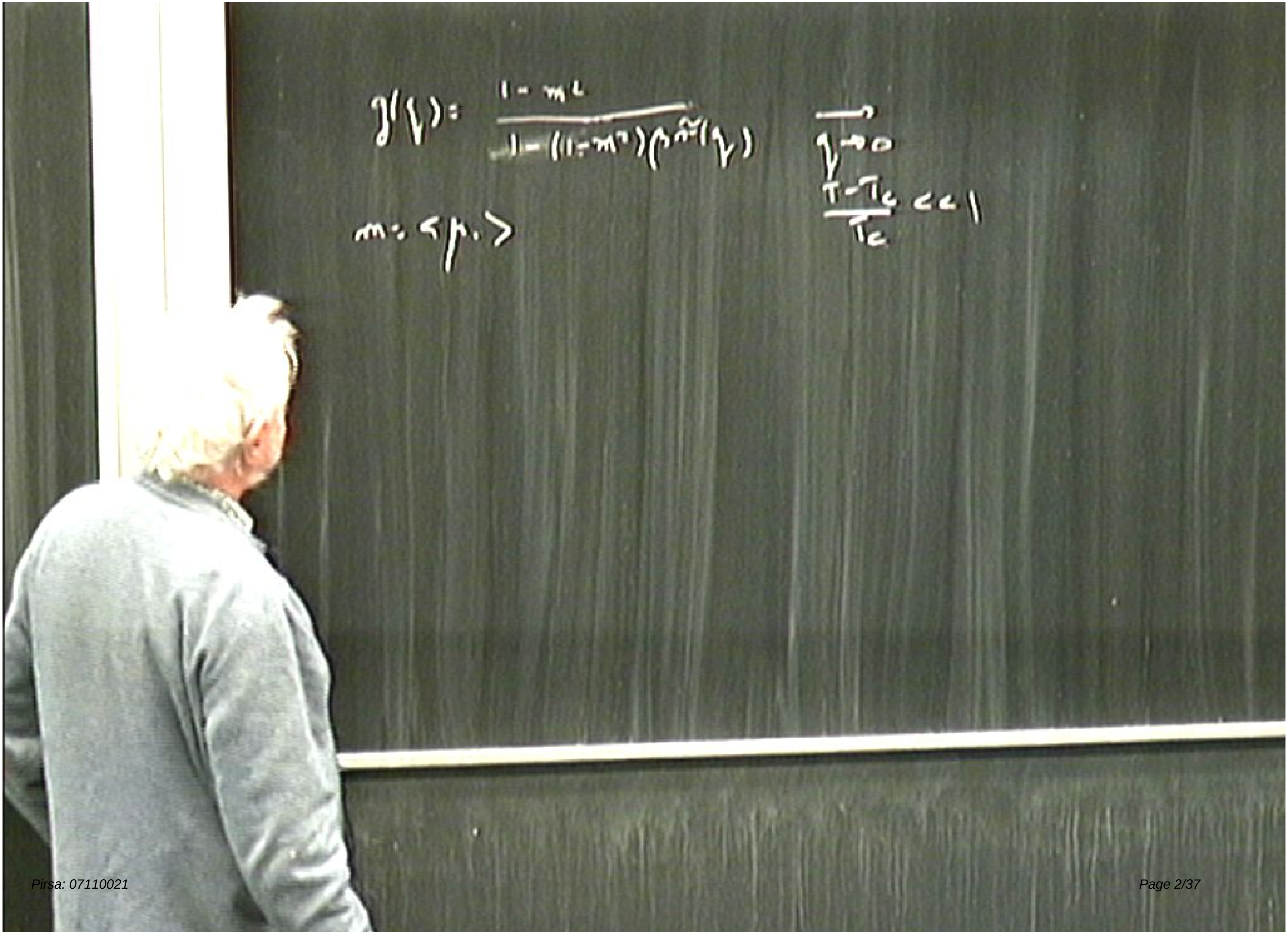
Title: Spontaneous Broken Symmetry 3A

Date: Nov 15, 2007 10:00 AM

URL: http://pirsa.org/07110021

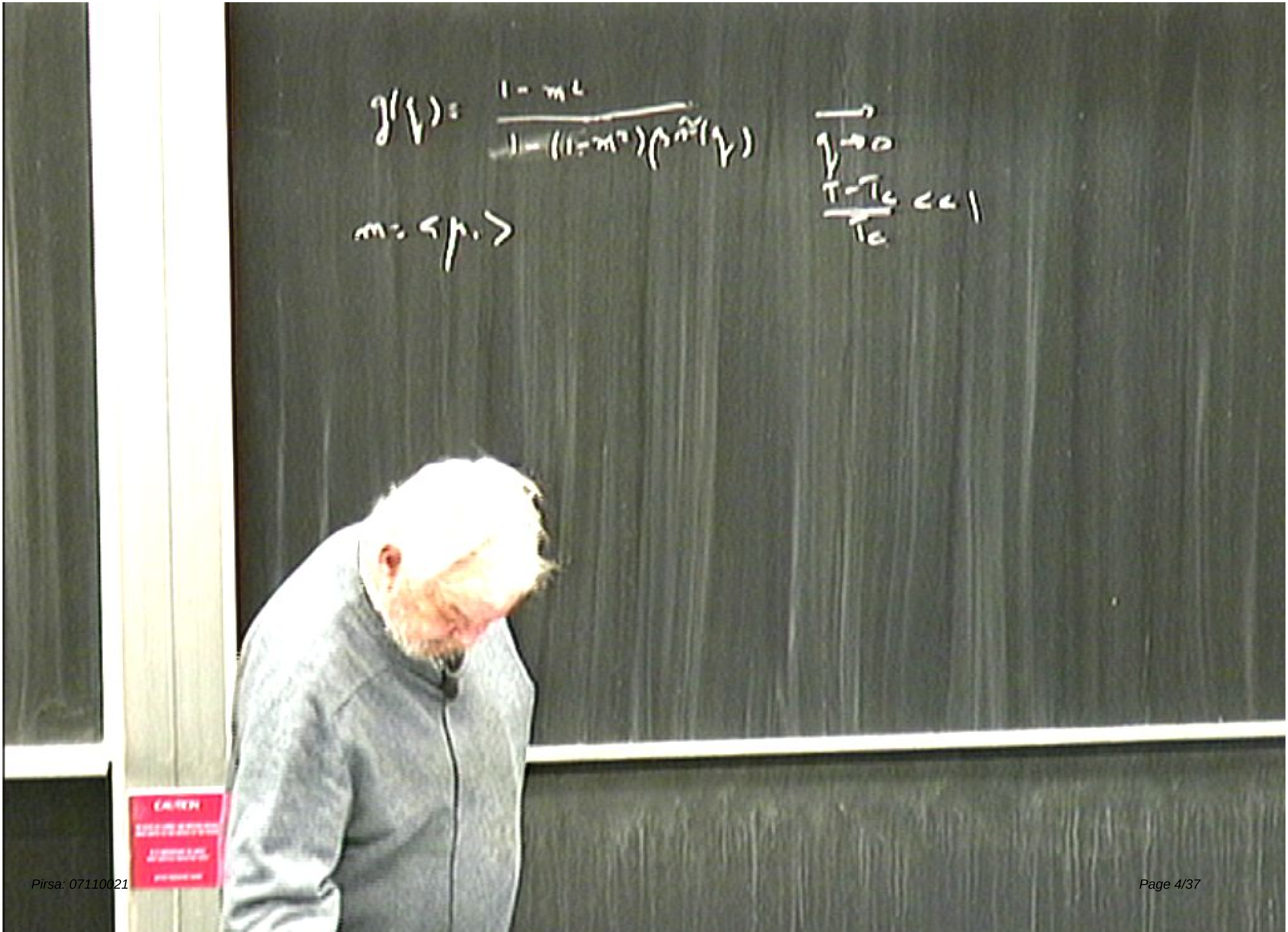
Abstract:


