

Title: Intro to Supersymmetry 16

Date: Nov 08, 2007 12:30 PM

URL: <http://pirsa.org/07110012>

Abstract:

$W_{\text{micro}} \rightarrow$ microscopis superpotentia (classical)

e_{eff}

$W_{\text{micro}} \rightarrow$ microscopic superpotential (classical)



$W_{\text{eff}} = ?$

$W_{\text{micro}} \rightarrow$ microscopic superpotential (classical)

\downarrow
 $\rightarrow W_{\text{eff}} - ?$

$W_{\text{micro}}(\lambda)$
 \downarrow
 \rightarrow

$W_{\text{micro}} \rightarrow$ microscopical superpotential (classical)

$W_{\text{eff}} = ?$

$W_{\text{micro}}(\lambda)$

λ is an expectation value of

$\hookrightarrow W_{\text{eff}}$

$W_{\text{micro}}(\lambda)$

\hookrightarrow

λ is an expectation value of some

$$\langle x \rangle = \lambda$$

We can contain only XSP

can contain only 2sf

$\langle X \rangle$

quantum
→ correction

Woff

Weff can contain only XSF

$$W(\langle X \rangle)$$

quantum
correction
→

$$\frac{\partial W_{\text{eff}}}{\partial \langle X \rangle} = 0$$

0

W

Wave can contain only xs

$$W(\langle X \rangle)$$

quantum
correction
→

$$\frac{\partial W_{off}}{\partial \langle X \rangle}$$

$$\langle X \rangle$$



$$\delta W_{off}$$

Weff can contain only χ^2

$W(\langle X \rangle)$ $\xrightarrow{\text{quantum correction}}$

$$\frac{\partial W_{\text{eff}}}{\partial \langle X \rangle} = 0$$



$$\frac{\partial W_{\text{eff}}}{\partial \langle X \rangle} = 0$$

Weff can contain only xsf

$$W_{\text{eff}}(\langle X \rangle)$$

quantum
correction
→

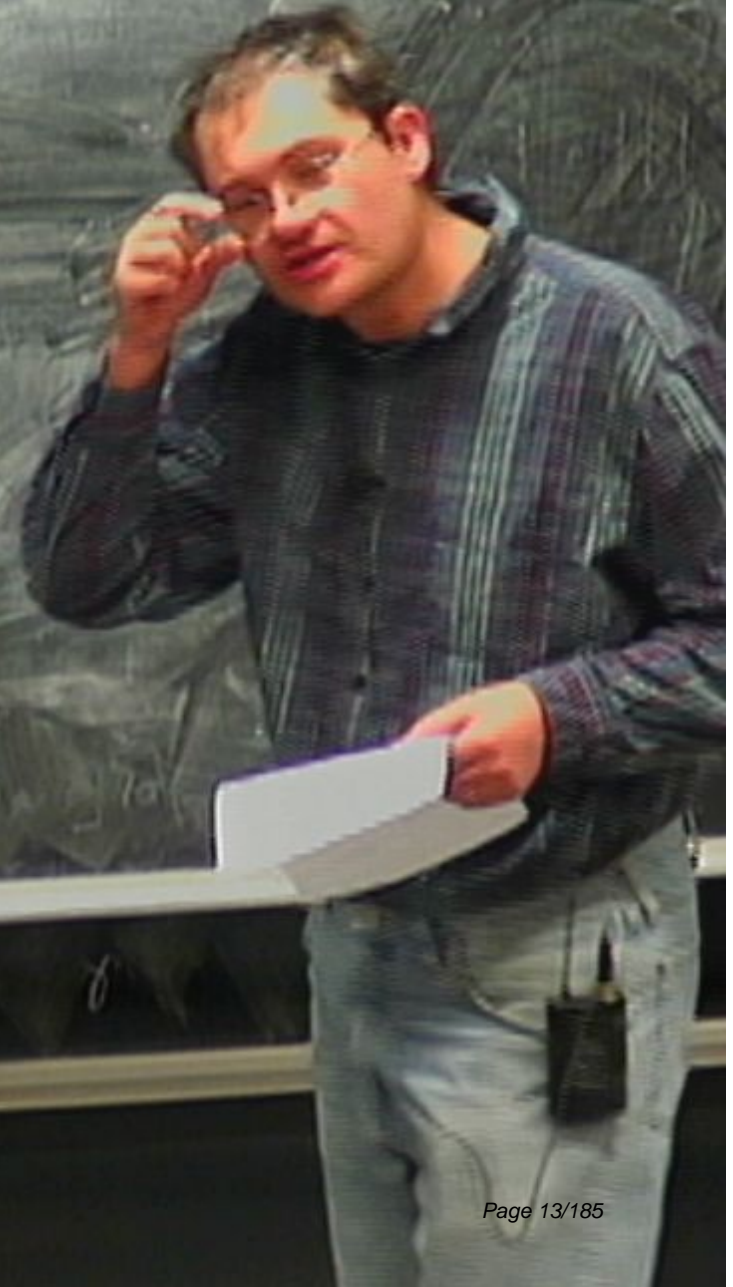
$$\frac{\partial W_{\text{eff}}}{\partial \langle X \rangle} = 0$$



$$\frac{\partial W_{\text{eff}}}{\partial \langle X \rangle} = 0$$

Example

$$W_{mic} = \frac{1}{2} m \omega^2 + \frac{1}{3} \sqrt{g}^3$$



Example

$$W_{mic} = \frac{1}{2} m \omega^2 + \frac{1}{3} \sqrt{g} \omega^3$$

↳ W_{eff}

Example

$$W_{mic} = \frac{1}{2} m \dot{\varphi}^2 + \frac{1}{3} \rho_0^3$$

$$\hookrightarrow W_{eff} = W_{eff}(\varphi)$$

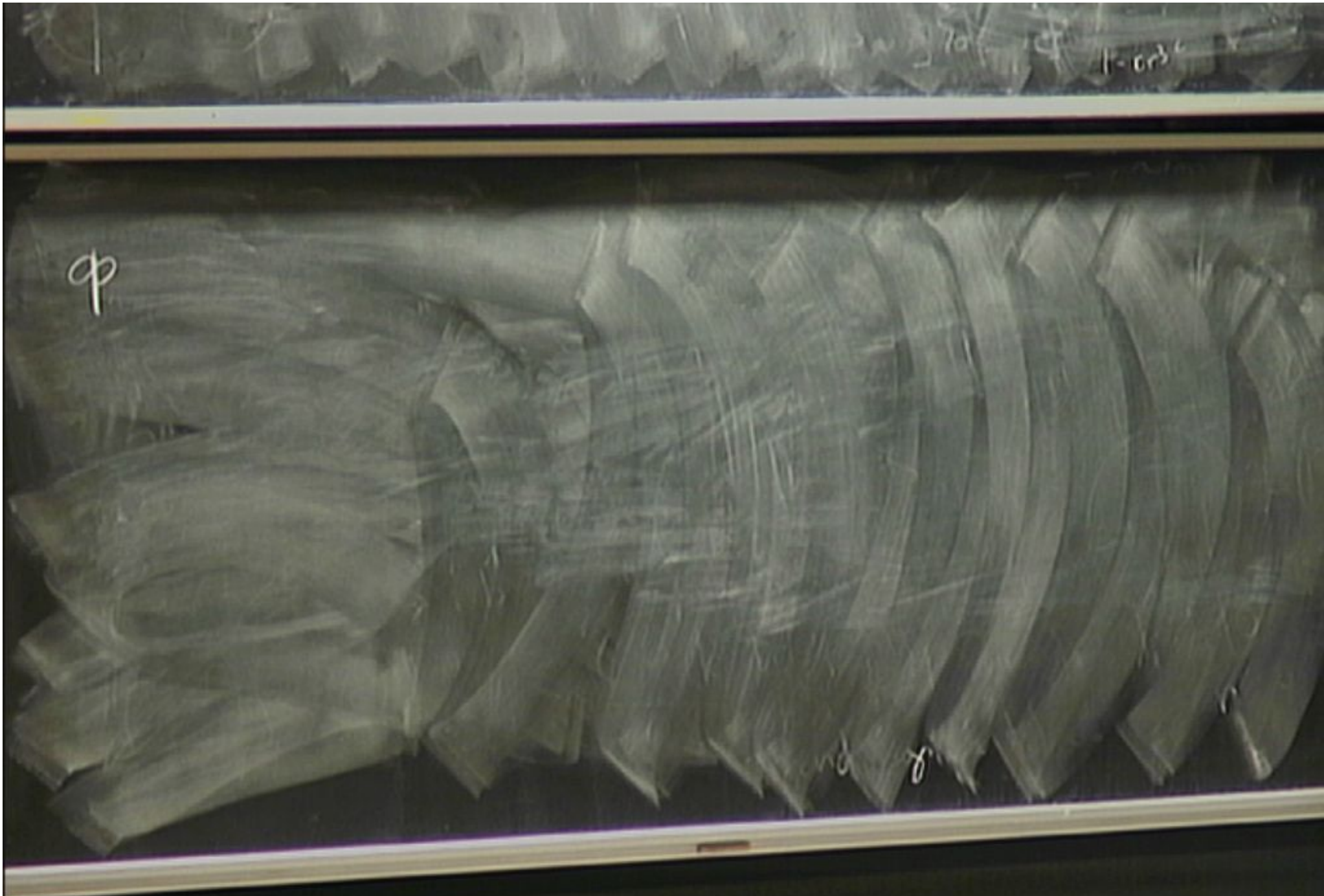
Example

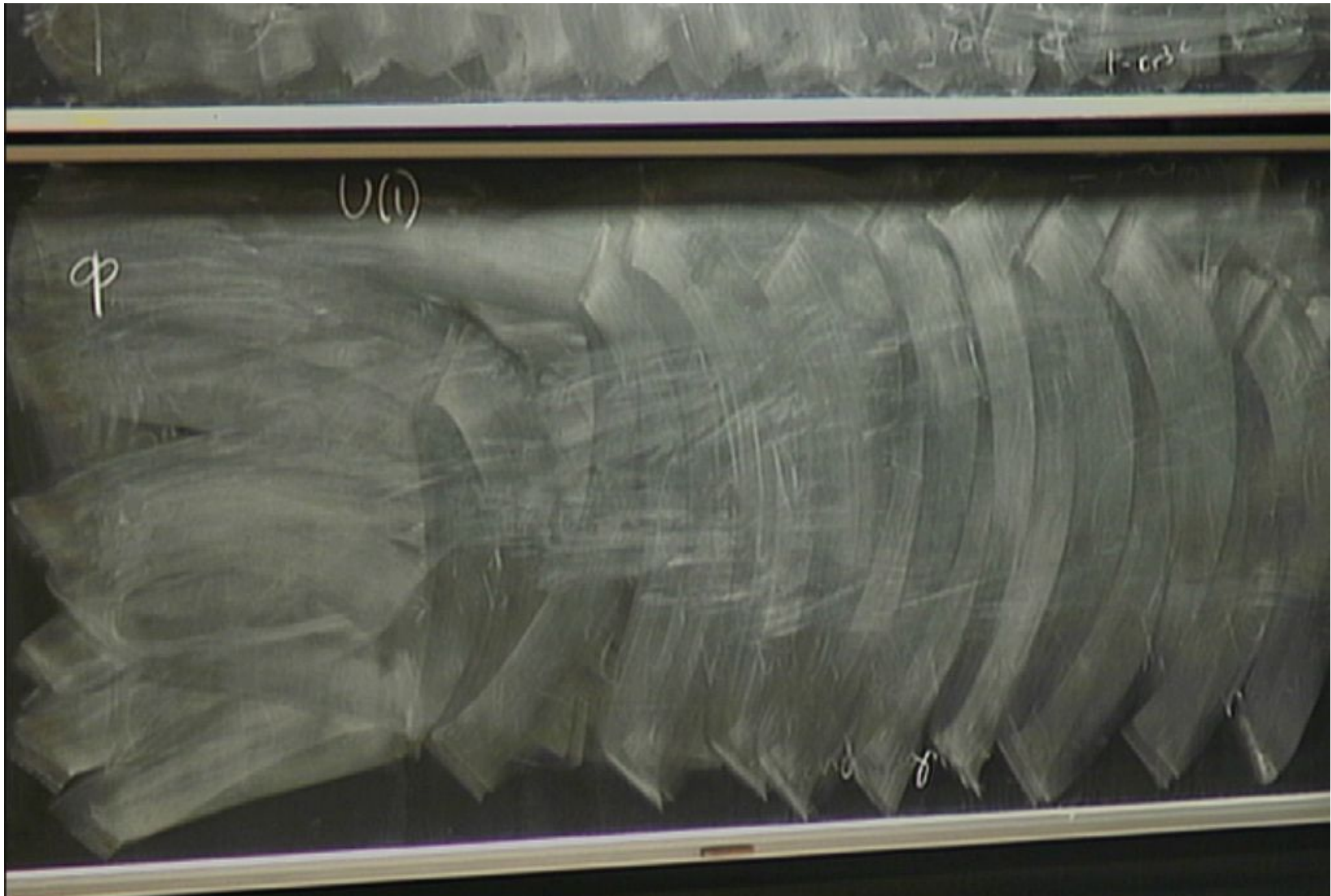
$$W_{\text{mic}} = \frac{1}{2} m \varphi^2 + \frac{1}{3} \lambda \varphi^3$$

