Title: Mass-generating mechanism for Nambu-Goldstone bosons in emergent spacetime and its application for quantum gravity phenomenology

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URL: http://pirsa.org/07110042

Abstract: Effective field theories (EFTs) have been widely used as a framework in order to place constraints on the Planck suppressed Lorentz violations predicted by various models of quantum gravity. There are however technical problems in the EFT framework when it comes to ensuring that small Lorentz violations remain small -- this is the essence of the \'naturalness\' problem. Herein we present an \'emergent\' space-time model, based on the \'analogue gravity\'\' programme, by investigating a specific condensed-matter system that is in principle capable of simulating the salient features of an EFT framework with Lorentz violations. Specifically, we consider the class of two-component BECs subject to laser-induced transitions between the components, and we show that this model is an example for Lorentz invariance violation due to ultraviolet physics. Furthermore our model explicitly avoids the \'naturalness problem\', and makes specific suggestions regarding how to construct a physically reasonable quantum gravity phenomenology.

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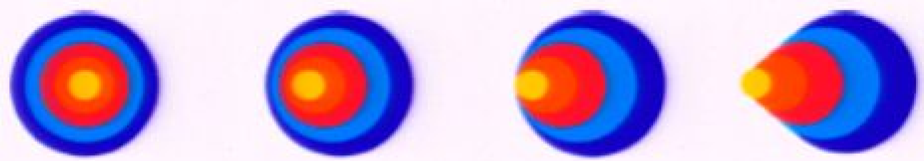
Mass-generating mechanism for Nambu-Goldstone bosons in emergent spacetime and its application for quantum gravity phenomenology

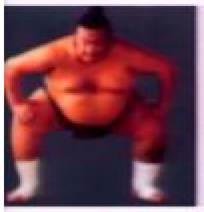


By Silke Weinfurtner

In collaboration with Stefano Liberati and Matt Visser

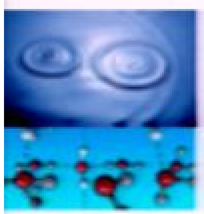
Outlook

Emergent spacetimes: 2 sides of a coin?



Generating a “mass” and Goldstone’s theorem?

Mono-metricity: A fine tuning fantasy?



Collective excitations & microscopic substructure?

Lessons learnt for QGP or a fancy CMS?

Introduction: Emergent spacetimes

[*] Concept of emergent spacetimes -
From micro- to macroscopic physics...

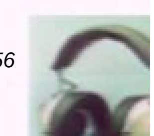
[*] Analogue models for gravity -
linearizing around some classical background...

[*] Inner and out observer -
2 sides of a coin...

The concept of emergence

Emergent spacetimes involve...

- A microscopic system of fundamental objects (e.g. strings, atoms or molecules);
- a dominant mean field regime, where the microscopic degrees of freedom give way to collective variables;
- a geometrical object (e.g. a symmetric tensor dominating the evolution of linearized classical and quantum excitations around the mean field;
- An emergent Lorentz symmetry for the long-distance behavior of the geometrical object;



Example BEC [microscopic degrees of freedom]

Emergent spacetimes from Bose-gas

- A microscopic system of fundamental objects:
  ultra-cold dilute gas of weakly interacting Bosons

Microscopic theory well understood:

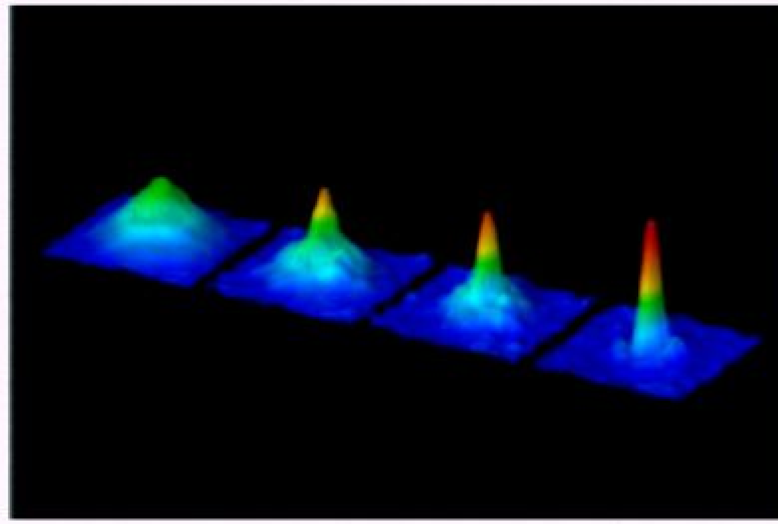
$$\hat{H} = \int dx \left(-\hat{\psi}^\dagger \frac{\hbar^2}{2m} \nabla^2 \hat{\psi} + \hat{\psi}^\dagger V_{ext} \hat{\psi} + \frac{U}{2} \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} \right)$$

Invariant under: $\theta \rightarrow \tilde{\theta} = \theta \exp(i\alpha)$

SO(2) – symmetry

Example BEC [macroscopic variables]

Emergent spacetimes from Bose-gas



➤ A dominant mean field regime:
Bose-Einstein condensate

Spontaneous symmetry breaking:

$$\langle\hat{\psi}(t,\mathbf{x})\rangle = \psi(t,\mathbf{x}) = \sqrt{n_0(t,\mathbf{x})}\,\exp\left(i\theta_0(t,\mathbf{x})\right) \neq 0$$

Example BEC [geometrical object]

- A geometrical object dominating excitations:
  Nambu-Goldstone bosons (massless fields)

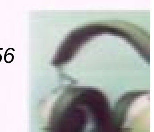
Effective covariant free-field equation for spin-0 massless particles:

$$\frac{1}{\sqrt{|g|}}\,\partial_a\left(\sqrt{|g|}\,g^{ab}\,\partial_b\hat{\theta}\right)=0$$

(Klein-Gordon equation)

Symmetric rank (d+1) tensor:

$$g_{\mu\nu}=\left(\frac{\rho}{c}\right)^{2/(d-1)}\left[\begin{array}{c|c} -\left(c^2-v^2\right) & -\vec{v}^T \\ -\vec{v} & \mathbf{I}_{d\times d}\end{array}\right]$$



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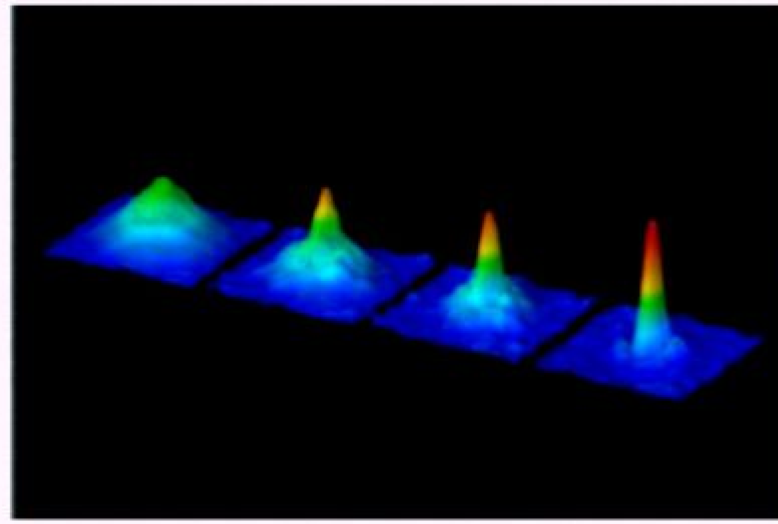

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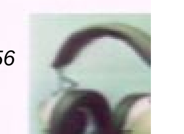

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Semi-classical quantum geometry

C. Barcelo, S. Liberati, and M. Visser. Analog gravity from field theory normal modes? *Class. Quant. Grav.*, 18:3595–3610, 2001.

Effective curved-spacetime quantum field theory description of the linearization process:

Small perturbations around some background solution $\phi_0(t, \mathbf{x})$

$$\phi(t, \mathbf{x}) = \phi_0(t, \mathbf{x}) + \epsilon\, \phi_1(t, \mathbf{x}) + \frac{\epsilon^2}{2}\, \phi_2(t, \mathbf{x})$$

In a generic Lagrangian $\mathcal{L}(\partial_a \phi, \phi)$, depdending only a single Scalar field and its first derivatives yields an effective Spacetime geometry

$$g_{ab}(\phi_0) = \left[-\det \left(\frac{\partial^2 \mathcal{L}}{\partial(\partial_a \phi)\, \partial(\partial_b \phi)} \right) \right]^{\frac{1}{d-1}} \Bigg|_{\phi_0} \left(\frac{\partial^2 \mathcal{L}}{\partial(\partial_a \phi)\, \partial(\partial_b \phi)} \right)^{-1} \Bigg|_{\phi_0}$$

For the classical/ quantum fluctuations. The equation of Motion for small perturbations around the background Are then given by $\left(\Delta_{g(\phi_0)} - V(\phi_0)\right) \phi_1 = 0$

Kinematics versus dynamics!

