Title: How well constrained is the primordial power spectrum?

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Abstract: I will discuss a new method of inflaton potential reconstruction that combines the flow formalism, which is a stochastic method of inflationary model generation, with an exact numerical calculation of the mode equations of quantum fluctuations. This technique allows one to explore regions of the inflationary parameter space yielding spectra that are not well parameterized as power-laws. We use this method to generate an ensemble of generalized spectral shapes that provide equally good fits to current CMB and LSS as data as do simpler power-law spectra.
Within this ensemble are spectra that exhibit a strong running on large angular scales (where cosmic variance is large) that turns off on small scales. Such strongly running spectra are accompanied by large tensor
components that lie outside the 1 and 2 sigma limits of current WMAP3
and SDSS data. This demonstrates that the generalization of the spectral shape adversely impacts our ability to constrain key inflationary observables. The inflationary models giving rise to such spectra are characterized by an initially fast rolling inflaton, in marked contrast to the dominant paradigm of slow roll inflation.

How well constrained is the primordial power spectrum?

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November 6, 2007

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The power spectrum as of 2 p.m. Nov. 6, 2007

A typical parameterization of the power spectrum:

$$P(k) = A\left(\frac{k}{k_0}\right)^{n_s - 1 + \frac{1}{2}\frac{dn_s}{d\ln k}\ln\left(\frac{k}{k_0}\right)}$$

Constraints from WMAP3 and SDSS[1]:

$$0.97 < n_s < 1.21$$

$$-0.13 < dn_s/d\ln k < 0.007$$

[1] Kinney, Kolb, Melchiorri, Riotto, astro-ph/0605338

Motivation

- We are *assuming* a form for the spectrum because:
  1) it's simple.
  2) it's what's expected from single field slow roll inflation.
  3) it's the best we can do with current data.
- Data is not perfect: cosmic variance, instrumental noise, foregrounds, oh my...

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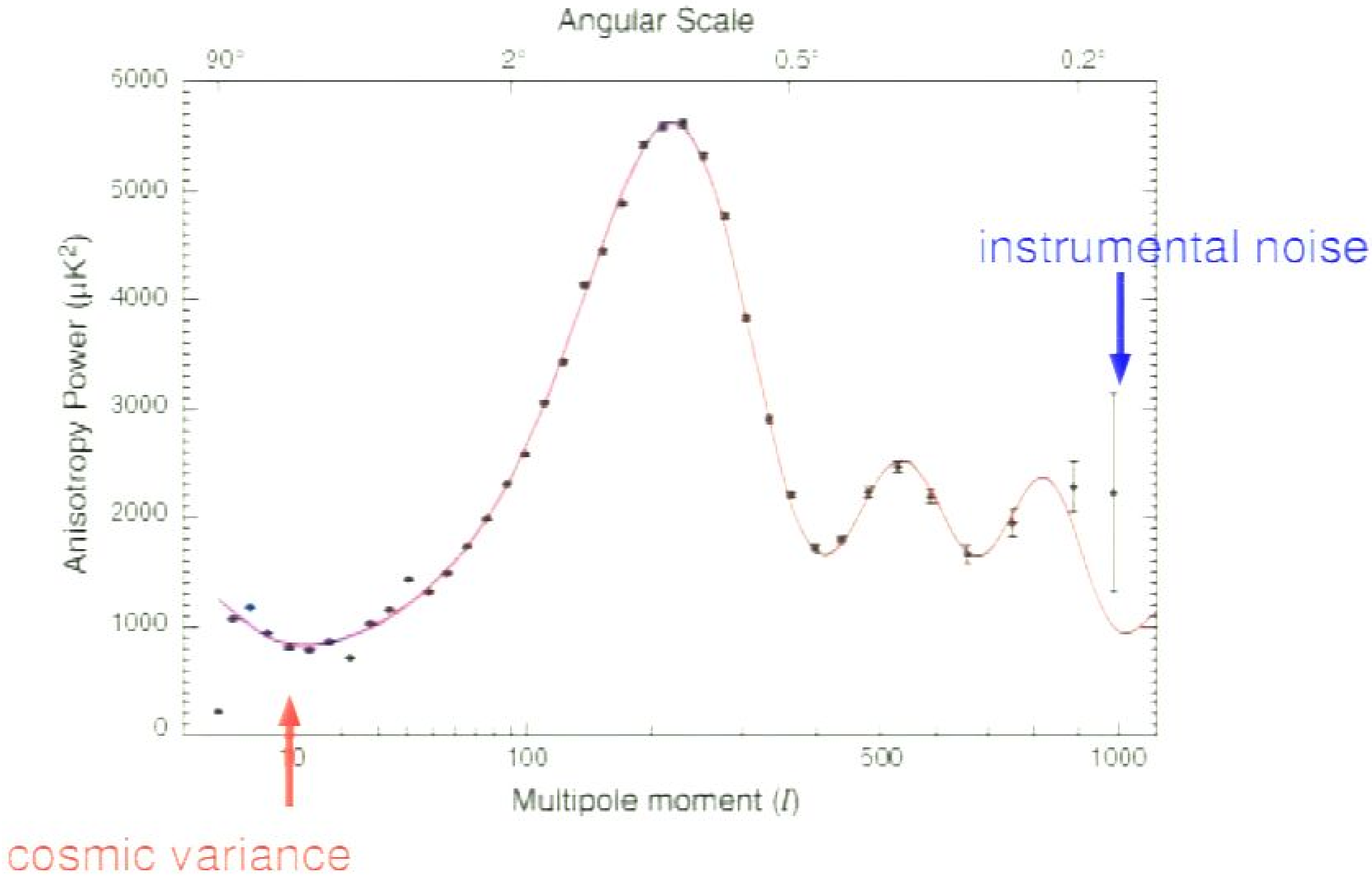
WMAP3



Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
Anisotropy Power (μK²)
10
100
500
1000
Multipole moment (l)
instrumental noise
cosmic variance

Motivation

- It's entirely possible that the spectrum is not even approximately of power law form.
- There might be a complicated potential driving inflation.
- There might be nontrivial initial conditions.

Challenging slow-roll inflation

- We want to test a wide variety of spectral shapes to see which ones yield satisfactory fits to current data.

- We take a purely phenomenological approach – we want to keep things as general as possible.

- We will develop a method for generating parameterization-independent spectra.

- This method must be able to connect these spectra to the underlying inlfationary evolution.

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Outline

- Inflationary basics
- Generalizing the potential
  - Flow formalism
  - Primordial perturbations
  - Methodology
  - Results
- Modifying the initial conditions

Inflation

- Accelerated expansion of the early universe.
- It makes the universe extremely flat, homogeneous and isotropic.
- It generically produces a spectrum of scalar (density) perturbations as well as tensor perturbations (gravitational waves).

Inflation

The universe is isotropic and homogeneous: FRW metric

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2 = a^2(\tau)(d\tau^2 - d\vec{x}^2)$$

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homogeneous scalar field

The scalar field obeys the Klein-Gordon equation:

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

Acceleration equation:

$$\frac{\ddot{a}}{a} = \frac{8\pi}{3m_{\mathrm{Pl}}^2}\left(V - \dot{\phi}^2\right)$$

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accelerated expansion: $\ddot{a} > 0$

$$V > \dot{\phi}^2$$

Change dynamical variable:

$$a(t) \to H(\phi)$$

$$[H'(\phi)]^2 - \frac{12\pi}{m_{\rm Pl}^2} H^2(\phi) = -\frac{32\pi}{m_{\rm Pl}^4} V(\phi)$$

$$\dot{\phi} = -\frac{m_{\rm Pl}^2}{4\pi} H'(\phi)$$

The condition for inflation, $\ddot{a} > 0$, becomes

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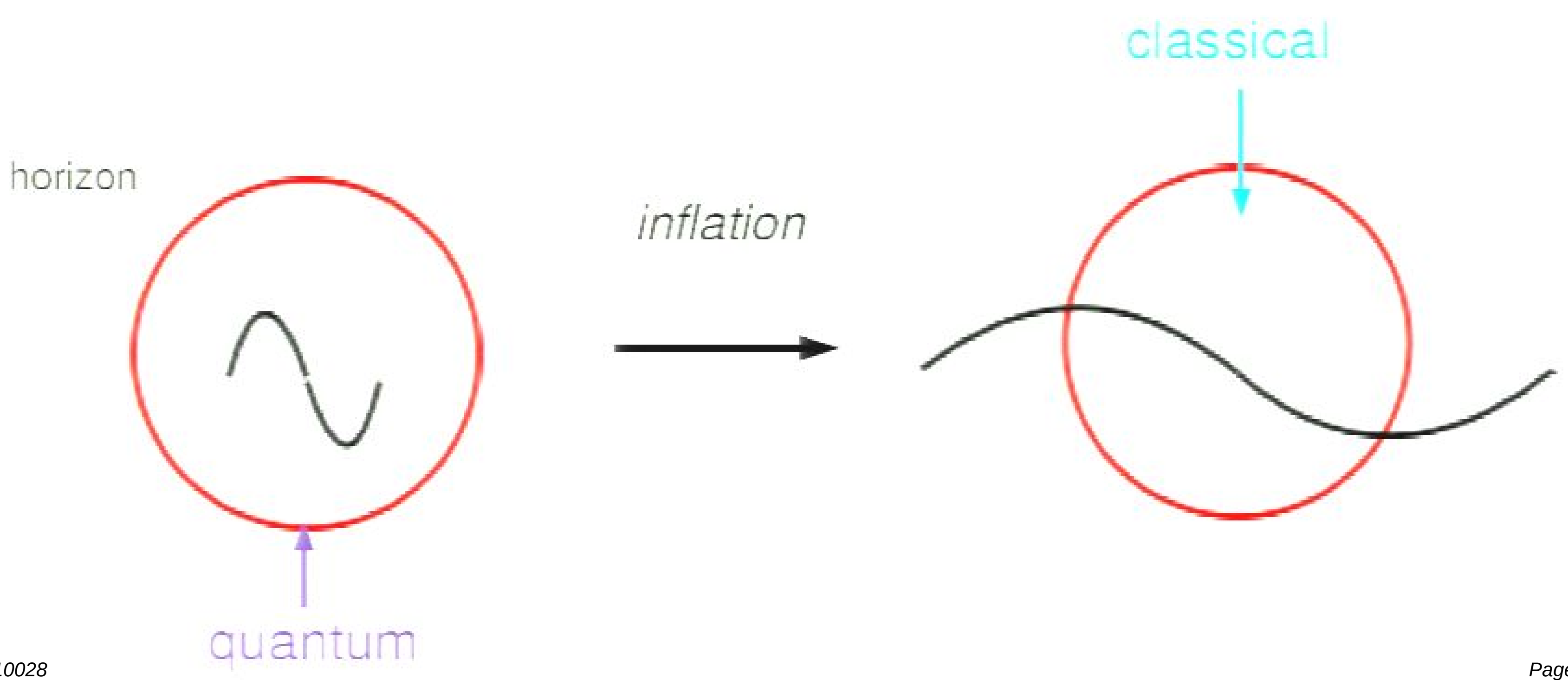

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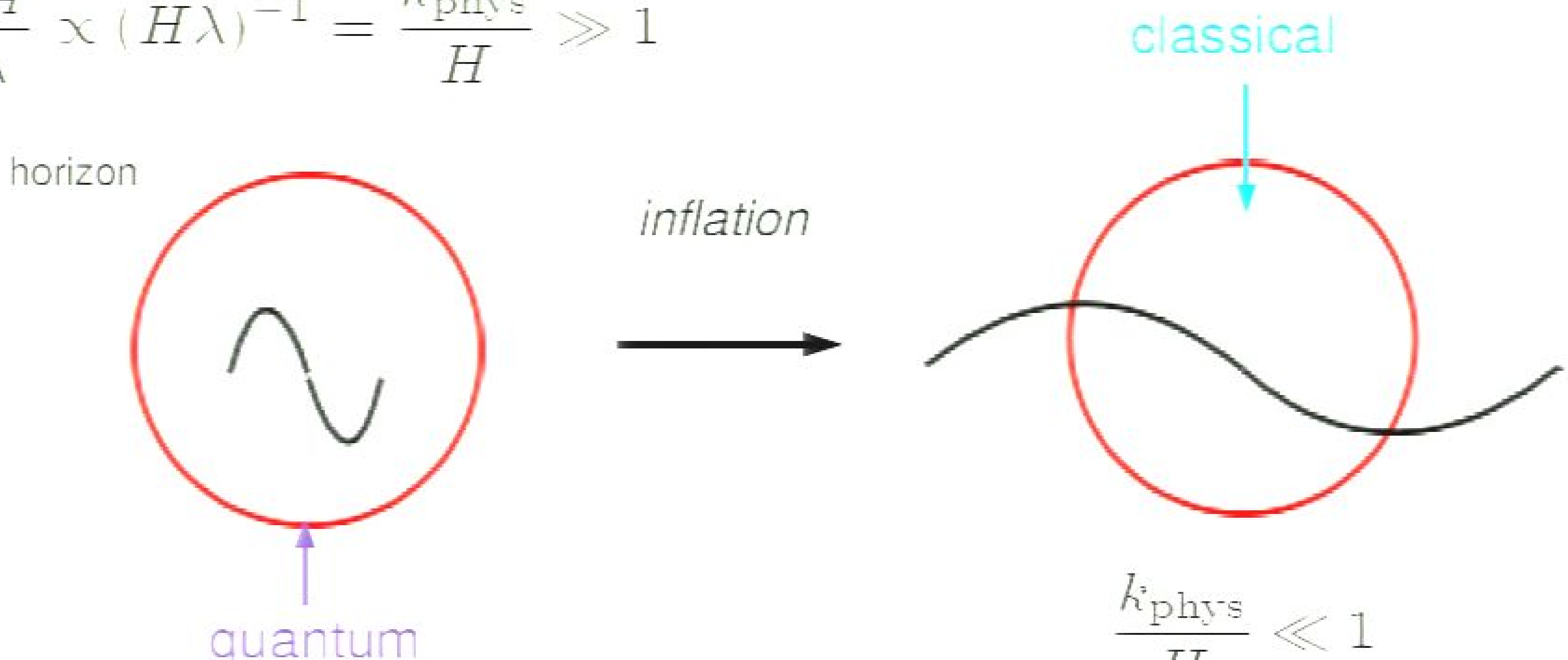
$$\frac{d_H}{\lambda} \propto (H\lambda)^{-1} = \frac{k_{\rm phys}}{H} \gg 1$$



$$\frac{k_{\rm phys}}{H} \ll 1$$

Power Spectra

Amplitudes at horizon crossing:

$$P_{\mathcal{R}} = \frac{H^2}{\pi\epsilon}\bigg|_{k=aH} \qquad P_T = \frac{16H^2}{\pi}\bigg|_{k=aH}$$

Scale dependence?

$$\frac{d\ln P_{\mathcal{R}}(k)}{d\ln k} = -\frac{1}{2\pi}\frac{\sqrt{\epsilon}}{1-\epsilon}\frac{d}{d\phi}\left(\frac{H^2}{\epsilon}\right) = 1 - 4\epsilon + 2\eta = n_s$$

where $\eta = \frac{m_{\rm Pl}^2}{4\pi}\left(\frac{H''}{H}\right)$

We then have

$$P_{\mathcal{R}}(k) \propto k^{n_s - 1} \qquad \text{with} \qquad n_s = 1 - 4\epsilon + 2\eta$$

$$k = \frac{k_{\text{phys}}}{a}$$

During slow roll,

$$\epsilon = \frac{m_{\text{Pl}}^2}{4\pi}\left(\frac{H'}{H}\right)^2 \longrightarrow \epsilon = \frac{m_{\text{Pl}}^2}{16\pi}\left(\frac{V'}{V}\right)^2$$

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$$P_{\mathcal{R}}(k_0),\ P_T(k_0),\ n_s(k_0) \longrightarrow V(\phi_0),\ V'(\phi_0),\ V''(\phi_0)$$

A large number of models are well described using the slow-roll approximation:

	Potentials	Notes
chaotic	$V(\phi) = \Lambda^4 \left(\frac{\phi}{\mu}\right)^p$	$n_s < 1.\ P_T/P_{\mathcal{R}} = r > 0.01$
SSB	$V(\phi) = \Lambda^4 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right]$	$n_s < 1.\ r < 0.01$
hybrid	$V(\phi) = \Lambda^4 \left[1 + \left(\frac{\phi}{\mu}\right)^p\right]$	$n_s > 1.\ r \approx 0$

Moving beyond slow-roll

- We need to move beyond the slow-roll approximation in order to study more general power spectra.
- We also do not want to work within the confines of any one particular class of models.
- We need a parameterization-independent approach to exploring the inflationary parameter space...

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Enter: The Flow Formalism

Flow Formalism[1]

The flow formalism is a method for stochastically generating large numbers of inflation models.

