Title: Phenomenology of Discrete Space: Possible Tests

Date: Nov 07, 2007 05:00 PM

URL: http://pirsa.org/07110051

Abstract: I will discuss possible tests of the grainularity of space including modified dispersion relations in the formation of white dwarfs and neutron stars and constraints on a stochastic direction field from atomic system tests.

Physics of Quantum Gravity?

Outline:

- Modified dispersion relations - threshold analysis
- Mass limit for white dwarfs (work in progress)
- Phenomenological modeling of discrete spatial geometry (work in progress)
- Loopy effects in quantum cosmology and primordial GW background (work in progress)

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Physics of Quantum Gravity?

Quantum geometry affects the propagation of fields:

Alfaro, Morales-Tecotl, Urrutia PRD 65 (2002) 103509; 66 (2002) 124006] suggested that states in LQG might modify the classical equations of motion e.g. "would be **semiclassical states**" defined by:

1. A characteristic scale $L >> \ell_p$
   For scales $>> L$ flat, continuous space
   For scales $<< L$ quantum geometry
2. Peaked on flat geometry and flat connections
3. Well-defined expectation values $\left\langle \hat{H}_{matter} \right\rangle$

Computed the ℓ_p/L expansion of $\left\langle \hat{H}_{matter} \right\rangle$

Physics of Quantum Gravity?

This expansion gives **Modified Dispersion Relations (MDR)**:
In the high energy limit:

- For **fermions**:

$$E_{\pm}^2 = p^2 + m^2 + \kappa_1 \left(\frac{\ell_p}{L}\right)^{\Upsilon+1} p^2 \mp \kappa_7 \left(\frac{\ell_p}{L}\right)^{\Upsilon} \frac{\ell_p^2 p^3}{L}$$

- For **photons**:

$$\omega_{\pm}^2 = k^2 + \theta_7 \left(\frac{\ell_p}{L}\right)^{2+2\Upsilon} k^2 \pm \theta_8 \ell_p k^3$$

Model parameters: κ, θ, and Υ are undetermined

Modified Dispersion Relations (MDR)

Nota Bene:

- **State is not Lorentz Invariant !**
- There is a preferred frame.
- The constants Υ, L are not fixed by the state
- Choice in L ??
- one choice is $L = 1/p$

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$$\Longrightarrow \boxed{E^2 = p^2 + m^2 + \kappa \frac{p^3}{E_P}}$$

- effects important when $p_{crit} \approx (m^2/\ell_p)^{1/3} \sim 10^{13}$ eV for electrons
- model limited by $p << E_P$

4-a

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How is it possible that a quantization of GR (LLI) gives modifications to LI?

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4-b

Process Thresholds with MDR

Jacobson, Liberati, Mattingly hep-ph/0110094; T. Konopka,SM New J. Phys. **4** (2002) 57

- κ order unity
- κ is positive or negative
- There is a preferred frame ! Special Relativity is modified!
- Effects important when $E_{crit} \approx (m^2 E_p)^{1/3} \sim 10^{13}$ and 10^{15} eV for electrons and protons
- Model limited by $p << E_P$

MDR take the leading order form

$$\boxed{E \approx p + \frac{m^2}{2p} + \kappa \frac{p^2}{2E_P}}$$

for $m << p << E_P$

DSR and Threshold Analysis

D. Heyman, F. Hinteleitner, SM PRD 69 (2004) 105016

Now energy-momentum relations are modified. Analysis simplified in Judes-Visser variables. At root the symmetry of SR is deformed so, not surprisingly there are no new threshold phenomenon. Instead thresholds are shifted.

Two incoming particles with masses m_1 and m_2, resulting in N outgoing particles with masses ($M := \sum_{i=3}^{N+2} m_i$ and $M^{(2)} = \sum_{i=3}^{N+2} m_i^2$). The SR threshold in the CM frame

$$E^*_{\mathrm{SR}} = \frac{m_1^2 - m_2^2 + M^2}{2M}.$$

Assuming that the composite particle relations do not differ significantly (!)

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$$E^*_{\mathrm{SR}} = \frac{m_1^2 - m_2^2 + M^2}{2M}.$$

Assuming that the composite particle relations do not differ significantly (!) The first order correction is, for Magueijo-Smolin DSR

$$E^* \approx E^*_{\mathrm{SR}} \left[1 - \lambda \left(E^*_{\mathrm{SR}} - \frac{4M(m_1^3 - m_2^3) - 2M^{(2)}(m_1^2 - m_2^2) + 2M^2 M^{(2)} - M^4}{2M(m_1^2 - m_2^2 + M^2)} \right) \right].$$

6-a

DSR and Threshold Analysis

The GZK process $p\gamma \to p\pi$ leading to

$$E^*_{\mathrm{SR}} = \frac{(m_p + m_\pi)^2 + m_p^2}{2(m_p + m_\pi)}.$$

For Magueijo-Smolin DSR

$$E^* = \frac{(\mu_p + \mu_\pi)^2 + \mu_p^2}{2(\mu_p + \mu_\pi) + \lambda[(\mu_p + \mu_\pi)^2 + \mu_p^2]}$$

