Title: Spontaneous Broken Symmetry 1

Date: Nov 01, 2007 10:00 AM

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Abstract:

1 Translation

1 Translations

2. Orientations in a ferromagnet

1 Translations

2. Orientation in a ferromagnet

3. Quantum phase of a quiescent superfluid

1 Translation

2. Orientation in a ferromagnet

3. Quantum phase of a quiescent superfluid < ^{4}He / superconductors,

4. Internal Symmetries — Chiral symmetry

1 Translations

2. Orientation in a ferromagnet

3. Quantum phase of a quiescent superfluid < ^{3}He, superconductors,

4. Internal Symmetries — Chiral symmetry

Point you special

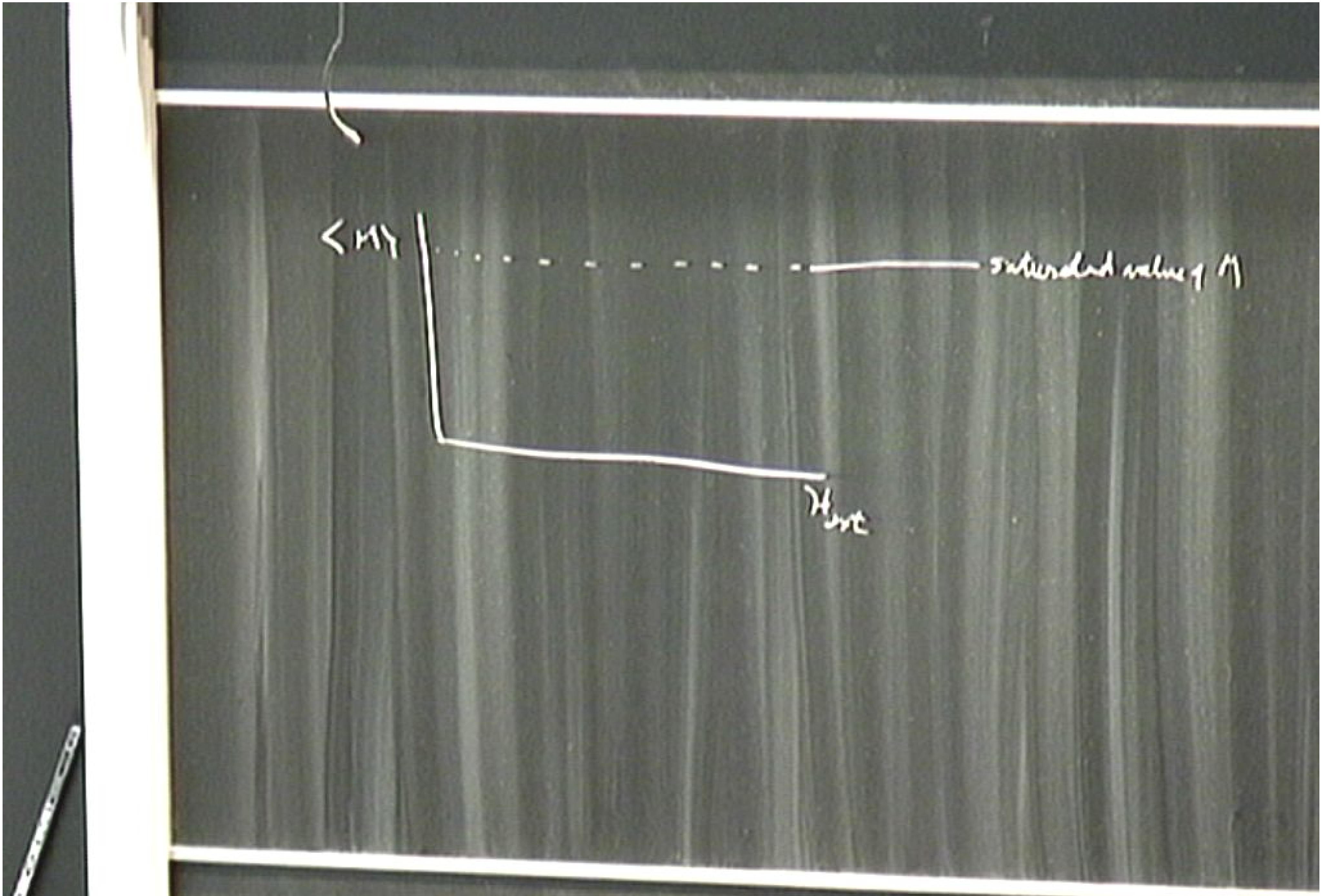
1 Translation

2. Orientation in a ferromagnet

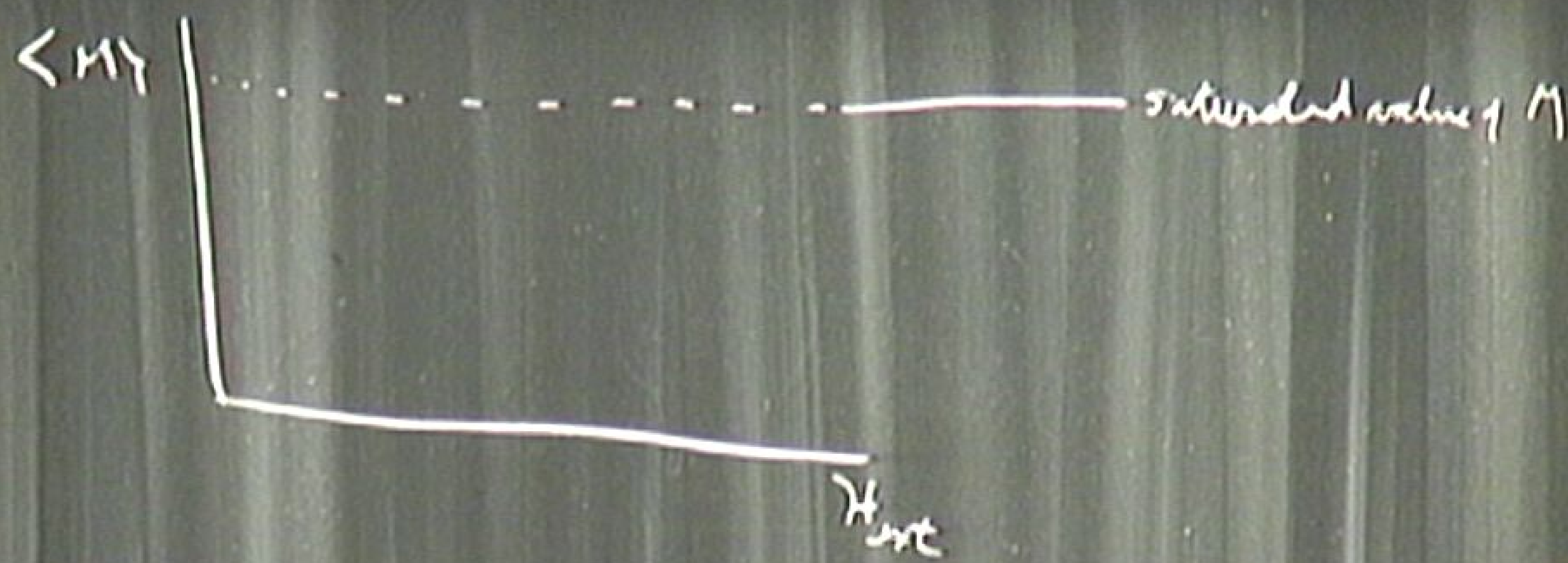
3. Quantum phase of a quiescent superfluid < ^{4}He, superconductors.

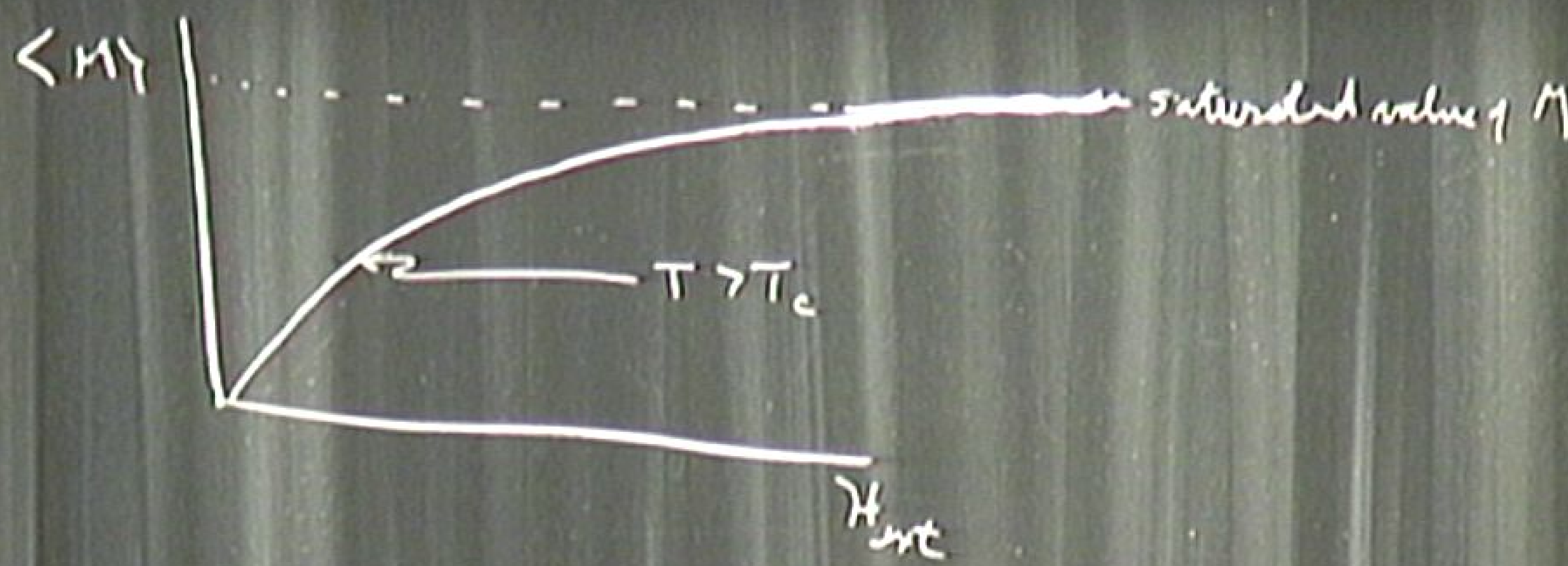
4. Internal Symmetries — Chiral symmetry

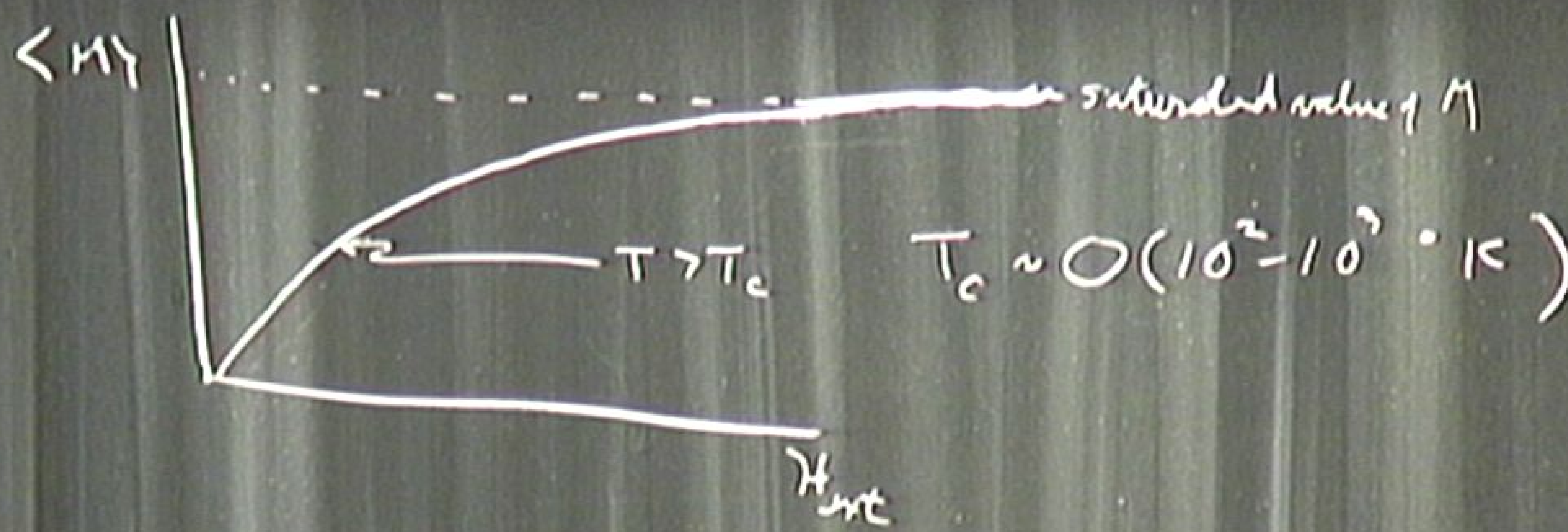
Point q'n special ... / gauge sym. (local sym)



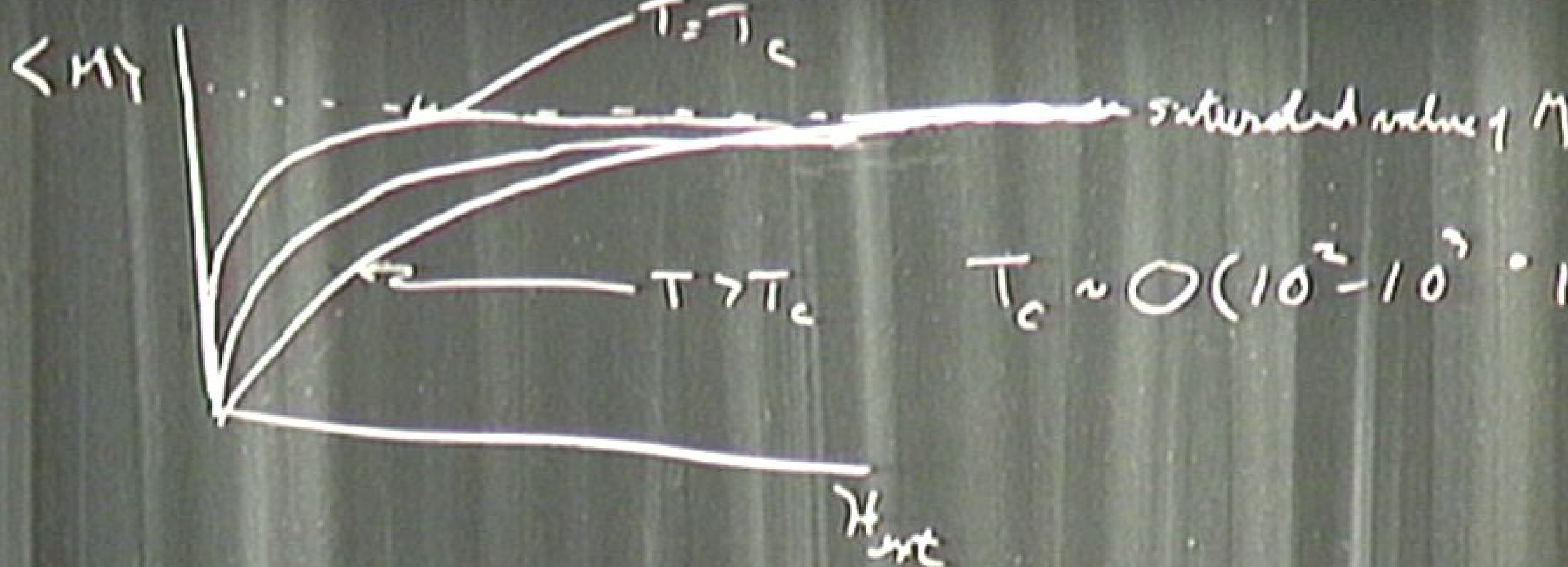
<M>
saturated value of M
H_ext


$\langle M \rangle$
saturated value of M
H_{ext}

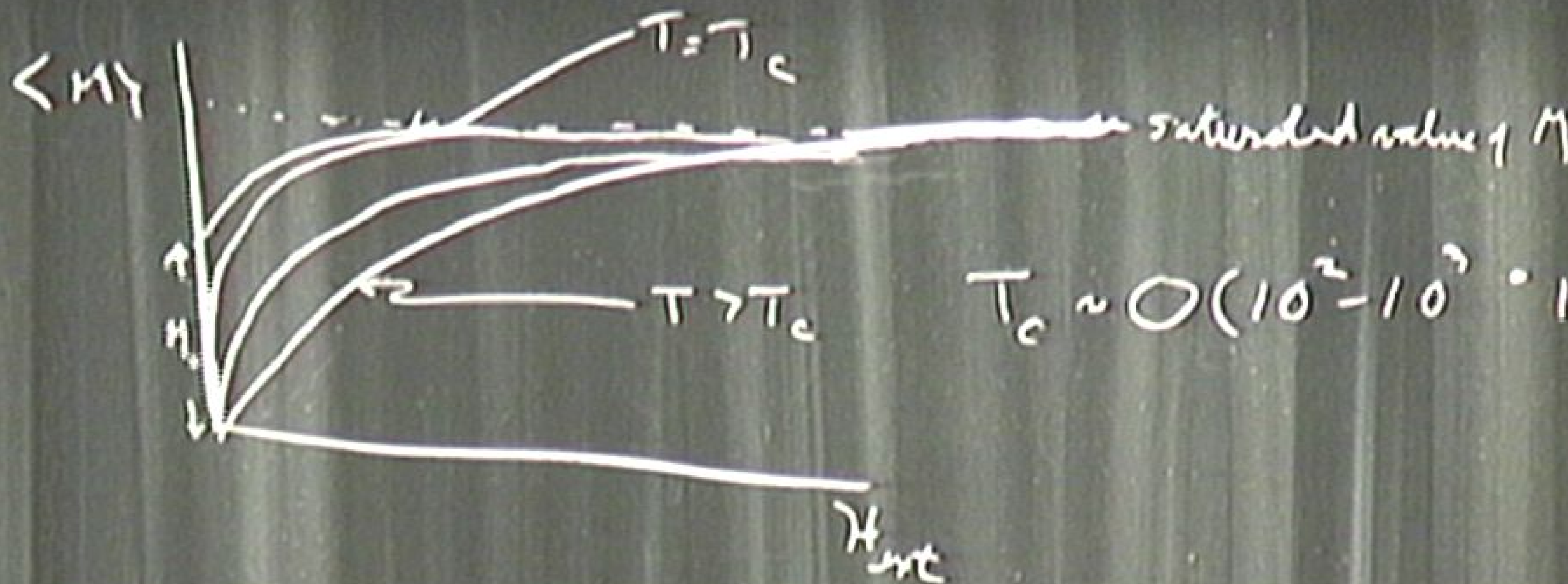

<M>
saturated value of M
T > T_c
H_ext



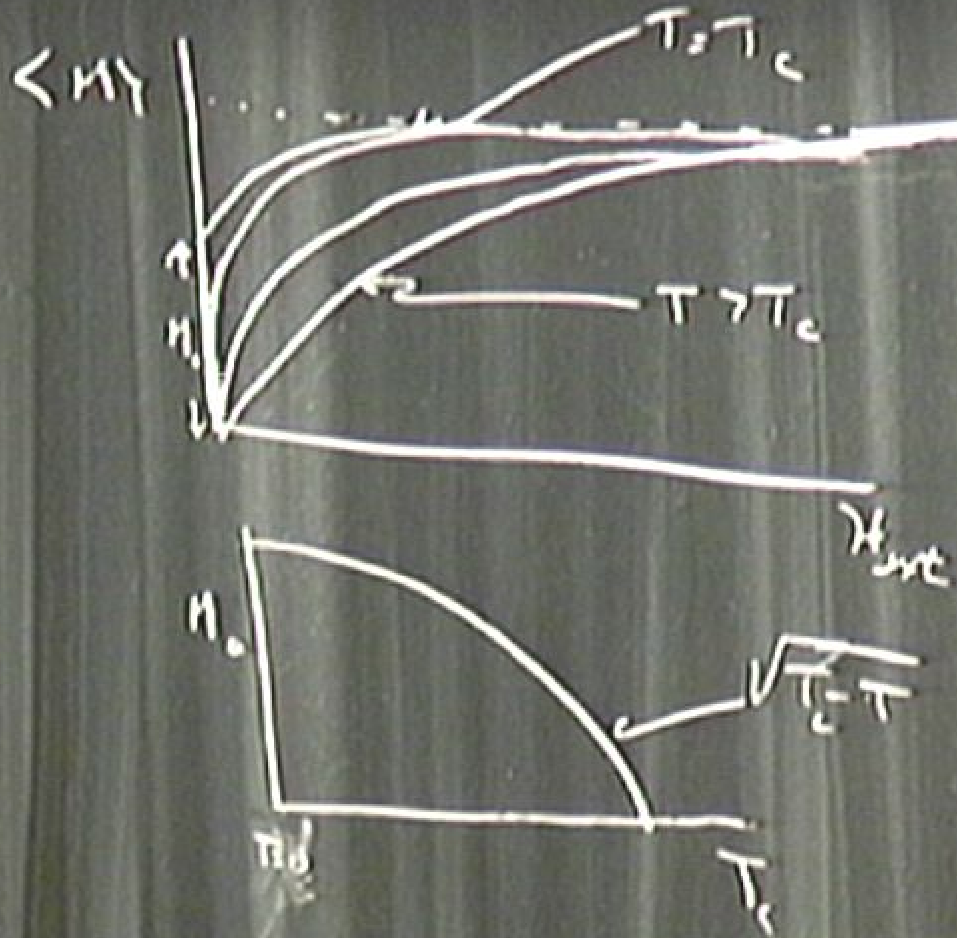
<M>
saturated value of M
T > T_c
$T_c \sim O(10^2 - 10^3\ °K)$
$\mathcal{H}_{ext}$