g(q) = (1 − m²) / (1 − (1 − m²) ρ̃(q))
q → 0
(T − T_c)/T_c << 1
m = <ρ_i>

$$g(q) = \frac{1 - m^2}{1 - (1 - m^2)\beta \tilde{J}(q)} \xrightarrow[q \to 0]{}$$

$$\frac{T - T_c}{T_c} \ll 1$$

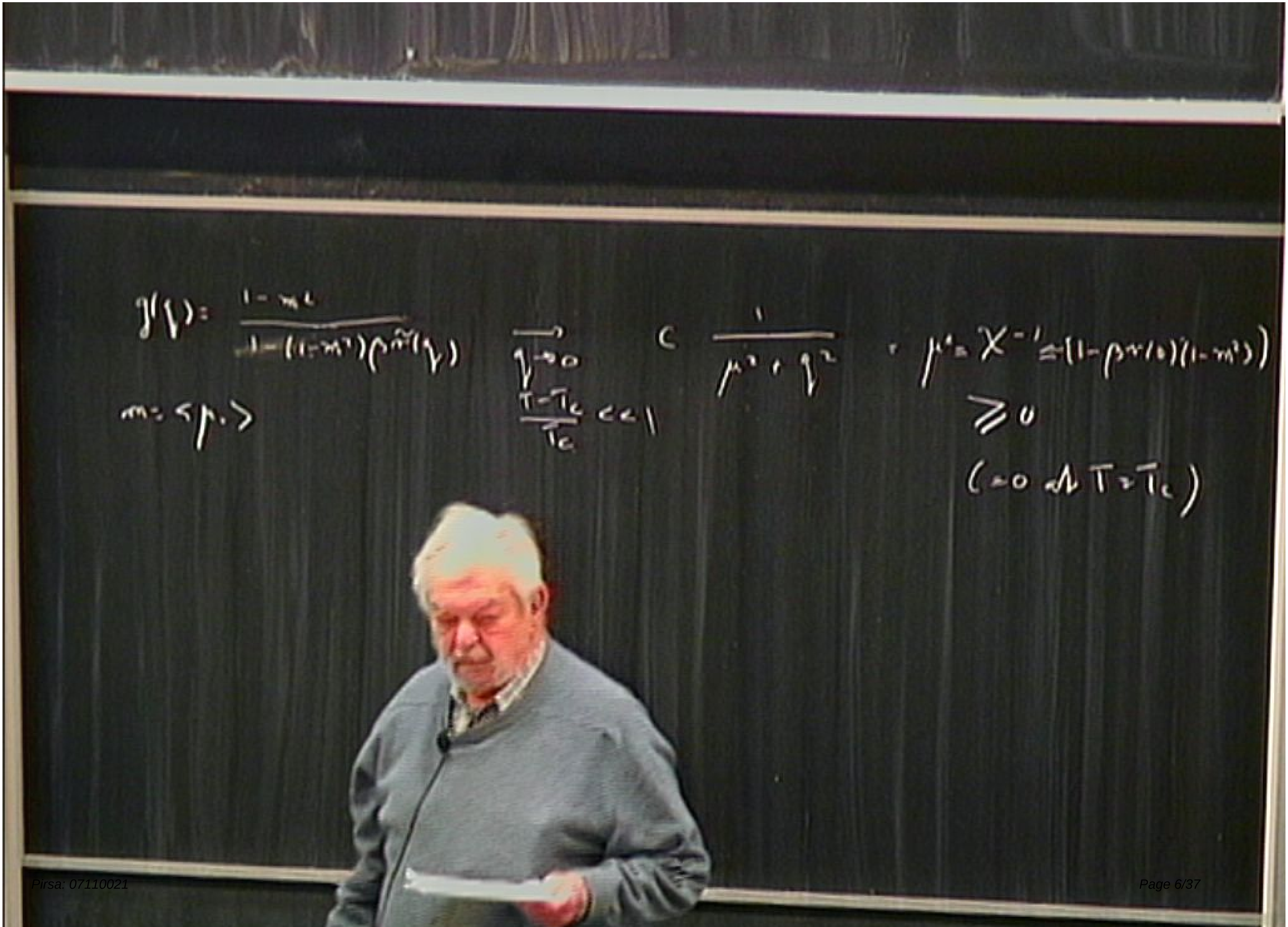
$$m = \langle \mu_i \rangle$$


g(q) = 1 - m² / 1 - ((1-m²)) ρ̃(q)
q→0
T-Tc / Tc << 1
m = ⟨μi⟩

$$g(q) = \frac{1 - m^2}{1 - (1 - m^2)\beta \tilde{J}(q)} \xrightarrow[q \to 0]{}$$

$$m = \langle \mu_i \rangle$$

$$\frac{T - T_c}{T_c} \ll 1$$


g(q) = (1 - m²) / (1 - (1 - m²) ρⁿⁿ(q)) → (q → 0) c 1/(μ² + q²), μ² = χ⁻¹ ≃ (1 - ρⁿⁿ(0))(1 - m²)
m = ⟨ρ₀⟩
(T - T_c)/T_c ≪ 1
≥ 0
(= 0 at T = T_c)

$$\cdots) \xrightarrow[\substack{q \to 0 \\ \frac{T-T_c}{T_c} \ll 1}]{} C \, \frac{1}{\mu^2 + q^2} \,, \quad \mu^2 = \chi^{-1} \simeq (1 - \beta v(0)(1-m^2))$$

