Title: DSR and Classical Gravity

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Abstract: The talk gives a brief overview over different versions of doubly or deformed special relativity (DSR) and its motivation, which comes from the occurrence of a fundamental invariant length in quantum gravity (QG). Despite its QG origin, DSR is a modification of flat space geometry without explicit notion of gravity. In the literature there is a considerable amount of work done to probe deformations of special relativity in classical and quantum mechanics and quantum field theory without taking into account intermediate steps between QG and flat space, like general relativity or quantum field theory in curved space. The more special part of this contribution makes one step into this gap by comparing the DSR modifications of simple quantum scattering of a particle in flat space with the modifications caused by a weak classical gravitational field.

Classical Gravity, (Loop) quantum gravity and deformations of Special relativity

Talk at the Perimeter Institute,

6 Nov. 2007

Motivation

- Approaches to Quantum Gravity – quantization of space-time geometry.
- Drastic changes at the Planck scale.
- Loop Quantum Gravity: Discrete spectrum of geometric quantities with an invariant length. → Ultraviolet cutoff.
- Is this compatible with Special Relativity?
- Does this necessarily distinguish a reference frame or is it possible to conciliate the existence of an invariant length with the principles of relativity?
- The latter is true. Relativistic theories with an invariant quantity in addition to c are called Deformed or Doubly Special Relativity.

DSR and Phenomenology

- In DSR and all the related attempts the key role is played by an invariant energy of the order of the Planck energy or an invariant length of the order of the Planck length,

$$m_P = \sqrt{\eta c / G} \approx 10^{19} GeV, \quad l_P = \sqrt{\eta G / c} \approx 10^{-35} m.$$

- The expected corrections are at most linear in the ratios E / m_P or l_P / l, or of higher order.
- Unless the invariant levels are shifted, by the existence of higher dimensions, DSR & Co. do not explain effects “as large as” the GZK cutoff at $10^{20} - 10^{21} eV$ or TeV photons

Program of the talk

- DSR in momentum space
- Dispersion relations
- DSR in space-time
- Formulation in De Sitter space
- Velocities
- Hopf algebras
- Other modifications
- DSR and gravity
- Classical gravity
- LQG and QFT in curved space
- Discreteness and Lorentz invariance

DSR in Momentum Space

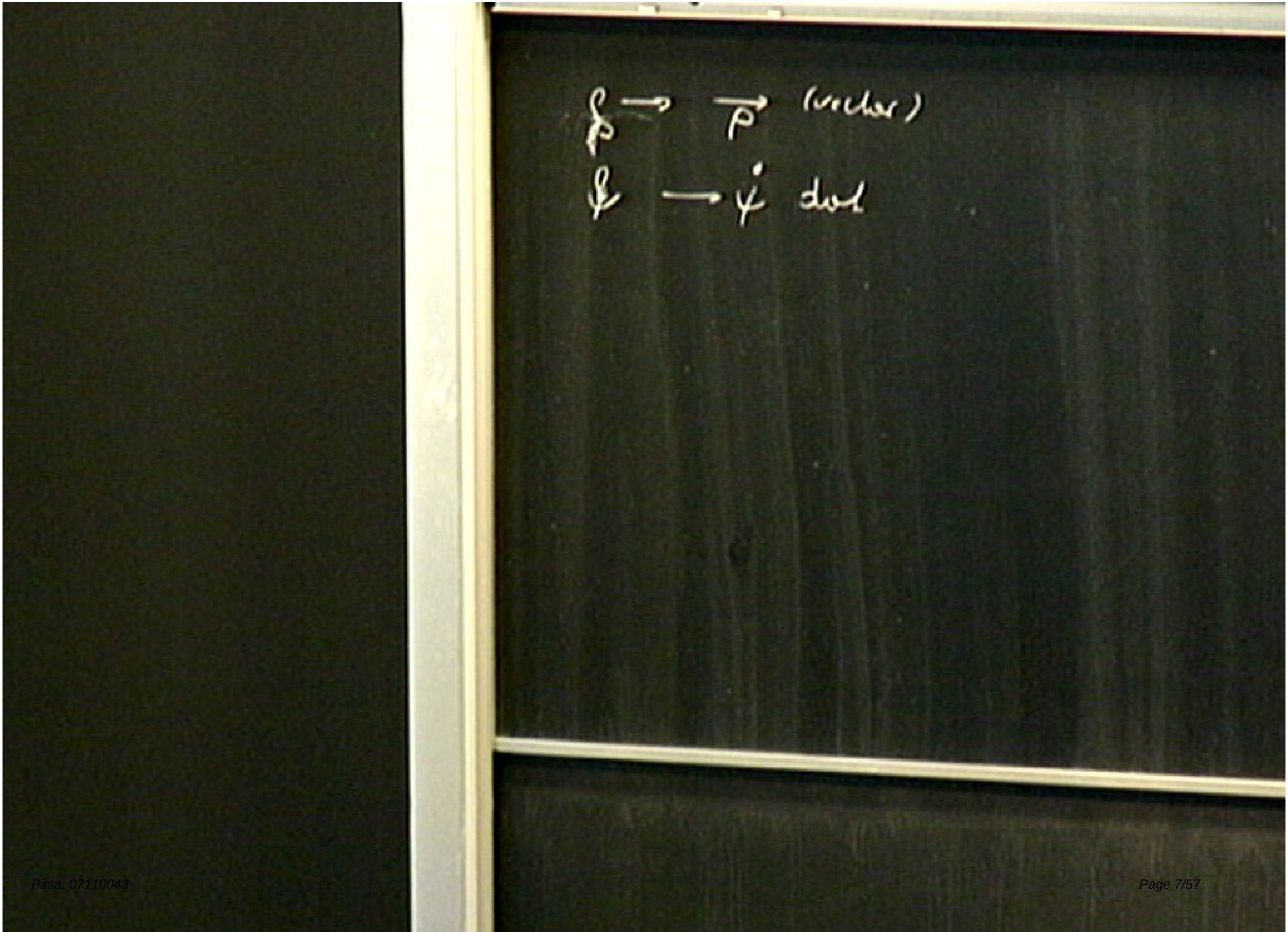
- 1947 Snyder showed that Lorentz transformations are compatible with a discrete length spectrum.
- ~ 2000: DSR in Momentum space by Amelino-Camelia and Magueijo & Smolin. Physical energy and momentum of a particle are nonlinearly transformed to so-called pseudo-variables,

$$(\varepsilon, \vec{\pi}) = U(E, \vec{p}),$$

  which the standard Lorentz transformations act on. Afterwards the inverse transformation leads back to the physical variables, so that in sum there is a nonlinear (or deformed) Lorentz transformation

