

Title: Symmetry and global independence in classical and quantum theories

Date: Nov 07, 2007 04:00 PM

URL: <http://pirsa.org/07110027>

Abstract: Renner's global quantum de Finetti theorem establishes that if the state of a quantum system is invariant under permutations of its systems, then almost all of its subsystems are almost in the same state and independent of each other. Motivated by this result, we show that the most straightforward classical analogue of Renner's theorem is false.

Joint work with Matthias Christandl (Cambridge).

Symmetry & Global Independence in Classical + Quantum Theories

+ Mathias

Symmetry & Global Independence in Classical + Quantum Theories

+ Matthias

Symmetry & Global Independence in Classical + quantum Theories

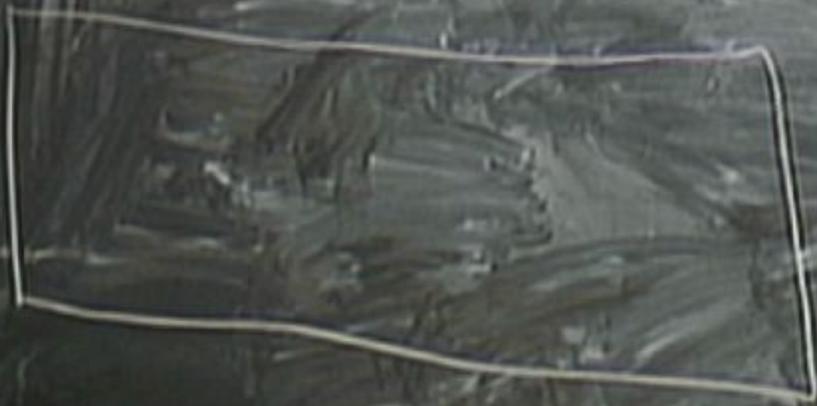
+ Matthias

Ronald Renner, Nature Physics 3, 645 (2007)

Symmetry & Global Independence in Classical + quantum Theories

+ Matthias

Ronald Renner, Nature Physics 3, 645 (2007)



Symmetry & Global Independence in Classical + quantum Theories

+ Matthias

Renato Renner, Nature Physics 3, 645 (2007)



Symmetry & Global Independence in Classical + Quantum Theories

+ Matthias

Renato Renner, Nature Physics 3, 645 (2007)



State
(subsystem)

Classical

$x_i \in \mathcal{X}$

$P[x_i]$

$P[x_1, x_2]$

Quantum

State
(subsystem)

Classical	Quantum
$x_i \in X$	
$P[x_i]$	
$P[x_1, \dots, x_n]$	
$P[x_1, \dots, x_n] = P[X]$	

Classical

Quantum

State
(subsystem)

$x_i \in \mathcal{X}$

$P[x_i]$

$P[x_1, x_2]$

$P[x_1, \dots, x_n] = P[X]$

State



Classical

Quantum

State
(subsystem)

$$x_i \in \mathcal{X}$$

$$P[x_i]$$

$$P[x_i = x_i']$$

$$P[x_1, \dots, x_n] = P[X]$$

State
(entire)

Classical

Quantum

State
(subsystem)

$$x_i \in \mathcal{X}$$

$$P[x_i]$$

$$P[x_i, x_j]$$

State
(universe)

$$P[x_1, \dots, x_n] = P[X]$$

	Classical	Quantum
State (subsystem)	$x_i \in X$	ρ
	$P[X_i]$	
	$P[X_i, x_i]$	
State (universe)	$P[X_1, \dots, X_n] = P[X]$	ρ^n

	Classical	Quantum
State (subsystem)	$x_i \in X$	ρ on \mathcal{H}
	$P[X_i]$	
	$P[X_i, x_i]$	
State (universe)	$P[X_1, \dots, X_n] = P[X]$	ρ^n on $\mathcal{H}^{\otimes n}$

