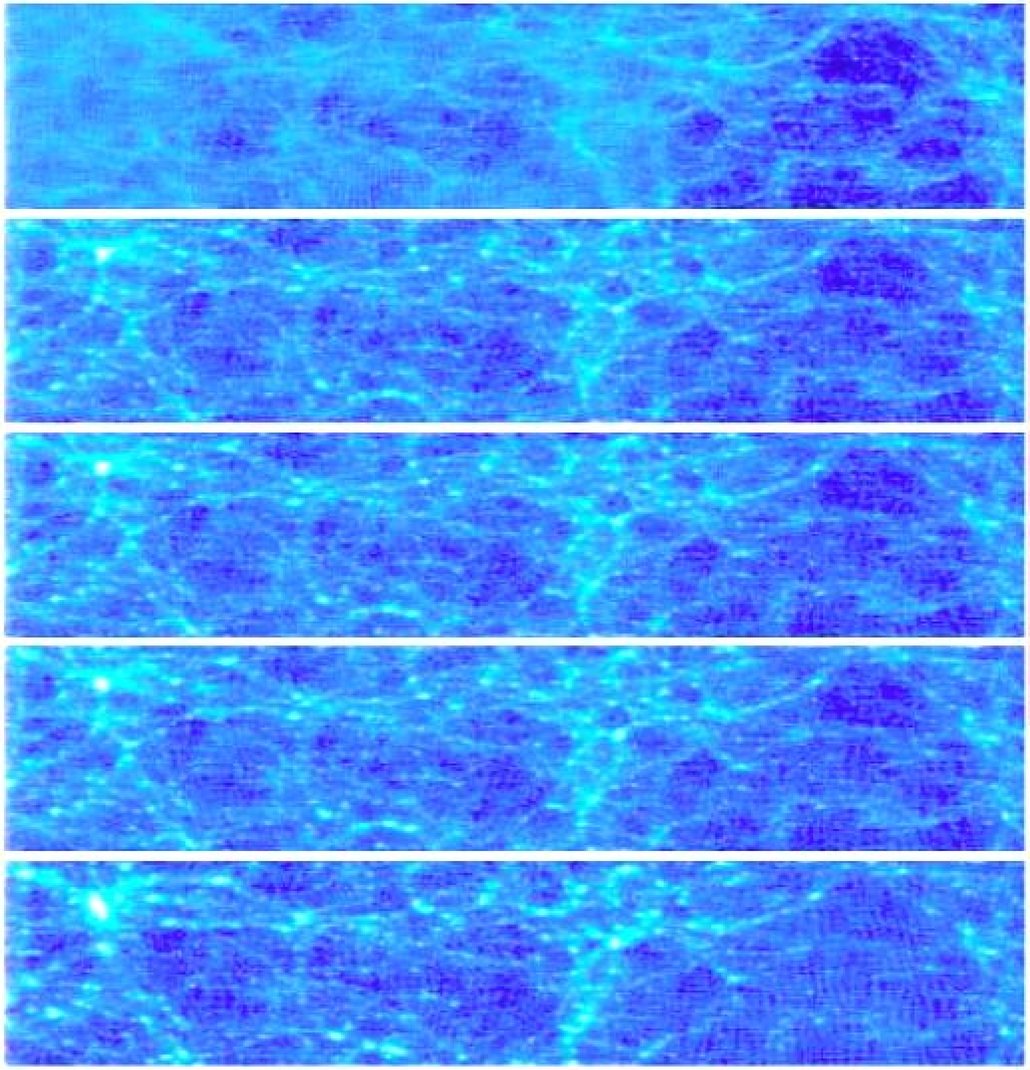

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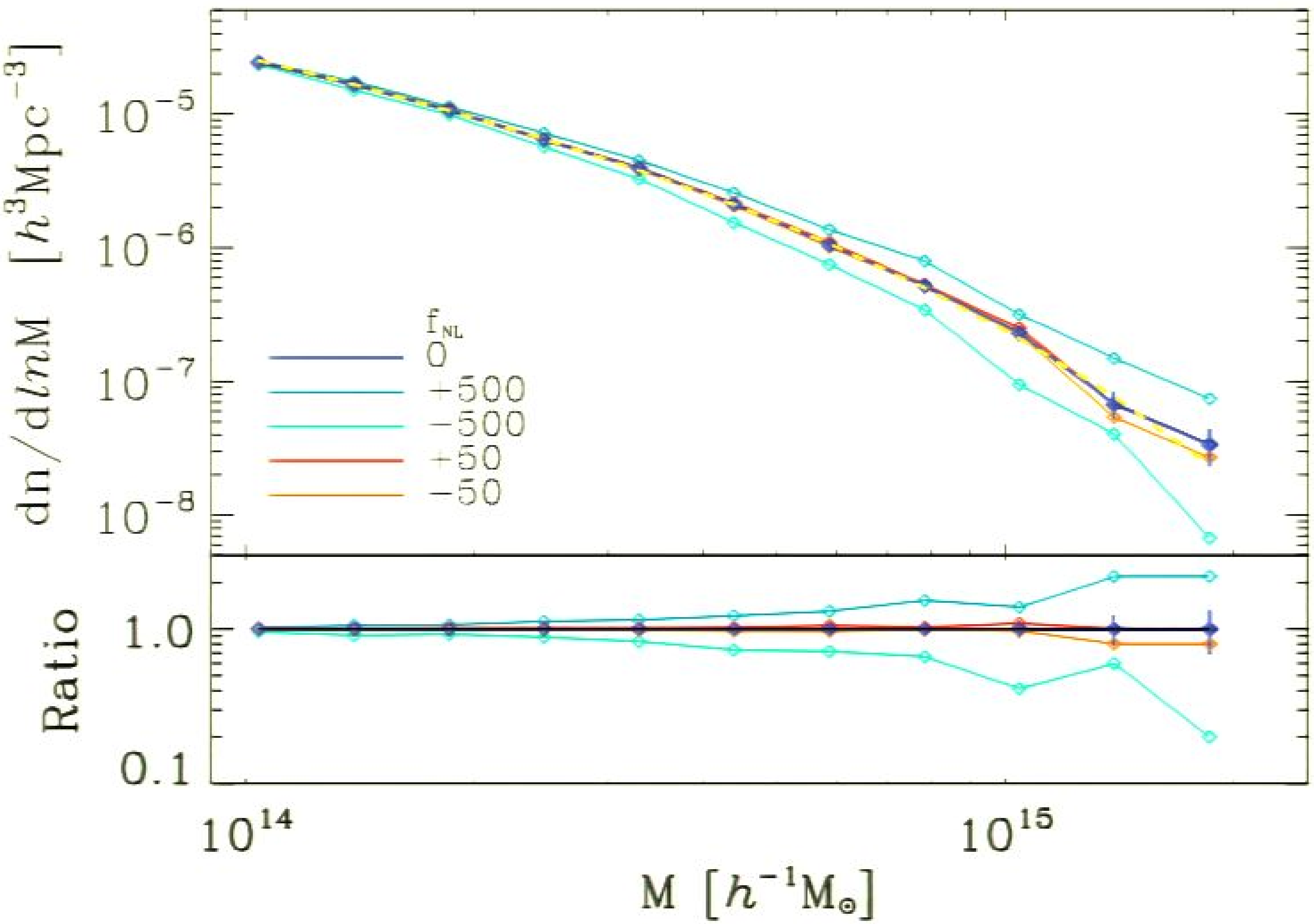

