Title: Spontaneous Broken Symmetry 3B

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Abstract:

Low energy excitations in the quantum theory.

Heisenberg model $H = -\frac{1}{2} \sum J_{ij} [\vec{\sigma}_i \cdot \vec{\sigma}_j]$

spin $\frac{1}{2}$

$\vec{\sigma}_i = 2\vec{S}_i$ = Pauli matrices

Orbital w.f. of C

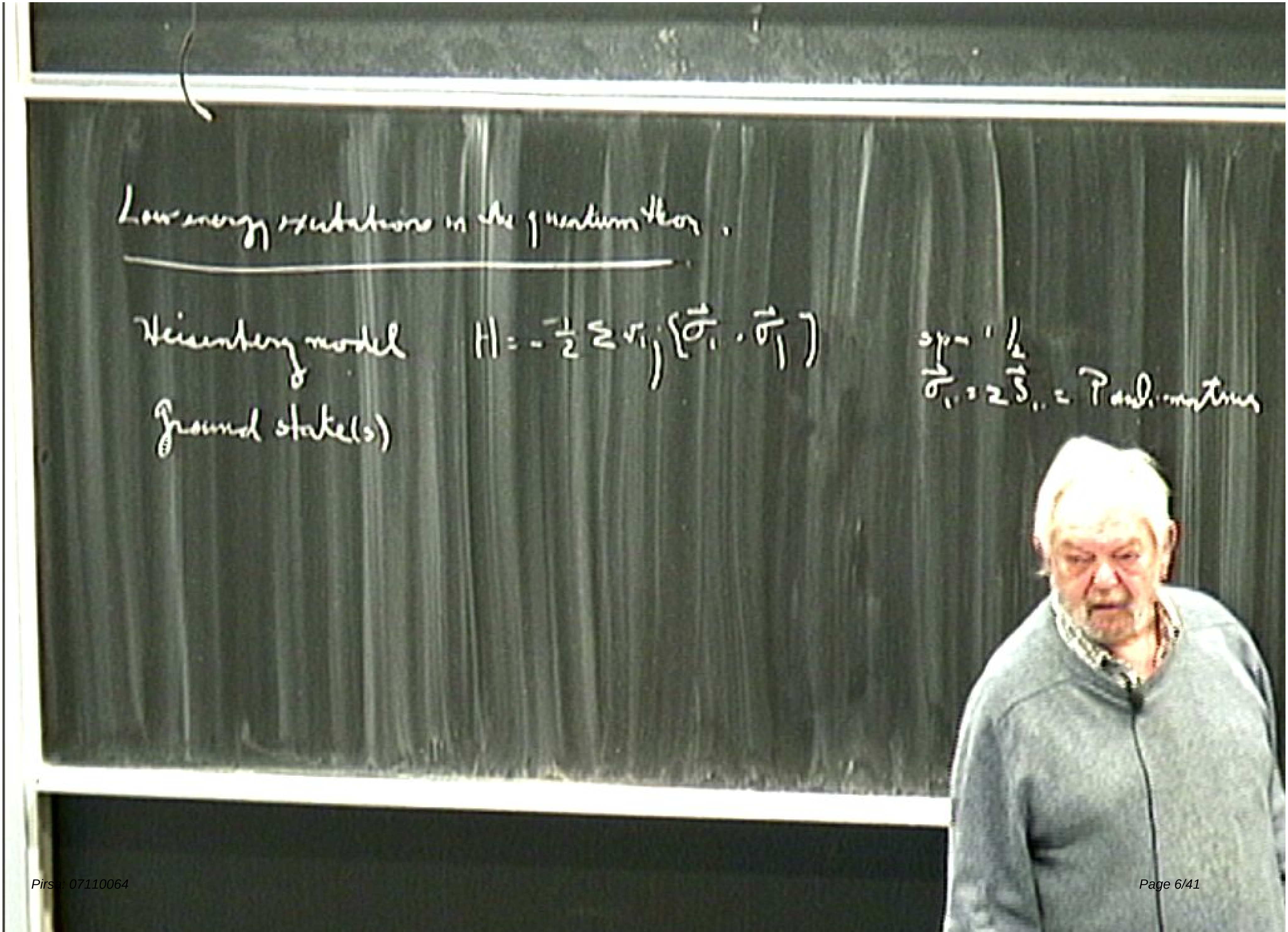
$$\left[p_x(r_1)\, p_y(r_2) - p_x(r_2)\, p_y(r_1) \right]$$

Orthonorm. of C

$$\left[P_x(n_1) P_y(n_2) - P_x(n_2) P_y(n_1) \right] \left[< \alpha \right]$$

Orbital wf. of 1C

$$\left[p_x(r_1)\, p_y(r_2) - p_x(r_2)\, p_y(r_1) \right] \begin{bmatrix} \alpha\alpha \\ \alpha\beta + \beta\alpha \\ \beta\beta \end{bmatrix}$$


Low energy excitations in the quantum theory.
Heisenberg model
H = - ½ Σ J_ij [σ_i · σ_j]
spin ½
σ_i = 2 S_i = Pauli matrices
Ground state(s)

Low energy excitations in the quantum theory.

Heisenberg model $H = -\frac{1}{2}\sum J_{ij}\,[\vec{\sigma}_i \cdot \vec{\sigma}_j]$

spin $\frac{1}{2}$

$\vec{\sigma}_i = 2\vec{S}_i$ = Pauli matrices

Ground state(s)

$|0\rangle = |\text{all spins up}\rangle$ $S^z(\text{Total}) = \frac{N}{2}$

Low energy excitations in the quantum theory.

Heisenberg model $H = -\frac{1}{2}\sum_{ij} J_{ij} \{\vec{\sigma}_i \cdot \vec{\sigma}_j\}$ spin $\frac{1}{2}$

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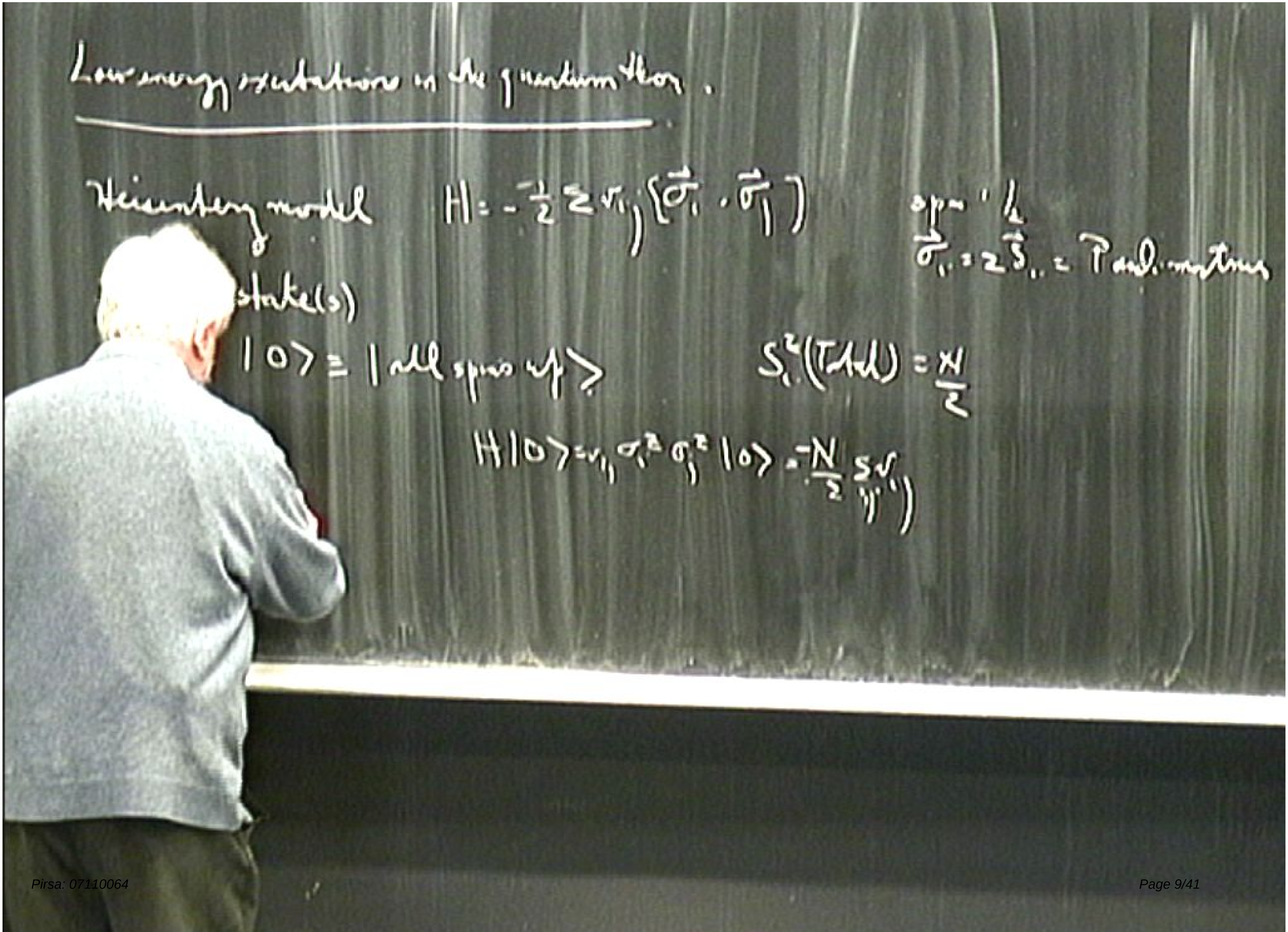
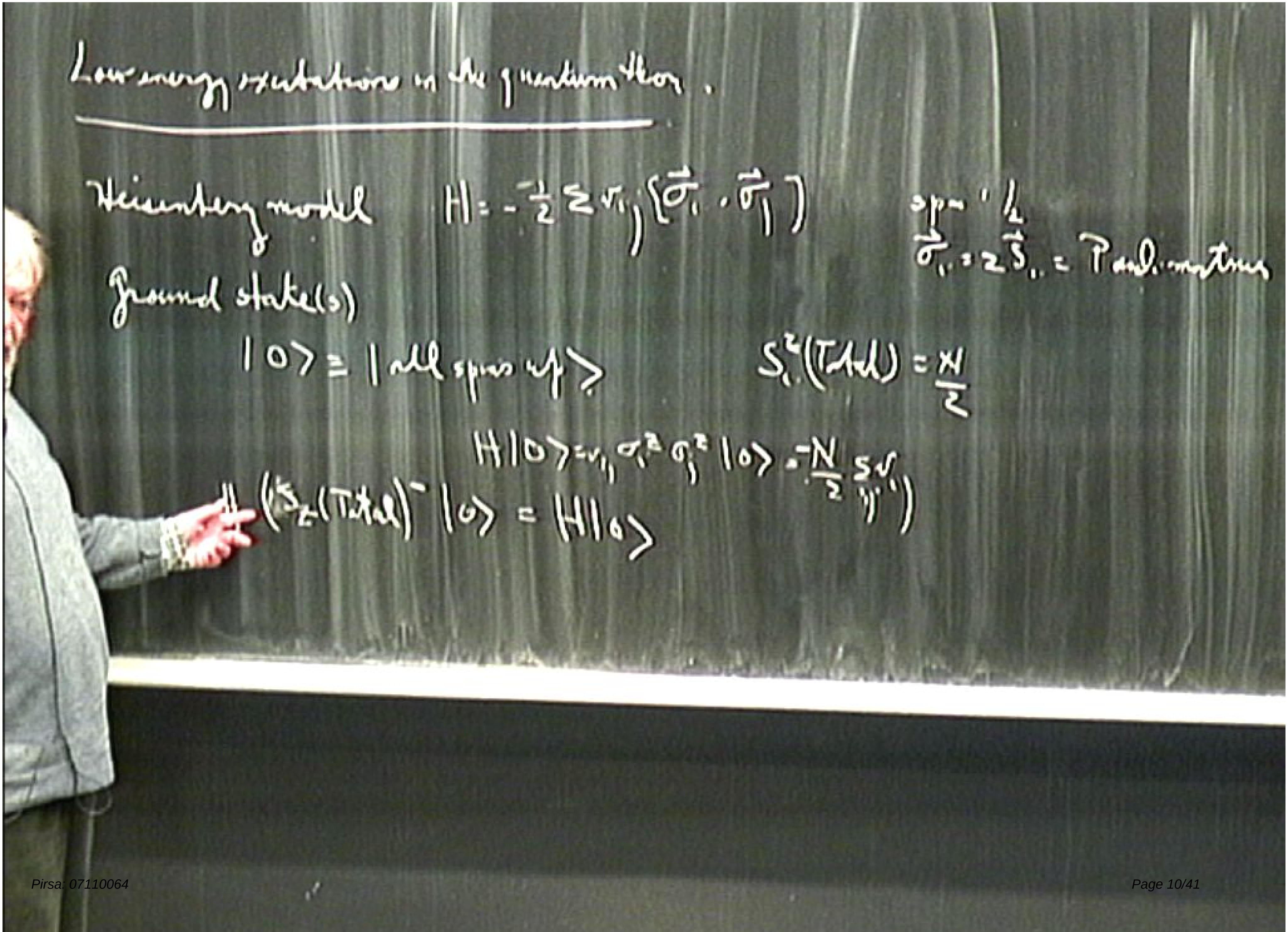
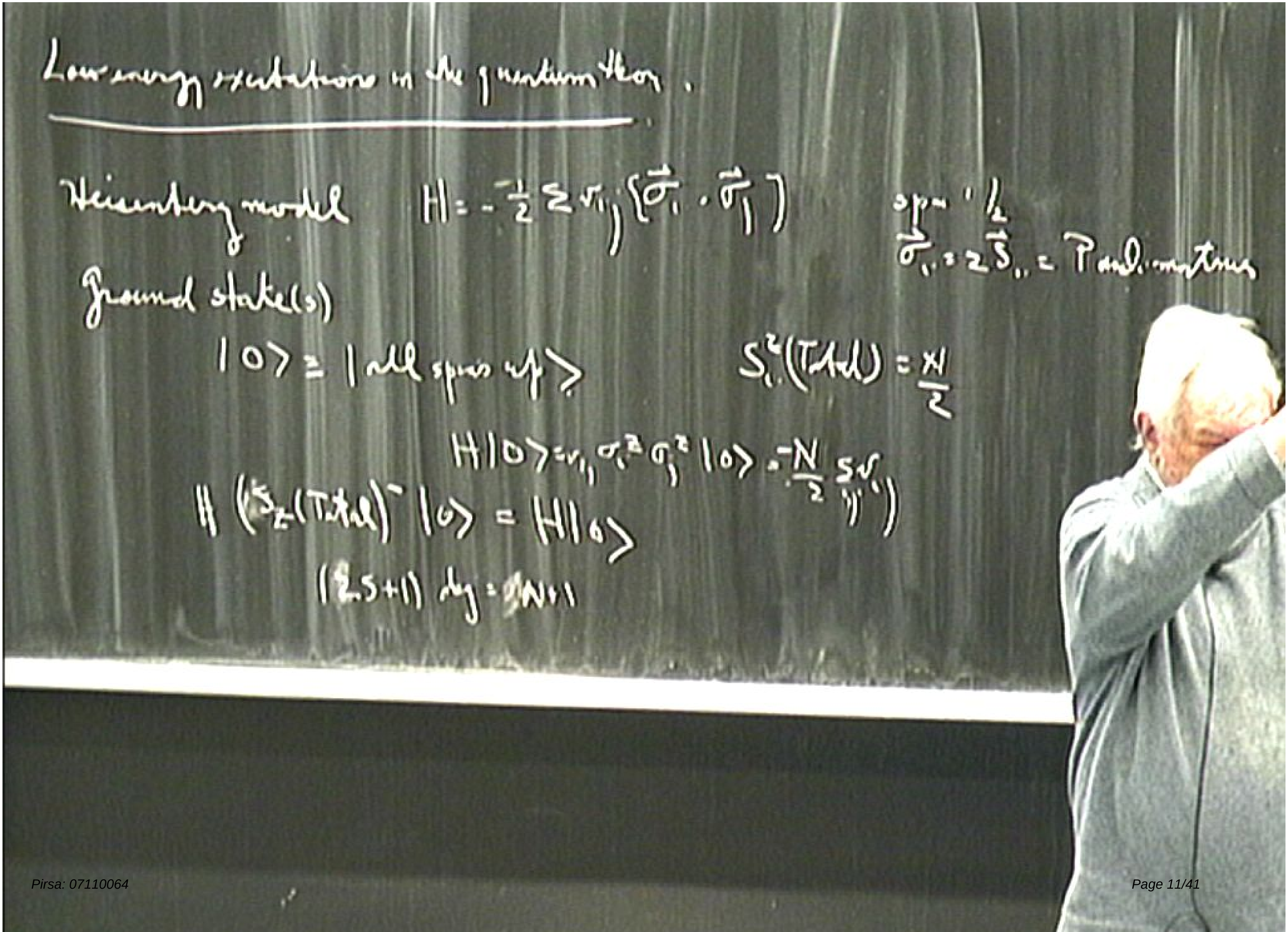

