Title: What if Quantum Thermodynamics were a fundamental extension of Quantum Mechanics?

Date: Nov 08, 2007 04:00 PM

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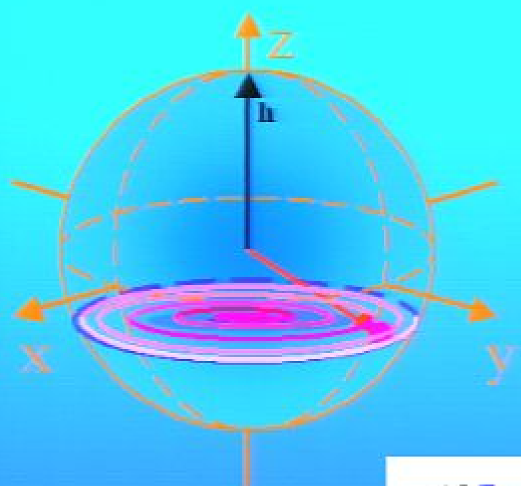
Abstract: What if the second law of thermodynamics, in the hierarchy of physical laws, were at the same level as the fundamental laws of mechanics, such as the great conservation principles? What if entropy were an intrinsic property of matter at the same level as energy is universally understood to be? What if irreversibility were an intrinsic feature of the microscopic dynamical law of all physical objects, including an individual qubit or qudit?

This talk will show how positive answers to these questions need not contradict any of the known results of quantum mechanics. We construct a logically consistent, mathematically sound and definite, physically intriguing, non-relativistic and non-statistical quantum theory, in which the second law of thermodynamics is embedded as a fundamental microscopical law. The theory hinges upon a nonlinear extension of unitary Hamiltonian dynamics which for uncorrelated and noninteracting systems reduces to the usual Schroedinger equation for the zero entropy states, but in general generates a group (not a semi group) of irreversible time evolutions, where the non-Hamiltonian entropy generating term in the evolution equation attracts the state towards the direction of maximal entropy increase. Various examples and features of this highly non-conventional dynamical theory are discussed. References available at http://www.quantumthermodynamics.org/

What if Quantum Thermodynamics were a fundamental extension of Quantum Mechanics?

Gian Paolo Beretta

Universita di Brescia, Italy

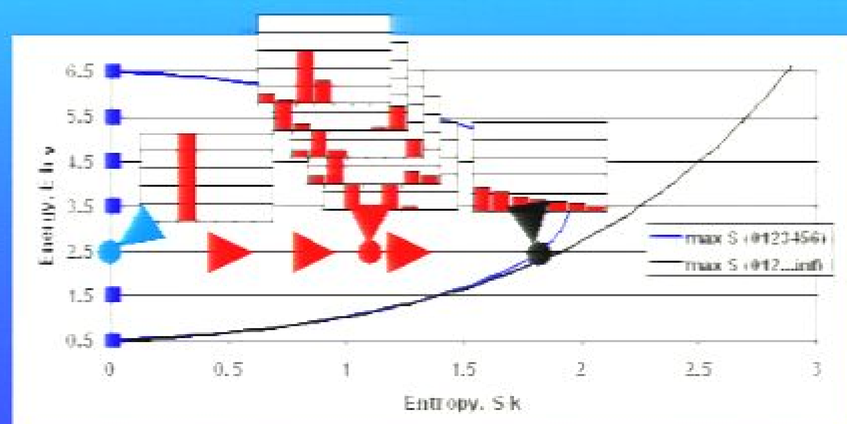


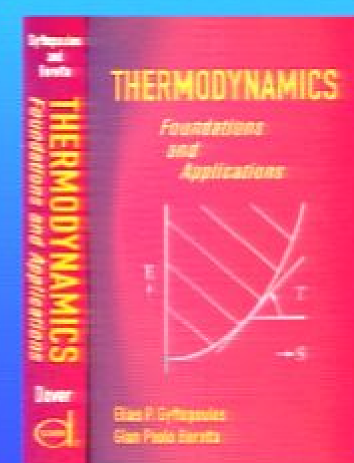

NATURE VOL. 316 4 JULY 1985

NEWS AND VIEWS

Uniting mechanics and statistics

An adventurous scheme which seeks to incorporate thermodynamics into the quantum laws of motion may end arguments about the arrow of time — but only if it works.



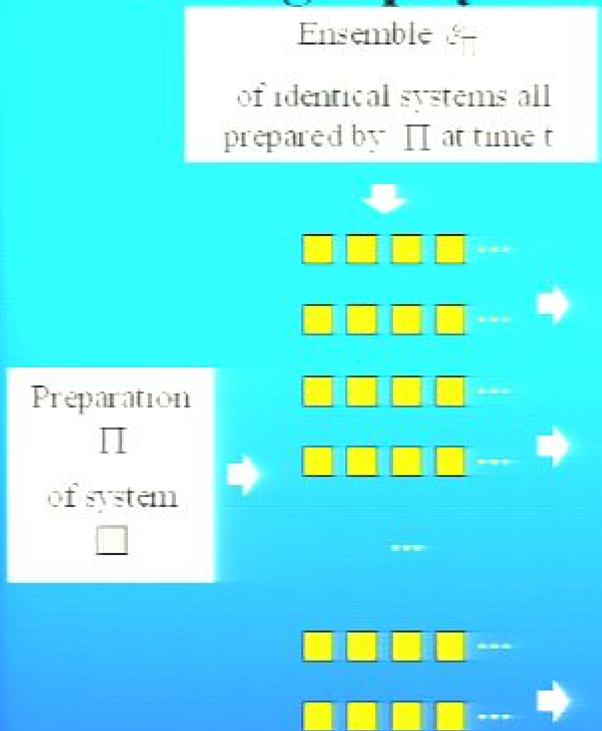


Outline

- Motivation for a new theory
- Ansatz #1: extension of Quantum Mechanics that builds the 2nd law of Thermodynamics directly into microscopic kinematics
- Ansatz #2: steepest-entropy-ascent quantum dynamics that builds irreversibility at the fundamental level, preserving standard QM
- Features of the resulting theory
  - equivalent variational formulation
  - numerical results for a 4-level system
  - stable and unstable equilibrium states and limit cycles
  - mathematical reversibilty and the arrow of time
  - time-energy and time-entropy uncertainty relations
  - Onsager relations
  - measurable results

"Tomography" of a preparation (or ensemble) at time *t*

Park and Band, FoundPhys.
1, 133 (1970); 1, 211 (1971);
1, 339 (1971)



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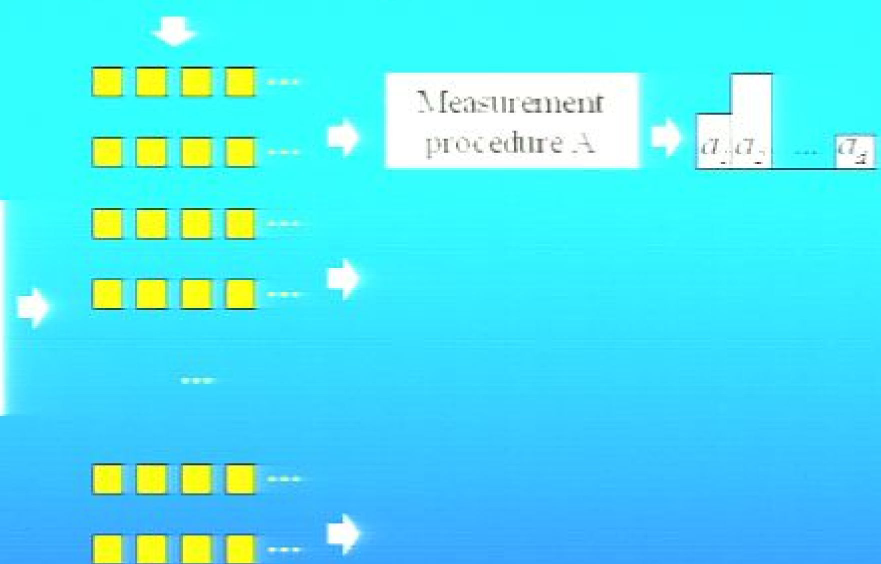
Ensemble $\mathcal{E}_\Pi$ of identical systems all prepared by Π at time t

Preparation Π of system □



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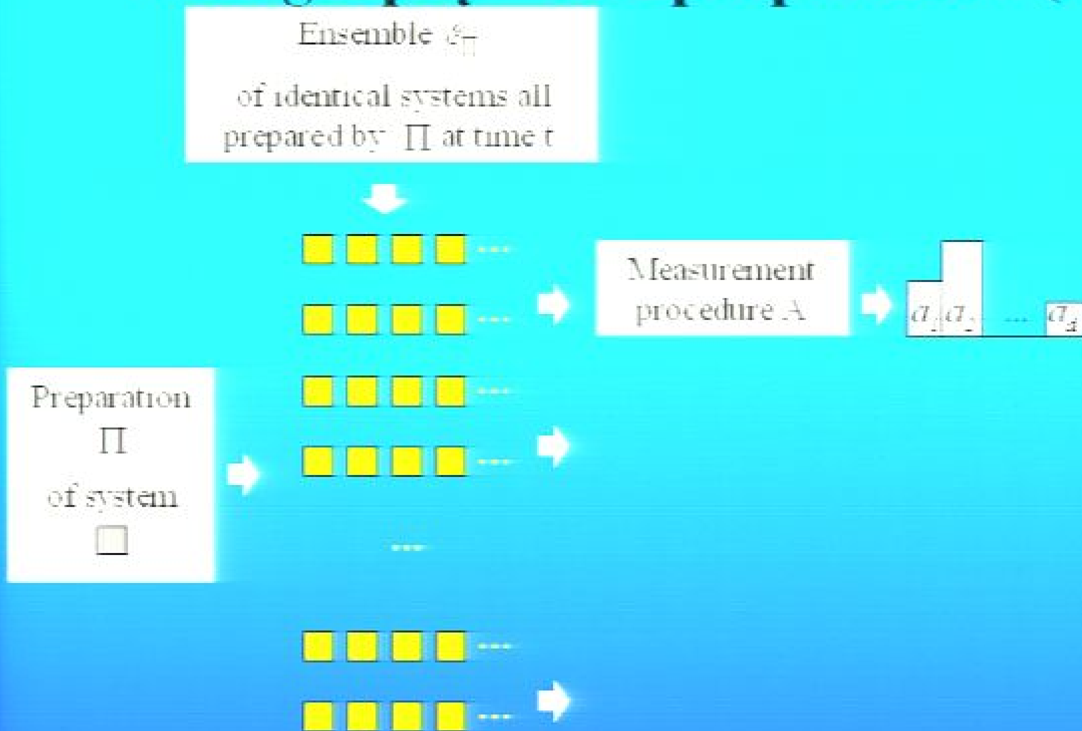