We begin by defining a hierarchy of *flow* parameters[2]:

$$\epsilon = \frac{m_{\rm Pl}^2}{4\pi}\left(\frac{H'}{H}\right)^2 \qquad \eta = \frac{m_{\rm PL}^2}{4\pi}\left(\frac{H''}{H}\right)$$

$$^{\ell}\lambda_H = \left(\frac{m_{\rm Pl}^2}{4\pi}\right)^{\ell} \frac{(H')^{\ell-1}}{H^{\ell}} \frac{d^{(\ell+1)}H}{d\phi^{(\ell+1)}}$$

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Flow Equations

$$\frac{dH}{dN} = \epsilon H$$

$$\frac{d\epsilon}{dN} = \epsilon(\sigma + 2\epsilon)$$

$$\frac{d({}^{\ell}\lambda_H)}{dN} = \left[\frac{1}{2}(\ell - 1)\sigma + (\ell - 2)\epsilon\right] {}^{\ell}\lambda_H + {}^{\ell+1}\lambda_H$$

where $\sigma = 2\eta - 4\epsilon$

- Truncation at order $\ell = M$ ensures that ${}^{\ell}\lambda_H = 0$ for all $\ell > M$
- The solution of the truncated system is *exact!*
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Implementation[1]

- Draw initial conditions randomly from prescribed ranges.
- Evolve the system forward in time until either inflation ends ($\epsilon > 1$) or the solution asymptotes to the attractor $\epsilon \to 0$.
- If $\epsilon > 1$, evolve the system backwards in time from $N = 0$ to $N \in [46, 60]$
- Reconstruct the inflaton potential[2]: $V = \frac{3m_{\rm Pl}^2}{8\pi} H^2 \left(1 - \frac{1}{3}\epsilon\right)$
- Repeat.

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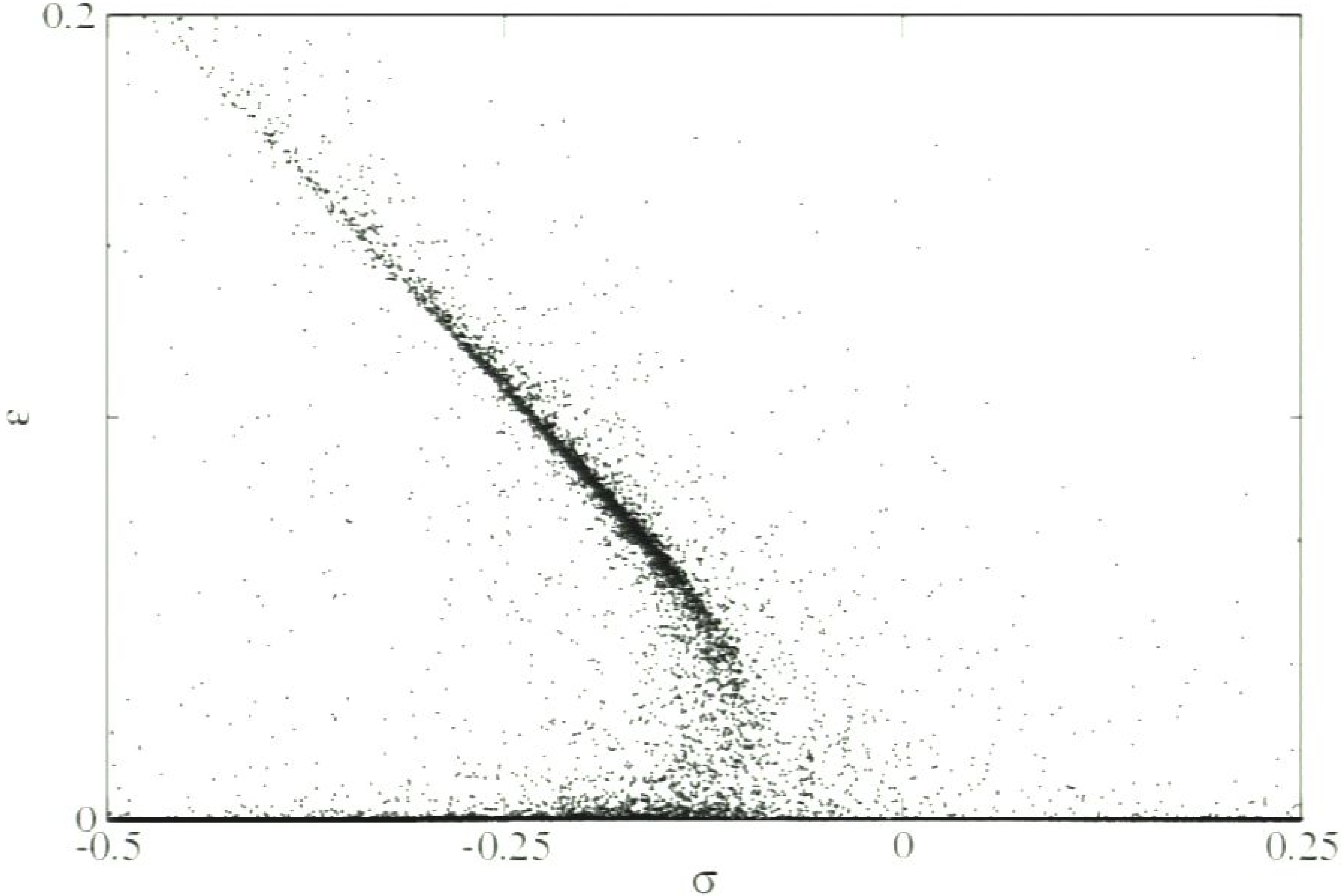
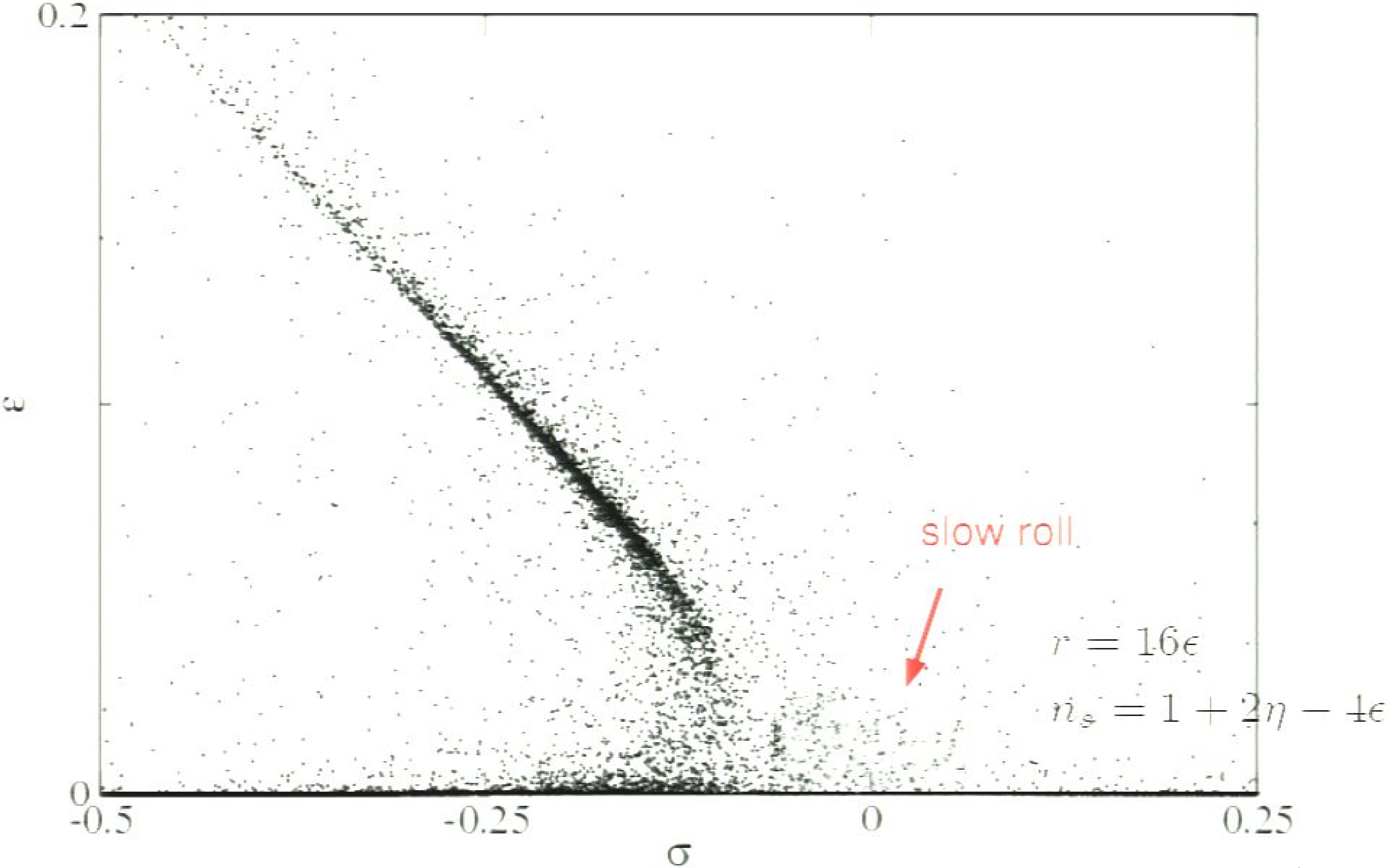

0.2
0
ε
-0.5
-0.25
0
0.25
σ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$r = 16\epsilon$
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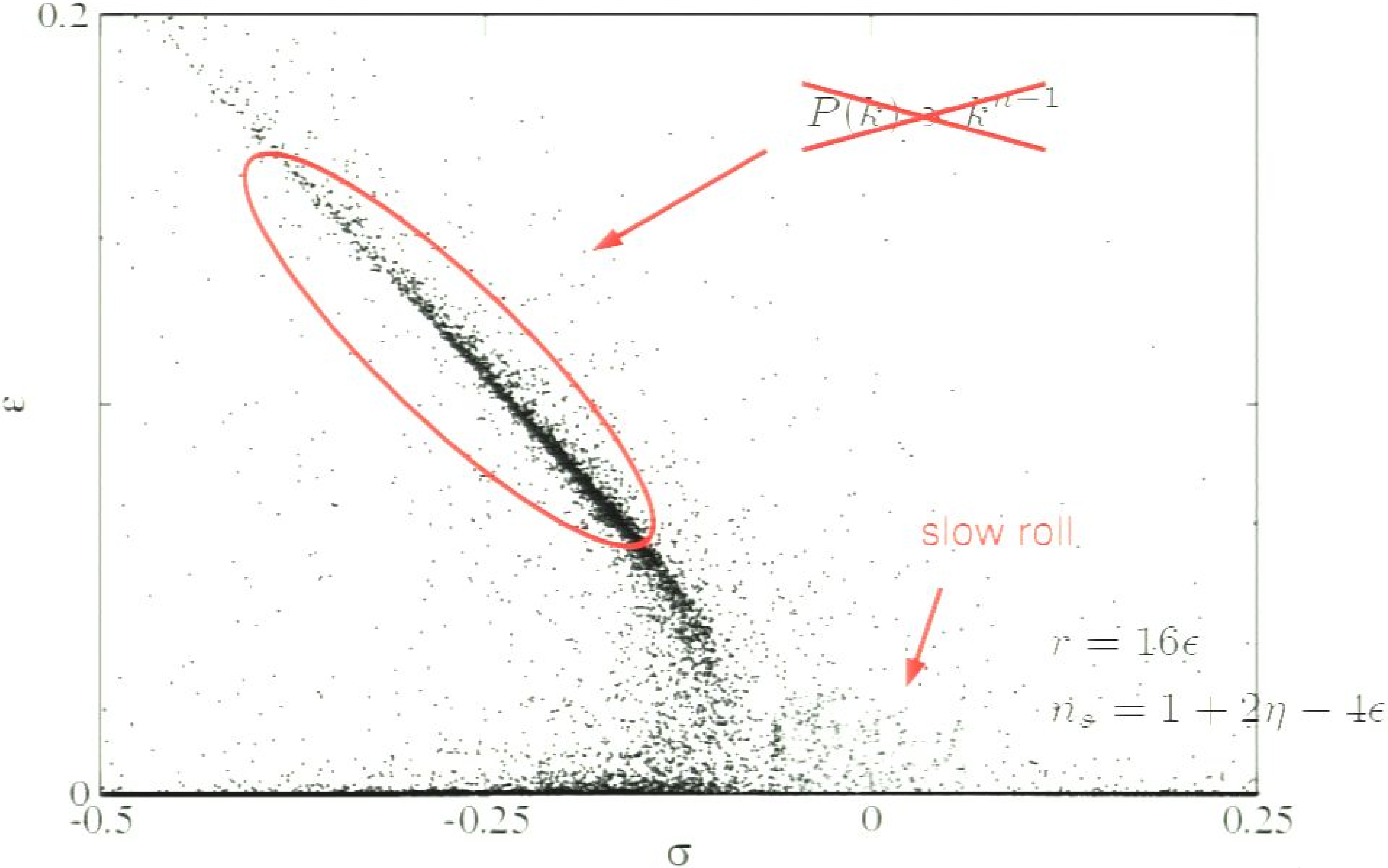

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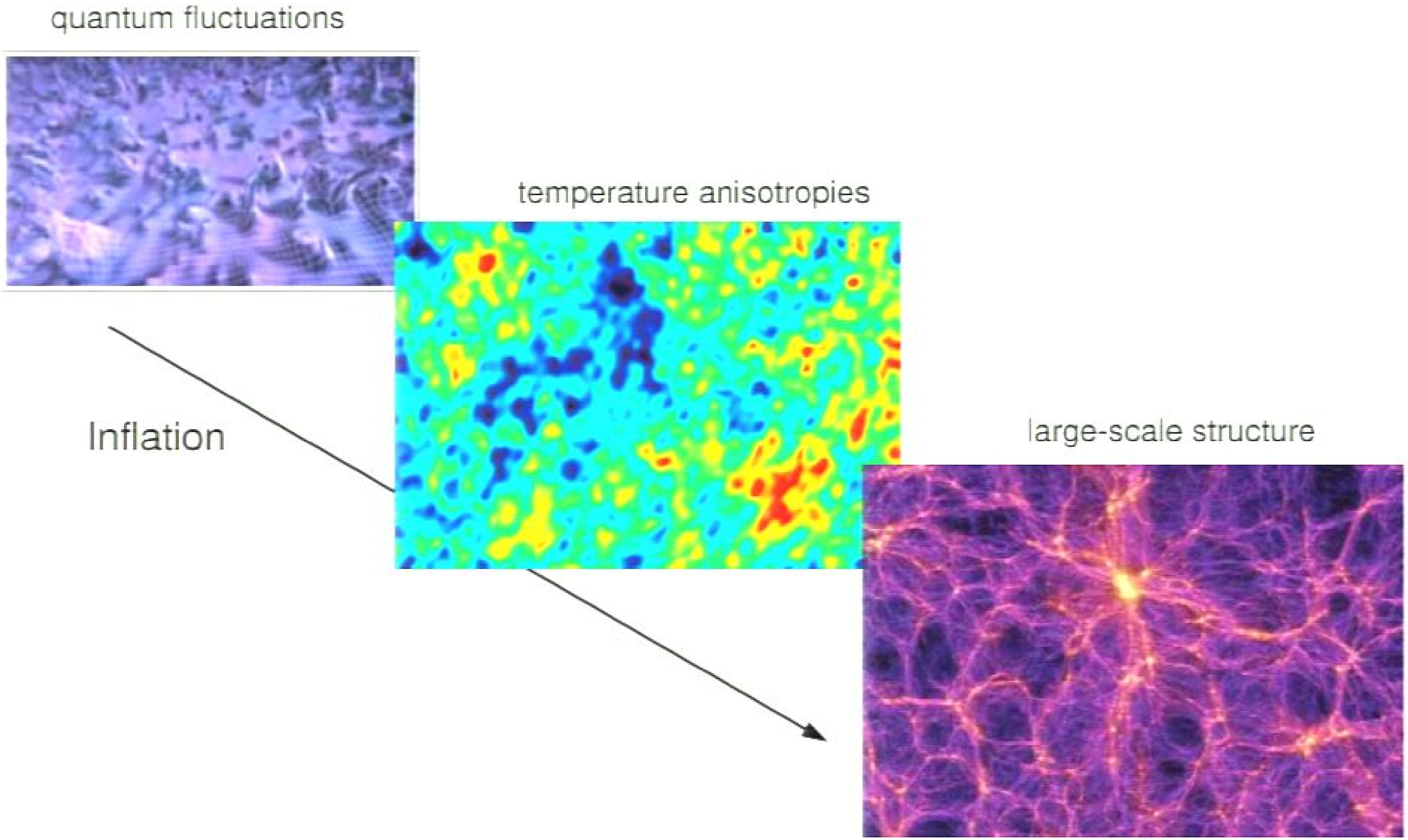
Primordial Perturbations