Low energy excitations in the quantum theory.
Heisenberg model $H = -\frac{1}{2}\sum J_{ij}(\vec{\sigma}_i \cdot \vec{\sigma}_j)$
spin 1/2
$\vec{\sigma}_i = 2\vec{S}_i$ = Pauli matrices
state(s)
$|0\rangle = |\text{all spins up}\rangle$
$S^z(\text{Total}) = \frac{N}{2}$
$H|0\rangle = J_{ij}\sigma_i^z\sigma_j^z|0\rangle = -\frac{N}{2}\sum J_{ij}$


Low energy excitations in the quantum theory.
Heisenberg model H = -1/2 Σ v_ij (σ_i · σ_j)
spin 1/2
σ_i = 2 S_i = Pauli matrices
Ground state(s)
|0> = |all spins up>
S_z (Total) = N/2
H|0> = v_ij σ_i^z σ_j^z |0> = -N/2 Σ v_ij
H (S_z(Total))^- |0> = H|0>


Low energy excitations in the quantum theory.
Heisenberg model
H = -1/2 Σ J_ij (σ_i · σ_j)
spin 1/2
σ_i = 2 S_i = Pauli matrices
Ground state(s)
|0> = |all spins up>
S_z(Total) = N/2
H|0> = J_ij σ_i^z σ_j^z |0> = -N/2 Σ J_ij
H (S_-(Total))^n |0> = H|0>
(2S+1) deg = N+1

Low energy excitations in the quantum theory.

Heisenberg model $H = -\frac{1}{2}\sum v_{ij} [\vec{\sigma}_i \cdot \vec{\sigma}_j]$

spin 1/2

$\vec{\sigma}_i = 2\vec{S}_i$ = Pauli matrices

Ground state(s)

$\rightarrow |0\rangle = |\text{all spins up}\rangle$ $S_z(\text{Total}) = \frac{N}{2}$

$H|0\rangle = v_{ij}\sigma_i^z \sigma_j^z |0\rangle = -\frac{N}{2}\sum_{ij} v_{ij}$)

$H\left((S_z(\text{Total}))^- |0\rangle\right) = H|0\rangle$

$(2S+1)$ deg $= N+1$

Excited States have 1 spin down.

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Try $\Psi_{esp.} = \sum a_i \sigma_i^- |0\rangle$