↳ $W_{\text{eff}} = W_{\text{eff}}(\varphi, \lambda, m)$

$$\frac{\partial W_{\text{eff}}}{\partial \lambda} = 0$$

$$\frac{\partial W_{\text{eff}}}{\partial m} = 0$$





ϕ

$U(1)_n$

$U(1)_R$

δ δ

φ

m

λ

$U(1)_n$

$U(1)_R$

$+1$

0

-1

$R[W_{mic}] = +2$

Somit global symmetrisch $U(1)_m$

φ

m

χ

$U(1)_R$

$+1$

0

-1

$R[W_{mic}] = +2$

Somewhat similar $U(1)_n$

$$+ 1$$

$U(1)_R$

$$+ 1$$

$$0$$

$$- 1$$

$$R[W_{mic}] = +2$$

ohne global symmetrisch $U(1)_m$

$$\varphi \quad +1$$

$$m \quad -2$$

χ

$U(1)_R$

$$+1$$

$$0$$

$$-1$$

$$R[W_{mic}] = +2$$

Some global symmetry $U(1)_G$

$$\phi \quad +1$$

$$m \quad -2$$

λ

$U(1)_R$

$$+1$$

$$0$$

$$-1$$

$$R[W_{mic}] = +2$$

$$G[W_{mic}] = 0$$

some global symmetry $U(1)_G$

$$\phi \quad +1$$

$$m \quad -2$$

λ

$U(1)_R$

$$+1$$

$$0$$

$$-1$$

$$R[W_{mic}] = +2$$

$$G[W_{mic}] = 0$$

some global symmetry $U(1)_B$

p

$+1$

m

-2

$U(1)_R$

$+1$

0

-1

$R[W_{mic}] = +2$

$G[W_{mic}] = 0$

some global symmetry $U(1)_G$

$$q \quad +1$$

$$m \quad -2$$

$$\lambda \quad -3$$

$U(1)_R$

$$+1$$

$$0$$

$$-1$$

$$R[W_{mic}] = +2$$

$$G[W_{mic}] = 0$$

Some global symmetry $U(1)_G$

$$\phi \quad +1$$

$$m \quad -2$$

$$\lambda \quad -3$$

$U(1)_R$

$$+1$$

$$0$$

$$-1$$

$$R[W_{mic}] = +2$$

$$G[W_{mic}] = 0$$

some global symmetry $U(1)_G$

$$q \quad +1$$

$$m \quad -2$$

$$\lambda \quad -3$$

$U(1)_R$

$$+1$$

$$0$$

$$-1$$

$$R[W_{mic}] = +2$$

$$G[W_{mic}] = 0$$

$U(1)_G \times U(1)_E$ survive Quantum correction.



$U(1)_C \times U(1)_E$ survive Quantum correction.

Wee (0s)



$U(1)_G \times U(1)_E$ survive Quantum correction.

$$W_{eff}(\phi \rightarrow 0) = W_{holonomy}$$

$\Rightarrow U(1)_G \times U(1)_E$ survive Quantum correction.
(no anomalies)

$W_{eff} (as \lambda \rightarrow 0) = W_{holomorphic}$

\Rightarrow limits of vanishing couplings -

$\Rightarrow U(1)_G \times U(1)_E$ survive Quantum correction.
(no anomalies)

$$W_{eff} (g_s \rightarrow 0) = W_{holonomy}$$

\Rightarrow limits of vanishing couplings are smooth

$$\langle x \rangle = x$$

\Rightarrow Construct most general monomials from \mathcal{P}, m, d invariant under all global symmetries,

$$\langle x \rangle = 1$$

⇒ Construct most general monomials from φ, m, d invariant under all global symmetries,

$$\varphi^{n_1} m^{n_2} d^{n_3}$$

$$\langle x \rangle \Rightarrow \lambda$$

on value of λ

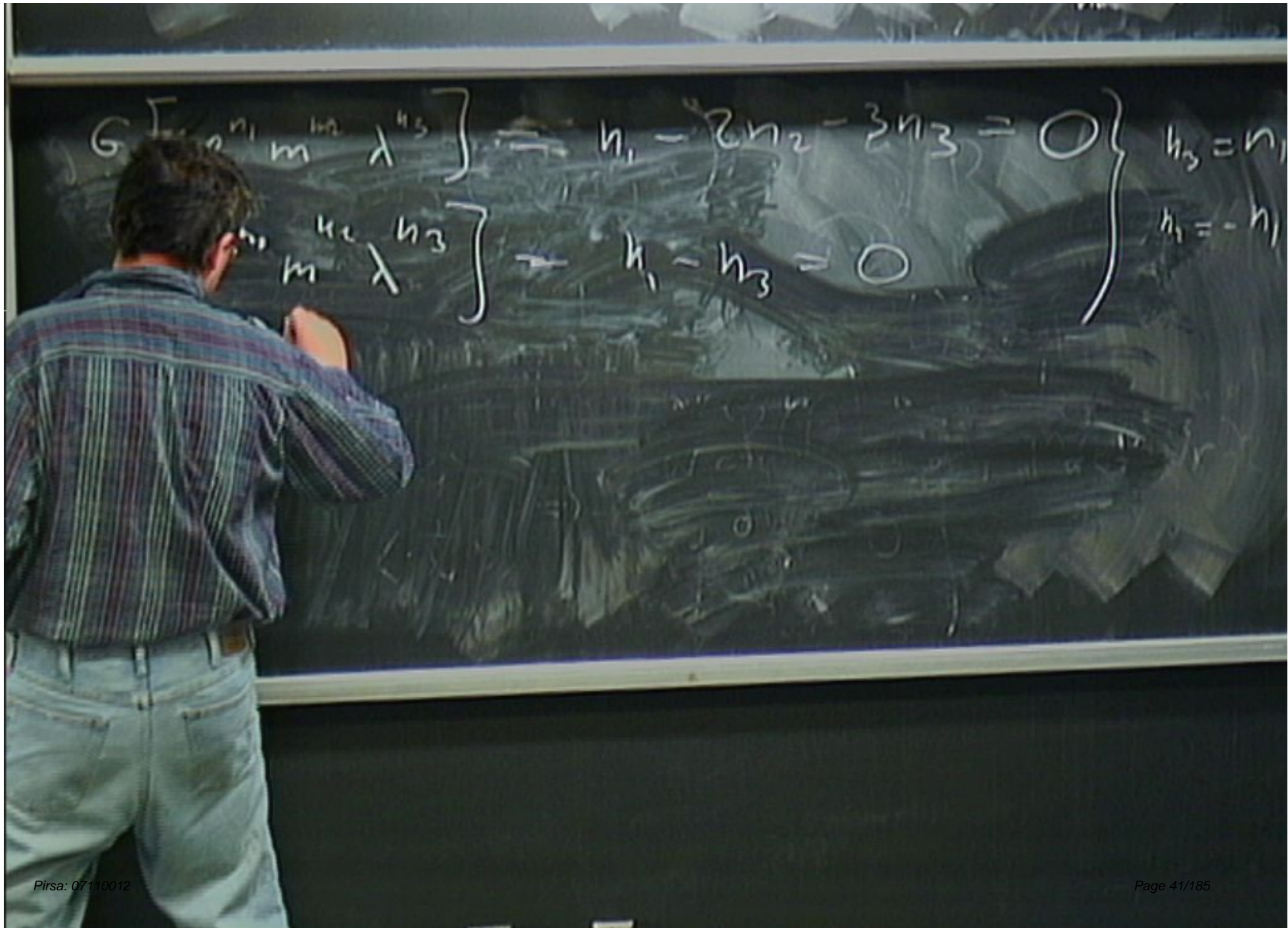
\Rightarrow Construct most general monomials from φ, m, λ invariant under all global symmetries,

$$R[\varphi^{n_1} m^{n_2} \lambda^{n_3}] = 0$$

$$G[\varphi^{n_1} m^{n_2} \lambda^{n_3}] = 0$$

$$G \left[\varphi^{n_1} m^{n_2} \lambda^{n_3} \right] = n_1 - 2n_2 - 3n_3 = 0$$

$$R \left[\varphi^{n_1} m^{n_2} \lambda^{n_3} \right] = n_1 - n_3 = 0$$



$$\left. \begin{aligned} G \left[\begin{array}{ccc} n_1 & n_2 & n_3 \\ m & m & \lambda \end{array} \right] &= n_1 - 2n_2 - 3n_3 = 0 \\ \left[\begin{array}{ccc} n_1 & n_2 & n_3 \\ m & m & \lambda \end{array} \right] &= n_1 - n_3 = 0 \end{aligned} \right\} \begin{aligned} h_3 &= n_1 \\ h_1 &= -n_1 \end{aligned}$$

$$G \left[\begin{array}{c} \varphi^{n_1} \\ m^{n_2} \\ \lambda^{n_3} \end{array} \right] \rightarrow \left. \begin{array}{l} n_1 - 2n_2 - 3n_3 = 0 \\ n_1 - n_3 = 0 \end{array} \right\} \dots$$

$$R \left[\begin{array}{c} \varphi^{n_1} \\ m^{n_2} \\ \lambda^{n_3} \end{array} \right] \rightarrow n_1 - n_3 = 0$$

$$\left(\frac{\lambda P}{m} \right)^{n_1}$$



$$\left. \begin{aligned}
 G \left[\begin{matrix} q^{n_1} & m^{n_2} & \lambda^{n_3} \end{matrix} \right] \rightarrow n_1 - 2n_2 - 3n_3 = 0 \\
 R \left[\begin{matrix} q^{n_1} & m^{n_2} & \lambda^{n_3} \end{matrix} \right] \rightarrow n_1 - n_3 = 0
 \end{aligned} \right\} \begin{aligned}
 n_3 = n_1 \\
 n_2 = -n_1
 \end{aligned}$$

$$\left(\frac{\lambda P}{m} \right)^{n_1} \text{ where } n_1 \text{ is any}$$

$$W_{eff} = m \omega^2 \int \left(\frac{\partial \phi}{\partial m} \right)$$

$$W_{\text{eff}} = m \omega^2 f\left(\frac{q}{m}\right)$$

an arbitrary function

$$W(f) = m \varphi^2 \quad f\left(\frac{\varphi}{m}\right)$$

$f(z)$ is that it is a holomorphic function,
an arbitrary function

$$W(f) = m \rho^2 \quad f\left(\frac{\rho}{m}\right)$$

an arbitrary function

$f(z)$ is that it is a holomorphic function.

