Title: Asymptotic safety

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Abstract: TBA

Asymptotic Safety

Perimeter Institute. November 29. 2007

Microsoft Windows xp

Professional

Asymptotic Safety

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The Problems of QG

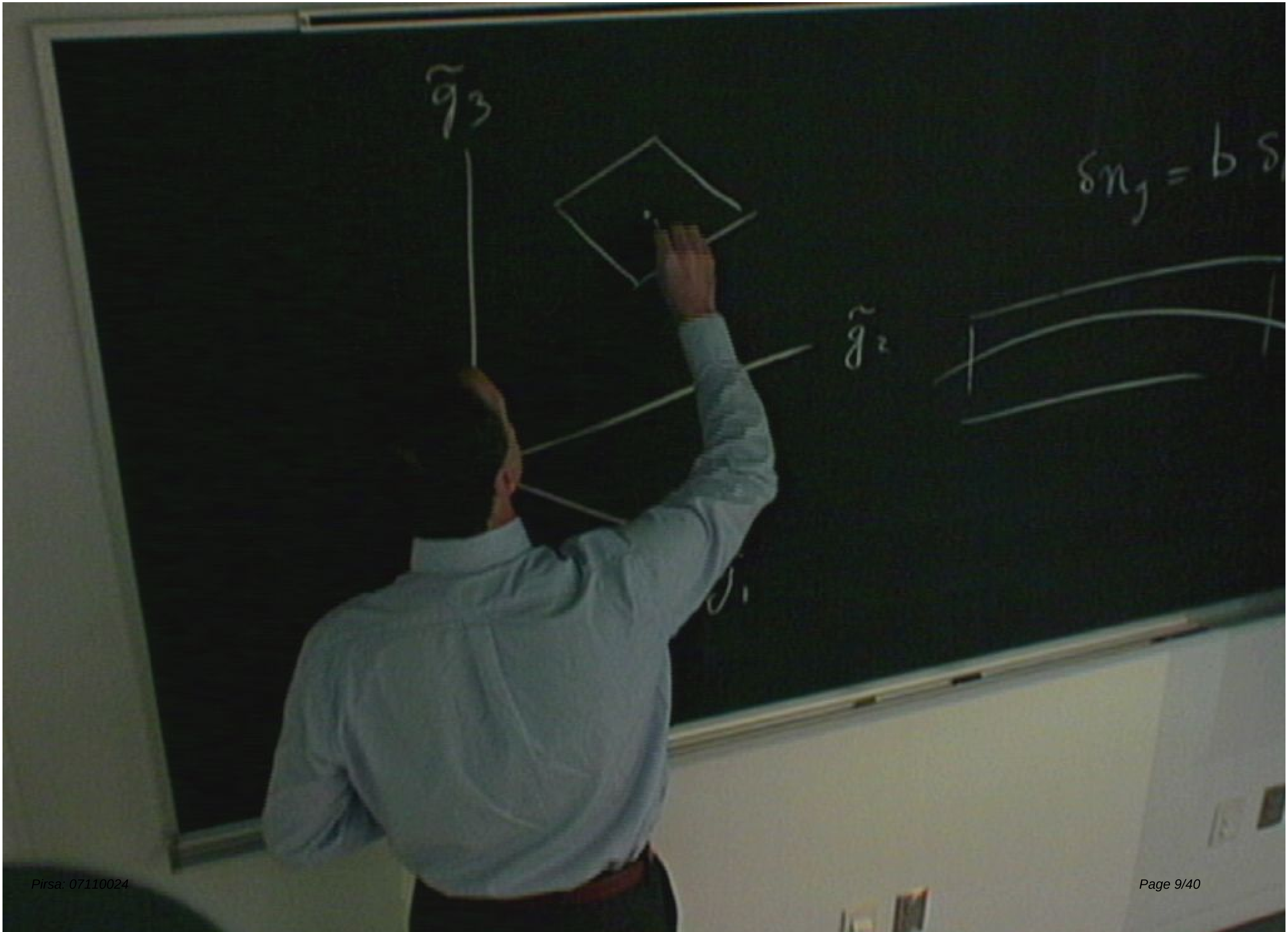
- interaction strength grows like $\tilde{G} = Gk^2$
- lack of predictivity