$$\geq 0$$

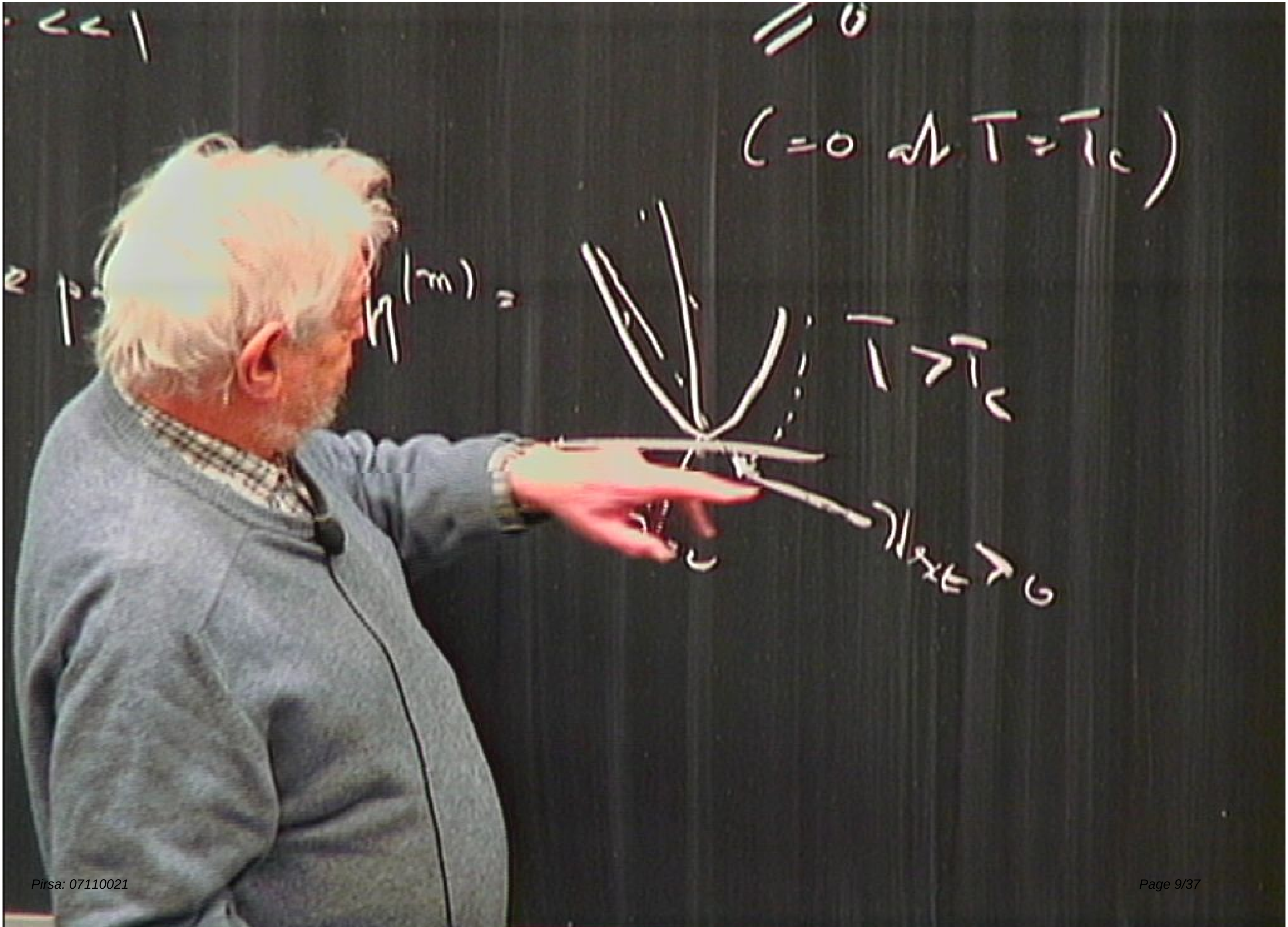
$$(= 0 \text{ at } T = T_c)$$

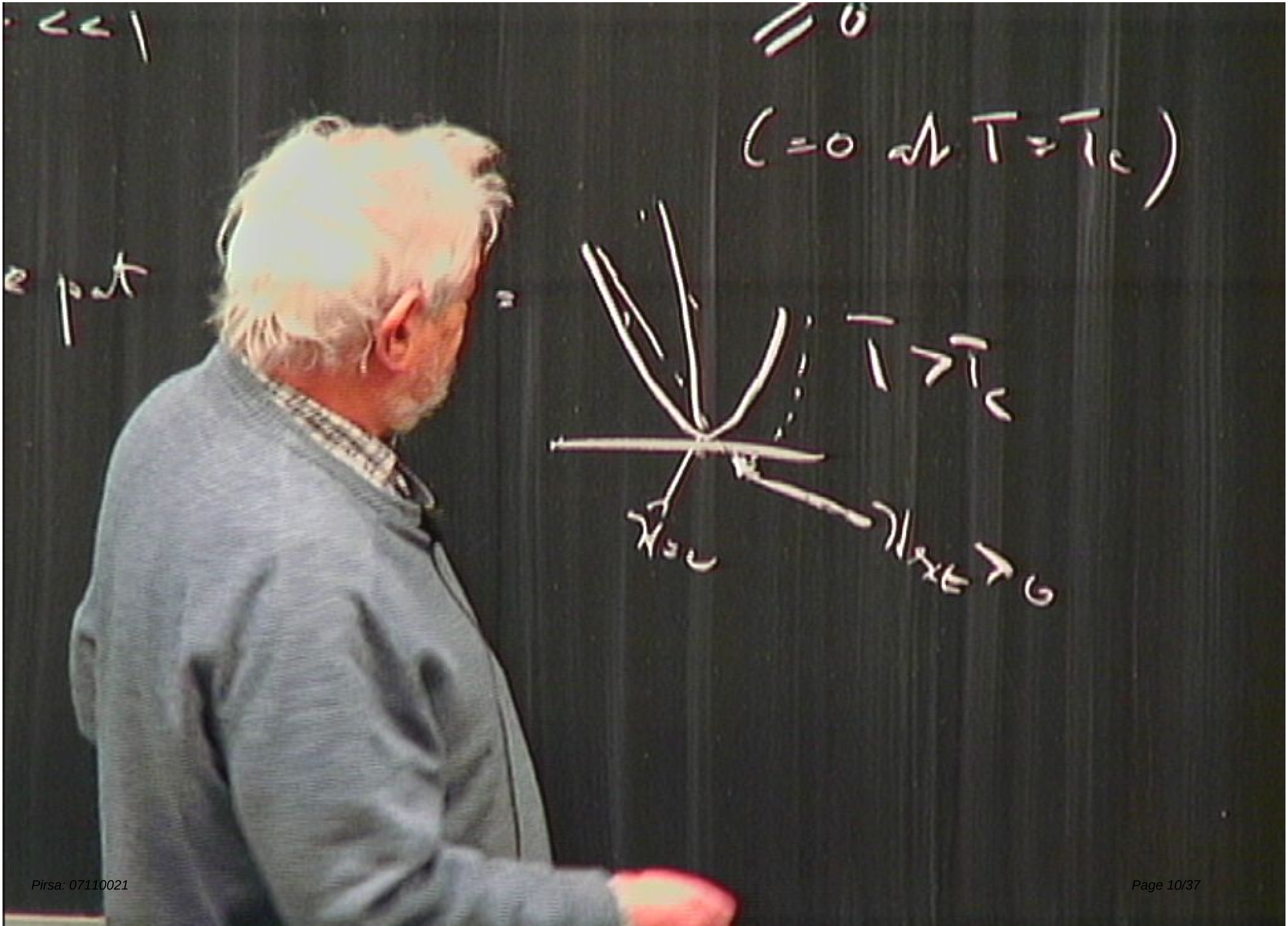
…l effective

$$g(q) = \frac{1-m^2}{1-(1-m^2)\beta\tilde{J}(q)} \xrightarrow[\substack{q\to 0 \\ \frac{T-T_c}{T_c} \ll 1}]{} C\,\frac{1}{\mu^2+q^2}$$

