

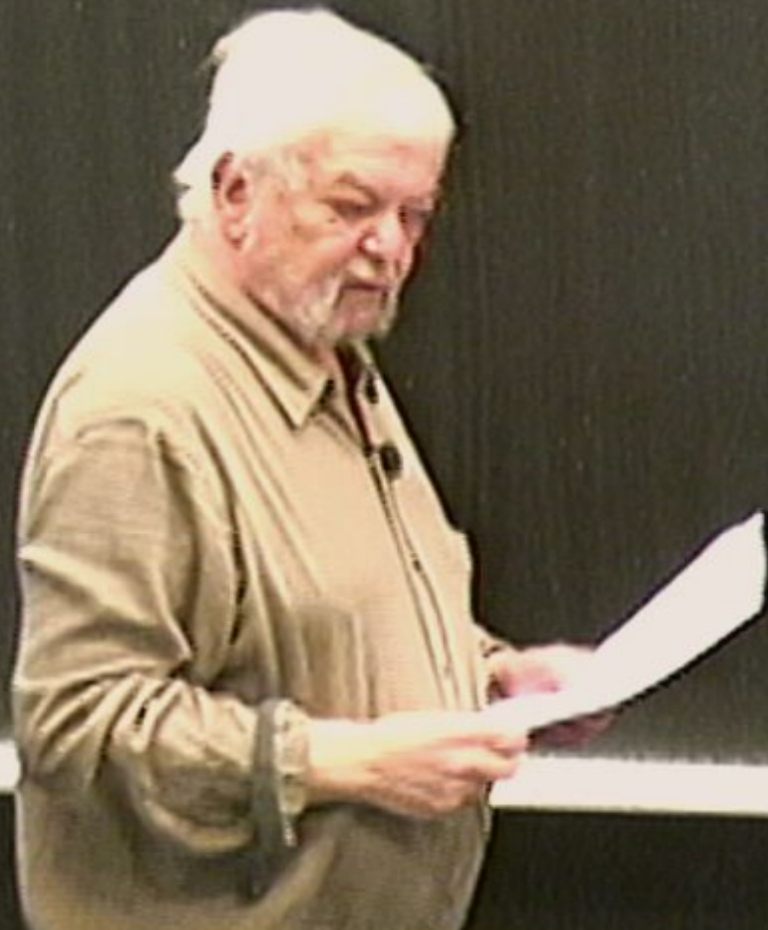
Title: Spontaneous Broken Symmetry 5B

Date: Nov 29, 2007 11:00 AM

URL: <http://pirsa.org/07110066>

Abstract:

$$2 \left(\frac{1}{2m} \right) \psi_k$$



$$2\left(\frac{\hbar^2}{2m}\right)\psi_k + \sum V_{k,k'}\psi_{k'}$$

$$\frac{\hbar^2 k^2}{2m} \psi_k + \sum_{k'} V_{kk'} \psi_{k'} = (2\varepsilon_F + E) \psi_k$$

E referred to $2\varepsilon_F$



$$\frac{\hbar^2 k^2}{2m} \psi_k + \sum_{k'} V_{kk'} \psi_{k'} = (2\epsilon_F + E) \psi_k$$

Cooper mode $V_{kk'} = -V$

$$E \text{ relative to } 2\epsilon_F = \hbar^2 k^2 / m$$



$$2\left(\frac{\hbar^2}{2m}\right)\psi_L + \sum V_{L L'} \psi_{L'} = (2\epsilon_F + E)\psi_L$$

$$E \text{ refer to } 2\epsilon_F = \hbar^2 n^2 / m$$

Cooper mode $V_{L L'} = -\frac{V}{L^3}$

$$L^3 = \text{volume}$$



$$2\left(\frac{\hbar^2}{2m}\right)\psi_L + \sum_{k'} V_{kk'} \psi_{k'} = (2\varepsilon_F + \varepsilon)\psi_L$$

$$\varepsilon \text{ refers to } 2\varepsilon_F = \hbar^2 k^2/m$$

Cooper mode $V_{kk'} = -\frac{V}{L^3}$

$L^3 = \text{volume}$

(L^3 volume,
 1st order $\sum_{k'} \sim \frac{L^3}{(2\pi)^3} \int d^3 k'$)



$$2\left(\frac{\hbar^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'} \psi_{k'} = (2E_F + E)\psi_k$$

$$E \text{ measured to } 2E_F = \hbar^2 k^2 / m$$

Cooper mode $V_{kk'} = -\frac{V}{L^3}$ (L^3 w-norm, 1 it cancels $\sum_{k'} \sim \frac{L^3}{(2\pi)^3} \int d^3 k'$)




convention: $-V$ means $V > 0$ is attractive

$$2\left(\frac{\hbar^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'} \psi_{k'} = (2E_F + E)\psi_k$$

E relative to $2E_F = \hbar^2 k^2/m$

Cooper pair $V_{kk'} = -\frac{V}{L^3}$ (L^3 w-norm, 1 it connects $\sum_{k'} \sim \frac{L^3}{(2\pi)^3} \int d^3 k'$)



then: $-V$ means $V > 0$ is attractive

but $V \neq 0$ if \vec{k}, \vec{k}' are in a shell ω_D in the Fermi sea

$$2\left(\frac{\hbar^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'} \psi_{k'} = (2\epsilon_F + \epsilon) \psi_k$$

