Title: Entanglement of assistance and random distillation

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Abstract: A class of operations distinct to entangled states shared between more than two parties is their conversion to entangled states shared between fewer parties. The extent to which these can be achieved in the regime of local operations and classical communication provides an operational characterisation of multiparty states, for example in the \"entanglement of assistance\" and related quantities. I will give a brief overview of this topic and discuss our results showing qualitatively different behaviour when the parties receiving the final state are not chosen beforehand.

Entanglement of assistance and random distillation[1]

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Perimeter Institute, 21st November 2007

[1] Phys. Rev. Lett. 98, 260501 (2007), arXiv:0709.4059 (2007)

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Outline

Introduction

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Does how we choose the parties matter?

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For a three-party pure state $|\Psi\rangle_{ABC}$, entanglement measure E, the entanglement of assistance $E_a(\rho_{AB}) = \max(p_i E(\psi_i))$, maximised over decompositions $\rho_{AB} = \mathrm{tr}_C|\Psi_{ABC}\rangle\langle\Psi_{ABC}| = \sum_i p_i \psi_i$.

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Highest expected entanglement of Alice and Bob after Charlie makes a measurement and broadcasts his result. (Not general LOCC).

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2. Can be calculated for pure-state concurrence of assistance of two qubits (Laustsen, Verstraete, Van Enk (2003)), or for asymptotic distillable entanglement (Smolin et al, (2005))
3. Not a bipartite measure or tripartite monotone (Gour & Spekkens (2006))

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3. For > 3 parties:
   For E=distillable entanglement, pure state, many copies, $C = \min\{S(AT), S(B\overline{T})\}$ (Horodecki, Oppenheim & Winter (2005))

Definitions - chosen parties

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$$\psi \underbrace{\rightarrow}_{LOCC} |\Phi\rangle^{\otimes M}_{P_iP_j}$$

$$E_{A_{ij}} \equiv \max(\langle M \rangle)$$

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$$\text{Asymptotically } \psi^{\otimes N} \rightarrow |\Phi\rangle_{P_i P_j}^{\otimes M}, \quad M, N \rightarrow \infty$$

$$E_{A_{ij}}^{\infty} \equiv \max \frac{M}{N}$$

Specified entanglement

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Maximising over all pairs - "specified entanglement" E_{sp}.

$$E_{sp}(\psi) \equiv \max_{ij} E_{A_{ij}}$$

$$E_{sp}^{\infty}(\psi) \equiv \max_{ij} E_{A_{ij}}^{\infty}$$

Bounds on E_A

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Multiple copies

$$E^{\infty}_{A_{ij}}(\psi) \leq \min(S(P_i), S(P_j))$$

Hence $E^{\infty}_{sp}(\psi) \leq$ second largest $S(P_i)$.

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Single copy

For a single copy E_A can be bound using the concurrence of assistance G_A.

$$E_{A_{ij}}(\psi) \leq G_{A_{ij}}(\psi)$$

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$$|\psi\rangle^{\otimes N}_{P_1 \dots P_m} \longrightarrow \bigotimes_{ij} |\Phi\rangle^{\otimes N_{P_i P_j}}_{P_i P_j}$$

$$E_t^{\infty}(\psi) = \sup \frac{\sum_{ij} N_{P_i P_j}}{N}$$

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Definitions - summary

1. "Assisted entanglement" E_A^∞ - how many EPRs we can make through LOCC between two chosen parties from a multiparty state.
2. "Specified entanglement" E_{sp}^∞ - the assisted entanglement, maximised over all pairs.
3. "Total entanglement" E_t^∞ - how many EPRs we can make through LOCC from a multiparty state, irrespective of who shares them.

2 New results - random distillation and the W state

- EPR from W
- Bounds

Specified parties

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Can't reliably make 1 EPR per W for specified parties.

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Can always make an EPR between *random* parties.
(Optimal for a single copy, conjectured to be optimal for many copies)

The protocol 1

Alice, Bob and Charlie each apply the unitary

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then

$$|W\rangle_{ABC} \longrightarrow (1-\epsilon^2)|W\rangle + \frac{\epsilon}{\sqrt{3}}(|021\rangle + |201\rangle + |012\rangle + |210\rangle + |102\rangle + |120\rangle) + O(\epsilon^2)$$

The protocol 2

All 3 parties perform the projection

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In limit of many rounds, small ϵ, 2 *random* parties always get an EPR.