$$m = \langle \mu_0 \rangle$$

χ^{-1} = curvature of effective pot


(= 0 at T = T_c)
T > T_c


(=0 at T=Tc)
T>Tc

$\frac{T - T_c}{T_c} << 1$

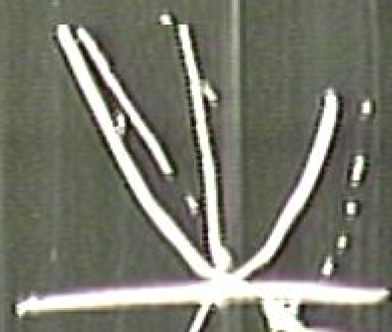
tive part

$$C \; \frac{1}{\mu^2 + q^2} \qquad \mu^2 = \chi^{-1} \simeq (1 - \beta v(0)(1 - m^2))$$

$$\geq 0$$

$$(= 0 \text{ at } T = T_c)$$

$V_{eff}(m) =$



$T > T_c$

$\rightarrow$

$\rightarrow 0$

$\frac{T - T_c}{T_c} << 1$

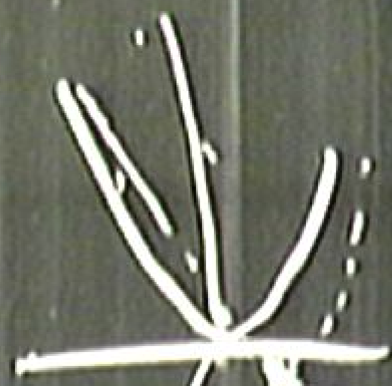
$C \, \frac{1}{\mu^2 + q^2}$ $\quad \mu^2 = \chi^{-1} \simeq (1 - \beta v(0)(1 - m^2))$