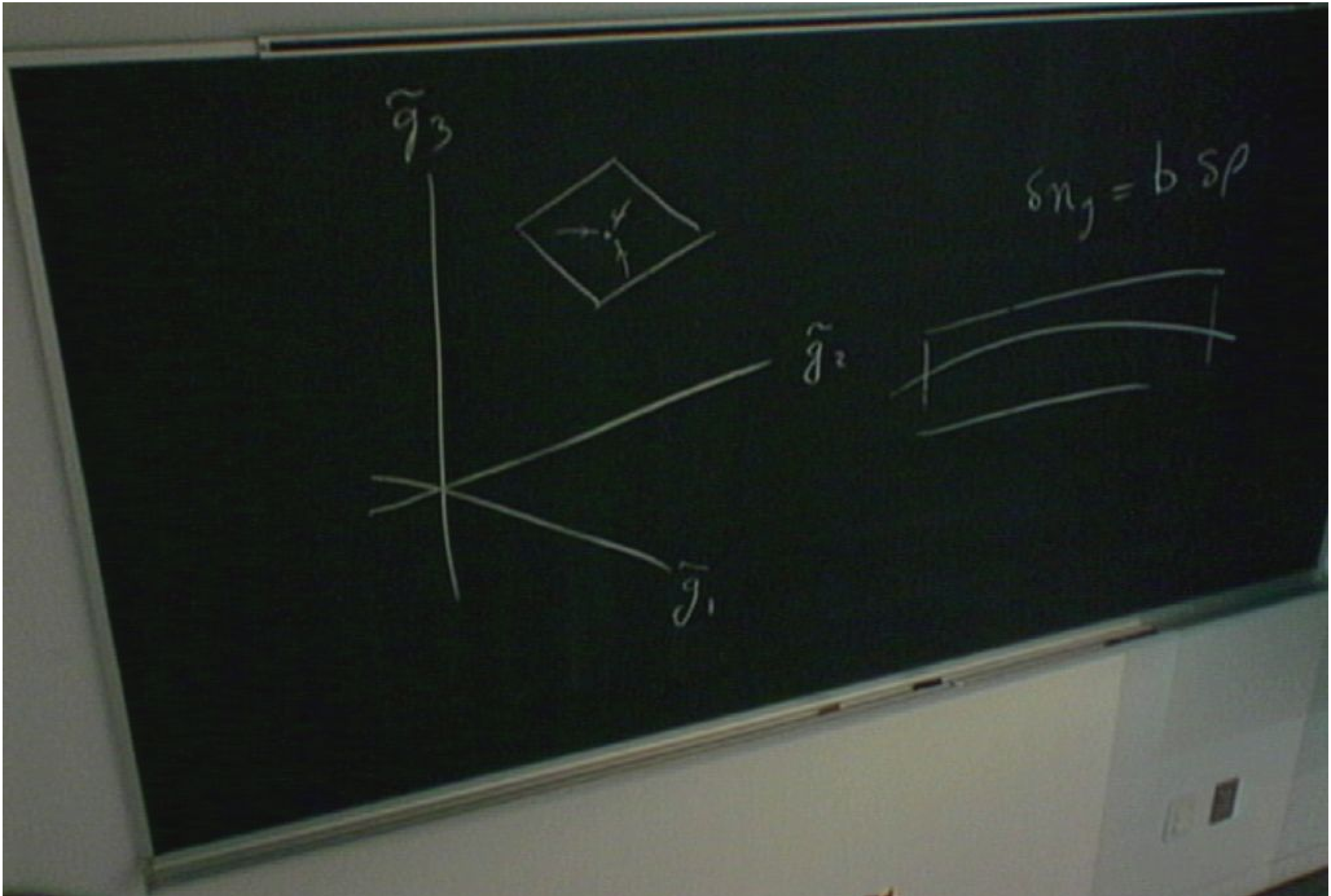
Possible RG solution

- $\tilde{G} = G(k)k^2 \to \tilde{G}_*$ (Fixed Point)
- more generally. $\tilde{g}_i \to \tilde{g}_{i*}$ where $\tilde{g}_i = k^{-d_i} g_i$
- predictivity requires that only a finite number of couplings is arbitrary
- The RG trajectories that flow into the FP for $t = \log \frac{k}{k_0} \to \infty$ form the UV critical surface.

