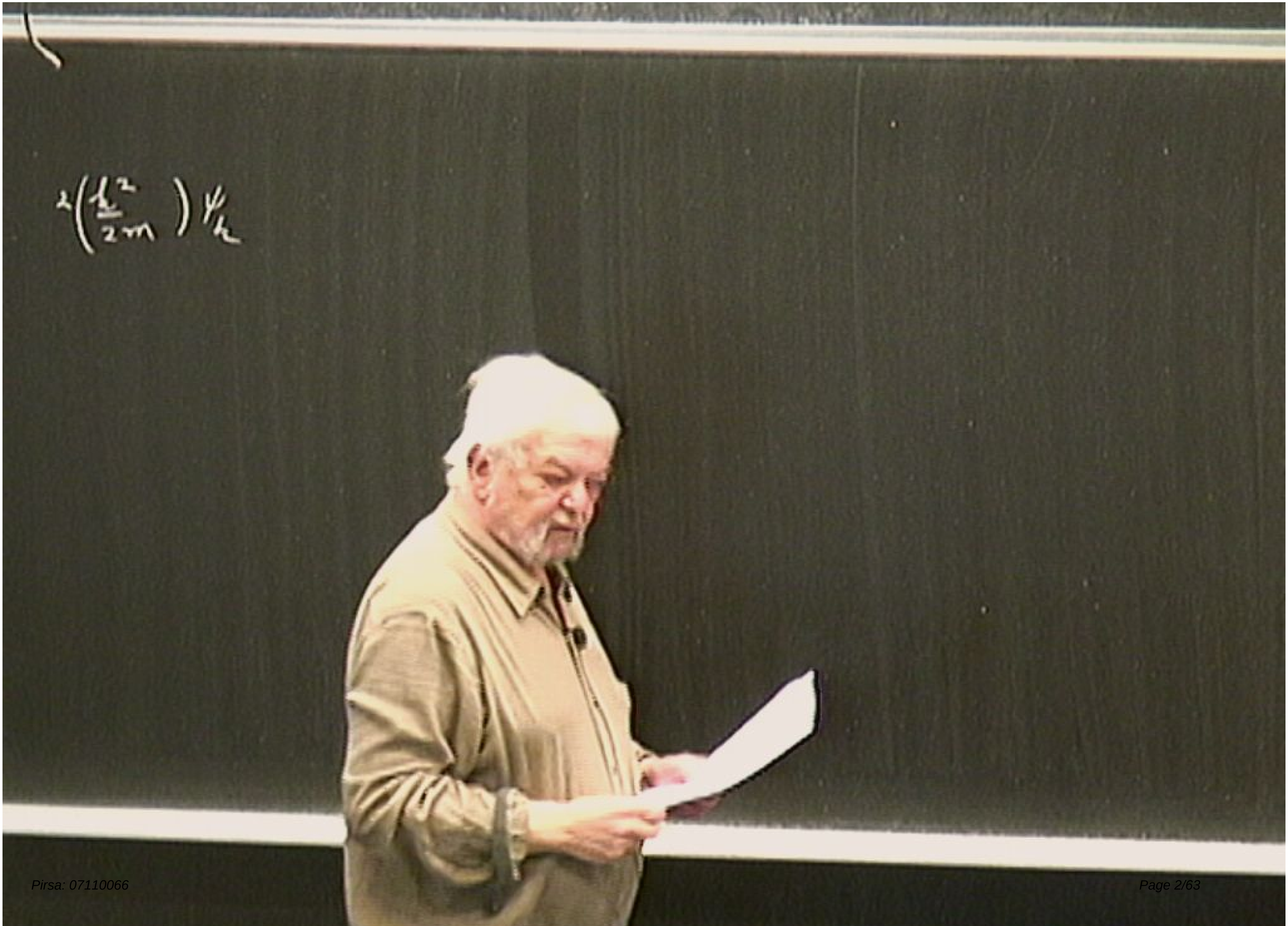
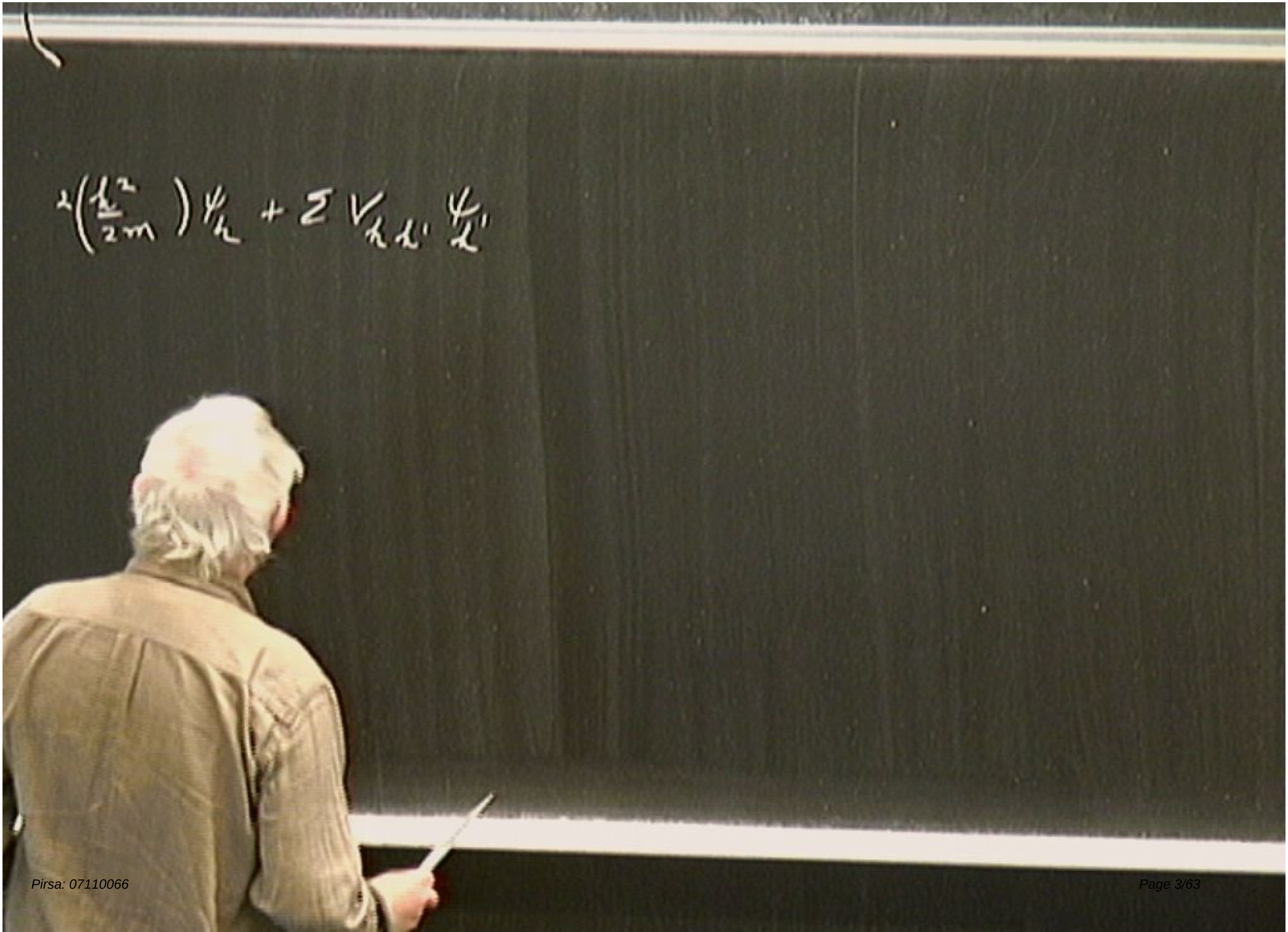
Title: Spontaneous Broken Symmetry 5B

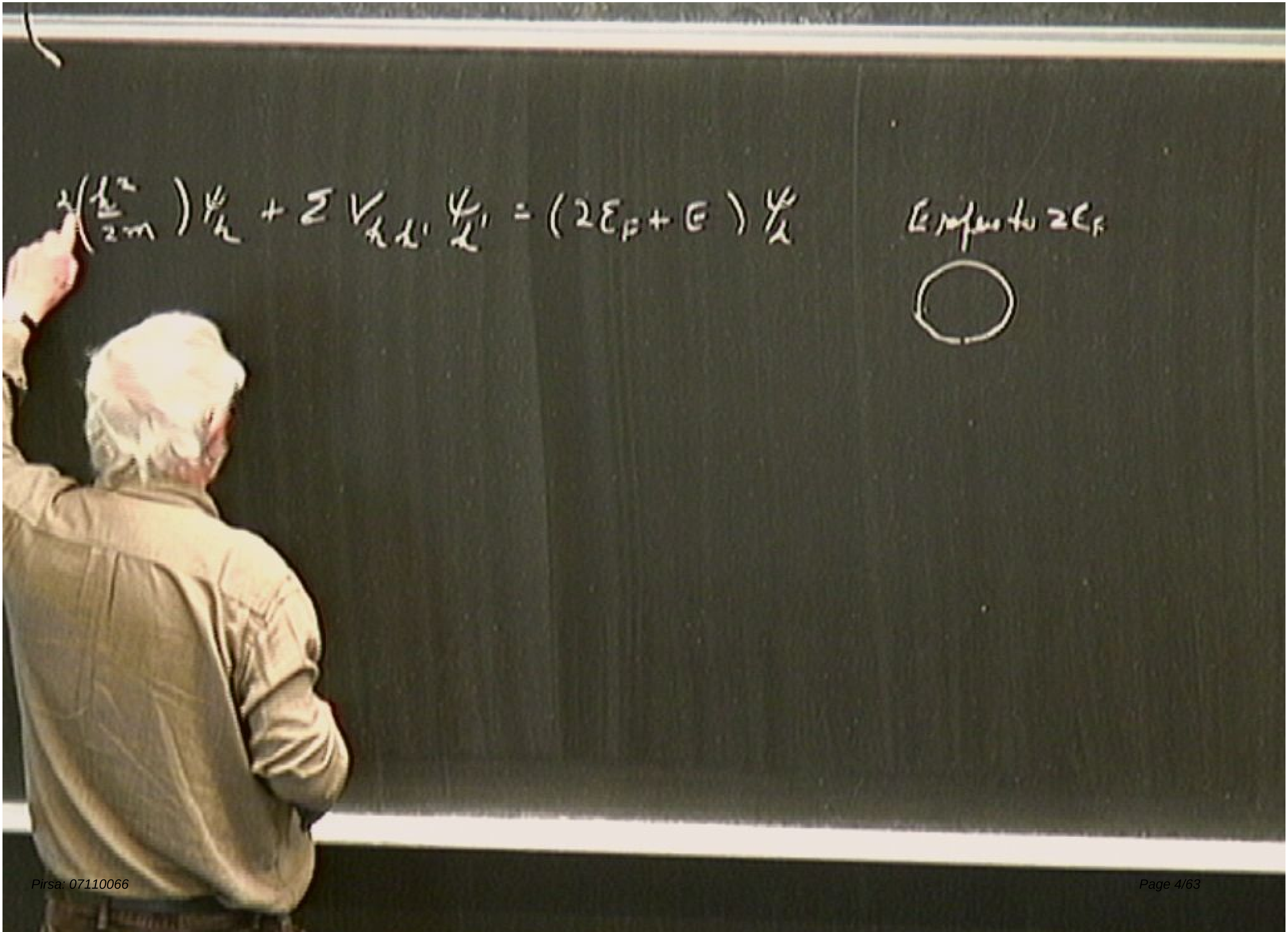
Date: Nov 29, 2007 11:00 AM

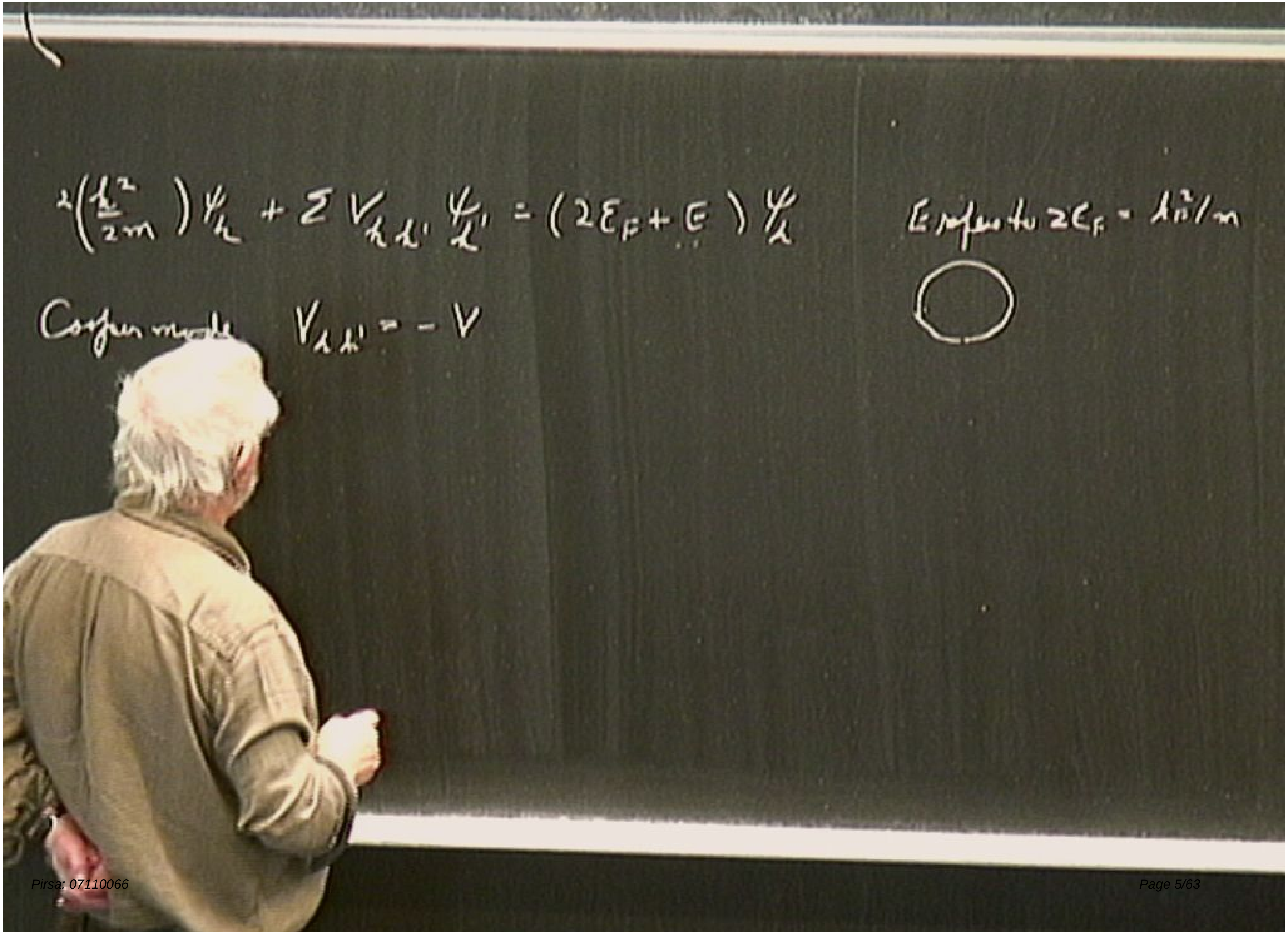
URL: http://pirsa.org/07110066

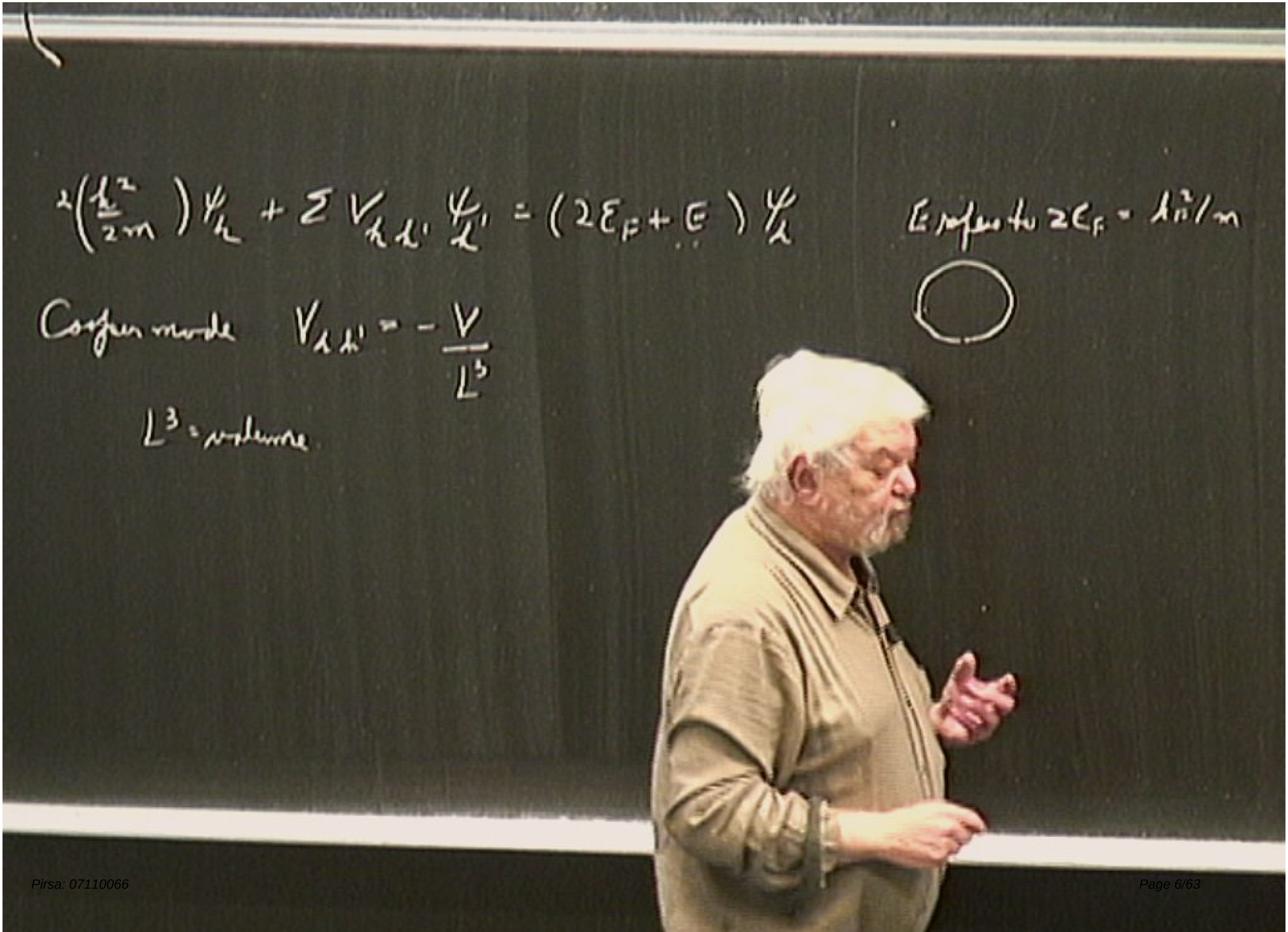
Abstract:




$\left(\frac{\hbar^2 k^2}{2m}\right)\psi_k + \sum V_{k k'} \psi_{k'}$


$2\left(\frac{\hbar^2 k^2}{2m}\right)\psi_k + \Sigma V_{kk'}\psi_{k'} = (2E_F + E)\psi_k$
E refers to $2E_F$