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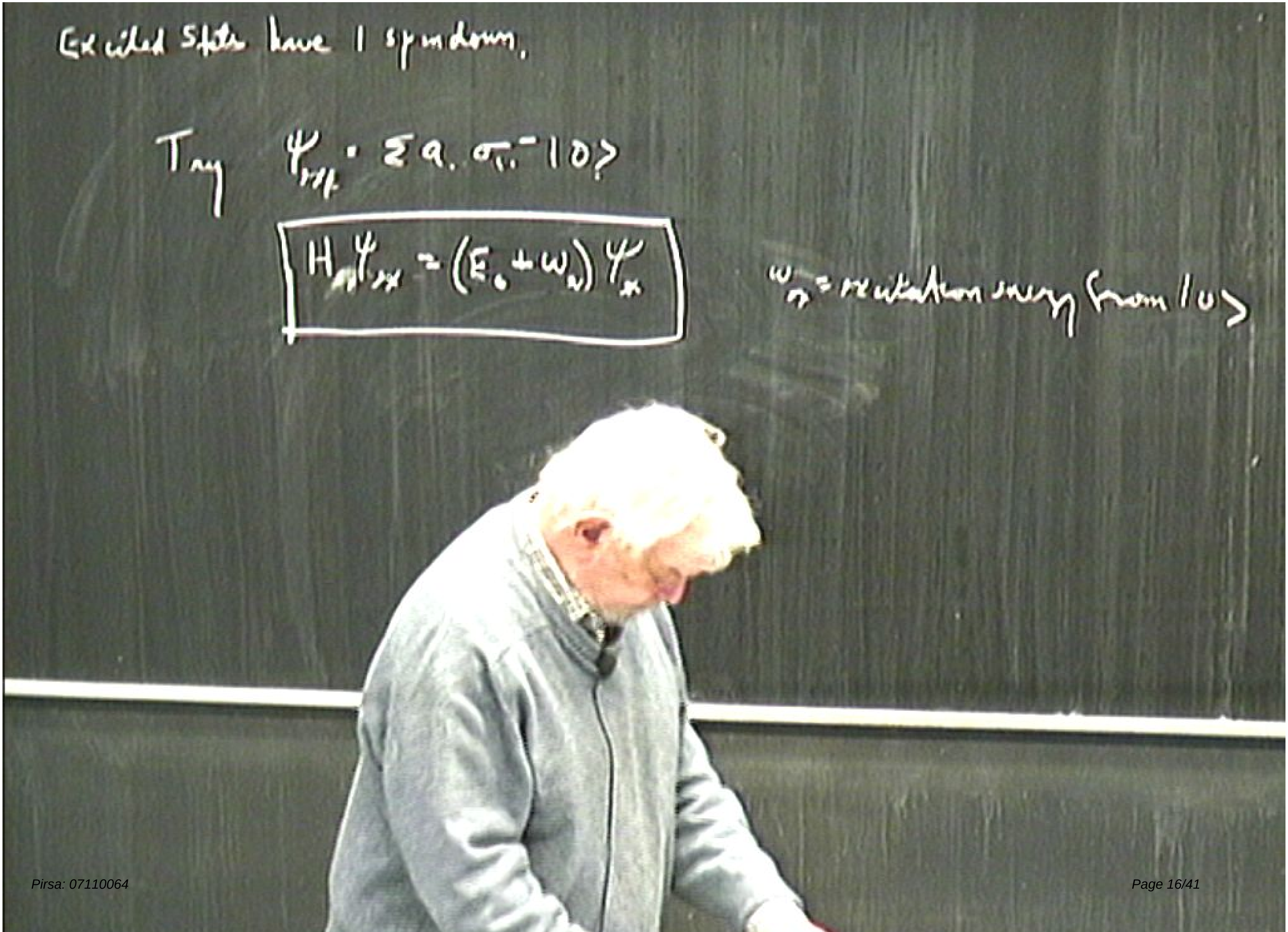
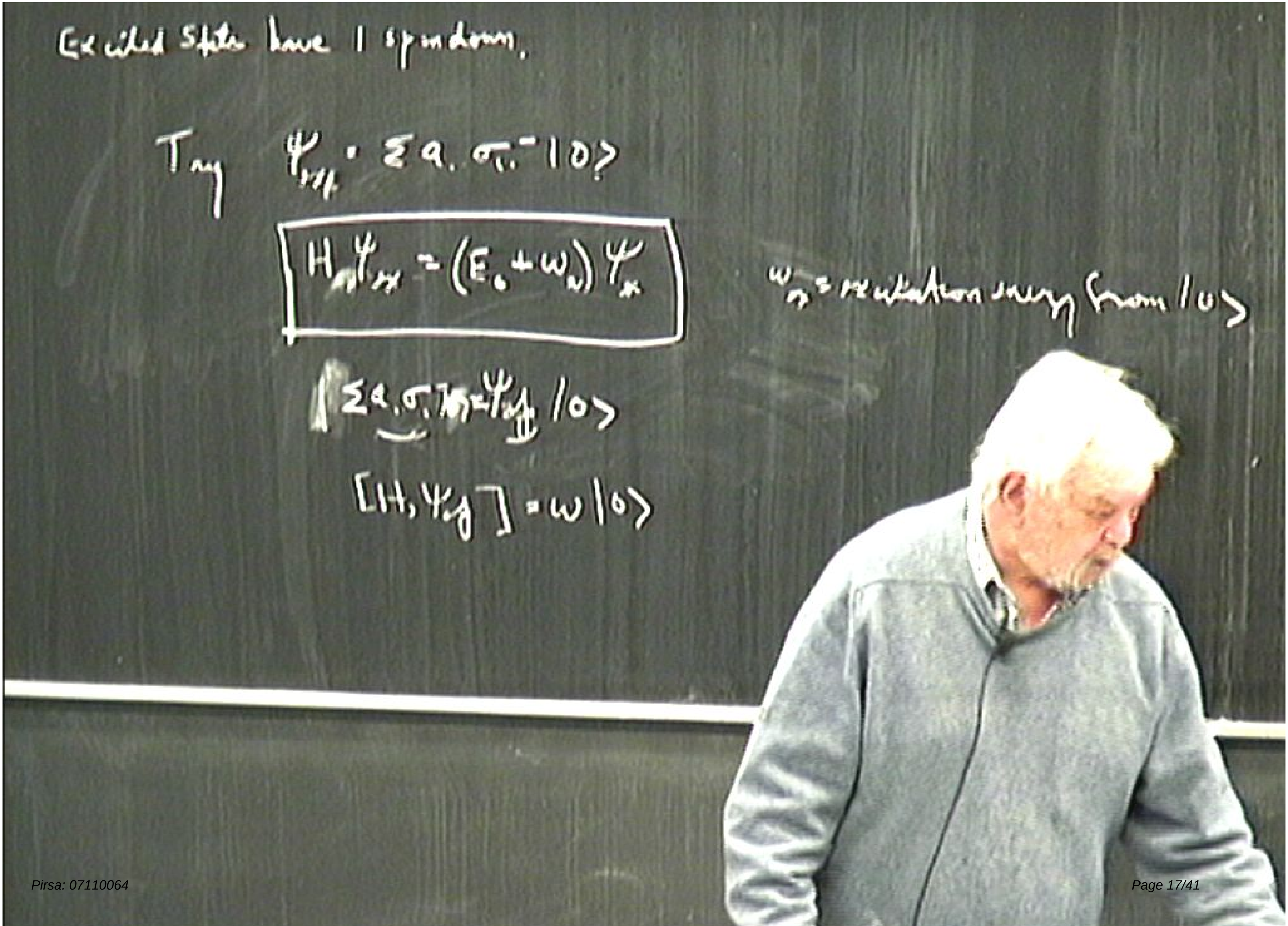
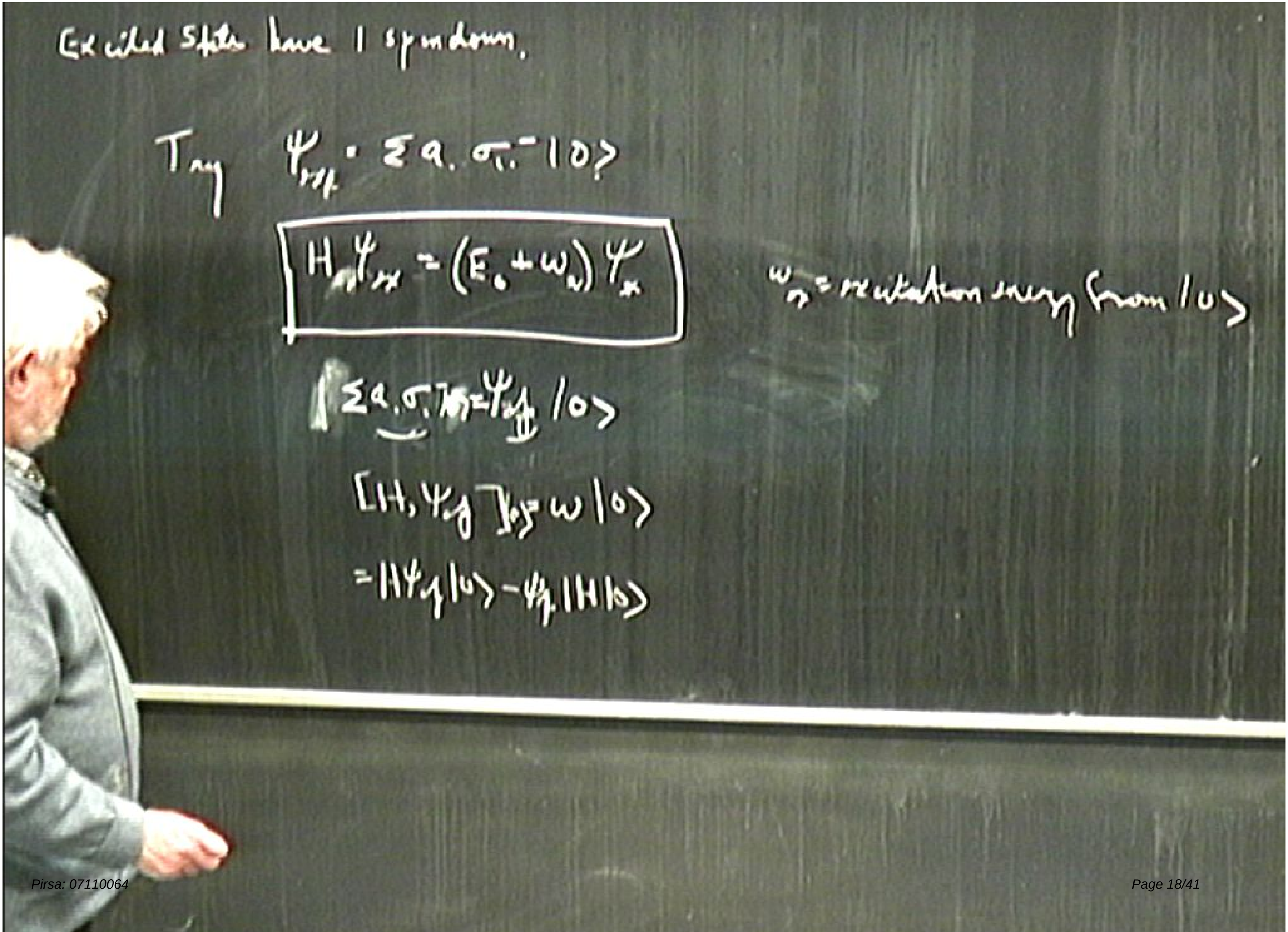
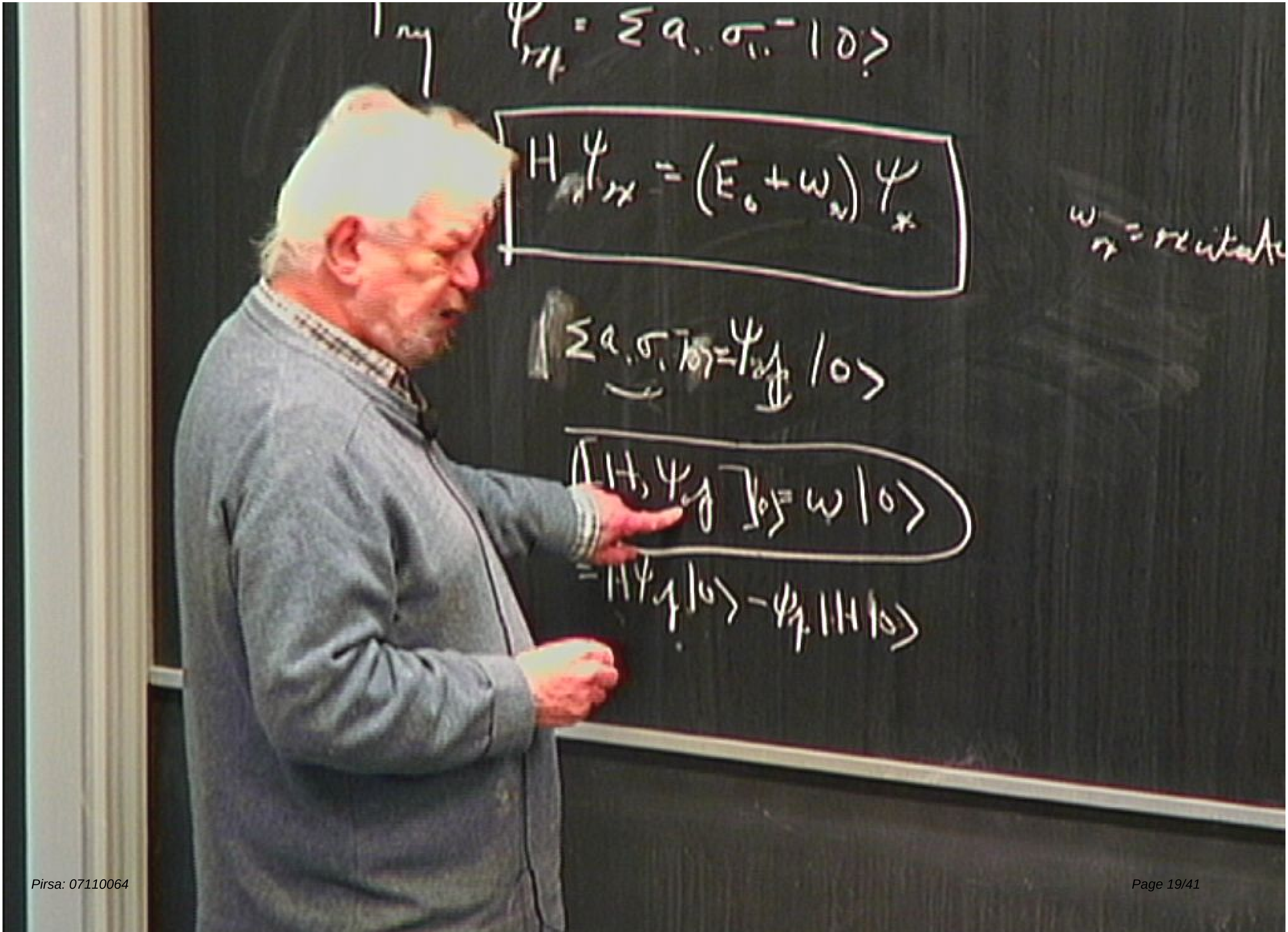
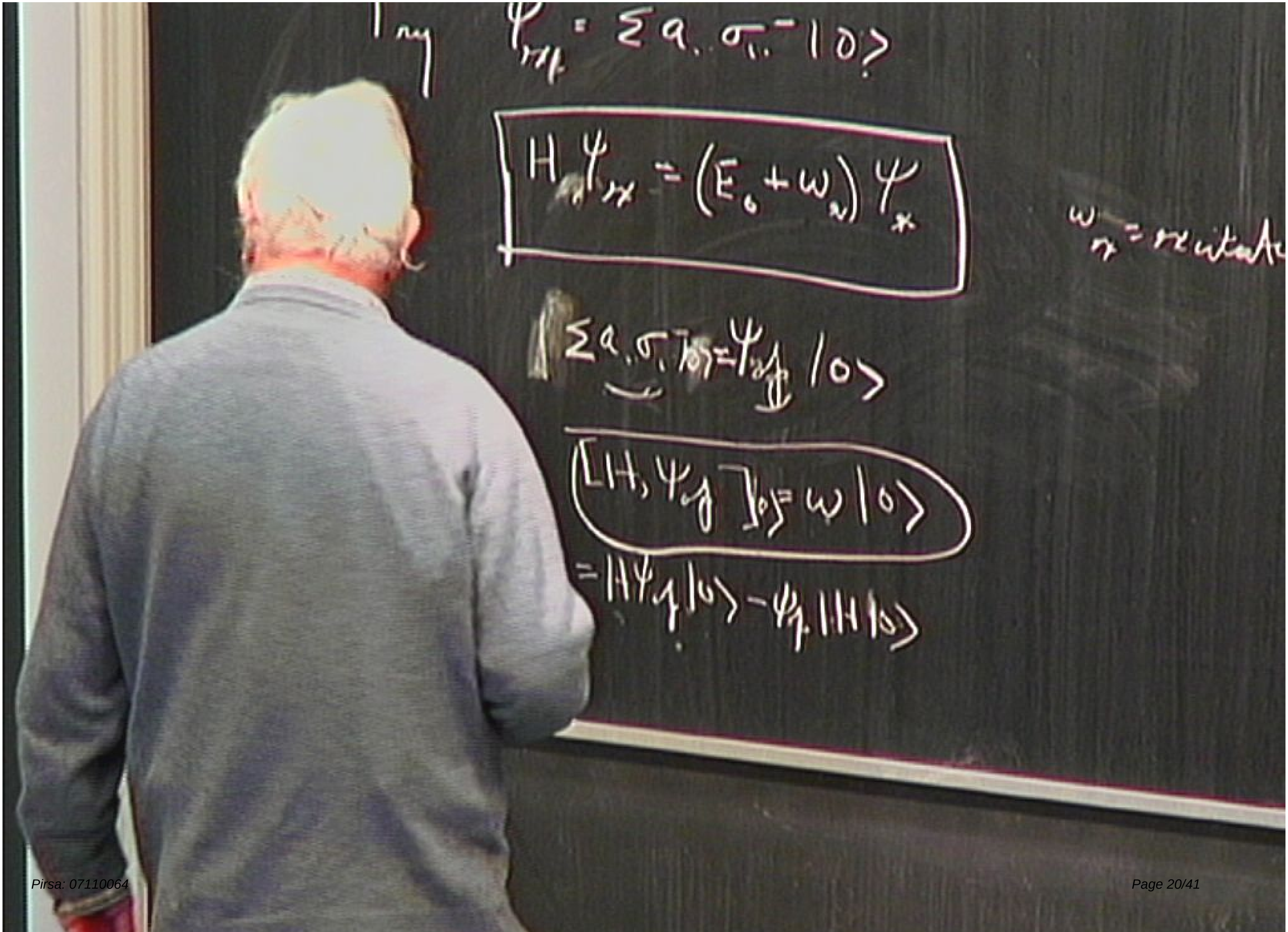
$$\text{Try} \quad \Psi_{exp} = \sum a_i \, \sigma_i^- |0\rangle$$


