Title: Automorphisms in Loop Quantum Gravity

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Abstract: There is a deep relation between Loop Quantum Gravity and notions from category theory, which have been pointed out by many researchers, such as Baez or Velhinho. Concepts like holonomies, connections and gauge transformations can be naturally formulated in that language. In this formulation, the (spatial) diffeomorphisms appear as the path grouopid automorphisms. We investigate the effect of extending the diffeomorphisms to all such automorphisms, which can be viewed as \"distributional diffeomorphisms\". We also give a notion of \"categorial holonomy-flux-algebra\", and present the construction of the automorphism-invariant Hilbert space for abelian gauge groups, which will be entirely combinatorial.

Distributional extension of diffeomorphisms

Several suggestions for $\overline{\mathrm{Diff}(\Sigma)}$:

- Diffeomorphisms which are smooth up to finitely many points
  Fairbairn, Rovelli, [arXiv:gr-qc/0403047]
- C^n diffeomorphisms, which are analytic up to lower dimensional submanifolds Ashtekar, Lewandowski, [arXiv:gr-qc/0404018]
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- piecewise analytic diffeomorphisms Zapata [gr-qc/9703038]
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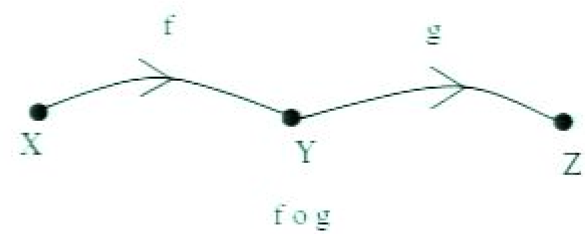
Category language

- A *category* $\mathscr{C}$ consists of
  - *objects*: $X, Y \in |\mathscr{C}|$
  - *morphisms* $f : X \longrightarrow Y$ from one object X to another Y



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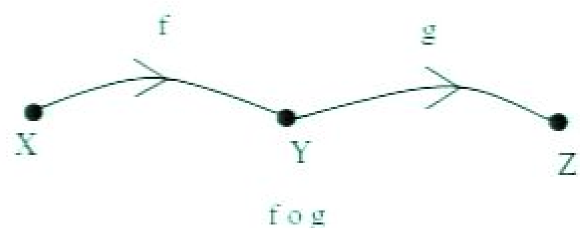

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- A *functor* $F : \mathscr{C} \longrightarrow \mathscr{D}$ between categories

$$X \longrightarrow F(X)$$
$$f : X \to Y \longrightarrow F(f) : F(X) \to F(Y)$$

such that

$$F(f \circ g) = F(f) \circ F(g)$$

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- A *natural transformation* between functors $F, G : \mathscr{C} \to \mathscr{D}$ for each object $X \in |\mathscr{C}|$ a morphism $g_X : F(X) \to G(X)$, so that for $f : X \to Y$

$$F(f) \circ g_Y = g_X \circ G(f)$$

The path groupoid category $\mathcal{P}$

- Objects in $\mathcal{P}$ $=$ points in space Σ

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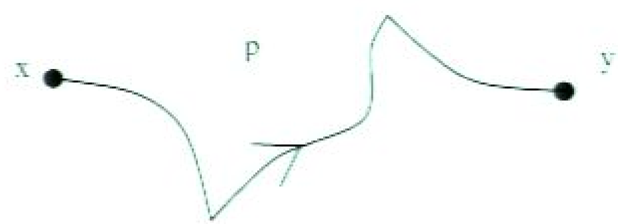
- Objects in $\mathcal{P}$ = points in space Σ
- Morphisms between two points $x, y \in \Sigma$: paths from x to y (piecewise analytic curves modulo reparametrization and retracing)

Composition of paths



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- In category language: A is a *functor* from $\mathcal{P}$ to the gauge group G. $(\overline{\mathcal{A}} = \mathrm{Hom}(\mathcal{P}, G))$

Gauge transformations are natural transformations

- Two connections A_1, A_2 can be related by a gauge transformation, if there is a smooth map $g : \Sigma \longrightarrow G$ such that, for every path p from x to y:

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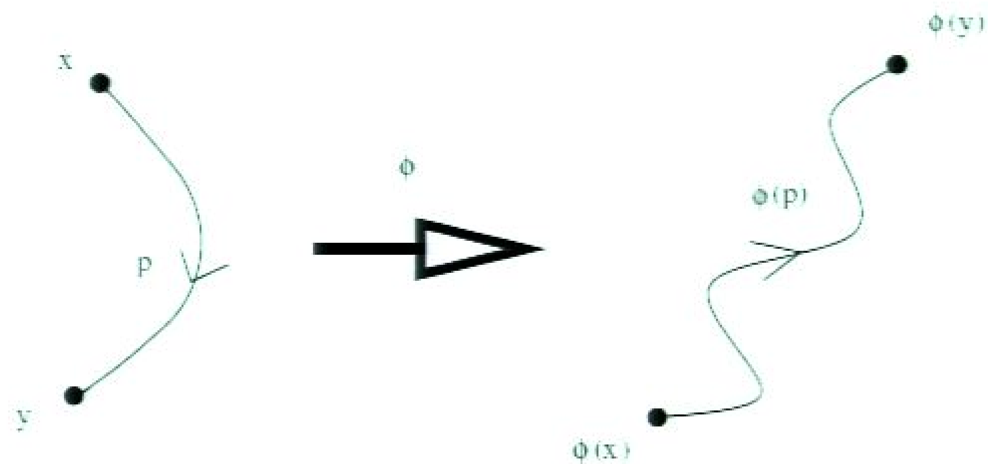
Diffeomorphisms are automorphisms of $\mathcal{P}$

