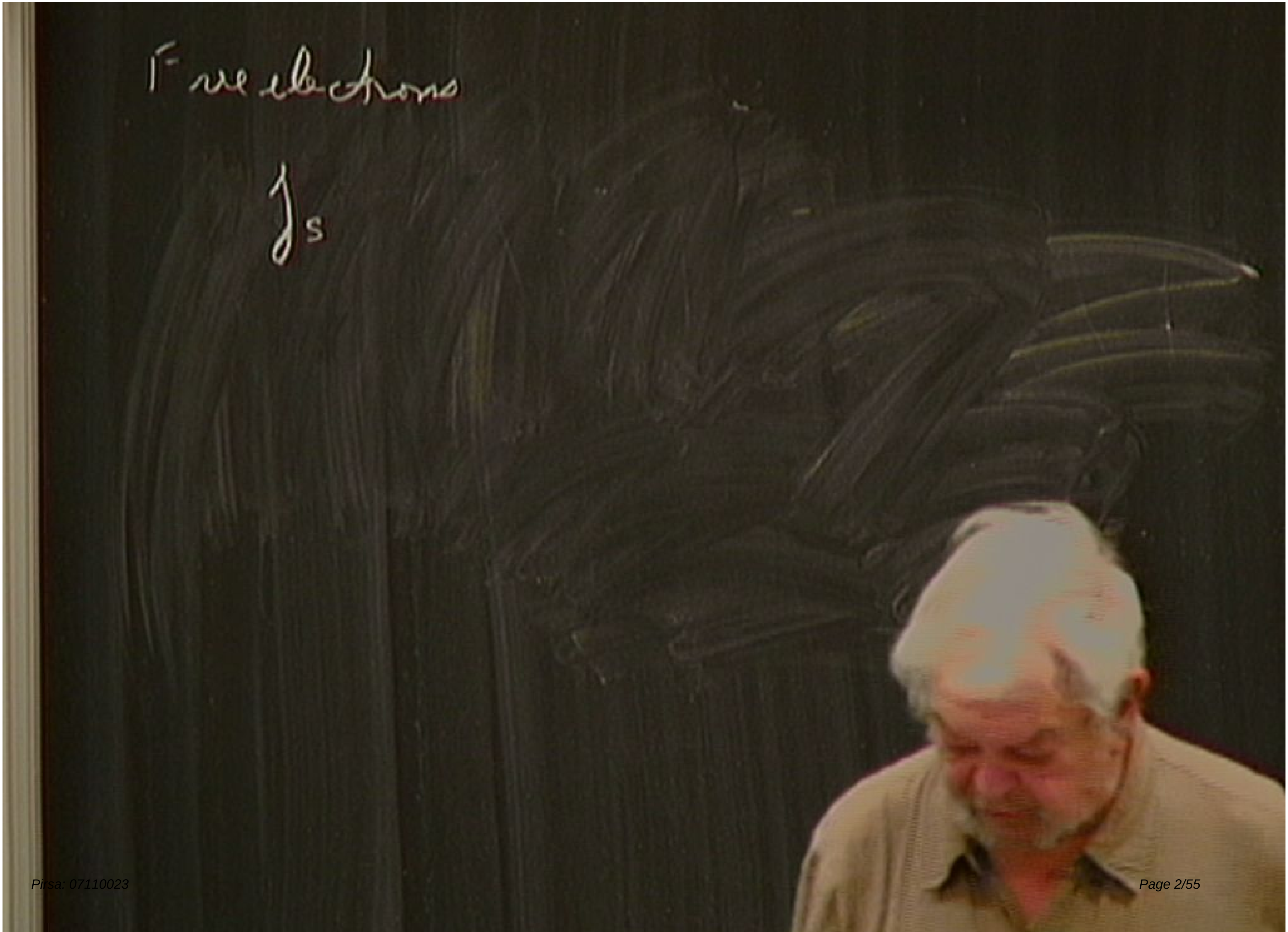
Title: Spontaneous Broken Symmetry 5A

Date: Nov 29, 2007 10:00 AM

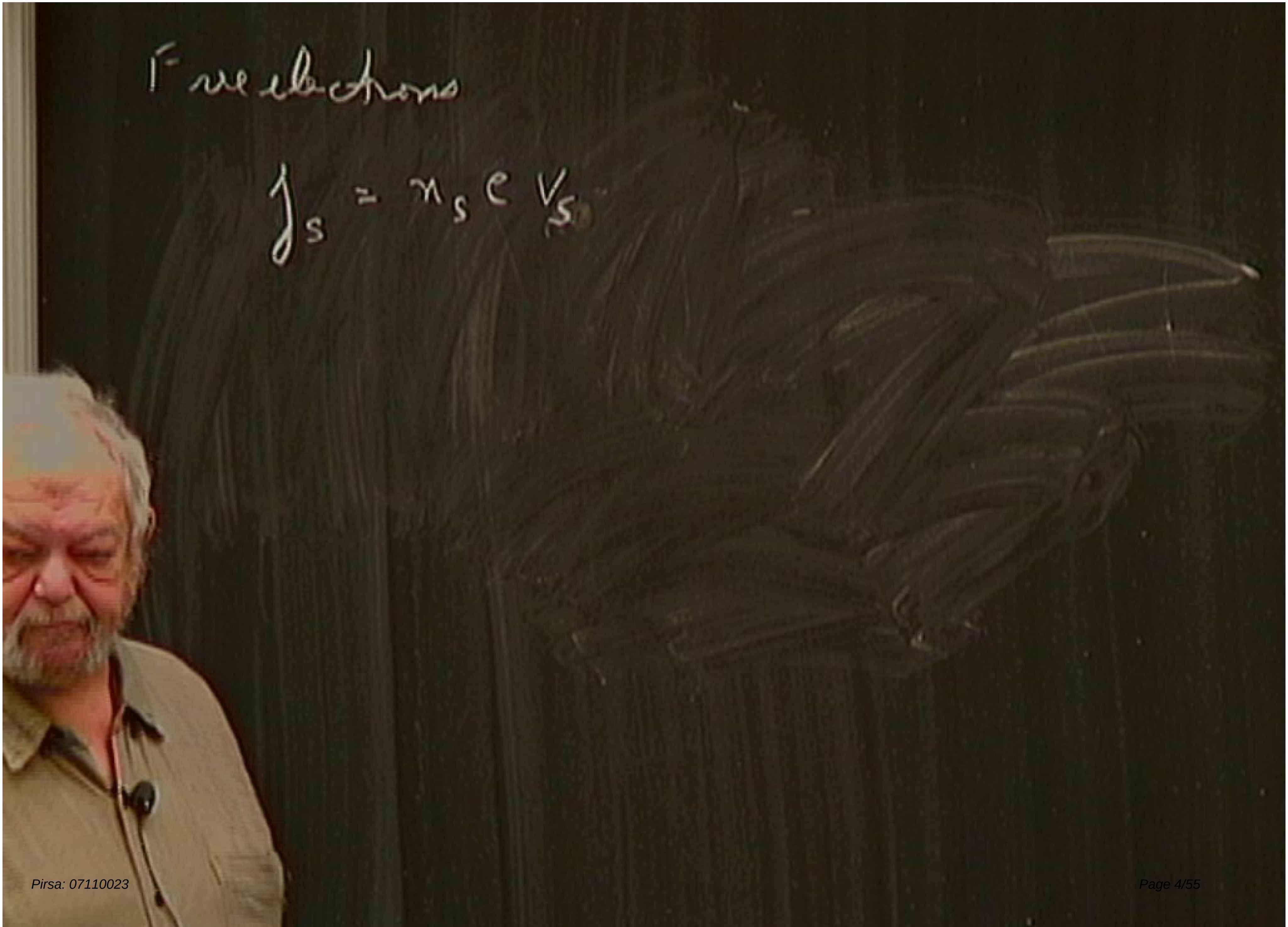
URL: http://pirsa.org/07110023

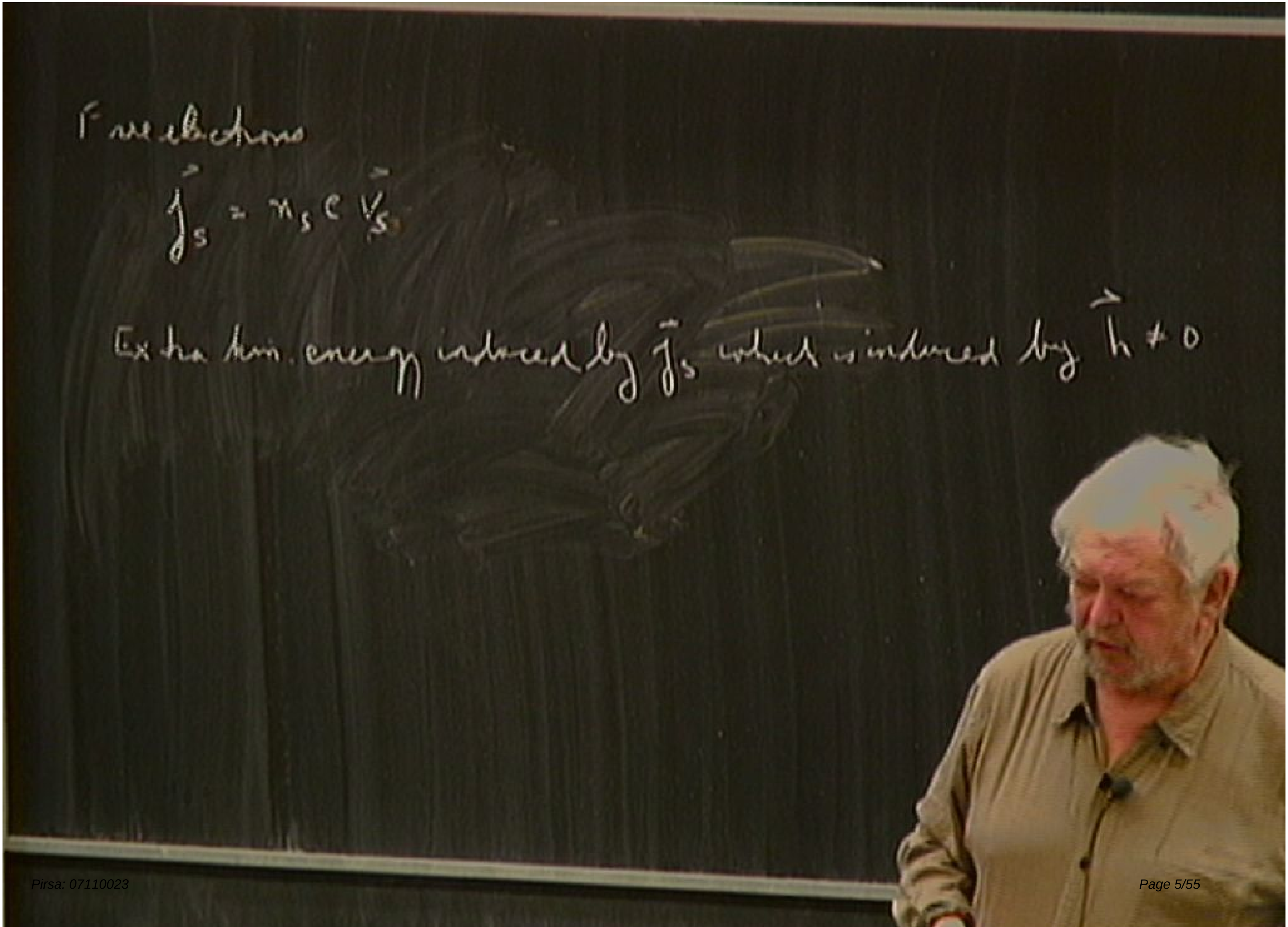
Abstract:

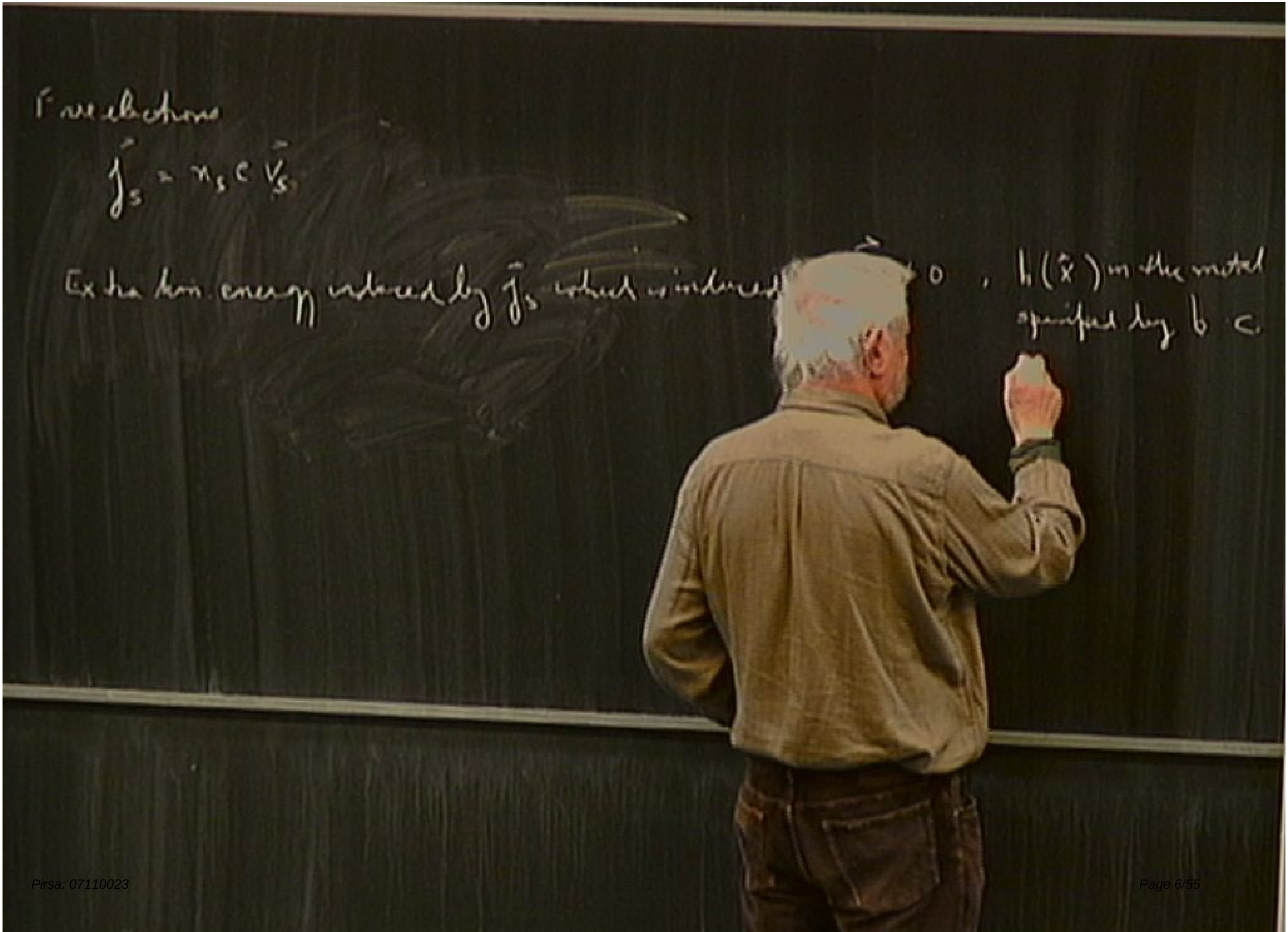

Free electrons
js

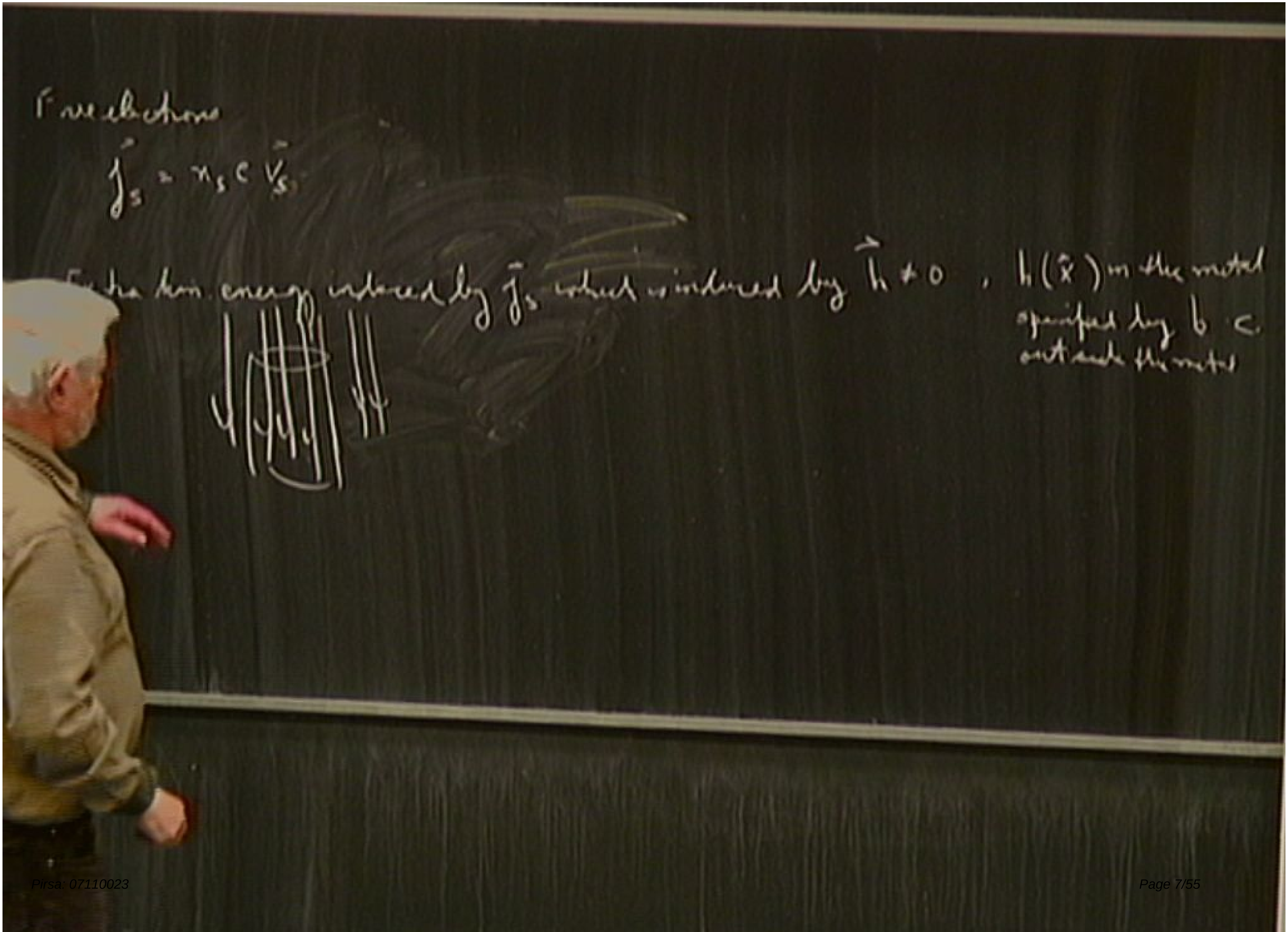
Free electrons

$$j_s = n_s$$


Free electrons
$j_s = n_s e v_s$


Free electrons
$\vec{j}_s = n_s e \vec{v}_s$
Extra kin. energy induced by $\vec{j}_s$ which is induced by $\vec{h} \neq 0$