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Because of this coupling, we employ the gauge invariant potential,

$$\hat{u} = a\delta\phi + \frac{\phi'}{H}\psi = a\delta\phi + z\psi$$

Fourier decomposing,

$$\hat{u} = \int \frac{d^3k}{(2\pi)^{3/2}} \left(u_k(\tau)\hat{a}_k e^{ikx} + u_k^*(\tau)\hat{a}_k^\dagger(\tau) e^{-ikx} \right)$$

the mode function obeys the wave equation,

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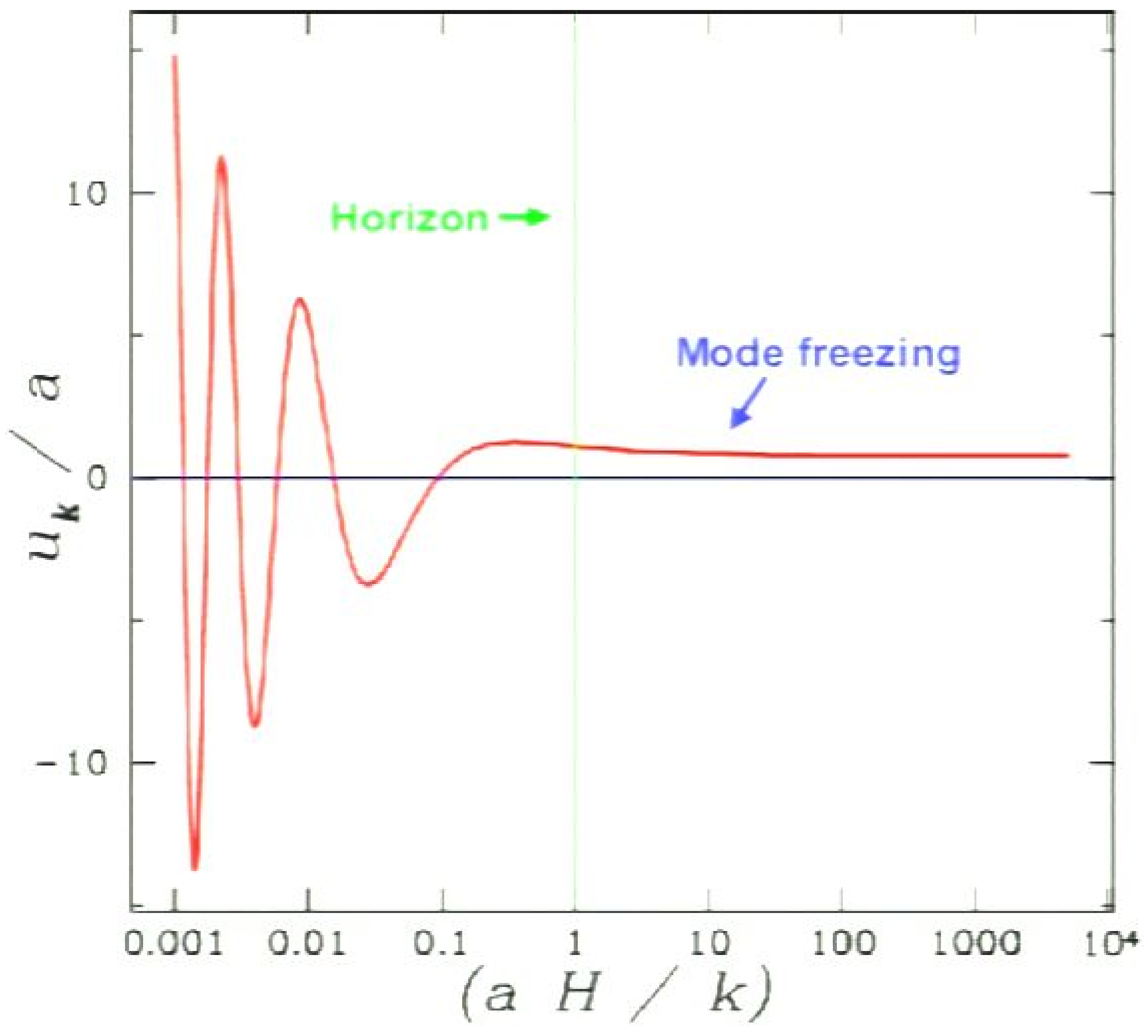

10
0
-10
Horizon
Mode freezing
u_k / a
0.001
0.01
0.1
1
10
100
1000
10^4
$(a H / k)$

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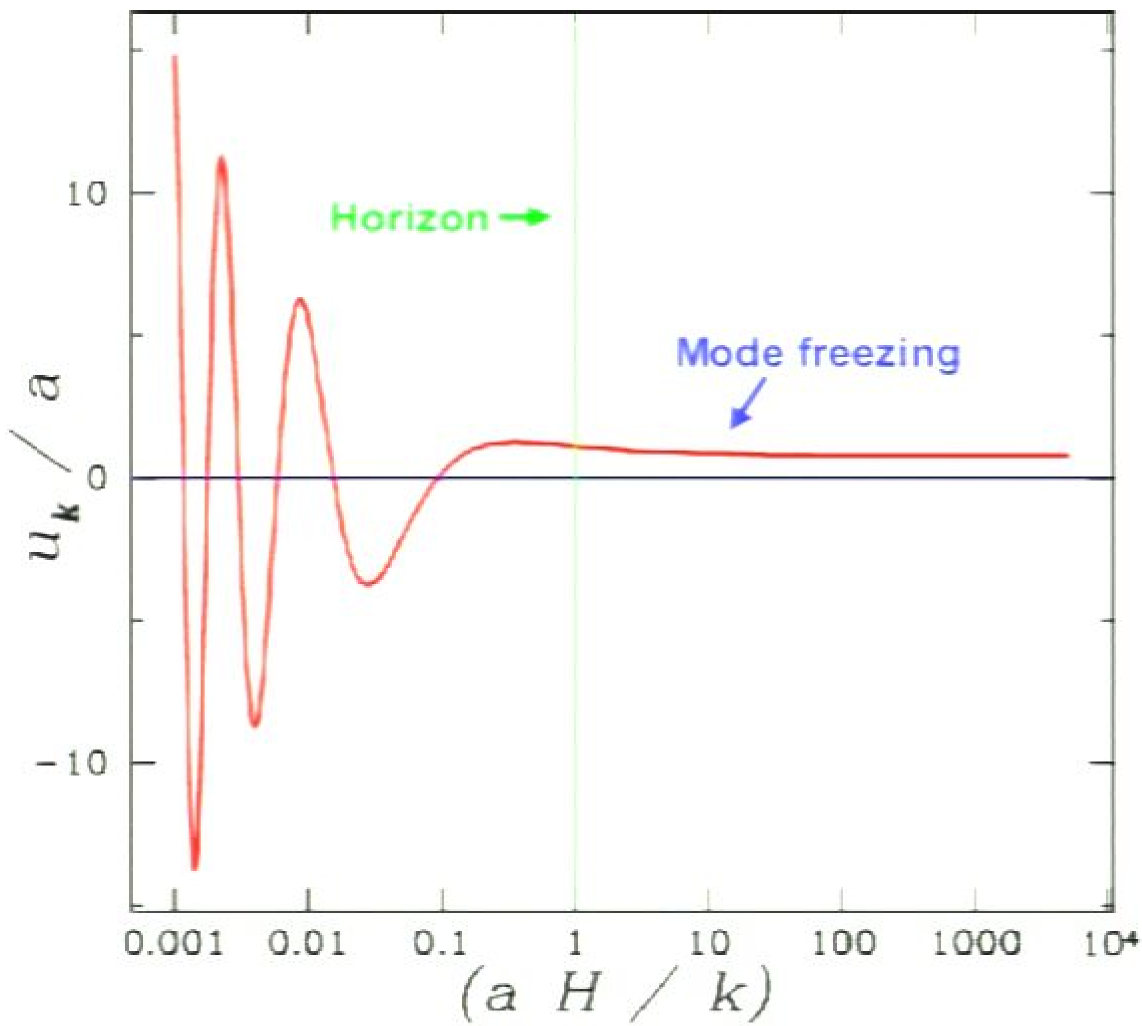
$k\tau \to -\infty$ $\qquad$ $k\tau \to 0$

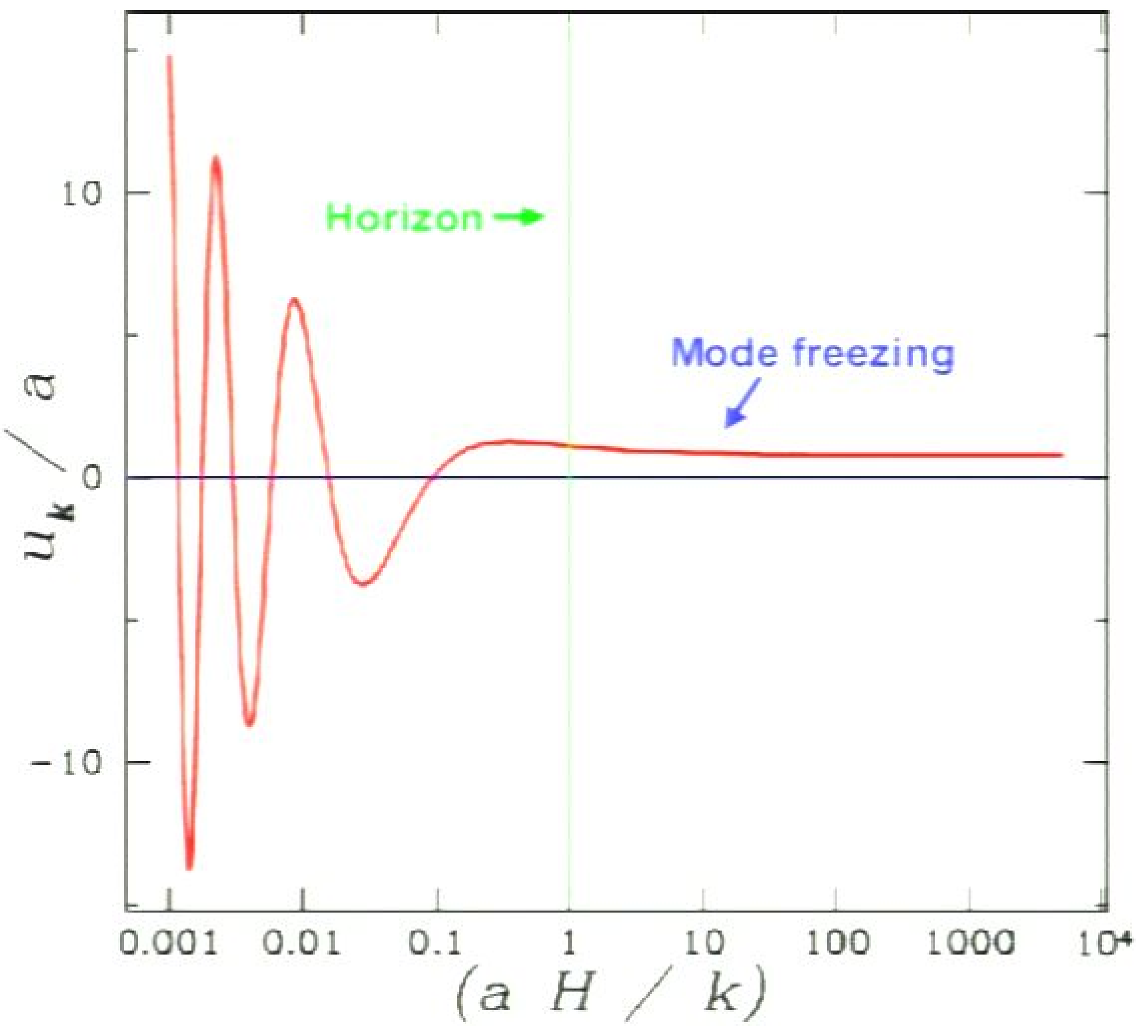
$u_k \propto e^{-ik\tau}$ $\qquad$ $u_k = \text{const}$



Horizon
Mode freezing
10
0
−10
u_k / a
0.001
0.01
0.1
1
10
100
1000
10^4
$(a\ H\ /\ k)$

Comoving Curvature Perturbation

Scalar curvature perturbation: $\mathcal{R} = \psi + \frac{1}{3}E$



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Fourier decomposing,

$$\hat{u} = \int \frac{d^3k}{(2\pi)^{3/2}} \left(u_k(\tau)\hat{a}_k e^{ikx} + u_k^*(\tau)\hat{a}_k^\dagger(\tau) e^{-ikx} \right)$$

the mode function obeys the wave equation,

$$u_k'' + \left(k^2 - \frac{z''}{z} \right) u_k = 0$$

$\frac{z''}{z} \approx$ constant

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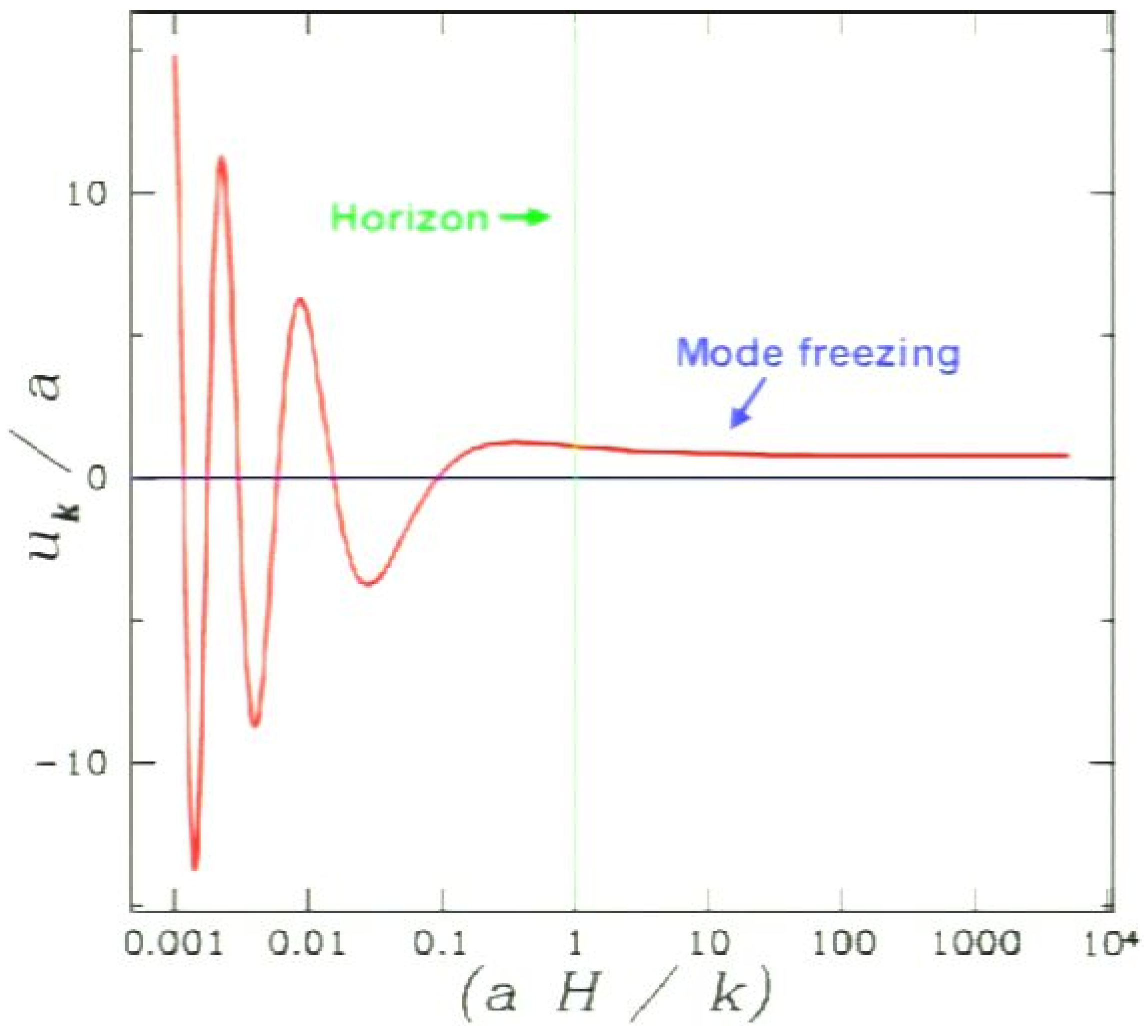

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Gauge invariant potential: $\hat{u} = a\delta\phi + z\psi$

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Gauge choices!!

comoving hypersurface

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Comoving Curvature Perturbation

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Power spectrum: $$P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\left|\mathcal{R}_k\right|^2$$

$$= \frac{k^3}{2\pi^2}\left|\frac{u_k}{z}\right|^2$$

Tensor Perturbations

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{ij}$$

Linearized Einstein's Eqs:

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} + k^2 h_{ij} = 0$$

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Obtain the background evolution:

Solve the flow equations

$$\frac{dH}{dN} = \epsilon H$$

$$\frac{d\epsilon}{dN} = \epsilon(\sigma + 2\epsilon)$$

$$\frac{d({}^{\ell}\lambda_H)}{dN} = \left[\frac{1}{2}(\ell - 1)\sigma + (\ell - 2)\epsilon\right] {}^{\ell}\lambda_H + {}^{\ell+1}\lambda_H$$

at 6th order for randomly drawn initial conditions:

$\epsilon_0, \sigma_0, ..., {}^{M}\lambda_{H0}$

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We denote the likelihood of this model $\mathcal{L}_0$

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$$\mathcal{L}_0(n_s, r) \longleftarrow \text{2 free parameters}$$

Apples and oranges. How do we compare these two?

