Title: Symmetry deformation from quantum relational observables

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Abstract: Observables in (quantum) General Relativity can be constructed from (quantum) reference frame -- a physical observable is then a relation between a system of interest and the reference frame. A possible interpretation of DSR can be derived from the notion of deformed reference frame (cf Liberati-Sonego-Visser). We present a toy model and study an example of such quantum relational observables. We show how the intrinsic quantum nature of the reference frame naturally leads to a deformation of the symmetries, comforting DSR to be a good candidate to describe the QG semi-classical regime.

OUTLINE

- Flat semi-classical space time: modification of symmetries → notion of reference frame revised. Is this effect due to a possible quantum nature of the reference frame?
- Why a quantum reference frame is natural from the Quantum Gravity perspective?
- Quantum reference frame in a toy model.

PART I:

There is now hope to probe semi-classical QG effects in experiments: need to construct a theory and in this context provide different experimental sets-up together with predictions.

Ideally this theory should be derived from:

$$\int [d\phi_M][dg]e^{i\int \mathcal{L}_M(\phi_M,g)+\mathcal{L}_{GR}(g)},$$

$\mathcal{L}_M(\phi_M, g)$, $\mathcal{L}_{GR}(g)$, lagrangians for resp. matter and grav. dof.

This is the "conservative" point of view. Introducing a semi-classical state for flat space-time, we obtain the effective lagrangian for matter.

$$\int [d\phi_M]e^{i\int \tilde{\mathcal{L}}_M(\phi)}$$

To regularize this expression, we could use for example the spinfoam approach.

If we take a "liberal" point of view, we can implement extra-dimensions, other fundamental objects (eg strings, D-particles...)...

- In 3d, the calculation can be done exactly, one obtains Deformed Special Relativity *(Freidel, Livine, hep-th/0512113)*.
- In 4d, it can't be done yet, so one models by hand the effective theory reproducing the QG fluctuations around flat space-time:
- $\rightsquigarrow$ Modify flat space-time (ie Special Relativity) to incorporate QG effects (eg Planck scale) at the kinematical level.

Special Relativity is characterized by the Poincaré symmetries.

⇝ If there is a new fundamental scale (M_P), Poincaré symmetries can be

- Unmodified: need to understand why it is so (eg discrete structure consistent with Lorentz symmetries *(causal sets, Snyder...)*).
- Broken: this is very much constrained at this time (cf Stefano's talk tomorrow).
- Deformed: there is still a (modified) relativity principle: symmetries are deformed to be consistent with the new scale. This is Deformed (or Doubly) Special Relativity.
- To distinguish between the different possibilities, one needs either a full derivation or some experiments.

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DEFORMED SPECIAL RELATIVITY (DSR)

We modify the action of the boost on momentum, but not the Lorentz algebra:
$\leadsto$ Non linear realization of the Lorentz group.

π_μ the "platonic" momentum variable carrying linear representation (therefore unbounded).

$p_\mu = F_\mu(\pi, M_P)$ the "physical" momentum bounded by M_P. F is a non linear invertible map.

$$\tilde{\Lambda}\cdot p = F\left(\Lambda \cdot F^{-1}(p)\right), \quad [\tilde{\Lambda}_{\mu\nu}, \tilde{\Lambda}_{\rho\sigma}] = \eta_{\mu\rho}\tilde{\Lambda}_{\nu\sigma} - \eta_{\mu\sigma}\tilde{\Lambda}_{\nu\rho} - \eta_{\nu\rho}\tilde{\Lambda}_{\mu\sigma} + \eta_{\nu\sigma}\tilde{\Lambda}_{\mu\rho}.$$

This leaves the modified mass shell invariant:

$$\pi^2 = m^2 \to E^2 = m^2 + p^2 + \sum_{n=1}^{\infty} \alpha_n(p, M_P)$$

DSR

There are various versions on how momenta add, how space-time is reconstructed, ...

$$\{x^\mu, x_\nu\} = \delta^\mu_\nu + g(p, M_P)$$
$$\{x^\mu, p_\nu\} = \delta^\mu_\nu + f(p, M_P)$$
$$p_1 \oplus p_2 = ?$$

DSR

This is not simply a rewriting of Special Relativity in a funny choice of coordinates if we choose carefully a new momenta addition for example, consistent with the deformed symmetries.

