Title: Gravitational collapse in quantum gravity

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Abstract: I will describe work aimed at understanding the dynamics of gravitational collapse in a fully quantum setting. Its emphasis is on the role played by fundamental discreteness. The approach used suggests modifications of a black hole\'s mass loss rate and thermodynamical properties. Numerical simulations of collapse with quantum gravity corrections indicate that black holes form with a mass gap.

Outline

1. Motivation and approach
2. Classical collapse: a model
3. Quantization and qg corrected equations
4. Numerical simulation
5. Conclusions and outlook

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Some basic questions

What is a quantum black hole?

How does it form?

What role is played by fundamental discreteness?

How does Hawing radiation show up in a suitable approximation?

. . .

Some approaches

In classical theory: Metric $g_{\mu\nu}$ and matter fields ϕ.

*Non-perturbative: background independent

$$g,\ \phi \rightarrow (q, \pi)\ (\phi, P_\phi) \qquad H(q, \pi, \phi, P_\phi) \rightarrow \hat{H}$$

– attempt to follow evolution of a matter-geometry initial state

* Perturbative: fix background

$$g = g_0 + h, \qquad \phi = \phi_0 + \chi$$

$$h \rightarrow \hat{h}, \qquad \chi \rightarrow \hat{\chi}$$

– compute $\langle \hat{h}(x)\hat{h}(x')....\rangle$, $\langle \hat{h}(x)\hat{h}(x')\hat{\chi}(x'')....\rangle$

* AdS/CFT: so far no approach to bh formation – a first step is to study gravitational collapse with qg corrections in asymptotically AdS spacetimes.

* Other: g, ϕ are "emergent" collective degrees of freedom and shouldn't be quantized ... so a collective motion ansatz such as cooper pairs, Laughlin wavefunction, BE condensate needed ... for a "fundamental" QG Hamlitonian.

We use a canonical background independent approach with a notion of fundamental discreteness.

A model

$$G_{ab} = 8\pi T_{ab}$$

$$T_{ab} = \partial_a\phi\partial_b\phi - \frac{1}{2}(\partial\phi)^2 g_{ab}$$

$$ds^2 = -f^2(r,t)dt^2 + g^2(r,t)dr^2 + r^2 d\Omega^2$$

* $\phi = 0 \rightarrow$ flat space or Schwarzschild metric.

* $\phi(r,t)$ is the source of local degrees of freedom.

* complicated 2d field theory

* no known analytic collapse solutions that are asymptotically flat

* solvable collapse models (Oppenheimer-Snyder, Vaidya, CGHS, and variations) have only matter inflows

scalar field model is much richer

PROBLEM

Find the quantum theory of this model

Classical results

* There are two classes of initial data $\phi(r, t = 0)$:

Weak data → no black hole formation in the long time limit.

Strong data → black holes form above threshold initial data parameters.

– Result of hard analysis (Christdoulou 1976)

* Details of transition weak → strong done by numerical simulation. (Choptuik 1993)

– with $\pm\Lambda$ (VH, M. Olivier, G. Kunstatter ... (2001))

Simulation procedure

* Specify $\phi(r, t=0) = ar^2 e^{-(r-r_0)^2/\sigma^2}$, $P_\phi(r, t=0) = 0$.

* Geometry data (q_{ab}, π^{ab}) determined by constraints.

* Evolve data and check for trapped surface formation at each time step: compute light expansions $\theta_\pm = D_a l^a_\pm$ on spheres S^2 embedded in time slice Σ_t.

$$\theta_\pm(\text{data on slice}) = \theta_\pm(r, t)$$

Normal: $\theta_+ > 0$, $\theta_- < 0$

Marginally trapped: $\theta_+ > 0$, $\theta_- < 0$

Trapped: $\theta_\pm < 0$

* Look for roots $\theta_+(r, t) = 0$ as simulation proceeds. Search for outermost root: this gives location of evolving horizon

$$r_H(t)$$

Results

$$M_{BH} = 2r_H(a, \sigma, r_0)$$

$a > a_* : M_{BH} \sim (a - a_*)^\gamma$

$a = a_*$: critical solution – naked singularity

$a < a*$: no horizon forms.

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In QG we expect fundamental discreteness, and singularity avoidance:

How are these results modified by quantum effects?
Are there potential experimental signatures?

Quantization

Use an ADM variables: phase space variables (q_{ab}, π^{ab}) for geometry and (ϕ, P_ϕ) for matter.

$$S = \int d^3x dt \left(\pi^{ab}\dot{q}_{ab} + P_\phi \dot{\phi} - NH - N^a C_a\right)$$

* Realize constraints as self-adjoint operators.
  H is Hamiltonian constraint $\rightarrow \hat{H}$
  C_a diffeomorphism constraint $\rightarrow \hat{C}_a$
* Compute $\langle\psi|\hat{H}|\psi\rangle$, $\langle\psi|\hat{C}_a|\psi\rangle$ for states $|\psi\rangle$ such that

$$H^{qg} \equiv \langle\psi|\hat{H}|\psi\rangle = H_{\rm classical}(q,\pi,\phi,P_\phi) + \left(\frac{l_P}{L}\right)^k f(q,\pi,\phi,P_\phi) + \cdots$$

* State $|\psi\rangle$ is peaked on the phase space point q, π, ϕ, P_ϕ, and L is a scale in the state – its width.

(semiclassical states for cosmology : VH, O. Winkler gr-qc)

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Evolve initial data using H^{qg} and C_a^{qg}

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$$\left(\widehat{\frac{1}{q}}\right)_\lambda = \left(\frac{1}{i\lambda}\left[\widehat{\sqrt{|q|}}, T_\lambda\right] T_\lambda^\dagger\right)^2$$