Statistical Inference

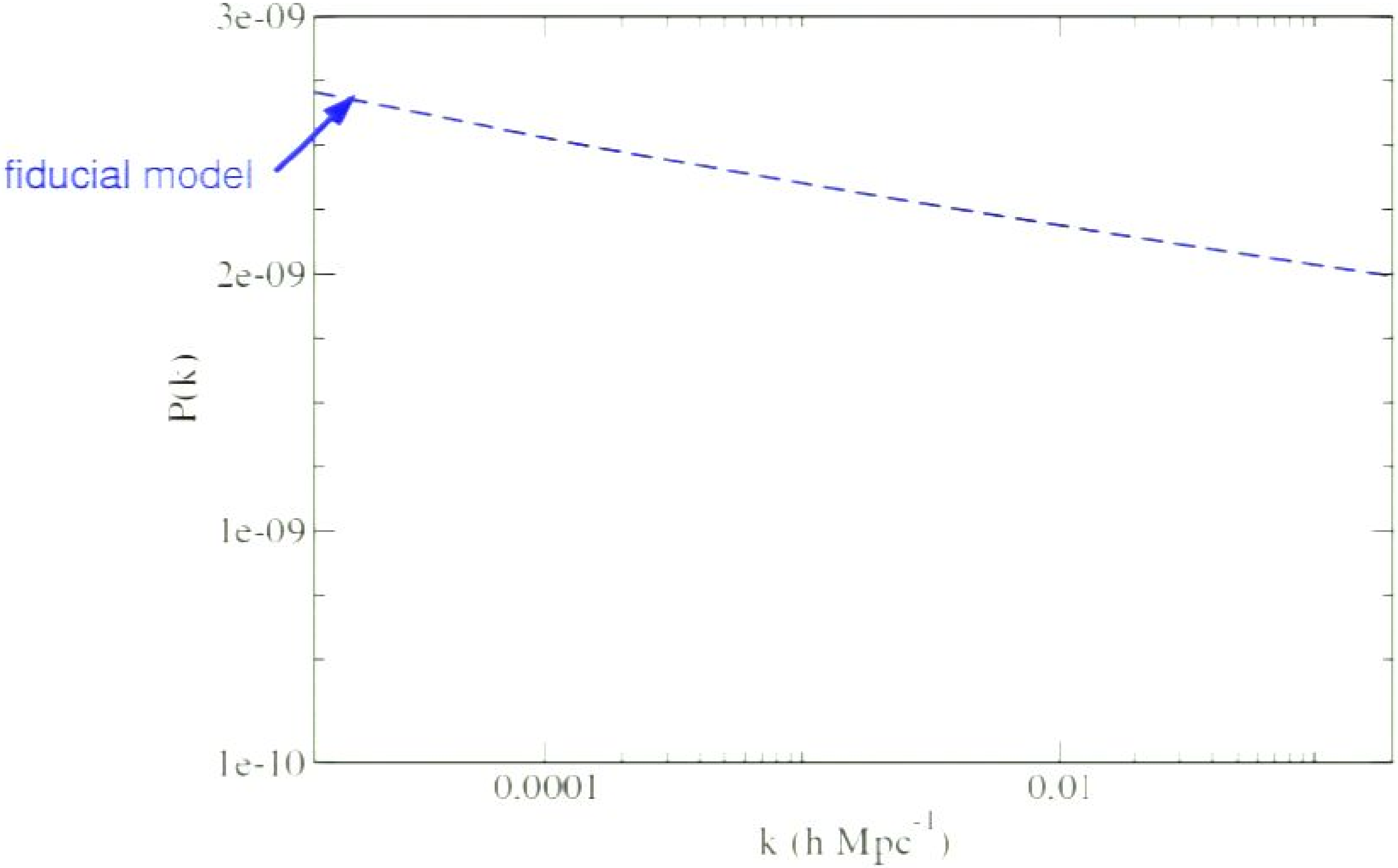
Make the identification $-2\ln\mathcal{L} = \chi^2_{eff}$.

Compute the p-value associated with this χ^2_{eff}.

$$p_\nu = \int_{\chi^2}^{\infty} P_\nu(y)dy$$

If $|\Delta p| = |p_0 - p_{\text{flow}}| \leq 0.01$, we consider this model indistinguishable from the fiducial model.

Results



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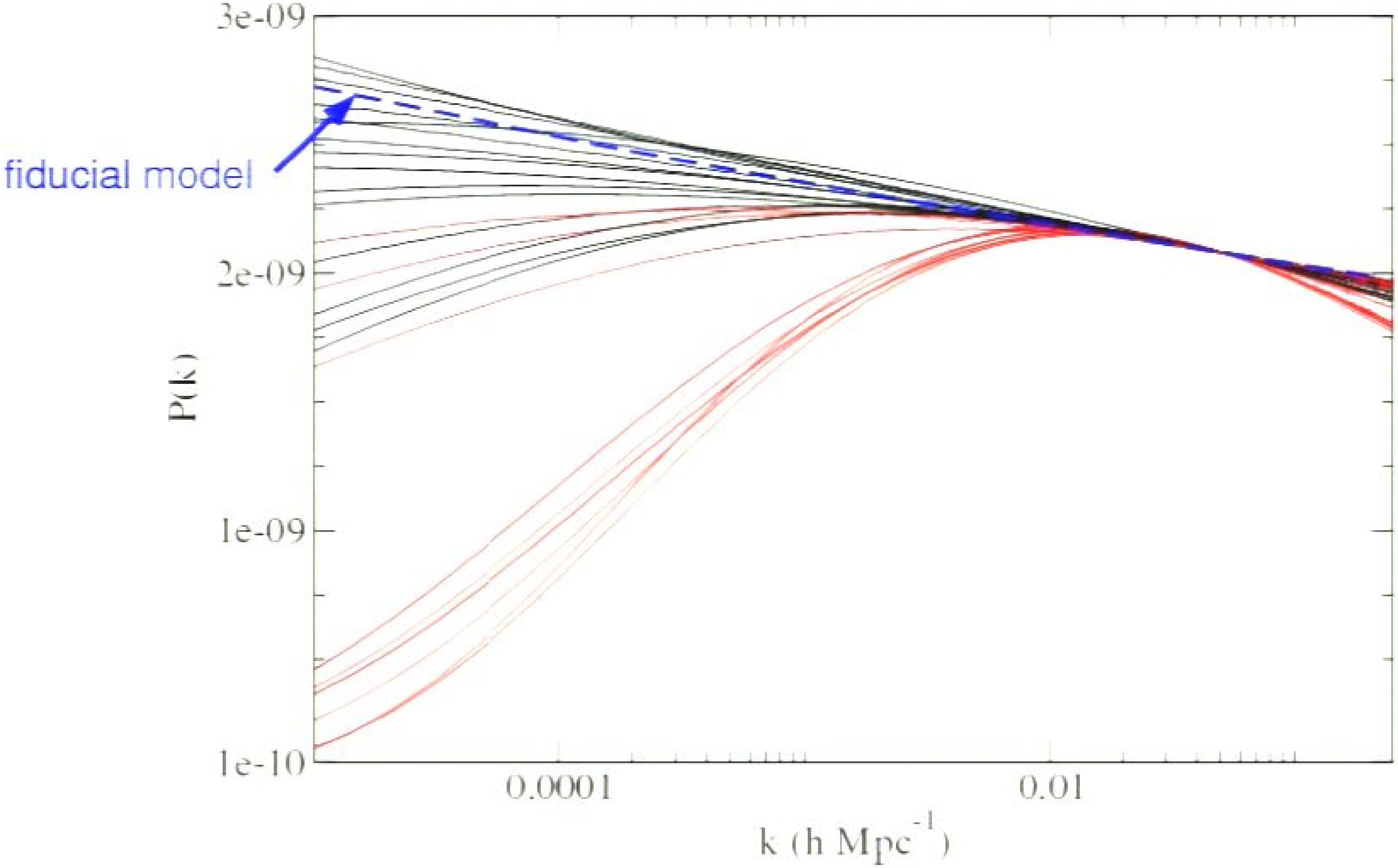

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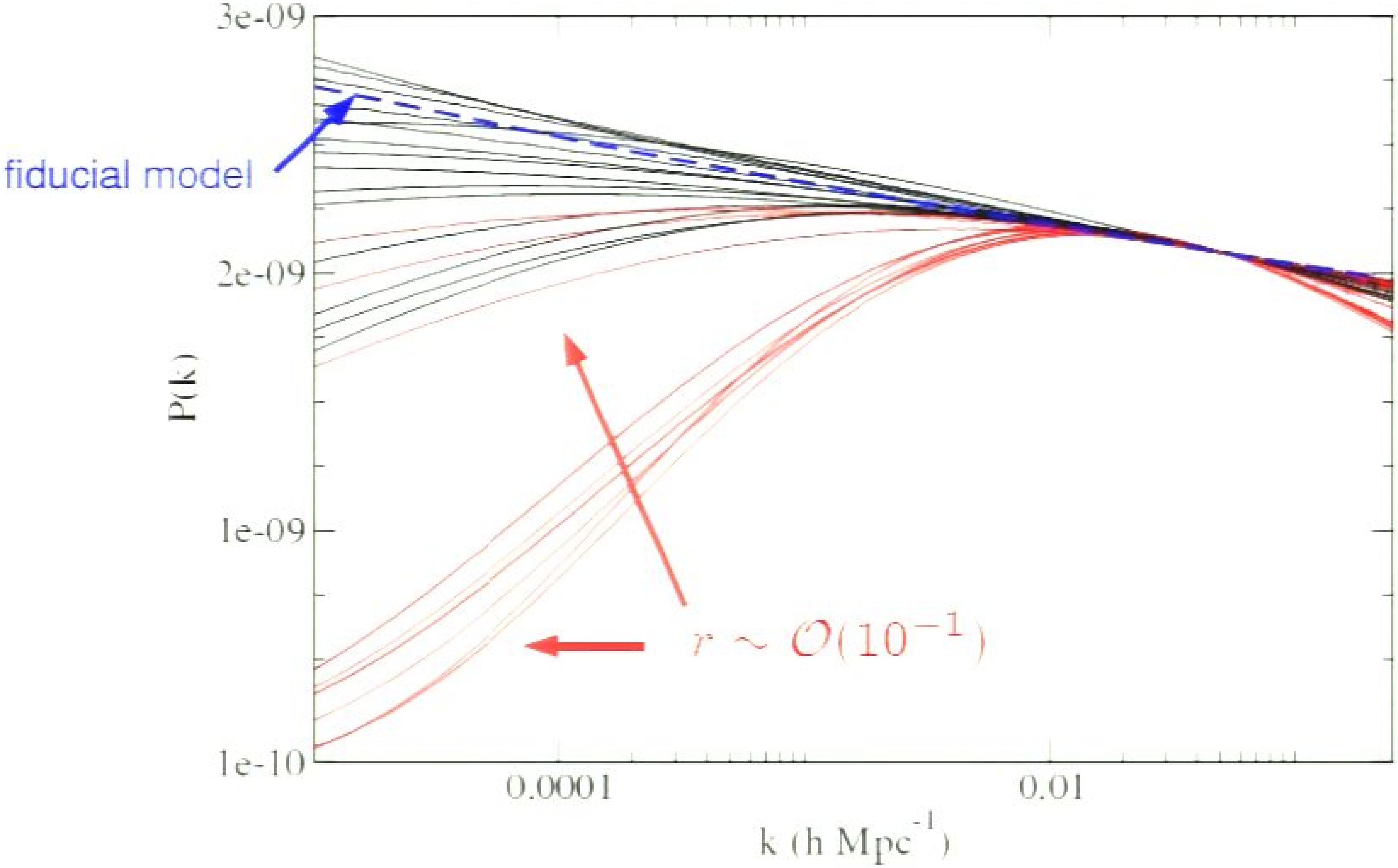

3e-09
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P(k)
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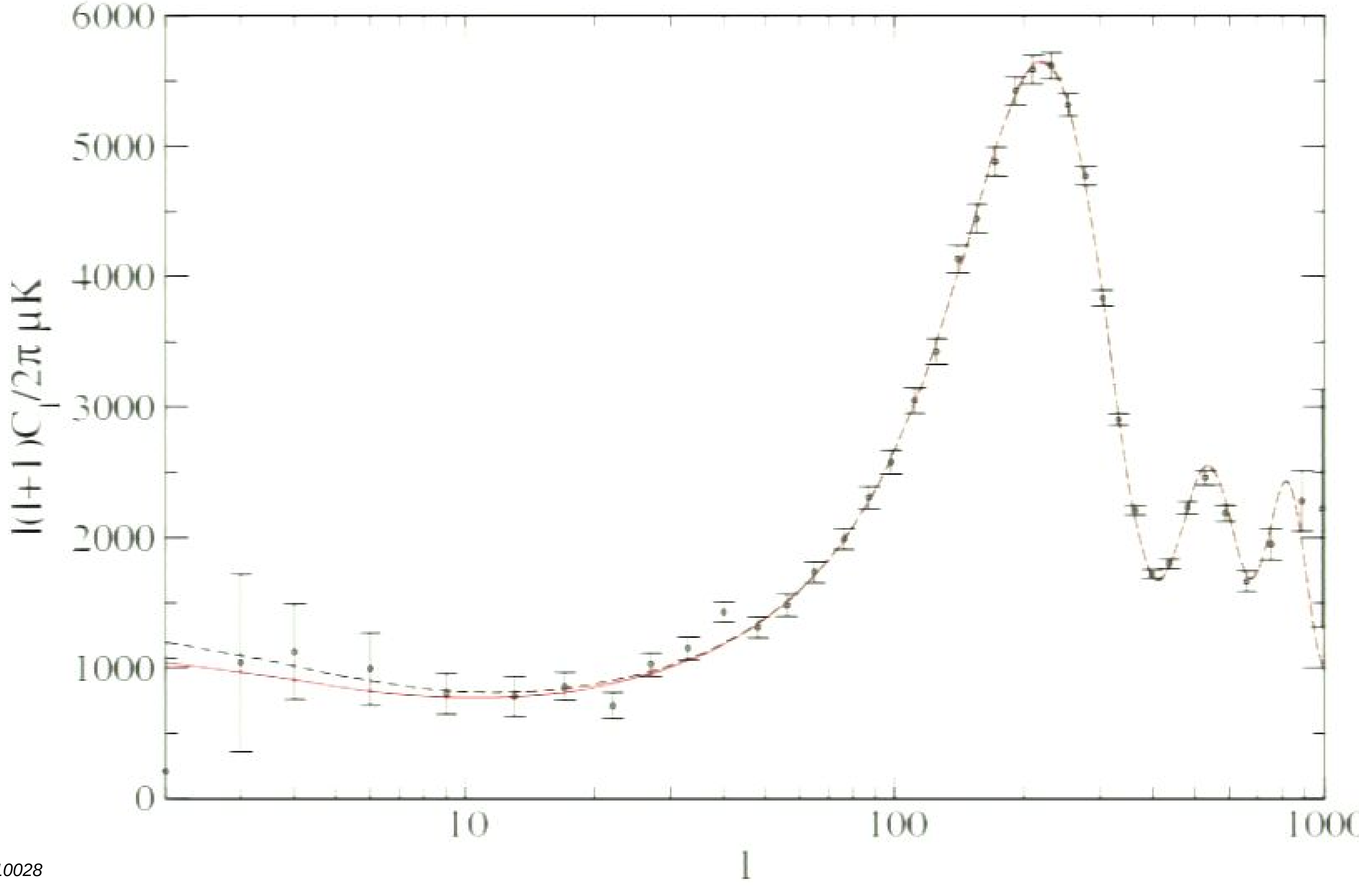
0.01
k (h Mpc^{-1})
fiducial model
$r \sim \mathcal{O}(10^{-1})$