$f = \text{rank}$

Some global symmetry $U(1)_G$

$U(1)_R$

$R[\] = +2$

q + 1

+ 1

m - 2

0

λ - 3

- 1



\Rightarrow poles

\mathcal{L}_2

\Rightarrow branch points

\Rightarrow

\Rightarrow poles

\mathcal{L}_2

\Rightarrow branch points

\Rightarrow essential singularity.

⇒ poles

L_2

⇒ branch points

⇒ essential singularity.

pole
a

↓
b

pole

$1 - \cos^2$

→ poles

→ branch points

→ essential singularity.

\mathbb{C}

pole
a

↓
limit

pole

$1 - \cos^2$

⇒ poles

⇒ branch points

⇒ essential singularity.

\mathbb{C}

pole
a

↓
limit

pole
a

Assumption

⇒ poles

⇒ branch points

⇒ essential singularity.

z

pole

↓
limit

pole

Assumption

$f(z)$ is an analytic function around $z=0$

⇒ poles

⇒ branch points

⇒ essential singularity.

Lz $f(z)$

pole

$z=0$

↓ limit

pole

Assumption

$f(z)$ is an analytic function around $z=0$

$$G \left[\begin{array}{ccc} p^{n_1} & m & \lambda^{n_3} \\ \dots & \dots & \dots \end{array} \right] \rightarrow \left. \begin{array}{l} h_1 - 2h_2 - 3h_3 = 0 \\ \dots \\ \dots \end{array} \right\} \begin{array}{l} h_3 = n_1 \\ h_1 = -h_1 \end{array}$$

$$R \left[\begin{array}{ccc} p^{n_1} & m & \lambda^{n_3} \\ \dots & \dots & \dots \end{array} \right] \rightarrow h_1 - h_3 = 0$$

$$\left(\frac{\lambda p}{m} \right)^{n_1}$$

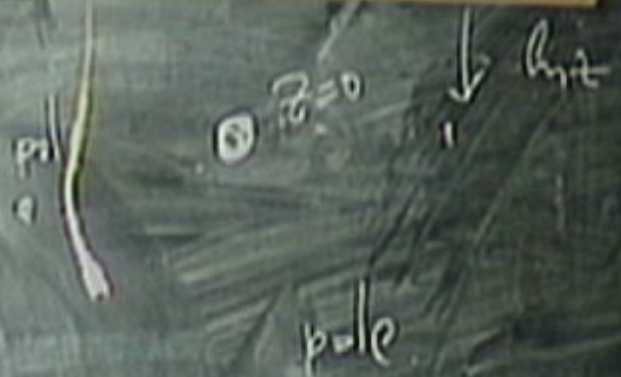
where n_1 is any

⇒ poles

⇒ branch points

⇒ essential singularity.

$\mathcal{L}z$



Assumption

$f(z)$ is an analytic function around $z=0$

$$f(z) = \sum_{n=0}^{\infty} f_n z^n$$

- ⇒ poles
- ⇒ branch points
- ⇒ essential singularity.

Assumption

$f(z)$

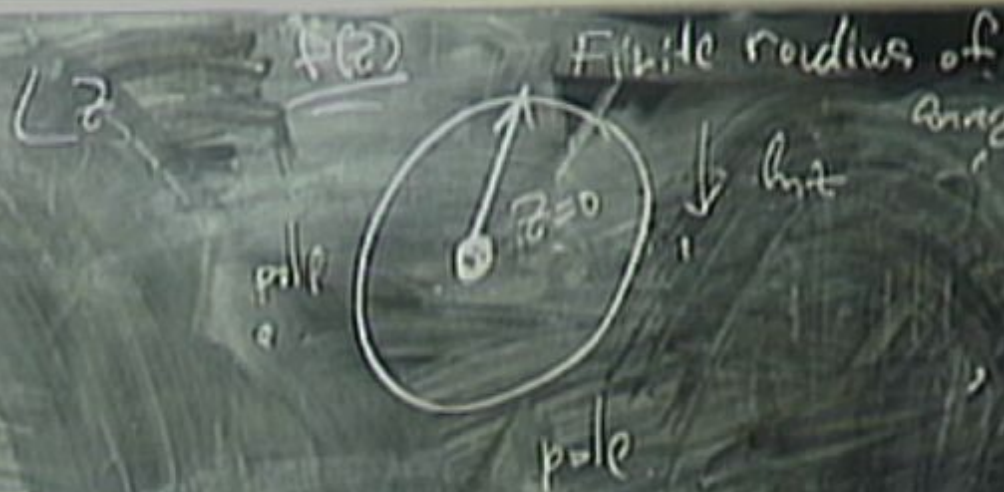
$$f(z) = \sum_{n=-\infty}^{\infty} f_n z^n$$

... around $z=0$

⇒ poles

⇒ branch points

⇒ essential singularity.



Assumption

$f(z)$ is an analytic function around $z=0$

$$f(z) = \sum_{n=0}^{\infty} f_n z^n \Rightarrow$$

$$W_{\text{eff}} = m_1 q^2$$

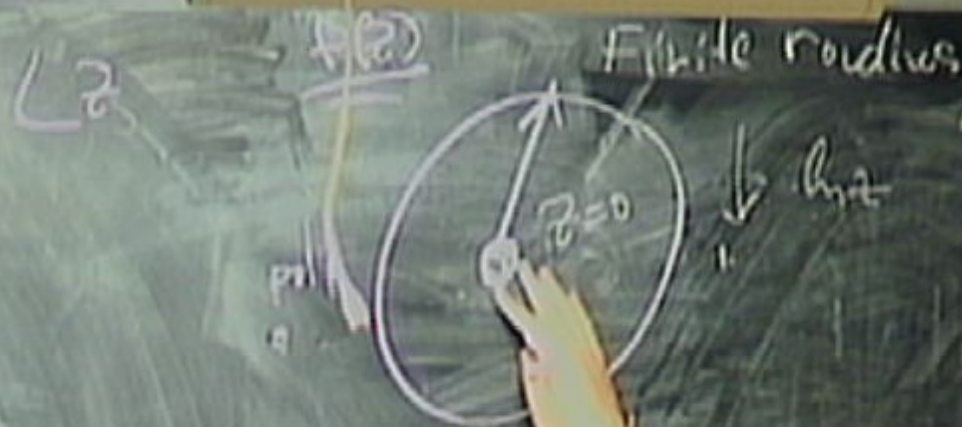
$$W_{\text{eff}} = m_1 \dot{\phi}^2$$

$\phi = \phi_0 \cos(\omega t)$
 $\dot{\phi} = -\phi_0 \omega \sin(\omega t)$
 $\dot{\phi}^2 = \phi_0^2 \omega^2 \sin^2(\omega t)$
 $W_{\text{eff}} = m_1 \phi_0^2 \omega^2 \sin^2(\omega t)$

$$W_{\text{eff}} = m \varphi^2 \sum_{h=0}^{\infty} \frac{1}{h!} \left(\frac{d}{dx} \right)^h \varphi^h$$

$$= \sum_{h=0}^{\infty} \frac{1}{h!} \varphi^{n+2} \left(\frac{d}{dx} \right)^h \varphi^h$$

- ⇒ poles
- ⇒ branch points
- ⇒ essential singularity.



Assumption

$f(z)$ is an analytic function

$$f(z) = \sum_{n=0}^{\infty} f_n z^n \Rightarrow$$

$$= \sum_{n=0}^{\infty} f_n z^n$$

⇒ poles

⇒ branch points

⇒ essential singularity.

$\mathbb{C}z$



Assumption

$f(z)$ is an analytic function around $z=$

$$f(z) = \sum_{n=0}^{\infty} f_n z^n \Rightarrow$$

Smaller assumption

$$= \sum_{l=0}^{\infty} f_n \rho^{nl} \lambda^n m$$

⇒ poles

⇒ branch points

⇒ essential singularity.

\mathbb{C}^2



Assumption

$f(z)$ is an analytic function around $z=$

$$f(z) = \sum_{n=0}^{\infty} f_n z^n \Rightarrow$$

Smaller assumption

$z=0$ is not a singular point.

$$= \sum_{h=0}^{\infty} f_n \rho^{n+2} \lambda^n m^{1-n}$$

Assumption

$f(z)$ is an analytic function around z_0 pole.

$$f(z) = \sum_{n=-\infty}^{\infty} f_n z^n \Rightarrow$$

Smaller assumption

$z_0 = 0$ is not a singular point

$$= \sum_{n=0}^{\infty} f_n z^{n+2} \lambda^n \mu^{1-n}$$

Smaller assumption

$\varphi=0$ is not a singular point

$$W_{\text{eff}} = m \varphi^2 \int \mathcal{L}_n \varphi^{\frac{n}{2}} d^4x m^{-n}$$
$$= \int \mathcal{L}_n \varphi^{n+2} d^4x m^{-n}$$

Smaller assumption

$\varphi=0$ is not a singular point

$$W_{\text{eff}} = m \varphi^2 \sum_{-\infty}^{\infty} f_n \varphi^{2n} \frac{1}{m^{-n}}$$

f_n are some constants

$$= \sum_{-\infty}^{\infty} f_n \varphi^{2n+2} \frac{1}{m^{-n}}$$

Smaller assumption

$q=0$ is not a singular point

$$W_{\text{eff}} = m q^2 \sum_{-\infty}^{\infty} f_n p^n d^n m^{-n}$$

f_n are some constants

$$= \sum_{-\infty}^{\infty} f_n p^{n+2} \lambda^n m^{k-n}$$

$\Rightarrow \lambda \rightarrow 0, m \rightarrow 0$ smooth

$\rightarrow 0$, m kept fixed

$\lambda \rightarrow 0$, m kept fixed

5

0

$\lambda \rightarrow 0$, m kept fixed

$\lambda \ll \tau$

λ

τ

$\lambda \rightarrow 0$, m kept fixed

$$\chi^2 \Rightarrow \boxed{n \gg 0}$$

$\lambda \rightarrow 0$, m kept fixed

$$\lambda^n \Rightarrow \boxed{n \geq 0}$$

$m \rightarrow 0$, λ kept fixed

$\lambda \rightarrow 0$, m kept fixed

$$\lambda^n \Rightarrow \boxed{n \geq 0}$$

$m \rightarrow 0$, λ kept fixed

$$m \sim n$$

$\lambda \rightarrow 0$, m kept fixed

$$\lambda^n \Rightarrow \boxed{n \geq 0}$$

$m \rightarrow 0$, λ kept fixed

$$m \rightarrow n$$

$$\Rightarrow p - n \geq 0$$

$$\boxed{n \leq 1}$$

χ^2 ∞
 f_n p_n d_n m_n
 f_n are some const

f_n p_n d_n m_n $=$

$m \rightarrow 0$ smooth

$$N_{\text{eff}} = m \rho^2 \sum_{n=0}^{\infty} f_n \rho^{n+2} \lambda^n m^{-n}$$

f_n are some constants

$$= \sum_{n=0}^{\infty} f_n \rho^{n+2} \lambda^n m^{-n}$$

$$\Rightarrow \lambda \rightarrow 0, m \rightarrow 0 \text{ small}$$

only surviving term $n=0$
 $n=1$

$$N_{\text{eff}} = m \rho^2 \sum_{n=0}^{\infty} f_n \rho^{2n} \lambda^n m^{-n}$$

f_n are some constants

$$= \sum_{n=0}^{\infty} f_n \rho^{n+2} \lambda^n m^{-n}$$

$$= f_0 \rho^2 m$$

$\Rightarrow \lambda \rightarrow 0, m \rightarrow 0$ small

only surviving term $n=0$
 $n=1$

$$N_{\text{eff}} = m \varphi^2 \sum_{-\infty}^{\infty} f_n \varphi^{n+2} \lambda^{-n} m^{-n}$$

f_n are some constants

$$= \sum_{-\infty}^{\infty} f_n \varphi^{n+2} \lambda^{-n} m^{-n} \Rightarrow f_0 \varphi^2 m + f_1 \varphi^3 \lambda$$