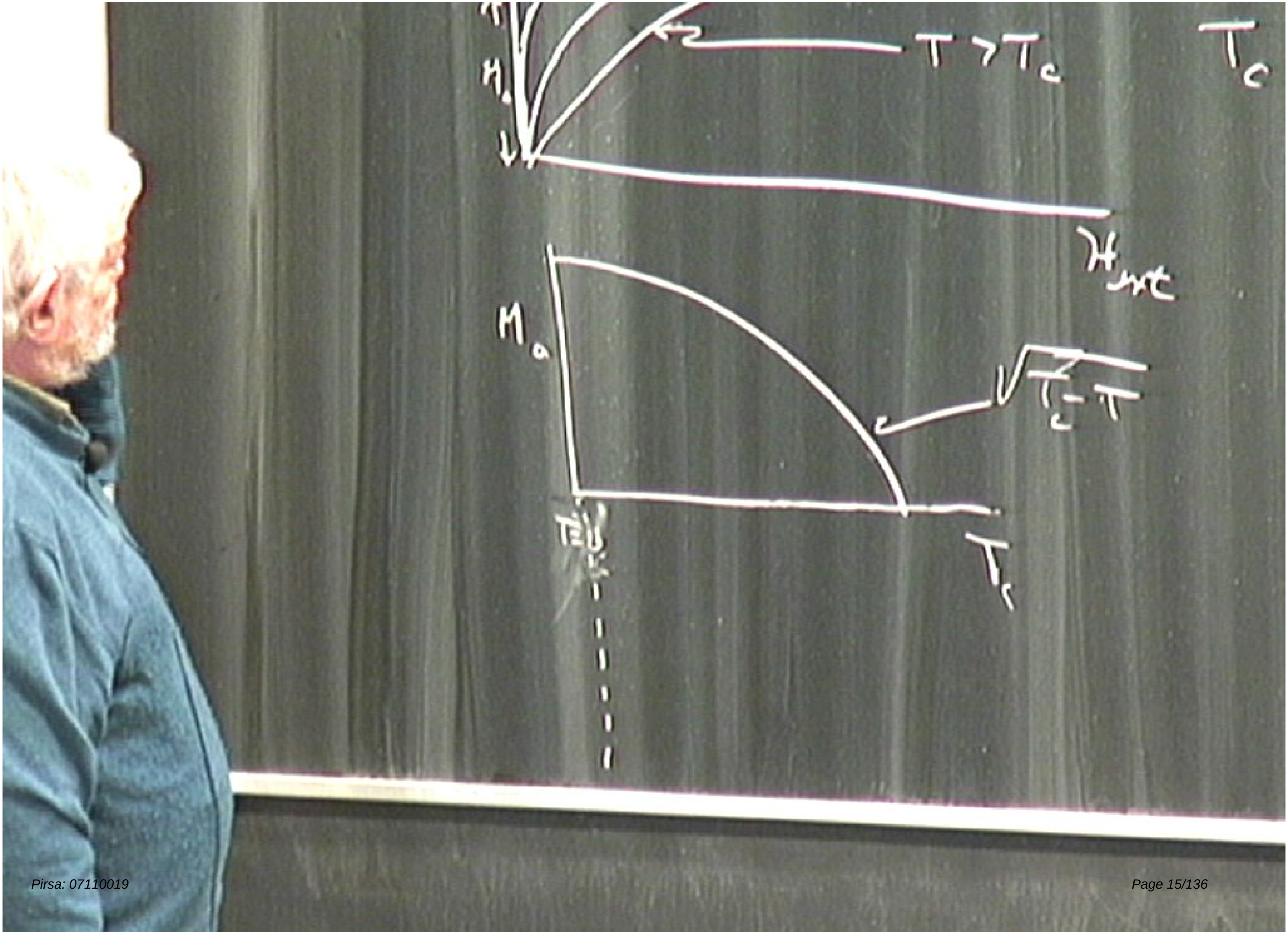
$T_c \sim O(10^2 - 10^3\ ^\circ K)$

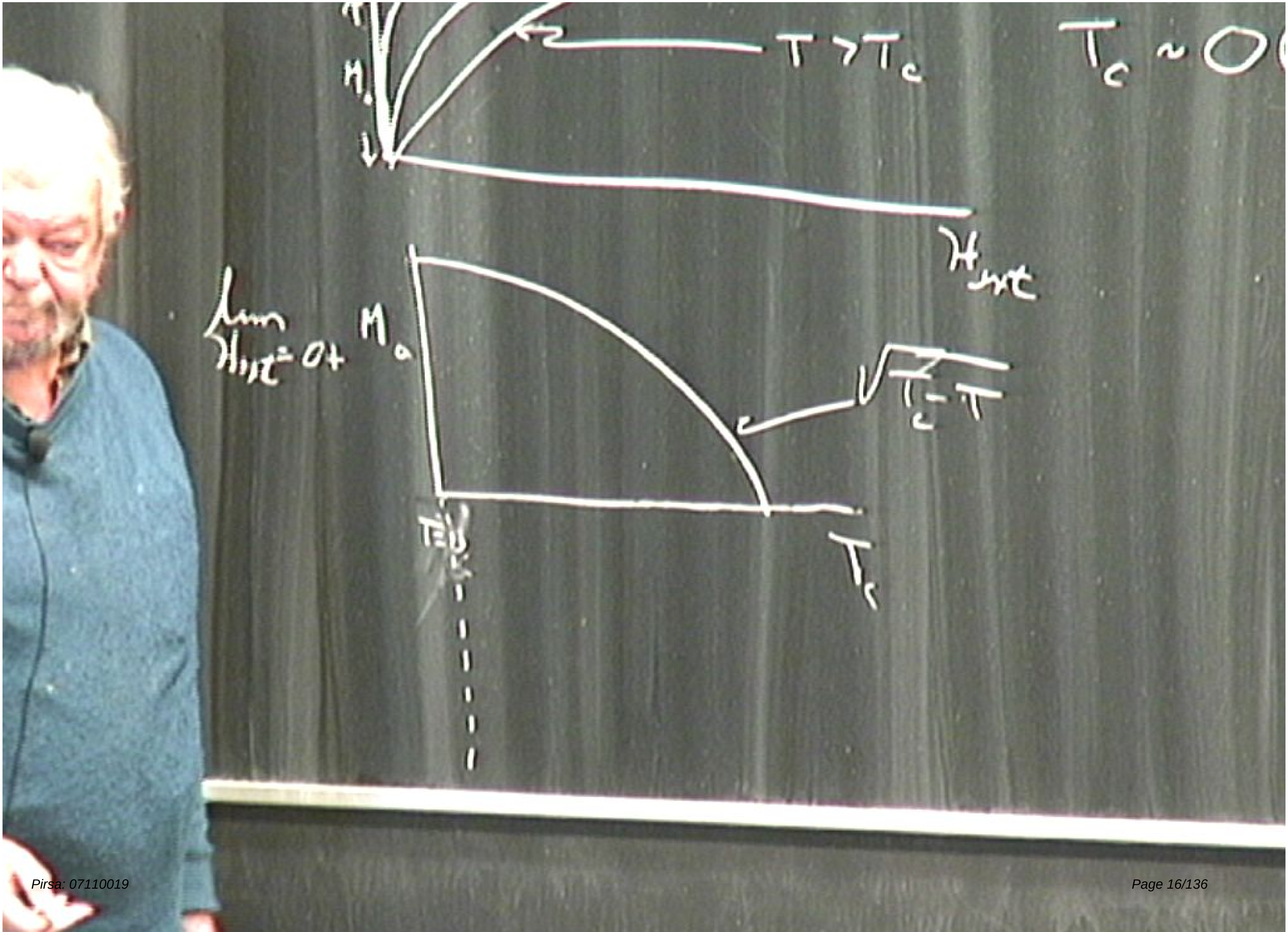


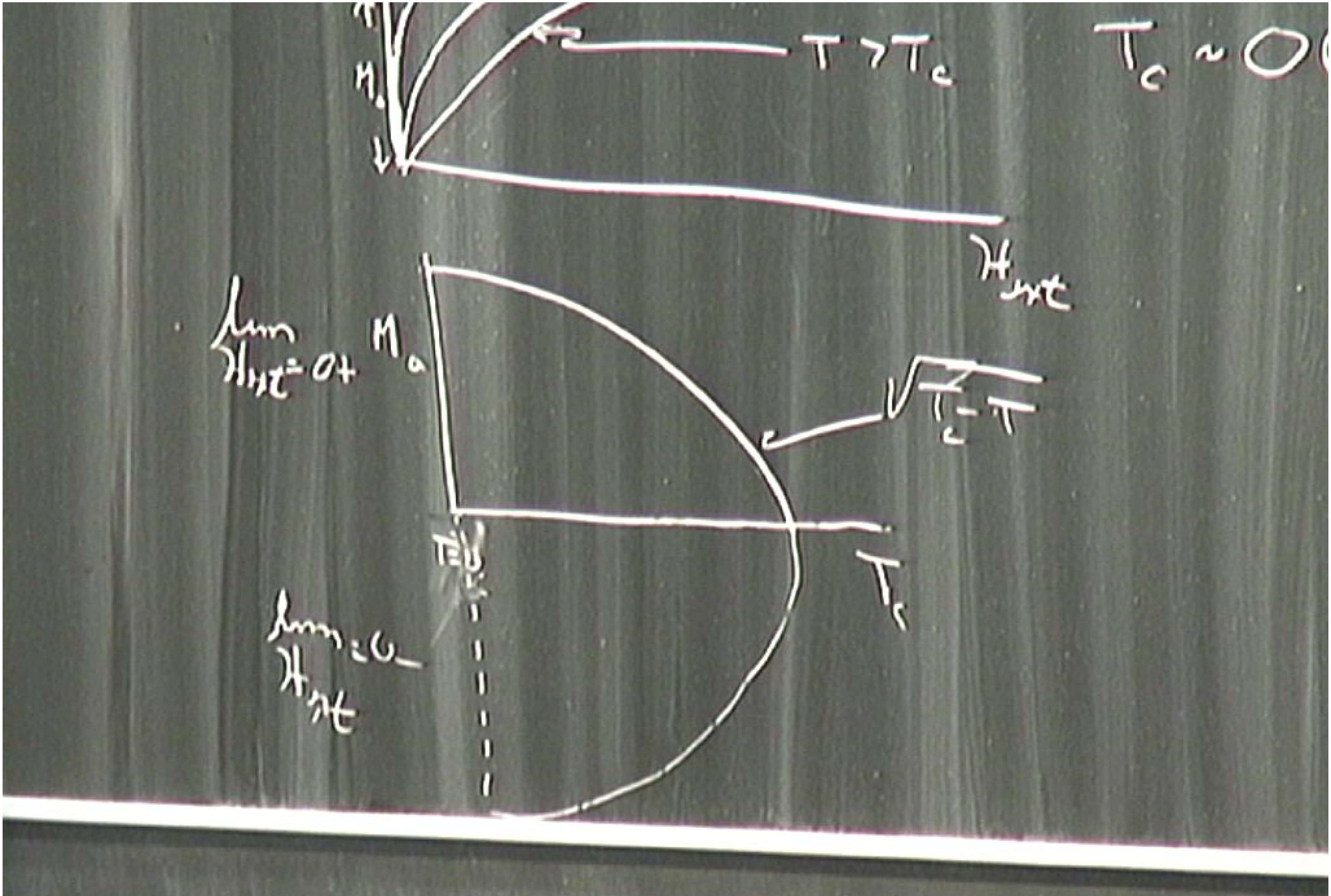
$T_c \sim O(10^2 - 10^3\ ^\circ K)$



$T_c \sim O(10^2 - 10^3\ ^\circ K)$


$T > T_c$
T_c
H_{ext}
M_0
$\sqrt{T_c - T}$
T_c


$T > T_c$
$T_c \sim O$
M_0
$\mathcal{H}_{ext}$
$\lim_{\mathcal{H}_{ext} = 0+} M_0$
$\sqrt{T_c - T}$
T_c


T > Tc
Tc ~ O
M0
Hext
lim
Hext = 0+
√Tc − T
Tc
lim = 0−
Hext

Weiss phenomenology

Internal field due to a spin surrounded by others

Weiss phenomenology

Internal field due to a spin surrounded by other

$\mu_0 \vec{H}_{ext} \vec{S}$

Weiss phenomenology

Internal field due to a spin surrounded by other

$$\mu_0 \vec{H}_{ext} \langle \vec{S} \rangle + \mu_0 \vec{H}_{int} \langle \vec{S} \rangle$$

$$\vec{H}_{int} =$$

Weiss phenomenology

Internal field due to a spin surrounded by other

$$\mu_0 \vec{H}_{ext} \langle \vec{S} \rangle + \mu_0 \vec{H}_{int} \langle \vec{S} \rangle$$

$$\vec{H}_{int} = \alpha \langle \vec{S} \rangle$$

Weiss phenomenology

Internal field due to a spin surrounded by other

$$\mu_0 \vec{H}_{ext} \langle \vec{S} \rangle + \mu_0 \vec{H}_{int} \langle \vec{S} \rangle$$

$$\vec{H}_{int} = \alpha \langle \vec{S} \rangle$$

If $H_{ext} = 0$

$$\langle M \rangle = B($$

Weiss phenomenology

Internal field due to a spin surrounded by other

$g\vec{H}_{ext}\langle\vec{S}\rangle + g\vec{H}_{int}\langle\vec{S}\rangle$ $\qquad \mathcal{H} = \text{energy}$

$\vec{H}_{int} = \alpha\langle\vec{S}\rangle$

If $\mathcal{H}_{int} = 0$

$\langle M\rangle = B(\mathcal{H}_{ext}$

Weiss phenomenology

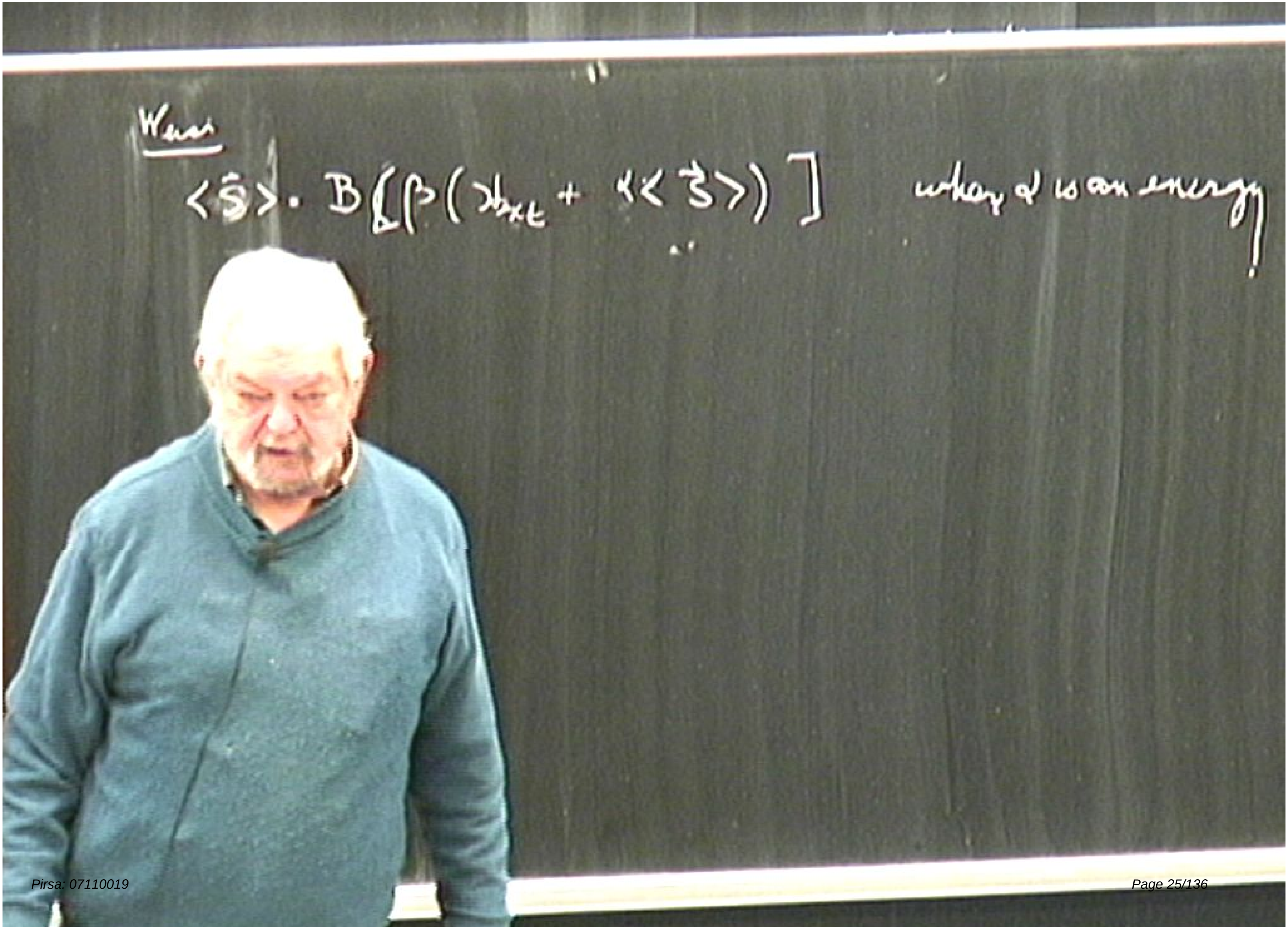
Internal field due to a spin surrounded by others

$$\mu \vec{H}_{ext}\langle \vec{S} \rangle + \mu \vec{H}_{int} \langle \vec{S} \rangle \qquad \mathcal{H} = \text{energy} \;.\; \beta = \frac{1}{k_T}$$

$$\vec{H}_{int} = \alpha \langle \vec{S} \rangle$$

If $\mathcal{H}_{int} = 0$

$$\langle M \rangle = B\left(\beta \mathcal{H}_{ext}\right)$$


Weiss
$\langle \vec{S} \rangle = B[\beta(h_{ext} + \alpha\langle \vec{S} \rangle)]$
where α is an energy

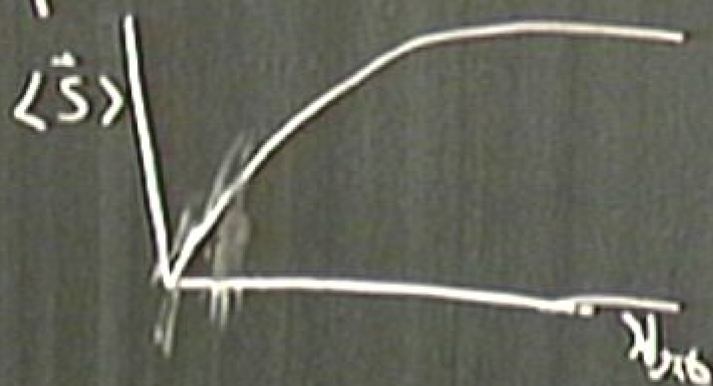
Weiss

$\langle \hat{S} \rangle = B_S[\beta(\mathcal{H}_{ext} + \alpha\langle \vec{S} \rangle)]$ where α is an energy!

Weiss

$$\langle \vec{S} \rangle = B[\beta(\mathcal{H}_{ext} + \alpha \langle \vec{S} \rangle)]$$ where α is an energy

Small β



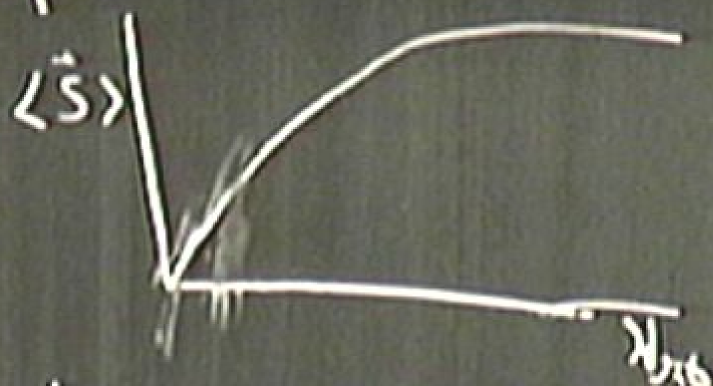
$$\longrightarrow \quad \langle \vec{S} \rangle = \beta \mathcal{H}_{ext} + \beta \alpha \vec{S} + \dots$$

$$\langle \vec{S} \rangle = \frac{\beta \mathcal{H}_{ext}}{1 - \beta \alpha}$$

Weiss

$$\langle \vec S \rangle = B[\beta(\mathcal{H}_{ext} + \alpha\langle \vec S \rangle)]$$ where α is an energy

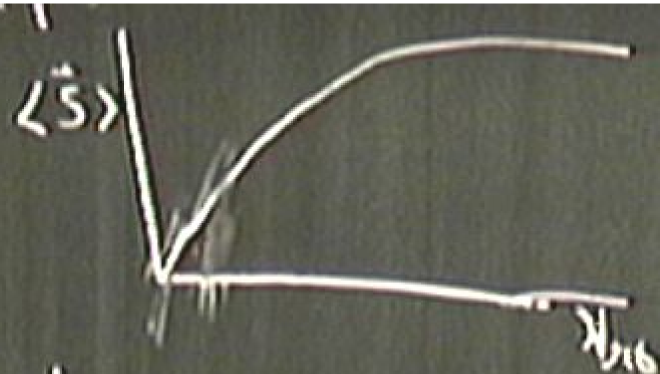
Small β



$$\langle \vec S \rangle = \beta \mathcal{H}_{ext} + \beta \alpha \vec S + \cdots$$

$$\longrightarrow \quad \langle \vec S \rangle = \frac{\beta \mathcal{H}_{ext}}{1 - \beta\alpha}$$

Identify α: $H_{int} = -\frac{1}{2} \sum V_{ij} \vec S_i \cdot \vec S_j$



$$\langle \vec{S} \rangle = \frac{\beta \mathcal{H}_{ext}}{1 - \beta\kappa}$$

Identify κ: $H_{int} = -\frac{1}{2}\sum V_{ij}\, \vec{S}_i \cdot \vec{S}_j$

$V_{ij} > 0$

i, j are like receptors

$1 - \beta\alpha$

$$\begin{cases} v_{ij} > 0 \\ ij \text{ are like neighbors} \end{cases}$$

$\mathcal{H}_{ext}$

α : $H_{int} = -\frac{1}{2} \sum V_{ij} \vec{S}_i \cdot \vec{S}_j$)

$\vec{\mathcal{H}}_{int} = - \sum_j V_{ij} \vec{S}_j$

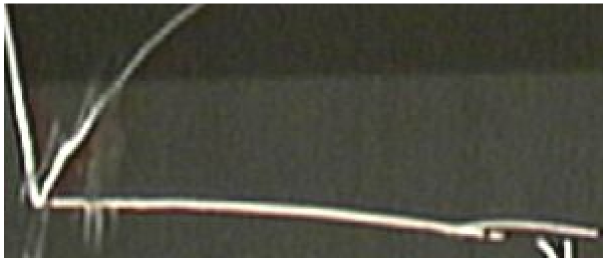
where α is an energy

$$\langle \vec{S} \rangle = \beta \mathcal{H}_{ext} + \beta \alpha \langle \vec{S} \rangle + \cdots$$

$$\rightarrow \langle \vec{S} \rangle = \frac{\beta \mathcal{H}_{ext}}{1 - \beta \alpha}$$

$\vec{S}_i \cdot \vec{S}_j$ $\quad$ $\sum_{ij} V_{ij} > 0$

ij are like neighbors



$$\langle S \rangle = \frac{\beta \mu_B H_{ext}}{1 - \beta\alpha}$$

$\mathcal{H}_{ext}$

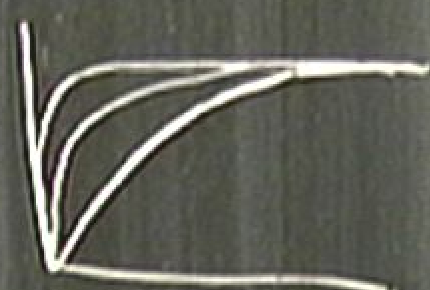
α : $H_{int} = -\frac{1}{2}\sum V_{ij}\, \vec{S}_i \cdot \vec{S}_j$; $\begin{cases} V_{ij} > 0 \\ i,j \text{ are like neighbors} \end{cases}$

$\vec{\mathcal{H}}_{int} = -\sum_j V_{ij}\, \vec{S}_j$

$\alpha = -\sum_j V_{ij}$; $V_{ij} = V(\vec{R}_i - \vec{R}_j)$ $\therefore$ α is ind. of $\vec{R}_i$

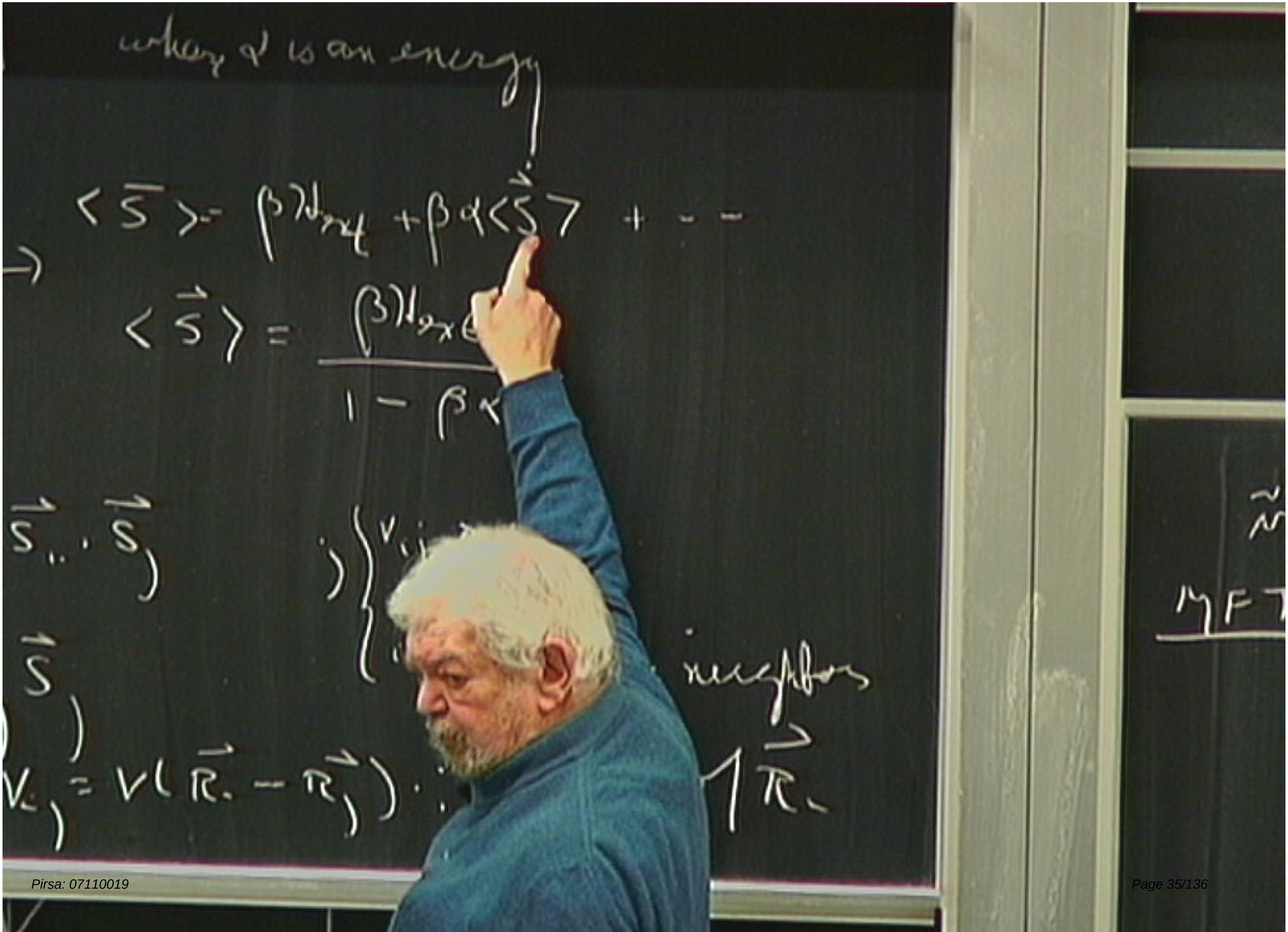
$\mathcal{H}_{int} = \alpha \langle \vec{S} \rangle$

$(\beta \mathcal{H}_{ext})$



$\eta\ H_{int} = 0$

$$\tilde{n}(q) = \sum n(R_i - R_j)\, e^{i q \cdot (\vec{R}_i - \vec{R}_j)}$$


where α is an energy
⟨S⟩ = βH_ext + βα⟨S⟩ + - - -
⟨S⟩ = βH_ext / 1 − βα
neighbors
V_ij = V(R_i − R_j)

$$\tilde{n}(q) = \sum n(R_i - R_j)\, e^{iq\cdot(\vec{R}_i - \vec{R}_j)}$$

MFT

$$\langle M \rangle = B\left(\beta \mathcal{H}_{ext} + \tilde{n}(0)\langle M \rangle\right)$$

$$\tilde{v}(q) = \sum v(R_i - R_j)\, e^{iq\cdot(\vec{R}_i - \vec{R}_j)}$$

MFT

$$\langle M \rangle = B\left(\beta[\mathcal{H}_{ext} + \tilde{v}(0)\langle M \rangle]\right)$$

Specializing: Ising Model $\vec{S}_i = \mu_i = \pm 1$

$$\tilde{\nu}(q) = \sum \nu(R_i - R_j)\, e^{iq\cdot(\vec{R}_i - \vec{R}_j)}$$

MFT

$$\langle M \rangle = B\left(\beta[H_{ext} + \tilde{\nu}(0)\langle M \rangle]\right)$$

Specializing: Ising Model $\vec{S}_i = \mu_i = \pm 1$

$$\tilde{v}(q) = \sum v(R_i - R_j)\, e^{i q\cdot(\vec{R}_i - \vec{R}_j)}$$

MFT

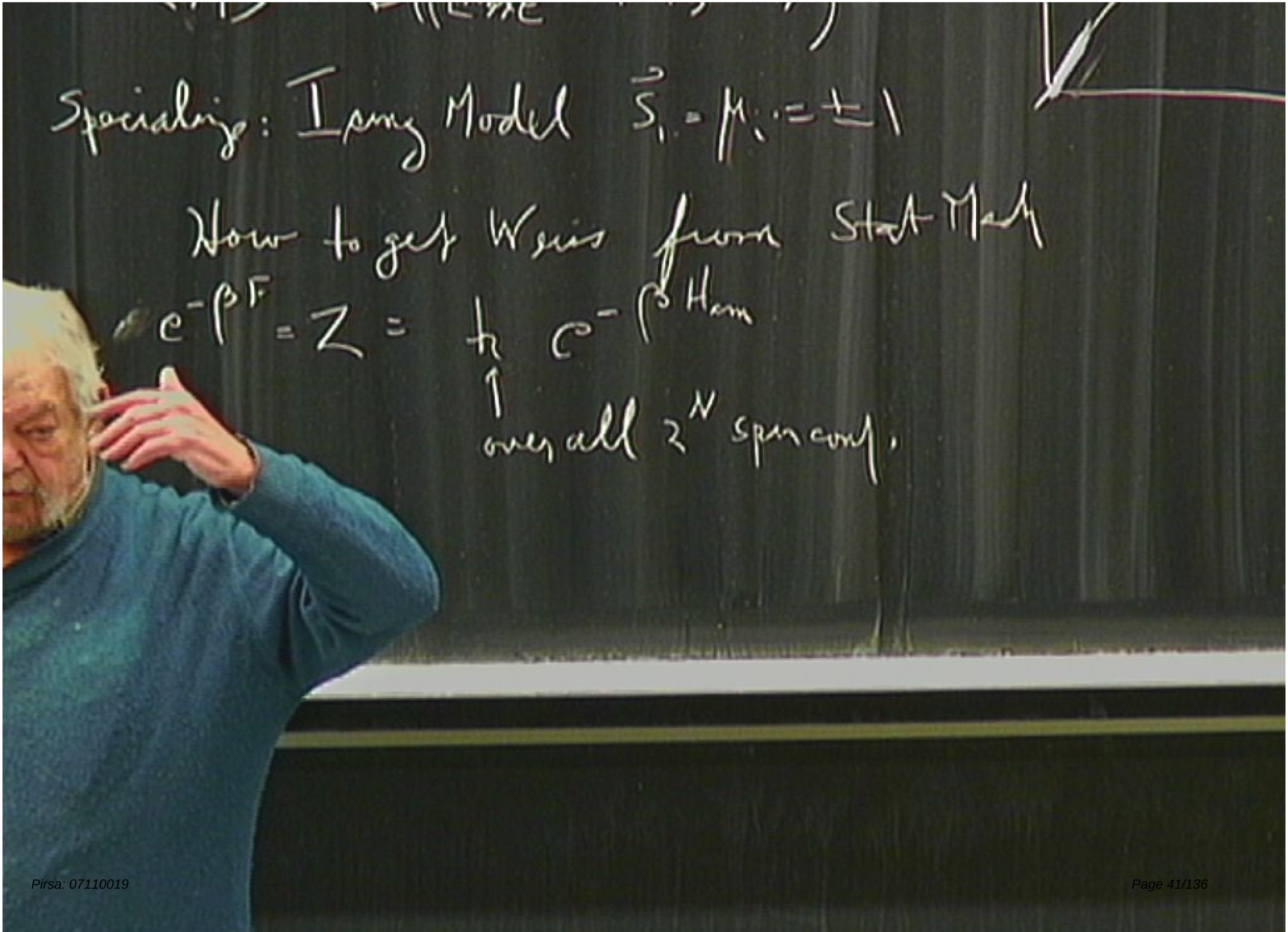
$$\langle M \rangle = B\left(\beta H_{ext} + \tilde{v}(0)\langle M\rangle\right)$$