V = - V

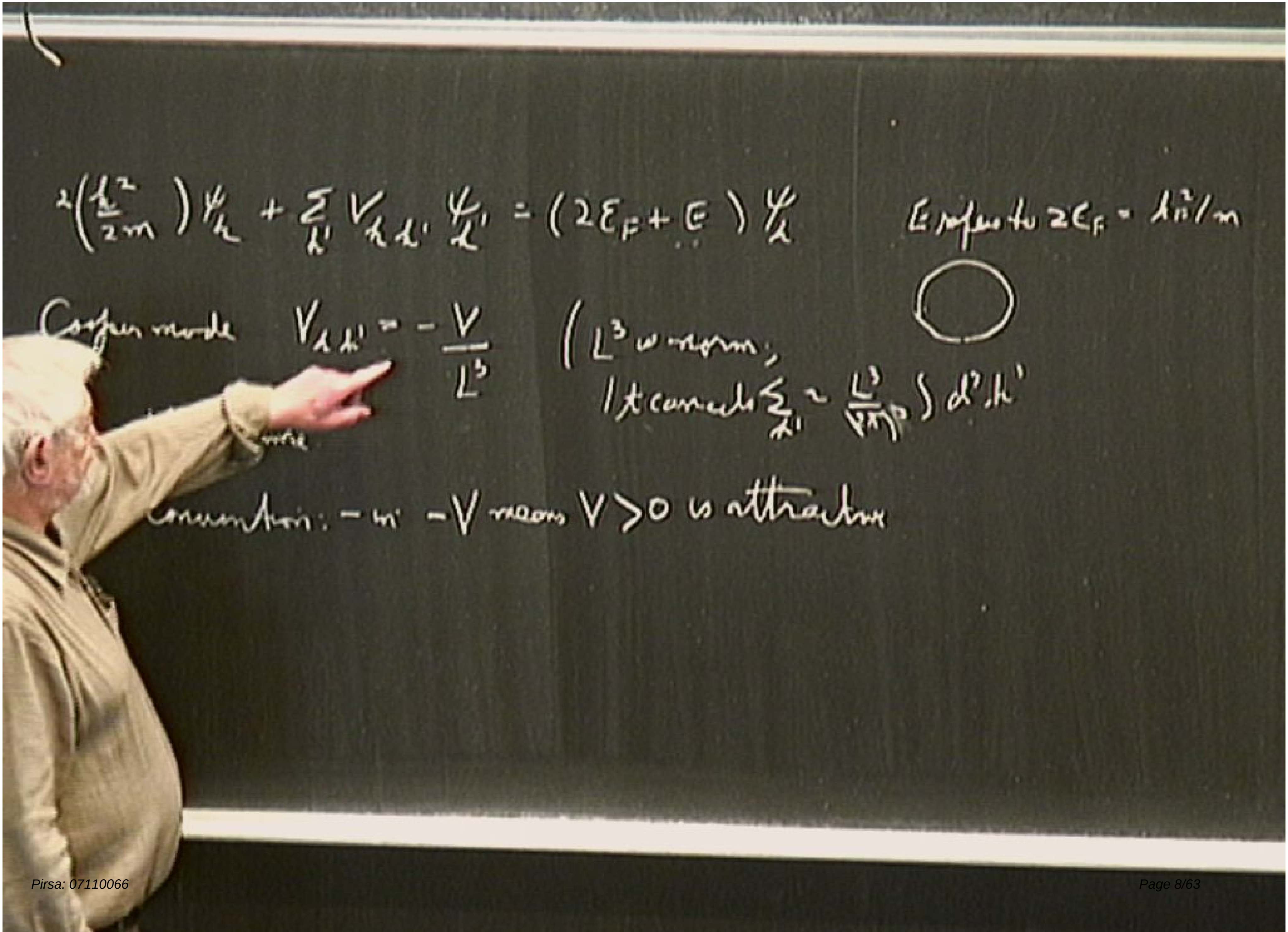

2(ℏ²k²/2m) ψ_k + Σ V_kk' ψ_k' = (2E_F + E) ψ_k
E refers to 2E_F = ℏ²k_F²/m
Cooper model V_kk' = − V/L³
L³ = volume

$$2\left(\frac{k^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'}\,\psi_{k'} = (2\epsilon_F + E)\,\psi_k$$

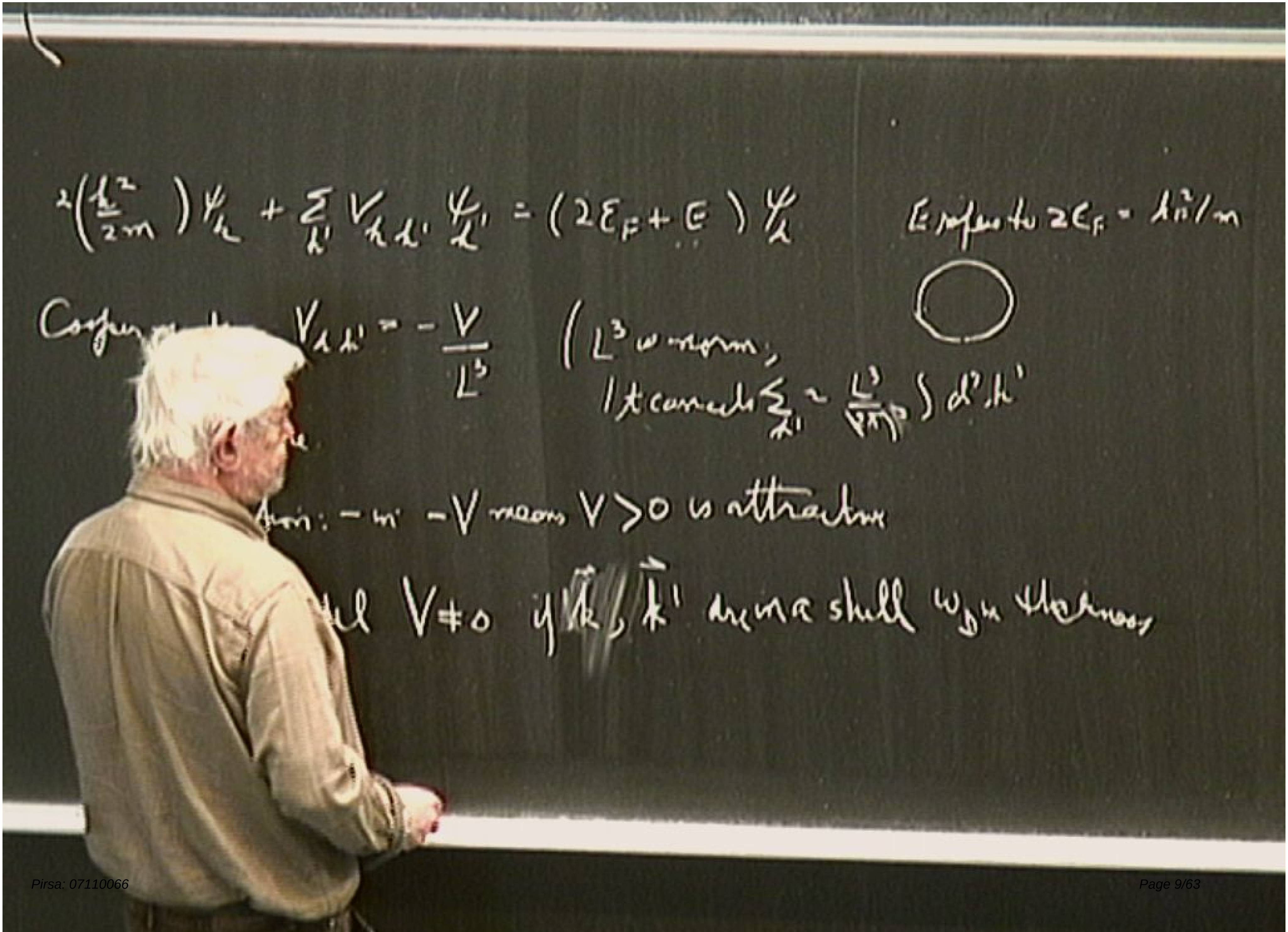
E refers to $2\epsilon_F = k_F^2/m$

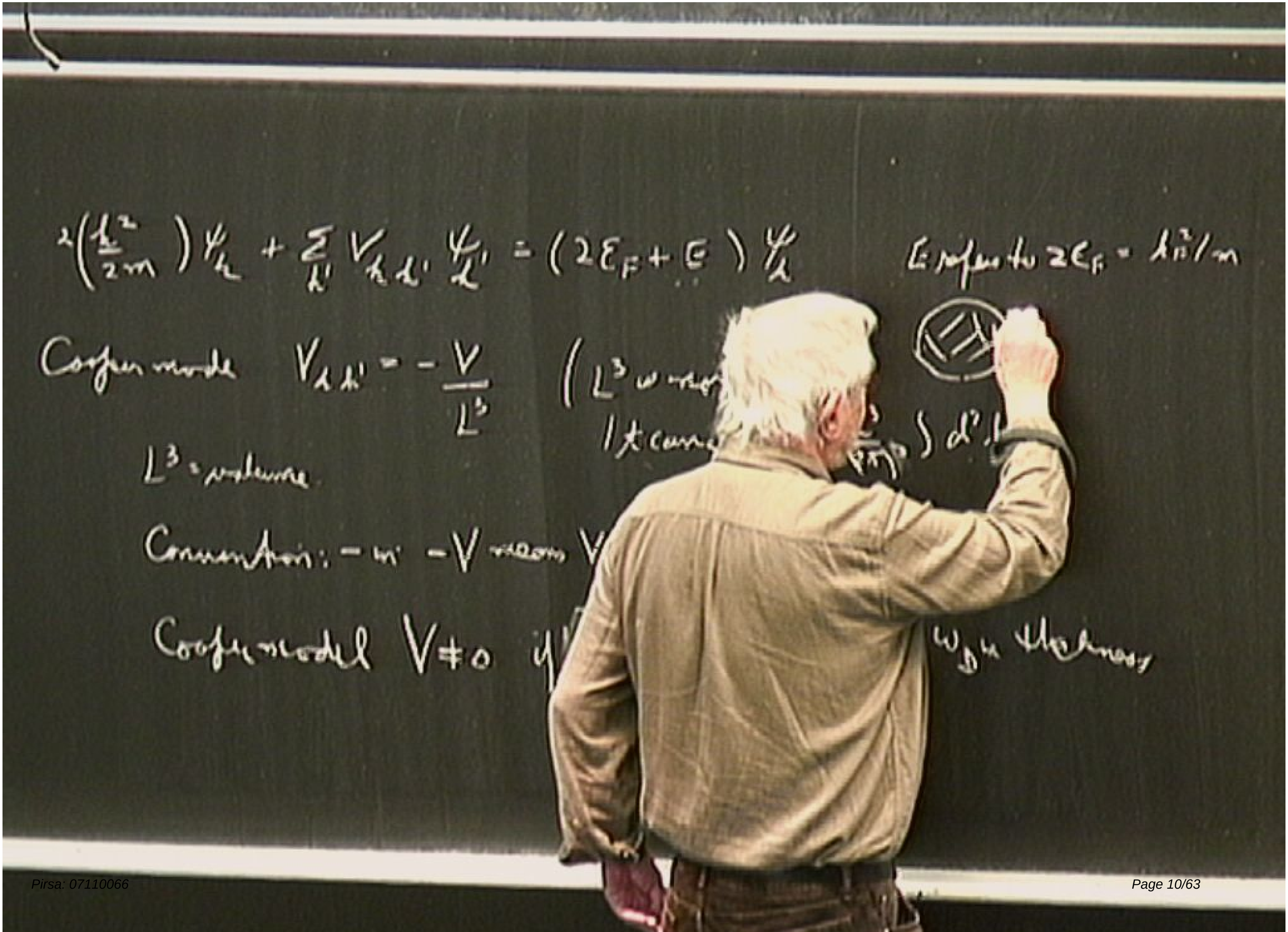
Cooper model $V_{kk'} = -\frac{V}{L^3}$ (L^3 w-norm; / k continuum $\sum_{k'} \approx \frac{L^3}{(2\pi)^3}\int d^3k'$)

L^3 = volume

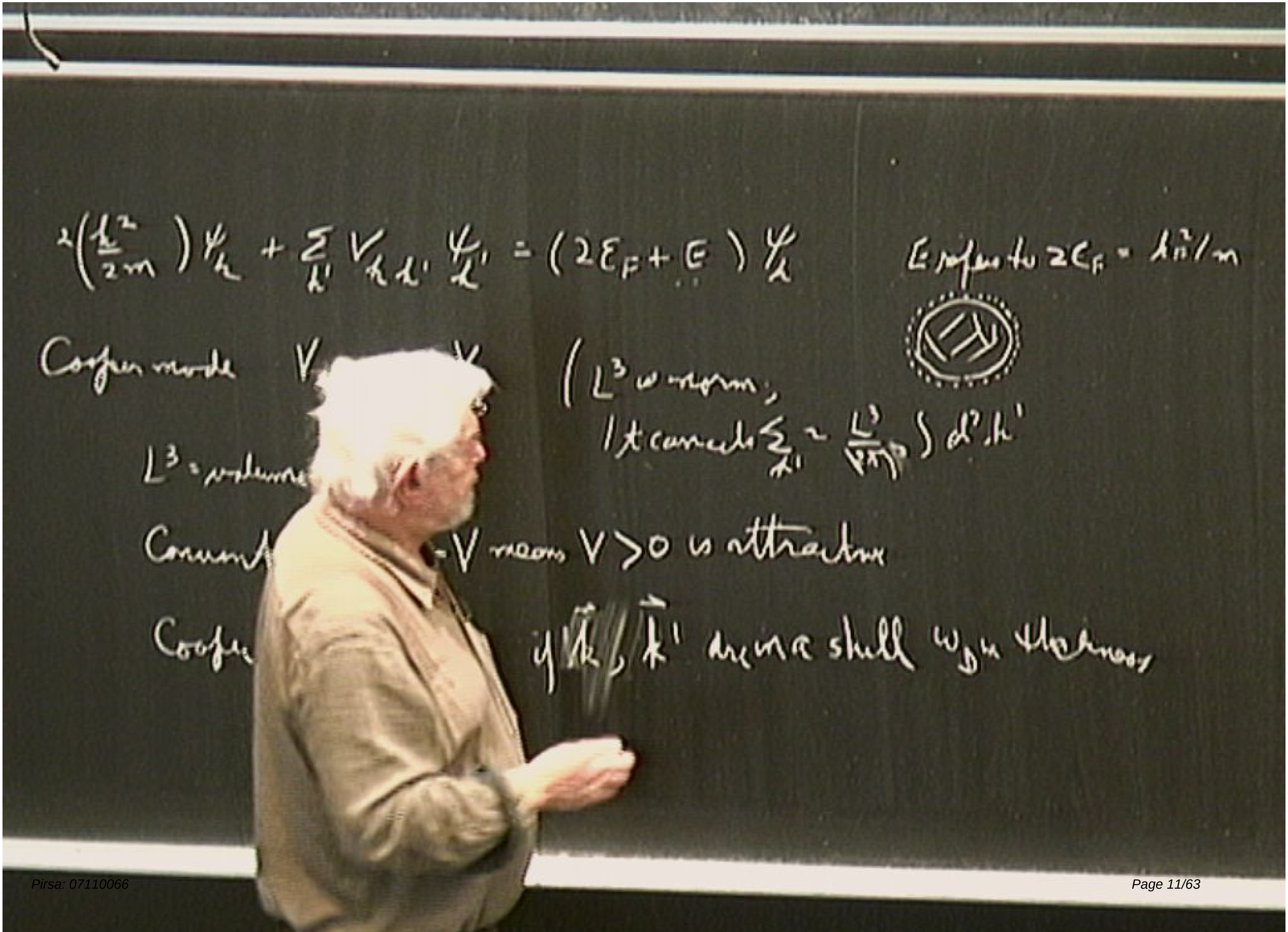


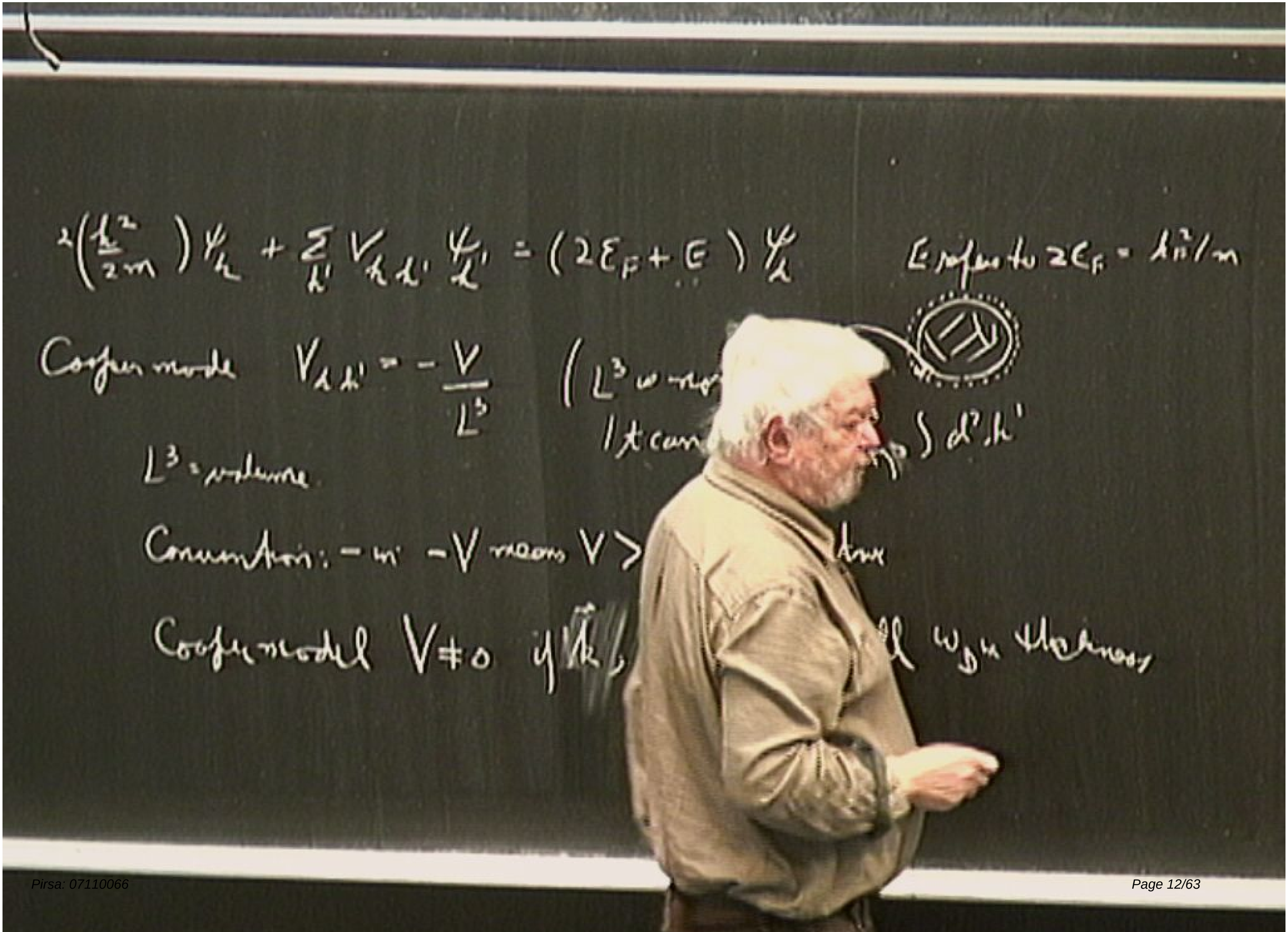
2(ħ²k²/2m) ψ_k + Σ_k' V_kk' ψ_k' = (2ε_F + E) ψ_k
E refers to 2ε_F = ħk_F²/m
Cooper model V_kk' = − V/L³
(L³ is norm;
it cancels Σ_k' = L³/(2π)³ ∫ d³k'
convention: − in −V means V > 0 is attractive


−V means V > 0 is attractive
V ≠ 0



2(k^2/2m) ψ_k + Σ_k' V_kk' ψ_k' = (2ε_F + E) ψ_k
E refers to 2ε_F = k_F^2/m
Cooper model V_kk' = -V/L^3
L^3 = volume
Convention: -ve -V means
Cooper model V≠0


2(ħ²k²/2m) ψ_k + Σ_k' V_kk' ψ_k' = (2E_F + E) ψ_k
E refers to 2E_F = ħk_F²/m
Cooper model V
L³ = volume
V means V > 0 is attraction
k, k' are in a shell