Asymptotic Safety I

- If (1) a FP exist. and (2) UV critical surface is finite-dimensional. then (a) reaction rates remain finite (in units of k) for $t \to \infty$ and (b) theory is predictive


$\tilde{g}_3$
$\tilde{g}_2$
$\delta n_j = b\,\delta$


$\tilde{g}_3$
$\delta n_J = b\,\delta\rho$
$\tilde{g}_2$
$\tilde{g}_1$

Asymptotic Safety II

- Linearize flow around FP: $M_{ij}\big|_* = \frac{\partial \tilde{\beta}_i}{\partial \tilde{g}_j}\big|_*$
- positive critical exponent = negative eigenvalue = UV attractive = relevant
- negative critical exponent = positive eigenvalue = UV repulsive = irrelevant
- Example: QCD. Gaußian Fixed Point at $\tilde{g}_{i*} = 0$.
- $\tilde{\beta}_i = \partial_t \tilde{g}_i = -d_i \tilde{g}_i + k^{-d_i} \beta_i$
- $M_{ij}\big|_* = -d_i \delta_{ij}$
- UV critical surface=span{renormalizable couplings}.

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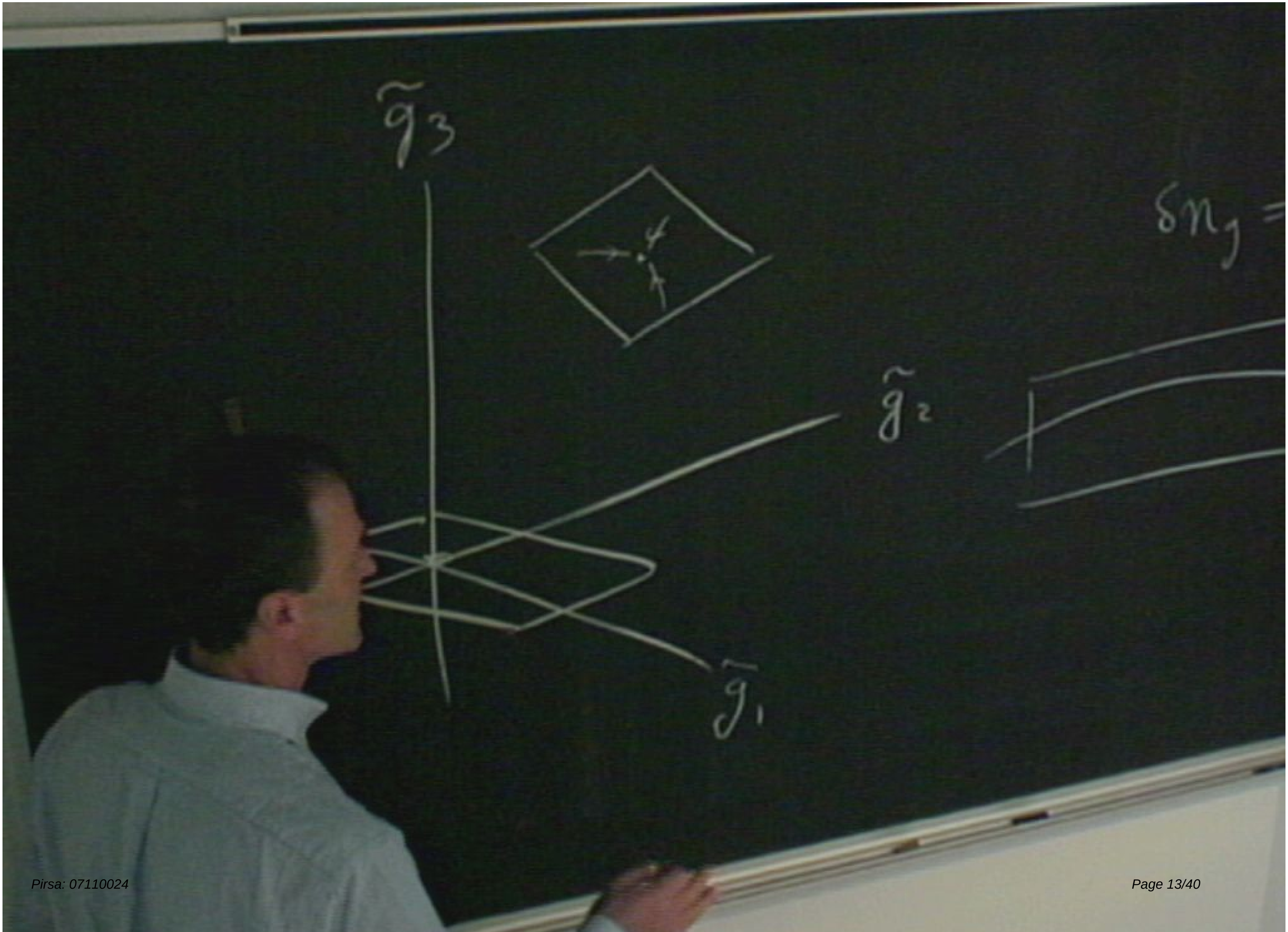