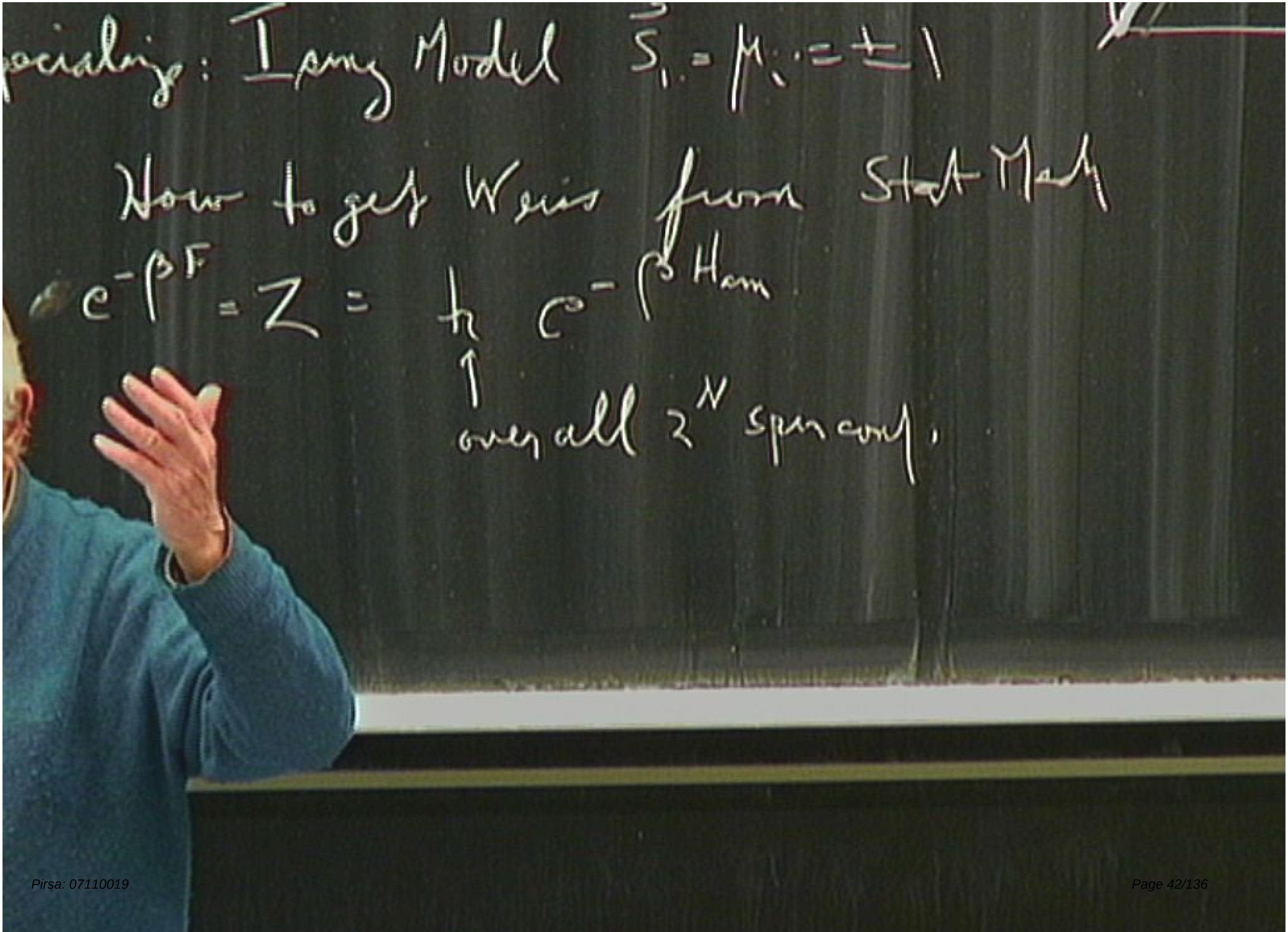
Specializing: Ising Model $\vec{S}_i = \mu_i = \pm 1$

Specializing: Ising Model $S_i = \mu_i = \pm 1$

How to get Weiss from Stat Mech

$e^{-\beta F} = Z =$


Specializing: Ising Model $\vec{S}_i = \mu_i = \pm 1$
How to get Weiss from Stat Mech
$e^{-\beta F} = Z = \mathrm{tr}\, e^{-\beta H_{am}}$
over all 2^N spin conf.


pecializing: Ising Model $\vec{S}_i = \mu_i = \pm 1$
How to get Weiss from Stat Mech
$e^{-\beta F} = Z = \mathrm{tr}\; e^{-\beta H_{am}}$
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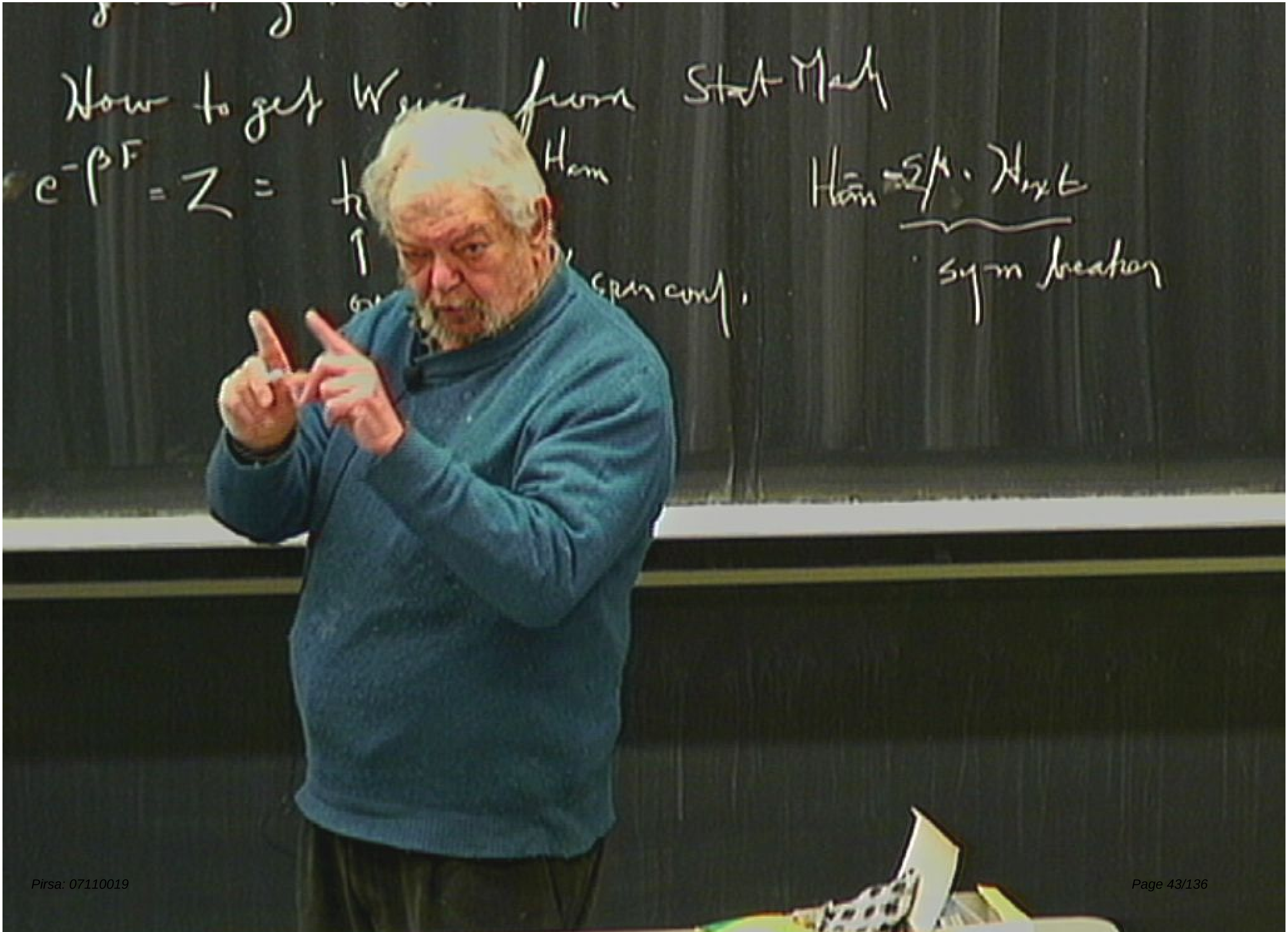
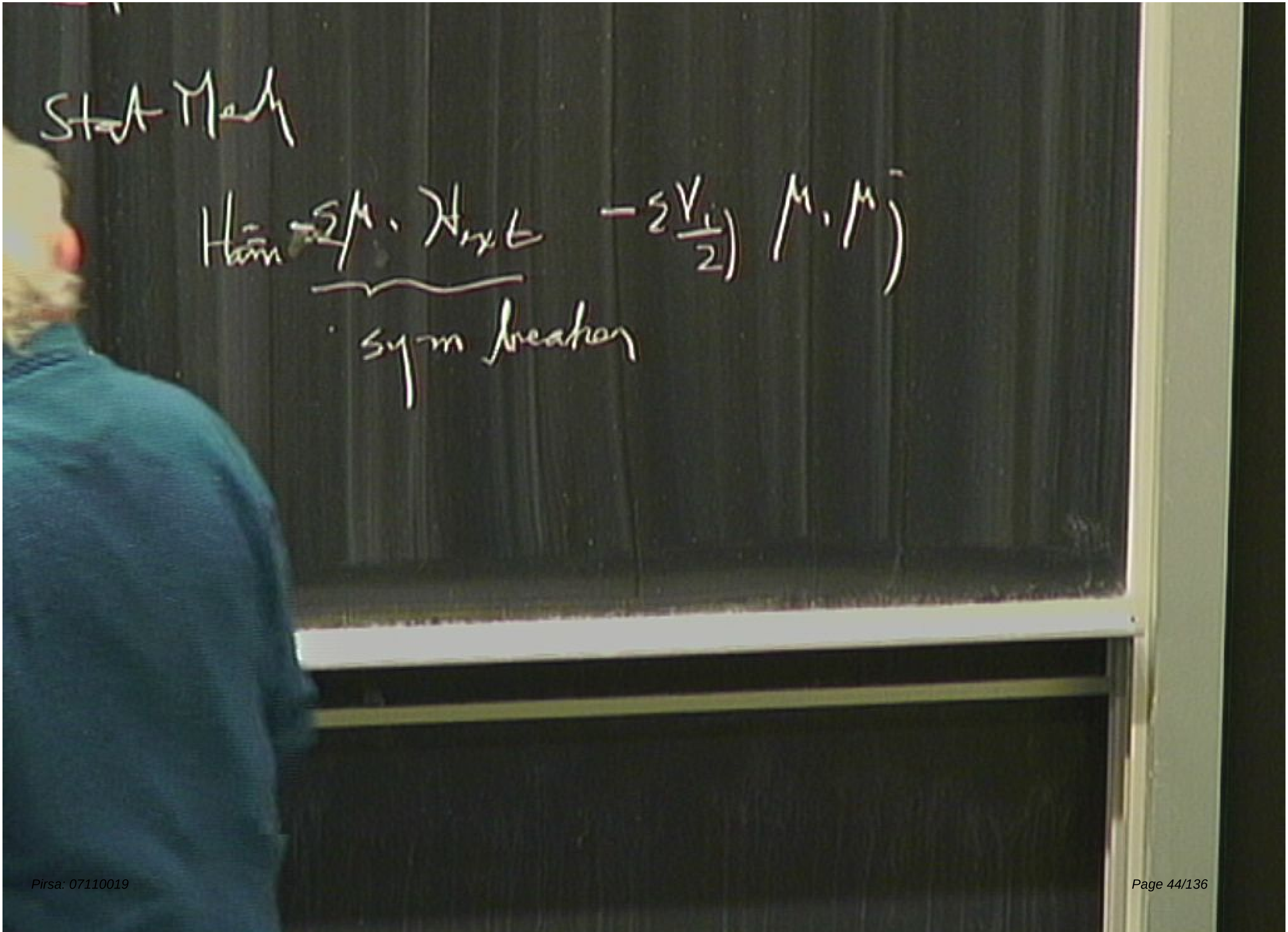

How to get W from Stat Mech
e^{-βF} = Z =
spin conf.
sym breaking


Stat Mech
$H = -\sum \mu_i \cdot H_{ext} - \sum \frac{V_{ij}}{2} \mu_i \cdot \mu_j$
sym breaking

Stat Mech

$$H_{am} = -\sum \mu_i \cdot H_{ext} - \sum \frac{V_{ij}}{2} \mu_i \cdot \mu_j$$

sym breaking $\qquad V_{ij} > 0$

conf.

Specializing: Ising Model $\vec{S}_i = \mu_i = \pm 1$

How to get Wsuis from Stat Mech

$$e^{-\beta F} = Z = \underbrace{\mathrm{tr}\ e^{-\beta H_{sim}}}_{}$$

over all 2^N spin conf.

$$H_{sim} = \underbrace{\sum \mu_i\, \mathcal{H}_{ext}}_{\text{sym breaking}} - \sum \frac{V_{ij}}{2}\, \mu_i \mu_j$$

$V_{ij} > 0$

Peel the onion skin

$$Z = \sum_{M} Z_M$$

Peel the onion skin

$$Z = \sum_M Z_M$$

$M = N^{\underline{o}}$ of up spins $= \sum p_i$

$-N \le M \le +N$

Peel the onion skin

$$Z = \sum_M Z_M$$

$M = N^{\underline{o}}$ of up spins $= \sum p_i$

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Peel the onion skin

$$Z = \sum_M Z_M$$

$M =$ N° of up spins $= \sum \rho_i$

$-N \leq M \leq +N$

To prove $\ln Z = \ln Z_{M^*} + O(\ln N)$

$$\left.\frac{\partial \ln Z_M}{\partial M}\right|_{M=M^*} = 0$$

$$Z = \sum_M Z_M$$

$M =$ N° of up spins $= \sum \uparrow$

$$-N \leq M \leq +N$$

To prove I: $\ln Z = \ln Z_{M^*} + O(\ln N)$

$$\left.\frac{\partial \ln Z_M}{\partial M}\right|_{M=M^*} = 0$$

$$Z_M = \frac{1}{W_M} \mathrm{tr}_M \, e^{-\beta H_{am}}$$

$$W_M = \mathrm{tr}_M \, 1 = \mathrm{tr}\, \delta(M - \Sigma$$

$$W_M = \binom{N}{M}$$

Peel the onion skin

$$Z = \sum_M Z_M$$

$M = N^{\circ}$ of up spins $= \sum \mu_i$

$-N \leq M \leq +N$

To prove $\ln Z = \ln Z_{M^*} + O(\ln N)$

$$\left.\frac{\partial \ln Z_M}{\partial M}\right|_{M=M^*} = 0$$

$$Z_M = \frac{1}{W_M} \mathrm{tr}_M\, e^{-\beta H_{am}} \qquad W_M = \mathrm{tr}_M 1 = \mathrm{tr}\, \delta(M - \sum \mu_i)$$

$$W_M = \binom{N}{M} \quad \ln W_M = N \ln 2 - \frac{1+m}{2} \ln \frac{1+m}{2} - \frac{1-m}{2} \ln \left(\frac{1-m}{2}\right)$$

$$+ O(\ln N)$$

$$-N \leq M \leq +N$$

$$\ln Z = \ln Z_{M^*} + O(\ln N)$$

$$\left.\frac{\partial \ln Z_M}{\partial M}\right|_{M=M^*} = 0$$

$$Z_M = \frac{1}{W_M} \mathrm{tr}_M\, e^{-\beta H_{am}} \qquad W_M = \mathrm{tr}_M 1 = \mathrm{tr}\, \delta\left(M - \sum \mu_i\right)$$

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$$+ O(\ln N)$$

$$Z = \sum_M Z_M$$

$M = N^{\underline{o}}$ of up spins =

$-N \le M \le +N$

To prove $\ln Z = \ln Z_{M^*} + O(\ln N)$

$$\left.\frac{\partial \ln Z_M}{\partial M}\right|_{M=M^*} = 0$$

$$Z_M = \frac{1}{W_M} \mathrm{tr}_M\, e^{-\beta H_{am}}$$

$W_M = \mathrm{tr}_M\, 1 = \mathrm{tr}\, S$

$$W_M = \binom{N}{M}$$

$\ln W_M = N \ln 2 - \frac{1+m}{2}$

Peel the onion skin

$$Z = \sum_M Z_M$$

$M = N^{o}$ of up spins $= \sum \mu_i$

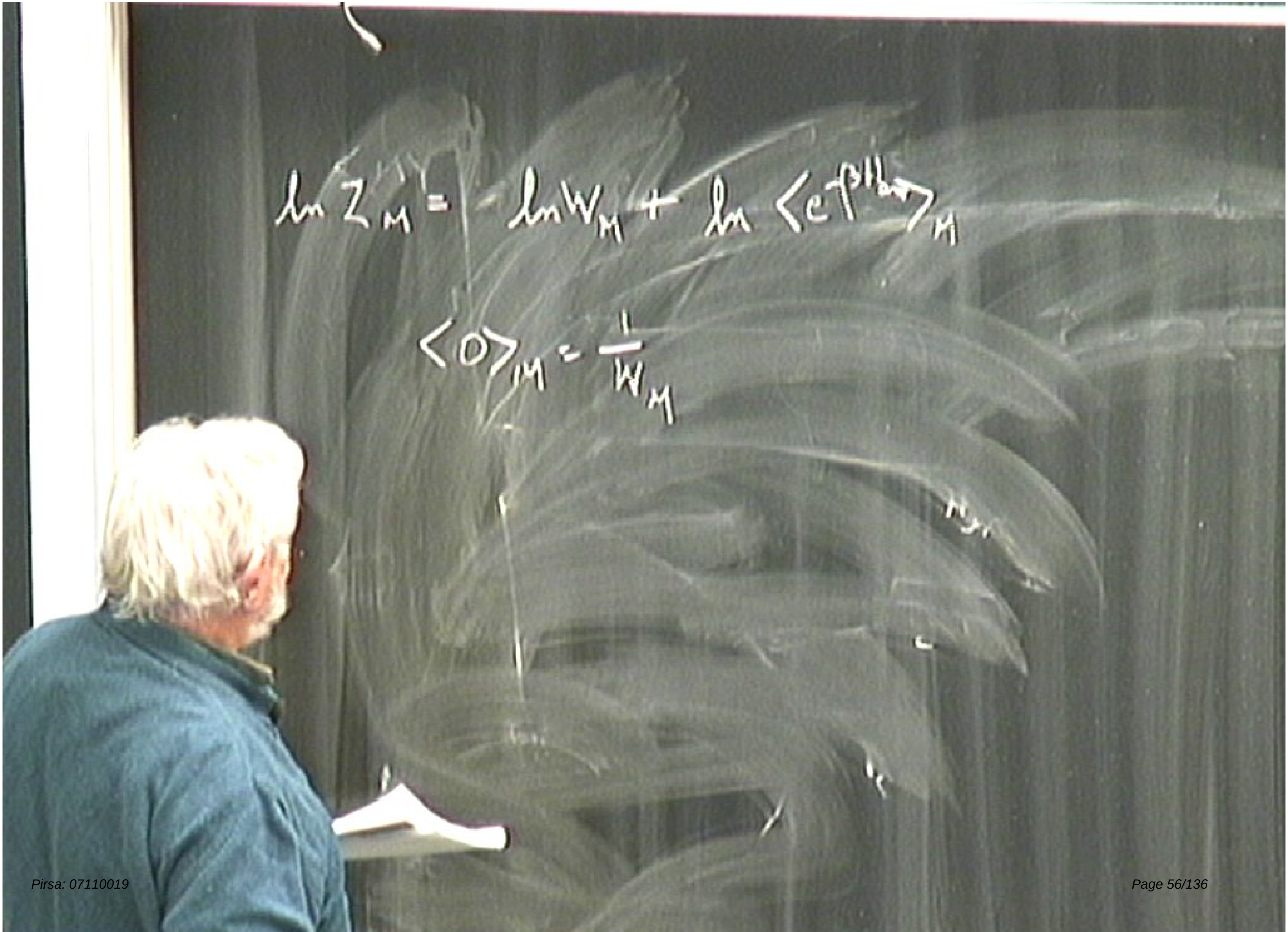
$-N \le M \le +N$

To prove $\ln Z = \ln Z_{M^*} + O(\ln N)$

$$\left.\frac{\partial \ln Z_M}{\partial M}\right|_{M=M^*} = 0$$

$$Z_M = \frac{1}{W_M} \mathrm{tr}_M\, e^{-\beta H_{am}} \qquad W_M = \mathrm{tr}_M 1 = \mathrm{tr}\, \delta(M - \sum \mu_i)$$

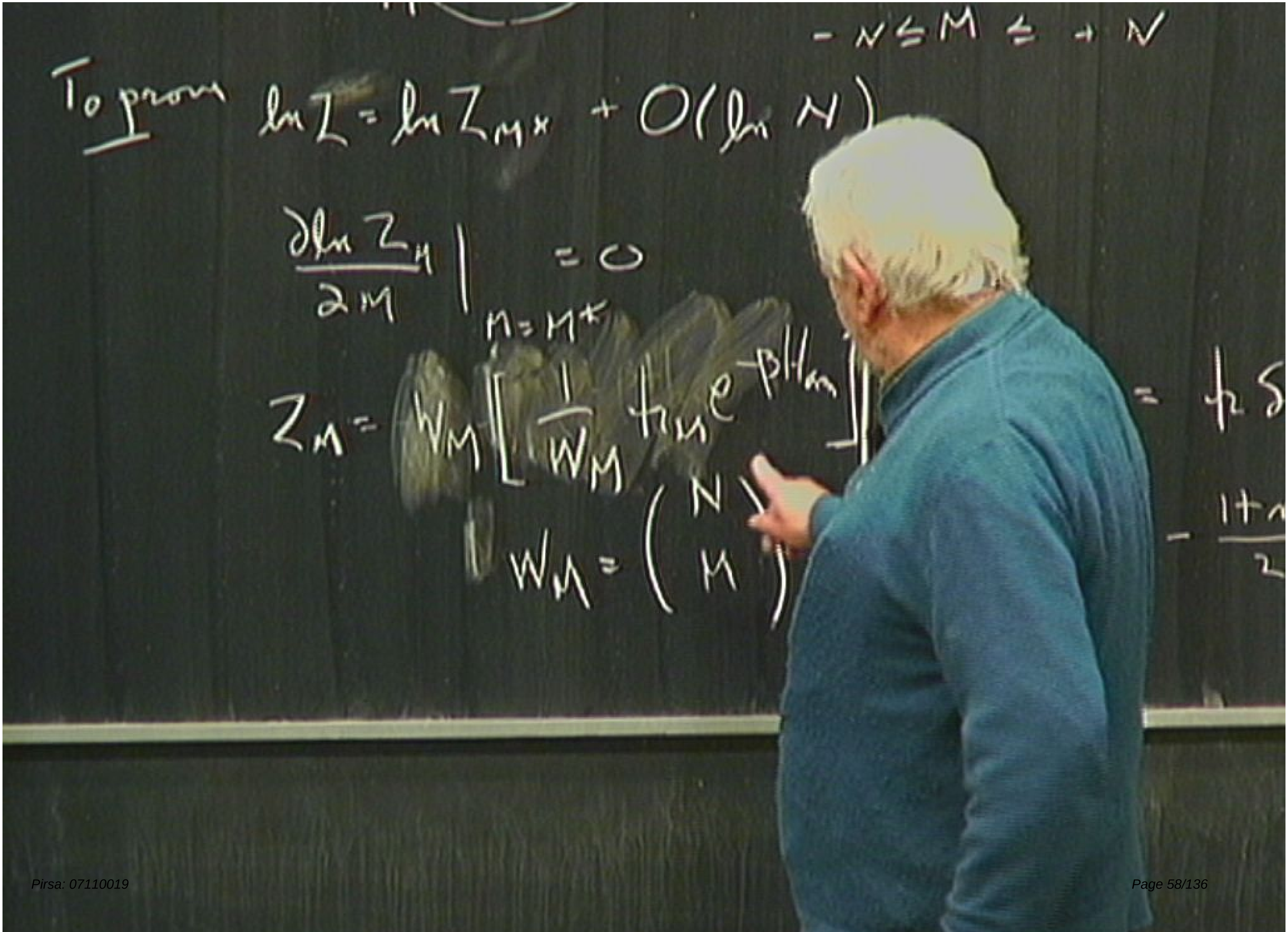
$$W_M = \binom{N}{M} \qquad \ln W_M = N\left[\ln 2 - \frac{1+m}{2} \ln \frac{1+m}{2} - \frac{1-m}{2} \ln\left(\frac{1-m}{2}\right)\right] + O(\ln N)$$



$\ln Z_M' = \ln W_M + \ln \langle e^{-\beta H'} \rangle_M$
$\langle O \rangle_M = \frac{1}{W_M}$

$$\ln Z_M' = \ln W_M + \ln \langle e^{-\beta H_{int}} \rangle_M$$

$$\langle O \rangle_M = \frac{1}{W_M} \operatorname{tr}_M O$$


- N ≤ M ≤ + N
To prove ln Z = ln Z_M* + O(ln N)
∂ ln Z_M / ∂M | M=M* = 0
Z_M = W_M [1/W_M tr_M e^-βH_m]
W_M = (N M)