ϵ relative to $2\epsilon_F = \hbar^2 k^2/m$

Cooper model $V_{kk'} = -\frac{V}{L^3}$ (L^3 volume)

1st order



$L^3 = \text{volume}$

Concentration: $-w$ $-V$ \rightarrow $\text{room } V$

Cooper model $V \neq 0$ if

w_D thickness

$$2\left(\frac{\hbar^2 k^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'} \psi_{k'} = (2E_F + E)\psi_k$$

$$E \text{ relative to } 2E_F = \hbar^2 k^2/m$$



Cooper mode ψ_k

(L^3 w-norm,

$$1 \text{ if connected } \sum_{k'} \sim \frac{L^3}{(2\pi)^3} \int d^3 k'$$

$L^3 = \text{volume}$

Concave

$-V$ norm, $V > 0$ is attractive

Cooper

if \vec{k}, \vec{k}' are in a shell w_D in thickness

$$2\left(\frac{\hbar^2}{2m}\right)\psi_{\mathbf{k}} + \sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \psi_{\mathbf{k}'} = (2\varepsilon_F + E)\psi_{\mathbf{k}}$$

E relative to $2\varepsilon_F = \hbar^2 k^2/m$

Cooper mode $V_{\mathbf{k}\mathbf{k}'} = -\frac{V}{L^3}$ (L^3 volume)

1st com



$d^3 k'$

$L^3 = \text{volume}$

Conventions: $-u_i - V \text{ room } V >$ $\hbar \omega_{\mathbf{k}}$


Cooper model $V \neq 0$ if $\hbar \omega_{\mathbf{k}}$ all $\omega_{\mathbf{k}}$ the lines

$$2\left(\frac{\hbar^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'} \psi_{k'} = (2E_F + E)\psi_k \quad E \text{ relative to } 2E_F = \hbar^2 k^2/m$$

Cooper mode $V_{kk'} = -\frac{V}{L^3}$ (L^3 is volume, $V > 0$ is attractive)

$L^3 = \text{volume}$

It cancels $\sum_{k'} \sim \frac{L^3}{(2\pi)^3} \int d^3 k'$




Assumption: $-V$ is attractive $V > 0$ is attractive

Cooper mode $V \neq 0$ if \vec{k}, \vec{k}' are in a shell ω_D thickness

$$2\left(\frac{\hbar^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'} \psi_{k'} = (2E_F + E)\psi_k \quad E \text{ relative to } 2E_F = \hbar^2 k^2/m$$

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$L^3 = \text{volume}$


Convention: $-V$ means $V > 0$ is attractive

Cooper model $V \neq 0$ if \vec{k}, \vec{k}' are in a shell W_D thickness

$$2\left(\frac{\hbar^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'} \psi_{k'} = (2E_F + E)\psi_k \quad E \text{ relative to } 2E_F = \hbar^2 k^2/m$$

Cooper mode $V_{kk'} = -\frac{V}{L^3}$ (L^3 is volume, $V > 0$ is attractive)

It connects $\sum_{k'} \approx \frac{L^3}{(2\pi)^3} \int d^3 k'$



Assumption: $-W_D$ $-V$ is attractive $V > 0$ is attractive


Cooper mode $V \neq 0$ if \vec{k}, \vec{k}' are in a shell W_D is thickness

$$2\left(\frac{\hbar^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'} \psi_{k'} = (2E_F + E)\psi_k \quad E \text{ relative to } 2E_F = \hbar^2 k^2/m$$

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$L^3 = \text{volume}$

It connects $\sum_{k'} \sim \frac{L^3}{(2\pi)^3} \int d^3 k'$



Convention: $-w$ $-V$ means $V > 0$ is attractive

Cooper model $V \neq 0$ if \vec{k}, \vec{k}' are in a shell w_D thickness
 For simplicity $V = \text{const}$ in the shell

$$\left[-\frac{\hbar^2}{m} + 2E_F + 4 \right] \psi_{\frac{1}{2}} = -\frac{V}{E_F}$$