CMB Spectra



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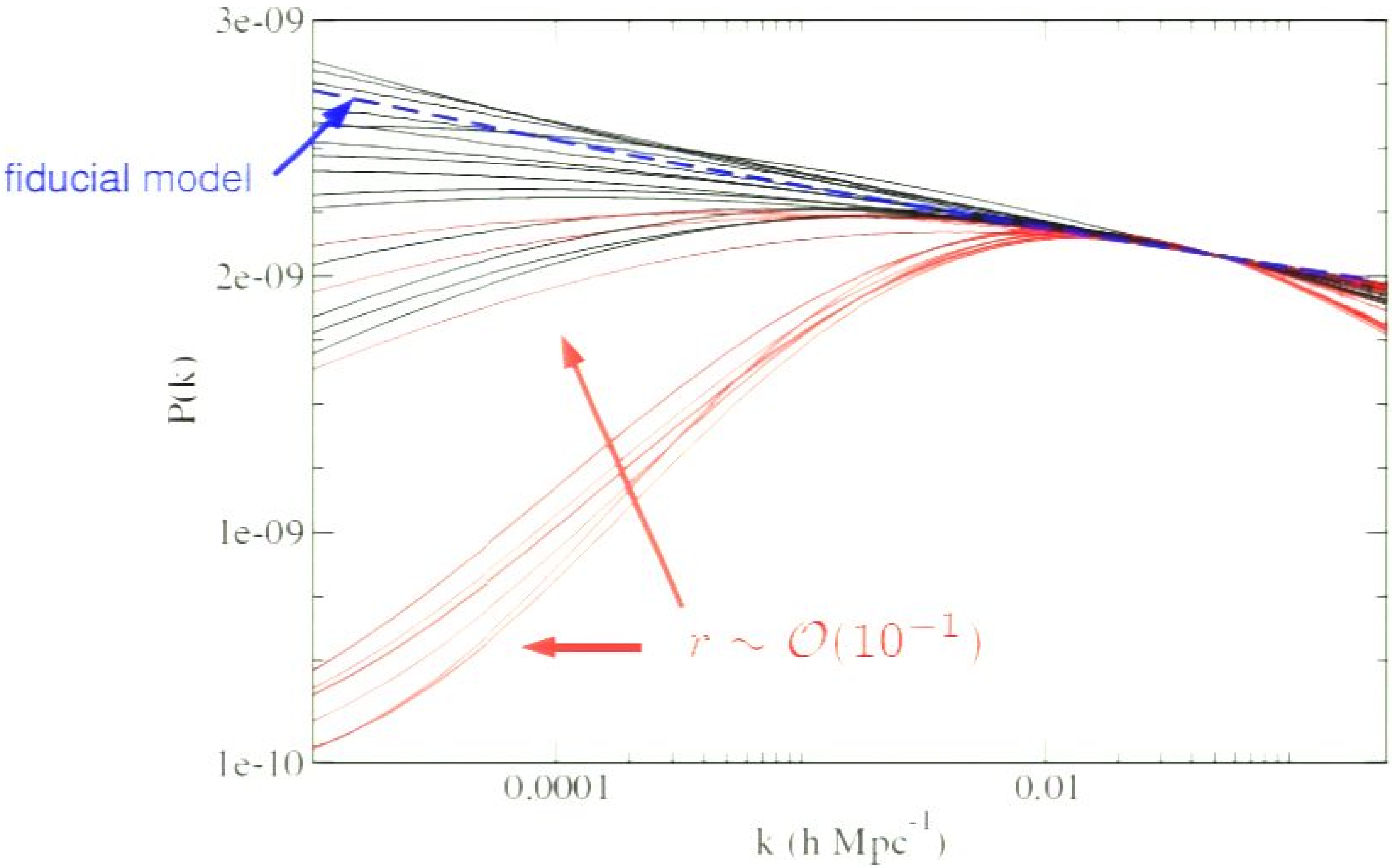

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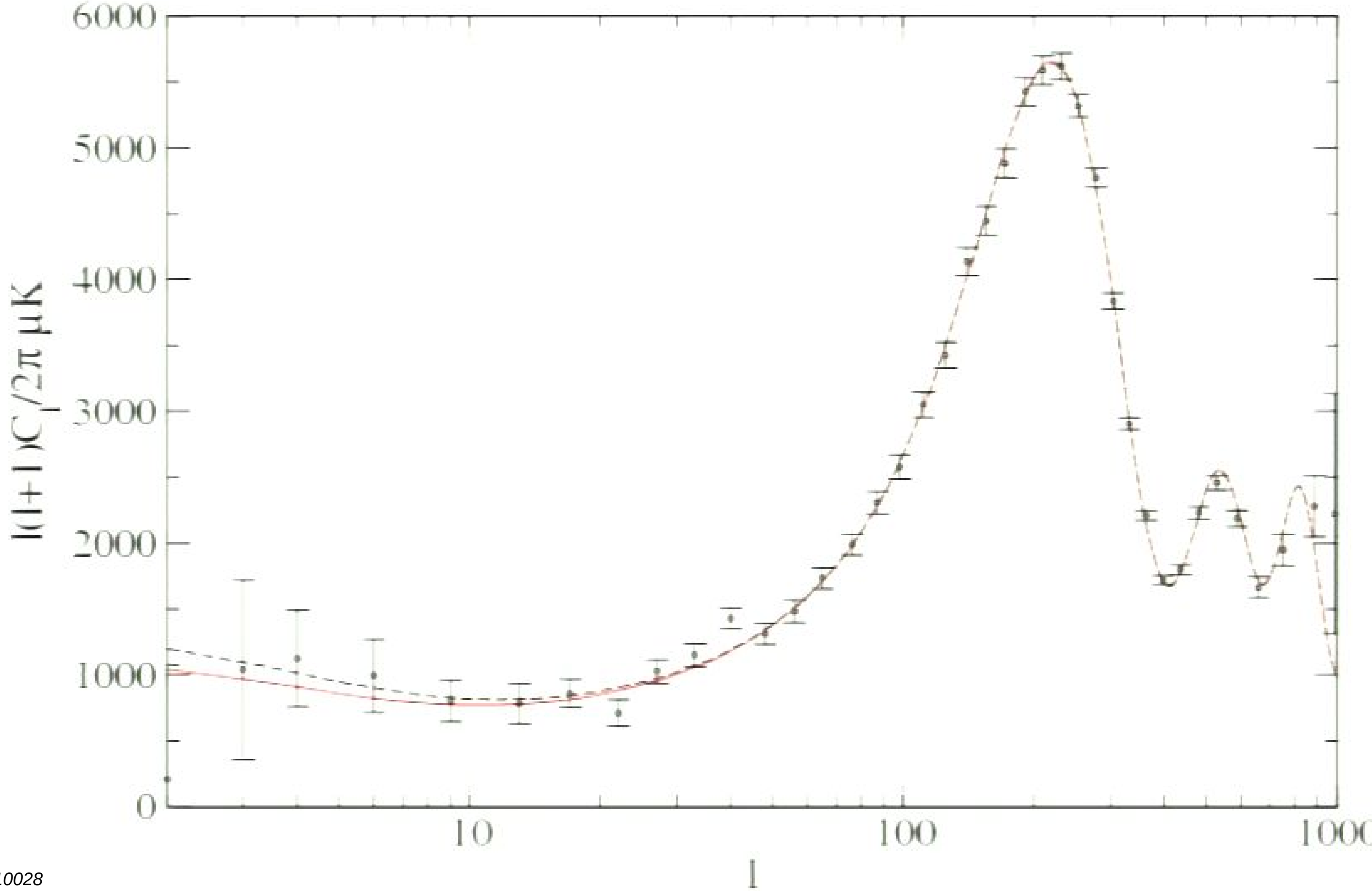

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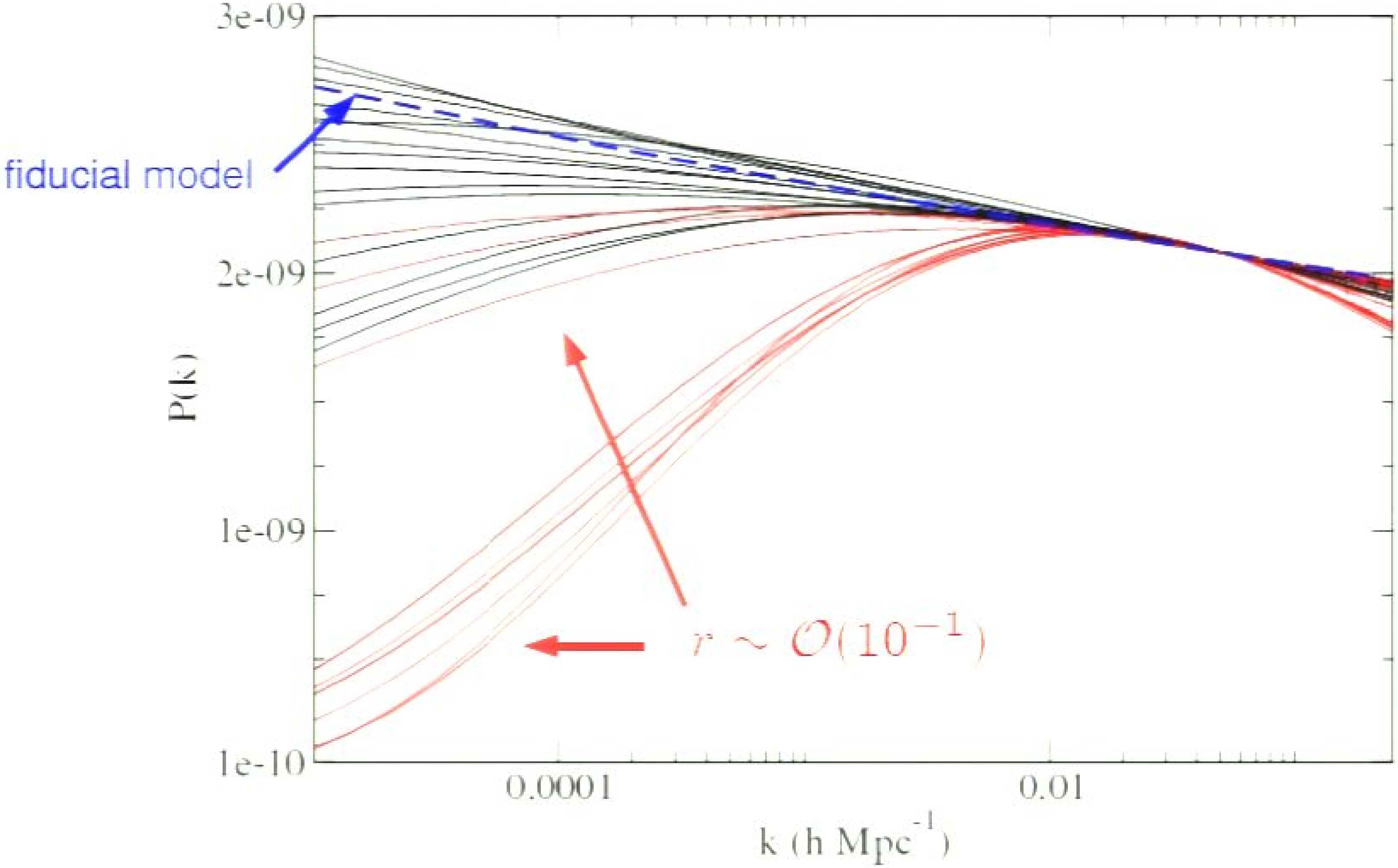

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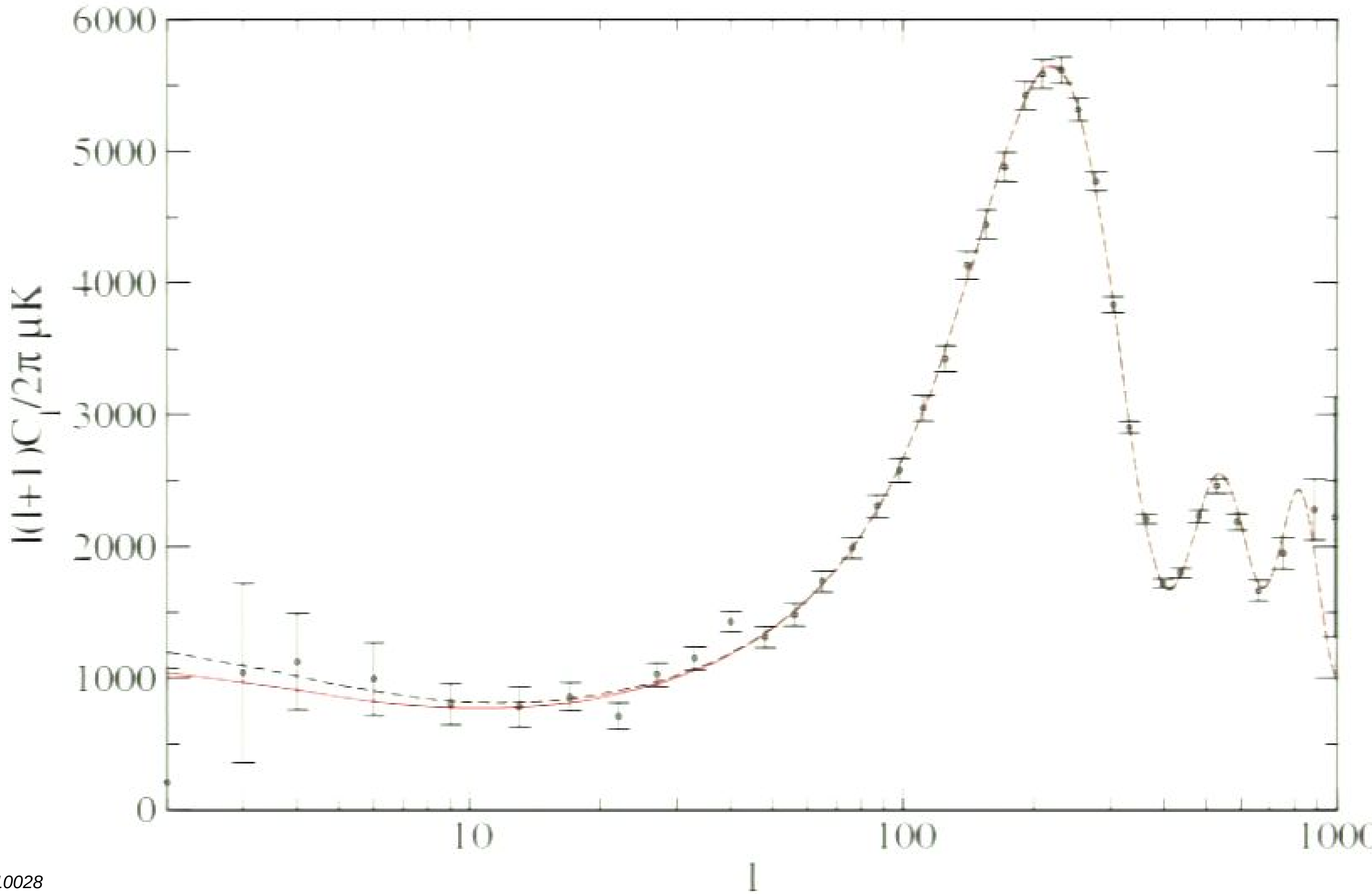
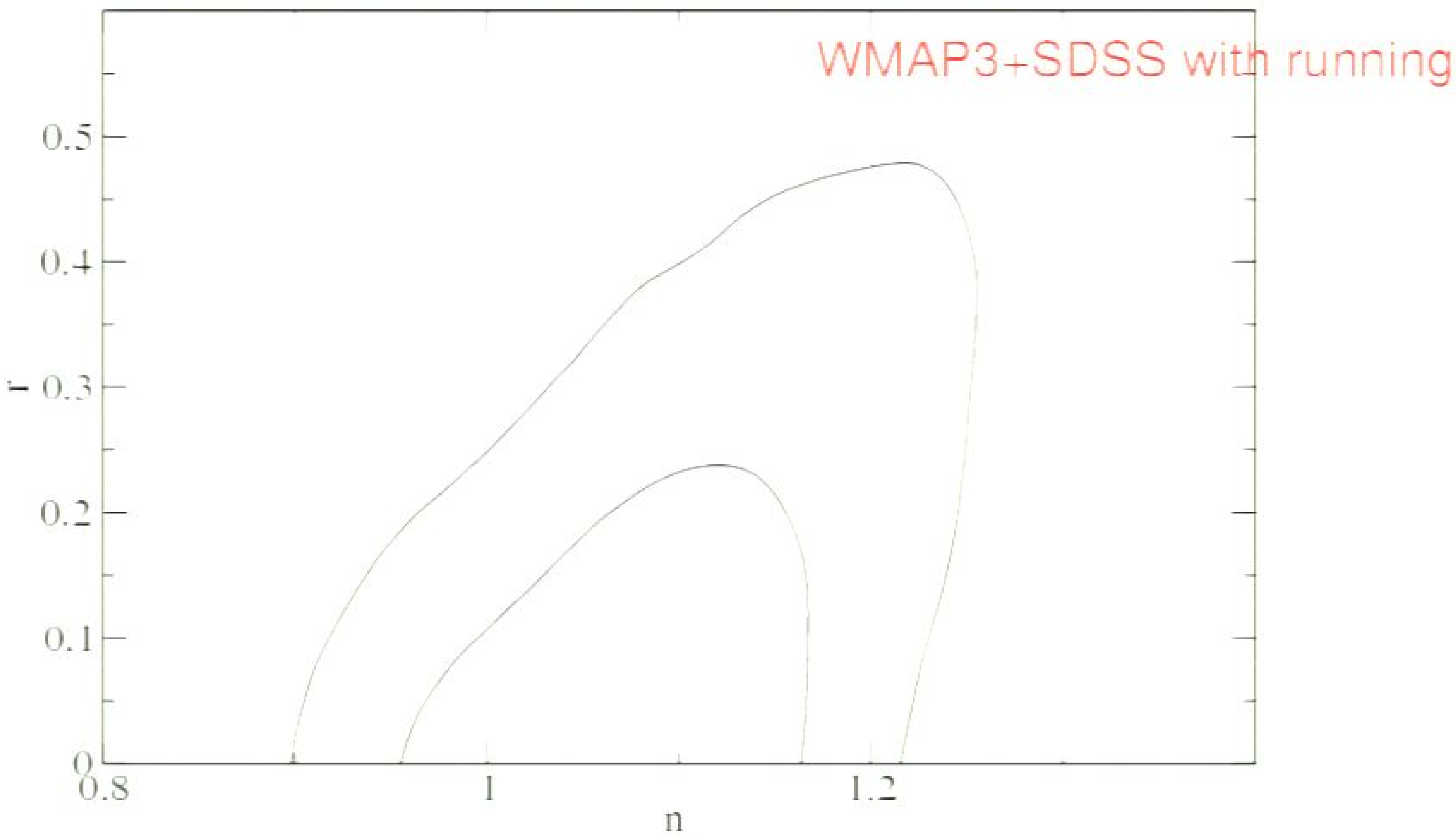