$\geqslant 0$

$(= 0$ at $T = T_c)$

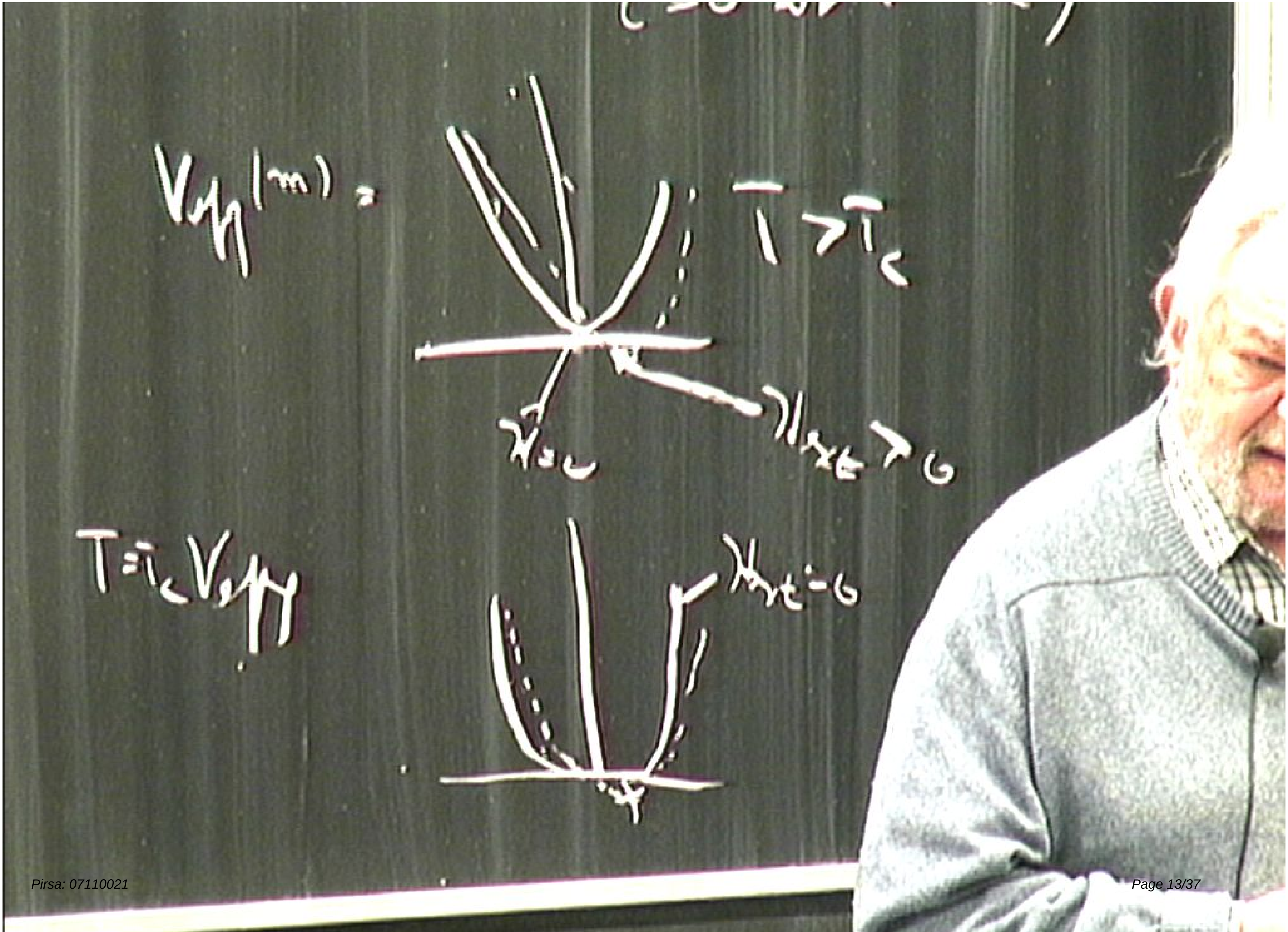
...tive part

$V_{eff}(m) =$



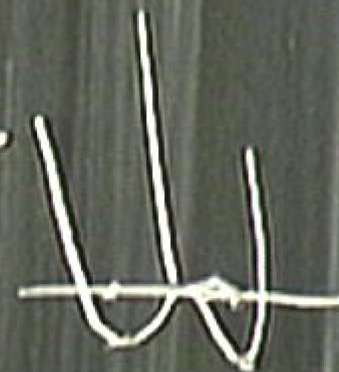
$T > T_c$

$\gamma = 0$ $\quad \gamma|_{x_c} > 0$


V_eff(m) =
T > T_c
λ = 0
λ_ext > 0
T = T_c V_eff
λ_ext = 0

χ^{-1} = curvature of effective pot

$H_{ext} = 0$
$T < T_c$



$T > T_c$

χ^{-1} = curvature of effective

$H_{ext} = 0$

$T < T_c$

$q \to 0$

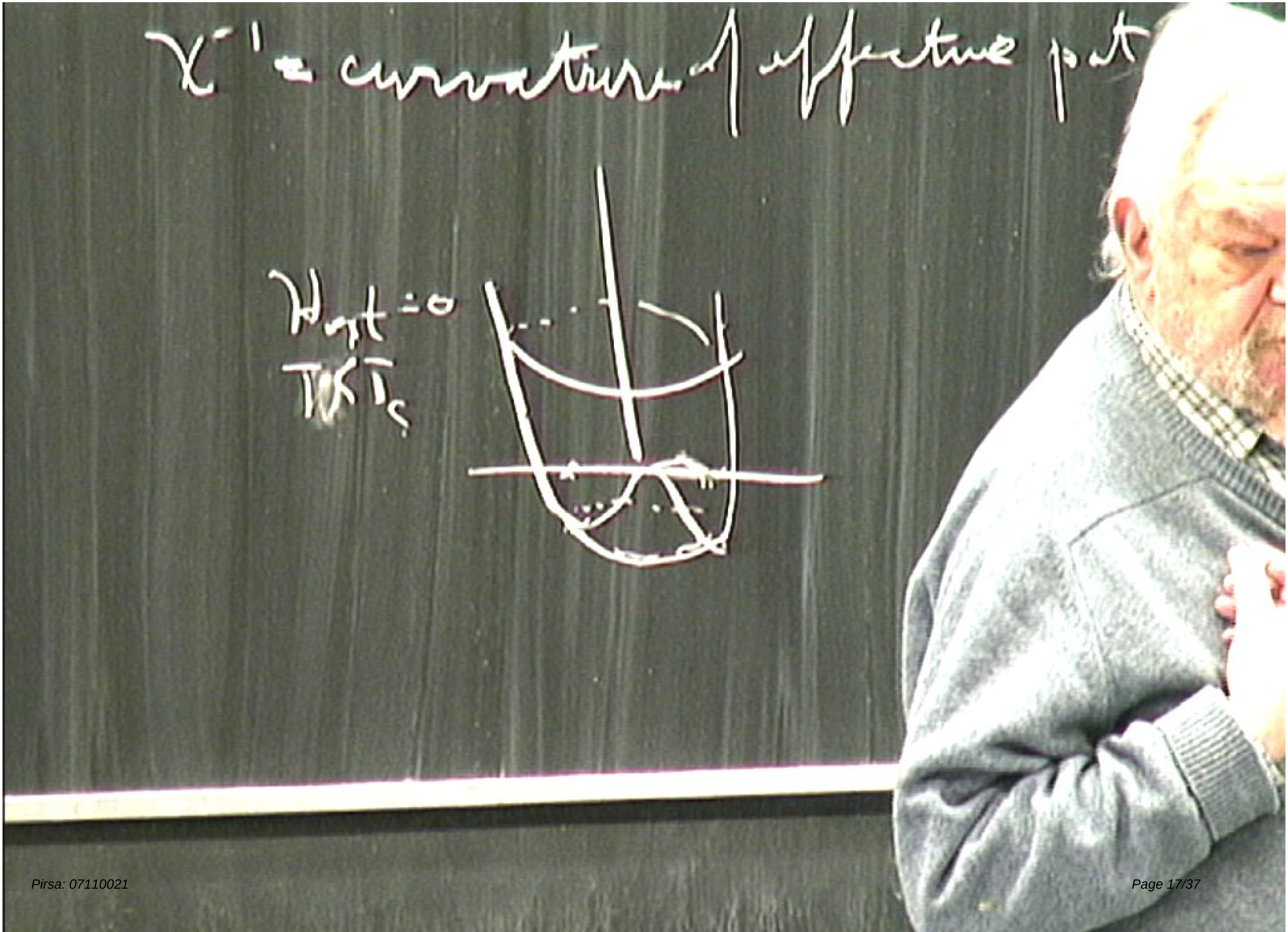
$\frac{T-T_c}{T_c} \ll 1$

≥ 0

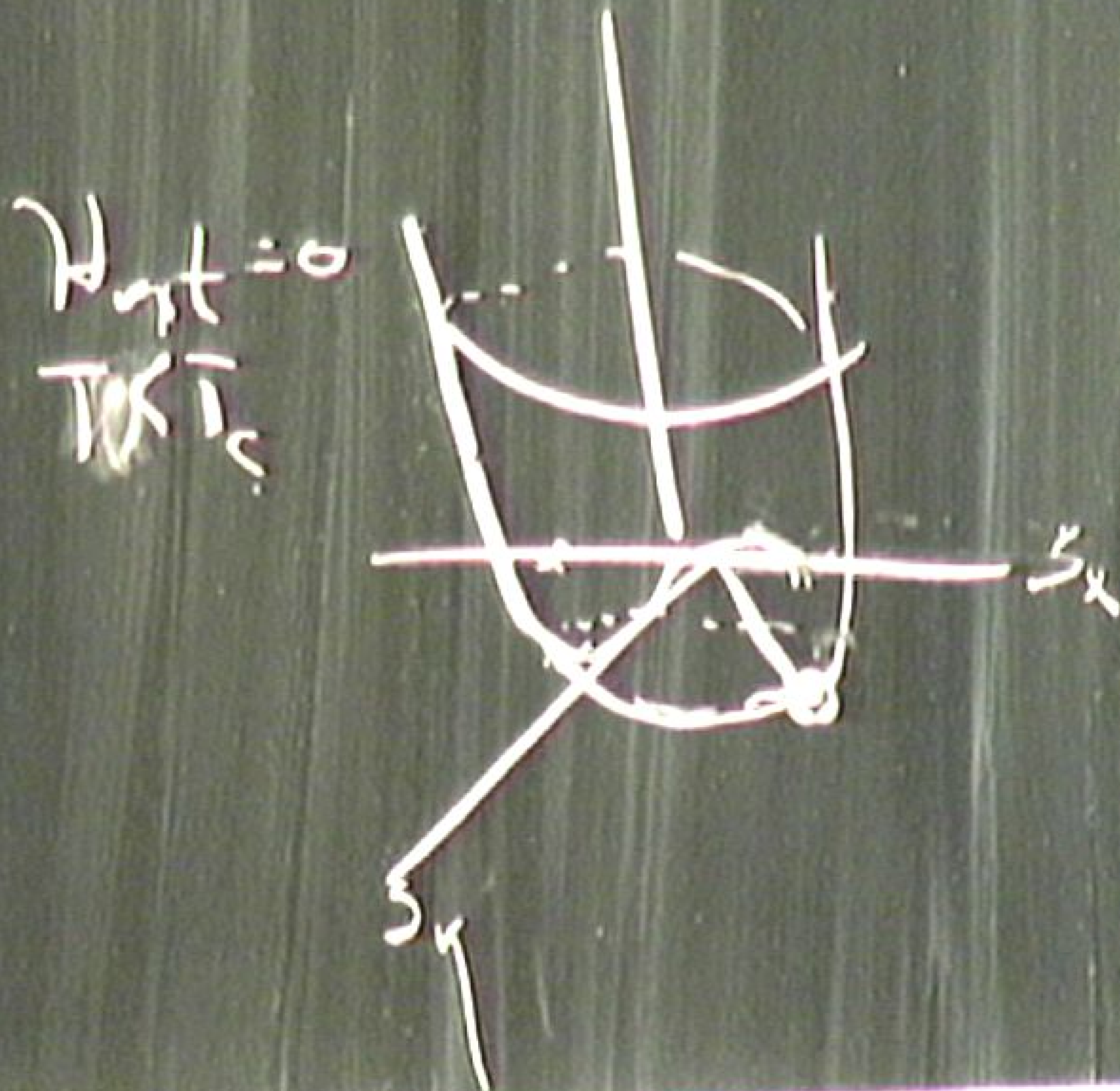
$(=0$ at $T=T_c)$

ffective pot $\quad V_{eff}(m) =$

$T \leq T_c \; V_{eff}$


χ^{-1} = curvature of effective pot
$H_{ext} = 0$
$T < T_c$

χ = curvature of effective pot



Transform of Ising model to a Field Theory

$$Z = \mathrm{tr}\, e^{\beta \underline{\mu V \mu}} = \int \mathcal{D}\phi$$

$\mu = \mu_1 \cdots \mu_N$ $\qquad \phi = \phi_1 \cdots \phi_N$

$V = v_{ij}$ $\qquad \mathcal{D}(\phi) = \prod d\phi_1 \cdots d\phi_N$

Transform of Ising model to a Field Theory

$$Z = \mathrm{tr}\, e^{\beta \frac{\mu V \mu}{2}} = \int \mathcal{D}\phi$$