To first order in λ this is

$$E^*_{\mathrm{ISR}} \approx E^*_{\mathrm{SR}} - \lambda \frac{m_\pi^2(6m_p^2 - m_\pi^2)}{4(m_p + m_\pi)^2},$$

apparently lowers the GZK threshold. Likewise for Amelino-Camelia DSR

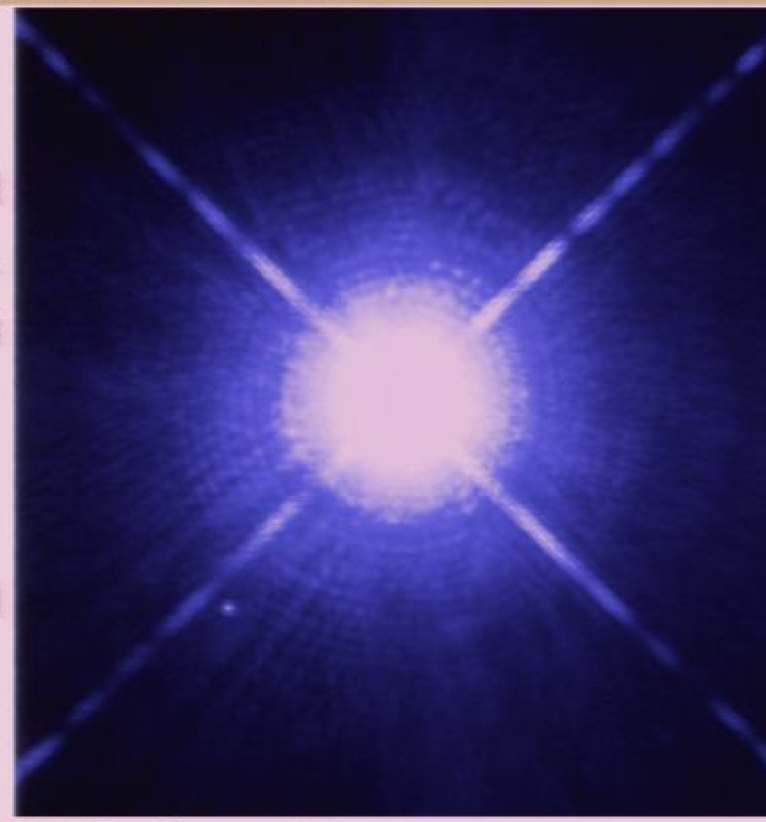
$$E^* \approx E^*_{\mathrm{SR}} - \frac{\lambda}{2}((E^*_{\mathrm{SR}})^2 - m_p^2)$$

MDR and Chandrasekhar Mass

Gravitational attraction supported by electron degeneracy pressure. Chandrasekhar found a maximum mass for white dwarfs

$$M_{ch} = 1.43 M_{SUN}.$$

See, T. Padmanabhan, Theoretical Astrophysics Vol II



Hubble Space Telescope

MDR and Chandrasekhar Mass

- What is the result for MDR? As in the threshold analysis, $p >> m$ and $p << E_P$ and

$$E^2 = p^2 + m^2 + \xi \frac{p^3}{E_P}$$

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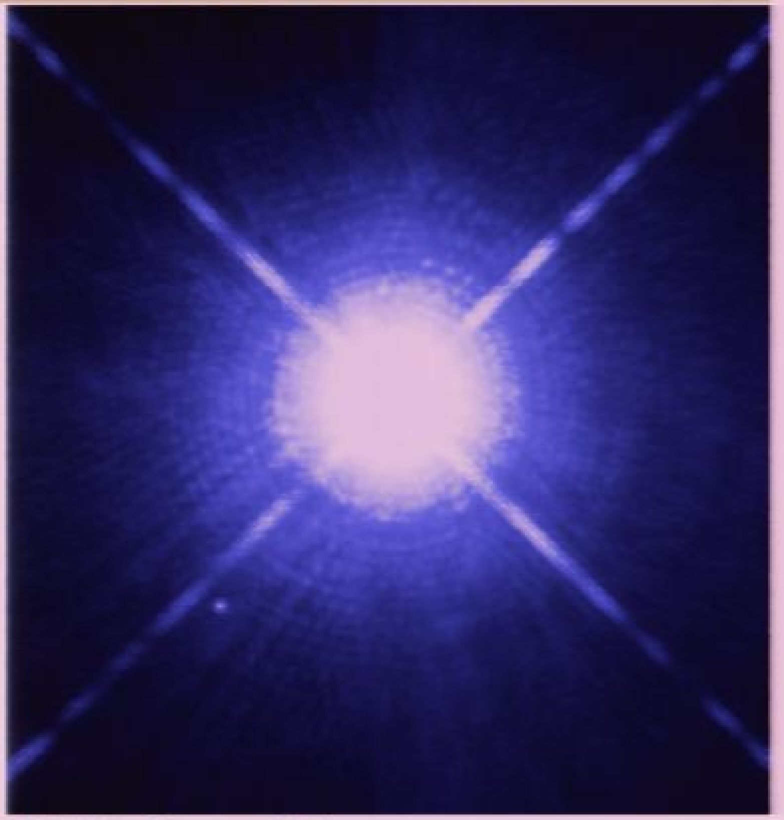

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$$P = \frac{8\pi m_e^4}{3h^3} \int_0^{pF} \frac{x^4 \left(1 + \frac{3}{2}\delta x\right)}{\sqrt{1 + x^2 + \delta x^3}} dx$$

with $\delta = \xi m_e / E_P \sim 10^{-23}$.

