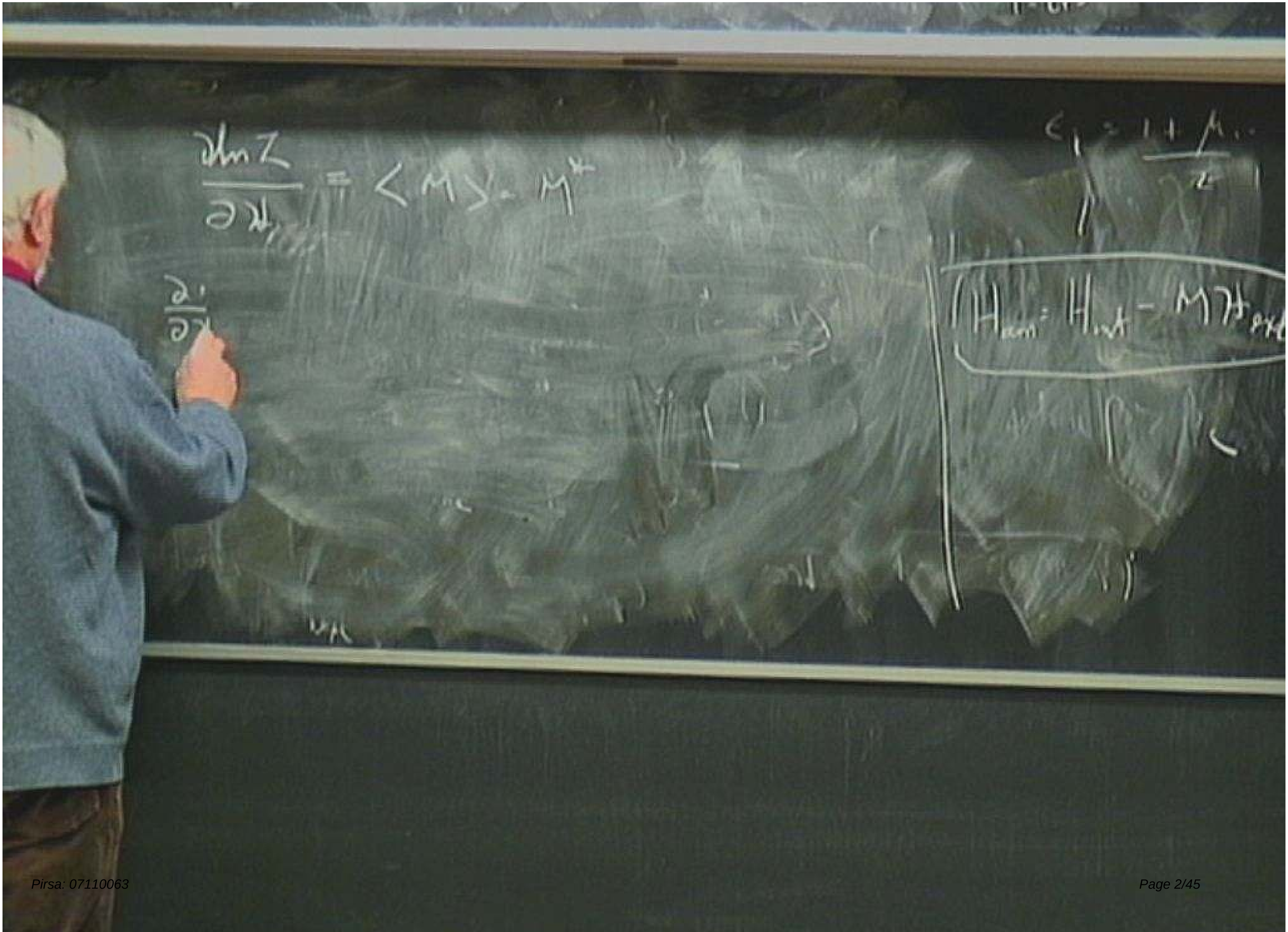
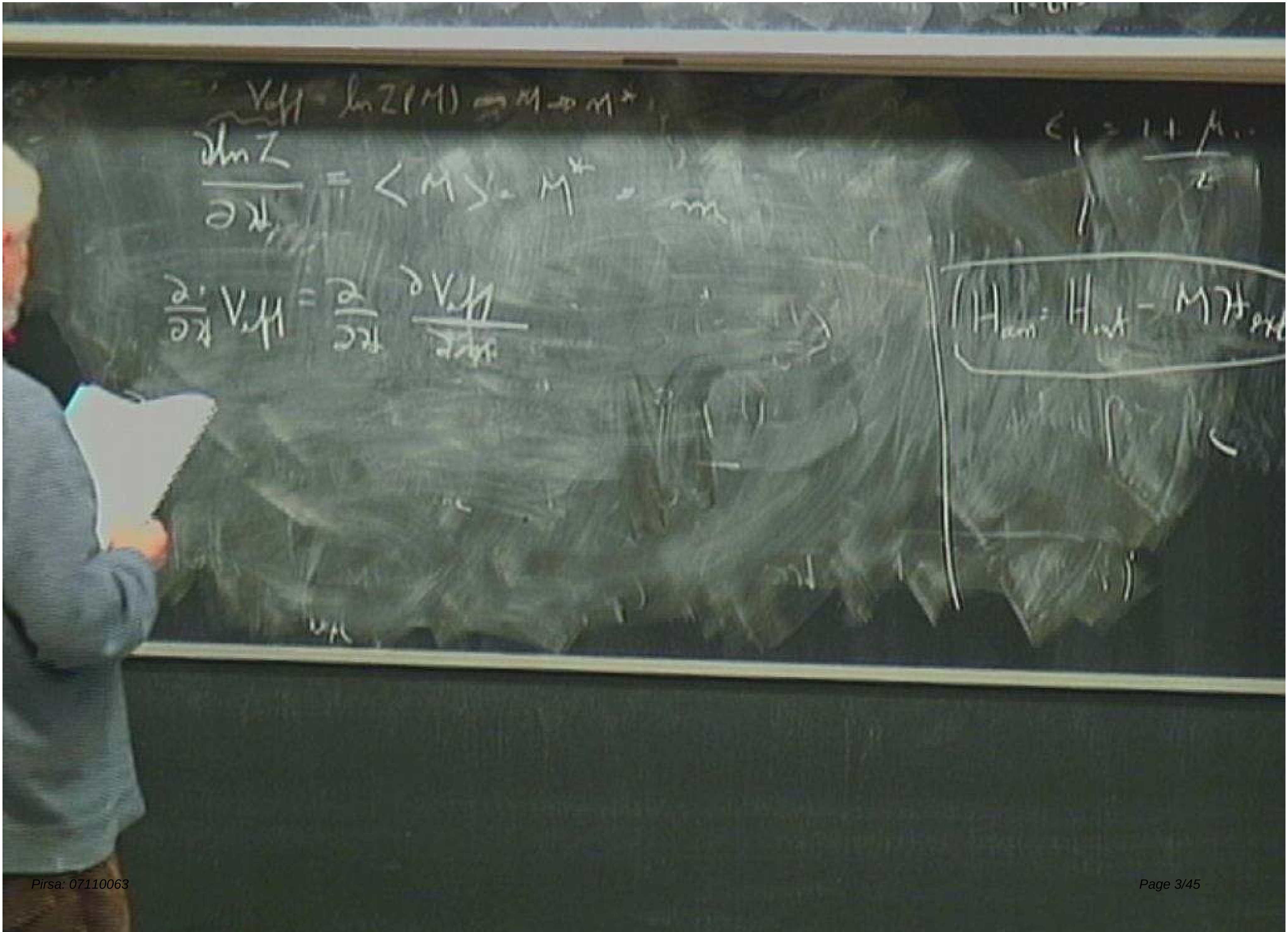
Title: Spontaneous Broken Symmetry 2B

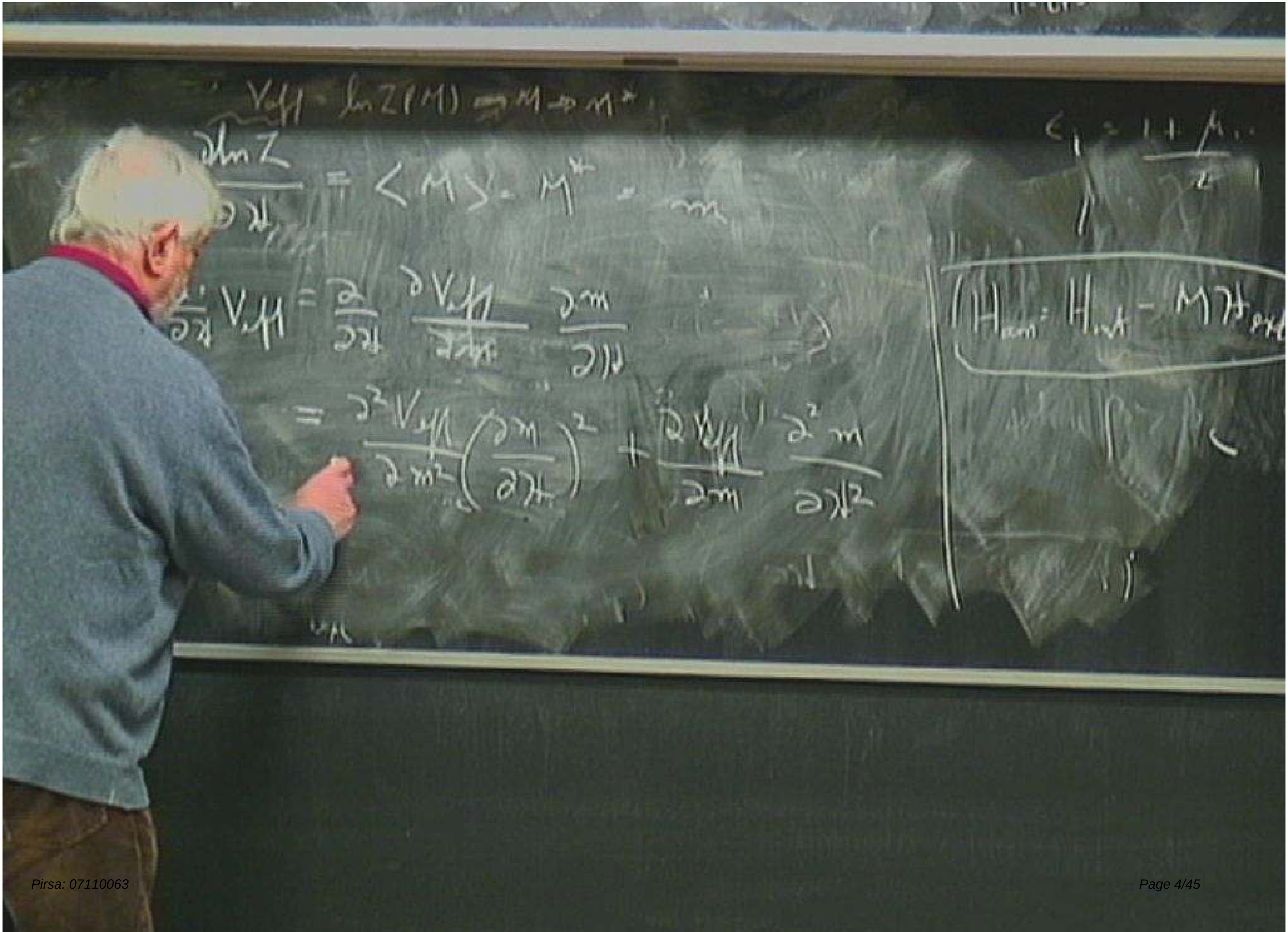
Date: Nov 08, 2007 11:00 AM

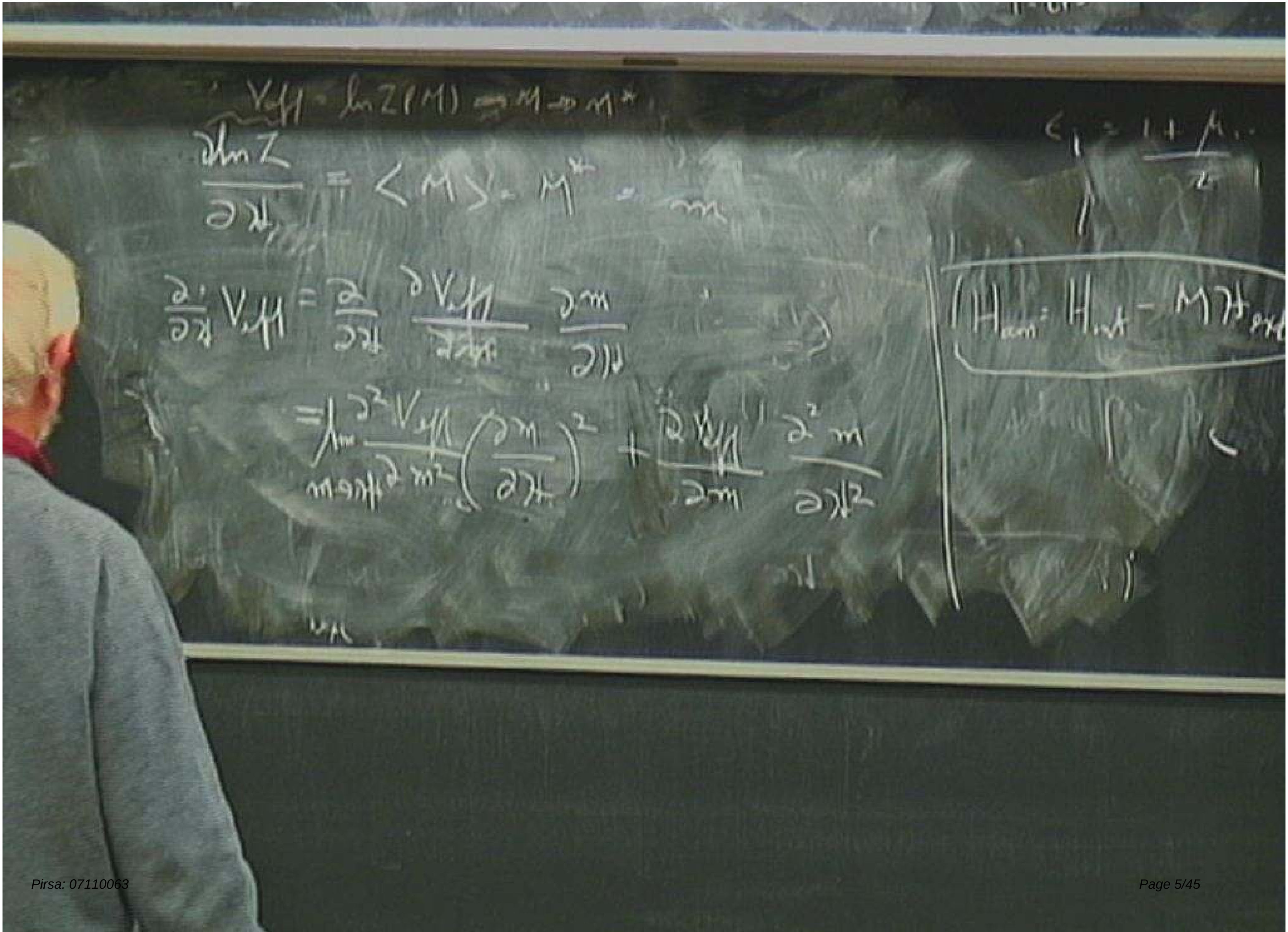
URL: http://pirsa.org/07110063

Abstract:






Veff = ln Z(M) → M → M*
∂ln Z / ∂h = <M> = M* = m
∂/∂h Veff = ∂/∂h ∂Veff/∂m ∂m/∂h
= ∂²Veff/∂m² (∂m/∂h)² + ∂Veff/∂m ∂²m/∂h²
H = Hint − M h ext


$V_{eff} = \ln Z(M)$
$\frac{\partial \ln Z}{\partial h} = \langle M \rangle = M^* = m$
$\frac{\partial}{\partial h} V_{eff} = \frac{\partial}{\partial h} \frac{\partial V_{eff}}{\partial m} \frac{\partial m}{\partial h}$
$H_{new} = H_{int} - M h_{ext}$