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Protocol beats $E_{sp}(W)$ for $N \geq 3$, beats $E_{sp}^{\infty}(W)$ for $N \geq 12$.

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- Teleport $|\alpha\rangle_{A}^{\otimes \frac{N}{6}} \to |\alpha\rangle_{ABC}^{\otimes \frac{N}{6}}$, likewise for Bob and Charlie.

Upper-bounding the EPR yield - 3 parties

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Linden *et al* (2005)

For LOCC on pure ρ_{ABC}:

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leading to

Theorem 2.

$$E_t^\infty(\sigma_{ABC}) \leq \min\{S(\sigma_{BC}) + E_r^\infty(\sigma_{BC}), S(\sigma_{AC}) + E_r^\infty(\sigma_{AC}), S(\sigma_{AB}) + E_r^\infty(\sigma_{AB})\}$$

(E_r =relative entropy of entanglement)

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(follows from Plenio & Vedral (2001))

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"Lower bound on upper bound" (Galvão, Plenio & Virmani, 2000)

$$E_r^{\infty}(W) \geq \log_2(3) - 5/9 \approx 1.03$$

(Can't prove conjecture using E_r alone)

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No advantage to random distillation.

3. More general states
   - 3-qubit classes
   - More parties

W and GHZ class

Dür *et al*, (2000)

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What random distillation properties do these classes have?

Symmetric states - simple protocol

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1. For 3-qubit symmetric state, pick a basis where Alice measuring $|0\rangle$ corresponds to large Bob-Charlie entanglement.

$$|\psi_1\rangle_{ABC} = |0\rangle_A(\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle)_{BC} + |1\rangle_{BC}(\ldots)$$

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Protocol:filter to symmetric state, then do random distillation.

Results

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For all W–class states

$$|W'\rangle = (\alpha|100\rangle + \beta|010\rangle + \gamma|001\rangle + \delta|000\rangle)$$

($\{\alpha, \beta, \gamma, \delta\} \in \mathbb{R}$, and $\alpha, \beta, \gamma > 0, \delta \geq 0$.)

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Random distillation is always advantageous for the W class.

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$$E_t^\infty(W) \geq E_{sp}^\infty(W)$$

Random distillation is always advantageous for the W class.
For GHZ class, random distillation is sometimes advantageous.
E.g. for

$$|\psi_G\rangle = \alpha(|100\rangle + |010\rangle + |001\rangle) + \epsilon|111\rangle, \quad \epsilon = \sqrt{1 - 3\alpha^2}, \alpha \geq 7/32$$

Multi-party W state

$$|W_M\rangle = \frac{1}{\sqrt{M}}\left(|\underbrace{00\ldots0}_{M-1}1\rangle + (\text{permutations})\right)$$

$(W_3 = W,\ W_2 = EPR)$

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$$E_{sp}^{\infty}(W_M) = H_2(1/M)$$
$$E_t^{\infty}(W_M) \geq 1$$

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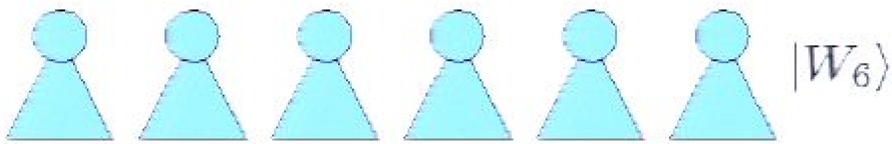
Can always get an EPR between random parties even though probability between specified parties is arbitrarily small.