9-a

MDR and Chandrasekhar Mass

The density of the star ρ, $\rho \propto p^3$, or

$$\rho = 9.8 \times 10^5 x^3 gcm^{-3}$$

Equilibrium occurs when the degeneracy pressure gradient balances the gravitational attraction

$$\frac{dP}{dr} = -\frac{G\rho(r)m(r)}{r^2}$$

Or,

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho(r)}\frac{dP}{dr}\right) = -4\pi G\rho(r)$$

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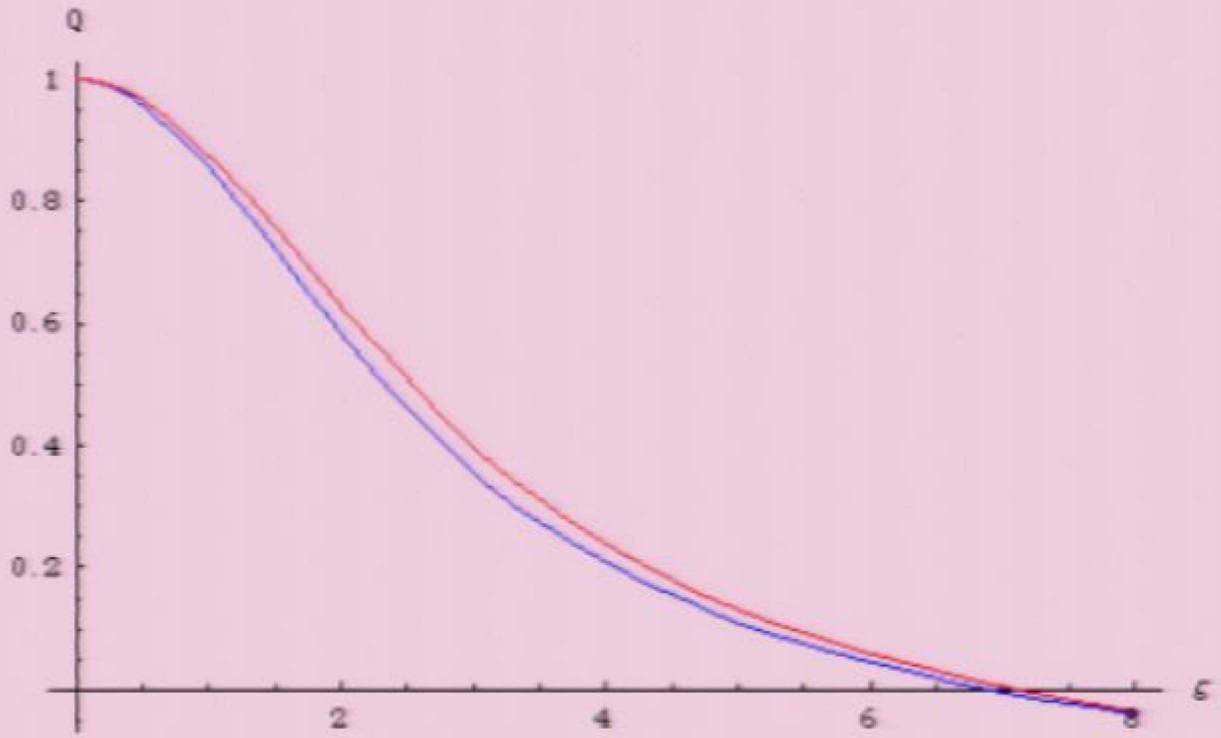
ODE more easily solved in terms of energy (per unit mass) normalized to the energy ($z_c = E_c/m_e$)center of the star Q.