	Classical	Quantum
State (subsystem)	$x_i \in X$	ρ_i on \mathcal{H}
	$P\{x_i\}$	
	$P\{x_1, \dots, x_n\}$	
State (universe)	$P\{x_1, \dots, x_n\} = P\{X\}$	ρ^n on $\mathcal{H}^{\otimes n}$

State
(subsystem)

$x \in X$

$P[X]$

$P[x_1, x_2]$

$P[x_1, \dots, x_n] = P[X]$

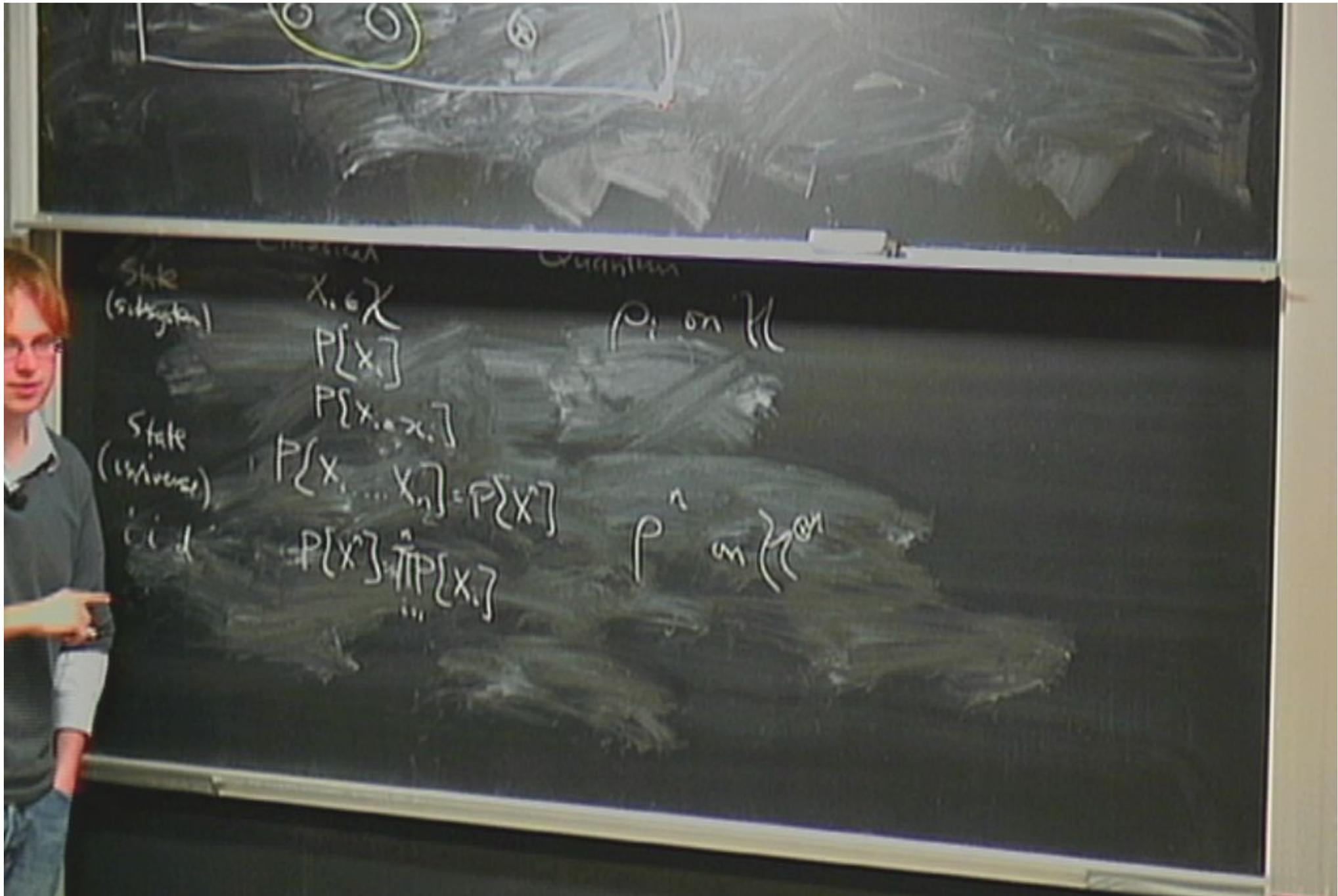
Quantum

ρ_i on \mathcal{H}

ρ^n on $\mathcal{H}^{\otimes n}$

State
(universe)