(Thiemann-like trick)

$$\frac{\partial}{\partial x} f(x)$$

$$\left(\frac{\partial}{\partial x}\sqrt{x}\right)^2 \sim \left(\frac{1}{\sqrt{x}}\right)^2$$

$$\rightarrow \quad \frac{f_{n+1} - f_{n-1}}{\Delta x}$$

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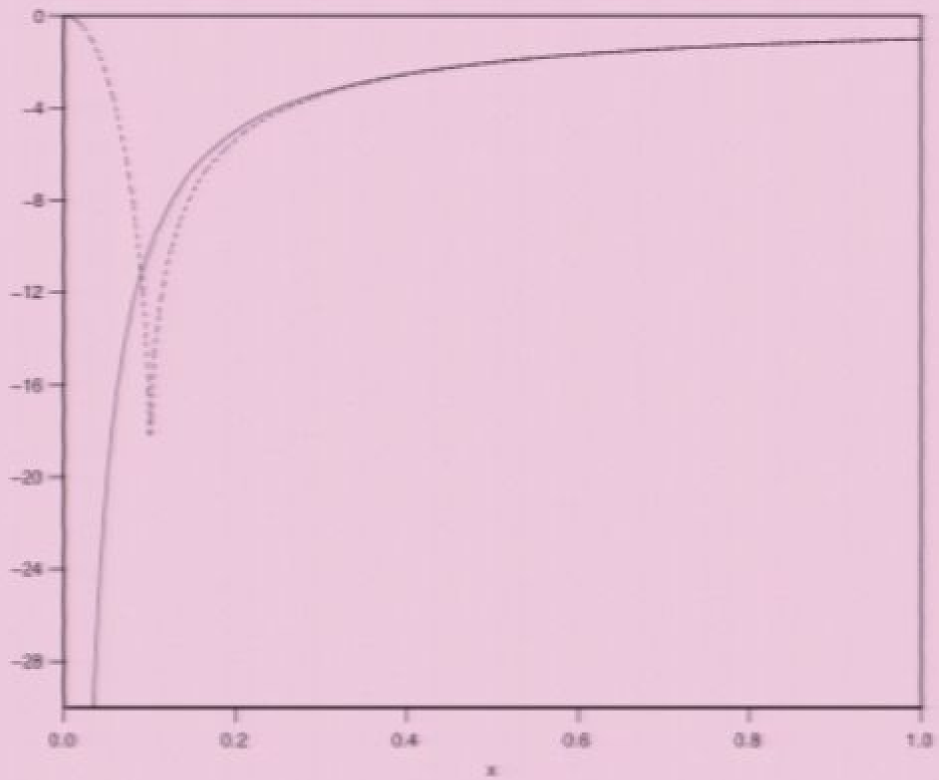

Classical $-1/x$ and the eigenvalue of $-1/x$ operator for $\lambda = 0.1$.

*Force changes sign – repulsion near the origin due to fundamental discreteness: QG "Fermi" pressure.

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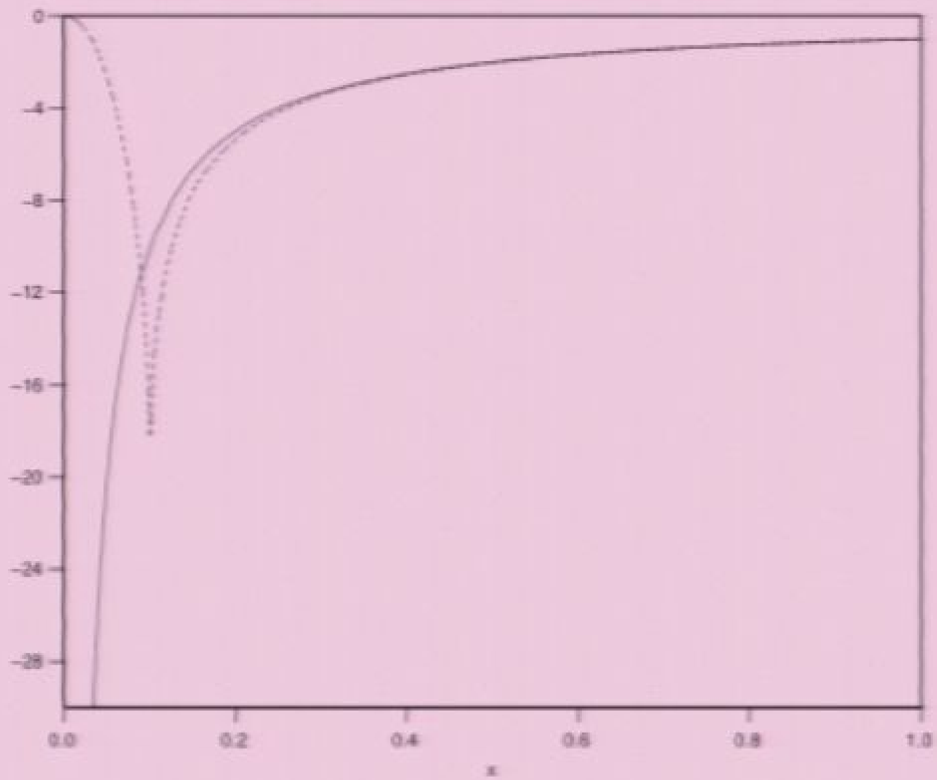

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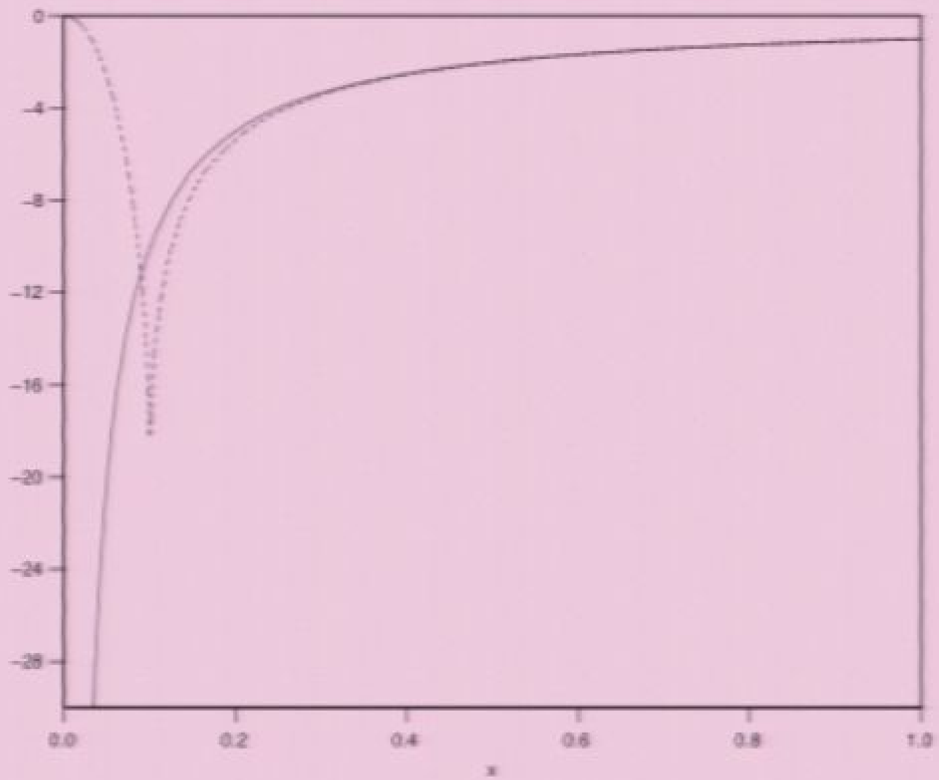

