Title: Symmetry and global independence in classical and quantum theories

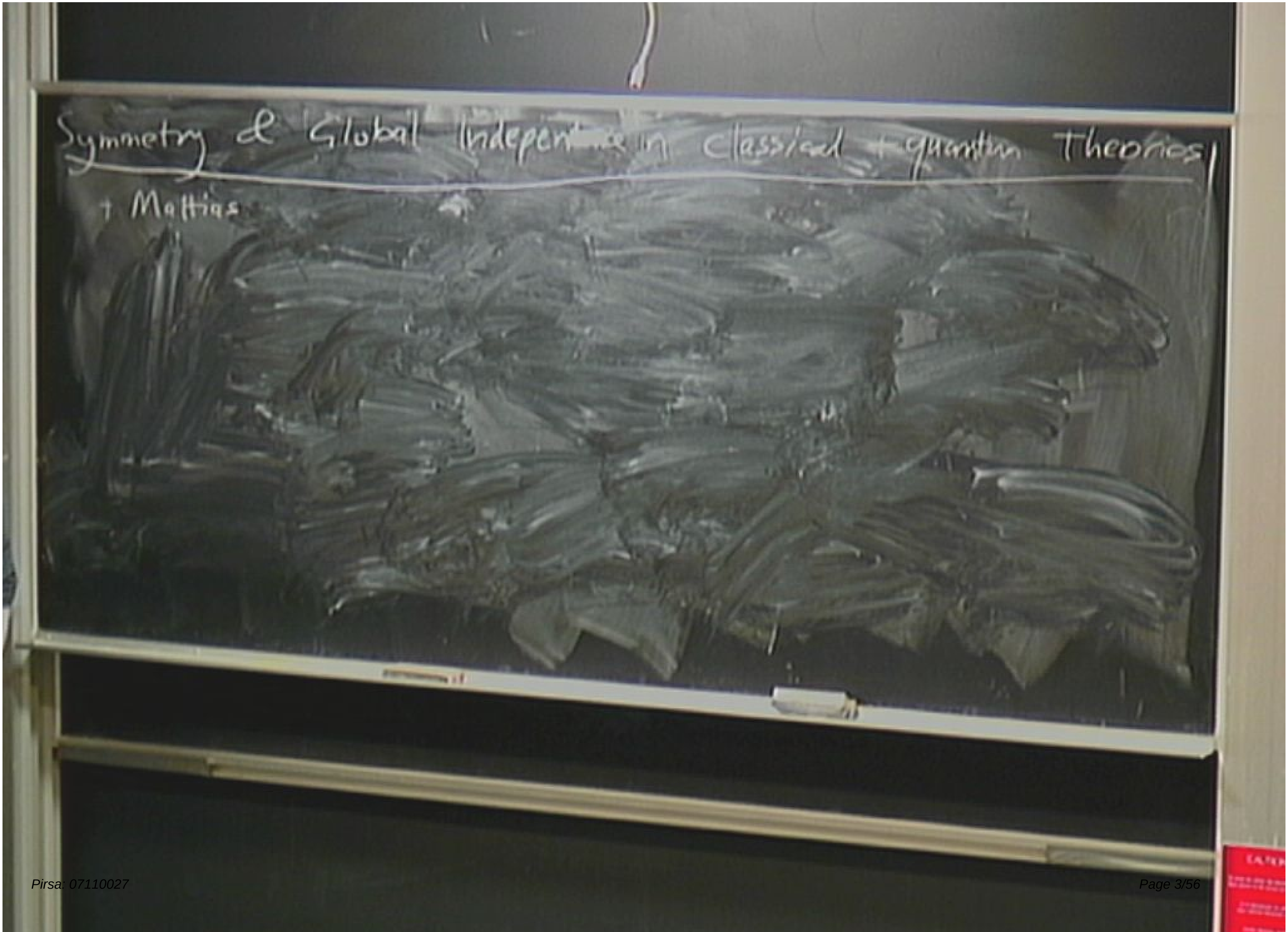
Date: Nov 07, 2007 04:00 PM

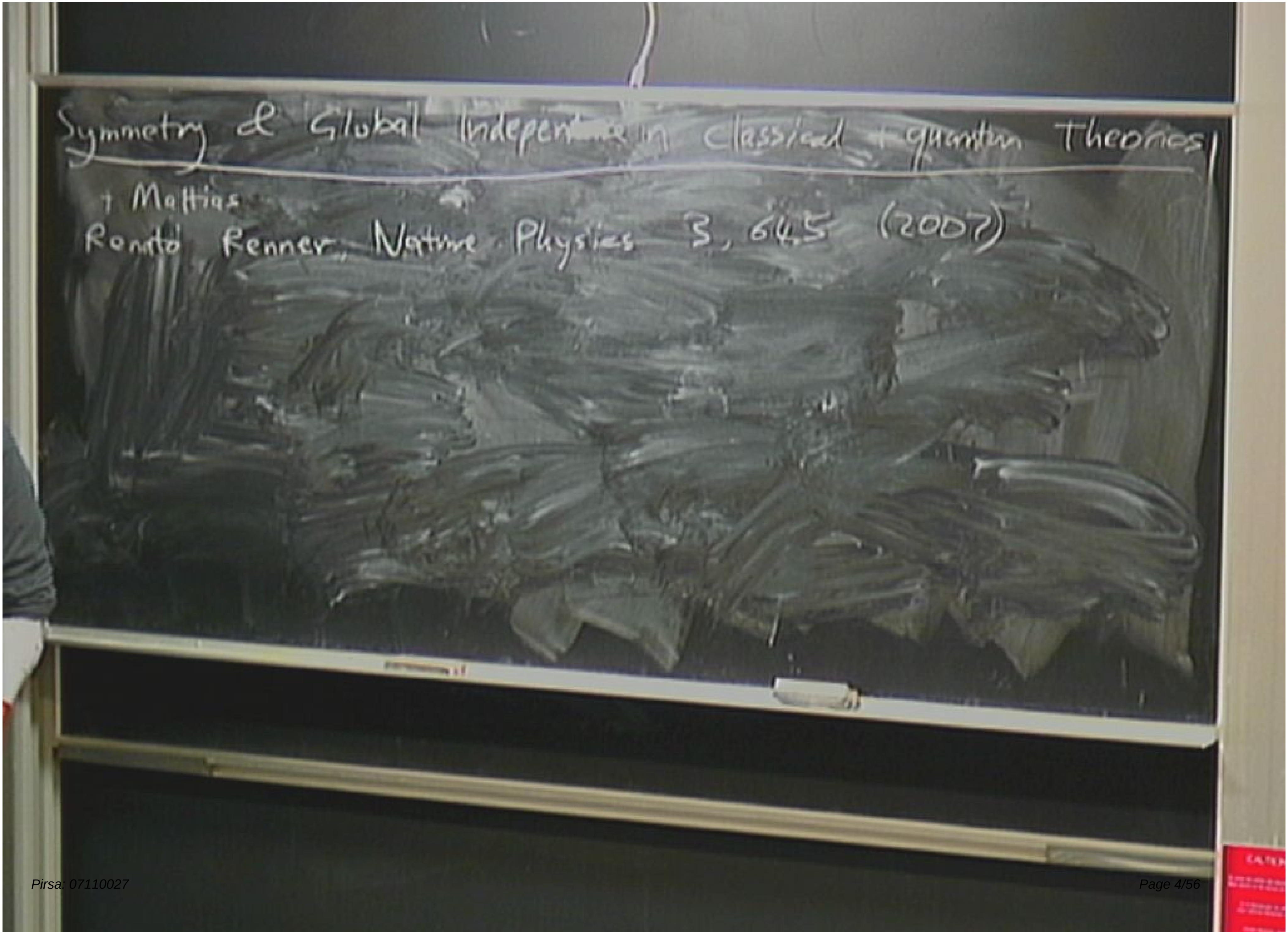
URL: http://pirsa.org/07110027

Abstract: Renner\'s global quantum de Finetti theorem establishes that if the state of a quantum system is invariant under permutations of its systems, then almost all of its subsystems are almost in the same state and independent of each other. Motivated by this result, we show that the most straightforward classical analogue of Renner\'s theorem is false.
Joint work with Matthias Christandl (Cambridge).