$$p_{a_j} = \langle Q_{a_j} \rangle = \lim_{N_A \to \infty} N_{a_j} / N_A$$

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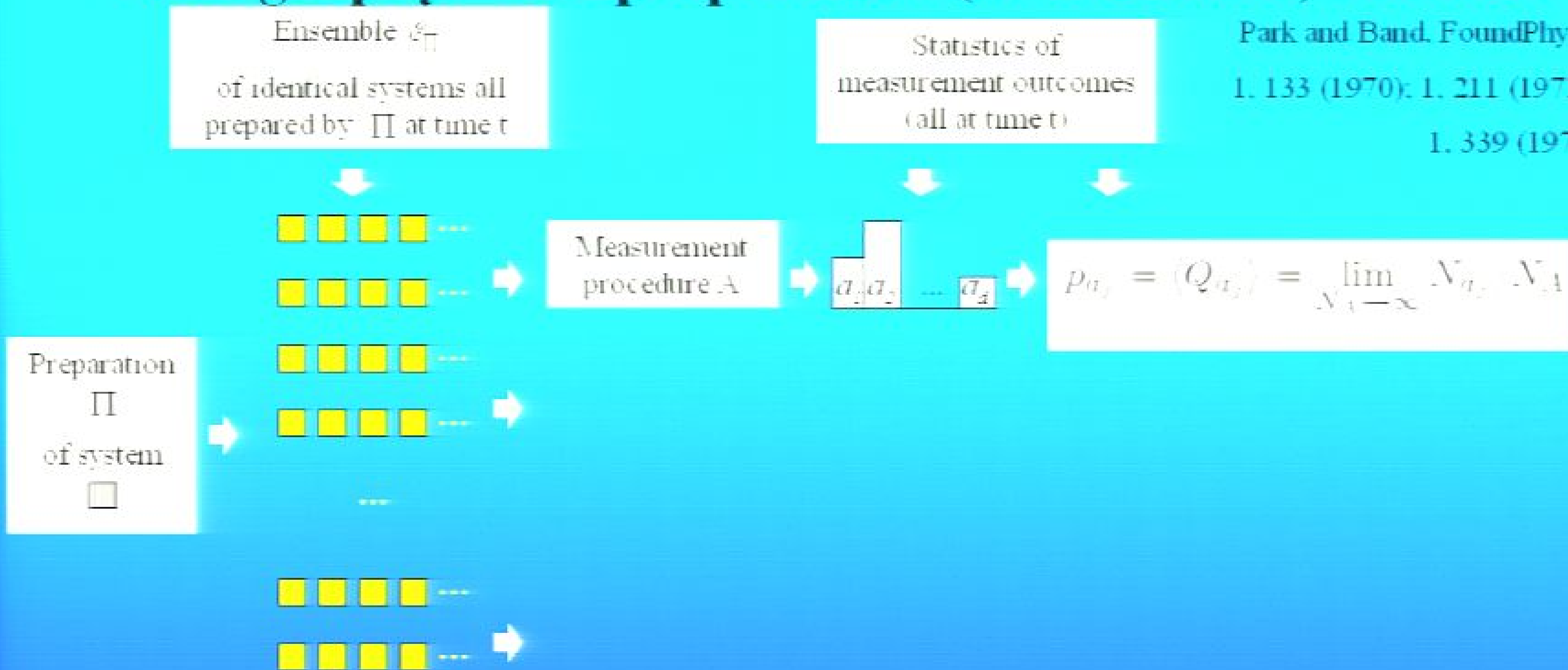


"Tomography" of a preparation (or ensemble) at time *t*

Ensemble $\mathcal{E}_\Pi$

of identical systems all prepared by Π at time t

Statistics of measurement outcomes (all at time t)

Park and Band, Found.Phys. 1, 133 (1970); 1, 211 (1971); 1, 339 (1971)

Preparation Π of system $\square$

Measurement procedure A

Measurement procedure B

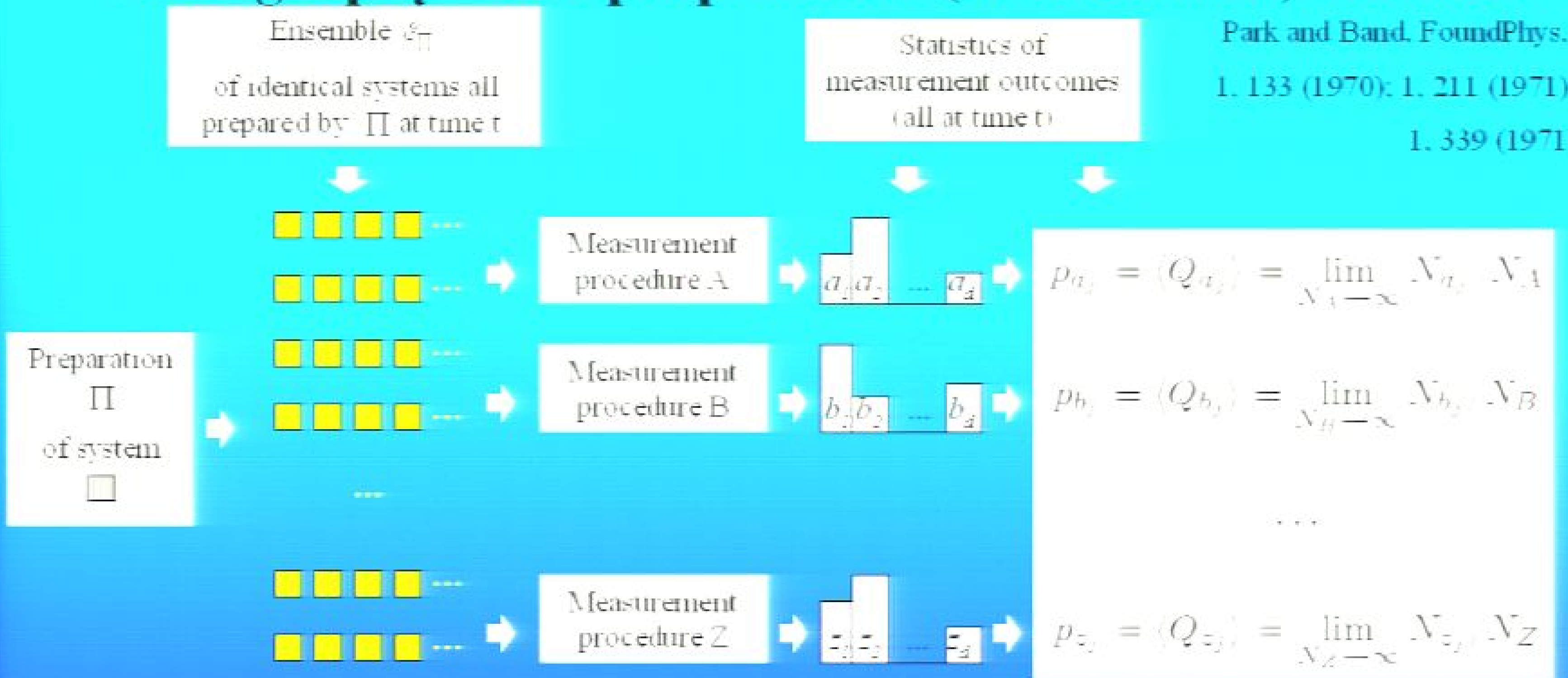
$a_1\ a_2\ \ldots\ a_d$

$b_1\ b_2\ \ldots\ b_d$

$$p_{a_i} = \langle Q_{a_i} \rangle = \lim_{N_A \to \infty} N_{a_i} / N_A$$

$$p_{b_j} = \langle Q_{b_j} \rangle = \lim_{N_B \to \infty} N_{b_j} / N_B$$

"Tomography" of a preparation (or ensemble) at time *t*



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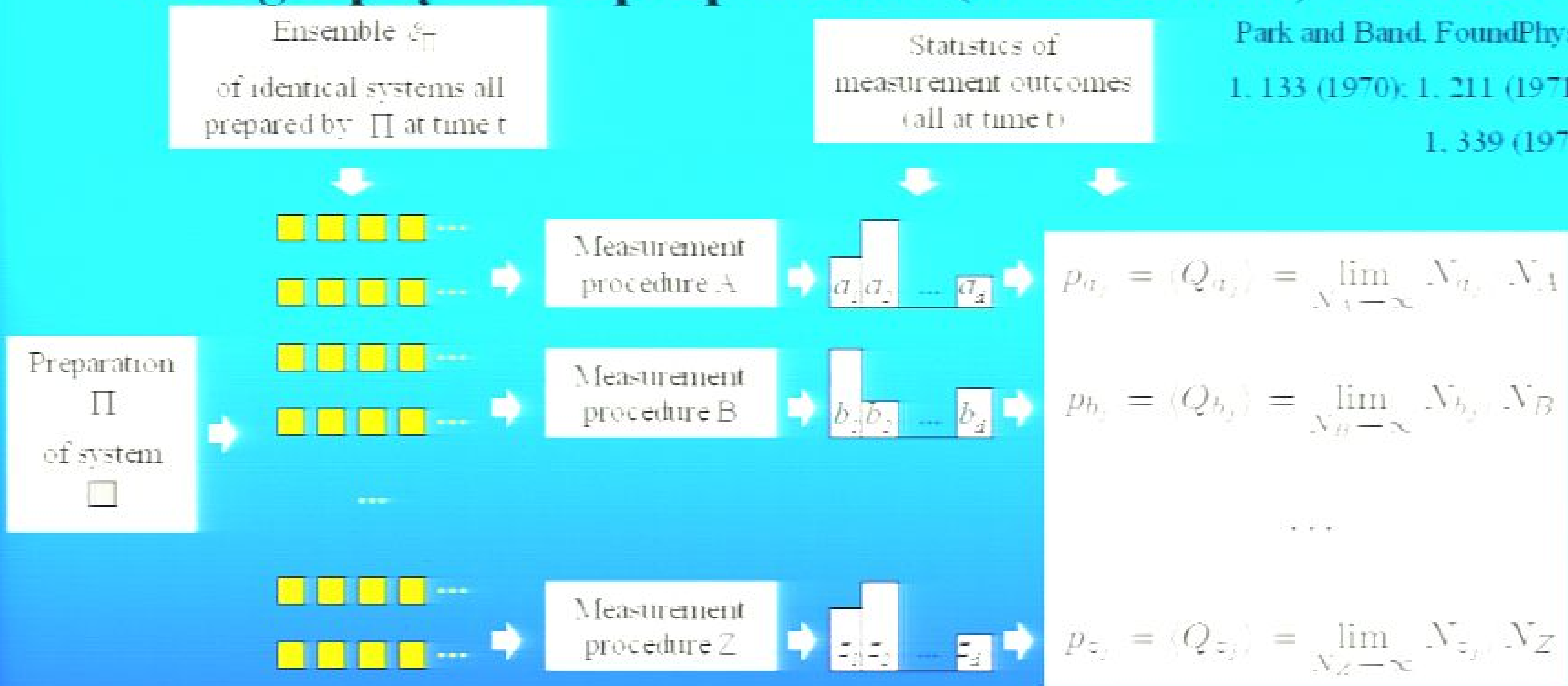
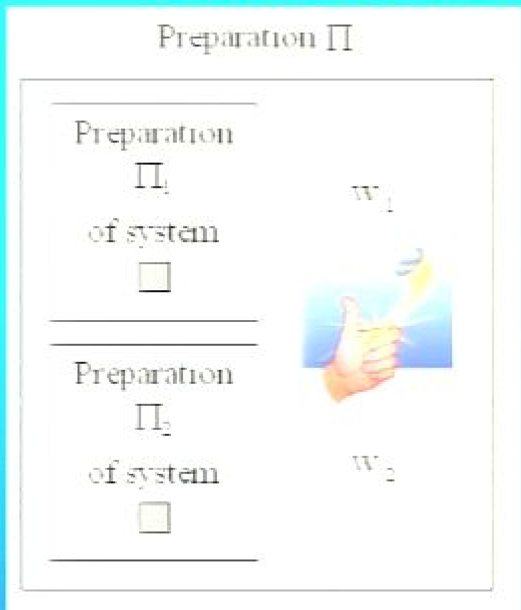


If $A, B, \ldots, Z$ are all the *conceivable* measurements, then preparation Π (and hence ensemble $\mathcal{E}_\Pi$) is completely characterized (at time t) by the set of numbers:

$$\Pi = \mathcal{E}_\Pi = \{p_{a_1}, \ldots, p_{a_a}, p_{b_1}, \ldots, p_{b_4}, \ldots, p_{z_1}, \ldots, p_{z_d}\}$$

Statistical mixing of preparations (or ensembles)



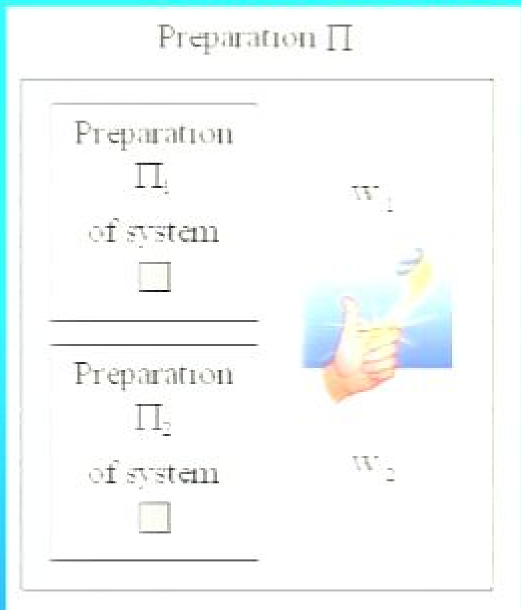
For any observable A, the statistics (at time t) is

$$\langle A\rangle_{\Pi} = w_1 \langle A\rangle_{\Pi_1} + w_2 \langle A\rangle_{\Pi_2}$$

and, therefore, the full statistics is

$$\Pi = w_1 \Pi_1 + w_2 \Pi_2$$

Statistical mixing of preparations (or ensembles)



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Homogeneous vs Heterogeneous preparations (or ensembles)

Given a preparation Π, we may look for *all conceivable decompositions*. We say that Π is *homogeneous* (von Neumann) iff there is no way to obtain the same full statistics from the statistical mixture of two *different* preparations, i.e.,

$$\text{iff } \Pi = w_1\Pi_1 + w_2\Pi_2 \text{ with } w_1, w_2 > 0 \text{ implies } \Pi_1 = \Pi_2$$

Homogeneous preparations (or ensembles) and "states"

For a homogeneous preparation, Π^0, no subdivision into different subensembles is conceivable and, therefore, the entire statistics Π^0 can be assigned to (is an intrinsic feature of) each individual member of the ensemble.