Multi-party W



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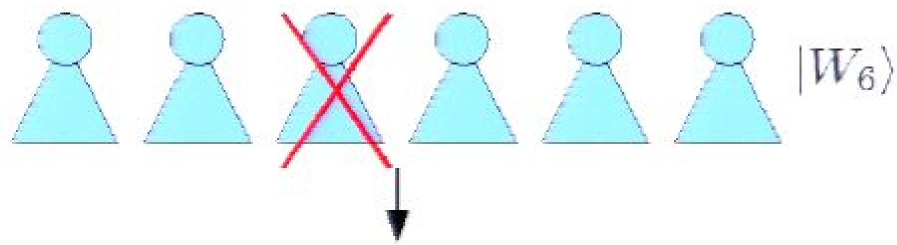

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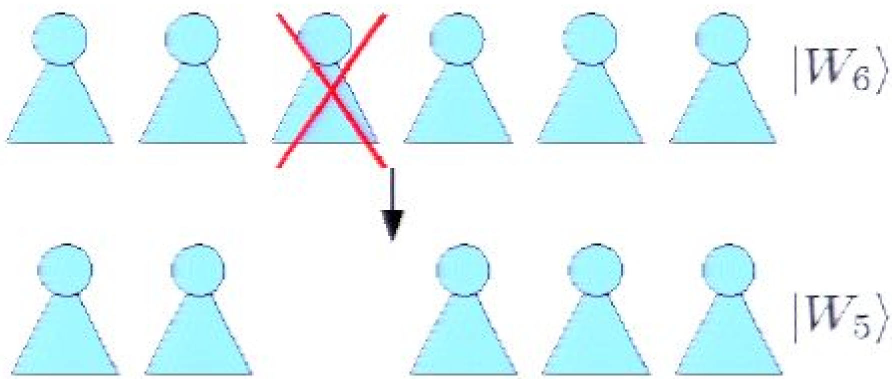

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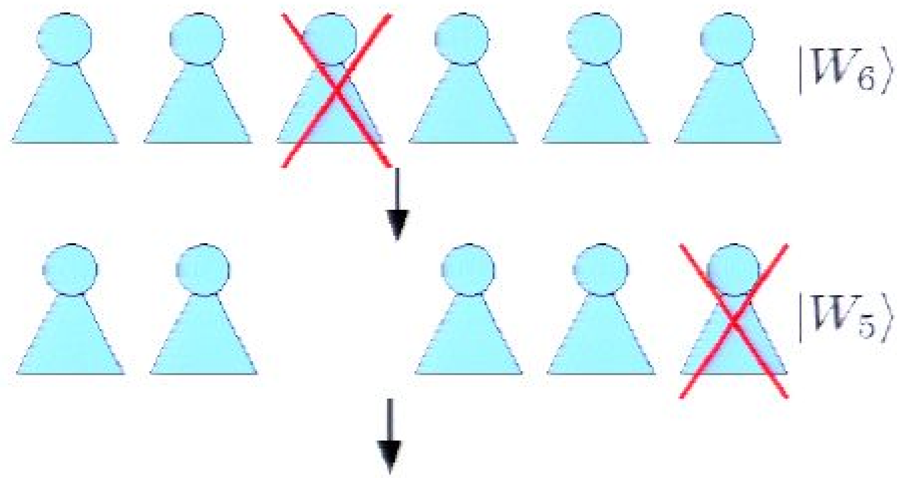

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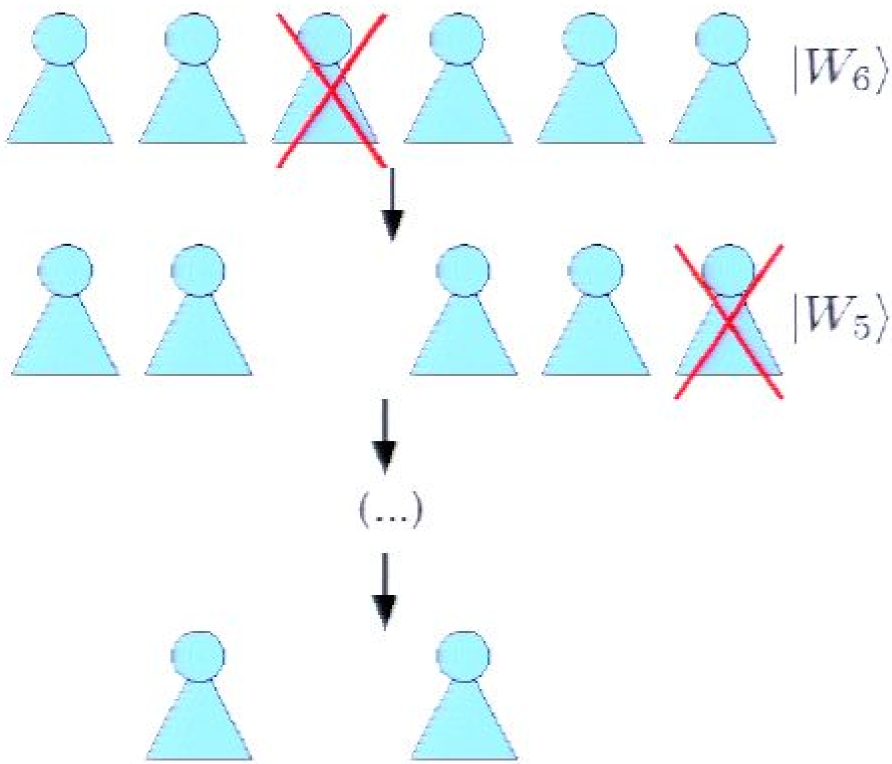