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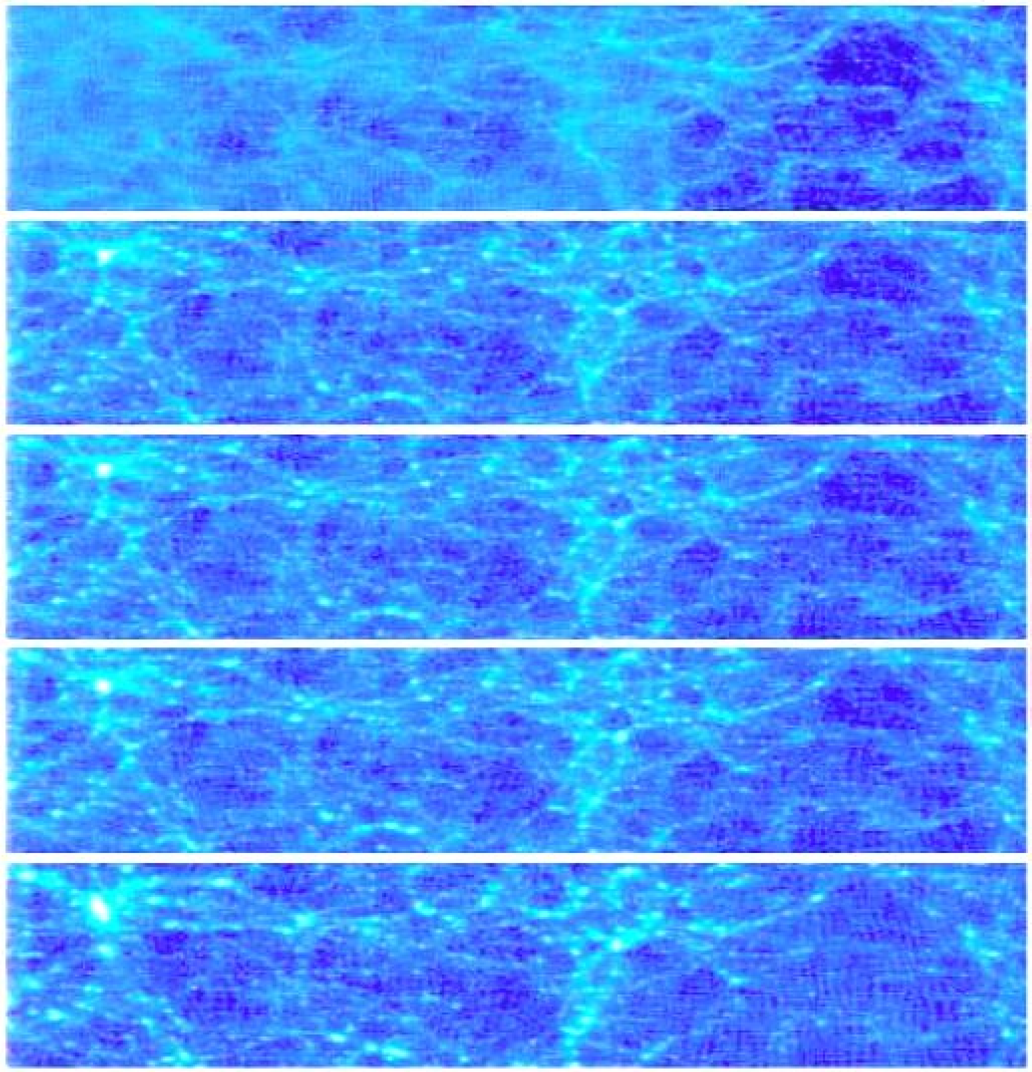

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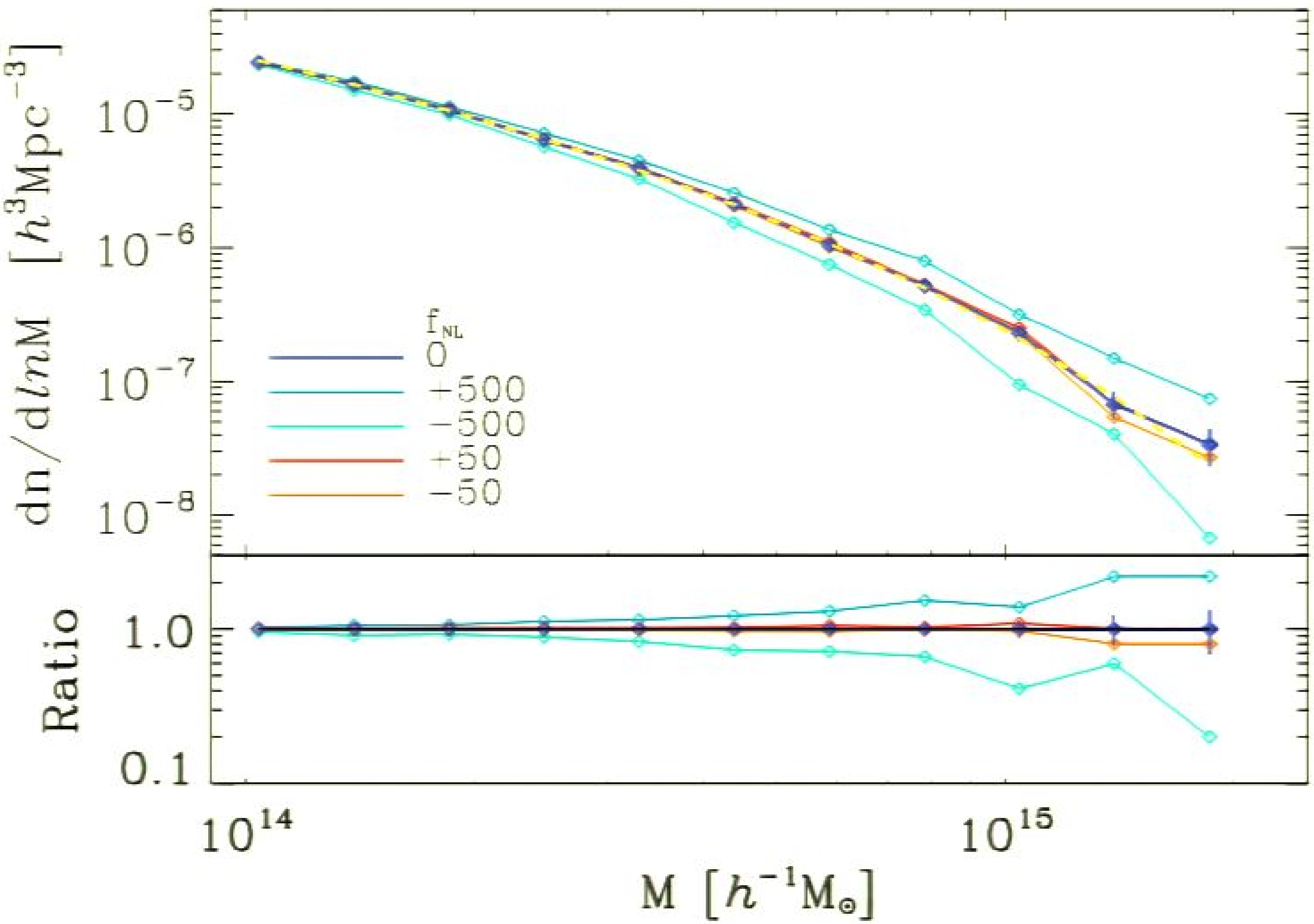
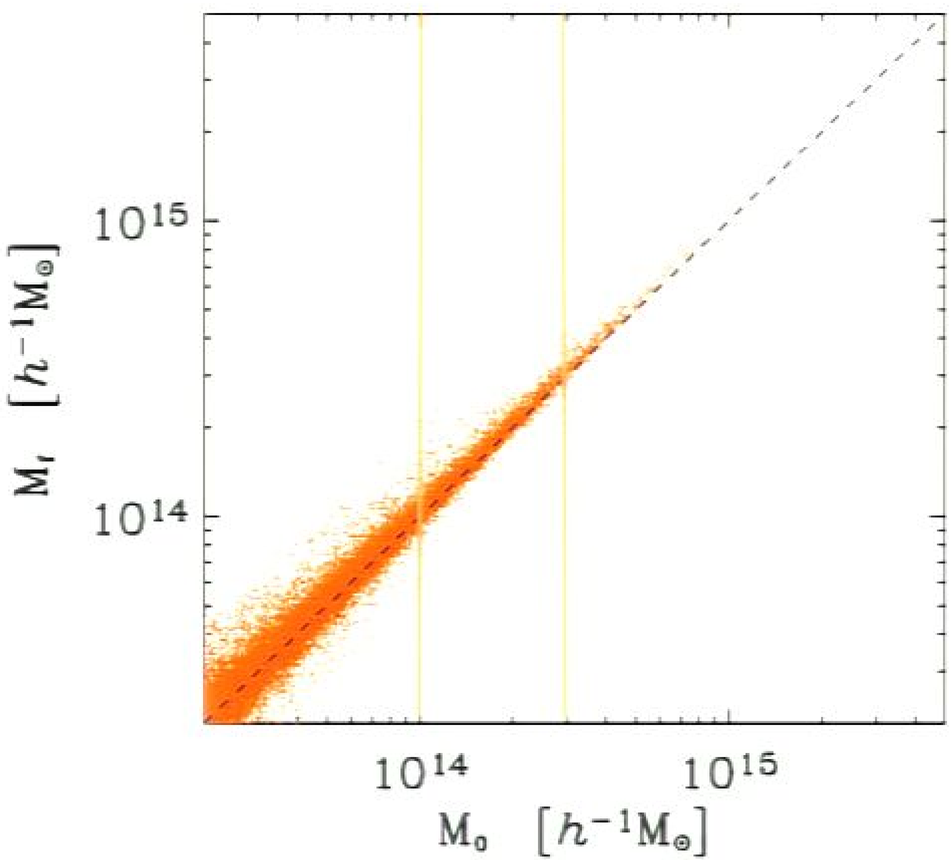