$2\left(\frac{k^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'}\psi_{k'} = (2E_F + E)\psi_k$
E refers to $2E_F = k_F^2/m$
Cooper model $V_{kk'} = -\frac{V}{L^3}$
L^3 = volume
Convention: − in −V means V >
Cooper model V ≠ 0

$$2\left(\frac{k^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'}\,\psi_{k'} = (2\epsilon_F + E)\,\psi_k$$

E refers to $2\epsilon_F = k_F^2/m$

Cooper model $V_{kk'} = -\dfrac{V}{L^3}$ (L^3 is norm; 1st correct $\sum_{k'} = \frac{L^3}{(2\pi)^3}\int d^3k'$)

L^3 = volume

Convention: − in −V means $V > 0$ is attraction

Cooper model $V \neq 0$ if $\vec{k}, \vec{k}'$ are in the shell ω_D in thickness

$$2\left(\frac{k^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'}\,\psi_{k'} = (2\epsilon_F + E)\,\psi_k$$

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Convention: − in −V means $V > 0$ is attractive

Cooper model $V \neq 0$ if $\vec{k}, \vec{k}'$ are in a shell ω_D in thickness

$$2\left(\frac{k^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'}\,\psi_{k'} = (2\epsilon_F + E)\,\psi_k$$

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Cooper model $V_{kk'} = -\frac{V}{L^3}$ (L^3 is norm., it cancels $\sum_{k'} = \frac{L^3}{(2\pi)^3}\int d^3k'$)

L^3 = volume

Convention: − in −V means V > 0 is attraction

Cooper model V ≠ 0 if $\vec{k}, \vec{k}'$ are in a shell ω_D in thickness

$$2\left(\frac{k^2}{2m}\right)\psi_k + \sum_{k'} V_{kk'}\,\psi_{k'} = (2\epsilon_F + E)\,\psi_k$$

E refers to $2\epsilon_F = k_F^2/m$

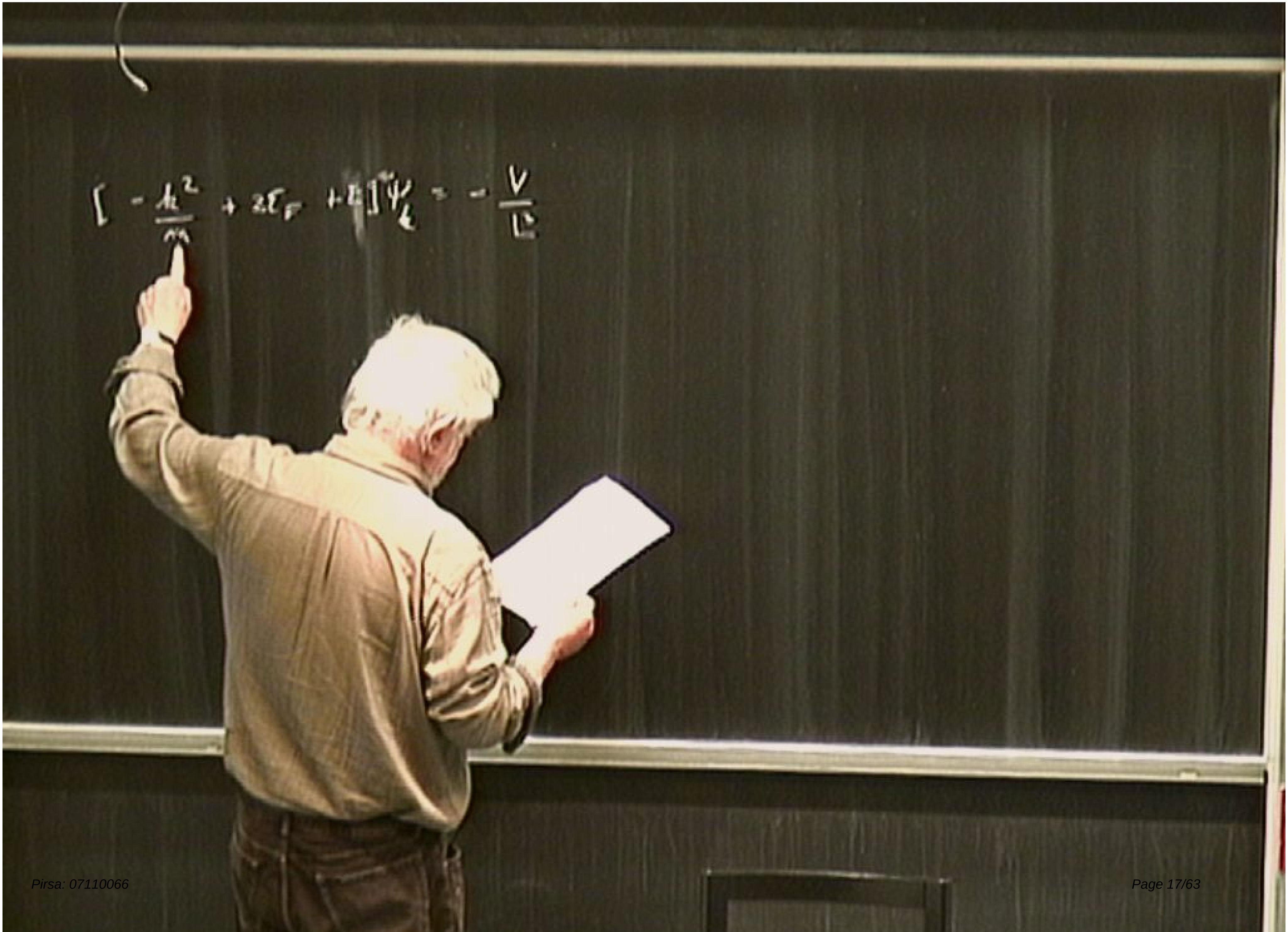
Cooper model $V_{kk'} = -\dfrac{V}{L^3}$ (L^3 is norm; at continuum $\sum_{k'} = \frac{L^3}{(2\pi)^3}\int d^3k'$)

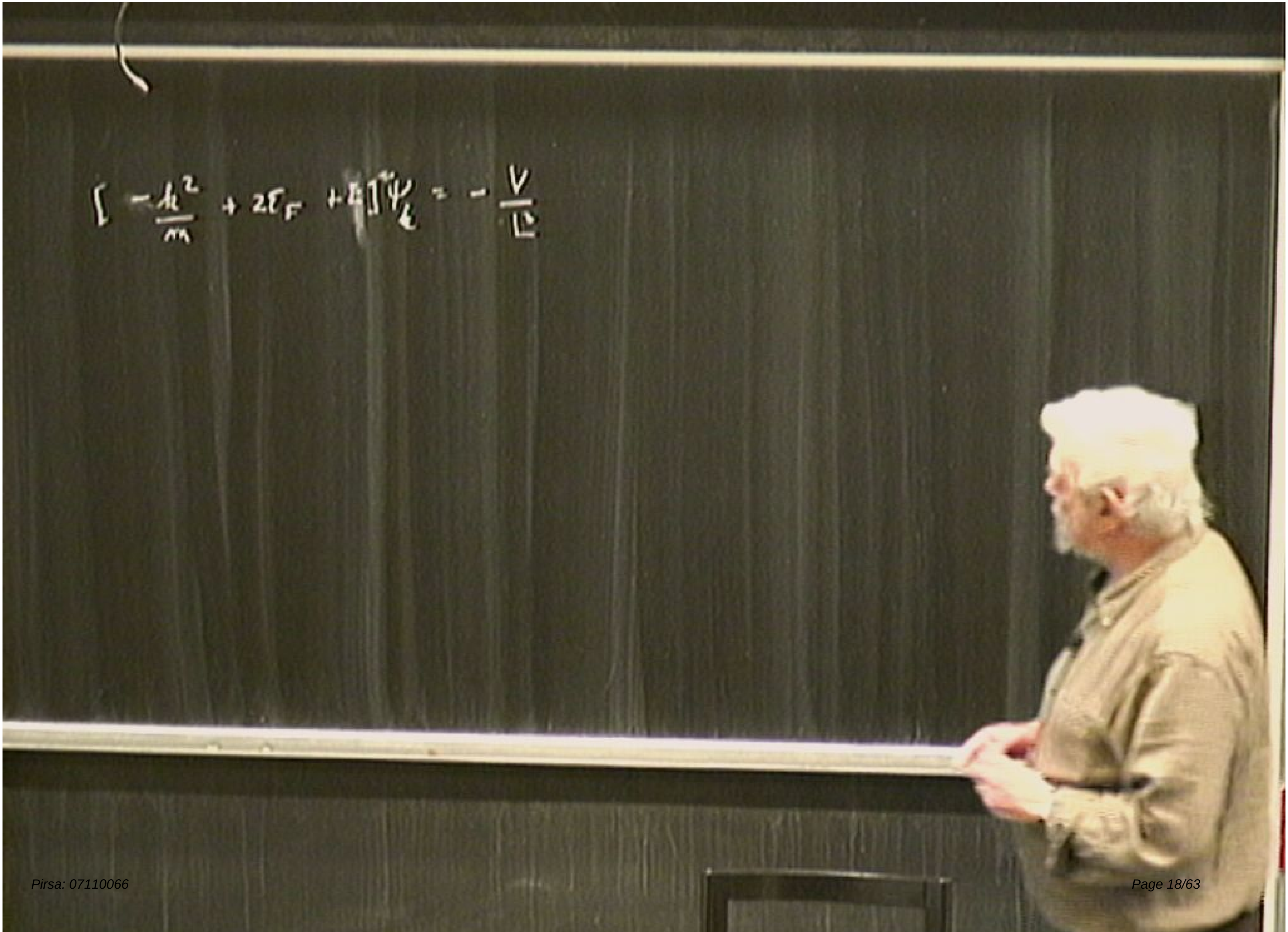
L^3 = volume

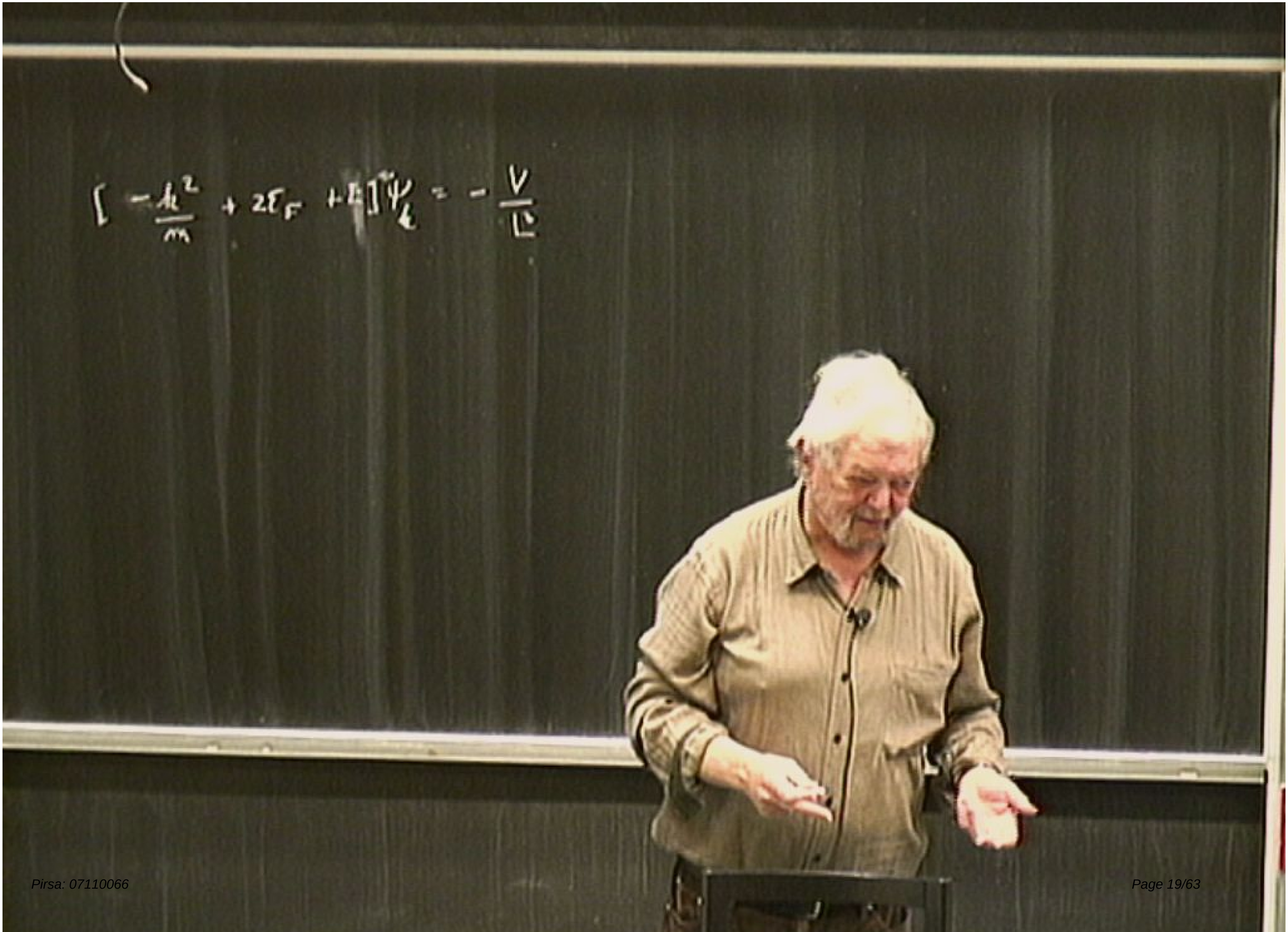
Convention: − in −V means V > 0 is attraction

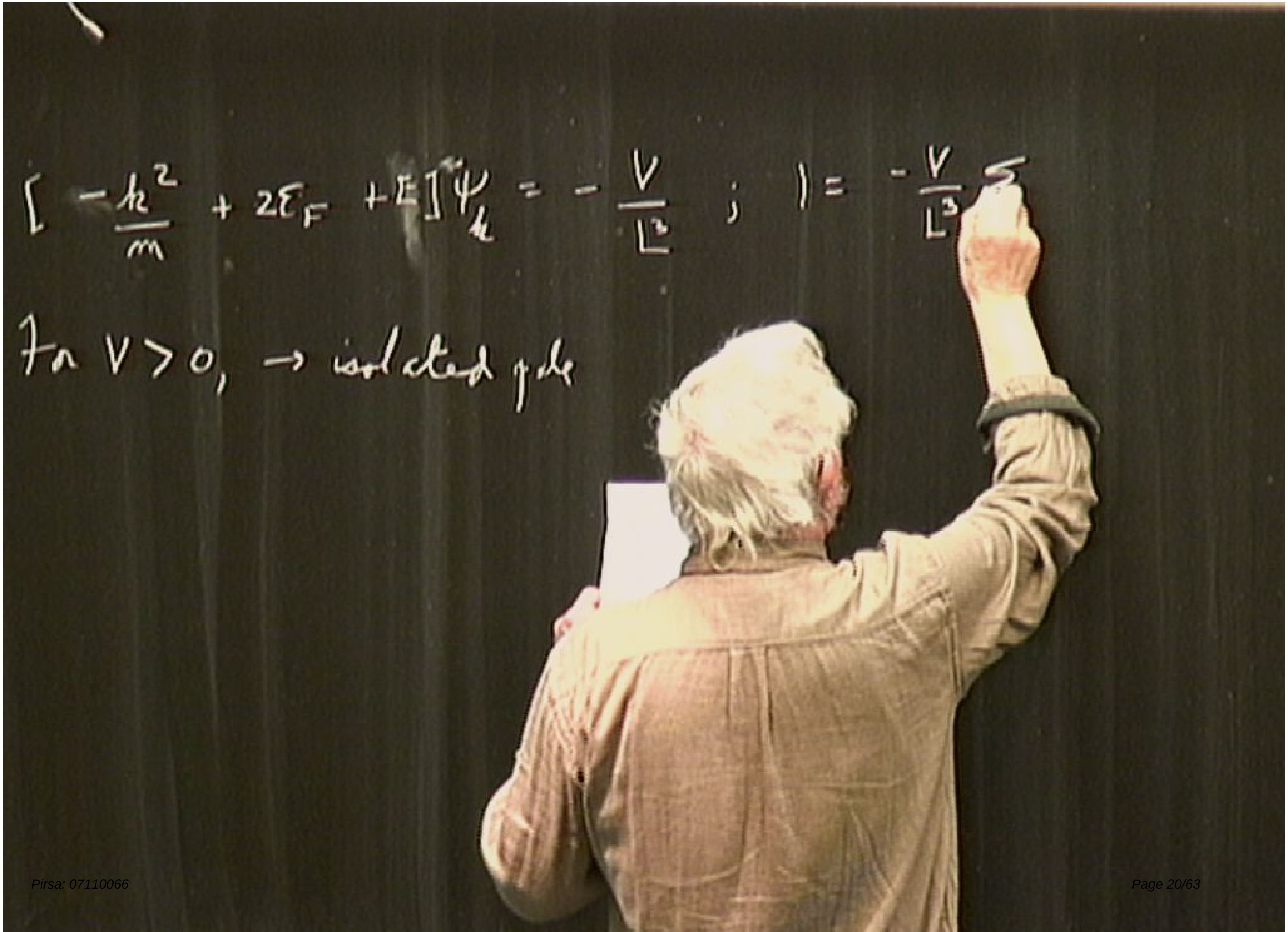
Cooper model $V \neq 0$ if $\vec{k}, \vec{k}'$ are in a shell ω_D in thickness