$$L = U^{-1} \Lambda U,$$

  acting on energy and momentum.
- The invariant energy or momentum is transformed to infinity before the Lorentz matrix acts on it.
- The energy and/or momentum (of particles) becomes bounded. Soccerball problem for macroscopic bodies from nonlinear addition.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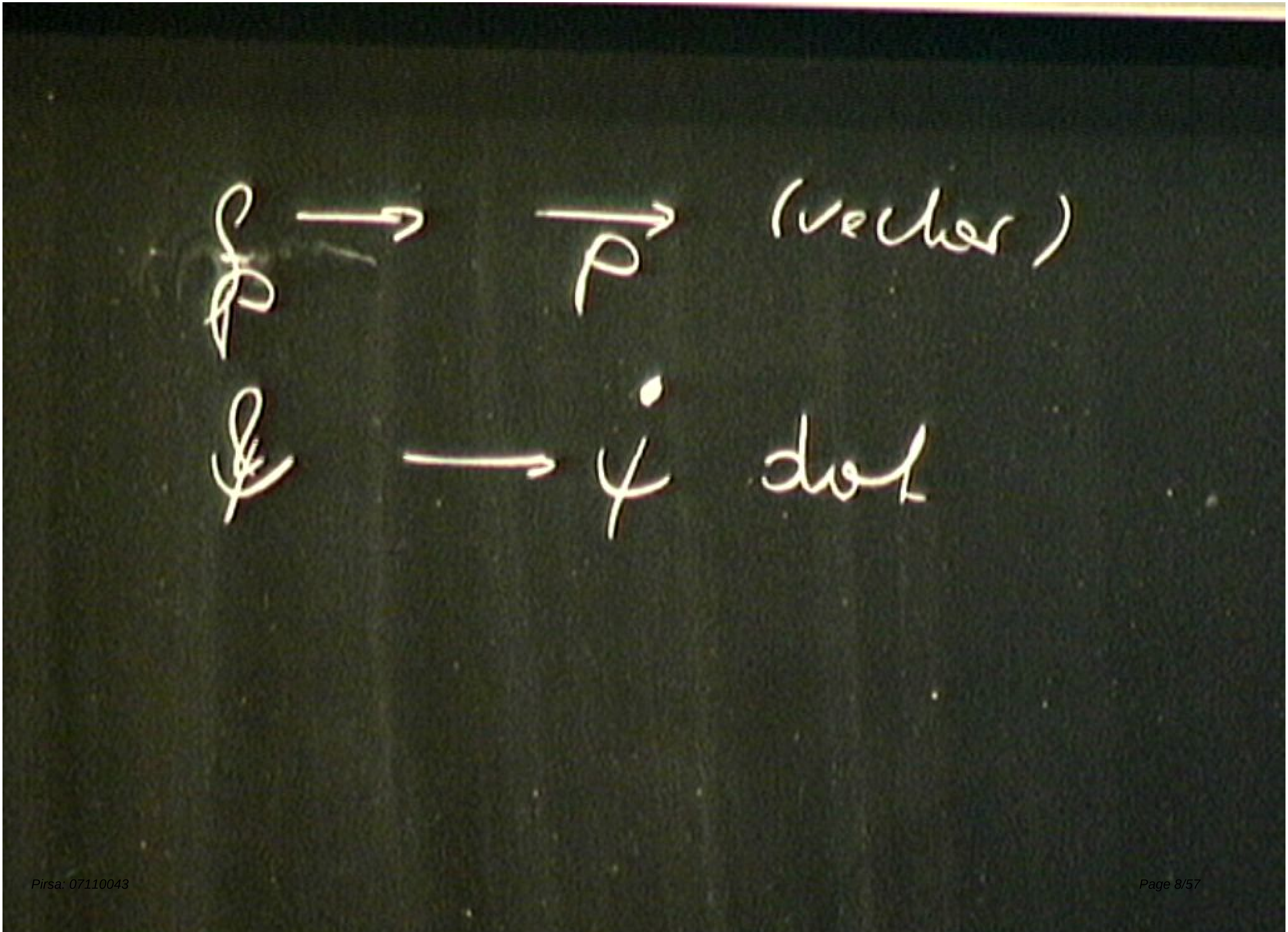
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Examples

- DSR 1 (Amelino-Camelia)

$$\varepsilon = \frac{e^{\lambda E} - \cosh(\lambda m)}{\lambda \cosh(\lambda m/2)}, \qquad \vec{\pi} = \frac{\vec{p}\, e^{\lambda E}}{\cosh(\lambda m/2)}.$$

- DSR 2 (Magueijo & Smolin)

$$\varepsilon = \frac{E}{1-\lambda E}, \qquad \vec{\pi} = \frac{\vec{p}}{1-\lambda E}.$$

m is the rest mass of a particle under consideration and λ is the inverse of the invariant energy (~ Planck length).

Dispersion Relations

- In DSR the standard quadratic energy-momentum relations apply to the pseudo-variables, so the relations between physical energy and momentum become modified. In DSR 1 they are

$$\cosh(\lambda E) = \cosh(\lambda m) + \frac{1}{2}\lambda^2 \vec{p}^2 e^{\lambda E},$$

- in DSR 2

$$E^2 - \vec{p}^2 = m^2\left(\frac{1-\lambda E}{1-\lambda m}\right)^2.$$

- For a possible detection only the lowest-order corrections are of interest, that is

$$E^2 - \vec{p}^2 = m^2 + \lambda E \vec{p}^2 + \dots$$

and

$$E^2 - \vec{p}^2 = m^2 - 2\lambda m^2 (E - m) + \dots$$

for DSR 1 and DSR 2, respectively.

DSR in Space-Time

- Direct construction: Snyder; Kimberly, Magueijo, Madeiros: invariant minimal space interval, no translation invariance, no plane wave solutions of linear fields.
- Constructions from momentum space transformations by the postulate of plane waves, with linear contractions of space-time and energy-momentum in the sense that

$$\mathrm{d}x^a p_a = \mathrm{d}x^0 p_0 + \mathrm{d}x^i p_i$$

  is invariant.
- Canonical constructions, so that the modified Lorentz transformations in momentum space and in space-time together are a canonical transformation in a particle's phase space.
- More mathematical constructions: κ - Minkowski space and Hopf algebras.

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Space-Time Metric

- In the nonlinear space-time construction there is no invariant quadratic metric. The metric disappears for small distances.
- In the plane-wave construction nonlinear momentum space induces an energy-dependent space-time metric.
- In the canonical construction the metric is energy-dependent, too.
- If an invariant space-time metric is postulated, the canonical structure (Poisson brackets of phase space variables) has to be amended. → Non-commutative space-time.

Interpretations of energy-dependent metrics

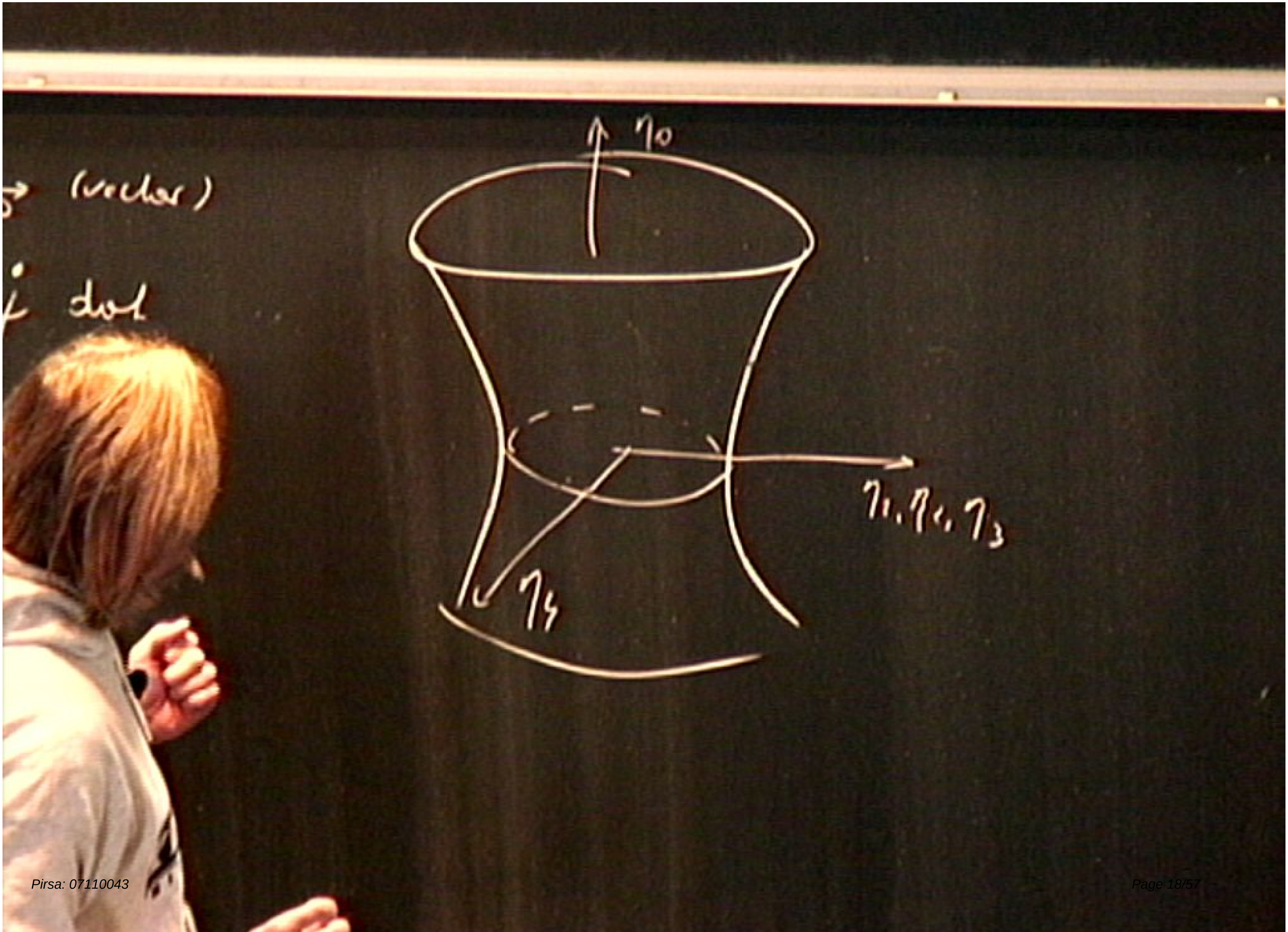
- Energy-dependent metrics are associated locally to each physical object according to its energy and momentum. → Position-dependent metric ↔ General Relativity?
- For each object space has a different metric. In a weak gravitational field these metrics are flat, but with different length and time scales. "Rainbow metric" of flat space = totality of all these flat metrics.
- This concept can be generalized to curved space: "Gravity's rainbow" (Magueijo & Smolin).