$-N \le M \le +N$

To prove $\ln Z = \ln Z_{M^*} + O(\ln N)$

$$\left.\frac{\partial \ln Z_M}{\partial M}\right|_{M=M^*} = 0$$

$$Z_M = W_M \left[\frac{1}{W_M} \mathrm{tr}_M\, e^{-\beta H_M} \right] \quad W_M = \mathrm{tr}_M 1 = \mathrm{tr}\, S$$

$$W_M = \binom{N}{M} \quad \ln W_M = N \ln 2 - \frac{1+\dots}{2}$$

$$\ln Z_M = \ln W_M + \ln \langle e^{-\beta H_{int}} \rangle_M$$

$$\langle O \rangle_M = \frac{1}{W_M} \mathrm{tr}_M O$$

$$\ln \int dx\, p(x) e^{-\alpha x}$$

$$\ln Z_M = \ln W_M + \ln \langle e^{-\beta H_0} \rangle_M$$

$$\langle O \rangle_M = \frac{1}{W_M} \mathrm{tr}_M O$$

$$\ln \int dx\, p(x) e^{-\alpha x} = \sum \frac{(-\alpha)^m}{m!} C_m$$

$C_1 = \langle x \rangle$

$C_2 = \langle x^2 \rangle - \langle x \rangle^2$

$\vdots$

$$\ln Z_M = \ln W_M + \ln \langle e^{-\beta H_1} \rangle_M$$

$$\langle O \rangle_M = \frac{1}{W_M} \, \mathrm{Tr}_M \, O$$

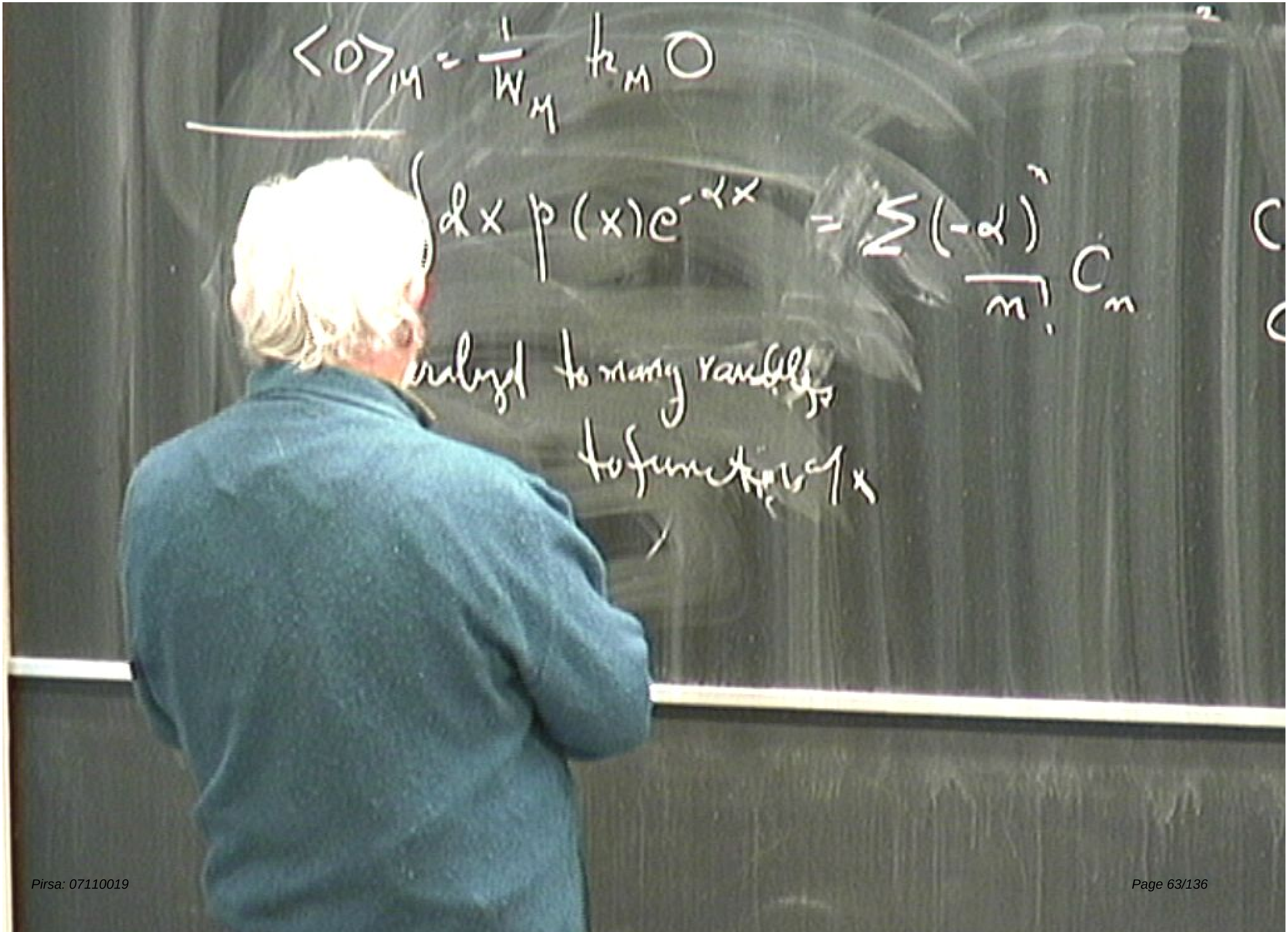
$$\ln \int dx \, p(x) e^{-\alpha x} = \sum \frac{(-\alpha)^n}{n!} C_n$$

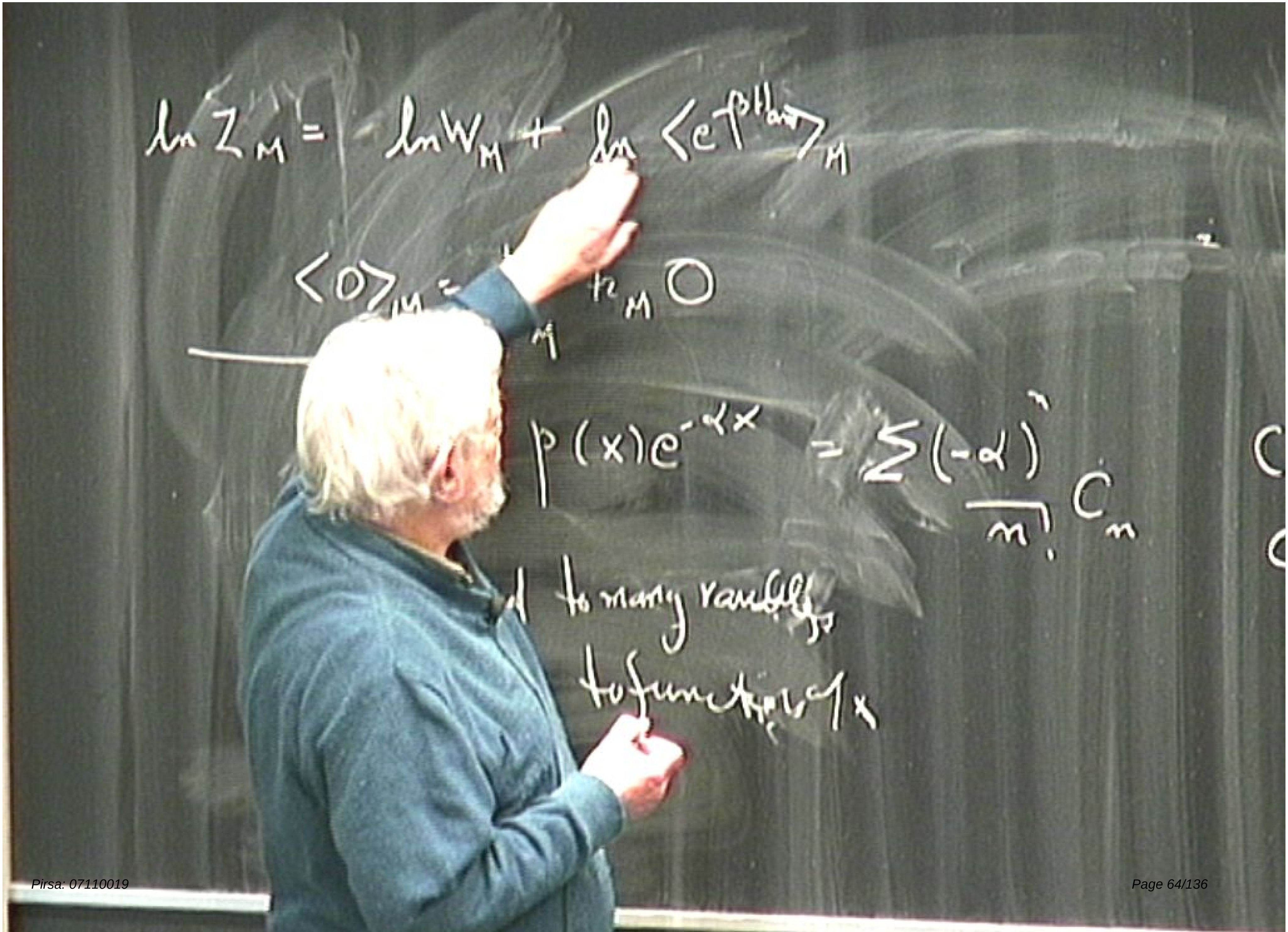
$$C_1 = \langle x \rangle$$

$$C_2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$\vdots$$

Generalized to many variables,
to functions of x


$\langle O \rangle_M = \frac{1}{W_M} \mathrm{tr}_M O$
$\int dx\, p(x) e^{-\alpha x} = \sum \frac{(-\alpha)^n}{n!} C_n$
to many variables


$\ln Z_M = \ln W_M + \ln \langle e^{-\beta H_{int}} \rangle_M$
$\langle O \rangle_M = \; \mathrm{tr}_M \, O$
$p(x) e^{-\alpha x} = \sum \frac{(-\alpha)^n}{n!} C_n$
to many variables
to functions of x

$$\ln Z_M = \ln W_M + \ln \langle e^{\beta \ln} \rangle_M$$

$$\langle O \rangle_M = \frac{1}{W_M} \operatorname{tr}_M O$$

$$\ln \int dx\, p(x) e^{-\alpha x} = \sum \frac{(-\alpha)^n}{n!} C_n$$

$C_1 = \langle x \rangle$

$C_2 = \langle x^2 \rangle - \langle x \rangle^2$

$\vdots$

Generalized to many variables,

to functions of x

$$\ln Z_M = \sum \frac{(-\beta)^n}{n!} C_n + \ldots$$

$$C_1 = \langle H_{am} \rangle$$

$$C_2 = \langle H_{am}^2 \rangle - \langle H_{am} \rangle^2$$

$\langle \vec{S}$

$$\ln Z_M = \sum \frac{(-\beta)^n}{n!} C_n + \left(\ldots \right) \Big]$$

$$C_1 = \langle H_{am} \rangle$$

$$C_2 = \langle H_{am}^2 \rangle - \langle H_{am} \rangle^2$$

MFT uses only C_1

$$= \langle H_{om} \rangle$$

$$_2 = \langle H_{om}^2 \rangle - \langle H_{om} \rangle^2$$

MFT uses only $C_1 = -\frac{1}{2} \langle V_{ij} \rangle \langle \mu_i \mu_j \rangle_M$

$$\langle \mu_i \mu_j \rangle_M = \left(\frac{M}{N}\right)^2 + O\left(\frac{1}{N}\right)$$

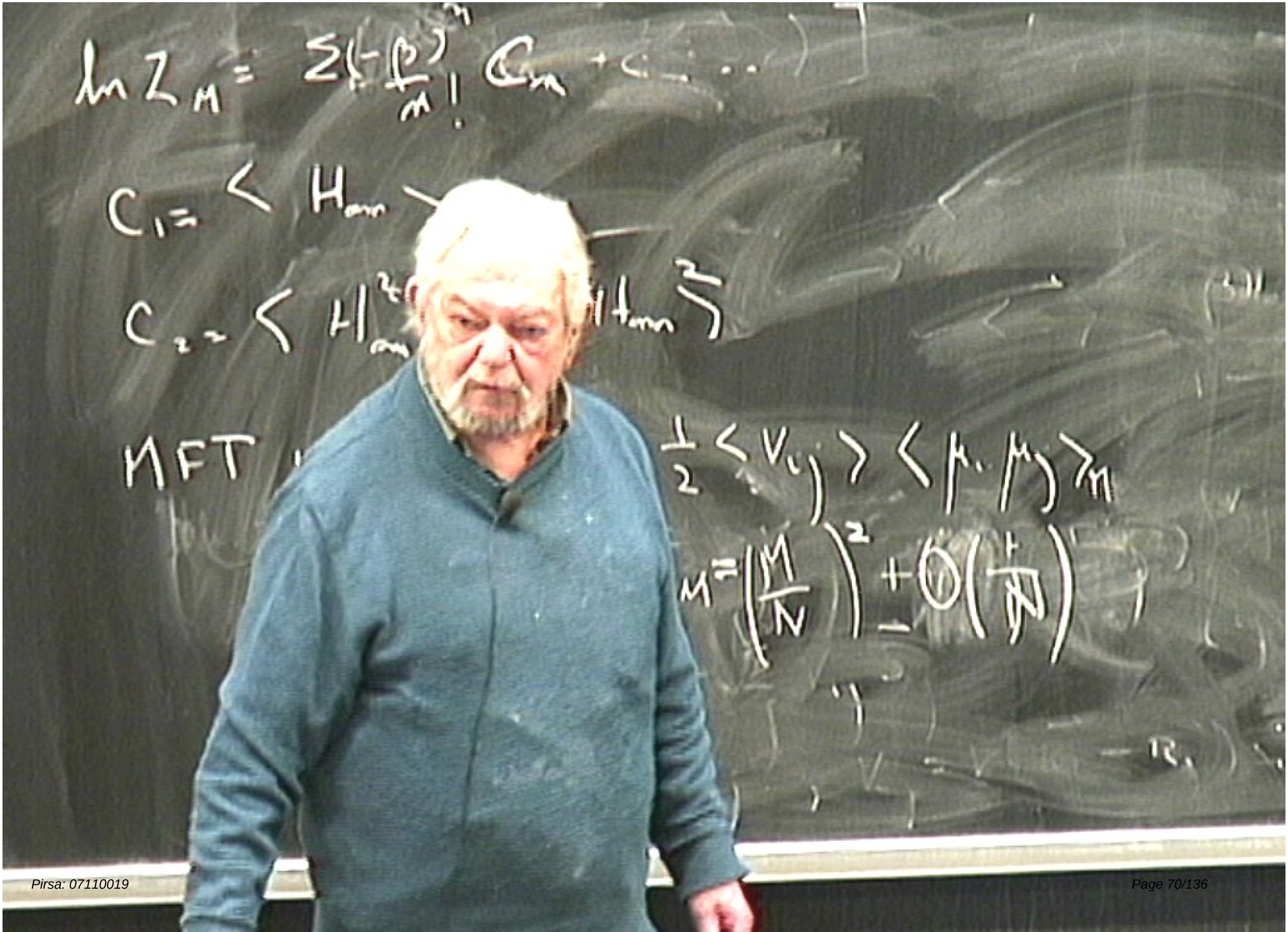
$$\ln Z_M = \sum \frac{(-\beta)^n}{n!} C_n + \ldots$$

$$C_1 = \langle H_{cm} \rangle$$

$$C_2 = \langle H_{cm}^2 \rangle - \langle H_{cm} \rangle^2$$

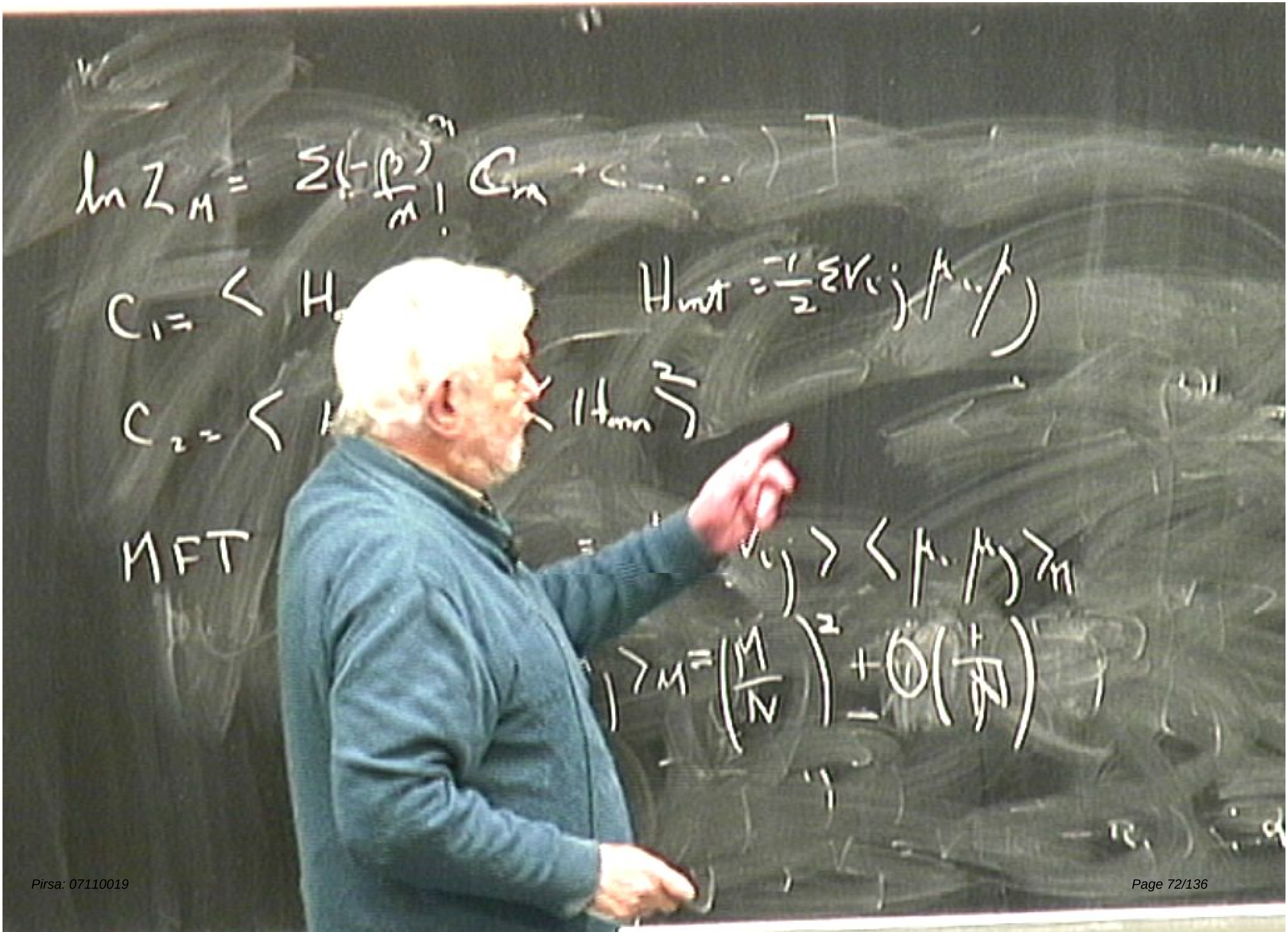
MFT uses only $C_1 = -\frac{1}{2} \langle V_{ij} \rangle \langle \mu_i \mu_j \rangle_M$

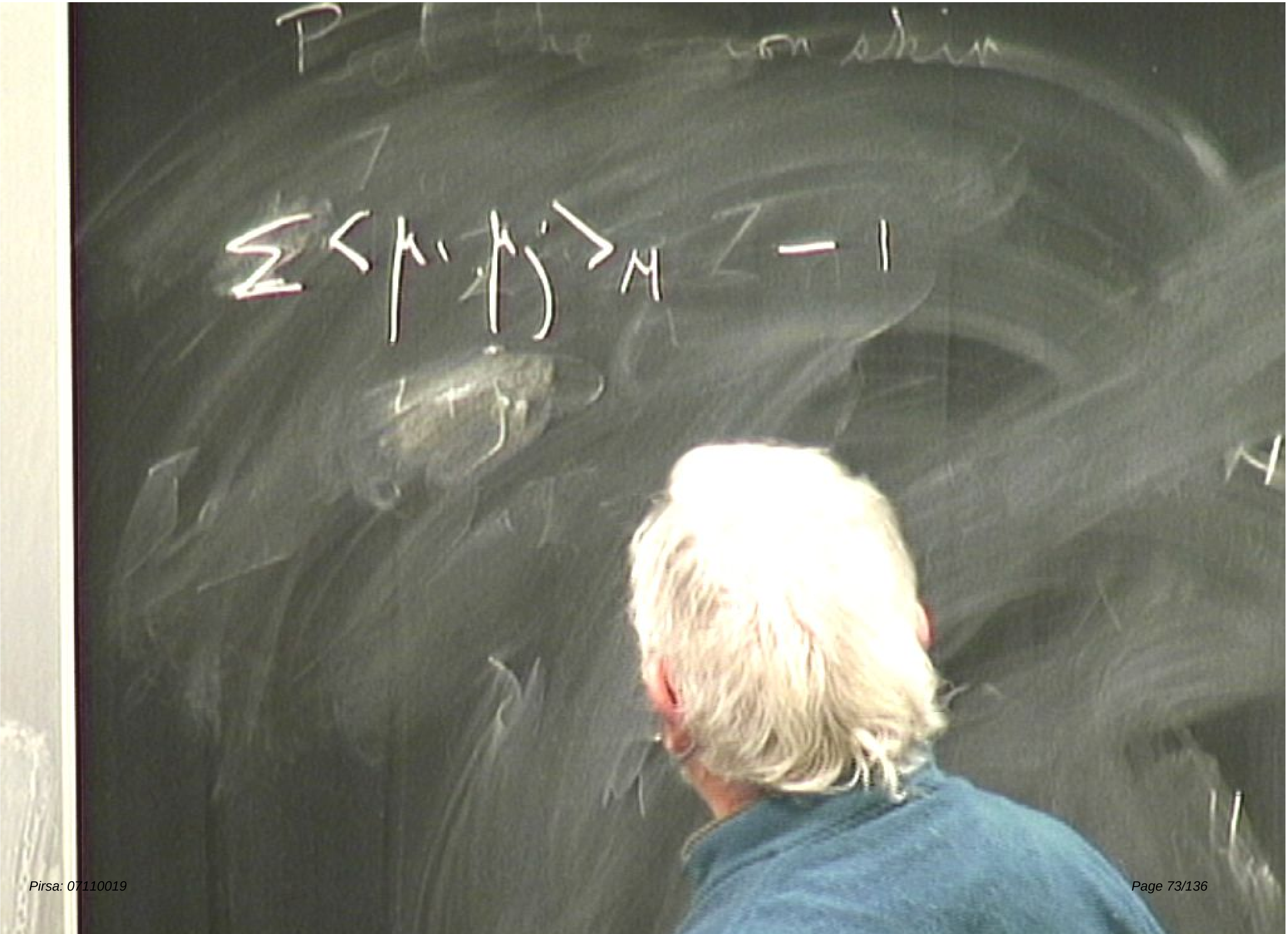
$$\langle \mu_i \mu_j \rangle_M = \left(\frac{M}{N}\right)^2 + O\left(\frac{1}{N}\right)$$