$\mu = \mu_1 \cdots \mu_N$ $\quad \phi = \phi_1 \cdots \phi_N \quad -\infty \le \phi_i \le +\infty$

$V = V_{ij}$ $\quad \mathcal{D}(\phi) = \prod d\phi_1 \cdots d\phi_N$

Transform of Ising model to a Field Theory

$$V = \beta\nu \quad \Big| \quad Z = \mathrm{tr}\, e^{\frac{\mu V \mu}{2}} = \mathrm{tr} \int \mathcal{D}\phi \; e^{-\mu\phi} \, e^{\phi \frac{V^{-1}}{2} \phi}$$

$$\mu = \mu_1 \cdots \mu_N \qquad \phi = \phi_1 \cdots \phi_N \qquad -\infty \le \phi_i \le +\infty$$

$$V = V_{ij} \qquad \mathcal{D}(\phi) = \prod d\phi_1 \cdots d\phi_N$$

...e Field Theory

$$e^{h}\int \mathcal{D}\phi \; \underline{e^{-\mu\phi}} \; e^{\phi \frac{V^{-1}}{2} \phi} = c \int \mathcal{D}(\phi)\, d[\phi V^{-1} \phi + M(\phi)]$$

$$\phi = \phi_1 \cdots \phi_N \qquad -\infty \le \phi_i \le +\infty$$

$$\mathcal{D}(\phi) = \prod d\phi_1 \cdots d\phi_N$$

Field Theory

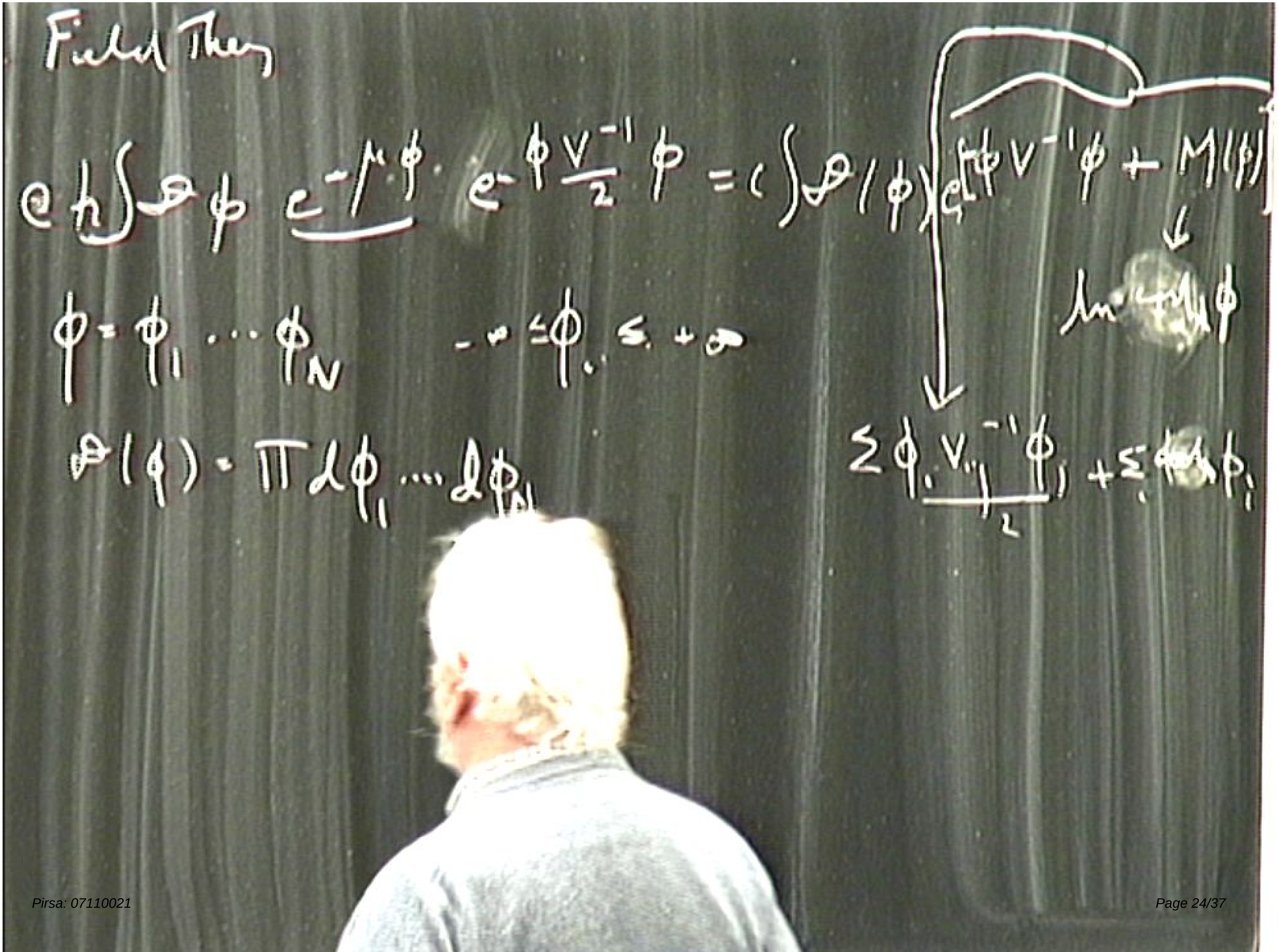
$$e\,\hbar\int \mathcal{D}\phi\;\underline{e^{-\mu\phi}}\; e^{-\phi\frac{V^{-1}}{2}\phi} = c\int \mathcal{D}(\phi)\, e^{-\left[\phi V^{-1}\phi + M(\phi)\right]}$$