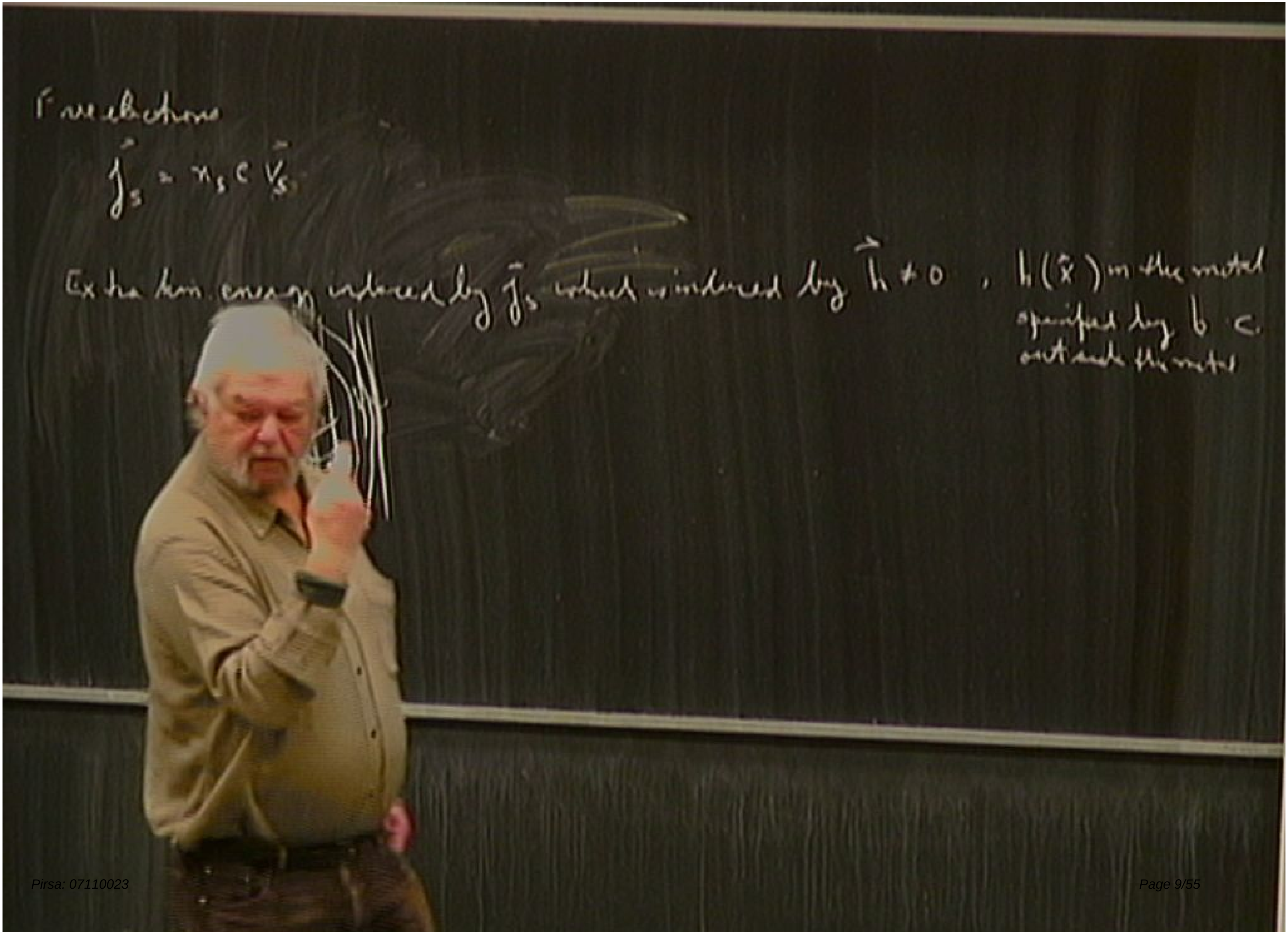

Free electrons
$\vec{j}_s = n_s e \vec{v}_s$
Extra kin. energy induced by $\vec{j}_s$ which is induced
0 , $b(\vec{x})$ in the metal
specified by b. c.

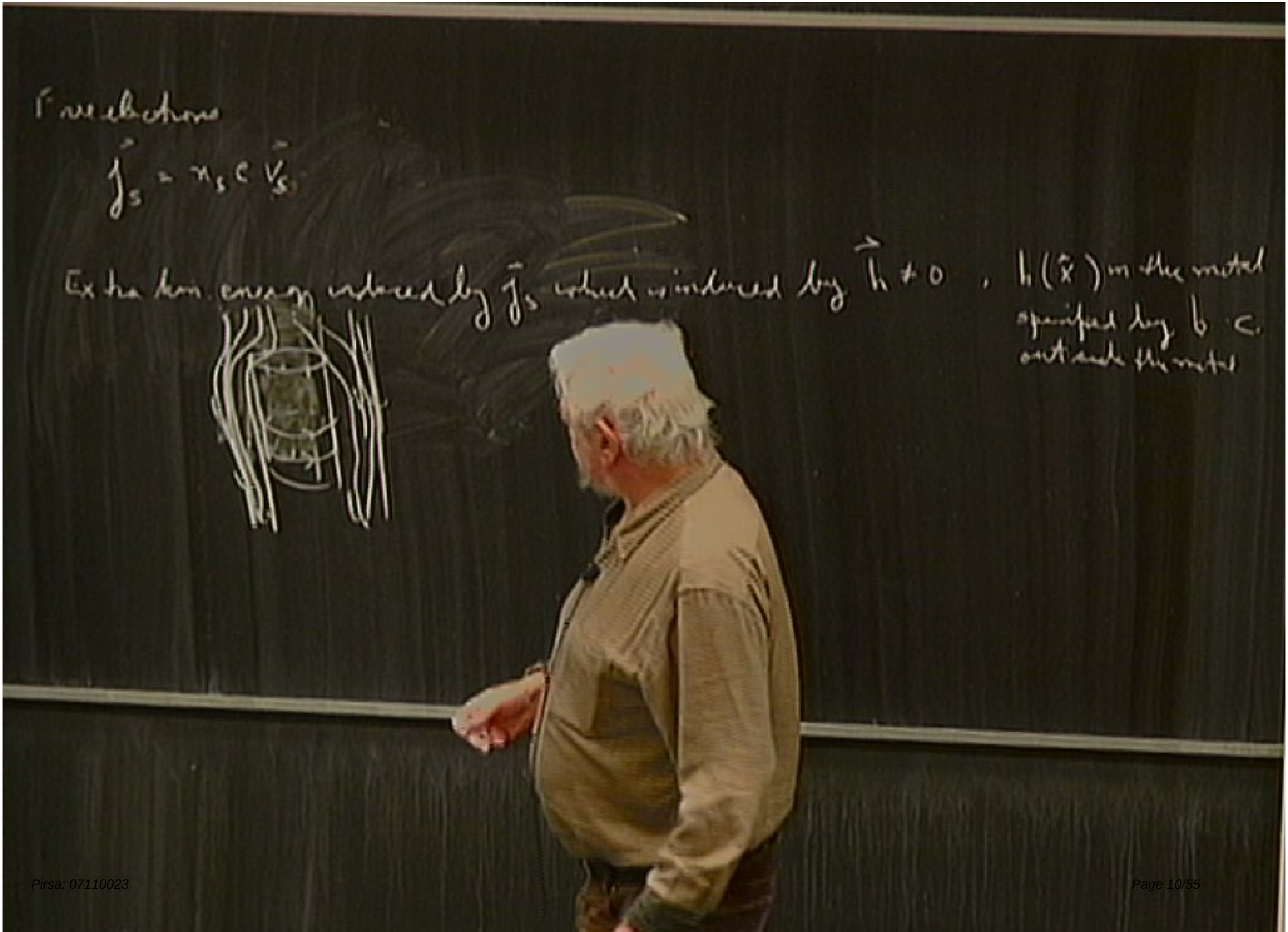

Free electrons
$\vec{j}_s = n_s e \vec{v}_s$
Extra kin. energy induced by $\vec{j}_s$ which is induced by $\vec{h} \neq 0$, $h(\vec{x})$ in the metal
specified by b.c.
outside the metal