\mathcal{H}



Classical
State (subsystem)

x, p, X
 $P[X]$

Quantum

ρ_i on K

State (universe)

$P[X_1, \dots, X_n]$
 $P[X_1, \dots, X_n] = P[X]$

ρ^n on $K^{\otimes n}$

and

$P[X] = \prod P[X_i]$
...

State
(subsystem)

$x \in X$

$$P\{x\}$$

$$P\{x_1, x_2\}$$

$$P\{x_1, \dots, x_n\} = P\{X\}$$

$$P\{x_1\} \cdot P\{x_2\} \cdot \dots$$

State
(universe)

i.i.d

Quantum

ρ_i on \mathcal{H}

$$\rho^n$$

$$\rho^n = \rho^{\otimes n}$$

State
(subsystem)

$$x_i \in X$$
$$P\{x_i\}$$

P_i on \mathcal{K}

$$P\{x_i, x_j\}$$

State
(universe)

$$P\{x_1, \dots, x_n\} = P\{X\}$$

i.i.d.

$$P\{x_i\} = P\{x_j\}$$

$$P^n \text{ on } \mathcal{K}^{\otimes n}$$
$$P^n = P^{\otimes n}$$

(Subgroup)

$$P\{x_i\}$$

$$P\{x_i, x_j\}$$

$$P\{x_1, \dots, x_n\} = P\{X\}$$

State (universe)

$$P\{X^i\} = P\{X^j\}$$

i.i.d symmetric

$$P\{x_i, \dots, x_j\} = P\{x_{\sigma(i)}, \dots, x_{\sigma(j)}\} \quad \forall \sigma \in S_n$$

$$P^n = \underbrace{P \otimes \dots \otimes P}_n$$

$$P^n = P^{\otimes n}$$

(subsystem)

$$P\{x_i\}$$

$$P\{x_i, x_j\}$$

$$P\{x_1, \dots, x_n\} = P\{X\}$$

State
(universe)