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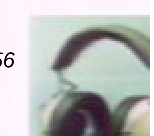
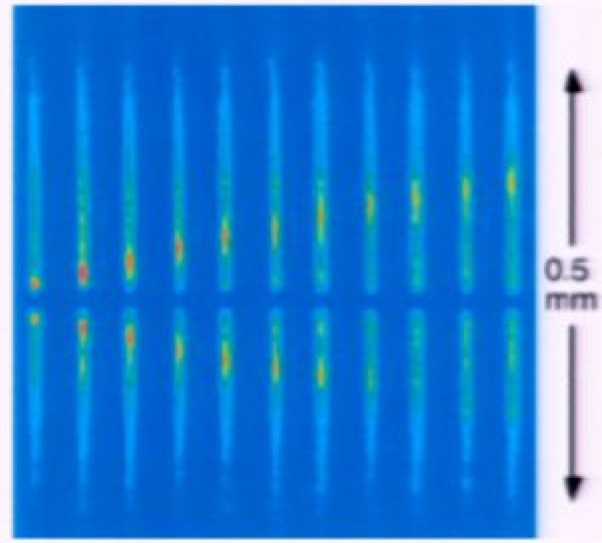
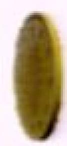
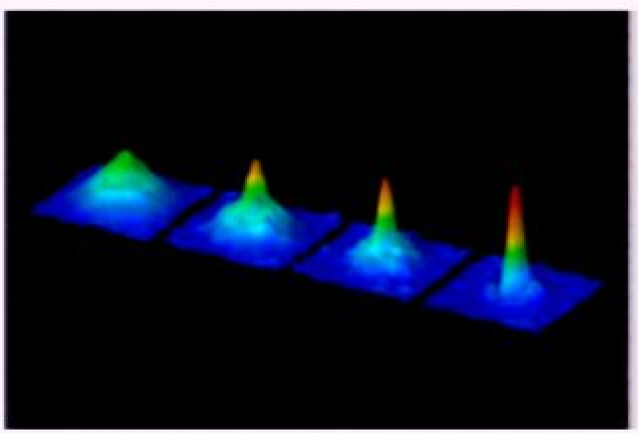


Inner and out observer/ absolute time

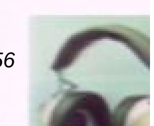
Inner observer:
Small excitations in the system experience an effective spacetime geometry represented by the macroscopic mean–field variables!







Outer observer:
Live in the preferred frame – the laboratory frame, such that the condensate parameters are functions of lab–time (absolute time).



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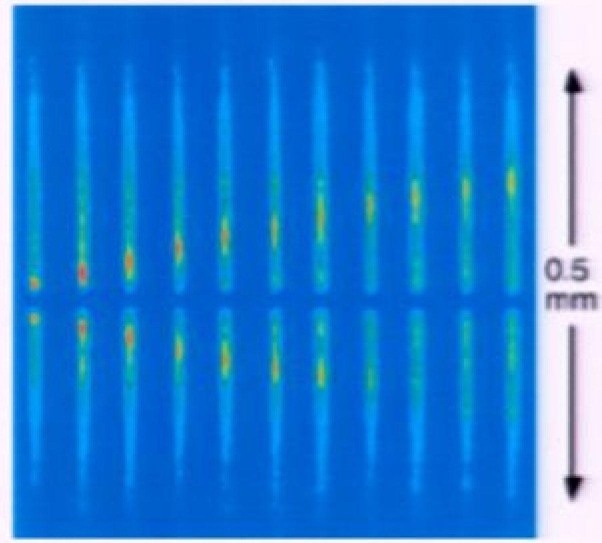

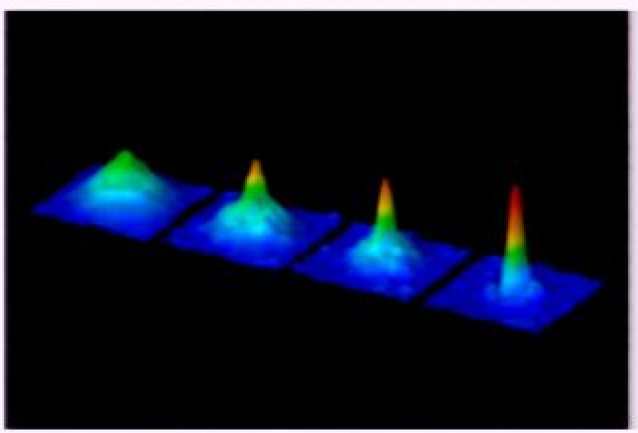

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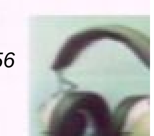


Generating a *mass* and Goldstone's theorm

[*] Back to the equation of motion -

Can we extend the class of fields...

[*] Goldstone's theorem -

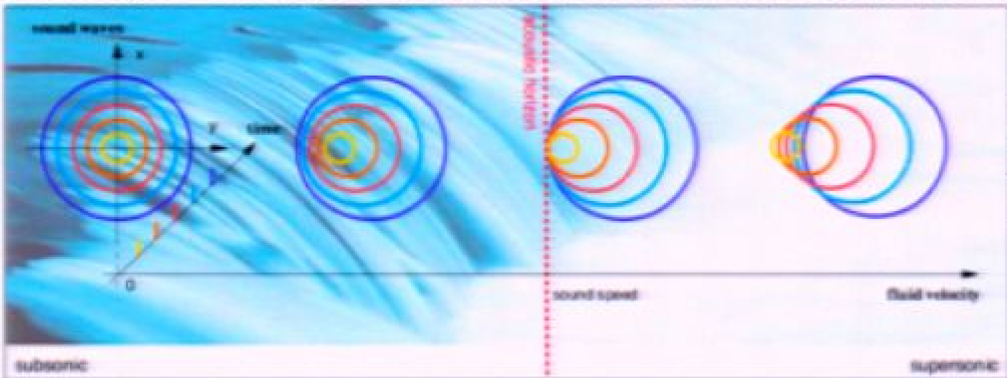
Spontaneous symmetry breaking...

[*] Mass generating mechnism -

Explicit symmetry breaking...

Analouge models for **massless** minimally coupled **scalar field**

A simple example: Sound waves in a fluid flow

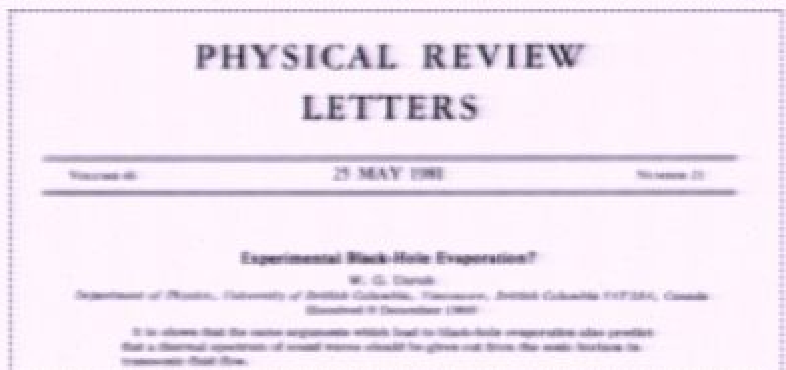


Exact analogy to a massless minimally coupled scalar field in an effective curved spacetime:

$$g_{ab} \propto \begin{bmatrix} -(c^2 - v^2) & -v_j \\ -v_i & \delta_{ij} \end{bmatrix}$$

The kinematic equations for small - classical or quantum - perturbations (i.e., sound waves) in barotropic, invicid and irrotational fluid are given by

$$\frac{1}{\sqrt{-g}}\partial_a\left(\sqrt{-g}g^{ab}\partial_b\phi\right) = 0$$


PHYSICAL REVIEW LETTERS

25 MAY 1981

Experimental Black-Hole Evaporation?

Is it possible to find AN analogue model for **massive** scalar fields?

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Goldstone's theorem…

"… whenever a continuos symmetry is spontaneously broken, massless fields, known as Nambu-Goldstone bosons, emerge." [Quantum Field Theory in a Nutshell, A. Zee]

Per definition Bose-Einstein condensation always (spontaneously) breaks SO(2) symmetry of many-body Hamiltonian!!!

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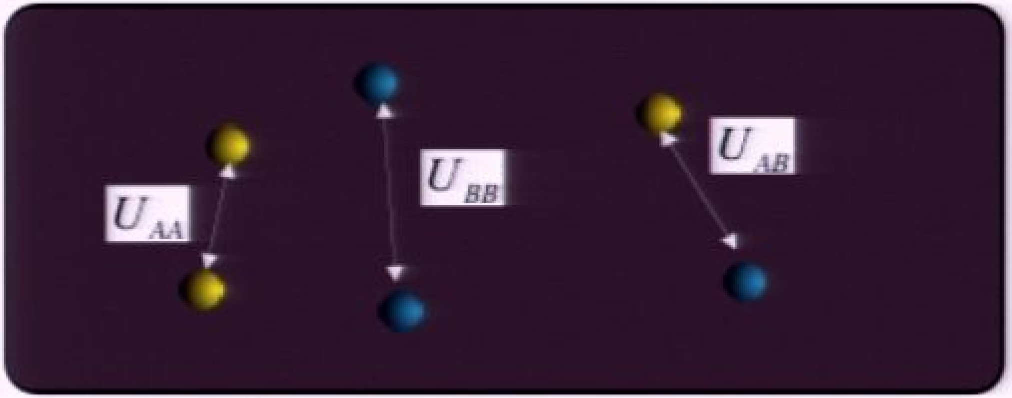
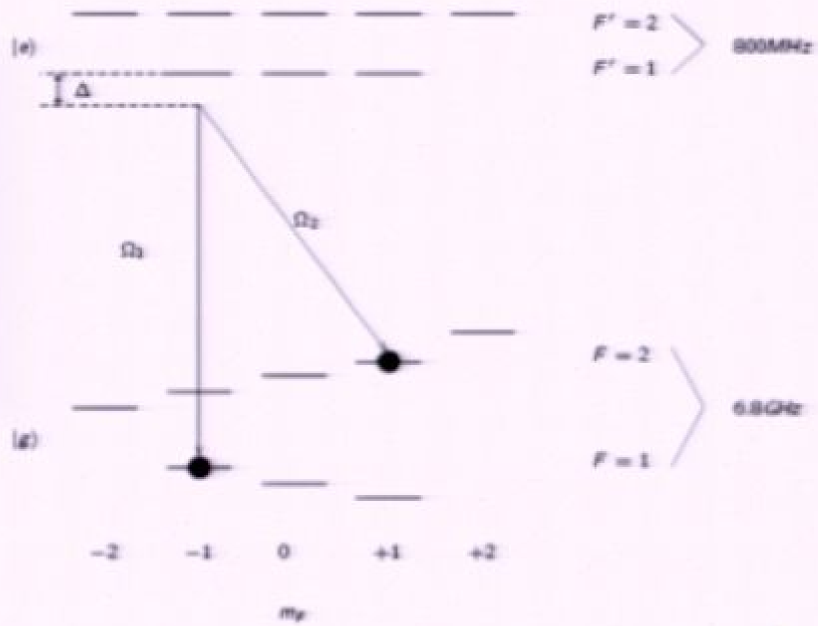
Mass generating mechnism - all about symmetries