Comparison with previous studies



[1] Kinney, Kolb, Melchiori, Riotto astro-ph/0605338

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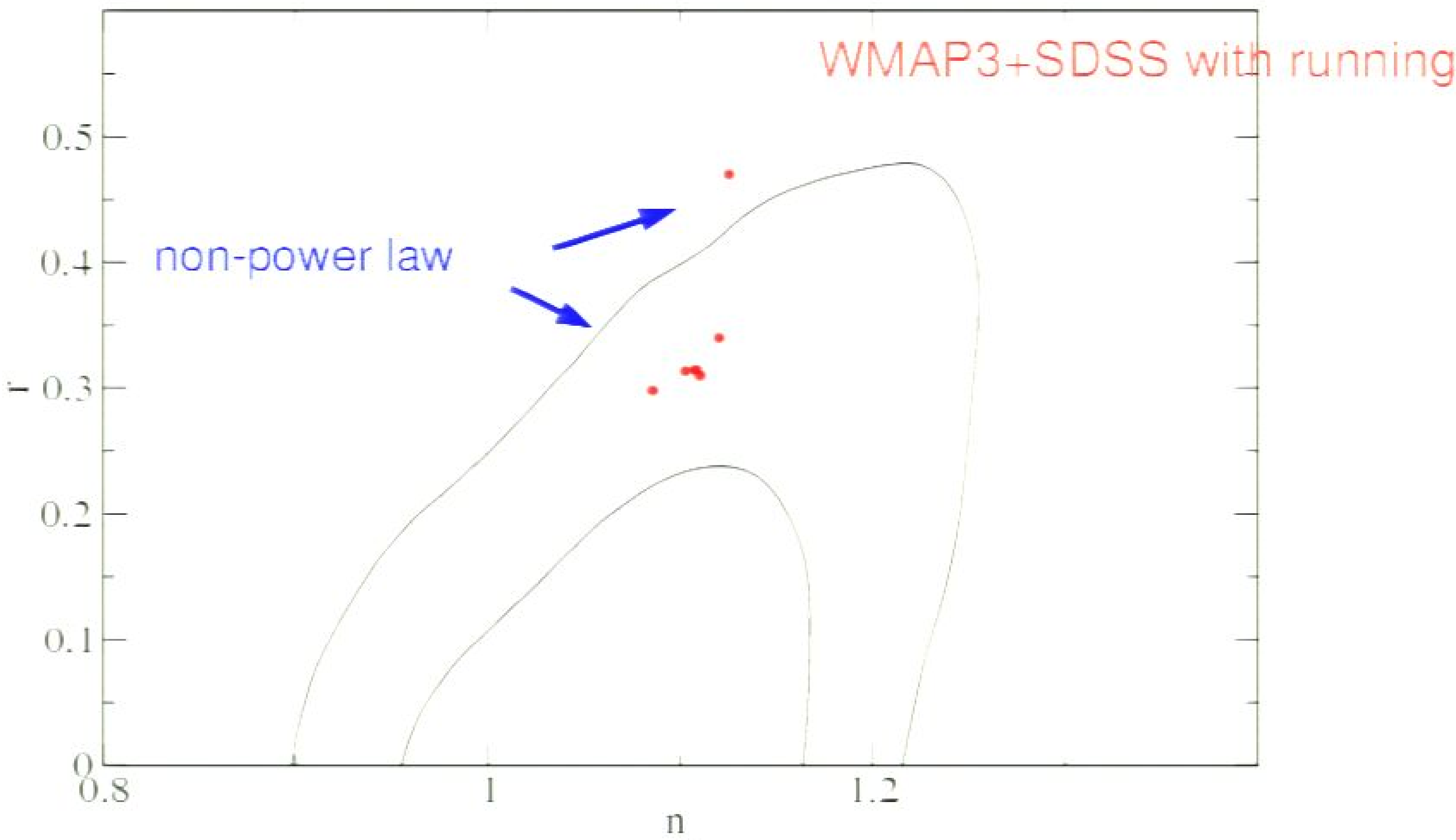

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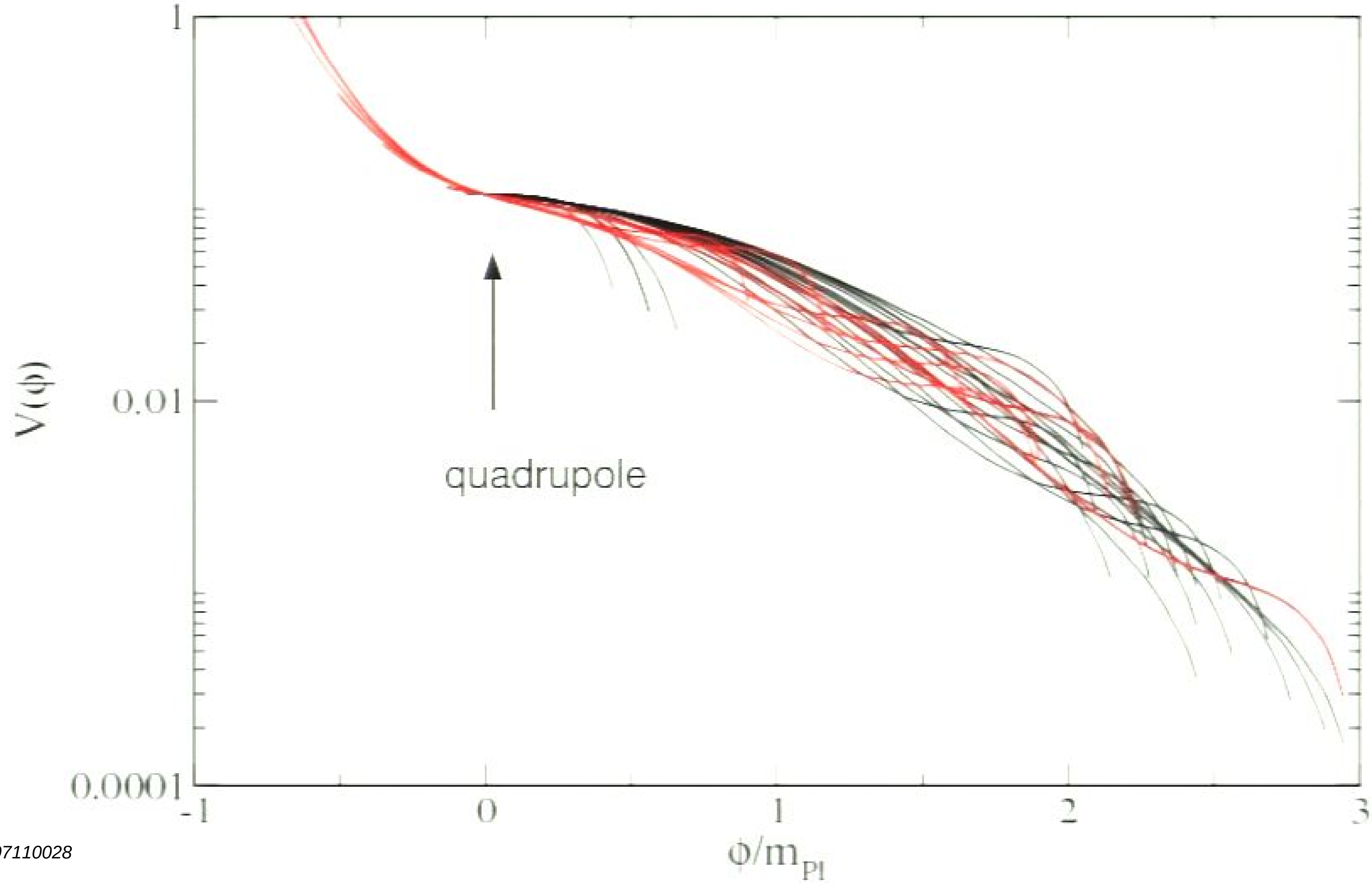
Inflaton Potentials



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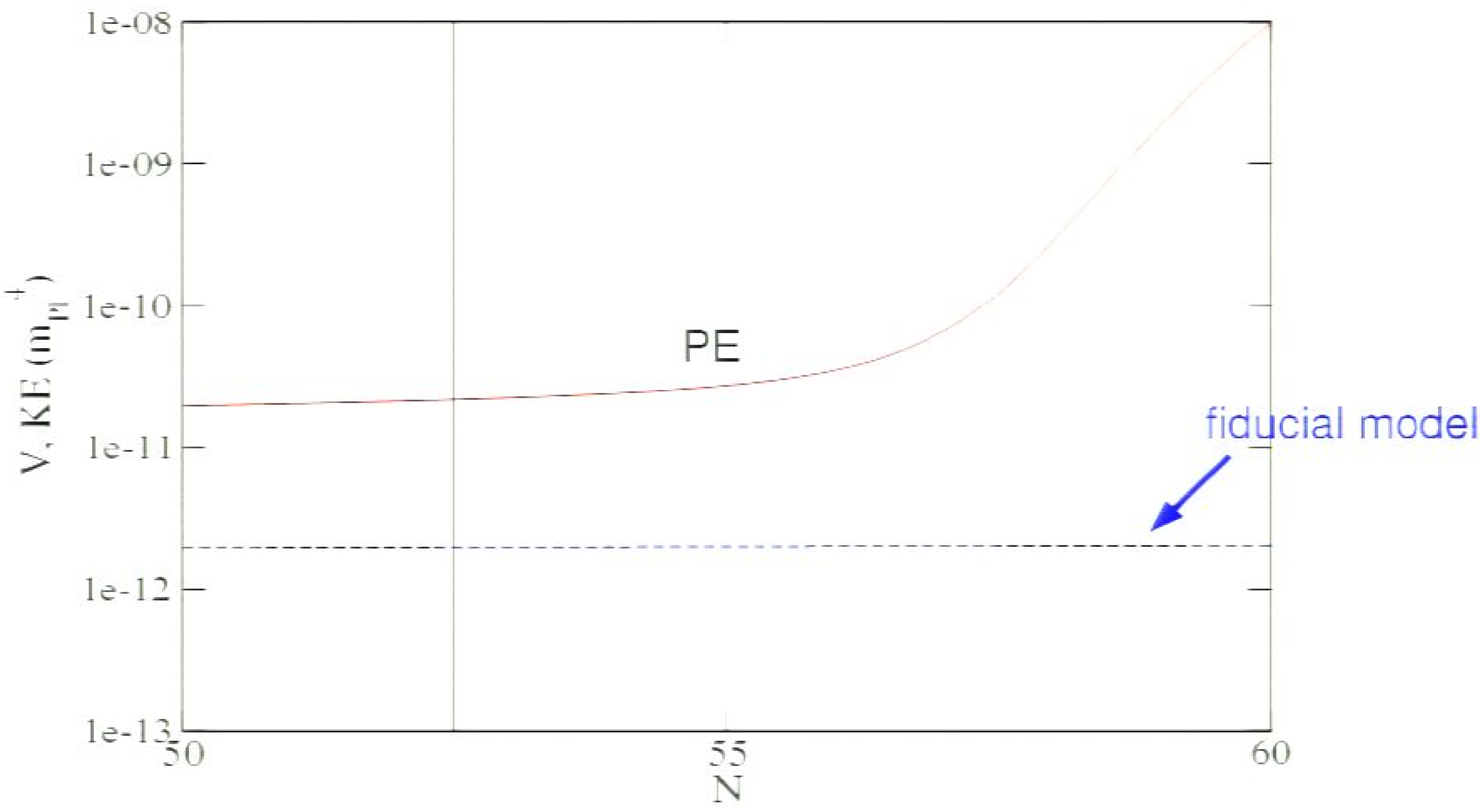

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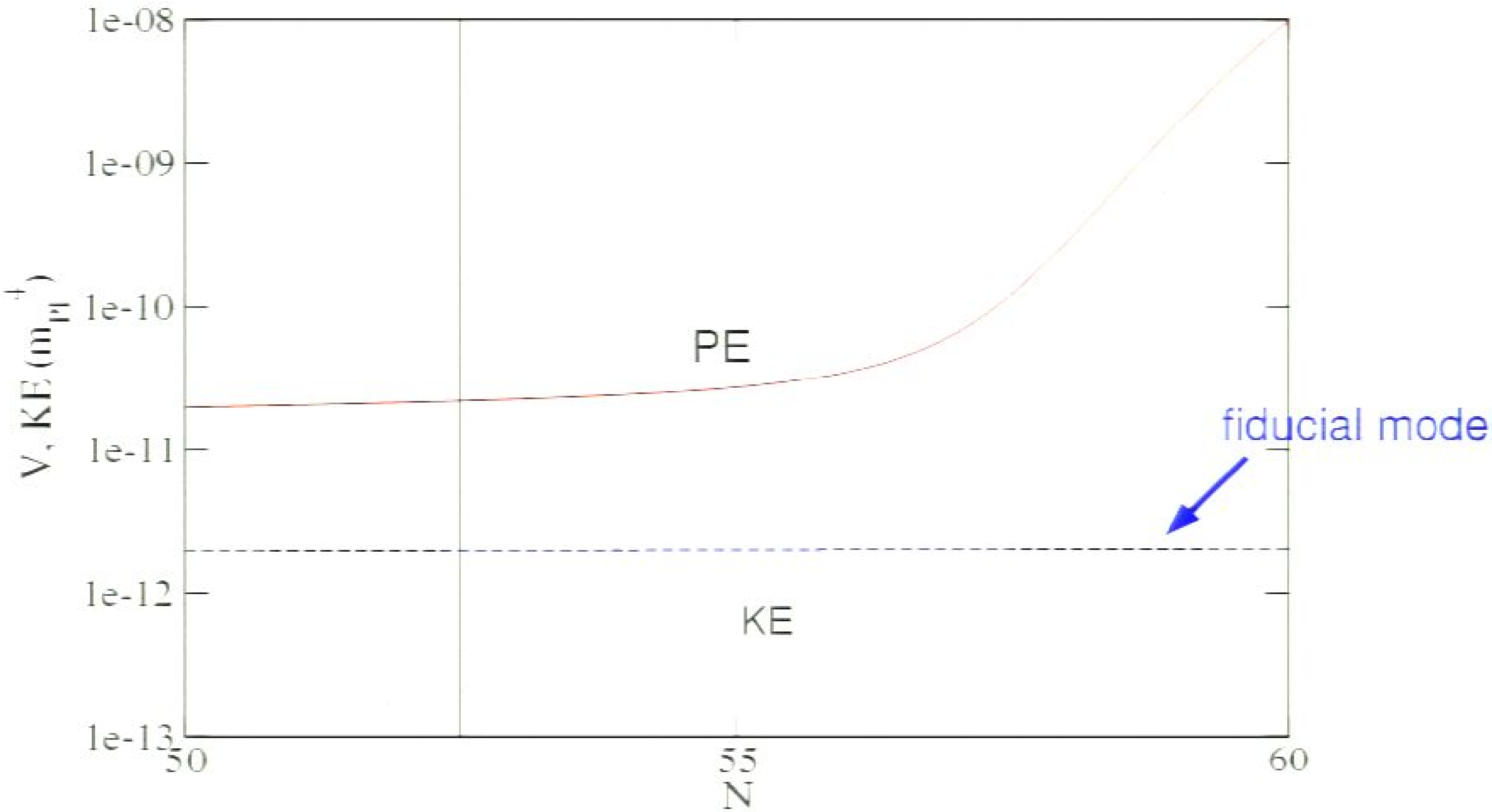