# BUILDING THE $P(M_F|M_O)$ DISTRIBUTION

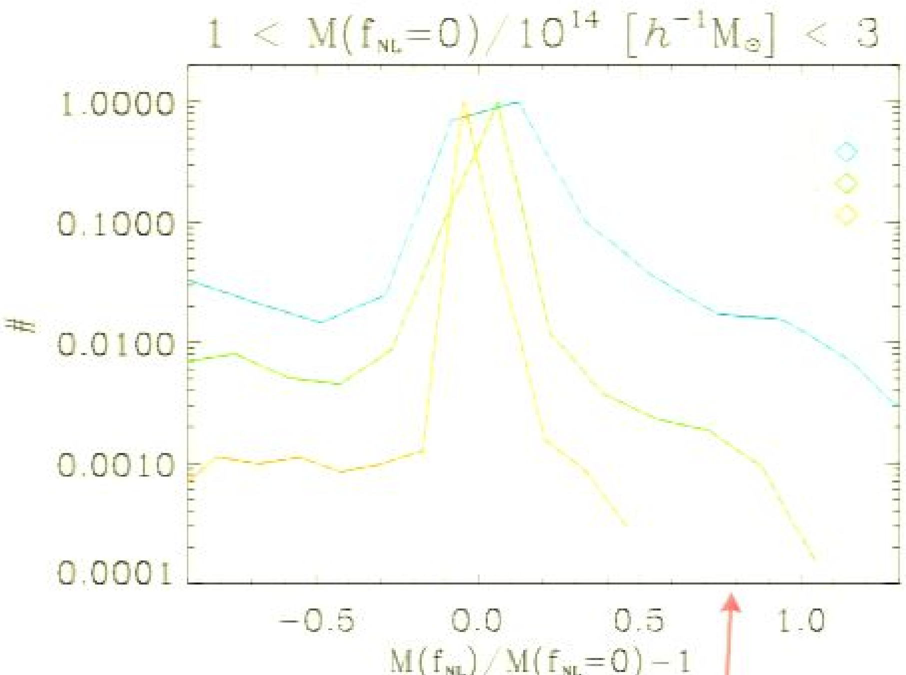
- USING A SIMPLE PARTICLE BASED PROCEDURE WE CAN MATCH CLUSTERS (MASS $M_O$) IN THE $F_{NL}$=0 SIMULATIONS TO THE CORRESPONDING ONE IN THE $F_{NL}$ SIMULATION (MASS $M_F$)

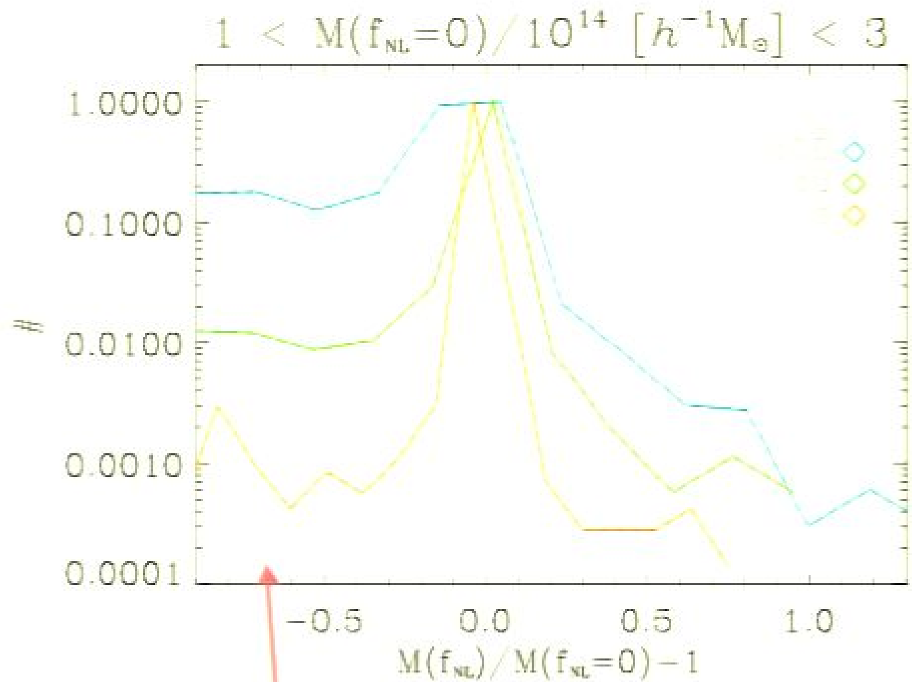



$F_{NL}$=500

# STATISTICAL APPROACH

## P($M_F$|$M_O$) BINNED IN $M_O$




SMALL CLUSTERS MERGE TO FORM BIGGER ONES

MASSIVE CLUSTERS SPLIT IN SMALLER ONES FOR $F_{NL}<0$

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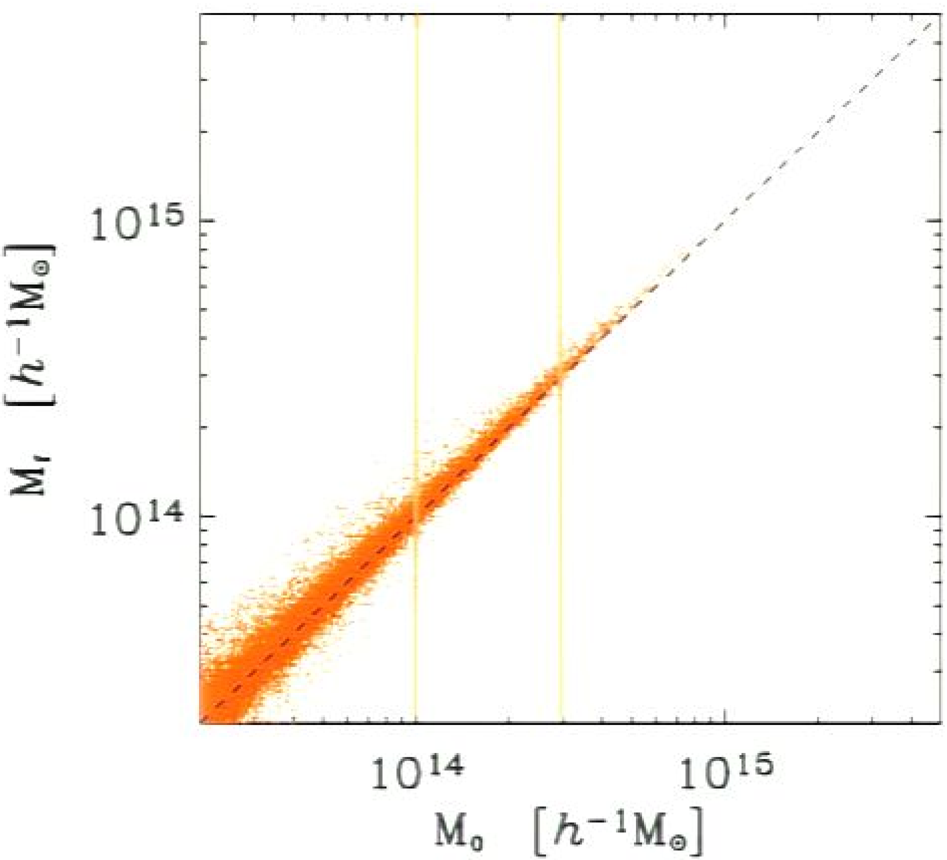
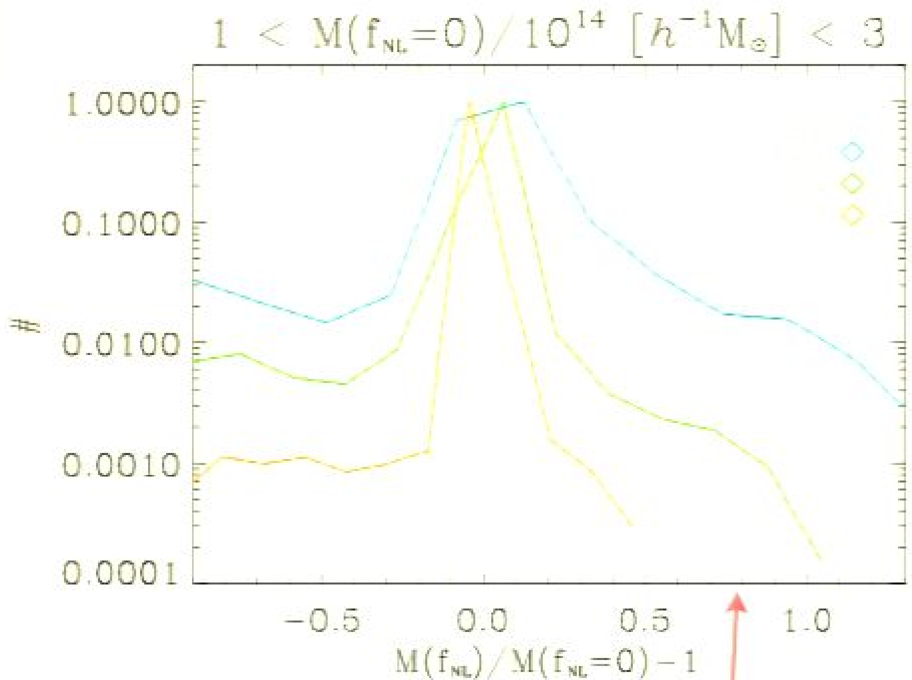
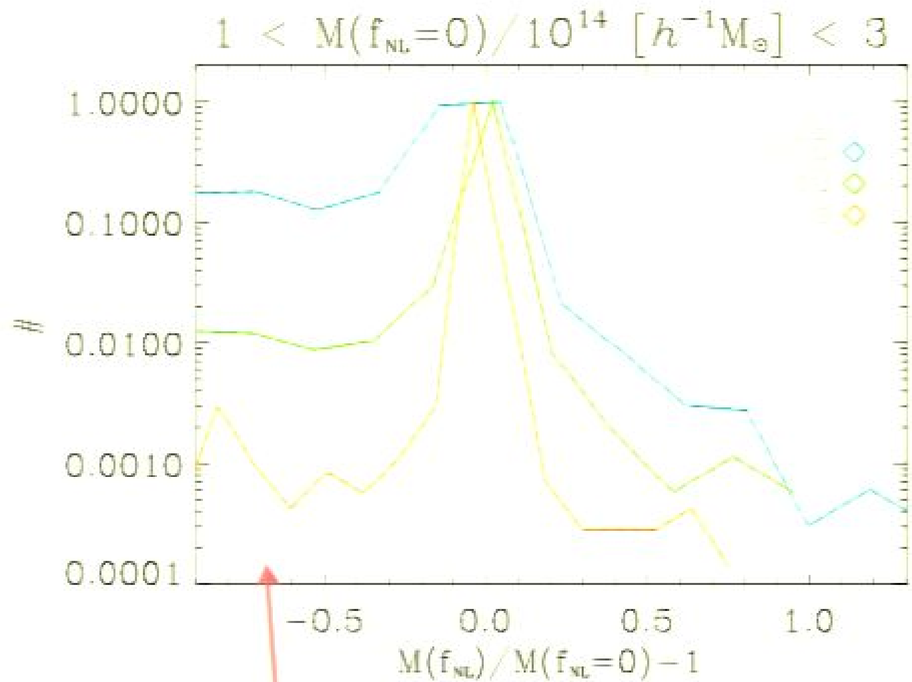