$\Rightarrow \lambda \rightarrow 0, m \rightarrow 0$ small

only surviving term $n=0$
 $n=1$

If Quantum corrections do not break

$$U(1)_B \times U(1)_R$$

If Quantum corrections do not break
 $U(1)_E \times U(1)_K$ (susy is
unbroken)
+ $\boxed{\varphi=0}$ is a nonsingular point in a field
space.

$$W_{\text{eff}} = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} \lambda \phi^4 \right]$$

the limit $\lambda \rightarrow 0$

$$W_{\text{eff}} = W_{\text{micro}}$$

$$W_{\text{eff}} = f_0 m \phi^2 + f_1 \lambda \phi^3$$

⇒ In the limit $\lambda \rightarrow 0$

$$W_{\text{eff}} = W_{\text{micro}}$$

$$f_0 = \frac{1}{2}$$

$$f_1 = \frac{1}{3}$$

$$\underline{W_{\text{eff}}} = f_0 m \varphi^2 + f_1 \lambda \varphi^3$$

⇒ In the limit $\lambda \rightarrow 0$

$$W_{\text{eff}} = W_{\text{micro}}$$

$$f_0 = \frac{1}{2}$$

$$f_1 = \frac{1}{3}$$

$$W_{\text{micro}} = \mu_1 \rho^2 + \mu_2 \rho^2 + \dots + \mu_n \rho^n$$

$$W_{\text{micro}} = \mu_1 q^2 + \mu_2 q^2 + \dots + \mu_n q^n$$

	$U_e(n)$	$U_k(n)$
q_1	$+1$	$+1$
μ_n	$-n$	$2-n$

$$W_{\text{micro}} = \mu_1 q_1^2 + \mu_2 q_2^2 + \dots + \mu_n q_n^2$$

	$U_1(l_1)$	$U_2(l_2)$	
q_1	$+1$	$+1$	
M_n	$-n$	$2-n$	

$$W_{\text{eff}} \equiv W_{\text{micro}}$$

$$W_{\text{micro}} = \mu_1 q_1^2 + \mu_2 q_2^2 + \dots + \mu_n q_n^2$$

	$U_a(l)$	$U_b(l)$
q_1	$+1$	$+1$
M_h	$-h$	$2-h$

$w_{\text{eff}} \equiv W$

smaller assumption

$\varphi=0$ is not a singular point

W_{eff}

$$= m \varphi^2 \sum_{-\infty}^{\infty} f_n \varphi^{n+2} \lambda^{\frac{n+2}{m}}$$

f_n are some constants

$$= \sum_{-\infty}^{\infty} f_n \varphi^{n+2} \lambda^{\frac{n+2}{m}}$$

$$= f_0 \varphi^2 m + f_1 \varphi^3$$

only surviving term $n=0$
 $n \geq 1$

$\Rightarrow \lambda \rightarrow 0, m \rightarrow 0$ smooth

$W \rightarrow$ know exactly of SUSY QFT.

⇒ we know exactly the quantum vacuum of He theory

⇒ H to study Fluctuation?

$W \rightarrow$ know exactly \mathcal{R} SUSY QFT.

\Rightarrow we know exactly the quantum vacuum of the theory

\Rightarrow How to study Fluctuation?

\Downarrow need to know kinetic terms.

spac

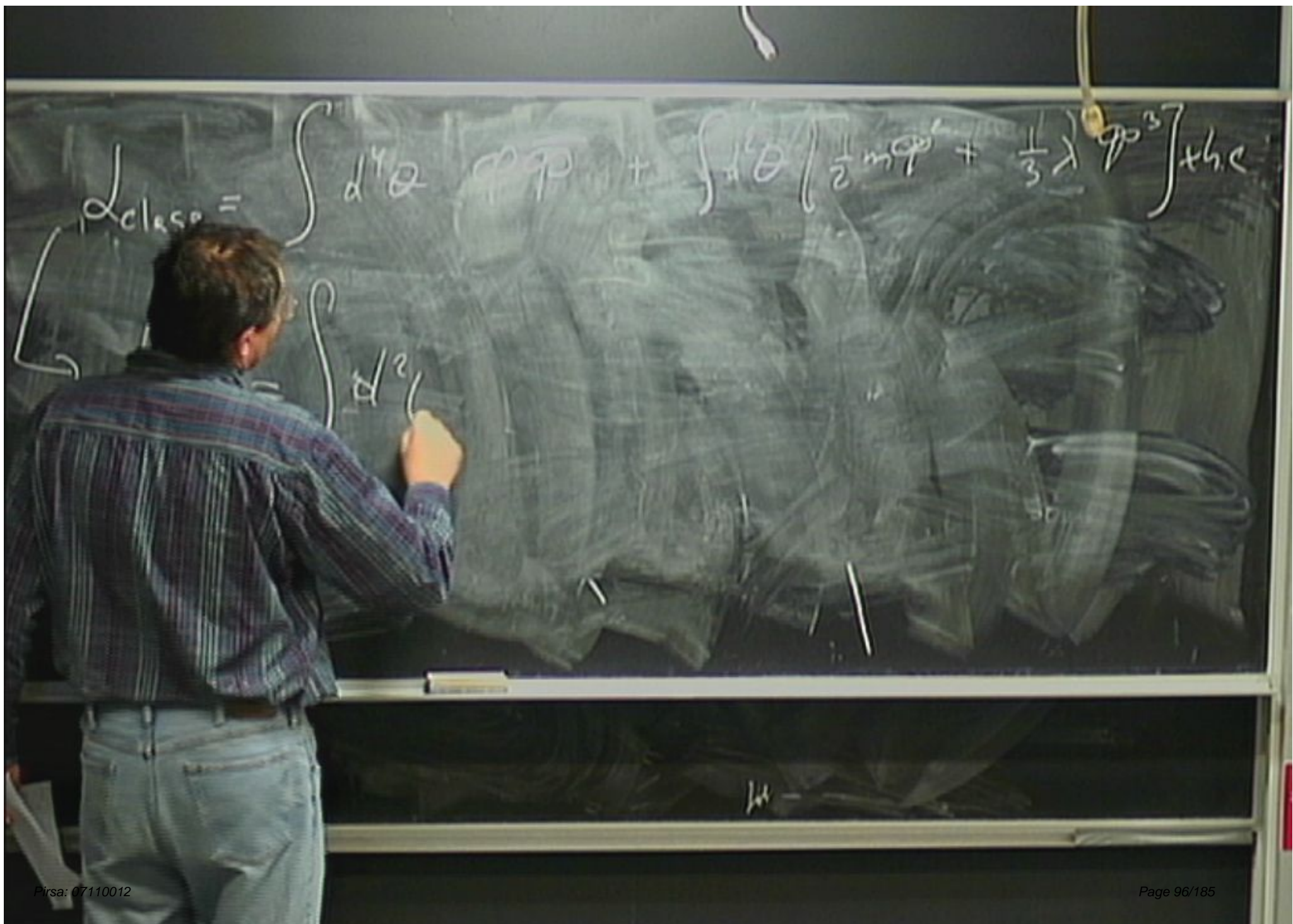
⇒ How to study Fluctuation?

⇓ need to know kinetic forms.

$K(q, \bar{q}) \rightarrow$ can not rely on holomorphicity

spac

$$\mathcal{Z}_{\text{clRSP}} = \int d^4\theta \quad \varphi \varphi$$



$$L_{classical} = \int d^4\theta [\dots] + \int d^4\theta \left[\frac{1}{2}m\phi^2 + \frac{1}{3}\lambda\phi^3 \right] + h.c.$$

$$\int d^2\theta$$

$$\mathcal{L}_{\text{class}} = \int d^4\theta \varphi \bar{\varphi} + \int d^4\theta \left[\frac{1}{2} m \varphi^2 + \frac{1}{3} \lambda \varphi^3 \right] + \text{h.c.}$$

$$\mathcal{L}_{\text{eff}} = \int d^4\theta \varphi \bar{\varphi} Z(m, \lambda, \Lambda) + \int d^4\theta \left[\frac{1}{2} m \varphi^2 + \frac{1}{3} \lambda \varphi^3 \right] + \text{h.c.}$$

wave-function
 renormalization

$$\mathcal{L}_{\text{class}} = \int d^4\theta \quad \varphi\varphi + \int d^4\theta \left[\frac{1}{2} m \varphi^2 + \frac{1}{3} \lambda \varphi^3 \right] + h.c.$$

$$\mathcal{L}_{\text{eff}} = \int d^4\theta \quad \varphi\varphi \sum_{(n, \bar{n}, \lambda, \bar{\lambda})} \left(\frac{1}{2} m \varphi^2 + \frac{1}{3} \lambda \varphi^3 \right) + h.c.$$

wave-functions
 renormalization

wave function
normalization

$$\int_{-\infty}^{\infty} \psi^* \psi dx = 1$$
$$\int_{-\infty}^{\infty} \psi^* \psi dx = 1$$

wave-function
renormalization

50

$$\hat{\phi} = \psi z^{1/2}$$
$$\hat{\phi} = \bar{\psi} z^{1/2}$$

$$K_{off} \hat{\phi} \hat{\phi}$$

wave-lengths
vibrations

$$\hat{\phi} = \frac{1}{2} \phi$$
$$\hat{\phi} = \frac{1}{2} \phi$$

$$k_{\text{eff}} = \frac{1}{2} k$$

$$W_{\text{eff}} = \frac{1}{2} W$$

wave function
renormalized

50

$$\hat{\phi} = \phi z^{1/2}$$
$$\hat{\phi} = \bar{\phi} z^{1/2}$$

$$K_{eff} \hat{\phi} \hat{\phi}$$

$$W_{eff} = \frac{1}{2} \frac{m \hat{\phi}^2}{z} + \frac{\lambda}{3 z^{3/2}} \hat{\phi}^3$$

\Rightarrow certain ratios of coupling can be protected
against Quantum corrections in k

\Rightarrow certain ratios of coupling can be protected
against Quantum corrections in k

$$W_{\text{eff}} = f_0 \phi^2 + f_1 \phi^3$$

\Rightarrow certain ratios of coupling can be protected against Quantum corrections in k

$$W_{\text{eff}} = f_0 \phi^2 + f_1 \phi^3$$

$\frac{d}{d\ln\mu} = 0$

⇒ certain ratios of coupling can be protected against Quantum corrections in k

$$W_{\text{eff}} = f_0 \phi^2 + f_1 \phi^3$$

$$\frac{f_1}{f_0} = \frac{1}{2} m^2 + \frac{1}{3} \lambda \mu$$

classically

⇒ certain ratios of coupling can be protected against Quantum corrections in k

$$W_{eff} = f_0 \phi^2 + f_1 \phi^3$$

$$\stackrel{\substack{\Rightarrow \\ \frac{1}{2} m \dot{\phi}^2}}{\text{classically}} = \frac{\left(\frac{1}{2} m\right)^3}{\left(\frac{1}{3} \lambda\right)^2} \stackrel{\substack{\Rightarrow \\ \lambda m}}{\text{classically}}$$

$$\frac{1}{2} m \dot{\phi}^2 + \frac{1}{3} \lambda \phi^3$$

Perturbative computation of ζ

Perturbative computation of Z

→ There is Yukawa interaction

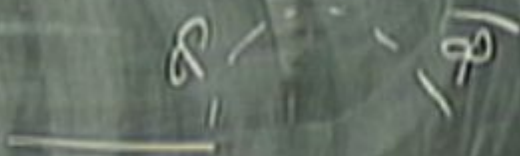
$$\lambda \psi^2 \psi \phi$$

Perturbative computation of Z

→ There is Yukawa interaction

$$\lambda \psi^2 \psi \phi$$

↑ cubic coupling.



classically
Perturbative computation of Z

→ There is Yukawa interaction

$\lambda \psi^2 \psi \phi$
↑
cubic coupling.