DSR

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EXAMPLE: "BICROSSPRODUCT" BASIS

This is the best mathematically defined and most studied example. Taking the platonic variables $\pi^A \in \mathbb{R}^5$ we have

- Momentum: $p_0 \equiv M_P \ln \frac{\pi_4 - \pi_0}{M_P}, \quad p_i \equiv M_P \frac{\pi_i}{\pi_0 - \pi_4}$
  $\rightarrow (2M_P \sinh \frac{p_0}{2M_P})^2 - \overrightarrow{p}^2 e^{-\frac{p_0}{M_P}} = m^2$
- κ-Minkowski spacetime:
  $\{x_0, x_i\} = -\frac{1}{M_P} x_i, \quad \{x_i, x_j\} = 0$
- Poisson brackets:
  $\{x_0, p_0\} = 1, \ \{x_i, p_j\} = -\delta_{ij}, \{x_0, p_i\} = -\frac{1}{M_P} p_i$
- Momentum addition:
  $(p_1 \oplus p_2)^0 = p_1^0 + p_2^0, \quad (p_1 \oplus p_2)^i = e^{-p_2^0/M_P} p_1^i + p_2^i$

- People study this mathematically well-defined theory: Quantum field theory on κ-Minkowski.
- There is a number of physical concrete questions arising from the general approach.
- Can we explore this theory from a different angle? Symmetries relate reference frames: modified symmetries means a modified notion of reference frame?

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- There is a number of physical concrete questions arising from the general approach.
  - What does it mean to have a non-commutative sum of momenta?
  - There is the "soccer ball" problem: eg $|\vec{p}| < M_P c \rightarrow |\vec{p}_1 \oplus \vec{p}_2| < M_P c$
  - What is the physical meaning of π?
  - ...
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SYMMETRY AND FRAME

- Introduce a reference frame $e^{\mu}{}_{\alpha}$, with $\mu \equiv$ space-time coordinates, α labels vectors.
- Let π_{μ} be the momentum of the particle.
- Then $p_{\alpha} = \pi_{\mu} e^{\mu}{}_{\alpha}$ is the measurement outcome (or coordinates) of π in e. In Minkowski space-time, $e^{\mu}{}_{\alpha} \sim \delta^{\mu}{}_{\alpha}$ so $\pi \sim p$.
- Consider another reference frame, $e^{\mu}{}_{\alpha} \to \bar{e}^{\mu}{}_{\alpha} = \Lambda^{\beta}{}_{\alpha} e^{\mu}{}_{\beta}$, Λ is a Lorentz transformation.

$$p'_{\alpha} = \pi_{\mu} \bar{e}^{\mu}{}_{\alpha} = \Lambda^{\beta}{}_{\alpha} e^{\mu}{}_{\beta} \pi_{\mu} = \Lambda^{\beta}{}_{\alpha} p_{\beta}.$$

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SYMMETRY AND FRAME

To get a deformation of symmetry, one needs a nonlinear function of reference frame and the system ("effective reference frame"). (Liberati, Sonego, Visser, grcq/0410113)

$$e^{\mu}{}_{\alpha} \to E^{\mu}{}_{\alpha} = \mathcal{U}(e^{\mu}{}_{\alpha}, \pi_{\mu}, M_P).$$

For example:

$$\mathcal{U}(e^{\mu}{}_{\alpha}, \pi_{\mu}, M_P) = \frac{1}{\sqrt{1 - \frac{\pi_{\mu} e^{\mu}{}_0}{M_P}}} e^{\mu}{}_{\alpha}, \quad p_{\alpha} = \frac{1}{\sqrt{1 - \frac{\pi_{\mu} e^{\mu}{}_0}{M_P}}} e^{\mu}{}_{\alpha} \pi_{\mu}$$

The fundamental variable is the frame $e^{\mu}{}_{\alpha}$. We have then a non-linear transformation of the effective frame:

$$p_{\alpha} = E^{\mu}{}_{\alpha} \pi_{\mu} \xrightarrow{\Lambda} p'_{\alpha} = \mathcal{U}\left(\Lambda \cdot \mathcal{U}^{-1}(E)\right) \pi_{\mu}.$$

Such frame will be associated to a Finsler metric (cf my talk on Saturday) or Rainbow metric.