M. Visser and S. W. *Phys. Rev.*, D72:044020, 2005.





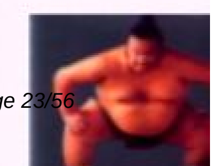
Explicit symmetry breaking through transitions in a 2-component system

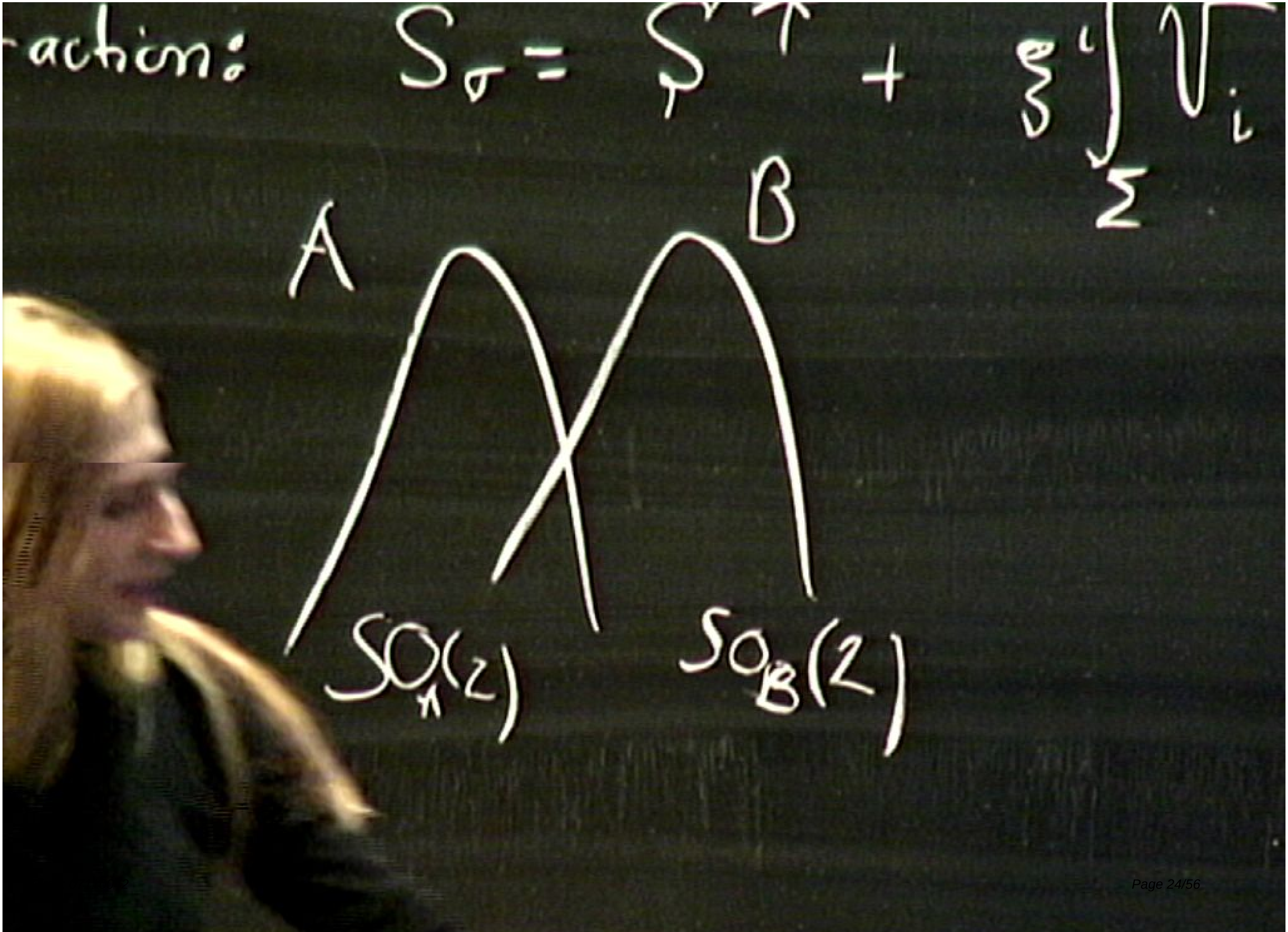
$$\hat{H} = \int d\mathbf{r} \left\{ \sum_{i=1,2} \left(-\hat{\Psi}_i^\dagger \frac{\hbar^2 \nabla^2}{2m_i} \hat{\Psi}_i + \hat{\Psi}_i^\dagger V_{ext,i}(\mathbf{r}) \hat{\Psi}_i \right) + \frac{1}{2} \sum_{i,j=1,2} \left(U_{ij} \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_i \hat{\Psi}_j + \lambda \hat{\Psi}_i^\dagger (\sigma_x)_{ij} \hat{\Psi}_j \right) \right\}$$

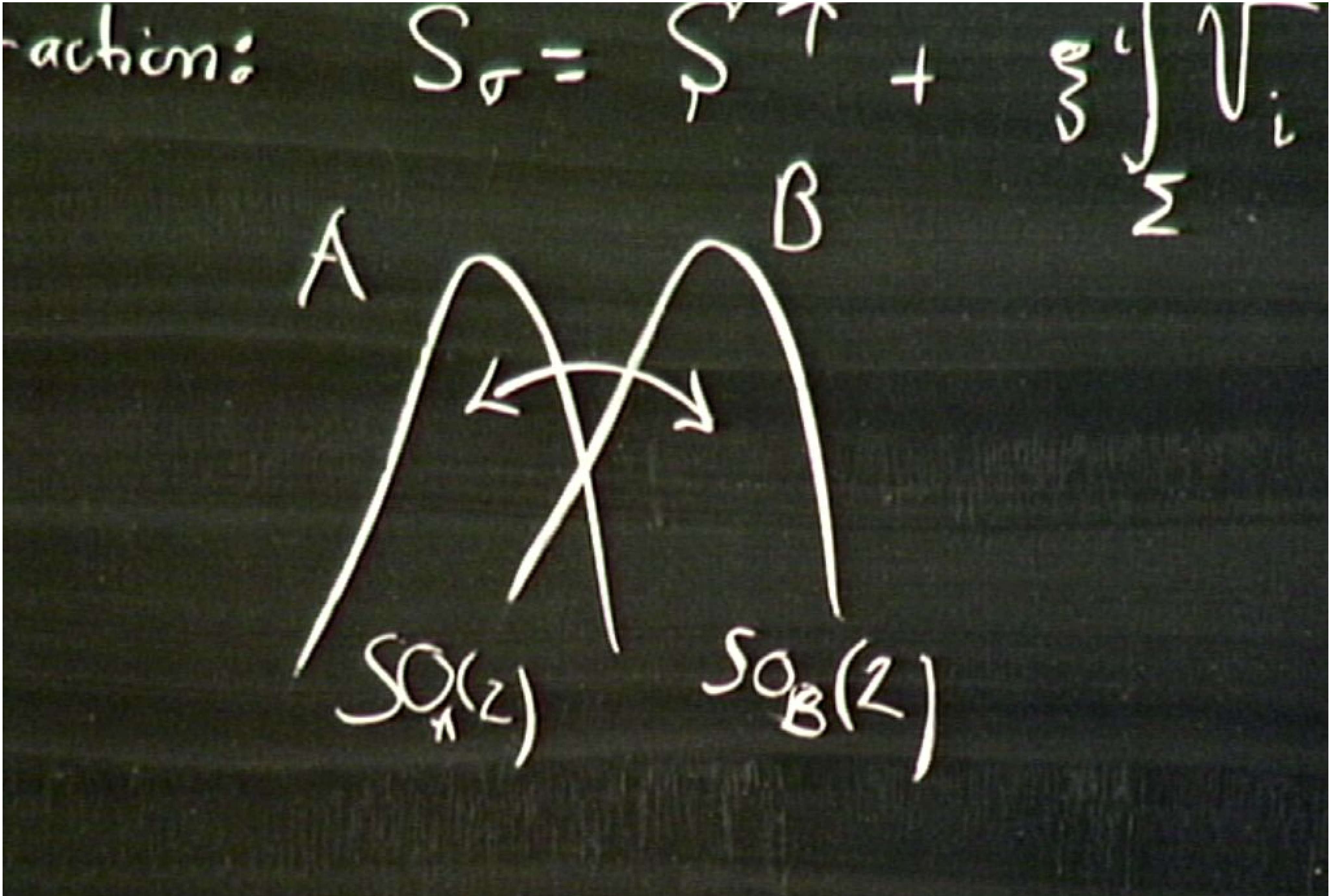
$$SO(2)_A \times SO(2)_B \rightarrow S0(2)_{AB}$$

$$\theta_A \rightarrow \tilde{\theta}_A = \theta_A \exp(+i\alpha) \qquad \theta_B \rightarrow \tilde{\theta}_B = \theta_B \exp(-i\alpha)$$

However, the fundamental Hamiltonian of the two-component system is a functional of $\vec{\theta} = (\theta_A, \theta_B)$. In the absence of transitions between the two fields the Hamiltonian exhibits an extra $SO(2)$ symmetry under which $\vec{\theta}$ transforms as a 2-component vector. This symmetry is explicitly broken for interacting fields, so that $SO(2)_A \times SO(2)_B \rightarrow S0(2)_{AB}$. The coupled system is only invariant under simultaneous transformations of the form $\theta_A \rightarrow \tilde{\theta}_A = \theta_A \exp(+i\,\alpha)$, and $\theta_B \rightarrow \tilde{\theta}_B = \theta_B \exp(-i\,\alpha)$. Thus the spontaneous symmetry breaking during the Bose–Einstein condensation relates to $S0(2)_{AB}$, instead of the individual symmetries. Altogether, linearizing around both fields yields two excitations, where one *has* to be a "Nambu–Goldstone Boson" (*i.e.*, a zero-mass excitation), while there are no constraints on the mass of the second quasi-particle.




action:
A
B
SO_A(2)
SO_B(2)


action: $S_\sigma = \sum + \xi^i \int_\Sigma V_i$
A
B
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Mass generating mechnism - all about symmetries