Formulation in de Sitter Space

- (J. Kowalski-Glikman). Momentum space has the geometry of de Sitter space, the position variables are considered as generators of translations on it.
- Due to the (constant) curvature of momentum space position variables are non-commutative. The parameter of non-commutativity is traditionally denoted by κ. It is of the order of magnitude of the Planck energy, $\kappa=1/\lambda$.
- κ - Minkowski space is characterized by the relation

$$\left\{x^0, x^i\right\} = x^i / \kappa.$$

- Different versions of DSR have the simple interpretation of different coordinates on de Sitter space.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η_0
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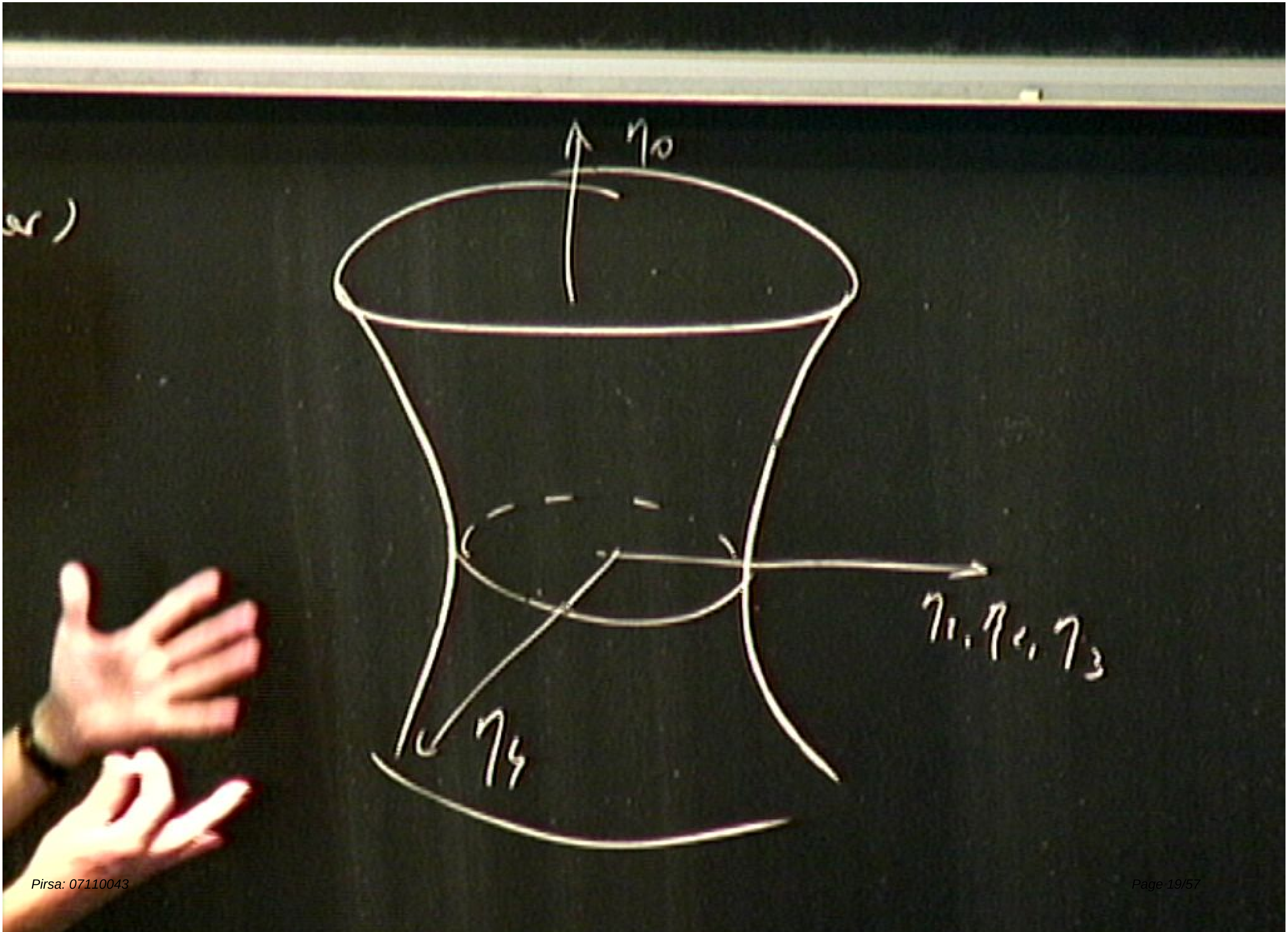
η_4



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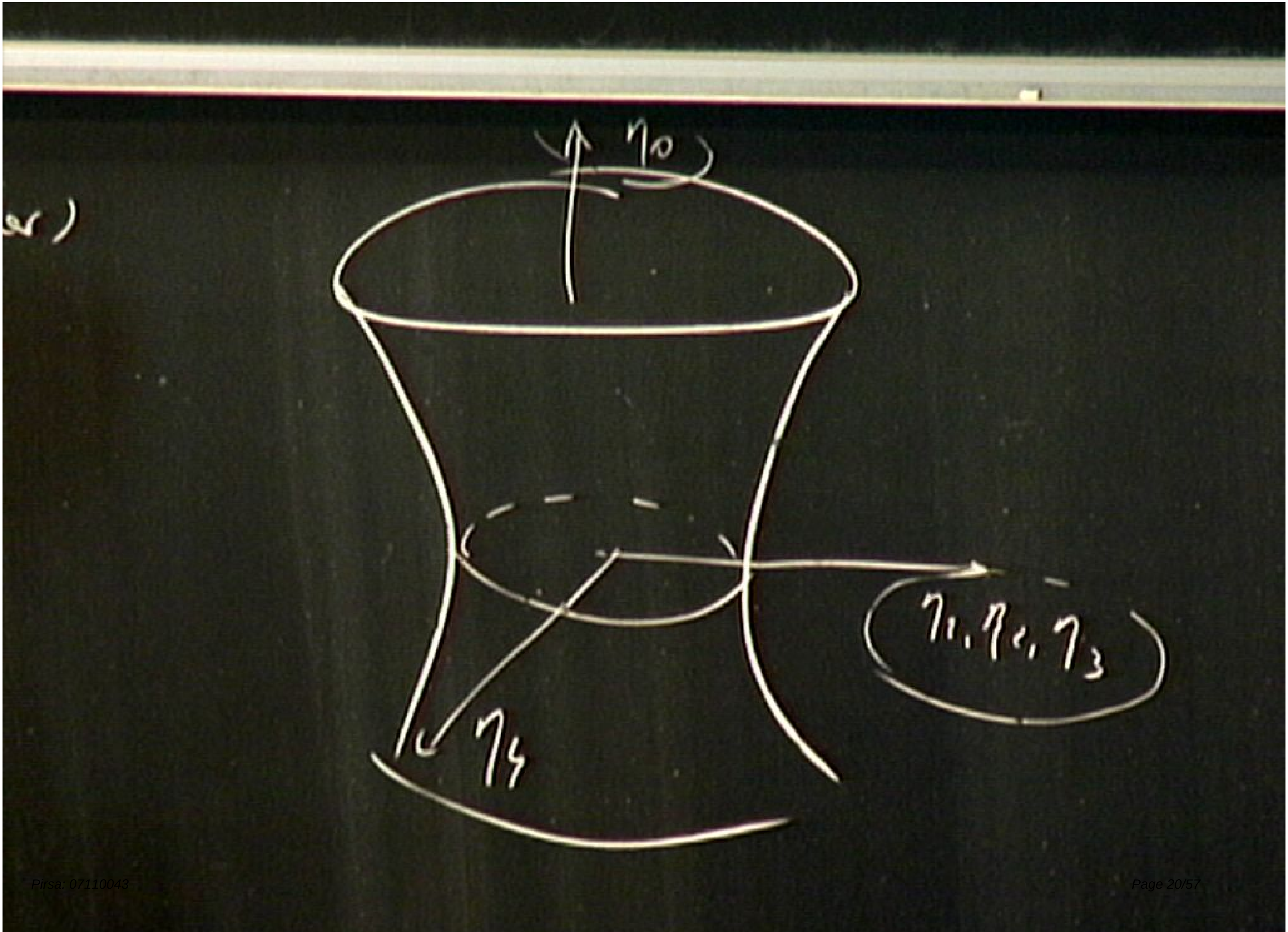