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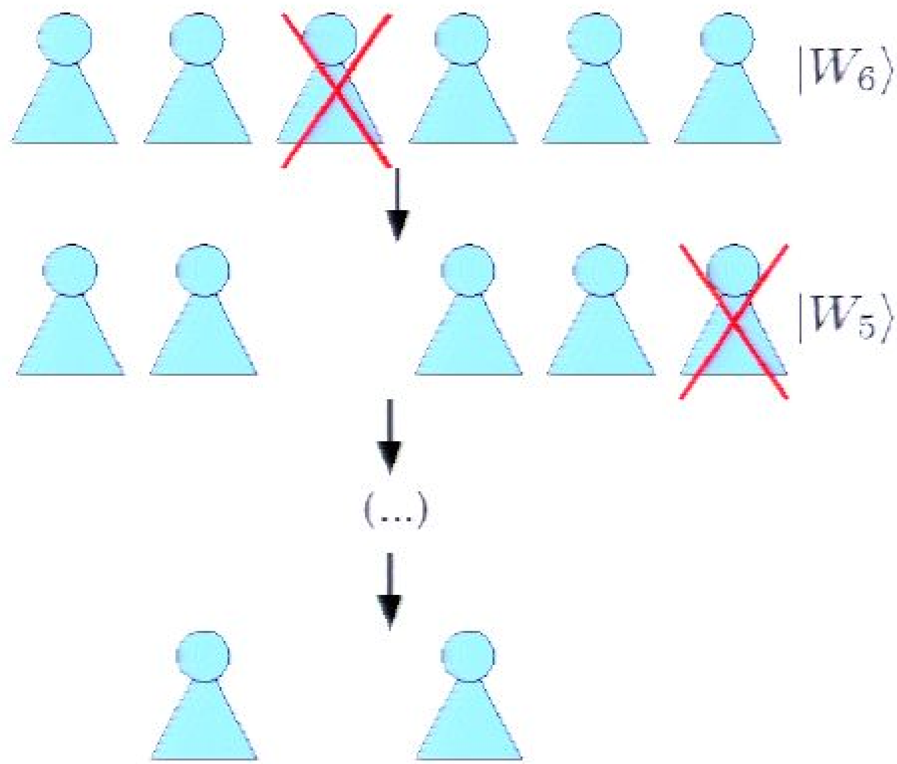

Symmetric Dicke states $\psi(M, N)$

Symmetric superposition of all states of M qubits, N in state 1 e.g.

$$|\psi(4,2)\rangle = \frac{1}{\sqrt{6}}\Big(|0011\rangle + |0110\rangle + |1100\rangle + |1001\rangle + |1010\rangle + |0101\rangle\Big)$$

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Random distillation - can force measurement of $|0\rangle$

$$|\psi(4,2)\rangle \rightarrow |\psi(3,2)\rangle$$

or $|1\rangle$

$$|\psi(4,2)\rangle \rightarrow |\psi(3,1)\rangle$$

Symmetric Dicke states - general result

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$$N' \geq (M'-M)+N$$

Not possible for chosen parties for $\binom{M'}{N'} > \binom{M}{N}$.