$$Q'' + \frac{2}{\zeta}Q' + Q^3(1 - \frac{3}{4}Qz_c\delta) = 0$$

MDR and Chandrasekhar Mass

ODE in $Q(\zeta)$



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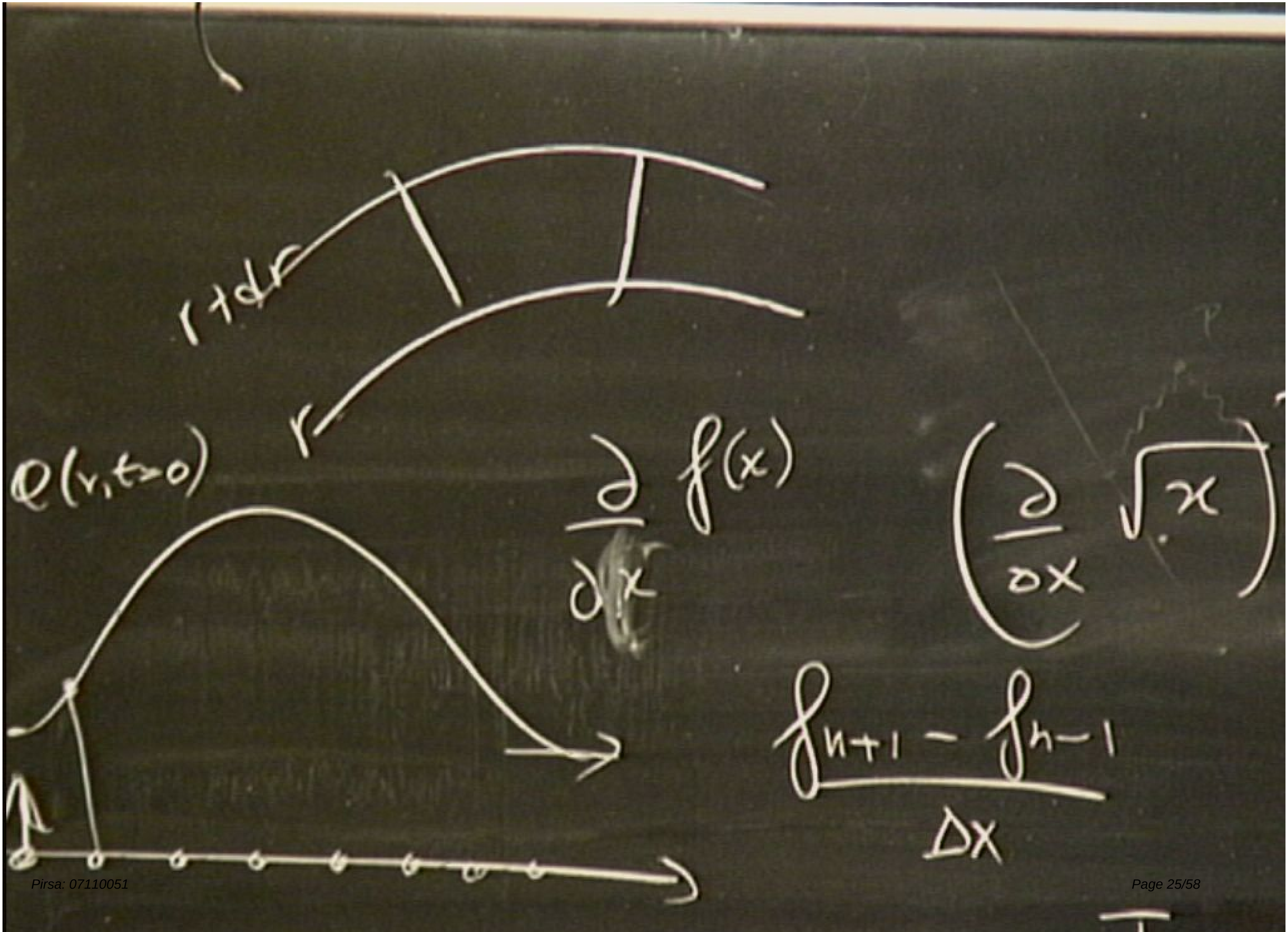
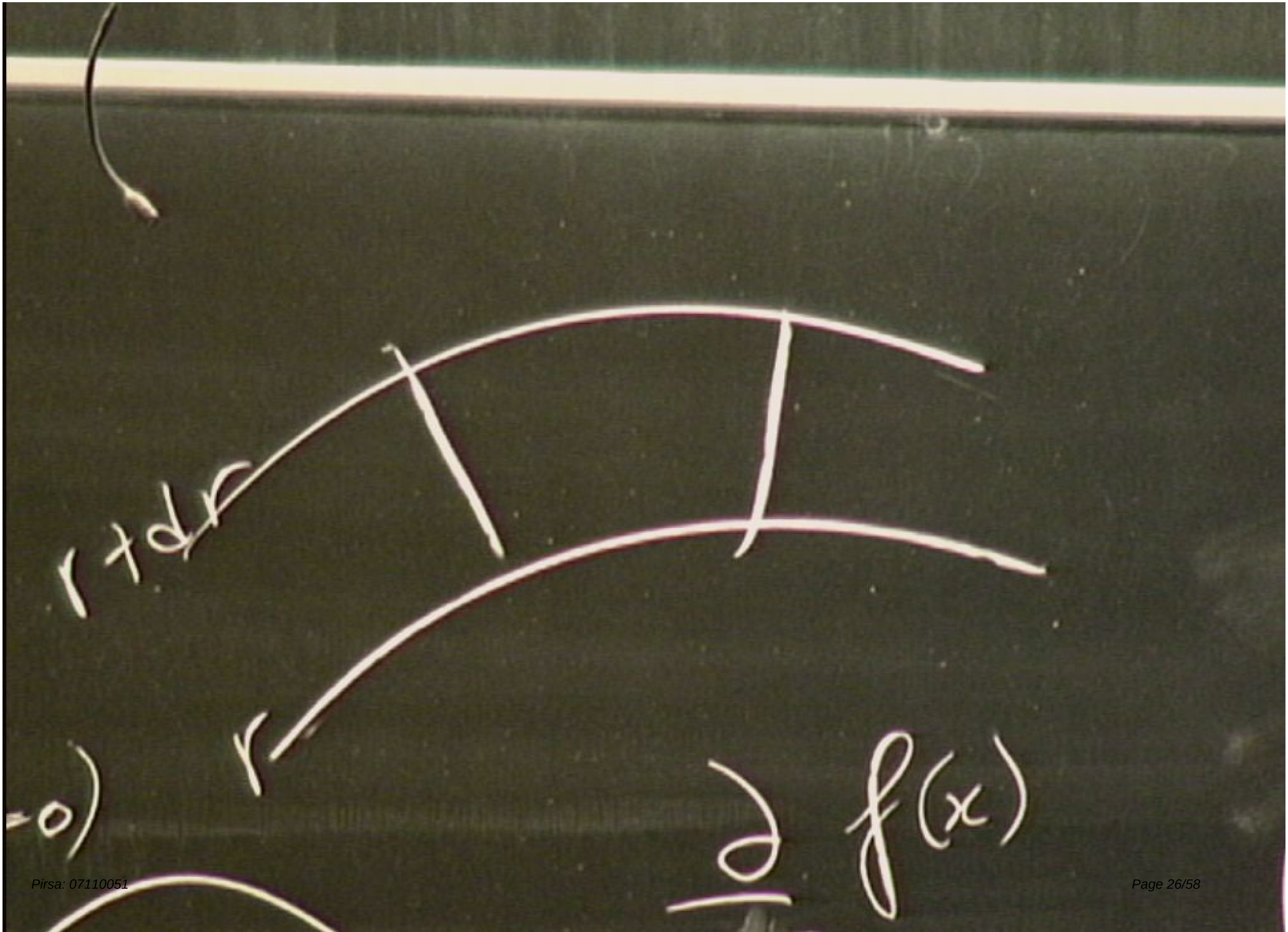
Numerical solution gives for $z_c = 10^{21}$ and $\xi = 1$