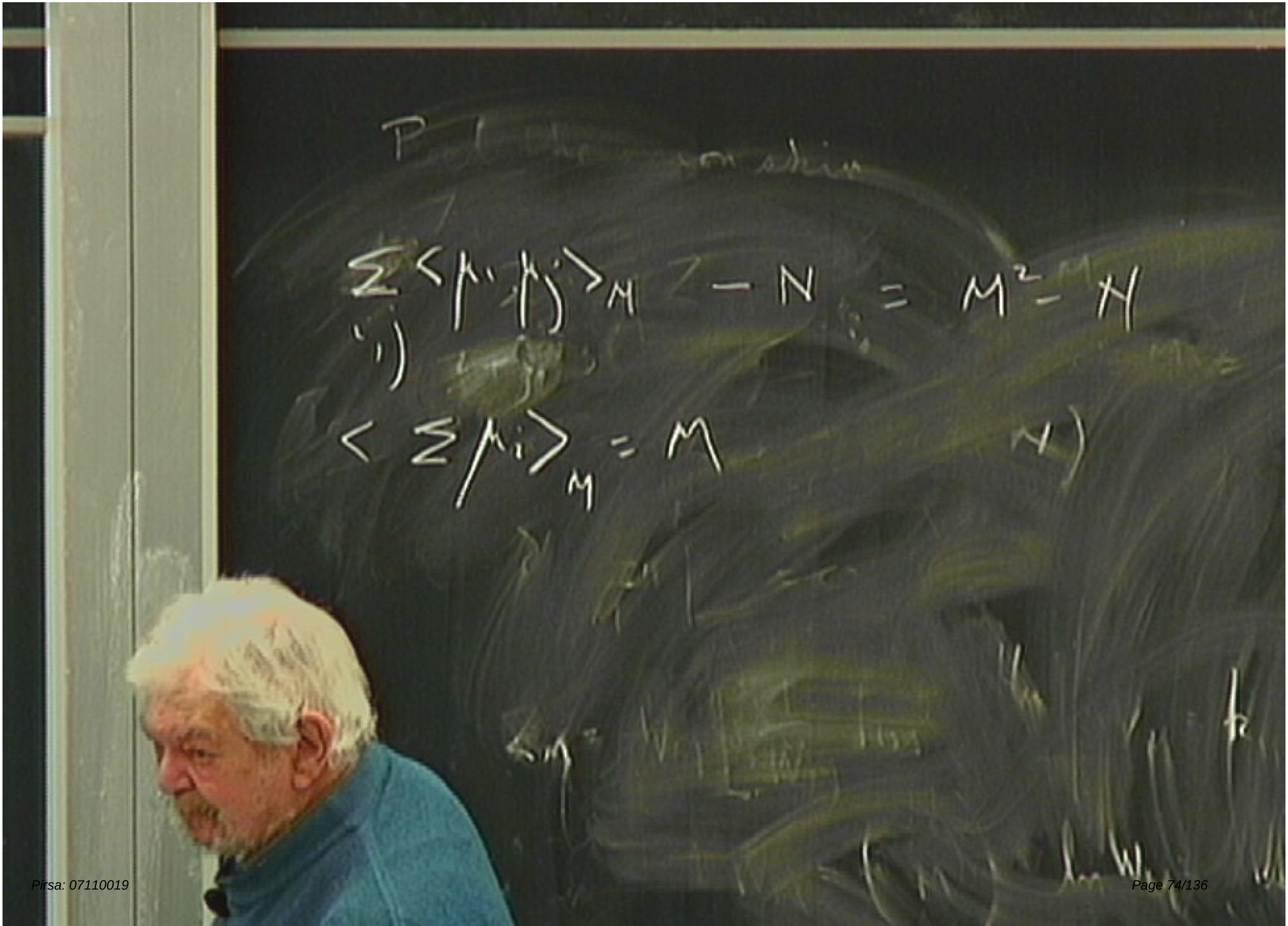

MFT

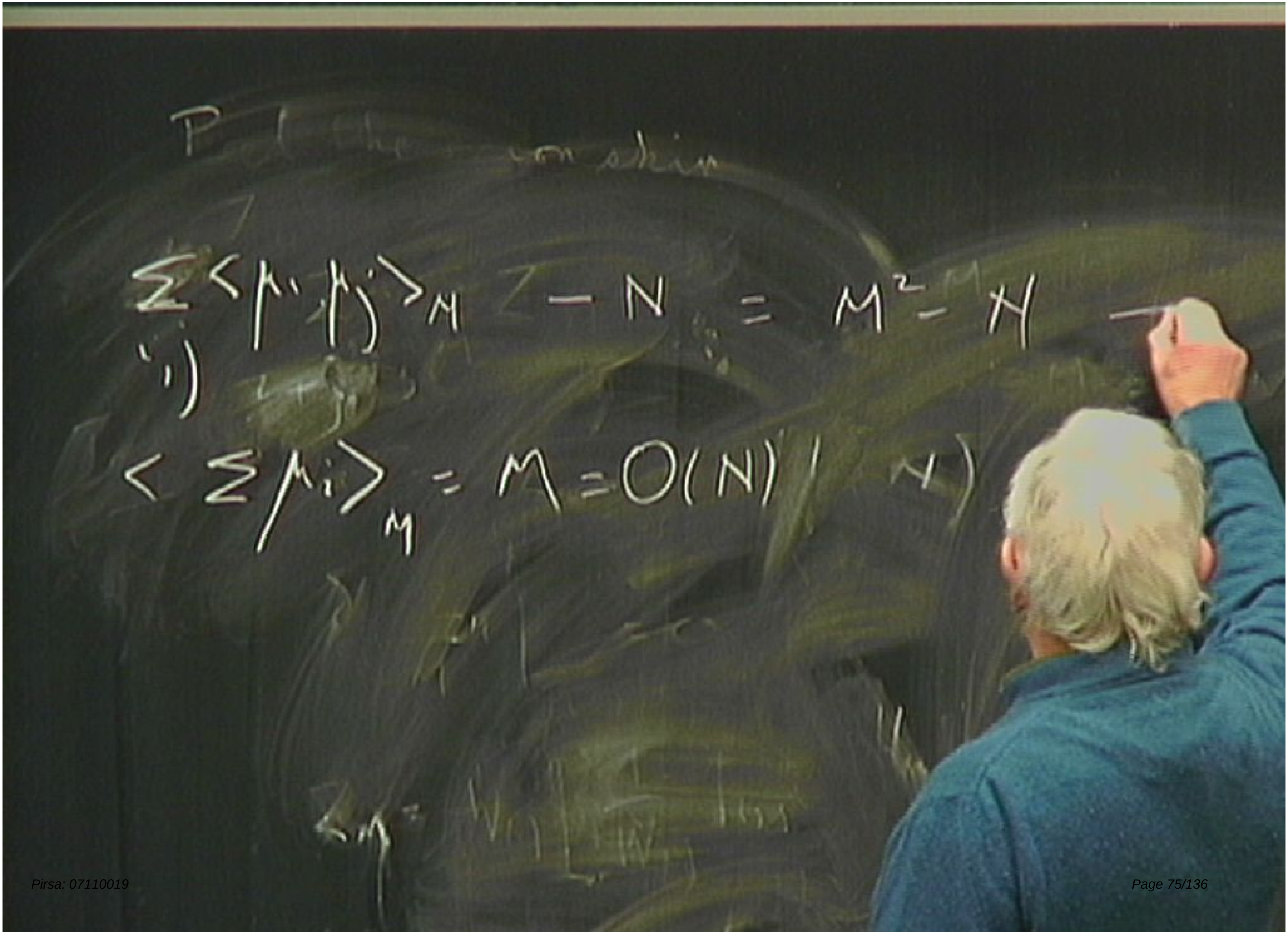
$$= -\frac{1}{2} \langle V_{ij} \rangle \langle \mu_i \mu_j \rangle_M$$

$$\langle \mu_i \mu_j \rangle_M = \left(\frac{M}{N}\right)^2 + O\left(\frac{1}{N}\right)$$


ln Z_M = Σ (−β)^n / n! C_n
H_int = −1/2 Σ V_ij μ_i μ_j
C_1 = < H
C_2 = < H_int^2 >
MFT
V_ij > < μ_i μ_j >_M
>_M = (M/N)^2 + O(1/N)




$\sum_{i,j} \langle \mu_i \mu_j \rangle_M - N = M^2 - N$
$\langle \sum \mu_i \rangle_M = M$


$\sum_{i,j} \langle \mu_i \mu_j \rangle_M - N = M^2 - M$
$\langle \sum \mu_i \rangle_M = M = O(N)$

$\sum \langle \mu_i \mu_j \rangle_M - N = M^2 - N \to M^2$ (thermodyn limit)

$\langle \sum \mu_i \rangle_M = M = O(N)$

$\frac{1-m}{2} \ln ($

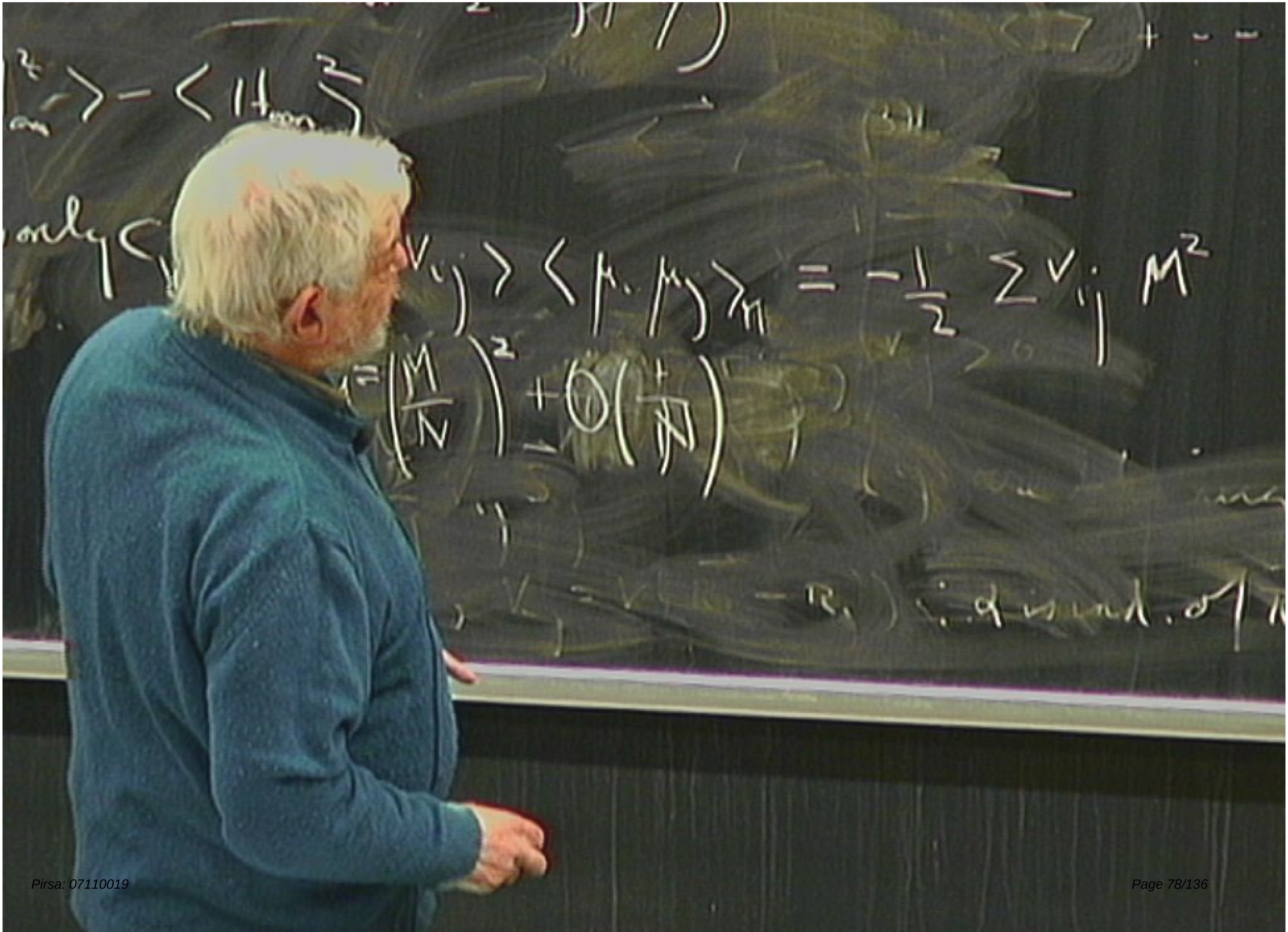
$+ O(N)$

$\sum \langle \mu_i \mu_j \rangle_M - N = M^2 - N \to M^2$ (thermodyn. limit

$\langle \sum \mu_i \rangle_M = M = O(N)$

$\frac{1-m}{2} \ln$

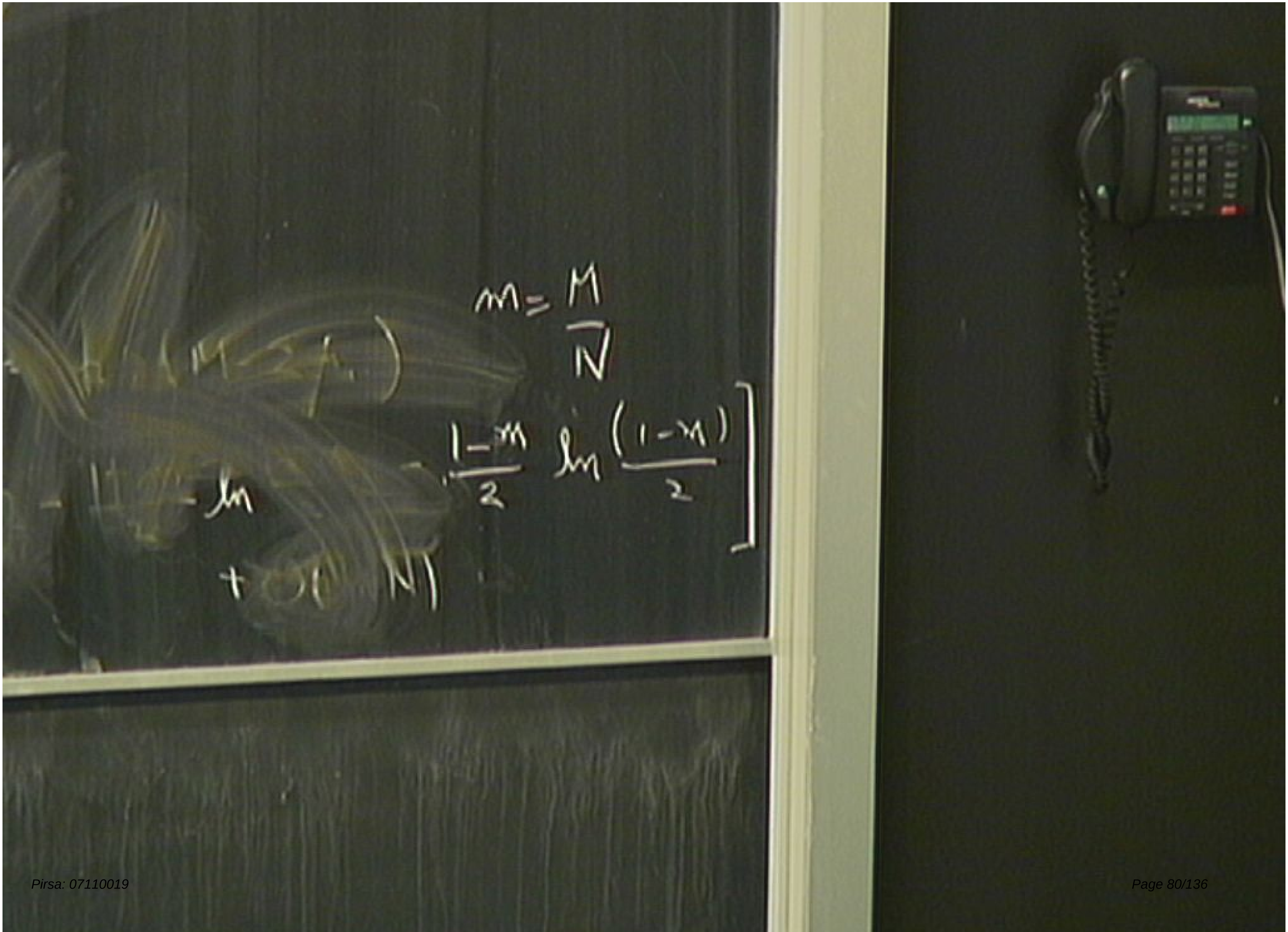
$+ O(N)$

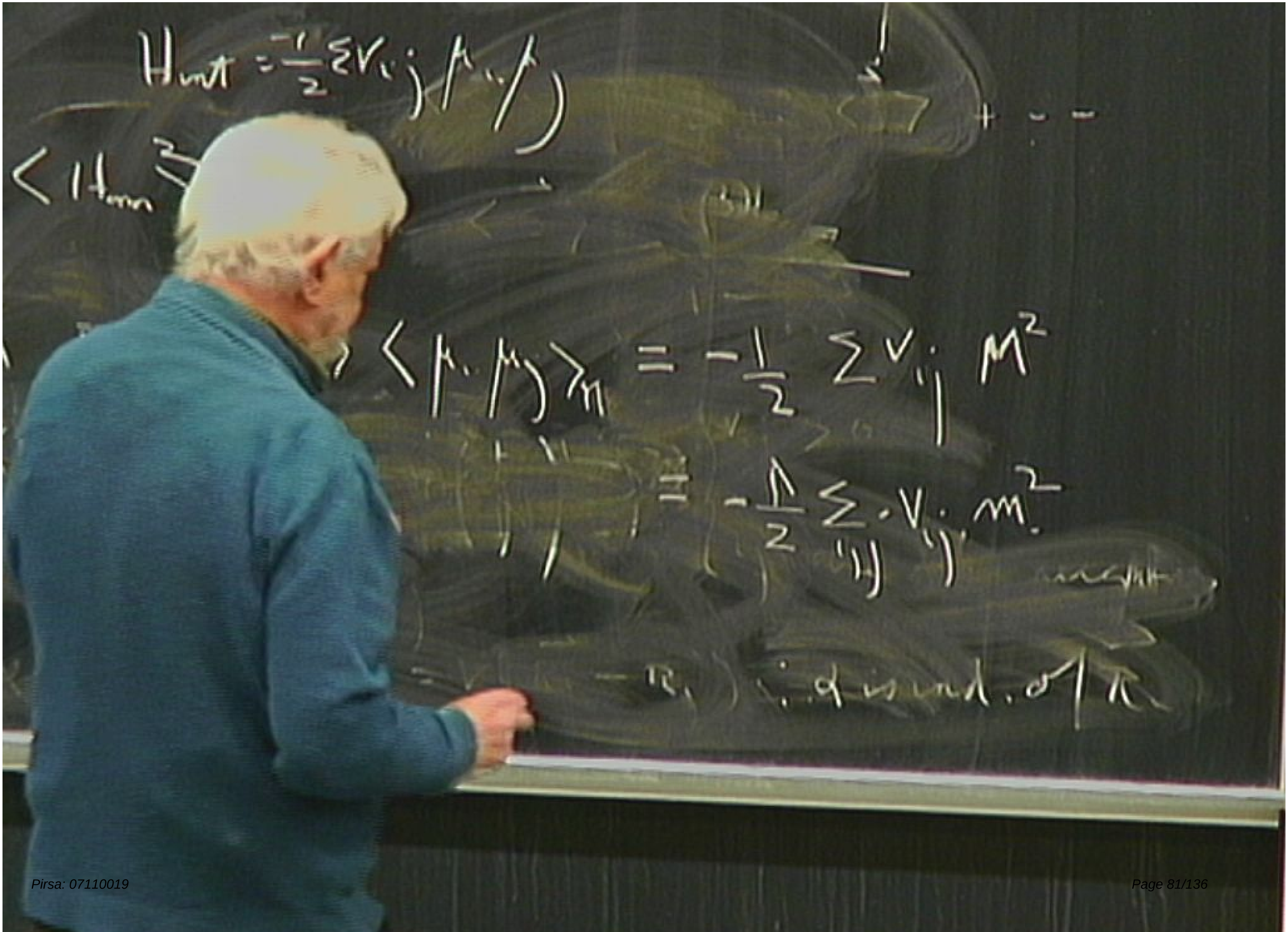

only
= -1/2 Σ V ij M²
+ O(1/N)

$$\rangle - \langle H_{mn}^2 \rangle$$

only $C_1 = -\frac{1}{2}\sum_{i,j} V_{ij} \langle \mu_i \mu_j \rangle_M = -\frac{1}{2}\sum V_{ij} M^2$

$$\langle \mu_i \mu_j \rangle_M = \left(\frac{M}{N}\right)^2 + O\left(\frac{1}{N}\right)$$


m = M/N
(1-m)/2 ln ((1-m)/2)
ln


$H_{int} = -\frac{1}{2} \sum V_{ij} \mu_i \mu_j$
$\langle \mu_i \mu_j \rangle_H = -\frac{1}{2} \sum V_{ij} M^2$
$= -\frac{1}{2} \sum_{ij} V_{ij} m^2$
i.e. is ind. of T

$C_1 = \langle H_{int} \rangle$ $\quad H_{int} = -\frac{1}{2}\sum V_{ij}\, \mu_i \mu_j \quad + \ldots$

$C_2 = \langle H_{int}^2 \rangle - \langle H_{int} \rangle^2$

MFT means only $C_1 = -\frac{1}{2}\sum_{\langle ij \rangle} V_{ij} \langle \mu_i \mu_j \rangle_M = -\frac{1}{2} \sum V_{ij} \frac{M^2}{N^2}$

$= -\frac{1}{2} \sum_{\langle ij \rangle} V_{ij}\, m^2$

$$\sum_{i,j} \langle \mu_i \mu_j \rangle_M - N = M^2 - M \longrightarrow$$

$$\left\langle \sum \mu_i \right\rangle_M = M = O(N)$$

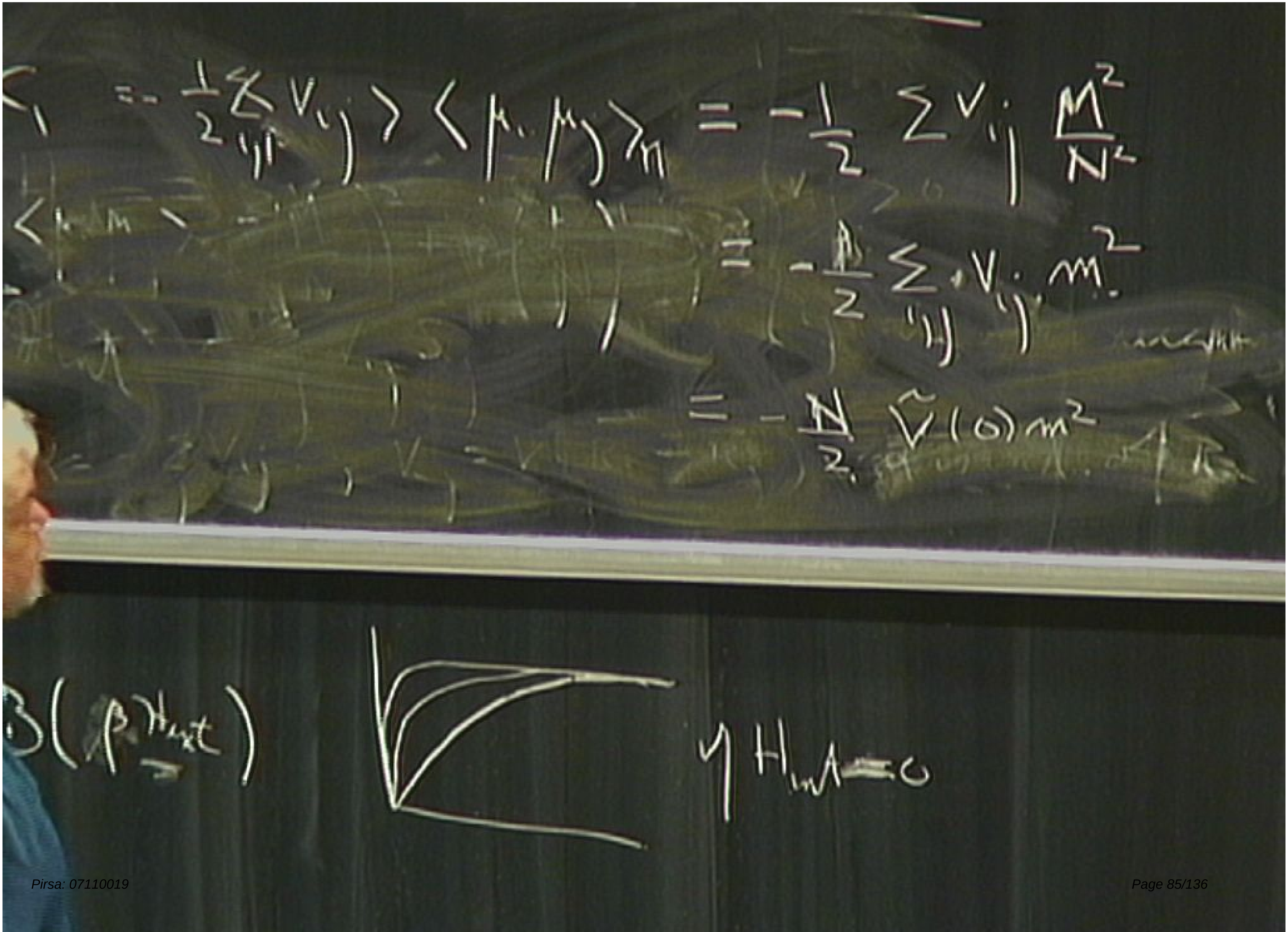
$$\langle \mu_i \rangle_M = \frac{M}{N}$$

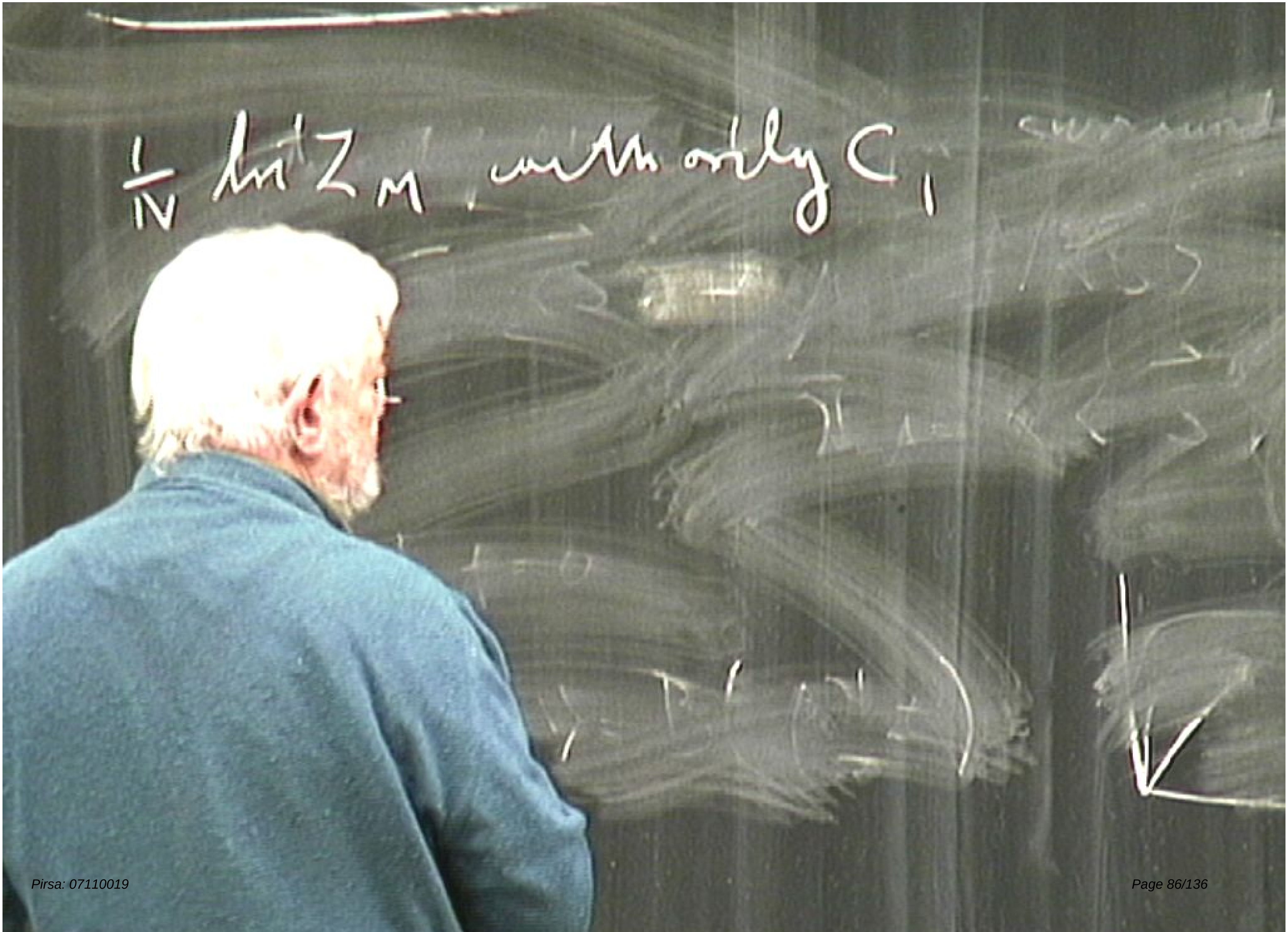
$H_{int} = -\frac{1}{2}\sum V_{ij}\,\mu_i\mu_j \qquad + \cdots$

$-\langle H_{int}\rangle$

$\langle\ \rangle = -\frac{1}{2}\sum_{ij} V_{ij}\,\langle\mu_i\mu_j\rangle_H = -\frac{1}{2}\sum V_{ij}\,\frac{M^2}{N^2}$

$= -\frac{1}{2}\sum_{ij} V_{ij}\,m^2$


$= -\frac{1}{2}\sum V_{ij} \langle \mu_i \mu_j \rangle_m = -\frac{1}{2}\sum V_{ij} \frac{M^2}{N^2}$
$= -\frac{1}{2}\sum_{ij} V_{ij}\, m^2$
$= -\frac{N}{2}\tilde{V}(0)\, m^2$
$H_{int} = 0$


1/N ln Z_M with only C_1

$\frac{1}{N} \ln^{t} Z_M$ with only C_1

$= \partial \cdot [\ 1-m$

$\frac{1}{N} \ln Z_M$, with only C_1

$$= -\left[\frac{1-m}{2} \ln \frac{1-m}{2} + \frac{(1+m)}{2} \ln \frac{1+m}{2} \right]$$