$F_{NL}$=500

# Statistical Approach

## P(M_F|M_O) binned in M_O

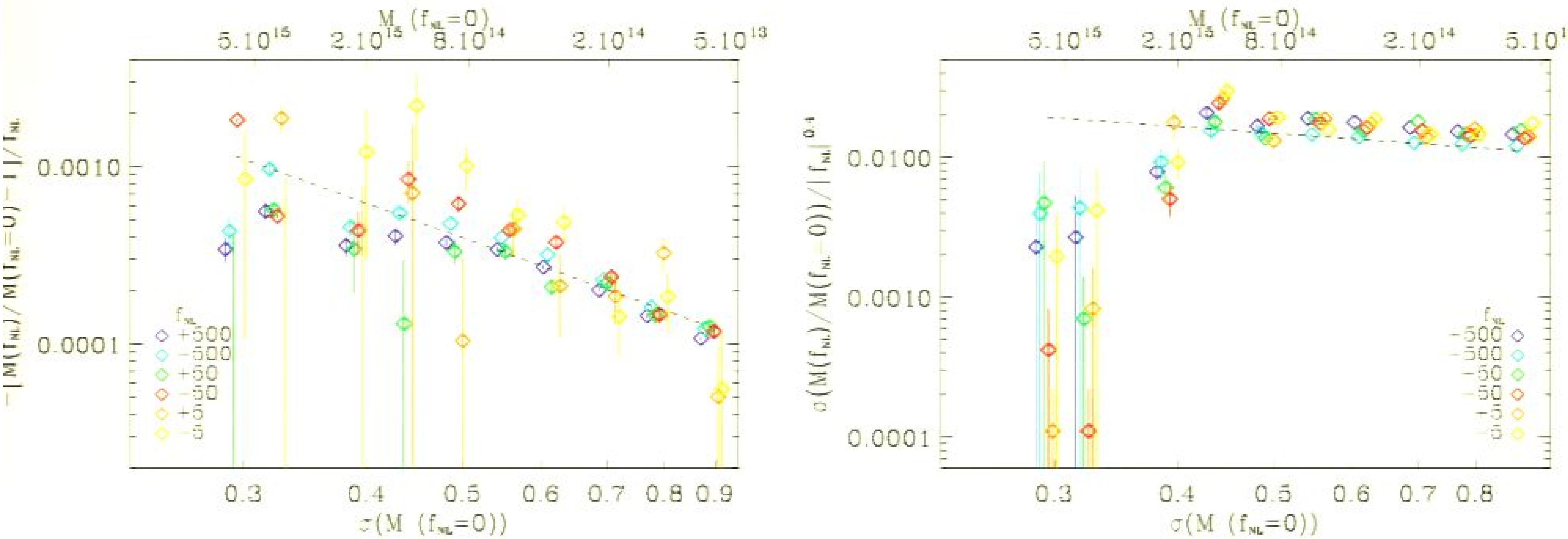



Small clusters merge to form bigger ones

Massive clusters split in smaller ones for $F_{NL}<0$

# DEFINING P(M$_F$|M$_O$)




DEFINING P(MF|MO) AS A GAUSSIAN, A GOOD FIT TO THE MEAN MASS SHIFT AND ITS VARIANCE IS GIVEN BY

$$\left\langle \frac{M_f}{M_0} \right\rangle - 1 = 1.3\ 10^{-4} f_{NL} \sigma_8\ \sigma(M_0, z)^{-2}$$

$$var\left(\frac{M_f}{M_0}\right) = 1.4\ 10^{-4} (f_{NL}\sigma_8)^{0.8}\ \sigma(M_0, z)^{-}$$

# A SIMPLE FITTING FUNCTION

- KNOWING THE MAPPING $M_O \to M_F$, IT IS THEN EASY TO BUILD A FITTING FUNCTION FOR DN/DM(M,Z,$F_{NL}$,...) STARTING FROM THE GAUSSIAN DN/DM, E.G. JENKINS ET AL.

$$\frac{dn}{dM_f} = \int dM_0 \frac{dn}{dM_0} \frac{dP(M_f|M_0)}{dM_f}(M_0)$$

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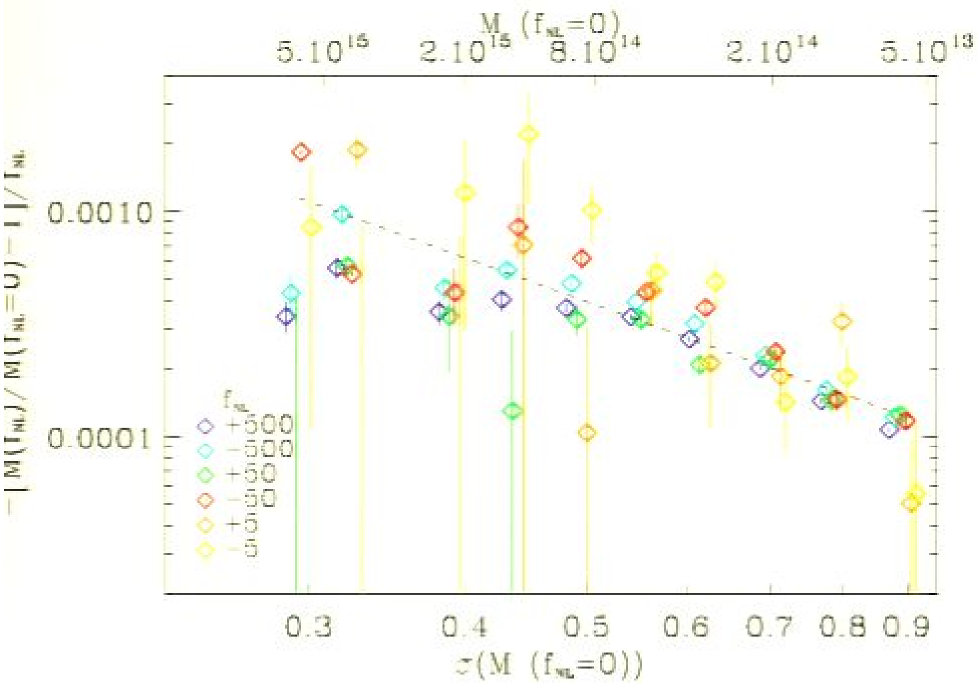