EXAMPLE

- In a couple of papers, Liberati et al introduced a stochastic component to the tetrad induced by QG effects. (gr-qc/0511031, gr-qc/0607024)
- A particle will probe the fluctuations of the tetrad up to the scale L $E^{\mu}{}_{\alpha}(L) = < e^{\mu}{}_{\alpha} >_L$. This scale can be naturally identified with the particle's de Broglie length $L = \hbar/p_0$.
- In this case, the effective reference frame becomes momentum dependent $E^{\mu}{}_{\alpha}(p_0)$.
- A similar result is obtained in the renormalization group approach (Girelli, Liberati, Percacci, Rahmede, gr-qc/0607030).
- It is however not clear how this extends in more general situations: multi-particles case...
- The key question to answer is how the reference frame becomes system independent.

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- The key question to answer is how the reference frame becomes system independent.

- When constructing the effective theory by hand, we had the choice: the symmetry is either broken (preferred frame) or untouched (usual frame?) or deformed (modified frame).
- Can we find a general argument from the QG perspective that will pinpoint one of these different possibilities?
- In QG, we need to talk about Quantum Reference Frame (QRF).
  $\rightsquigarrow$ Why is that?
  $\rightsquigarrow$ I will study QRF in a toy model that will generate some modification of the symmetry.

PART II:

Why Quantum Reference Frame in Quantum Gravity?

Why a RF?

- Gravity can be described as a constrained system (Dirac, Lectures on Quantum Mechanics)
- First class constraints $\mathcal{C}_i$ can be seen as encoding symmetries (in the General Relativity case, the diffeomorphsims).
- Observables $\mathcal{O}$ are phase space functions that commute with constraints.

$$\{\mathcal{O}, \mathcal{C}_i\} = 0$$

It is often hard to construct in general the set of observables, but using physical reference frame helps in general (cf Rovelli's book, Dittrich's papers, Giddings-Hartle-Marolf's paper).

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EXAMPLE 1

Relativistic particle:

$$\left.\begin{array}{c} \mathcal{S} = m\int \sqrt{\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}\, d\tau \\ p_\nu = m\frac{\eta_{\mu\nu}\dot{x}^\mu}{\sqrt{\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}} \\ (\delta_{\mu\nu} - \frac{\dot{x}_\mu \dot{x}_\nu}{\eta_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta})\ddot{x}^\mu = 0 \\ H d\tau = p_\mu \dot{x}^\mu - \mathcal{L} = 0 \end{array}\right) \underset{\leftrightsquigarrow}{\textbf{Legendre transform}} \left(\begin{array}{c} \mathcal{S} = \int(\dot{x}^\mu p_\mu - \lambda(p^2 - m^2))d\tau \\ \dot{x}^\mu = 2\lambda p^\mu \\ \dot{p}^\mu = 0 \\ p^2 = m^2 \end{array}\right.$$

- λ is a Lagrange multiplier encoding the mass-shell constraint.
- This constraint encodes the time reparametrization symmetry.
- Observables $\mathcal{O}$ are functions on phase space such that $\{\mathcal{O}, p^2 - m^2\} = 0$.
  $J_{\mu\nu}, p_\mu$ commute since p^2 is Poincaré Casimir, but $\{x^\mu, p^2 - m^2\} = p^\mu \neq 0$.

To define observable positions, **we introduce a clock** $\mathcal{F} = x^0$, a "time reference frame". Consider

$$X^\mu = x^\mu + \frac{p^\mu}{p^0}(x^0 - T) \Rightarrow \{X^\mu, p^2 - m^2\} = 0$$

so X^μ is observable. It is a relational coordinate:
When $x^0 = T$ then $X^\mu = x^\mu \rightsquigarrow$ **when the clock indicates** T**, the particle is at the position** x^μ.