$$\left[-\frac{\hbar^2}{m} + 2E_F + \dots \right] \psi = -\frac{V}{E}$$

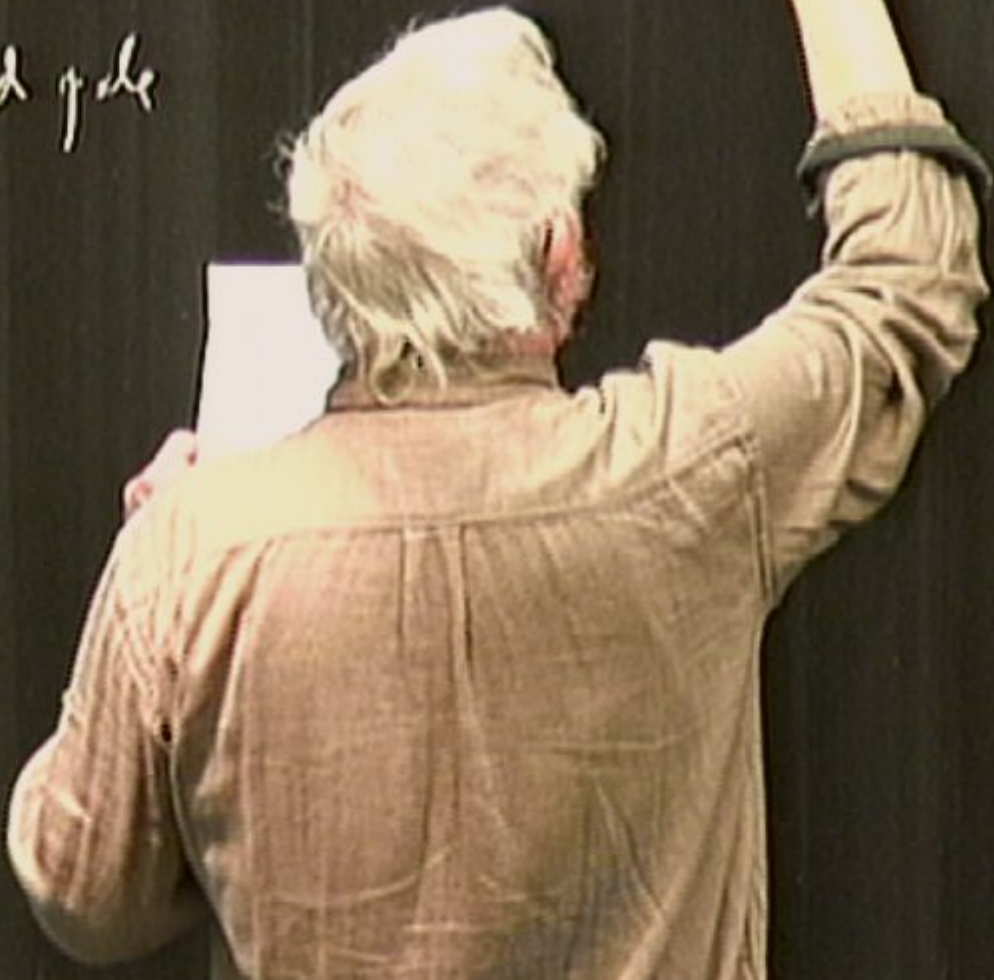


$$\left[-\frac{\hbar^2}{m} + 2E_F + 4 \right] \psi_k = -\frac{V}{E}$$



$$\left[-\frac{\hbar^2}{m} + 2E_F + E \right] \psi_k = -\frac{V}{L^3} ; \quad \psi = -\frac{V}{L^3} \sum$$

For $V > 0$, \rightarrow isolated gate



$$\left[-\frac{\hbar^2}{m} \nabla^2 + 2E_F + E \right] \psi_k = -\frac{V}{L^3} \sum_{k'} \psi_{k'} \quad ; \quad 1 = -\frac{V}{L^3} \sum_{k'} \frac{1}{\hbar^2/m + 2E_F + E}$$

For $V > 0$, \rightarrow isolated gap

$$\left[-\frac{\hbar^2}{m} + 2E_F + E \right] \psi_k = -\frac{V}{L^3} ; \quad 1 = -\frac{V}{L^3} \sum \frac{1}{-\hbar^2/m + 2E_F + E}$$

For $V > 0$, \rightarrow isolated state

$$\left[-\frac{\hbar^2}{m} + 2E_F + E \right] \Psi_k = -\frac{V}{L^3} ; \quad 1 = -\frac{V}{L^3} \sum \left(\frac{1}{-\hbar^2/m + 2E_F + E} \right)$$

For $V > 0$, \rightarrow isolated pole. A solution exists if $V > 0$ with $E < 0$.

$$\left[-\frac{\hbar^2 k^2}{m} + 2E_F + E \right] \Psi_k = -\frac{V}{L^3} ; \quad 1 = -\frac{V}{L^3} \sum_k \left(\frac{1}{-\hbar^2 k^2/m + 2E_F + E} \right)$$

For $V > 0$, \rightarrow isolated pole A solution exists if $V > 0$ with $E < 0$.
 No " " $V < 0$ ($E > 0$)

$$\left[-\frac{\hbar^2}{m} + 2E_F + E \right] \Psi_k = -\frac{V}{L^3} ; \quad \rho = -\frac{V}{L^3} \sum_k \left(\frac{1}{-\hbar^2/m + 2E_F + E} \right)$$

For $V > 0$, \rightarrow isolated pole A solution exists if $V > 0$ with $E < 0$.
 No " " $V < 0$ ($E > 0$)

$$\left[-\frac{\hbar^2}{m} + 2E_F + E \right] \psi_k = -\frac{V}{L^3} ; \quad \psi_k = -\frac{V}{L^3} \sum_k \left(\frac{1}{-\hbar^2/m + 2E_F + E} \right)$$