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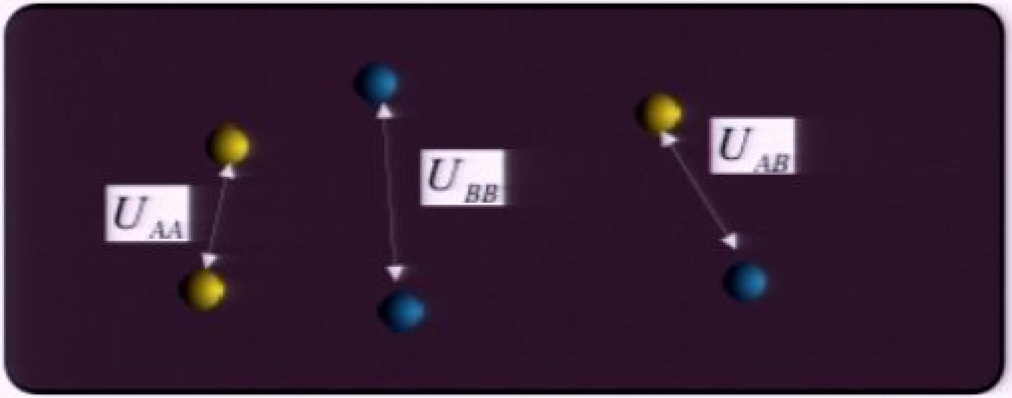
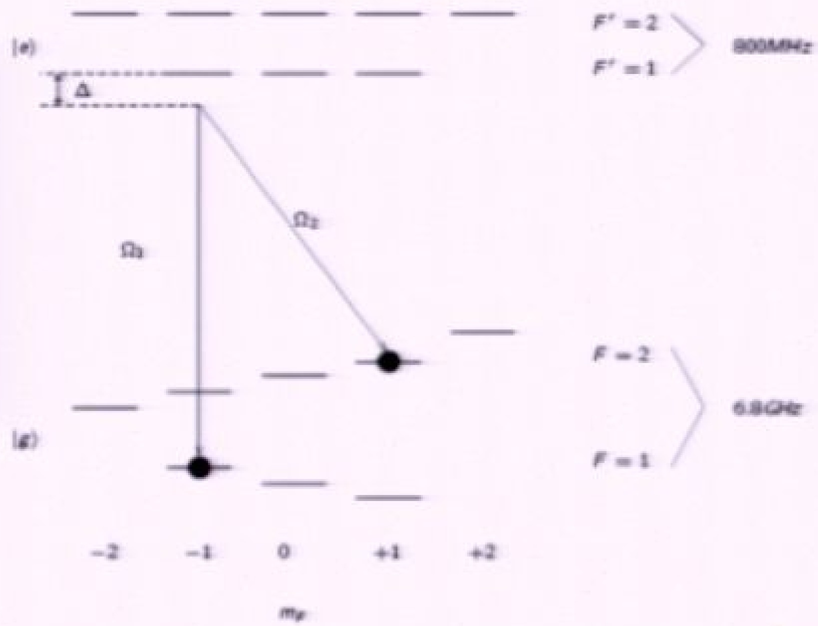


Explicit symmetry breaking through transitions in a 2-component system



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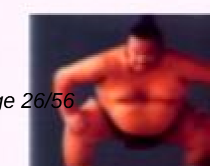
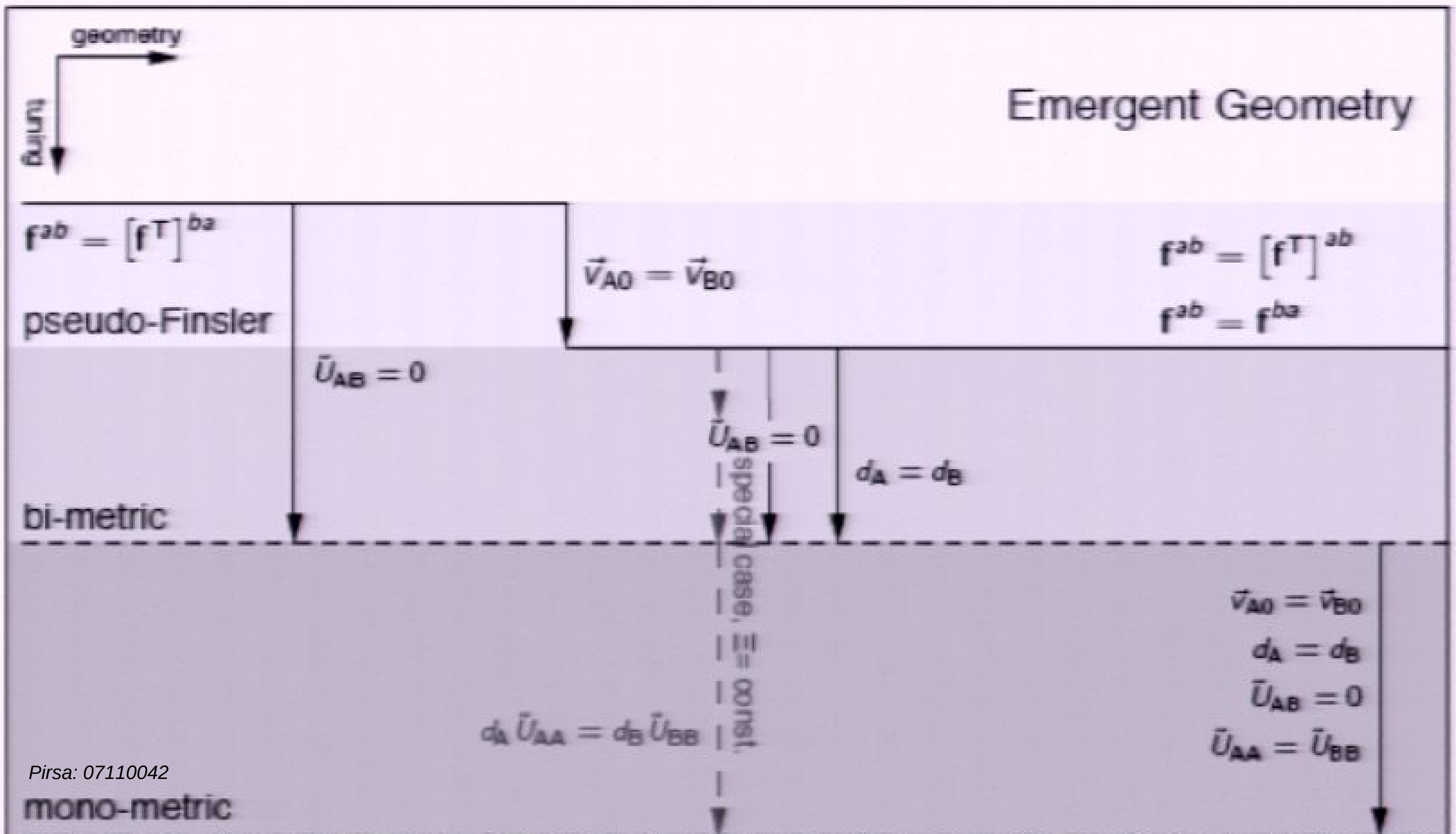


Mono-metricity

More complicated hyperbolic wave equation: $\partial_a\left(f^{ab}\,\partial_b\bar\theta\right) + (\Lambda + K)\,\bar\theta + \frac{1}{2}\left\{\Gamma^a\,\partial_a\bar\theta + \partial_a(\Gamma^a\bar\theta)\right\} = 0$

$$f^{ab} = \left[\begin{array}{c|c} \Xi_{11}^{-1}\left(\begin{array}{c|c} -1 & -\vec{v}_{A0}^{T} \\ \hline -\vec{v}_{A0} & \frac{d_A}{\Xi_{11}}\delta_{ij} - \vec{v}_{A0}\vec{v}_{A0}^{T}\end{array}\right) & \Xi_{12}^{-1}\left(\begin{array}{c|c} 1 & \vec{v}_{B0}^{T} \\ \hline \vec{v}_{A0} & \vec{v}_{A0}\vec{v}_{B0}^{T}\end{array}\right) \\ \hline \Xi_{21}^{-1}\left(\begin{array}{c|c} 1 & \vec{v}_{A0}^{T} \\ \hline \vec{v}_{B0} & \vec{v}_{B0}\vec{v}_{A0}^{T}\end{array}\right) & \Xi_{22}^{-1}\left(\begin{array}{c|c} -1 & -\vec{v}_{B0}^{T} \\ \hline -\vec{v}_{B0} & \frac{d_B}{\Xi_{22}}\delta_{ij} - \vec{v}_{B0}\vec{v}_{B0}^{T}\end{array}\right)\end{array}\right]$$

$$f^{ab} \sim \sqrt{-g}\,g^{ab}$$



$ds = \sqrt[4]{g_{abcd}\,dx^a dx^b dx^c dx^d}$

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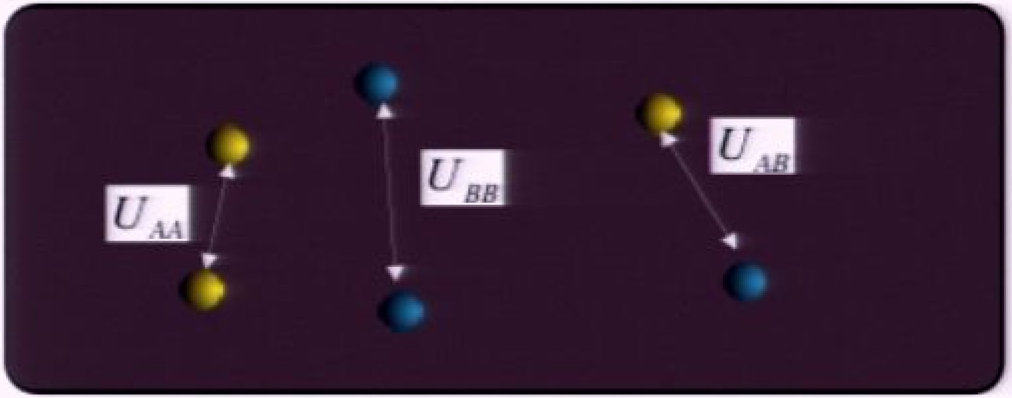