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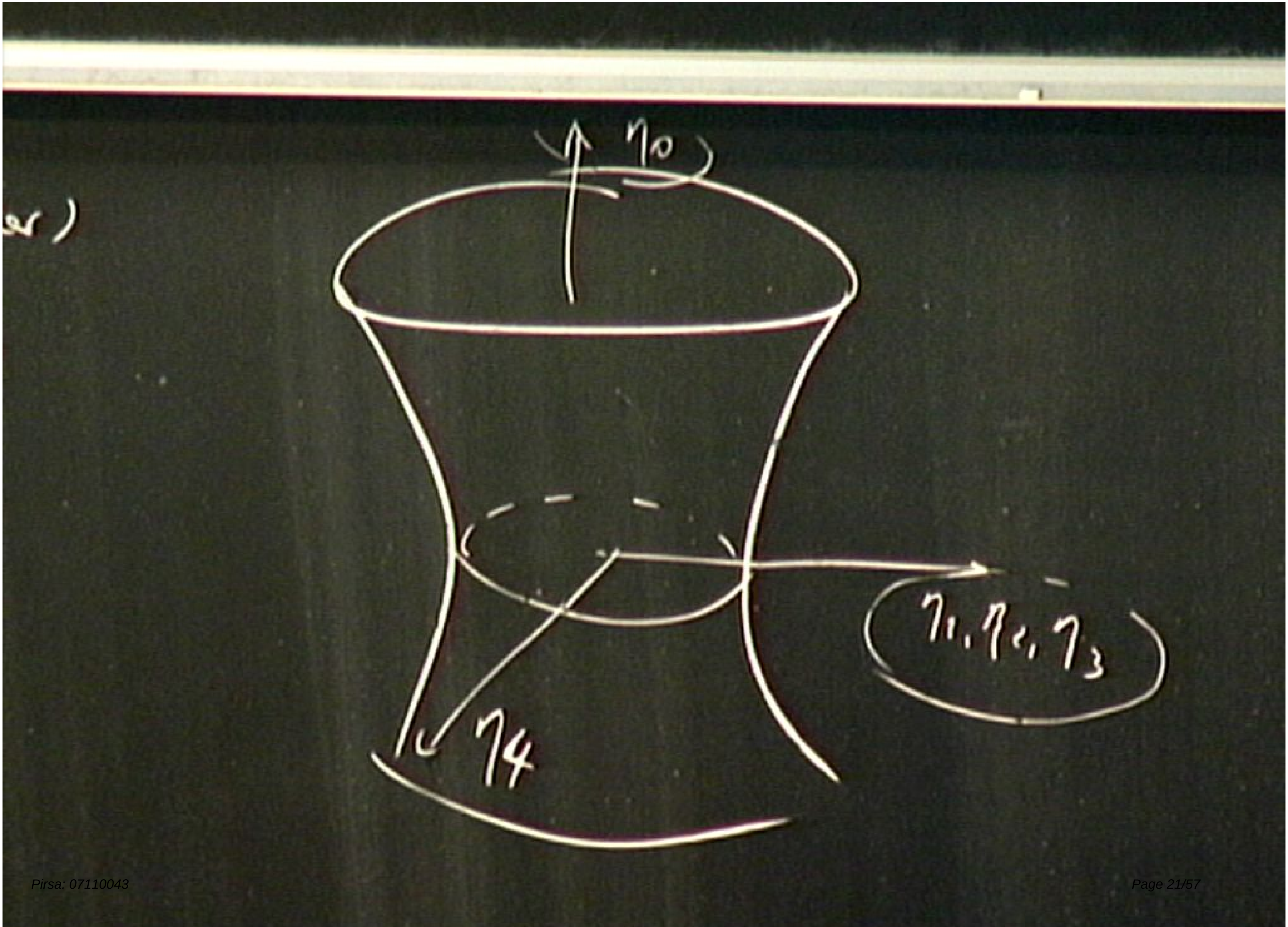

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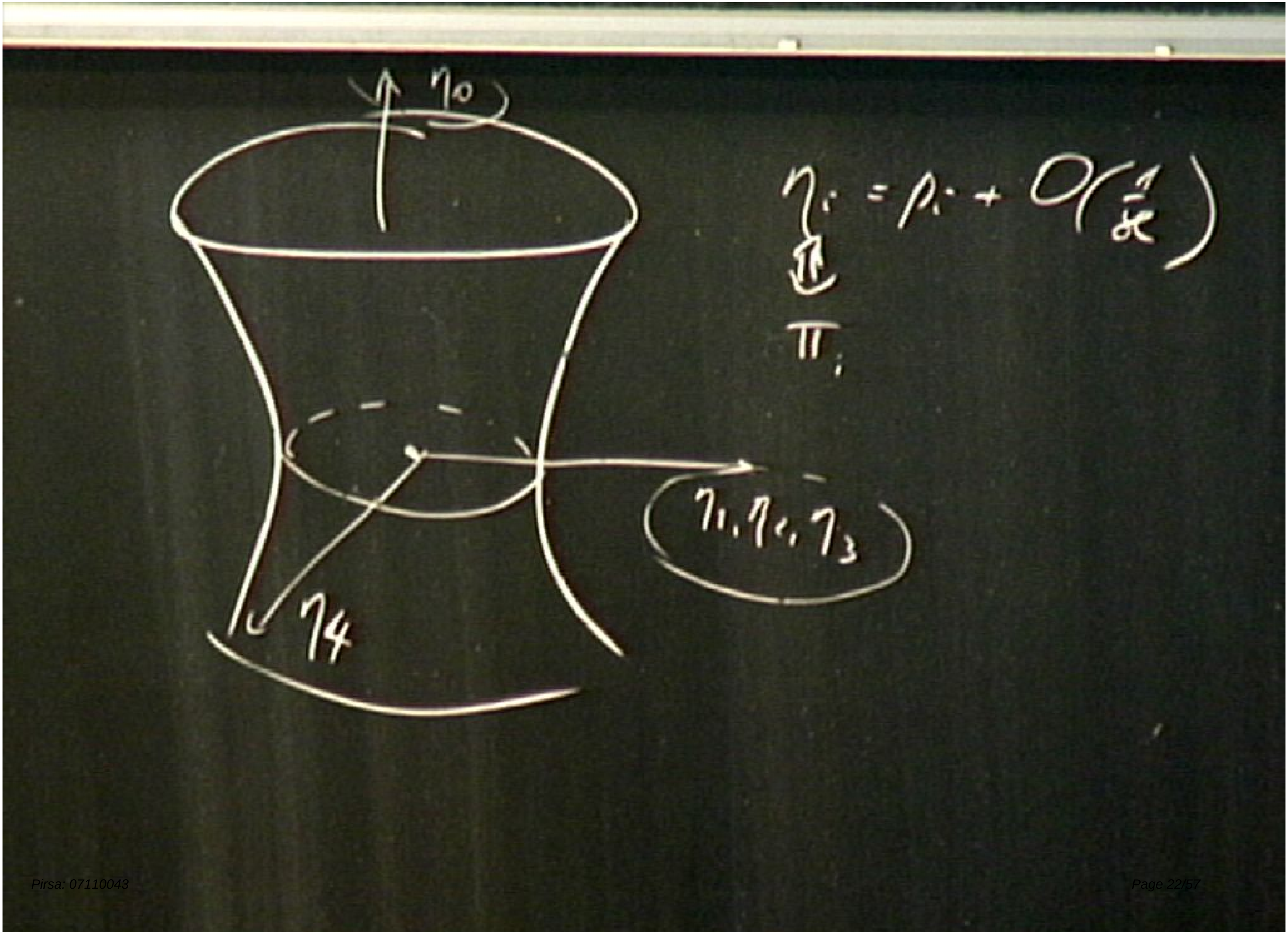
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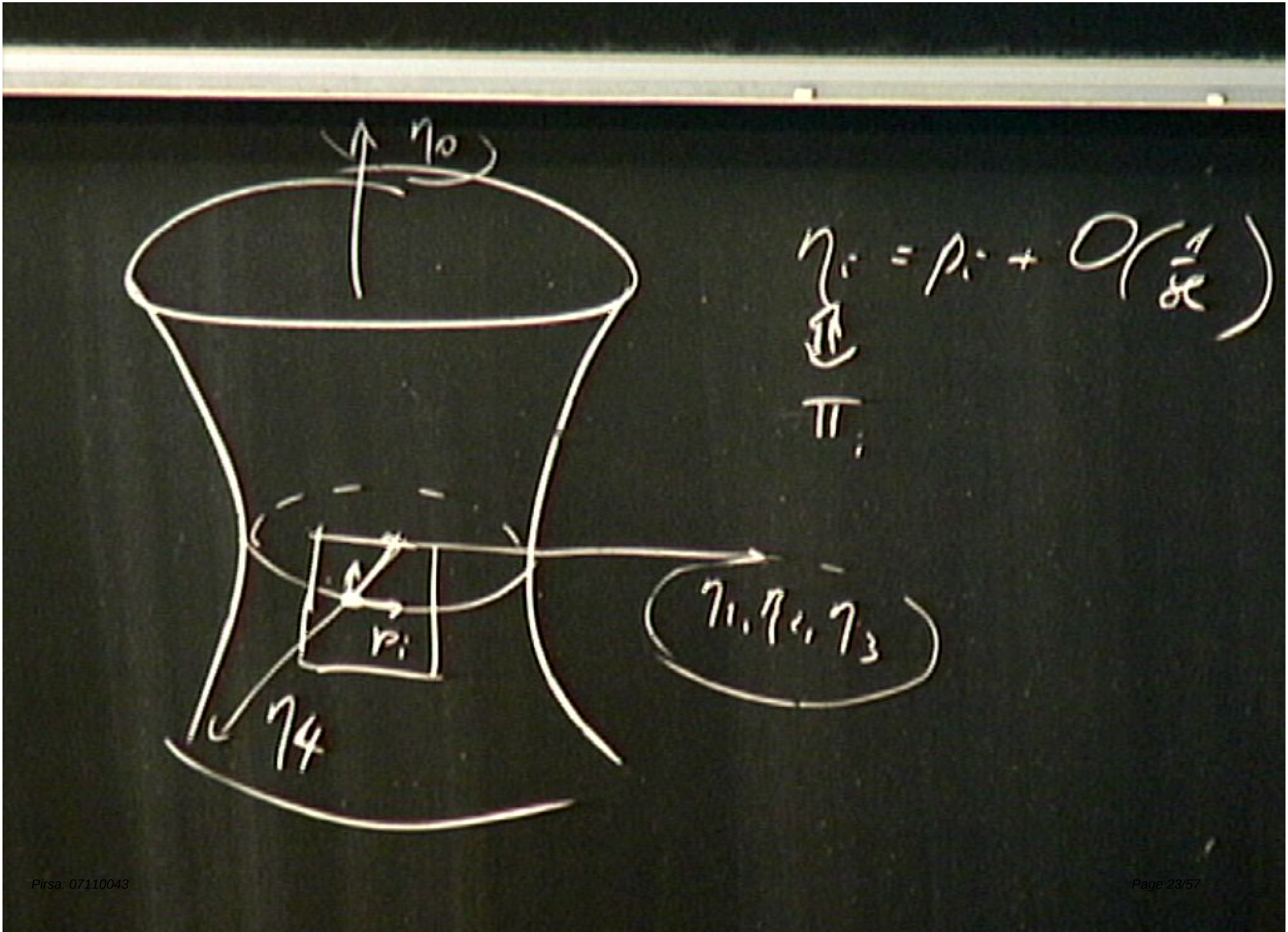
η0
ηi = pi + O(1/α)
πi
η1, η2, η3
pi
η4