For $V > 0$, \rightarrow isolated pole. A solution exists if $V > 0$ with $E < 0$.
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$$\left[-\frac{\hbar^2}{m} + 2E_F + E \right] \psi_k = -\frac{V}{L^3} ; \quad 1 = -\frac{V}{L^3} \sum_k \left(\frac{1}{-\hbar^2/m + 2E_F + E} \right)$$

For $V > 0$, \rightarrow isolated pole. A solution exists if $V > 0$ with $E < 0$.
 No " " " " $V < 0$ ($E > 0$)
 $-E_F$

density of states at $E = E_F = \frac{4\pi \hbar^2}{(2\pi)^3} \frac{dk}{dE_k}$

$$\left[-\frac{\hbar^2 k^2}{m} + 2E_F + E \right] \psi_k = -\frac{V}{L^3} ; \quad 1 = -\frac{V}{L^3} \sum_k \left(\frac{1}{-\hbar^2 k^2/m + 2E_F + E} \right)$$

For $V > 0$, \rightarrow isolated pole. A solution exists if $V > 0$ with $E < 0$.

N_0 " " $V < 0$ ($E > 0$)

$$z = \hbar^2 k^2 / 2m - E_F$$

$$N(z): \text{density of states at } \hbar = \hbar_F = \frac{4\pi \hbar^2}{(2\pi)^3} \frac{d\hbar}{dz_k} \approx \frac{4\pi \hbar_c^2}{(2\pi)^3} \left. \frac{d\hbar}{dz_k} \right|_{\hbar = \hbar_c}$$

$$= V \int_0^{\omega} \dots$$

$$\left[-\frac{\hbar^2 k^2}{m} + 2E_F + E \right] \Psi_k = -\frac{V}{L^3} ; \quad 1 = -\frac{V}{L^3} \sum_k \left(\frac{1}{-\hbar^2 k^2/m + 2E_F + E} \right)$$

For $V > 0$, \rightarrow isolated pole. A solution exists if $V > 0$ with $E < 0$.

N_0 " " $V < 0$ ($E > 0$)

$$z = \hbar^2 k^2 / 2m - E_F$$

$$N(z): \text{density of states at } \hbar^2 k^2 = \frac{4\pi \hbar^2}{(2\pi)^3} \frac{dk}{dz_k} \approx \frac{4\pi \hbar^2}{(2\pi)^3} \left. \frac{dk}{dz_k} \right|_{k=k_0}$$

$$= V \int_0^{\omega} \frac{1}{(z - E)} dz$$

$$\left[-\frac{\hbar^2 k^2}{m} + 2E_F + E \right] \psi_k = -\frac{V}{L^3} \sum_k \left(\frac{1}{-\hbar^2 k^2/m + 2E_F + E} \right)$$

For $V > 0$, \rightarrow isolated pole. A solution exists if $V > 0$ with $E < 0$.

N_0 " " $V < 0$ ($E > 0$)

$$z = \hbar^2 k^2 / 2m - E_F$$

$$N(z): \text{density of states at } \hbar^2 k^2 = \frac{4\pi \hbar^2}{(2\pi)^3} \frac{dk}{dz_k} = \frac{4\pi \hbar^2}{(2\pi)^3} \frac{dk}{dz_k} \Big|_{k=k_0}$$

$$= V \int_0^{\omega} \frac{N(L) dz}{(z - E)}$$

Variational basis for the eq. for h .

$$E = \omega_D$$

$$\nabla \times \nabla \times \mathbf{r} = \dots$$

unit model

characteristic

Variational basis for the eq. for k .

$$E = \omega_D e^{-1/N(0)}$$

unit model

characteristic

Variational basis for the eq. for ψ .

$$E = \omega_D e^{-1/N(0)V}$$

Variational base for the eq. for ψ .

$$E = -\omega_D e^{-1/(N|0\rangle V)}$$

$$\langle n^2 \rangle = \frac{\int |\psi(r)|^2 n^2 r^2 dr}{\int |\psi(r)|^2 r^2 dr}$$

$$\int |\psi(r)|^2 r^2 dr$$

$$\psi(r) = \sum_k e^{ik \cdot r} \psi_k$$

$$\psi_k =$$

character

$$\left[-\frac{\hbar^2}{m} + 2\varepsilon_F + E \right] \psi_k = -\frac{V \sum \psi_k}{L^3} - \frac{V}{L^3} \sum \left(\frac{1}{-\hbar^2/m + 2\varepsilon_F + E} \right)$$

For $V > 0$, \rightarrow pole A solution exists if $V > 0$ with $E < 0$.
 " " " " $V < 0$ ($E > 0$)

$$z = k^2$$

N/V

$$\frac{4\pi k^2}{(2\pi)^3} \frac{dk}{dz_k} = \frac{4\pi k^2}{(2\pi)^3} \frac{dk}{dz_k} \Big|_{k=k_0}$$

$$\left[-\frac{\hbar^2 k^2}{m} + 2E_F + E \right] \psi_k = -\frac{V}{L^3} \sum_k' \psi_k \left(\frac{1}{-\hbar^2/m + 2E_F + E} \right)$$