$$P\{X^*\} = P\{X\}$$

i.i.d.
symmetric

$$P\{x_1, \dots, x_n\} = P\{x_{\sigma(1)}, \dots, x_{\sigma(n)}\}$$

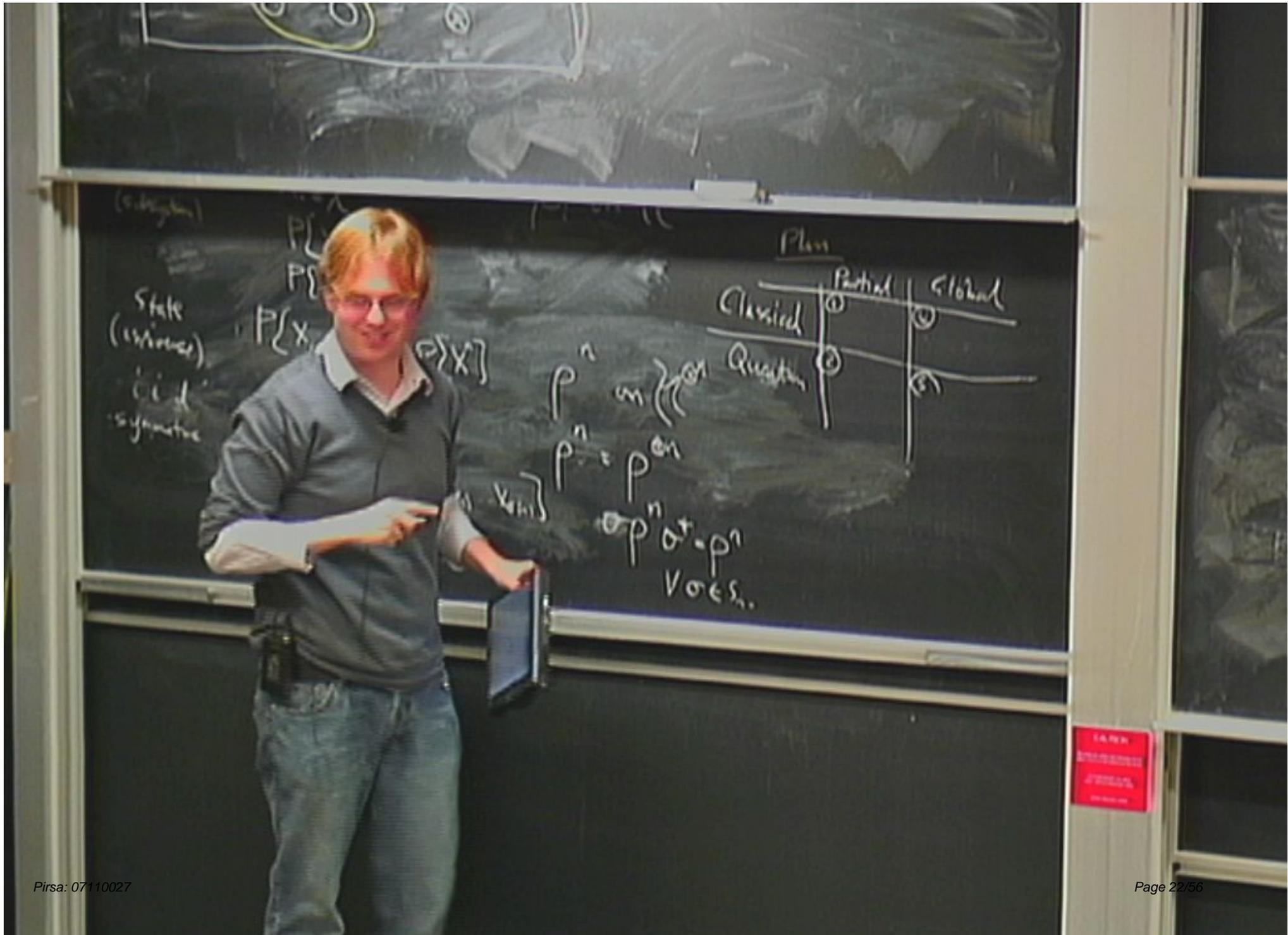
$\forall \sigma \in S_n$

$$P^n = \underbrace{P \otimes \dots \otimes P}_n$$

$$P^n = P^{\otimes n}$$

$$\sigma P^n \sigma^T = P^n$$

$\forall \sigma \in S_n$



(category)
State (matrix)
is symmetric

$P(x)$
 $P(x)$

$$P^n = P^n$$
$$\sigma P^n \sigma^T = P^n$$

Voeg.

Plan

	Partial	Global
Classical	(1)	(2)
Quantum	(3)	(3)

Th: (Diaconis & Freedman, 1980)
Suppose $P\{X^i\}$ sym.

Th: (Diaconis & Freedman, 1980)

Suppose $P\{X^i\}$ sym.

$\} p(\lambda)$

Th: (Diaconis & Freedman, 1980)

Suppose $P[X^k]$ sym.

$\int p(\lambda)$

$$\|P[X^k] - \int d\lambda p(\lambda) P_\lambda[X^k]\|_1 \leq \min\left(\frac{2k\|X\|}{n}, \frac{k(k-1)}{n}\right)$$

Th: (Diaconis & Freedman, 1980)

Suppose $P[X^*]$ sym.

$\int \mu(\lambda)$

$$\|P[X^*] - \int d\lambda \mu(\lambda) P_\lambda[X^*]\|$$

$$\leq \min\left(\frac{2k\|X\|}{n}, \frac{k(k-1)}{n}\right)$$

$$\|P(X)\| = \sum_{\omega} |P(\omega)|$$

Th. (Dixon & Freedman, 1980)

Suppose $P[X^k]$ sym.