Perturbative computation of Z

→ There is Yukawa interaction

$\lambda \psi^2 \psi \phi$
↑
cubic coupling.



1&M (3 2/2)

2 2

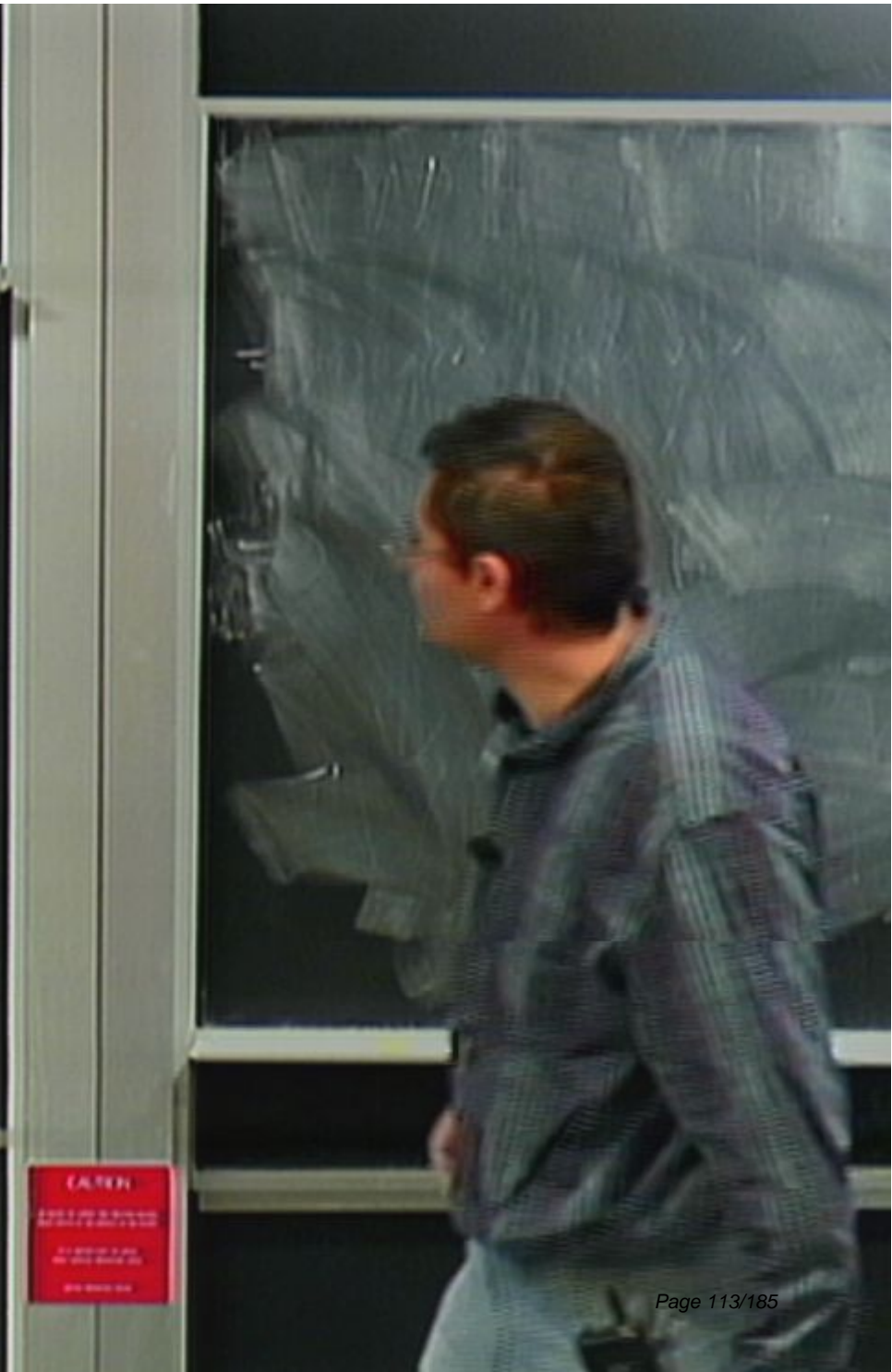
Change

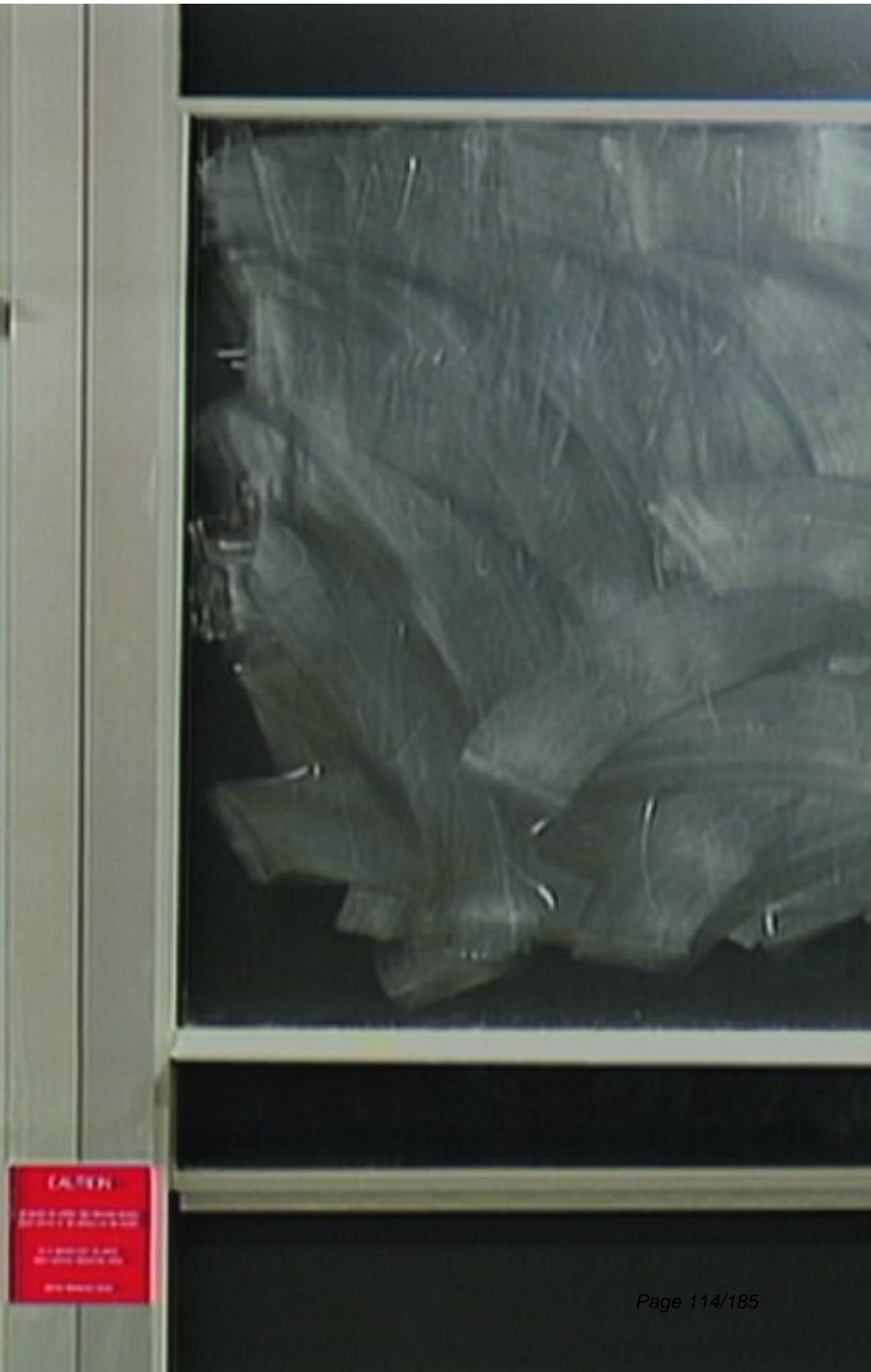
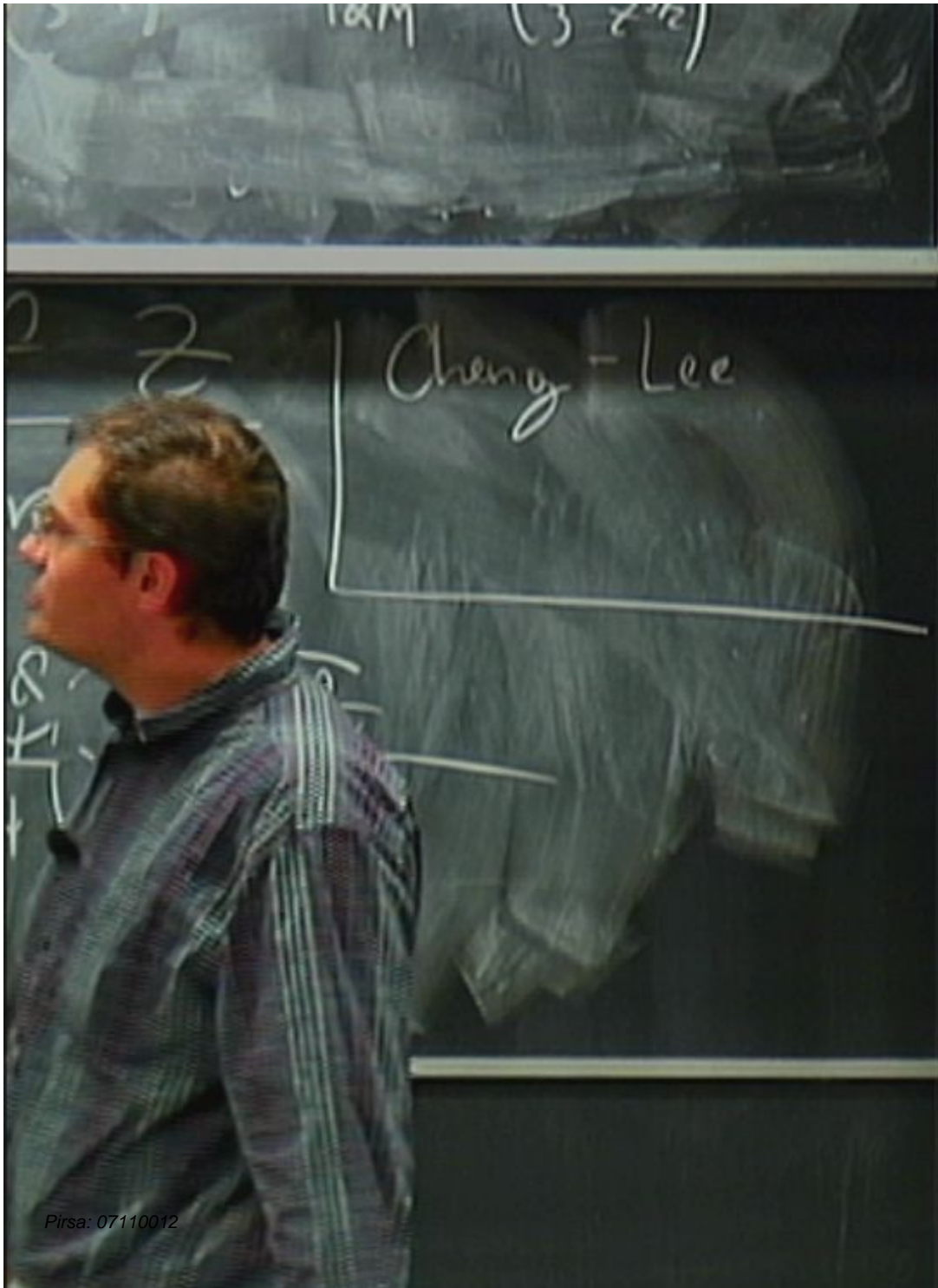
rotation