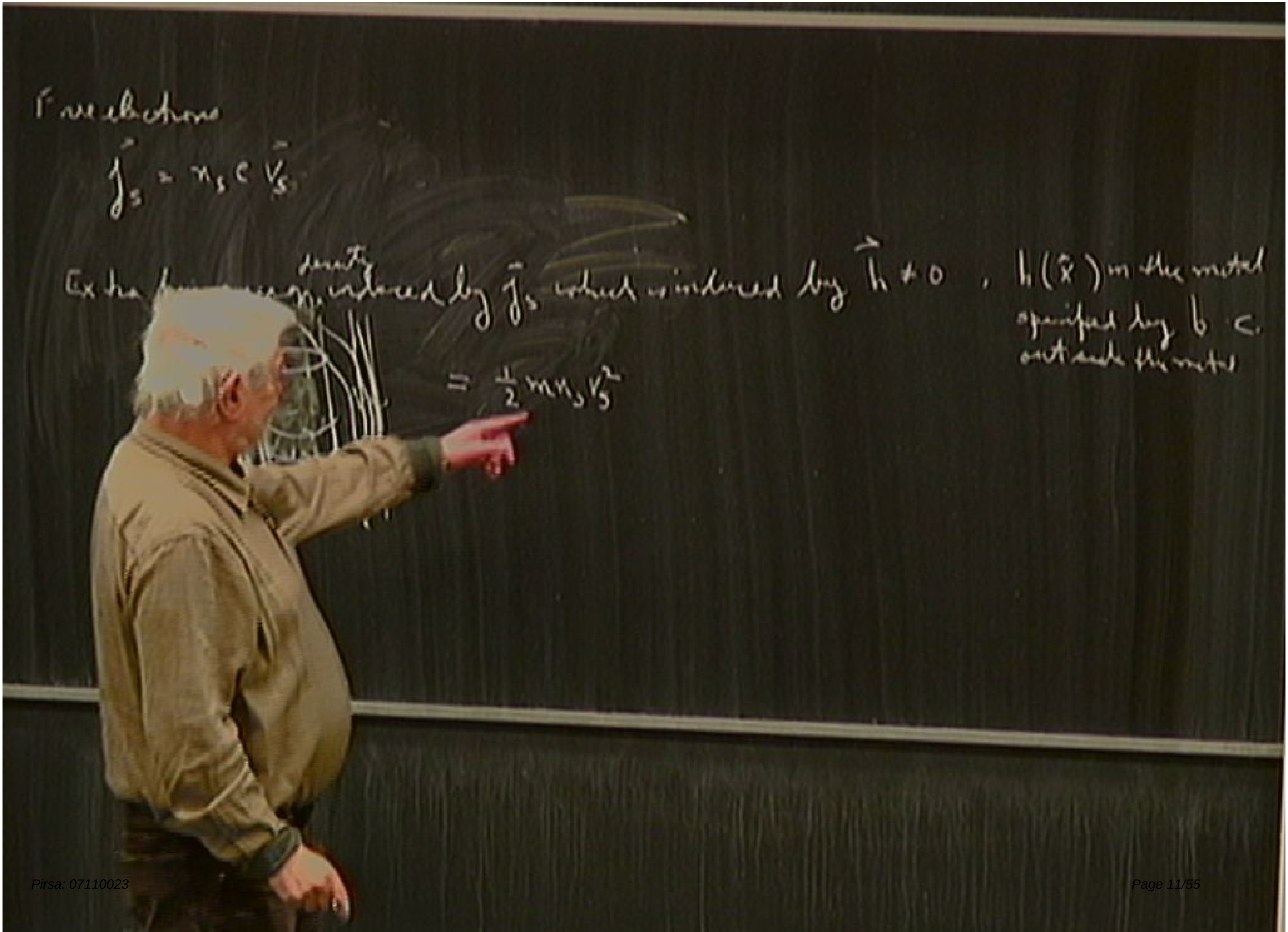
Free electrons

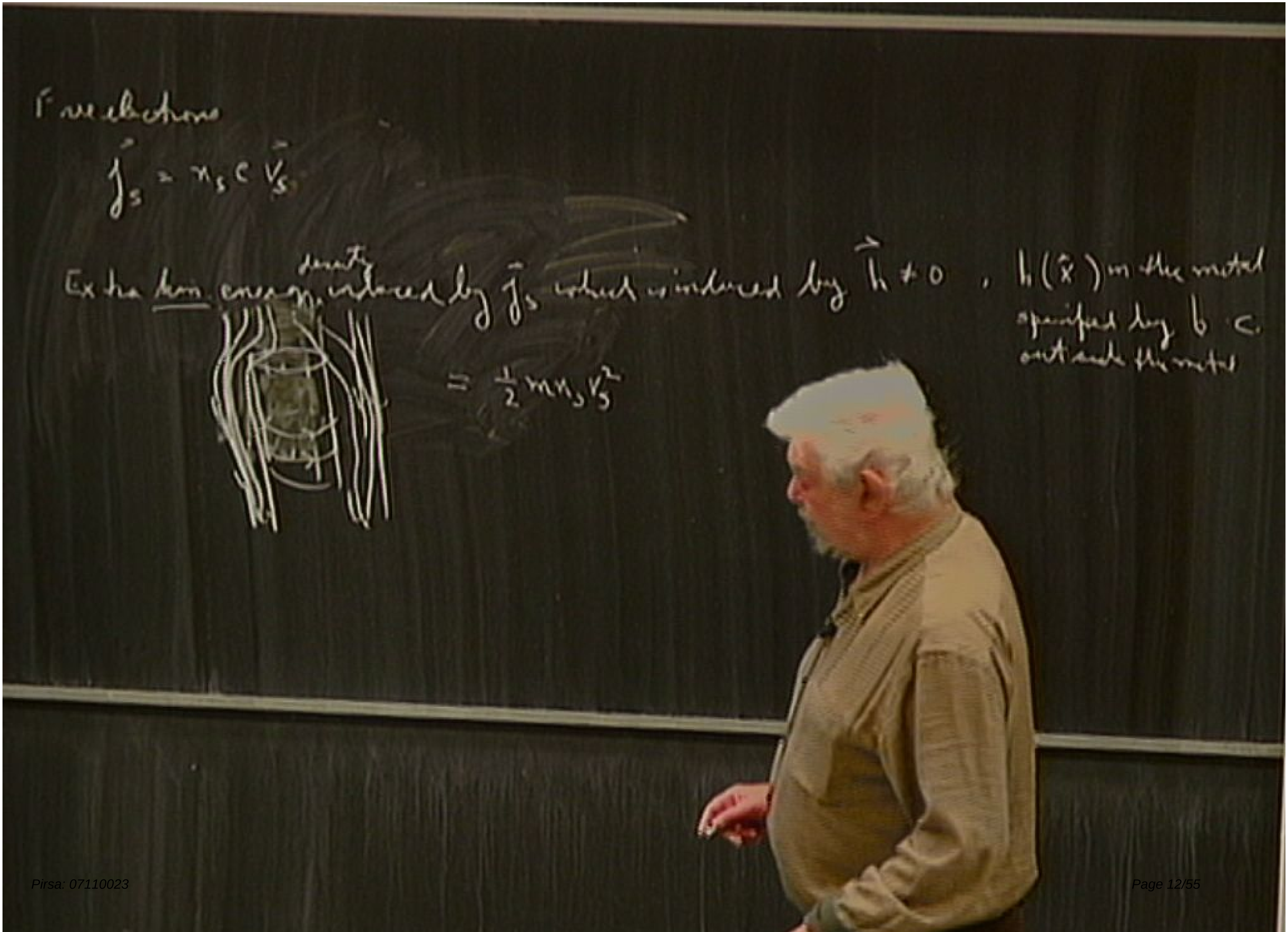
$$\vec{j}_s = n_s e \vec{v}_s$$

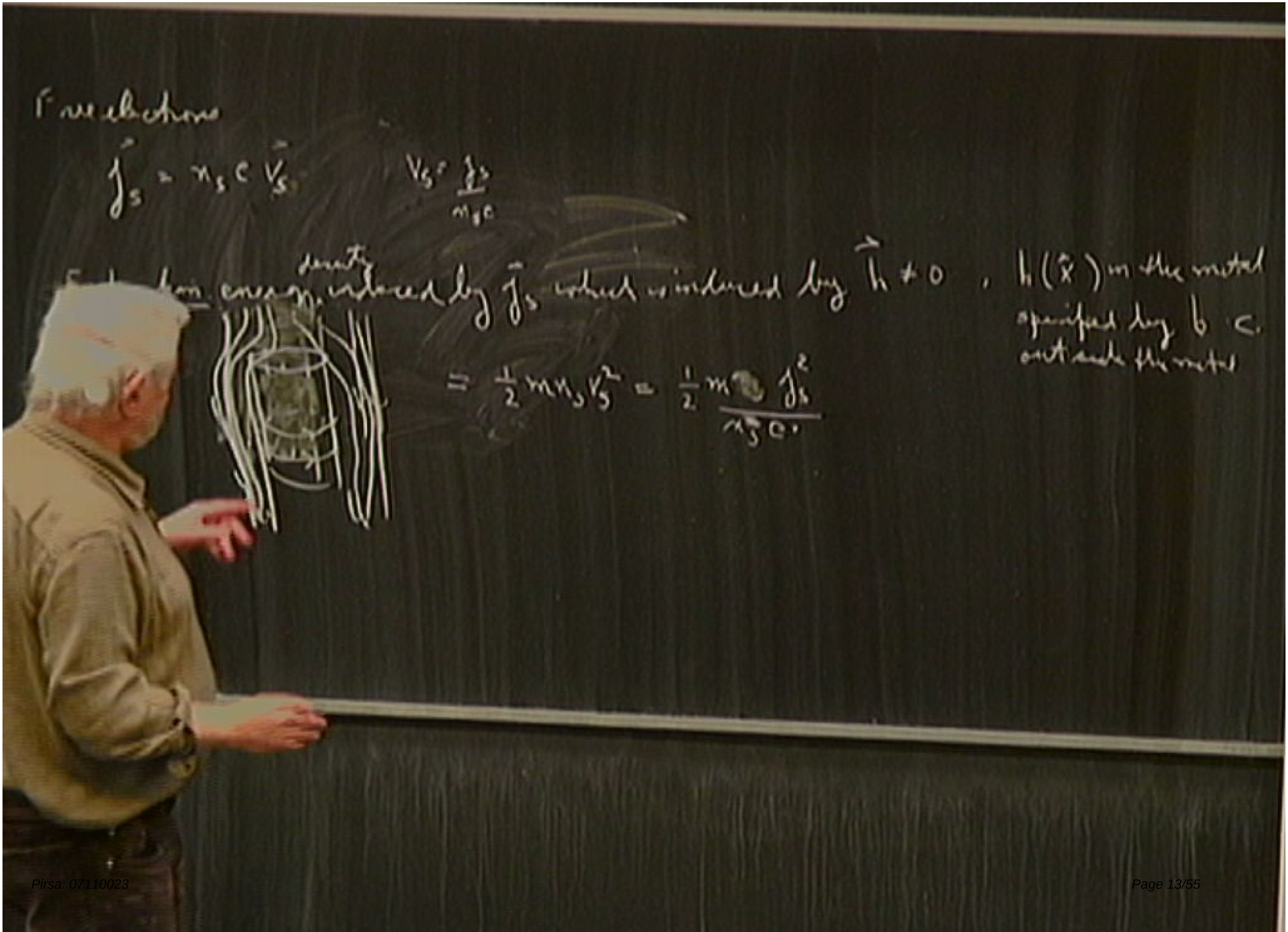
Extra kin. energy induced by $\vec{j}_s$ which is induced by $\vec{h} \neq 0$, $h(\vec{x})$ in the metal specified by b.c. outside the metal

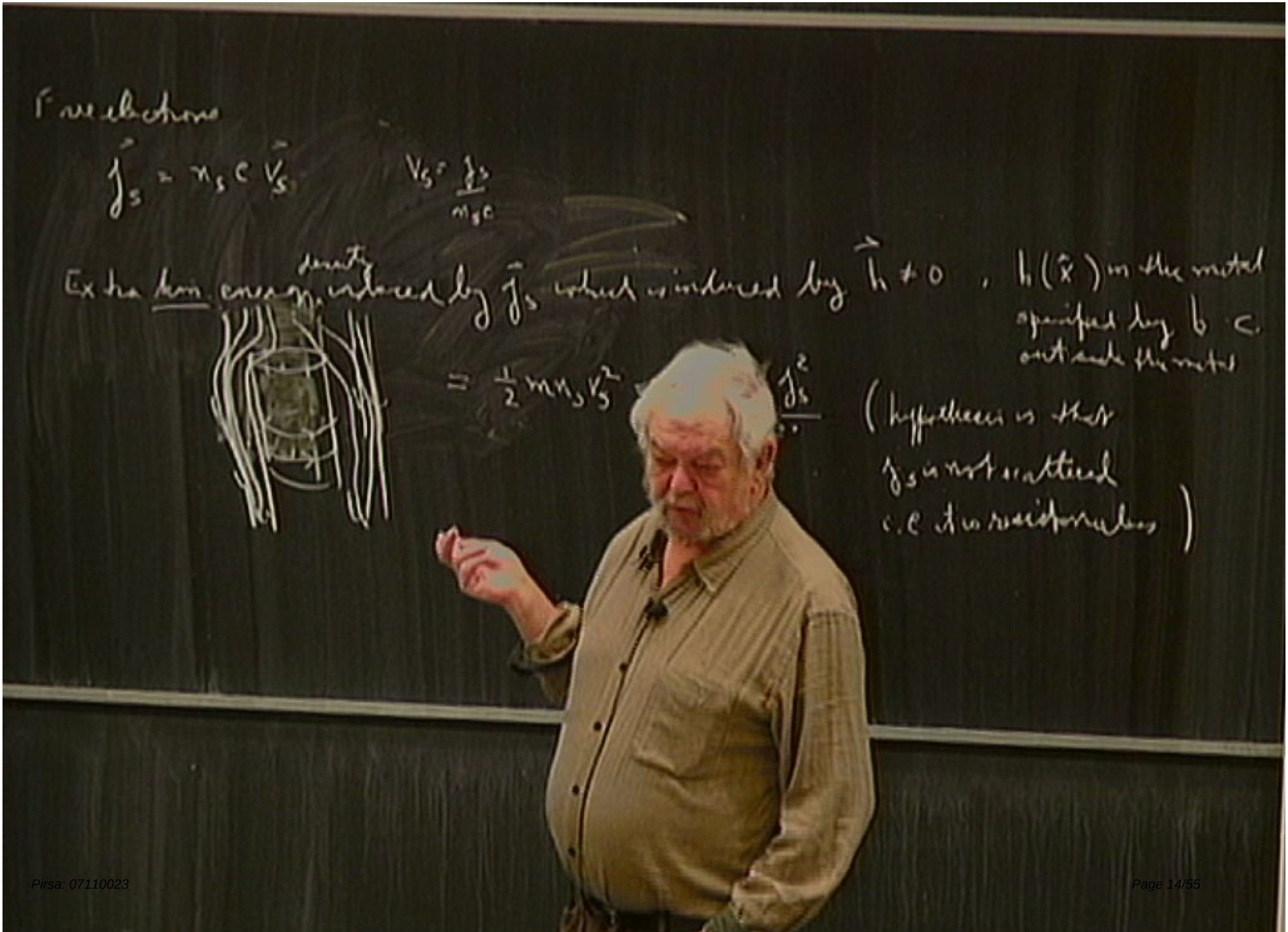

Free electrons
j_s = n_s e v_s
Extra kin. energy induced by j_s which is induced by h ≠ 0 , h(x) in the metal
specified by b.c.
outside the metal

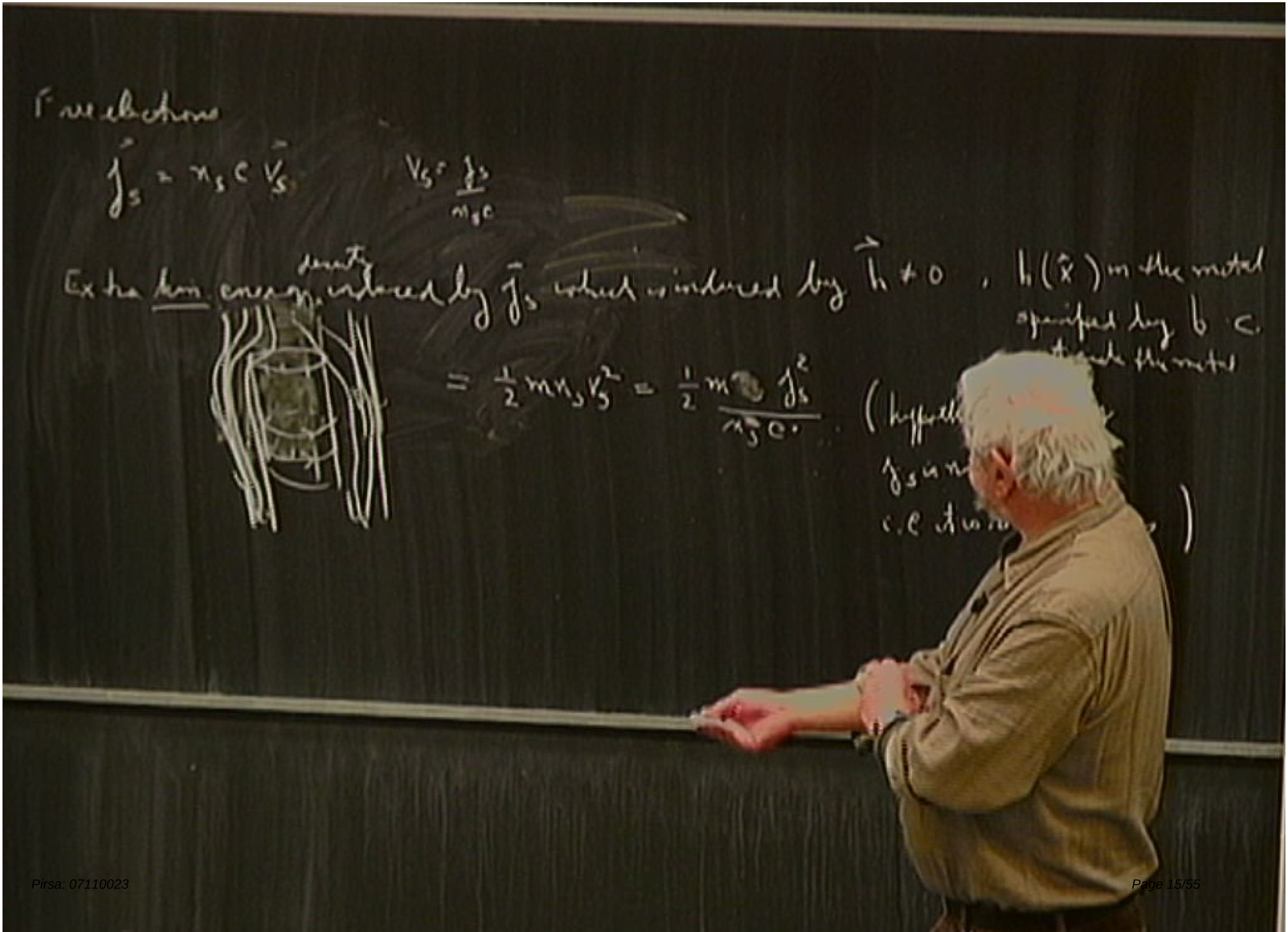

Free electrons
$\vec{j}_s = n_s e \vec{v}_s$
Extra kin. energy induced by $\vec{j}_s$ which is induced by $\vec{h} \neq 0$
$h(\vec{x})$ in the metal
specified by b.c.
outside the metal

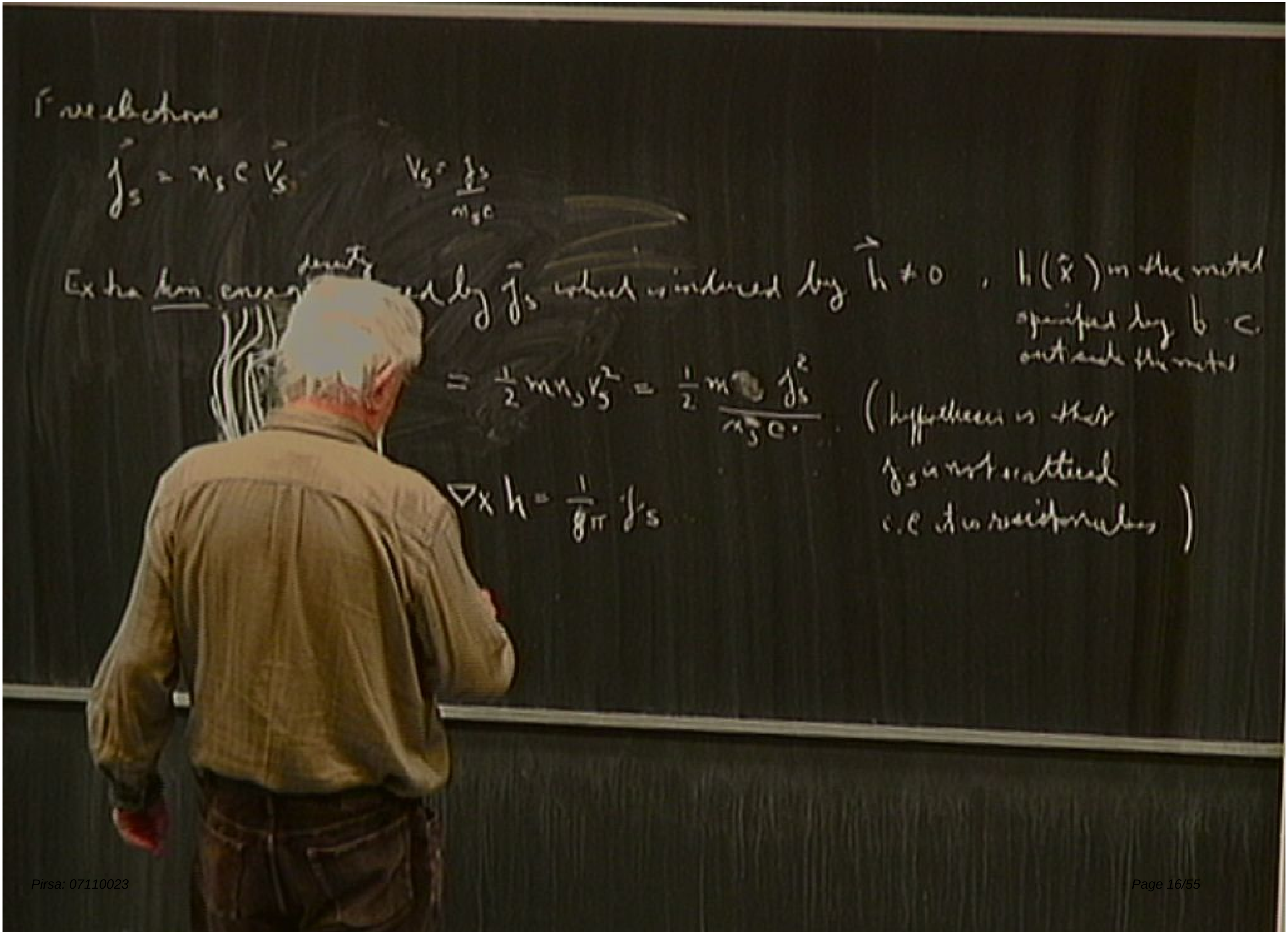

Free electrons
$\vec{j}_s = n_s e \vec{v}_s$
density
induced by $\vec{j}_s$ which is induced by $\vec{h} \neq 0$
$h(\vec{x})$ in the metal
outside the metal
$= \frac{1}{2} m n_s v_s^2$


Free electrons
$\vec{j}_s = n_s e \vec{v}_s$
Extra kin energy density induced by $\vec{j}_s$ which is induced by $\vec{h} \neq 0$, $h(\vec{x})$ in the metal specified by b.c. outside the metal
$= \frac{1}{2} m n_s v_s^2$


Free electrons
$\vec{j}_s = n_s e \vec{v}_s$
$v_s = \frac{j_s}{n_s e}$
energy density induced by $\vec{j}_s$ which is induced by $\vec{h} \neq 0$
$h(\vec{x})$ in the metal specified by b.c. outside the metal
$= \frac{1}{2} m n_s v_s^2 = \frac{1}{2} m \frac{j_s^2}{n_s e^2}$


Free electrons
$\vec{j}_s = n_s e \vec{v}_s$
$v_s = \frac{j_s}{n_s e}$
Extra kin. energy density induced by $\vec{j}_s$ which is induced by $\vec{h} \neq 0$, $h(\vec{x})$ in the metal specified by b.c. outside the metal
$= \frac{1}{2} m n_s v_s^2$
(hypothesis is that j_s is not scattered


Free electrons
j_s = n_s e v_s
v_s = j_s / n_s e
Extra kin energy density induced by j_s which is induced by h ≠ 0, h(x) in the metal specified by b.c.
through the metal
= 1/2 m n_s v_s^2 = 1/2 m j_s^2 / n_s e^2


Free electrons
$\vec{j}_s = n_s e \vec{v}_s$
$v_s = \frac{j_s}{n_s e}$
density
Extra kin energ ... ed by j_s which is induced by $\vec{h} \neq 0$, $h(\vec{x})$ in the metal
specified by b.c.
outside the metal
$= \frac{1}{2} m n_s v_s^2 = \frac{1}{2} m \frac{j_s^2}{n_s e^2}$
(hypothesis is that
j_s is not scattered
i.e. there is no resistance)
$\nabla \times h = \frac{1}{8\pi} j_s$