- Diffeomorphisms $\phi \in \mathrm{Diff}(\Sigma)$ map points to points and paths to paths:



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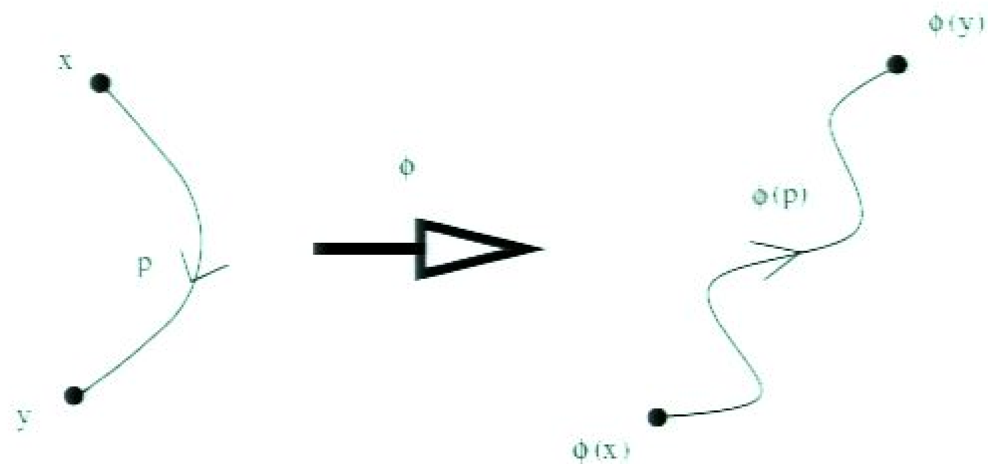

- In category language: ϕ is an invertible functor from the category $\mathcal{P}$ to itself: $\phi \in \text{Aut}(\mathcal{P})$

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- $\overline{\mathcal{G}}$ is the set of *all* natural transformations on functors A
- Every $\phi \in \mathrm{Diff}(\Sigma)$ induces an automorphism on $\mathcal{P}$
- Suggestion: Try $\overline{\mathrm{Diff}(\Sigma)} := \mathrm{Aut}(\mathcal{P})$ the set of *all* automorphisms on $\mathcal{P}$.

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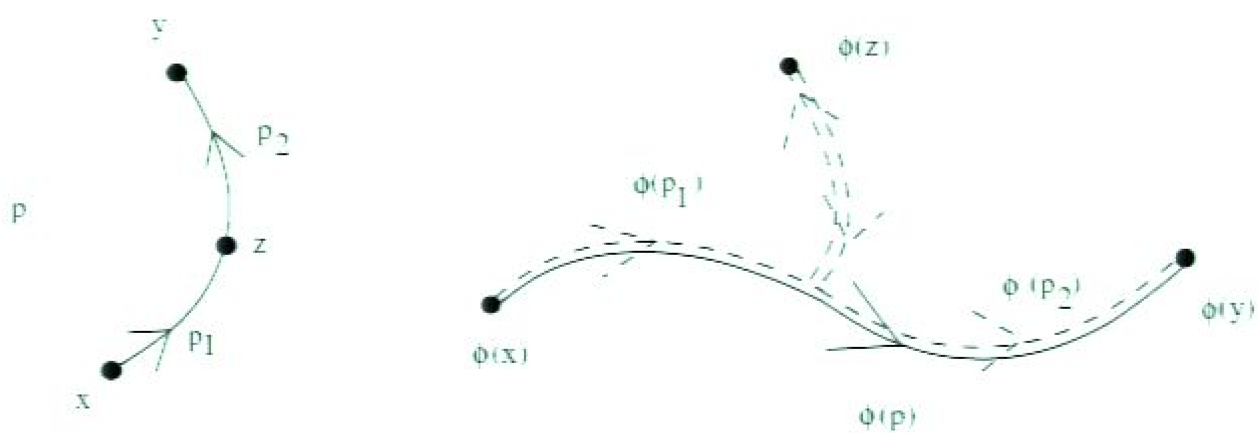
- But if z lies on p, $\phi(z)$ does not have to lie on $\phi(p)$!

Elements in $\mathrm{Aut}(\mathcal{P})$

Reason: The fact that retracings cancel



Here $p = p_1 \circ p_2$, and z lies on p. One has $\phi(p) = \phi(p_1) \circ \phi(p_2)$, but $\phi(z)$ does not lie on $\phi(p)$!