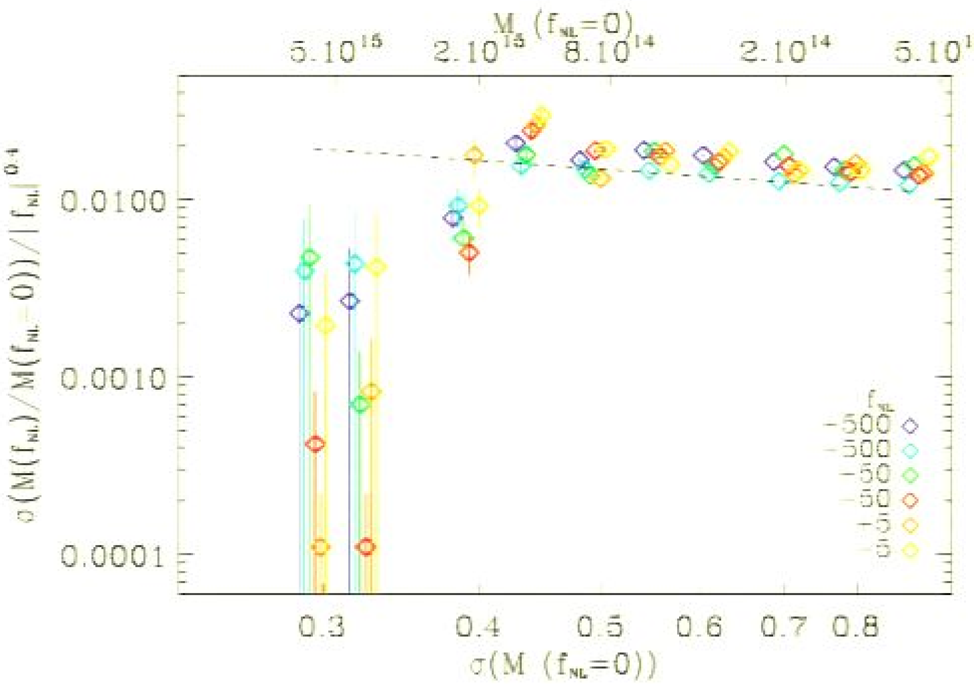


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$$\left\langle \frac{M_f}{M_0} \right\rangle - 1 = 1.3\ 10^{-4} f_{NL} \sigma_8\ \sigma(M_0,z)^{-2} \qquad var\left(\frac{M_f}{M_0}\right) = 1.4\ 10^{-4} (f_{NL}\sigma_8)^{0.8}\ \sigma(M_0,z)^{-}$$

# A simple fitting function

- Knowing the mapping $M_O \rightarrow M_F$, it is then easy to build a fitting function for $dn/dM(M, z, f_{NL}, \ldots)$ starting from the gaussian $dN/dM$, e.g. Jenkins et al.

$$\frac{dn}{dM_f} = \int dM_0 \frac{dn}{dM_0} \frac{dP(M_f|M_0)}{dM_f}(M_0)$$

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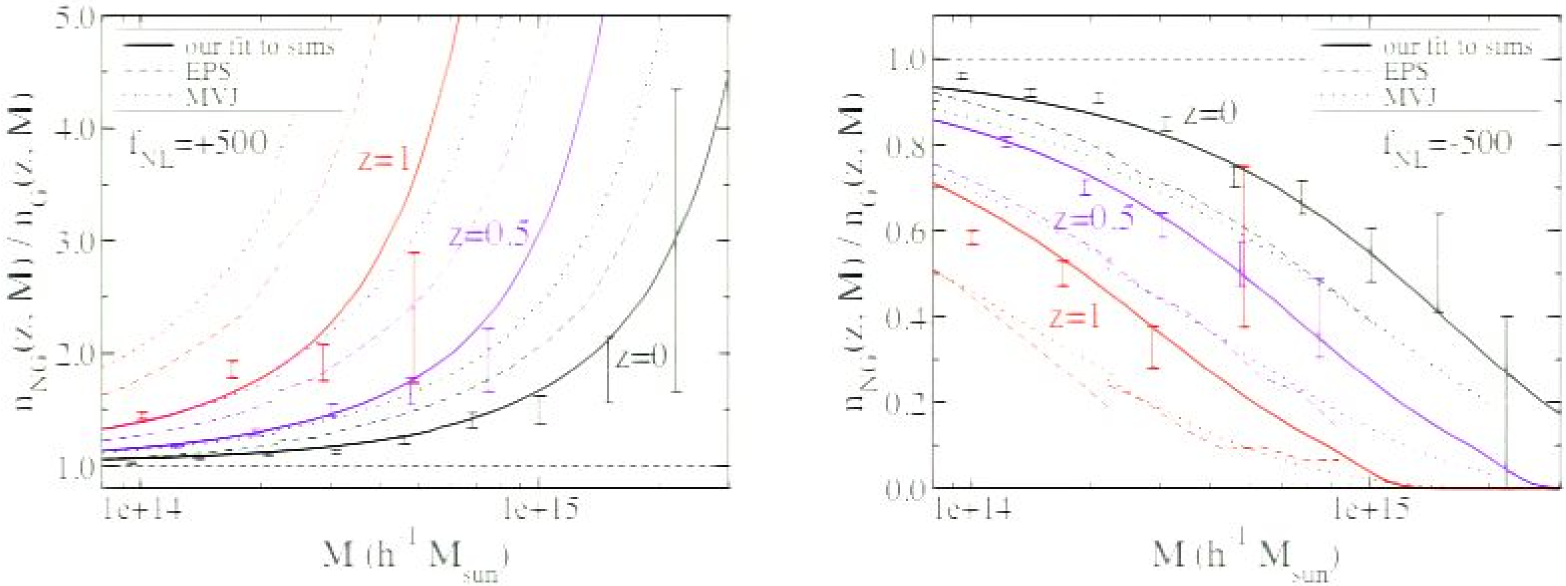



- RATIO OF NG TO GAUSSIAN DN/DM

- AGREEMENT TO WITHIN 10% OVER THE ENTIRE RANGE OF MASS AND Z
- ALTHOUGH NOT PERFECT, IT IS BASICALLY AS GOOD AS THE FITS TO GAUSSIAN (LUKIC ET AL. 07) SO NOT NEED TO REFINE MORE...

# Halo clustering: Analytical estimates I

- NG parametrized as

  $$\Phi = \phi + f_{NL}(\phi^2 - \langle \phi^2 \rangle)$$

  where $\Phi$ is a gaussian field

- It follows that

  $$\nabla^2 \Phi_{NG} = \nabla^2 \phi + 2 f_{NL} \left[ \phi \nabla^2 \phi + |\nabla \phi|^2 \right]$$

- Since $\Phi$ is a Gaussian field, we know the joint statistics of $\Phi$, $\nabla\Phi$ and $\nabla^2\Phi$ and using Poisson equation ($\nabla^2\Phi \propto \delta$) we can write down the PDF of $\delta_{NG}$ as a function of $\delta$. That leads to e.g.

  $$S_3 = \frac{\langle \delta_{NG}^3 \rangle}{\sigma^4} = 12 f_{NL} \frac{\langle \phi \delta \rangle}{\sigma^2}$$

- Near peaks, $|\nabla\Phi|^2$ is negligible and we get

  $$\delta_{NG} \simeq \delta(1 + 2 f_{NL} \phi)$$

# HALO CLUSTERING: ANALYTICAL ESTIMATES II

- WE THIS FORMULA.

$$\delta_{NG} \simeq \delta(1 + 2 f_{NL}\phi)$$

- IT IS EASY TO COMPUTE THE PEAK-PEAK CORRELATION FUNCTION ($\delta > \delta_{CRIT}$) (OR THE PEAK DENSITY) (À LA BBKS)

$$\xi_{pk} = b_L^2 \left[ \xi_{\delta\delta} + 4 f_{NL} \delta_{crit} \xi_{\phi\delta} \right]$$

- OR MORE INTERESTINGLY THE BIAS OF THE PEAK-PEAK CORRELATION (A LA BBKS)

$$P_{pk}(k) = b_L^2 P(k) \left[ 1 + 4 f_{NL} \delta_{crit} \frac{3\Omega_m}{2 a r_H^2 k^2} \right]$$