$V_{eff} = \ln Z(M)$ $M \to M^*$

$$\frac{\partial \ln Z}{\partial \mathcal{H}} = \langle M \rangle = M^* = m$$

$$\frac{\partial^2}{\partial \mathcal{H}^2} V_{eff} = \frac{\partial}{\partial \mathcal{H}} \frac{\partial V_{eff}}{\partial m} \frac{\partial m}{\partial \mathcal{H}}$$

$$= \frac{\partial^2 V_{eff}}{\partial m^2}\left(\frac{\partial m}{\partial \mathcal{H}}\right)^2 + \frac{\partial V_{eff}}{\partial m}\frac{\partial^2 m}{\partial \mathcal{H}^2}$$

$$\boxed{H_{tot} = H_{int} - M\mathcal{H}_{ext}}$$

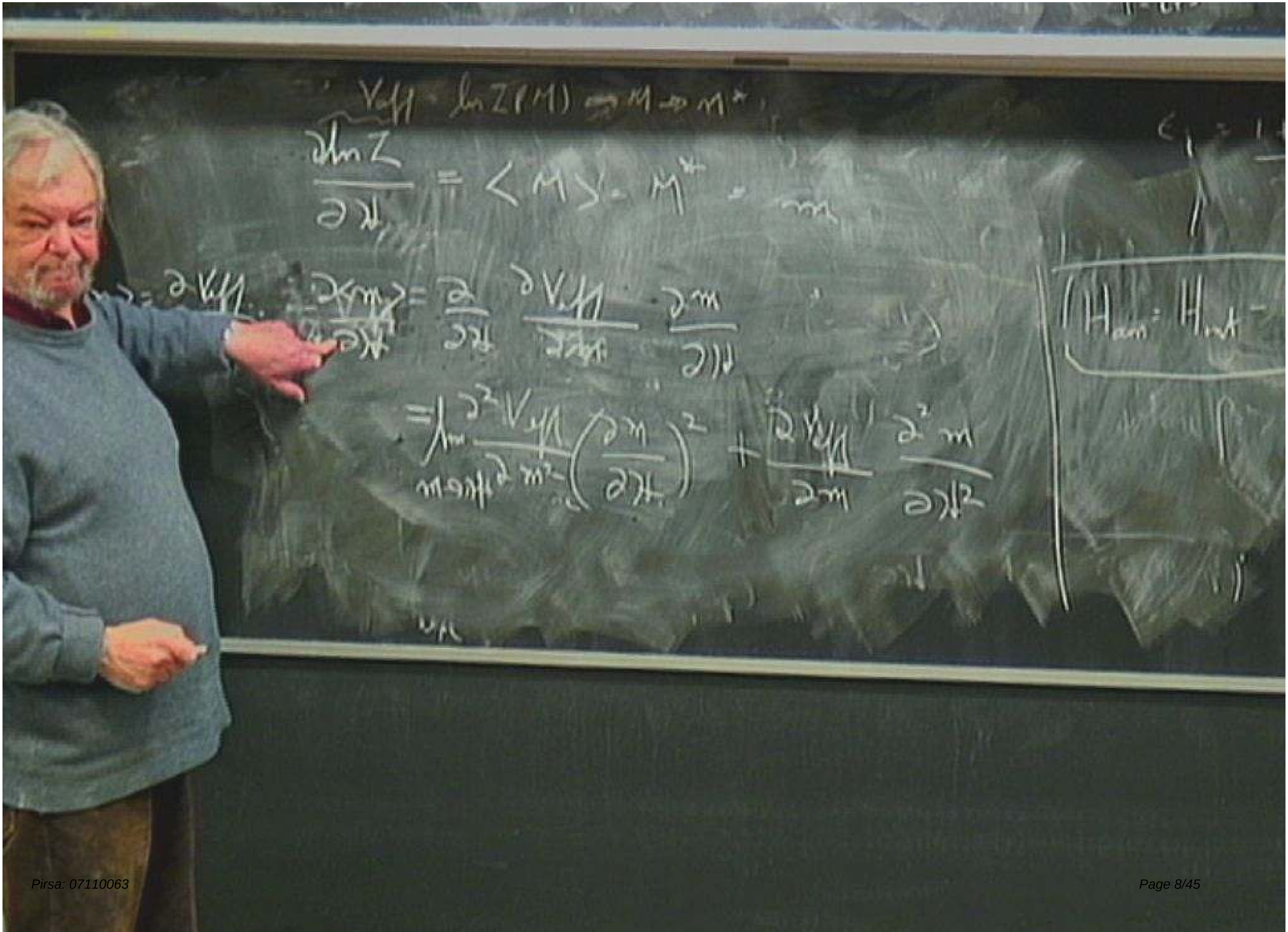
$$V_{eff} = \ln Z(M) \to M \to M^*$$

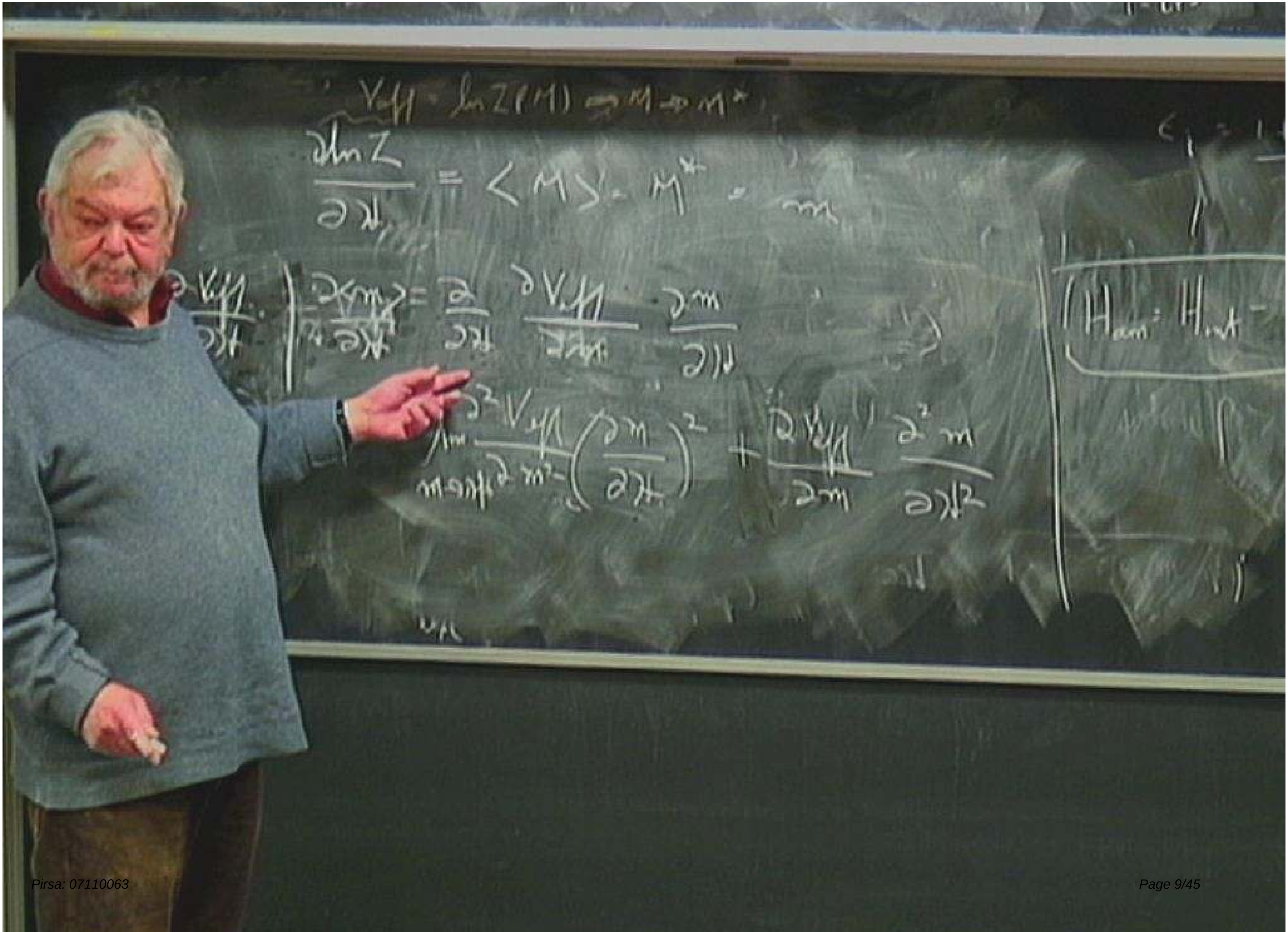
$$\frac{\partial \ln Z}{\partial h} = \langle M \rangle = M^* = m$$

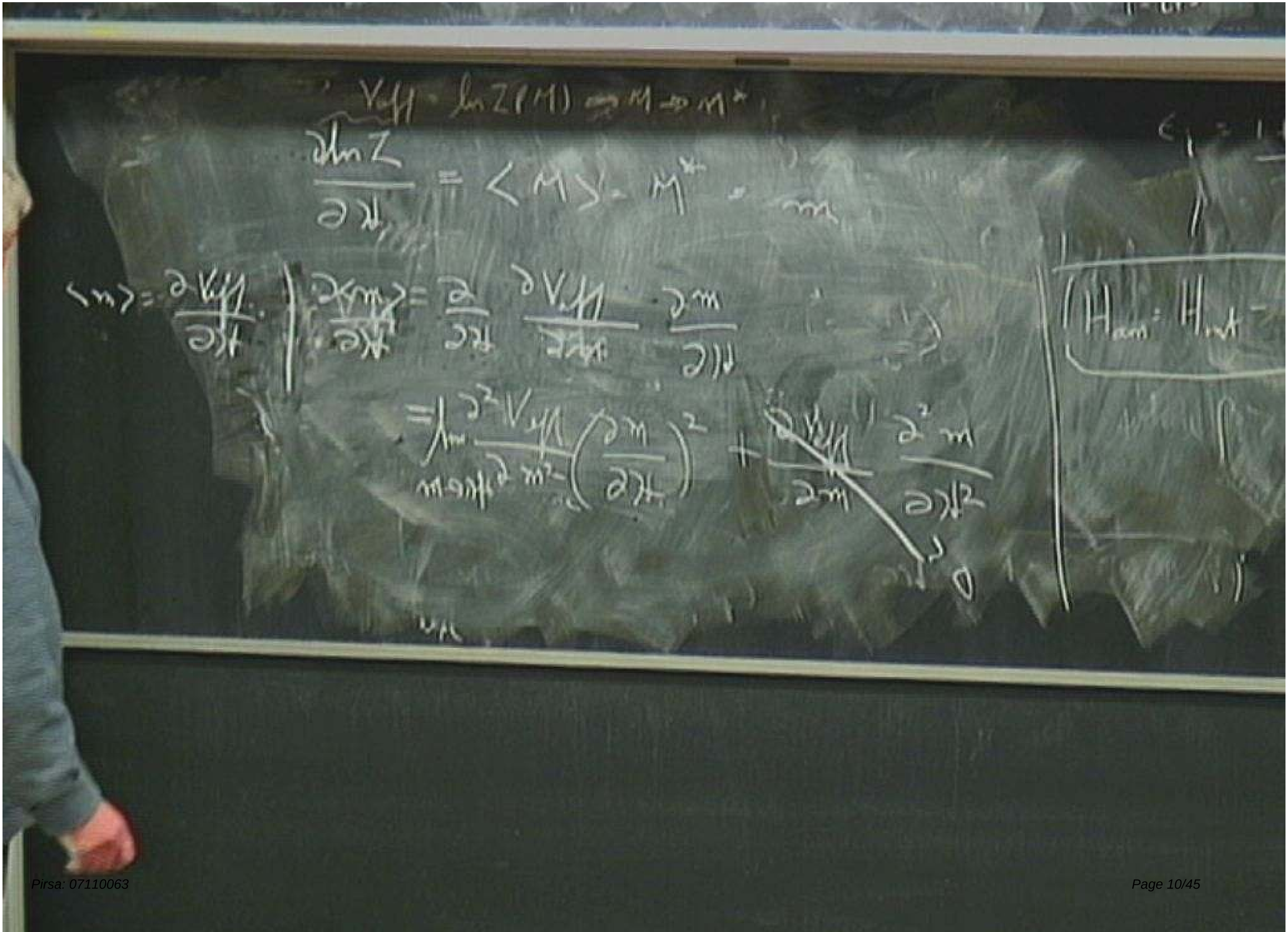
$$\frac{\partial}{\partial h} V_{eff} = \frac{\partial}{\partial h}\, \frac{\partial V_{eff}}{\partial m}\, \frac{\partial m}{\partial h}$$

$$= \lim_{m \to m^*} \frac{\partial^2 V_{eff}}{\partial m^2}\left(\frac{\partial m}{\partial h}\right)^2 + \frac{\partial V_{eff}}{\partial m}\, \frac{\partial^2 m}{\partial h^2}$$

$$\boxed{H_{eff} = H_{int} - M h_{ext}}$$







$V_{eff} = \ln Z(M) \Rightarrow M \to M^*$

$$\frac{\partial \ln Z}{\partial h} = \langle M \rangle = M^* = m$$

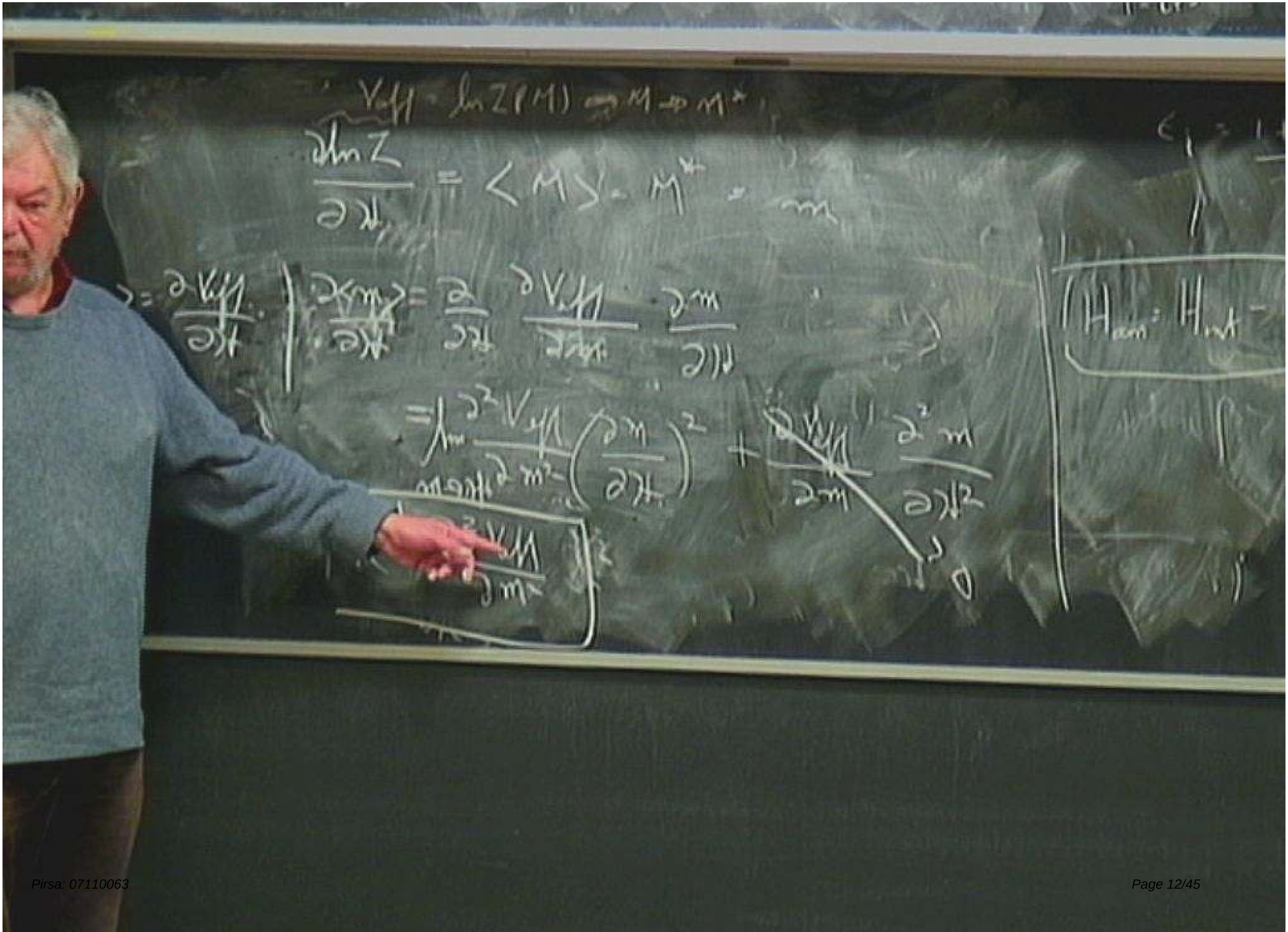
$$\langle m \rangle = \frac{\partial V_{eff}}{\partial h} \qquad \frac{\partial \langle m \rangle}{\partial h} = \frac{\partial}{\partial h}\frac{\partial V_{eff}}{\partial m}\frac{\partial m}{\partial h}$$