$\int \mu(x)$

$$\|P[X^k] - \int d\lambda \mu(\lambda) P_\lambda[X^k]\|_1 \leq \min\left(\frac{2k\chi}{n}, \frac{k(k-1)}{n}\right)$$

$$\|Q(x)\|_1 = \sum_x |Q(x)|$$

Th. (Diaconis & Freedman, 1980)

Suppose $P[X^*]$ sym.

$\int \mu(\lambda)$

$$\|P[X^*] - \int d\lambda \mu(\lambda) P_\lambda[X]\|_{\infty} \leq \min\left(\frac{2k\|X\|}{n}, \frac{k(k-1)}{n}\right)$$

$$|Q(y)| = \sum_x |Q(x)|$$

Th. (Dixon & Freedman, 1980)

Suppose $P[X^*]$ sym.

$\int p(\lambda)$

$$\|P[X^*] - \int d\lambda p(\lambda) P_\lambda[X^*]\| \leq \min\left(\frac{2k\|X\|}{n}, \frac{k(k-1)}{n}\right)$$

$$\|Q(X)\|_1 = \sum_X |Q(X)|$$

Th: (Diaconis & Freedman, 1980)

Suppose $P[X^*]$ sym.

$\int \mu(\lambda)$

$$\|P[X^*] - \int d\lambda \mu(\lambda) P_\lambda[X^*]\|_1$$

$$\|C\|_1 = \sum_x |C(x)|$$

$$\leq \min\left(\frac{2k}{n}, \frac{k(k-1)}{n}\right)$$

$\frac{2}{5}$

Th: (Dixon & Freedman, 1980)

Suppose $P[X^*]$ sym.

$\forall k \in \{1, \dots, n\}$

$$\|P[X^*] - \int d\lambda \mu(\lambda) P_\lambda[X^*]\|_1 \leq \min\left(\frac{2k}{n}, \frac{k(k-1)}{n}\right)$$

$$\|Q(X)\|_1 = \sum_X |Q(X)|$$

Th. (Dixon & Freedman, 1980)

Suppose $P[X^*]$ sym.

$\forall k \in \{1, \dots, k\}$

$$\|P[X^*] - \int d\lambda \mu(\lambda) P_\lambda[X^*]\|_1$$

$$\leq \min\left(\frac{2k}{n}, \frac{k(k-1)}{n}\right)$$

$$\|Q(x)\|_1 = \sum_x |Q(x)|$$



Th: (Dixon & Freedman, 1980)

Suppose $P[X^*]$ sym.

$\forall k \in \{1, \dots, n\}$

$$\|P[X^*] - \int d\lambda \mu(\lambda) P_\lambda[X^*]\|_1 \leq \min\left(\frac{2k}{n}, \frac{k(k-1)}{n}\right)$$

$$\|Q(x)\|_1 = \sum_x |Q(x)|$$



20
0

$P\{x\}$

$$P\{x\} \approx \int dP \, M(P) P_P\{x\}$$

20
0

$$P\{x\} \approx \int dP \, M(P) P_r\{x\}^k$$

20
0

$$P\{x\} \approx \int dP \underbrace{M(P)}_1 P_r\{x\}^{lc}$$

20
0

Quantum
 ρ^n symm
 $\rho^k = \text{tr}_{n+k} \rho^n$

20
0

Quantum

ρ^n sym.

$$p^k = \text{tr}_{n+k} \rho^n$$

$\mathbb{I} m(\sigma)$

$\|\rho^k\|$

Quantum

ρ^n sym.

$$p^k = \text{tr}_n \rho^n$$

$\exists m(\sigma)$

$$\|\rho^k - \int dm(\sigma) \sigma^{\otimes k}\|_1 \leq \frac{4kd^2}{n}$$

20
0

Quantum

ρ^n sym.