Symmetry & Global Independence in Classical + quantum theories

+ Mattias

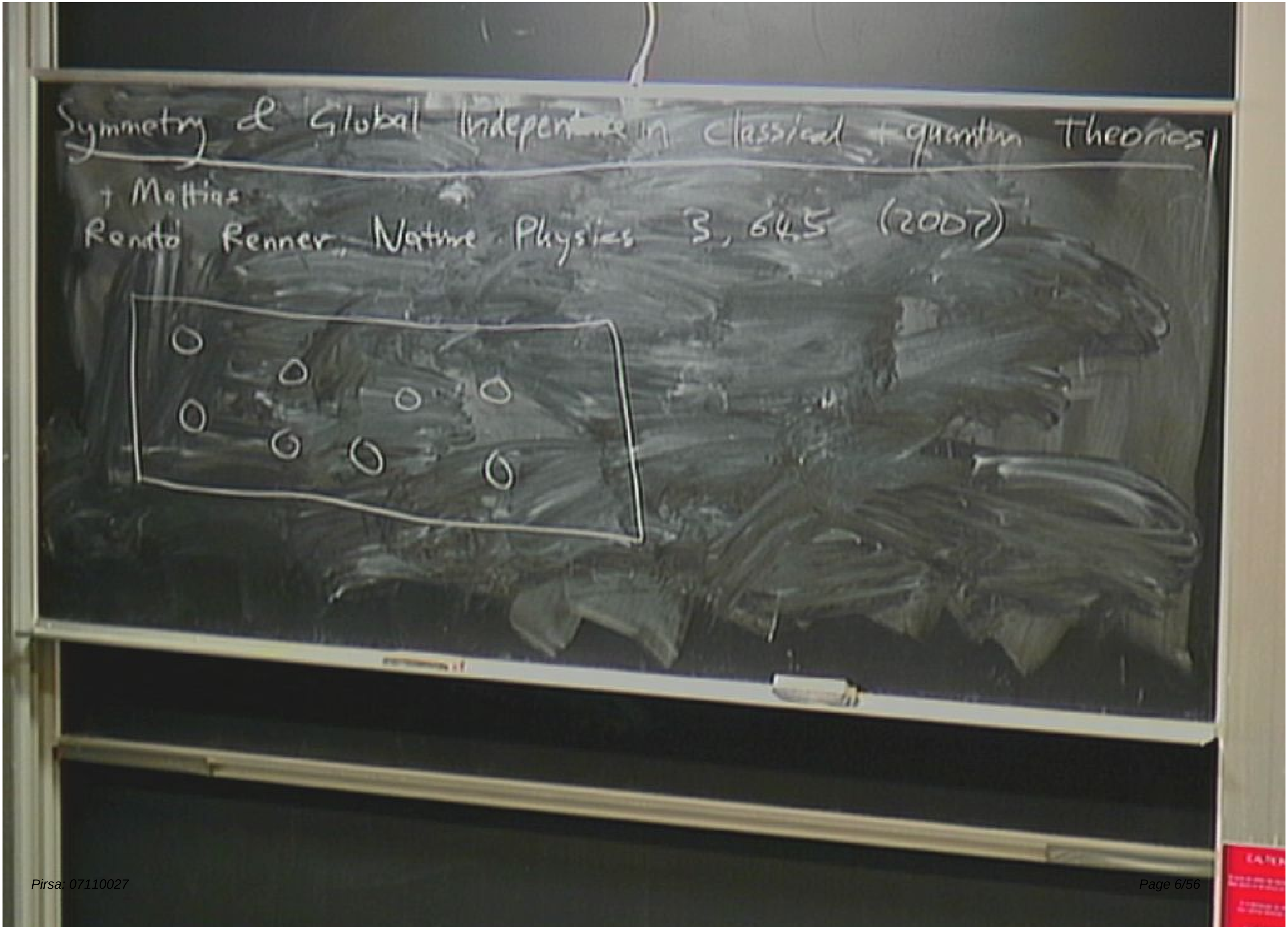

Symmetry & Global Independence in Classical + quantum theories
+ Mattias

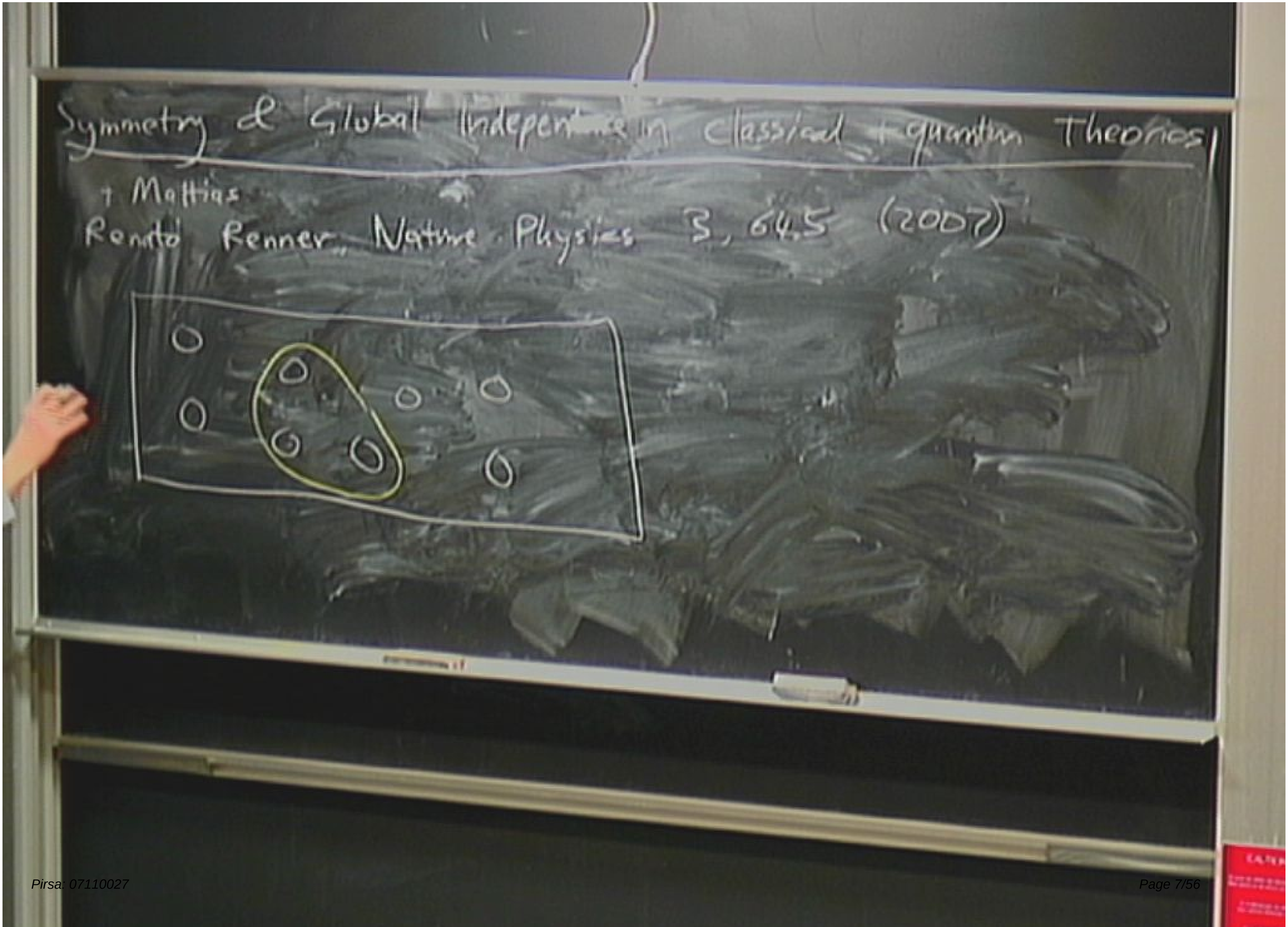

Symmetry & Global Indepen in Classical + quantum theories
+ Mattias
Renato Renner, Nature Physics 3, 645 (2007)

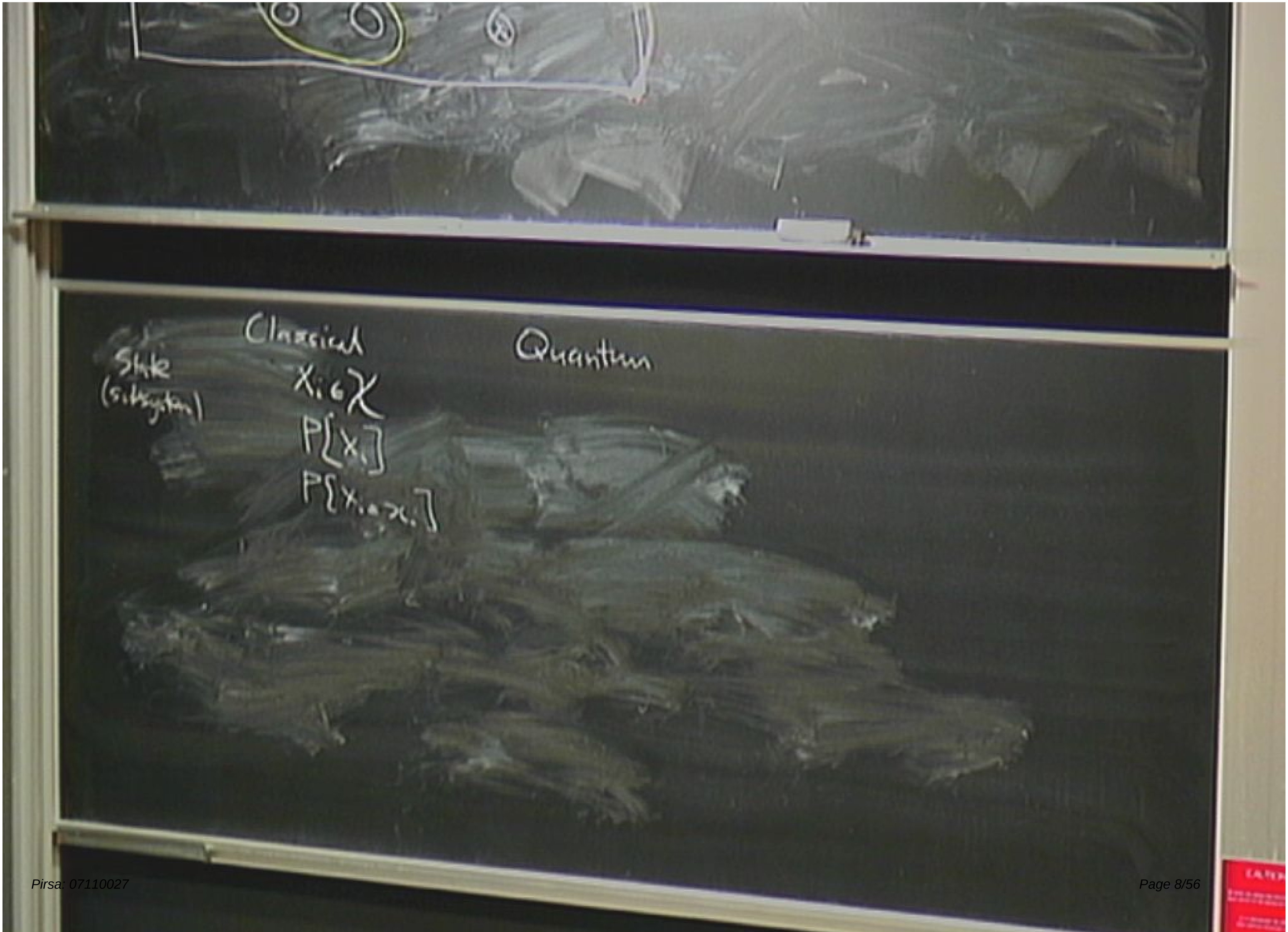
Symmetry & Global Independence in Classical + quantum theories

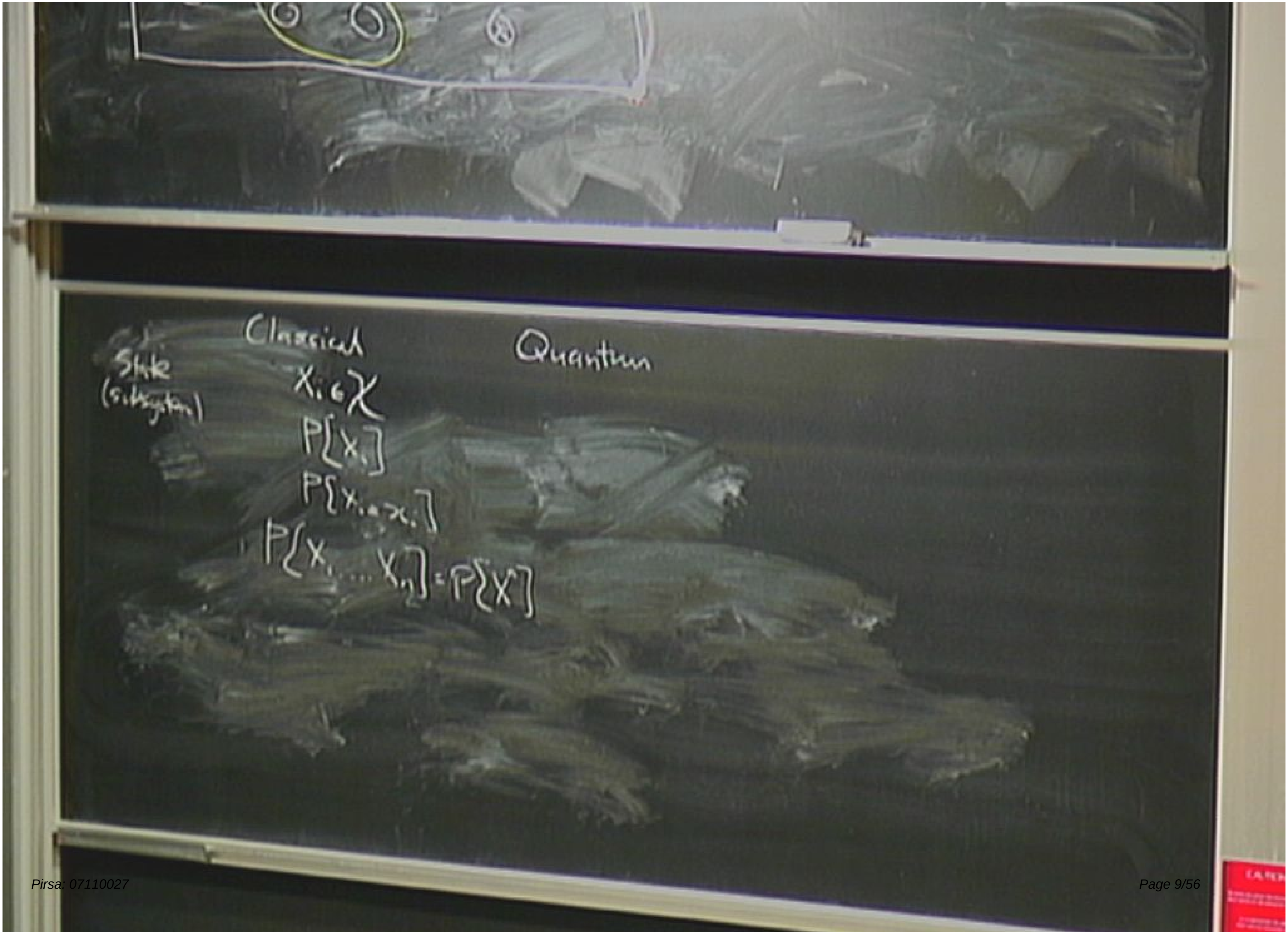
+ Mattias

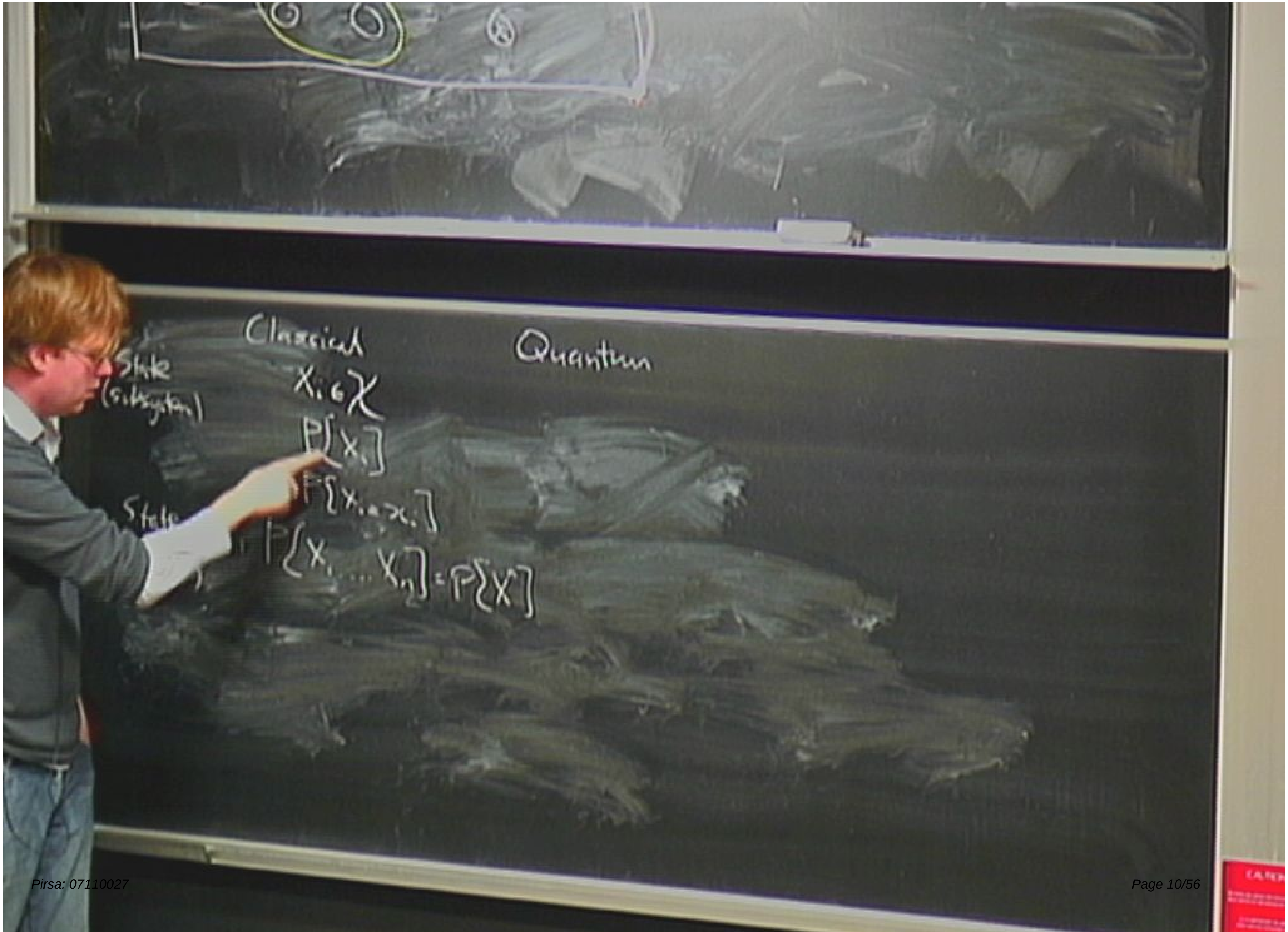
Renato Renner, Nature Physics 3, 645 (2007)