Von Neumann, book, engl.transl.1932

Homogeneous preparations (or ensembles) and "states"

For a homogeneous preparation, Π°, no subdivision into different subensembles is conceivable and, therefore, the entire statistics Π° can be assigned to (is an intrinsic feature of) each individual member of the ensemble.

But this qualifies to represent the concept of *individual state* only if every heterogeneous preparation (ensemble) admits one and only one decomposition into homogeneous components. Otherwise:

Von Neumann, book, engl.transl.1932

Schrödinger, PCPS, 32, 446 (1936)

ParkBand-FoundPhys-6-157-1976

quant-ph/0509116

ModPhysLettA-21-2799-2006

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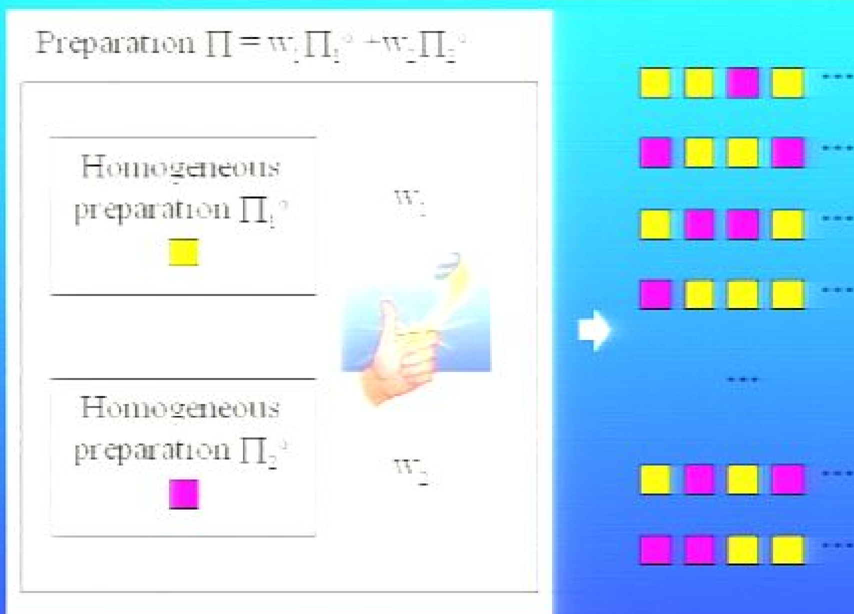

"State is either ■ or ■ "

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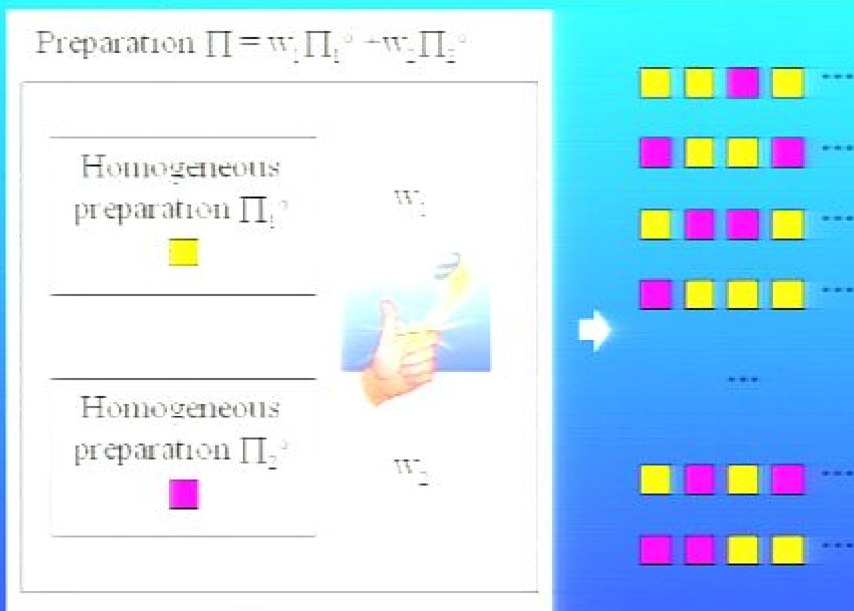

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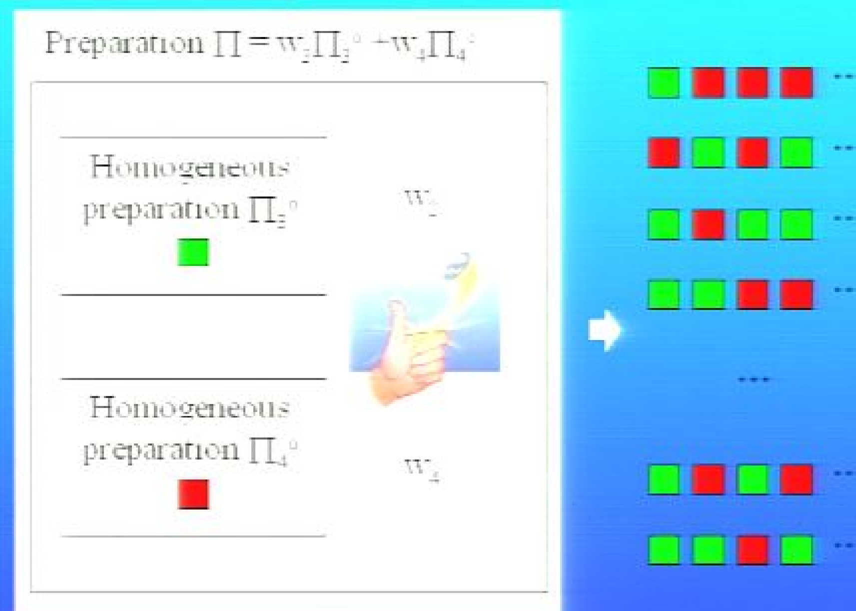

"State is either ■ or ■ "

Traditional construction

Mechanics: Classical Quantum

Homogeneous preparation of a system →

Construct	Classical representative	Quantal counterpart
I. System	Phase space	Hilbert space
II. State of system	Phase point (q, p)	Ray $\vert\Psi\rangle$
III. Observable	Function of phase	Hermitian operator with complete orthonormal eigenvector set

Park&Band-FoundPhys-6-157-1976

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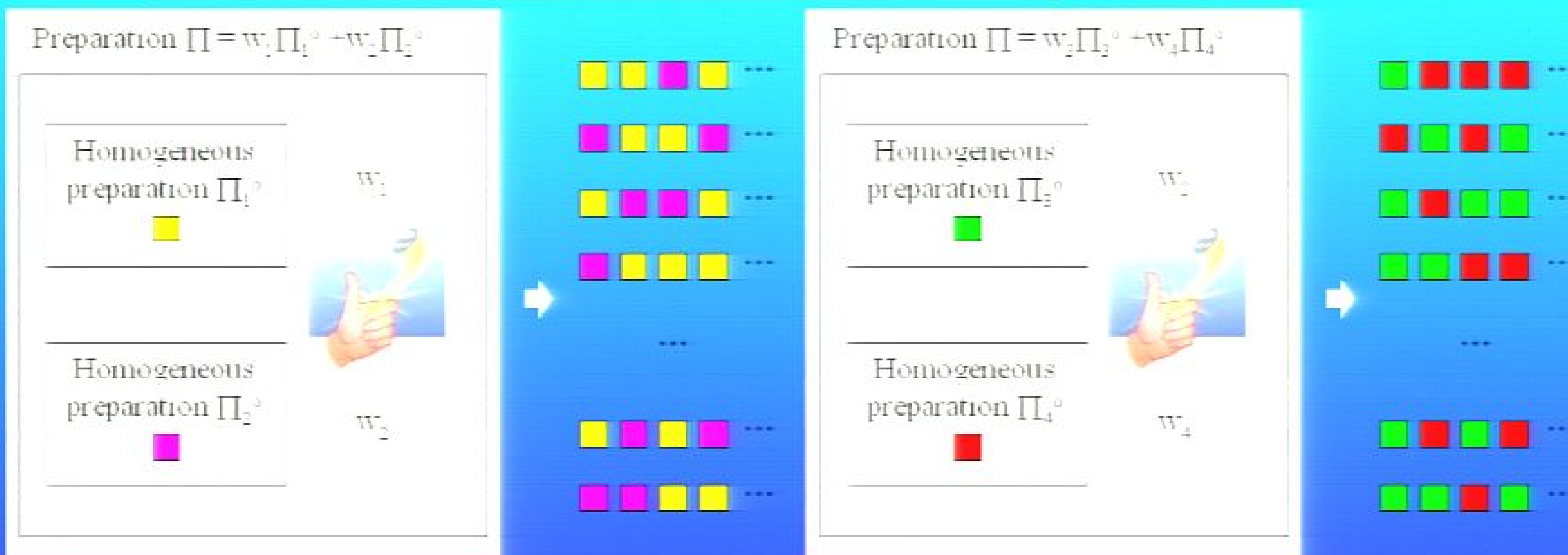

"State is either ■ or ■ " **contradicts** "State is either ■ or ■ "

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Traditional construction

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Mechanics: Classical Quantum

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Statistical Mechanics: Classical Quantum

Heterogeneous preparation of a system →

IV. Ignorance of true state	Gibbsian coefficient of probability of phase $\rho(q, p)$	Density operator $\rho = \sum_k w_k$ [illegible]

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Traditional construction

Mechanics: Classical / Quantum

Construct	Classical representative	Quantal counterpart
I. System	Phase space	Hilbert space
II. State of system	Phase point (q, p)	Ray $|\Psi\rangle$
III. Observable	Function of phase	Hermitian operator with complete orthonormal eigenvector set

Homogeneous preparation of a system → I–III

Statistical Mechanics: Classical / Quantum

Construct	Classical representative	Quantal counterpart
IV. Ignorance of true state	Gibbsian coefficient of probability of phase $\rho(q, p)$	Density operator $\rho = \sum$ [illegible]

Heterogeneous preparation of a system → IV

Unique decomposition?	YES	NO

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Mixed density operators have multiple decompositions into 'pure' ones