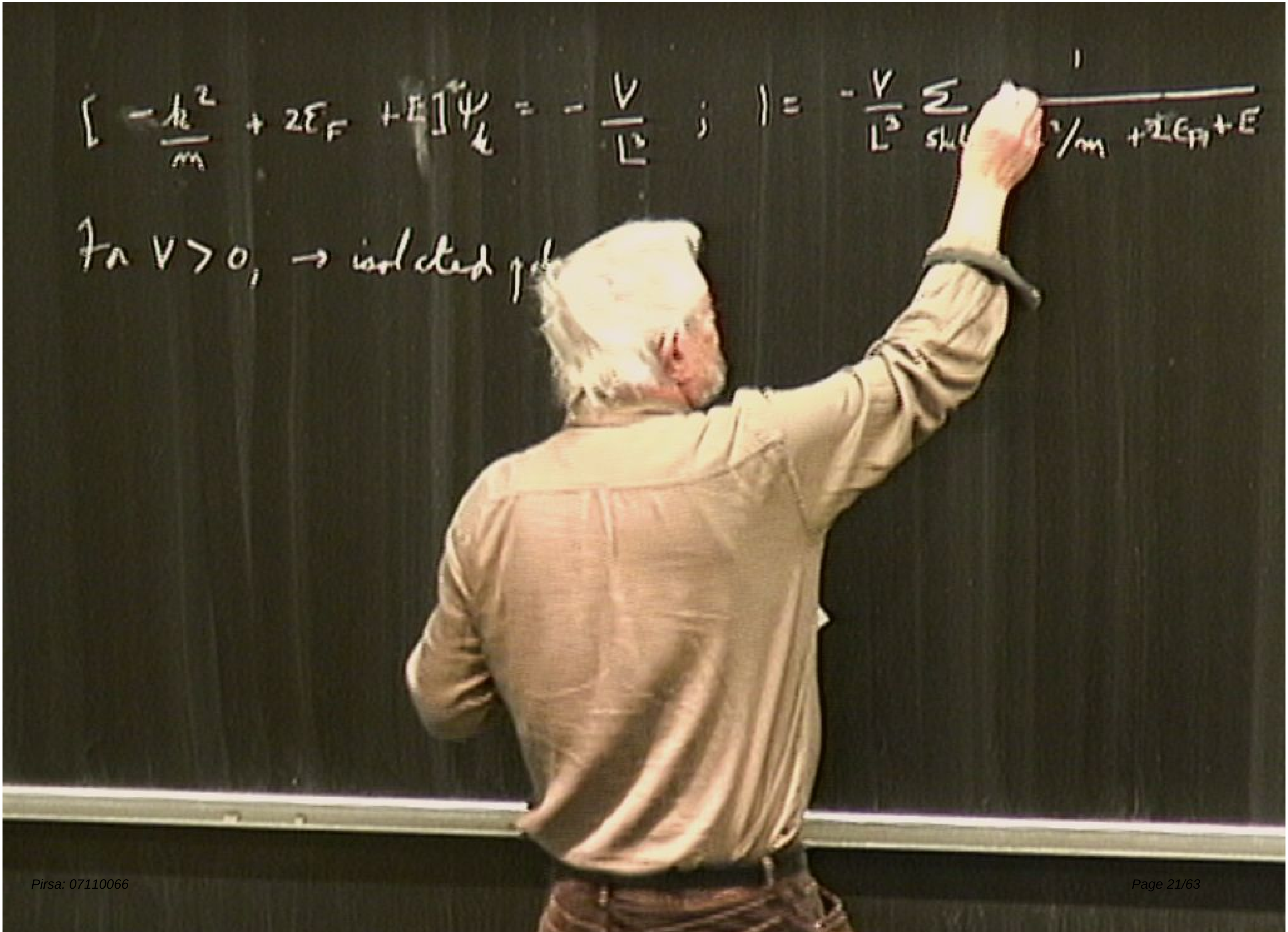
For simplicity V = const in the shell

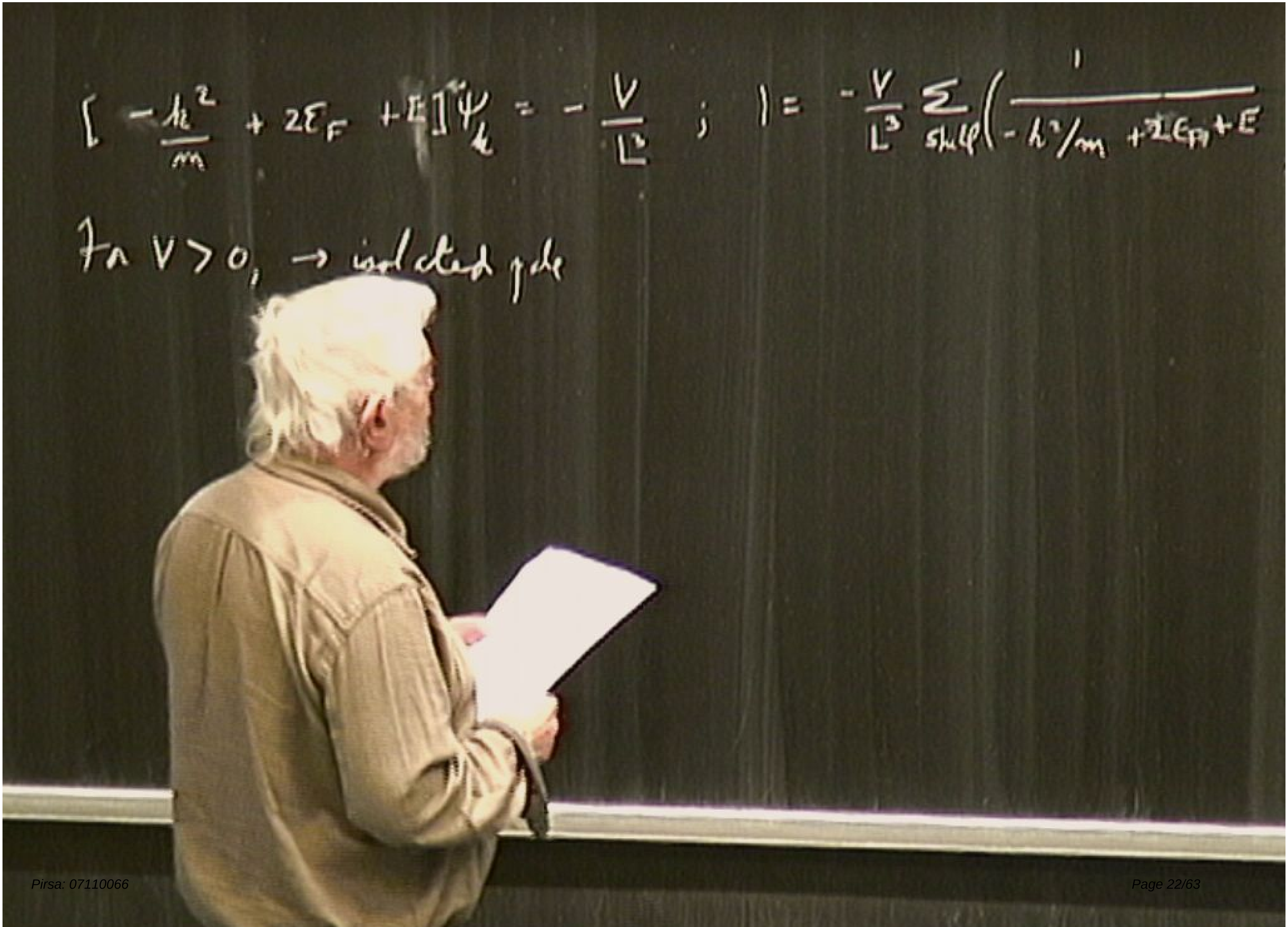

$\left[-\frac{k^2}{m} + 2\varepsilon_F + E \right] \psi_k = -\frac{V}{L}$


$\left[-\frac{k^2}{m} + 2E_F + E \right] \psi_k = - \frac{V}{L^3}$


$[-\frac{k^2}{m} + 2E_F + E] \psi_k = - \frac{V}{L}$


[−k²/m + 2ε_F + E] ψ_k = − V/L³ ;) = − V/L³ Σ
For V > 0, → isolated pole


[− k²/m + 2E_F + E] ψ_k = − V/L³ ; 1 = − V/L³ Σ
for V > 0, → isolated


[− k²/m + 2E_F + E] ψ_k = − V/L³ ; 1 = − V/L³ Σ_shell (1 / (− k²/m + 2E_F + E)
for V > 0, → isolated pole

$$\left[-\frac{k^2}{m} + 2E_F + E \right] \psi_k = -\frac{V}{L^3} \; ; \; 1 = -\frac{V}{L^3} \sum_{\text{shell}} \left(\frac{1}{-k^2/m + 2E_F + E} \right)$$

For $V > 0$, → isolated pole A solution exists if $V > 0$ with $E < 0$.

$$\left[-\frac{k^2}{m} + 2\varepsilon_F + E \right] \psi_k = -\frac{V}{L^3} \; ; \quad 1 = -\frac{V}{L^3} \sum_{\text{shell}} \left(\frac{1}{-k^2/m + 2\varepsilon_F + E} \right)$$

For $V > 0$, → isolated pole A solution exists if $V > 0$ with $E < 0$

No " " $V < 0$ $(E > 0)$

$$\left[-\frac{k^2}{m} + 2E_F + E \right] \psi_k = -\frac{V}{L^3} \; ; \quad 1 = -\frac{V}{L^3} \sum_{\text{shell}} \left(\frac{1}{-k^2/m + 2E_F + E} \right)$$

For $V > 0$, → isolated pole

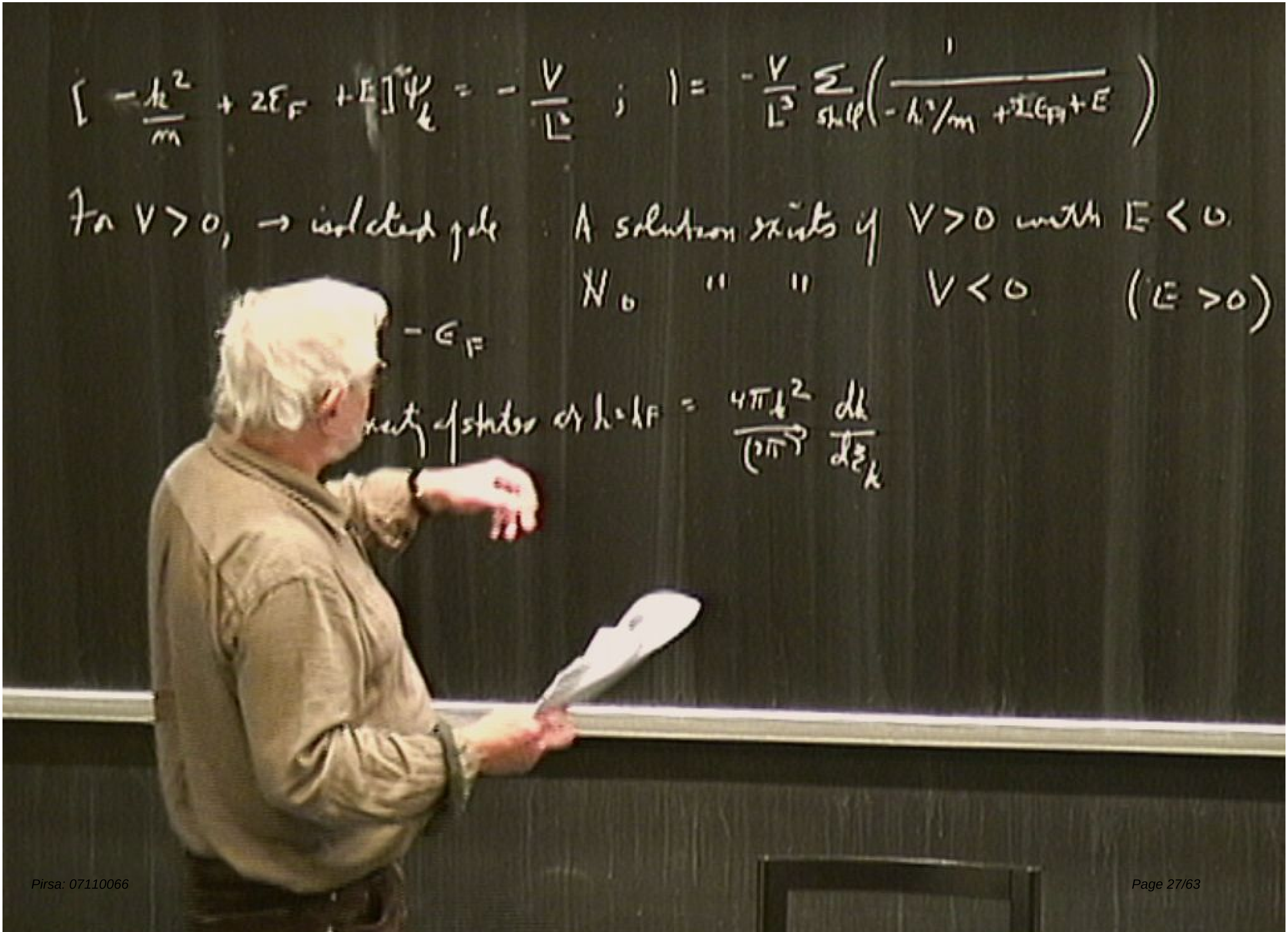
A solution exists if $V > 0$ with $E < 0$

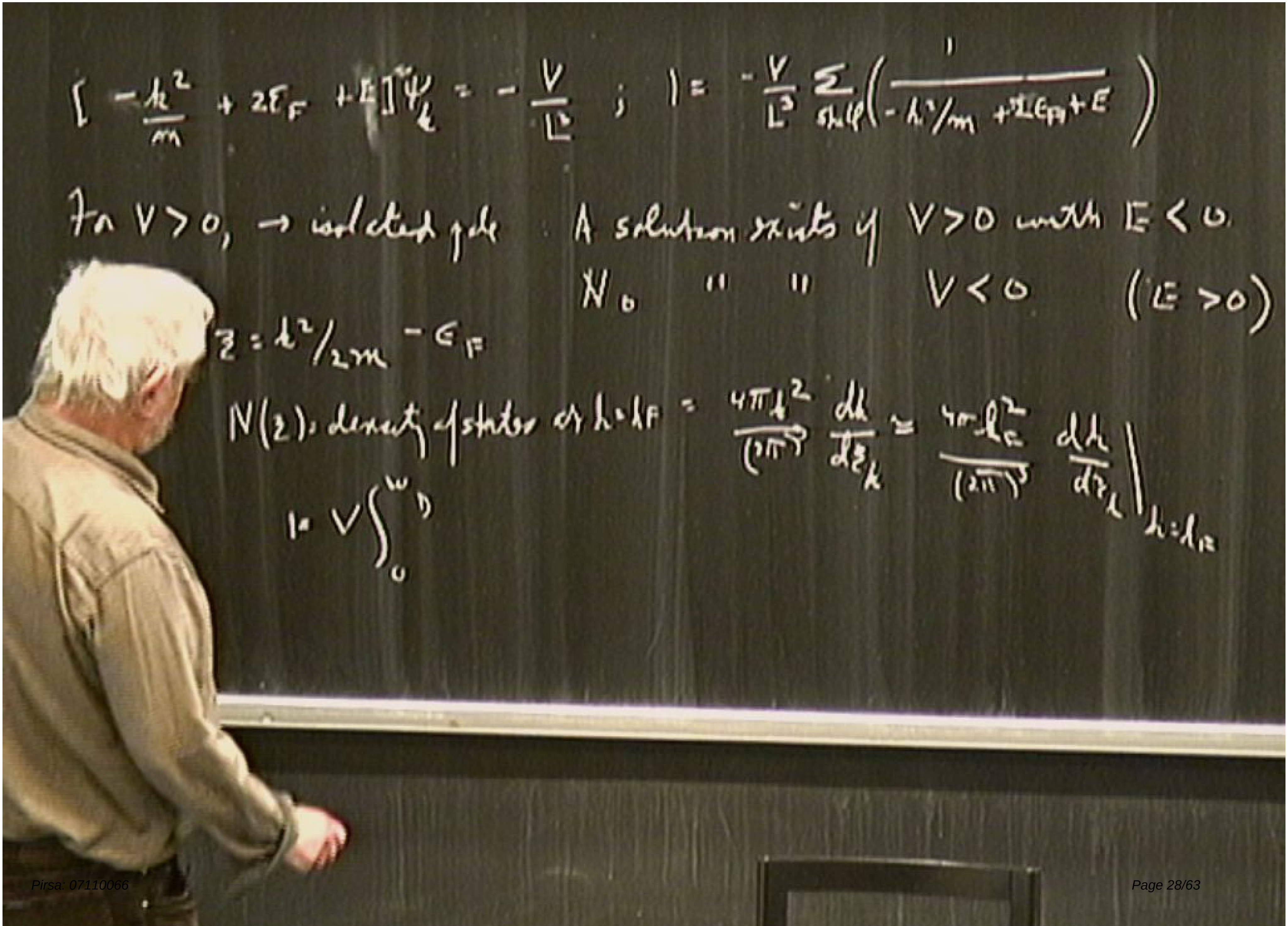
No " " $V < 0$ $(E > 0)$

$$\left[-\frac{k^2}{m} + 2\varepsilon_F + E \right] \psi_k = -\frac{V}{L^3} \; ; \quad 1 = -\frac{V}{L^3} \sum_{\text{shell}} \left(\frac{1}{-k^2/m + 2\varepsilon_F + E} \right)$$

For $V > 0$, $\rightarrow$ isolated pole. A solution exists if $V > 0$ with $E < 0$.

No " " $V < 0$ $(E > 0)$


[−k²/m + 2ε_F + E] ψ_k = − V/L³ ; 1 = − V/L³ Σ_shell (1 / (−k²/m + 2ε_F + E))
For V > 0, → isolated pole
A solution exists if V > 0 with E < 0
N_0 " " V < 0 (E > 0)
− ε_F
of states at k = k_F = 4πk²/(2π)³ dk/dε_k


For V > 0, → isolated pole
A solution exists if V > 0 with E < 0
" " V < 0 (E > 0)
N(ξ) = density of states at k = k_F

$$\left[-\frac{k^2}{m} + 2\varepsilon_F + E\right]\psi_k = -\frac{V}{L^3} \; ; \quad 1 = -\frac{V}{L^3}\sum_{\text{shell}}\left(\frac{1}{-k^2/m + 2\varepsilon_F + E}\right)$$

For $V > 0$, $\rightarrow$ isolated pole. A solution exists if $V > 0$ with $E < 0$

$\not\exists$ " " $V < 0$ $(E > 0)$

$$\xi = k^2/2m - \varepsilon_F$$

$$N(\xi) = \text{density of states at } k = k_F = \frac{4\pi k^2}{(2\pi)^3}\frac{dk}{d\xi_k} = \frac{4\pi k_F^2}{(2\pi)^3}\left.\frac{dk}{d\xi_k}\right|_{k=k_F}$$

$$1 = V\int_0^{\omega_D}\frac{1}{(2\xi - E)}$$

$$\left[-\frac{k^2}{m} + 2\varepsilon_F + E\right]\psi_k = -\frac{V}{L^3} \quad ; \quad 1 = -\frac{V}{L^3}\sum_{\text{shell}}\left(\frac{1}{-k^2/m + 2\epsilon_F + E}\right)$$

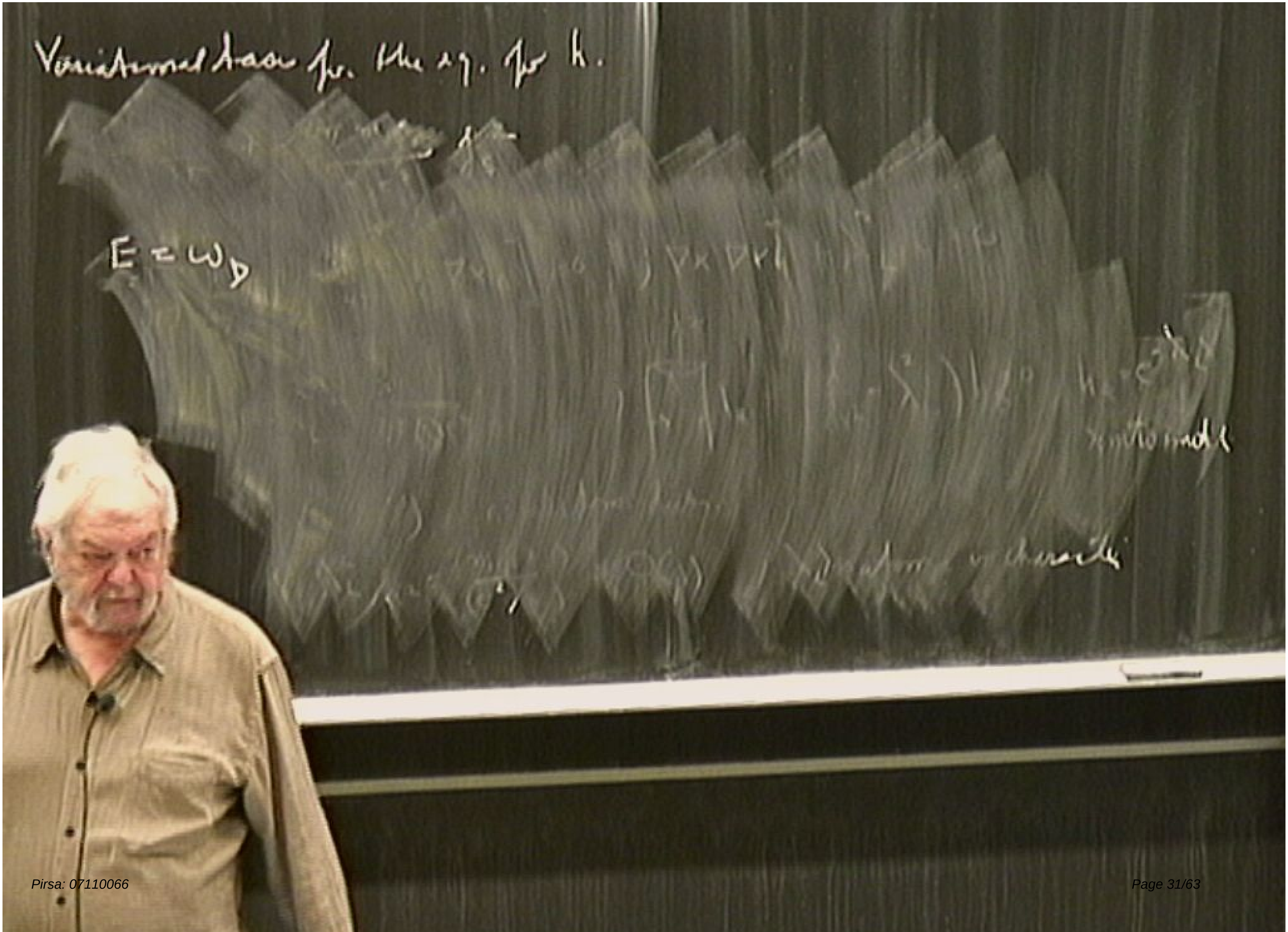
For $V > 0$, → isolated pole. A solution exists if $V > 0$ with $E < 0$