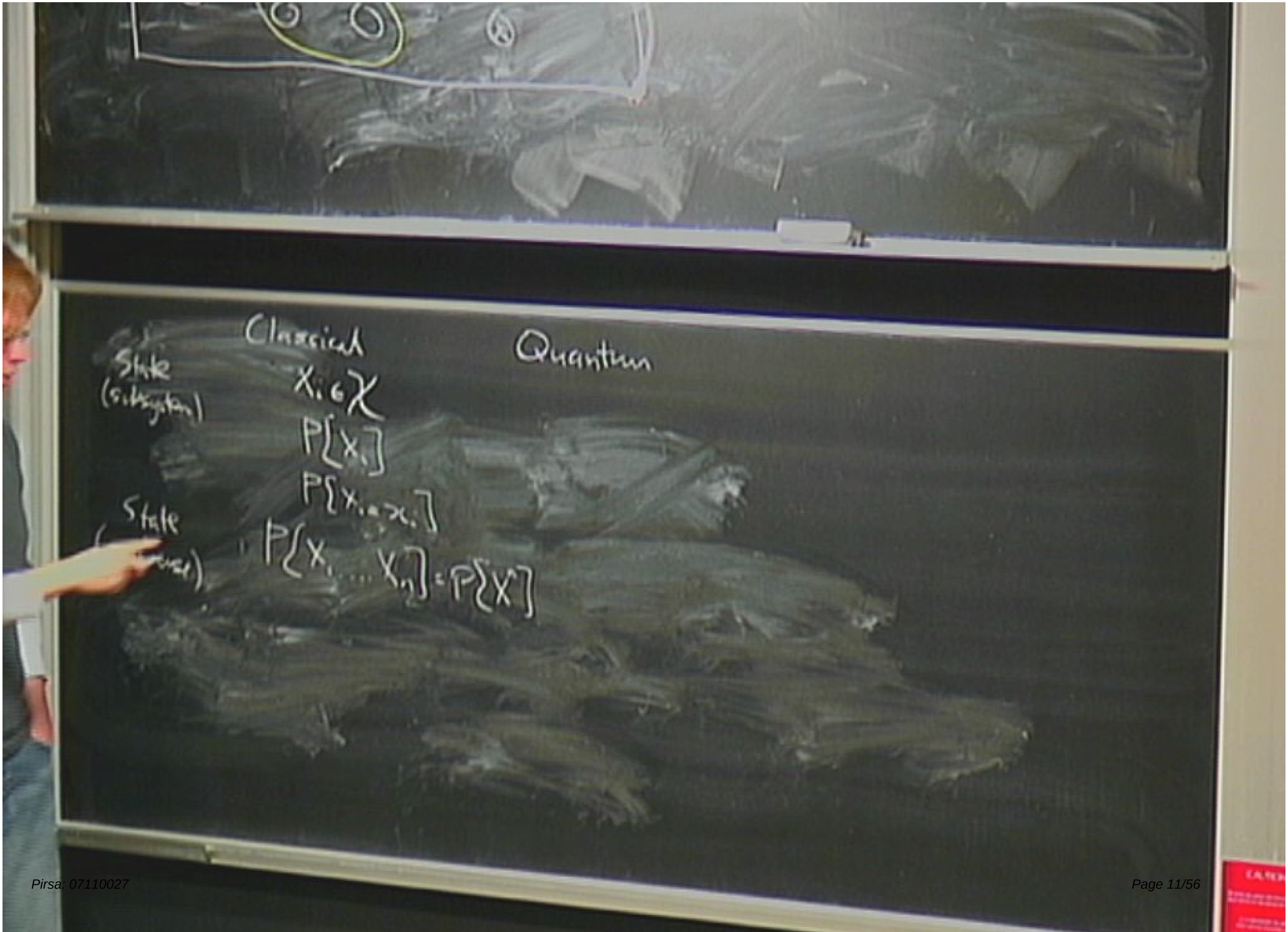

Symmetry & Global Indepen in Classical + quantum Theories
+ Mattias
Renato Renner, Nature Physics 3, 645 (2007)


Symmetry & Global Independence in Classical + quantum Theories
+ Mattias
Renato Renner, Nature Physics 3, 645 (2007)

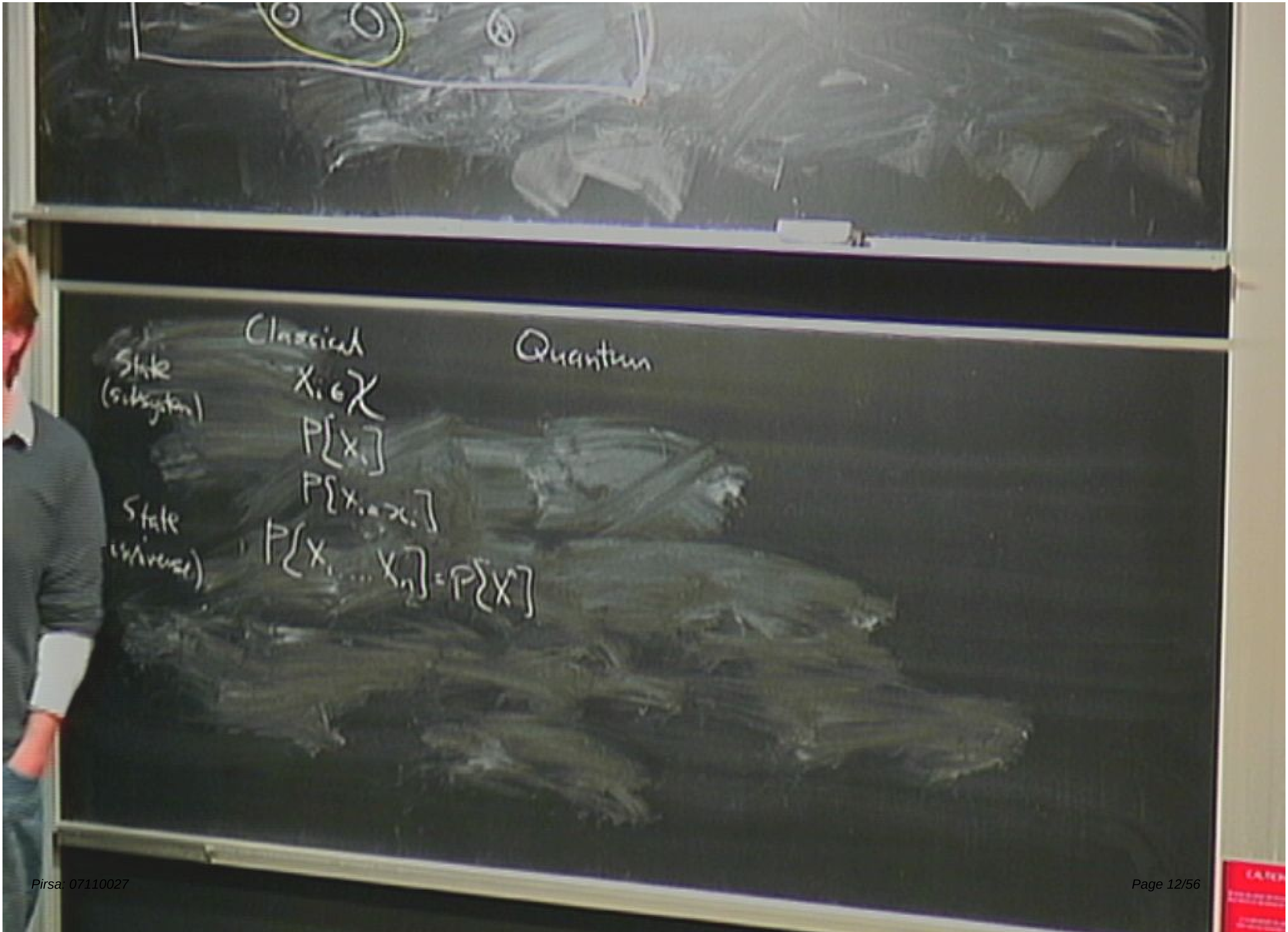

Classical
Quantum
State
(subsystem)
$X_i \in \mathcal{X}$
$P[X_i]$
$P[X_i = x_i]$


Classical
Quantum
State
(subsystem)
$X_i \in \mathcal{X}$
$P[X_i]$
$P[X_i = x_i]$
$P[X_1 \dots X_n] = P[X]$


Classical
Quantum
State



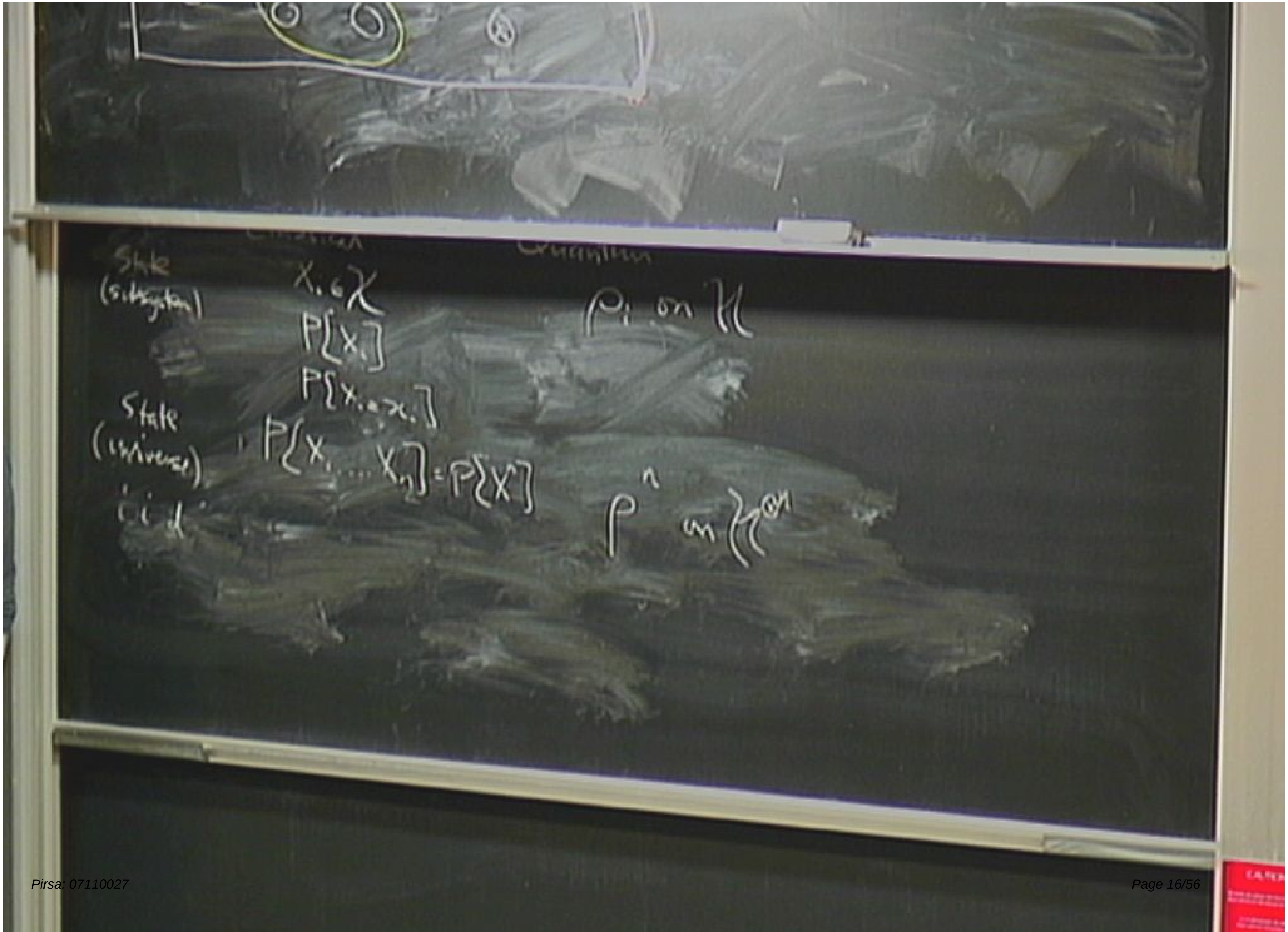
Classical
Quantum
State
(subsystem)
$X_1 \in \mathcal{X}$
$P[X_1]$
$P[X_1 = x_1]$
State
$P[X_1 \ldots X_n] = P[X]$