- WE OBTAIN THE FOLLOWING SCALE DEPENDENT BIAS

$$\Delta b(k) = 2 b_L f_{NL} \delta_{crit} \frac{3\Omega_m}{2 a r_H^2 k^2}$$

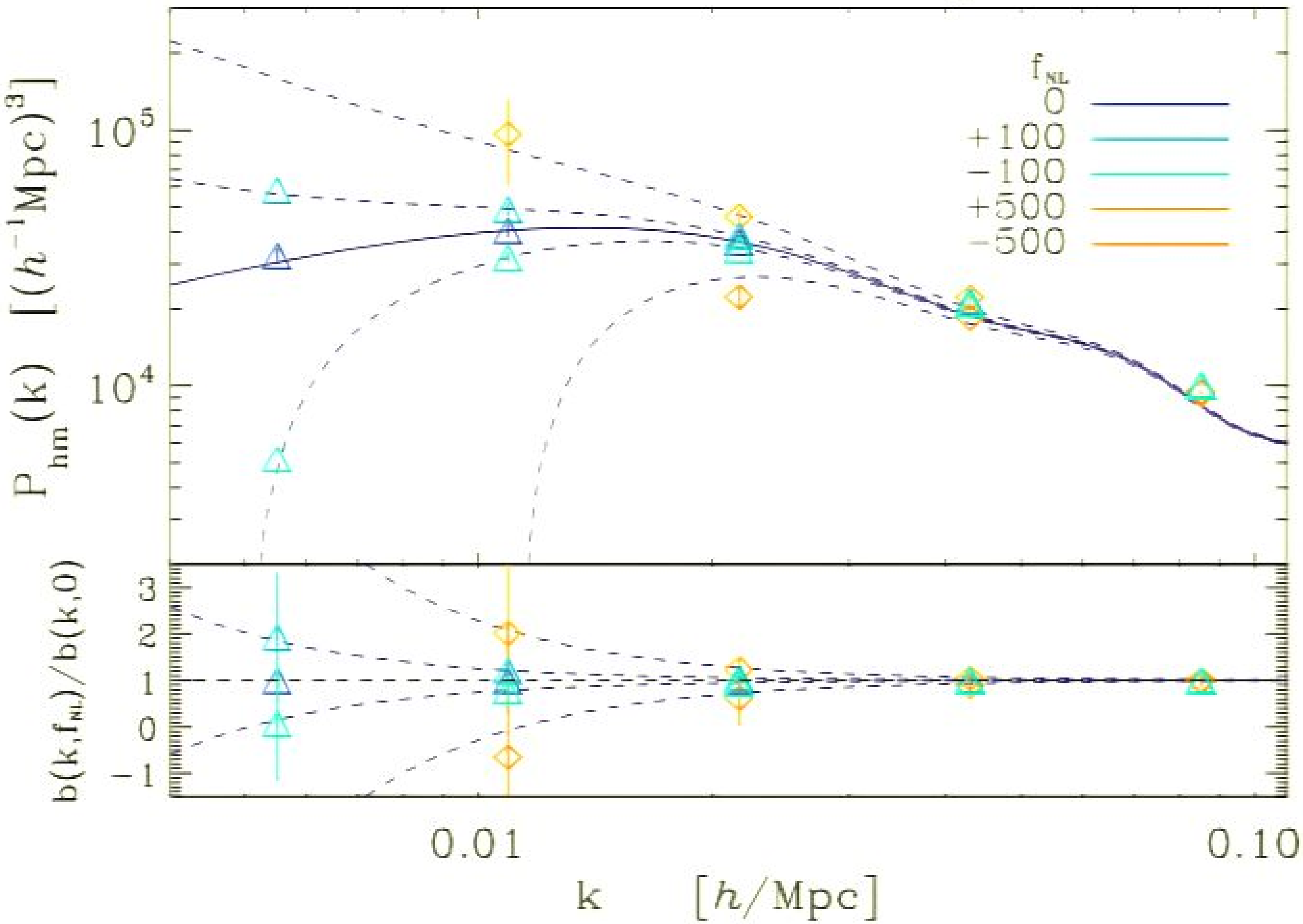
- THIS RESULT DIFFERS FROM THE USUAL CONSTANT LINEAR BIAS... BUT DERIVATION OF THIS RESULT GENERALLY ASSUMES LOCALITY OF THE "GALAXY" FORMATION PROCESS... WHEREAS HERE OUR NG TYPE IS NON LOCAL FOR $\delta$. IF WE WERE TO CONSIDER A NG OF THE TYPE $\delta_{NG} = \delta + f_{NL}\delta^2$, THEN WE WOULD NOT FIND ANY SCALE DEPENDENT BIAS

# Scale-dependent bias

$$\Delta b(k) = 2 b_L f_{NL} \delta_{crit} \frac{3\Omega_m}{2 a r_H^2 k^2}$$

Halo-Matter cross power spectrum for .6 $10^{13}$<M<3.2 $10^{13}$ M☉ b~2.3

Bias ratio




- Strong scale dependance in the linear regime
- Perfect agreement with sims and analytics (no free parameter)

# Constraints from P(k): Galaxy surveys, BAO, and ISW

- To measure the large scale bias offers a new opportunity to measure $f_{NL}$
- This effect should be easy to measure since there is a very specific k and z dependance on large scales
- A LRG survey ($b_L$=2, n=4. $10^{-5}$ $(h^{-1}Mpc)^{-3}$) out to z=0.7 could give $f_{NL} \leq 5\ f_{sky}^{-1/2}$
- This effect will also shift the first BAO peak at k=0.07h/Mpc by 0.4% at z=1 for $f_{NL}$=100, which would lead to a 1~2% bias in w if unaccounted for
- Ultimate ISW observations (~7$\sigma$) should yield $f_{NL} \leq 40$ (1$\sigma$) and current observations (~2-4$\sigma$) should imply around $f_{NL} \leq 120$ (1$\sigma$)
- It also opens the (unexplored yet) possibility to use void statistics to measure $f_{NL}$. Can we do that with current (Ly$\alpha$?) SDSS data?

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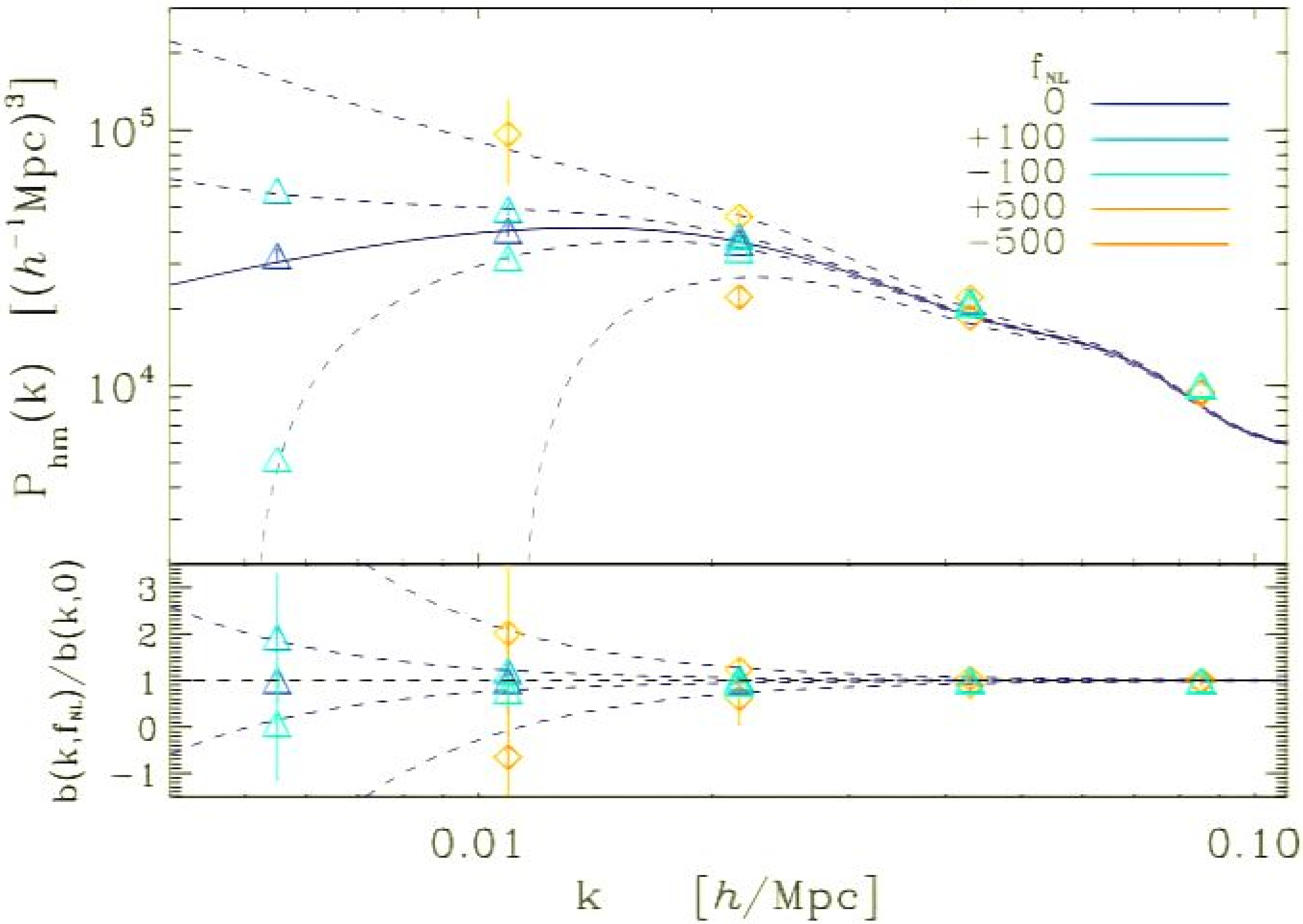