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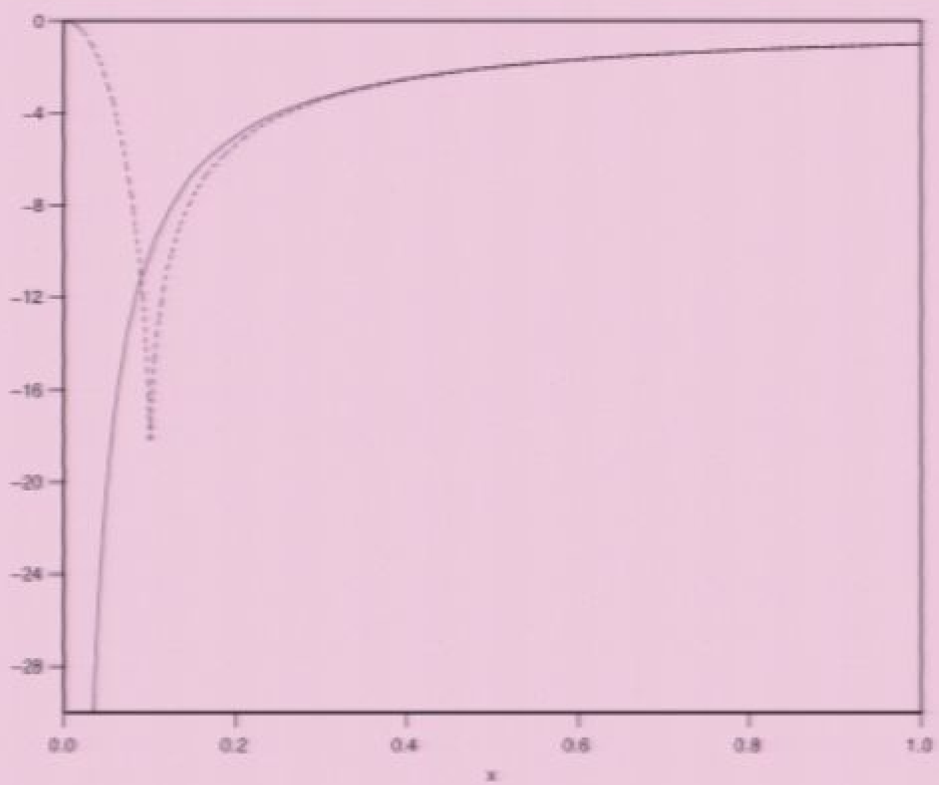

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Details

* In spherical symmetry the 3-metric is

$$ds^2 = \Lambda(r,t)dr^2 + R^2(r,t)d\Omega^2$$

so the geometry phase space variables are the pairs (R, P_R) and (Λ, P_Λ), and the matter variables are (ϕ, P_ϕ).

* Basic operators

$$\hat{R}(r_k, t)|a_1, a_2, \cdots a_n\rangle = a_k|a_1 \cdots a_N\rangle$$

$$e^{\widehat{i\lambda P_R(r_k,t)}}|a_1, a_2, \cdots a_n\rangle = |a_1, \cdots a_k + \lambda, \cdots a_n\rangle$$

Similar definitions of the other fields – LQG-like representation (VH, O. Winkler, gr-qc)

Numerical simulation

* A code to evolve equations implemented with quantum corrected constraints, and a choice of lapse and shift – (modification of a code used with G. Kunstater (2003))