Other choice of relational coordinates, ie other choice of clock $\mathcal{F} = x^\mu p_\mu$:

$$\tilde{X}^\mu = x^\mu + \frac{p^\mu}{p^2}(x^\mu p_\mu - T)$$

These relational coordinates become non-commutative: $\{\tilde{X}_\mu, \tilde{X}_\nu\} = \frac{1}{p^2} J_{\mu\nu}$

When we have (first class) constraints or symmetries, relational quantities provide a natural set of observables.

To construct a relational quantity, we introduce a reference frame and define a relation between the system of interest and the reference frame.

EXAMPLE 2

Spin systems: Consider a set of non interacting classical spin particles $\vec{S}^{(i)}$. We impose that the total spin is zero: we impose global rotational symmetry, through the constraints

$$S_x^{(tot)} = 0 \quad S_y^{(tot)} = 0, \quad S_z^{(tot)} = 0.$$

A relative angle is a natural observable when dealing with rotational symmetry: Consider the system $\vec{S}^{(k)} = \vec{S}$, pick one set of spins $\vec{J}^1$ as our reference axis, then

$$s^1 = \vec{S} \cdot \vec{J}^1 \Rightarrow \{s^1, S_a^{(tot)}\} = 0$$

To construct a 3d reference frame, we pick up another set of spins $\vec{J}^2$. We can then define $\vec{J}^3 = \vec{J}_1 \wedge \vec{J}_2$. We have then the physical relational coordinates:

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EXAMPLE 3

GPS coordinates: (cf Rovelli's book)

Consider a point P in (2d) space-time. We impose diffeomorphisms symmetry: P can have any arbitrary coordinates as parametrization of the manifold.

We can define physical system by considering two "satellites" with relativistic velocities $V_\mu{}^1, V_\mu{}^2$, emitting radio signals.

Then the physical coordinates of P with respect to the satellites are:

$$s^\alpha = x^\mu V_\mu{}^\alpha - \sqrt{(x^\mu V_\mu{}^\alpha)^2 - x^\mu x_\mu}.$$

This is the Einstein's "incidence point".