$$\ln \phi$$

$$\phi = \phi_1 \cdots \phi_N \qquad -\infty \le \phi_i \le +\infty$$

$$\sum \phi_i V_{ij}^{-1} \phi_j + \sum_i \phi_i$$

$$\mathcal{D}(\phi) = \prod d\phi_1 \cdots d\phi_N$$


Field Theory
$\int \mathcal{D}\phi \, e^{-\mu\phi} \, e^{-\phi \frac{V^{-1}}{2} \phi} = C \int \mathcal{D}(\phi) \, e^{-[\phi V^{-1} \phi + M(\phi)]}$
$\phi = \phi_1 \cdots \phi_N \quad -\infty \leq \phi_i \leq +\infty$
$\mathcal{D}(\phi) = \prod d\phi_1 \cdots d\phi_N$
$\sum \phi_i \frac{V_{ij}^{-1}}{2} \phi_j + \sum_i \mu_i \phi_i$

$$\ldots\phi = c \int \mathcal{D}(\phi)\, e^{-\left[\phi V^{-1}\phi + M(\phi)\right]}$$

$$\ln \text{[illegible]}\, \phi$$

$$\Sigma_i + \infty$$

$$\frac{\Sigma\, \phi_i V_{ij}^{-1} \phi_j}{2} + \Sigma_i \text{[illegible]}\, \phi_i$$