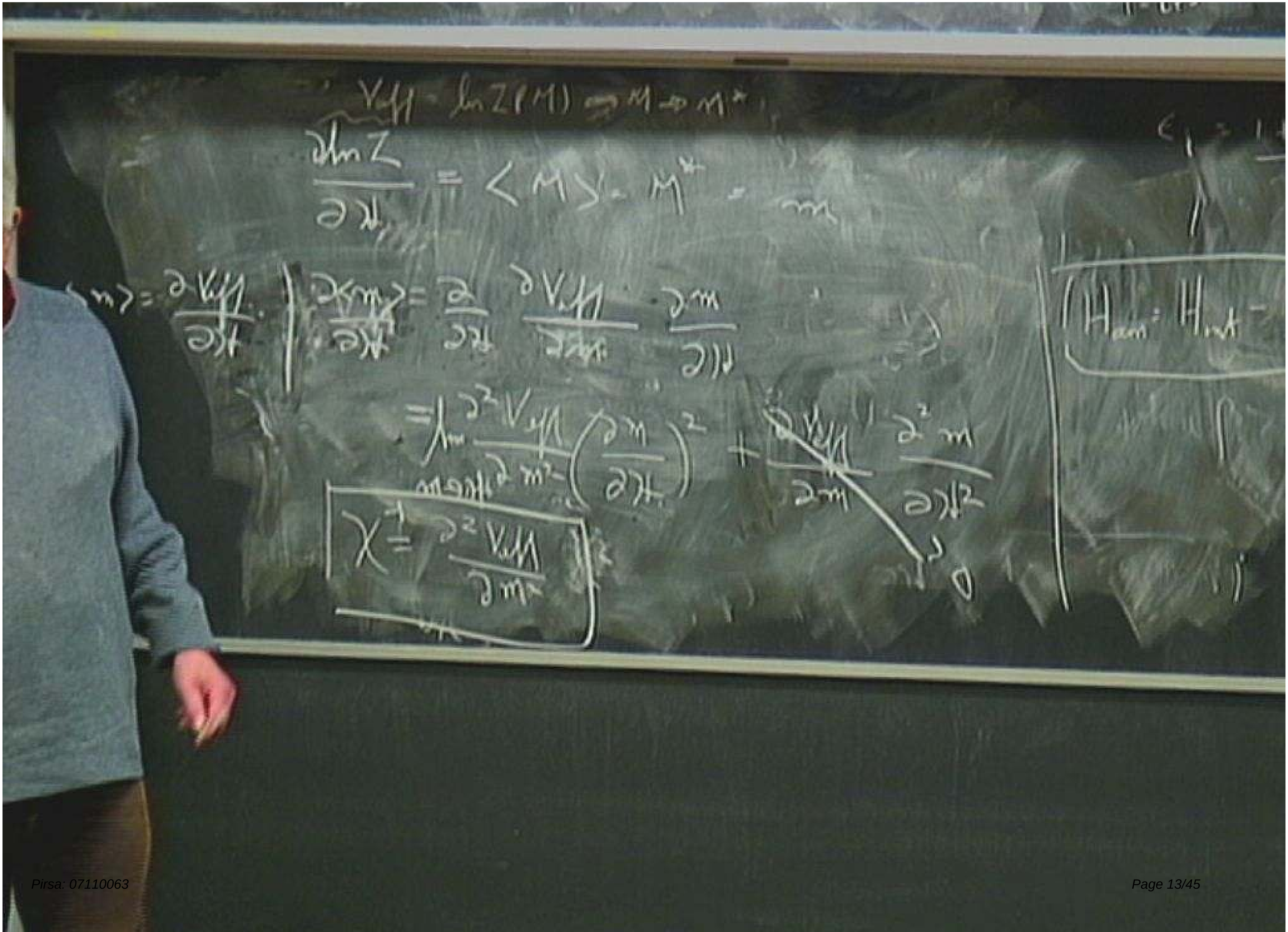
$$= \frac{\partial^2 V_{eff}}{\partial m^2}\left(\frac{\partial m}{\partial h}\right)^2 + \cancel{\frac{\partial V_{eff}}{\partial m}}\frac{\partial^2 m}{\partial h^2}$$

≈ 0

$$\chi = \frac{\partial^2 V_{eff}}{\partial m^2}\chi^2$$

$\boxed{H_{am} = H_{int} -}$




$V_{eff} = \ln Z(M) \Rightarrow M \to M^*$
$\frac{\partial \ln Z}{\partial h} = \langle M \rangle = M^* = m$
$\langle m \rangle = \frac{\partial V_{eff}}{\partial h}$
$\frac{\partial \langle m \rangle}{\partial h} = \frac{\partial}{\partial h} \frac{\partial V_{eff}}{\partial m} \frac{\partial m}{\partial h}$
$= \frac{\partial^2 V_{eff}}{\partial m^2} \left(\frac{\partial m}{\partial h}\right)^2 + \frac{\partial V_{eff}}{\partial m} \frac{\partial^2 m}{\partial h^2}$
$\chi^{-1} = \frac{\partial^2 V_{eff}}{\partial m^2}$
$H_{am} = H_{int}$

$V_{eff} = \ln Z(M) \Rightarrow M \Rightarrow M^*$

$$\frac{\partial \ln Z}{\partial H} = \langle M \rangle = M^* = m$$

$$\langle m \rangle = \frac{\partial V_{eff}}{\partial H}$$

$$\frac{\partial \langle m \rangle}{\partial H} = \frac{\partial}{\partial H} \frac{\partial V_{eff}}{\partial m} \frac{\partial m}{\partial H}$$

$$= \lambda_m \frac{\partial^2 V_{eff}}{\partial m^2} \left(\frac{\partial m}{\partial H}\right)^2 + \cancel{\frac{\partial V_{eff}}{\partial m}} \frac{\partial^2 m}{\partial H^2}$$

$\to 0$

$$\boxed{\chi^{-1} = \frac{\partial^2 V_{eff}}{\partial m^2}}$$

$(H_{am} = H_{int} -$

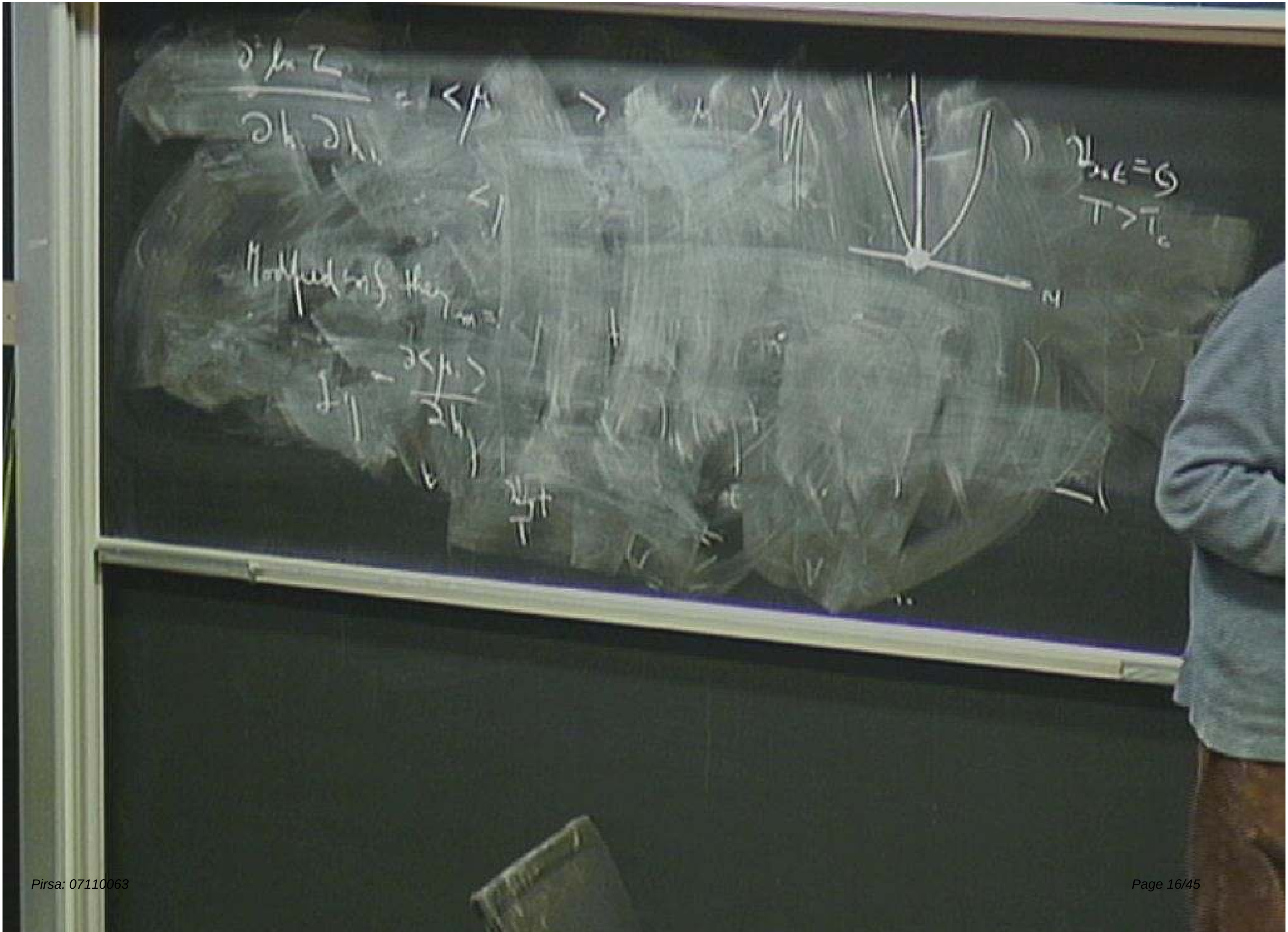
$$\frac{\partial^2 \ln Z}{\partial h_i \partial h_j} = \langle \mu \ \ \rangle$$

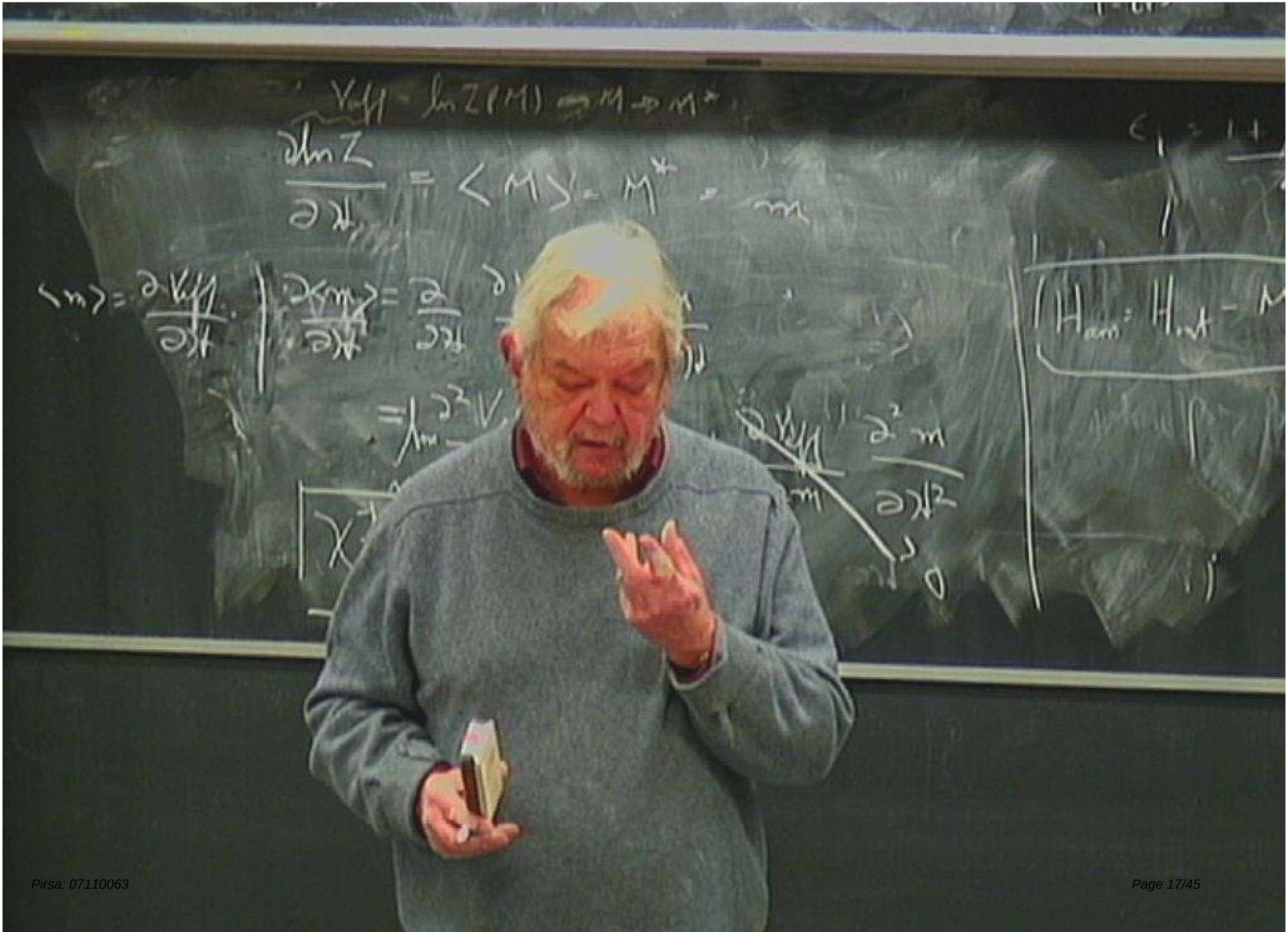
$h_{ext} = 0$

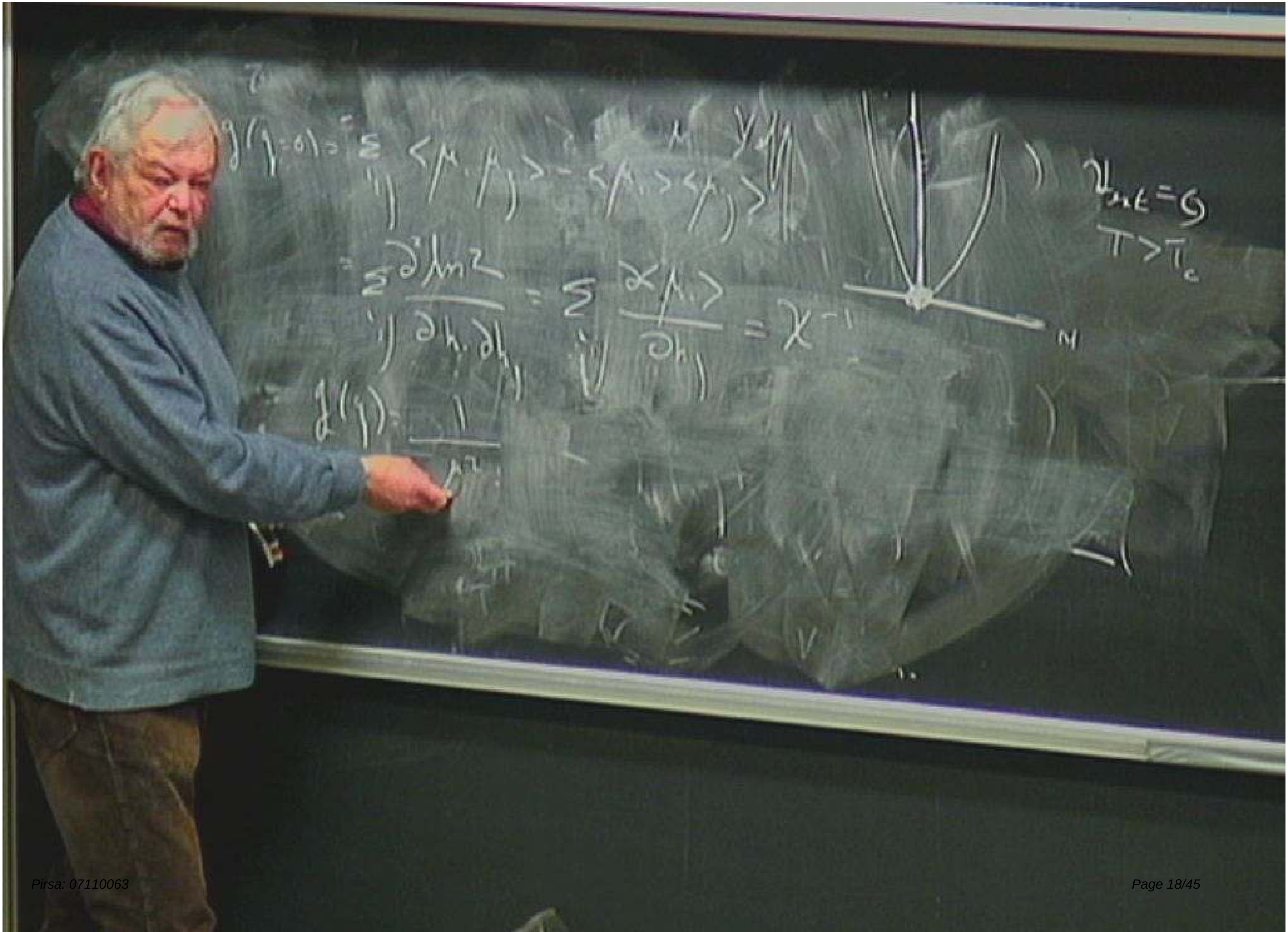
$T > T_c$

Mean field theory

$$\chi_{ij} = \frac{\partial \langle \mu_i \rangle}{\partial h_j}$$







$$g(q=0) = \sum_{ij} \langle \mu_i \mu_j \rangle - \langle \mu_i \rangle \langle \mu_j \rangle$$

$$= \sum_{ij} \frac{\partial^2 \ln Z}{\partial h_i \partial h_j} = \sum_{ij} \frac{\partial \langle \mu_i \rangle}{\partial h_j} = \chi^{-1}$$