Free electrons

$$\vec{j}_s = n_s e \vec{v}_s \qquad v_s = \frac{j_s}{n_s e}$$

Extra kin. energy density induced by $\vec{j}_s$ which is induced by $\vec{h} \neq 0$, $h(\vec{x})$ in the metal specified by b.c. outside the metal

$$= \frac{1}{2} m n_s v_s^2 = \frac{1}{2} m \frac{j_s^2}{n_s e^2}$$

(hypothesis is that j_s is not scattered i.e. there's no resistance)

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Free electrons

$$\vec{j}_s = n_s e \vec{v}_s \qquad v_s = \frac{j_s}{n_s e}$$

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$$= \frac{1}{2} m n_s v_s^2 = \frac{1}{2} m \frac{j_s^2}{n_s e^2}$$

(hypothesis is that j_s is not scattered i.e. there's no resistance)

$$\nabla \times h = \frac{1}{8\pi} j_s$$

Variational base for the eq. for h.

$$F = \int \frac{h^2}{8\pi}$$

Variational base for the eq. for h.

$$F = \int \frac{h^2}{8\pi} + \frac{4\pi m}{n_s e^2}\, j_s \, j_s^{2}$$

$$= \frac{1}{8\pi}\int \left[h^2 + \frac{m}{4\pi n_s e^2}\, \nabla \times h \right]^2$$

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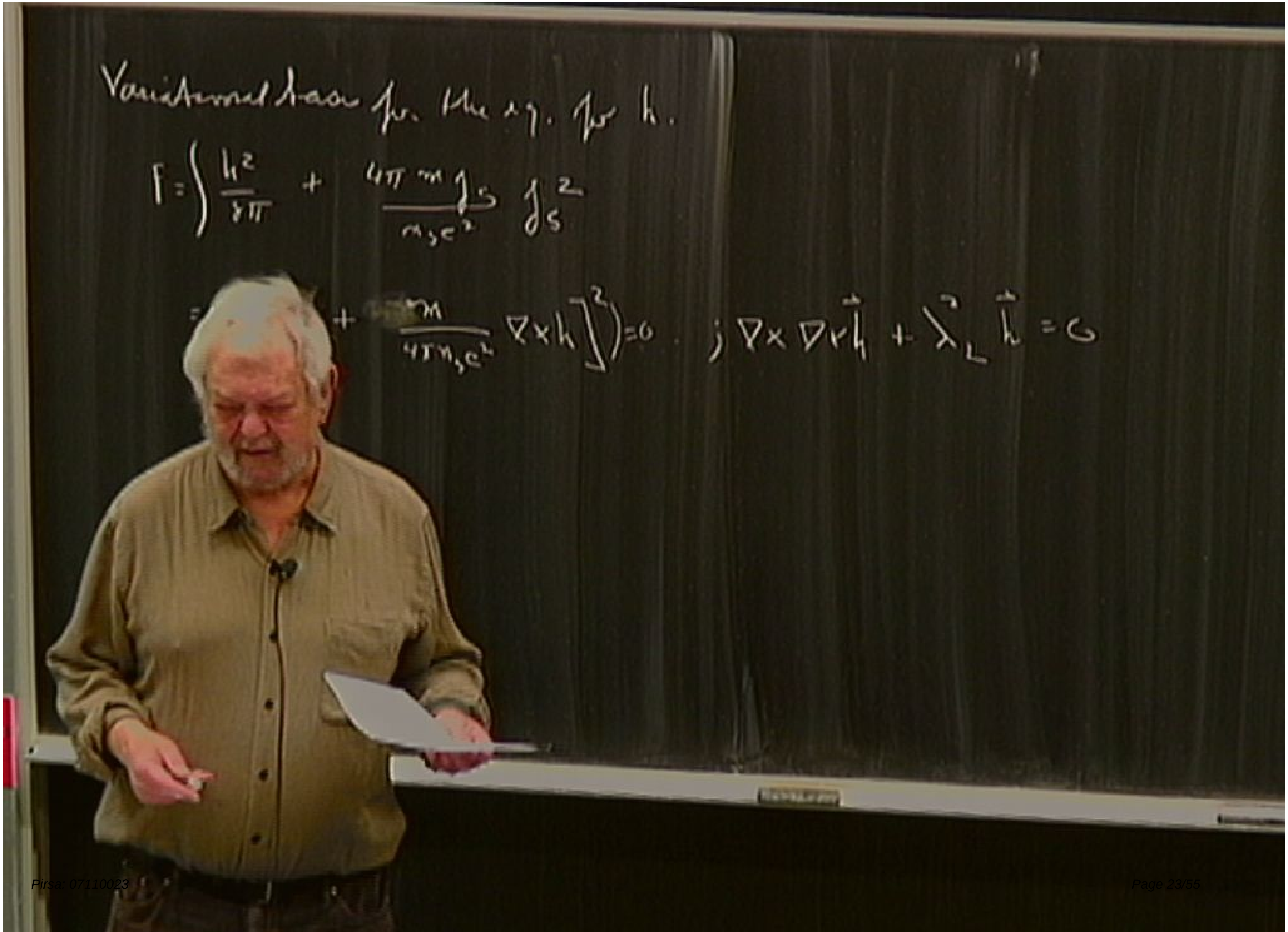
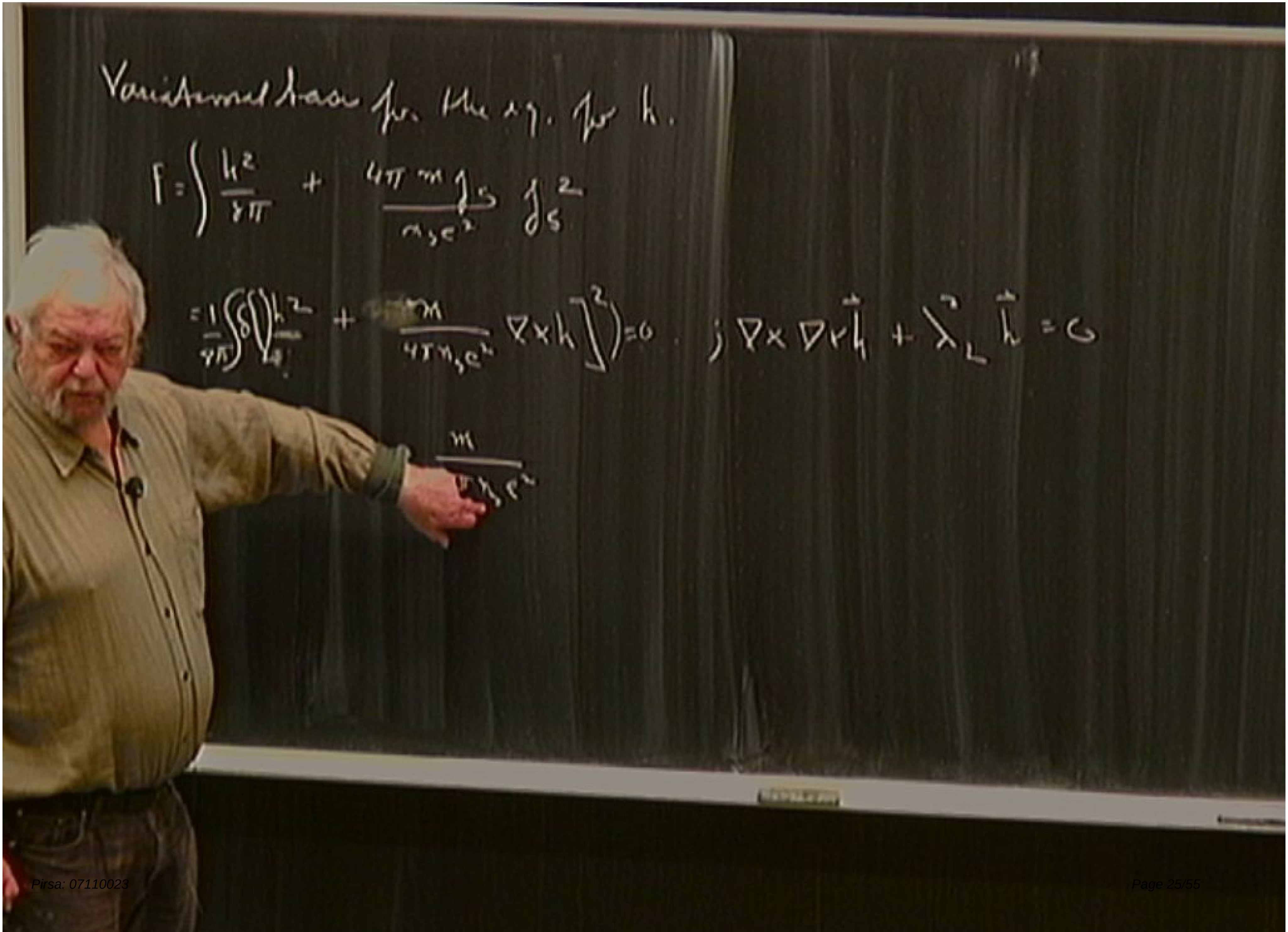
$$= \frac{1}{8\pi}\int \delta\left(h^2 + \frac{m}{4\pi n_s e^2}\left[\nabla \times h\right]^2 \right) = 0 \quad ; \quad \nabla \times \nabla \ldots$$


Variational basis for the eq. for h.
F = ∫ h²/8π + 4π m/(n_s e²) j_s j_s²
+ m/(4π n_s e²) ∇×h]²) = 0 ; ∇×∇×h + λ_L⁻² h = 0

Variational base for the eq. for h.

$$F = \int \frac{h^2}{8\pi} + \frac{4\pi m}{n_s e^2} j_s \, j_s^2$$

$$= \frac{1}{8\pi} \delta \left(\int \frac{h^2}{8\pi} + \frac{m}{4\pi n_s e^2} \left[\nabla \times h \right]^2 \right) = 0 \quad ; \quad \nabla \times \nabla \times \vec{h} + \lambda_L^{-2} \vec{h} = 0$$


Variational base for the eq. for h.
$F = \int \frac{h^2}{8\pi} + \frac{4\pi m}{n_s e^2} j_s \, j_s^2$
$= \frac{1}{8\pi} \int \delta \left(\frac{h^2}{8\pi} + \frac{m}{4\pi n_s e^2} [\nabla \times h]^2 \right) = 0$
$; \nabla \times \nabla \times \vec{h} + \lambda_L^{-2} \vec{h} = 0$
$\frac{m}{4\pi n_s e^2}$

Variational basis for the eq. for h.

$$F = \int \frac{h^2}{8\pi} + \frac{4\pi m}{n_s e^2} j_s \, j_s^2$$

$$= \frac{1}{4\pi}\int \delta\left(\frac{h^2}{2} + \frac{m}{4\pi n_s e^2}\left[\nabla\times h\right]^2\right) = 0 \quad ; \quad \nabla\times\nabla\times\vec{h} + \lambda_L^2\,\vec{h} = 0$$

$$\lambda_L^2 = \frac{m}{4\pi n_s e^2}$$

Variational basis for the eq. for h.

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$$\lambda_L^2 = \frac{m}{4\pi n_s e^2}$$

;  h_z $\quad \left(\frac{d^2}{dx^2} - \lambda_L^{-2}\right) h_z = 0 \quad h_z = e^{-\lambda_L x}$

x into middle

Variational basis for the eq. for h.

$$F = \int \frac{h^2}{8\pi} + \frac{4\pi m}{n_s e^2} j_s^2$$

$$= \frac{1}{8\pi}\int \delta \left(h^2 + \frac{m}{4\pi n_s e^2} [\nabla \times h]^2 \right) = 0 \quad ; \quad \nabla \times \nabla \times \vec{h} + \lambda_L^{-2} \vec{h} = 0$$

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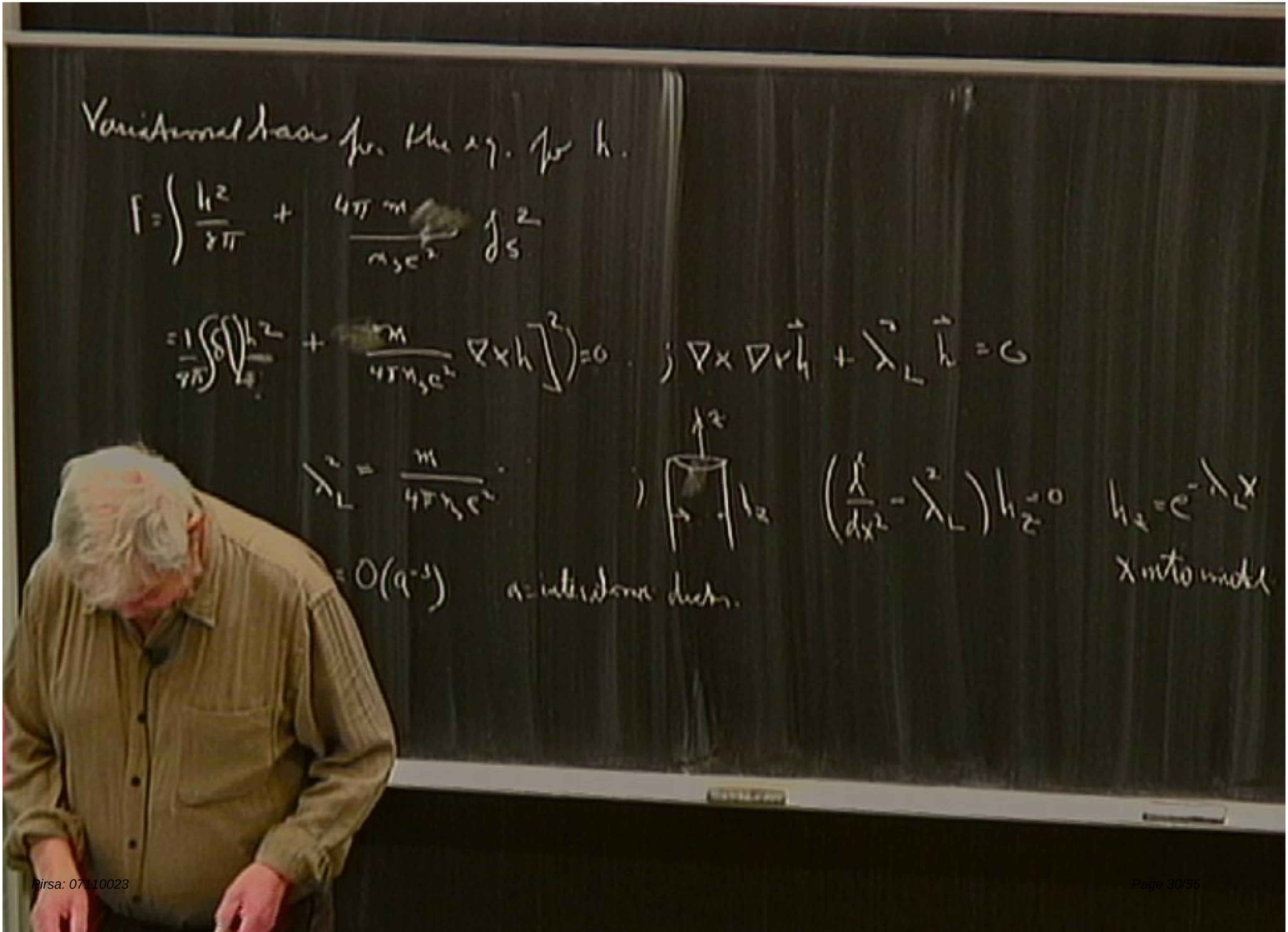
h_z $\quad \left(\frac{d^2}{dx^2} - \lambda_L^{-2} \right) h_z = 0 \quad h_z = e^{-\lambda_L x}$

x into middle


Variational basis for the eq. for h.
$F = \int \frac{h^2}{8\pi} + \frac{4\pi m}{n_s e^2} j_s^2$
$= \frac{1}{8\pi} \int \delta \left(h^2 + \frac{m}{4\pi n_s e^2} [\nabla \times h]^2 \right) = 0$; $\nabla \times \nabla \times \vec{h} + \lambda_L^{-2} \vec{h} = 0$
$\lambda_L^2 = \frac{m}{4\pi n_s e^2}$
$\left(\frac{d^2}{dx^2} - \lambda_L^{-2} \right) h_z = 0$ $h_z = e^{-\lambda_L x}$
$= O(a^{-3})$ a = interatomic dist.
x into metal

Variational basis for the eq. for h.

$$F = \int \frac{h^2}{8\pi} + \frac{4\pi m}{n_s e^2} j_s^2$$

$$= \frac{1}{8\pi}\int \delta\left(h^2 + \frac{m}{4\pi n_s e^2} [\nabla \times h]^2 \right) = 0 \quad ; \quad \nabla \times \nabla \times \vec{h} + \lambda_L^{-2} \vec{h} = 0$$