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many others later

quant-ph/0509116

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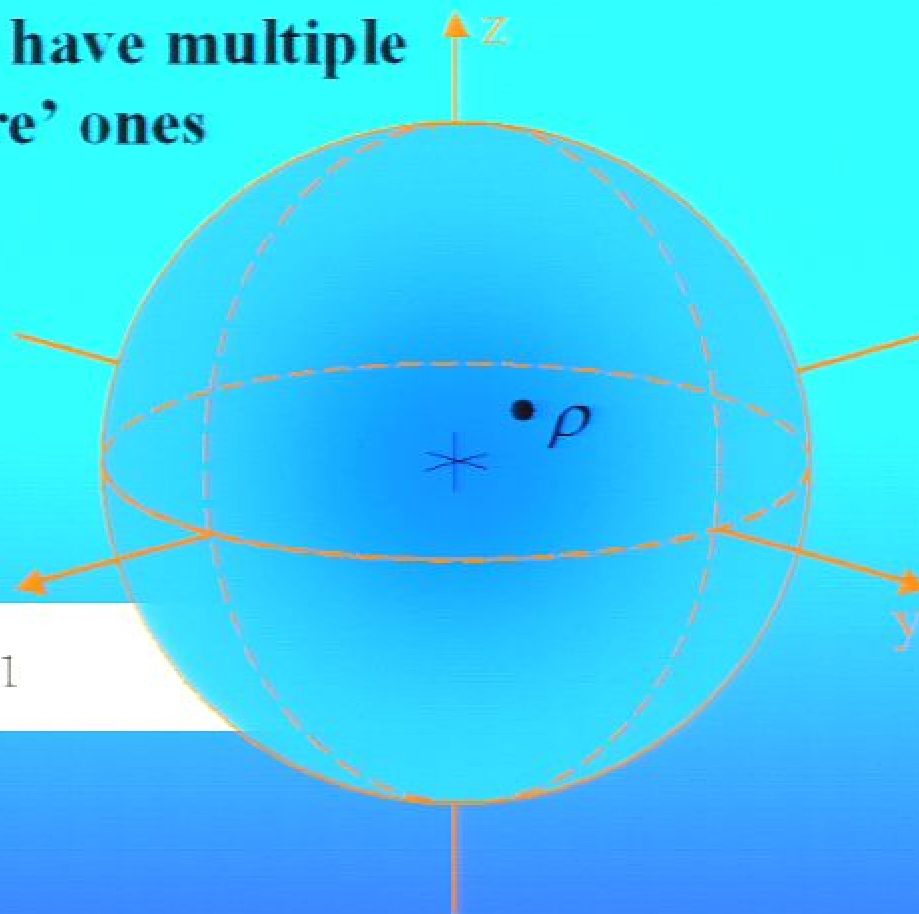

$$\rho = \frac{1}{2}(I + \underline{r}_\rho \cdot \underline{\sigma}) \quad \text{mixed if } r_\rho^2 < 1$$

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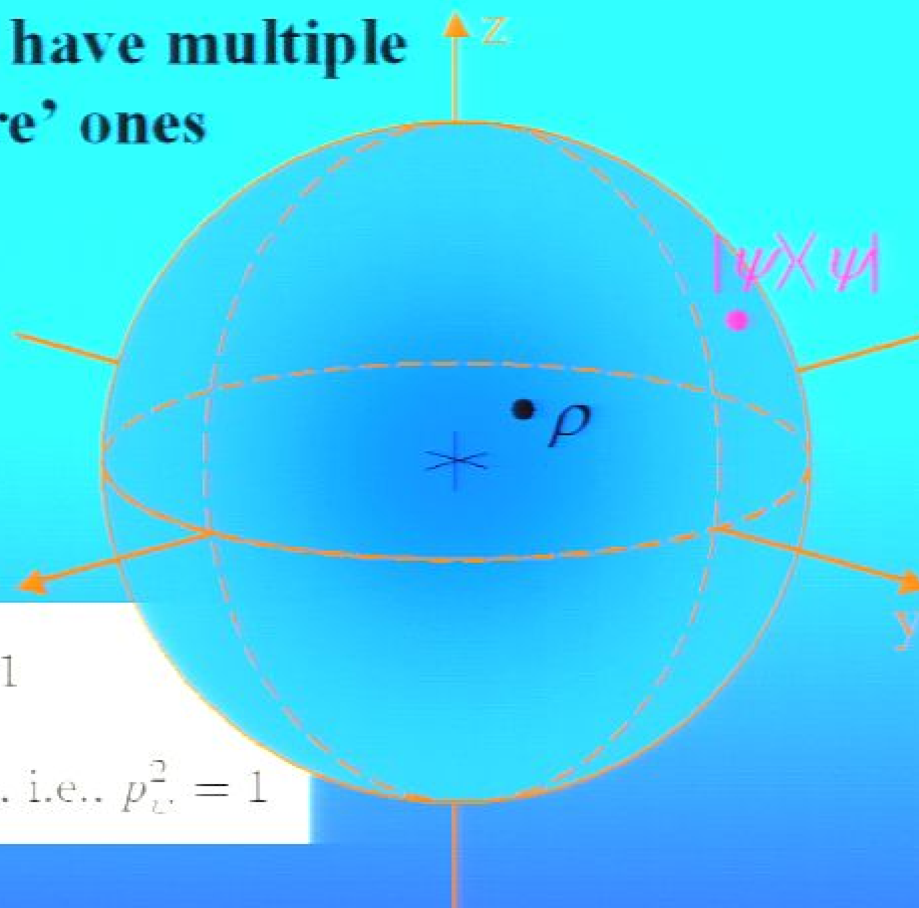

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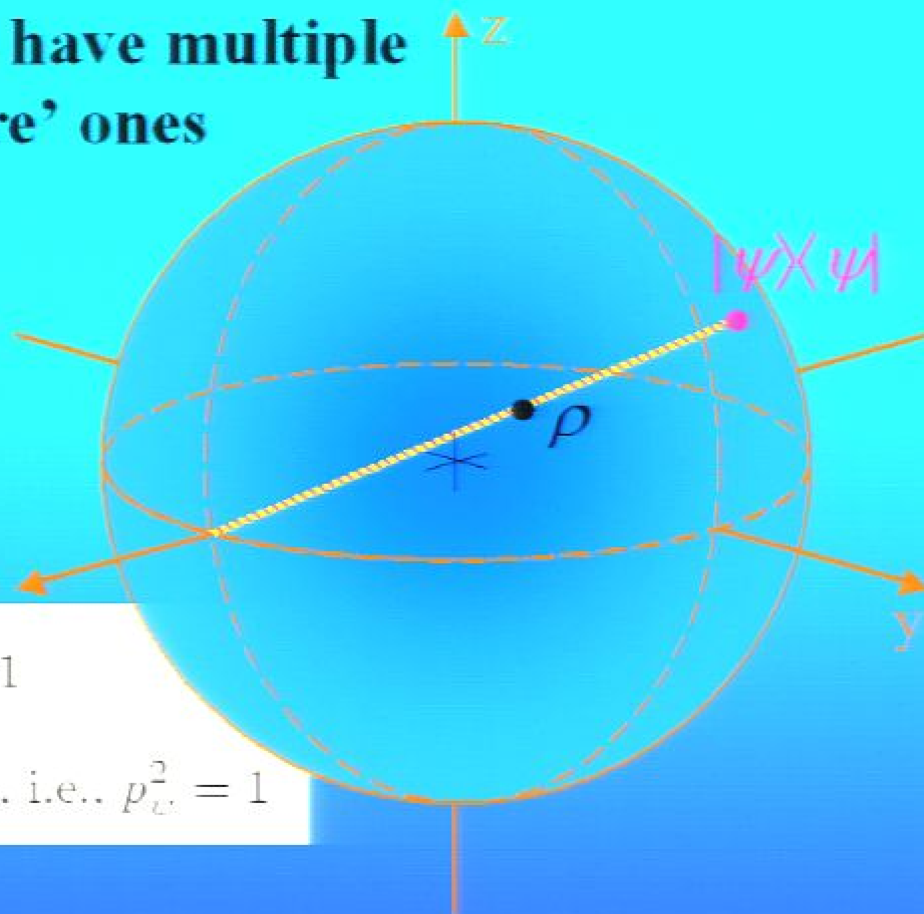

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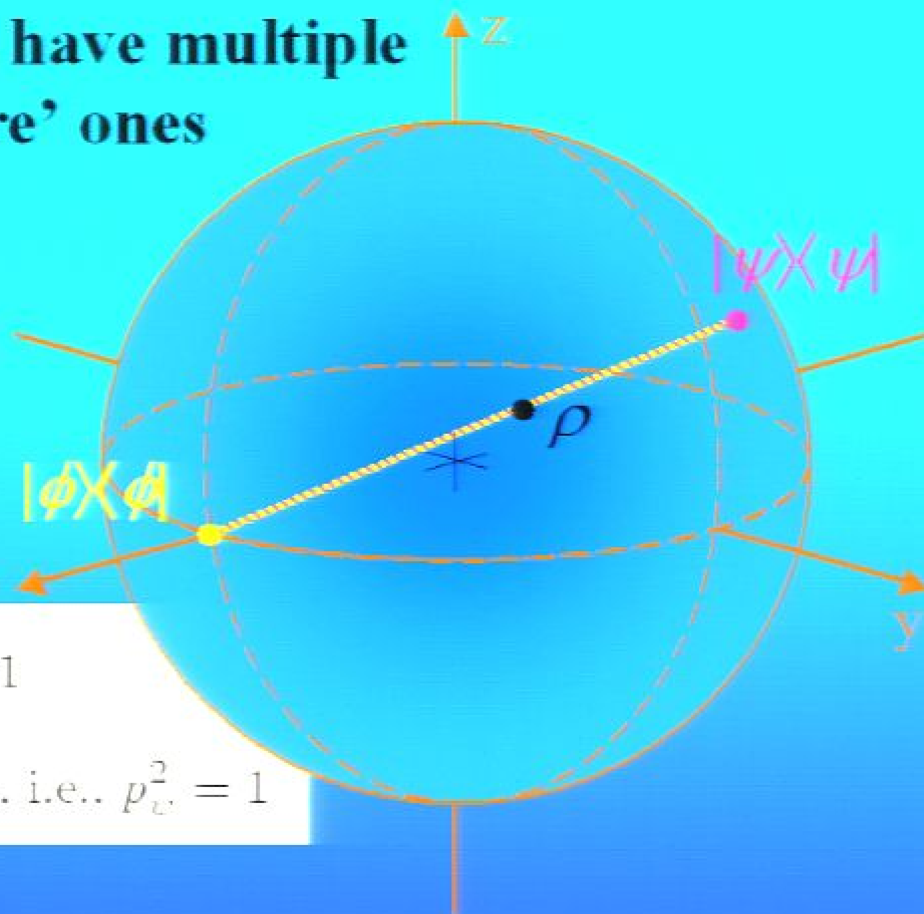

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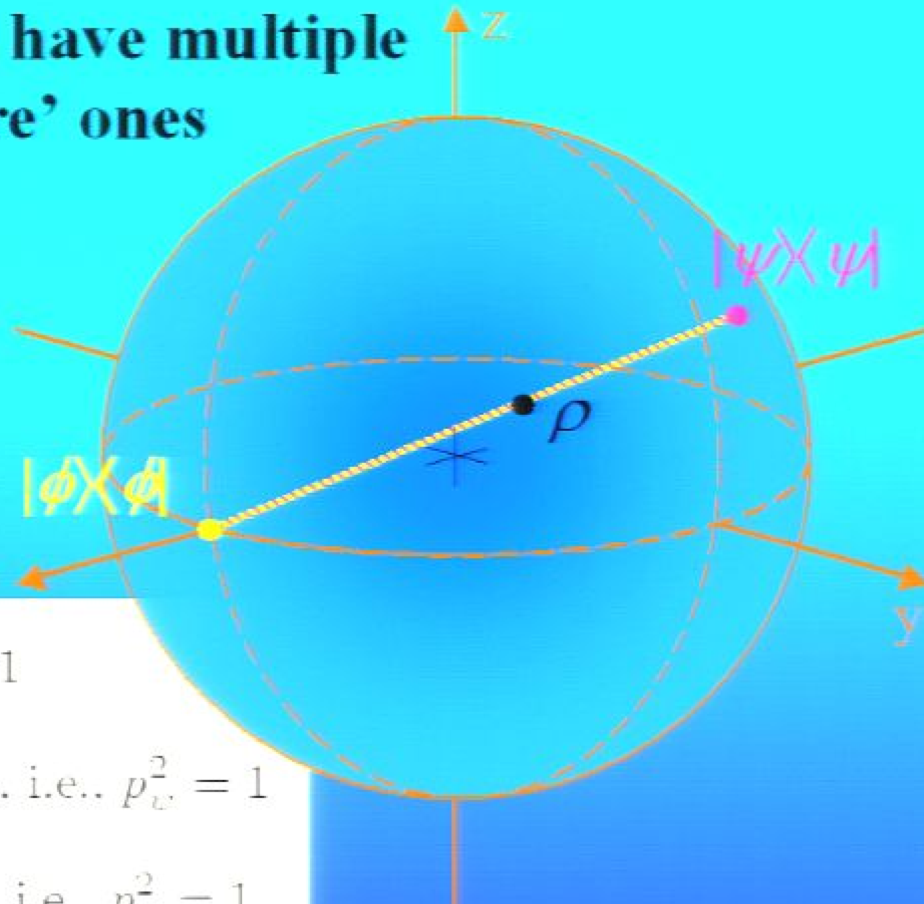