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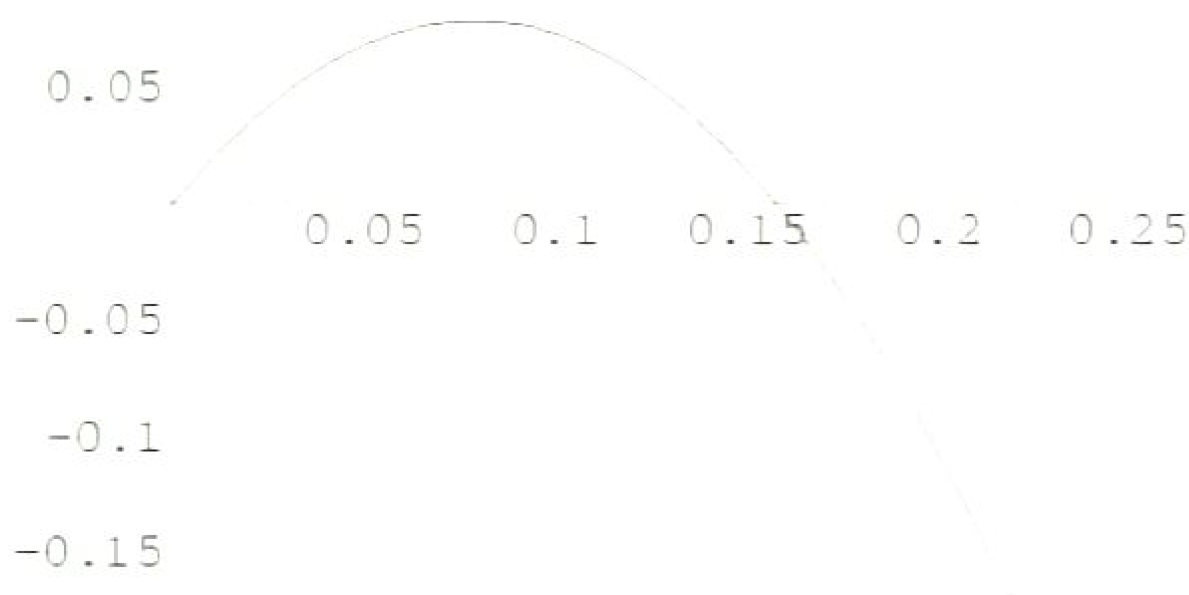
Gravity in $2+\epsilon$

$$\tilde{G} = Gk^{\epsilon}$$

$$\beta_{\tilde{G}} = \epsilon\tilde{G} - \frac{38}{3}\tilde{G}^2$$



Can we take $\epsilon \to 2$?