$$g(q) = \frac{1}{q^2}$$

$h_{ext} = 0$

$T > T_c$

$V_{eff} = \ln Z(M)$ $M \to M^*$

$$\frac{\partial \ln Z}{\partial h} = \langle M \rangle = M^* = m$$

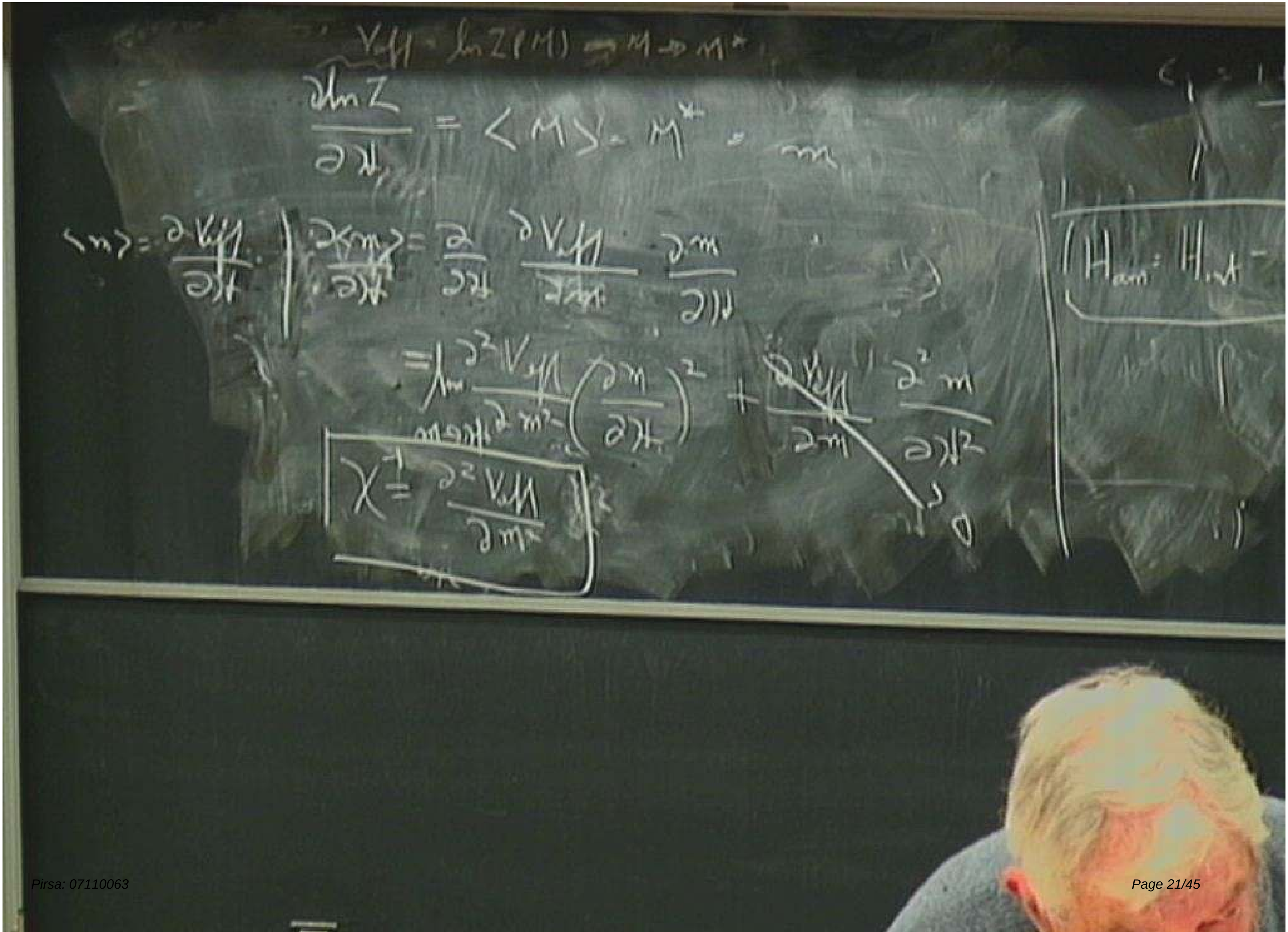
$$\langle m \rangle = \frac{\partial V_{eff}}{\partial h}$$

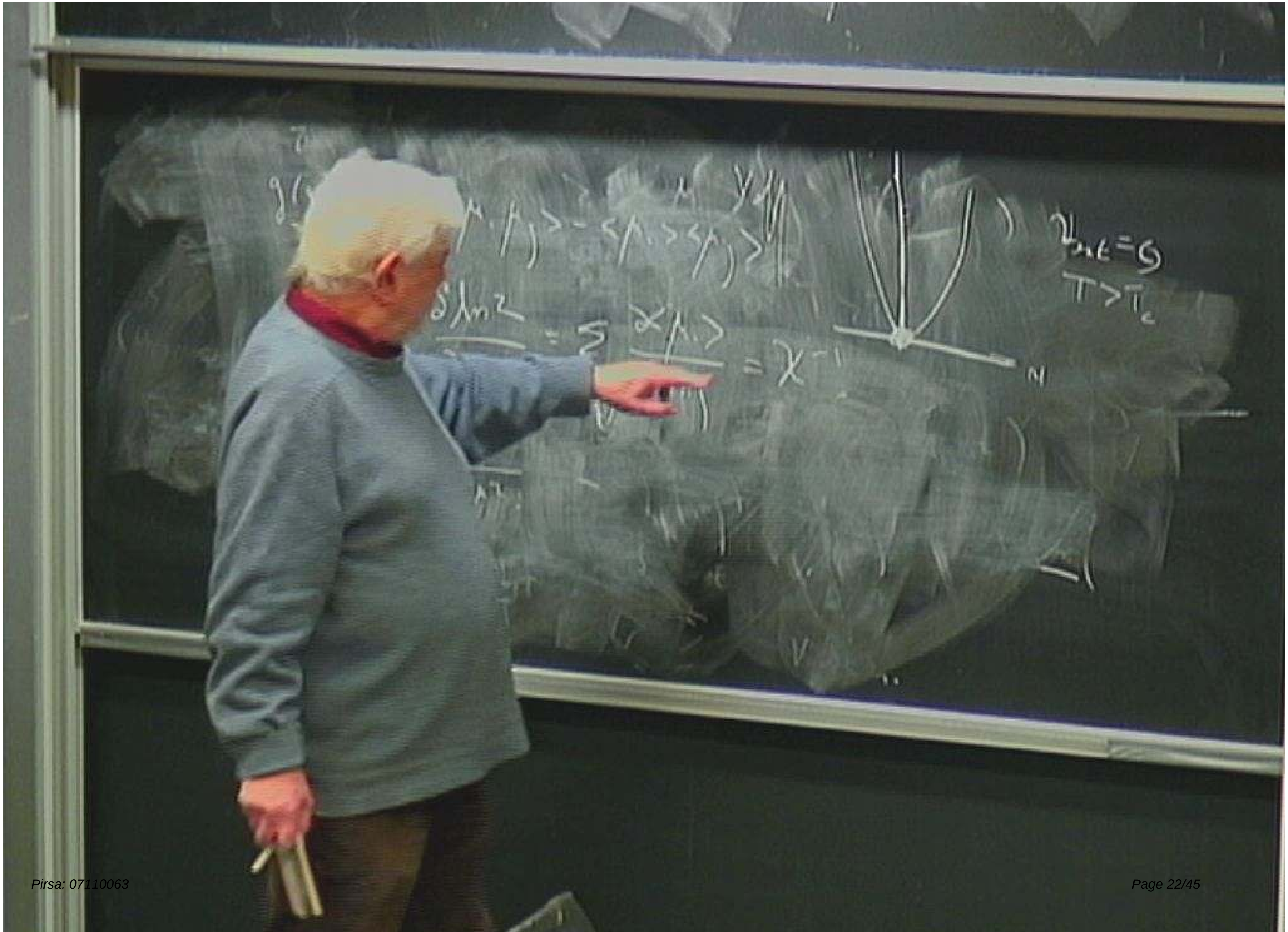
$$\frac{\partial \langle m \rangle}{\partial h} = \frac{\partial}{\partial h} \frac{\partial V_{eff}}{\partial m} \frac{\partial m}{\partial h}$$

$$= \frac{\partial^2 V_{eff}}{\partial m^2} \left(\frac{\partial m}{\partial h} \right)^2 + \cancel{\frac{\partial V_{eff}}{\partial m}} \frac{\partial^2 m}{\partial h^2} \to 0$$

$$\boxed{\chi^{-1} = \frac{\partial^2 V_{eff}}{\partial m^2}}$$

$(H_{em} = H_{int}$


$V_{eff} = \ln Z(M) \Rightarrow M \to M^*$
$\frac{\partial \ln Z}{\partial H} = \langle M \rangle = M^* = m$
$\langle m \rangle = \frac{\partial V_{eff}}{\partial H}$
$\frac{\partial \langle m \rangle}{\partial H} = \frac{\partial}{\partial H} \frac{\partial V_{eff}}{\partial m} \frac{\partial m}{\partial H}$
$= \frac{\partial^2 V_{eff}}{\partial m^2} \left(\frac{\partial m}{\partial H} \right)^2 + \cancel{\frac{\partial V_{eff}}{\partial m}} \frac{\partial^2 m}{\partial H^2}$
0
$\boxed{\chi^{-1} = \frac{\partial^2 V_{eff}}{\partial m^2}}$
$H_{am} = H_{int}$

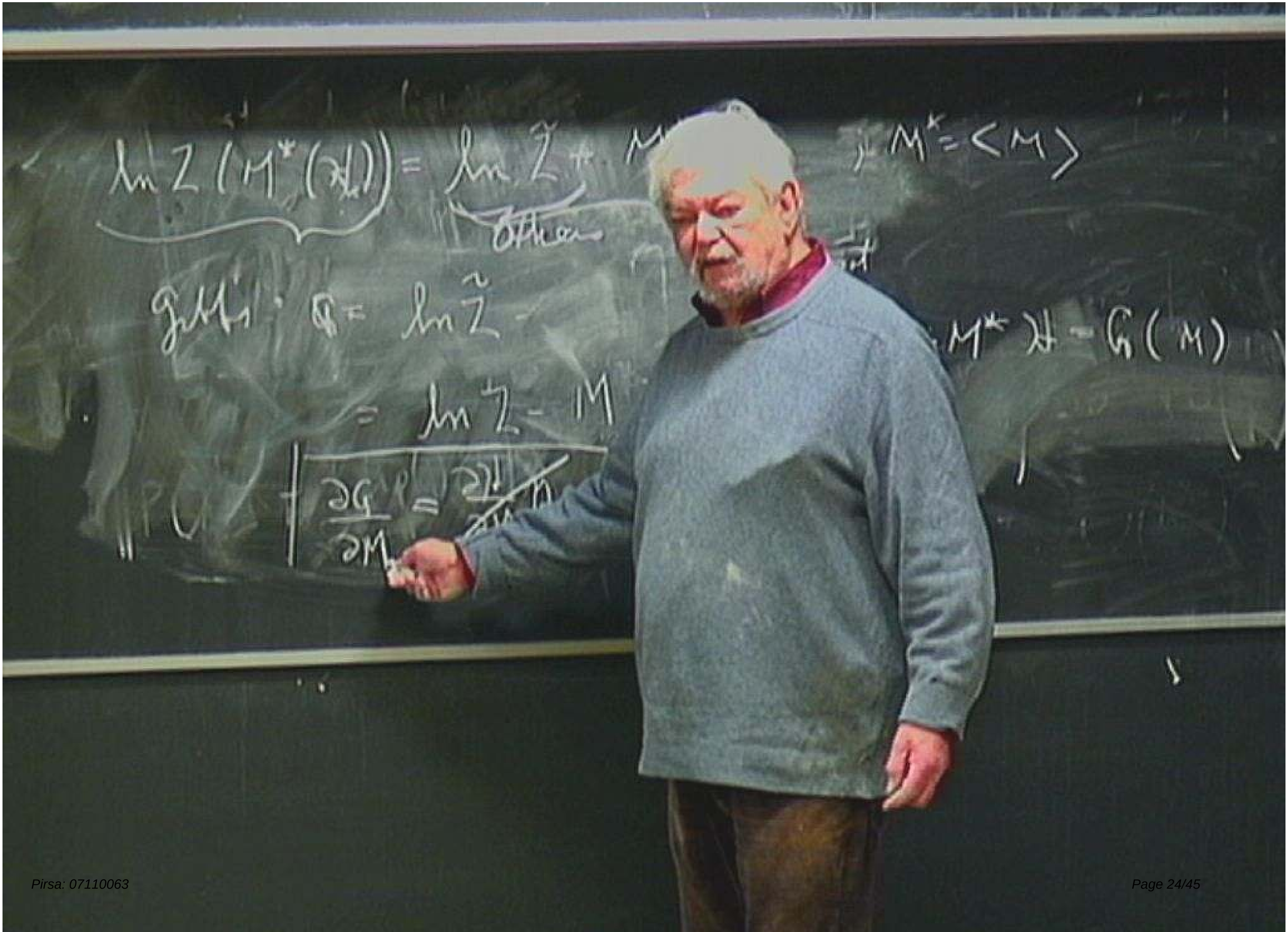


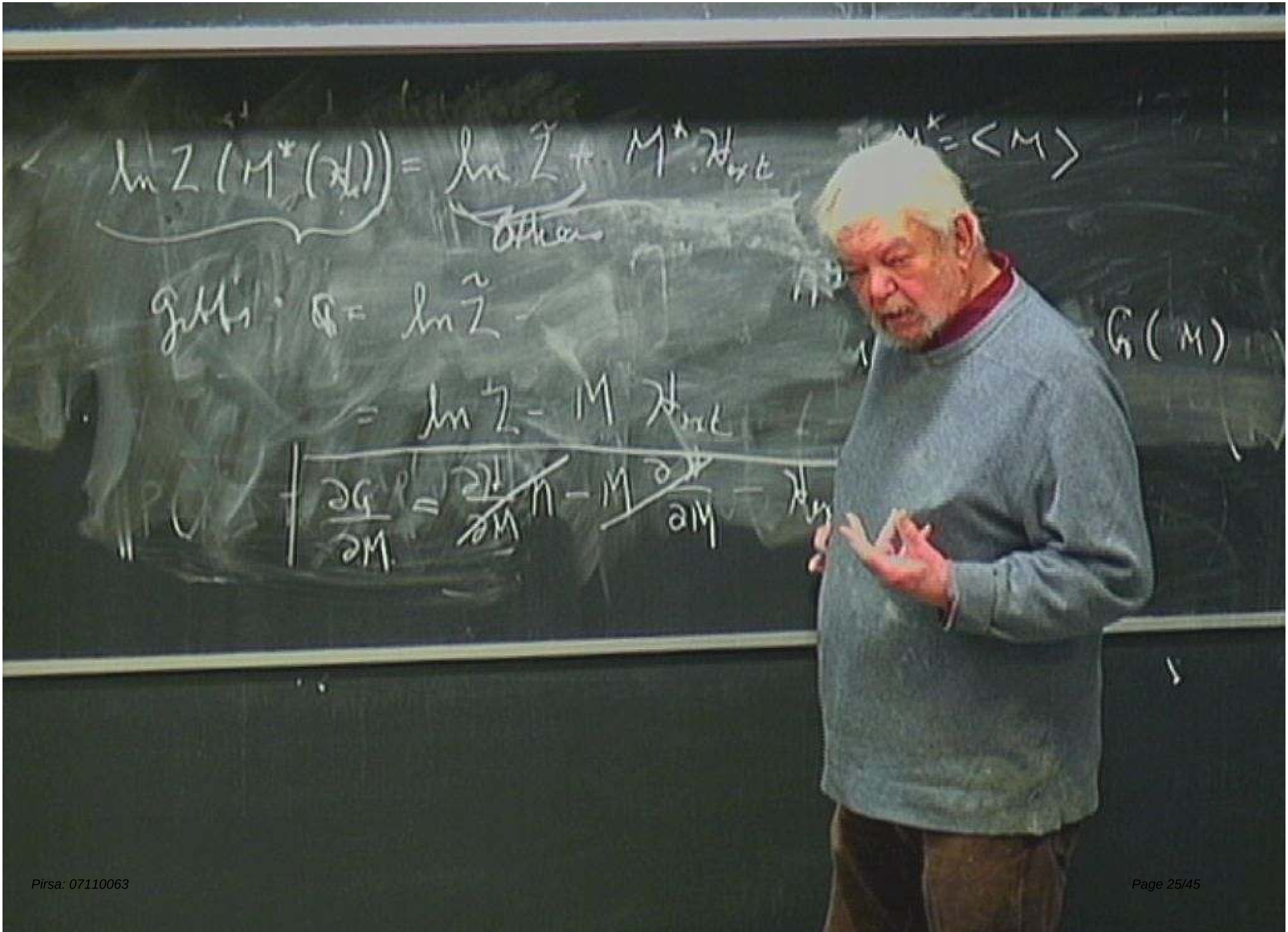
$$\ln Z(M^*(\mathcal{H})) = \underbrace{\ln Z}_{\text{Others}} + M^* \mathcal{H}_{ext} \quad ; \; M^* = \langle M \rangle$$

Gibbs: $G = \ln \tilde{Z}$

$$= \ln \tilde{Z} - M \mathcal{H}_{ext}$$

$$\ln Z = M^* \mathcal{H} - G(M)$$

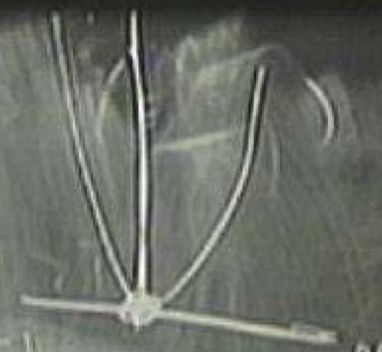




$$g(r_{ij}=0) = \sum_{ij} \langle \mu_i \mu_j \rangle - \langle \mu_i \rangle \langle \mu_j \rangle$$