Recap

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- We have identified a class of fast-roll inflation models that agree well with current data.
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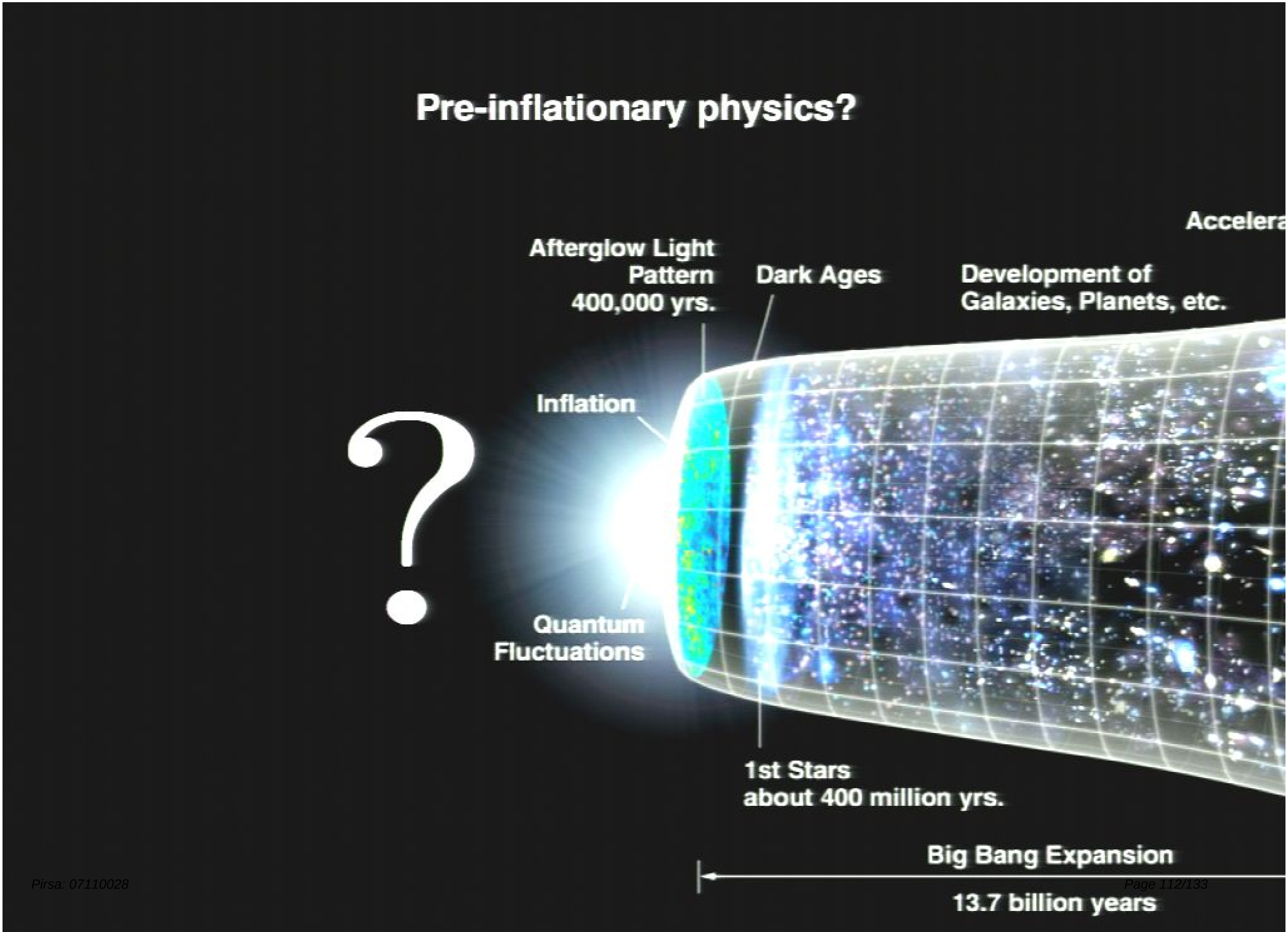


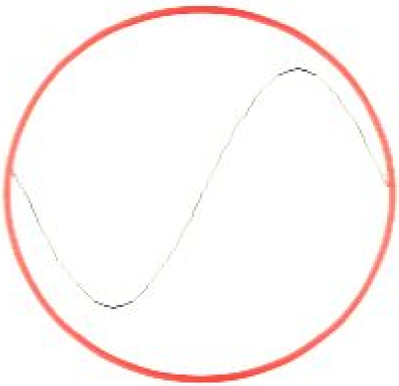
Pre-inflationary physics?
Accelera
Afterglow Light Pattern 400,000 yrs.
Dark Ages
Development of Galaxies, Planets, etc.
Inflation
?
Quantum Fluctuations
1st Stars about 400 million yrs.
Big Bang Expansion
13.7 billion years

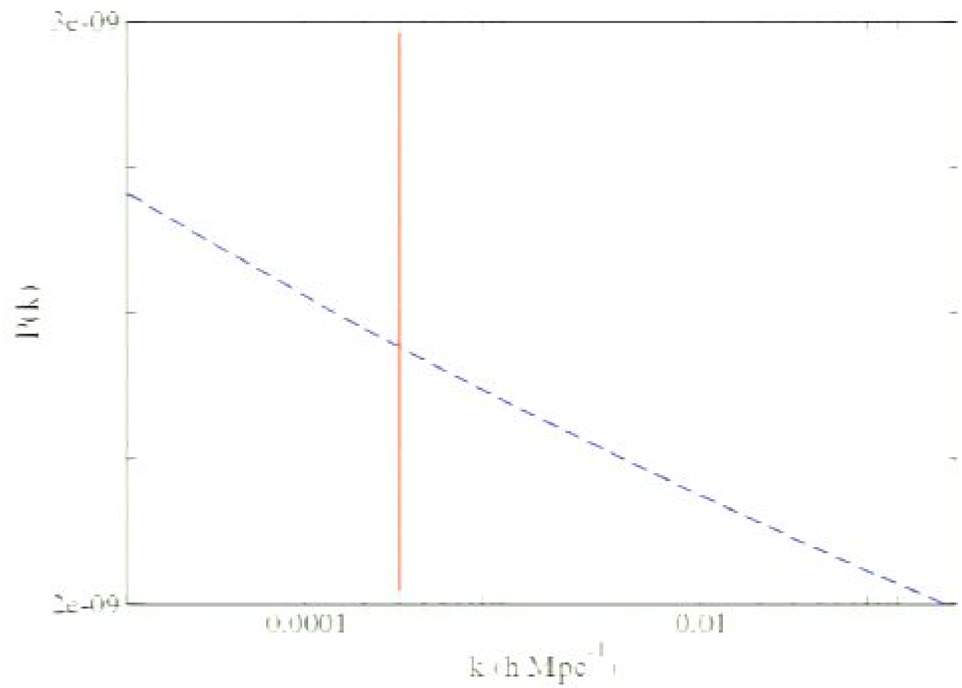
Pre-inflationary physics

- The nature of the universe prior to inflation is unknown: eternal? finite?
- Even if finite, inflation is typically presumed to have lasted many more than the 60 e-folds required to solve the flatness/horizon problems.
- Suppose instead that inflation lasts only just long enough, $N \approx 60$.
- Can we see signs of pre-inflationary physics in the CMB?

Why would we see anything?

$N = 60$





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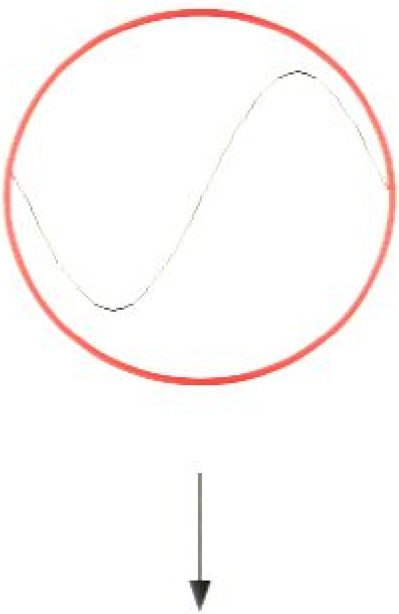
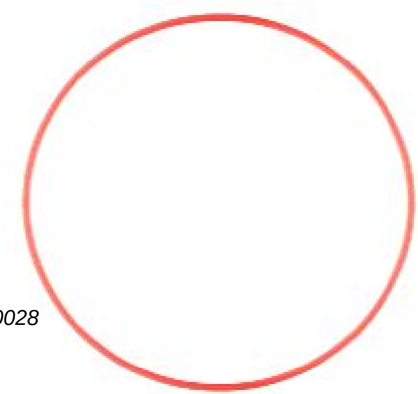
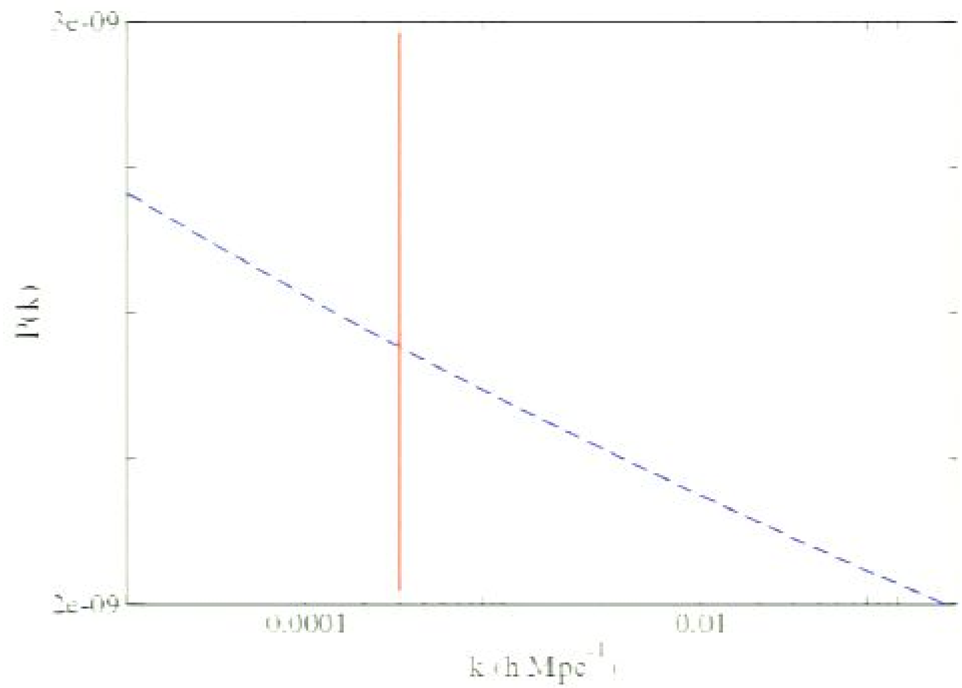


$N > 60$





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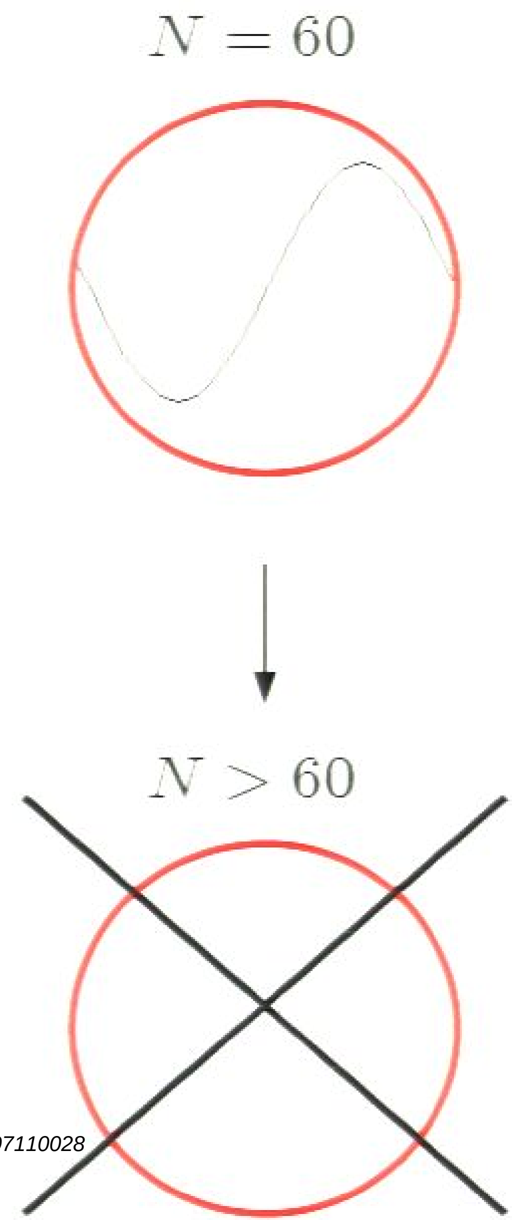

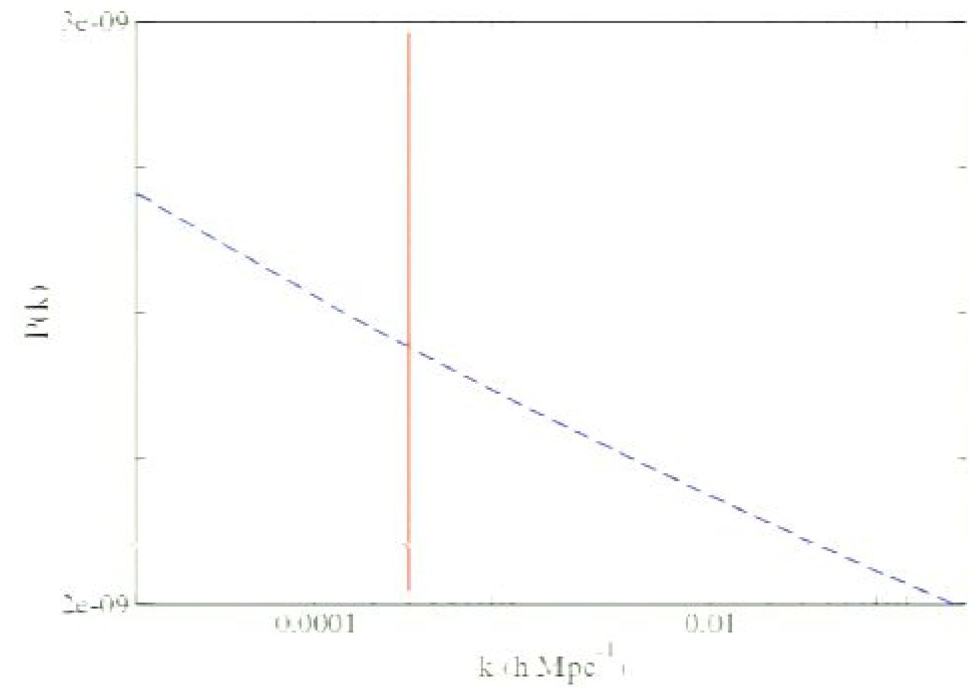

The details of the pre-inflationary phase are necessary!

A pre-inflationary radiation-dominated phase

- We consider a universe consisting of scalar field matter and radiation.

$$a(\tau) \propto \tau \quad (\tau > 0)$$

- How do modes evolve in such a background?

$$y = \frac{k}{aH} = k\tau$$

$$\frac{dy}{d\tau} > 0$$

- Modes on horizon scales at N = 60 were *never* in the UV limit.