Excited States have 1 spin down.
Try $\psi_{exp} = \Sigma a_i \sigma_i^- |0\rangle$
$H_{xy}\psi_{ex} = (E_0 + \omega_n)\psi_{ex}$
ω_n = excitation energy from $|0\rangle$


Excited States have 1 spin down.
Try ψ_eff = Σ a_i σ_i^- |0>
H_n ψ_ex = (E_0 + ω_n) ψ_ex
ω_n = excitation energy from |0>
[H, ψ_eff] = ω |0>

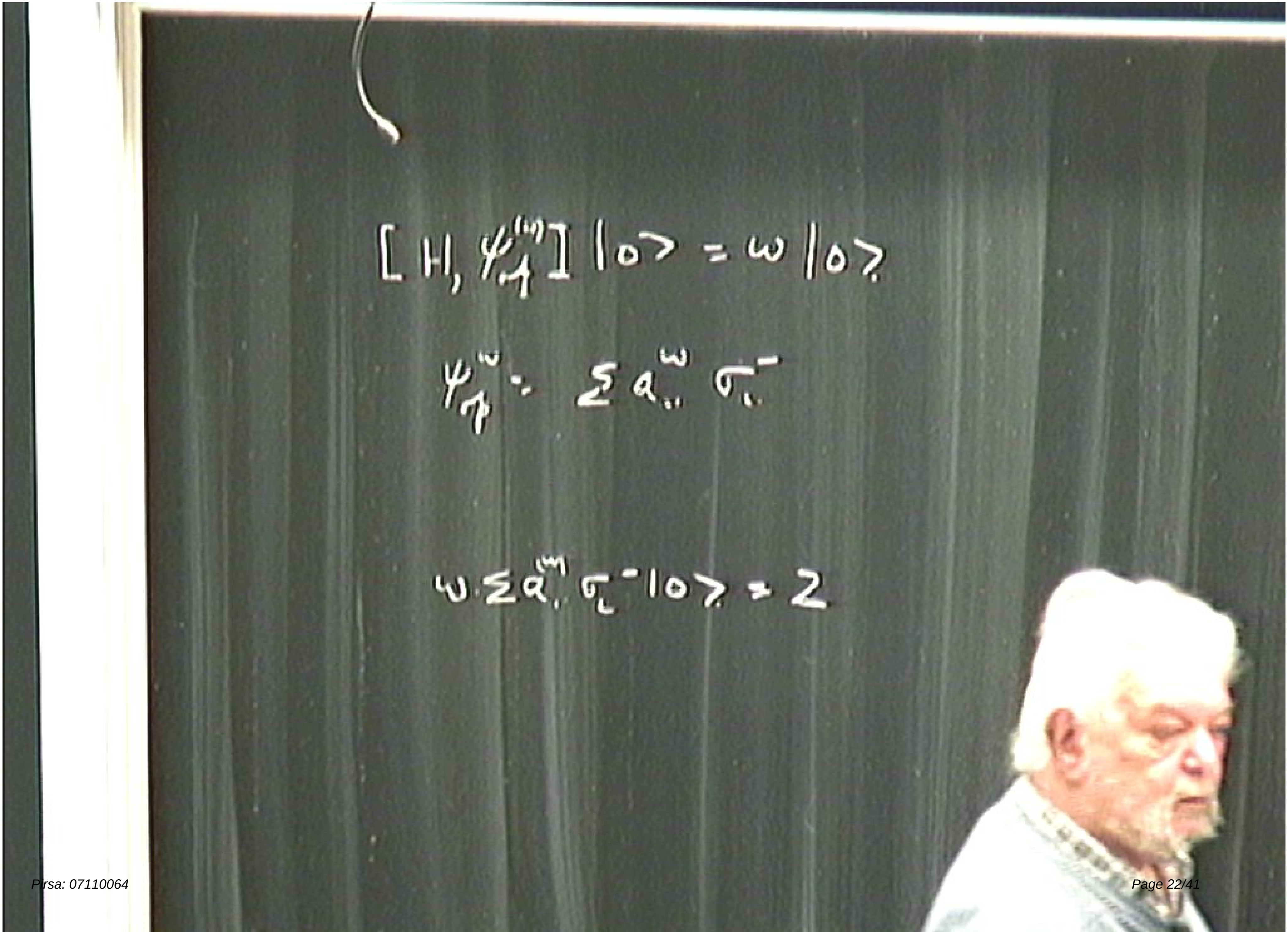

Excited States have 1 spin down.
Try Ψ_exc = Σ a_i σ_i^- |0>
H Ψ_ex = (E_0 + ω_n) Ψ_ex
ω_n = excitation energy from |0>
Σ a_i σ_i^- |0> = Ψ_eff |0>
[H, Ψ_eff] |0> = ω |0>
= H Ψ_eff |0> - Ψ_eff H |0>





$$[H, \psi_{A}^{(\omega)}] |0\rangle = \omega |0\rangle$$

$$\psi_{A}^{\omega} = \sum a_{ii}^{\omega} \sigma_i^-$$


$[H, \psi_q^{(\omega)}] |0\rangle = \omega |0\rangle$
$\psi_q^{\omega} = \sum a_i^{\omega} \sigma_i^-$
$\omega \sum a_i^{(\omega)} \sigma_i^- |0\rangle = Z$

$$[H, \psi_\eta^{(\omega)}]\,|0\rangle = \omega\,|0\rangle$$

$$\psi_\eta^{\omega} = \sum a_i^{\omega}\, \sigma_i^-$$

$$\omega \sum a_i^{\omega}\, \sigma_i^- |0\rangle = 2 \sum a_i\, v_{ij} \left[\sigma_i^- \overset{1}{\cancel{\sigma_j^z}} - \sigma_j^- \overset{1}{\cancel{\sigma_i^z}}\right] |0\rangle$$

$$[H, \psi_1^{(\omega)}]\,|0\rangle = \omega\,|0\rangle$$

$$\psi_1^{\omega} = \sum a_i^{\omega}\, \sigma_i^-$$

$$\omega \sum a_i^{\omega}\, \sigma_i^- |0\rangle = 2 \sum a_i^{\omega}\, V_{ij} \left[\sigma_i^- \frac{1}{\sigma_j^z} - \sigma_j^- \frac{1}{\sigma_i^z} \right] |0\rangle$$

$$= 2 \sum_{i,j} a_i^{\omega}\, V_{ij}\, \sigma_i^- |0\rangle - 2 \sum a_i^{\omega}\, V_{ij}\, \sigma_j^- |0\rangle$$

$$= 2 \sum_i a_i^{\omega}\, \sigma_i^-\, V_j(i)\, |0\rangle$$

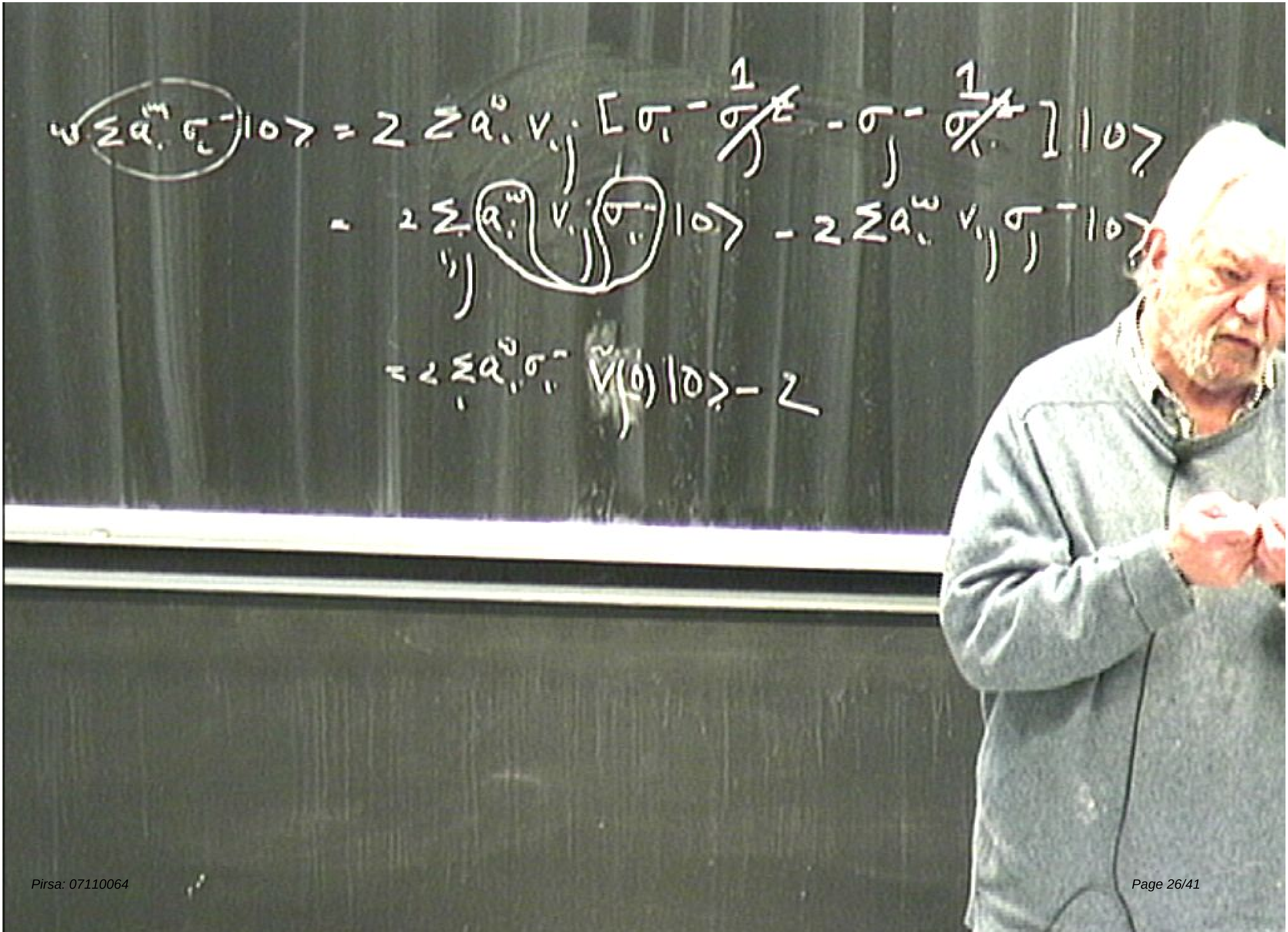
$$[H, \psi_1^\omega]|0\rangle = \omega|0\rangle$$