$\rho \in \mathcal{H}^n$

$$p^k = \text{tr}_{n-k} \rho^n$$

$$\dim(\mathcal{H}) = d$$

$\exists m(\sigma)$

$$\|\rho^k - \int dm(\sigma) \sigma^{\otimes k}\|_1 \leq \frac{4kd^2}{n}$$

+ Matthias
 Renato Renner, Nature Physics 3, 645 (2007)



$$P[x_1, \dots, x_n] = P[x_{\sigma(1)}, \dots, x_{\sigma(n)}]$$

$$\forall \sigma \in S_n$$

$$P = P$$

$$\sigma \rho^n \alpha^+ = \rho^n$$

$$\forall \alpha \in S$$

Renato Renner
(Zurich)

$$= P \left[X^n \right]$$

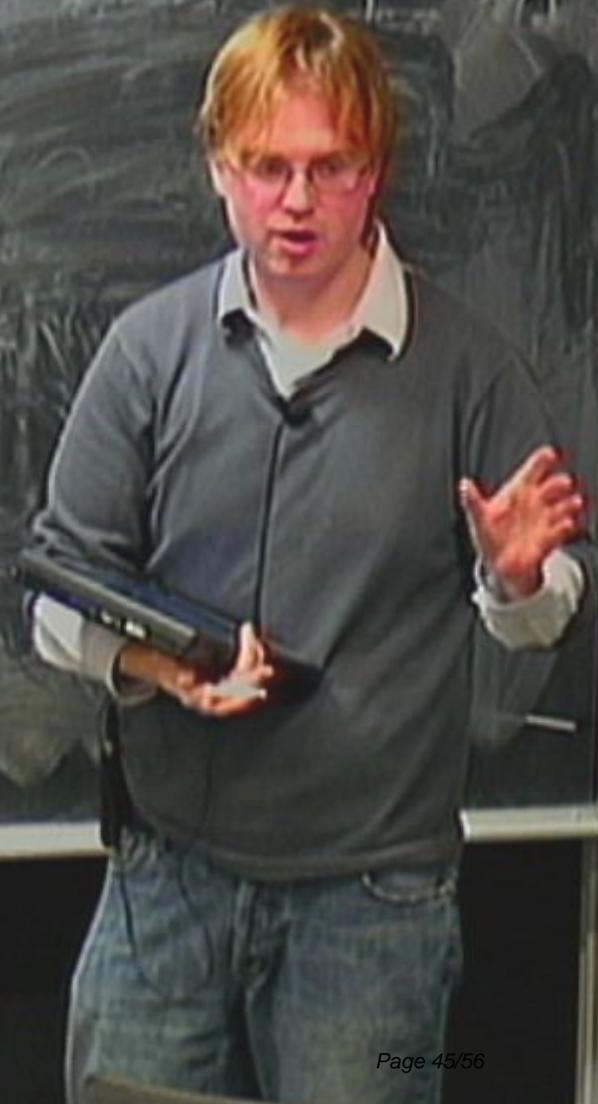
n
 P

\in $\left\{ \begin{array}{l} \circledast \\ \circledast \end{array} \right\}^n$

$$|\psi_0\rangle = \frac{1}{\sqrt{2}} (|\theta\rangle + |\psi\rangle)$$

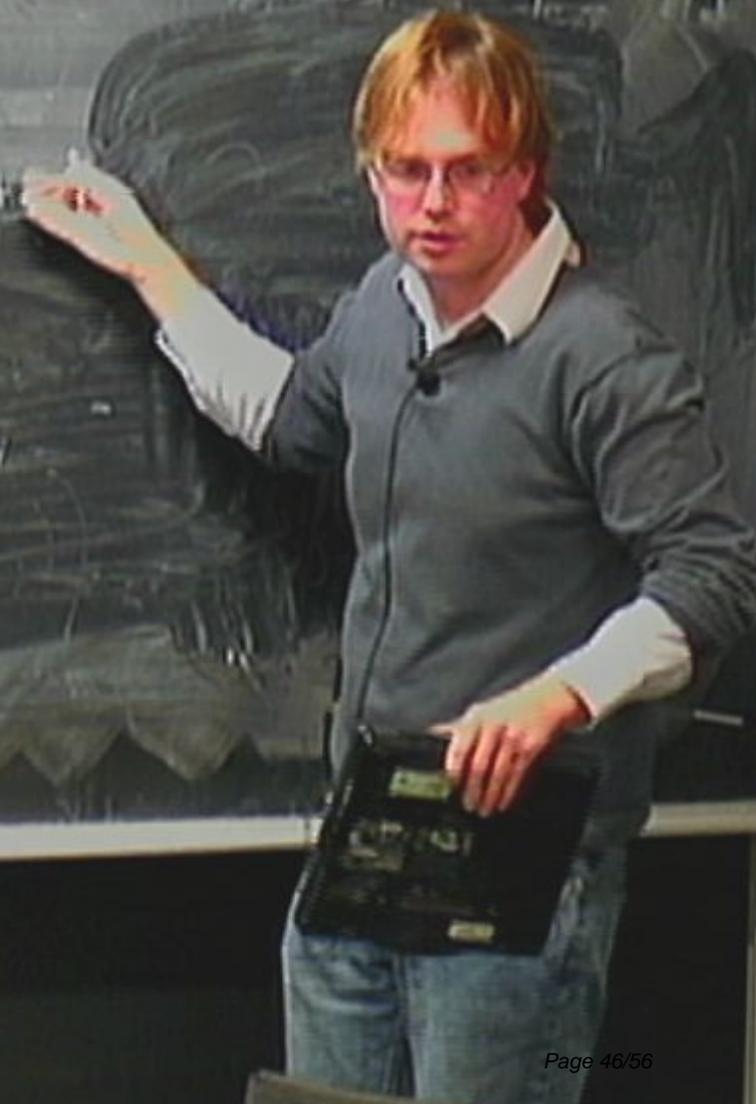
PIR
07110027

$$|\psi_0^*\rangle = \frac{1}{N} \sum_{n \in \text{res.}} t [a_{0n} |\theta\rangle^{\otimes k-n} |\psi_{n0}\rangle]$$



$$|\psi_0^k\rangle = \frac{1}{\sqrt{K}} \sum_{\alpha \in S_k} \pi [a_{\alpha} |\theta\rangle^{\otimes k-\pi} |\psi_{\alpha}\rangle]$$

$$\hat{\rho}_0^k = \frac{1}{K!} \sum_{\alpha \in S_k} \pi |\psi_0^k\rangle \langle \psi_0^k| \pi$$