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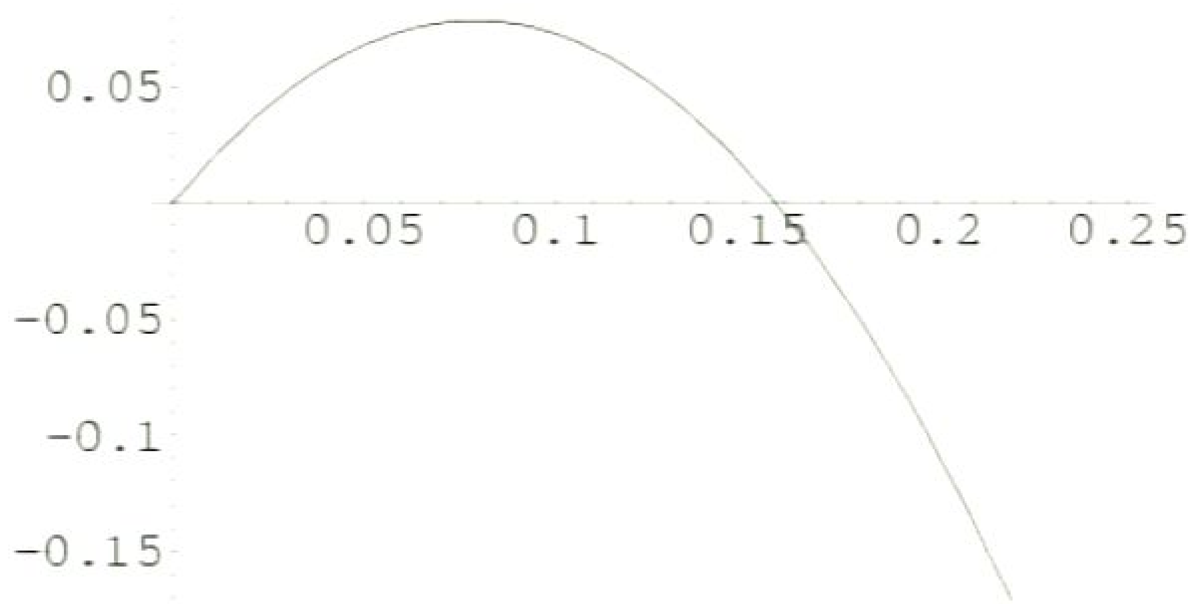

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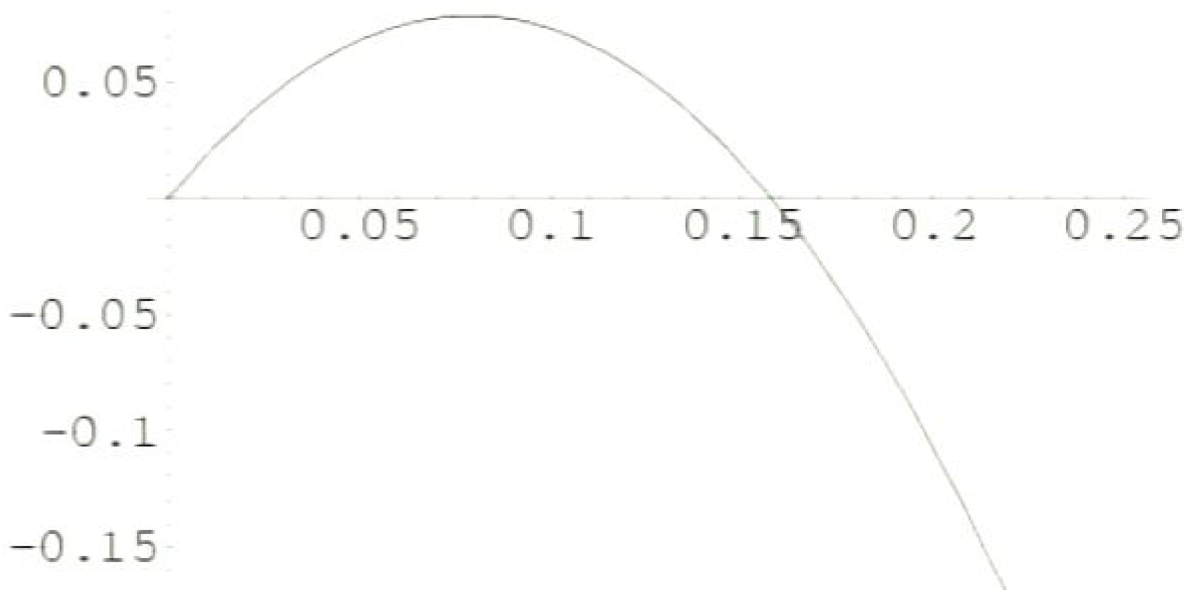

Can we take $\epsilon \to 2$?