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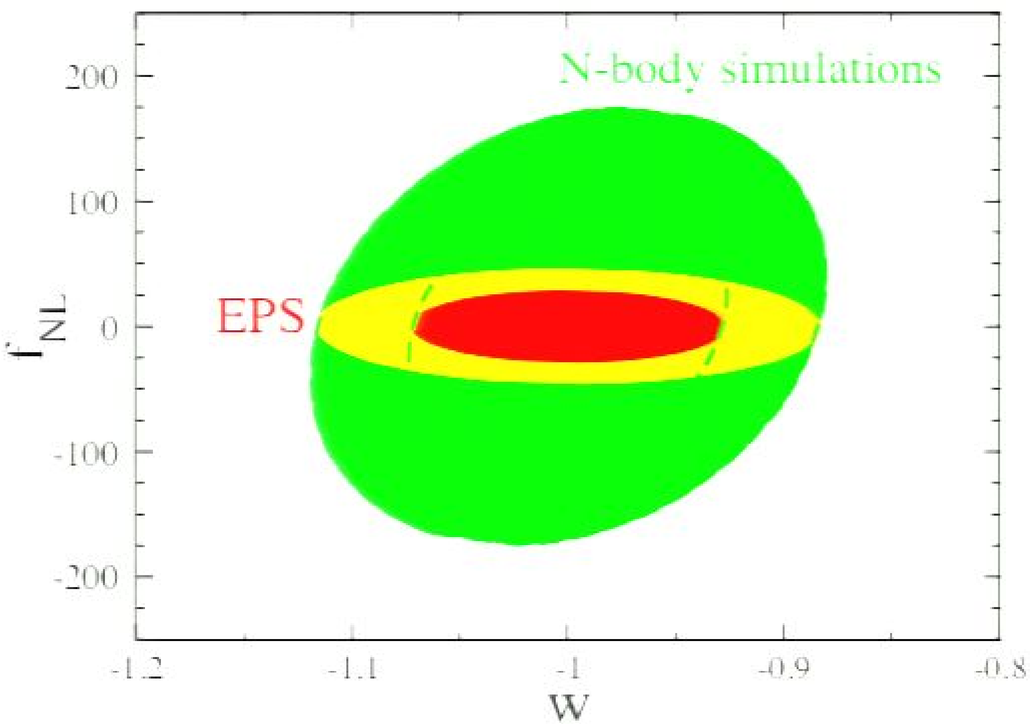
# Measuring $F_{NL}$ with dN/dM?




- SPT like survey, ie 4000 sq. deg up to z=1.5
- ~7000 clusters with M>2 $10^4$ $M_\odot$
- $|F_{NL}|$~< 100
- Not really a nuisance for w

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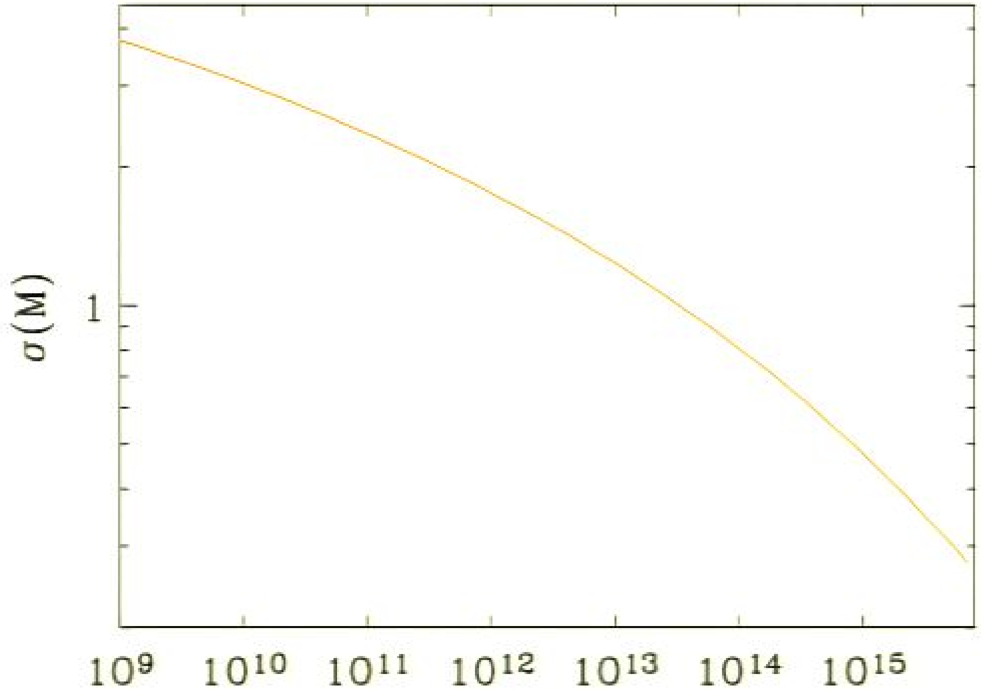
# MEASURING $F_{NL}$ WITH HIGH Z HALO COUNTS?

- IF OUR ANSATZ FOR THE MAPPING RELATION IS CORRECT THEN IT SUGGESTS THAT WHAT MATTERS IS $\sigma(M,z)^{-2}$ AND NOT $M/M^*(z)$
- FOR EXAMPLE, HIGH-Z QSOS OR GALAXIES, EVEN THOUGH THEY HAVE $M/M^*$ GREATER THAN 200 OR SO BELONG TO $10^{12}$ HALOS AND THUS LEAD TO A 2% EFFECT FOR $F_{NL}\sim500$ AT Z=6
- THIS IS TO BE COMPARED TO A 25% EFFECT AT Z=1 FOR A $10^{15}$ OBJECT



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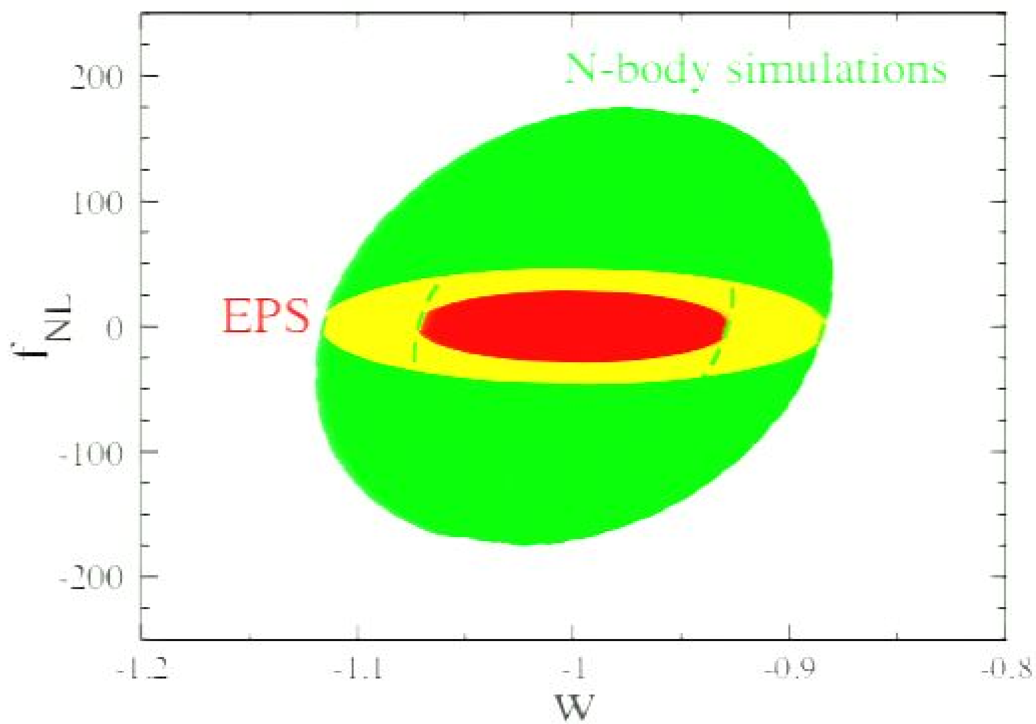