For $V > 0$, \rightarrow indet. A solution exists if $V > 0$ with $E < 0$.
 N_0 " " $V < 0$ ($E > 0$)

$\exists -E_F$

number of states at $E = E_F = \frac{4\pi k^2}{(2\pi)^3} \frac{dk}{dz_k} \approx \frac{4\pi k_F^2}{(2\pi)^3} \left. \frac{dk}{dz_k} \right|_{k=k_F}$

$$\frac{N(L) dz}{(2\pi - E)}$$

$$\left[-\frac{\hbar^2 k^2}{m} + 2E_F + E \right] \psi_k = -\frac{V \sum \psi_k}{L^3} - \frac{V}{L^3} \sum \left(\frac{1}{-\hbar^2/m + 2E_F + E} \right)$$

For $V > 0$, \rightarrow isolated pole. A solution exists if $V > 0$ with $E < 0$.

N_0 " " $V < 0$ ($E > 0$)

$$E = \hbar^2 k^2 / 2m - E_F$$

$$N(E): \text{density of states at } E = E_F = \frac{4\pi \hbar^2}{(2\pi)^3} \frac{dk}{dE} = \frac{4\pi \hbar^2}{(2\pi)^3} \frac{dk}{dE} \Big|_{E=E_F}$$

$$= V \int_0^{\omega} \frac{N(E) dE}{(2E - E_F)}$$

Variational basis for the eq. for ψ .

$$E = -\omega_D e^{-1/2(N/\omega_D)V}$$

$$\langle n^2 \rangle = \frac{\int |\psi(r)|^2 n^2 r^2 dr}{\int |\psi(r)|^2 n^2 dr}$$

$$\int |\psi(r)|^2 n^2 dr$$

$$\psi(r) = \sum_k e^{ik \cdot r} \psi_k$$

$$\psi_k = \frac{\text{const}}{E - \epsilon_k}$$

Variational ansatz for the eq. for ψ .

$$E = \omega_D e^{-\gamma |N| \omega_D}$$

$$v_F = \hbar v_F / m$$

$$\langle n^2 \rangle = \frac{\int |\psi(r)|^2 n^2 r^2 dr}{\int |\psi(r)|^2 r^2 dr} = \frac{4}{3} \frac{\lambda_F^2}{m^2 v_F^2} \sim N_F$$

$$\int |\psi(r)|^2 r^2 dr$$

$$\psi(r) = \sum_k e^{i k \cdot r} \psi_k$$

$$\psi_k =$$

$$\frac{v_F}{E - \epsilon_k}$$

Variational ansatz for the eq. for ψ .

$$E = \omega_D e^{-\gamma(N_0)/V}$$

$$n_{F0} = k_F/m$$

$$\langle n^2 \rangle = \frac{\int |\psi(r)|^2 n^2 r^2 dr}{\int |\psi(r)|^2 n^2 dr} = \frac{4}{3} \frac{k_F^2}{m^2 \epsilon^2} \sim \left(\frac{n_{F0}}{\epsilon} \right)^2$$

$$\int |\psi(r)|^2 n^2 dr$$

$$\psi(r) = \sum_{\lambda} e^{i \lambda \cdot r} \psi_{\lambda}$$

$$\psi_{\lambda} = \frac{\text{const}}{\epsilon - 2\lambda}$$

etc.

Variational ansatz for the eq. for ψ .

$$E = \omega_D e^{-\gamma(N/\omega) V}$$

$$v_F = \hbar v / m$$

$$\langle n^2 \rangle = \frac{\int |\psi(r)|^2 n^2 r^2 dr}{\int |\psi(r)|^2 n^2 dr} = \frac{4}{3} \frac{\lambda_F^2}{m^2 v^2} \sim \left(\frac{v_F}{v} \right)^2 = (\text{corr length})^2$$

$$\int |\psi(r)|^2 n^2 dr$$

$$\psi(r) = \sum_{\lambda} e^{i \lambda \cdot r} \psi_{\lambda}$$

$$\psi_{\lambda} = \frac{\text{const}}{E - \epsilon_{\lambda}}$$

$$\Psi = |BCS\rangle = \prod_{\mathbf{k}} [u_{\mathbf{k}} + v_{\mathbf{k}} b_{\mathbf{k}}^{\dagger}] |0\rangle$$

$$b_{\mathbf{k}} = a_{\mathbf{k}\uparrow} a_{-\mathbf{k}\downarrow}$$

$$\Psi = |BCS\rangle = \prod_{\mathbf{k}} [u_{\mathbf{k}} + v_{\mathbf{k}} b_{\mathbf{k}}^{\dagger}] |0\rangle$$

$$b_{\mathbf{k}}^{\dagger} = -a_{\mathbf{k}\uparrow}^{\dagger} a_{-\mathbf{k}\downarrow}^{\dagger}$$

$$\Psi = |BCS\rangle = \prod_{\mathbf{k}} [u_{\mathbf{k}} + v_{\mathbf{k}} b_{\mathbf{k}}^{\dagger}] |0\rangle$$