One Loop Corrections in Einstein's Theory

$$G(r) = G\left[1 - \frac{167}{30\pi}\frac{G\hbar}{r^2}\right]$$

[N.E.J. Bjerrum-Bohr, J.F. Donoghue and B.R. Holstein, Phys. Rev. D 68, 084005 (2003)]

$$\beta_{\tilde{G}} = 2\tilde{G} - \frac{167}{15\pi}\tilde{G}^2$$

ERGE I

$$e^{-W_k[J]} = \int (D\phi) \mathrm{e}xp(-(S + \Delta S_k + \int J\phi))$$

$$\Delta S_k(\phi) = \frac{1}{2} \int d^4 q \phi(-q) R_k(q^2) \phi(q)$$

modified inverse propagator $P_k(q^2) = q^2 + R_k(q^2)$

$$\tilde{\Gamma}_k[\phi] = W_k[J] - \int J\phi$$

$$\Gamma_k[\phi] = \tilde{\Gamma}_k[\phi] - \Delta S_k$$

ERGE II

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \left(\frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi} + R_k \right)^{-1} \partial_t R_k$$

[C. Wetterich, Phys. Lett. B **301** (1993) 90.]

[T. Morris, Phys. Lett. B **329** (1994) 241.]

ERGE III: One loop

$$\Gamma^{(1)} = \frac{1}{2} \ln \det \frac{\delta^2 S}{\delta\phi\delta\phi}$$

$$\partial_t \Gamma_k^{(1)} = \frac{1}{2} \mathrm{Tr} \left(\frac{\delta^2 S}{\delta\phi\delta\phi} + R_k \right)^{-1} \partial_t R_k$$

ERGE IV: Beta functions

$$\Gamma_k(\phi) = \sum_i g_i(k)\mathcal{O}_i(\phi)$$

$$\partial_t \Gamma_k = \sum_i \partial_t g_i \mathcal{O}_i = \sum_i \beta_{g_i} \mathcal{O}_i$$

compare with

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left(\frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi} + R_k\right)^{-1} \partial_t R_k$$

read off beta functions.

ERGE IVGRAVITY

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$S_{GF}(h, \bar{g}) = \int d^4x \sqrt{\bar{g}}\, \chi_\mu \bar{g}^{\mu\nu} \chi_\nu$$

$$\chi_\nu = \bar{\nabla}^\mu h_{\mu\nu} + \beta \bar{\nabla}_\nu h$$

$$\Delta S_k(h, \bar{g}) = \frac{1}{2} \int d^4x \sqrt{\bar{g}}\; h_{\mu\nu} \bar{g}^{\mu\rho} \bar{g}^{\nu\sigma} R_k(-\bar{\nabla}^2) h_{\rho\sigma}$$

$$\Gamma_k(\bar{g}_{\mu\nu}, \langle h_{\mu\nu} \rangle)$$

$$\Gamma_k(g_{\mu\nu}) = \Gamma_k(g_{\mu\nu}, 0)$$

[M. Reuter, Phys. Rev. D **57** 971(1998)]

[D. Dou and R. P., Class. and Quantum Grav. **15** 3449 (1998)]

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Heat Kernel Technique I

$$\mathrm{Tr}\, W(\Delta) = Q_{\frac{d}{2}}(W) B_0(\Delta) + Q_{\frac{d}{2}-1}(W) B_2(\Delta) + \ldots + Q_0(W) B_{2d}(\Delta) + \ldots$$

$$Q_n[W] = \frac{1}{\Gamma(n)} \int_0^\infty dz\, z^{n-1} W(z) \; ; \qquad Q_{-n}[W] = (-1)^n W^{(n)}(0) \; . \qquad n > 0$$

[A.H. Chamseddine and A. Connes, Comm. Math. Phys. **186**, 731 (1997)]