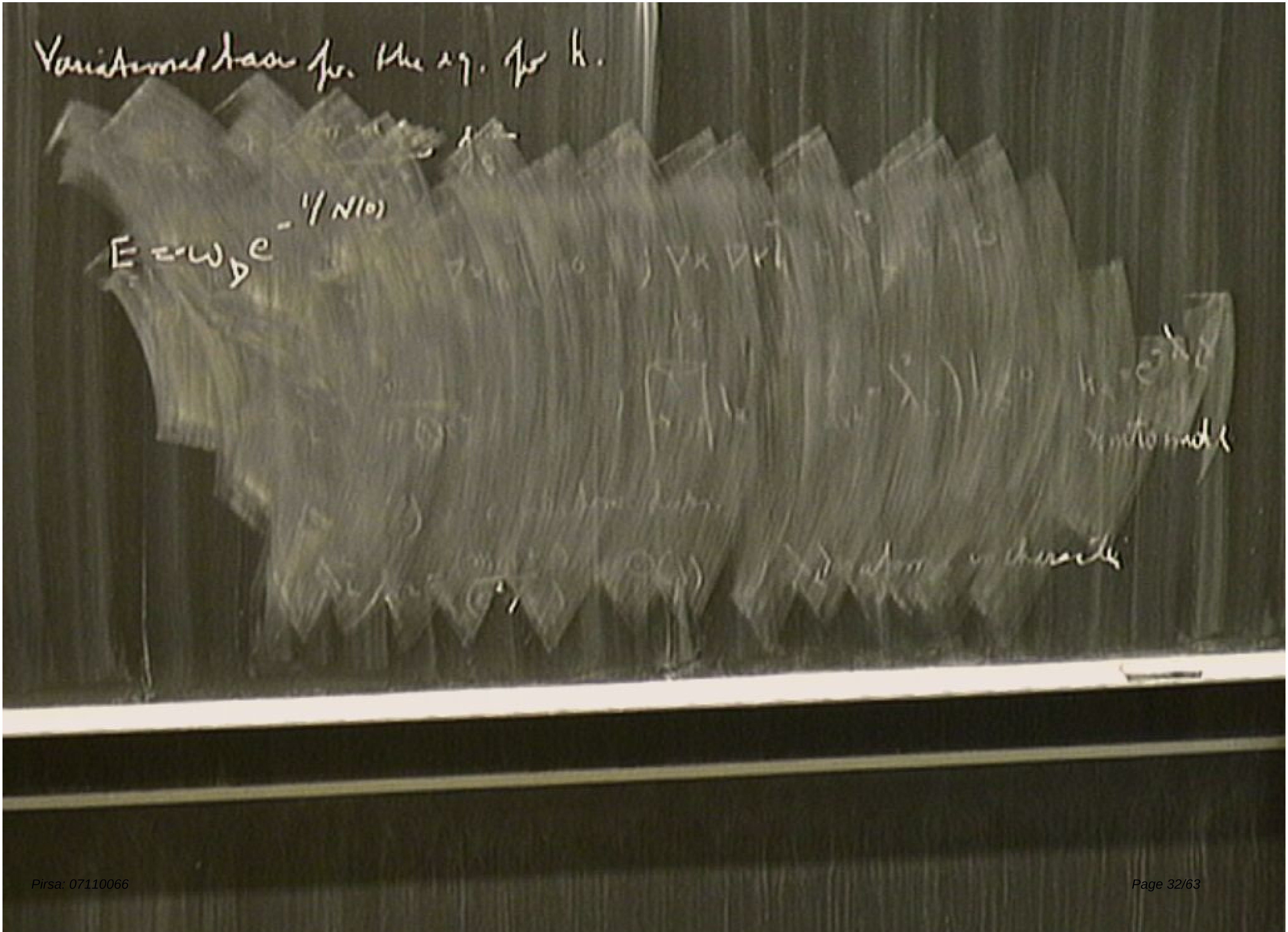
No " " $V < 0$ $\quad (E > 0)$

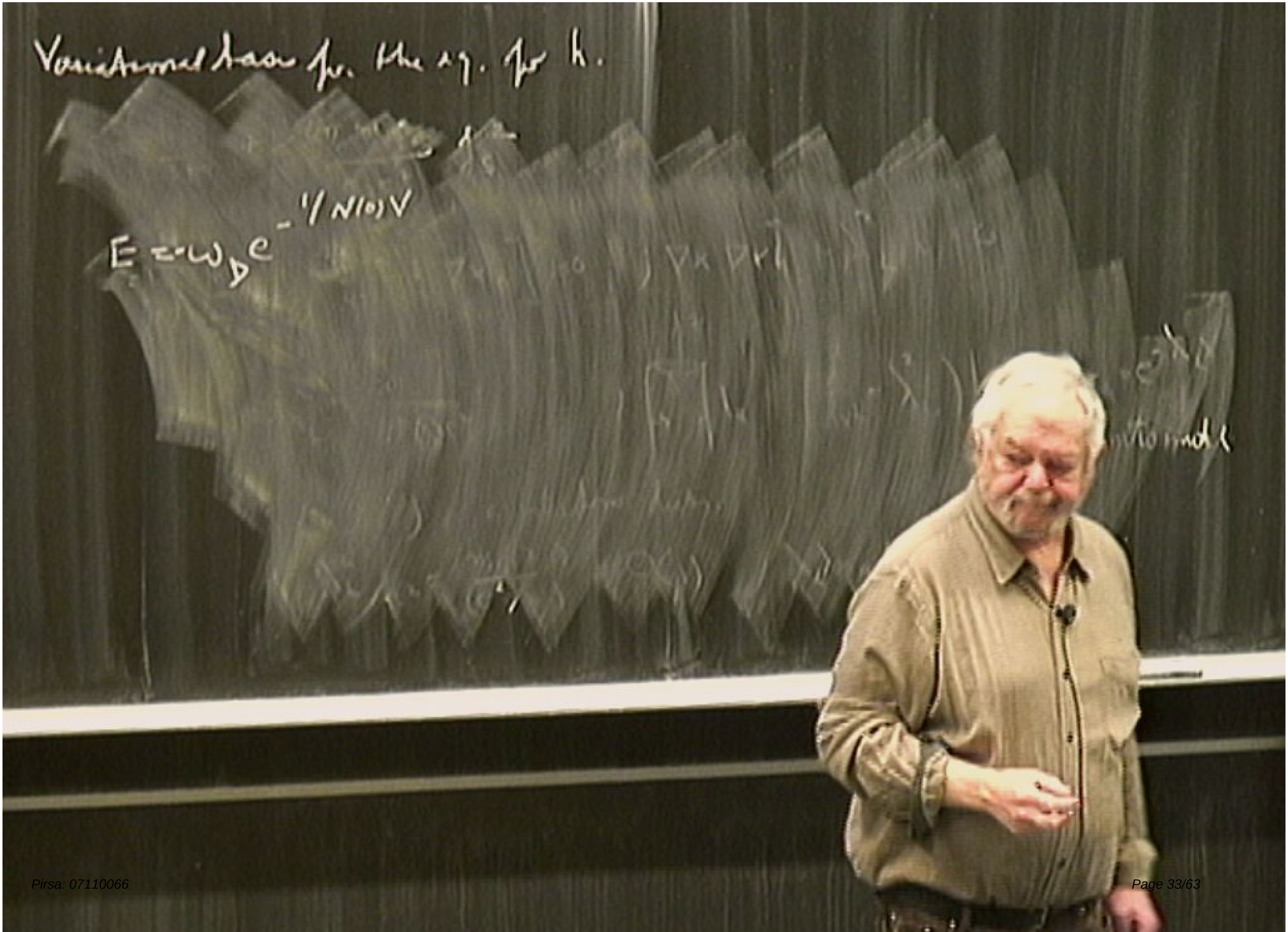
$$\xi = k^2/2m - \epsilon_F$$

$N(\xi)$: density of states at $k = k_F$ $= \frac{4\pi k^2}{(2\pi)^3}\frac{dk}{d\xi_k} = \frac{4\pi k_F^2}{(2\pi)^3}\frac{dk}{d\xi_k}\Big|_{k=k_F}$

$$1 = V\int_0^{\omega_D}\frac{N(\xi)\,d\xi}{(2\xi - E)}$$


Variational basis for the eq. for h.
E = ω_D


Variational base for the eq. for h.
$E = \omega_D e^{-1/N(0)}$

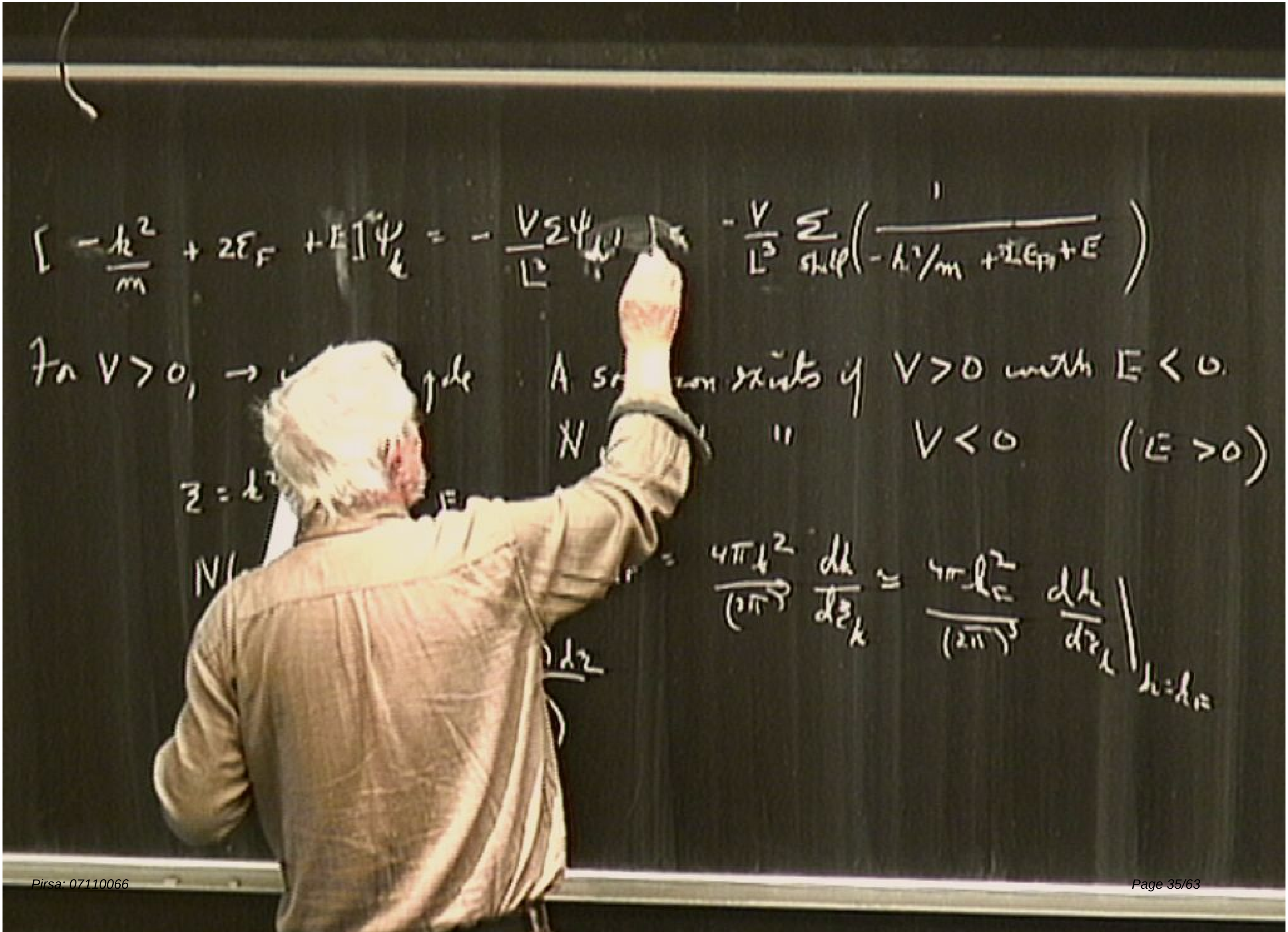

Variational base for the eq. for h.
E ≈ ω_D e^{-1/N(0)V}

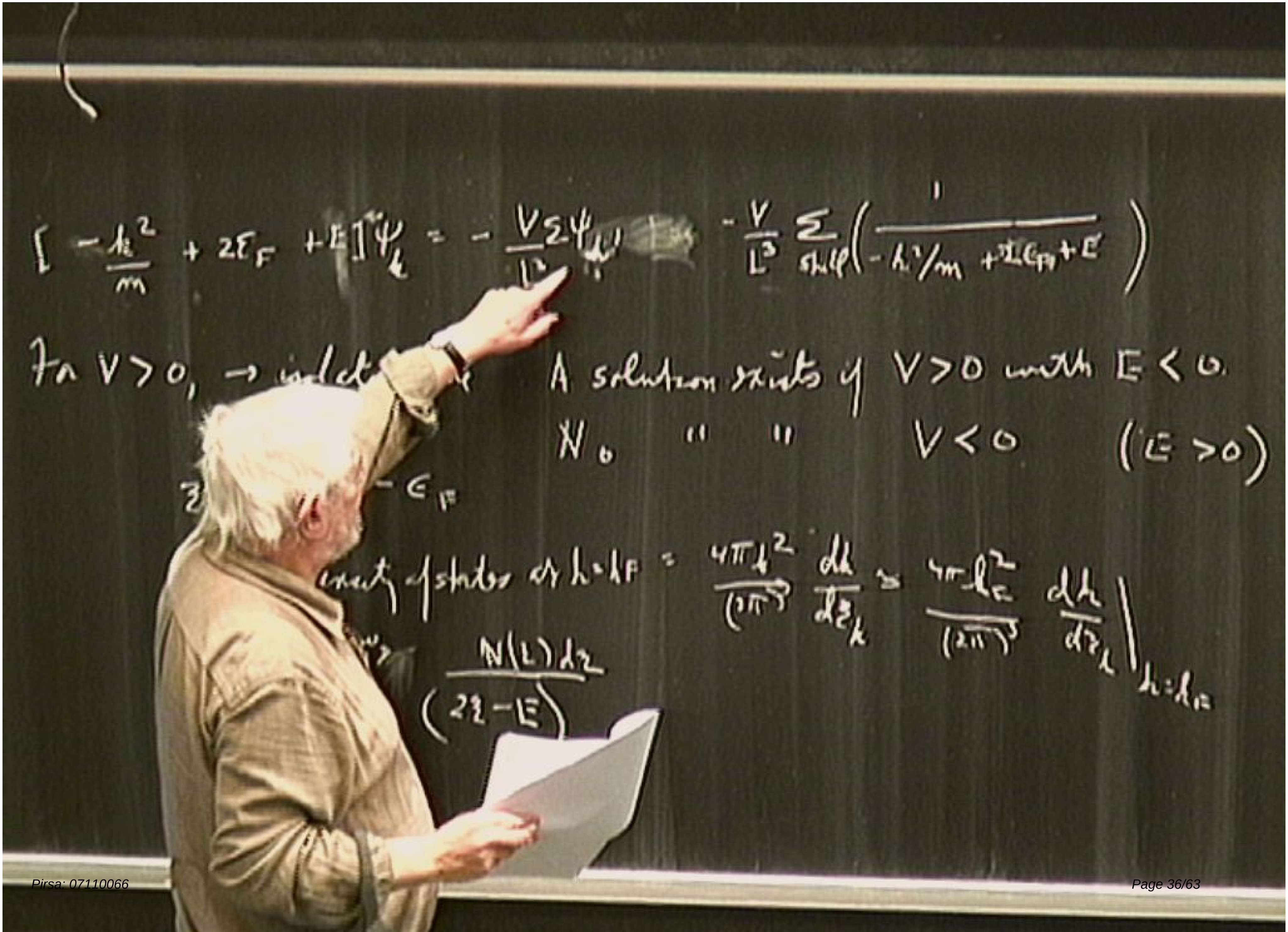
Variational basis for the eq. for h.

$$E \simeq -\omega_D e^{-1/N(0)V}$$

$$\langle r^2 \rangle = \frac{\int |\psi(r)|^2 r^2 r^2 dr}{\int |\psi(r)|^2 r^2 dr}$$

$$\psi(r) = \sum_k e^{ik\cdot r} \psi_k \quad , \quad \psi_k =$$


A solution exists if V > 0 with E < 0
V < 0 (E > 0)


A solution exists if V > 0 with E < 0.
No " " V < 0 (E > 0)

$$\left[-\frac{k^2}{m} + 2\epsilon_F + E\right]\psi_k = -\frac{V}{L^3}\sum\psi_{k'} \quad -\frac{V}{L^3}\sum_{\text{shell}}\left(\frac{1}{-k^2/m + 2\epsilon_F + E}\right)$$

For $V > 0$, → isolated pole : A solution exists if $V > 0$ with $E < 0$.