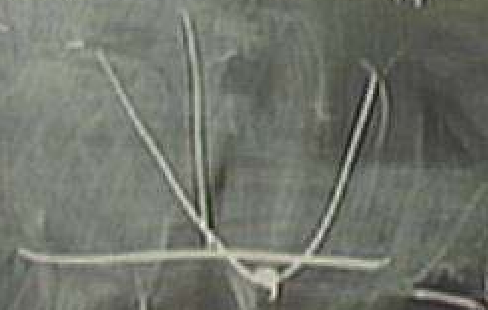
$$= \sum_{ij} \frac{\partial^2 \ln Z}{\partial h_i \partial h_j} = \sum_{ij} \frac{\partial \langle \mu_i \rangle}{\partial h_j} = \chi^{-1}$$

$$g(r) = \frac{1}{r^{\cdots}}$$



$h_{ext} = 0$

$T > T_c$

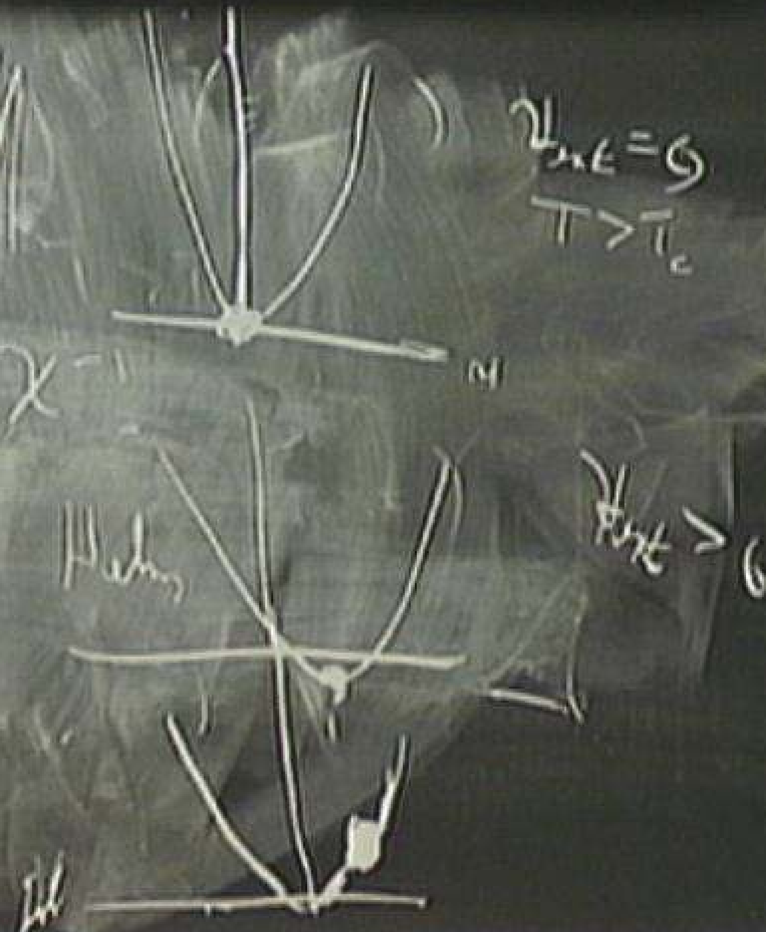


$h_{ext} > 0$

$$g(q=0) = \sum_{ij} \langle \mu_i \mu_j \rangle - \langle \mu_i \rangle \langle \mu_j \rangle$$

$$= \sum_{ij} \frac{\partial^2 \ln Z}{\partial h_i \partial h_j} = \sum_{ij} \frac{\partial \langle \mu_i \rangle}{\partial h_j} = \chi^{-1}$$

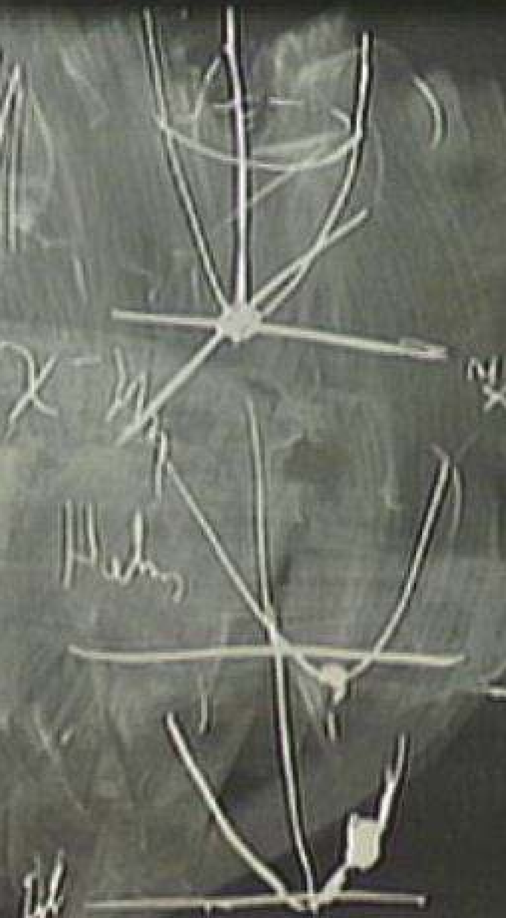
$$g(q) = \frac{1}{\mu^2}$$



$$g(q=0) = \sum_{ij} \langle \mu_i \mu_j \rangle - \langle \mu_i \rangle \langle \mu_j \rangle$$

$$= \sum_{ij} \frac{\partial^2 \ln Z}{\partial h_i \partial h_j} = \sum_{ij} \frac{\partial \langle \mu_i \rangle}{\partial h_j} = \chi$$