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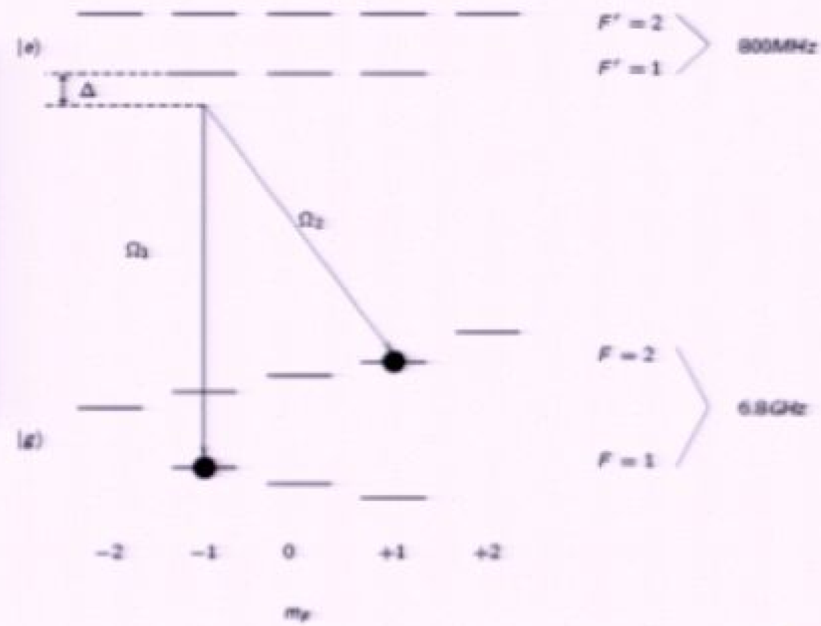

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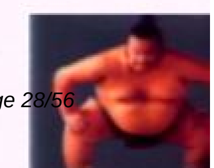

Mono-metricity

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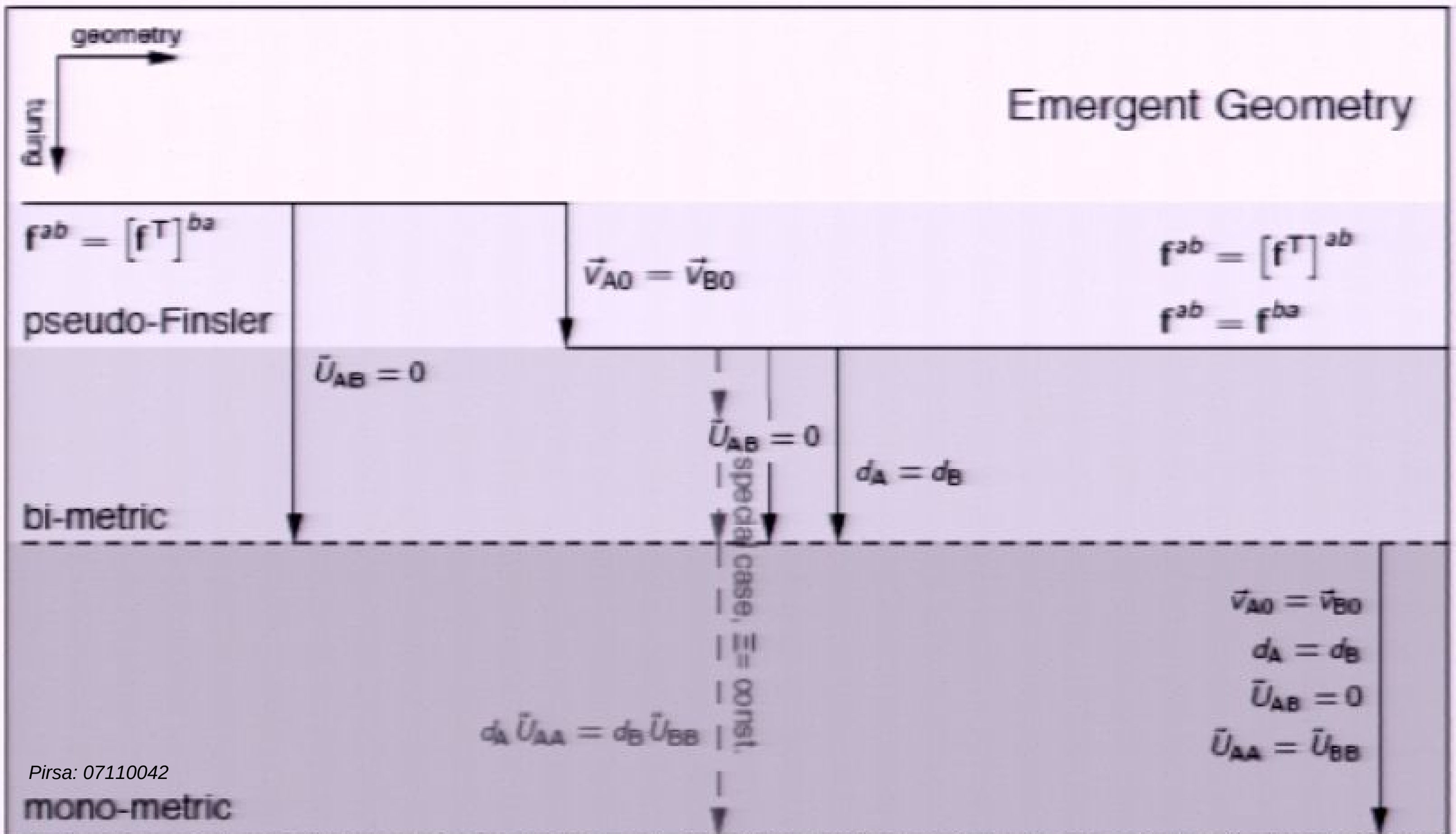
$$\mathsf{f}^{ab} = \left[\begin{array}{c|c} \Xi_{11}^{-1}\left(\begin{array}{c|c} -1 & -\vec{v}_{A0}^{\,T} \\ \hline -\vec{v}_{A0} & \frac{d_A}{\Xi_{11}^{-1}}\delta_{ij} - \vec{v}_{A0}\vec{v}_{A0}^{\,T} \end{array}\right) & \Xi_{12}^{-1}\left(\begin{array}{c|c} 1 & \vec{v}_{B0}^{\,T} \\ \hline \vec{v}_{A0} & \vec{v}_{A0}\vec{v}_{B0}^{\,T} \end{array}\right) \\ \hline \Xi_{21}^{-1}\left(\begin{array}{c|c} 1 & \vec{v}_{A0}^{\,T} \\ \hline \vec{v}_{B0} & \vec{v}_{B0}\vec{v}_{A0}^{\,T} \end{array}\right) & \Xi_{22}^{-1}\left(\begin{array}{c|c} -1 & -\vec{v}_{B0}^{\,T} \\ \hline -\vec{v}_{B0} & \frac{d_B}{\Xi_{22}^{-1}}\delta_{ij} - \vec{v}_{B0}\vec{v}_{B0}^{\,T} \end{array}\right) \end{array}\right] \qquad \mathsf{f}^{ab} \sim \sqrt{-g}\, g^{ab}$$

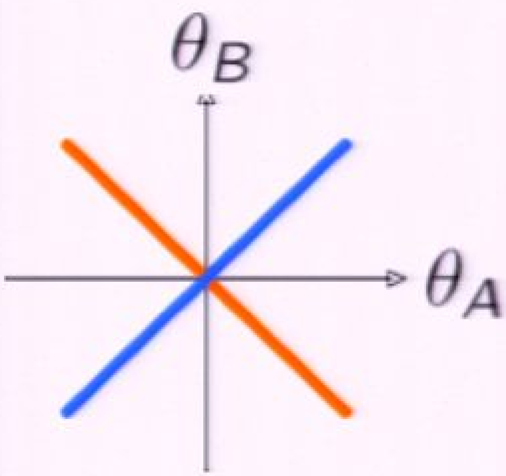


$ds = \sqrt[4]{g_{abcd}\, dx^a dx^b dx^c dx^d}$ ←

Emergent massive fields

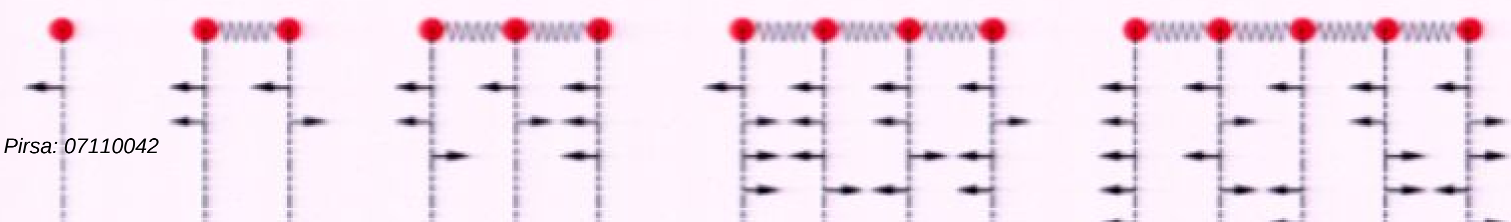
$$\frac{1}{\sqrt{-g_{I/II}}}\partial_a\left\{\sqrt{-g_{I/II}}\,(g_{I/II})^{ab}\,\partial_b\tilde{\theta}_{I/II}\right\}+\omega_{I/II}^2\,\tilde{\theta}_{I/II}=0$$

the acoustic metrics are given by $(g_{I/II})_{ab} \propto \left[\begin{array}{c|c} -(c^2 - v_0^2) & -\vec{v_0}^{\,T} \\ \hline -\vec{v_0} & \mathbf{I}_{d\times d}\end{array}\right]$



In-phase perturbtion (= mass zero particle): $\omega_I^2 = 0$

Anti-phase perturbtion (= mass non-zero particle): $\omega_{II}^2 \propto \lambda \neq 0$