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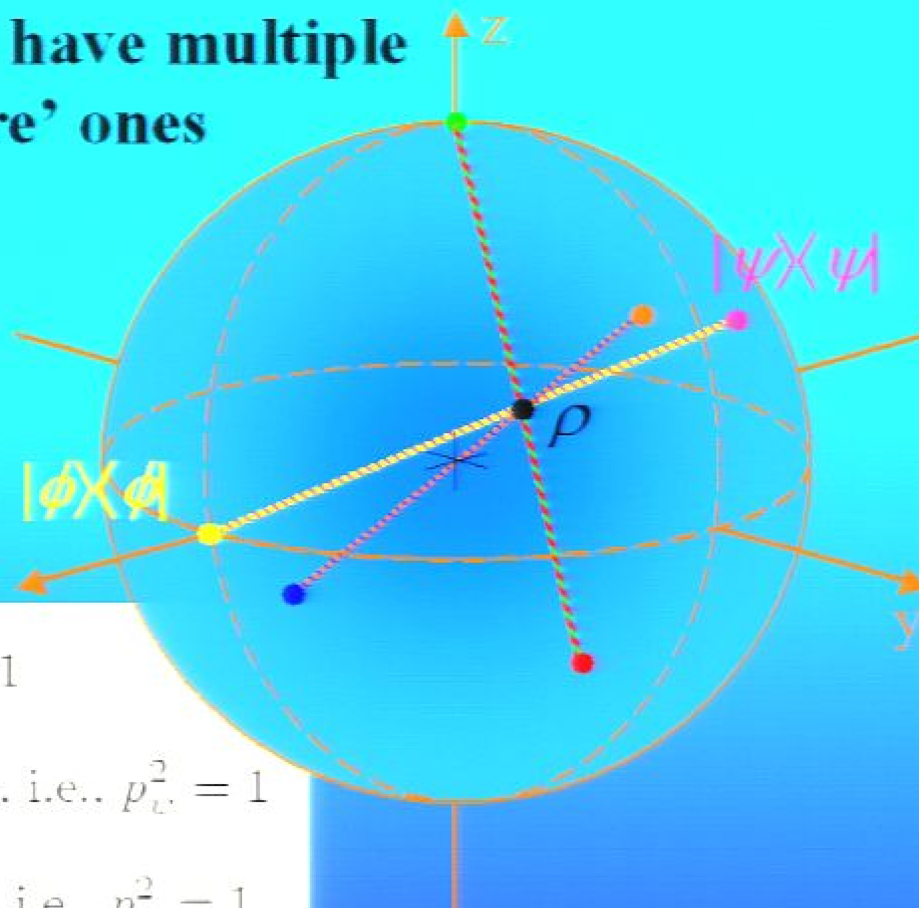

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Quantum Statistical Mechanics does not conform to the desiderata … but it is extremely successful in describing thermodynamic equilibrium data

Boltzmann, Fermi-Dirac, Bose-Einstein thermodynamic equilibrium statistics, regularized by:

"Canonical" equilibrium density operators

$$\rho_{ce} = \frac{\exp(-\beta H)}{\mathrm{Tr}\exp(-\beta H)}$$

max $-\mathrm{Tr}\rho\ln\rho$ subject to $\mathrm{Tr}\rho H = \langle E\rangle$ and $\mathrm{Tr}\rho = 1$.

"Grand canonical" equilibrium density operators

$$\rho_{gce} = \frac{\exp[-\beta(H - \sum_{j=1}^{r}\mu_j N_j)]}{\mathrm{Tr}\exp[-\beta(H - \sum_{j=1}^{r}\mu_j N_j)]}$$

max $-\mathrm{Tr}\rho\ln\rho$ for $\mathrm{Tr}\rho H = \langle E\rangle$, $\mathrm{Tr}\rho N_j = \langle N_j\rangle$, $\mathrm{Tr}\rho = 1$.

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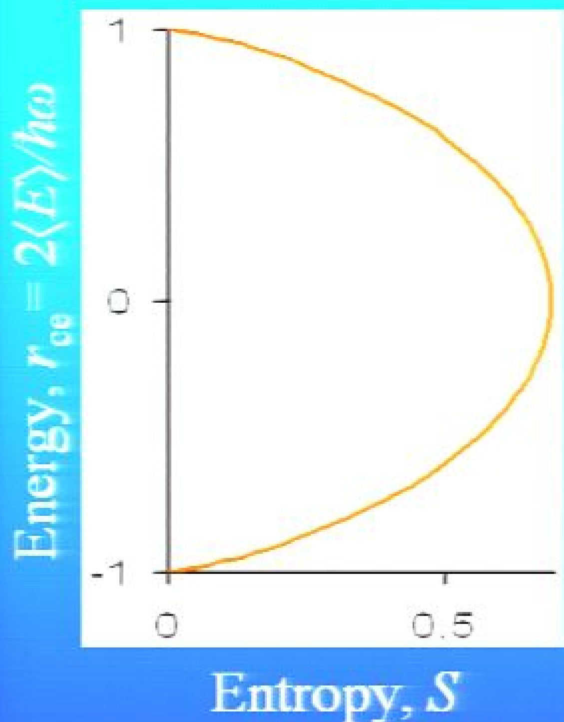
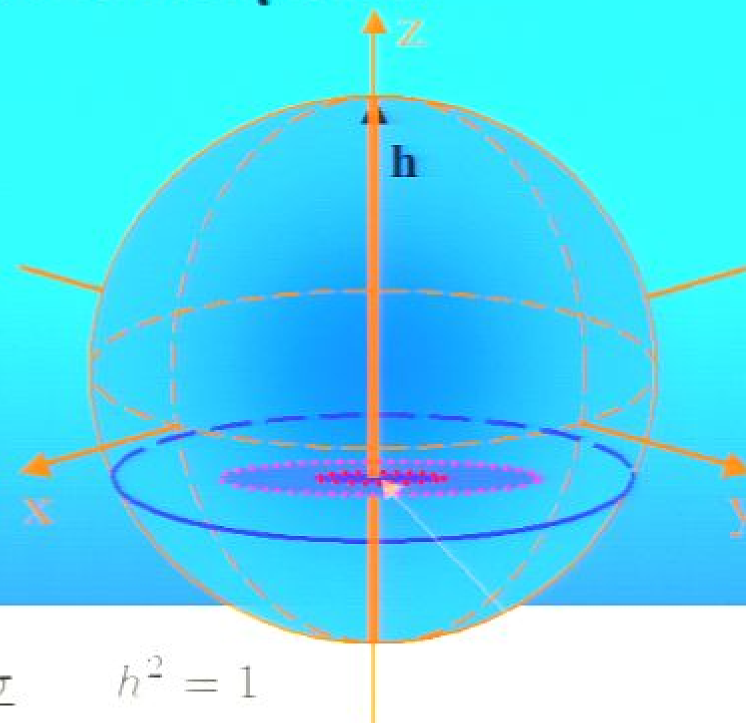
$\max -\mathrm{Tr}\rho\ln\rho$ for $\mathrm{Tr}\rho H = \langle E\rangle$, $\mathrm{Tr}\rho N_j = \langle N_j\rangle$, $\mathrm{Tr}\rho = 1$.

Quantum Statistical Mechanics, for a single isolated and uncorrelated two-level system





$$H = \frac{1}{2}\hbar\omega\underline{h}\cdot\underline{\sigma} \qquad h^2 = 1$$

$$\rho_{ee} = \frac{1}{2}(I + r_{ee}\underline{h}\cdot\underline{\sigma}) \qquad r_{ee} = \frac{\langle E\rangle}{\hbar\omega/2}$$

$$S_{ee} = -k_B\left(\frac{1+r_{ee}}{2}\ln\frac{1+r_{ee}}{2} + \frac{1-r_{ee}}{2}\ln\frac{1-r_{ee}}{2}\right)$$

Traditional construction

Mechanics: Classical Quantum

Homogeneous preparation of a system →

Construct	Classical representative	Quantal counterpart
I. System	Phase space	Hilbert space
II. State of system	Phase point (q, p)	Ray $\|\Psi\rangle$
III. Observable	Function of phase	Hermitian operator with complete orthonormal eigenvector set

Statistical Mechanics: Classical Quantum

Heterogeneous preparation of a system →

IV. Ignorance of true state	Gibbsian coefficient of probability of phase $\rho(q, p)$	Density operator $\rho = \sum_k$ [illegible]

Unique decomposition? **YES** **NO**

Park&Band-FoundPhys-6-157-1976

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Quantum Thermodynamics as a fundamental extension of Quantum Mechanics

GN Hatsopoulos and EP Gyftopoulos, Found.Phys., Vol. 6, 15, 127, 439, 561 (1976)

Mechanics:

	Classical Mechanics	Quantum Mechanics and Thermodynamics
Construct	Classical representative	Quantal counterpart
I. System	Phase space	Hilbert space
II. State of system	Phase point (q, p)	Density operator
III. Observable	Function of phase	Hermitian operator with complete orthonormal eigenvector set

Homogeneous preparation of a system → II. State of system

Statistical Mechanics:

	Classical	Quantum
IV. Ignorance of true state	Gibbsian coefficient of probability of phase $\rho(q, p)$	Density operator $\rho = \sum_n \ldots \rho_n \ldots$

Heterogeneous preparation of a system → IV. Ignorance of true state

Unique decomposition? **YES**

Park&Band-FoundPhys-6-157-1976

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Perimeter Institute, Waterloo, Canada, November 8, 2007 - **References available at: www.quantumthermodynamics.org**

Quantum Thermodynamics as a fundamental extension of Quantum Mechanics

GN Hatsopoulos and EP Gyftopoulos, Found.Phys., Vol. 6, 15, 127, 439, 561 (1976)

~~Mechanics~~: Classical Mechanics | Quantum Mechanics and Thermodynamics

Homogeneous preparation of a system →

Construct	Classical representative	Quantal counterpart
I. System	Phase space	Hilbert space
II. State of system	Phase point (q, p)	Density operator
III. Observable	Function of phase	Hermitian operator with complete orthonormal eigenvector set

Statistical Mechanics: Classical | Quantum

Heterogeneous preparation of a system →

IV. Ignorance of true state	Gibbsian coefficient of probability of phase	~~Subjective probability distribution defined over the density operators~~

Unique decomposition? — Classical: **YES** — Quantum: **YES**

Park&Band-FoundPhys-6-157-1976

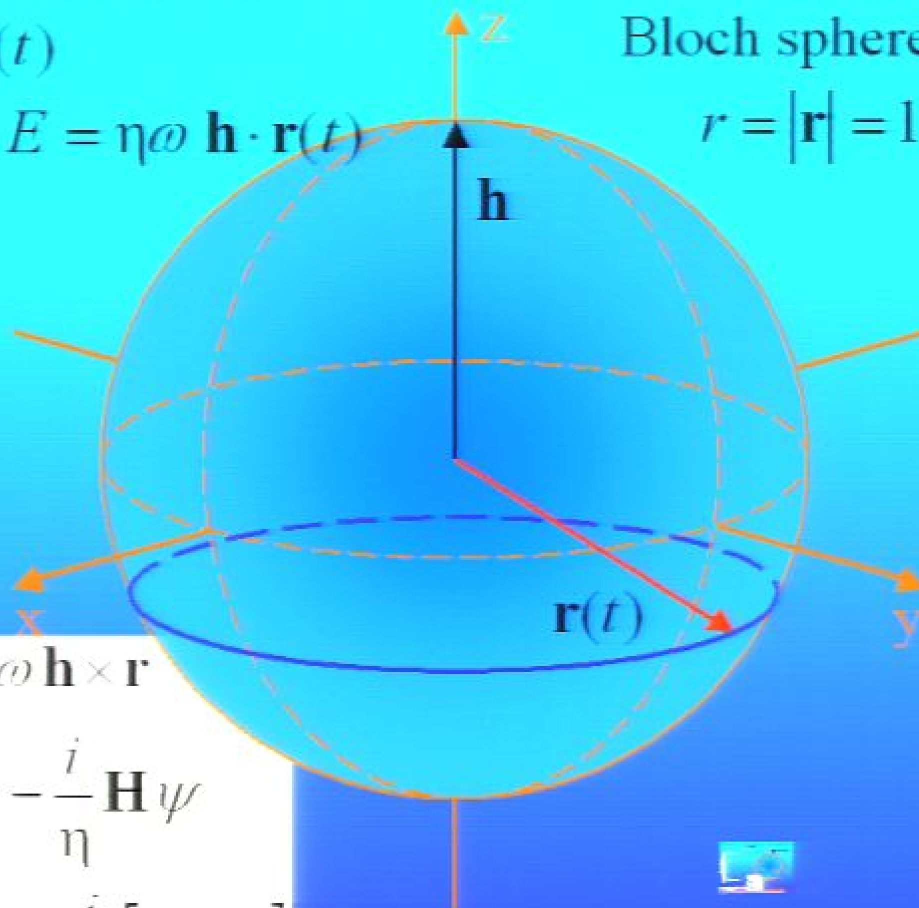
Isolated two-level system, standard Quantum Mechanics