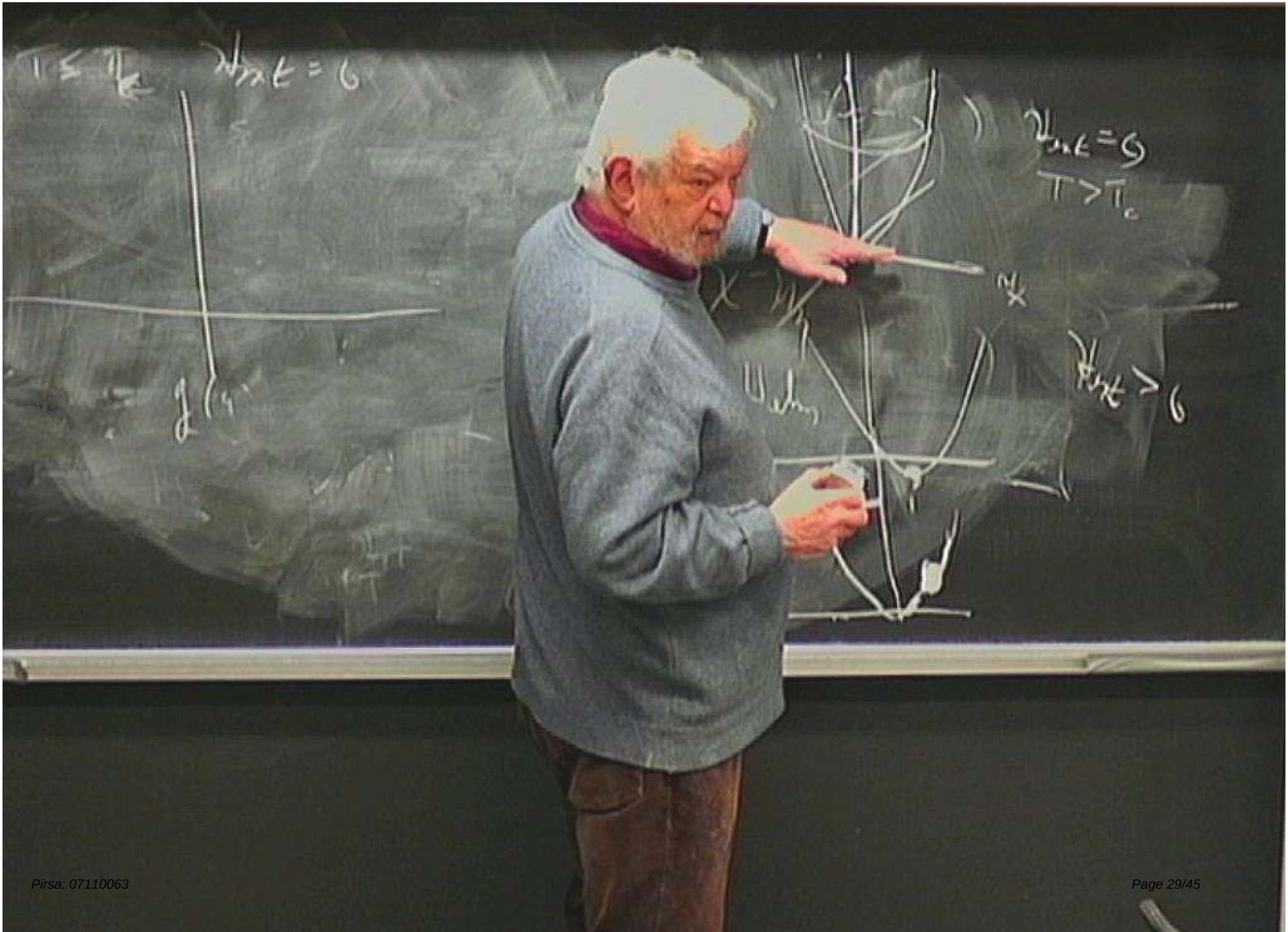
$$g(q) = \frac{1}{q^2}$$

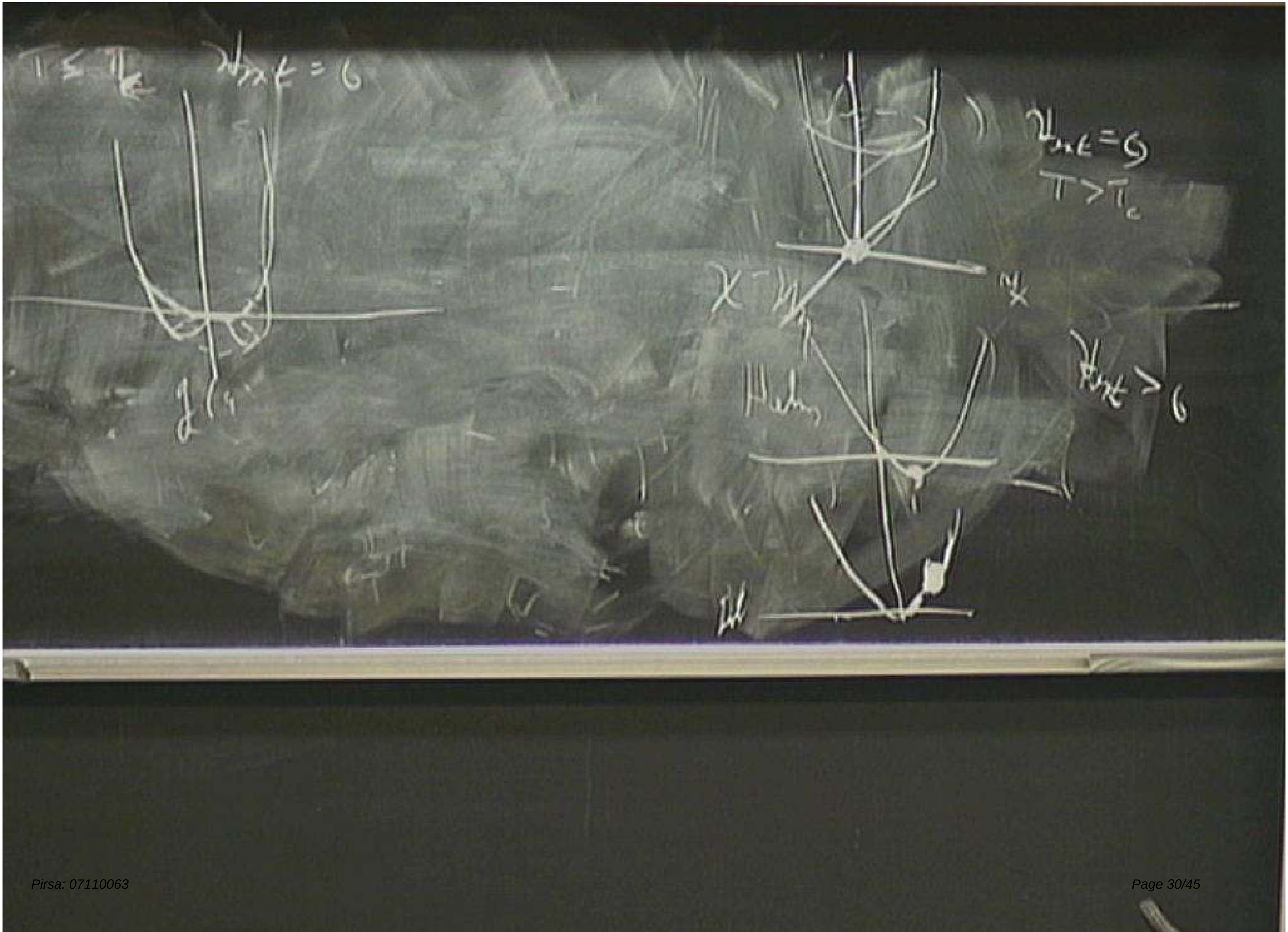


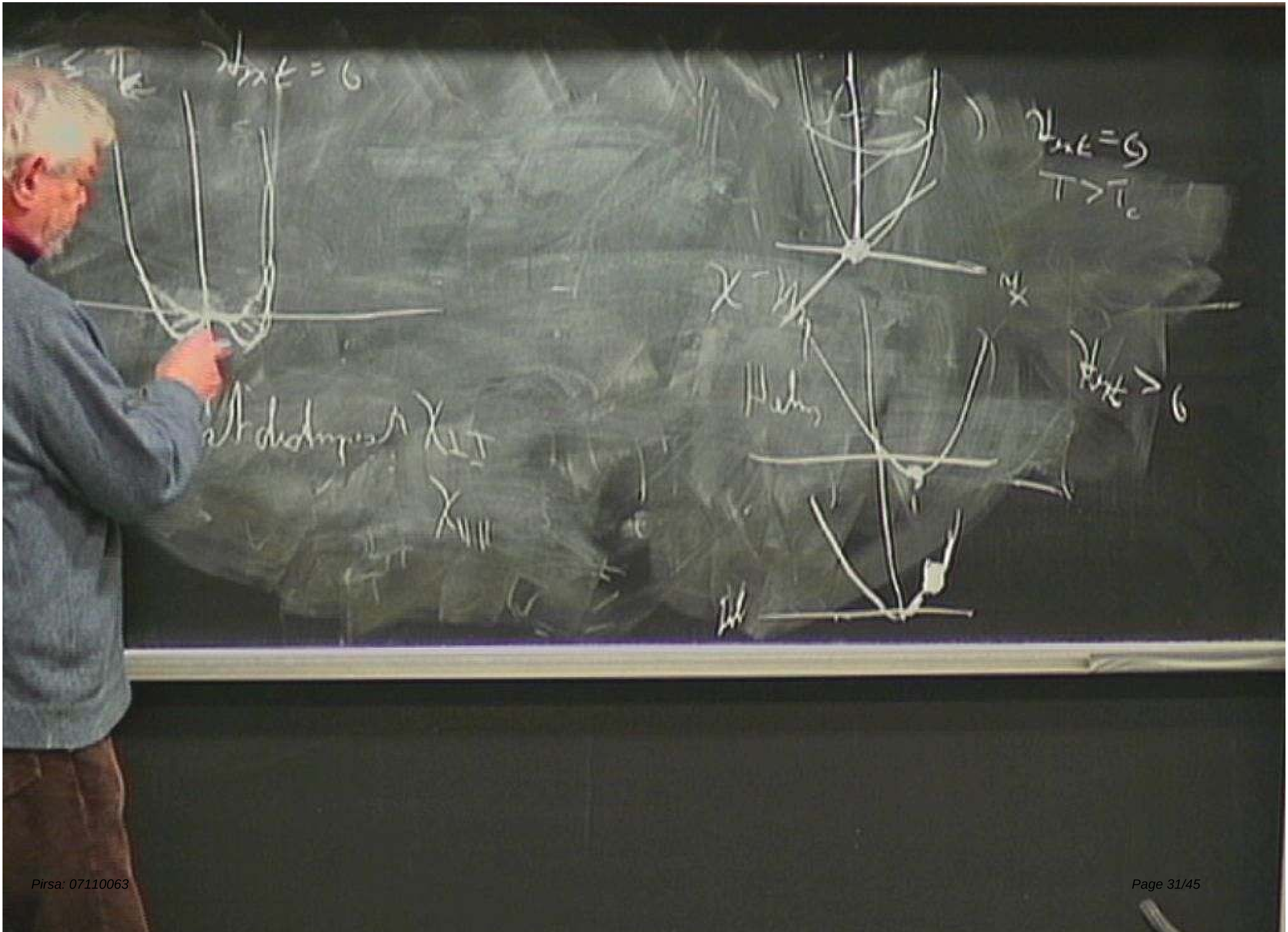
$h_{ext} = 0$

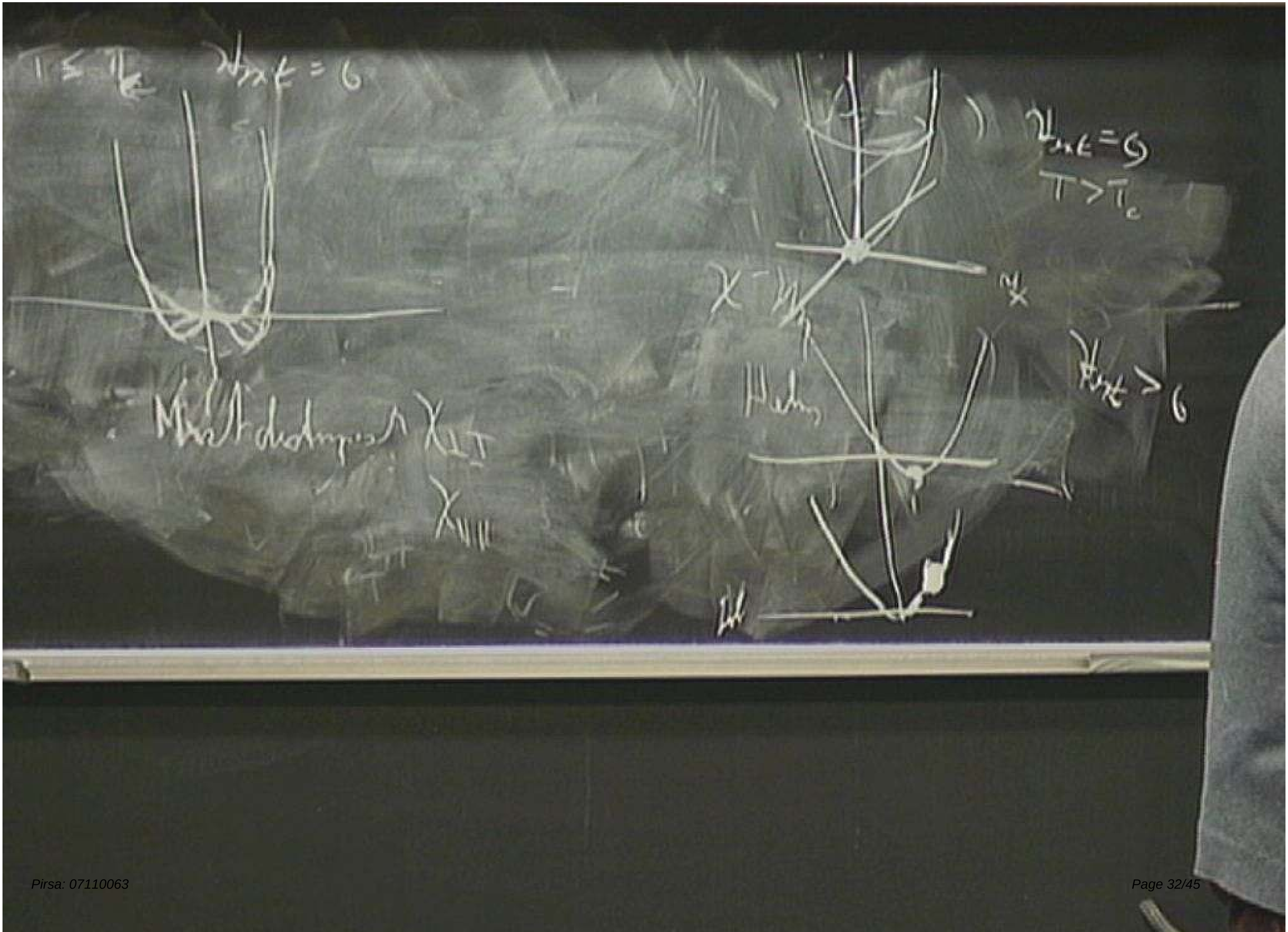
$T > T_c$

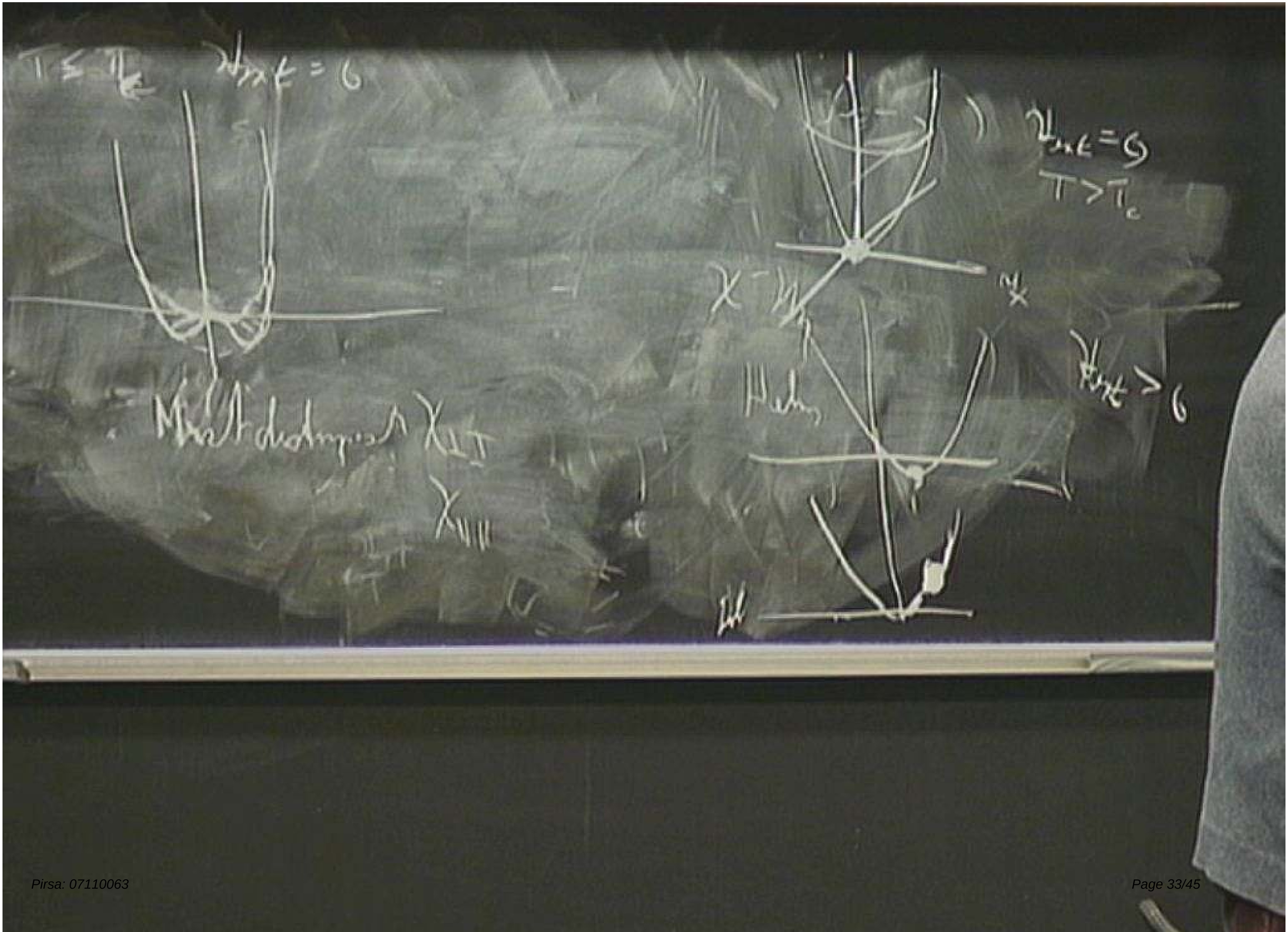
$h_{ext} > 0$

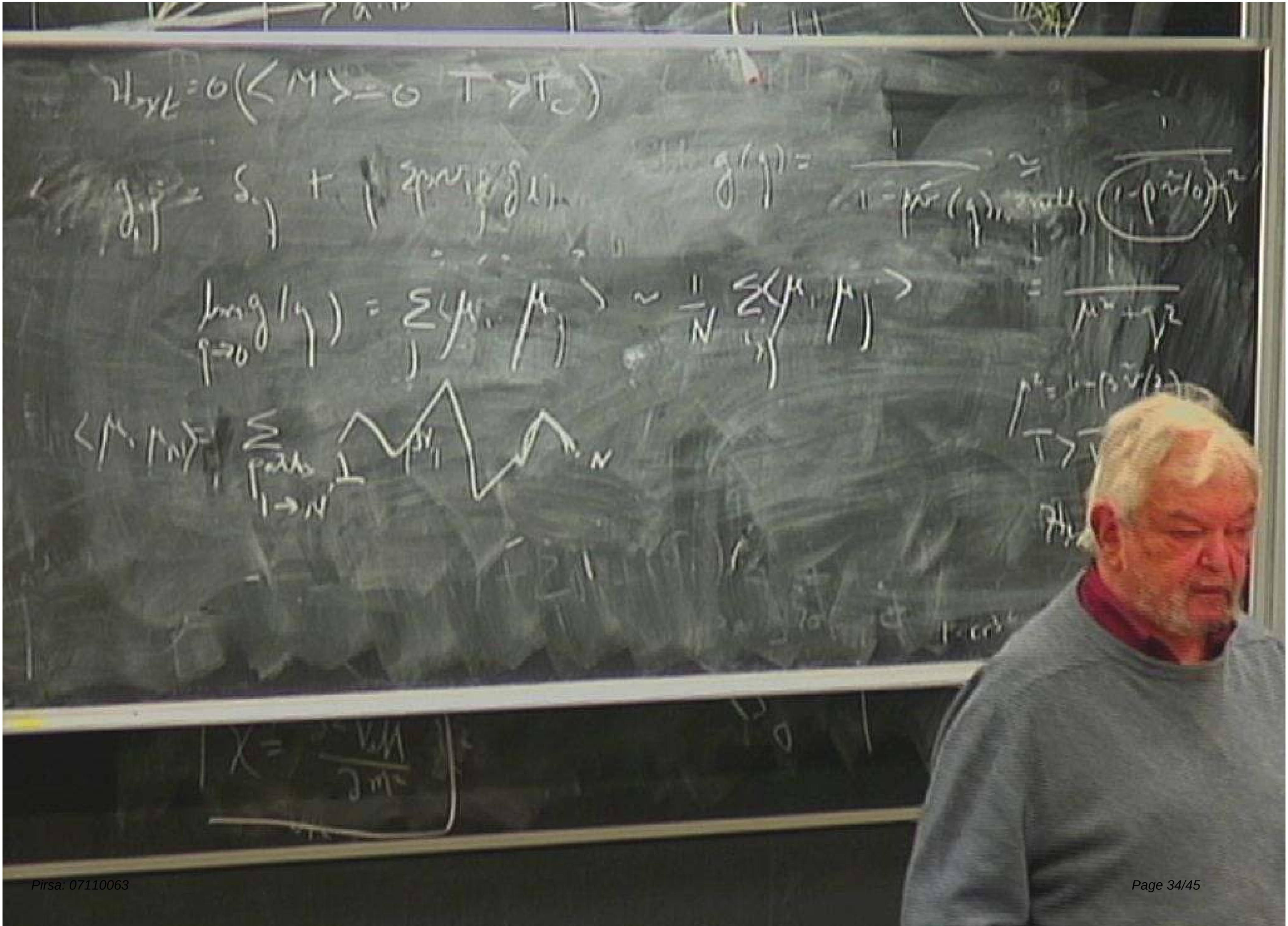