No " " $V < 0$ $(E > 0)$

$$\xi = k^2/2m - \epsilon_F$$

$$N(\xi) = \text{density of states at } k = k_F = \frac{4\pi k^2}{(2\pi)^3}\frac{dk}{d\xi_k} \simeq \frac{4\pi k_F^2}{(2\pi)^3}\left.\frac{dk}{d\xi_k}\right|_{k=k_F}$$

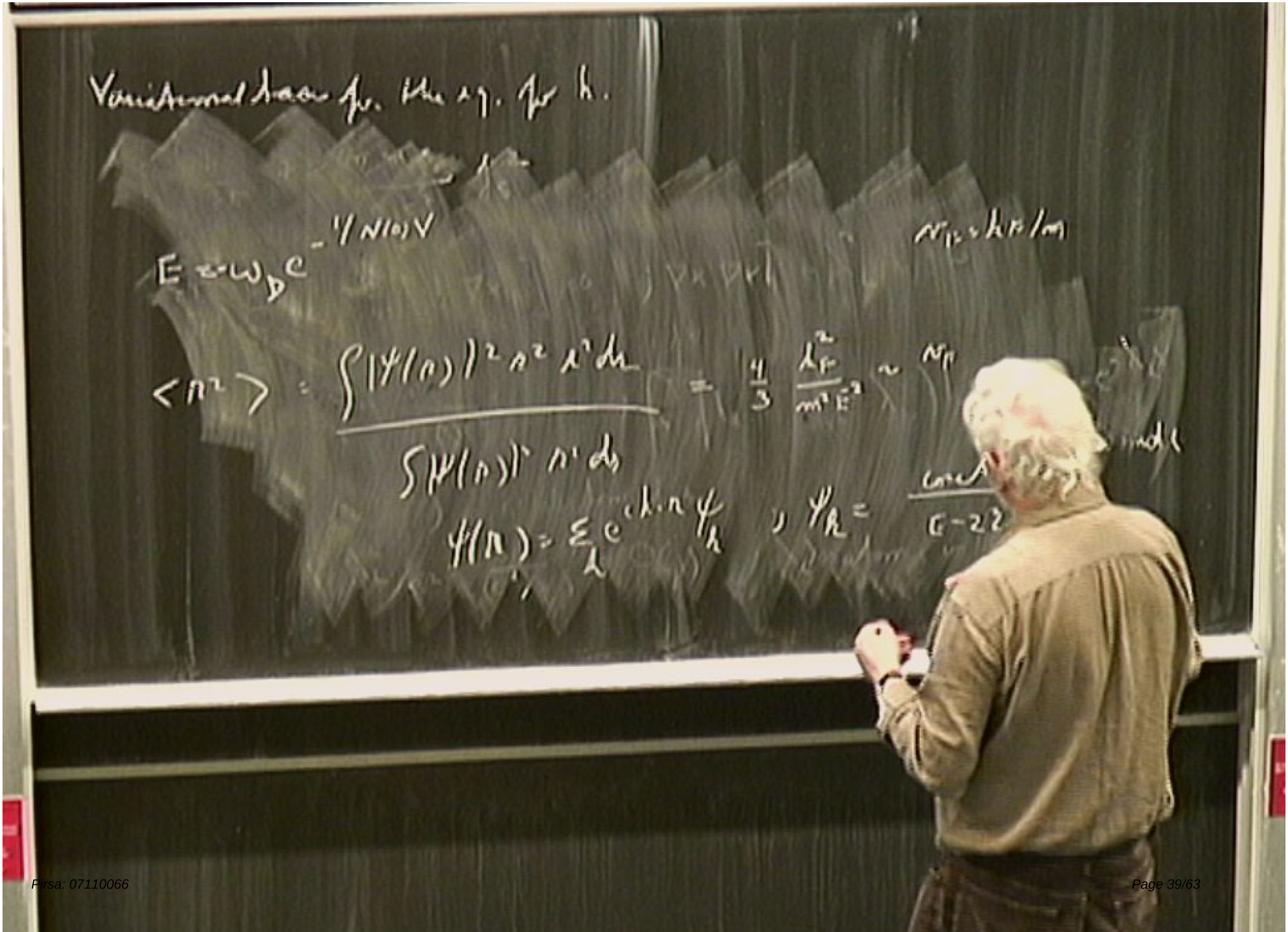
$$1 = V\int_0^{\omega_D}\frac{N(\xi)\,d\xi}{(2\xi - E)}$$

Variational basis for the eq. for h.

$$E = -\omega_D e^{-1/N(0)V}$$

$$\langle r^2 \rangle = \frac{\int |\psi(r)|^2 r^2 r^2 dr}{\int |\psi(r)|^2 r^2 dr}$$

$$\psi(r) = \sum_k e^{ik\cdot r} \psi_k \quad , \quad \psi_k = \frac{\text{const}}{E - 2\epsilon}$$

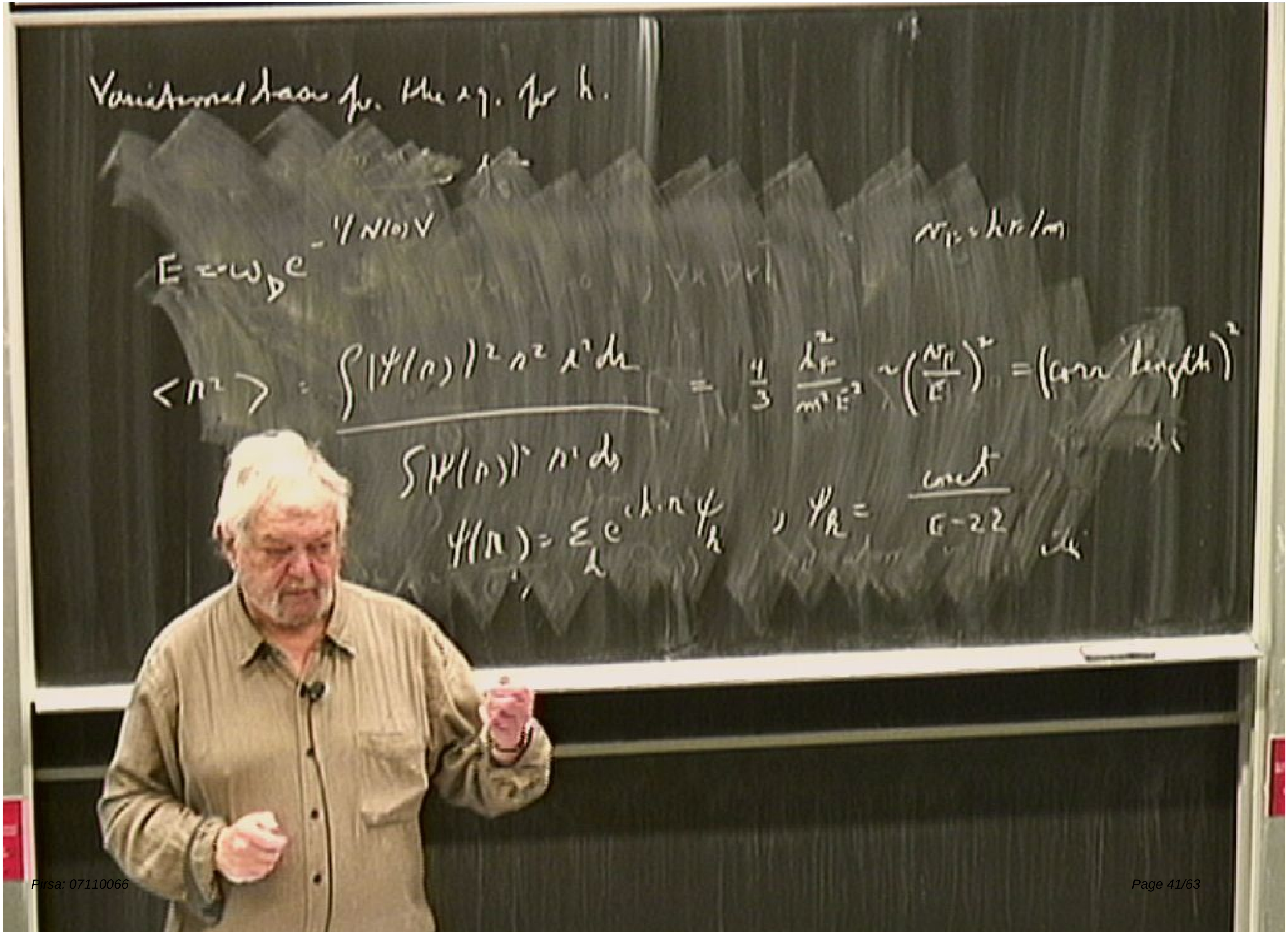

E ≈ ω_D e^{-1/N(0)V}
v_F = ħk_F/m
<r²> = ∫|ψ(r)|² r² r² dr / ∫|ψ(r)|² r² dr = 4/3 ħ²k_F²/m²E²
ψ(r) = Σ_k e^{ik·r} ψ_k

Variational bases for the eq. for h.

$E \approx 2\omega_D e^{-1/N(0)V}$ $\qquad v_F = \hbar k_F/m$

$$\langle r^2 \rangle = \frac{\int |\psi(r)|^2 r^2 r^2 dr}{\int |\psi(r)|^2 r^2 dr} = \frac{4}{3}\frac{k_F^2}{m^2 E^2} \sim \left(\frac{v_F}{E}\right)^2$$

$$\psi(r) = \sum_k e^{ik\cdot r}\psi_k \,, \quad \psi_k = \frac{\text{const}}{E - 2\epsilon}$$

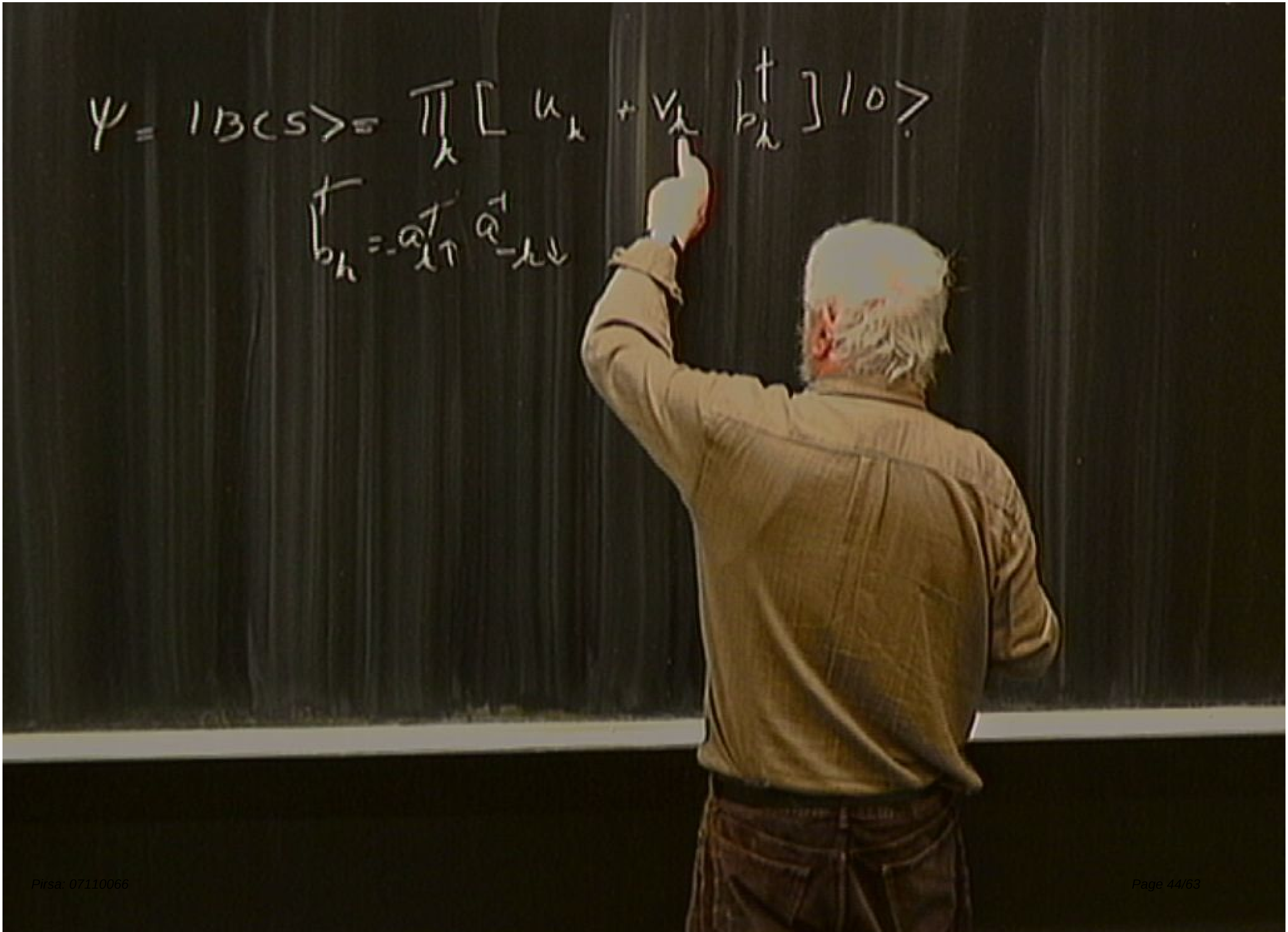

Variational basis for the eq. for h.
(corr. length)²

$$\Psi = |BCS\rangle = \prod_k \left[u_k + v_k \, b_k^\dagger \right] |0\rangle$$

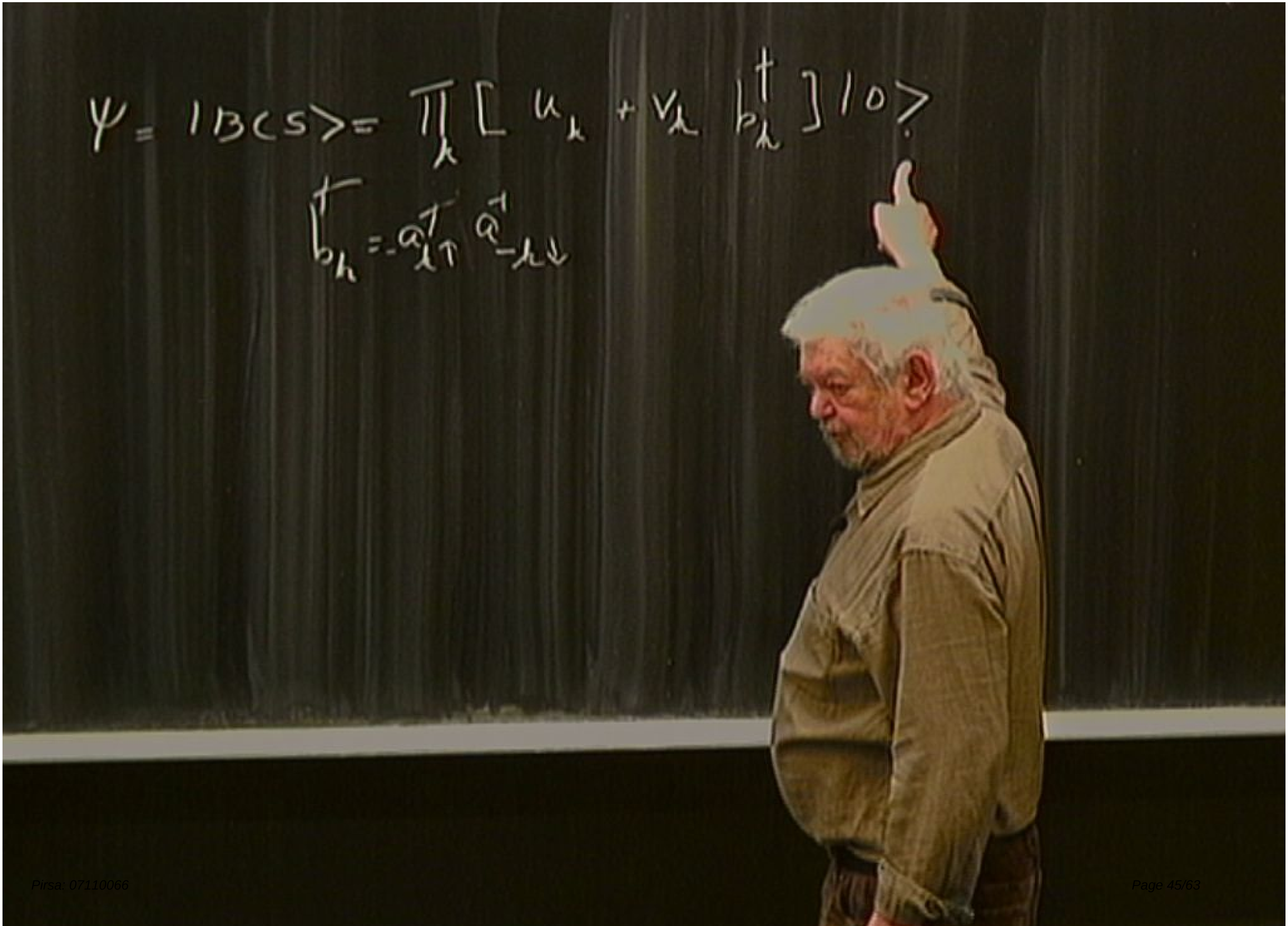
$$b_k = a_{k\uparrow} \, a_{-k\downarrow}$$

$$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$$

$$b_k^\dagger = a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger$$



$\Psi = |BCS\rangle = \prod_k [\, u_k + v_k \, b_k^\dagger \,] |0\rangle$
$b_k^\dagger = -a^\dagger_{k\uparrow} \, a^\dagger_{-k\downarrow}$

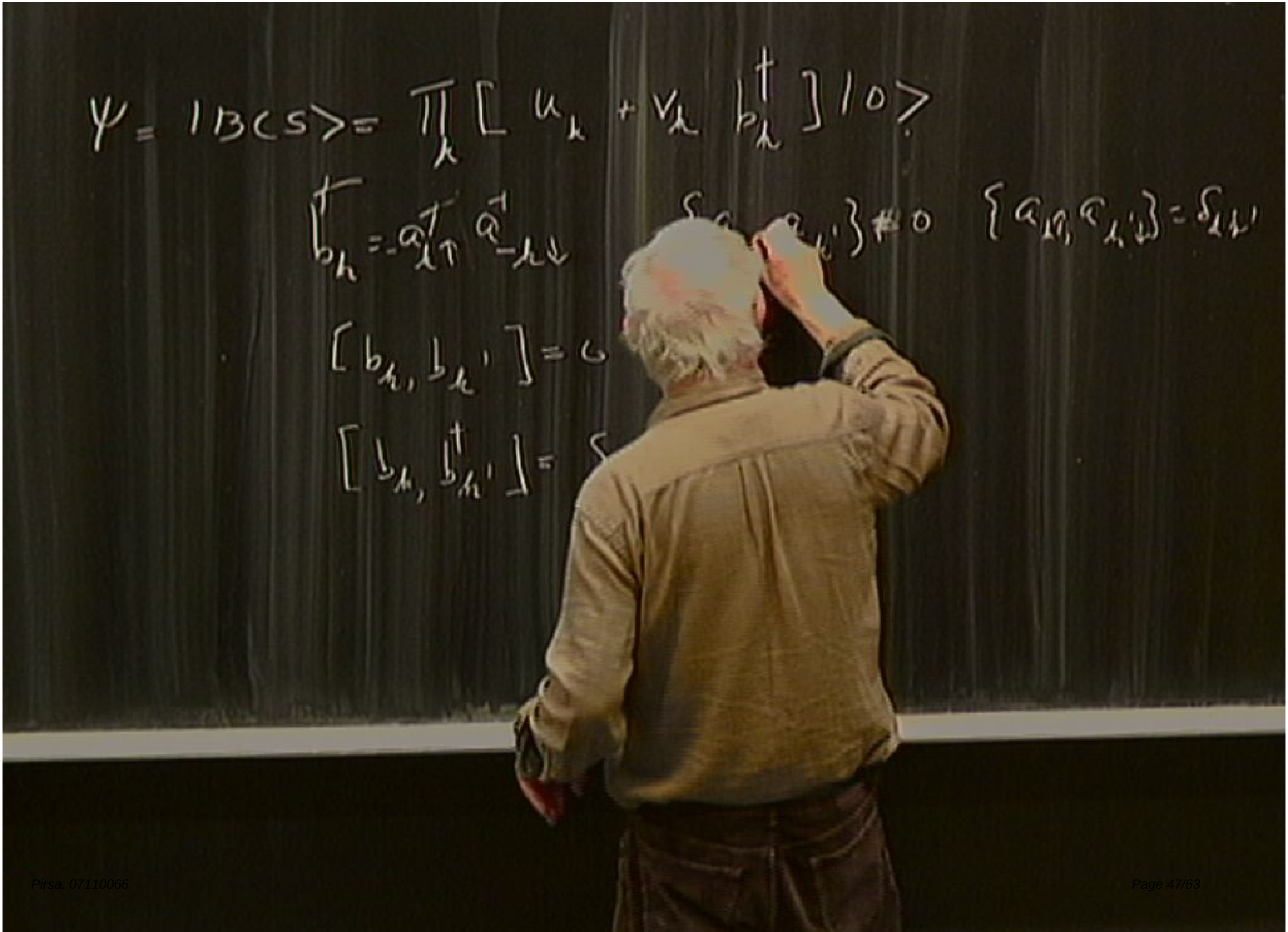

$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$
$b_k^\dagger = a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger$

$$\Psi = |BCS\rangle = \prod_k \left[u_k + v_k \, b_k^\dagger \right] |0\rangle$$