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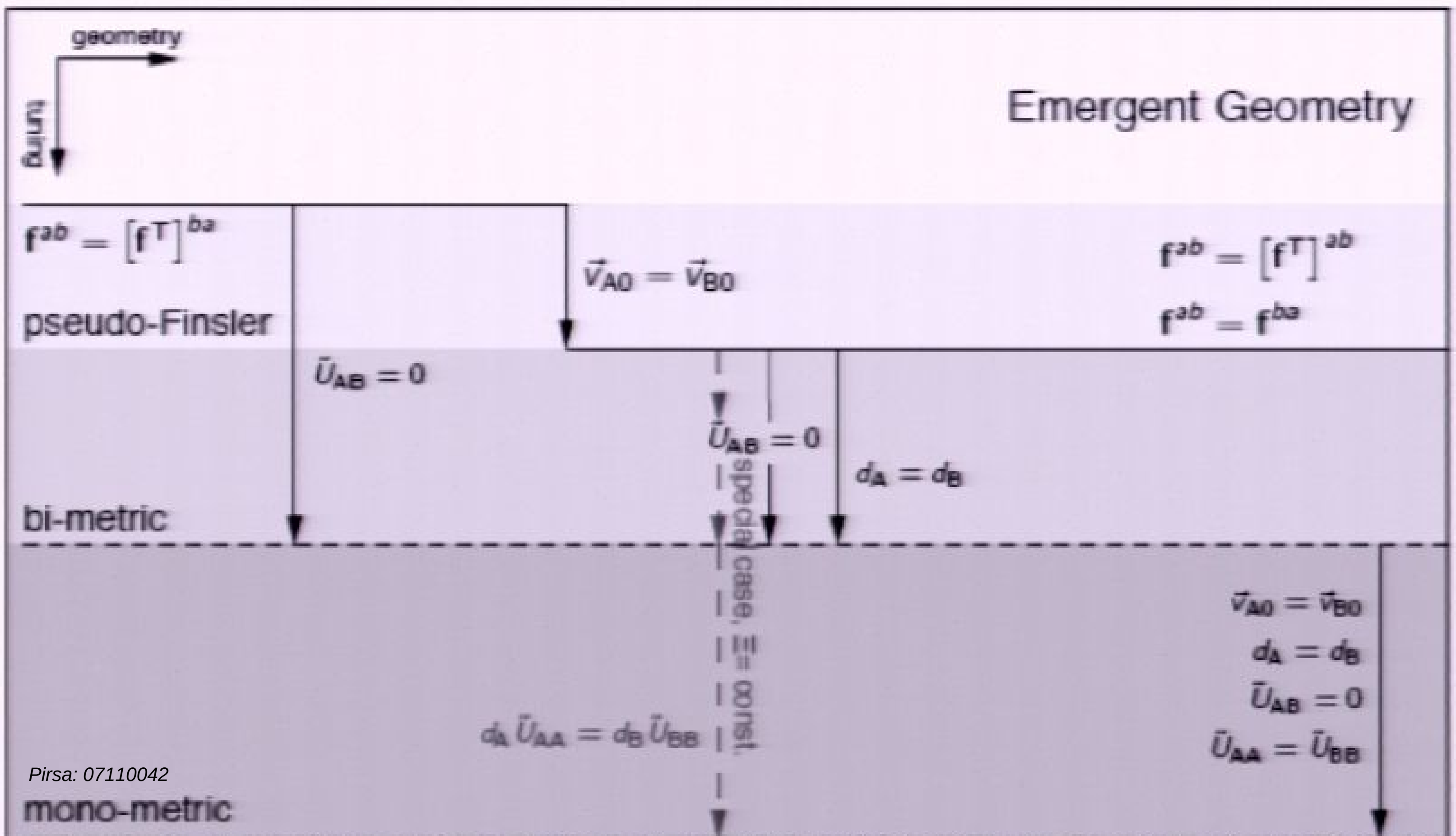

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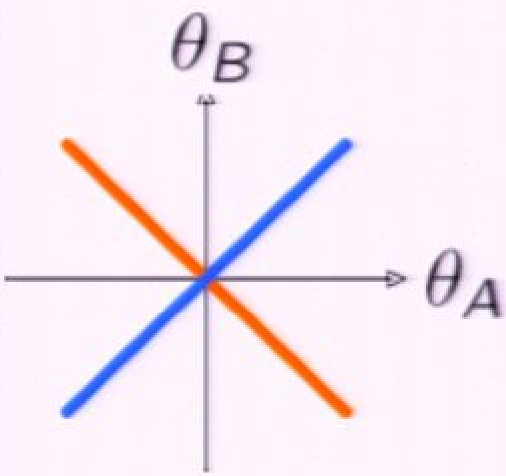

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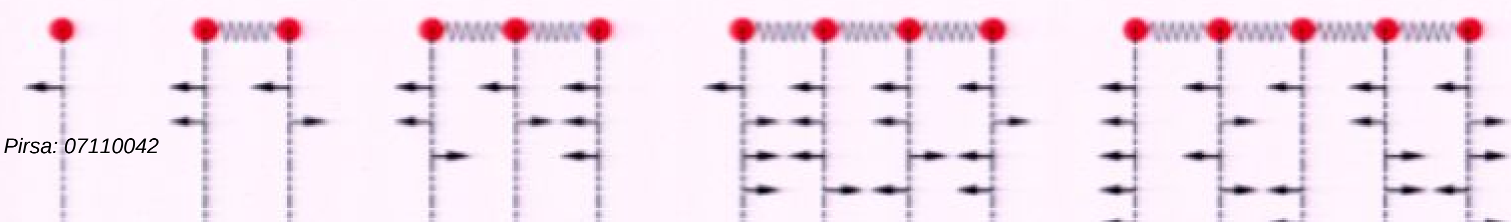

The concept of emergence

Emergent spacetimes involve...

- A microscopic system of fundamental objects (e.g. strings, atoms or molecules);
- a dominant mean field regime, where the microscopic degrees of freedom give way to collective variables;
- a geometrical object (e.g. a symmetric tensor dominating the evolution of linearized classical and quantum excitations around the mean field;
- An emergent Lorentz symmetry for the long-distance behavior of the geometrical object;

Collective excitations + microscopic fingerprints

[*] Quantum gravity phenomenology -
Lorentz symmetry breaking...

[*] Lorentz symmetry breaking -
Example BEC...

[*] LIV in emergent spacetimes -
Interpolating between the 2 sides of a coin...

Quantum gravity phenomenology [LIV]

QGP: Summarizes all possible **phenomenological consequences from quantum gravity**. While different quantum gravity candidates may have completely distinct physical motivation, they can yield similar observable consequences, e.g. **Lorentz symmetry breaking at high energies**.

1) Presense of a preferred frame;
2) All frames equal, but transformation laws between frames are modified.

D. Mattingly. Modern tests of Lorentz invariance. *Living Reviews in Relativity*, 8(5), 2005.

Collective excitations + microscopic fingerprints

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Example BEC [Emergent covariance]

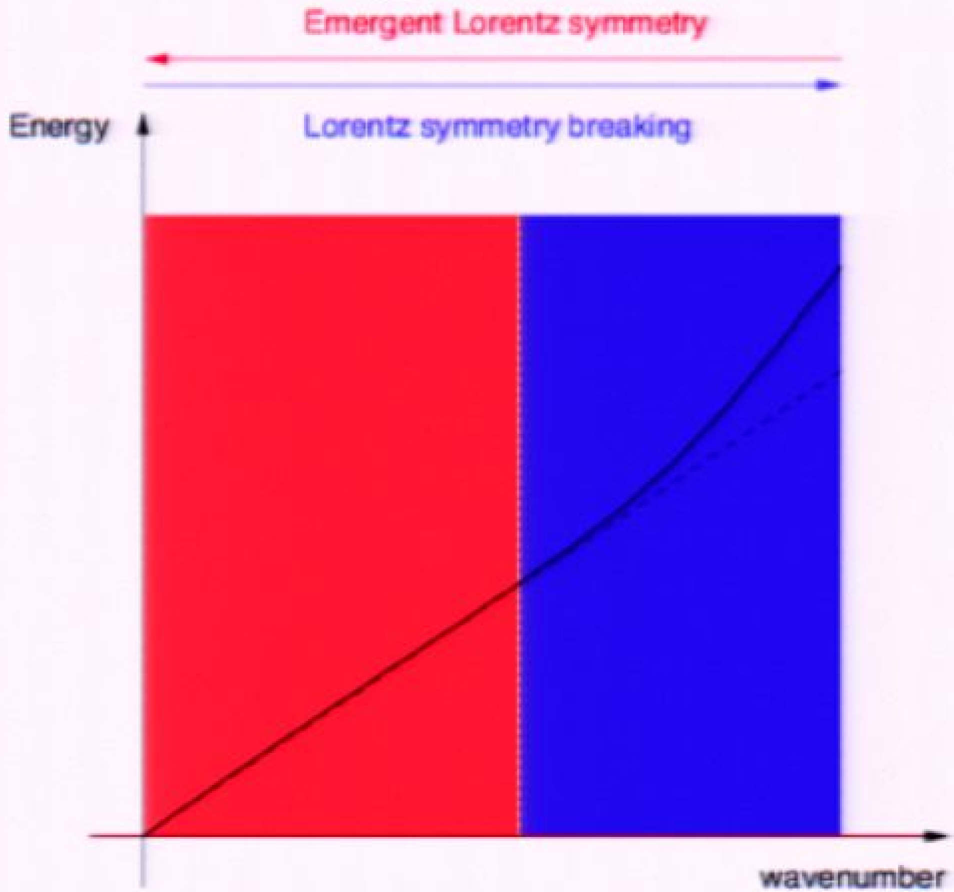
Emergent spacetimes from Bose-gas

- An emergent Lorentz symmetry: $\omega_k^2 = c_0^2 k^2 + \left(\frac{\hbar}{2}\right)^2 k^4$

Bogoliubov dispersion relation for exciations

Macroscopic variables

$\omega_k \approx c_0 k$



Microscopic variables

$\omega_k \approx \frac{\hbar}{2m} k^2$

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Analogue Lorentz symmetry breaking

Symmetry breaking mechanism in different analogue models lead to model-specific modifications:

Bose-Einstein condensate: $\omega_k^2 = c_0^2\, k^2 + \epsilon_{\rm qp}^2\, k^4$

Electromagnetic waveguide: $\omega_k^2 = \frac{4}{L\,C} \sin^2\left(\frac{k\,\Delta x}{2}\right) \approx c^2\, k^2 - \frac{\Delta x^4}{12\,L\,C}\, k^4.$

Despite all fundamental differences similar modifications:

$$\Delta\omega_k^2 \sim \pm k^4.$$

Any emergent spacetimes based on analogue models per definition have a preferred frame: The external observer.