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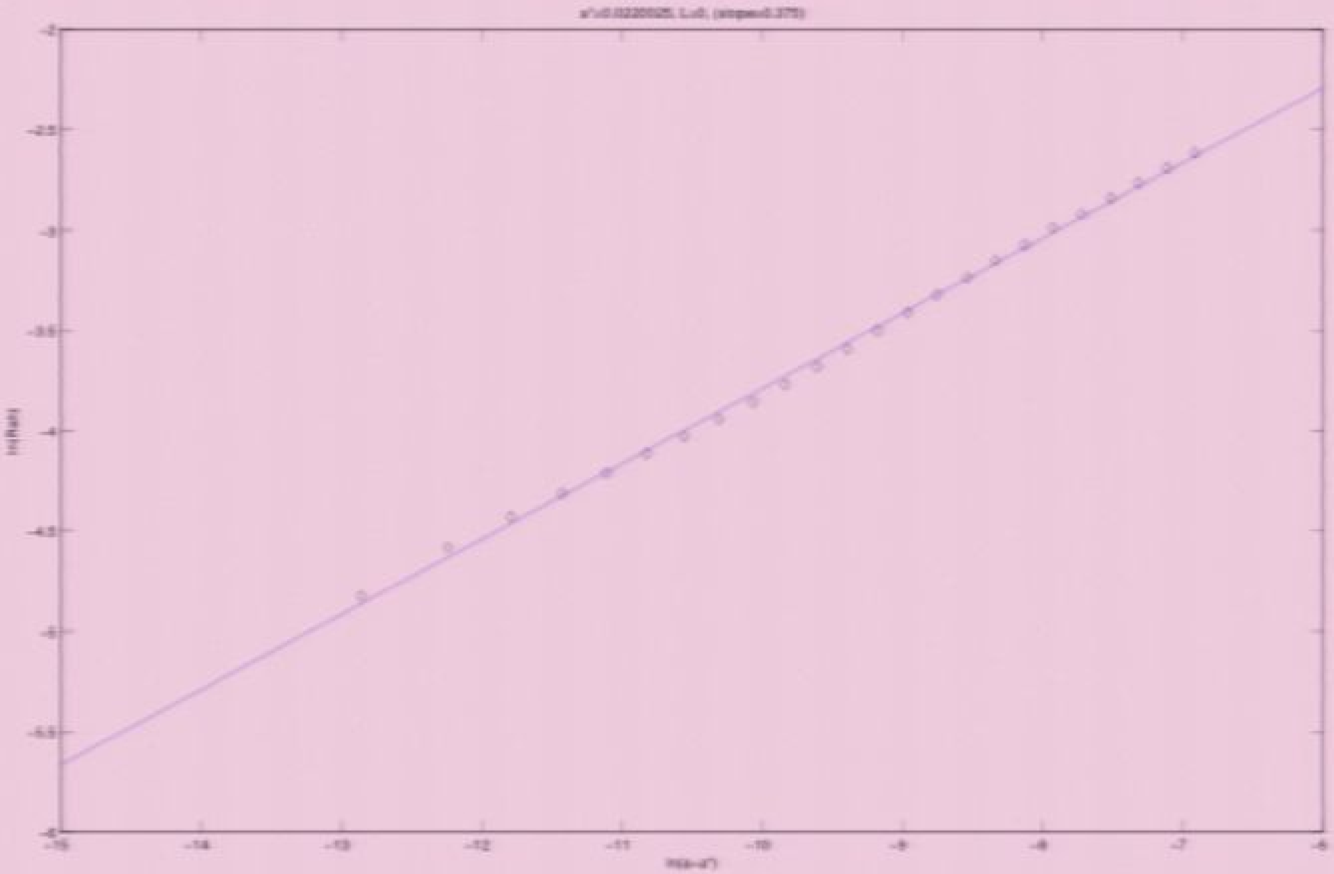
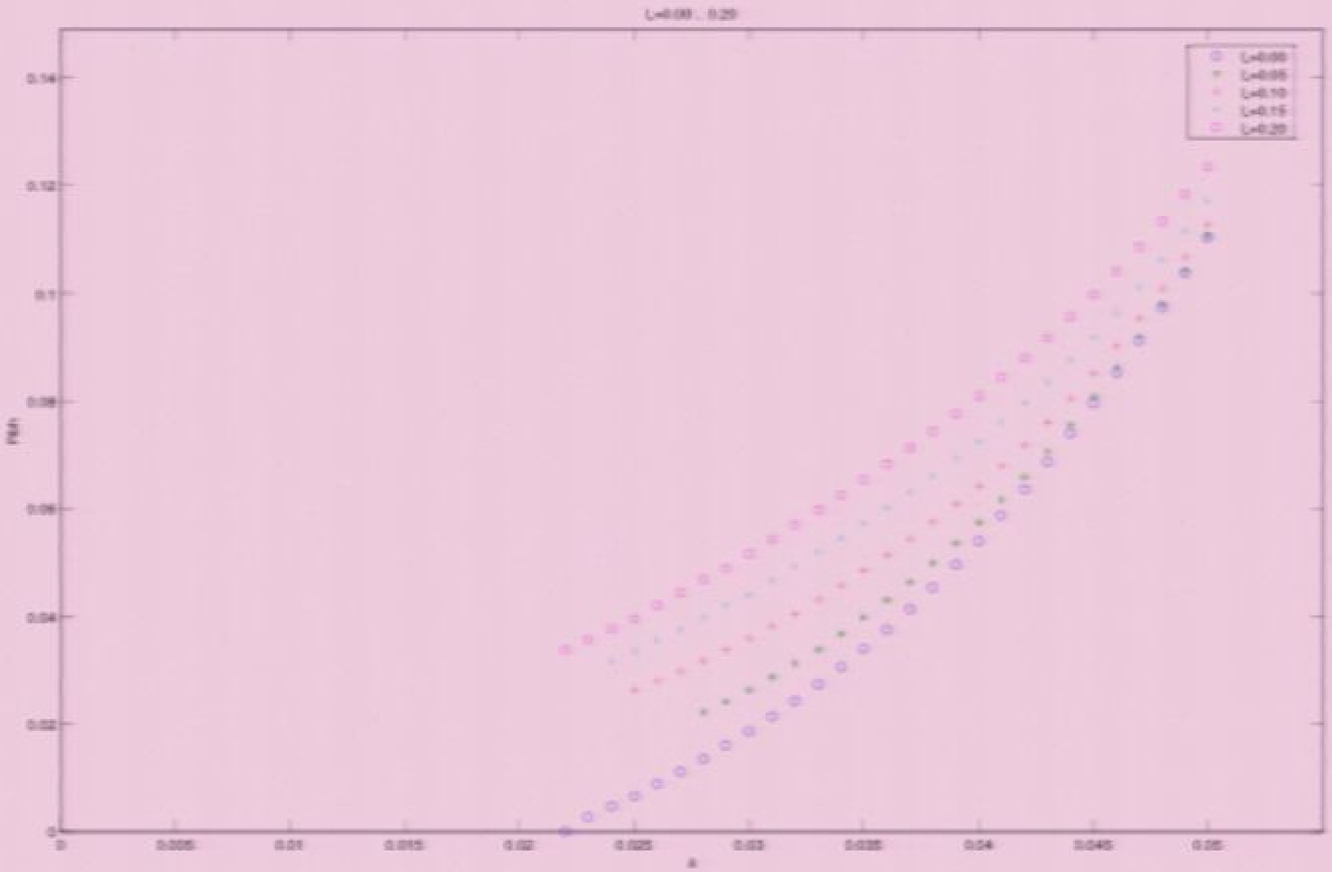
* Horizon detection using same procedure: compute $\theta_{\pm}$ at each time step of simulation.
* Initial data is scalar field profile $\phi(r,t=0) = ar^2 e^{-(r-r_0)^2/\sigma^2}$



$\lambda = 0$: this is the known classical result $M_{BH} = k(a - a*)^{0.37}$



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- points converge to classical case for large amplitude data
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Summary