A Hamiltonian in de Sitter Space

- De Sitter space is described as a hyperboloid in 5-dimensional Minkowski space,

$$-\eta_0{}^2 + \eta_1{}^2 + \eta_2{}^2 + \eta_3{}^2 + \eta_4{}^2 = \kappa^2.$$

- The coordinates $\eta_0, \ldots, \eta_3$ figure as deformed momenta, for low energies $\eta_i \approx p_i + O(1/\kappa)$ and

$$\eta_4{}^2 = \kappa^2 + p_0{}^2 - \vec{p}^2 + O(1/\kappa).$$

- Therefore

$$\kappa\eta_4 = \kappa^2 + (p_0{}^2 - \vec{p}^2)/2 + O(1/\kappa)$$

is a natural choice for a Hamiltonian.

A Hamiltonian in de Sitter Space

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Velocities in DSR

- Early definitions of velocity in momentum space DSR alone are rather arbitrary and of little significance.
- Different constructions of DSR in momentum and position space yield different definitions of velocities, particularly an energy-dependent speed of light.
- In κ - Minkowski space there is a unique definition: The Poisson bracket of the space-time coordinates with the Hamiltonian.
- In the result the limiting speed of massive particles, like the speed of massless particles, is equal to *c* for all versions of DSR.

Hopf algebras

- (S. Majid) In a given algebra *A* with unity a *coproduct* $\Delta: A \to A \otimes A$ is defined, dual to the product.
- In the case of the Poincaré Lie algebra the co-algebra sector with the trivial coproduct

$$\Delta u = u \otimes 1 + 1 \otimes u$$

concerns the action of the Lie algebra on space-time functions as derivative operators.

- Hopf algebras provide strict rules connecting deformations of the algebraic relations with deformations of the coproduct. They are an algebraic equivalent to the geometric construction in de Sitter space.

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$\wp \longrightarrow \vec{p}$ (vector)

$\wp \longrightarrow \dot{\varphi}$ dot

$\dot{x} = \{H, x\}$

$\frac{\dot{x}}{i}$

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Other Modifications

- Split between energy & momentum and frequency & wave vector of a particle (S. Hossenfelder).
- Whereas $(E, \vec{p})$ are unbounded, like DSR pseudo-variables, $(\omega, \vec{k})$ play formally the role of the physical variables in DSR with upper bounds.
- This leads immediately to a space-time formulation with space and/or time intervals bounded from below.
- Formal equivalence to DSR, but with a different physical interpretation. There are no merely auxiliary "pseudo-variables", all variables have a physical meaning.

Consequence: varying constants

- In the most general case

$$(\omega, \vec{k}) = (E f(E, \vec{p}), \vec{p} g(E, \vec{p})).$$

- For low energies, of course, $(E, \vec{p}) = \eta(\omega, \vec{k})$. Assuming these relations for arbitrary energies leads to an energy/momentum dependence of the speed of light and of Planck's constant,

$$\tilde{c}(E, \vec{p}) = c \frac{f(E, \vec{p})}{g(E, \vec{p})} \quad \text{and} \quad \tilde{\eta}(E, \vec{p}) = \frac{1}{f(E, \vec{p})}.$$

- The variable Planck constant leads to generalized uncertainty relations in quantum mechanics.

DSR and Gravity

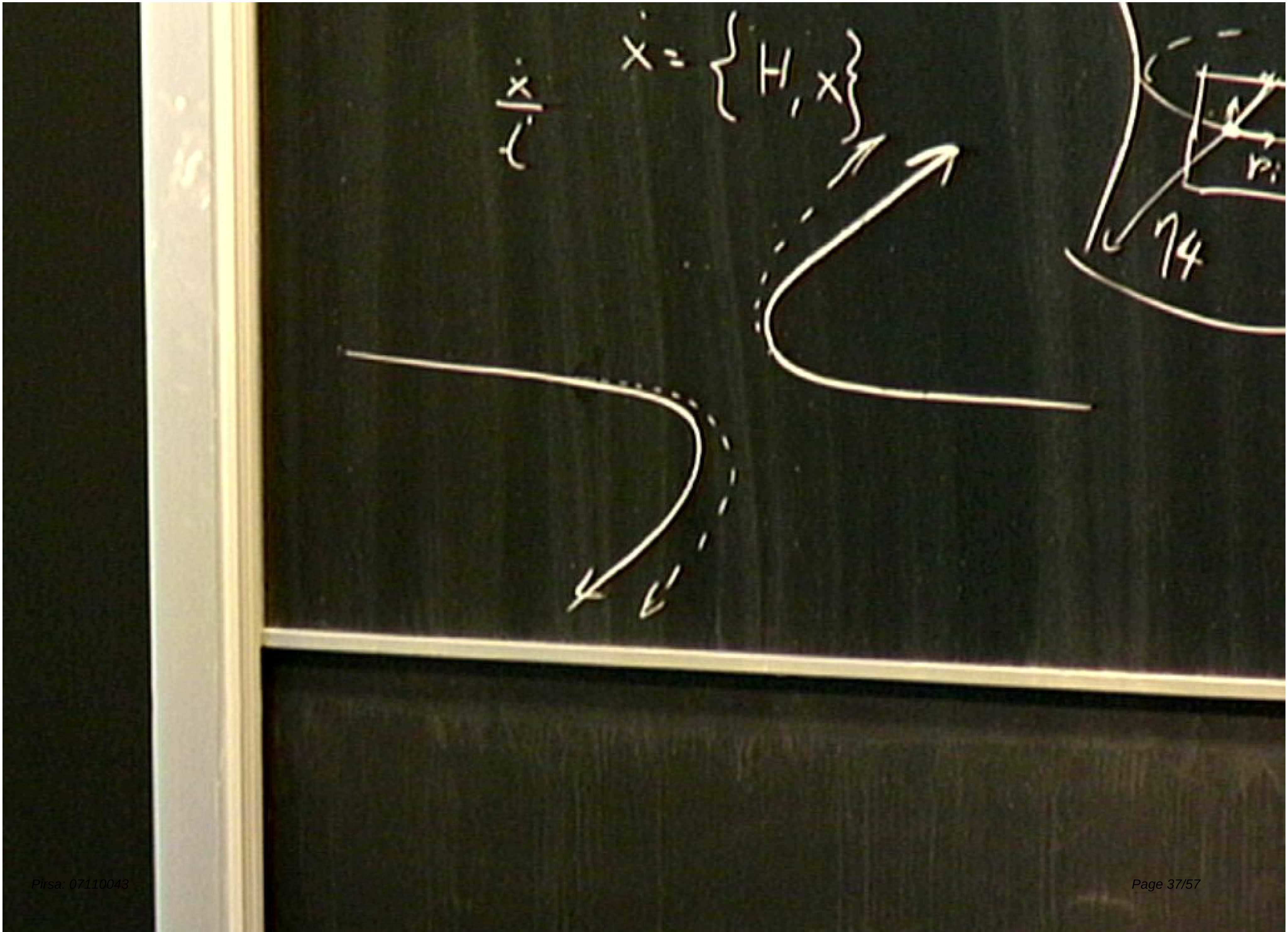
- Gravity is the driving force behind DSR and similar efforts, but formally the deformations of SR have been introduced as independent assumptions.
- A direct relation between quantum gravity and space-time non-commutativity exists, by definition, in the non-commutative geometry approach to quantum gravity.
- Again, by definition, the closest relation between gravity and DSR exists in the Hopf algebra attempt to quantize gravity.
- In LQG and string theory there are rather heuristic "derivations" of deformed dispersion relations or non-commutativity from basic principles and fundamental results.