$\frac{1}{N} \ln Z_M$, with only C_1

$$= -\left[\frac{1-m}{2} \ln \frac{1-m}{2} + \frac{(1+m)}{2} \ln \frac{1+m}{2} \right]$$

$$+ H_{ext}\, m$$

$\frac{1}{N} \ln Z_M$ with only C_1

$$= -\left[\frac{1-m}{2} \ln \frac{1-m}{2} + \frac{(1+m)}{2} \ln \frac{1+m}{2} \right]$$

$$+ \beta H_{ext}\, m + \frac{\beta}{2} \tilde{V}(0)\, m^2$$

$\frac{1}{N}\ln Z_M$ with only C_1

$$= -\left[\frac{1-m}{2}\ln\frac{1-m}{2} + \frac{(1+m)}{2}\ln\frac{1+m}{2} \right]$$

$$+\beta H_{ext}\, m + \frac{\beta}{2}\tilde{V}(0)\, m^2$$

$$\frac{\partial \ln Z_M}{\partial M}\Big|_N = 0 \qquad m = \tanh\beta\left[H_{ext} + \tilde{V}(0)\, m \right]$$

$\ln Z_M$ with only C_1

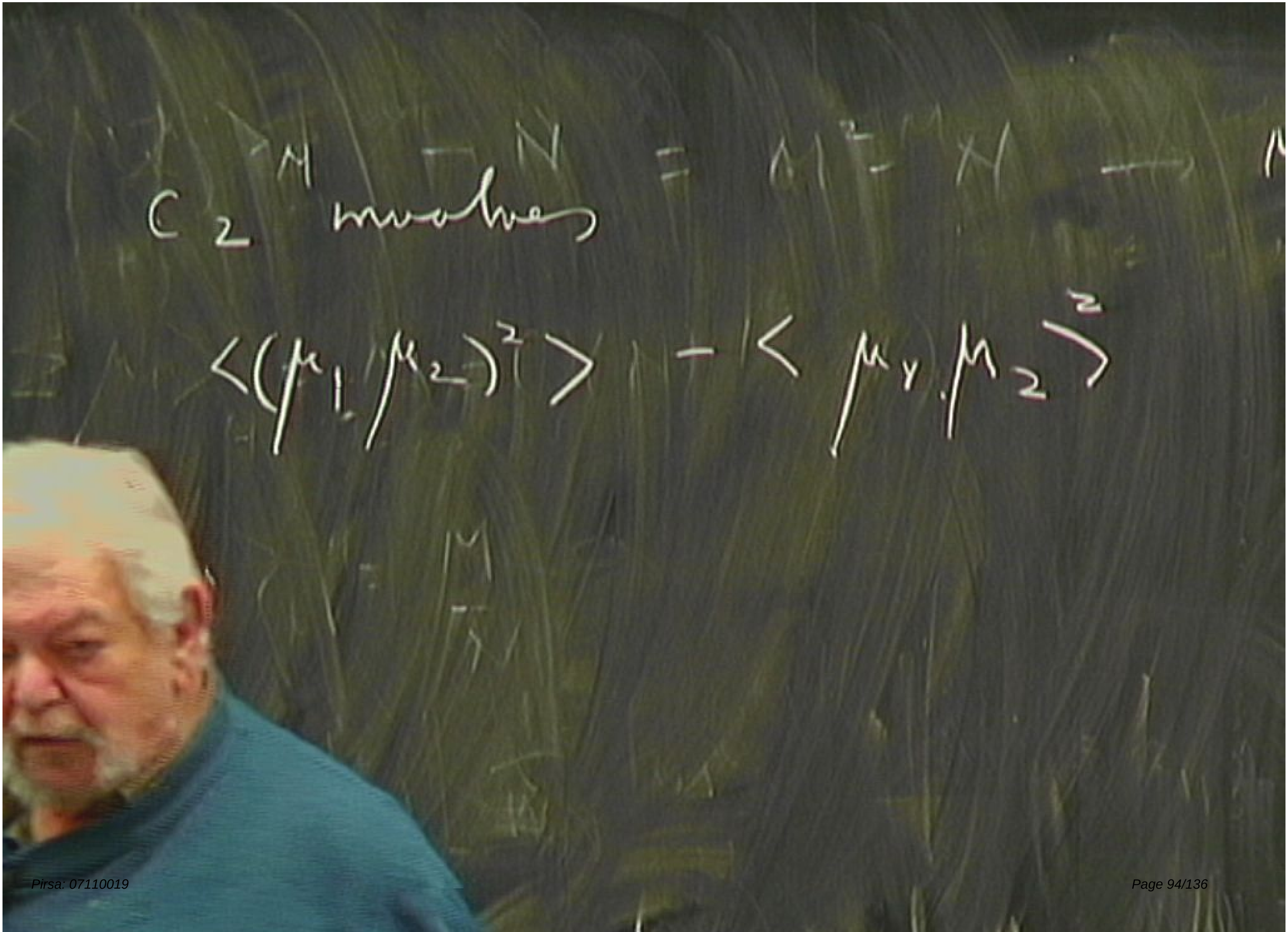
$$= -\left[\frac{1-m}{2} \ln \frac{1-m}{2} + \frac{(1+m)}{2} \ln \frac{1+m}{2} \right]$$

$$+ \beta H_{ext}\, m + \frac{\beta}{2} \tilde{V}(0)\, m^2$$

$$\left.\frac{\partial \ln Z_M}{\partial M}\right|_N = 0 \qquad m = \tanh \beta \left[H_{ext} + \tilde{V}(0)\, m \right] \qquad = 0$$

C_2 involves

$$\langle (\mu_1 \cdot \mu_2)^2 \rangle$$


C_2 involves
$\langle (\mu_1 \cdot \mu_2)^2 \rangle - \langle \mu_1 \cdot \mu_2 \rangle^2$

C_2 involves

$$\langle (\mu_1 \cdot \mu_2)^2 \rangle_M - \langle \mu_1 \cdot \mu_2 \rangle_M^2$$

$$= 1 - (m^2)^2$$

C_2 involves

$$\langle (\mu_1 \cdot \mu_2)^2 \rangle_M - \langle \mu_1 \cdot \mu_2 \rangle_M^2$$

$$= 1 - (m^2)^2$$

C_2 involves

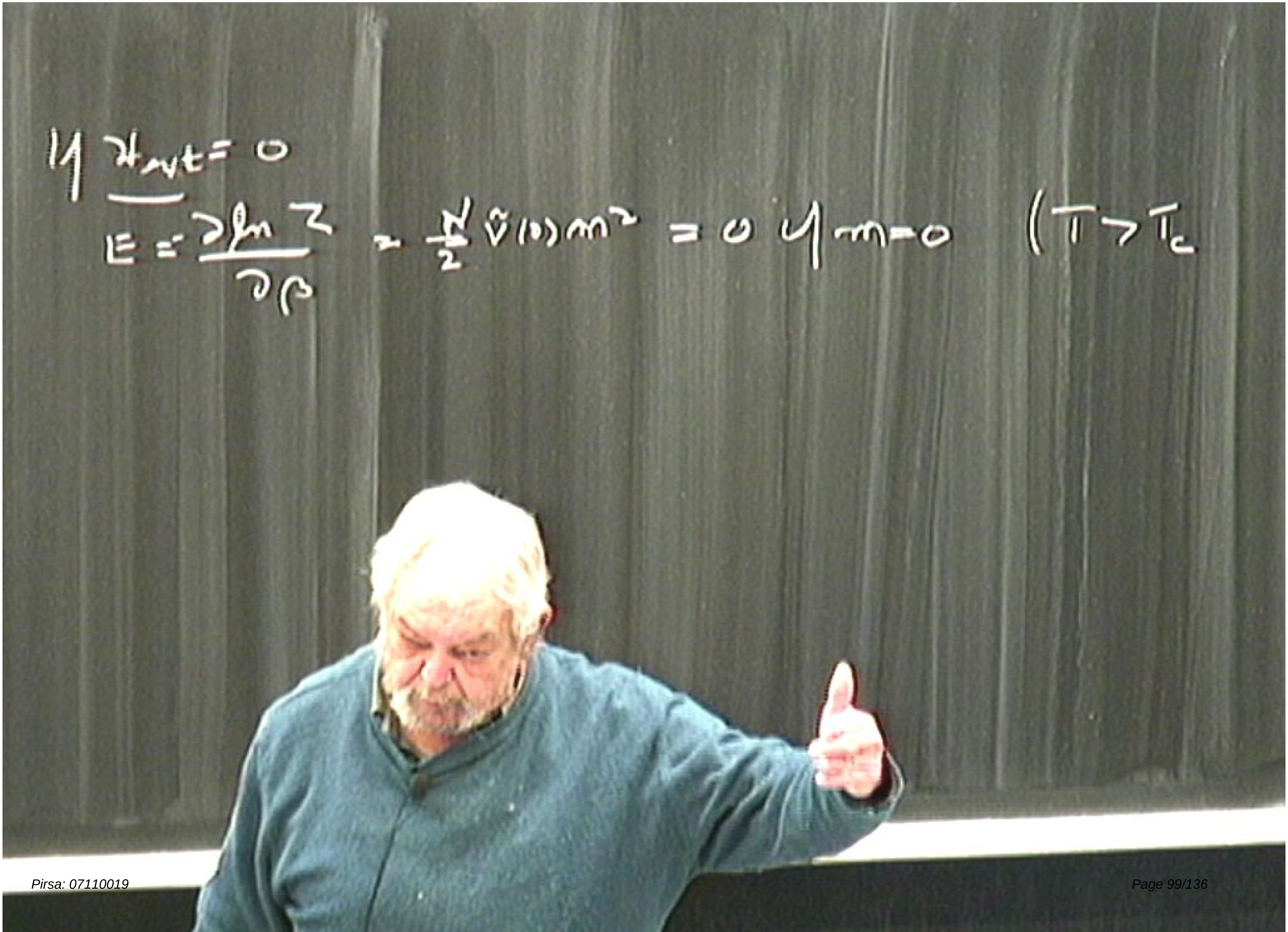
$$\langle(\mu_1 \cdot \mu_2)^2\rangle_M - \langle \mu_1 \cdot \mu_2 \rangle_M^2$$

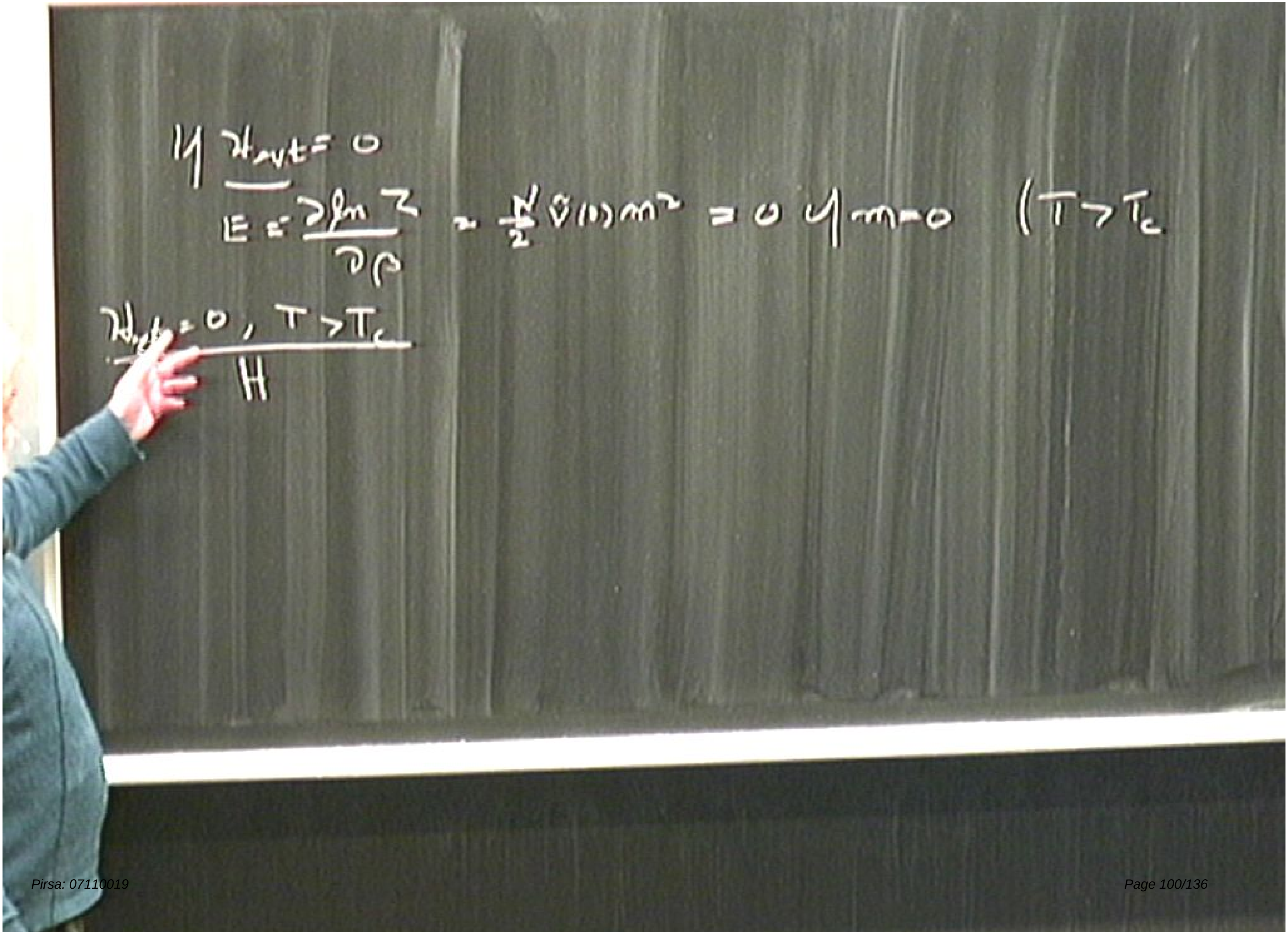
$$= 1 - (m^2)^2$$

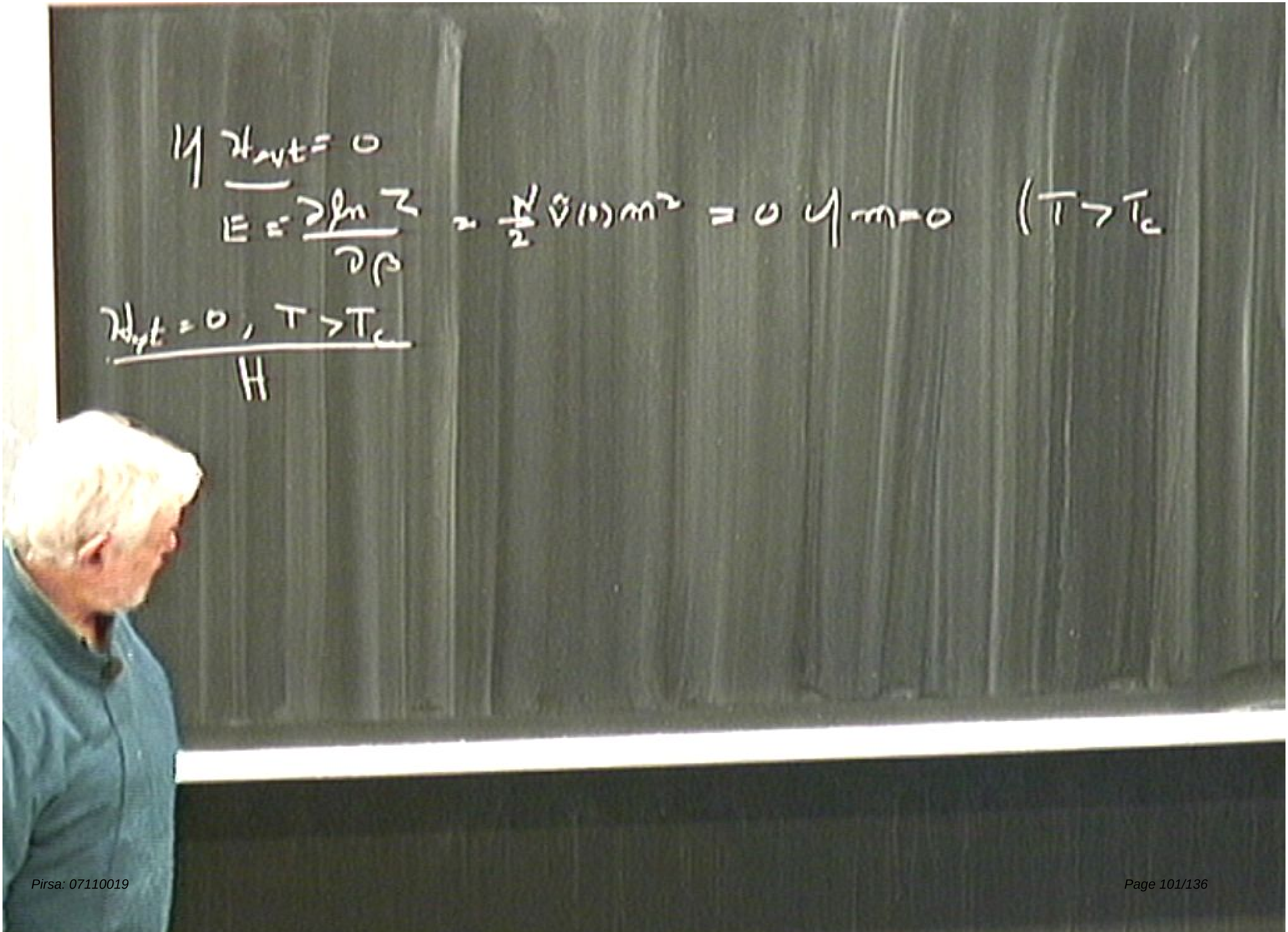
$\langle \mu_1 \mu_2 \rangle$ = Average value of μ_2 if μ_1 is up

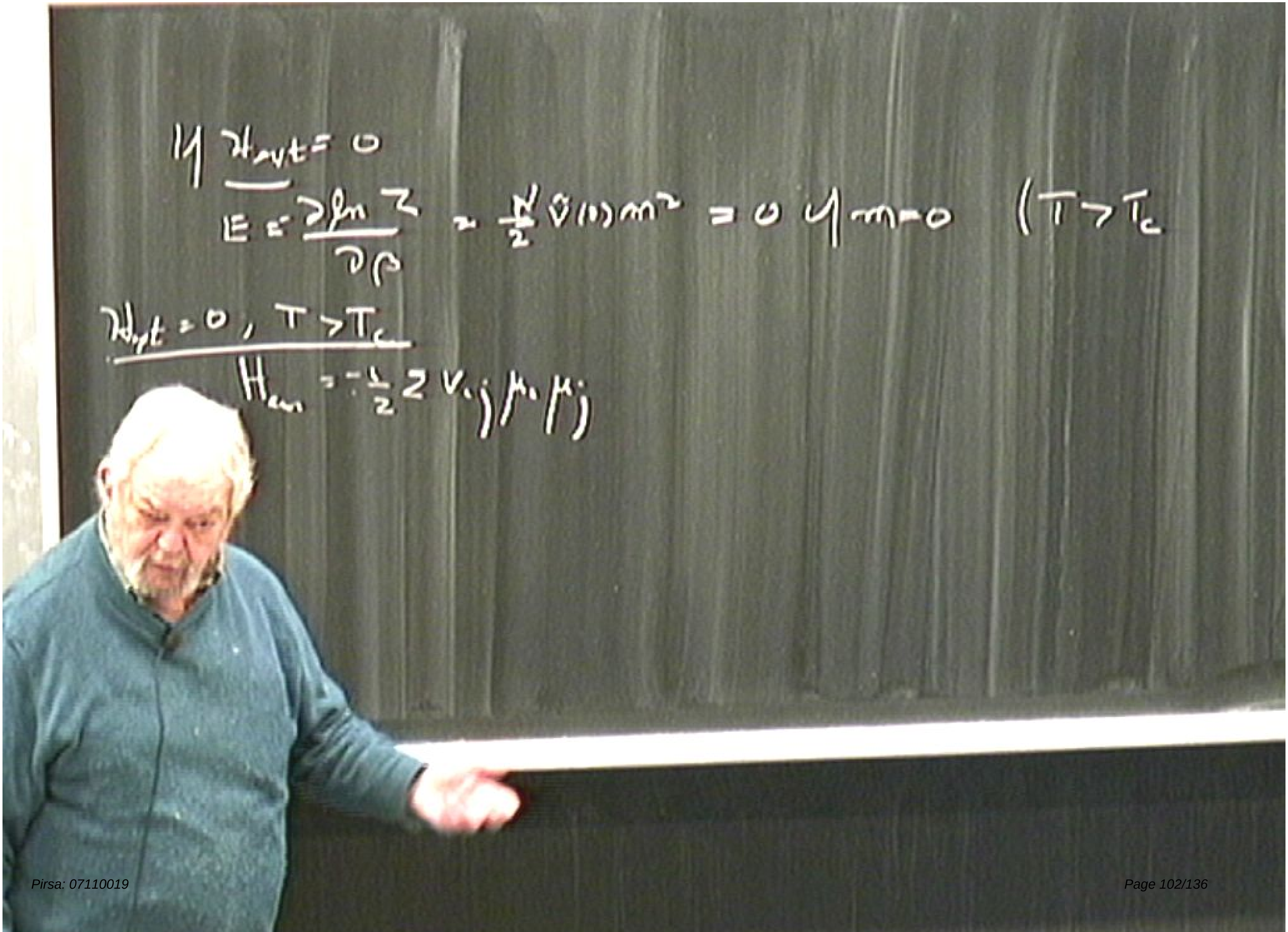
If $\underline{\mathcal{H}_{ext}} = 0$

$$E = \frac{\partial \ln Z}{\partial \beta} = \frac{N}{2}\tilde{v}(0)\, m^2 = 0 \text{ if } m = 0$$


If $\mathcal{H}_{ext} = 0$
$E = -\frac{\partial \ln Z}{\partial \beta} = -\frac{N}{2}\tilde{v}(0)m^2 = 0$ if $m = 0$ $(T > T_c$


1) $\mathcal{H}_{ext} = 0$
$E = \frac{\partial \ln Z}{\partial \beta} = \frac{N}{2}\hat{v}(0) m^2 = 0$ if $m = 0$ $(T > T_c$
$\mathcal{H}_{ext} = 0,\ T > T_c$
$\mathcal{H}$


If $H_{ext} = 0$
$E = \frac{\partial \ln Z}{\partial \beta} = -\frac{N}{2}\tilde{V}(0)m^2 = 0$ if $m = 0$ $(T > T_c$
$H_{ext} = 0,\ T > T_c$
H

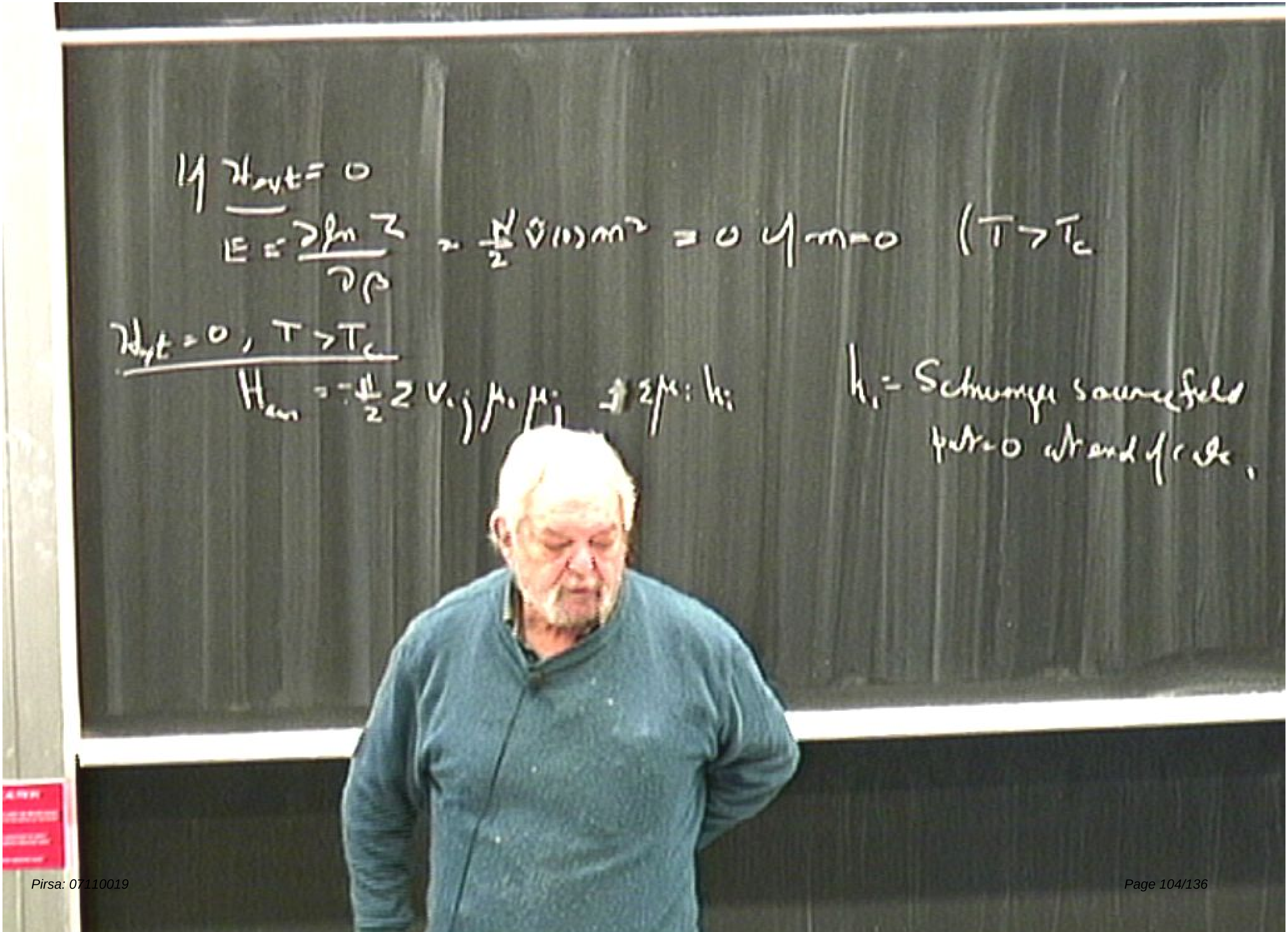

If $\mathcal{H}_{ext} = 0$
$E = \frac{\partial \ln Z}{\partial \beta} = \frac{N}{2}\tilde{v}(0)m^2 = 0$ if $m = 0$ $(T > T_c$
$\mathcal{H}_{ext} = 0,\ T > T_c$
$H_{int} = -\frac{1}{2}\sum v_{ij}\mu_i\mu_j$

If $\underline{\mathcal{H}_{ext} = 0}$

$$E = \frac{\partial \ln Z}{\partial \beta} = -\frac{N}{2}\hat{v}(0)m^2 = 0 \text{ if } m = 0$$

$\underline{\mathcal{H}_{ext} = 0,\ T > T_c}$

$$H_{am} = -\frac{1}{2}\sum v_{ij}\mu_i\mu_j + \sum \mu_i h_i$$


If $\mathcal{H}_{ext} = 0$
$E = \frac{\partial \ln Z}{\partial \beta} = \frac{N}{2}\hat{v}(0)m^2 = 0$ if $m = 0$ $(T > T_c$
$\mathcal{H}_{ext} = 0,\ T > T_c$
$H_{eff} = -\frac{1}{2}\sum v_{ij}\mu_i\mu_j + \sum \mu_i h_i$
h_i = Schwinger source field
put = 0 at end of calc.