$$|\psi_0^k\rangle = \frac{1}{\sqrt{k!}} \sum_{\pi \in S_k} \pi \left[a_{\pi\theta} |\theta\rangle^{\otimes k-r} |\psi_{\pi\theta}\rangle \right]^2$$

$$\tilde{\rho}_0^k = \frac{1}{k!} \sum_{\pi \in S_k} \pi |\psi_0^k\rangle \langle \psi_0^h| \pi^\dagger$$

$$\rho^k = \int d\theta p(\theta) \tilde{\rho}_0^k$$

$$|\psi_0^k\rangle = \frac{1}{\sqrt{k!}} \sum_{\pi \in S_k} \pi [a_{r0} |\theta\rangle^{\otimes k-r} |\psi_{r0}\rangle]^2$$

$$\hat{\rho}_0^k = \frac{1}{k!} \sum_{\pi \in S_k} \pi |\psi_0^k\rangle \langle \psi_0^k| \pi^\dagger$$

$$\rho^k = \int d\theta p(\theta) \hat{\rho}_0^k$$

$$\text{Th: } \|\rho^k - \hat{\rho}_0^k\|_1 \leq 8 \exp \left[-\frac{(n-k)(r+1)}{2n} + \frac{d^2}{2} \ln(n-k) \right]$$

$$|\psi_0^k\rangle = \frac{1}{\sqrt{\binom{n}{k}}} \sum_{\pi \in S_n} \pi [a_{r_0} |\theta\rangle^{\otimes k-r_0} |\psi_{r_0}\rangle^{\otimes r_0}]$$

$$\hat{\rho}_0^k = \frac{1}{\binom{n}{k}} \sum_{\pi \in S_n} \pi |\psi_0^k\rangle \langle \psi_0^k| \pi^\dagger$$

$$\rho^k = \int d\theta p(\theta) \hat{\rho}_0^k$$

$$\text{Th: } \|\rho^k - \hat{\rho}_0^k\|_1 \leq 8 \exp \left[-\frac{(n-k)(r+1)}{2n} + \frac{d^2}{2} \ln(n-k) \right]$$