Collective excitations + microscopic fingerprints

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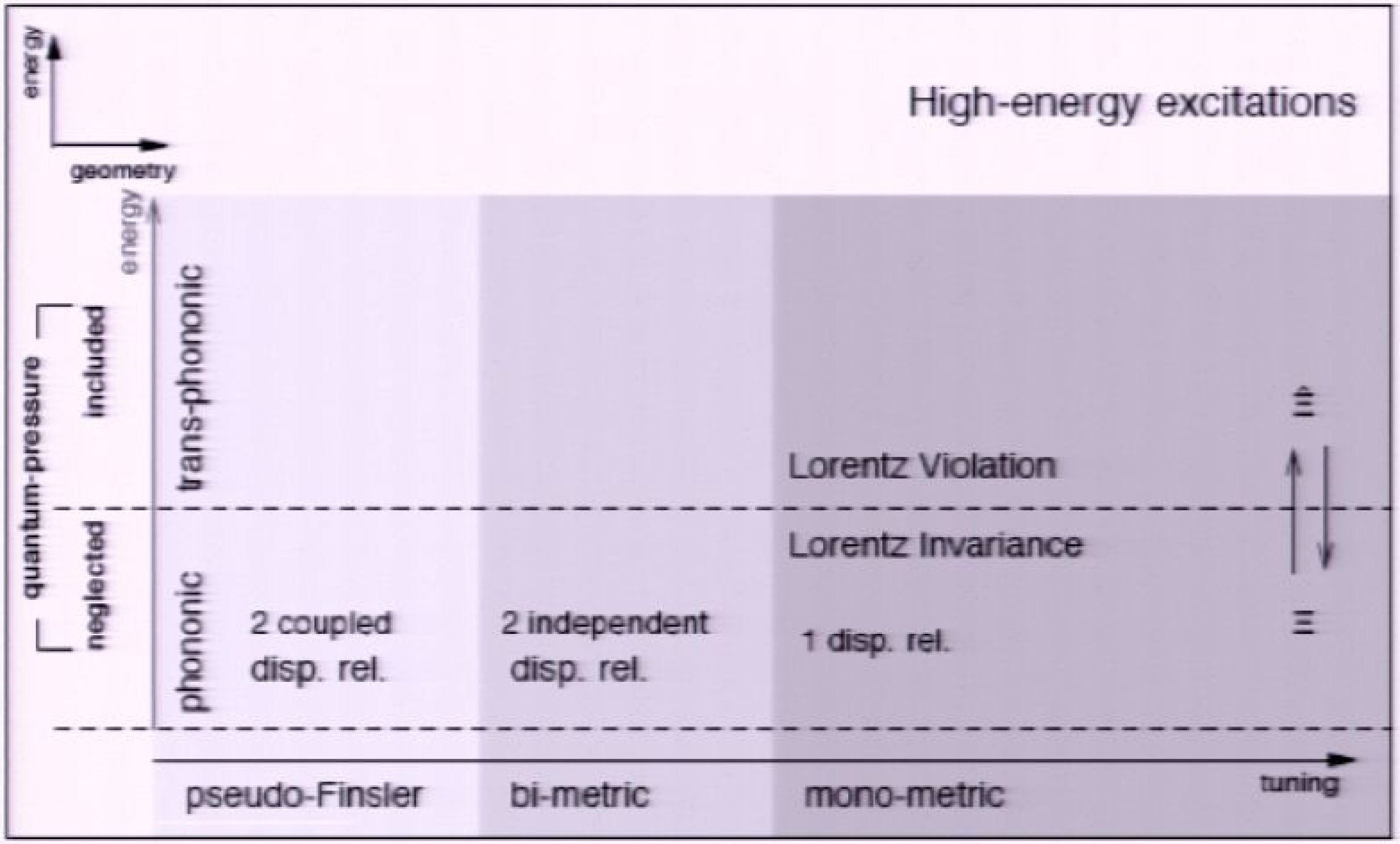
Collective excitations + microscopic fingerprints

Beyond the hydrodynamic approximation...



1] S. Liberati, M. Visser, and S. W.. *Class. Quant. Grav.*, 23:3129–3154, 2006.

Bogoliubov dispersion realtion for 2-comp. sys.

$$\omega_k^2 = \omega_0^2 + (1+\eta_2)\, c^2 k^2 + \eta_4 \left(\frac{\hbar}{M_{LIV}}\right)^2 k^4 + \cdots$$

$$\eta_{4,X} \approx 1; \qquad \mathbf{X} = \mathrm{I}, \mathrm{II}$$

$$\eta_{2,X} \approx \left(\frac{m_X}{M_{eff}}\right)^2 = \left(\frac{\text{mass scale of quasiparticle}}{\text{effective Planck scale}}\right)^2; \qquad \mathbf{X} = \mathrm{I}, \mathrm{II}$$

Conclusions: Lessons for QGP or fancy CMS..?

[*] Nature of non-perturbative corrections -
Quantum pressure term...

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Independent smallness of quadratic correction...

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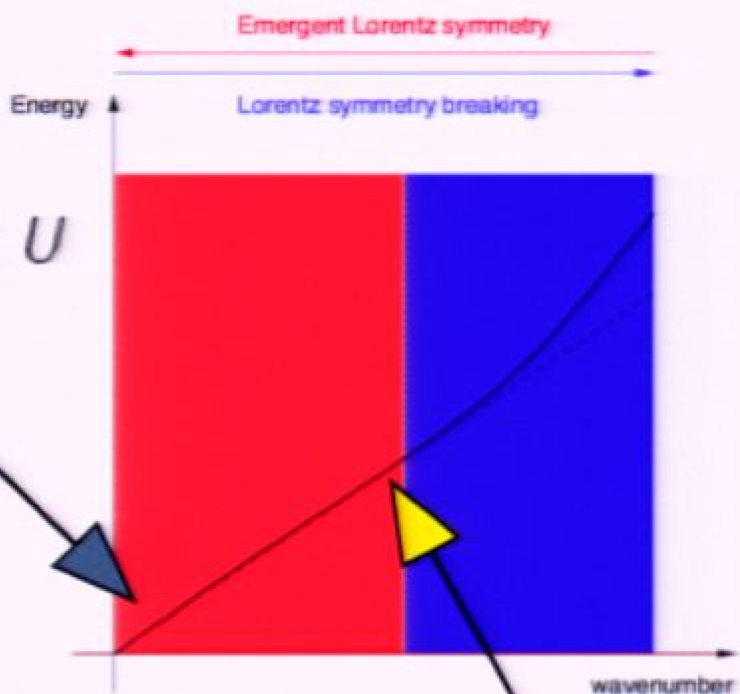
Nature of non-perturbative corrections

ydrodynamic approximation: Variations in the kinetic energy f the condensate are considered to be negligible, compared o the internal potential of the Boson:

$$\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{n_0+\hat{n}}}{\sqrt{n_0+\hat{n}}} \ll U$$



$$c_k^2 = \frac{U_k n_0}{m} = c_0^2 + \epsilon_{qp}^2 k^2$$

$$\epsilon_{qp} = \frac{\hbar}{2m}$$

healing length:

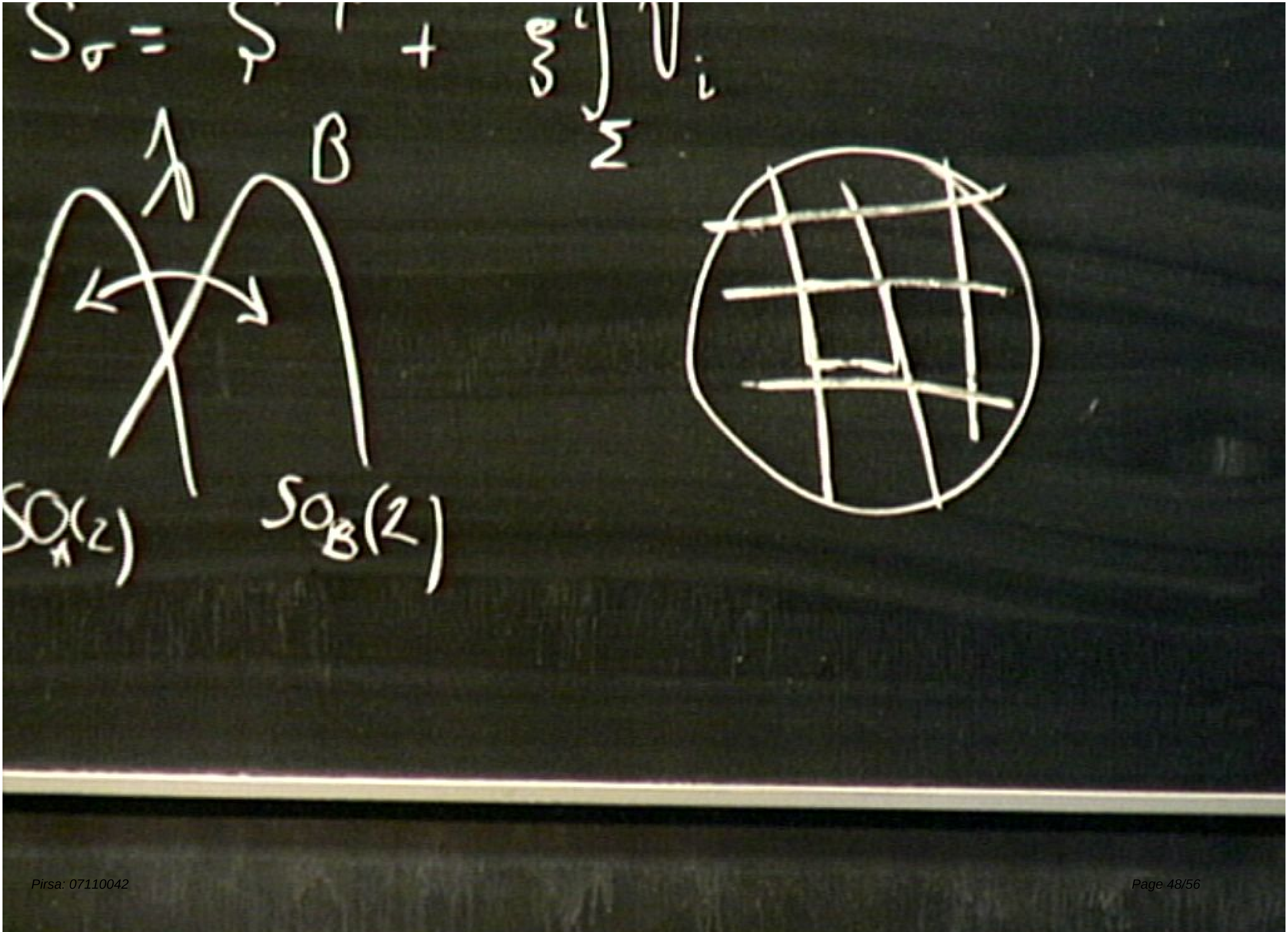
$$\xi^2 = \frac{1}{2}\frac{\epsilon_{qp}^2}{c_0^2}$$