$m = 0 \quad (T > T_c$

h_i = Schwinger source field

put = 0 at end of calc.

If $\mathcal{H}_{ext} = 0$

$$E = \frac{\partial \ln Z}{\partial \beta} = \frac{N}{2} \tilde{v}(0)\, m^2 = 0 \text{ if } m = 0$$

$\mathcal{H}_{ext} = 0, \; T > T_c$

$$H_{am} = -\frac{1}{2} \sum v_{ij} \mu_i \mu_j \quad h_i \qquad h_i =$$

$$Z = \mathrm{tr}\, e^{-\beta H_{am}}$$

$$\frac{\partial \ln Z}{\partial h_i} = \langle \mu_i \rangle$$

If $\mathcal{H}_{ext} = 0$

$$E = \frac{\partial \ln Z}{\partial \beta} = \frac{N}{2} \tilde{V}(0)\, m^2 = 0 \text{ if } m = 0$$

$\mathcal{H}_{ext} = 0,\ T > T_c$

$$H_{em} = -\frac{1}{2} \sum V_{ij} \mu_i \mu_j - \sum \mu_i h_i \qquad h_i =$$

$$Z = \mathrm{tr}\, e^{-\beta H_{em}}$$

$$\frac{\partial \ln Z}{\partial h_i} = \langle \mu_i \rangle \quad ; \quad \partial^2 \ln$$

If $\mathcal{H}_{ext} = 0$

$$E = \frac{\partial \ln Z}{\partial \beta} = \frac{N}{2} \tilde{v}(0) m^2 = 0 \text{ if } m = 0$$

$\mathcal{H}_{ext} = 0, \ T > T_c$

$$H_{am} = -\frac{1}{2} \sum v_{ij} \mu_i \mu_j - \sum \mu_i h_i \qquad h$$

$$Z = \mathrm{tr}\, e^{-\beta H_{am}}$$

$$\frac{\partial \ln Z}{\partial h_i} = \langle \mu_i \rangle \ ; \ \frac{\partial^2 \ln Z}{\partial h_i \partial h_j}$$

If $\mathcal{H}_{ext} = 0$

$$E = \frac{\partial \ln Z}{\partial \beta} = \frac{N}{2} \tilde{v}(0) m^2 = 0 \text{ if } m = 0 \quad (T > T_c$$

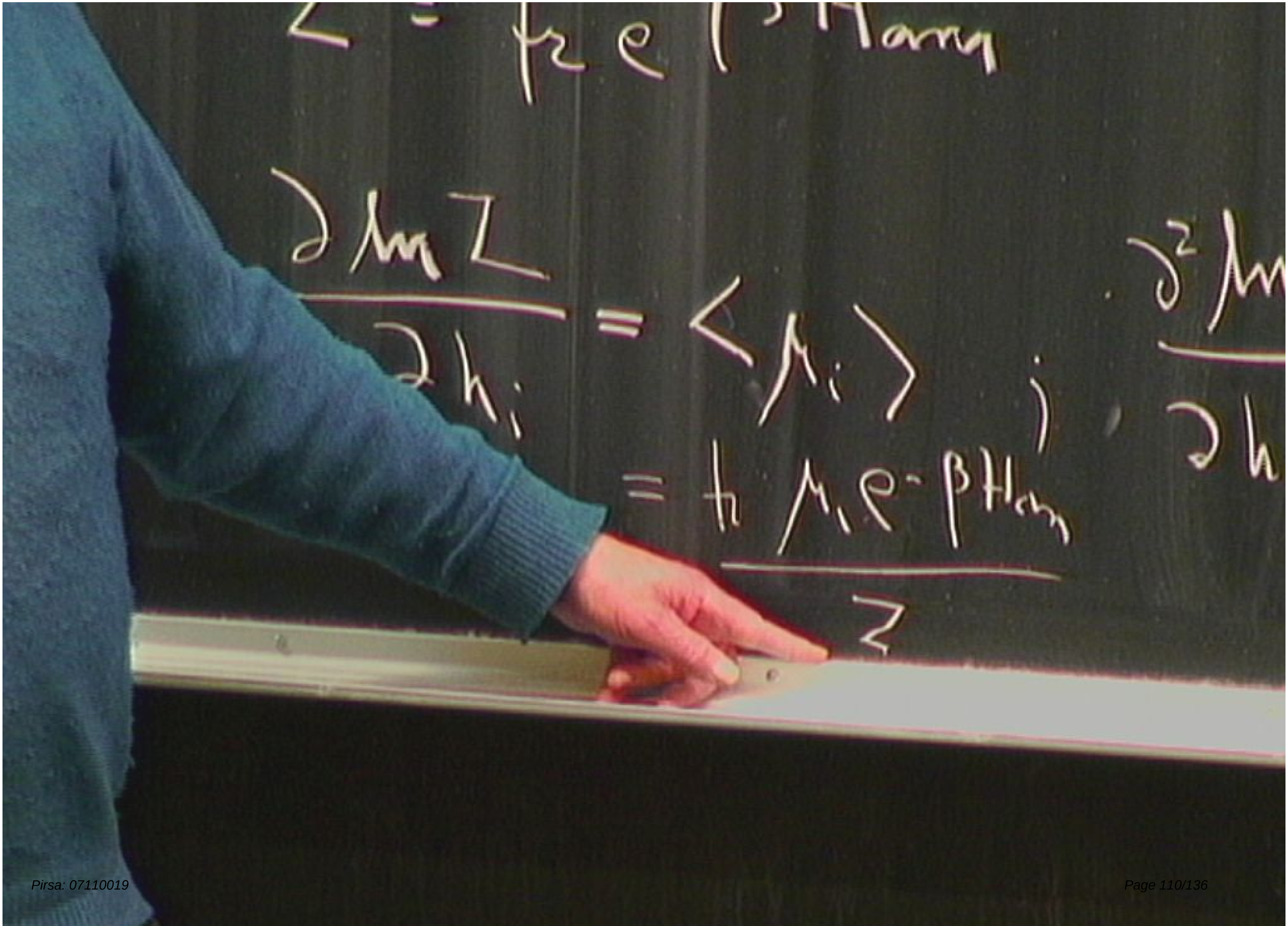
$\mathcal{H}_{ext} = 0, \; T > T_c$

$$H_{em} = -\frac{1}{2} \sum v_{ij} \mu_i \mu_j - \sum \mu_i h_i$$

h_i - Schwinger so... put $= 0$ at e...

$$Z = \operatorname{tr} e^{-\beta H_{em}}$$

$$\frac{\partial \ln Z}{\partial h_i} = \langle \mu_i \rangle \; ; \; \frac{\partial^2 \ln Z}{\partial h_i \partial h_j} = \langle \mu_i \mu_j \rangle - \langle \mu_i \rangle \langle \mu_j \rangle.$$



$Z = \mathrm{tr}\, e^{-\beta H_{array}}$
$\frac{\partial \ln Z}{\partial h_i} = \langle \mu_i \rangle$;
$\frac{\partial^2 \ln}{\partial h}$
$= \frac{\mathrm{tr}\, \mu_i e^{-\beta H_{array}}}{Z}$

$$H_{ann} = -\frac{1}{2}\sum V_{ij}\mu_i\mu_j - \sum \mu_i h_i$$

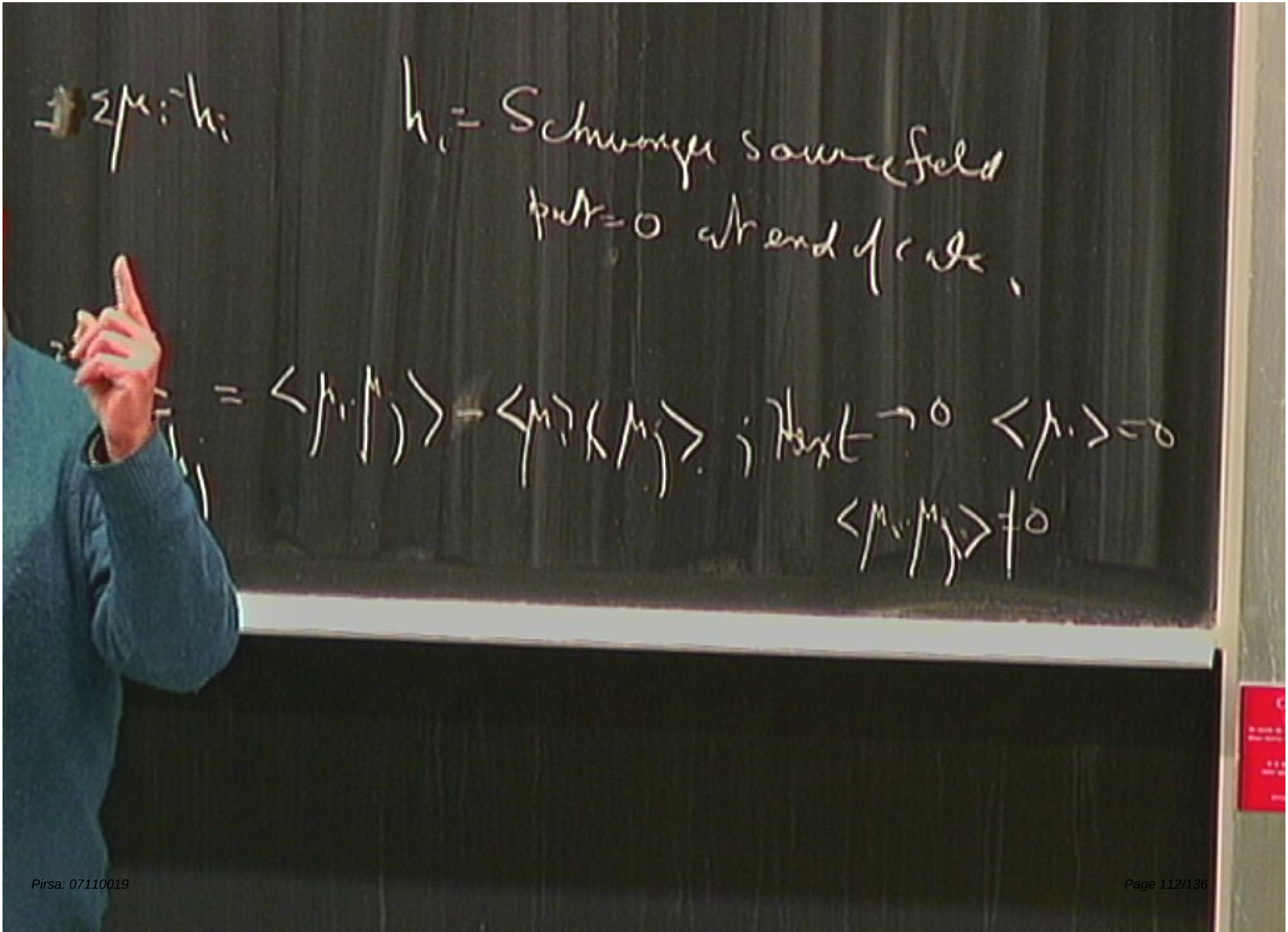
h_i = Schwinger s

pert = 0 at

$$Z = \mathrm{tr}\, e^{-\beta H_{ann}}$$

$$\frac{\partial \ln Z}{\partial h_i} = \langle \mu_i \rangle \;; \quad \frac{\partial^2 \ln Z}{\partial h_i \partial h_j} = \langle \mu_i \mu_j \rangle - \langle \mu_i \rangle \langle \mu_j \rangle$$

$$= \frac{\mathrm{tr}\, \mu_i e^{-\beta H_{ann}}}{Z}$$


hi = Schwinger source field
put = 0 at end of calc.
= <μi μj> − <μi><μj> ; Hext → 0 <μi> = 0
<μi μj> ≠ 0

Calc of $\langle \rho_i \rho_j \rangle$ by a modified MFT

Calc of $\langle \mu_i \mu_j \rangle$ by a modified MFT

$$\mu_{\vec{q}} = \sum \mu_i e^{i\vec{q}\cdot\vec{R}_i}$$

If $\mathcal{H}_{ext} = 0$

$$E = \frac{\partial \ln Z}{\partial \beta} = \frac{N}{2}\hat{v}(0) m^2 = 0 \text{ if } m = 0 \quad (T > T_c$$

$\mathcal{H}_{ext} = 0,\ T > T_c$:

$$H_{an} = -\frac{1}{2}\sum V_{ij}\mu_i\mu_j \ \sum \mu_i h_i$$

h_i = Schwinger source fie[ld]
put = 0 at end of c[alc]

$$Z = \mathrm{tr}\, e^{-\beta H_{an}}$$

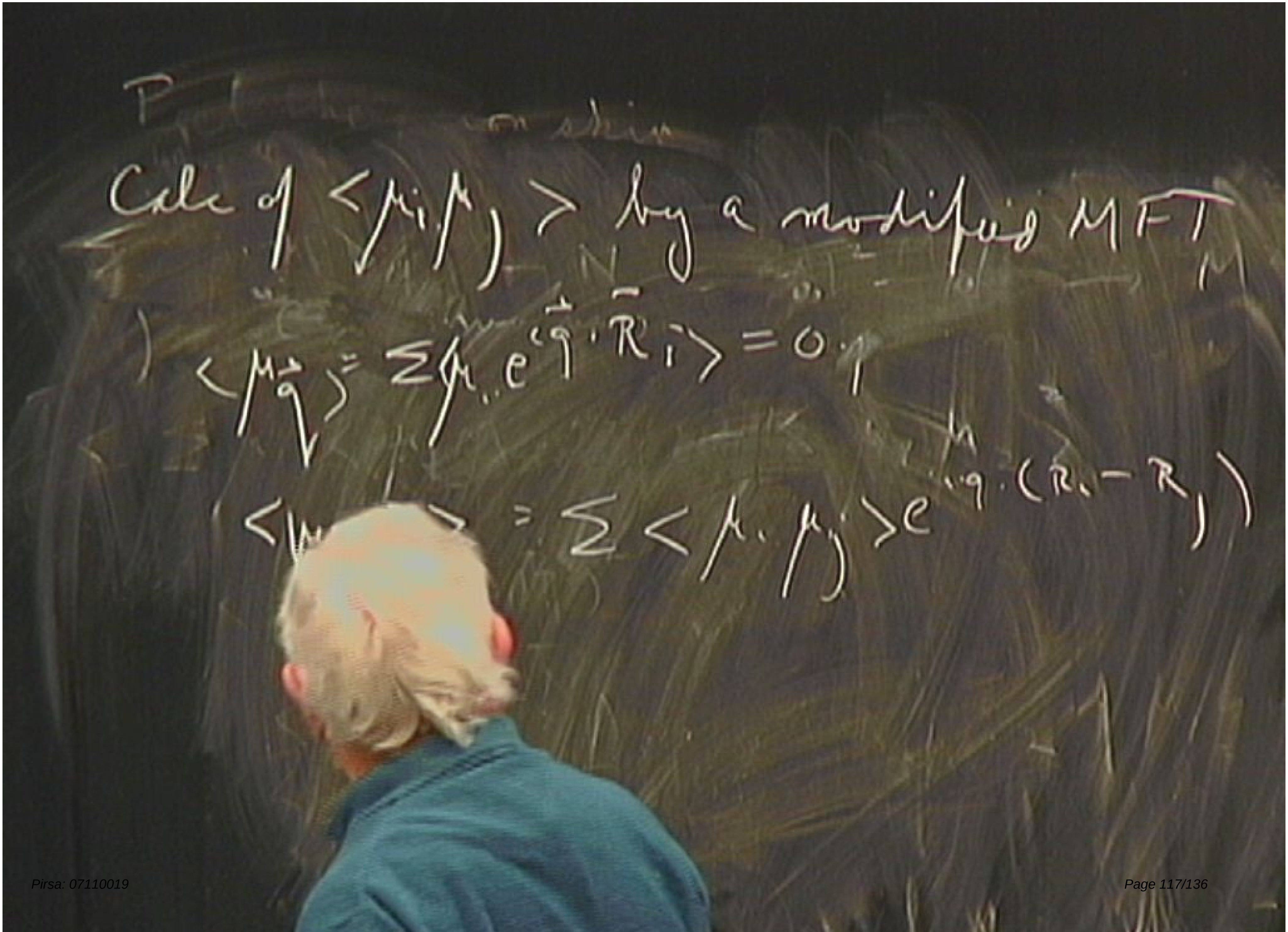
$$\frac{\partial \ln Z}{\partial h_i} = \langle \mu_i \rangle = \frac{\mathrm{tr}\, \mu_i e^{-\beta H_{an}}}{Z} \ ; \ \frac{\partial^2 \ln Z}{\partial h_i \partial h_j} = \langle \mu_i \mu_j \rangle - \langle \mu_i \rangle \langle \mu_j \rangle \ ; \ \mathcal{H}_{ext} \to 0$$

$\langle \mu_i \mu_j \rangle$

Calc of $\langle \mu_i \mu_j \rangle$ by a modified MFT

$$\mu_{\vec{q}} = \sum \mu_i e^{i\vec{q}\cdot\vec{R}_i} = 0$$

$$\langle |\mu_{\vec{q}}|^2 \rangle = \sum \langle \mu_i \mu_j \rangle e^{iq\cdot(R_i - R_j)}$$


Calc of $\langle \mu_i \mu_j \rangle$ by a modified MFT
$\langle \mu_{\vec q} \rangle = \sum \langle \mu_i e^{i\vec q \cdot \vec R_i} \rangle = 0$
$\langle \mu_{q} \mu_{-q} \rangle = \sum \langle \mu_i \mu_j \rangle e^{iq\cdot(R_i - R_j)}$

Calc of $\langle \mu_i \mu_j \rangle$ by a modified MFT

$$\langle \mu_{\vec{q}} \rangle = \sum \mu_i \, e^{i\vec{q}\cdot\vec{R}_i} \rangle = 0.$$

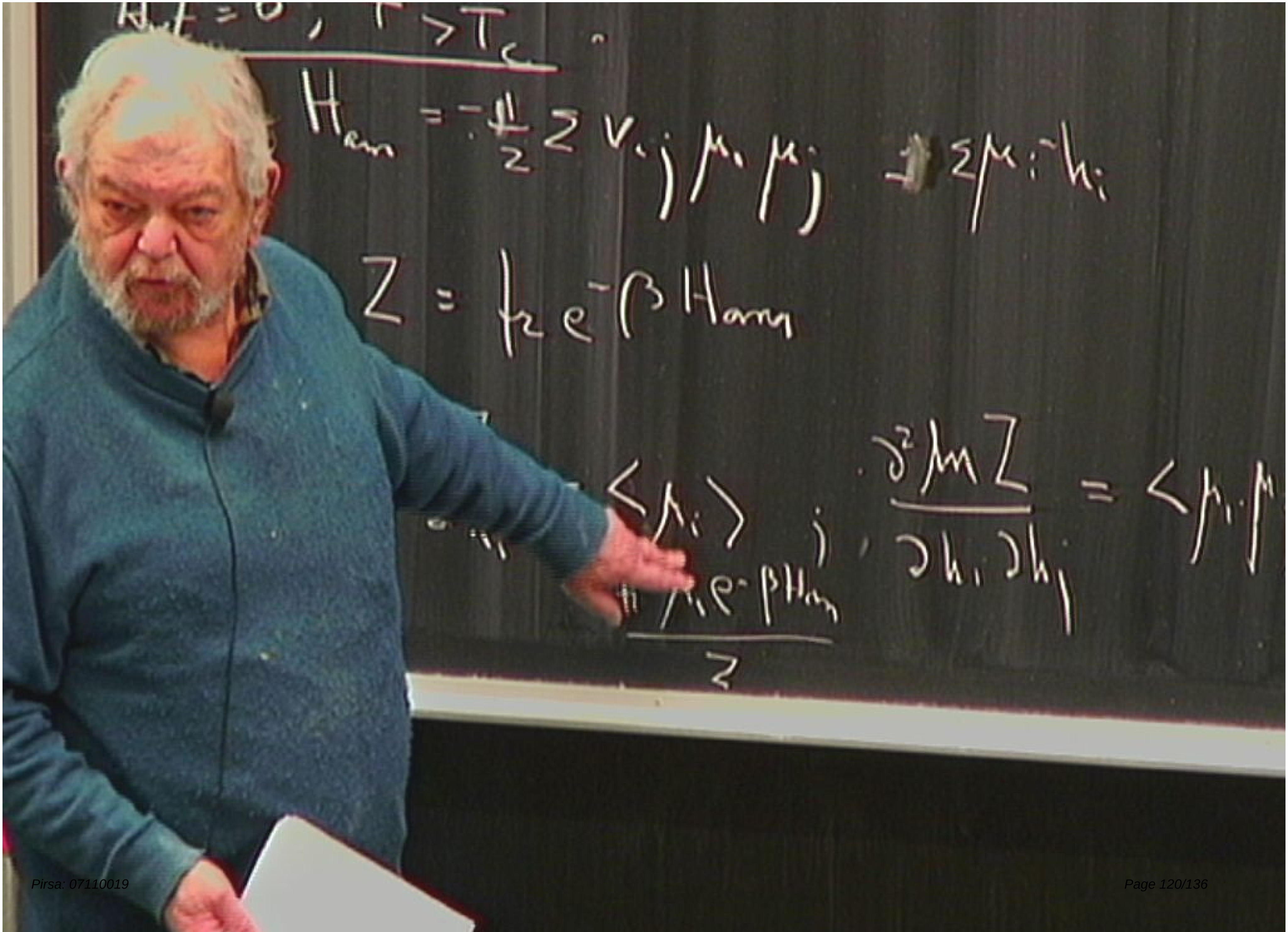
$$\langle |\mu_{\vec{q}}|^2 \rangle = \sum \langle \mu_i \mu_j \rangle e^{i\vec{q}\cdot(\vec{R}_i - \vec{R}_j)}$$

Calc of $\langle \mu_i \mu_j \rangle$ by a modifying MFT

$$\langle \mu_{\vec{q}} \rangle = \sum \mu_i \, e^{i\vec{q}\cdot\vec{R}_i} \rangle = 0$$

$$\langle |\mu_{\vec{q}}|^2 \rangle = \sum \langle \mu_i \mu_j \rangle e^{i\vec{q}\cdot(\vec{R}_i - \vec{R}_j)}$$

$$\langle \mu_i \mu_j \rangle - \langle \mu_i \rangle \langle \mu_j \rangle = \frac{\partial \langle \mu_i \rangle}{\partial h_j}$$



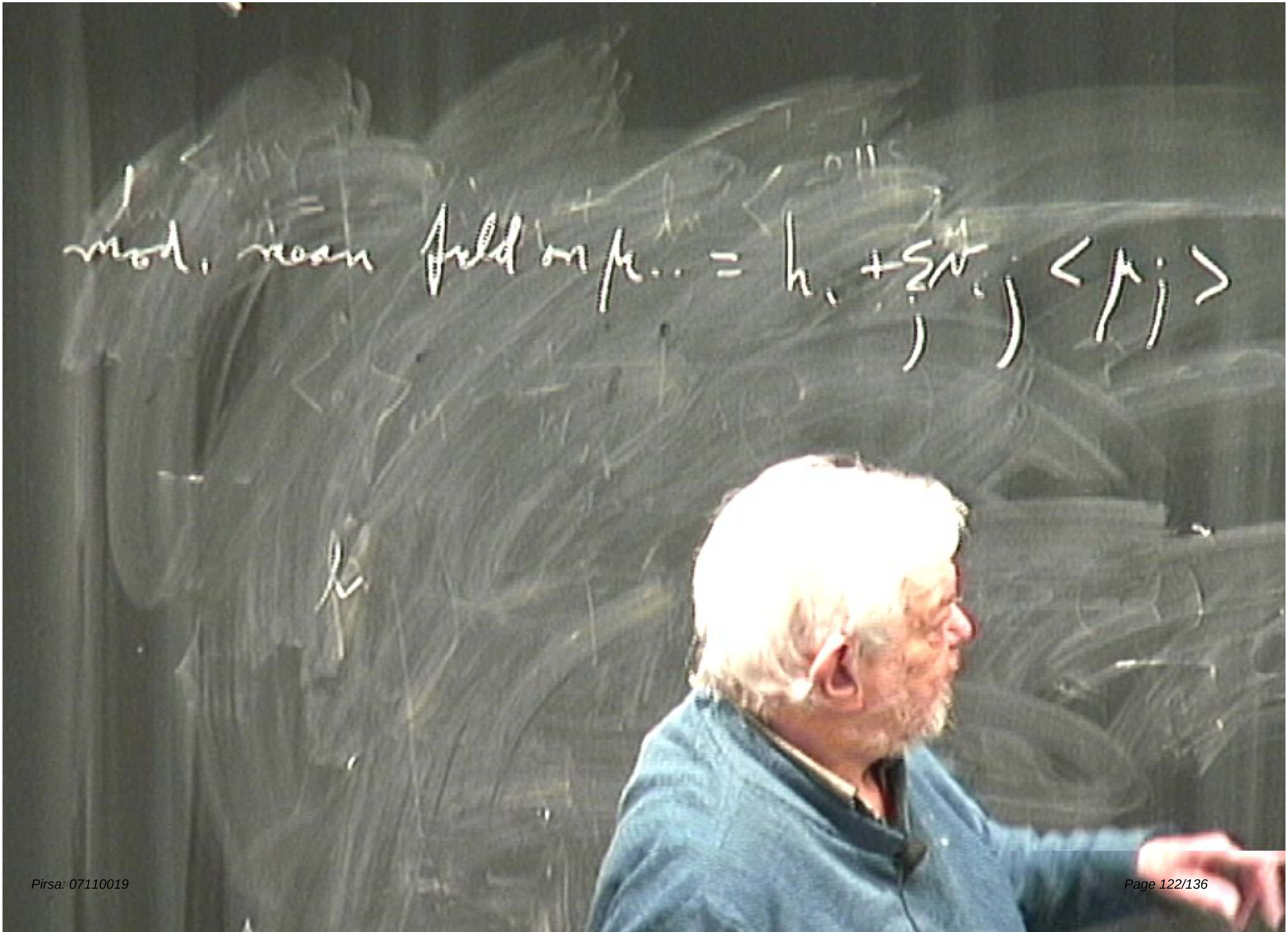
Calc of $\langle \mu_i \mu_j \rangle$ by a modifying MFT