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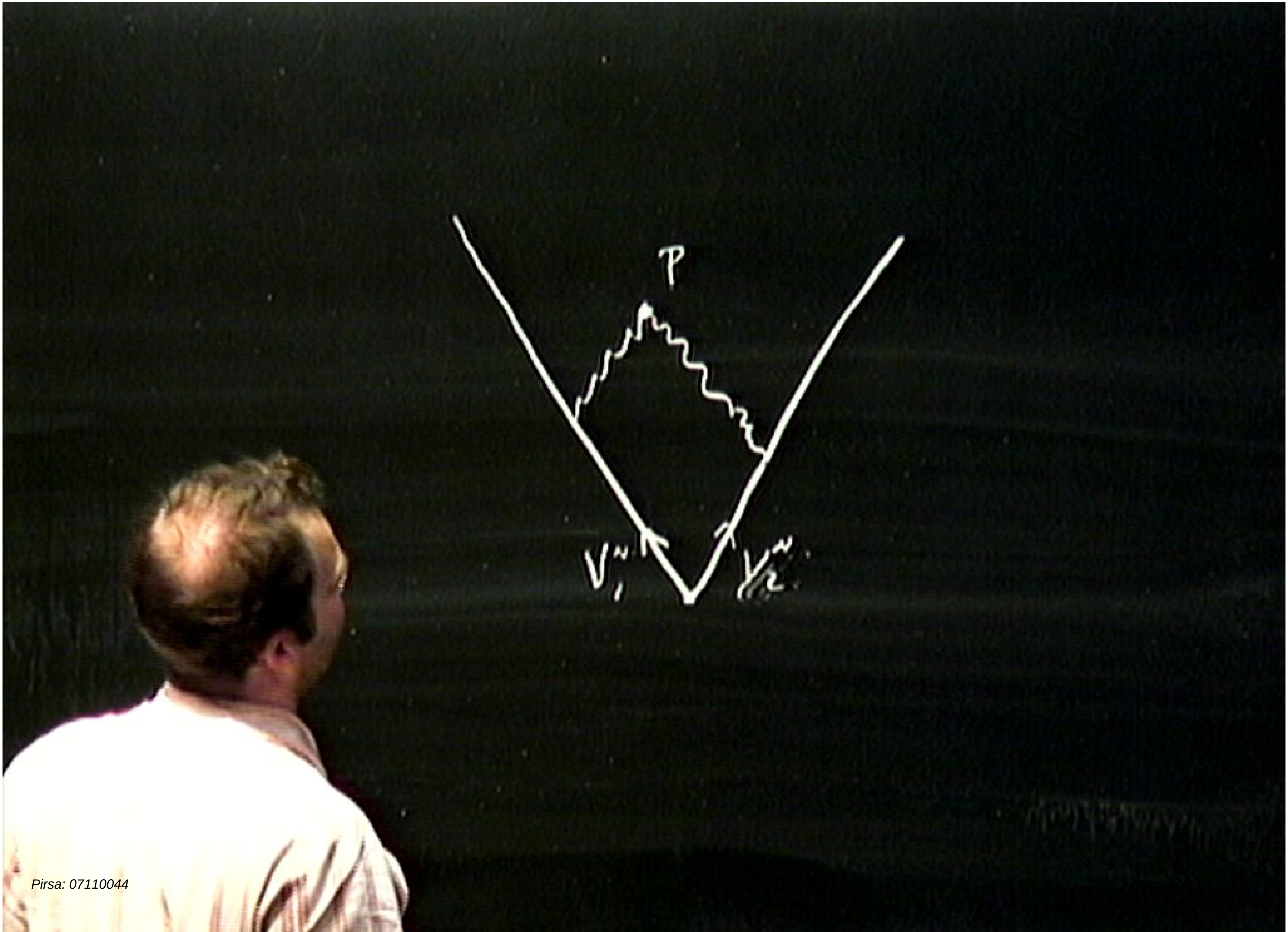
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QUANTIZATION

One follows Dirac canonical procedure:

- Phase space → "kinematical" Hilbert space
- $C_i \to \hat{C}_i$ (careful with ordering ambiguities...)
- Kernel of $\hat{C}_i$ is the physical Hilbert space.
- Observables: $[\hat{\mathcal{O}}, \hat{C}_i] = 0$. It is now natural to look at quantum relational observables, constructed out of a quantum reference frame.

EXAMPLE OF A QRF

Spin systems revisited: Consider a set of non interacting quantum spin particles $\vec{\xi}^{(i)}$. The quantum constraints are obtained in a straightforward manner

$$\hat{\xi}_x^{(tot)} = 0 \quad \hat{\xi}_y^{(tot)} = 0, \quad \hat{\xi}_z^{(tot)} = 0.$$

Quantum relational observables:

$$\mathfrak{s}^i = \vec{\xi} \cdot \vec{J}^i \Rightarrow \{\mathfrak{s}^i, \hat{\xi}_a^{(tot)}\} = 0$$

These are quantum relational observables and $J^\mu{}_\alpha$ is a quantum reference frame.

- Quantum Gravity is a quantum constrained system. Observables can be constructed as relational in terms of a quantum reference frame.
- We need to understand the physical implication of having quantum reference frames.

QUESTIONS TO EXPLORE:

- What is the precision of a QRF?
- What is the robustness of a QRF?
- Do we get non-commutative coordinates?
- What is the notion of symmetry?
- How do we coarse-grain relational observables (multi-particles state)?

These questions are of interest for both the Quantum Information Theory and QG community. In particular, the QIT people would implement that in concrete experiments.