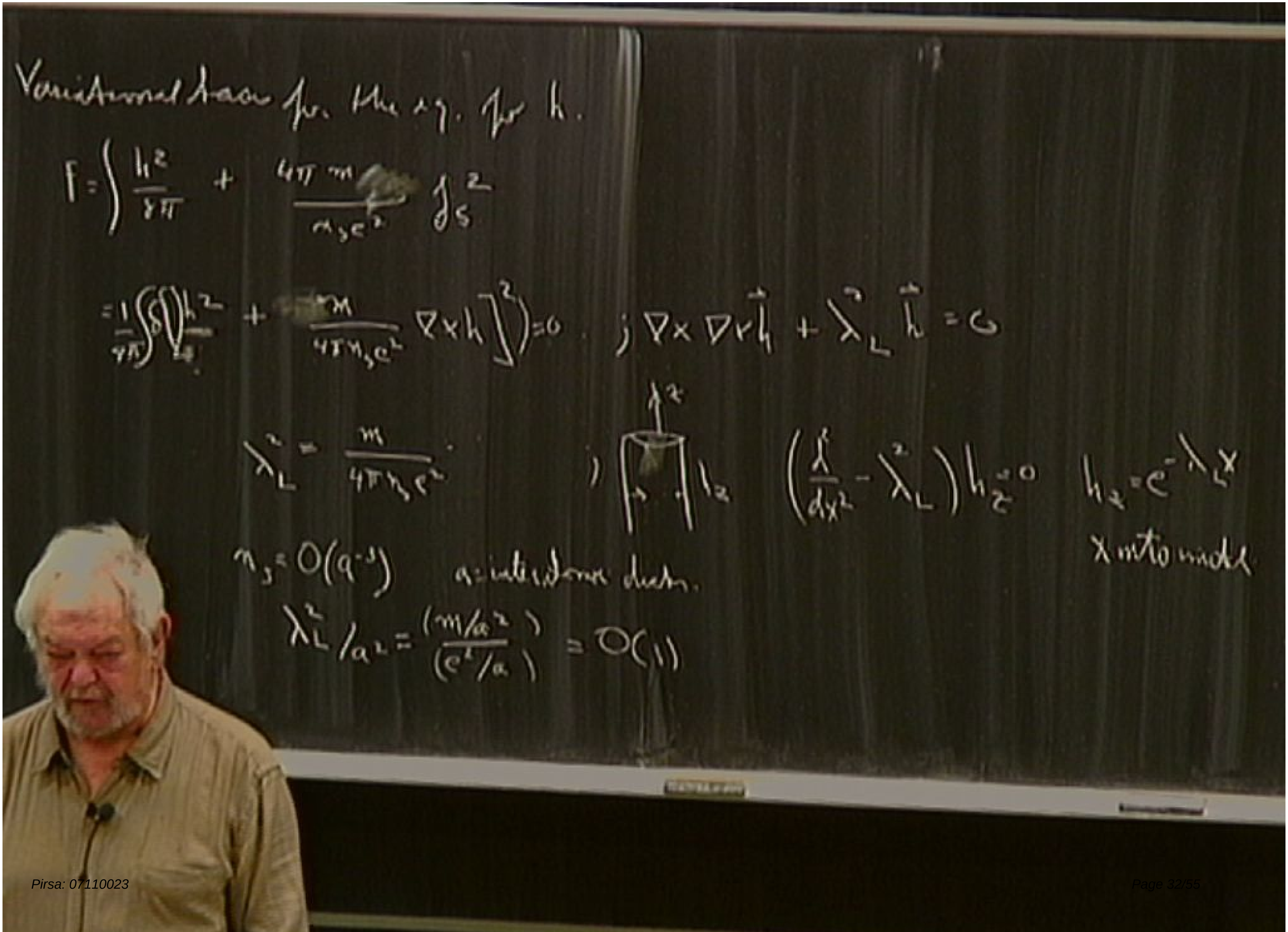
$$\lambda_L^2 = \frac{m}{4\pi n_s e^2}.$$

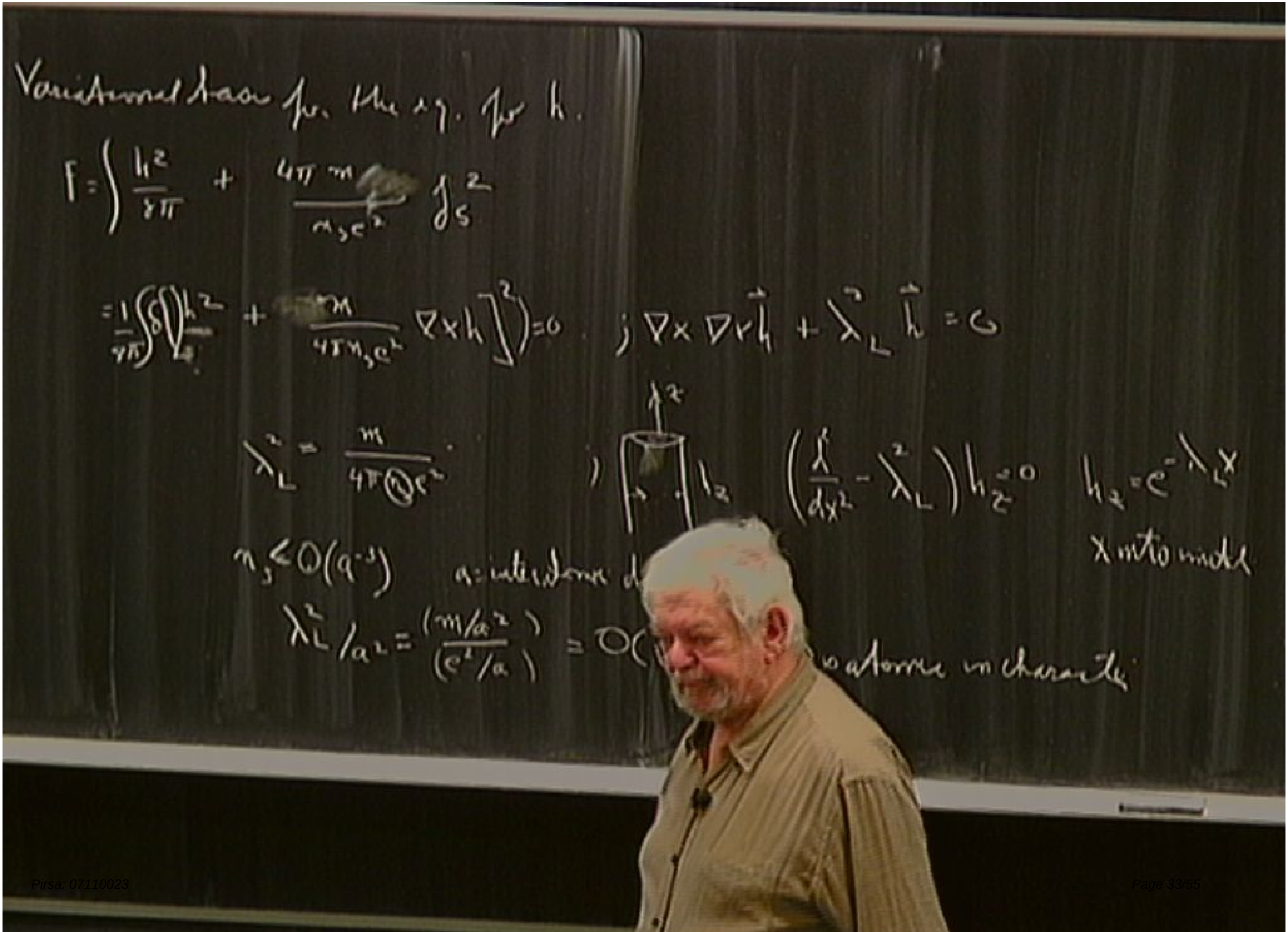
$$\left(\frac{d^2}{dx^2} - \lambda_L^{-2}\right) h_z = 0 \qquad h_z = e^{-\lambda_L x}$$

x into middle

$n_s = O(a^{-3})$ a: interatomic dist.

$$\lambda_L^2 / a^2 = \frac{(m/a^2)}{(e^2/a)}$$


Variational basis for the eq. for h.
F = ∫ h²/8π + 4π m/(n_s e²) j_s²
= 1/8π ∫ δ (h² + m/(4π n_s e²) [∇×h]²) = 0 ; ∇×∇×h + λ_L h = 0
λ_L² = m/(4π n_s e²)
z
h_z
(d²/dx² − λ_L) h_z = 0
h_z = e^(−λ_L x)
x into model
n_s = O(a⁻³)
a = interatomic dist.
λ_L² / a² = (m/a²) / (e²/a) = O(1)


Variational base for the eq. for h.
x into medium

Variational basis for the eq. for h.

$$F = \int \frac{h^2}{8\pi} + \frac{4\pi m}{n_s e^2} j_s^2$$

$$= \frac{1}{8\pi}\int \delta \left(h^2 + \frac{m}{4\pi n_s e^2} [\nabla\times h]^2 \right) = 0 \quad ; \nabla\times\nabla\times\vec{h} + \lambda_L^{-2}\,\vec{h} = 0$$

$$\lambda_L^2 = \frac{m}{4\pi n_s e^2} \quad ;$$

$$\left(\frac{d^2}{dx^2} - \lambda_L^{-2}\right) h_z = 0 \qquad h_z = e^{-\lambda_L x}$$

x into middle

$n_s \leq O(a^{-3})$ a: interatomic dist.

$$\lambda_L^2/a^2 = \frac{(m/a^2)}{(e^2/a)} = O(1)$$; $\lambda_L \gtrsim$ atomic in character

Variational basis for the eq. for h.

$$F = \int \frac{h^2}{8\pi} + \frac{4\pi m}{n_s e^2} j_s^2$$

$$= \frac{1}{8\pi}\int \delta\left(\int h^2 + \frac{m}{4\pi n_s e^2}\left[\nabla\times h\right]^2\right) = 0 \quad ; \nabla\times\nabla\times\vec{h} + \lambda_L^{-2}\vec{h} = 0$$

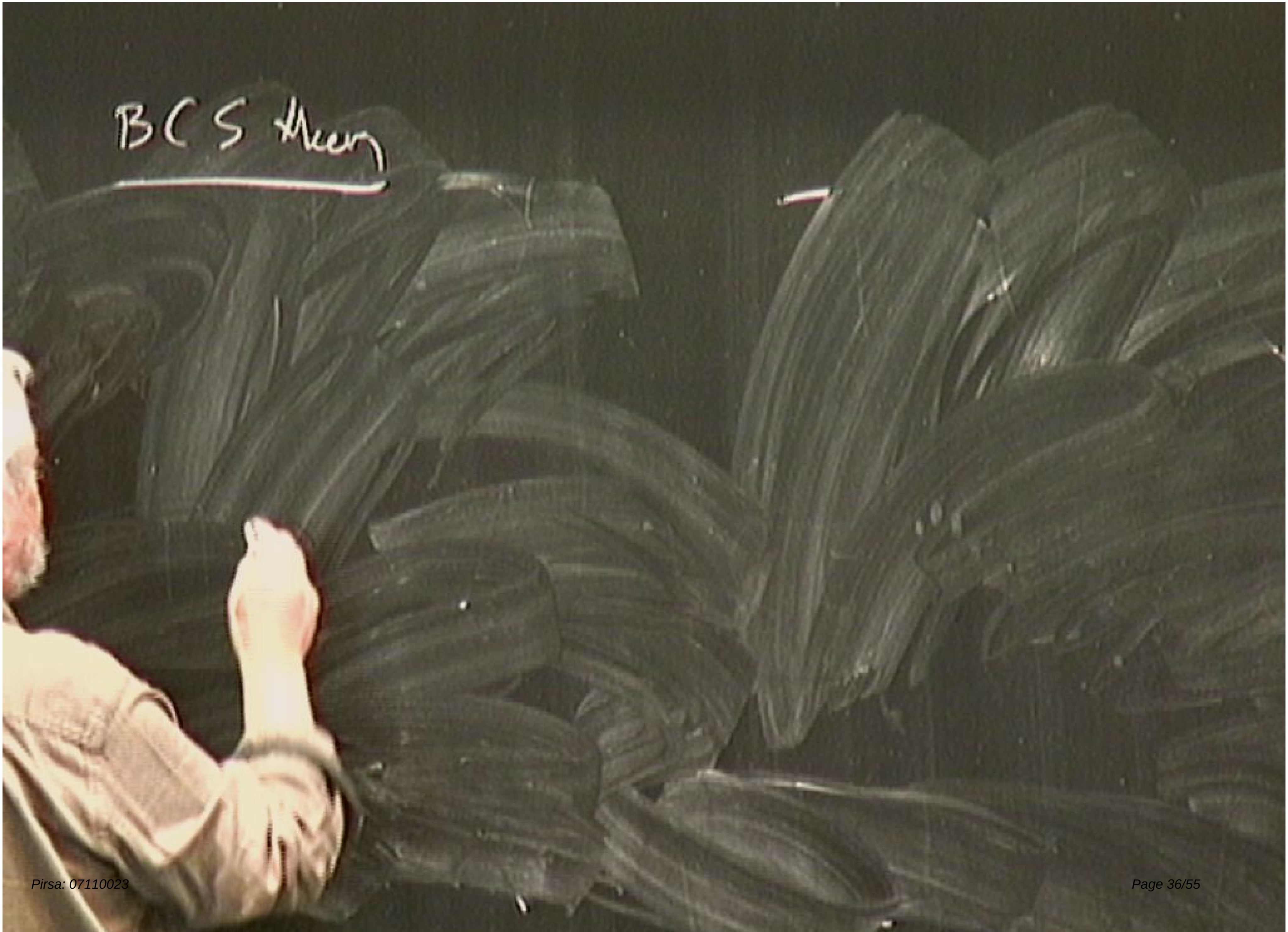
$$\lambda_L^2 = \frac{m}{4\pi n_s e^2}$$;

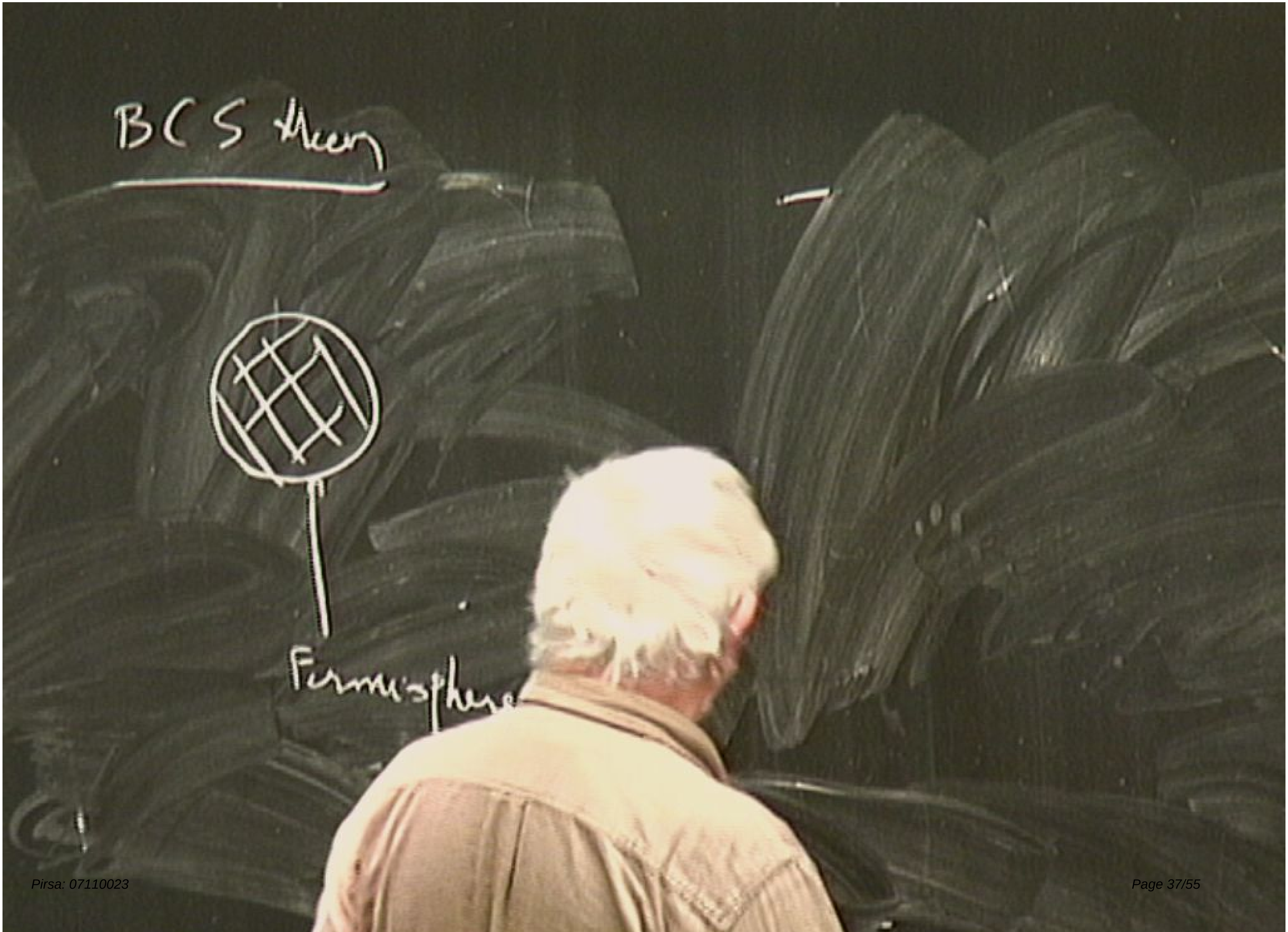
$$\left(\frac{d^2}{dx^2} - \lambda_L^{-2}\right) h_z = 0 \qquad h_z = e^{-\lambda_L x}$$