$$M_{ch} = 1.55 M_{SUN}$$

with smaller radius. For $\xi = -1$

$$M_{ch} = 1.38 M_{SUN}$$


r+dr
r
ρ(r,t=0)
∂/∂x f(x)
(∂/∂x √x)
f_{n+1} − f_{n−1}
Δx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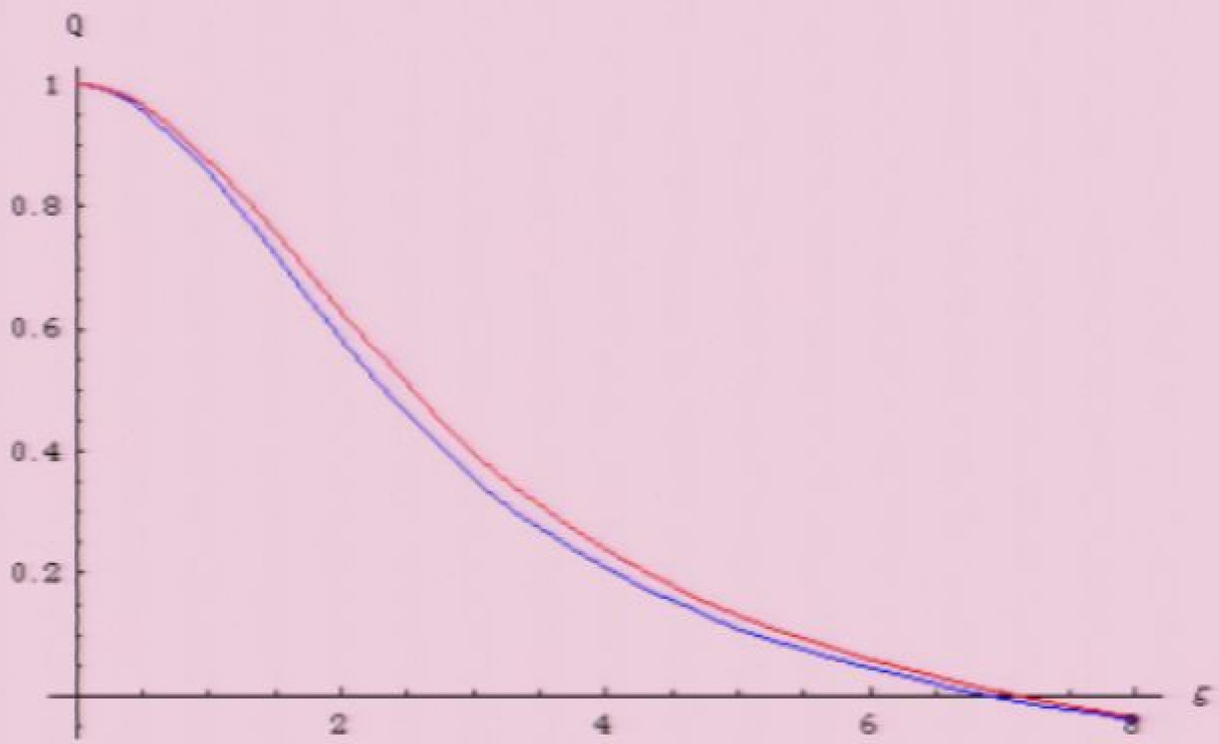