Hamiltonian, $\eta\omega\,\mathbf{h}$

State, $\mathbf{r}(t)$

Energy, $E = \eta\omega\,\mathbf{h}\cdot\mathbf{r}(t)$

Bloch sphere, $r = |\mathbf{r}| = 1$



$$\dot{\mathbf{r}} = \omega\,\mathbf{h}\times\mathbf{r}$$

$$\dot{\psi} = -\frac{i}{\eta}\mathbf{H}\,\psi$$

$$\dot{P}_{\psi} = -\frac{i}{\eta}\left[\mathbf{H}, P_{\psi}\right]$$

Isolated two-level system, Quantum Thermodynamics

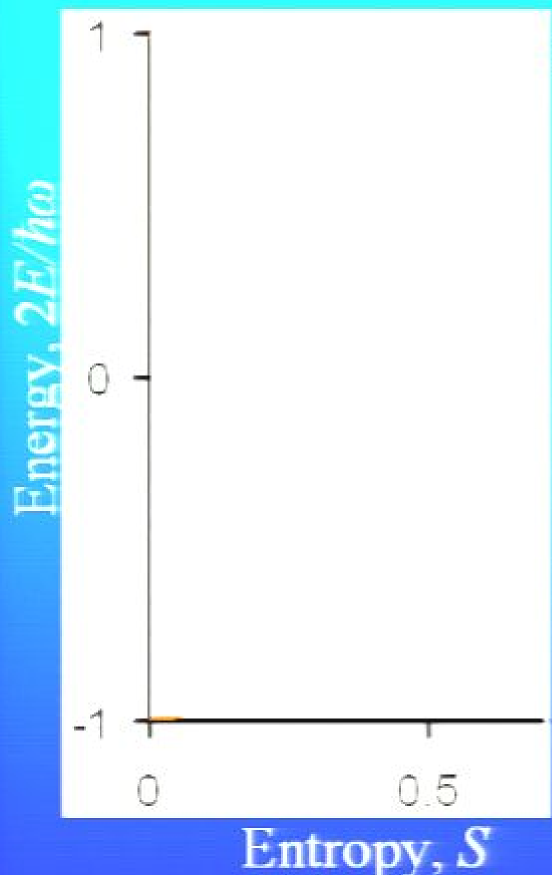
Hamiltonian, $\eta\omega\,\mathbf{h}$

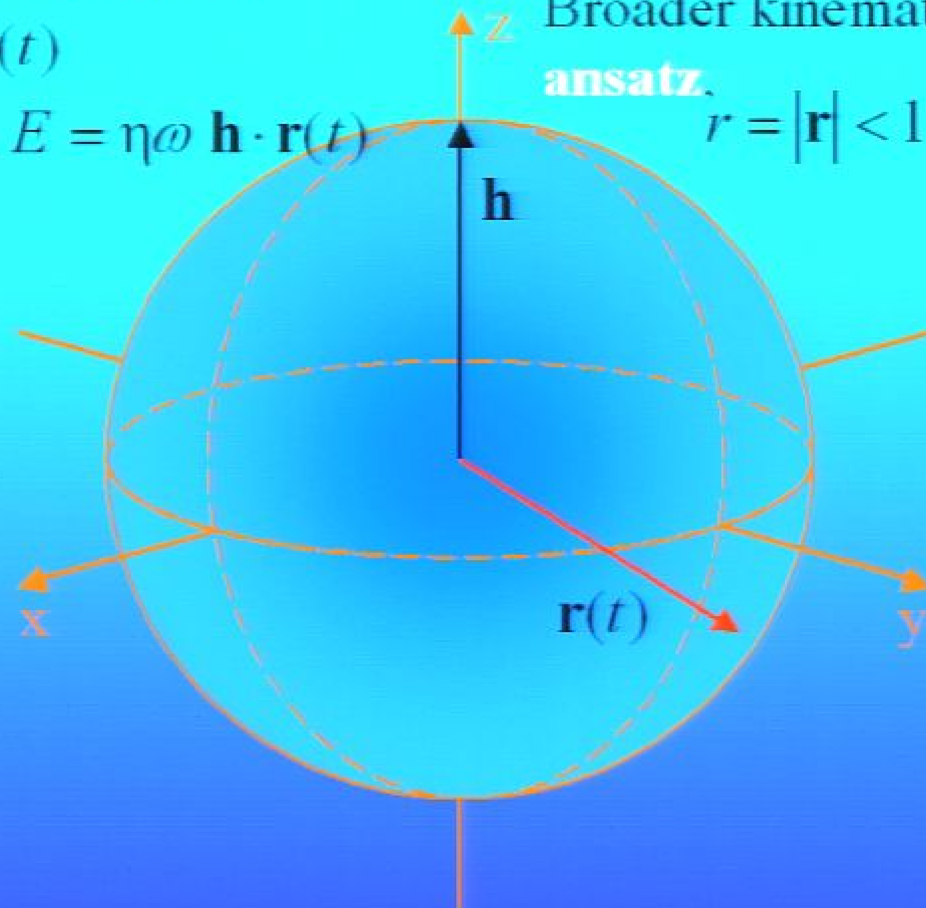
State, $\mathbf{r}(t)$

Energy, $E = \eta\omega\,\mathbf{h}\cdot\mathbf{r}(t)$

Broader kinematics

ansatz, $r = |\mathbf{r}| < 1$





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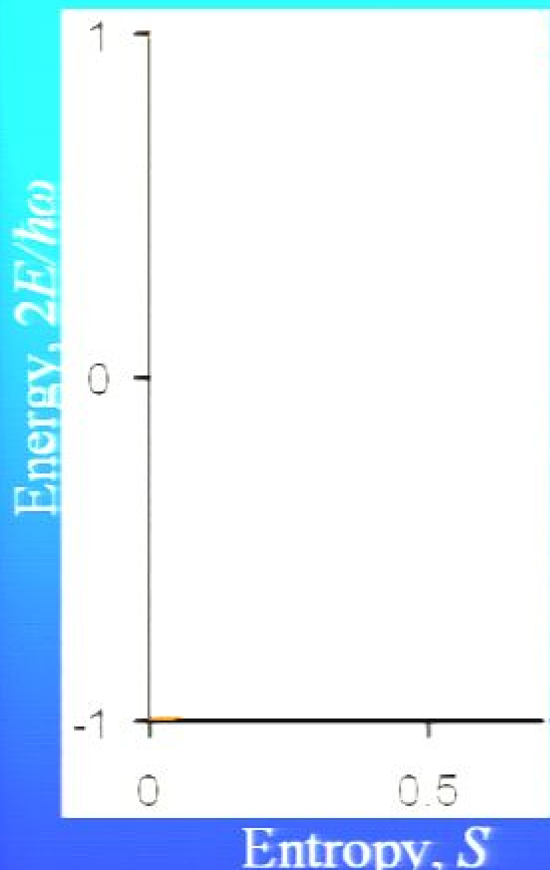
Isolated two-level system, Quantum Thermodynamics