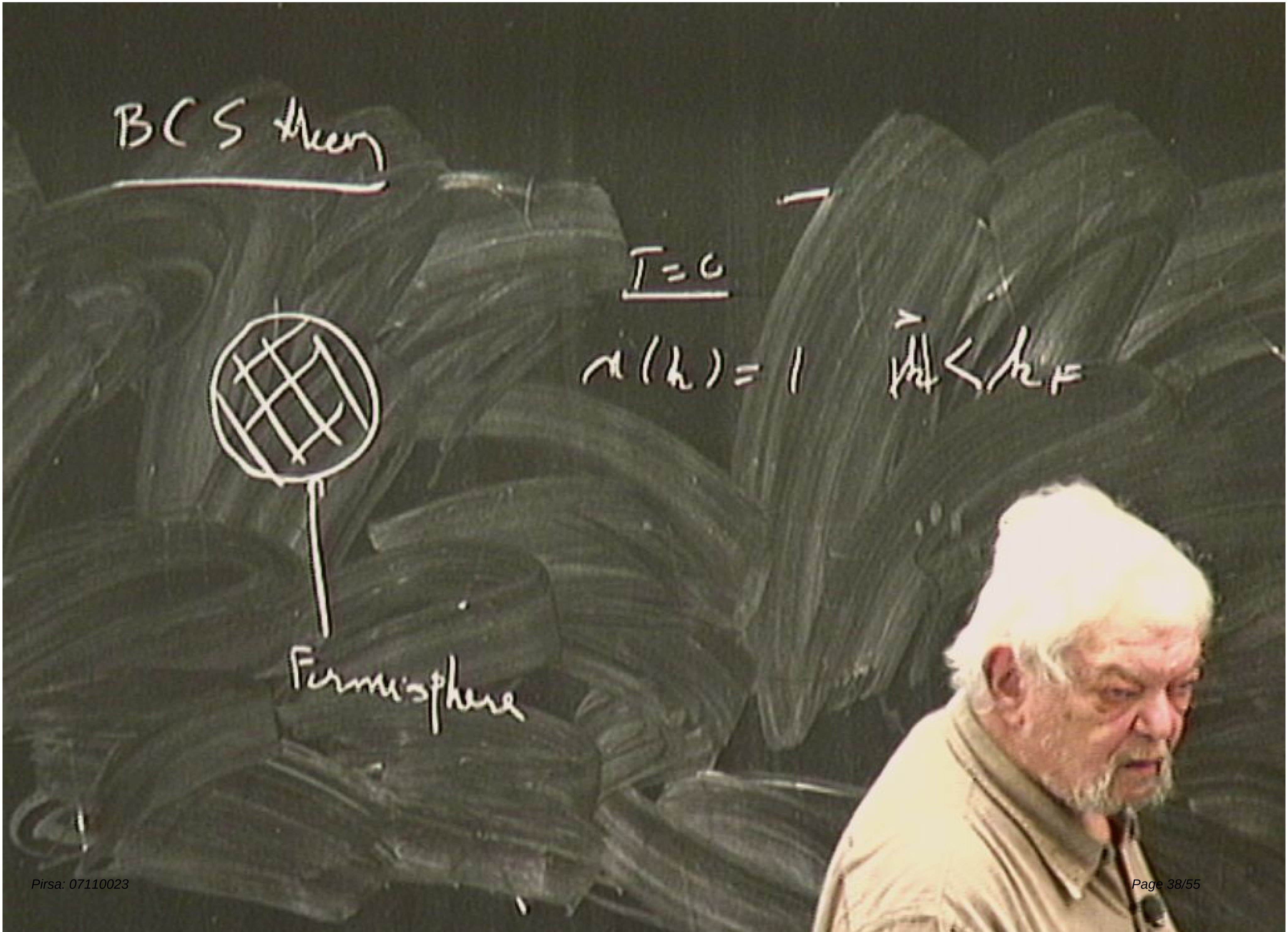
x into metal

$n_s \leq O(a^{-3})$ a: interatomic dist.

$$\lambda_L^2 / a^2 = \frac{(m/a^2)}{(e^2/a)} = O(1)$$; λ_L is atomic in character


BCS theory


BCS theory
Fermisphere


BCS theory
T = 0
n(k) = 1 |k| < k_F
Fermi sphere

BCS theory

Fermi sphere

$T = 0$

$$n(k) = 1 \qquad |\vec{k}| < k_F$$

$$\frac{V k_F^3}{(2\pi)} \frac{4\pi}{3} = N_e$$

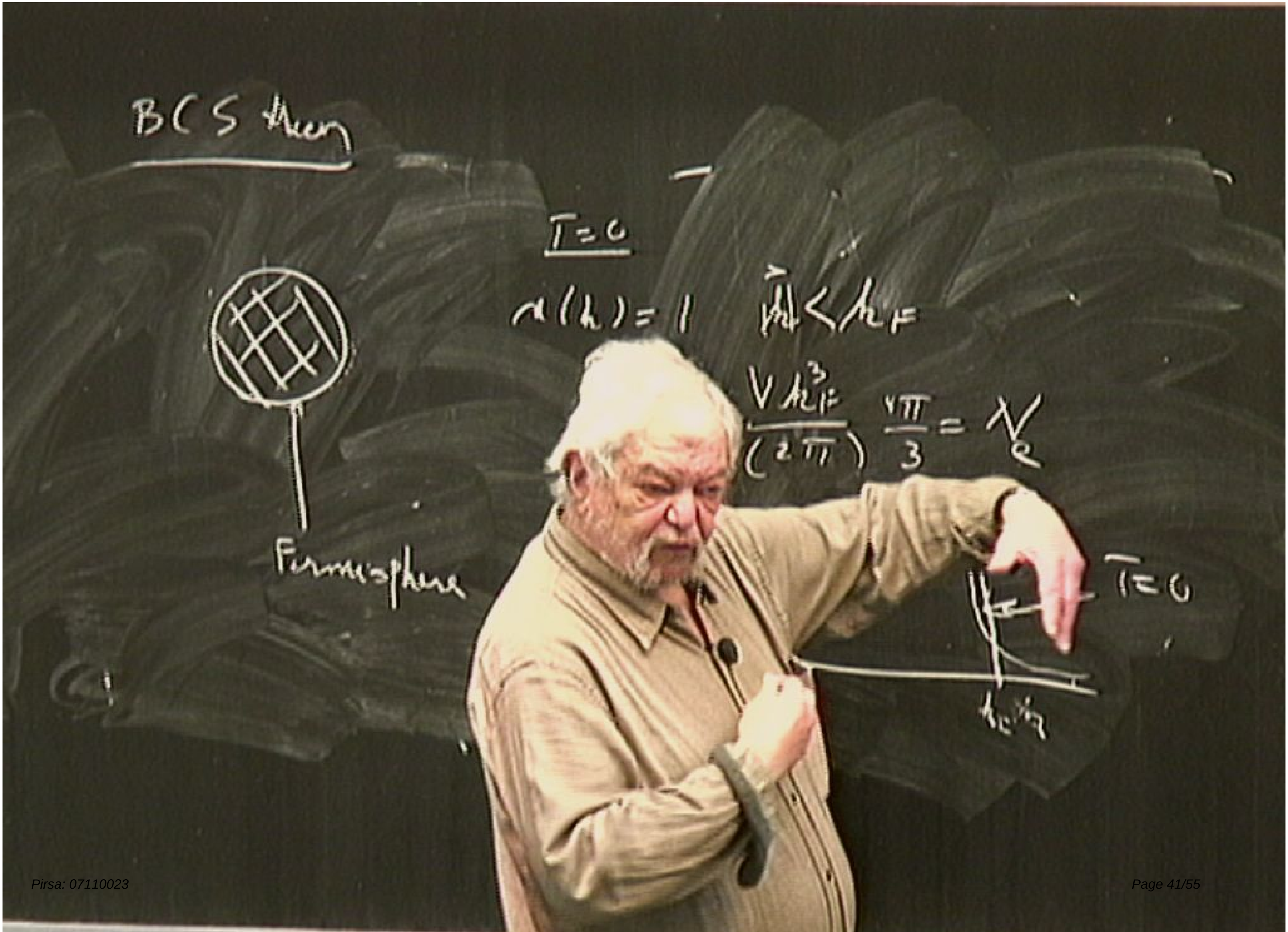
BCS theory

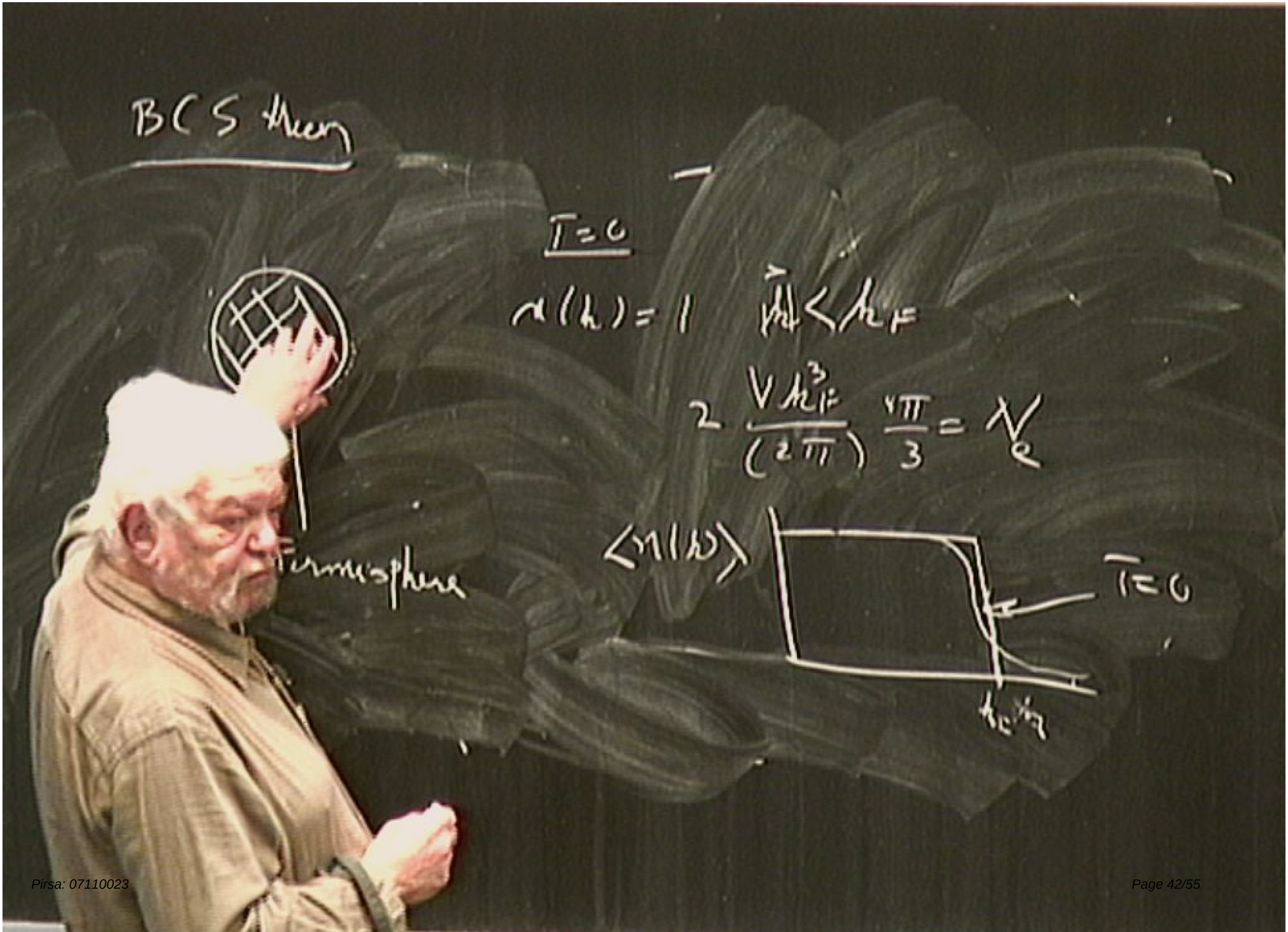
Fermi sphere

$T = 0$

$n(k) = 1 \quad |\vec{k}| < k_F$

$$2 \frac{V k_F^3}{(2\pi)^3} \frac{4\pi}{3} = N_e$$


BCS theory
T = 0
n(k) = 1 |k| < k_F
V k_F^3 / (2π)^3 · 4π/3 = N_e
Fermi sphere
T = 0
k_F


BCS theory
T=0
n(k) = 1 |k| < k_F
2 V k_F^3 / (2π) 4π/3 = N_e
⟨n(k)⟩
T=0
Fermi sphere

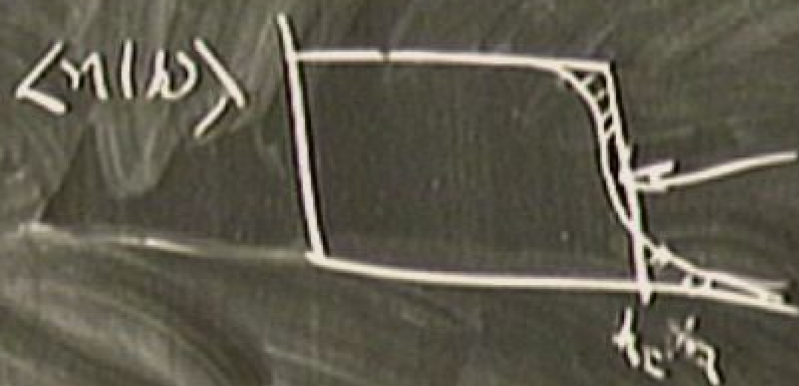
BCS theory

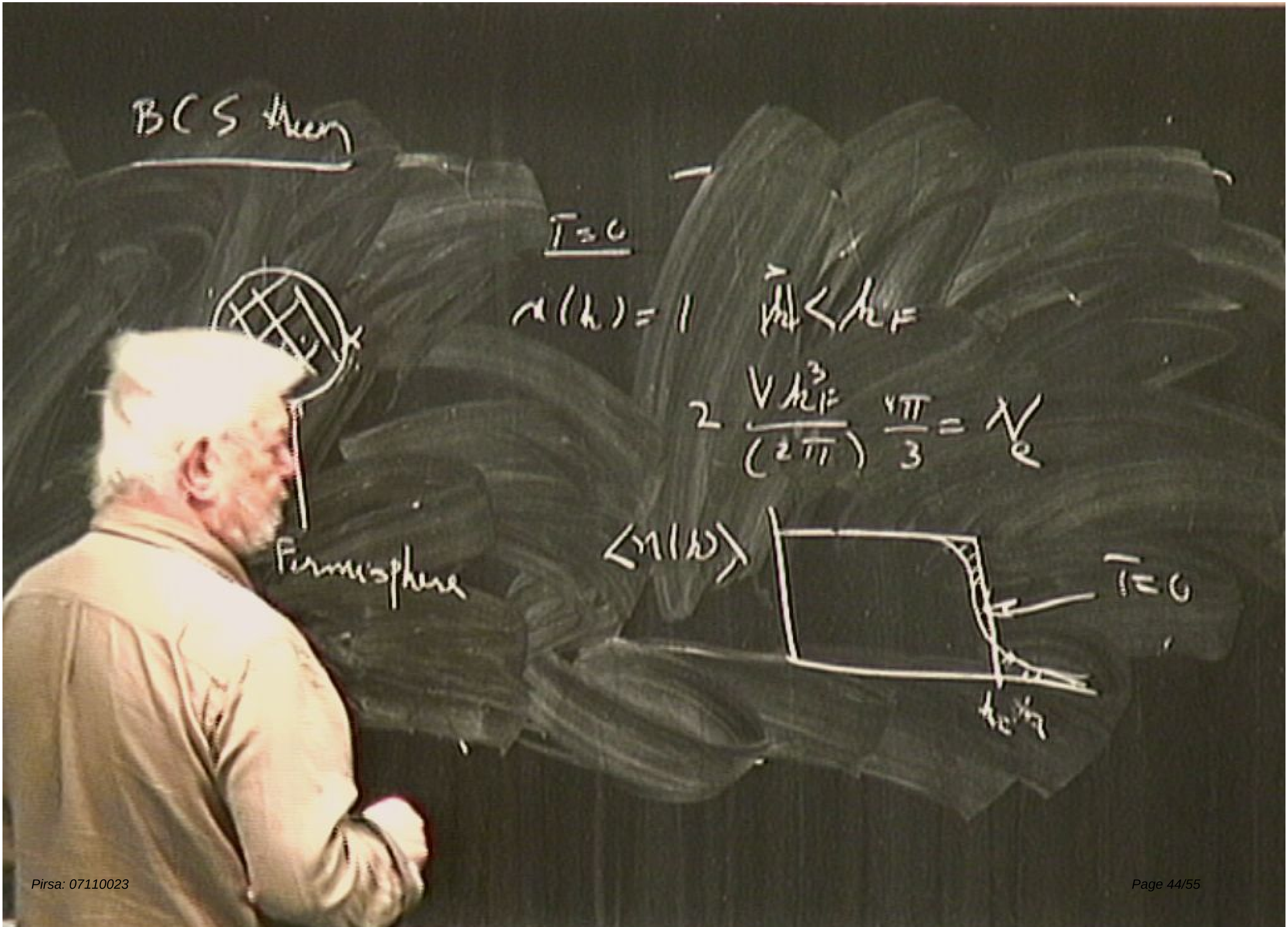
T = 0

$n(k) = 1 \quad |\vec{k}| < k_F$

$$2 \frac{V k_F^3}{(2\pi)^3} \frac{4\pi}{3} = N_e$$

Fermi sphere




BCS theory
T = 0
$n(k) = 1 \quad |\vec{k}| < k_F$
$2 \frac{V k_F^3}{(2\pi)^3} \frac{4\pi}{3} = N_e$
Fermisphere
$\langle n(k) \rangle$
T = 0
k_F