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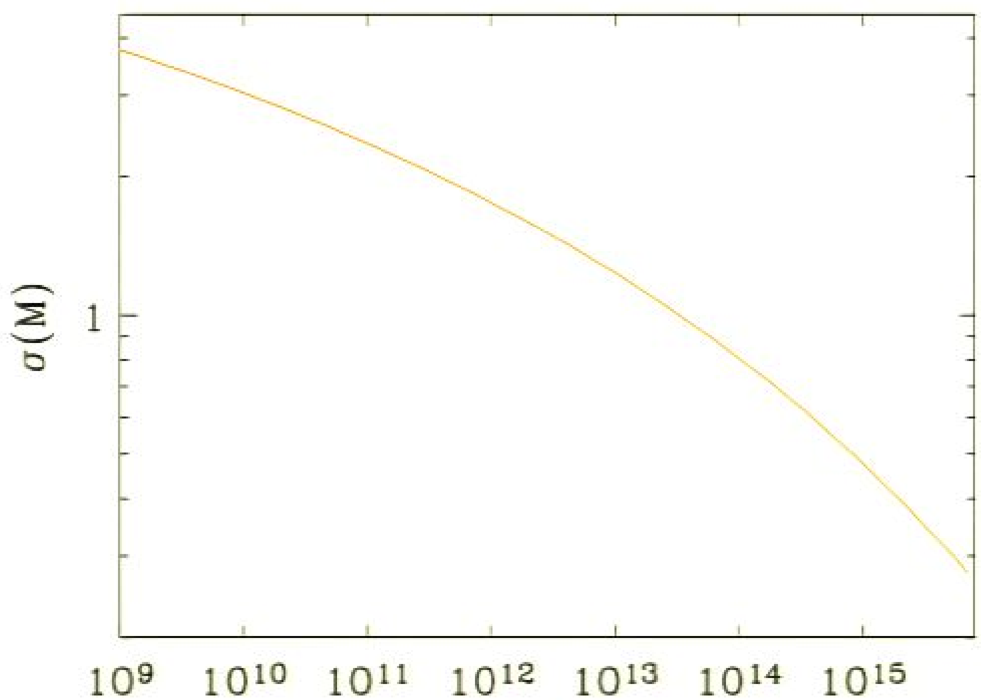
# MEASURING $F_{NL}$ WITH HIGH Z HALO COUNTS?

- If our ansatz for the mapping relation is correct then it suggests that what matters is $\sigma(M,z)^{-2}$ and not $M/M^*(z)$
- For example, high-z QSOs or galaxies, even though they have $M/M^*$ greater than 200 or so belong to $10^{12}$ halos and thus lead to a 2% effect for $F_{NL}$~500 at z=6
- This is to be compared to a 25% effect at z=1 for a $10^{15}$ object



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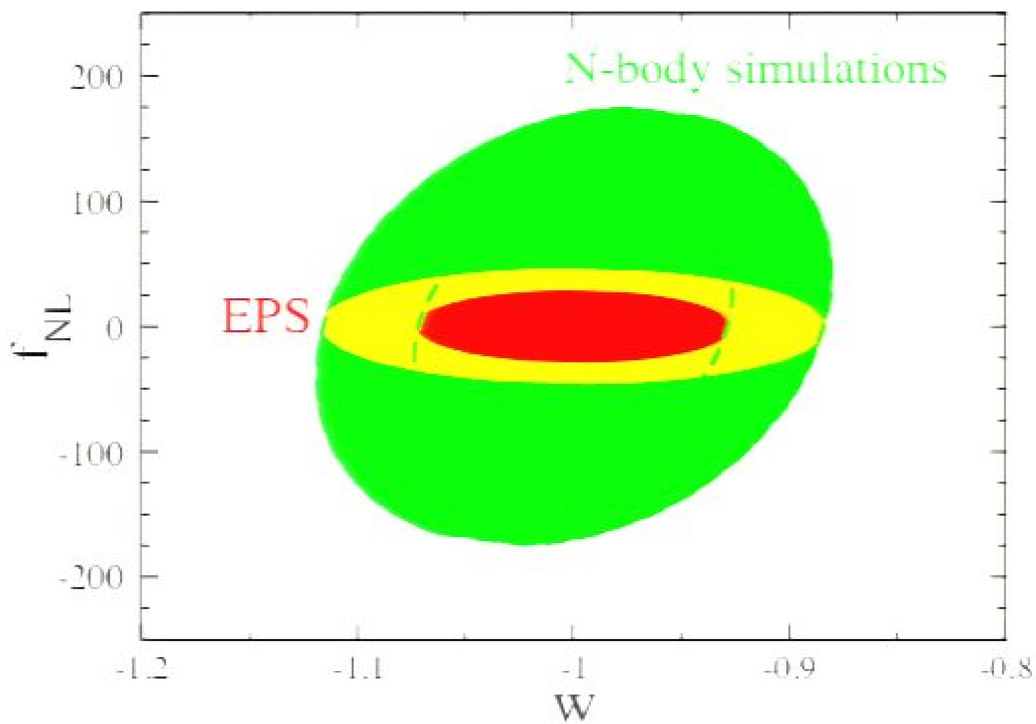


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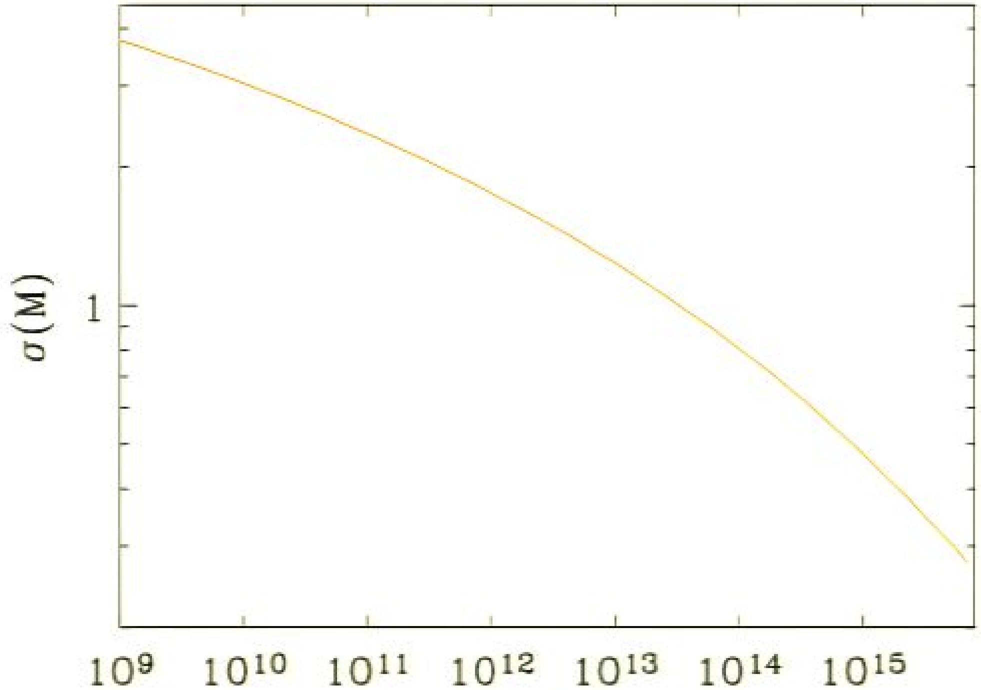

# Summary

- We proposed a new and measurable observational signature of primordial NG (constant $f_{NL}$ type)
- We proposed a simple and accurate fitting formula for the halo mass function as a function of $f_{NL}$
- We showed how to calculate peak statistics for $f_{NL}$ cosmologies and in particular the bias shape
- We tested our predictions against N-body simulations and fund a remarkable agreement
- As a consequence, the measure of the large scale halo bias should give $f_{NL}$ constraints comparable to Planck, i.e. $f_{NL} < O(10)$
- All those conclusions rely on one type of "local NG", what happens if we take into account $f_{NL}(k)$ for example (eg LoVerde et al. 07) remains to be studied