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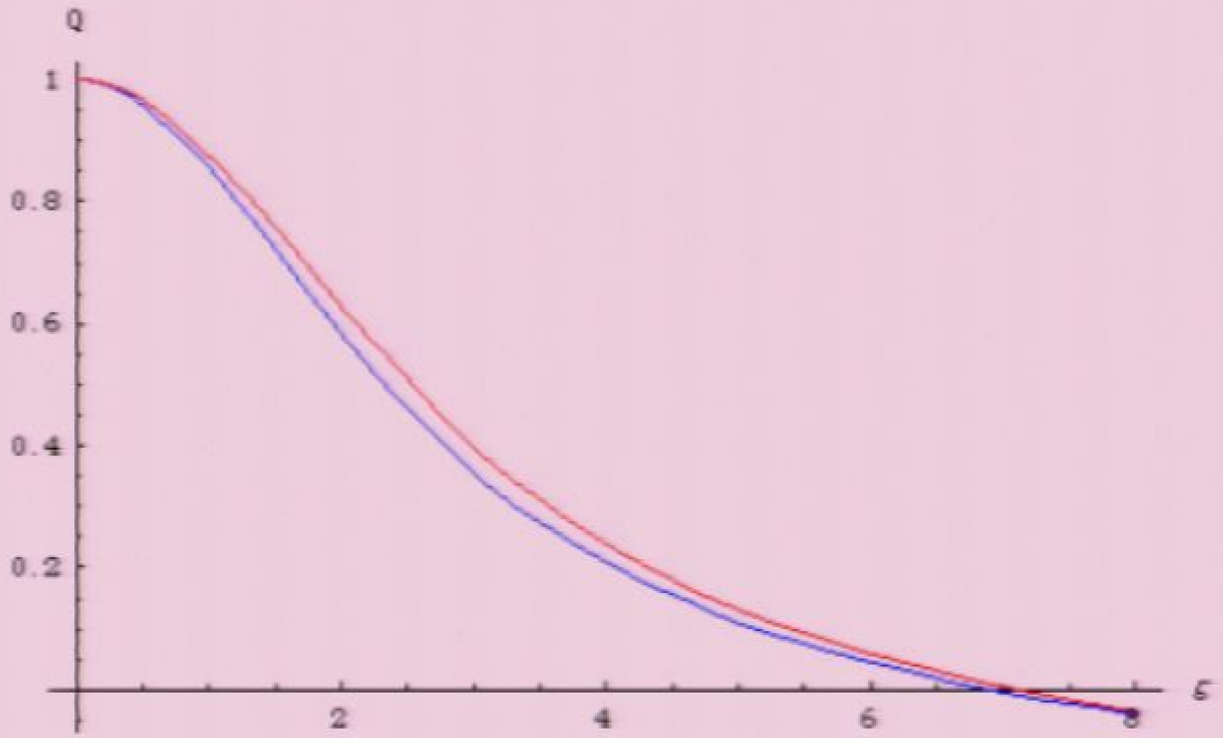

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But

-Camacho "allows us to discard" $\xi > 0$

CQG 23 (2006) 7355

-Corrections are small

-Parameters are already constrained further

-Other physical processes (neutronisation, GR corrections) occur at lower densities

$$\rho \approx 10^6 z_c^3$$

For He, C white dwarfs GR corrections become important at 3×10^{10} g cm^{-3} (stability).

- 'Secondary effects'? e.g. accretion and Type Ia supernovae - champagne supernova?