DSR as low energy limit of gravity

- Correspondence principle (Magueijo & Smolin):
  "In the low energy limit a modified theory must accord with General Relativity".
- One approach: The low energy limit of QG is taken by $G \to 0$ and $\eta \to 0$, such that $\kappa = \sqrt{\eta / G}$ remains constant → DSR (J. Kowalski-Glikman).
  $G \to 0,\ \eta = \text{const.},\ \kappa \to \infty$: → SR.
- Physically the local d.o.f. of gravity are switched off, only the topological ones remain, like in 2+1 gravity (which may be formulated as DSR).
- Influence of the cosmological constant on the deformations of the Poincaré algebra.

DSR and Classical Gravity I

- In the foregoing approach DSR has nothing to do with local gravity, e.g. the attraction of two bodies.
- On the other hand: The low energy limit of GR is Newtonian gravity, so one might compare it, in a different interpretation of the correspondence principle, with the lowest order approximation of DSR.
- In a simple scattering process of two particles with a repulsive potential Newtonian gravity is introduced as small perturbation (provided the particles remain point-like, when they come very close) and compared with DSR corrections.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DSR and Classical Gravity 2

- Correction of the scattering angle (ϑ_0 is the unperturbed angle without gravity) corrected by Newtonian Gravity:

$$\cos\frac{\vartheta_0}{2} = \left[1 + \alpha\,\frac{m\,p}{m_{\mathrm{P}}^{\;2}}\left(1 - \cos\frac{\vartheta_0}{2}\right) + \beta\,\frac{p^2}{m_{\mathrm{P}}^{\;2}}\left(1 - \cos\frac{\vartheta_0}{2}\right)^2\right]\cos\frac{\vartheta_0}{2}.$$

- Correction due to DSR with $\pi = p\left[1 + \kappa\left(p/m_{\mathrm{P}}\right)^n\right]$:

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DSR and Classical Gravity 3

- Possible reasons for this mismatch:

- DSR corrections arise from quantum properties of flat space and are not directly connected with the local gravitational field; confirmation of the hypothesis that DSR = topological gravity.
- The identification pseudo-variables = unperturbed variables in the absence of gravity and physical variables = gravitationally corrected variables is wrong.
- The simple DSR ansatz in momentum space is too naïve.

LQG and QFT in Curved Space

- H. Sahlmann and T. Thiemann: QFT in curved space as effective theory from full quantum theory of gravity and matter in a state of geometry close to a classical geometry.
- $\hat{H}$ is the Hamiltonian constraint of the coupled system, $\psi_{\text{grav}}(m)$ represents geometry (flat space for DSR).

$$\left\langle \psi_{\text{matter}}, \hat{H}^{\text{eff}}_{\text{matter}}(m)\psi'_{\text{matter}} \right\rangle = \left\langle \psi_{\text{grav}}(m) \otimes \psi_{\text{matter}}, \hat{H}\,\psi_{\text{grav}}(m) \otimes \psi'_{\text{matter}} \right\rangle$$

m represents the metric variables in the matter operator.

- For the purpose of DSR we consider only free fields, because quantum fluctuations of matter do not influence Poincaré invariance.
- The Hamiltonian contains a huge number of vertex expressions $q(v)$ from the spin network states representing Minkowski space.

Continuous approximation

- Continuous limit of the vertex functions $q(v) \to q(\vec{x}(v))$
- Functions at different vertices, connected with $q(v)$ are Taylor-expanded:

$$q(\vec{x}(v')) = q(\vec{x}(v)) + (x^i(v') - x^i(v)) \frac{\partial}{\partial x^i} q(\vec{x}(v)) + \ldots$$

- The first term is a local contribution at $\vec{x}(v)$, in the remainder originally nonlocal terms are replaced by higher-order derivatives.
- In the sequel this gives rise to modifications of the dispersion relations, when q are energy and momentum functions.

A Model for Matter

- Matter is represented by a scalar field (M. Bojowald, H. Morales-Técotl, H. Sahlmann) with a classical Hamiltonian

$$H = \int_{\Sigma} \mathrm{d}^3x \left(\frac{1}{2} N \left[(\det q)^{-\frac{1}{2}} p_{\phi}{}^{2} + \sqrt{\det q}\, q^{ab} \partial_a \phi \partial_b \phi \right] \right)$$

where q^{ab} is the spatial metric, N is the lapse function, Σ a spacelike hypersurface and p_{ϕ} the canonical field momentum.

- LQG: Discretization → patches α of equal size and expectation values of the metric quantities.

Quantization

- The geometry of the patches α is assumed to be isotropic with the geometric operators borrowed from isotropic Loop Quantum Cosmology. There the only relevant quantity is the scale factor a, or the densitized triad component $p = a^2$, respectively.
- The partial derivatives are replaced by lattice derivatives, where $\alpha \pm e_I$ are neighbouring patches in the direction e_I.

$$\phi_{\alpha+e_I} = \phi(x_\alpha) + \partial_I \phi(x_\alpha) + \tfrac{1}{2}\partial_I^2 \phi(x_\alpha) + \mathrm{K}$$

- → Discrete, isotropic matter Hamiltonian.

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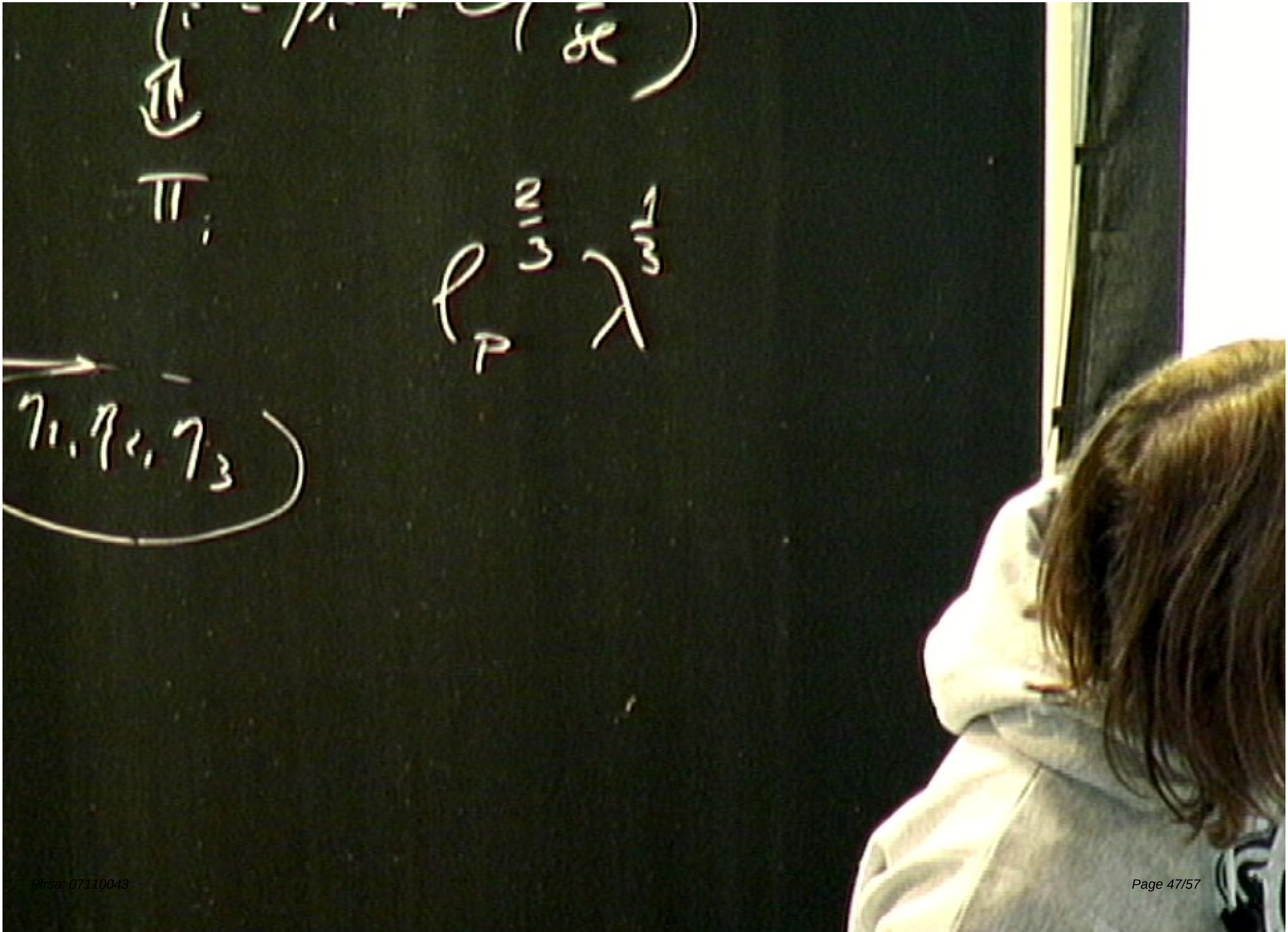
Quantitative Estimate