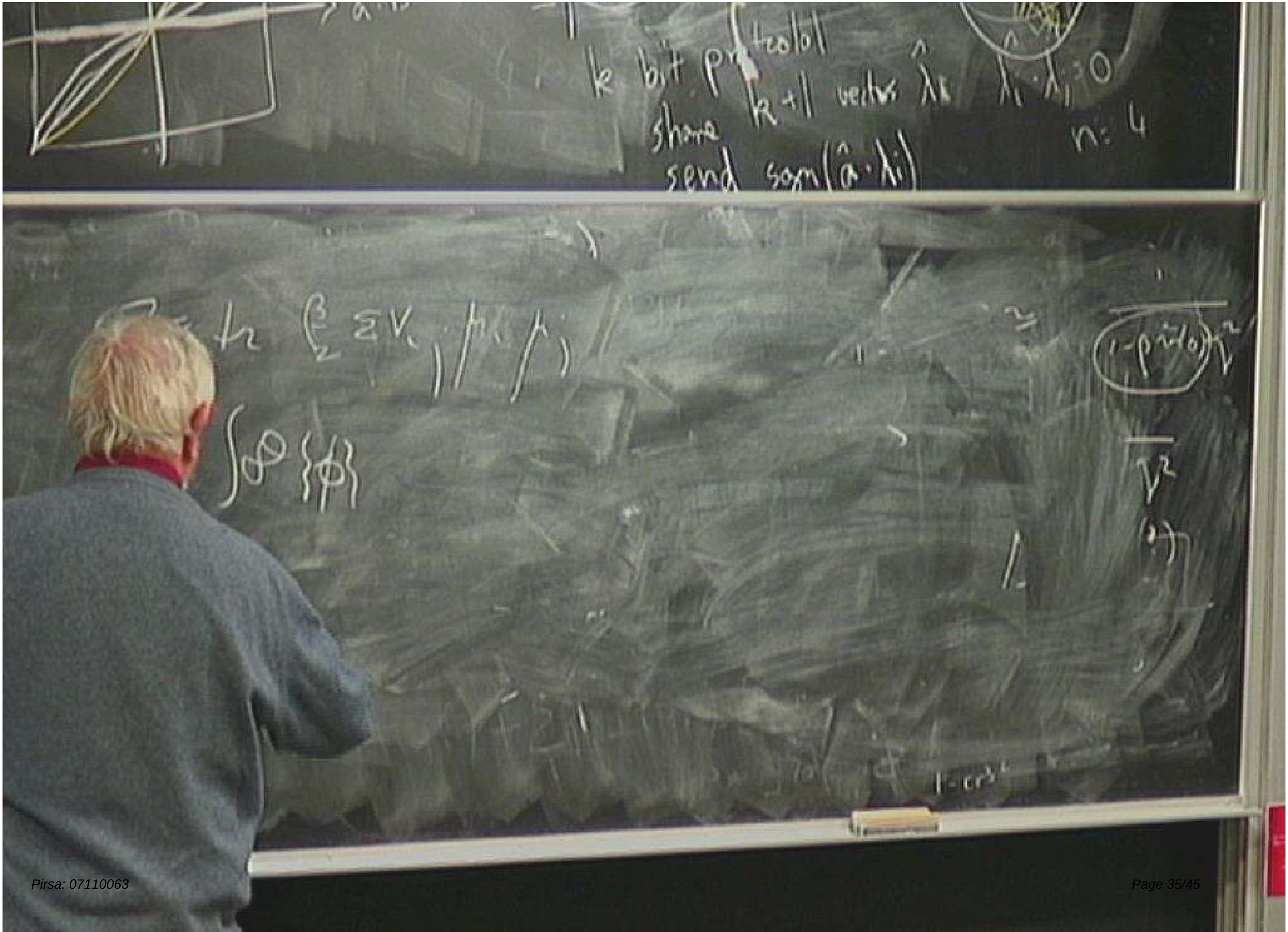


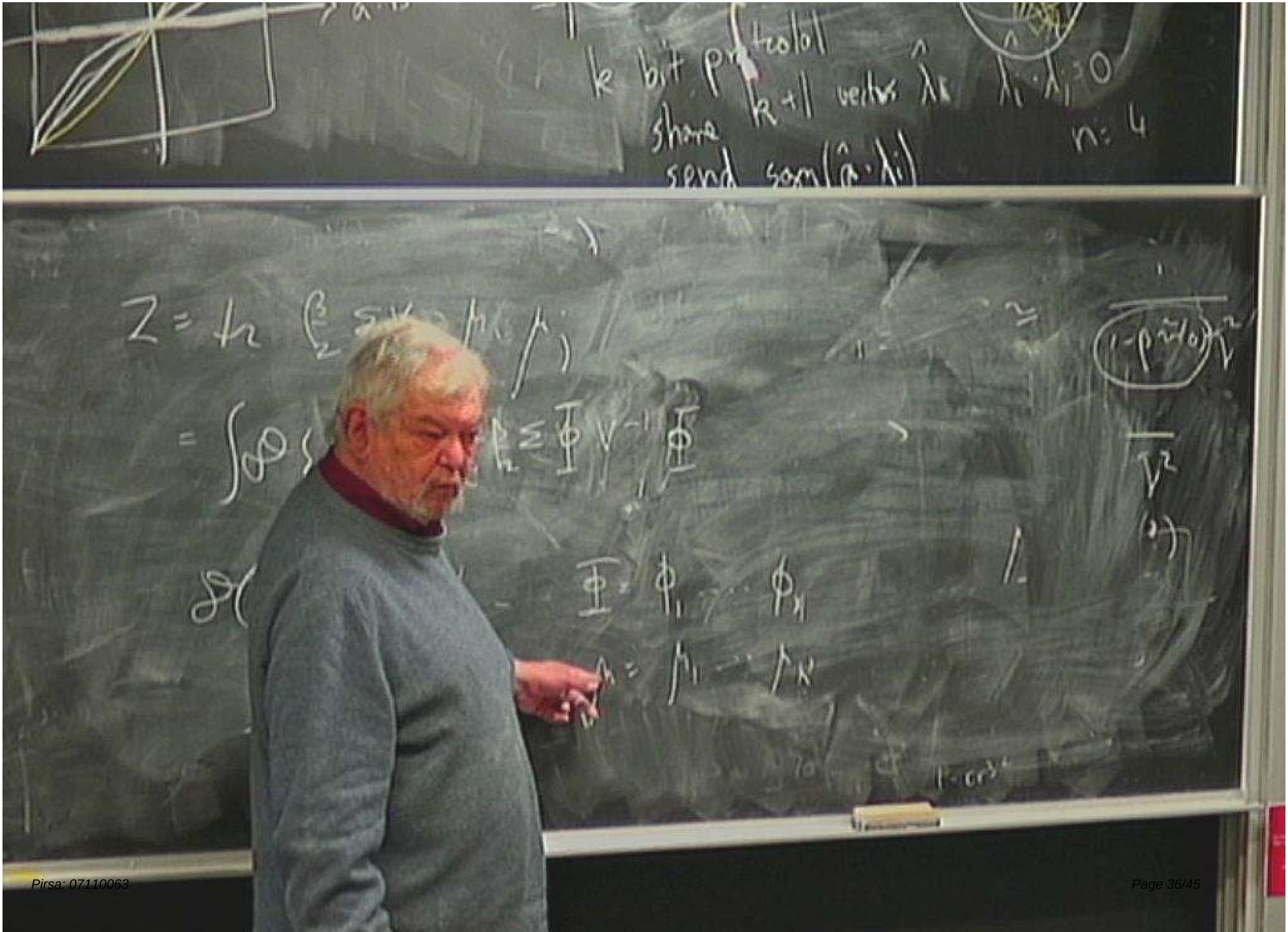

T > T_c

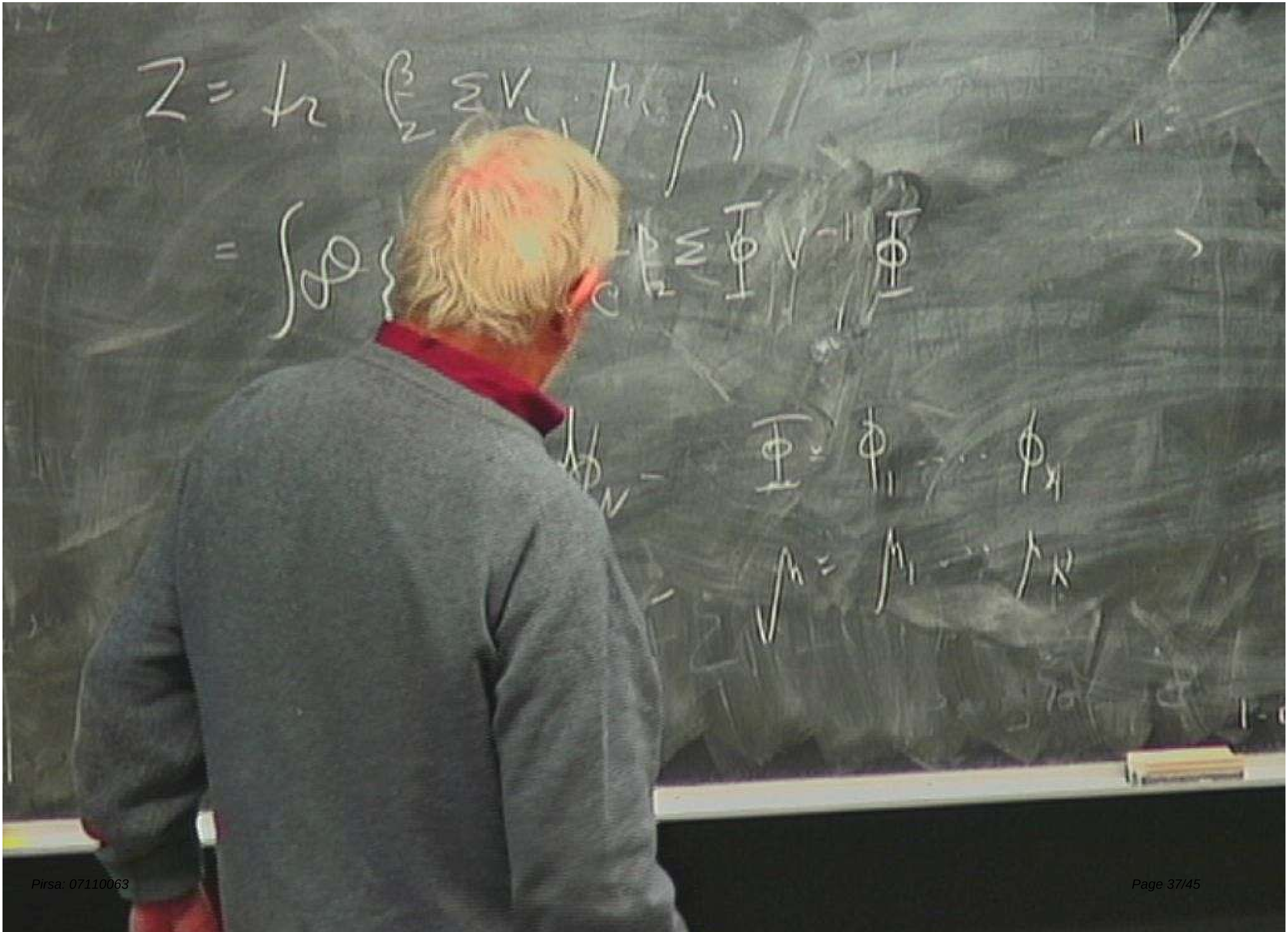

T > Tc

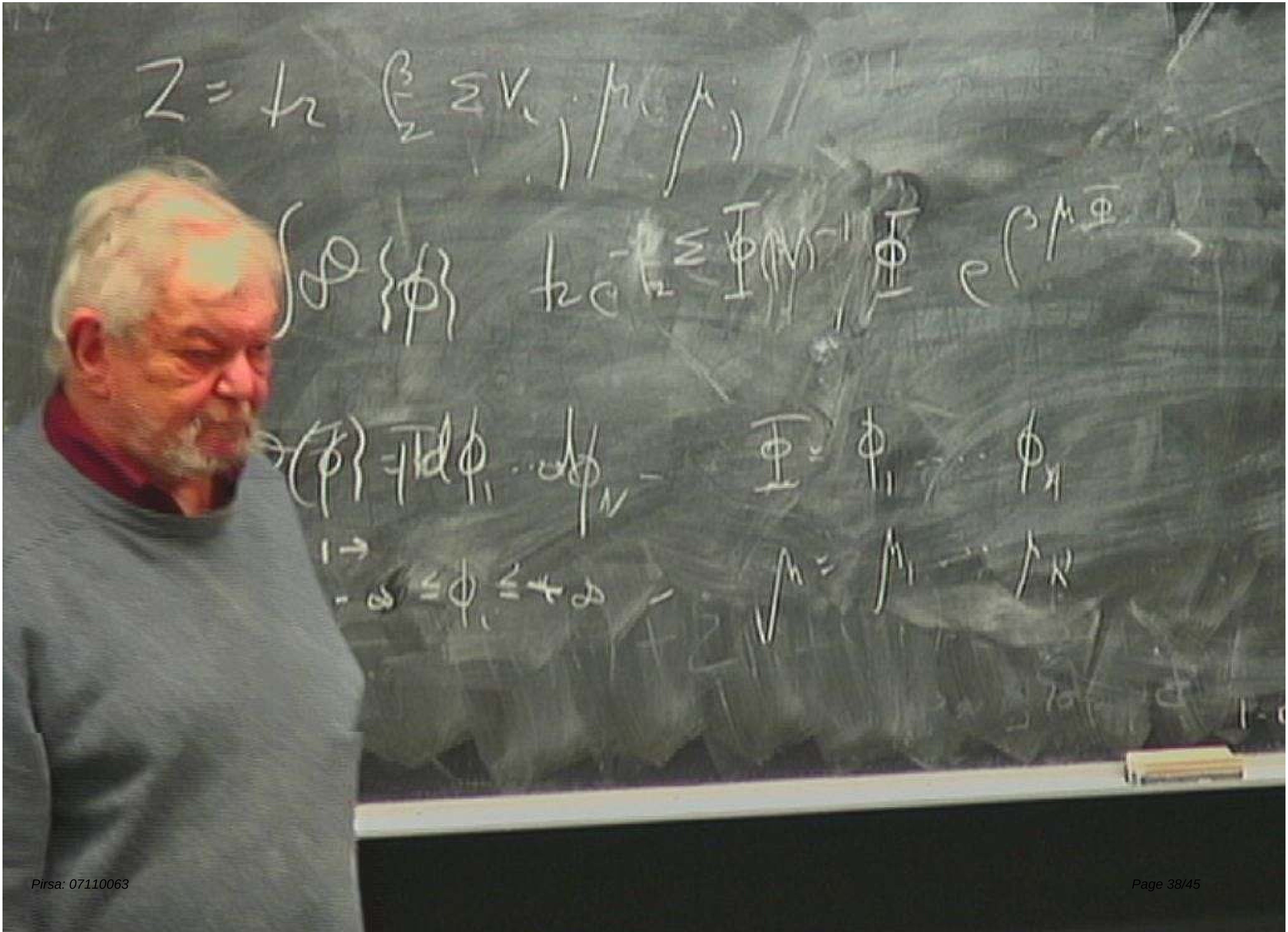

T ≤ Tc
T > Tc

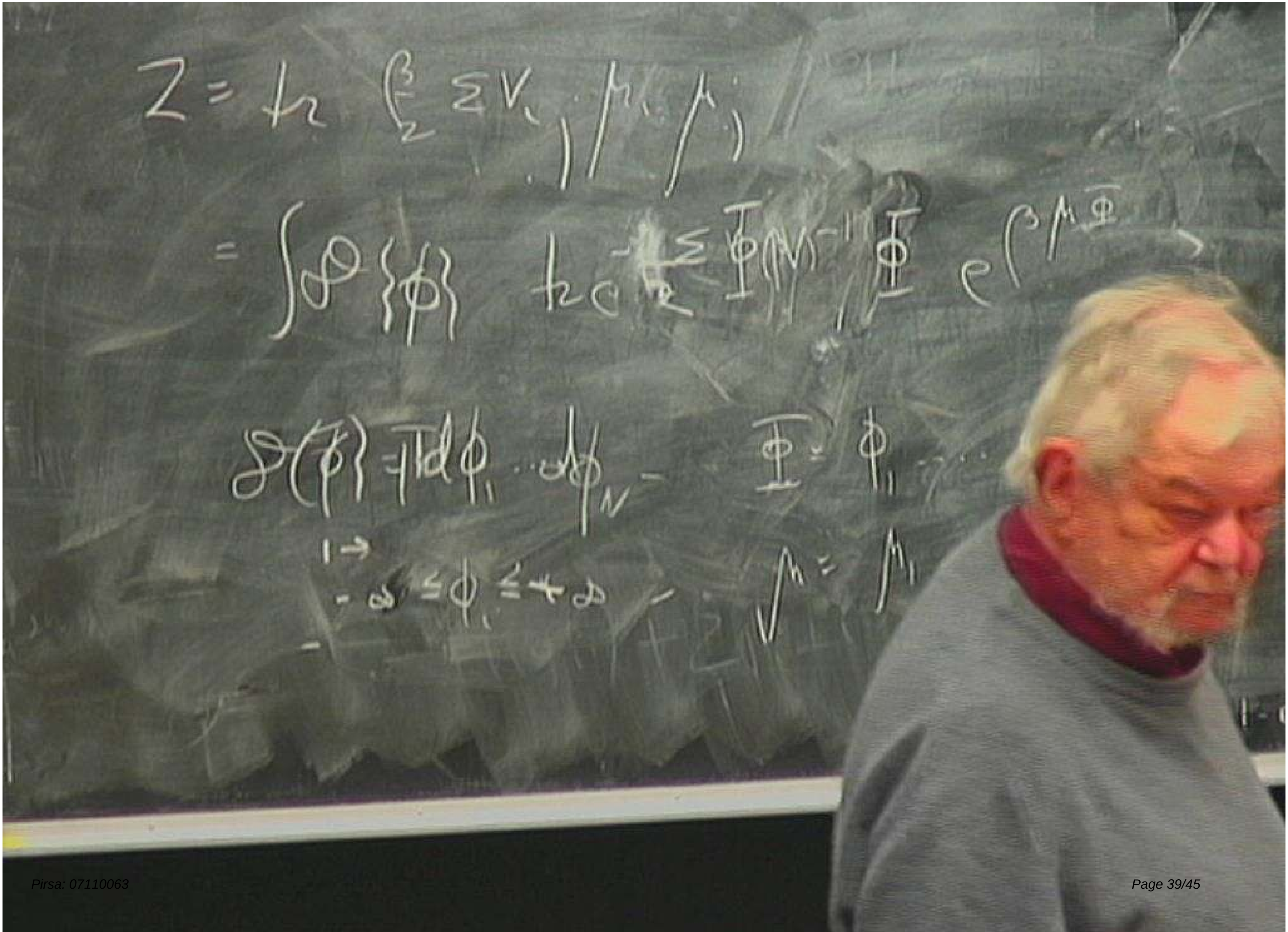


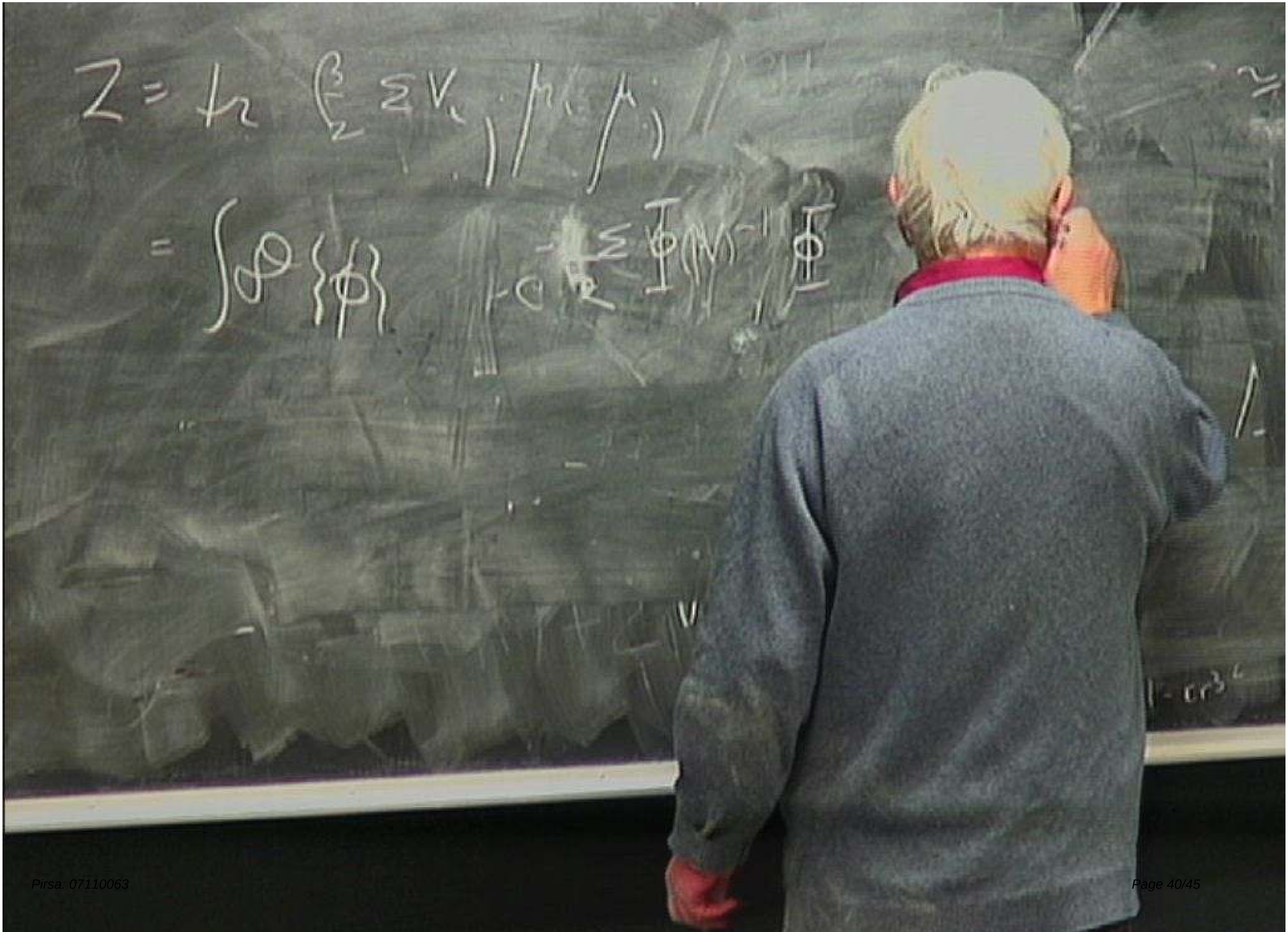

k bit protocol