180°

180°

180°





10/19 (3-2-12)

2
Lion

Cheng - Lee
(Section 2)
 $m \mathbb{Z}^2 + \lambda \mathbb{T}^2$

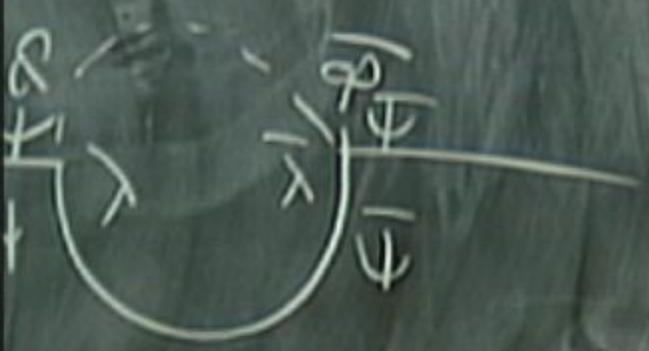
\mathbb{R}^2
 \mathbb{Z}^2
 \mathbb{T}^2

CAUTION
DO NOT OPEN OR MOVE THIS
DOOR WITHOUT THE
APPROPRIATE KEY
OR AUTHORITY

10/11 (3-2/12)

2 2
vrotation

Cheng - Lee
(Section 2)
 $mT^2 + \lambda T^4$



[The chalkboard on the right is mostly obscured by a large, dark, textured object, possibly a piece of fabric or a large sheet of paper, which is draped over the board. Only some faint lines and shapes are visible through the fabric.]

CAUTION
DO NOT TOUCH THE BOARD
OR THE BOARD IS HOT

$$z^{i/2} =$$

=

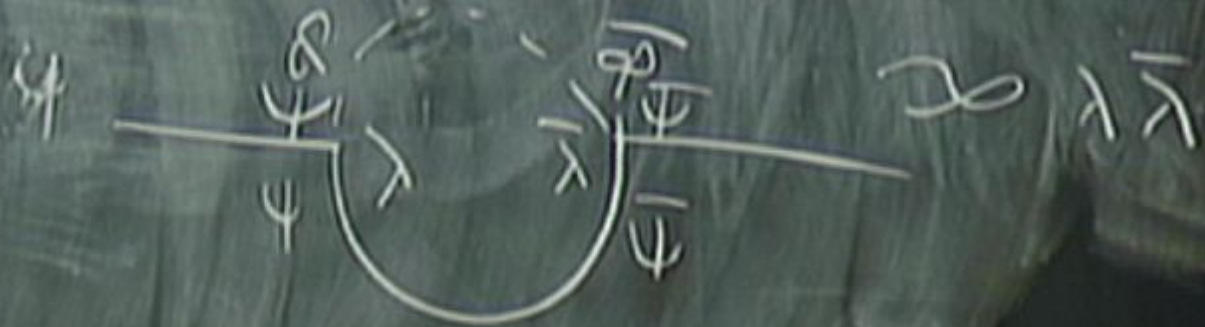
perturbative computation of Z

Cheng-Lee
(Section 2)
 $m^2 \psi^2 + \lambda \psi^4$

There is Yukawa interaction

$$\lambda \psi^2 \psi \phi$$

↑
cubic coupling.

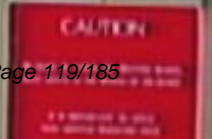
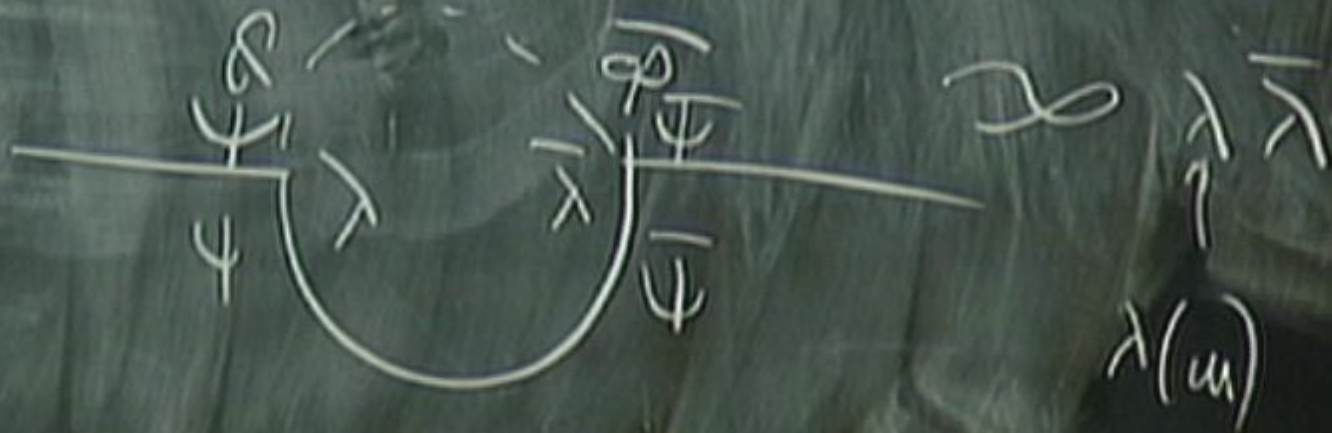


Representation of Z

Cheng-Lee
(Section 2)
 $m \tau^2 + \lambda \tau^4$

Kaon interaction

ϕ
 ϕ



$$z^{1/2} = 1 + \lambda^{(n)}$$

$$\mathbb{Z}^{1/2} = \mathbb{Z} \oplus \mathbb{Z}$$

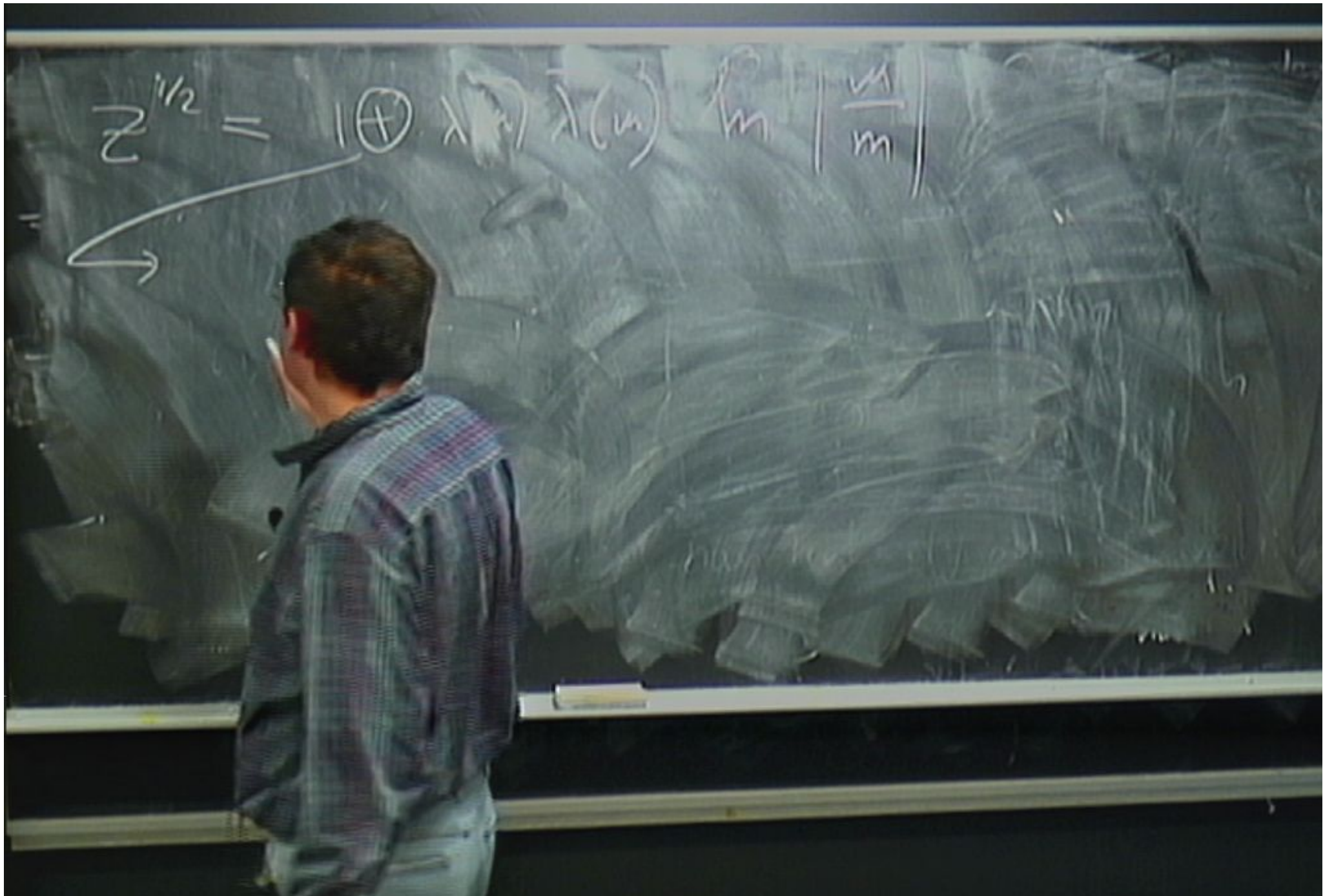
$$\oplus$$

$$\mathbb{Z} \oplus \mathbb{Z}$$

$$\mathbb{Z}$$

$$\mathbb{Z}$$





$$Z^{1/2} = \oplus$$



$$\left(\begin{array}{c} \sqrt{m} \\ \sqrt{m} \end{array} \right)$$

$$Z^{1/2} = \bigoplus_{\lambda \in \lambda(u)} \begin{matrix} m \\ \lambda \\ m \end{matrix}$$