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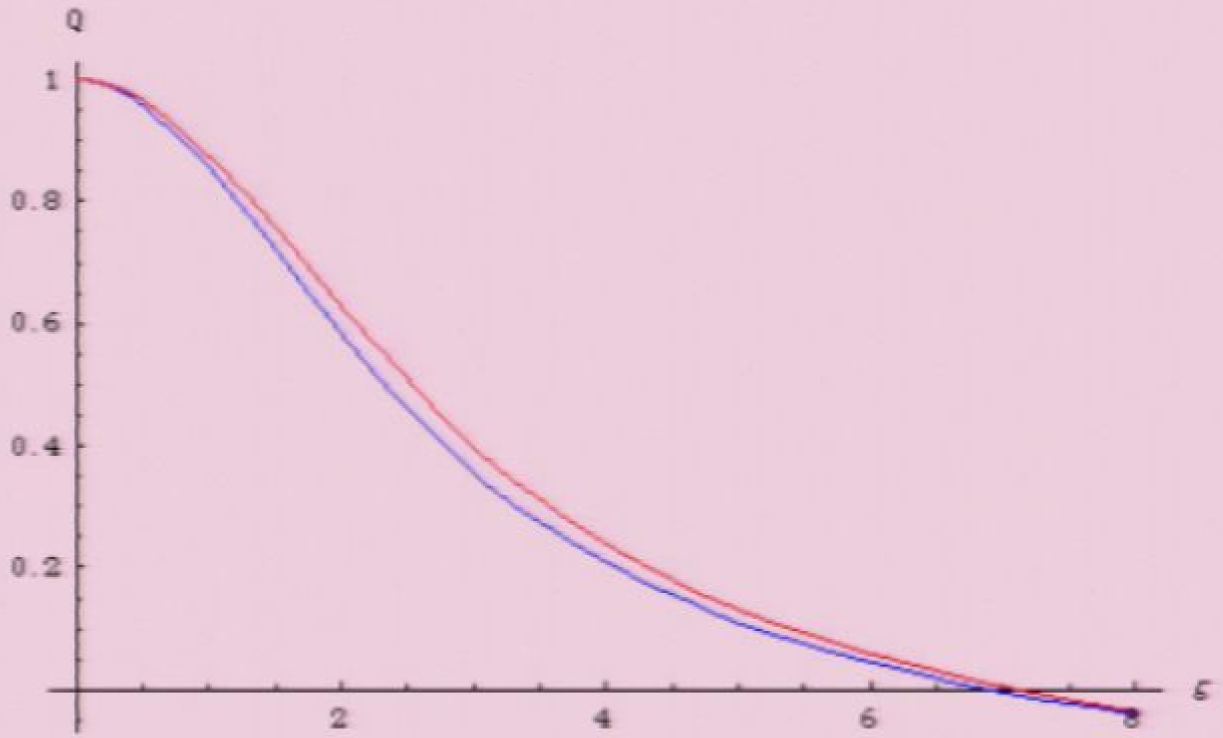

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Discrete Geometry Phenomenology

Ted Jacobson, Thermodynamics of Spacetime: The Einstein Equation of State

Phys.Rev.Lett. 75 (1995) 1260, Phys.Rev.Lett. 96 (2006) 121301

"Einstein equation can be derived from the requirement that the Clausius relation $dS = dQ/T$ hold for all local acceleration horizons through each spacetime point, where dS is one quarter the horizon area change in Planck units, and dQ and T are the energy flux across the horizon and Unruh temperature seen by an accelerating observer just inside the horizon"

GR is (only) a macroscopic theory.

Deep geometry has no metric structure

Discrete Geometry Phenomenology

Deep geometry has no metric structure

- Mass, metric emerge
- Equivalence principle is violated, holds for "macroscopic" quantities
- Local Lorentz invariance is "broken" (LLI)
- Local position invariance is "broken" (LPI)
- Rotational invariance is "broken"
- Effects exist in flat space, curvature is not necessary

Remnants of deep spatial geometry have physical effects

Discrete Geometry Phenomenology

Observed geometry is stable

- no inflating extra dimensions
- no sign of signature change
- no change in orientation
- Planck temperature 10^{32} K
- Fluctuations small, long range

Deep geometry stable and discrete

Discrete Geometry Phenomenology

A "phenomenological test theory" for possible signature of discrete spatial geometry: there is a local, dynamic preferred direction. u^μ with stochastic dynamics

The average vanishes, what is the variance? Are there effects?

- spatial (vs. Jacobson and Mattingly)
- local preferred direction
- stochastic dynmaics

Continuum approximation

- use Effective Field Theory
- remnant effects of discrete geometry are tiny - use PT

Goal is to constrain parameters and explore tests:

- LV leads to the possibility of strong constraints, e.g. bounds on modifications to dispersion relations
- Violation of rotational invariance leads to low energy effects

Discrete Geometry Phenomenology

Effective Field Theory model for fermions: - flat metric $\eta_{\mu\nu}$

All possible dimension 4 operators linear in the modifications,

constructed from u^μ

- linear in model parameters
- u dynamics not included in the field theory

Relativistic lagrangian, free spin-1/2 fermion ψ

$$\mathcal{L} = i\bar{\psi}\Gamma^\mu\partial_\mu\psi + \bar{\psi}M\psi$$

with

Discrete Geometry Phenomenology

$$\Gamma^\mu = \gamma^\mu + \alpha u^\mu u^\nu \gamma_\nu + \beta u^\mu u^\nu \gamma_5 \gamma_\nu + \delta u^\mu + i\epsilon \gamma_5 u^\mu$$
$$M = m + \zeta u^\mu \gamma_\mu + \eta \gamma_5 \gamma_\mu u^\mu$$

-The parameters ζ and η have mass dimension 1 while the rest of the parameters are dimensionless.