$$b_k^\dagger = a_{k\uparrow}^\dagger \, a_{-k\downarrow}^\dagger$$

$$[\, b_k , b_{k'} \,] = 0$$

$$[\, b_k , b_{k'}^\dagger \,] =$$

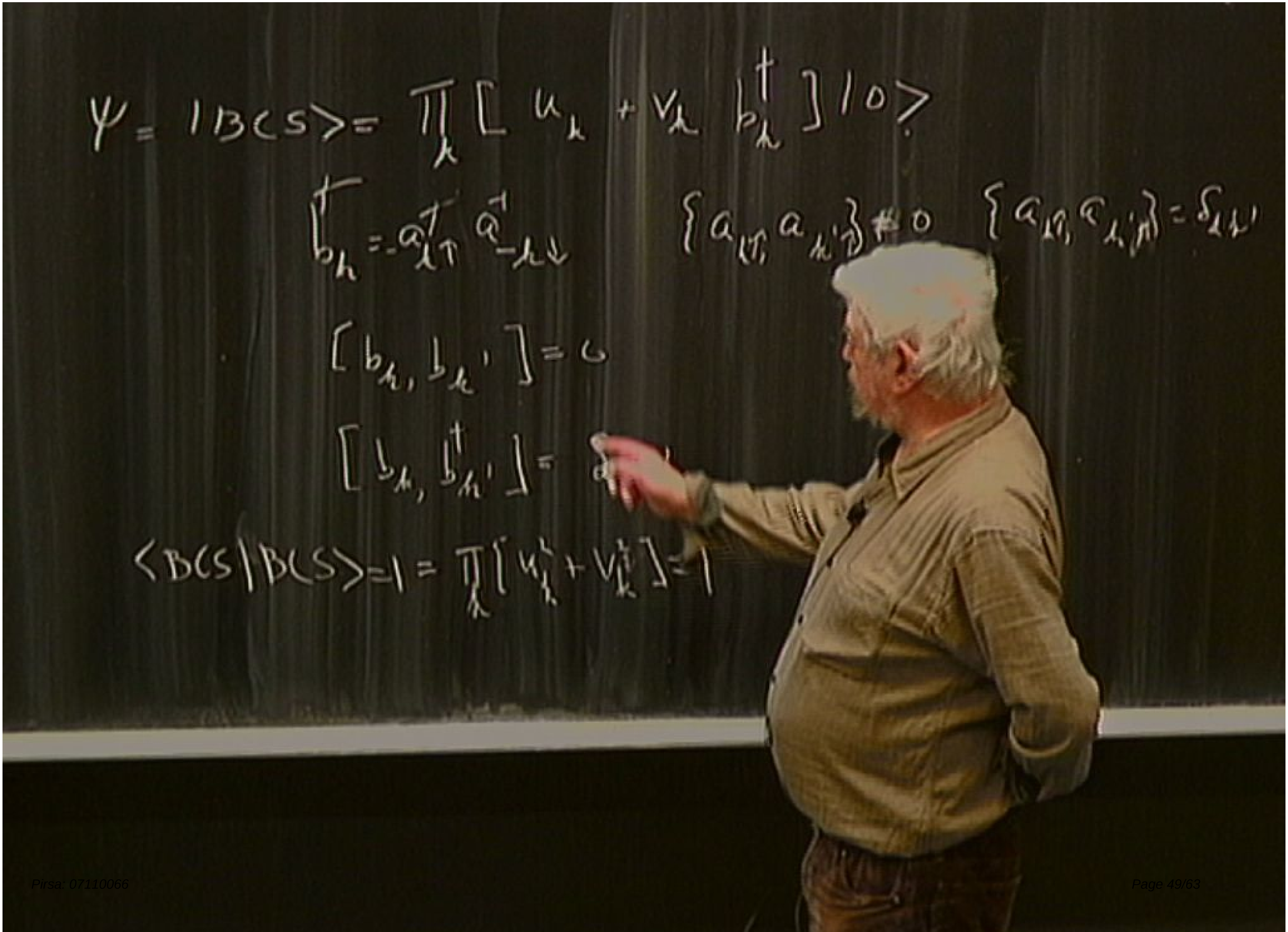

ψ = |BCS> = ∏_k [u_k + v_k b_k†] |0>
b_k† = a_k↑† a_-k↓†
≠ 0
{a_kσ, a_k'σ'†} = δ_kk'
[b_k, b_k'] = 0
[b_k, b_k'†] =

$$\Psi = |BCS\rangle = \prod_k \left[u_k + v_k \, b_k^\dagger \right] |0\rangle$$

$$b_k^\dagger = a_{k\uparrow}^\dagger \, a_{-k\downarrow}^\dagger \qquad \{a_{k\uparrow}, a_{k'\uparrow}\} = 0 \qquad \{a_{k\uparrow}, a_{k'\uparrow}^\dagger\} = \delta_{kk'}$$

$$[b_k, b_{k'}] = 0$$

$$[b_k, b_{k'}^\dagger] = \delta_{kk'}$$


Ψ = |BCS> = ∏_k [u_k + v_k b_k†] |0>
b_k† = -a_k↑† a_-k↓†
{a_kσ, a_k'σ'} = 0
{a_kσ, a_k'σ'†} = δ_kk'
[b_k, b_k'] = 0
[b_k, b_k'†] =
<BCS|BCS> = 1 = ∏_k [u_k² + v_k²] = 1

$$\Psi = |BCS\rangle = \prod_k [\, u_k + v_k \, b_k^\dagger \,] |0\rangle$$

$$b_k^\dagger = a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow} \qquad \{a_{k\sigma}, a_{k'\sigma'}\} = 0 \qquad \{a_{k\sigma}, a^\dagger_{k'\sigma'}\} = \delta_{kk'}$$

$$[b_k, b_{k'}] = 0$$

$$[b_k, b^\dagger_{k'}] = \delta_{kk'}$$

$$\langle BCS|BCS\rangle = 1 = \prod_k [u_k^2 + v_k^2] = 1$$

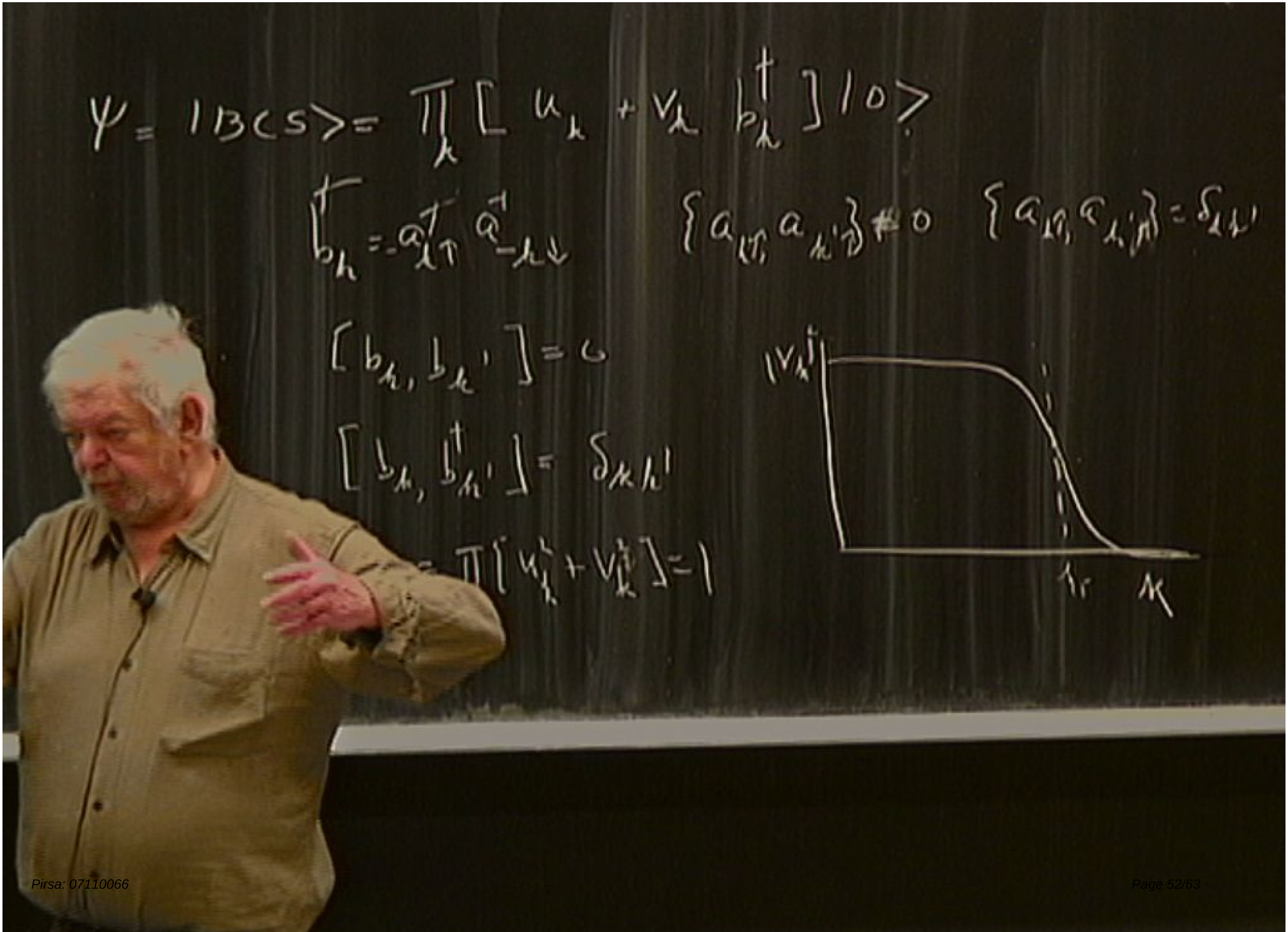
$$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$$

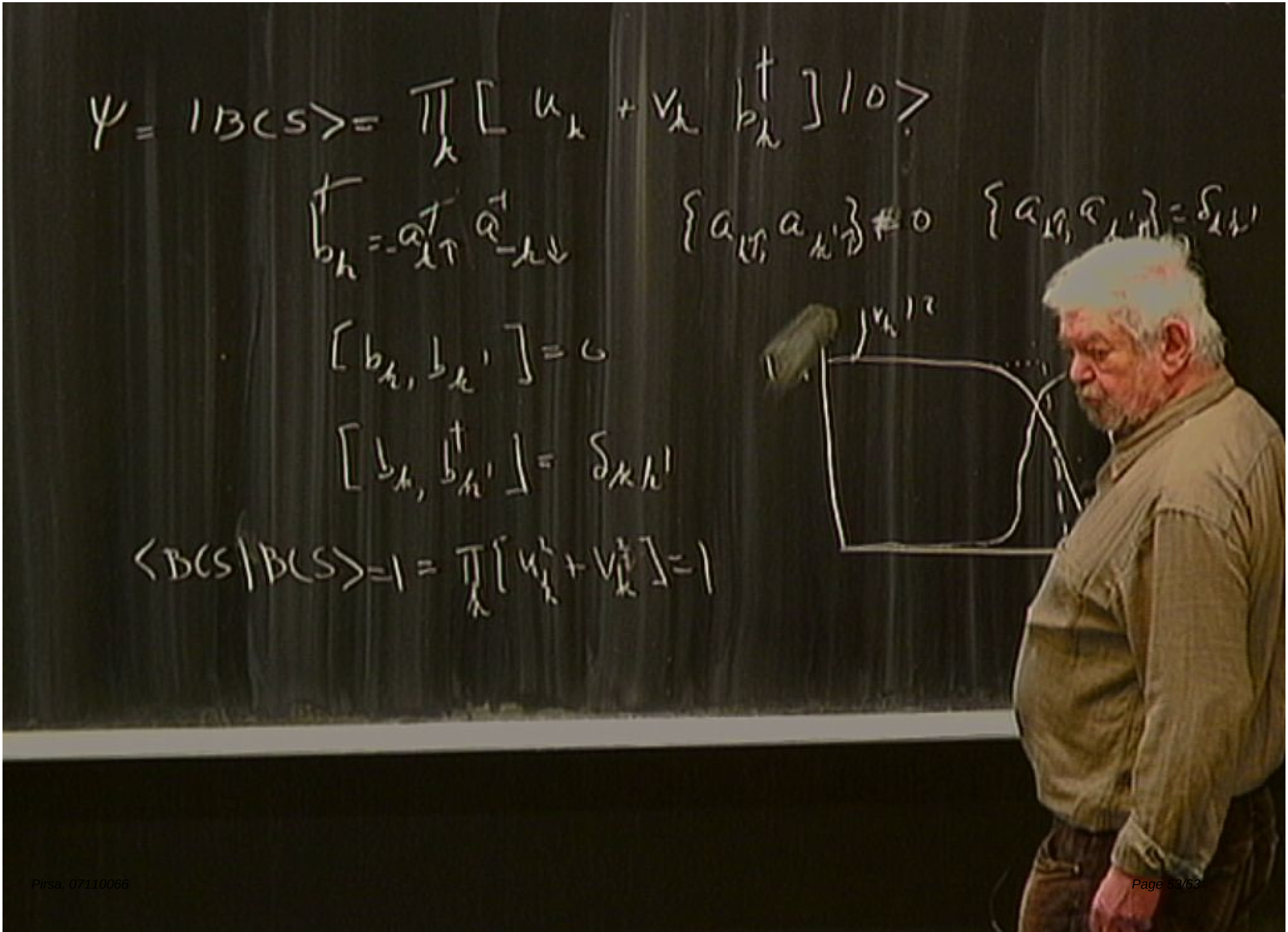
$$b_k^\dagger = a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger \qquad \{a_{k\uparrow}, a_{k'\uparrow}\} \neq 0 \qquad \{a_{k\uparrow}, a_{k'\uparrow}^\dagger\} = \delta_{kk'}$$

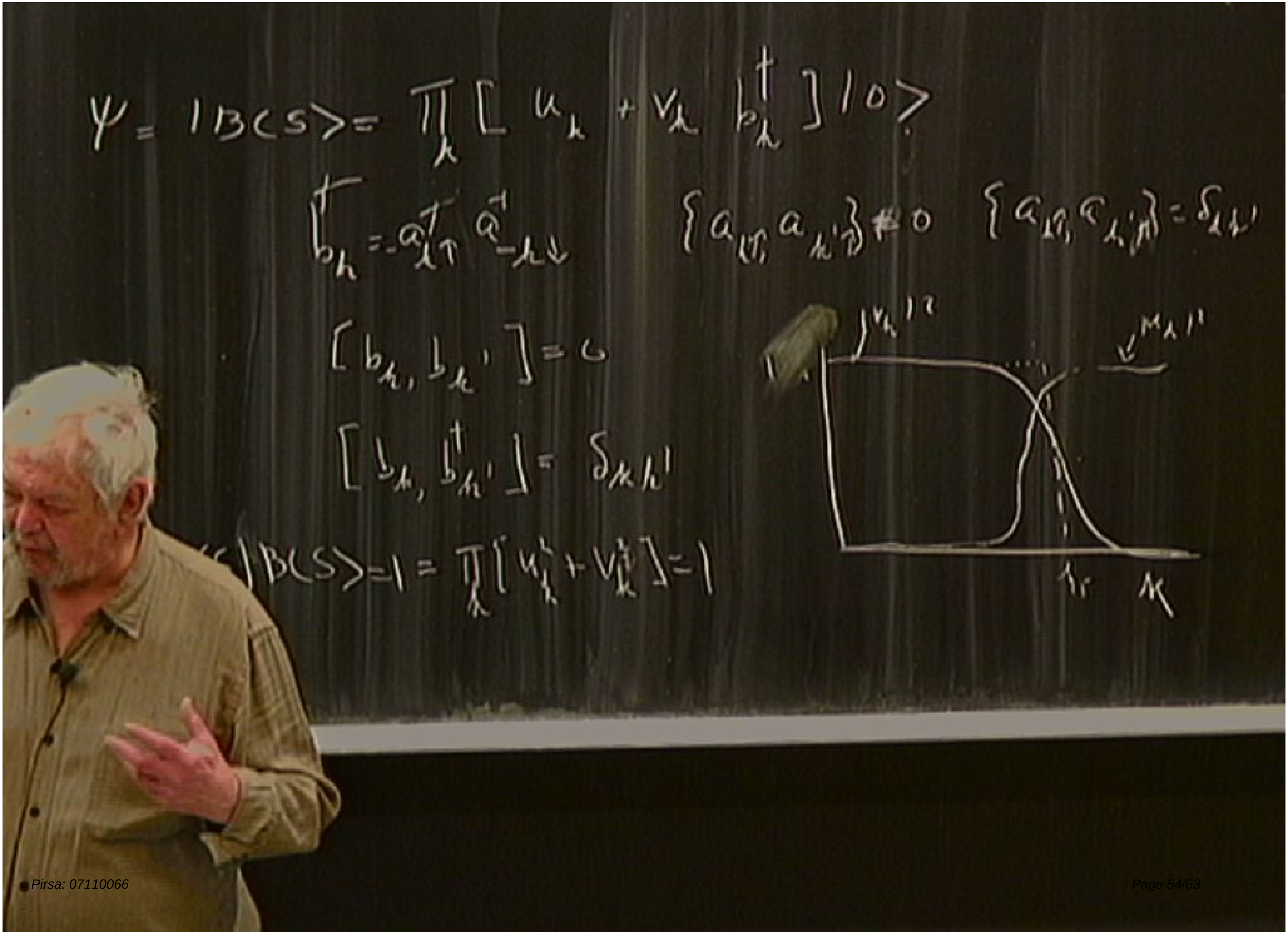
$$[b_k, b_{k'}] = 0$$

$$[b_k, b_{k'}^\dagger] = \delta_{kk'}$$

$$\langle BCS | BCS\rangle = 1 = \prod_k [u_k^2 + v_k^2] = 1$$


$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$
$b_k^\dagger = -a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow}$
$\{a_{k\uparrow}, a_{k'\uparrow}\} = 0$
$\{a_{k\uparrow}, a^\dagger_{k'\uparrow}\} = \delta_{kk'}$
$[b_k, b_{k'}] = 0$
$[b_k, b^\dagger_{k'}] = \delta_{kk'}$
$\prod [u_k^2 + v_k^2] = 1$
$|v_k|^2$
k_F
k


$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$
$b_k^\dagger = a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger$
$\{a_{k\uparrow}, a_{k'\uparrow}\} = 0$
$\{a_{k\uparrow}, a_{k'\uparrow}^\dagger\} = \delta_{kk'}$
$[b_k, b_{k'}] = 0$
$[b_k, b_{k'}^\dagger] = \delta_{kk'}$
$\langle BCS|BCS\rangle = 1 = \prod_k [u_k^2 + v_k^2] = 1$


$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$
$b_k^\dagger = a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger$
$\{a_{k\uparrow}, a_{k'\uparrow}\} = 0$
$\{a_{k\uparrow}, a_{k'\uparrow}^\dagger\} = \delta_{kk'}$
$[b_k, b_{k'}] = 0$
$[b_k, b_{k'}^\dagger] = \delta_{kk'}$
$\langle BCS|BCS\rangle = 1 = \prod_k [u_k^2 + v_k^2] = 1$
$|v_k|^2$
$|u_k|^2$
k_F
k

$$\Psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$$

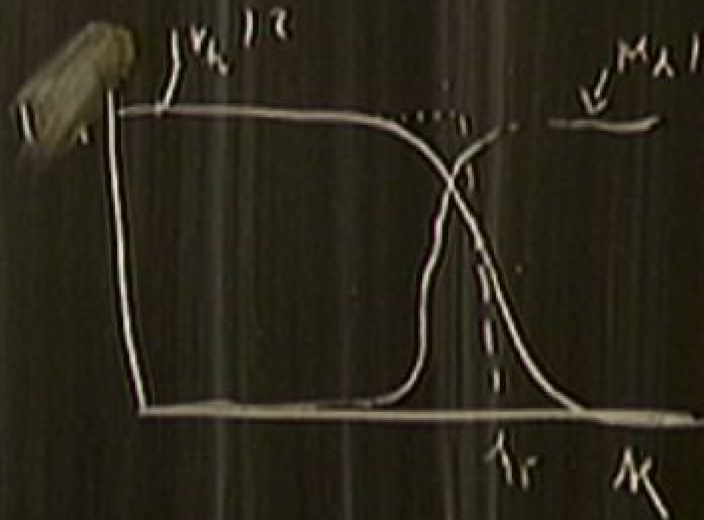
$$b_k^\dagger = a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger$$