Our theory is an effective FT

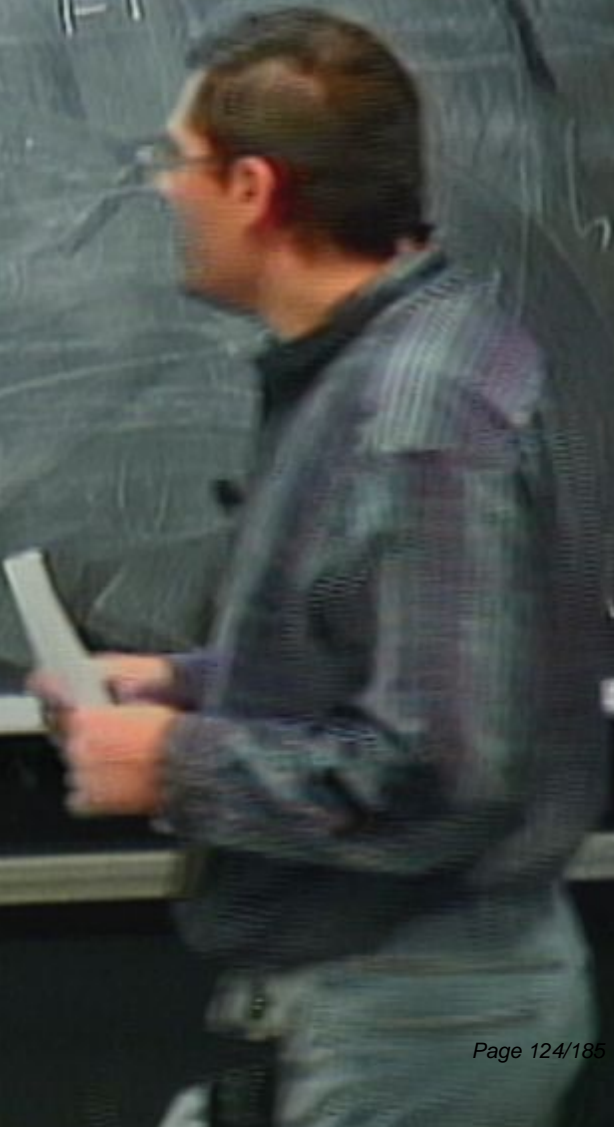
$$E \ll m$$

$$Z^{1/2} = \bigoplus_{\lambda(m)} \lambda(m) \left| \frac{m}{m} \right|$$

Our theory is an effective FT

$$E \ll |m|$$

$$m \rightarrow 0$$



$$Z^{1/2} = \left(\oplus_{\lambda(m)} \lambda(m) \right) \left(\frac{1}{m} \right) \left(\frac{1}{m} \right)$$

Our theory is an effective FT

$$E \ll |m|$$

$m \rightarrow 0$ (m is kept fixed)

$$Z^{1/2} = \left| \oplus \lambda(\mu) \lambda(\mu) \left| m \right| \frac{\mu}{m} \right|$$

theory is an effective FT

$$E \ll |m|$$

(μ is kept fixed)

$\rightarrow 0$ (equivalent to low energy approximation)

$$Z^{1/2} = \left| \oplus \lambda(u) \bar{\lambda}(u) \right|_{m} \left| \frac{u}{m} \right|$$

Our theory is an effective FT

$$E \ll |m|$$

$$m \rightarrow 0 \quad (\mu \text{ is kept fixed})$$

$$\Downarrow \quad E \rightarrow 0 \quad (\text{equivalent to low energy approx})$$

fermion $n=0$
 $n=1$

The physical field is ψ plus

term $n=0$
 $n=1$

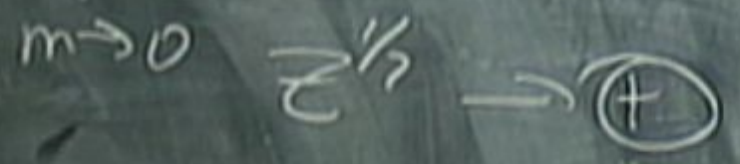
The physical field is of

\rightarrow as $F \rightarrow 0$ theory becomes weakly coupled
in term of

term $n=0$
 $n=1$

The physical field is $\hat{\phi}$

\rightarrow as $F \rightarrow 0$ theory becomes weakly coupled
in term of $\hat{\phi}$



term $n=0$
 $n=1$

The physical field is $\hat{\phi}$

\rightarrow as $F \rightarrow 0$ theory becomes weakly coupled
in term of $\hat{\phi}$

$$m \rightarrow 0 \quad \frac{1}{2} \rightarrow \oplus \|\vec{\lambda}\|^2 \ln\left(\frac{\mu}{0}\right)$$

$$\hat{\phi} = \alpha z^{1/2}$$

$$\hat{\phi} = \bar{\alpha} z^{1/2}$$

$$K_{\text{eff}} = \hat{\phi} \frac{1}{\bar{\phi}}$$

$$W_{\text{eff}} = \frac{1}{2} \frac{m \hat{\phi}^2}{z} + \frac{\lambda}{3} \hat{\phi}^3$$

$$\hat{\phi} = \alpha z^{1/2}$$

$$\hat{\phi} = \bar{\alpha} z^{1/2}$$

$$K_{\text{eff}} = \hat{\phi} \frac{1}{\bar{\phi}}$$

$$W_{\text{eff}} = \frac{1}{2} \frac{m \hat{\phi}^2}{z} + \frac{1}{3} \frac{\hat{\phi}^3}{z^{3/2}}$$



The physical field is ϕ

\rightarrow as $F \rightarrow 0$ theory becomes weakly coupled
in term of $\hat{\phi}$

$$m \rightarrow 0 \quad \frac{1}{2} \rightarrow \oplus \quad |\vec{\lambda}|^2 \ln\left(\frac{\mu}{0}\right) \rightarrow +\infty$$

Wef $\rightarrow 0$

surviving
term $n=0$
 $n=1$

The physical field is $\hat{\phi}$

\rightarrow as $F \rightarrow 0$ theory becomes weakly coupled
in term of $\hat{\phi}$

$$m \rightarrow 0 \quad \frac{1}{2} \rightarrow \oplus \|\vec{\lambda}\|^2 \ln\left(\frac{\mu}{0}\right) \rightarrow +\infty$$

$W_{eff} \rightarrow 0$

$$B^{(n)} = \langle \varphi(x_1), \dots, \varphi(x_n) \rangle$$

$$G^{(n)} = G^{(n)}(A)$$

\Rightarrow prove $\frac{\partial G^{(n)}}{\partial A}$

$$B^{(n)} = \langle \varphi(x_1), \dots, \varphi(x_n) \rangle$$

$$G^{(n)} = G^{(n)}(\lambda)$$

\Rightarrow prove $\frac{\partial G^{(n)}}{\partial \lambda} \Big|_{\lambda=1} = 0$

$$B^{(n)} = \langle \varphi(x_1), \dots, \varphi(x_n) \rangle$$

zsf 15

$$G^{(n)} = G^{(n)}(\lambda)$$

$$\Rightarrow \text{prove } \frac{\partial G^{(n)}}{\partial \lambda} = 0$$

Moduli spaces and their applications in moduli spaces!

Moduli spaces and deformation theory

CASE A (not generic).

Moduli spaces and singularities on moduli spaces!

CASE A (not generic).

$$W = \frac{1}{2} m \varphi^2 + \frac{1}{3} \lambda \varphi^3$$

Moduli spaces and singularities on moduli spaces!

CASE A (not generic).

$$W = \frac{1}{2} m \varphi^2 + \frac{1}{3} \lambda \varphi^3$$

\Rightarrow Find the space of SUSY vacua

If $\vec{W} = W(\eta)$

Then SUSY vacua

If $\vec{W} = W(\varphi_i)$

then SUSY vacua are determined

$$F$$

If $\bar{W} = W(\phi_i)$

Then SUSY vacua are determined

$$F_i = \frac{\partial W}{\partial \phi_i} = 0, \text{ for any } i$$

$\frac{dL}{dC}$

$$\frac{dL}{Mc} = 0 \Rightarrow m\dot{\rho} + \lambda \dot{\rho}^2$$

$$\frac{dL}{dR} = \frac{M}{C} = 0 \Rightarrow mR + \lambda R^2 \Rightarrow \mathcal{L} = \left\{ 0, \frac{1}{5} \right\}$$

$$\frac{\partial W}{\partial \rho} = m\rho + \lambda \rho^2 \Rightarrow \rho = \left\{ 0, \frac{-1}{5\lambda} \right\}$$

⇒ discrete values

$$\frac{\partial W}{\partial \varphi} = m\varphi + \lambda\varphi^3 \Rightarrow \varphi = \left\{ 0, \pm \sqrt{\frac{\lambda}{m}} \right\}$$

\Rightarrow discrete vacua \rightarrow usually arise from breaking of some discrete symmetry

$$\vec{0} = \frac{\partial W}{\partial \varphi} = m\varphi + \lambda\varphi^2 \Rightarrow \varphi_{\pm} = \left\{ 0, -\frac{\lambda}{m} \right\}$$

\Rightarrow discrete vacuum \rightarrow usually arise from breaking of some discrete symmetry

\exists some hidden \mathbb{Z}_2 symmetry in the model

turn $v=0$

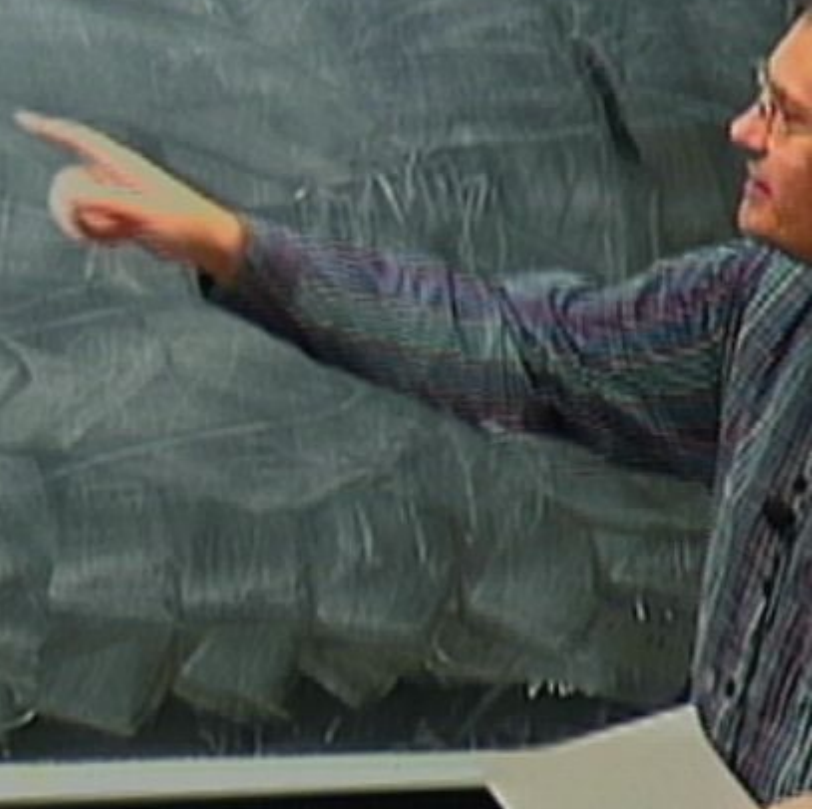
$$\frac{\partial W}{\partial \phi} = m\phi + \lambda\phi^2 \Rightarrow \phi = \left\{ 0, -\frac{\lambda}{m} \right\}$$

discrete vau \rightarrow usually arise from breaking of some discrete symmetry

} some hidden Z_2 symmetry in the model

$$\varphi = \frac{\lambda}{2\pi} + h$$

\uparrow const. \uparrow new xsp.



$$\varphi = \pm \frac{\lambda}{2h} \pm h$$

\uparrow constant \uparrow new xsp.

$$h = \pm \frac{\lambda}{2h}$$

$h \leftrightarrow -h$

$$\varphi = \pm \frac{\lambda}{2m} + h$$

↑
const.