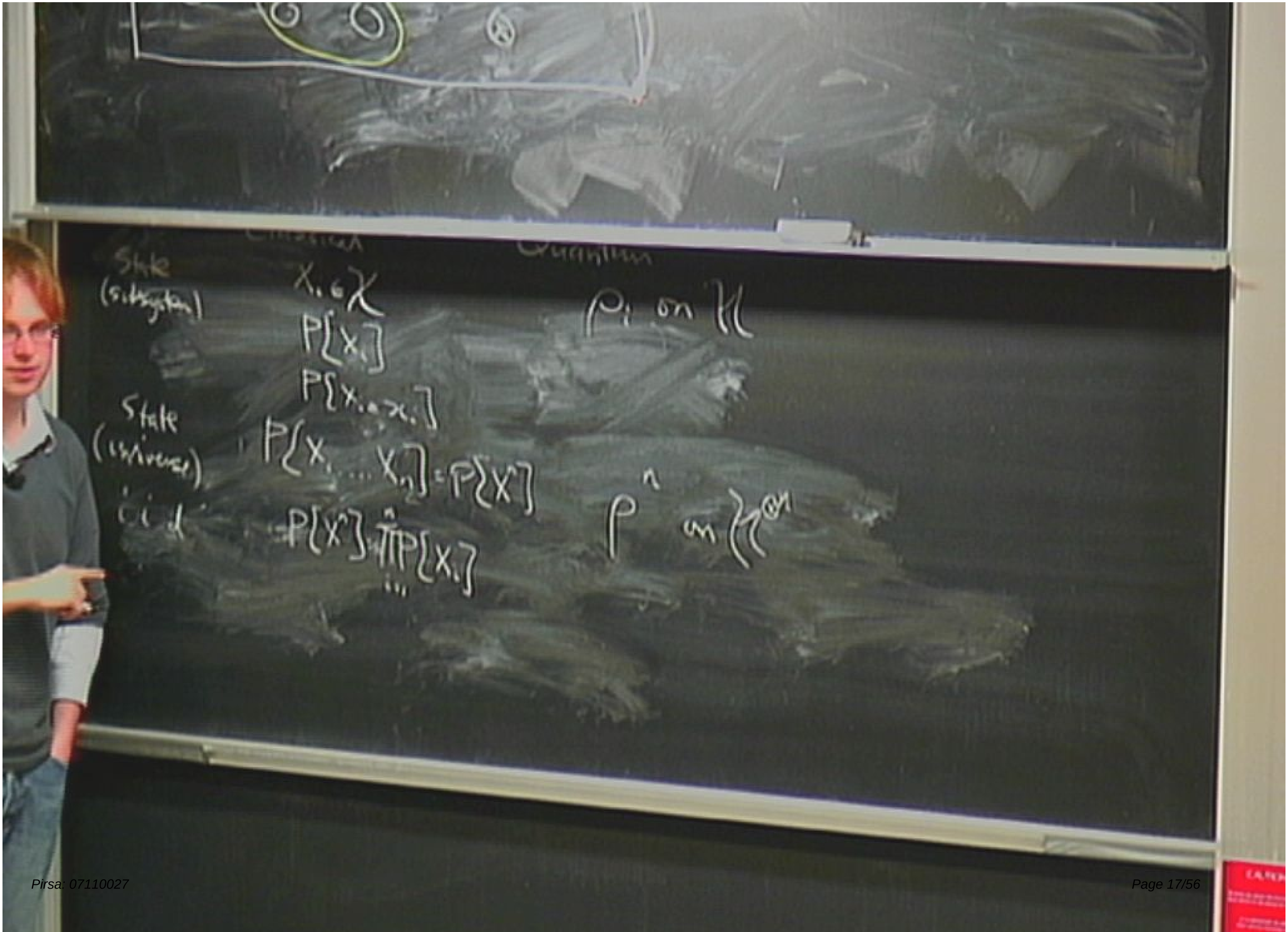

Classical
Quantum
State
(subsystem)
$X_i \in \mathcal{X}$
$P[X_i]$
$P[X_i = x_i]$
State
(universe)
$P[X_1 \ldots X_n] = P[X]$

	Classical	Quantum
State (subsystem)	$X_i \in \mathcal{X}$	ρ
	$P[X_i]$	
	$P[X_i = x_i]$	
State (universe)	$P[X_1 \ldots X_n] = P[X^n]$	ρ^n

	Classical	Quantum
State (subsystem)	$X_i \in \mathcal{X}$	ρ on $\mathcal{H}$
	$P[X_i]$	
	$P[X_i = x_i]$	
State (universe)	$P[X_1 \ldots X_n] = P[X^n]$	ρ^n on $\mathcal{H}^{\otimes n}$

	Classical	Quantum
State (subsystem)	$x_i \in \mathcal{X}$	ρ_i on $\mathcal{H}$
	$P[x_i]$	
	$P[X_i = x_i]$	
State (universe)	$P[X_1 \ldots X_n] = P[\vec{X}]$	ρ^n on $\mathcal{H}^{\otimes n}$


State
(subsystem)
$X_i \in \mathcal{X}$
ρ_i on $\mathcal{H}$
$P[X_i]$
$P[X_i = x_i]$
State
(universe)
$P[X_1 \ldots X_n] = P[X]$
i.i.d.
ρ^n on $\mathcal{H}^{\otimes n}$


Classical
Quantum
State (subsystem)
X_i ∈ 𝒳
P[X_i]
ρ_i on H
P[X_1 … X_i]
State (universe)
P[X_1 … X_n] = P[X^n]
i.i.d.
= P[X^n] = ∏ P[X_i]
ρ^n on H^{⊗n}

	Classical	Quantum
State (subsystem)	$x_i \in \mathcal{X}$	ρ_i on $\mathcal{H}$
	$P[x_i]$	
	$P[x_i, x_j]$	
State (universe)	$P[x_1 \ldots x_n] = P[\vec{x}]$	ρ^n on $\mathcal{H}^{\otimes n}$
i.i.d.	$P[\vec{x}] = \prod_{i=1}^{n} P[x_i]$	$\rho^n = \rho^{\otimes n}$

	Classical	Quantum
State (subsystem)	$X_i \in \mathcal{X}$	ρ_i on $\mathcal{H}$
	$P[X_i]$	
	$P[X_i = x_i]$	
State (universe)	$P[X_1 \ldots X_n] = P[X^n]$	ρ^n on $\mathcal{H}^{\otimes n}$
i.i.d.	$P[X^n] = P[X]^n$	$\rho^n = \rho^{\otimes n}$

(subsystem)

$P[x_1]$

$P[x_1 \ldots x_1]$

State (universe)

$P[X_1 \ldots X_n] = P[X^n]$ $\quad \rho^n$

i.i.d. $\quad P[X^n] = P[X]^n \quad \rho^n = \rho^{\otimes n}$

symmetric

$P[X_1 \ldots X_n] = P[X_{\sigma(1)} \ldots X_{\sigma(n)}] \quad \rho^n = \rho^{\otimes n}$

$\forall \sigma \in S_n$

(subsystem)

$P[X_1]$

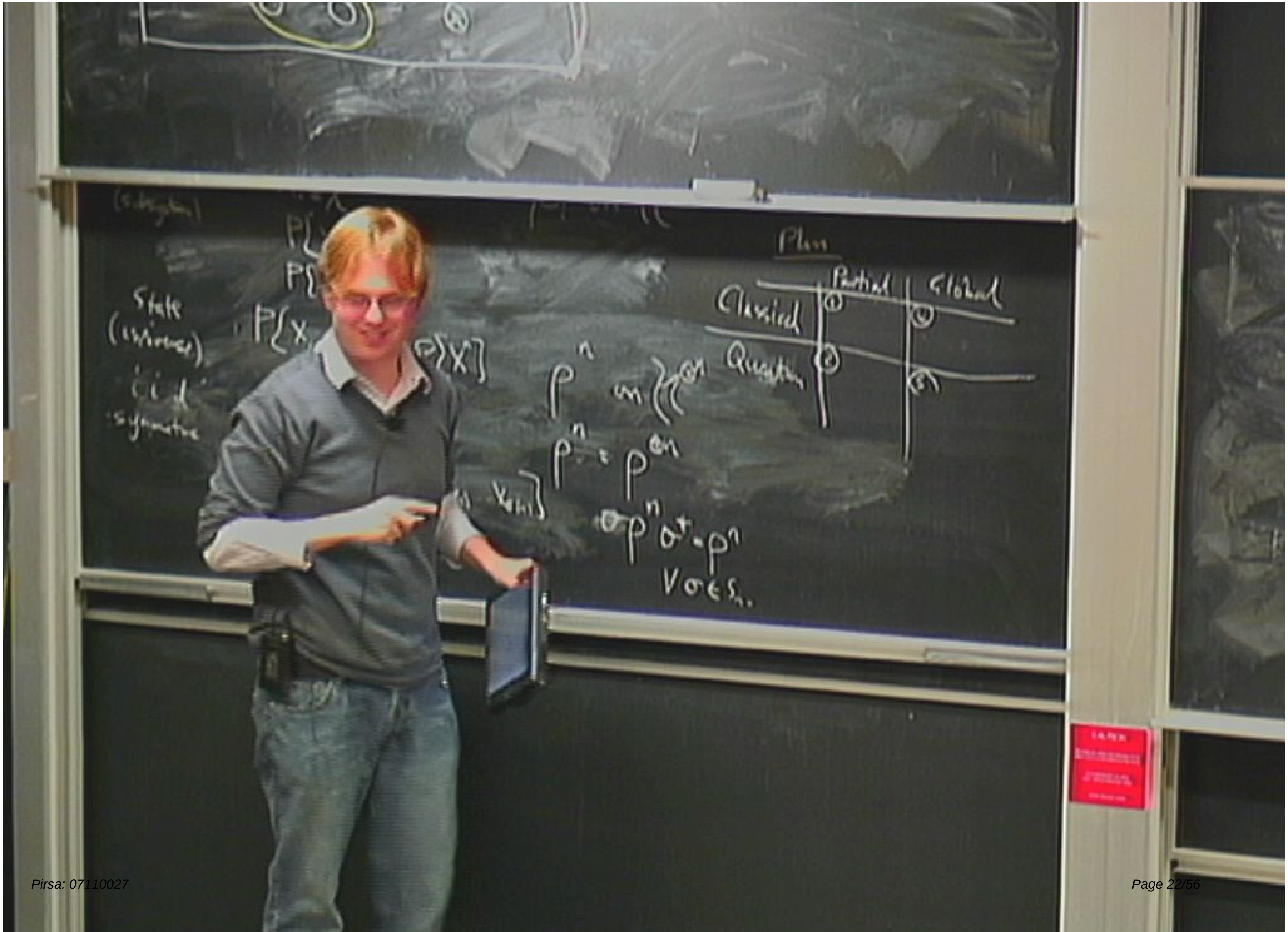
$P[X_1 \ldots X_n]$

State (universe) $\quad P[X_1 \ldots X_n] = P[X^n] \qquad \rho^n$

i.i.d. $\quad P[X^n] = P[X]^n \qquad \rho^{\otimes n}$

symmetric $\quad P[X_1 \ldots X_n] = P[X_{\sigma(1)} \ldots X_{\sigma(n)}] \quad \forall \sigma \in S_n \qquad \rho^n = \rho^{\otimes n}$

$\sigma \rho^n \sigma^\dagger = \rho^n \quad \forall \sigma \in S_n.$


Plan
Partial
Global
Classical
Quantum
State
i.i.d.
symmetric

Th: (Diaconis & Freedman, 1980)

Suppose $P[X^n]$ sym.

Th: (Diaconis & Freedman, 1980)

Suppose $P[X^n]$ sym.

$\exists\ \mu(\lambda)$

Th: (Diaconis & Freedman, 1980)

Suppose $P[X^n]$ sym.

$\exists\ \mu(\lambda)$

$$\left\| P[X^k] - \int d\lambda\, \mu(\lambda)\, P_\lambda[X]^k \right\|_1 \leq \min\left(\frac{2k|\chi|}{n}, \frac{k(k-1)}{n} \right)$$

Th: (Diaconis & Freedman, 1980)

Suppose $P[X^n]$ sym.