$$b_{\mathbf{k}}^{\dagger} = -a_{\mathbf{k}\uparrow}^{\dagger} a_{-\mathbf{k}\downarrow}^{\dagger}$$

$$\Psi = |BCS\rangle = \prod_{\lambda} [u_{\lambda} + v_{\lambda} b_{\lambda}^{\dagger}] |0\rangle$$

$$b_{\lambda}^{\dagger} = -a_{\lambda\uparrow}^{\dagger} a_{-\lambda\downarrow}^{\dagger}$$



$$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$$

$$b_k^\dagger = -a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger$$

$$[b_k, b_{k'}] = 0$$

$$[b_k, b_{k'}^\dagger] =$$

$$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$$

$$b_k^\dagger = -a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger$$

$$\{a_{k\uparrow}, a_{k'\downarrow}\} \neq 0 \quad \{a_{k\uparrow}, a_{k'\downarrow}\} = \delta_{kk'}$$

$$[b_k, b_{k'}] = 0$$

$$[b_k, b_{k'}^\dagger] = \delta_{kk'}$$

$$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$$

$$b_k^\dagger = -a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger \quad \{a_{k\uparrow}, a_{k'\downarrow}\} \neq 0 \quad \{a_{k\uparrow}, a_{k'\uparrow}\} = \delta_{kk'}$$

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$$\Psi = |BCS\rangle = \prod_{\mathbf{k}} [u_{\mathbf{k}} + v_{\mathbf{k}} b_{\mathbf{k}}^{\dagger}] |0\rangle$$

$$b_{\mathbf{k}}^{\dagger} = -a_{\mathbf{k}\uparrow}^{\dagger} a_{-\mathbf{k}\downarrow}^{\dagger}$$

$$\{a_{\mathbf{k}\uparrow}, a_{\mathbf{k}'\downarrow}\} \neq 0 \quad \{a_{\mathbf{k}\uparrow}, a_{\mathbf{k}'\uparrow}\} = \delta_{\mathbf{k}\mathbf{k}'}$$

$$[b_{\mathbf{k}}, b_{\mathbf{k}'}] = 0$$

$$[b_{\mathbf{k}}, b_{\mathbf{k}'}^{\dagger}] = \delta_{\mathbf{k}\mathbf{k}'}$$

$$\langle BCS | BCS \rangle = 1 = \prod_{\mathbf{k}} [u_{\mathbf{k}}^2 + v_{\mathbf{k}}^2] = 1$$

$$\Psi = |BCS\rangle = \prod_{\lambda} [u_{\lambda} + v_{\lambda} b_{\lambda}^{\dagger}] |0\rangle$$

$$b_{\lambda}^{\dagger} = -a_{\lambda\uparrow}^{\dagger} a_{-\lambda\downarrow}^{\dagger} \quad \{a_{\lambda\uparrow}, a_{\lambda'\downarrow}\} \neq 0 \quad \{a_{\lambda\uparrow}, a_{\lambda'\uparrow}\} = \delta_{\lambda\lambda'}$$

$$[b_{\lambda}, b_{\lambda'}] = 0$$

$$[b_{\lambda}, b_{\lambda'}^{\dagger}] = \delta_{\lambda\lambda'}$$

$$\langle BCS | BCS \rangle = 1 = \prod_{\lambda} [u_{\lambda}^2 + v_{\lambda}^2] = 1$$

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$$b_{\mathbf{k}}^{\dagger} = -a_{\mathbf{k}\uparrow}^{\dagger} a_{-\mathbf{k}\downarrow}^{\dagger}$$

$$\{a_{\mathbf{k}\uparrow}, a_{\mathbf{k}'\downarrow}\} \neq 0 \quad \{a_{\mathbf{k}\uparrow}, a_{\mathbf{k}'\uparrow}\} = \delta_{\mathbf{k}\mathbf{k}'}$$

$$[b_{\mathbf{k}}, b_{\mathbf{k}'}] = 0$$

$$[b_{\mathbf{k}}, b_{\mathbf{k}'}^{\dagger}] = \delta_{\mathbf{k}\mathbf{k}'}$$

$$\langle BCS | BCS \rangle = 1 = \prod_{\mathbf{k}} [u_{\mathbf{k}}^2 + v_{\mathbf{k}}^2] = 1$$

$$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$$

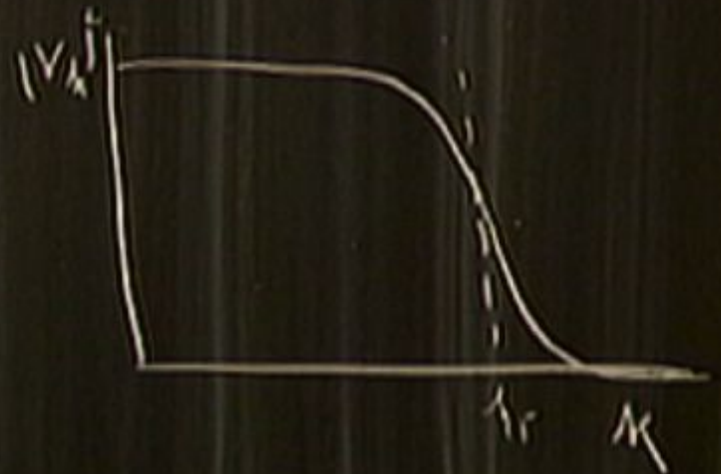
$$b_k^\dagger = -a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger$$