PART III:

Quantum Reference Frame in a toy model

TOY MODEL

We consider the set of quantum spin particles. The QRF axis $\vec{J}^1, \vec{J}^2$ are made of spin ℓ and satisfy

$$[J^\mu{}_\alpha, J^\nu{}_\beta] = \begin{cases} 0 & \text{if } \alpha \neq \beta \\ \frac{i}{\sqrt{\ell(\ell+1)}} \epsilon_\eta{}^{\mu\nu} J^\eta{}_\alpha & \text{if } \alpha = \beta \end{cases}$$

We introduce also

$$\vec{J}_3 = \vec{J}_1 \wedge \vec{J}_2, \quad \mathfrak{s}_3 = \vec{\xi} \cdot \vec{J}_3$$

The quantum system $\vec{\xi}$ is a spin j. The relational coordinates satisfy the modified algebra: analog to the emergence of "non-commutativity".

$$[\mathfrak{s}_1, \mathfrak{s}_2] = i\mathfrak{s}_3$$

$$[\mathfrak{s}_2, \mathfrak{s}_3] = i\mathfrak{s}_1 + -iJ^\mu{}_2 J_{\mu 1} \mathfrak{s}_2 + i\frac{\mathfrak{s}_1\mathfrak{s}_2 - \left(\vec{\xi} \cdot \vec{\xi}\right) J^\mu{}_2 J_{\mu 1}}{\sqrt{\ell(\ell+1)}}$$

$$[\mathfrak{s}_3, \mathfrak{s}_1] = i\mathfrak{s}_2 + -i\mathfrak{s}_1 J^\mu{}_2 J_{\mu 1} + i\frac{\mathfrak{s}_1\mathfrak{s}_2 - \left(\vec{\xi} \cdot \vec{\xi}\right) J^\mu{}_2 J_{\mu 1}}{\sqrt{\ell(\ell+1)}}$$

We obtain the "semi-classical relational" coordinates by taking the RF in a semi-classical state $\rho = \rho_1 \otimes \rho_2$

$$\xi_\alpha^{semi-classical} = \mathrm{Tr}_{RF}\left(\mathfrak{s}_\alpha \rho\right),$$

taking each of the axis $a = 1, 2$ of the reference frame in the adequate coherent state $|\ell_a, \ell_a\rangle$ and $\ell_a \to \infty$.

$$\langle \ell_a, \ell_a | J^\mu{}_a | \ell_a, \ell_a \rangle = \delta_{a\mu} \frac{\ell_a}{\sqrt{\ell_a(\ell_a+1)}} \stackrel{\ell_a \to \infty}{\to} \delta_{a\mu},$$

$$\xi_\alpha^{semi-classical} \stackrel{\ell_a \to \infty}{\to} \xi_\alpha$$

We recover the standard $su(2)$ algebra.

$$[\mathfrak{s}_i, \mathfrak{s}_j] \quad \to \quad [\xi_i, \xi_j] = i\epsilon_{ij}^k \xi_k$$

The semi-classical coordinates can be decomposed into a sum of $2j+1$ orthogonal projectors P_α^m, $m = -j, \ldots, j$ associated with distinct eigenvalues m.

MEASUREMENTS: PROJECTORS

σ state of the source particle, ρ state of reference frame. The state $\rho \otimes \sigma$ of s^α is in $j \otimes \ell \sim \oplus_{k=\ell-j}^{\ell+j} k$.
Spectral theorem tells us that:

$$\mathfrak{s}_\alpha = \sum_{k=\ell-j}^{\ell+j} \lambda^k \Pi_\alpha^k \text{ with eigenvalues } \lambda^k = \frac{k(k+1) - \ell(\ell+1) - j(j+1)}{2\sqrt{\ell(\ell+1)}}$$

and projector

$$\Pi_\alpha^k = \frac{1}{N^k} \prod_{k' \neq k} (\lambda^{k'} - \mathfrak{s}_\alpha),$$

where the normalization factor is

$$N^k = \prod_{k' \neq k} (\lambda^{k'} - \lambda^k).$$

Π_α^k are non-linear functions of the relational coordinates $\mathfrak{s}_\alpha$ (polynomials of degree $2j$).