$\exists\ \mu(\lambda)$

$$\left\| P[X^k] - \int d\lambda\, \mu(\lambda)\, P_\lambda[X]^k \right\|_1 \leq \min\left(\frac{2k|\mathcal{X}|}{n}, \frac{k(k-1)}{n} \right)$$

$\|P(x)\| = \sum_x |P(x)|$

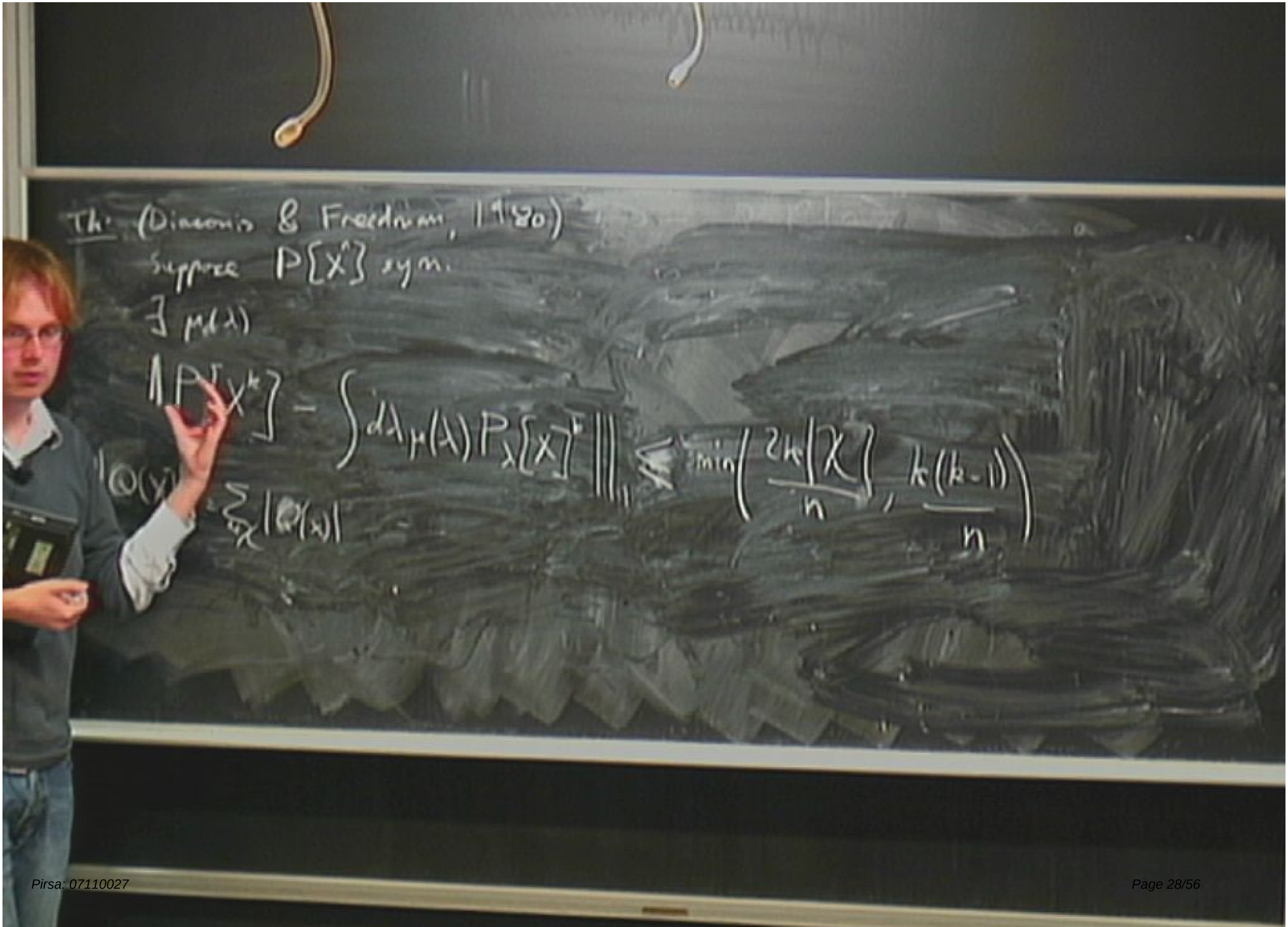
Th: (Diaconis & Freedman, 1980)

Suppose $P[X^n]$ sym.

$\exists\ \mu(\lambda)$

$$\left\| P[X^k] - \int d\lambda\, \mu(\lambda)\, P_\lambda[X]^k \right\|_1 \leq \min\left(\frac{2k|X|}{n}, \frac{k(k-1)}{n} \right)$$

$\|Q(x)\|_1 = \sum_x |Q(x)|$


Thm (Diaconis & Freedman, 1980)
Suppose $P[X^n]$ sym.
$\exists\, \mu(\lambda)$
$\| P[X^k] - \int d\lambda\, \mu(\lambda)\, P_\lambda[X]^k \|_1 \leq \min\left(\frac{2k|X|}{n}, \frac{k(k-1)}{n} \right)$
$\| Q(x) \|_1 = \sum_x |Q(x)|$

Th: (Diaconis & Freedman, 1980)

Suppose $P[X^n]$ sym.

$\exists\ \mu(\lambda)$

$$\left\| P[X^k] - \int d\lambda\, \mu(\lambda)\, P_\lambda[X]^k \right\|_1 \leq \min\left(\frac{2k|\mathcal{X}|}{n}, \frac{k(k-1)}{n} \right)$$

$$\|Q(x)\|_1 = \sum_x |Q(x)|$$

Thm (Diaconis & Freedman, 1980)

Suppose $P[X^n]$ sym.

$\exists\ \mu(\lambda)$

$$\left\| P[X^k] - \int d\lambda\, \mu(\lambda)\, P_\lambda[X]^k \right\|_1 \leq \min\left(\frac{2k|X|}{n}, \frac{k(k-1)}{n} \right)$$

$\|Q\|_1 = \sum_x |Q(x)|$

$\frac{ck}{n}$

Th: (Diaconis & Freedman, 1980)

Suppose $P[X^n]$ sym.

$\forall k \ \exists \mu(\lambda)$

$$\left\| P[X^k] - \int d\lambda \, \mu(\lambda) P_\lambda[X]^{k} \right\|_1 \leq \min\left(\frac{2k|X|}{n}, \frac{k(k-1)}{n} \right)$$

$$\frac{ck}{n}$$

$$\|Q(x)\|_1 = \sum_x |Q(x)|$$

Th: (Diaconis & Freedman, 1980)

Suppose $P[X^n]$ sym.

$\forall k \ \exists \mu(\lambda)$

$$\left\| P[X^k] - \int d\lambda\, \mu(\lambda)\, P_\lambda[X]^k \right\|_1 \leq \min\left(\frac{2k|\chi|}{n}, \frac{k(k-1)}{n} \right)$$

$$\frac{ck}{n}$$

$$\|Q(x)\|_1 = \sum_x |Q(x)|$$

Th: (Diaconis & Freedman, 1980)

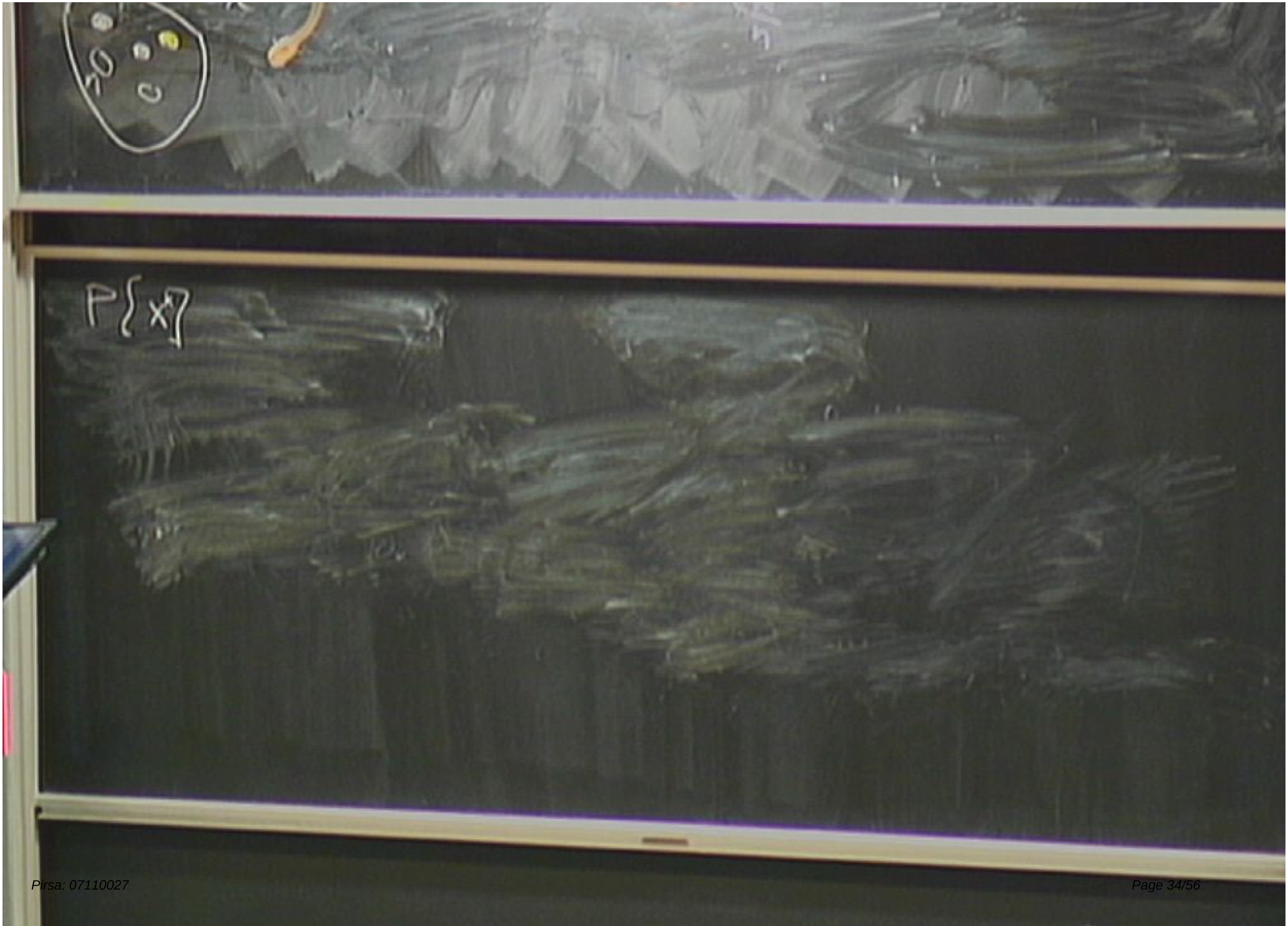
Suppose $P[X^n]$ sym.

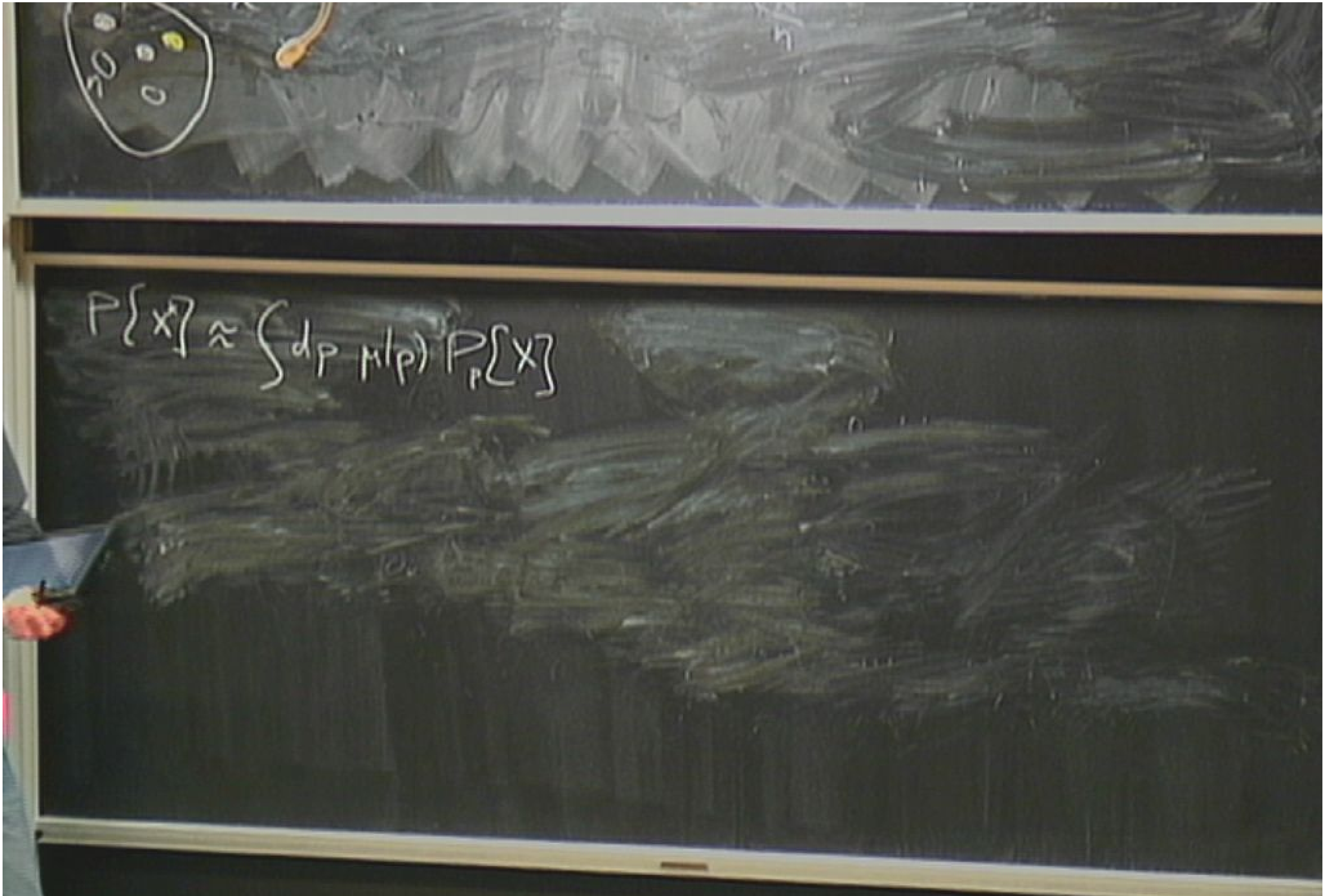
$\forall k \ \exists \mu(\lambda)$

$$\left\| P[X^k] - \int d\lambda \, \mu(\lambda) P_\lambda[x]^k \right\|_1 \leq \min\left(\frac{2k|\chi|}{n}, \frac{k(k-1)}{n} \right)$$