Elements in $\mathrm{Aut}(\mathcal{P})$

There are strange elements in $\mathrm{Aut}(\mathcal{P})$:

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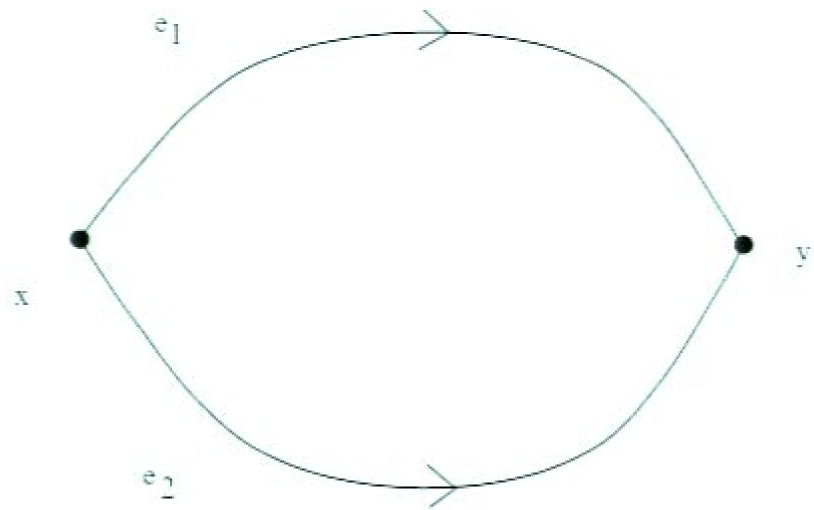
- Automorphisms $\phi \in \mathrm{Aut}(\mathcal{P})$ which permute the points in Σ arbitrarily, but leave the paths essentially invariant ("natural transformations of the identity").
- Automorphisms $\phi \in \mathrm{Aut}(\mathcal{P})$ which swap two edges $e_1, e_2 : x \longrightarrow y$, but leave *all* points invariant, as well as *all* other paths that intersect with e_1, e_2 in at most finitely many points ("edge-interchanger").

Edge-interchanger



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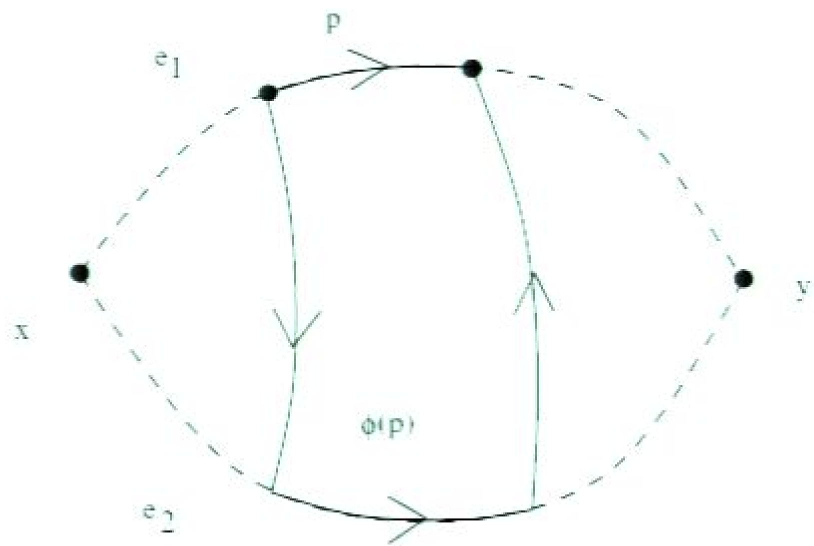

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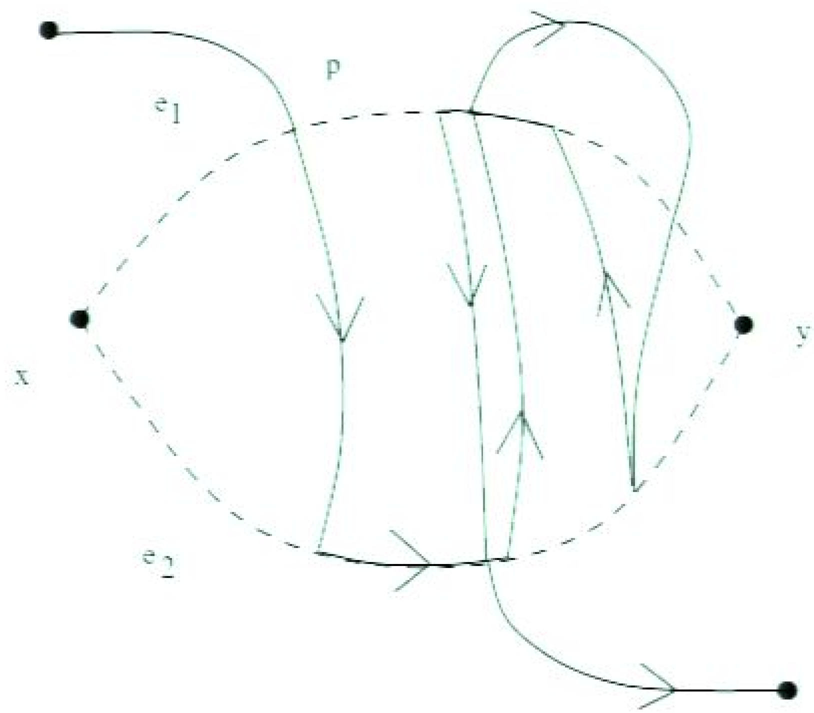