↑
a new
exp.

$$h = \pm \frac{\lambda}{2m}$$

$$h \leftrightarrow -h$$

$$W(h) = \frac{m^3}{12\lambda^2} - \frac{m^2}{4\lambda} h + \frac{1}{3} \lambda h^3$$

$$\varphi = \pm \left[-\frac{\lambda}{2m} + h \right]$$

↑
ground

↑
a new
exp.

$$h_{\pm} = \pm \frac{\lambda}{2m}$$

$$h \leftrightarrow -h$$

$$W(h) = \int d^2\theta W$$

$$= \frac{m^3}{12\lambda} - \frac{m^2}{4\lambda} h + \frac{1}{3} \lambda h^3 \sim -\frac{m^2}{4\lambda} h^2 + \frac{1}{3} \lambda h^3$$

$$\varphi = \frac{1}{2m} \left(-\frac{\lambda}{2m} + h \right)$$

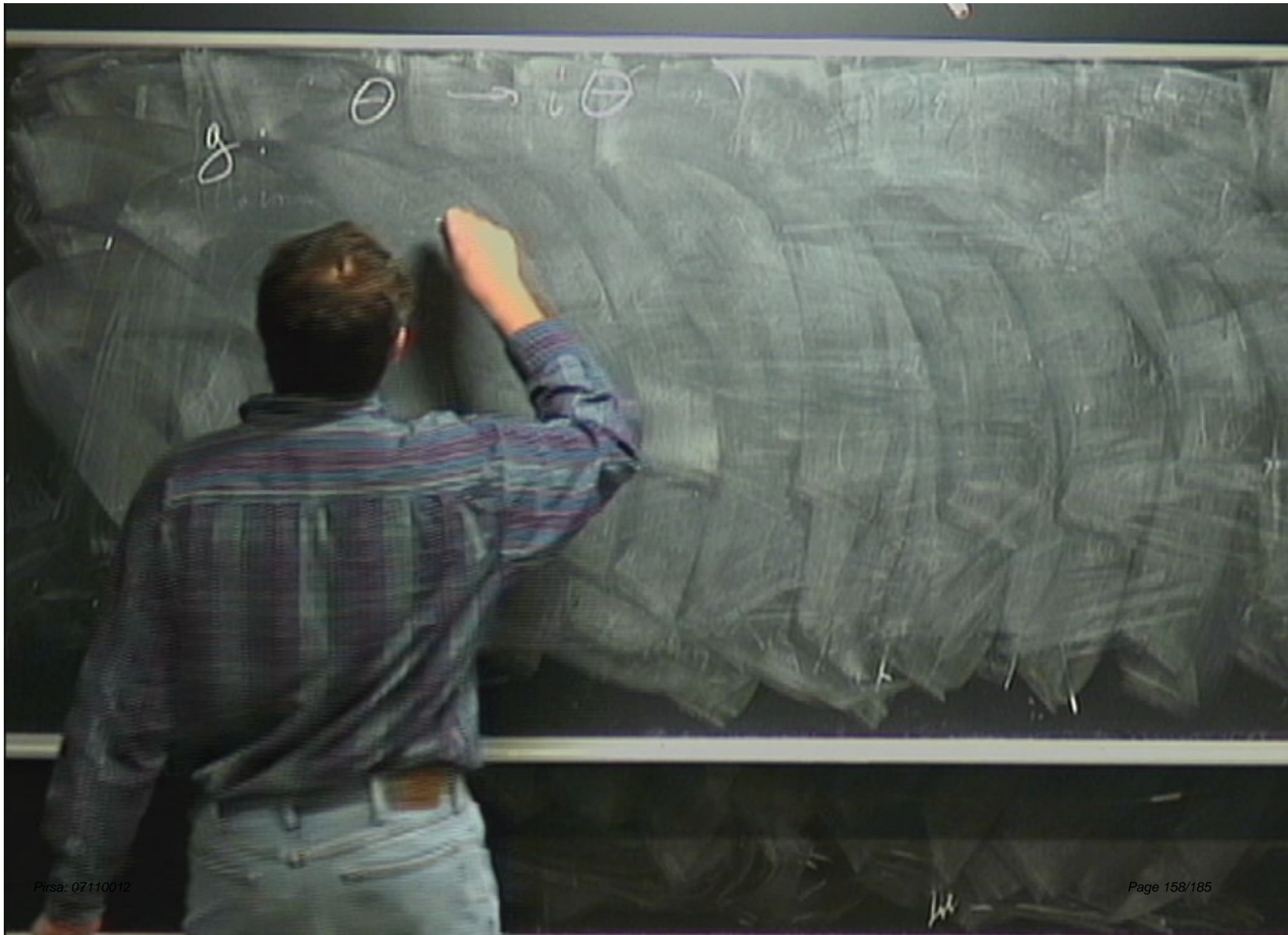
\uparrow \uparrow
 const \uparrow \uparrow
 a new \uparrow
 xpr.

$$h_{\pm} = \pm \frac{\lambda}{2m}$$

$$h \leftrightarrow -h$$

$$W(h) = \frac{m^2}{4\lambda} h + \frac{1}{3} \lambda h^3 \sim -\frac{m^2}{4\lambda} h + \frac{1}{3} \lambda h^3$$

$\int d^2\theta W$



$$g: \begin{matrix} \theta \rightarrow i\theta \\ h \rightarrow -h \end{matrix} \quad \Leftrightarrow \quad \begin{matrix} \theta \rightarrow -i\theta \\ h \rightarrow -h \end{matrix}$$

g
 \uparrow
 a generator
 of the
 discrete
 symmetry

$$w(h) =$$

$$g: \begin{array}{l} \theta \rightarrow -\theta \\ h \rightarrow -h \end{array} \Leftrightarrow \begin{array}{l} d^2\theta \rightarrow -d^2\theta \\ W(h) \rightarrow -W(h) \end{array}$$

a generator
of the
discrete
symmetry

$$\int d^2\theta W(h) \rightarrow \int d^2\theta W(h)$$

$$h \rightarrow -h$$

a generator
of the
discrete

symmetry why \mathbb{Z}_2

$$W(h) \rightarrow -W(h)$$

$$\int d^2\theta W(h) \rightarrow \int d^2\theta W(h)$$

and not \mathbb{Z}_4 ?

$$g: \begin{matrix} \theta \rightarrow -\theta \\ h \rightarrow -h \end{matrix} \iff \begin{matrix} d^2\theta \rightarrow -d^2\theta \\ W(h) \rightarrow -W(h) \end{matrix}$$

a generator
of the
discrete

$$\int d^2\theta W(h) \rightarrow \int d^2\theta W(h)$$

Symmetry why \mathbb{Z}_2 and not \mathbb{Z}_4 ?

$$g^2: \begin{matrix} \theta \rightarrow -\theta \\ h \rightarrow h \end{matrix}$$

$$g^4: \begin{matrix} \theta \rightarrow \theta \\ h \rightarrow h \end{matrix}$$

Generic case B

$$W = \frac{1}{2} \lambda L H^2 \Rightarrow$$

$$\frac{\partial W}{\partial L} = \frac{1}{2} \lambda H^2 = 0$$

$$\frac{\partial W}{\partial H} = \lambda L H = 0$$

Generic case B

$$W = \frac{1}{2} \lambda L H^2 \quad \rightarrow$$

$$\rightarrow \frac{H=0}{L}$$

$$\frac{\partial W}{\partial L} = \frac{1}{2} \lambda H^2 = 0$$

$$\frac{\partial W}{\partial H} = \lambda L H = 0$$

- Generic case B

$$W = \frac{1}{2} \lambda L H^2 \quad \rightarrow$$

$$\begin{aligned} \langle H \rangle &> 0 \\ \rightarrow \langle L \rangle & - \text{anything} \end{aligned}$$

$$\frac{\partial W}{\partial L} = \frac{1}{2} \lambda H^2 = 0$$

$$\frac{\partial W}{\partial H} = \lambda L H = 0$$

a Δ -complex dimensional space of P_n vacua
 $\Rightarrow \mathbb{C}^1$ moduli space

Δ -complex dimensional space of P_n vacua

$\Rightarrow \mathbb{C}^1$ moduli space

finite discrete vacua where equivalent

... of several manifolds of dimension n
a Δ -complex dimensional space of P_n vacua

$\Rightarrow \mathbb{C}^1$ moduli space

\Rightarrow before discrete vacua were equivalent

Construct a set of several manifolds from
a Δ -complex dimensional space of P_n vacua

$\Rightarrow \mathbb{C}^1$ moduli space

\Rightarrow before discrete vacua were equivalent

\rightarrow has is equivalent vacua

for $n=0$

\Rightarrow or point on a moduli space leads to $P_{0,1}$?

\Rightarrow a point on a moduli space leads to a characterist spectrum

for $n=0$

for $n=0$

\Rightarrow on point on a moduli space looks like \mathbb{P}^1 characteristic spectrum

$$H = \langle H \rangle + \gamma$$

$$L = \langle L \rangle + X$$

for $n=0$

\Rightarrow on point on a moduli space leads to Poincaré characteristic spectrum

$$H = \langle H_i \rangle + y = y$$

$$L = \langle L \rangle + X = \langle L \rangle + X$$

for $n=0$

\Rightarrow in point on a model space looks like a characteristic system

$$H = \langle H \rangle + y = y$$

$$W(x,y) = \frac{\Delta}{2} [\langle L \rangle + x] y^2$$

$$L = \langle L \rangle + X = \langle L \rangle + X$$

for $n=0$

\Rightarrow on point on a model space leads to the characteristic spectrum

$$H = \langle H \rangle + y = y$$

$$W(x,y) = \frac{\Delta}{2} [\langle L \rangle + x] y^2$$

$$L = \langle L \rangle + X = \langle L \rangle + X$$

for $n=0$

\Rightarrow at point on a moduli space looks like for characteristic spectrum

$$H = \langle H_i \rangle + y = y$$

$$W(x,y) = \frac{\lambda}{2} [\langle L \rangle + x] y^2$$

$$L = \langle L \rangle + X = \langle L \rangle + X$$

$$= \frac{\lambda \langle L \rangle}{2} y^2 + \frac{\lambda}{2} x y^2$$

\Rightarrow or point on a model space leads to the characteristic spectrum

$$H = \langle H_i \rangle + y = y$$

$$W_{PSE}(x, y) = \frac{\lambda}{2} \left[\langle L \rangle + x \right] y^2$$

$$L = \langle L \rangle + X = \langle L \rangle + X$$

$$= \frac{\lambda \langle L \rangle}{2} y^2 + \frac{\lambda}{2} x y^2$$

+x

$$\mathbb{H} = \frac{\lambda \langle L \rangle}{2} y^2 + \frac{\lambda}{2} x y^2$$

mass term

$$\frac{\partial \phi}{\partial x} \Big|_{x=0} = \frac{1}{\sqrt{A}}$$



$$\Delta \propto \left| \frac{\partial \phi}{\partial t} \right|^2$$

$$W \propto \rho \dot{\phi}^2$$



$$\sqrt{\lambda} \propto \left| \frac{d\phi}{dx} \right|^2$$

$$W \propto m\phi^2$$



$$\sqrt{\lambda}$$

$$\propto$$

$$|m| \phi^2$$

$\frac{d\phi}{dt} \propto \sqrt{V}$
 $W \propto m \phi^2 \rightarrow \sqrt{V} \propto \frac{1}{m} \sqrt{p^2}$
 $\Rightarrow \phi$ is massive



$$\sqrt{V} \propto \left| \frac{d\phi}{dx} \right|^2$$

$$W \propto m\phi^2 \Rightarrow$$

$$\sqrt{V}$$

$$\propto$$

$$|m|^2 \phi^2$$

ϕ is a massive

superfield

$$m^2 =$$

$$|\langle \gamma \rangle|^2$$

$$V \propto \left| \frac{\partial W}{\partial \phi} \right|^2$$

$$W \propto m \phi^2 \Rightarrow V \propto |m \phi|^2$$

\Rightarrow ϕ is a massive superfield $m^2 = \langle \dots \rangle^2$
 X is massless.



$$V \propto \left| \frac{\partial \phi}{\partial t} \right|^2$$

$$W \propto m \phi^2 \Rightarrow V \propto |m \phi|^2$$

\Rightarrow y is a massive superfield $m^2 = |\langle L \rangle|^2$
 x is massless, (no ferm $X^2 \neq$)