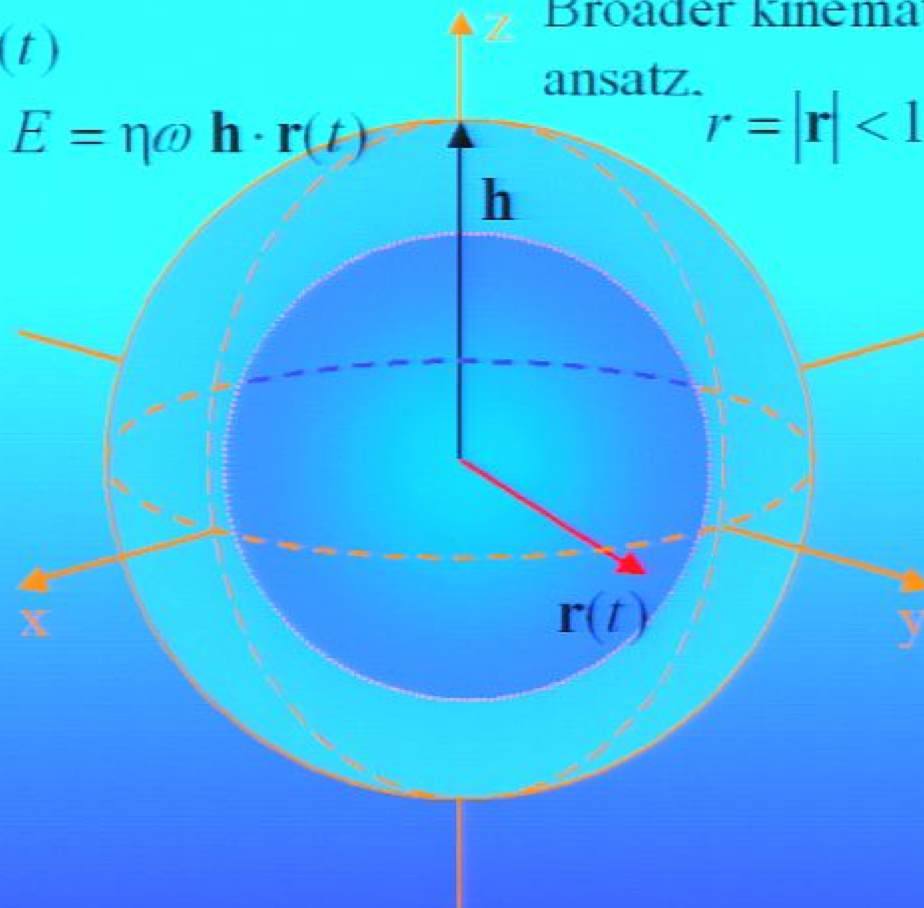
Hamiltonian, $\eta\omega\ \mathbf{h}$

State, $\mathbf{r}(t)$

Energy, $E = \eta\omega\ \mathbf{h}\cdot\mathbf{r}(t)$

Broader kinematics ansatz. $r = |\mathbf{r}| < 1$





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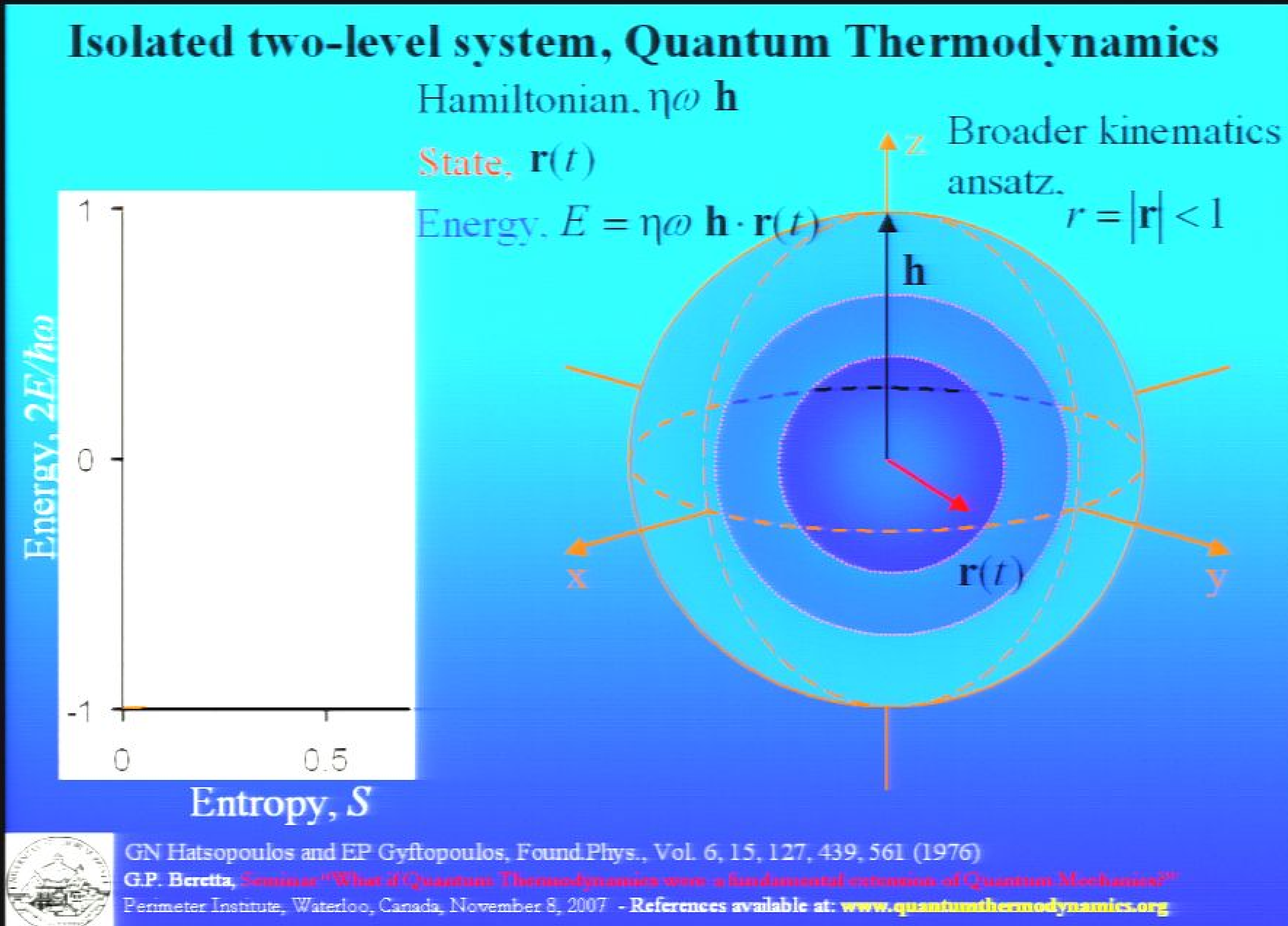
Isolated two-level system, Quantum Thermodynamics
Hamiltonian, ηω h
State, r(t)
Energy, E = ηω h · r(t)
Broader kinematics ansatz.
r = |r| < 1
z
h
x
y
r(t)
Energy, 2E/hω
1
0
-1
0
0.5
Entropy, S
GN Hatsopoulos and EP Gyftopoulos, Found.Phys., Vol. 6, 15, 127, 439, 561 (1976)
G.P. Beretta, Seminar "What if Quantum Thermodynamics were a fundamental extension of Quantum Mechanics?"
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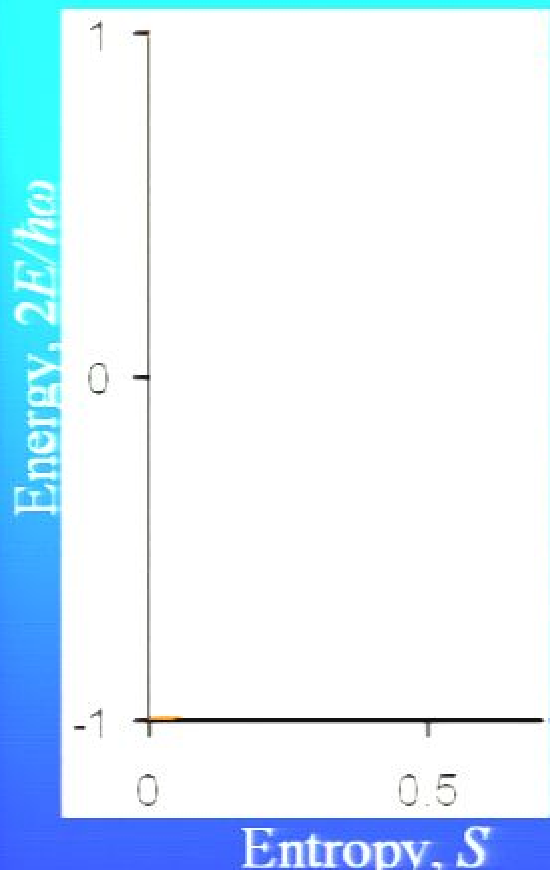
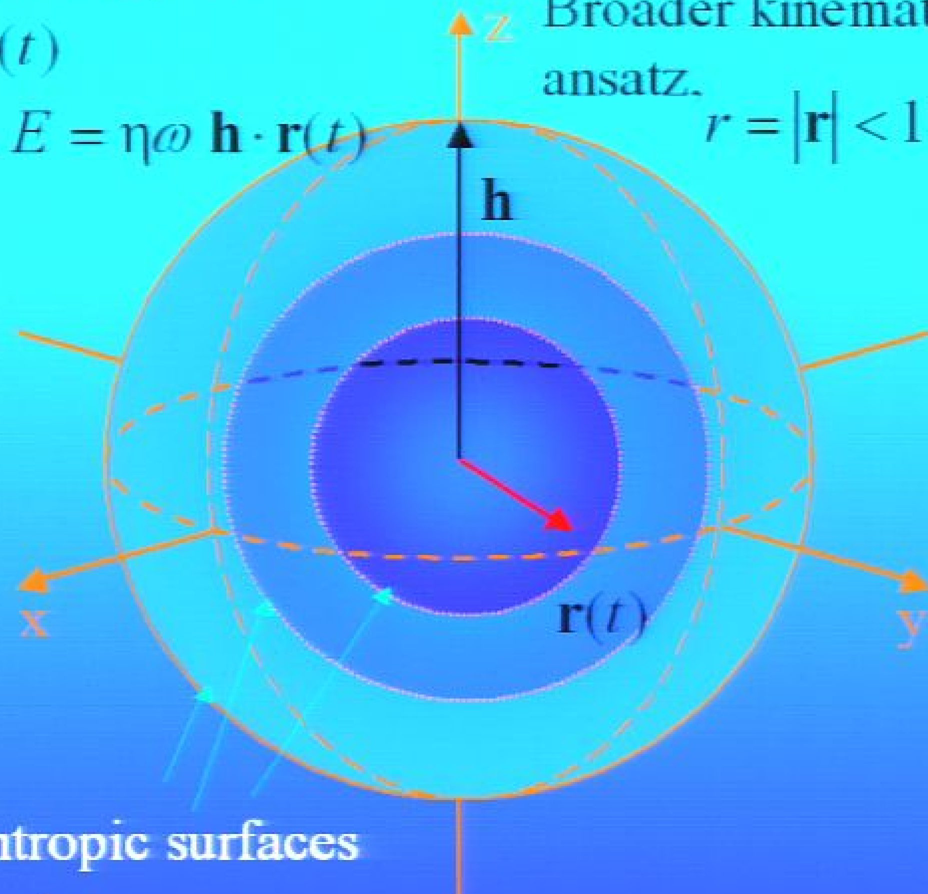
Broader kinematics ansatz, $r = |\mathbf{r}| < 1$





$$S = -k_B\left(\frac{1+r}{2}\ln\frac{1+r}{2} + \frac{1-r}{2}\ln\frac{1-r}{2}\right)$$

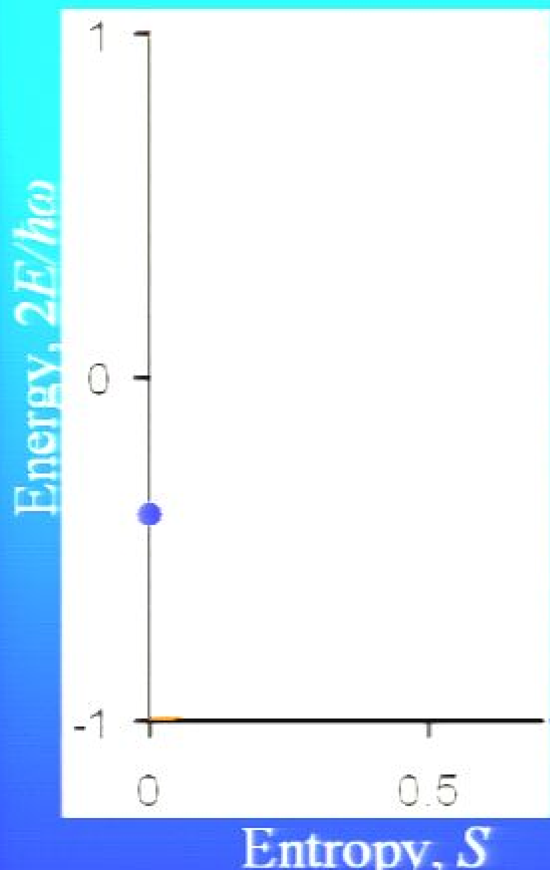
Isolated two-level system, Quantum Thermodynamics