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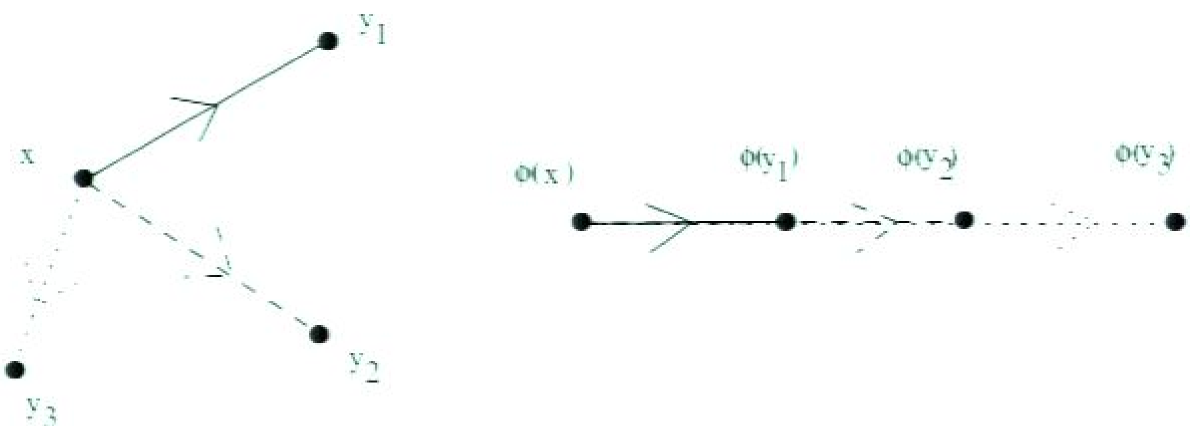
- In fact more general: Also true for hyphs.

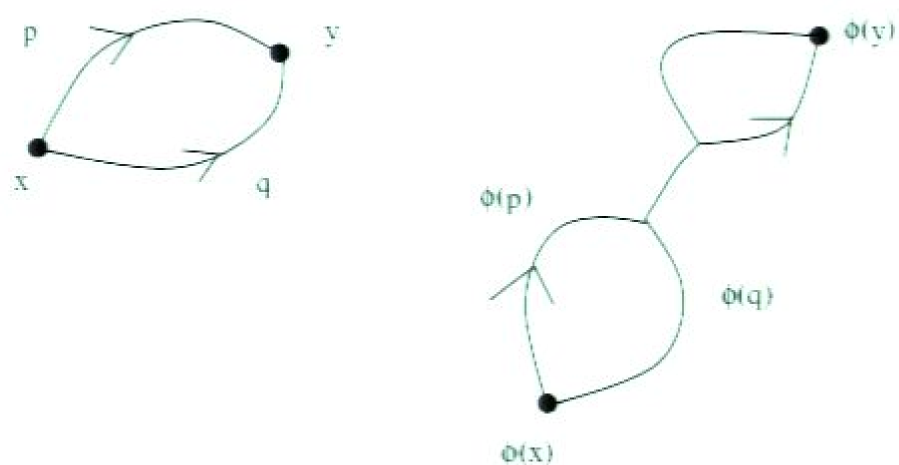
Graph combinatorics

Automorphisms only respect the fact that parallel transports along paths are independent, but not how these paths are embedded in space:



Orbits of the Automorphisms in $\mathcal{H}_{\mathrm{kin}}$

- What is the action of an automorphism $\phi \in \mathrm{Aut}(\mathcal{P})$ on a function f cylindrical over a graph γ?



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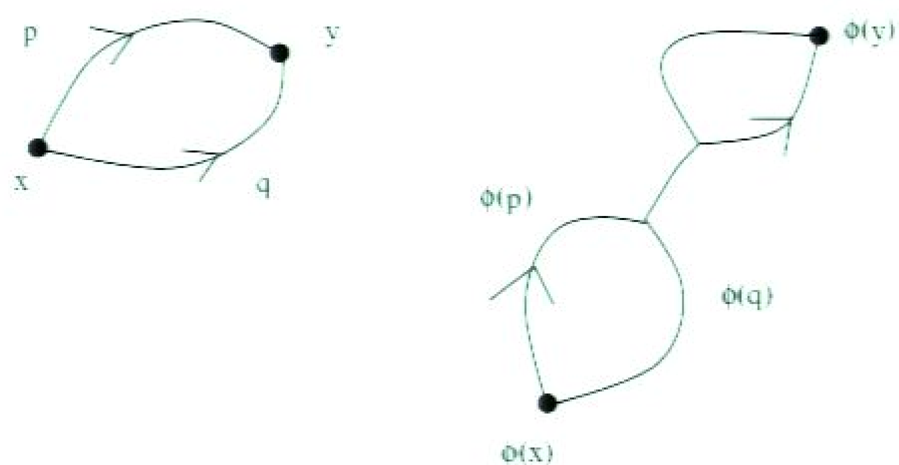
- Warning: γ a graph, but $\phi(\gamma)$ is no graph in general!



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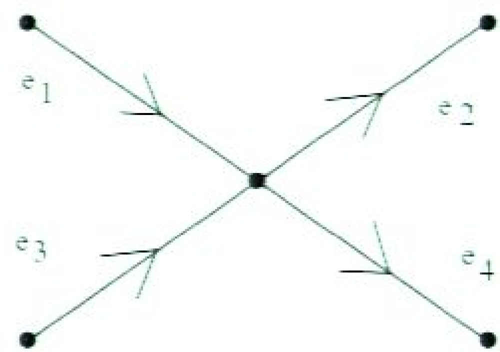
Consider a function $f : \overline{\mathcal{A}} \to \mathbb{C}$ which is cylindrical over the following graph:

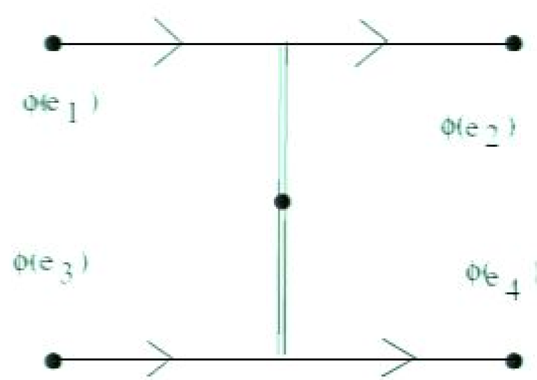


i.e. $f(A) = F(A(e_1), A(e_2), A(e_3), A(e_4))$ for some function $F : G^4 \to \mathbb{C}$. Choose F such that

$$F(h_1, h_2, h_3, h_4) = F'(h_1 h_2, h_3 h_4)$$

Orbits of the Automorphisms in $\mathcal{H}_{\mathrm{kin}}$

Then $\hat{U}(\phi)f(A) := f(\alpha_\phi A)$ is cylindrical over the following hyph:



But since f depends only on the parallel transport along $e_1 \circ e_2$ and $e_3 \circ e_4$, $\hat{U}(\phi)f$ depends only on the parallel transports along $\phi(e_1) \circ \phi(e_2)$ and $\phi(e_3) \circ \phi(e_4)$.

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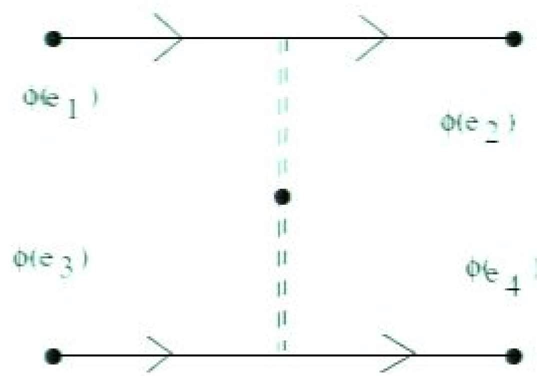

which consists of the two edges $\phi(e_1) \circ \phi(e_2)$ and $\phi(e_3) \circ \phi(e_4)$. But only because of the peculiar dependence of f on the parallel transports in γ!

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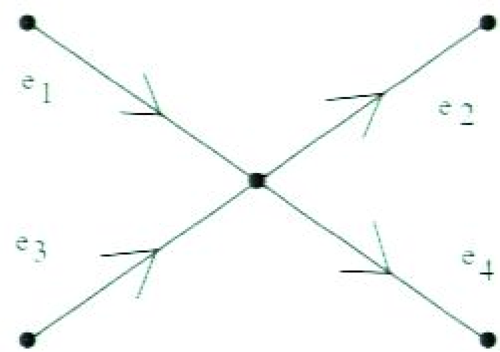

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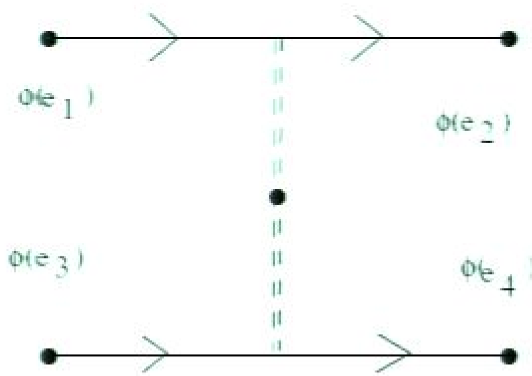

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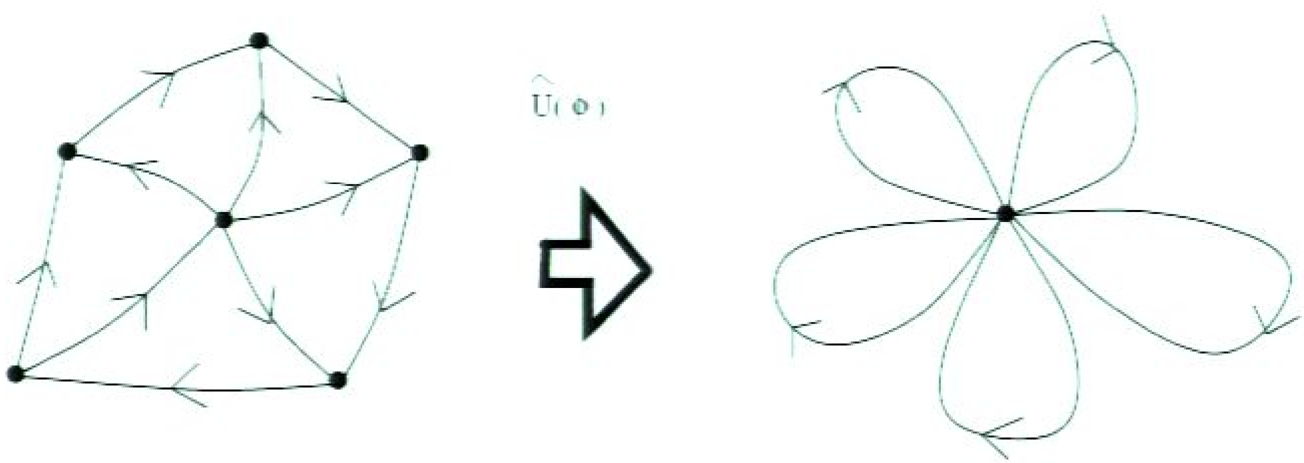
Action of Aut($\mathcal{P}$) on gauge-invariant functions