$$= c \int \mathcal{D}(\phi)\, e^{-[\phi V^{-1} \phi + M(\phi)]}$$

$$\ln \ldots \phi$$

$$\sum \phi_i V^{-1}_{ij} \phi_j + \sum \ldots \phi_i$$

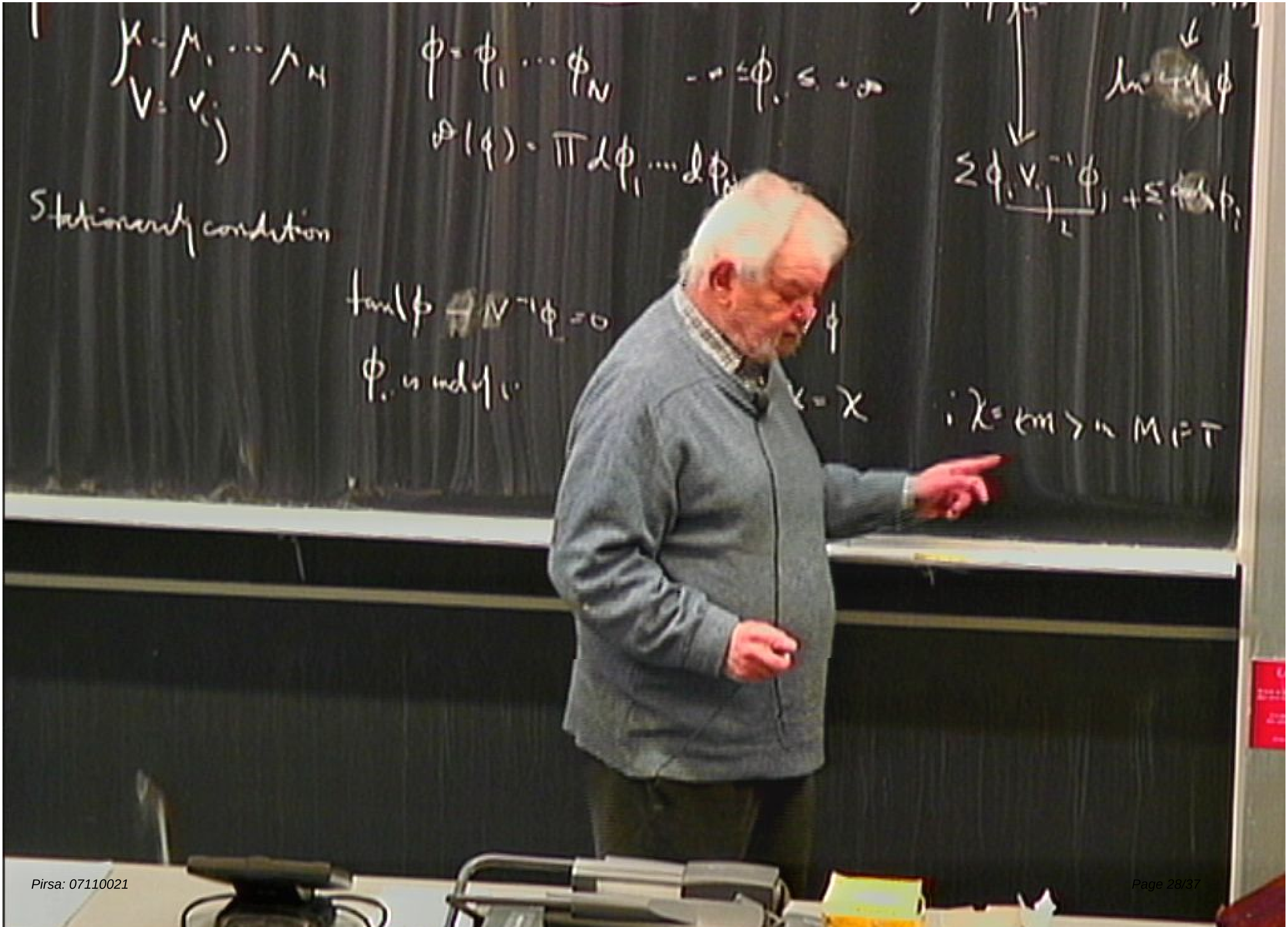
$V_i V_j$

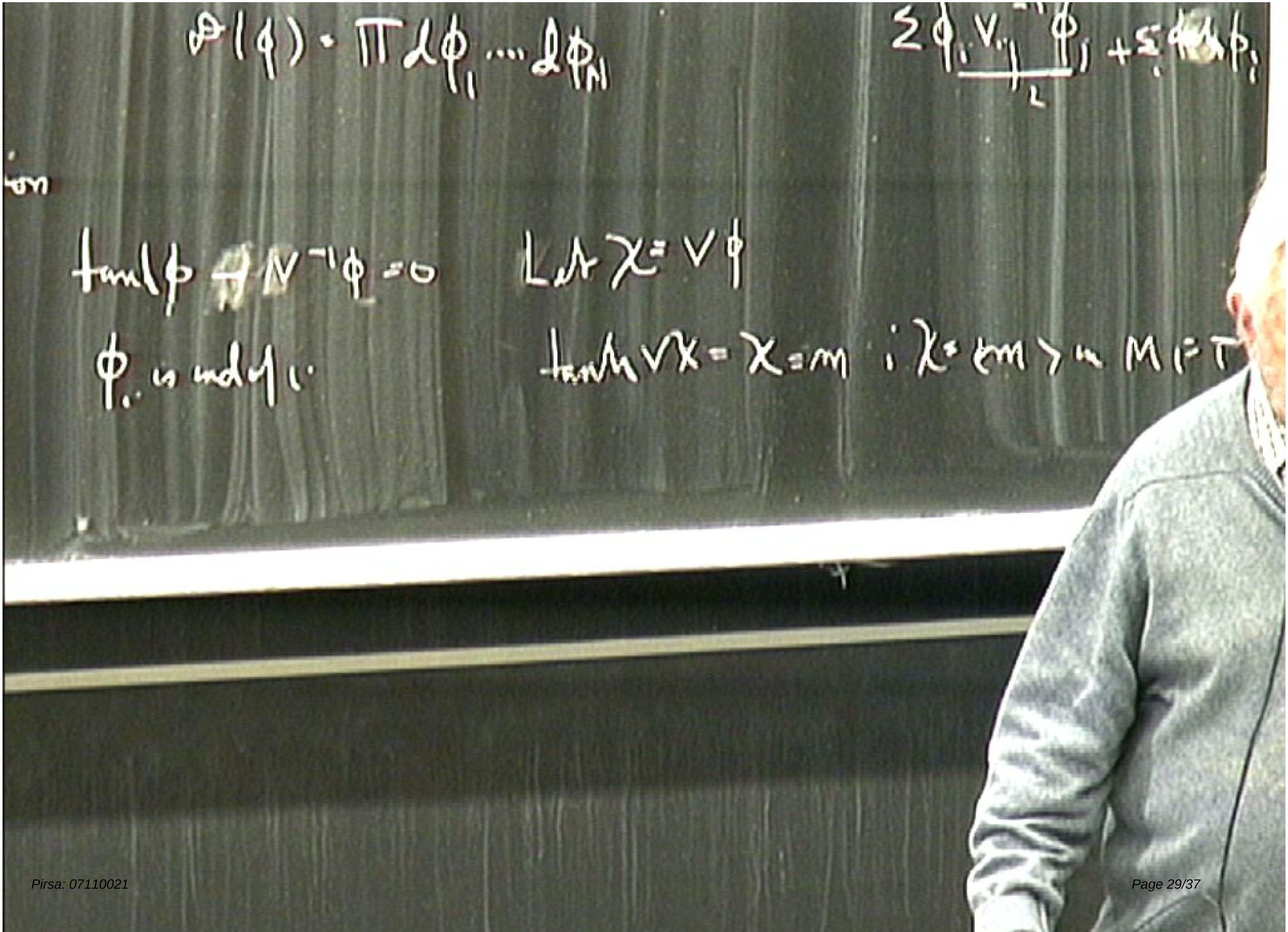
$\mathcal{D}(\phi) = \prod d\phi_1 \cdots d$

Stationarity condition

$\tanh \phi - V^{-1}\phi = 0$

ϕ_i is indep of i.


Stationarity condition


$\mathcal{D}(\phi) = \prod d\phi_1 \cdots d\phi_N$
Let $\chi = V\phi$

$\mathcal{D}(\phi) = \prod d\phi_1 \cdots d\phi_N$ $\qquad$ $\frac{\Sigma \phi_i V_{ij} \phi_j}{2} + \Sigma_i h_i \phi_i$

ion

$\tanh \phi - V^{-1}\phi = 0$ $\qquad$ Let $\chi = V\phi$

ϕ_i is indep of i. $\qquad$ $\tanh V\chi = \chi = m$; $\chi = \langle m \rangle$ in MFT

Eigenvalues of V are $V(\gamma)$

Eigenvalues of V are $V(q_v)$

" V^{-1} are $V^{-1}(q_v)$

$$\frac{T - T_c}{T_c}$$

$$\frac{1}{V(q)} = \frac{1}{V(0) + c\,q^2}$$

$$= \frac{1}{\beta v(0)}$$

φ_i is ... ... $\chi = m$, λ ...

Long distance (small q) , $\frac{T-T_c}{T_c}$ small ($\beta v(0)$ is near 1)

$$\frac{1}{V(q)} = \frac{1}{V(0) + q^2}$$

$$= \frac{1}{\beta v(0) - q^2} = \frac{1}{\beta v(0)}\left[1 + \frac{q^2}{\beta v(0)}\right]$$

$$\simeq \frac{1}{\beta v(0)}\left[1 + q^2\right] \quad \text{(because } \beta v(0) \simeq 1\text{)}$$

$$\ln\cosh\phi = \frac{\phi^2}{2} - \frac{\phi^4}{12} + \cdots$$

Action

$$-\frac{1}{2}\sum|\phi(q)|^2\left[(\beta\tilde{v}(0)-1)+q^2\right]+\lambda\phi^4$$

$$\ln\cosh\phi = \frac{\phi^2}{2} - \frac{\phi^4}{12} + \cdots$$

Action

$$-\frac{1}{2}\sum|\phi(q)|^2\left[\left(\beta\tilde{J}(0)-1\right) + q^2\right] + \lambda\phi^4$$

$$= -\int d^dx\left[\mu^2\phi^2 + (\nabla\phi)^2 + \lambda\phi^4 + \cdots\right]$$

$$Z = \int\mathcal{D}(\phi)\, e^{L(\phi)}$$

$$\ln \cosh \phi = \frac{\phi^2}{2} - \frac{\phi^4}{12} + \cdots$$

Action

$$= \frac{1}{2} \sum |\phi(q)|^2 \left[\left(\beta \tilde{J}(0) - 1 \right) + q^2 \right] + \lambda \phi^4$$

$$= -\int dr \left[\mu^2 \vec{\phi}^2 + (\nabla \vec{\phi})^2 + \lambda (\vec{\phi}^2)(\vec{\phi}^2) \right]$$

$$Z = \int \mathcal{D}(\phi) e^{L(\phi)}$$