- Assume a solution of the field equations of the form
$$\phi = \exp\left[i\left(akx + N\varpi t\right)\right].$$
- The discretization corrections, which grow with the size of the patches, contain the wavelength of ϕ.
- The corrections coming from the expectation values become larger for smaller patches.
- One can find a patch size a leading to minimal corrections:
$$a = \sqrt[6]{\frac{c}{8\pi d} l_{\mathrm{P}}^4 \lambda^2}$$
- d is a numerical constant.


Π_i
ℓ_P^{2/3} λ^{1/3}
η_1, η_2, η_3

Discreteness of LQG

- LQG indicates spatial discreteness.
- Time discreteness? Indications from Loop quantum Cosmology.
- LQG is formulated in Hamiltonian guise – time is by definition a continuous parameter.
- Spin foams, the space-time covariant counterpart, do not yet yield results concerning Lorentz invariance.
- Apparent Lorentz symmetry violations by higher-order spatial derivative terms may be an artefact.
- The perturbative expansion and the Legendre transform to the covariant Lagrange formalism do not commute.

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Lagrange and Hamilton Formalism

- In an exact theory the Lagrange and the Hamilton formalism are equivalent, but not so in a perturbative expansion.
- A Legendre transform of a Lagrangian with space and time perturbative corrections leads to a Hamiltonian that is non-analytic in the expansion parameter.
- A Legendre transform of a perturbative Hamiltonian does not contain higher-order time derivative corrections.
- A Lagrangian constructed in this way might apparently be Lorentz-violating, only because the Hamiltonian formalism "does not see" time corrections.

A Simple Example - Lagrange

- Scalar field, whose Lorentz invariant Lagrangian

$$L = -\frac{1}{2}\int\left(\psi\left(\Box + \varepsilon\Box^2\right)\psi + m^2\psi^2\right)$$

- leads to Lorentz invariant field equations

$$-\left(\Box + \varepsilon\Box^2\right)\psi = m^2\psi.$$

- The dispersion relations for a plane wave

$$E^2 = \vec{k}^2 + m^2 - \varepsilon\, m^4 + O(\varepsilon^2)$$

are Lorentz invariant.

A Simple Example - Hamilton

- A Legendre transform of the above Lagrangian requires two canonical momenta, π_ψ and $\pi_{\dot\psi}$ and contains a term $-\frac{1}{2}\varepsilon^{-1}\pi_{\dot\psi}^{\;2}$.
- A perturbation of the according Hamiltonian yields

$$H = \frac{1}{2}\left(\pi_\psi^{\;2} - \psi\,\Delta\psi + m^2\psi^2 + \varepsilon(2\pi_\psi\Delta\pi_\psi + \psi\,\Delta^2\psi)\right),$$

the non-Lorentz invariant field equations

$$\ddot\psi = \Delta\psi - m^2\psi + \varepsilon\left(\Delta^2\psi - 2m^2\Delta\psi\right)$$

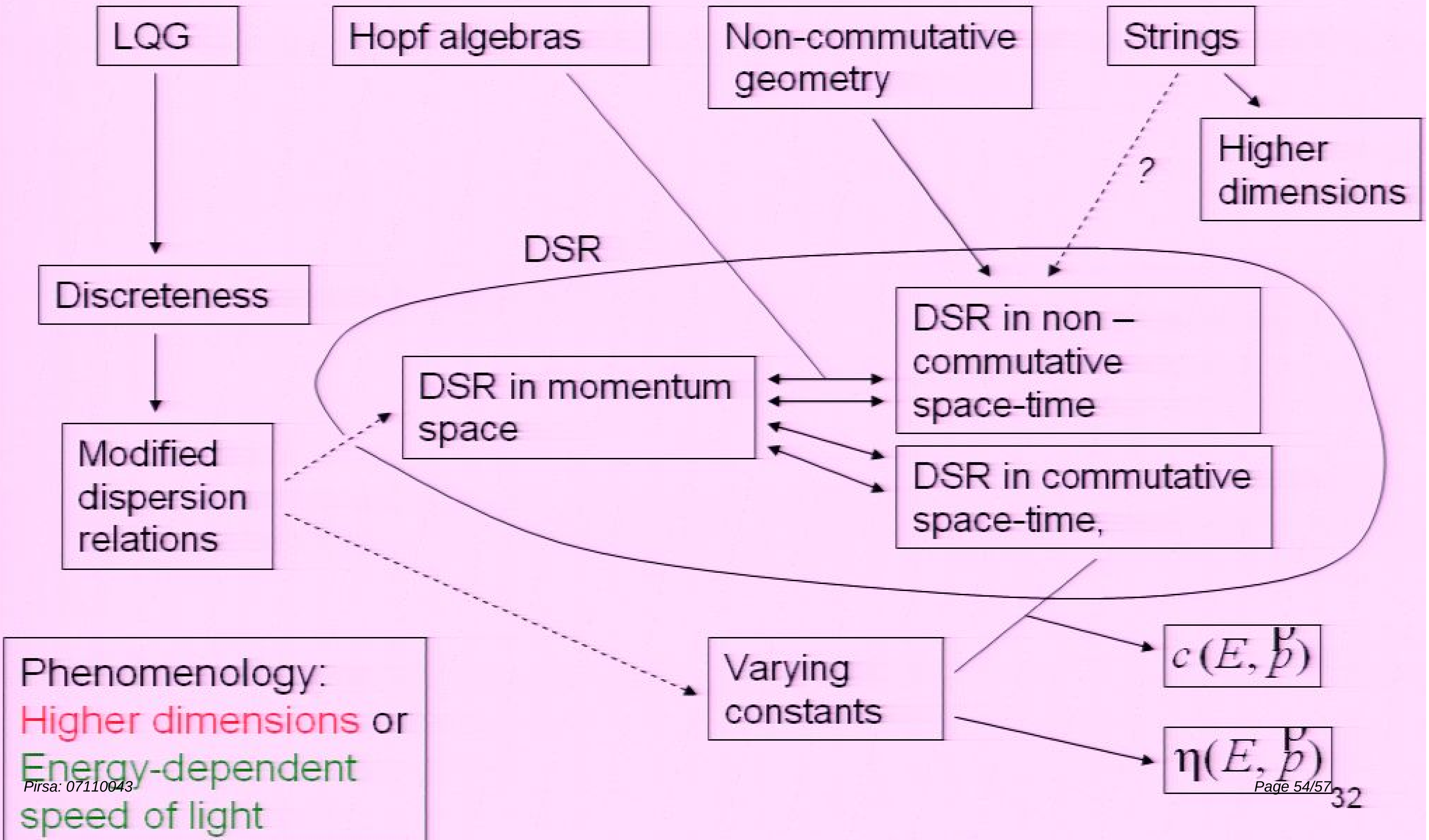
and the non-Lorentz invariant dispersion relations