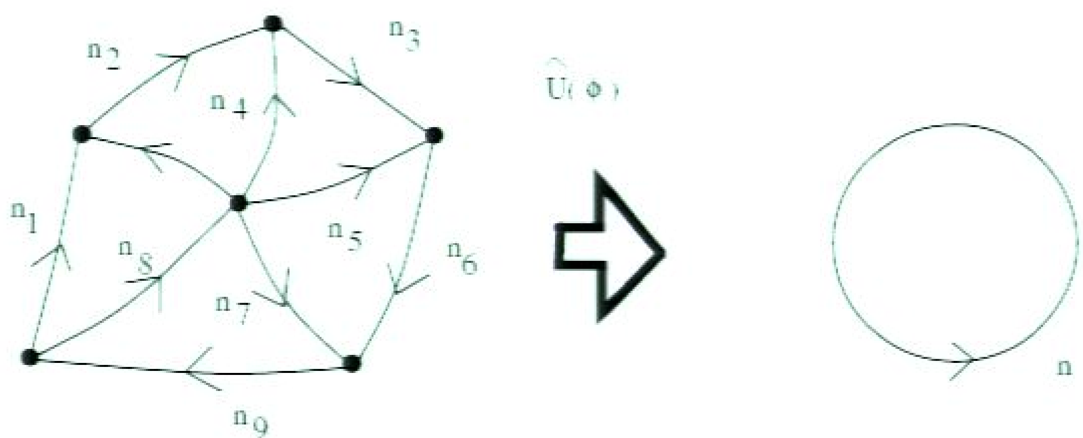
This has the following consequence:
Let γ be a graph f be a gauge-invariant function on γ. Then there is an $\phi \in \mathrm{Aut}(\mathcal{P})$ such that $\hat{U}(\phi)f$ is cylindrical over a flower graph.



$\mathcal{H}_{\mathrm{Aut}}$ for $G = U(1)$

Abelian gauge group: For each (gauge-invariant) charge-network function $T_{\gamma,\bar{n}}$ there is an automorphism ϕ mapping $T_{\gamma,\bar{n}}$ on a Charge-Wilson loop

$$\hat{U}(\phi)T_{\gamma,\bar{n}} = \bigcirc_n$$



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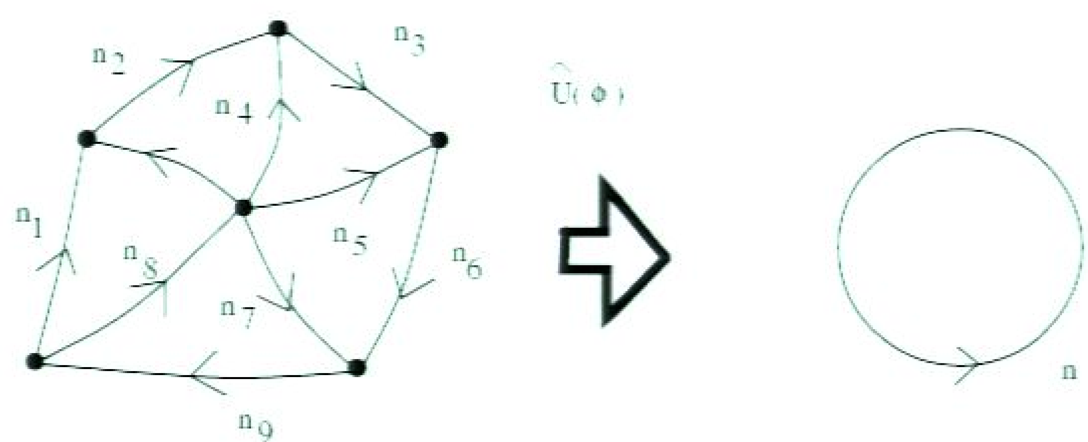


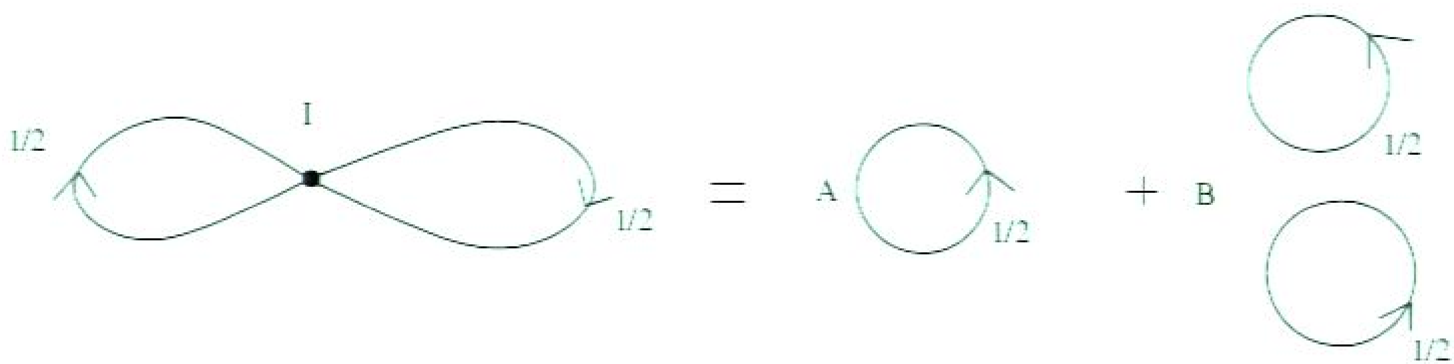
$\mathcal{H}_{\mathrm{Aut}}$ for $G = U(1)$