- A procedure for computing quantum gravity corrections to gravitational collapse.
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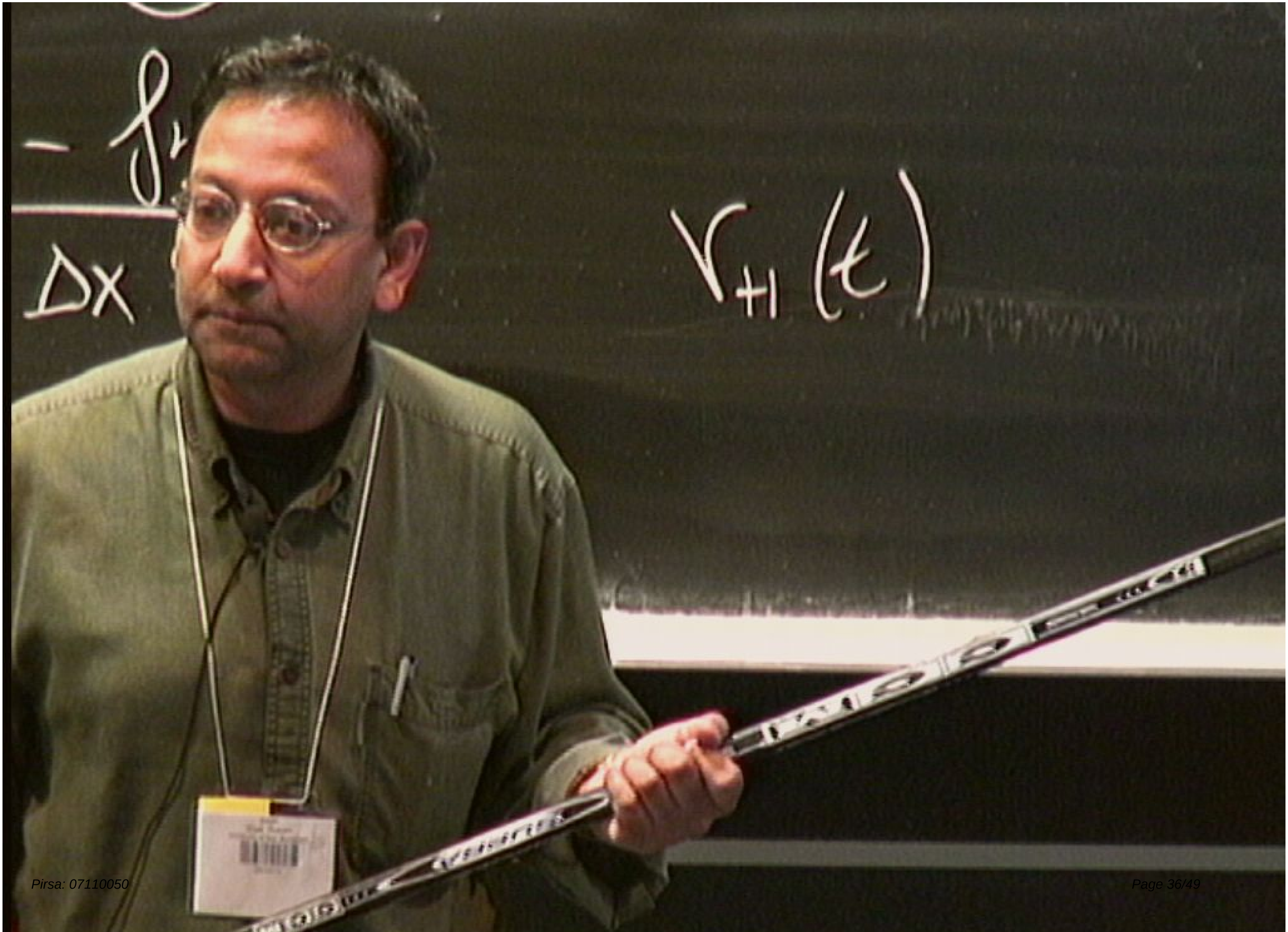
  (Mass gap known in the homogeneous case of Oppenheimer-Snyder model (Bojowald, Maartens, Singh), but no critical behaviour)
- Long to do list: put in P_λ corrections, continue evolution beyond horizon formation (do horizons begin to shrink?), derive qg corrected KG equation in a similar way, $\cdots$.



A comment on holography

– For the spatial metric $ds^2 = \Lambda^2(r,t)dt^2 + R^2(r,t)d\Omega^2$, in gauge $\Lambda = 1$, volume $V \sim \int_0^r R^2(r',t)dr'$.

– This becomes an operator $\hat{V} = L\sum_k \hat{R}^2(r_k)$, where L is a coordinate length interval in the chosen gauge.

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– For a fixed coordinate length L, the unit quantum volume is kLl_P^2, so the number of such units in a sphere is

$$N = \frac{4\pi L^3}{3kLl_P^2} \sim \frac{L^2}{l_P^2}$$

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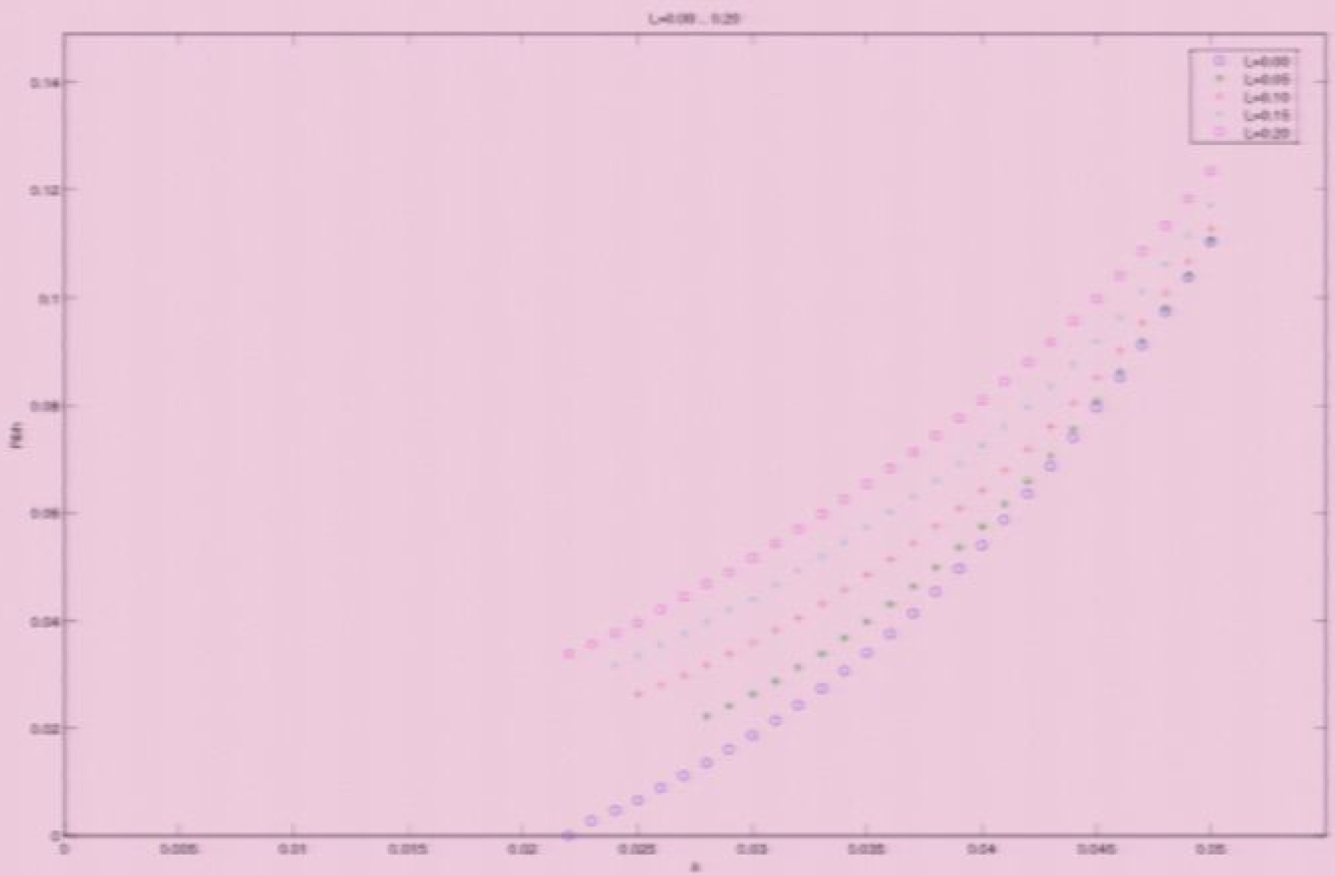