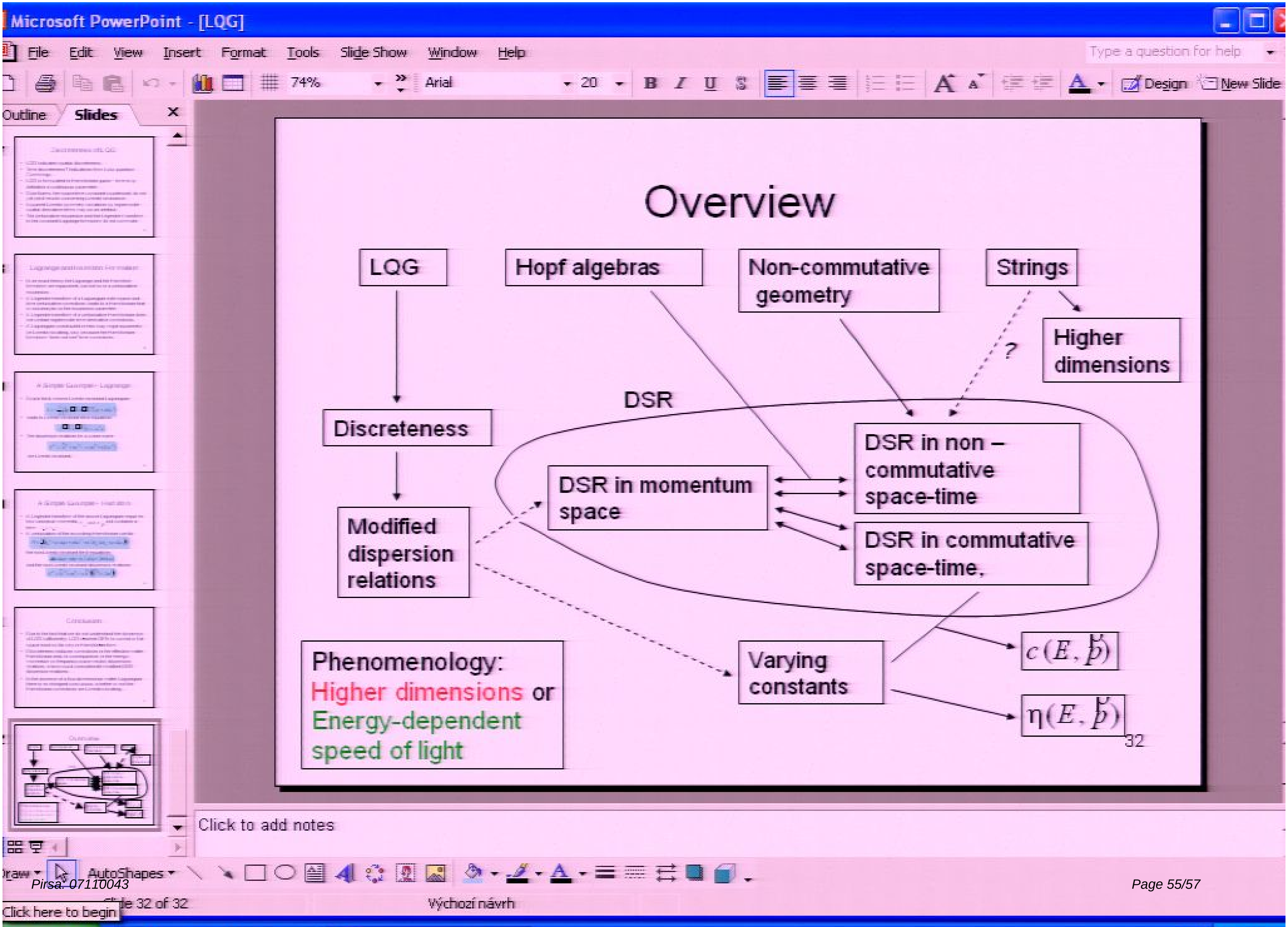
$$E^2 = \vec{k}^2 + m^2 - \varepsilon\,\vec{k}^2\left(\vec{k}^2 + 2m^2\right).$$

Conclusion

- Due to the fact that we do not understand the dynamics of LQG sufficiently, LQG inspired QFTs in curved or flat space exist so far only in Hamiltonian form.
- Discreteness induces corrections in the effective matter Hamiltonian and, in consequence, in the energy-momentum (or frequency-wave vector) dispersion relations, which could coincide with modified DSR dispersion relations.
- In the absence of a four-dimensional matter Lagrangian there is no stringent conclusion, whether or not the Hamiltonian corrections are Lorentz-violating.

Overview


LQG
Hopf algebras
Non-commutative geometry
Strings
Higher dimensions
?
DSR
Discreteness
Modified dispersion relations
DSR in momentum space
DSR in non – commutative space-time
DSR in commutative space-time,
Varying constants
c (E, p)
η(E, p)
Phenomenology:
Higher dimensions or
Energy-dependent
speed of light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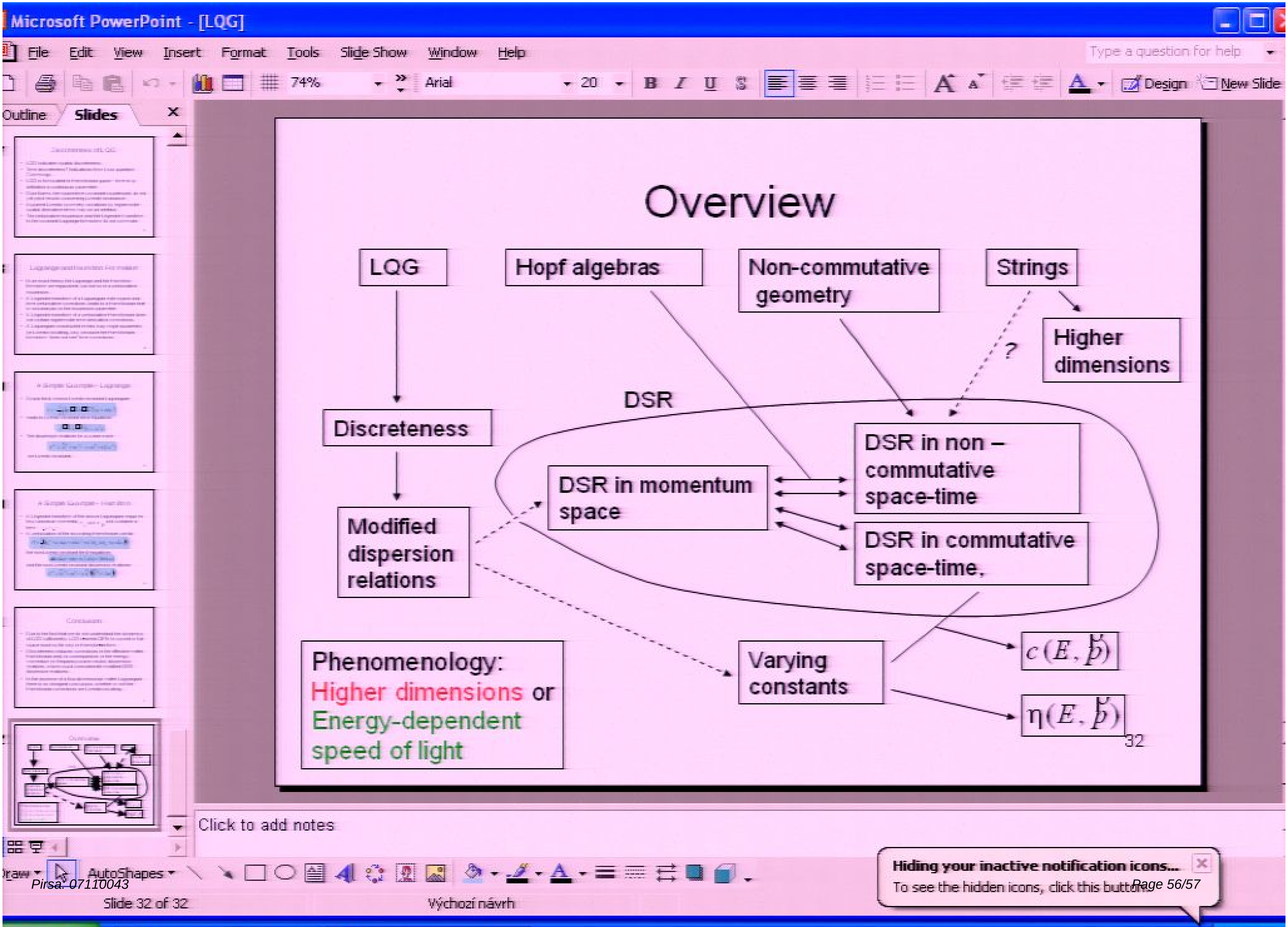
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