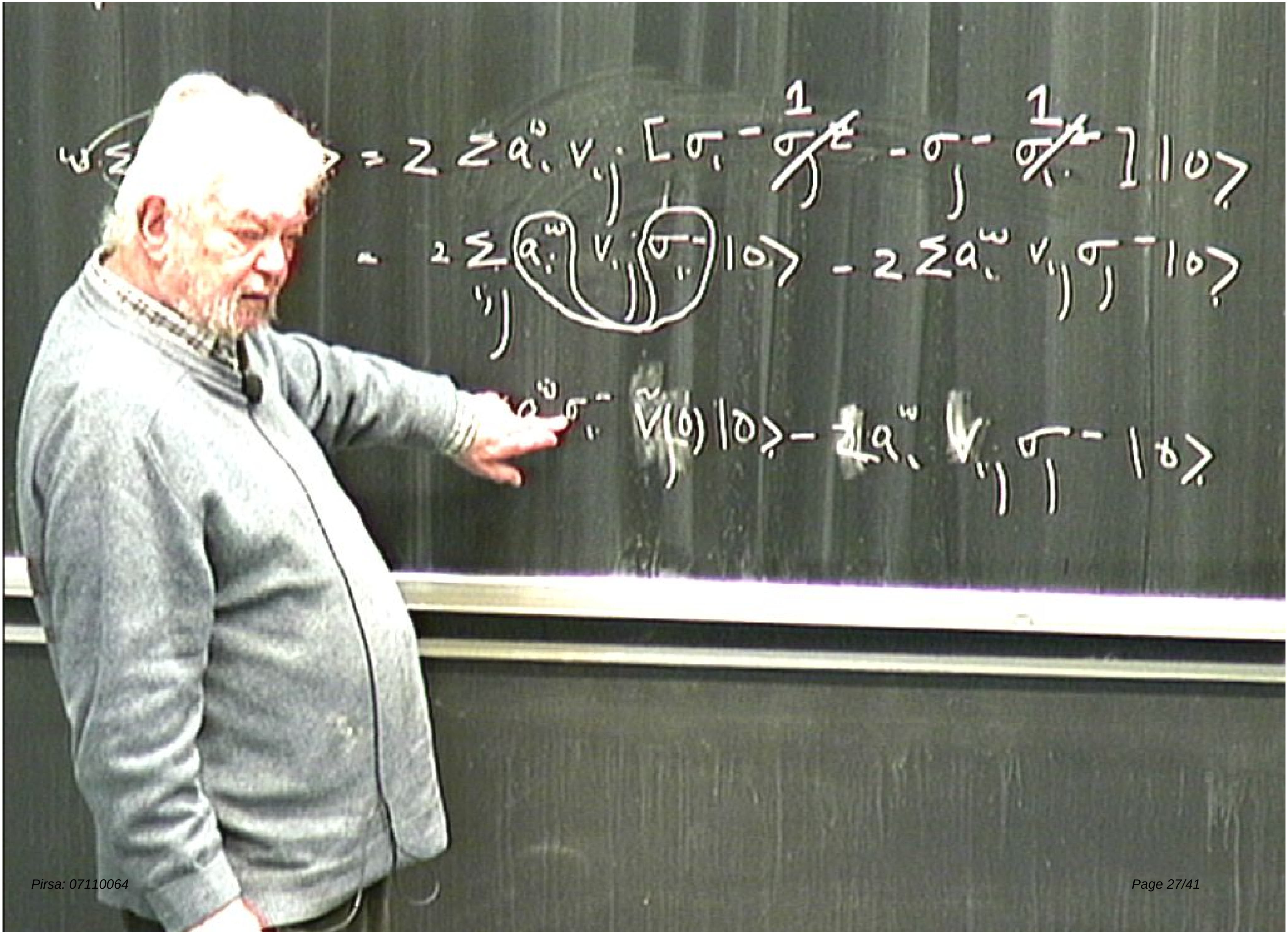
$$\psi_1^\omega = \sum a_i^\omega \sigma_i^-$$

$$\omega \sum a_i^\omega \sigma_i^- |0\rangle = 2 \sum a_i^\omega v_{ij} \left[\sigma_i^- \tfrac{1}{\sigma_j^z} - \sigma_j^- \tfrac{1}{\sigma_i^z}\right]|0\rangle$$

$$= 2 \sum_{i,j} a_i^\omega v_{ij} \sigma_i^- |0\rangle - 2 \sum a_i^\omega v_{ij} \sigma_j^- |0\rangle$$

$$= 2 \sum_i a_i^\omega \sigma_i^- \tilde{V}(0)|0\rangle$$

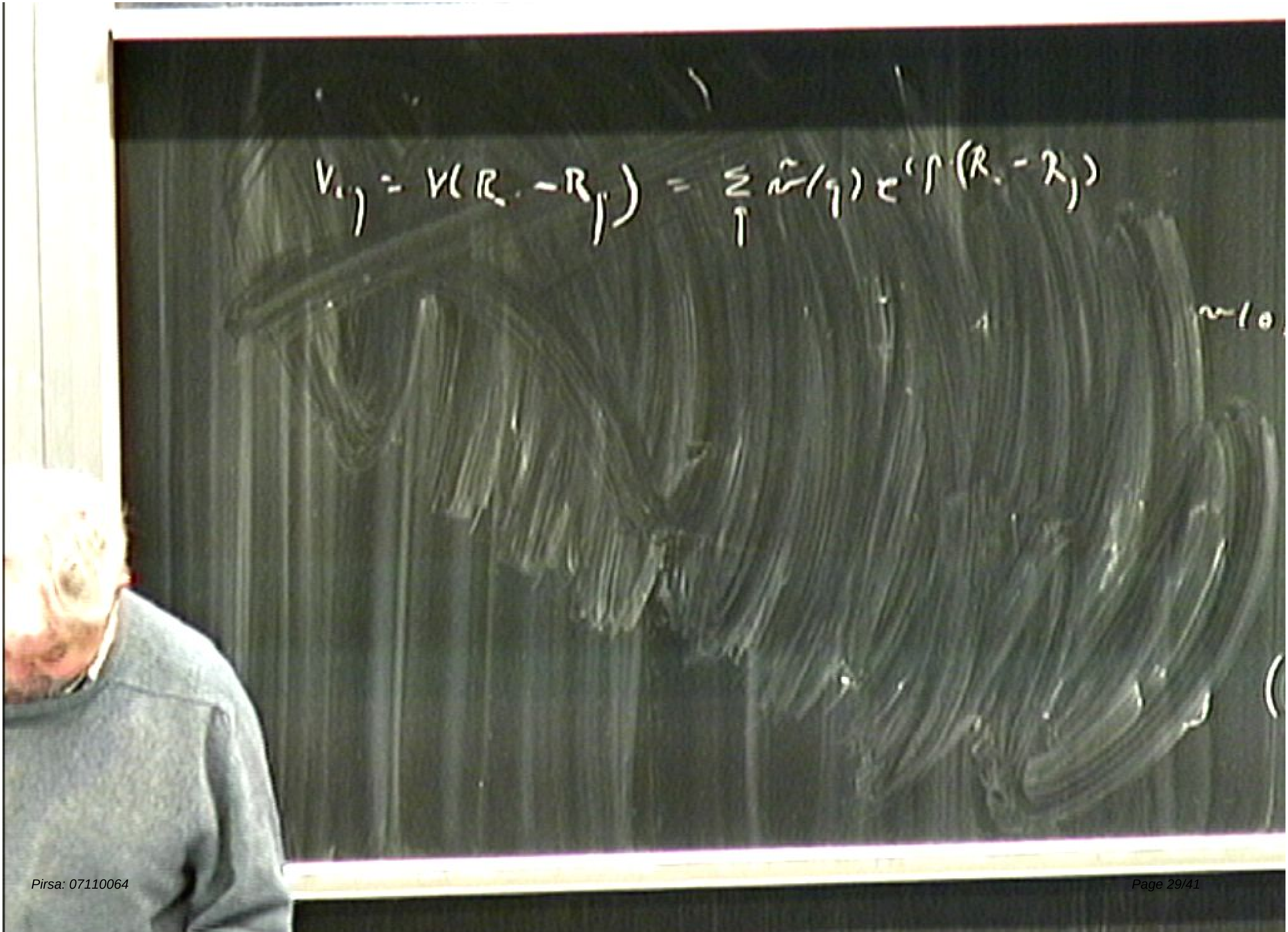

$\omega \sum a_i^\omega \sigma_i^- |0\rangle = 2 \sum a_i^\omega V_{ij} [\sigma_i^- \frac{1}{\sigma_j} - \sigma_j^- \frac{1}{\sigma_i}] |0\rangle$
$= 2 \sum_{ij} a_i^\omega V_{ij} \sigma_i^- |0\rangle - 2 \sum a_i^\omega V_{ij} \sigma_j^- |0\rangle$
$= 2 \sum_i a_i^\omega \sigma_i^- \tilde{V}_j(0) |0\rangle - 2$



$$\omega \sum a_i^{\omega} \sigma_i^- |0\rangle = 2 \sum a_i^{\omega} V_{ij} \left[\sigma_i^- \overset{1}{\cancel{\sigma_j^z}} - \sigma_j^- \overset{1}{\cancel{\sigma_i^z}} \right] |0\rangle$$

$$= 2 \sum_{ij} a_i^{\omega} V_{ij} \sigma_i^- |0\rangle - 2 \sum a_i^{\omega} V_{ij} \sigma_j^- |0\rangle$$

$$= 2 \sum_i a_i^{\omega} \sigma_i^- \tilde{V}_j(0) |0\rangle - 2 \sum_{ij} a_i^{\omega} V_{ij} \sigma_j^- |0\rangle$$