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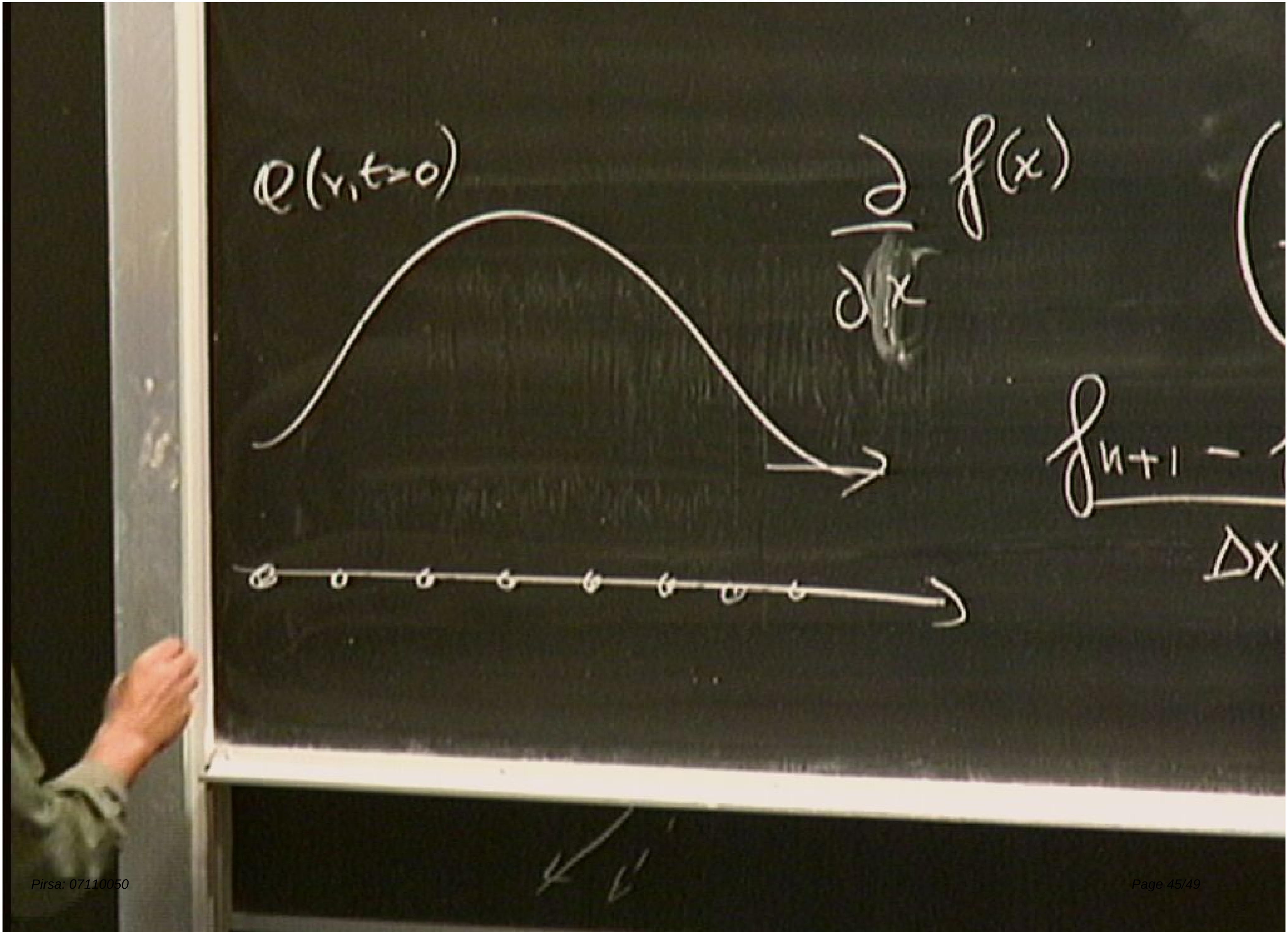
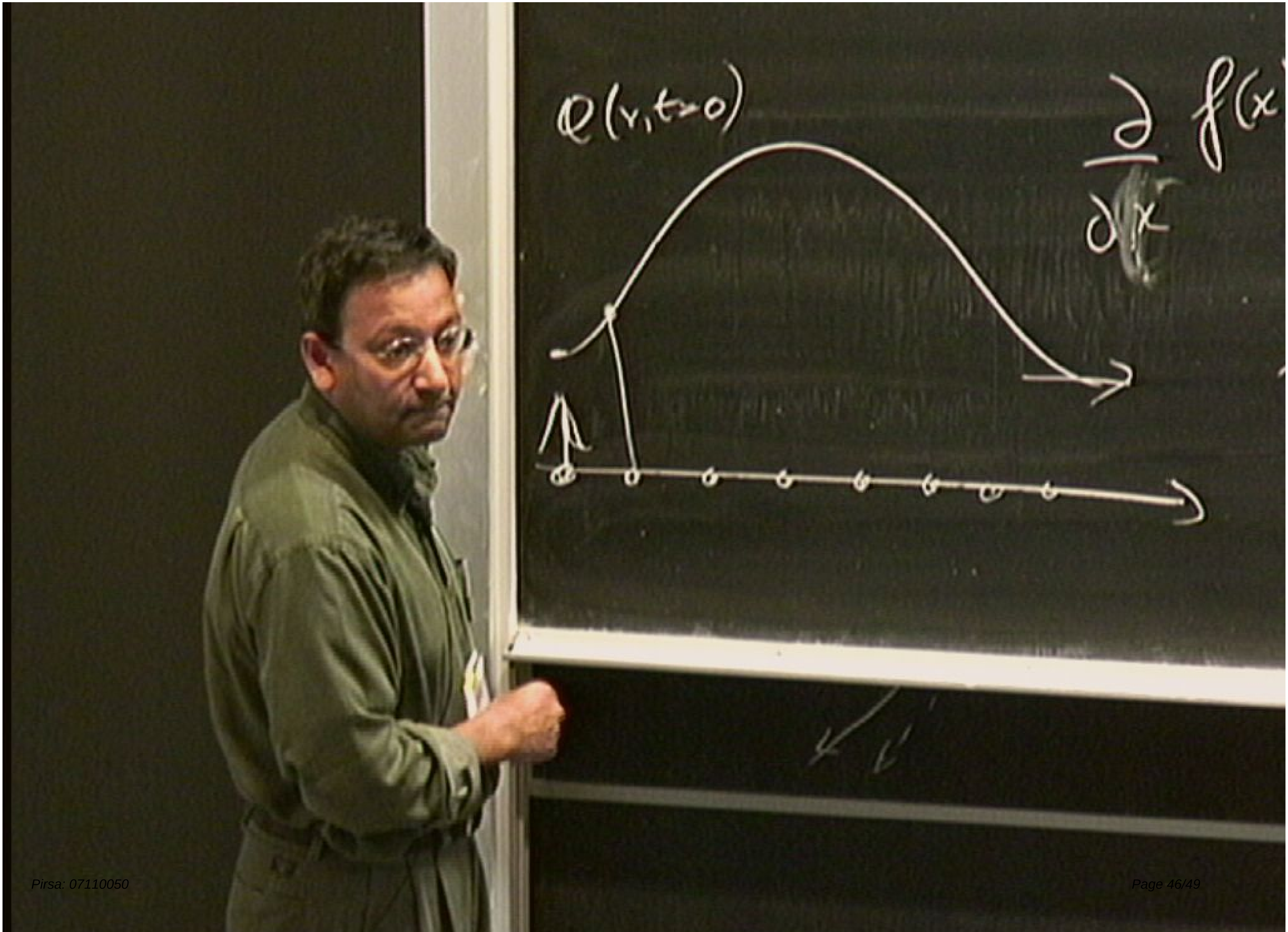