$$\{a_{k\uparrow}, a_{k'\downarrow}\} \neq 0 \quad \{a_{k\uparrow}, a_{k'\uparrow}\} = \delta_{kk'}$$

$$[b_k, b_{k'}] = 0$$

$$[b_k, b_{k'}^\dagger] = \delta_{kk'}$$

$$= \prod_k [v_k^2 + u_k^2] = 1$$



$$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$$

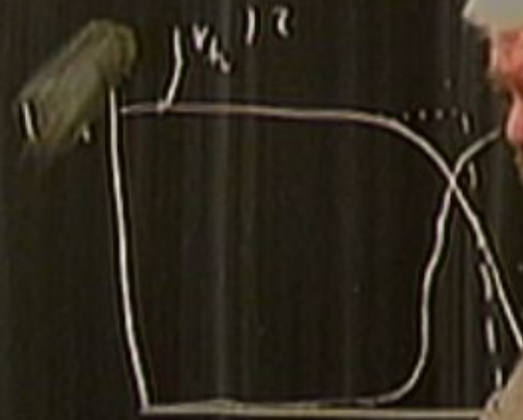
$$b_k^\dagger = -a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger$$

$$\{a_{k\uparrow}, a_{k'\downarrow}\} \neq 0 \quad \{a_{k\uparrow}, a_{k'\downarrow}\} = \delta_{kk'}$$

$$[b_k, b_{k'}] = 0$$

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$$\langle BCS | BCS \rangle = 1 = \prod_k [u_k^2 + v_k^2] = 1$$



$$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$$

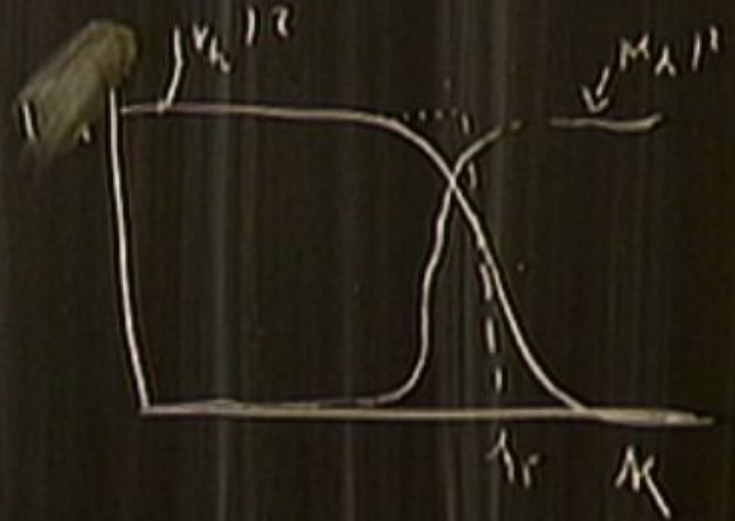
$$b_k^\dagger = -a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger$$

$$\{a_{k\uparrow}, a_{k'\downarrow}\} \neq 0 \quad \{a_{k\uparrow}, a_{k'\uparrow}\} = \delta_{kk'}$$

$$[b_k, b_{k'}] = 0$$

$$[b_k, b_{k'}^\dagger] = \delta_{kk'}$$

$$\langle 0 | BCS \rangle = 1 = \prod_k [u_k^2 + v_k^2] = 1$$



$$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$$

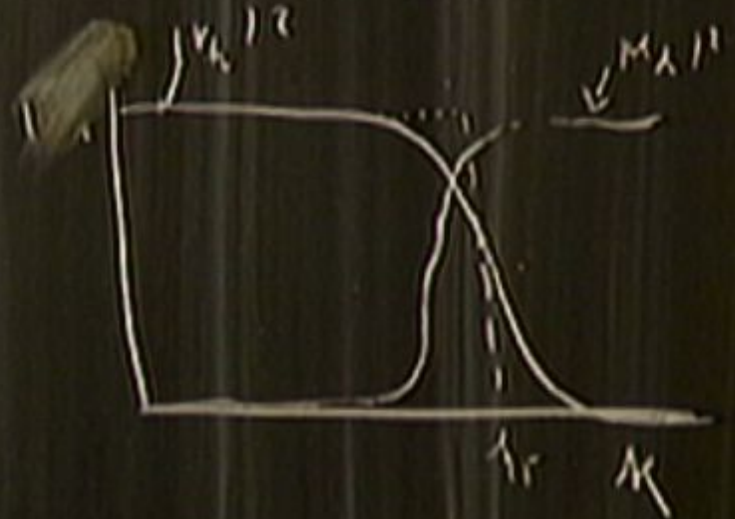
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$$\{a_{k\uparrow}, a_{k'\downarrow}\} \neq 0 \quad \{a_{k\uparrow}, a_{k'\uparrow}\} = \delta_{kk'}$$

$$[b_k, b_{k'}] = 0$$

$$[b_k, b_{k'}^\dagger] = \delta_{kk'}$$

$$\langle BCS | BCS \rangle = 1 = \prod_k [u_k^2 + v_k^2] = 1$$



$$\left[-\frac{\hbar^2}{m} + 2E_F + E \right] \psi_k = -\frac{V}{L^3} \sum_{k'} \psi_{k'} \left(\frac{1}{-\hbar^2/m + 2E_F + E} \right)$$

For $V > 0$, \rightarrow isolated pole. A solution exists if $V > 0$ with $E < 0$.