So for $G = U(1)$, one can compute the automorphism-invariant Hilbert space:

$$\psi_{\mathrm{Aut}} = \sum_{n=0}^{\infty} c_n \left[\bigcirc_n \right]$$

$\mathcal{H}_{\rm Aut}$ for $G = SU(2)$

Observation: Finitely many separate Wilson loops build basis for lower intertwiner spaces on flowers, i.e.:

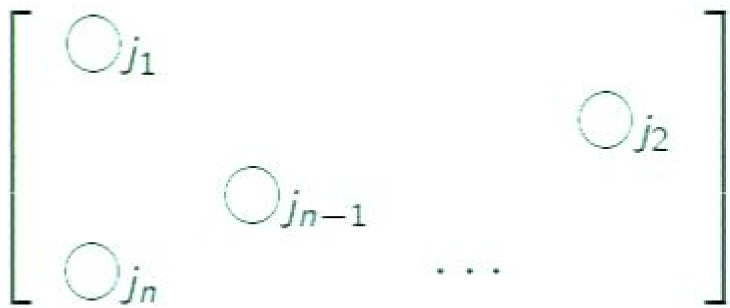


$$F(h_1, h_2) \quad = \quad A\,\mathrm{tr}_{\frac{1}{2}}(h_1 h_2) \quad + \quad B\,\mathrm{tr}_{\frac{1}{2}}(h_1)\,\mathrm{tr}_{\frac{1}{2}}(h_2)$$

$\mathcal{H}_{\mathrm{Aut}}$ for $G = SU(2)$

- Conjecture: These are already enough, i.e.

$$\psi_{\mathrm{Aut}} = \sum_{n=0}^{\infty} \sum_{j_1,\dots,j_n \in \frac{1}{2}\mathbb{N}} c_{n,\vec{j}}$$



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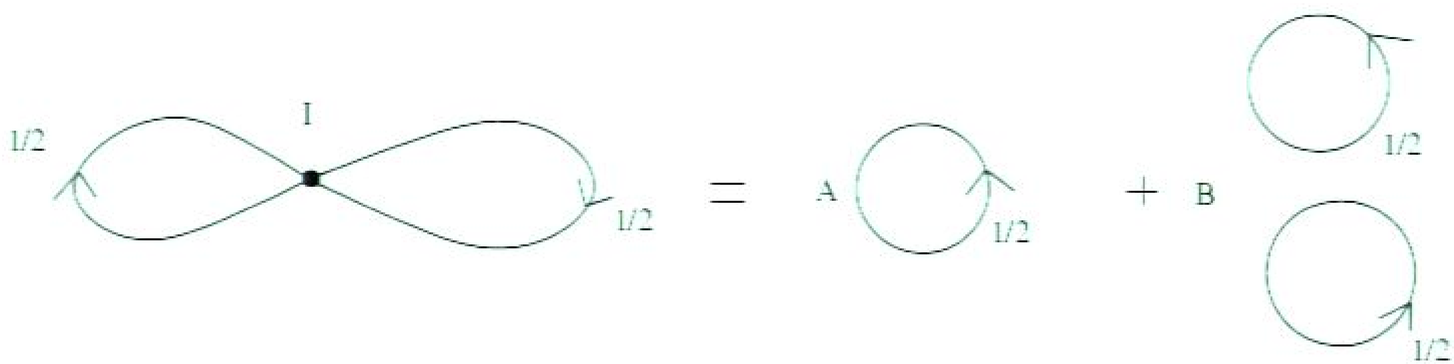

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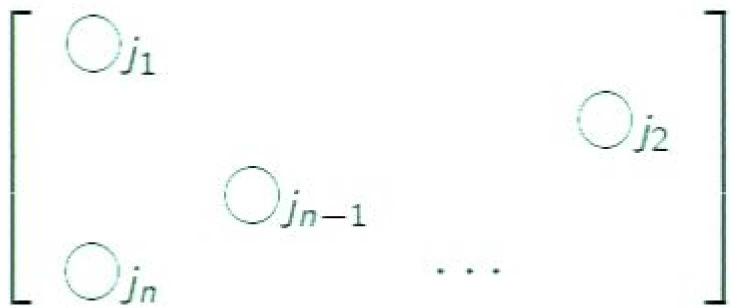

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- Just basis, not orthonormal. (Use Gram-Schmidt-Procedure to get ONB)

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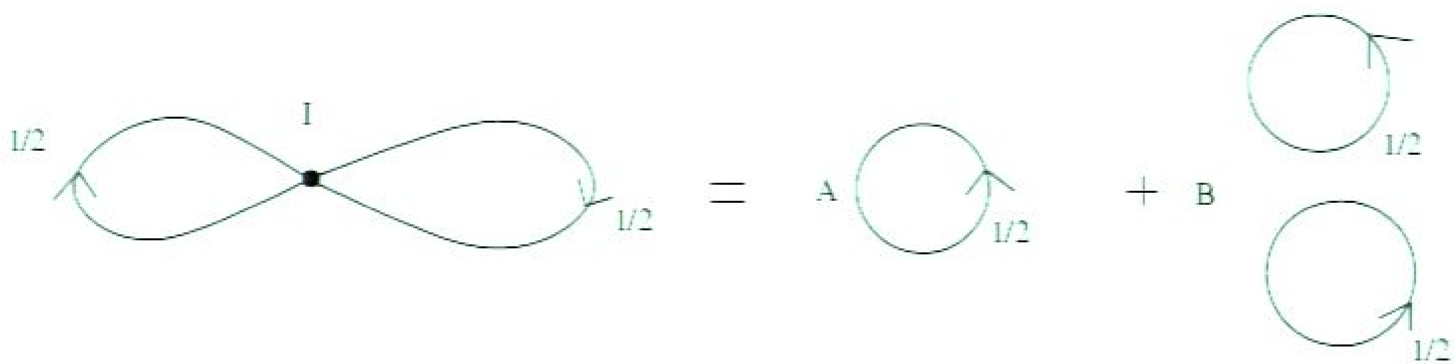