ρ(r, t=0)
∂/∂x f(x)
f n+1 −
Δx



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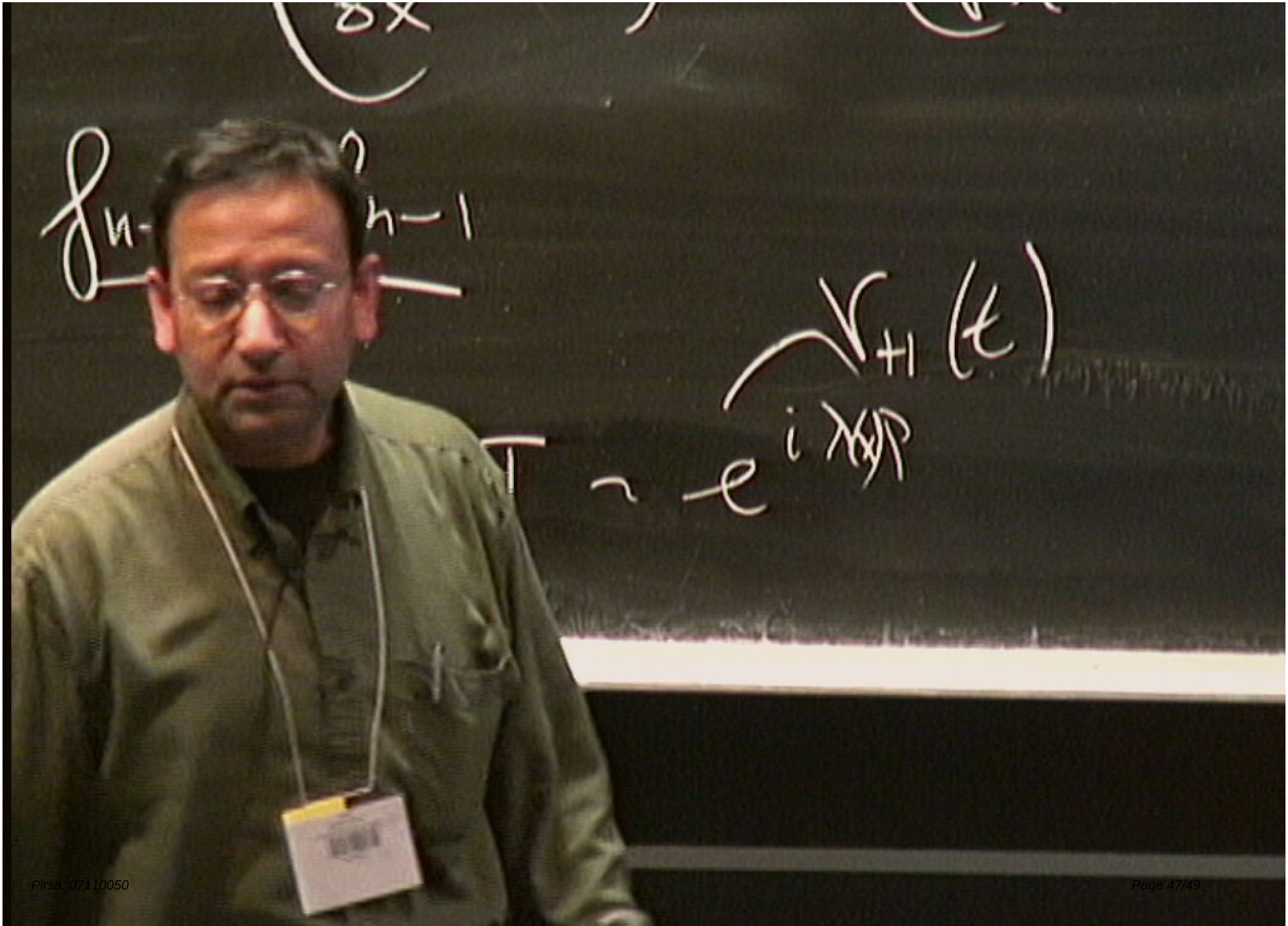