" " " " $V < 0$ ($E > 0$)

$$z = \hbar^2 / 2m - \epsilon$$

$$N(z) \text{ derivative} = \frac{4\pi \hbar^2}{(2\pi)^3} \frac{dk}{dz_k} = \frac{4\pi \hbar^2}{(2\pi)^3} \frac{dk}{dz_k} \Big|_{k=k_0}$$

$$= V \int \dots$$

$$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$$

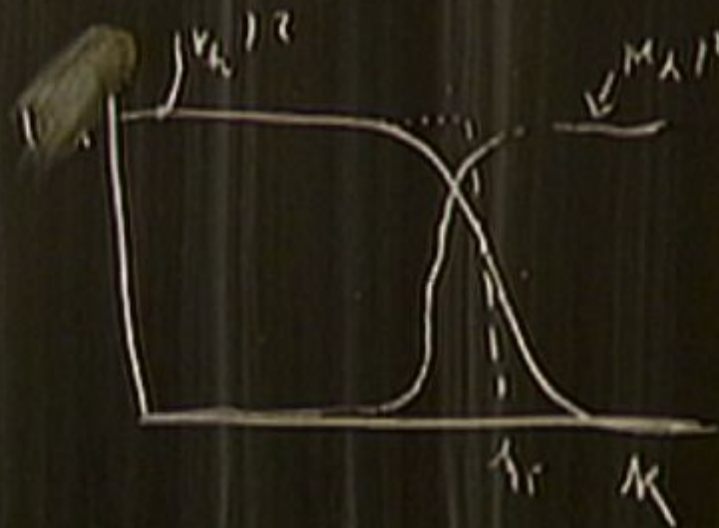
$$b_k^\dagger = -a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger$$

$$\{a_{k\uparrow}, a_{k'\downarrow}\} \neq 0 \quad \{a_{k\uparrow}, a_{k'\uparrow}\} = \delta_{kk'}$$

$$[b_k, b_{k'}] = 0$$

$$[b_k, b_{k'}^\dagger] = \delta_{kk'}$$

$$1 = \prod_k [u_k^2 + v_k^2] = 1$$



$$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$$

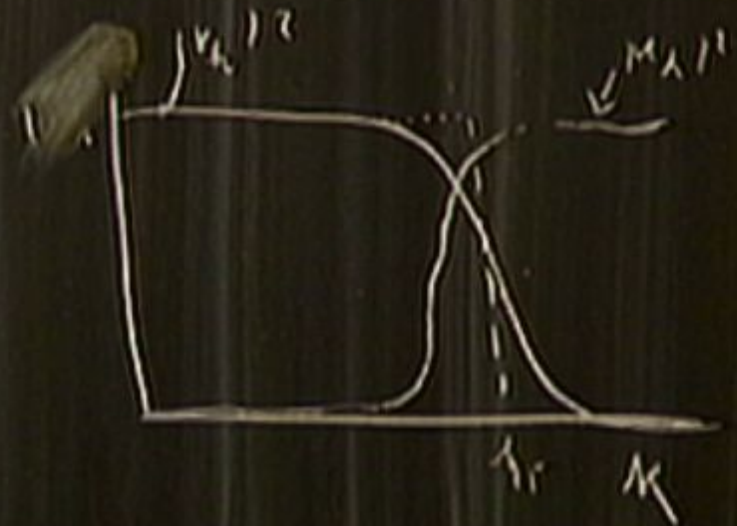
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$$[b_k, b_{k'}] = 0$$

$$[b_k, b_{k'}^\dagger] = \delta_{kk'}$$

$$\langle S \rangle = 1 = \prod_k [u_k^2 + v_k^2] = 1$$



$\langle b_1 \rangle$

$$\langle b_\lambda \rangle = \langle BCS | b_\lambda | BCS \rangle$$

$$\langle b_{\lambda} \rangle = \langle \text{BCS} | b_{\lambda} | \text{BCS} \rangle$$
$$= u_{\lambda} v_{\lambda}.$$

$$\langle b_k \rangle = \langle \text{BCS} | b_k | \text{BCS} \rangle$$
$$= u_k v_k.$$

Violates conservation of N

$$\langle b_k \rangle = \langle \text{BCS} | b_k | \text{BCS} \rangle$$

$$= u_k v_k.$$

Violates conservation of N

$|\text{BCS}\rangle$ is not an e.f. of $N (= \sum_k a_k^\dagger a_k)$