MEASUREMENTS OF $\mathfrak{s}_\alpha$

Measurement of $\mathfrak{s}_\alpha$ will produce the outcomes λ^k with probability

$$Pr(\lambda^k) = \mathrm{Tr}\{\Pi^k_\alpha \rho \otimes \sigma\}$$

Induced measurements on system

$$Pr(\lambda^k) = \mathrm{Tr}\{\Pi^k_\alpha \rho \otimes \sigma\} = \mathrm{Tr}\{\Lambda^k_\alpha \sigma\}$$

with

$$\Lambda^k_\alpha = \mathrm{Tr}_{RF}\{\Pi^k_\alpha \rho\}.$$

Λ^k_α can be seen as an approximation of the measure of the "semi-classical relational" coordinates.

$$\Lambda^k_\alpha = (1-\epsilon^k) P^k_\alpha + \sum_{k' \neq k} \epsilon^{k',k} P^{k'}_\alpha,$$

where $\epsilon^k, \epsilon^{k',k}$ depend on ρ and are of order ℓ^{-1}. We can then play on ρ to determine which ρ provides the optimum measurement: this is one way to encode the precision of the QRF.

INDUCED MEASUREMENTS ON RF: BACK-ACTION ON QRF

First note that

$$[J^{\mu}{}_{\alpha}, \mathfrak{s}_{\alpha}] = \frac{i}{\ell}\epsilon^{\mu}{}_{\nu\eta}J^{\nu}{}_{\alpha}\xi^{\eta}.$$

By the uncertainty principle, a measurement of $\mathfrak{s}_{\alpha}$ will thus alter the value of J_{α}, and so disturb any future measurement that make use of that frame.

After measurement, we have the (normalized) state:

$$\frac{\Pi_\alpha^k(\rho \otimes \sigma)\Pi_\alpha^k}{Pr(\lambda^k)}.$$

We are interested at the dynamics of the QRF induced by the measurements.
After a measurement, we sum over the different outcomes and trace out the system $\sigma = \sum_i s_i |\phi_i\rangle\langle\phi_i|$

$$\rho' = \sum_k \mathrm{Tr}_S\{\Pi_\alpha^k(\rho \otimes \sigma)\Pi_\alpha^k\} = \mathcal{E}_\alpha(\rho) = \sum_b K_b \rho K_b^\dagger$$

with $a = (k, i, i')$ and the Kraus operators $K_b = \sqrt{s_j}\langle\phi_{i'}|\Pi_\alpha^k|\phi_i\rangle$ satisfying $\sum_b K_b^\dagger K_b = 1$.

Example

If $\vec{\xi}$ is a spin $\frac{1}{2}$, assuming the QRF in coherent state, then at first order in ℓ^{-1}

$$\begin{aligned} \mathcal{E}_\alpha(\rho_\alpha) &\approx \rho_\alpha - \frac{i}{\ell}\epsilon_{\mu\nu\eta}\mathcal{J}^\mu{}_\alpha\langle\xi^\nu\rangle \sin\theta[J^\eta{}_\alpha, \rho_\alpha] \\ &= \mathcal{U}^\dagger_\alpha(J, \xi, 1/\ell)(\rho_\alpha) \end{aligned}$$

with

$$\mathcal{J}^\mu{}_\alpha = \mathrm{Tr}\,(J^\mu{}_\alpha\rho), \quad \langle\xi^\nu\rangle = \mathrm{Tr}\,(\xi^\nu\sigma), \quad \cos\theta = \vec{\mathcal{J}}_\alpha \cdot \langle\vec{\xi^\nu}\rangle$$

The reference frame axis undergoes a rotation around the axis $\vec{\mathcal{J}}_\alpha \wedge \langle\vec{\xi}\rangle$ of angle θ.
This is in the Schrodinger picture, we can move to the Heinsenberg picture.

$$J^\mu{}_\alpha \xrightarrow{n} \mathcal{U}^n_\alpha(J^\mu{}_\alpha, \xi, 1/\ell),$$

after n measurements. Non-unitary effects (noise) will appear in general at higher orders.

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- The evolution can be decomposed into an invertible part and some non-invertible part (noise). When $\ell \gg n$ the number of measurements then the invertible part is dominant. This is a simplified example of the "decoupling theorem".