Discrete Geometry Phenomenology

Low energy, high precision LLI tests - non-relativistic theory

- Foldy-Wouthuysen transformation expansion in p/m

Kostelecky and Lane hep-th/9909542

To order p/m and linear in model parameters:

$$
\begin{aligned}
H_{NR} \;=\;& m + \frac{1}{2m}\left(\delta^{ij} + \delta m^{ij} + \delta m_l^{ij}\sigma^l\right)(p_i + eA_i)(p_j + eA_j) \\
+\;& \left(\delta n^i + \delta_l^i \sigma^l - \beta u_0^2 \sigma^i\right)(p_i + eA_i) \\
+\;& \frac{e}{2m}(1 + 2\alpha u_0^2)\sigma^i B_i + \frac{e\alpha}{2m}\epsilon^{ijk}\sigma_i u_l u_k \partial_j A_l + \delta m + \delta m^i \sigma_i - e\phi
\end{aligned}
$$

Effects:

- Anisotropic inertial mass
- Spin-coupled mass

Discrete Geometry Phenomenology

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\begin{aligned}
\delta m_{ij} &= -2\alpha u_i u_j \\
\delta m^l_{ij} &= -2i\alpha\epsilon^l_{jk} u_i u^k \\
\delta n^i &= -\delta u^i + i\alpha\frac{\partial^j(u_{(i}u_{j)})}{2m} - \frac{\zeta u^i}{m} \\
\delta n^l_i &= \beta u_i u^l - \alpha\frac{\epsilon^{ljk}\partial_j u_i u_k}{2m} - \epsilon\frac{\partial^l u_i}{2m} \\
\delta m &= i\zeta\frac{\partial_j u^j}{2m} \\
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- frame with $u^0 = 0$ effects still present.

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Discrete Geometry Phenomenology

Summary - A field theory model with a stochastic, spatial direction field

- Tests:
  - Spin-polarized torsion pendula
  - Penning trap (single electron)
  - Clock comparison experiments
  - High energy tests
  - non-systematic dispersion relations
  - particle production
- Differs from other models
  - extended standard model (Kostelecky et. al.) "dynamical" breaking due to (stochastic) field
  - non-metric test theories such as the $TH\epsilon\mu$
  - Ford et. al. model uses metric fluctuations

Loopy effects and GW background

Loopy effective theory and primordial gravitational wave background $h(\eta)$
Focus on method

Observation: - Density operator corrected in early universe

$$\left\langle \frac{1}{a} \right\rangle \neq \frac{1}{\langle a \rangle}$$

- Background dynamics modified $a(\eta)$

PGW Model (so far!):
In LQC and Husain-Winkler model, leading order corrections to inverse scale factor

$$\left\langle \frac{1}{a} \right\rangle \approx \frac{1}{a}\left(1 + \frac{\epsilon a_*^n}{a^n}\right)$$

$\epsilon < 1, n = 1, 2, 4$

Loopy effects and GW background

Starting with action

$$S = \frac{1}{16\pi G}\left\langle \int_V \sqrt{-g} R d^4x + \int_{\partial V} \sqrt{q} k d^3x \right\rangle$$

usual split $g_{ab} = a^2(e_{ab} + h_{ab})$. Track kinematic factors to find

$$\left\langle\frac{1}{a}\right\rangle^6 \langle a\rangle^4 h'' + \left[4\langle a\rangle' \langle a\rangle^3 \left\langle\frac{1}{a}\right\rangle^6 + 2\left\langle\frac{1}{a}\right\rangle^5 \left\langle\frac{1}{a}\right\rangle' \langle a\rangle^4\right] h' + \left\langle\frac{1}{a}\right\rangle^6 \langle a\rangle^4 k^2 h = 0$$

These kinematic effects then give the effective equation for the modes

$$\ddot{h} + \left[k^2 - \frac{\ddot{a}}{a}\left(1 + \epsilon\frac{a_*^n}{a^n}\right) + \frac{(n-1)\epsilon a_*^n \dot{a}^2}{a^{n+2}}\right] h = 0$$

Loopy effects and GW background

Loopy effective theory and primordial gravitational wave background $h(\eta)$
Focus on method

Observation: - Density operator corrected in early universe

$$\left\langle \frac{1}{a} \right\rangle \neq \frac{1}{\langle a \rangle}$$

- Background dynamics modified $a(\eta)$

PGW Model (so far!):
In LQC and Husain-Winkler model, leading order corrections to inverse scale factor

$$\left\langle \frac{1}{a} \right\rangle \approx \frac{1}{a}\left(1 + \frac{\epsilon a_*^n}{a^n}\right)$$

$\epsilon < 1, n = 1, 2, 4$

Discrete Geometry Phenomenology

Summary - A field theory model with a stochastic, spatial direction field

- Tests:
  - Spin-polarized torsion pendula
  - Penning trap (single electron)
  - Clock comparison experiments
  - High energy tests
  - non-systematic dispersion relations
  - particle production
- Differs from other models
  - extended standard model (Kostelecky et. al.) "dynamical" breaking due to (stochastic) field
  - non-metric test theories such as the $TH\epsilon\mu$
  - Ford et. al. model uses metric fluctuations

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Summary: Discrete Space and Physics

- MDR with broken LI yields remarkable constraints via process threshold analysis. Not so in case of deformed symmetries.

- Despite the ultrarelativistic nature of the Chandrasekhar mass calculation the corrections are only 10 % in usual regime. Astrophysically not relevant?

- A model of an oscillating direction field. Formulation in terms of low-energy physics for equivalence principle tests.

- A nascent model of loopy effects on primordial gravitational wave background