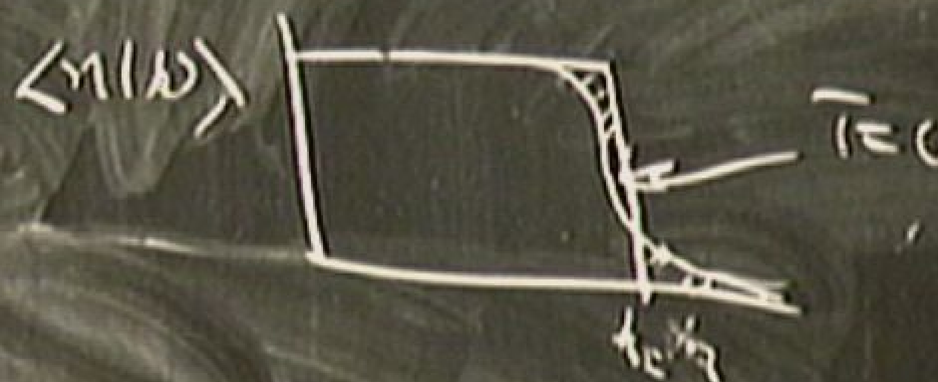
BCS theory

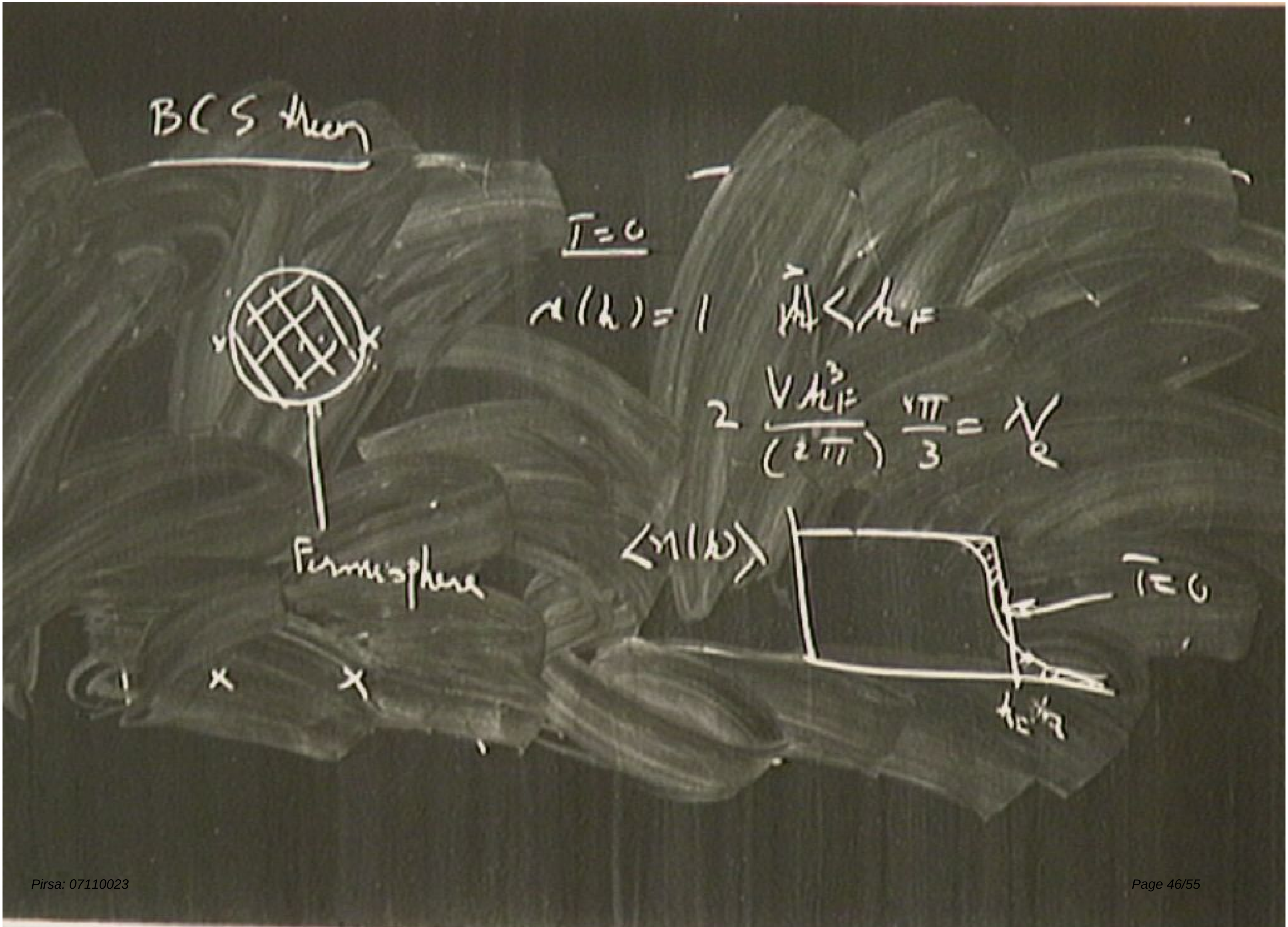
$\underline{T=0}$

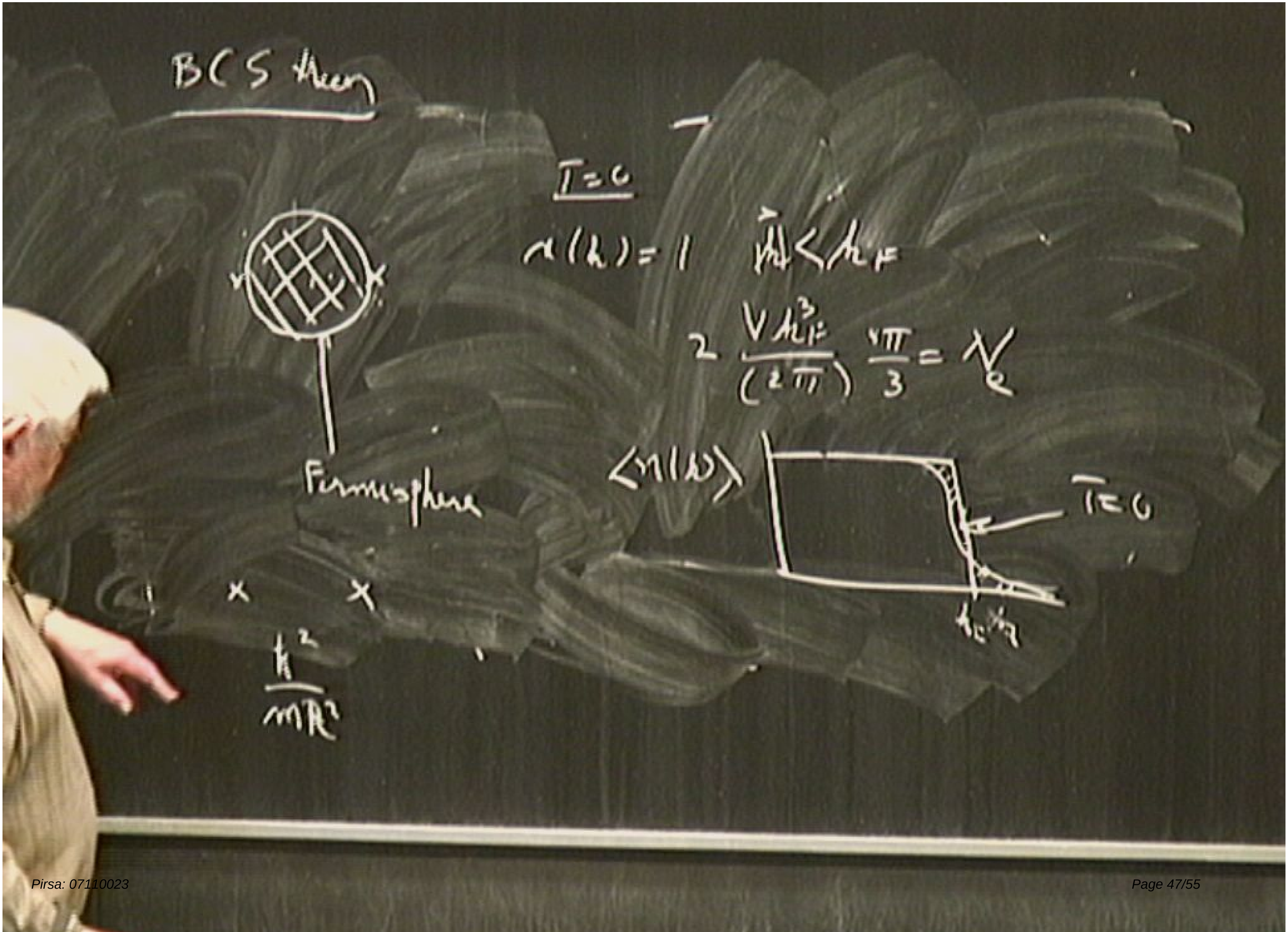
$$n(k) = 1 \qquad |\vec{k}| < k_F$$

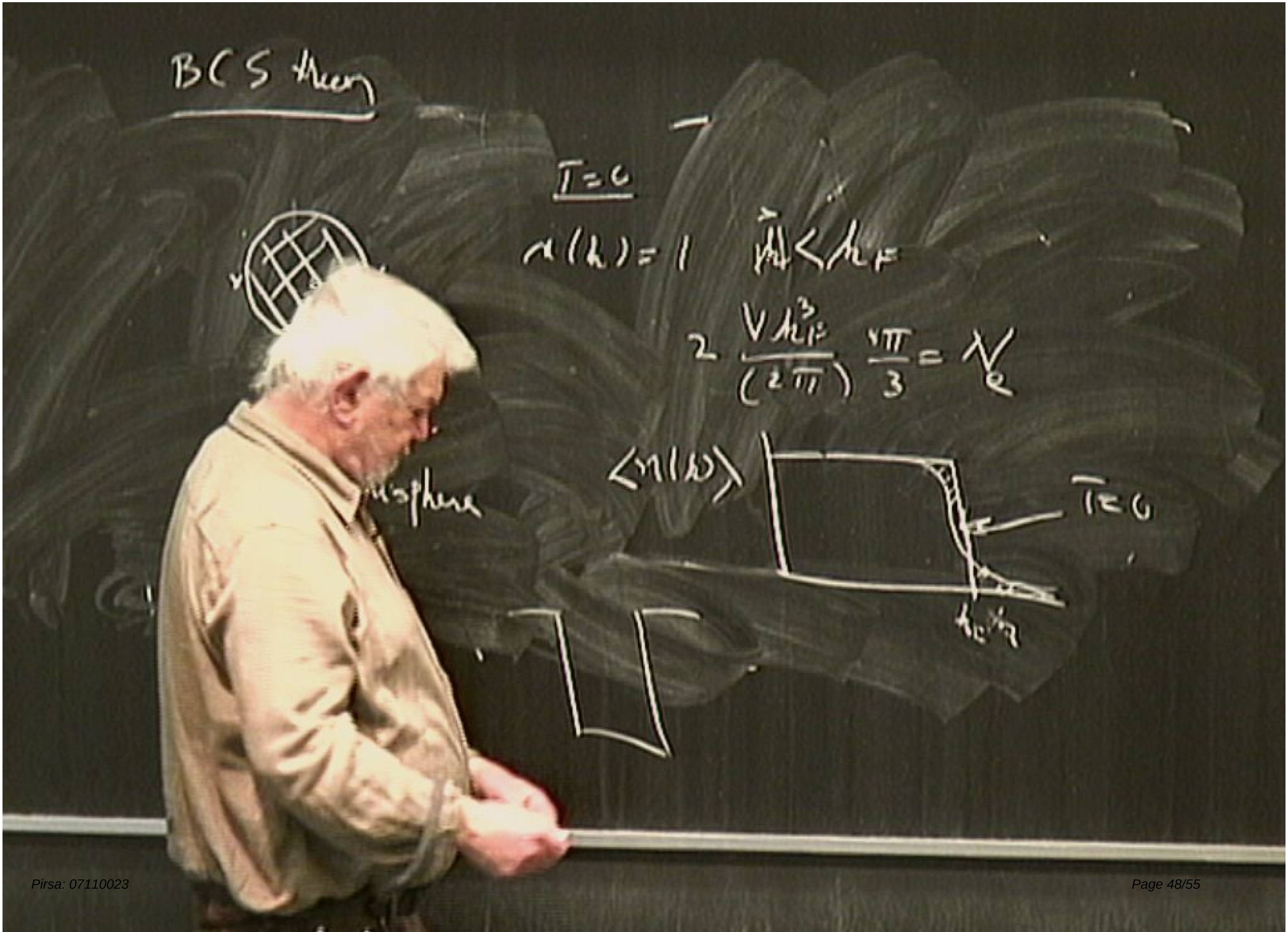
$$2 \frac{V k_F^3}{(2\pi)^3} \frac{4\pi}{3} = N_e$$

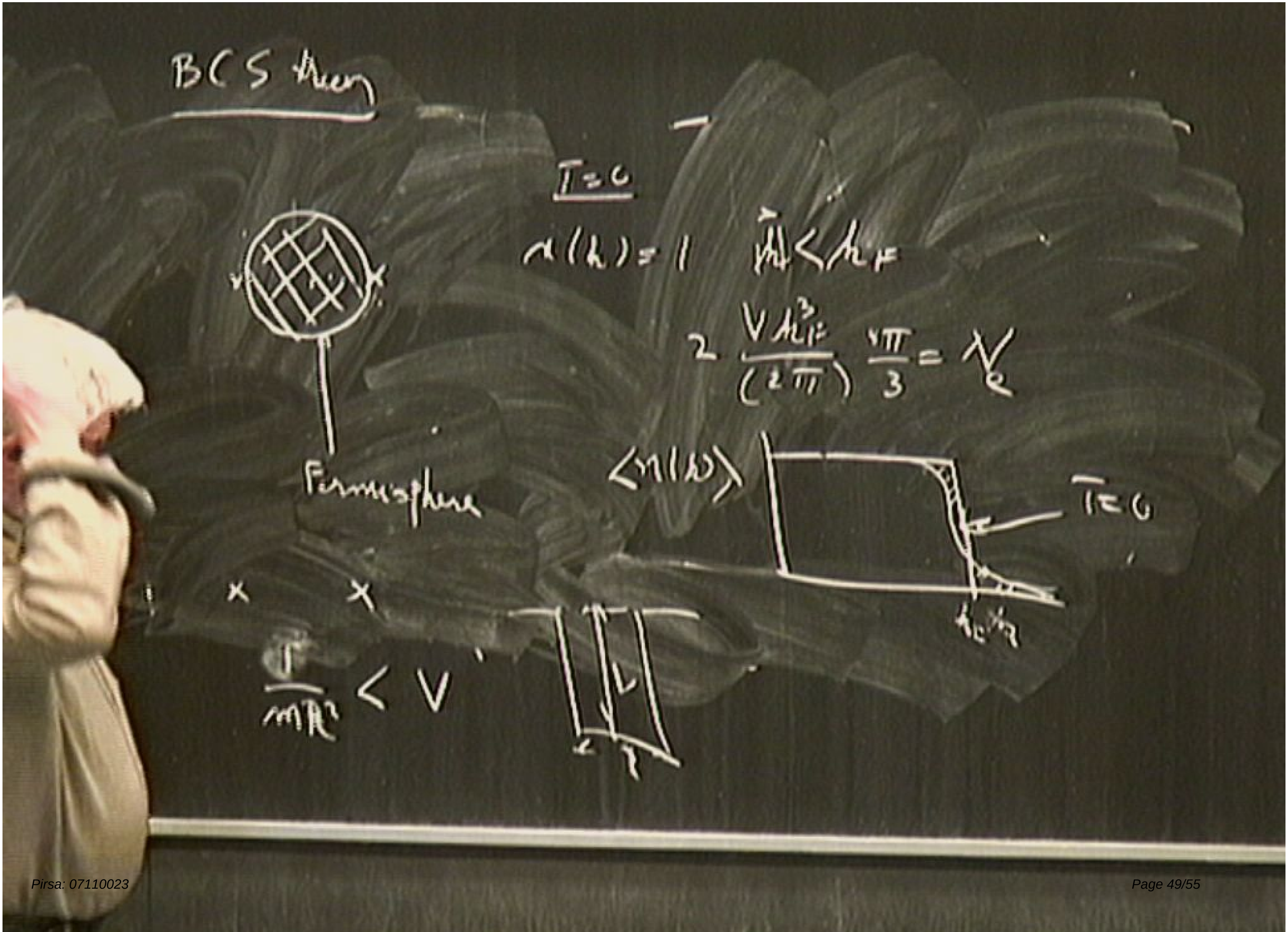
Fermi sphere




BCS theory
T = 0
n(k) = 1   k < k_F
2 V k_F^3 / (2π)^3 · 4π/3 = N_e
Fermi sphere
<n(k)>
T = 0
k_F


BCS theory
T=0
n(k) = 1 |k| < k_F
2 V k_F^3 / (2π)^3 · 4π/3 = N_e
Fermi sphere
⟨n(k)⟩
T≠0
ħ^2 / mR^2


BCS theory
T=0
n(k) = 1   k < k_F
T≠0


BCS theory
T=0
n(k) = 1 k < k_F
2 V k_F^3/(2π)^3 · 4π/3 = N/2
⟨n(k)⟩
T≠0
Fermi sphere