share
k+1 vectors
send sgn
n: 4

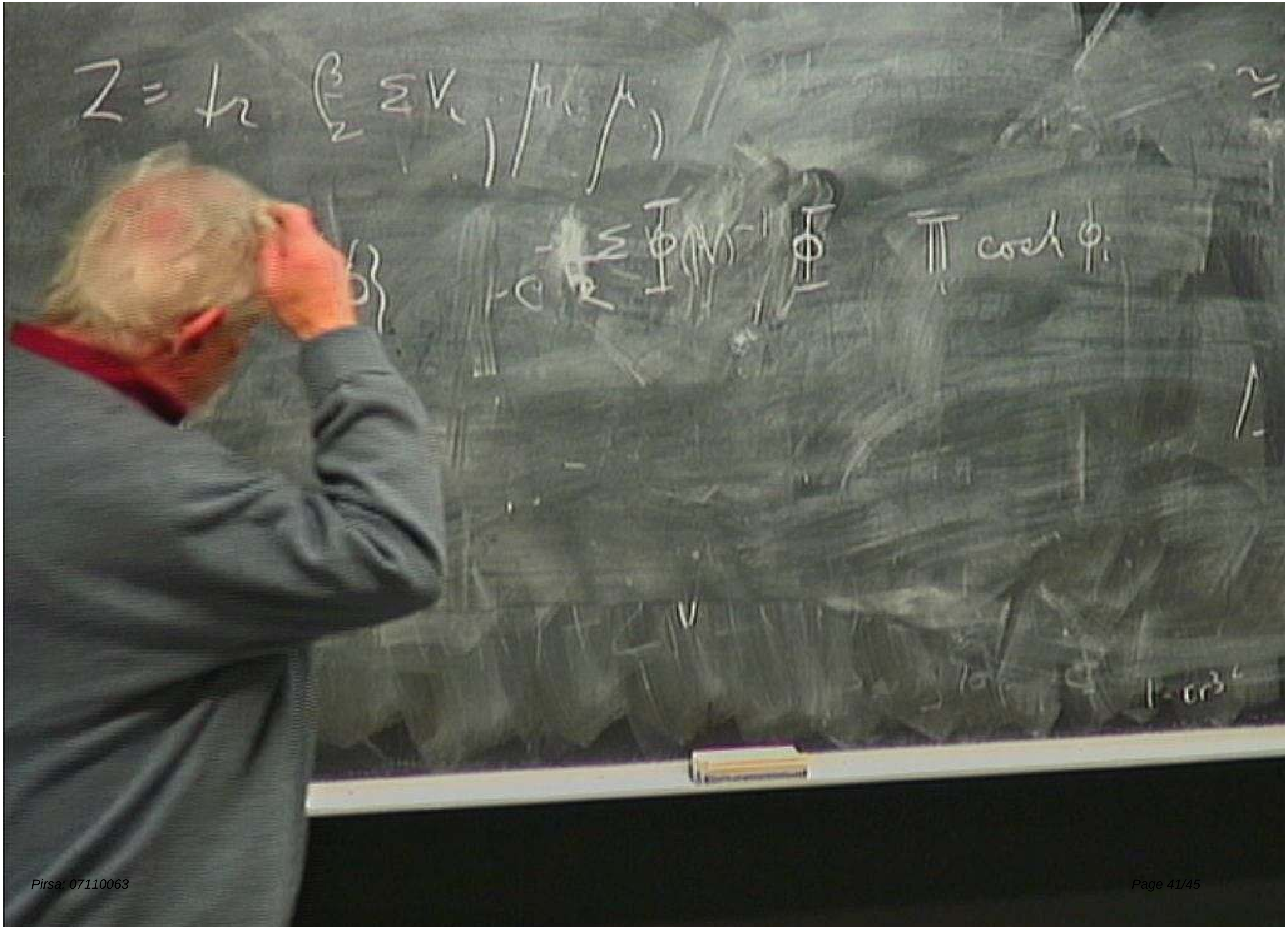

k bit protocol
share
send
n: 4

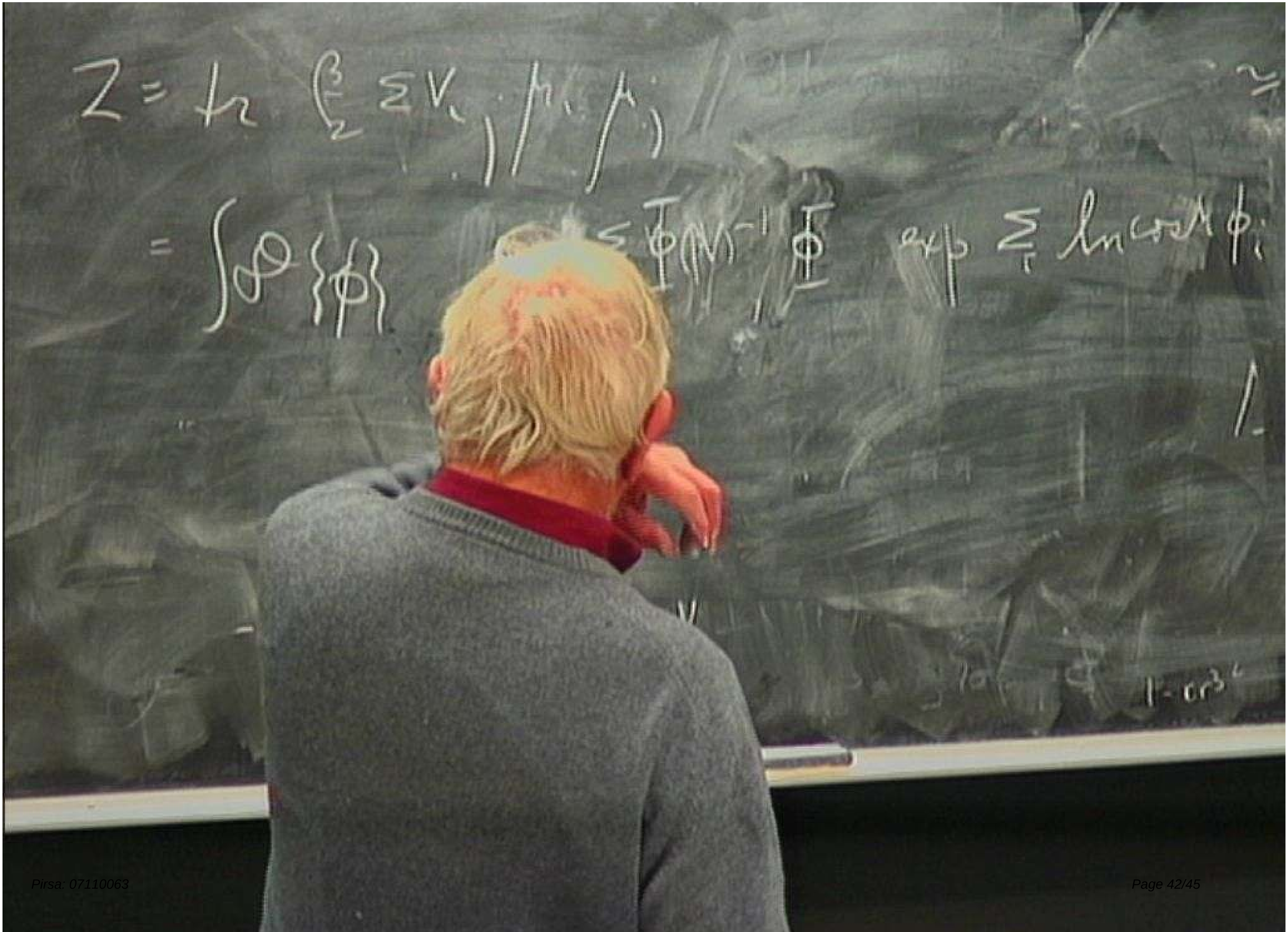


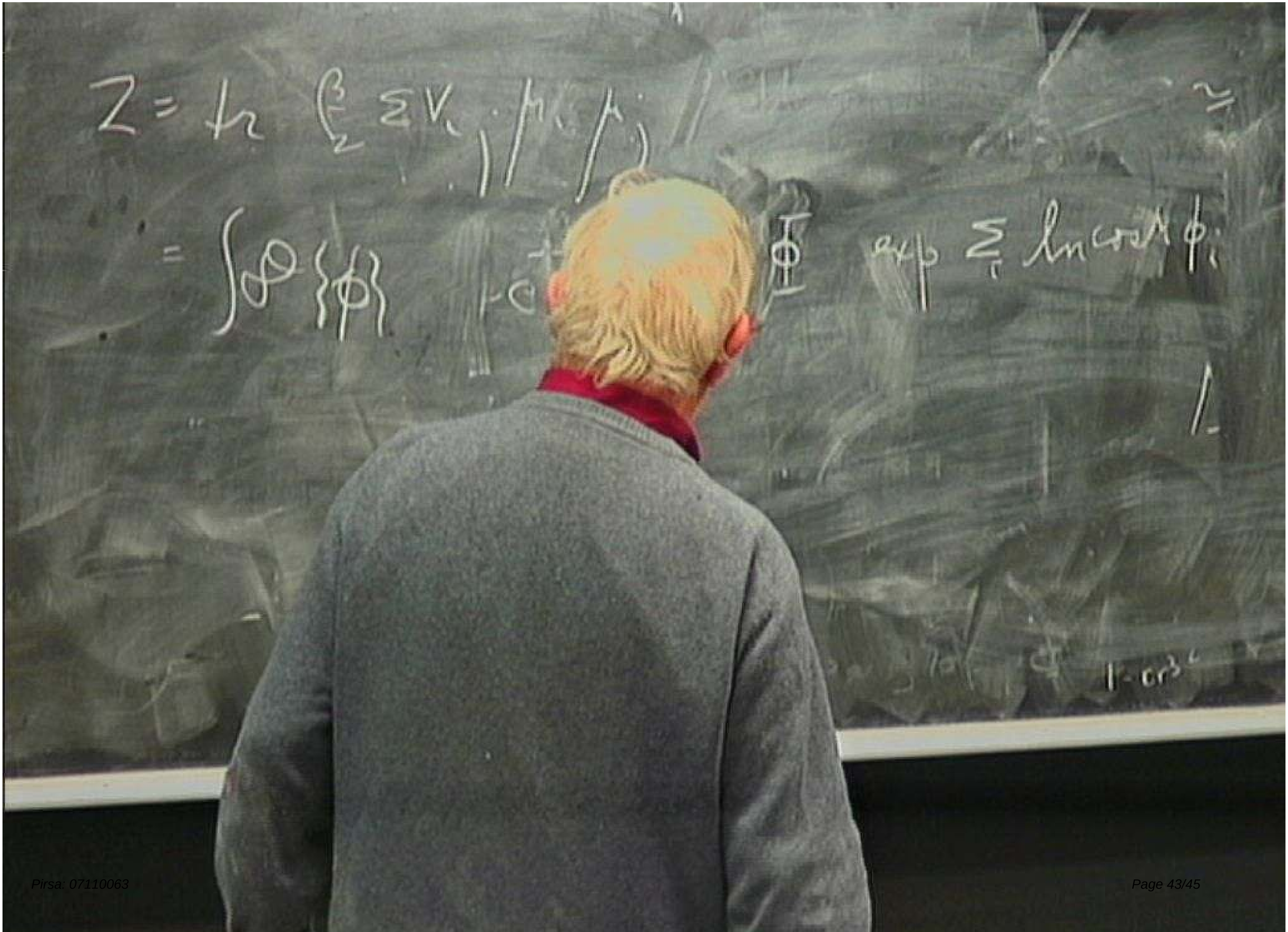


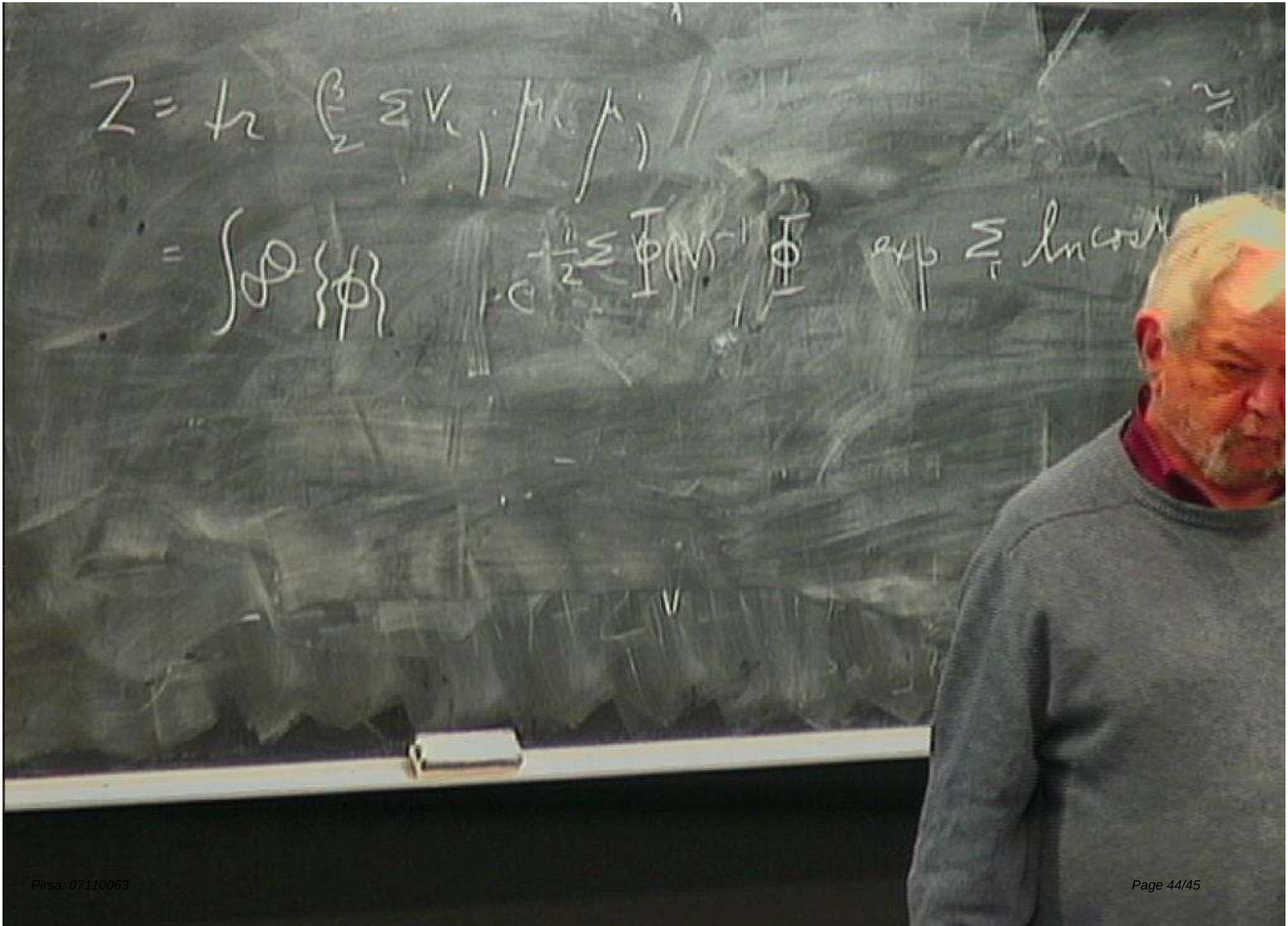


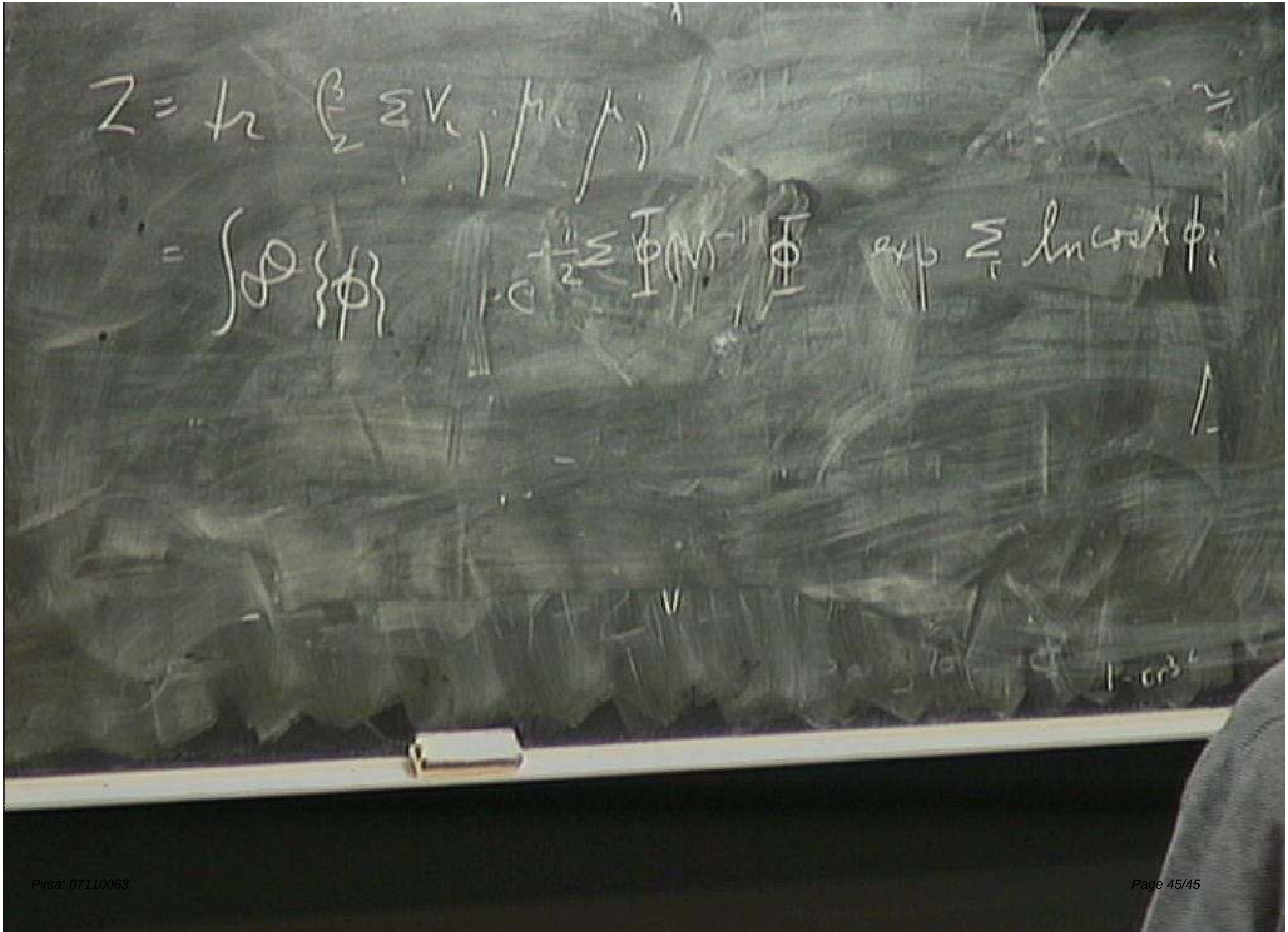











Z = tr
exp Σ ln cosh φ