$V_{ij} = V(R_i - R_j) = \sum_q \tilde{v}(q) e^{iq(R_i - R_j)}$
$\tilde{v}(0)$

$$V_{ij} = V(R_i - R_j) = \sum_q \tilde{v}(q)\, e^{iq\cdot(R_i - R_j)}$$

$$a_i^{\omega} \sim e^{i\vec{q}\cdot\vec{R}_i}$$

$$\omega(q)\,\sigma_q^- |0\rangle = 2\left[\tilde{v}(0) - \tilde{v}(q)\right]\sigma_q^- |0\rangle$$

$$\sigma_q^- = \sum_i e^{i\vec{q}\cdot\vec{R}_i}\sigma_i^-$$

$$V_{ij} = V(R_i - R_j) = \sum_q \tilde{v}(q)\, e^{iq\cdot(R_i - R_j)}$$

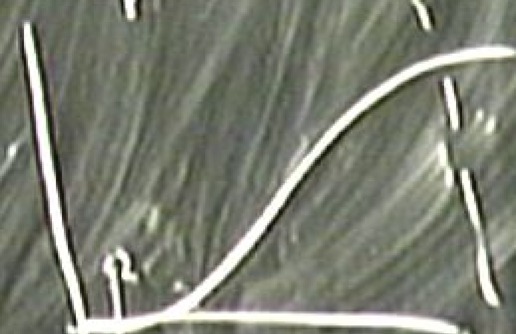
$$q_i^{\omega} \sim e^{i\vec{q}\cdot\vec{R}_i}$$

$\tilde{v}(0)$ is

$$\omega(q)\,\sigma_q^-|0\rangle = 2[\tilde{V}(0) - \tilde{v}(q)]\,\sigma_q^-|0\rangle$$

$$\sigma_q^- = \sum_i e^{iq\cdot R_i}\,\sigma_i^-$$

$$\omega(q) = \tilde{V}(0) - \tilde{v}(q) \quad ; \quad \omega($$



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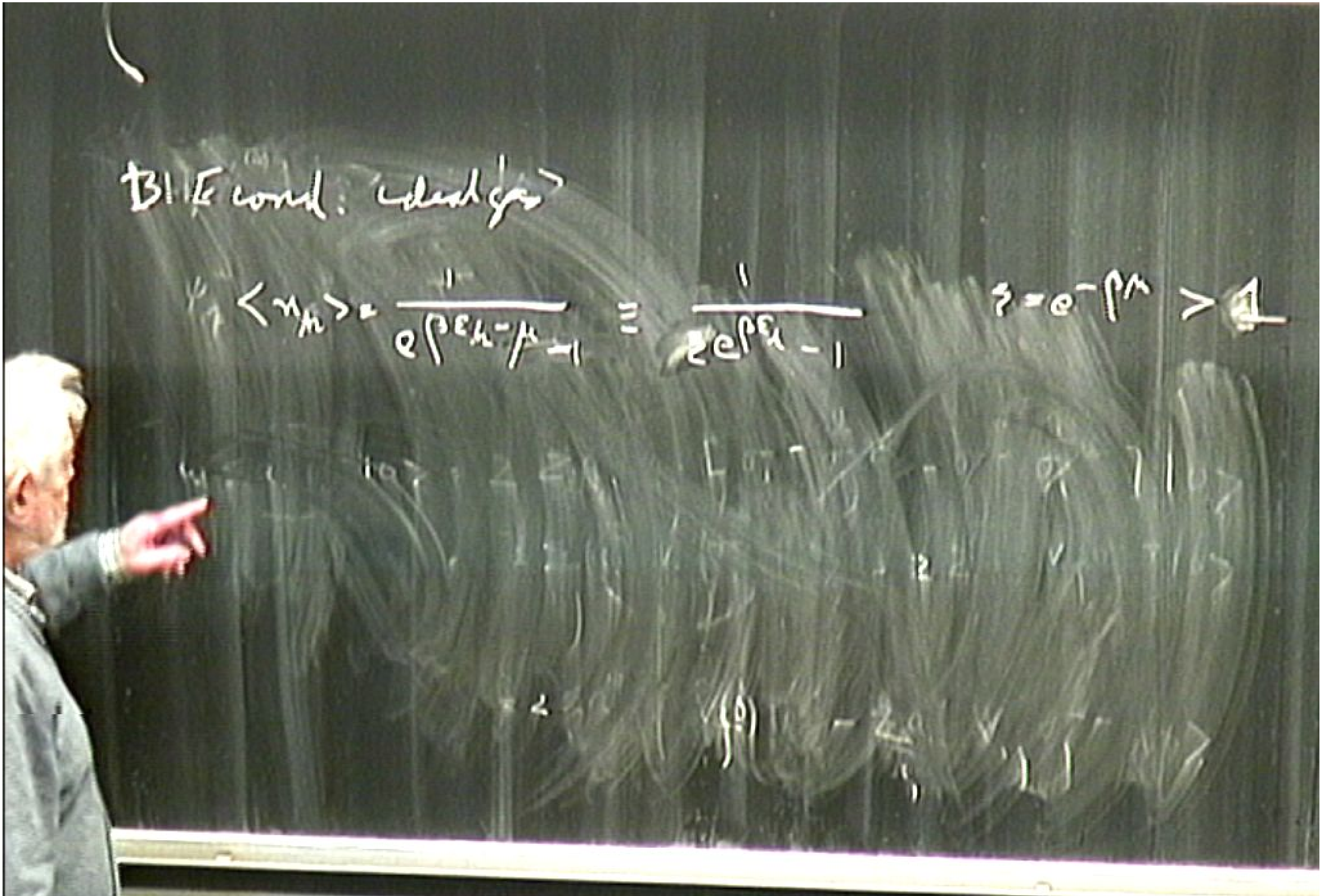
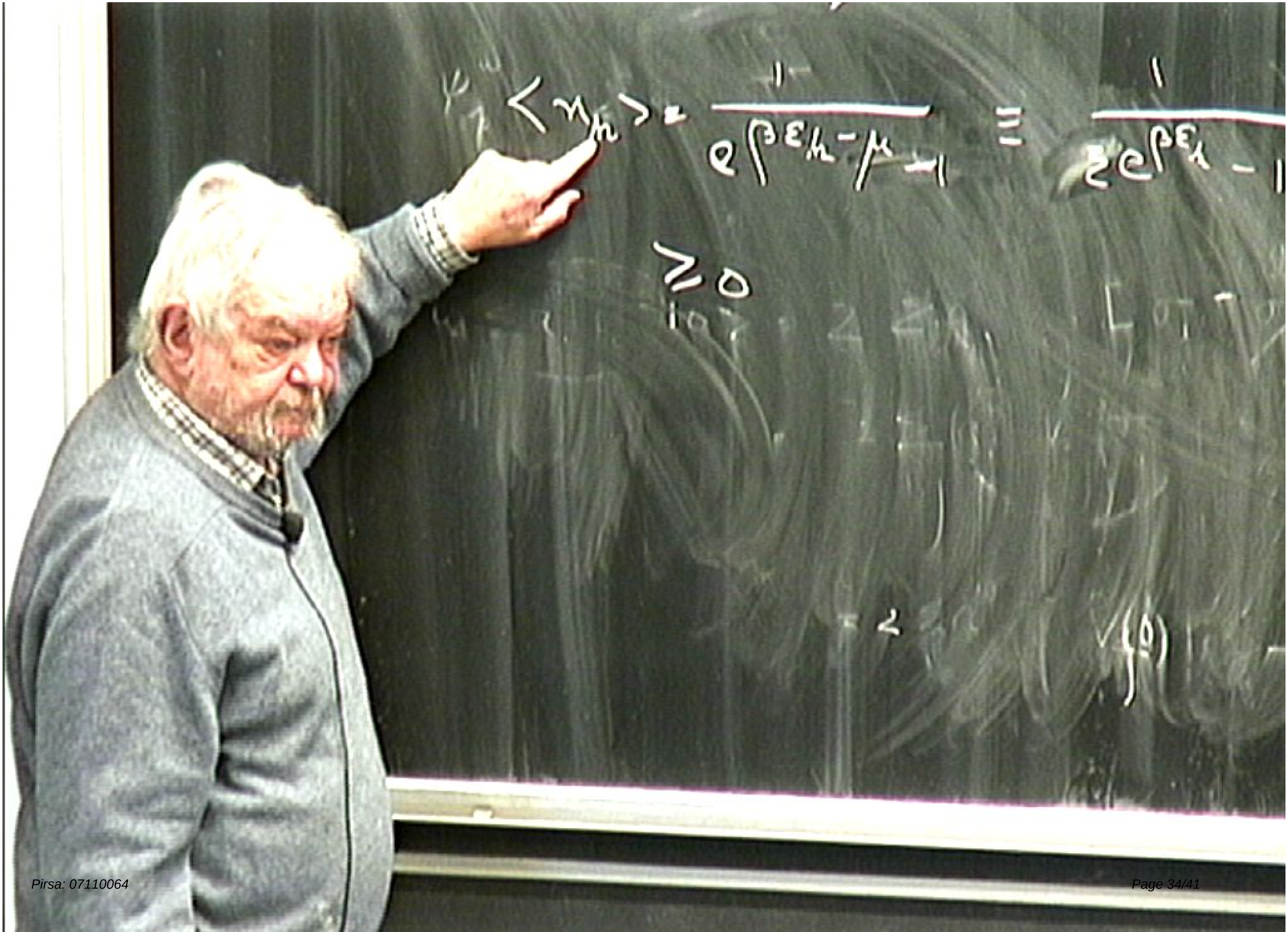
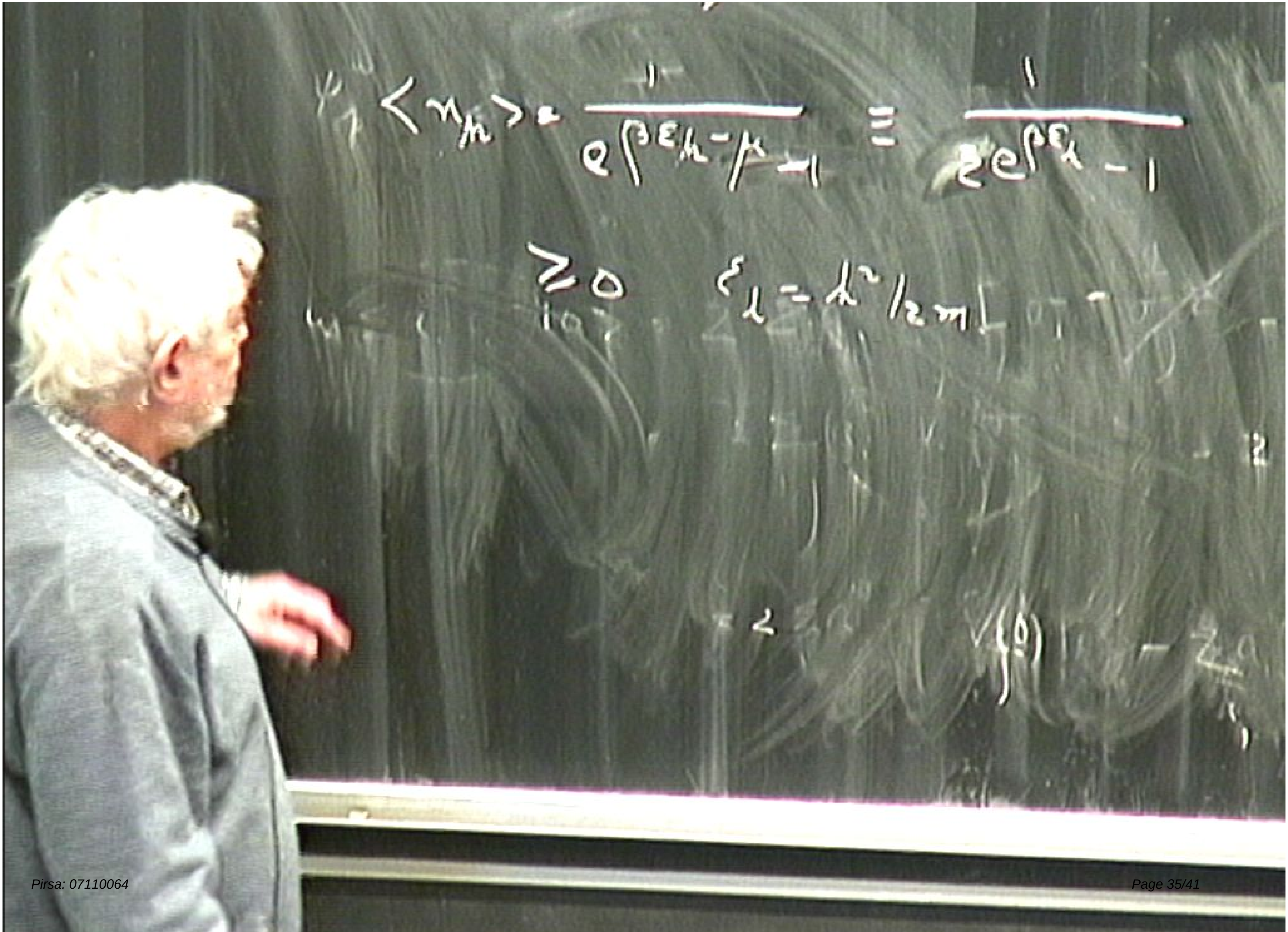
BEC cond: ideal gas

$$\langle n_k \rangle = \frac{1}{e^{\beta(\epsilon_k - \mu)} - 1} \equiv \frac{1}{z e^{\beta \epsilon_k} - 1} \qquad z = e^{-\beta\mu} > 0$$