- The measurement of the relational quantum coordinate of a source particle induces a back-action on the RF, which is in general a non-linear function $\mathcal{E}_\alpha$ of the quantum relational coordinate p_α.

$$J^\mu{}_\alpha \to \mathcal{U}^n_\alpha(J^\mu{}_\alpha, \xi, 1/\ell).$$

This is to be compared with the DSR case:

$$e^\mu{}_\alpha \to \mathcal{U}(e^\mu{}_\alpha, \pi, M_P)$$

- One can then ask for which state the QRF will be the most robust, and possibly the most robust *and* the most precise as there might be a trade-off between the two.

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MULTI-PARTICLES:

Remember for DSR we have eg:

$$p_0^{(tot)} = p_0^{(1)} + p_0^{(2)}, \quad p_i^{(tot)} = e^{-p_0^{(2)}/M_P} p_i^{(1)} + p_i^{(2)}.$$

Within the reference frame interpretation, $\pi^{(tot)} = \pi^{(1)} + \pi^{(2)}$ is the total intrinsic momentum. The physical momentum is naturally defined then as

$$p_\alpha^{(tot)} = \left(\pi^{(1)} + \pi^{(2)}\right)_\mu e^\mu{}_\alpha = p_\alpha^{(1)} + p_\alpha^{(2)}.$$

If the frame is "deformed"

$$p_\alpha^{(tot)} = \left(\pi^{(1)} + \pi^{(2)}\right)_\mu E^\mu{}_\alpha(e, (\pi^{(1)} + \pi^{(2)}), \gamma M_P),$$

where I added γ to solve the "soccer ball" problem.

Can we get something analog in the toy model?

The total intrinsic spin is

$$\vec{\xi}^{(tot)} = \vec{\xi}^{(1)} + \vec{\xi}^{(2)}, \quad [\xi_\mu^{(1)}, \xi_\nu^{(2)}] = 0, \quad [\xi_\mu^{(tot)}, \vec{\xi}_\mu^{(k)}] = 0$$

The relational coordinate is therefore

$$\mathfrak{s}_\alpha^{(tot)} = \xi_\mu^{(tot)} J^\mu{}_\alpha.$$

But then

$$[\mathfrak{s}_\alpha^{(1)}, \mathfrak{s}_\alpha^{(2)}] = \frac{\epsilon_\rho{}^{\mu\nu}}{\sqrt{\ell(\ell+1)}} \xi_\mu^{(1)} \xi_\nu^{(2)} J_\alpha{}^\rho,$$

that is the components of the physical total spin are not commuting anymore. Also we have

$$[\mathfrak{s}_\alpha^{(k)}, \mathfrak{s}_\alpha^{(tot)}] \neq 0 \qquad k = 1, 2,$$

that is measuring the total relational spin of two particles differs from measuring their individual relational spins and adding the outcomes.

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ANALOG TO THE SOCCER-BALL PROBLEM?

- Measurements are defined in terms of the projector Π_α^k, with $k = |\ell - j|, ..., \ell + j$. There are therefore $2\ell + 1$ projectors.
- The system $\vec{\xi}$ is defined in terms of $2j + 1$ projectors.
- If $j < \ell$, then we have enough projectors to specify a priori the information encoded in the system.
- If $j > \ell$, then we do not have enough projectors to specify completely the information encoded in the system. ℓ is comparable to the Planck scale.
- But there is not a fundamental problem a priori: I should just take a bigger reference frame such that $\ell' > j$.

CONCLUSIONS

- **Quantum Reference Frames are essential in QG to construct some observables.**
- I used a toy model to explore the consequences of dealing with a QRF: interesting structures such as non-commutativity, symmetry deformation, unusual coarse-graining appeared.
- These structures also appeared in the context of effective description of semi-classical flat space-time, such as Deformed Special Relativity. The analogy suggests therefore a new argument favoring DSR as the natural QG flat semi-classical limit.

- Physics of a QRF can also be of interest for QIT community: precision, robustness of the QRF to encode information in relational quantities.
- **New models need to be explored**. The toy-model concentrated here on spin, ie $su(2)$. For the QG flat semi-classical limit, we need to deal with Poincaré symmetries. We should aim at constructing for example a quantum reference frame using quantum fields.

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