BCS theory

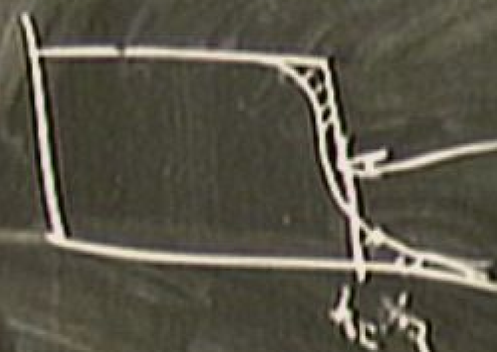
Fermi sphere

$T=0$

$n(k) = 1 \quad \vec{k} < k_F$

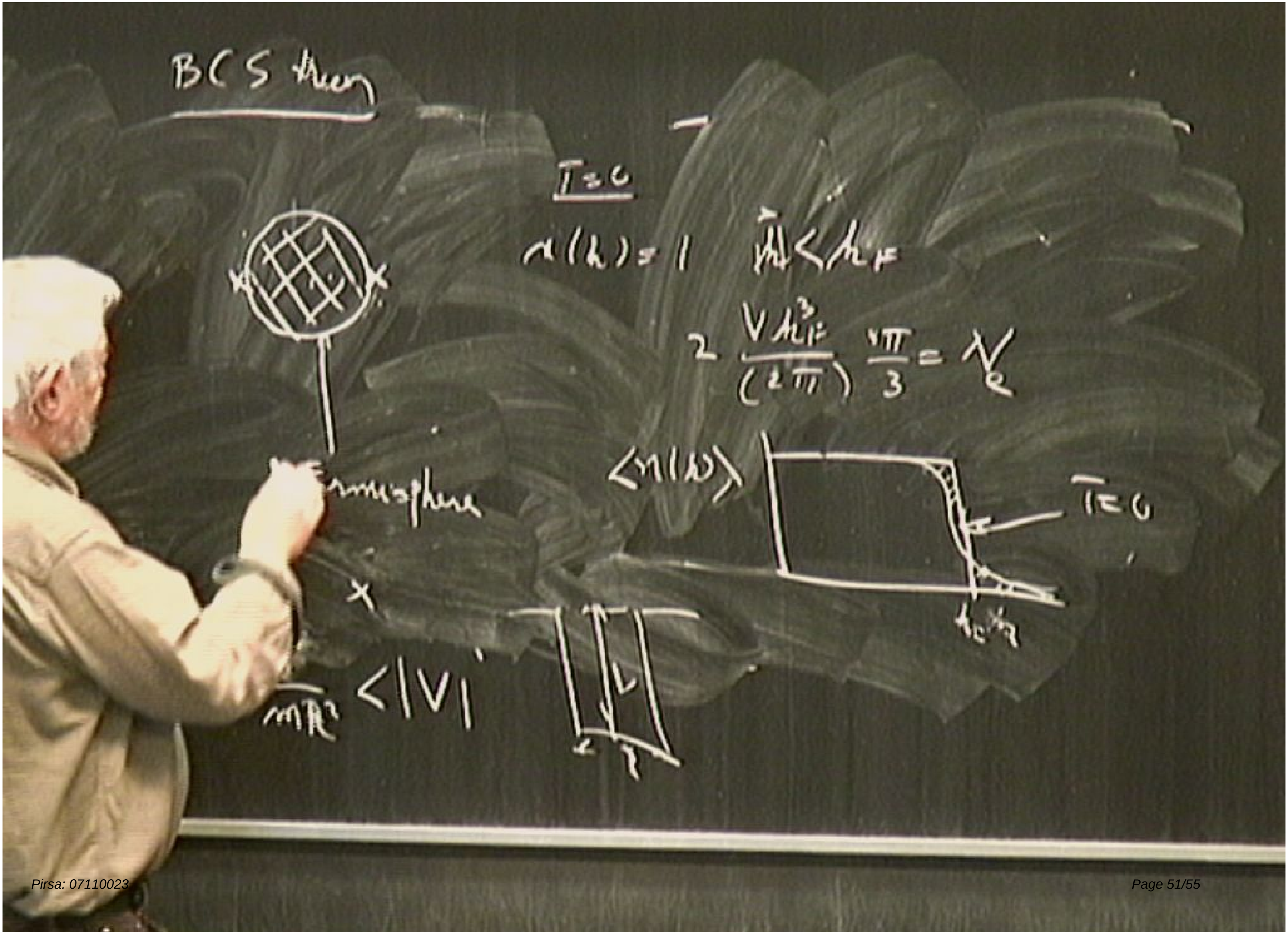
$$2\,\frac{V k_F^3}{(2\pi)^3}\,\frac{4\pi}{3} = N_e$$

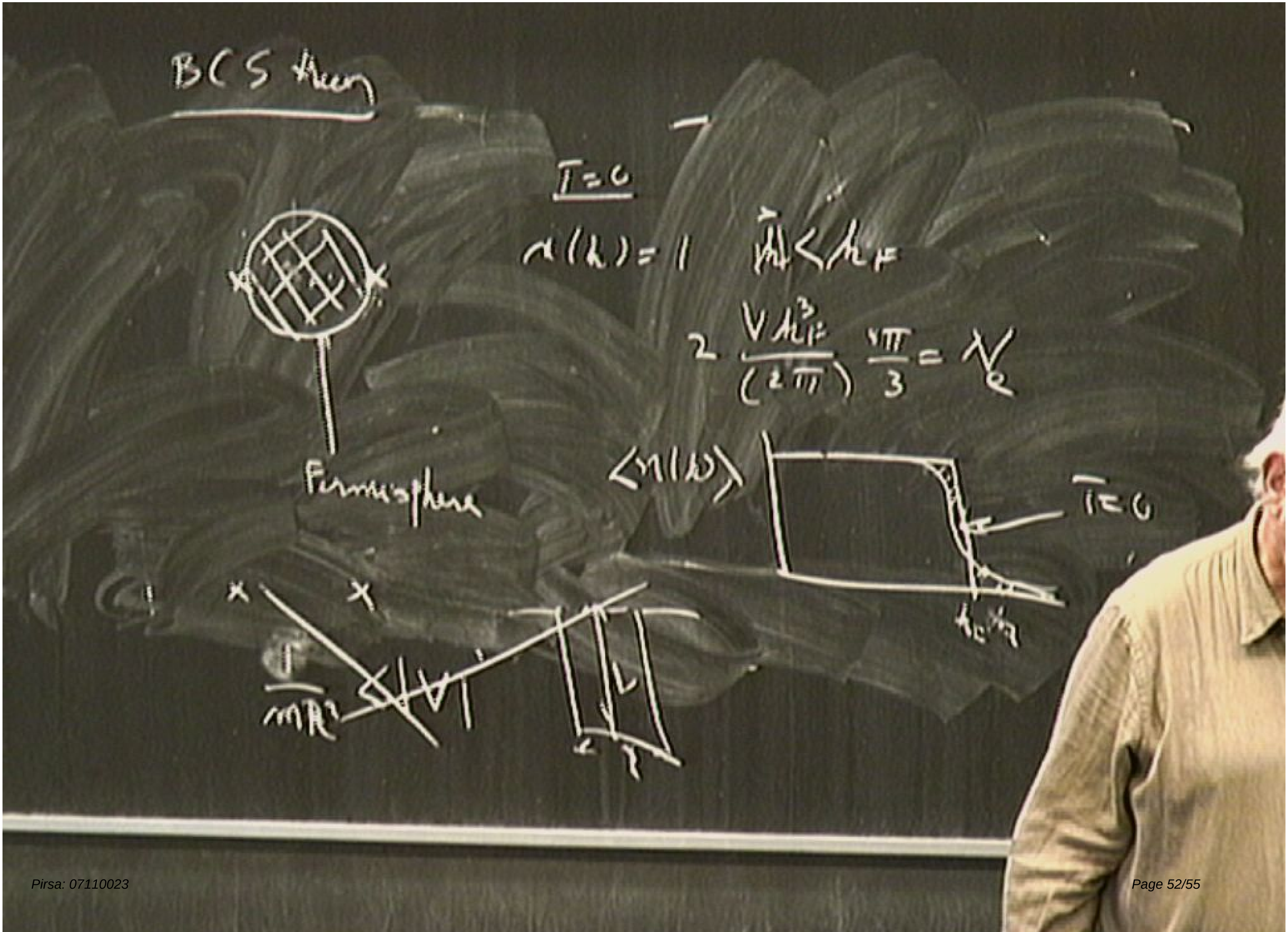
$\langle n(k) \rangle$

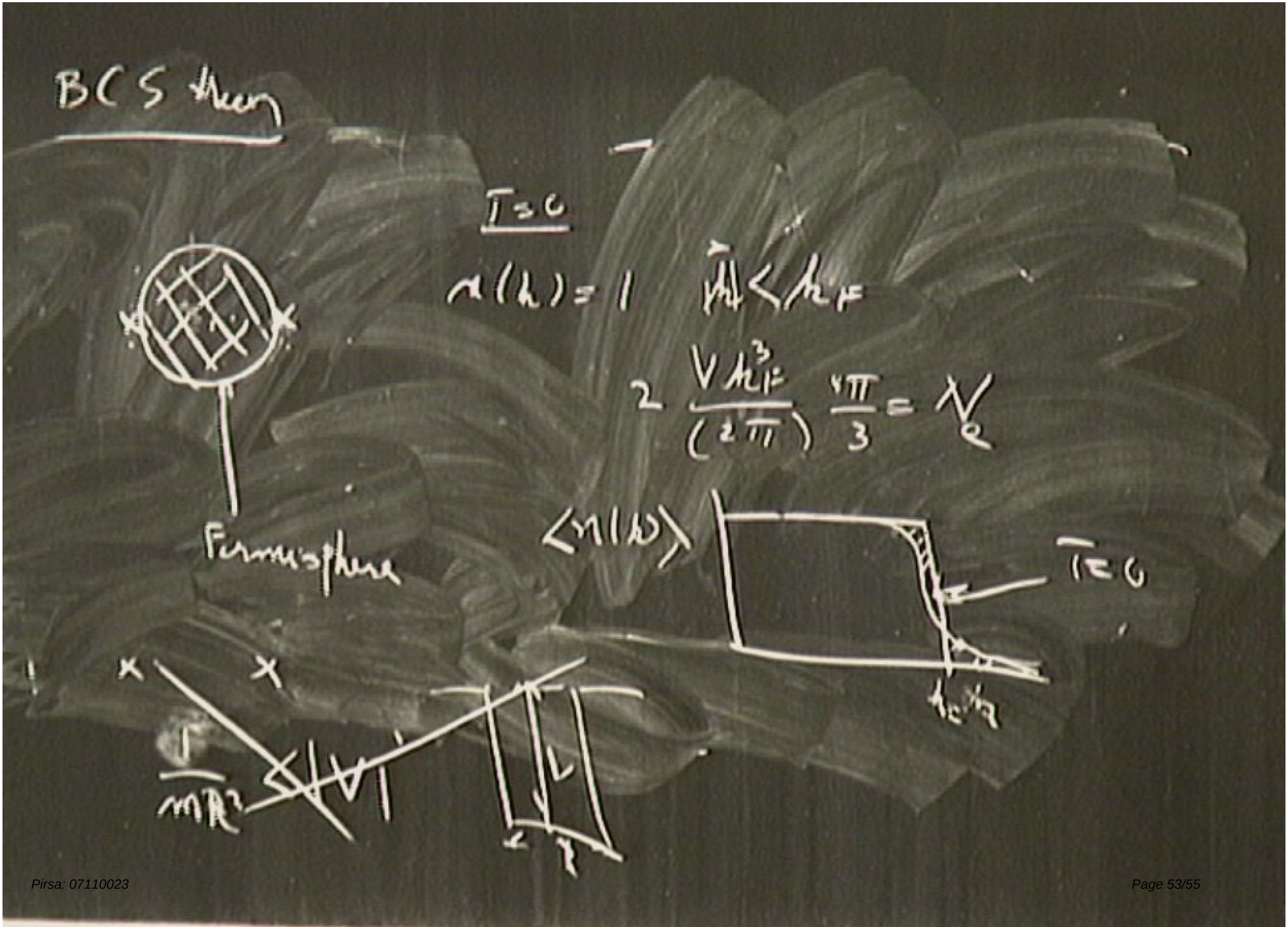


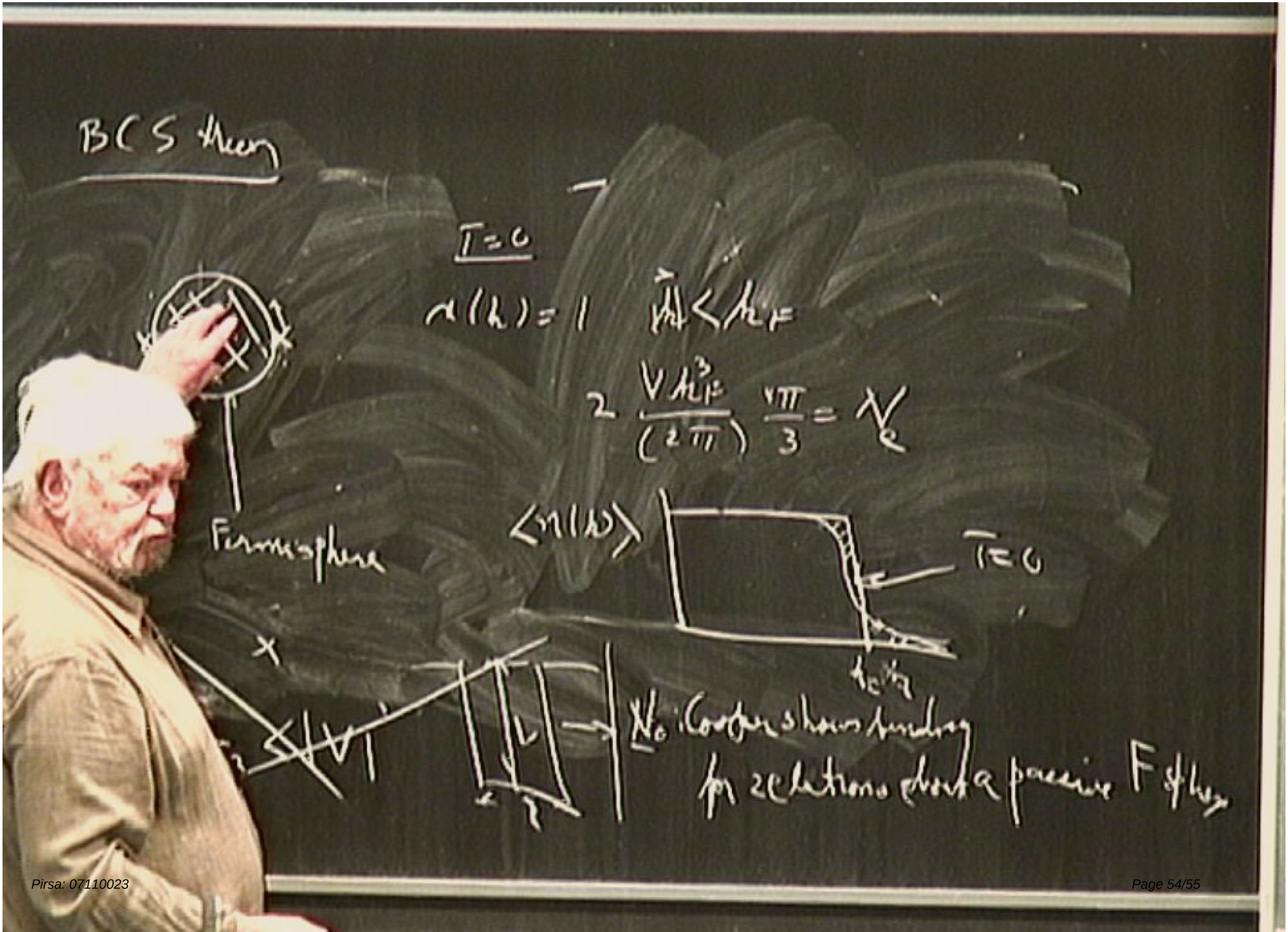
$\times \quad \times \quad \times$

$\frac{\hbar^2}{mR^2} < |V|$


BCS theory
T=0
n(k) = 1 |k| < k_F
2 (V k_F^3/(2π)^3)(4π/3) = N_e
⟨n(k)⟩
T=0
Fermi sphere
<|V|


BCS theory
T=0
n(k) = 1   k < k_F
2 V k_F^3 / (2π)^3 · 4π/3 = N/2
⟨n(k)⟩
T=0
Fermisphere


BCS theory
T=0
n(k) = 1 |k| < k_F
2 V k_F^3 / (2π)^3 · 4π/3 = N_e
⟨n(k)⟩
T≠0
Fermi sphere
k_F
1/(mR^2)


BCS theory
T = 0
n(k) = 1   k < k_F
2 V k_F^3 / (2π)^3 · 4π/3 = N_e
⟨n(k)⟩
T = 0
Fermi sphere
Cooper shows binding
for 2 electrons above a passive F sphere

BCS theory

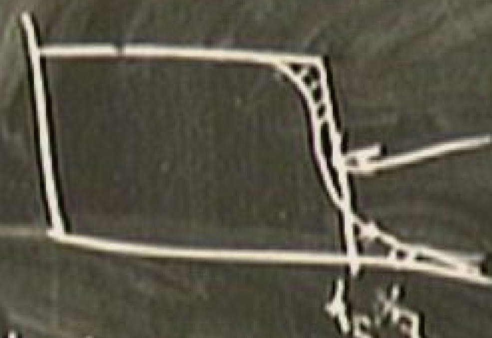
$\underline{T = 0}$

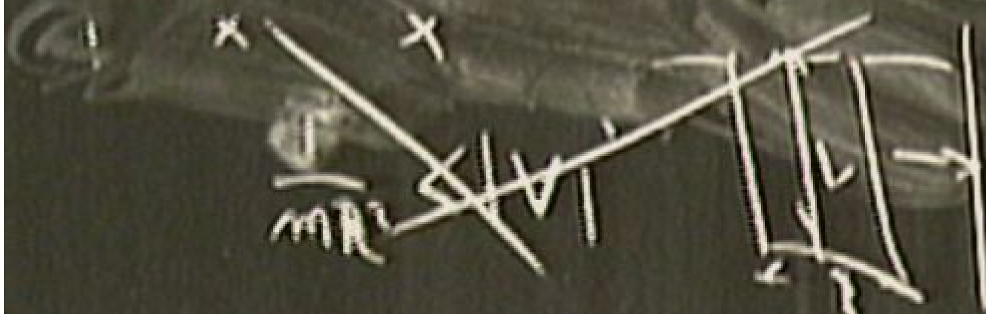
$n(k) = 1 \quad |\vec{k}| < k_F$

$$2\,\frac{V k_F^3}{(2\pi)^3}\,\frac{4\pi}{3} = N_e$$

Fermi sphere

$\langle n(k) \rangle$





N_0: Cooper shows binding for 2 electrons above a passive F.S.