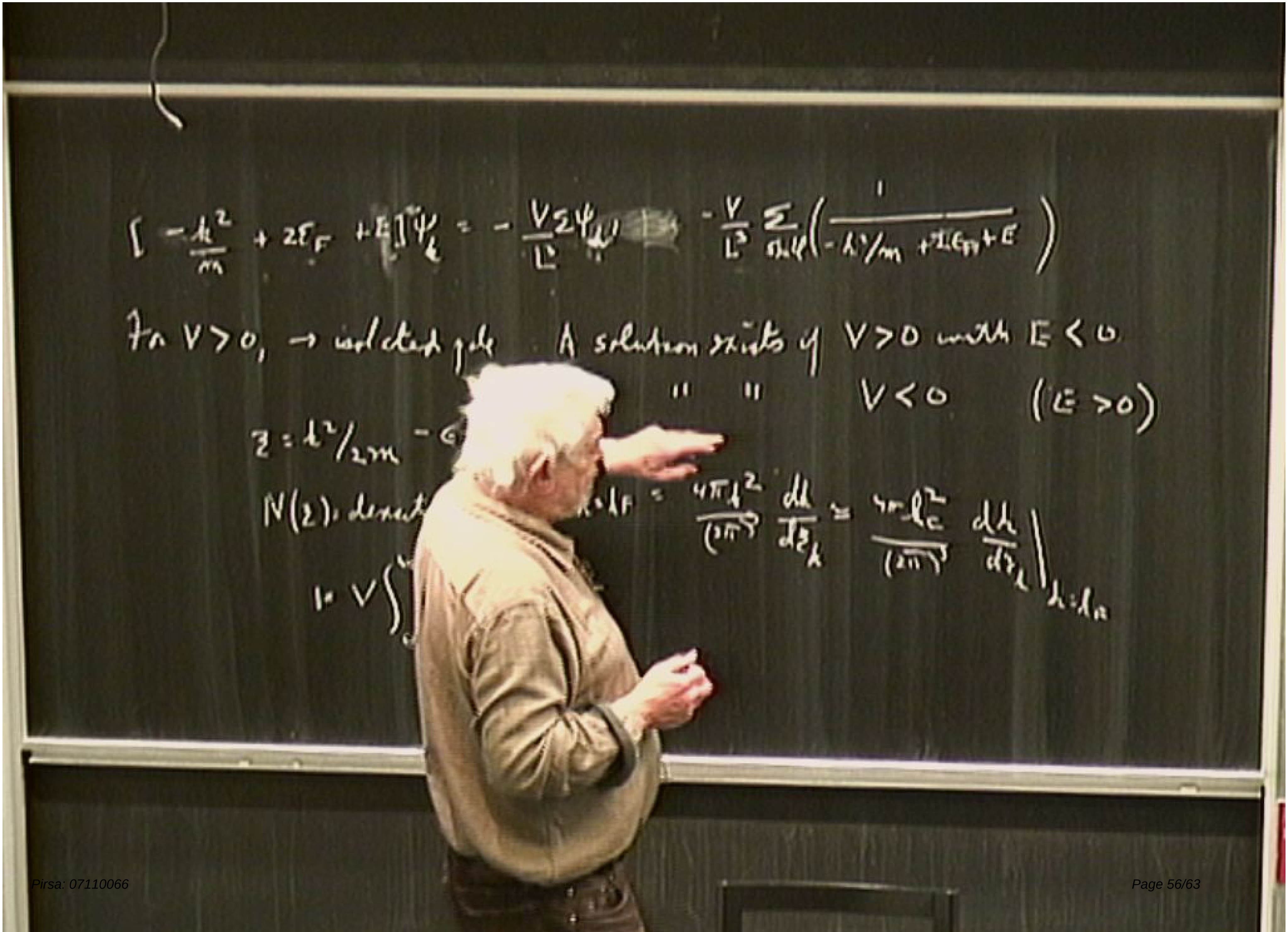
$$\{a_{k\uparrow}, a_{k'\uparrow}\} = 0 \qquad \{a_{k\uparrow}, a^\dagger_{k'\uparrow}\} = \delta_{kk'}$$

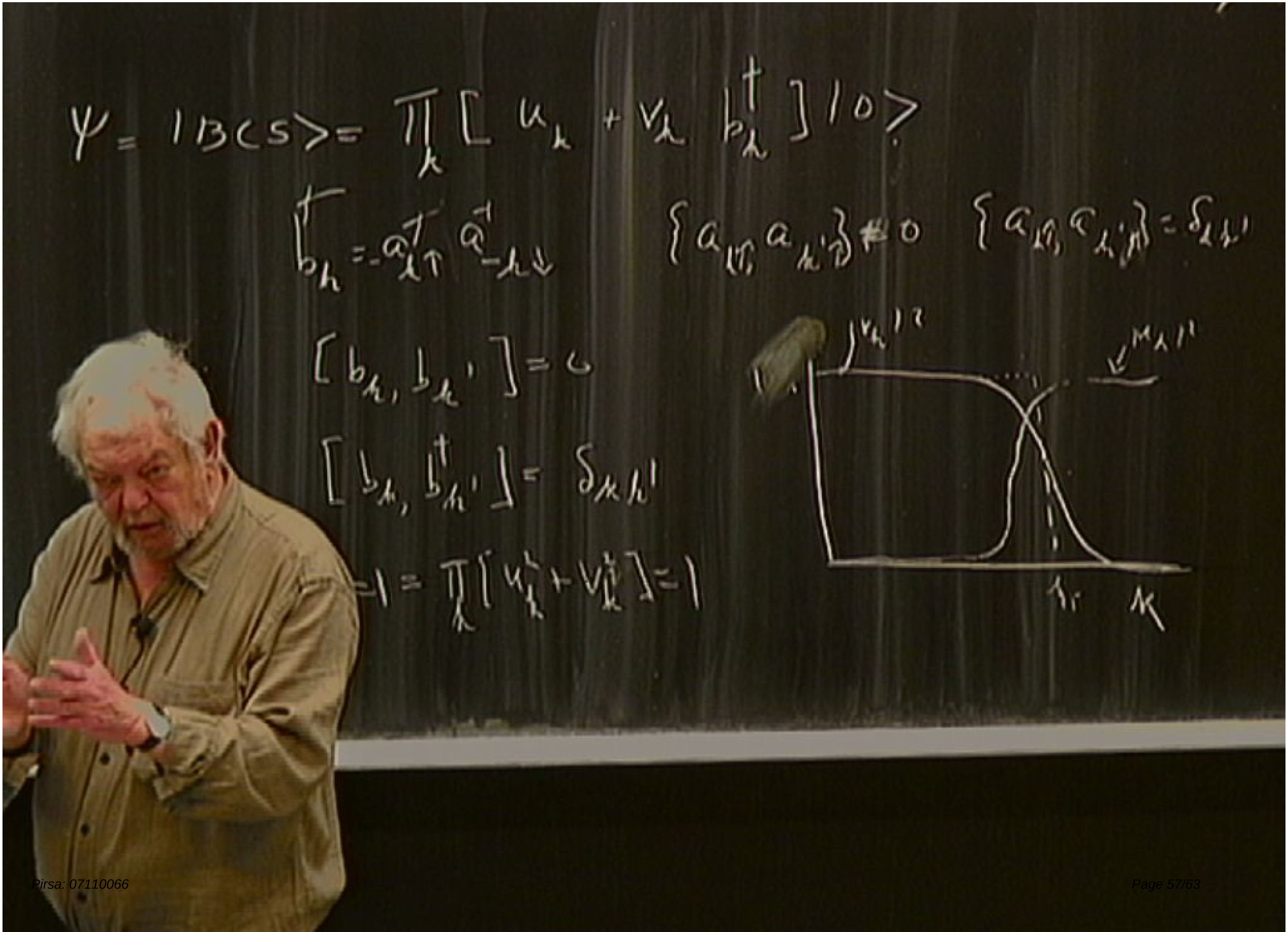
$$[b_k, b_{k'}] = 0$$

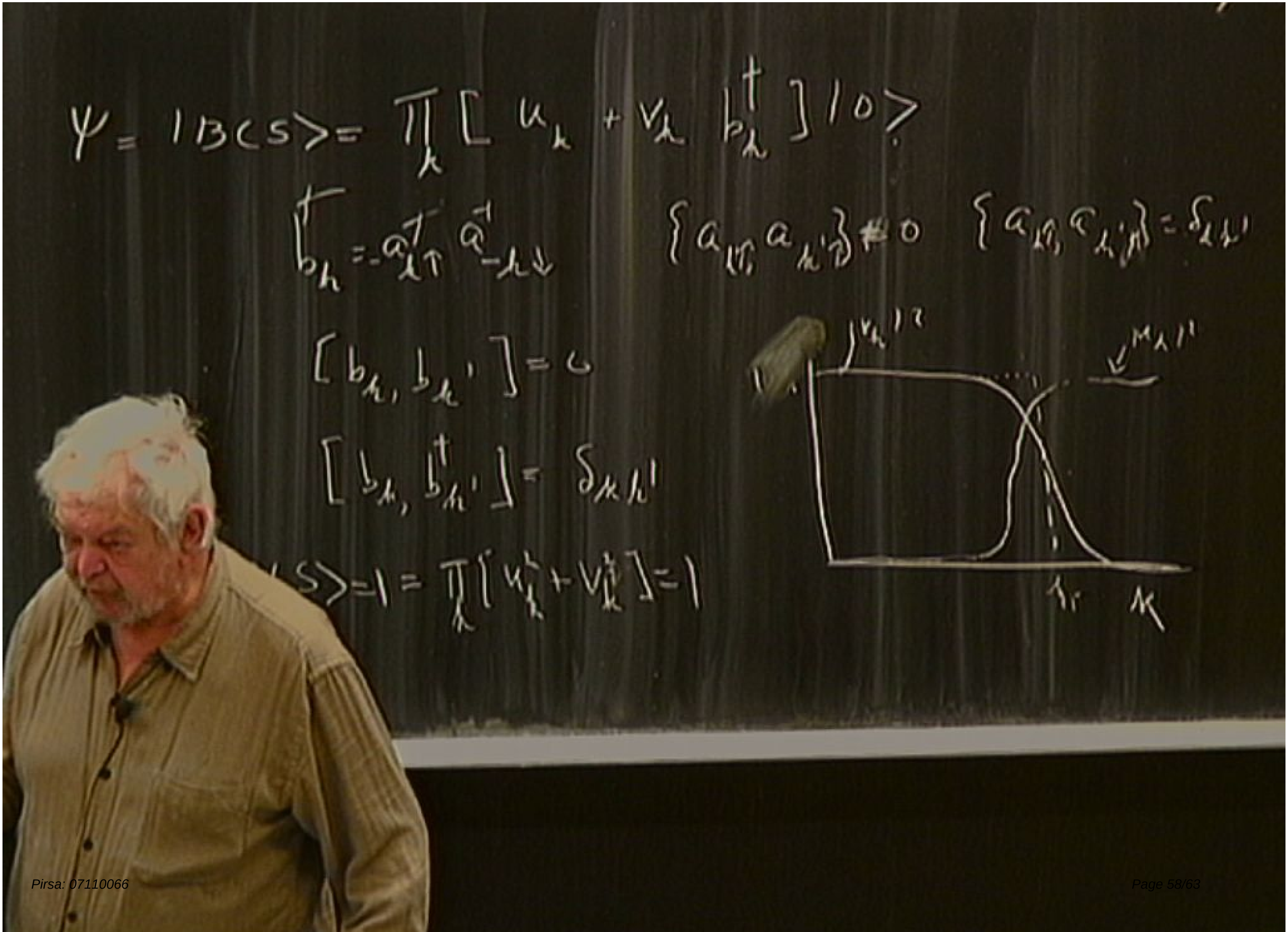
$$[b_k, b_{k'}^\dagger] = \delta_{kk'}$$

$$\langle BCS|BCS\rangle = 1 = \prod_k [u_k^2 + v_k^2] = 1$$

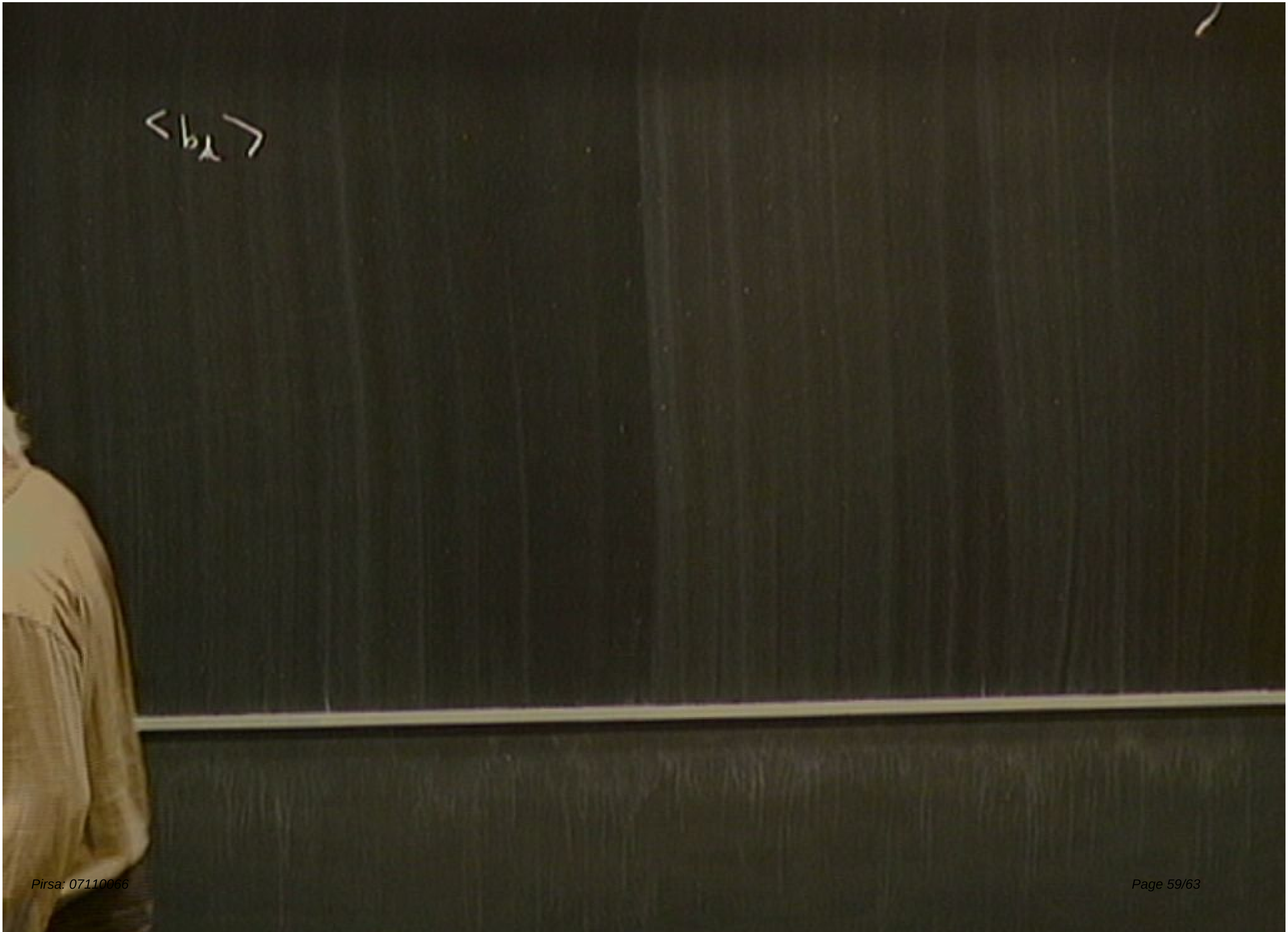


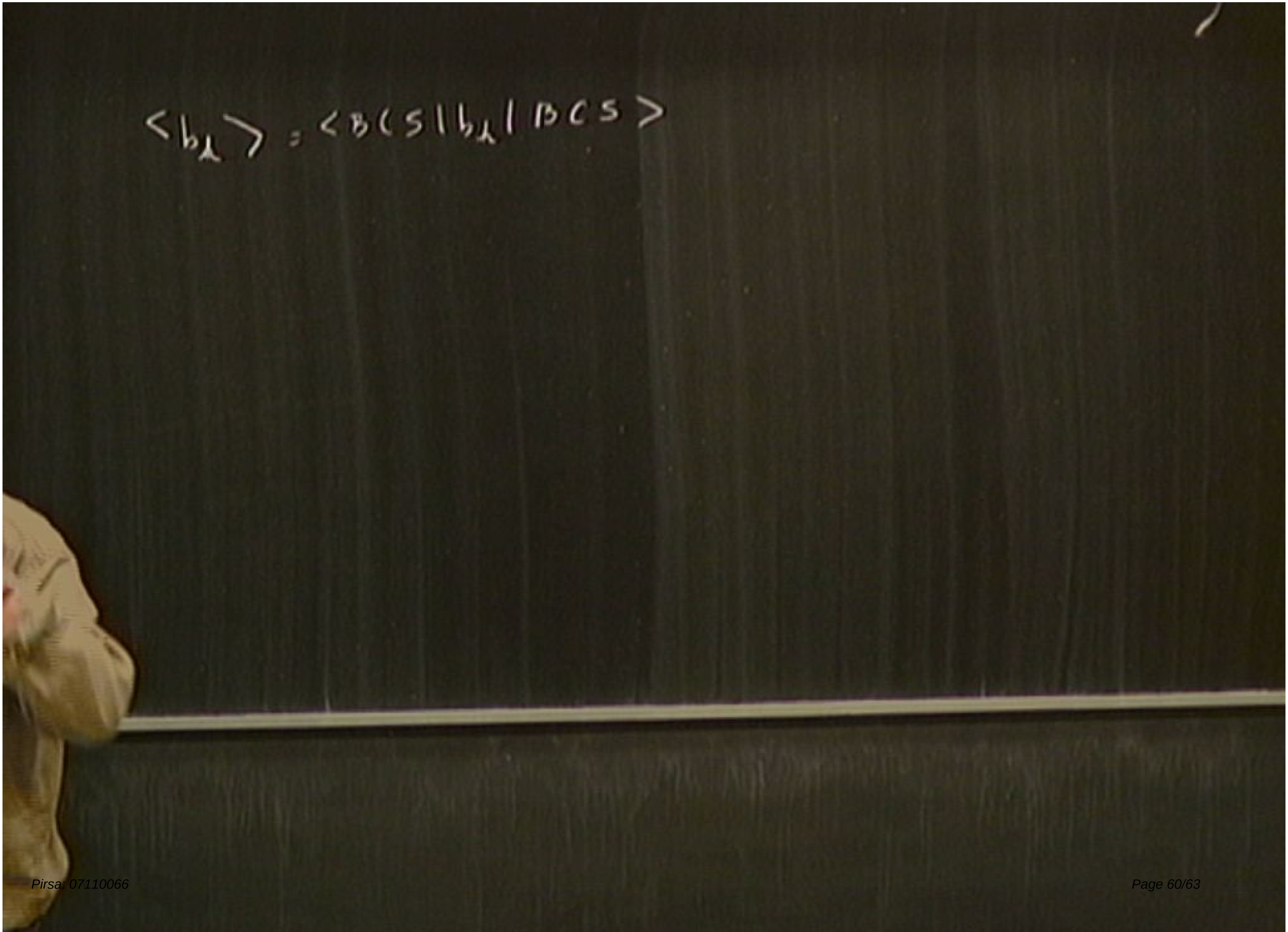

[-k²/m + 2E_F + E] ψ_k = - V/L³ Σ ψ_k' - V/L³ Σ (1 / (-k²/m + 2E_F + E))
For V > 0, → isolated pole
A solution exists if V > 0 with E < 0
" " V < 0 (E > 0)
ξ = k²/2m
N(ξ) = 4πk²/(2π)³ dk/dξ_k ≈ 4πk_F²/(2π)³ dk/dξ_k |k=k_F
1 = V ∫


$\psi = |BCS\rangle = \prod_k [u_k + v_k b_k^\dagger] |0\rangle$
$b_k^\dagger = a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger$
$\{a_{k\sigma}, a_{k'\sigma'}\} = 0$
$\{a_{k\sigma}, a^\dagger_{k'\sigma'}\} = \delta_{kk'}$
$[b_k, b_{k'}] = 0$
$[b_k, b_{k'}^\dagger] = \delta_{kk'}$
$= 1 = \prod_k [u_k^2 + v_k^2] = 1$
$|v_k|^2$
$|u_k|^2$
k_F
k



Ψ = |BCS> = ∏ₖ [uₖ + vₖ bₖ†] |0>
bₖ† = a†ₖ↑ a†₋ₖ↓
[bₖ, bₖ'] = 0
[bₖ, bₖ'†] = δₖₖ'


$\langle h_\lambda \rangle$


$\langle b_\lambda \rangle = \langle BCS | b_\lambda | BCS \rangle$

$$\langle b_k \rangle = \langle BCS | b_k | BCS \rangle$$

$$= u_k v_k .$$

$$\langle b_k \rangle = \langle BCS | b_k | BCS \rangle$$

$$= u_k v_k .$$

Violates conservation of N

$$\langle b_k \rangle = \langle BCS | b_k | BCS \rangle$$

$$= u_k v_k .$$

Violates conservation of N

$|BCS\rangle$ is not an e.f. of $N \; (= \sum a^\dagger_k a_k$