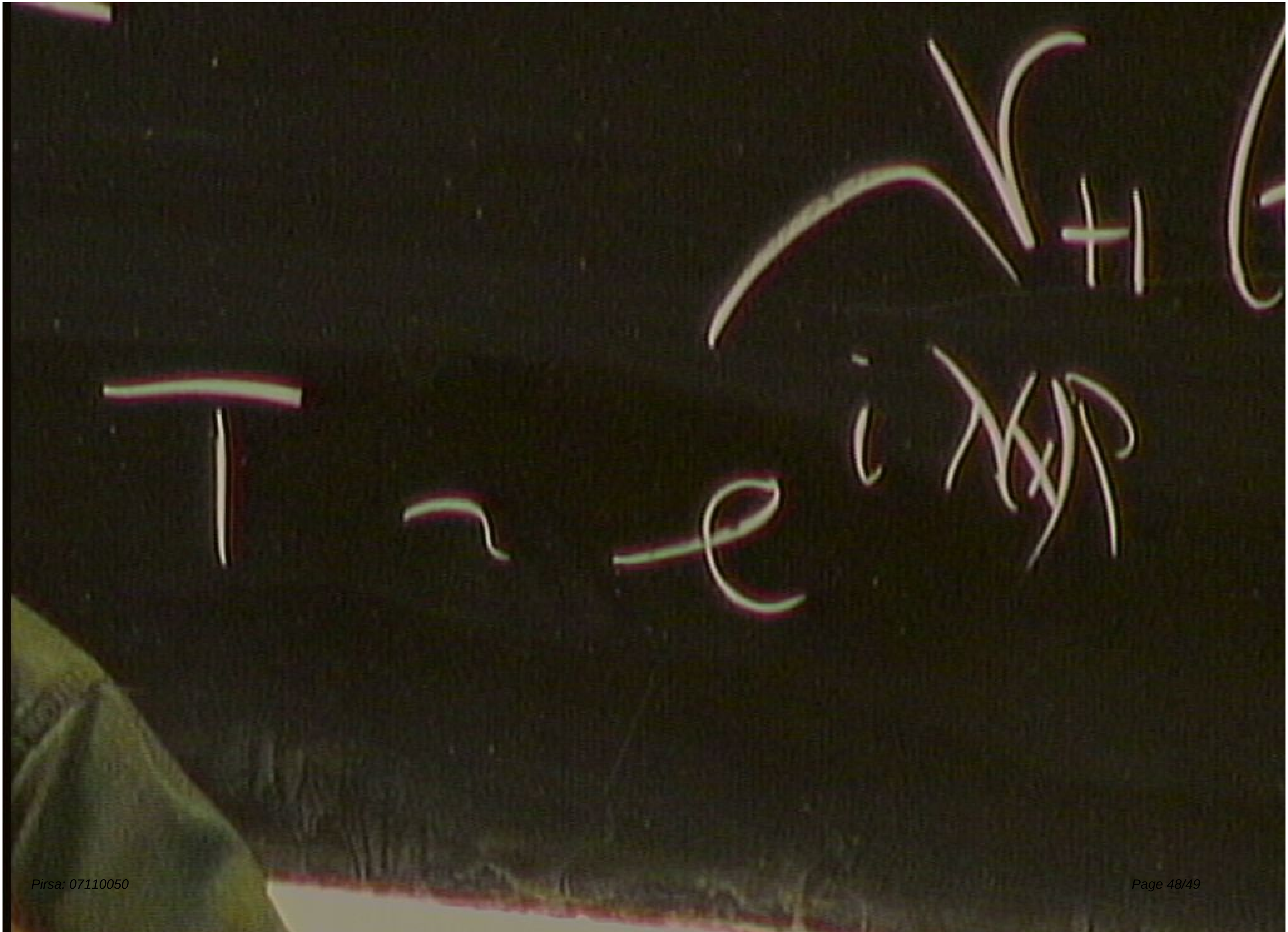

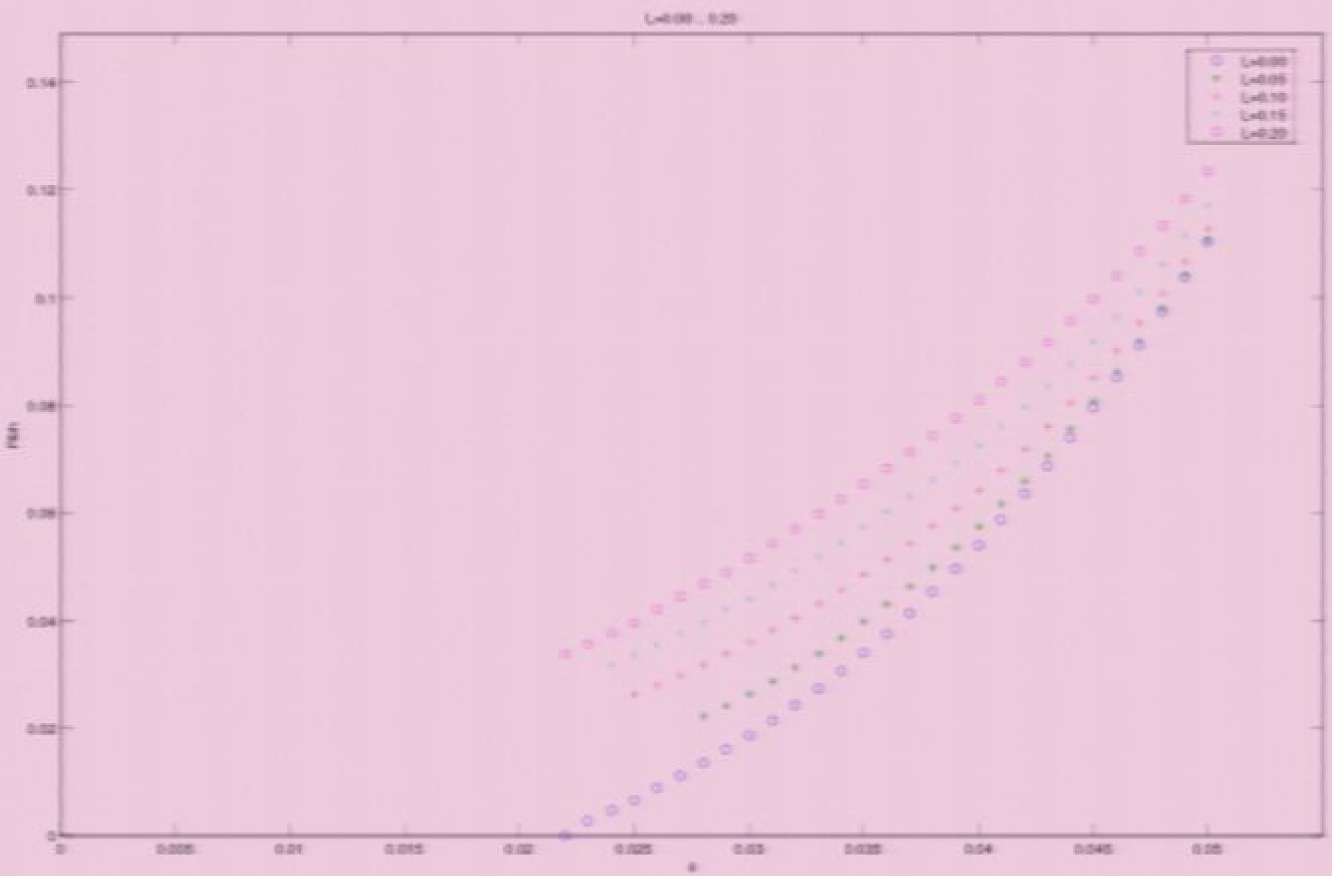

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