$\frac{k}{n} = 1 - o(1), \quad \frac{r}{k} = o(1)$

$$|\psi_0^k\rangle = \frac{1}{\sqrt{\binom{n}{k}}} \sum_{\pi \in S_n} \pi [a_{01} |\theta\rangle^{\otimes k-r} |\psi_{k-r}\rangle]^2$$

$$\hat{\sigma}_0^k = \frac{1}{\binom{n}{k}} \sum_{\pi \in S_n} \pi |\psi_0^k\rangle \langle \psi_0^k| \pi^\dagger$$

$$\sigma_{e^n}^k = \int d\theta p(\theta) \tilde{\rho}_0^k$$

The

$$\|\rho^k - \sigma_{e^n}^k\|_1 \leq 8 \exp \left[-\frac{(n-k)(r+1)}{2n} + \frac{d^2}{2} \ln(n-k) \right]$$

$\frac{k}{n} = 1 - o(1)$, $\frac{r}{k} = o(1)$



$$\text{Th: } \|\rho^k - \sigma^k\|_1 \leq k \|\rho - \sigma\|_1$$

Defn: Almost iid distributions

$$Q_r[A^{nr}]$$

Defn: Almost iid distributions

$$Q_r[X^n] = \int \prod_{i=1}^n Q(x_i) R[A^n]$$

Defn: Almost iid distributions

$$Q_r^\lambda[X^n] = \text{Sym} \left[Q^\lambda[X]^{k-r} R[A^r] \right]$$

Thm: $\exists P[X^n]$ sym,
s.t. $\forall \lambda$
(r) d

$$\text{Th: } \|p^k - q^k\|_1 \leq \dots \frac{1}{k} = o(1)$$

Defn: Almost iid distributions

$$Q_r^\lambda[X^k] = \text{Sym} \left[Q_r^\lambda[X]^{k-n} R_r^\lambda[A^r] \right]$$

Thm $\exists P[X^n]$ sym.

s.t. $\forall P(\lambda)$

$$\|P[X^n] - \int d\lambda P(\lambda) Q_r^\lambda[X^k]\| \geq \frac{1}{100} \left(\frac{k}{n} - 2 \frac{r}{k} \right) + 10^{-19} + o\left(\frac{1}{k}\right)$$

$$\frac{1}{n} \|\rho^k - q^k\| \leq \dots$$

Def n: Almost iid distributions

$$Q_r^{\lambda}[X^k] = \text{Syn} [Q_r^{\lambda}[X]^{k-r} R_r^{\lambda}[A^r]]$$

Thm: $\exists P[X^k]$ sym.

s.t. $\forall P(\lambda)$

$$\|P[X^k] - \int d\lambda P(\lambda) Q_r^{\lambda}[X^k]\| > \frac{1}{100} \left(\frac{k}{n} - 2 \frac{r}{k} \right) 10^{-19} + O\left(\frac{1}{k}\right)$$

$$\text{Th: } \|p^k - q^k\|_1 \leq \frac{1}{k} = o(1)$$

Defn: Almost iid distributions

$$Q_r^{\lambda}[X^k] = \text{Syn} \left[Q_r^{\lambda}[X]^{k-r} R_r^{\lambda}[A^r] \right]$$

Thm: $\exists P[X^k]$ sym.

s.t. $\forall P(\lambda)$

$$\|P[X^k] - \int d\lambda P(\lambda) Q_r^{\lambda}[X^k]\| \geq \frac{1}{100} \left(\frac{k}{n} - 2 \frac{r}{k} \right) 10^{-19} + o\left(\frac{1}{k}\right)$$