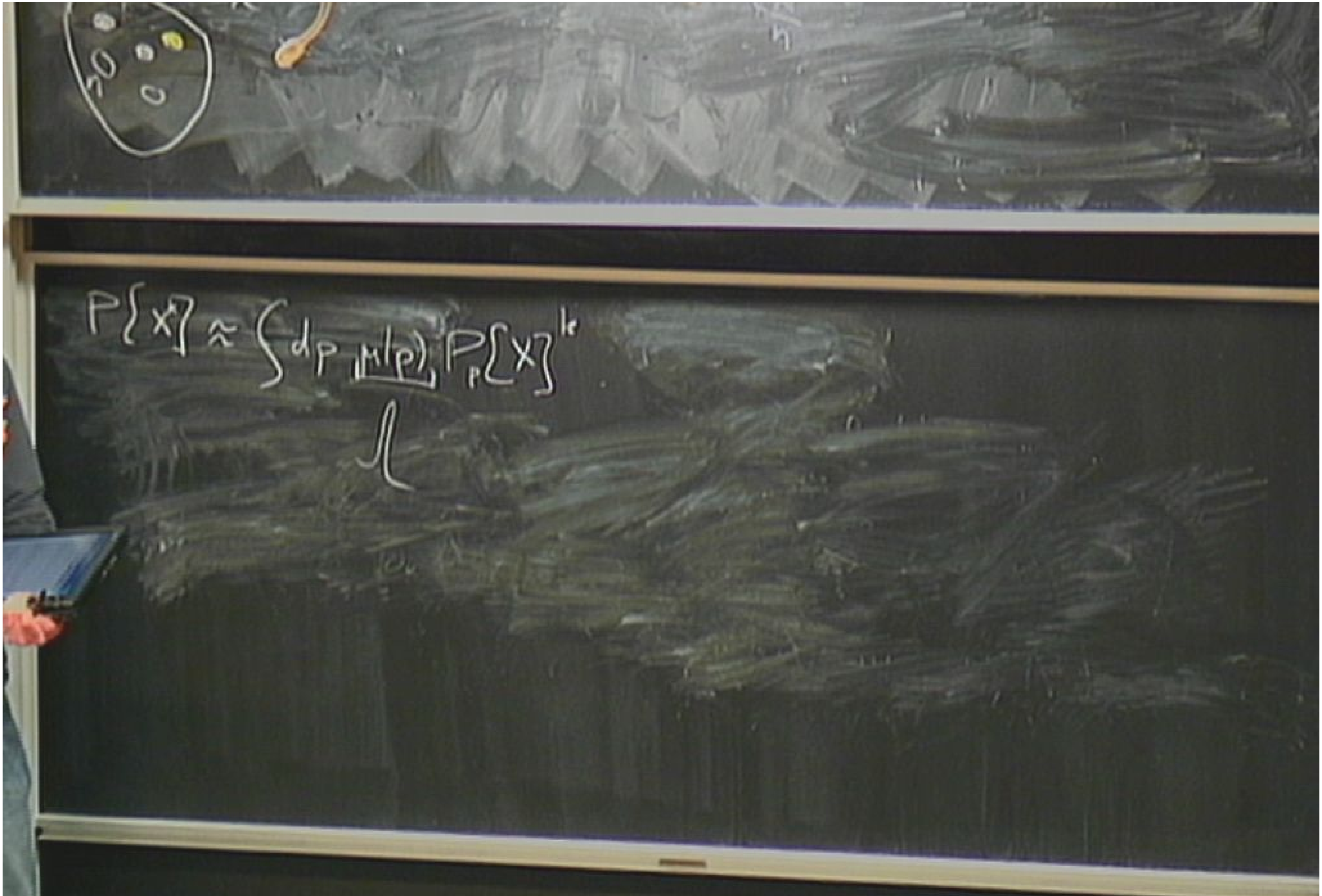
$$\frac{ck}{n}$$

$$\|Q(x)\|_1 = \sum_x |Q(x)|$$


P{x}


$P[x] \approx \int dp \, \mu(p) \, P_p[x]$

$$P[x^k] \approx \int dp\, \mu(p)\, P_p[x]^k$$


$P[x^k] \approx \int dp\, \underbrace{\mu(p)}\, P_p[x]^k$

<u>Quantum</u>

ρ^n sym.

$\rho^k = \mathrm{tr}_{n-k}\, \rho^n \ldots$

Quantum

ρ^n sym.

$\rho^k = \mathrm{tr}_{n-k}\,\rho^n$

$\exists\, m(\sigma)$

$\|\rho^k$

Quantum

ρ^n sym.

$\rho^k = \mathrm{tr}_{n-k}\, \rho^n$

$\exists\, m(\sigma)$

$$\left\| \rho^k - \int dm(\sigma)\, \sigma^{\otimes k} \right\|_1 \leq \frac{4kd^2}{n}$$

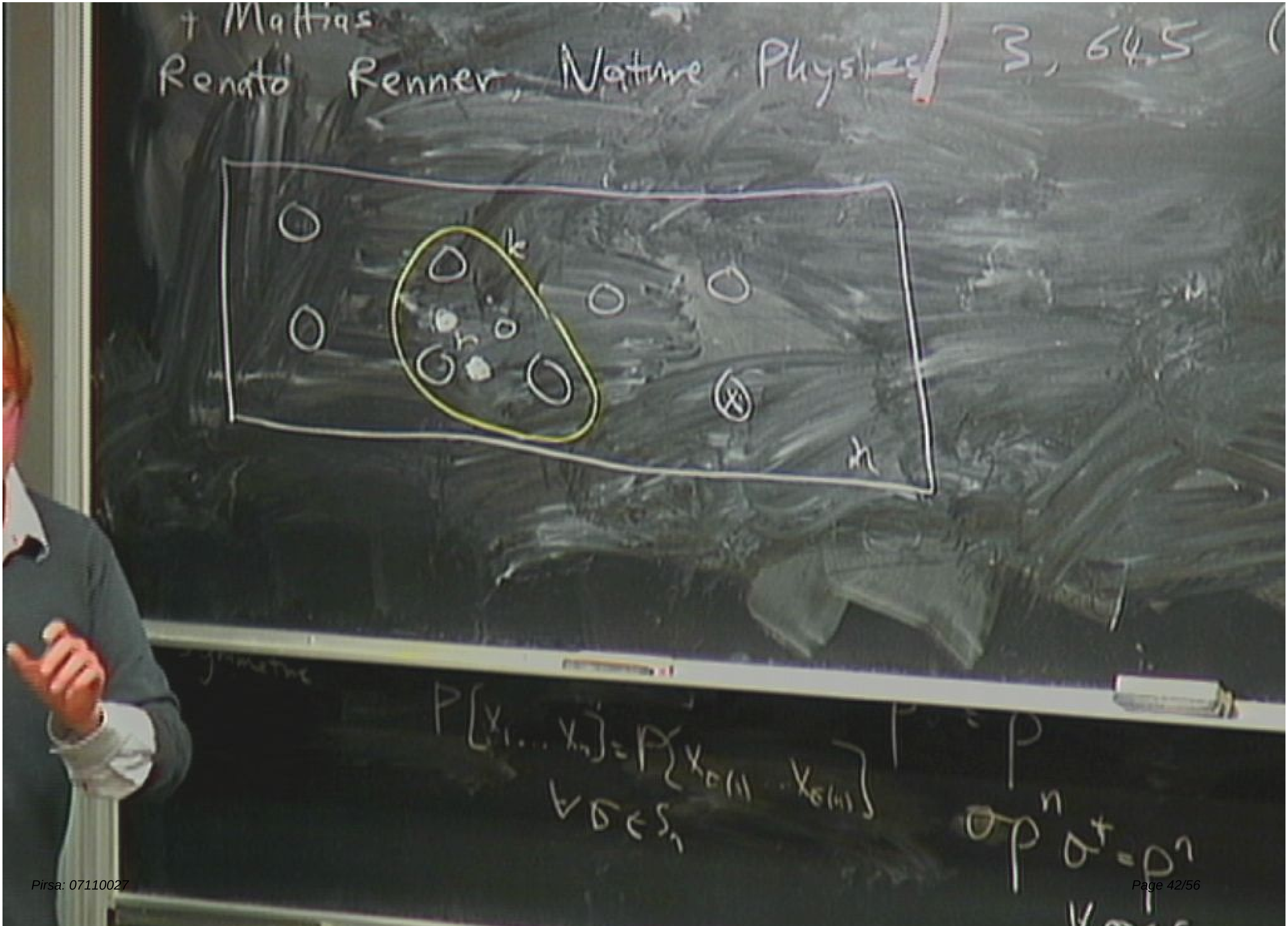
Quantum

ρ^n sym. $\quad \rho \in \mathcal{H}^n$

$\rho^k = \mathrm{tr}_{n-k}\, \rho^n \quad \dim(\mathcal{H}) = d$

$\exists\, m(\sigma)$

$$\left\| \rho^k - \int dm(\sigma)\, \sigma^{\otimes k} \right\|_1 \le \frac{4kd^2}{n}$$

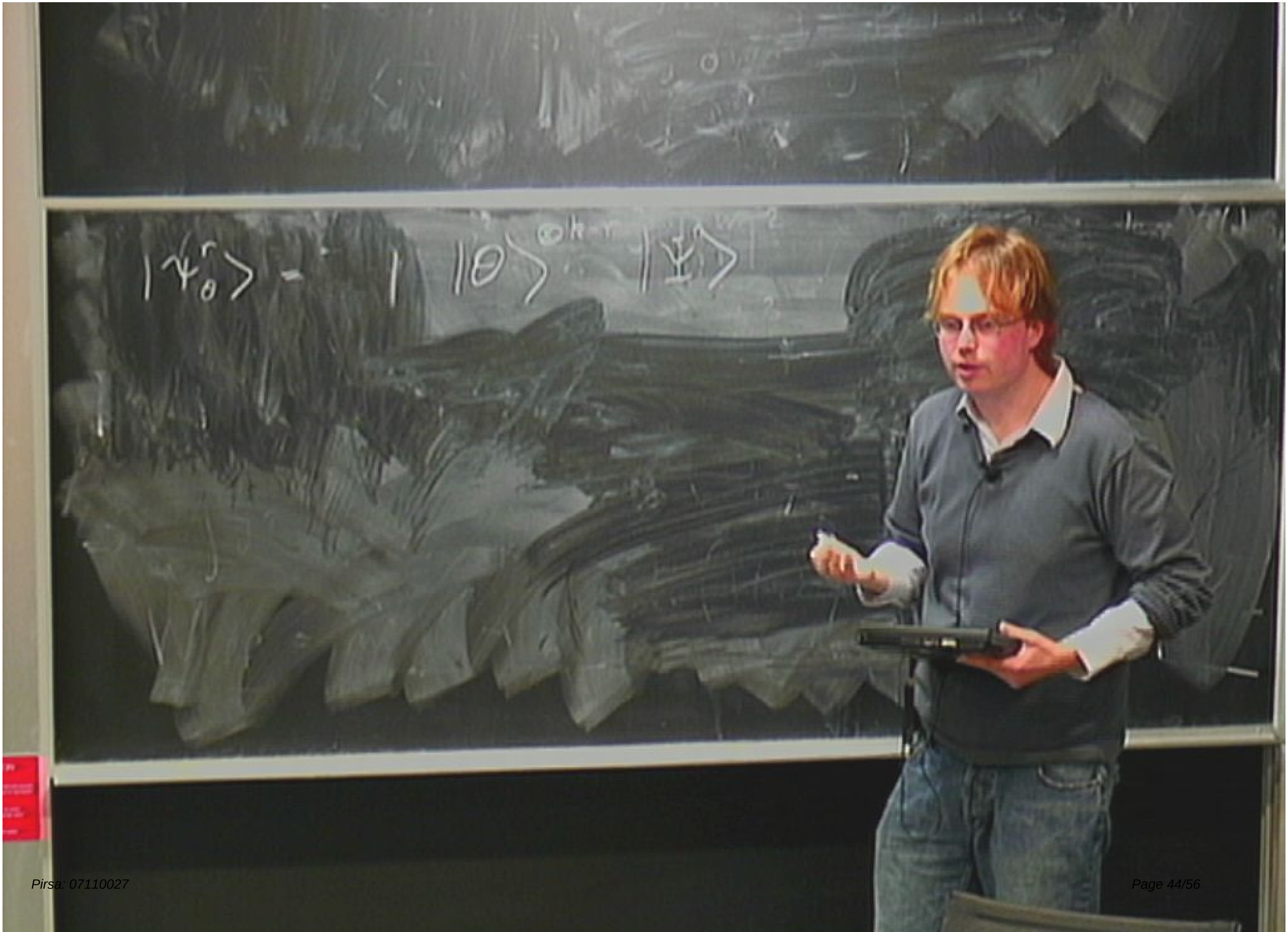

+ Mattias
Renato Renner, Nature Physics 3, 645
k
n
P[x1 ... xn] = P{xσ(1) ... xσ(n)}
∀σ∈Sn
σρ^n σ† = ρ^n