BE cond: ideal gas
⟨n_k⟩ = 1/(e^{β(ε_k−μ)} − 1) ≡ 1/(z e^{βε_k} − 1)
z = e^{−βμ} > 1



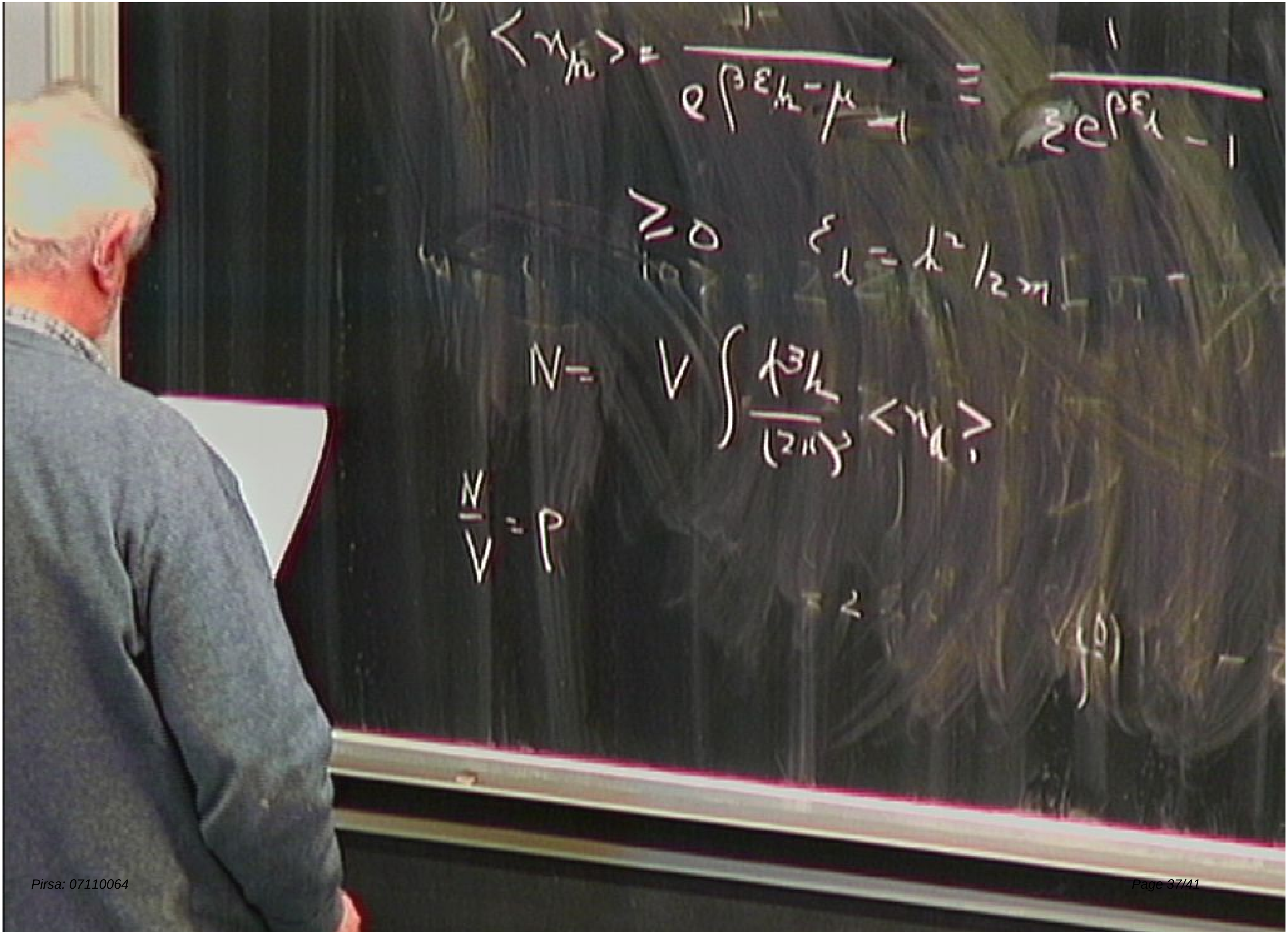
<n_k> = 1 / (e^{βε_k - μ} - 1) ≡ 1 / (z e^{βε_k} - 1)
≥ 0


$\langle n_k \rangle = \frac{1}{e^{\beta(\epsilon_k - \mu)} - 1} \equiv \frac{1}{z e^{\beta \epsilon_k} - 1}$
$\geq 0 \quad \epsilon_k = k^2/2m$

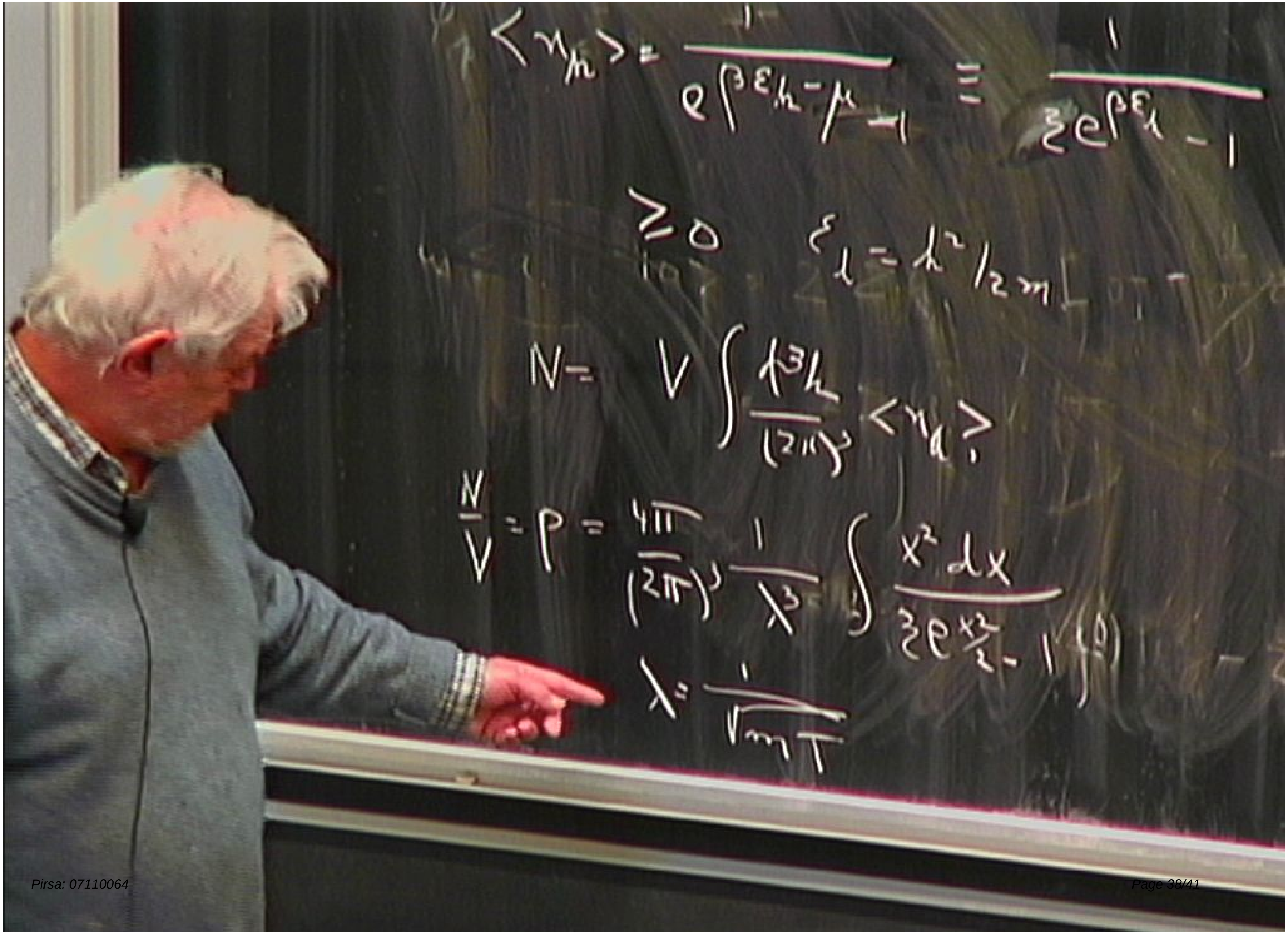
BE cond: ideal gas

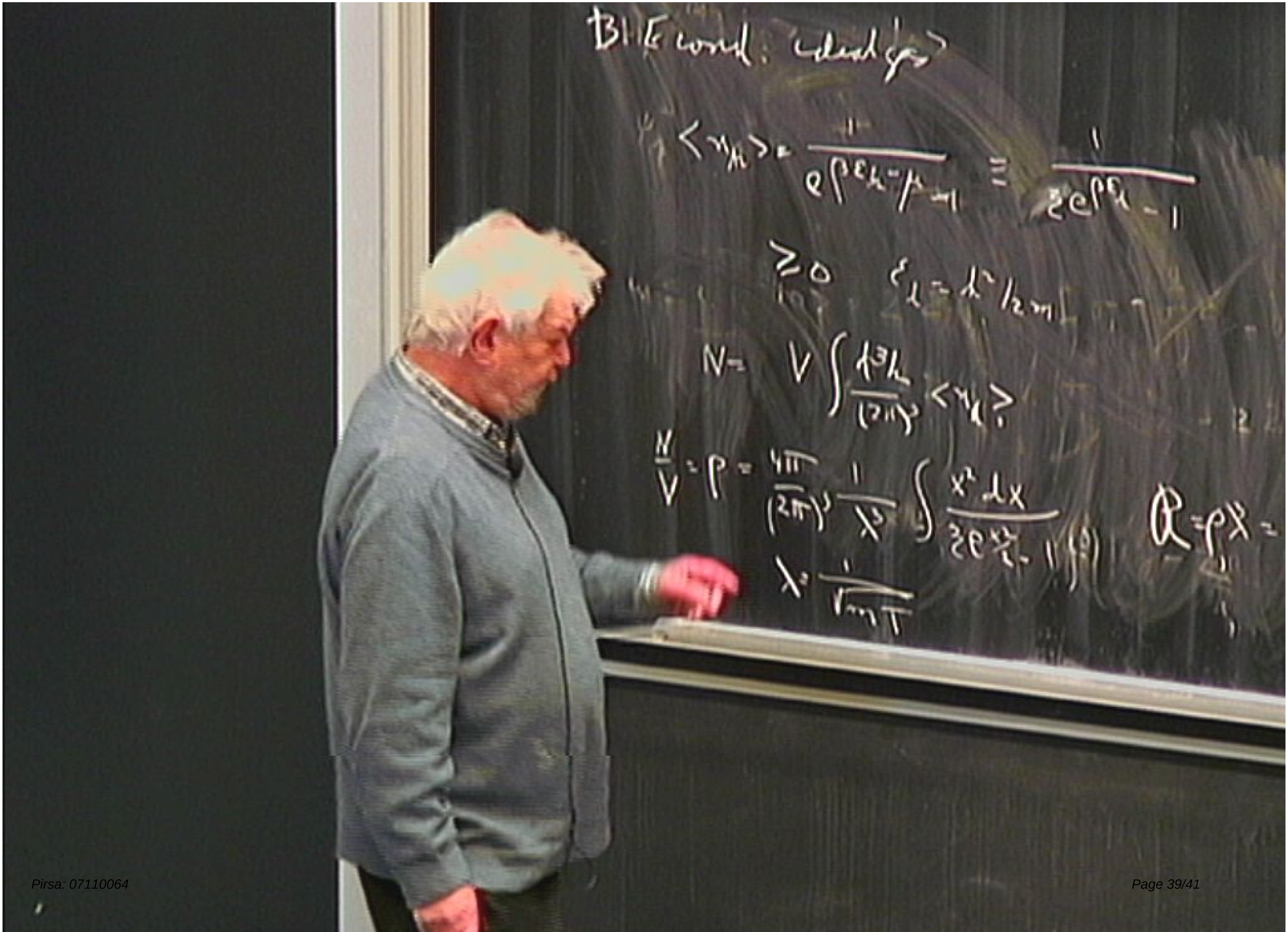
$$\langle n_k \rangle = \frac{1}{e^{\beta(\epsilon_k - \mu)} - 1} \equiv \frac{1}{z\, e^{\beta\epsilon_k} - 1} \qquad z = e^{-\beta\mu} > 1$$