4 Redefinition

Problems with using E_t

The condition $E_t > E_{sp}$ can be trivially satisfied in some systems. E.g.

$$|\psi\rangle_{ABC} = |\Phi\rangle_{AB} \otimes |\Phi\rangle_{BC}$$

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For $\psi_i \underbrace{\rightarrow}_{LOCC} \{\psi_f, p_f\}$:

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"The maximum (over protocols) expected maximum (over pairs) assisted entanglement"

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2. "Specified entanglement" E_{sp}^∞ - the assisted entanglement, maximised over all pairs.
3. "Total entanglement" E_t^∞ - how many EPRs we can make through LOCC from a multiparty state, irrespective of who shares them.
4. "Random entanglement" E_{rnd}^∞ - how many EPRs we can make through LOCC between a pair of parties not *a priori* specified.

E_{rnd} and multiple copies

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- Very small advantage, soon lost for more copies (with this protocol).

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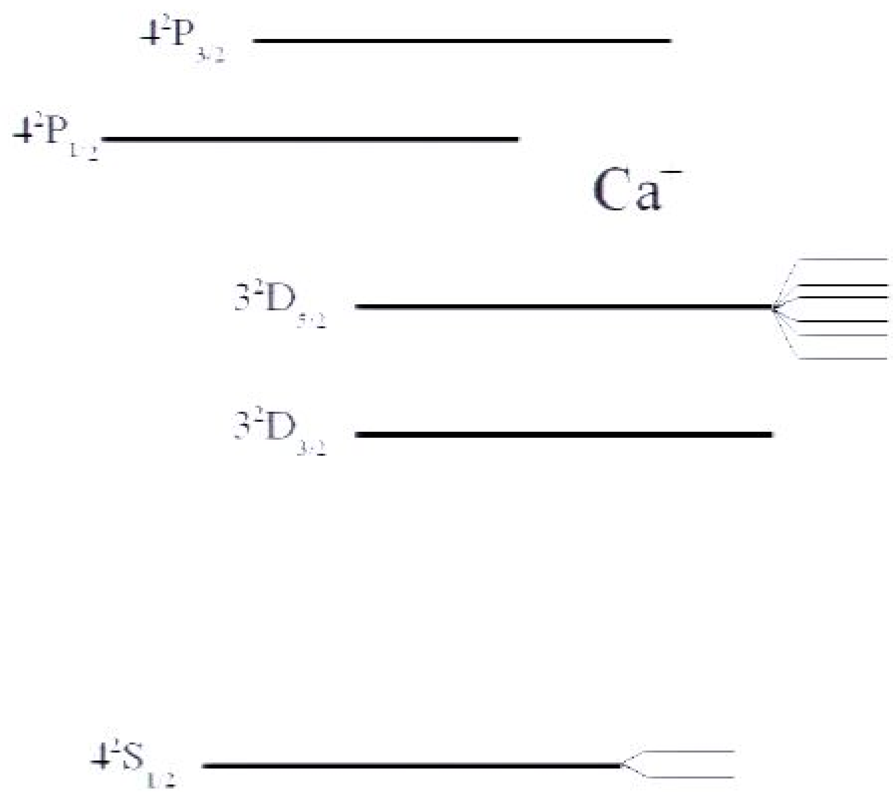
Proof.

(Lo & Popescu (1999))
Probability for a party to increase their distillable EPR pairs through LOCC $\to 0$ as $N \to \infty$.
Hence no increase in S or E_A for any party through LOCC - necessary for advantageous random distillation. □