[M. Reuter, Phys. Rev. D **57**, 971 (1998)]

Heat Kernel Technique II

$$
\begin{aligned}
\partial_t \Gamma_k &= \frac{1}{2}\mathrm{Tr}\left(\frac{\partial_t R_k(\Delta)}{P_k(\Delta)}\right) \\
&= \frac{1}{2}\left[Q_{\frac{d}{2}}\left(\frac{\partial_t R_k}{P_k}\right) B_0(\Delta) + Q_{\frac{d}{2}-1}\left(\frac{\partial_t R_k}{P_k}\right) B_2(\Delta) + \ldots\right]
\end{aligned}
$$

with optimized cutoff $R_k(z) = (k^2 - z)\theta(k^2 - z)$

[D.Litim, Phys.Rev.D64:105007,2001]

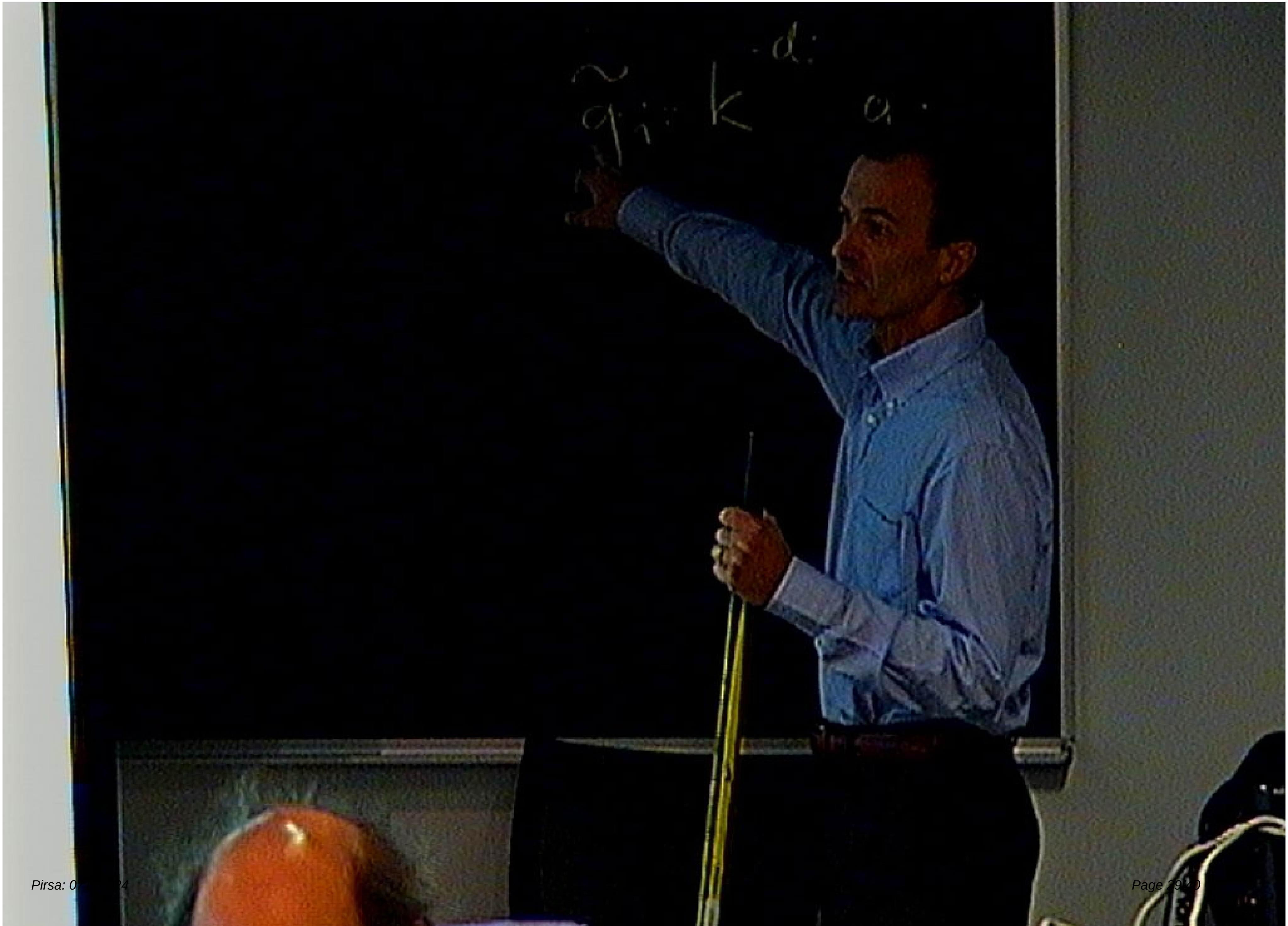
$$
Q_n\left(\frac{\partial_t P}{P^\ell}\right) = \frac{2}{n}k^{2(n-\ell+1)} \; ; \qquad Q_0\left(\frac{\partial_t P}{P^\ell}\right) = 2k^{2(-\ell+1)}
$$