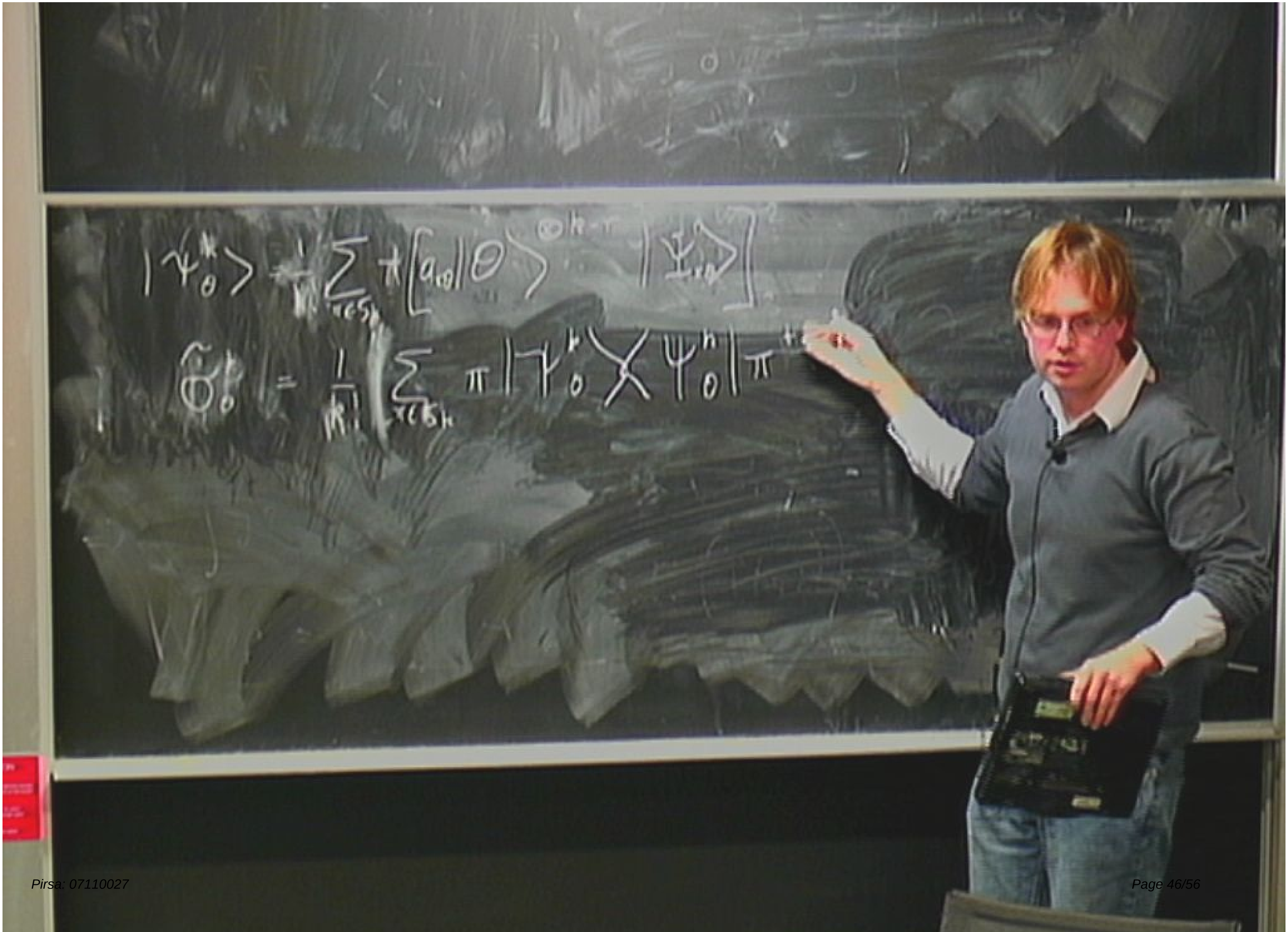
Renato Renner

(Zurich)

$= P[X^n]$

ρ^n on $\mathcal{H}^{\otimes n}$

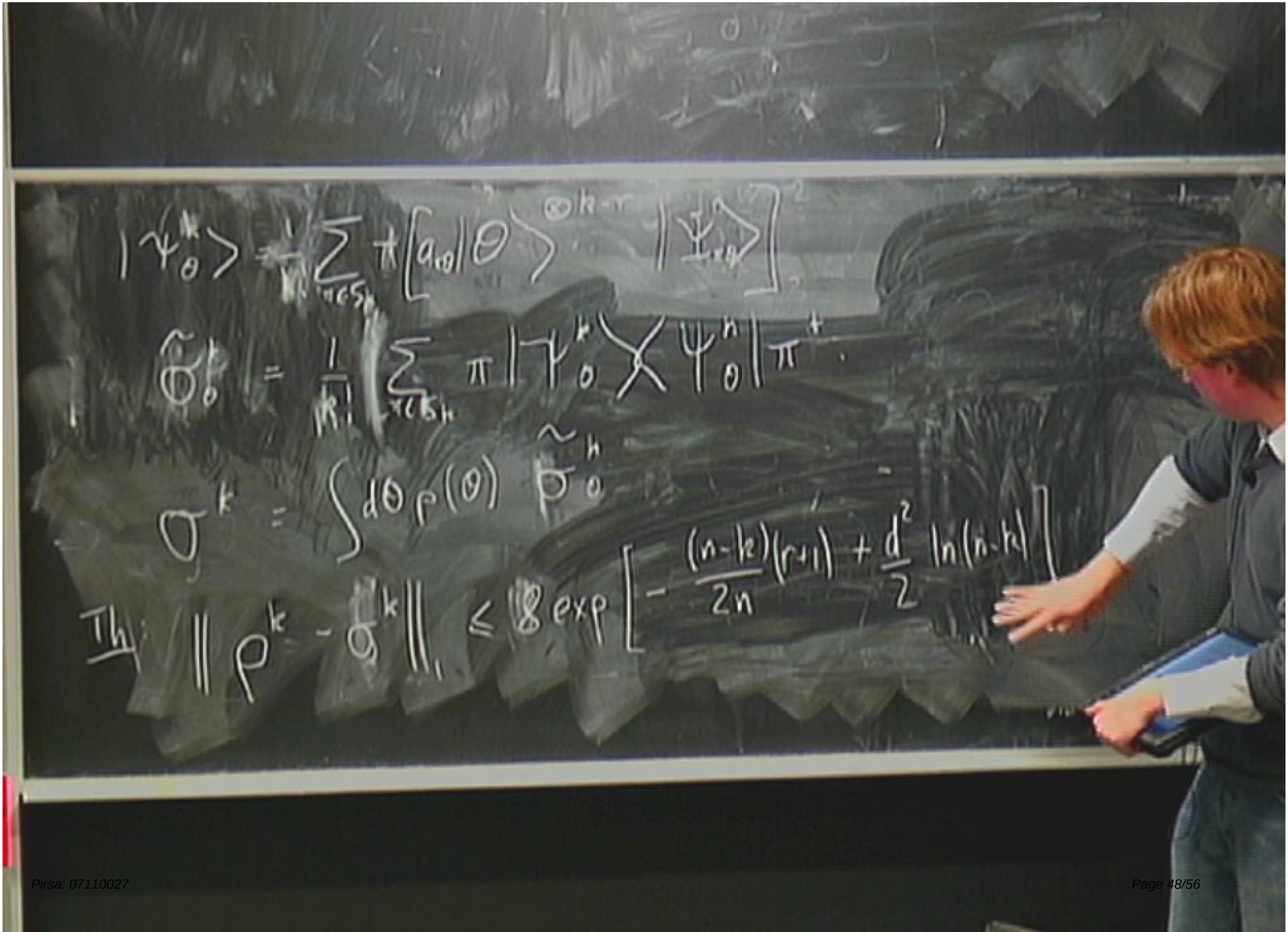



$|\psi_\theta^k\rangle = \frac{1}{k!}\sum_{\pi\in S_k} \pi\left[|a_\theta\rangle|\theta\rangle^{\otimes k-r}|\Psi_{r,\theta}\rangle\right]$
$\tilde{\rho}_\theta^k = \frac{1}{k!}\sum_{\pi\in S_k} \pi|\psi_\theta^k\rangle\langle\psi_\theta^k|\pi^{-1}$

$$|\tilde{\psi}_\theta^k\rangle = \frac{1}{k!}\sum_{\pi \in S_k} \pi \left[|a_{r\theta}\rangle |\theta\rangle^{\otimes k-r} |\Psi_{\pi\theta}\rangle \right]$$

$$\tilde{\sigma}_\theta^k = \frac{1}{k!}\sum_{\pi \in S_k} \pi |\tilde{\psi}_\theta^k\rangle\langle\psi_\theta^n| \pi^\dagger$$

$$\sigma^k = \int d\theta\, \rho(\theta)\, \tilde{\sigma}_\theta^k$$


$|\Psi_\theta^k\rangle = \frac{1}{\sqrt{k!}}\sum_{\pi\in S_k}\pi\left[a_\theta|\theta\rangle^{\otimes k-r}|\Psi_{r\theta}\rangle\right]$
$\tilde{\rho}_\theta^k = \frac{1}{k!}\sum_{\pi\in S_k}\pi|\Psi_\theta^k\rangle\langle\Psi_\theta^n|\pi^\dagger$
$\sigma^k = \int d\theta\, p(\theta)\, \tilde{\rho}_\theta^n$
Th: $\|\rho^k - \sigma^k\|_1 \le 8\exp\left[-\frac{(n-k)(r+1)}{2n} + \frac{d^2}{2}\ln(n-k)\right]$

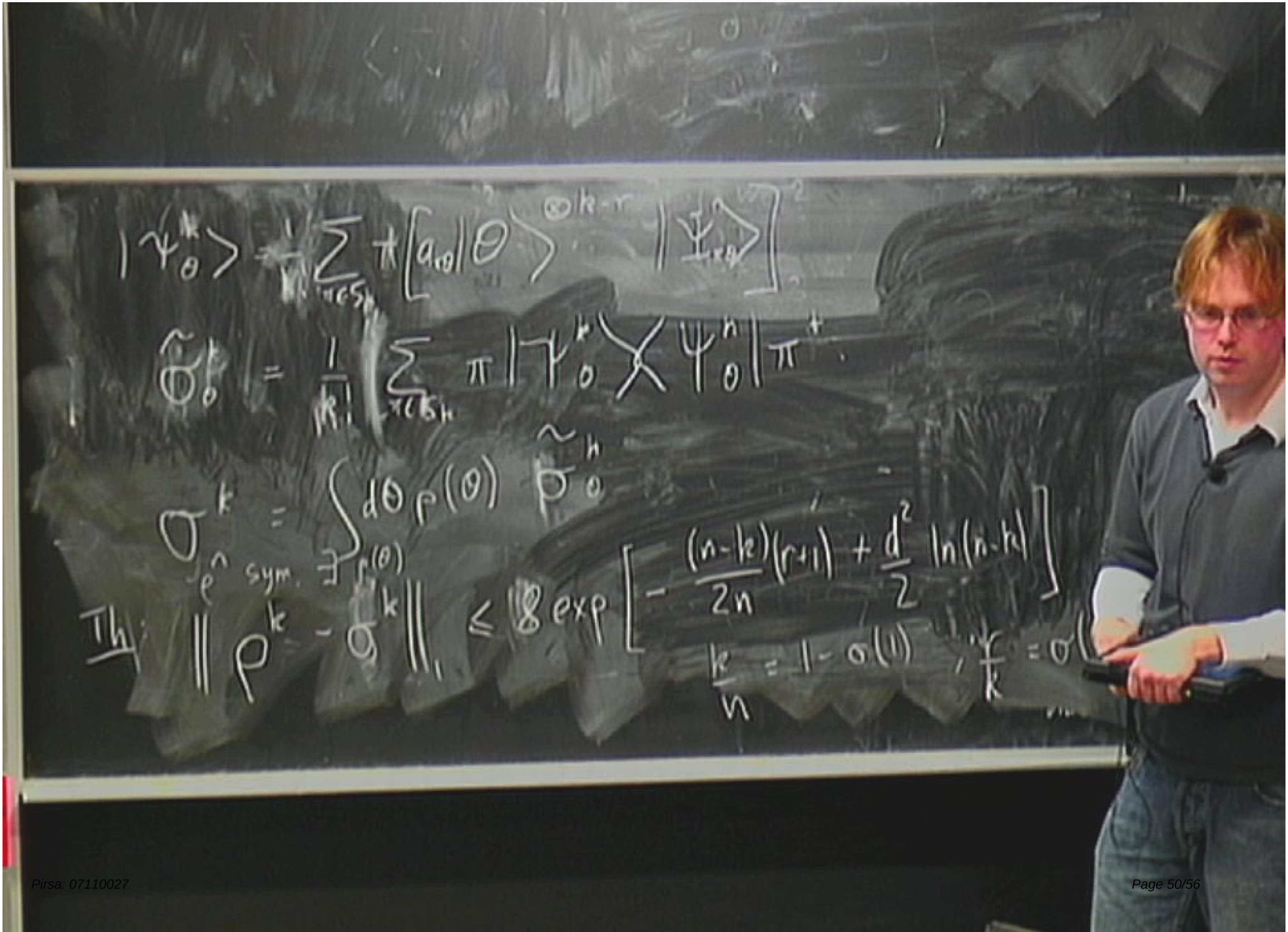
$$|\Psi^k_\theta\rangle = \frac{1}{\sqrt{k!}} \sum_{\pi \in S_k} \pi \left[|a_\theta\rangle |\theta\rangle^{\otimes k-r} |\Psi_{r,\theta}\rangle \right]$$

$$\tilde{\sigma}^k_\theta = \frac{1}{k!} \sum_{\pi \in S_k} \pi |\Psi^k_\theta\rangle\langle\Psi^k_\theta| \pi^\dagger$$