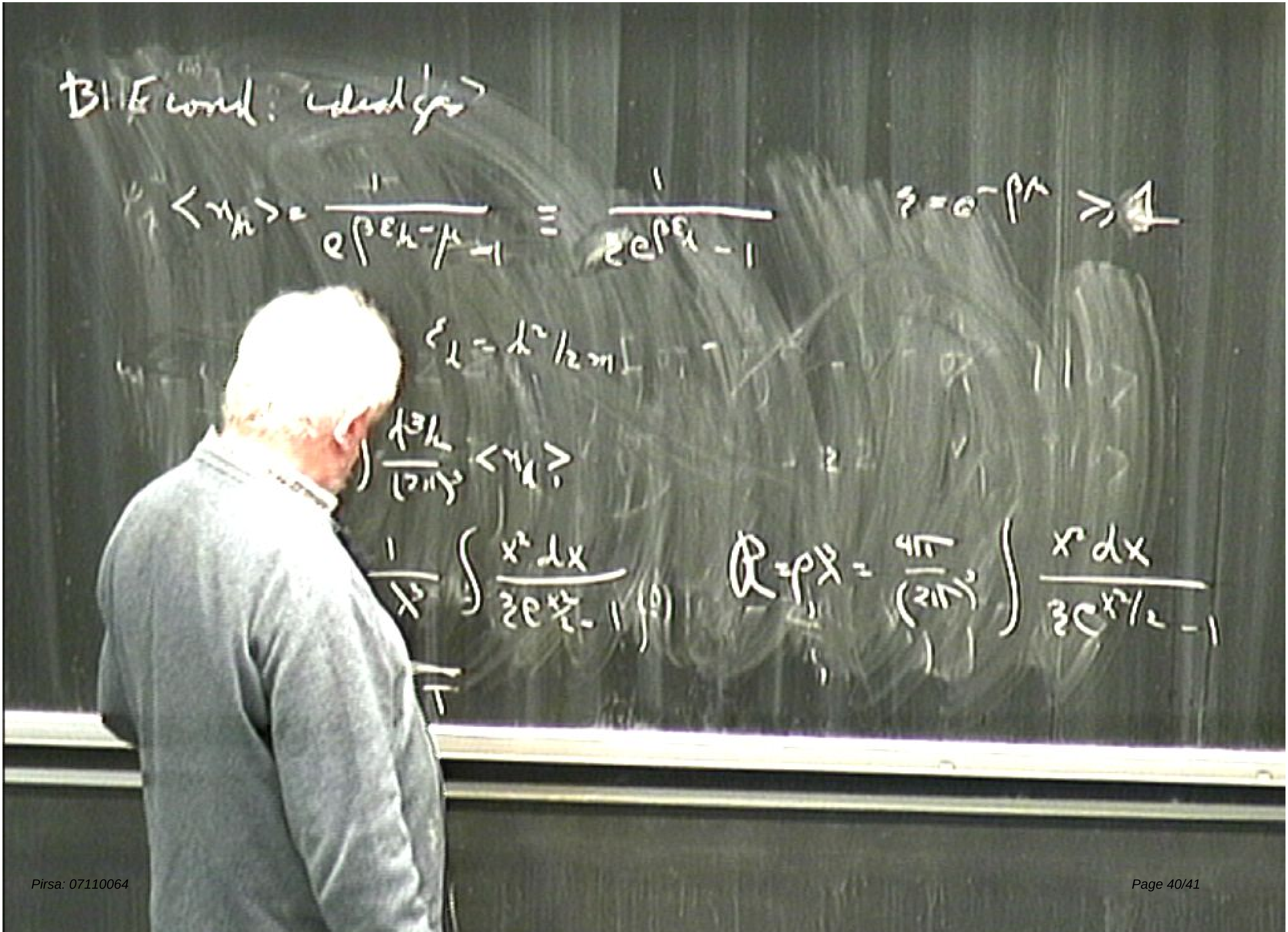
$$\geq 0 \qquad \epsilon_k = k^2/2m$$

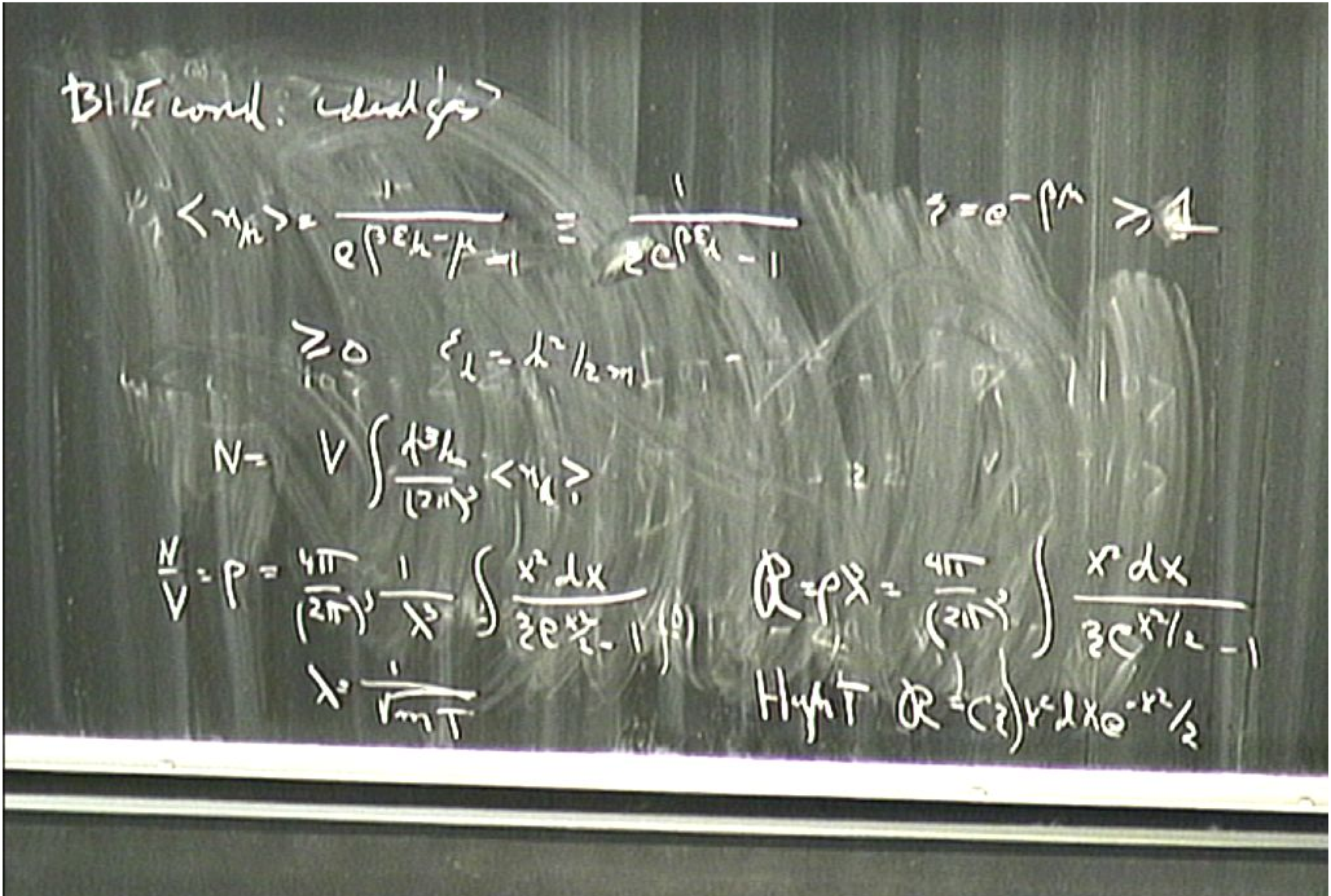


$\langle n_k \rangle = \frac{1}{e^{\beta(\varepsilon_k - \mu)} - 1} \equiv \frac{1}{\xi e^{\beta \varepsilon_k} - 1}$
≥ 0
$\varepsilon_k = k^2/2m$
$N = V \int \frac{d^3k}{(2\pi)^3} \langle n_k \rangle$
$\frac{N}{V} = \rho$


<n_k> = 1 / (e^{βε_k − μ} − 1) ≡ 1 / (ξ e^{βε_k} − 1)
ξ ≥ 0   ε_k = k²/2m
N = V ∫ d³k/(2π)³ <n_k>
N/V = ρ = 4π/(2π)³ · 1/λ³ ∫ x² dx / (ξ e^{x²/2} − 1)
λ = 1/√(mT)




BEC cont'd: ideal gas
$\langle n_k \rangle = \frac{1}{e^{\beta \varepsilon_k - \mu} - 1} \equiv \frac{1}{z e^{\beta \varepsilon_k} - 1}$
$z = e^{-\beta\mu} > 1$
$\varepsilon_k = k^2/2m$
$\int \frac{d^3k}{(2\pi)^3} \langle n_k \rangle$
$\frac{1}{\lambda^3} \int \frac{x^2 dx}{z e^{x^2/2} - 1}$
$R = \rho \lambda^3 = \frac{4\pi}{(2\pi)^3} \int \frac{x^2 dx}{z e^{x^2/2} - 1}$


BE cond: ideal gas
$\langle n_k \rangle = \frac{1}{e^{\beta\epsilon_k - \mu} - 1} \equiv \frac{1}{z e^{\beta\epsilon_k} - 1}$ $z = e^{-\beta\mu} > 1$
≥ 0 $\epsilon_k = k^2/2m$
$N = V \int \frac{d^3k}{(2\pi)^3} \langle n_k \rangle$
$\frac{N}{V} = \rho = \frac{4\pi}{(2\pi)^3} \frac{1}{\lambda^3} \int \frac{x^2\,dx}{z e^{x^2/2} - 1}$
$R = \rho\lambda^3 = \frac{4\pi}{(2\pi)^3} \int \frac{x^2\,dx}{z e^{x^2/2} - 1}$
$\lambda = \frac{1}{\sqrt{mT}}$
High T $R = \frac{1}{z} c \int x^2\,dx\, e^{-x^2/2}$