Experimental random distillation

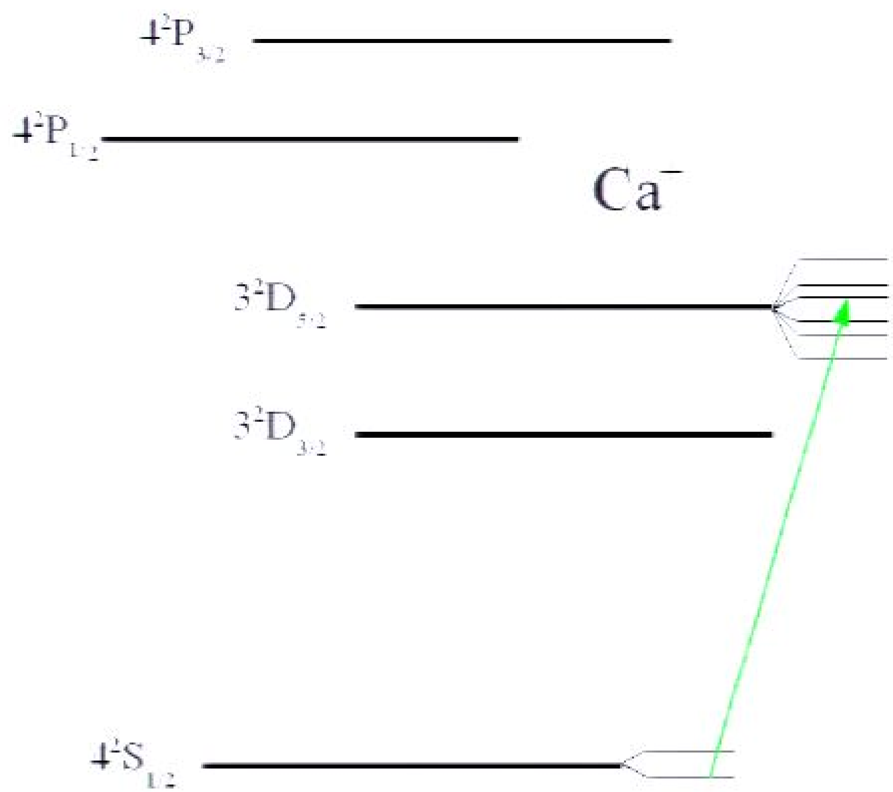
(Experimental QO, Uni. Innsbruck) Potential to implement the W random distillation protocol in trapped ions.

Random distillation scheme



$$\alpha|S\rangle + \beta|D\rangle$$

Random distillation scheme



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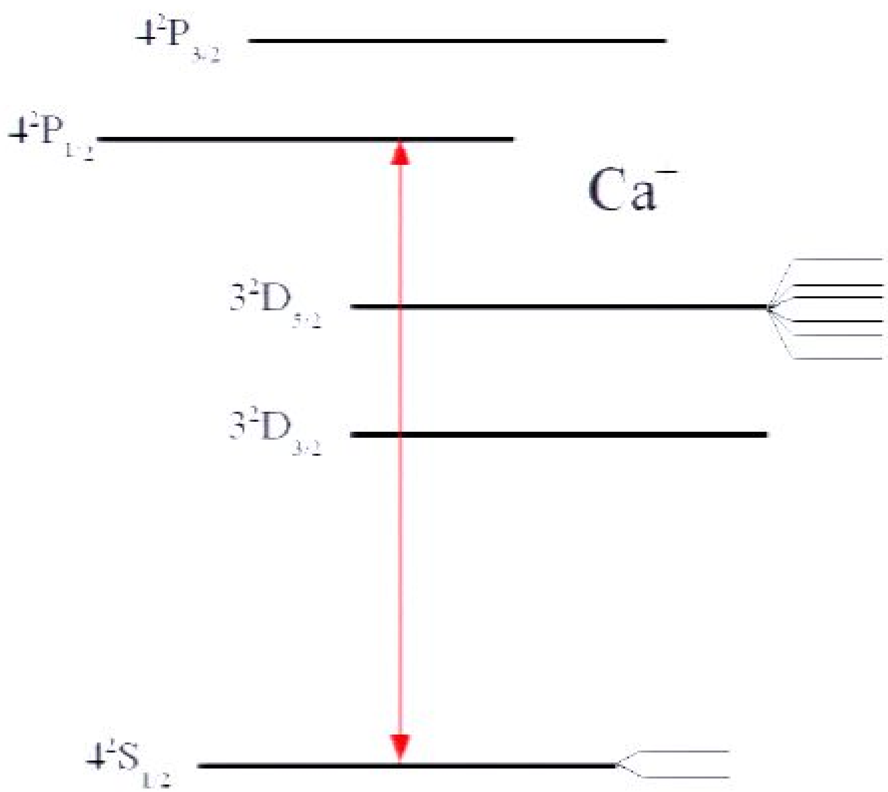

$$(S \to P), \quad |S\rangle\langle S|$$

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- Random distillation is always advantageous for W class, sometimes for GHZ class.
- Provably no advantage in the many-copy limit.

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