Einstein–Hilbert truncation I

$$\Gamma_k(g) = \frac{1}{16\pi G}\int dx\sqrt{g}(2\Lambda - R)$$

$$\beta_{\tilde{\Lambda}} = \frac{-2(1-2\tilde{\Lambda})^2\tilde{\Lambda} + \frac{36-41\tilde{\Lambda}+42\tilde{\Lambda}^2-600\tilde{\Lambda}^3}{72\pi}\tilde{G} + \frac{467-572\tilde{\Lambda}}{288\pi^2}\tilde{G}^2}{(1-2\tilde{\Lambda})^2 - \frac{29-9\tilde{\Lambda}}{72\pi}\tilde{G}}$$

$$\beta_{\tilde{G}} = \frac{2(1-2\tilde{\Lambda})^2\tilde{G} - \frac{373-654\tilde{\Lambda}+600\tilde{\Lambda}^2}{72\pi}\tilde{G}^2}{(1-2\tilde{\Lambda})^2 - \frac{29-9\tilde{\Lambda}}{72\pi}\tilde{G}}$$



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Summary and Conclusions

- Asymptotic Safety of gravity plausible
- Not exclusive
- Bottom up approach
- Relate to low–energy physics: reproduce Randall-Sundrum I model; applications to cosmology and astrophysics; identify RG trajectory corresponding to real world; predictions for cross sections at colliders.
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FAQs and Bibliography

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Scalar + gravity

$$\Gamma_k = \int d^4x \sqrt{g}\left(V(\phi^2) - F(\phi^2)R + \tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)$$

$$V(\phi^2) = \sum_{n=0}^{\infty}\lambda_{(2n)}\phi^{2n}$$

$$F(\phi^2) = \sum_{n=0}^{\infty}\xi_{(2n)}\phi^{2n}$$

$\lambda_0 = 2Z\Lambda$, $\xi_0 = Z = \frac{1}{16\pi G}$ $\lambda_2 = \frac{m^2}{2}$, $\xi_2 = \xi$, $\lambda_4 = \lambda$, ...

[R. P. and D. Perini, Phys. Rev. D **68**, 044018 (2003).]

Gaussian-Matter FP: $\tilde{\lambda}_{2n} = \tilde{\xi}_{2n} = 0$ for $n \geq 1$

$\tilde{G}_* = 0.359$ and $\tilde{\Lambda}_* = 0.320$

Elementary Consequence I

$$k' = k\sqrt{G} = \sqrt{\tilde{G}}$$

If $\tilde{G} \to \tilde{G}_*$, $k' \to k'_* = \sqrt{\tilde{G}_*}$

the cutoff is bounded in Planck units

Need not be true in other units

$k'' = \frac{k}{m}$.

there may be a FP where $\tilde{m} = \frac{m}{k} \to 0$.

No built-in cutoff in units of m.

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Elementary Consequence II

Since $Z_g = \frac{1}{16\pi G} = \tilde{Z}_g k^2$

$$\eta_g = \frac{1}{Z_g} k \frac{\partial Z_g}{\partial k} = \frac{1}{\tilde{Z}_g} k \frac{\partial \tilde{Z}_g}{\partial k} + 2$$

so at a FP

$$\eta_{g*} = 2$$

then inverse propagator $\approx q^{2+\eta} = q^4$

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