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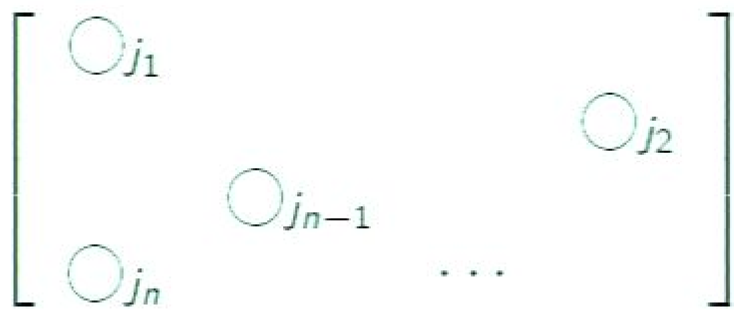

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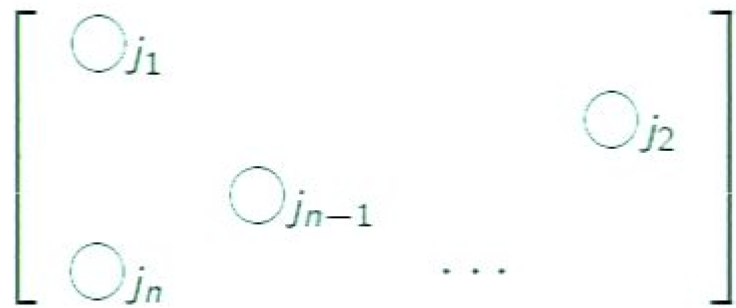
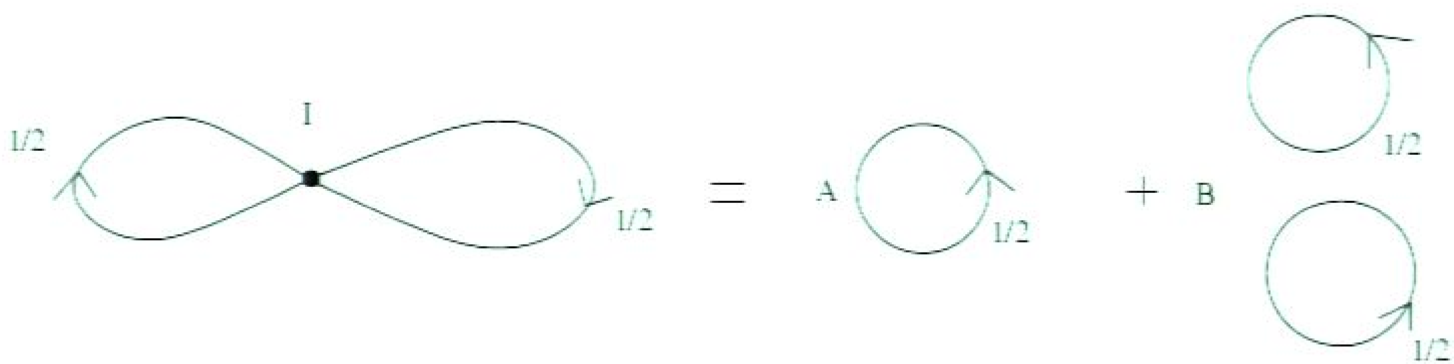


- Just basis, not orthonormal. (Use Gram-Schmidt-Procedure to get ONB)

$\mathcal{H}_{\mathrm{Aut}}$ for $G = SU(2)$

Observation: Finitely many separate Wilson loops build basis for lower intertwiner spaces on flowers, i.e.:



$$F(h_1, h_2) = A\,\mathrm{tr}_{\frac{1}{2}}(h_1 h_2) + B\,\mathrm{tr}_{\frac{1}{2}}(h_1)\,\mathrm{tr}_{\frac{1}{2}}(h_2)$$

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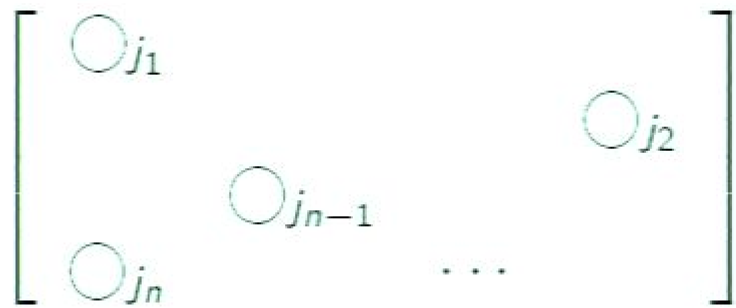

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