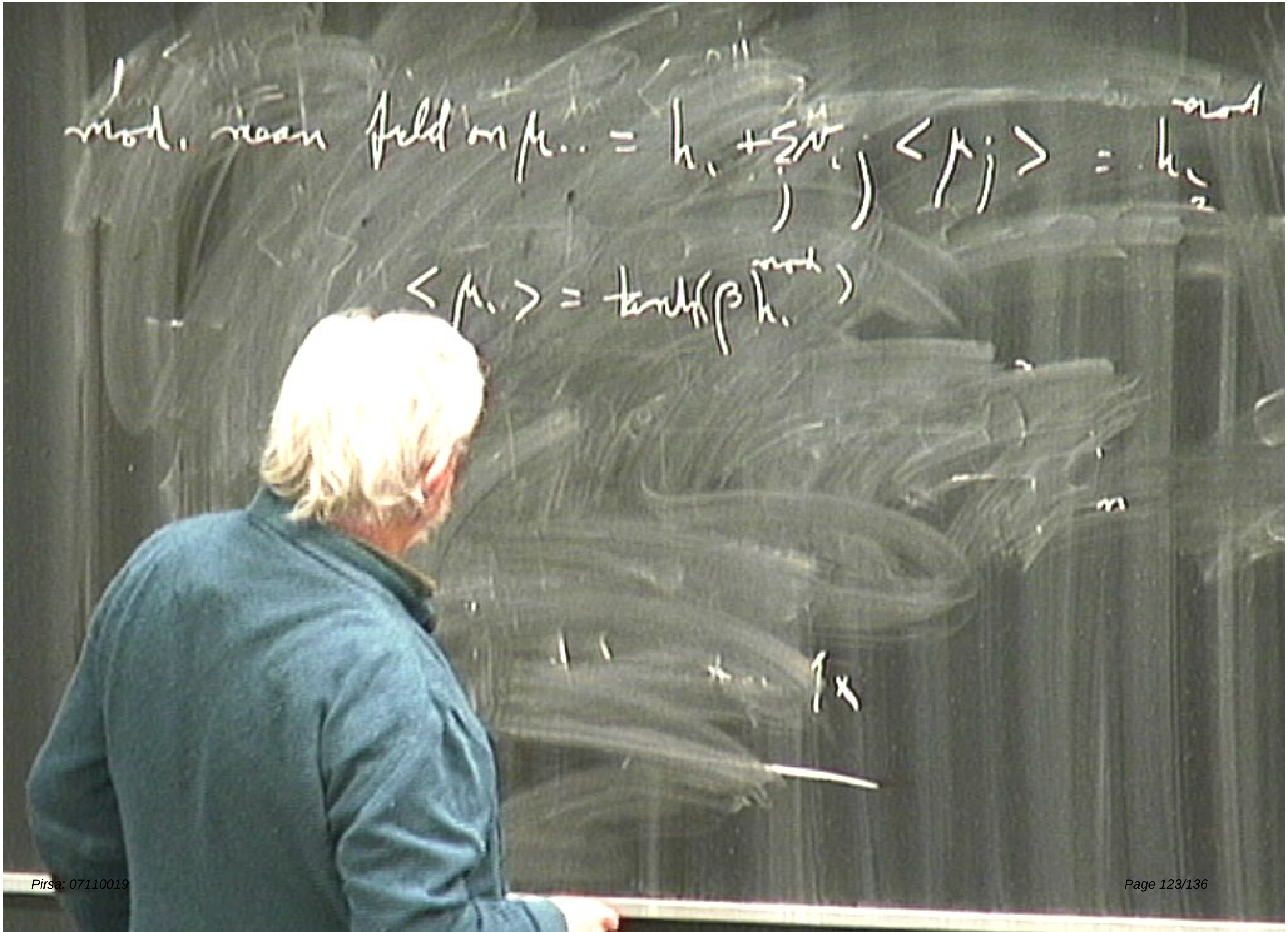
$$\langle \mu_{\vec{q}} \rangle = \sum \langle \mu_i e^{i\vec{q}\cdot\vec{R}_i} \rangle = 0$$

$$\langle |\mu_{\vec{q}}|^2 \rangle = \sum \langle \mu_i \mu_j \rangle e^{i\vec{q}\cdot(\vec{R}_i - \vec{R}_j)}$$

$$\langle \mu_i \mu_j \rangle - \langle \mu_i \rangle \langle \mu_j \rangle = \frac{\partial \langle \mu_i \rangle}{\partial h_j}$$

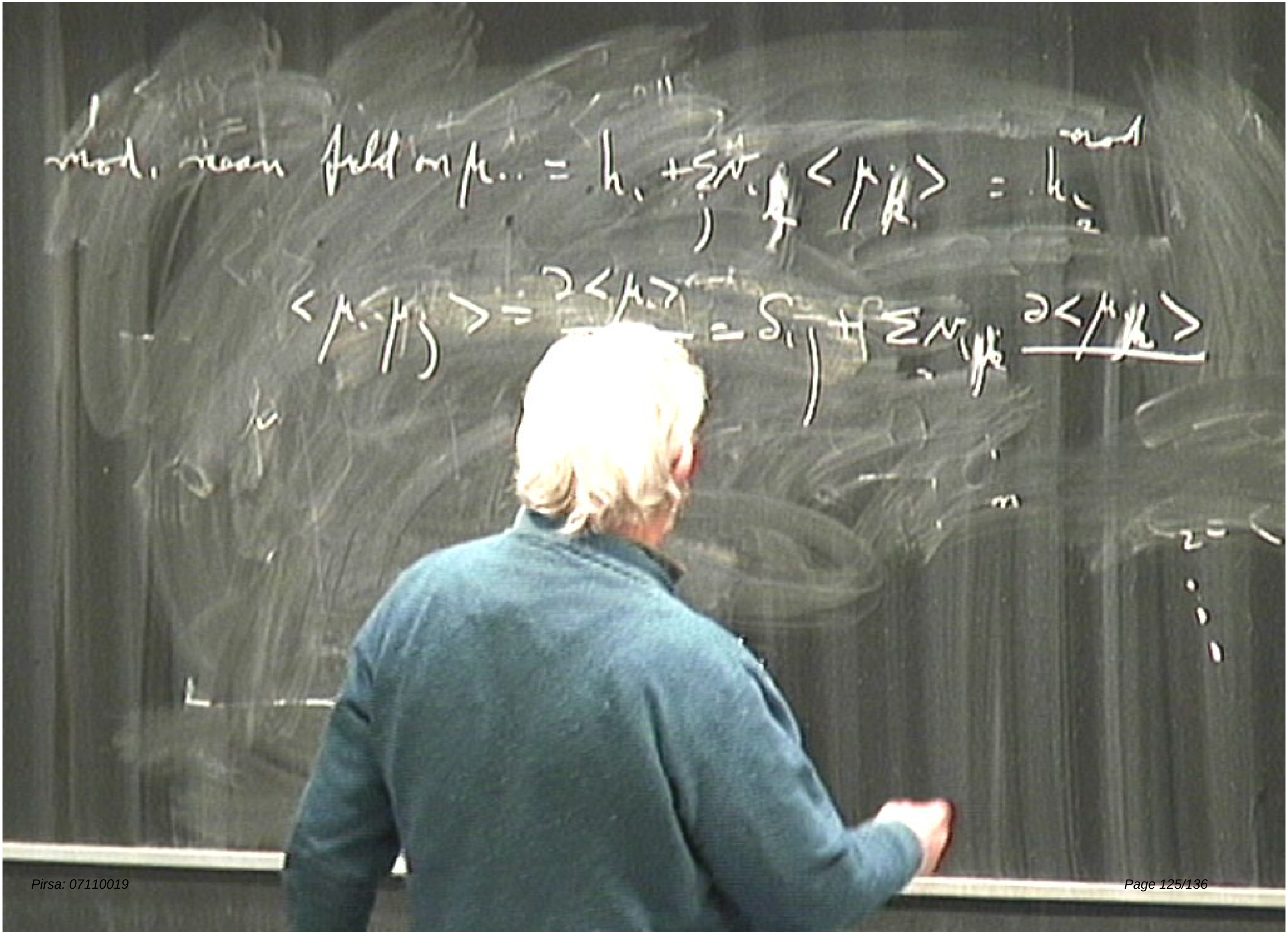
mean field modification

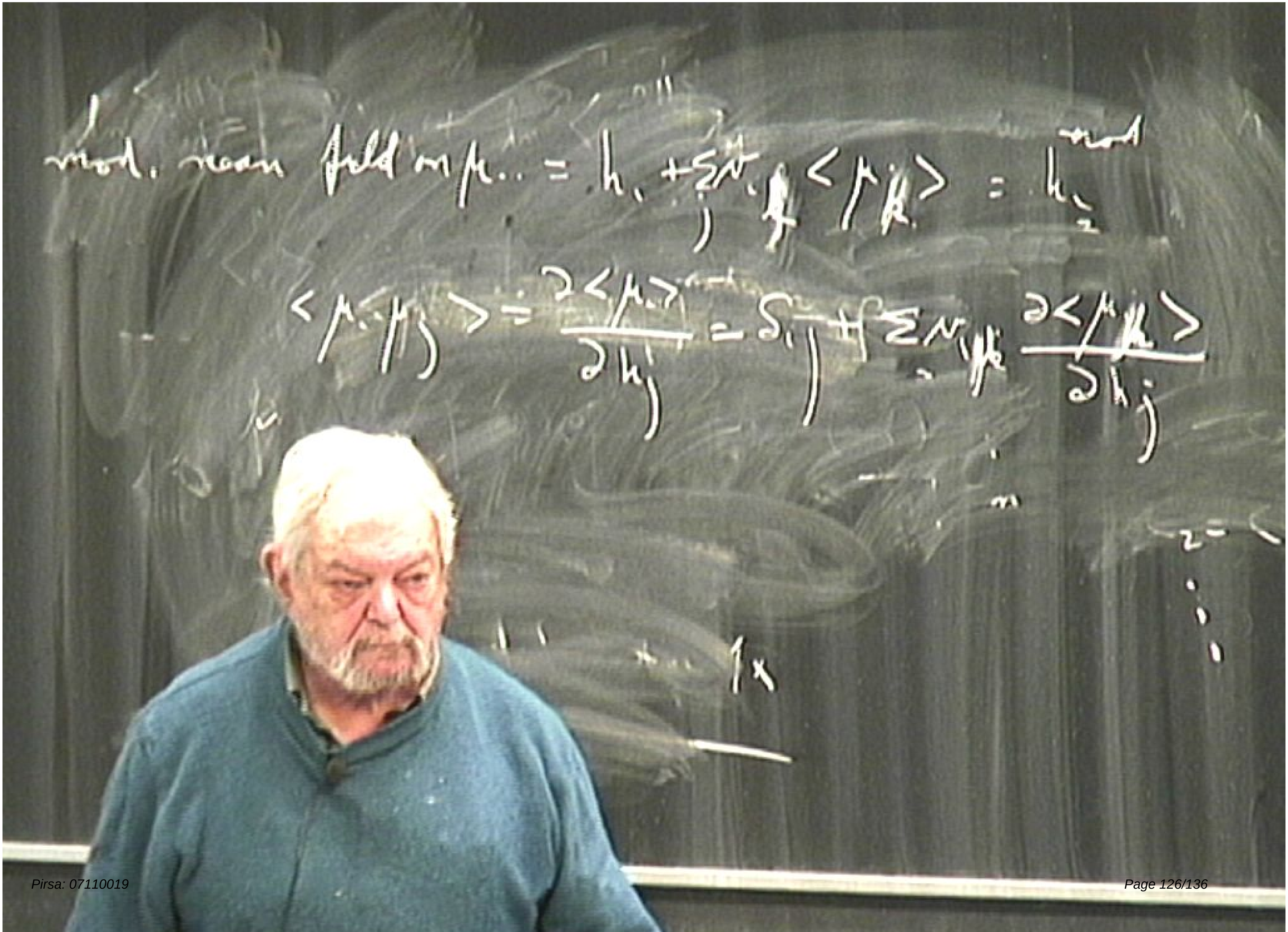

mol. mean field on $\mu_i = h_i + \sum_j J_{ij} \langle \mu_j \rangle$


mol. mean field on μ_i = $h_i + \sum_j v_{ij} \langle \mu_j \rangle$ = h_i^{mod}
$\langle \mu_i \rangle = \tanh(\beta h_i^{mod})$

mod. mean field on $\mu_i = h_i + \sum_j v_{ij} \langle \mu_j \rangle = h_i^{mod}$

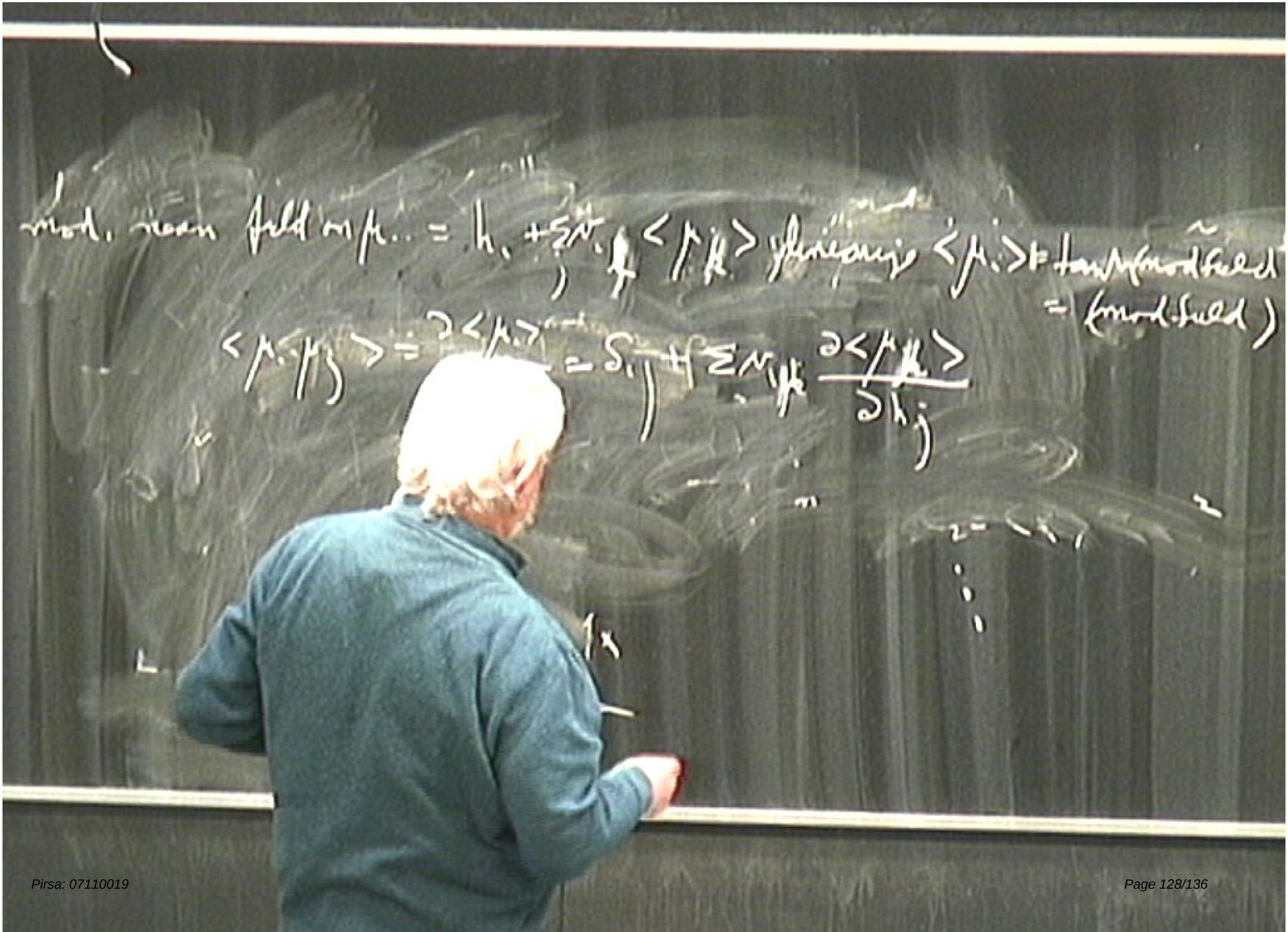
$$\langle \mu_i \rangle = \tanh(\beta h_i^{mod}) \simeq \beta$$

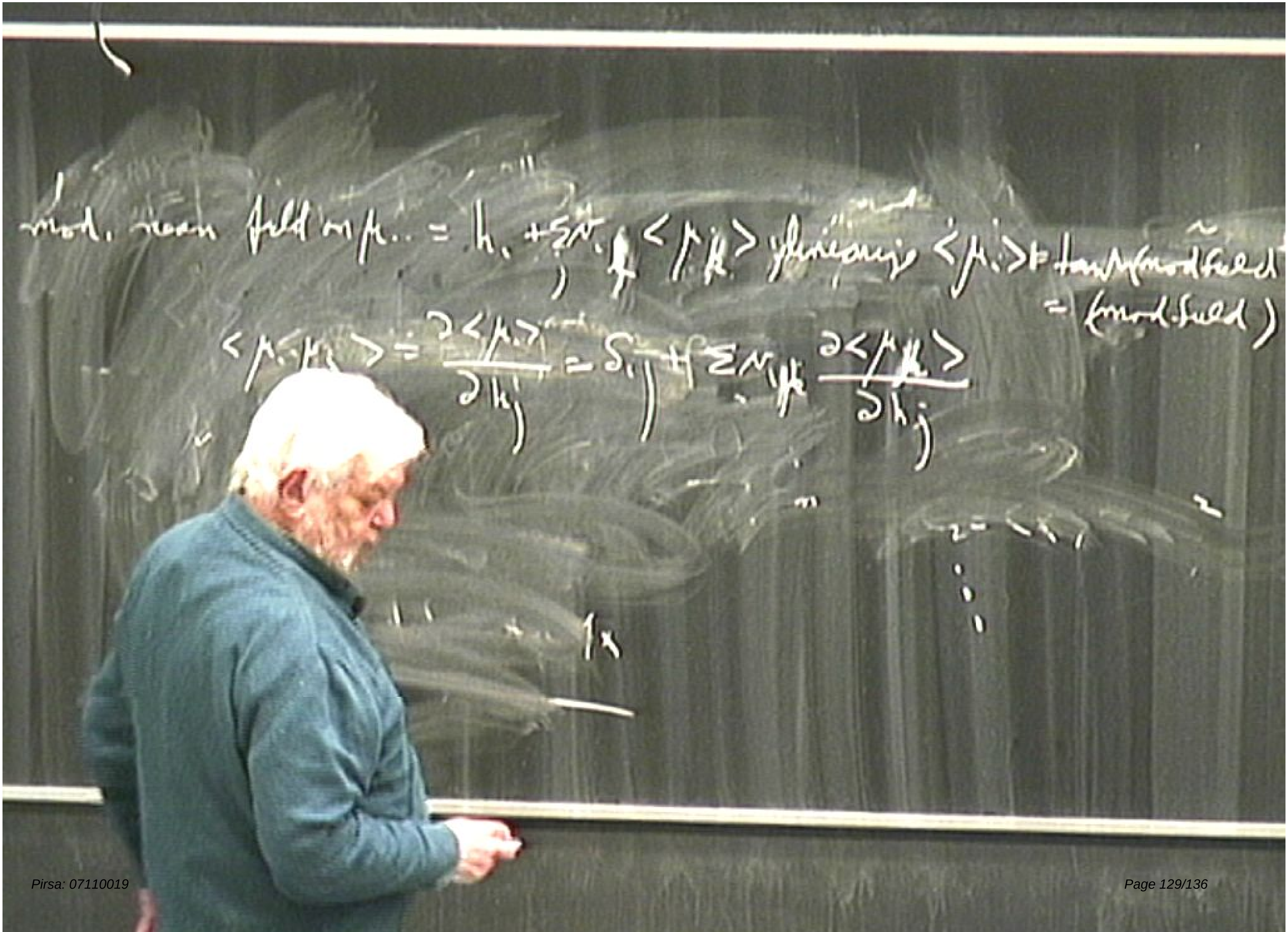

mod. mean field on μ_i = h_i + Σ_j N_ik <μ_k> = h_i^mod
<μ_i μ_j> = ∂<μ_i>/∂h_j = δ_ij + Σ N_ik ∂<μ_k>/∂h_j


mod. mean field on μ_i = h_i + Σ_j N_ij <μ_j> = h_i^mod
<μ_i μ_j> = ∂<μ_i>/∂h_j = δ_ij + Σ N_ik ∂<μ_k>/∂h_j

$N_{k} \langle \mu_k \rangle$; $\langle \mu_k \rangle$ = standard mod. non-field ≅ mod field

$= \delta_{ij} + \beta \sum N_{k} \frac{\partial \langle \mu_k \rangle}{\partial h_j}$




mod. mean field on μ_i = h_i + Σ_j N_ij ⟨μ_j⟩
= (mod. field)
⟨μ_i μ_j⟩ = ∂⟨μ_i⟩/∂h_j = δ_ij + Σ N_ik ∂⟨μ_k⟩/∂h_j

mod. mean field on $\mu_i = h_i + \sum_j N_{ij}\, \langle \mu_j \rangle$ ... $\langle \mu_i \rangle = \tanh(\text{mod. field})$

$= (\text{mod. field})$

$$\langle \mu_i \mu_j \rangle = \frac{\partial \langle \mu_i \rangle}{\partial h_j} = \delta_{ij} + \sum N_{ik} \frac{\partial \langle \mu_k \rangle}{\partial h_j}$$

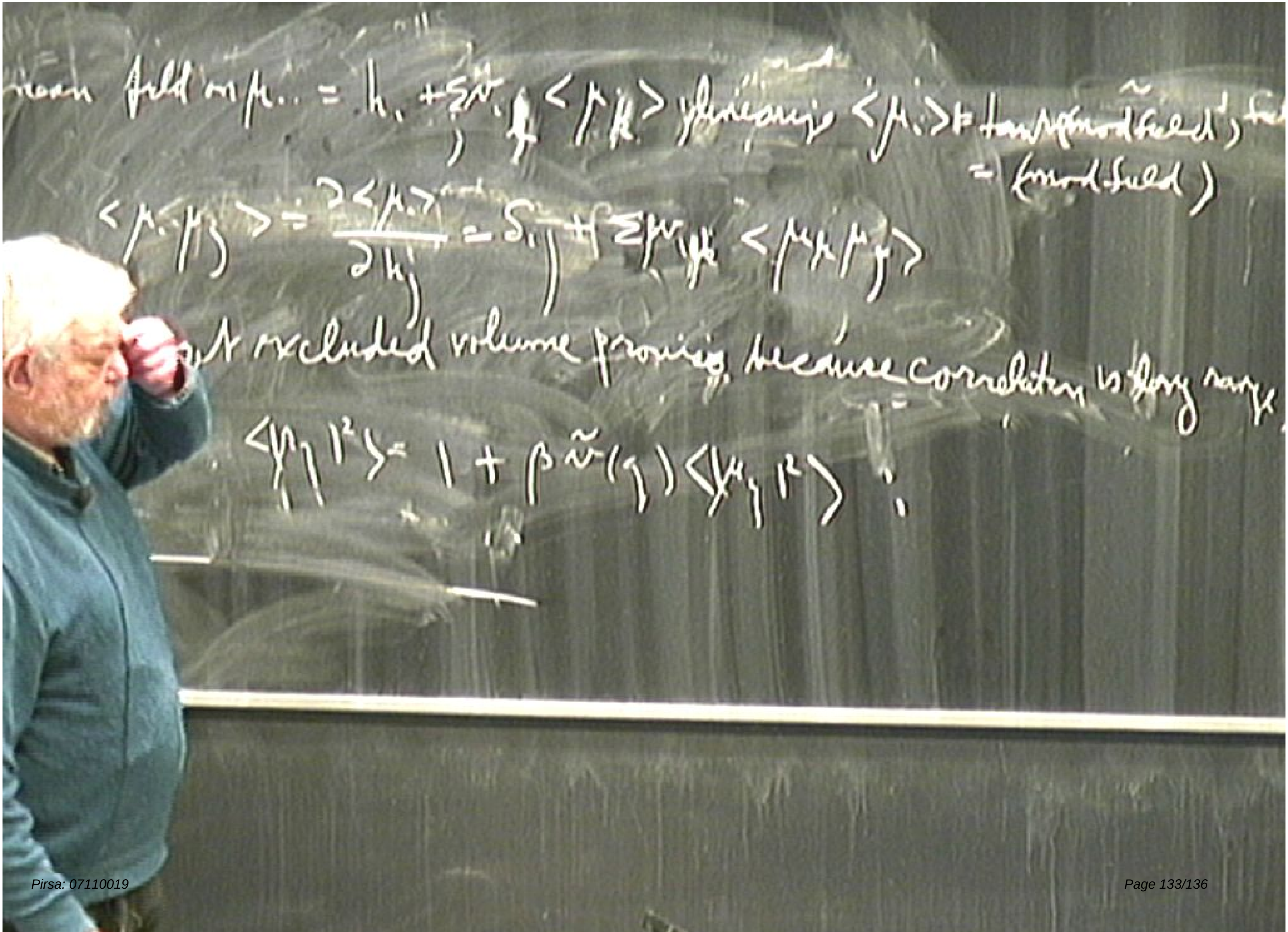
$+\sum_j N_{ik} \langle \mu_k \rangle$ linearising $\langle \mu_i \rangle = \tanh(\text{mod}$

$= (\text{mod Su}$

$\rangle = \delta_{ij} + \sum N_{ik} \langle \mu_k \mu_j \rangle$

field on μ_i .. $= h_i + \sum_j J_{ik} \langle \mu_k \rangle$ linearize $\langle \mu_i \rangle$

$$\langle \mu_i \mu_j \rangle = \frac{\partial \langle \mu_i \rangle}{\partial h_j} = \delta_{ij} + \beta \sum J_{ik} \langle \mu_k \mu_j \rangle$$


because correlation is long range

mean field on $\mu_i = h_i + \sum_j \tilde{v}_{ij} \langle \mu_k \rangle$ physically $\langle \mu_i \rangle$ + term (smoothed field), $= \langle$mod field$\rangle$

$$\langle \mu_i \mu_j \rangle = \frac{\partial \langle \mu_i \rangle}{\partial h_j} = \delta_{ij} + \beta \sum_k \tilde{v}_{ik} \langle \mu_k \mu_j \rangle$$

Forgot excluded volume proviso because correlation is long range,

$$\langle |\mu_q|^2 \rangle = 1 + \beta \tilde{v}(q) \langle |\mu_q|^2 \rangle$$

mean field on $\mu_i = h_i + \sum_j \tilde{v}_{ij} \langle \mu_j \rangle$ ... $\langle \mu_i \rangle$ ... (mod field)

= (mod field)

$$\langle \mu_i \mu_j \rangle = \frac{\partial \langle \mu_i \rangle}{\partial h_j} = \delta_{ij} + \sum_k \beta v_{ik} \langle \mu_k \mu_j \rangle$$

ought excluded volume promising because correlation is long range

$$\langle |\mu_q|^2 \rangle = 1 + \beta \tilde{v}(q) \langle |\mu_q|^2 \rangle$$

$$\langle |\mu_q|^2 \rangle = \frac{1}{1 - \beta \tilde{v}(q)}$$

$$\mu_j \rangle = \frac{\partial \langle \mu_i \rangle}{\partial h_j} = \delta_{ij} + \beta \sum v_{ik} \langle \mu_k \mu_j \rangle$$

...pt excluded volume promise because co...

$$\langle |\mu_q|^2 \rangle = 1 + \beta \tilde{v}(q) \langle |\mu_q|^2 \rangle \,.$$

$$\langle |\mu_q|^2 \rangle = \frac{1}{1 - \beta \tilde{v}(q)}$$