sound speed c(t), where t is lab–time!

$$\omega_k \approx c_0 k$$

Non-perturbative corrections for quantum modes comparable to the

$$\omega_k \approx \frac{\hbar}{2m}k^2$$


B
Σ
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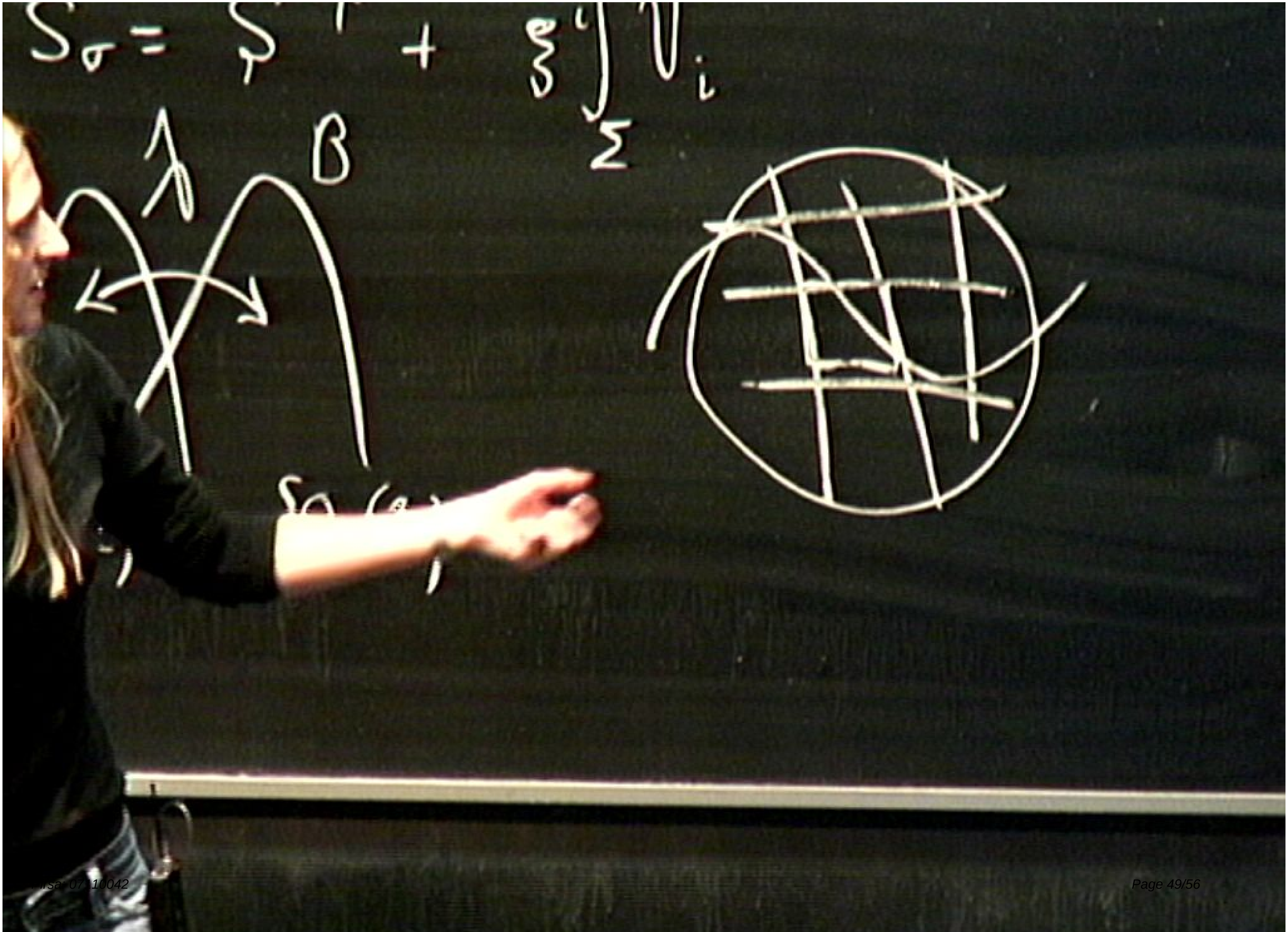
SO_B(2)



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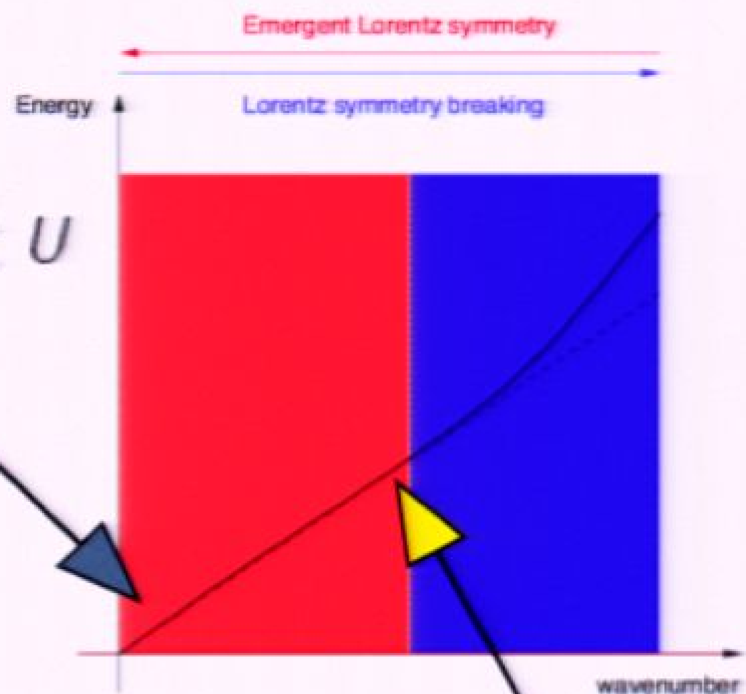

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Naturalness problem in emergent spacetime

Dispersion relation obtained from our CMS has the form:

$$\omega^2 = \omega_0^2 + (1+\eta_2)\, c^2\, k^2 + \eta_4 \left(\frac{\hbar}{M_{\rm LIV}}\right)^2 k^4 + \dots$$

* CPT invariant (LIV in the boost subgroup)
* has the form as suggested in many (non-renormalizable) effective field theory approaches
* **natural suppression of low-order modifications in our model!**

$$\eta_{2,\rm I/II} \approx \left(\frac{m_{\rm I/II}}{M_{\rm LIV}}\right)^2 = \left(\frac{\text{quasiparticle mass}}{\text{effective Planck scale}}\right)^2;$$
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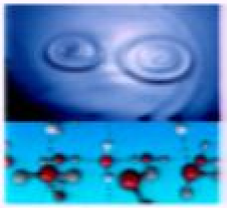
Mass-generating mechanism and LIV

Explicit symmetry breaking in our CMS:

* generates a massive excitation
* induces low-order LIV terms
* *exhibits a rich geometrical structure* *[Finsler, bi-metric, mono-metric]*



The end...

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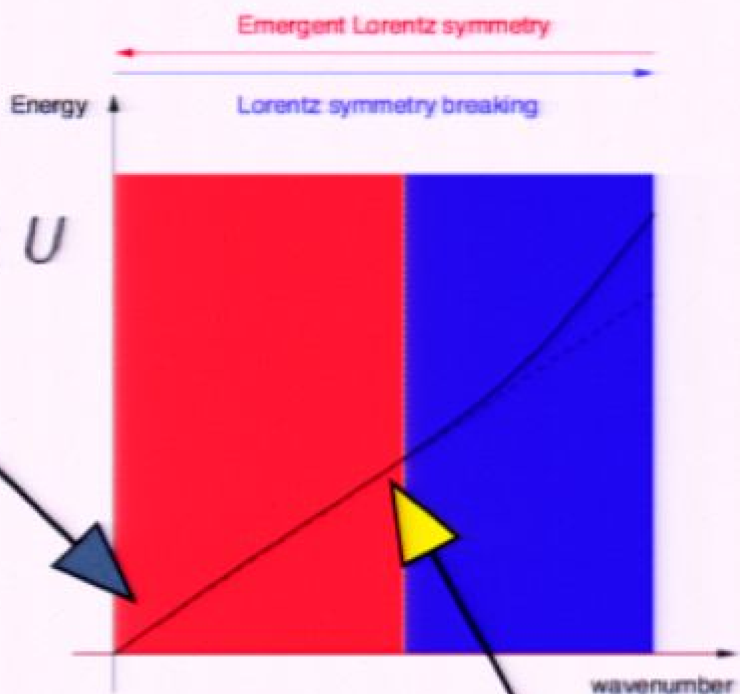

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