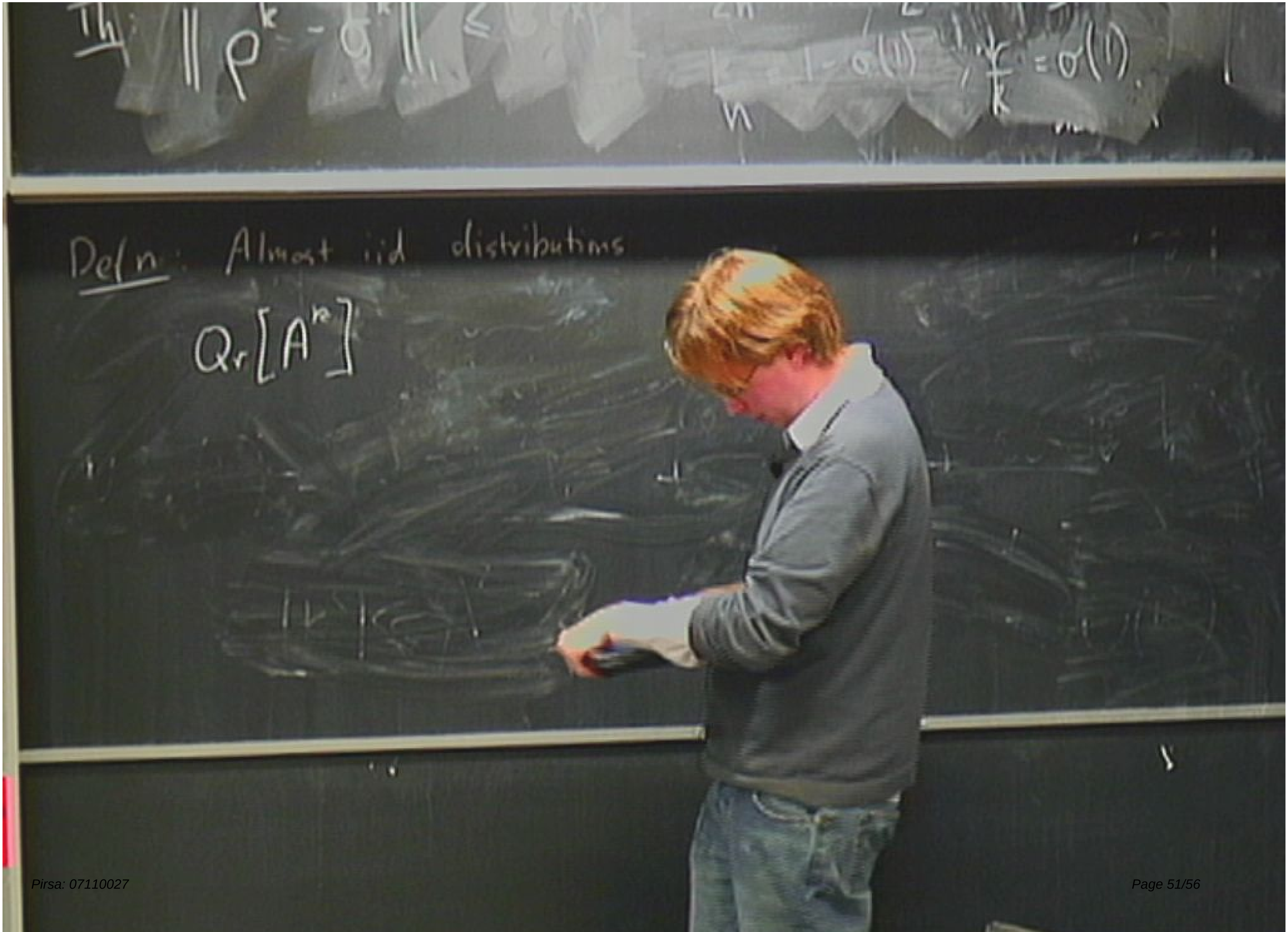
$$\sigma^k = \int d\theta\, p(\theta)\, \tilde{\sigma}^k_\theta$$

Th: $$\|\rho^k - \sigma^k\|_1 \le 8 \exp\left[-\frac{(n-k)(r+1)}{2n} + \frac{d^2}{2} \ln(n-k) \right]$$

$$\frac{k}{n} = 1 - o(1), \quad \frac{r}{k} = o(1)$$

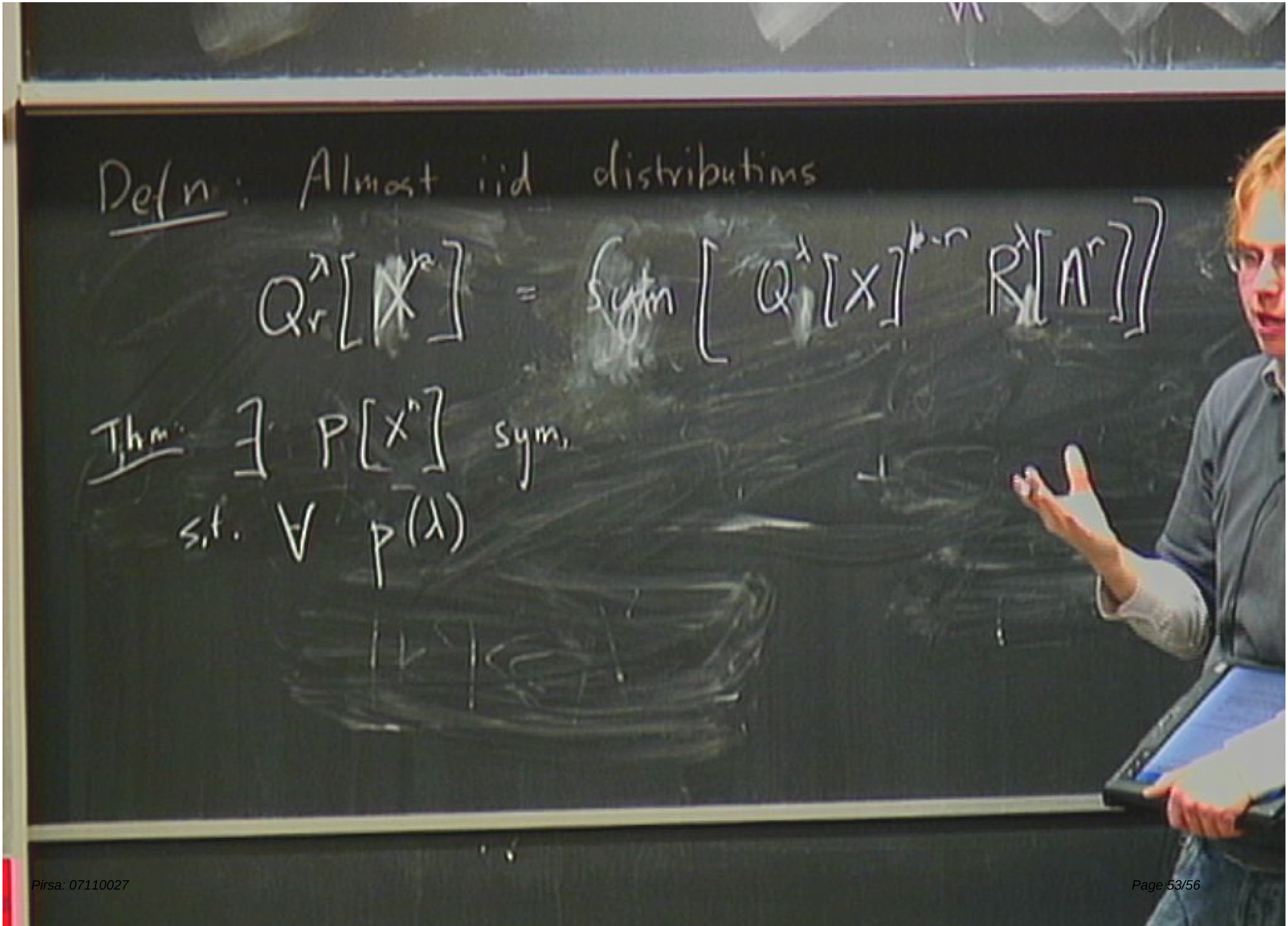


$|\Psi_\theta^k\rangle = \frac{1}{\sqrt{k!}} \sum_{\pi \in S_n} \pi\left[a_\theta |\theta\rangle^{\otimes k-r} |\Psi_{r\theta}\rangle \right]$
$\tilde{\sigma}_\theta^k = \frac{1}{k!} \sum_{\pi \in S_n} \pi |\Psi_\theta^k\rangle\langle\Psi_\theta^k| \pi^\dagger$
$\sigma^k = \int d\theta\, \rho(\theta)\, \tilde{\sigma}_\theta^k$
ρ^n sym. $\exists\, \rho(\theta)$
Th: $\|\rho^k - \sigma^k\|_1 \leq 8 \exp\left[-\frac{(n-k)(r+1)}{2n} + \frac{d^2}{2} \ln(n-k) \right]$
$\frac{k}{n} = 1 - o(1)$, $\frac{r}{k} = o($


Defn: Almost iid distributions
Qr[A^n]

<u>Defn</u>: Almost iid distributions

$$Q_r[X^n] = \mathrm{Sym}\left[Q[X]^{n-r} R[A^r] \right]$$


Defn: Almost iid distributions
$Q_r^{\lambda}[X^n] = \mathrm{Symm}\left[Q^{\lambda}[X]^{\otimes n-r}\, R^{\lambda}[\Lambda^r] \right]$
Thm: $\exists\ P[X^n]$ sym.
s.t. $\forall\ p(\lambda)$

Thm: $\|\rho^k - \sigma^{\otimes k}\|_1 \leq \ldots \quad \frac{k}{n} = o(1), \quad \frac{r}{k} = o(1).$

Defn: Almost iid distributions

$$Q_r^\lambda[X^n] = \mathrm{Sym}\left[\, Q^\lambda[X]^{n-r} \, R^\lambda[A^r] \,\right]$$

Thm: $\exists\ P[X^n]$ sym.

s.t. $\forall\ p(\lambda)$

$$\left\| P[X^k] - \int d\lambda\, p(\lambda)\, Q_r^\lambda[X^k] \right\| > \frac{1}{100}\left(\frac{k}{n} - 2\frac{r}{k}\right) + 10^{-19} + O\left(\frac{1}{k}\right).$$

Thm: $\|\rho^k - \sigma^{\otimes k}\|_1 \le \ldots \quad \frac{k}{n} = 1 - o(1), \quad \frac{r}{k} = o(1)$

Defn: Almost iid distributions

$$Q_r^\lambda[X^k] = \mathrm{Sym}\left[Q^\lambda[X]^{k-r} R^\lambda[A^r] \right]$$

Thm: $\exists\ P[X^n]$ sym.

s.t. $\forall\ p(\lambda)$

$$\left\| P[X^k] - \int d\lambda\, p(\lambda)\, Q_r^\lambda[X^k] \right\| > \frac{1}{100}\left(\frac{k}{n} - 2\frac{r}{k}\right) - 10^{-19} + O\left(\frac{1}{k}\right)$$

Thm: $\|\rho^k - \sigma^k\|_1 \leq \ldots = 1 - o(1), \quad \frac{k}{n} = o(1).$

Defn: Almost iid distributions

$$Q_r^\lambda[X^k] = \mathrm{Sym}\left[Q^\lambda[X]^{\otimes k-r}\, R^\lambda[A^r] \right]$$

Thm: $\exists\ P[X^n]$ sym.

s.t. $\forall\ p(\lambda)$

$$\left\| P[X^k] - \int d\lambda\, p(\lambda)\, Q_r^\lambda[X^k] \right\| > \frac{1}{100}\left(\frac{k}{n} - 2\frac{r}{k}\right) - 10^{-19} + O\left(\frac{1}{k}\right).$$