Large-scale quantum fluctuations

- We expect to have quantum fluctuations on all length scales, including super-horizon.
- The subsequent evolution of the fluctuations through the RD phase will lead to modified "initial conditions" when inflation begins.
- It can be shown[1] that thermal fluctuations and inflaton fluctuations evolve independently during RD provided that $\epsilon \ll 1$.
- This allows us to write the equation of motion for $u_k = a\delta\phi_k$,

$$u_k'' + k^2 u_k = 0$$

[1] BAP and Kinney, astro-ph/0612006

- The mode never evolves out of the vacuum:

$$u_k = c_1 e^{-ik\tau} + c_2 e^{ik\tau}$$

- What is this vacuum? We know nothing of the nature of the large-scale fluctuations, but we expect that as $k\tau \to 0$, the modes should reduce to the BD limit,

$$u_k = \sqrt{\frac{1}{2k}} e^{-ik\tau}$$

- This can be achieved for

$$c_1 = \sqrt{\frac{1}{2k}}, \; c_2 = 0$$

Radiation-dominated vacuum

- Vacuum choice for RD:

$$u_{k,i} = \sqrt{\frac{1}{2k}} e^{-ik\tau_i}$$

constant time: start of inflation

- After inflation starts, the modes evolve as usual according to

$$u_k'' + \left(k^2 - \frac{z''}{z} \right) u_k = 0$$

- The mode never evolves out of the vacuum:

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Large-scale quantum fluctuations

- We expect to have quantum fluctuations on all length scales, including super-horizon.
- The subsequent evolution of the fluctuations through the RD phase will lead to modified "initial conditions" when inflation begins.
- It can be shown[1] that thermal fluctuations and inflaton fluctuations evolve independently during RD provided that $\epsilon \ll 1$.
- This allows us to write the equation of motion for $u_k = a\delta\phi_k$,

$$u_k'' + k^2 u_k = 0$$

[1] BAP and Kinney, astro-ph/0612006

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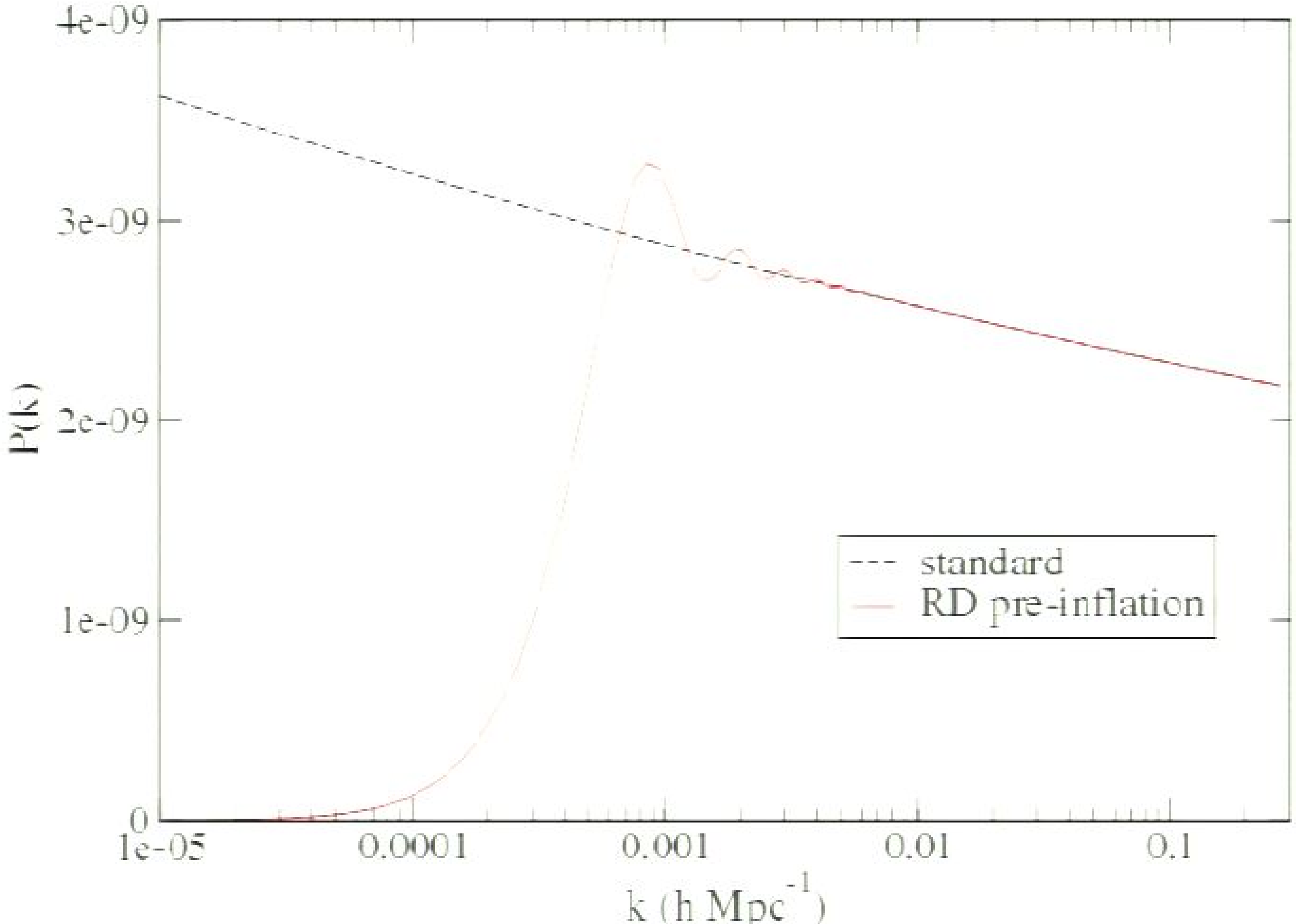
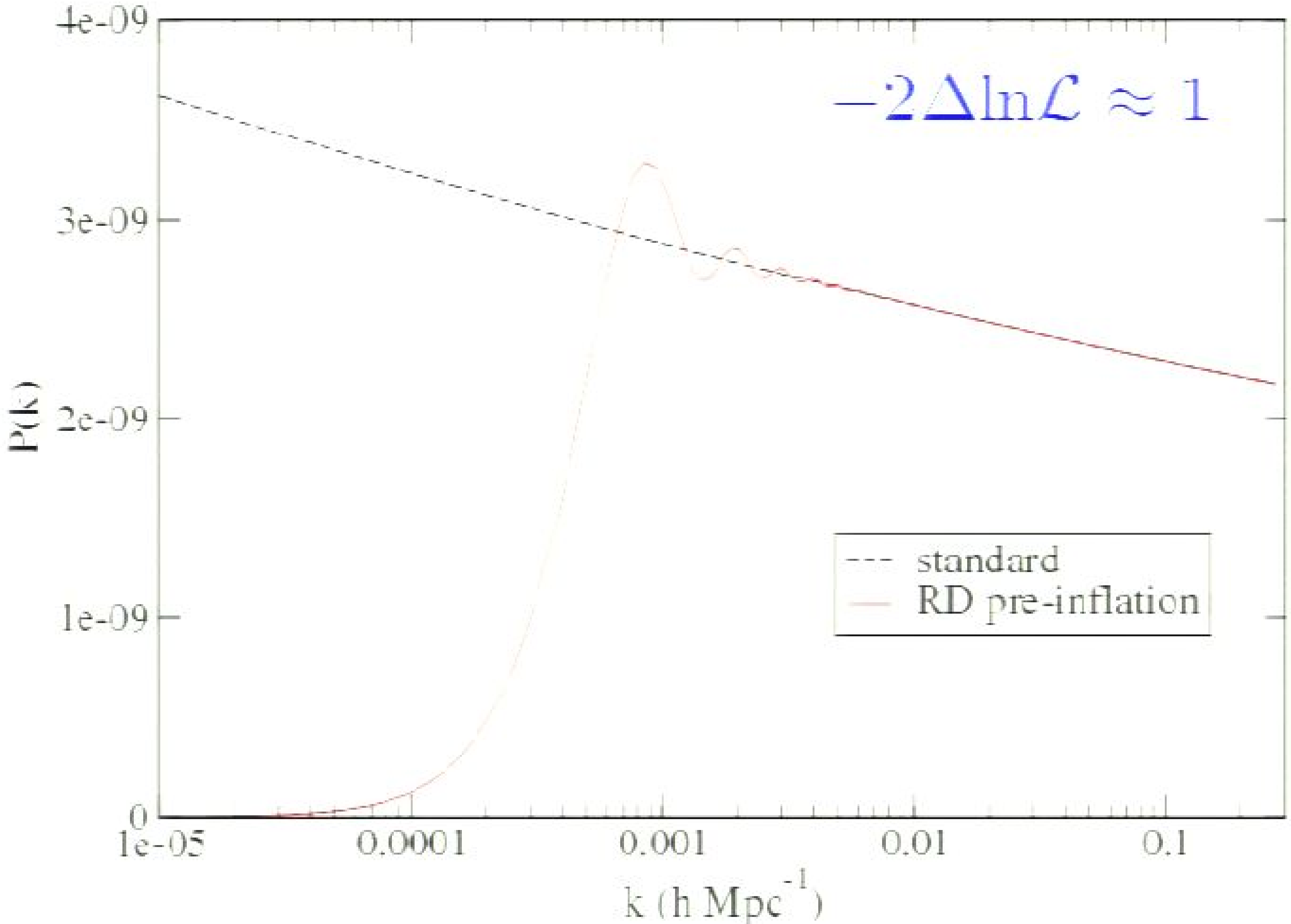
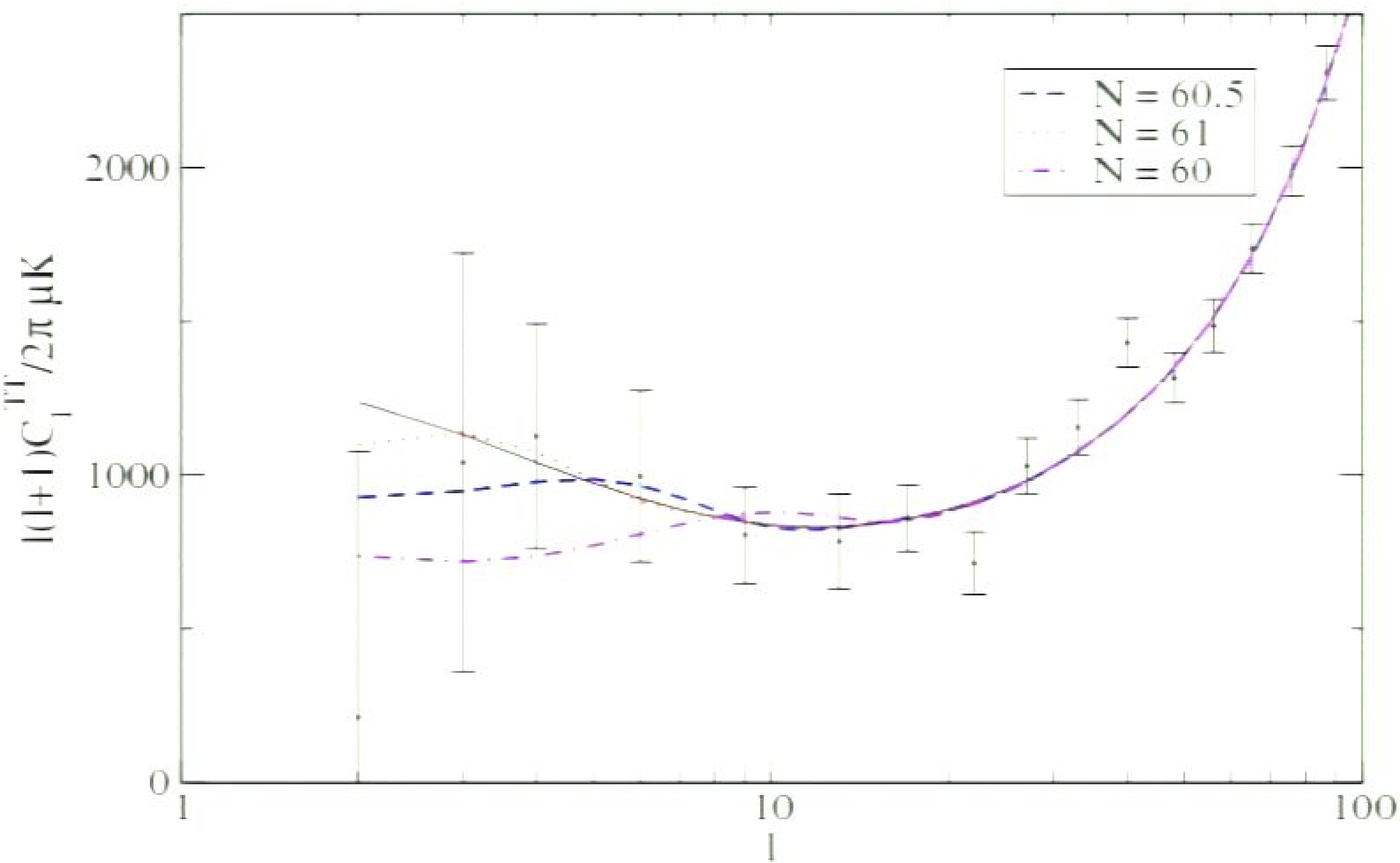
Power spectrum



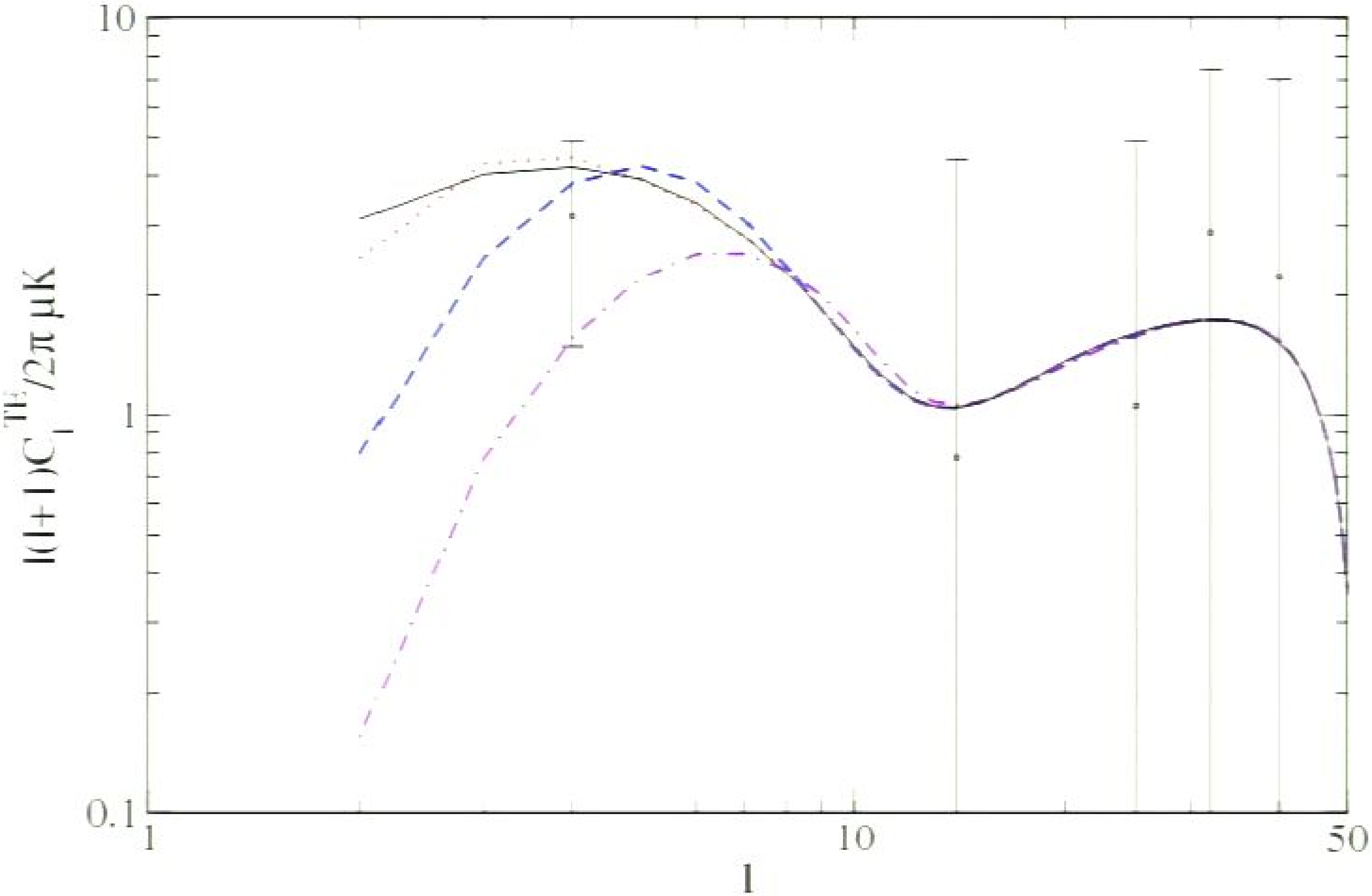
Power spectrum



Temperature (TT) spectrum



TE Spectra



Conclusions

- The spectrum is poorly constrained on the largest angular scales.
- The spectrum is very well constrained on scales

$$0.01h\text{Mpc}^{-1} \leq k \leq 0.1h\text{Mpc}^{-1}$$

- We have obtained varied forms for the power spectrum by considering nontrivial inflaton potentials as well as by modifying the initial conditions of the mode functions.
- While currently degenerate, improved polarization measurements in the near future will allow us to distinguish between these models.

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