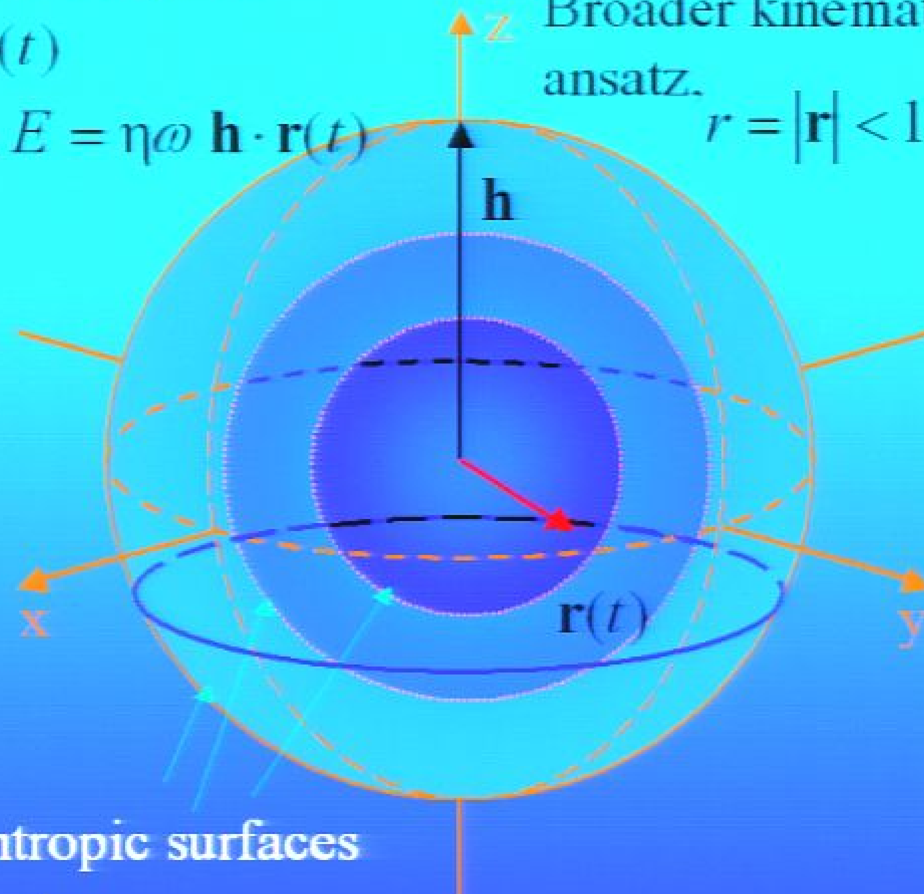
Hamiltonian, $\hbar\omega\, \mathbf{h}$

State, $\mathbf{r}(t)$

Energy, $E = \hbar\omega\, \mathbf{h}\cdot\mathbf{r}(t)$

Broader kinematics ansatz, $r = |\mathbf{r}| < 1$





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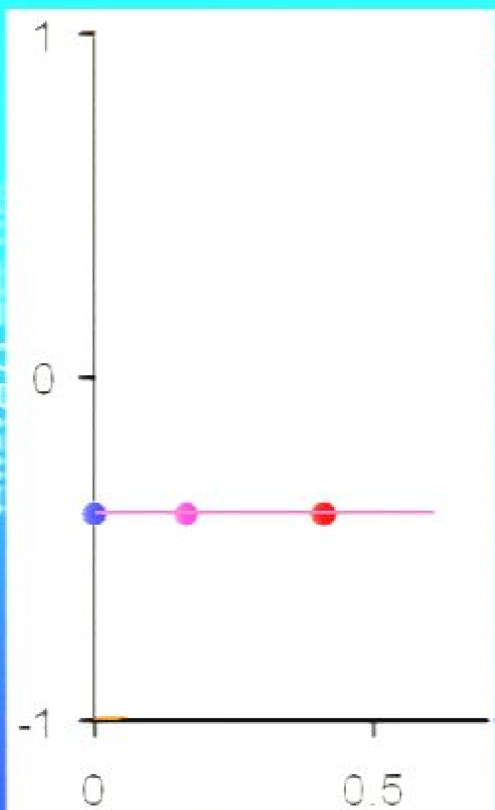
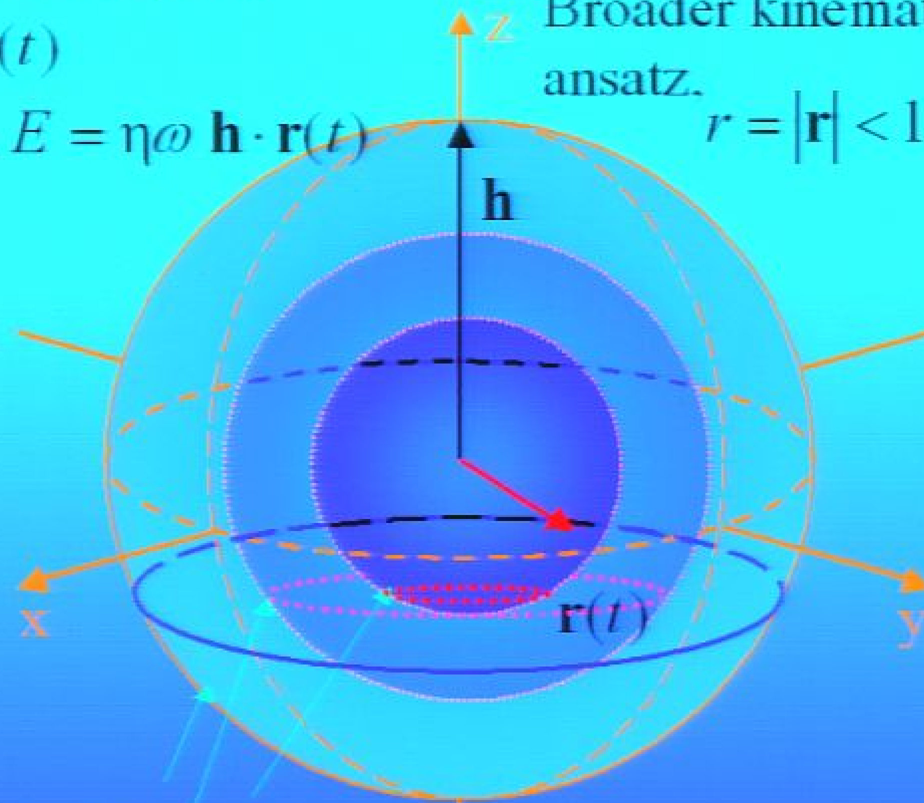


Entropy, S



Isoentropic surfaces

$$S = -k_B\left(\frac{1+r}{2}\ln\frac{1+r}{2} + \frac{1-r}{2}\ln\frac{1-r}{2}\right)$$

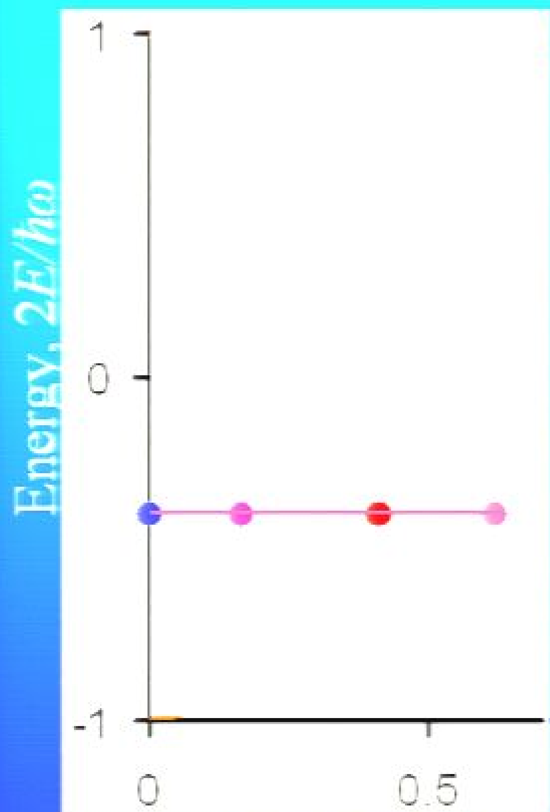
Isolated two-level system, Quantum Thermodynamics

Hamiltonian, $\eta\omega\ \mathbf{h}$

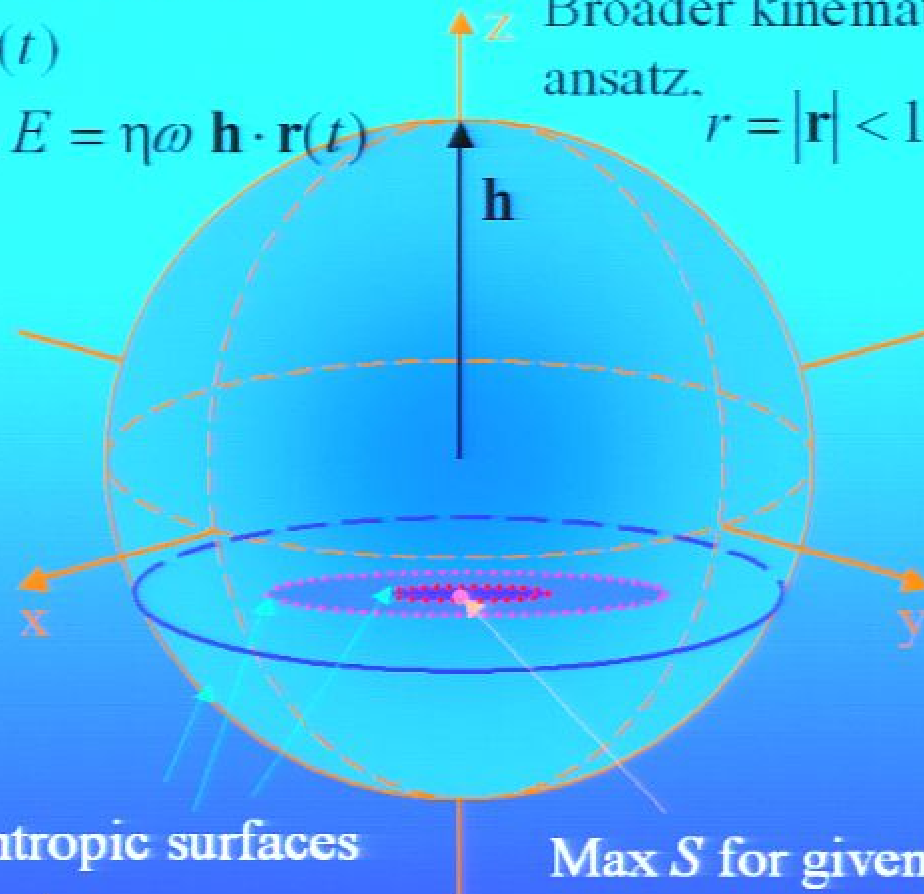
State, $\mathbf{r}(t)$

Energy, $E = \eta\omega\ \mathbf{h}\cdot\mathbf{r}(t)$

Broader kinematics ansatz, $r = |\mathbf{r}| < 1$



Entropy, S



Isoentropic surfaces

Max S for given E

$$S = -k_B\left(\frac{1+r}{2}\ln\frac{1+r}{2} + \frac{1-r}{2}\ln\frac{1-r}{2}\right)$$

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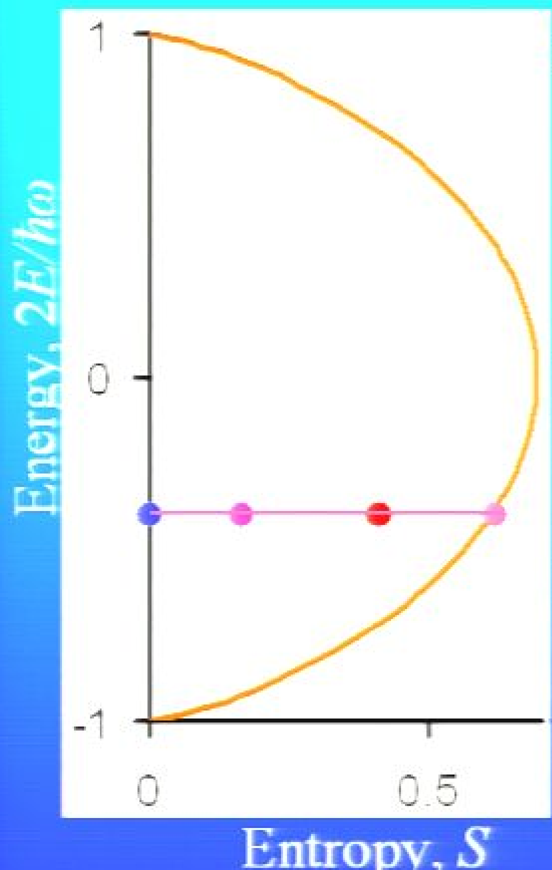

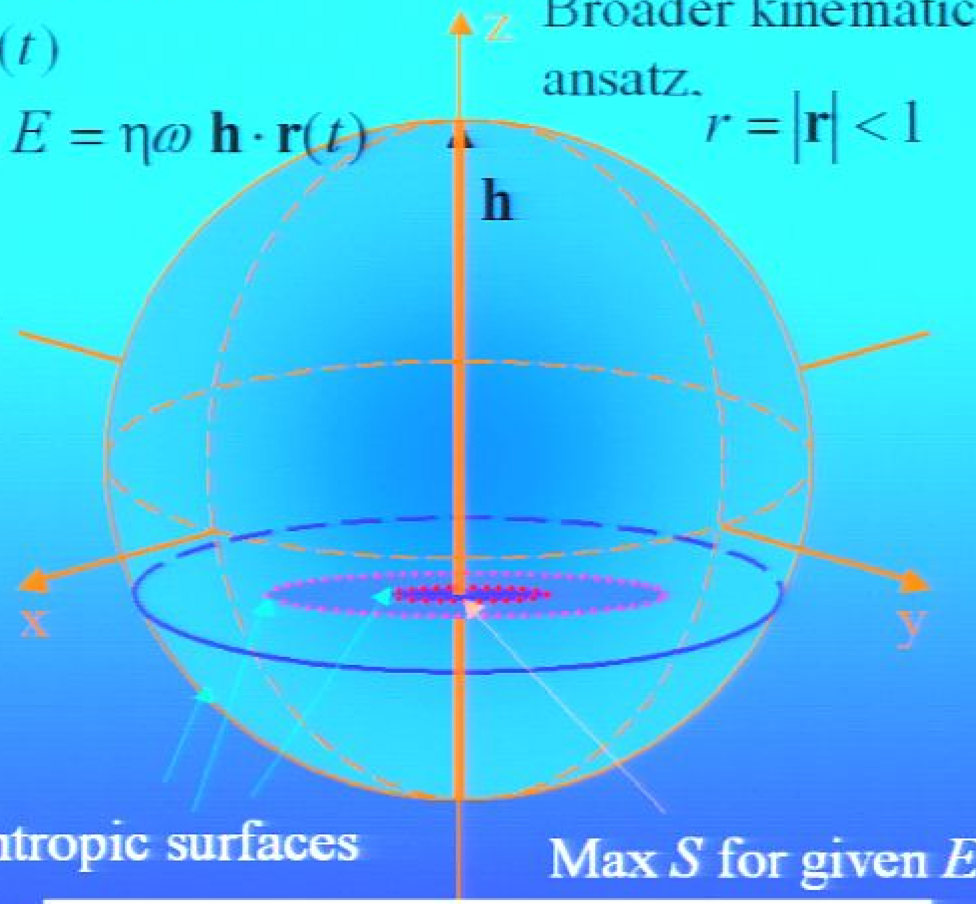

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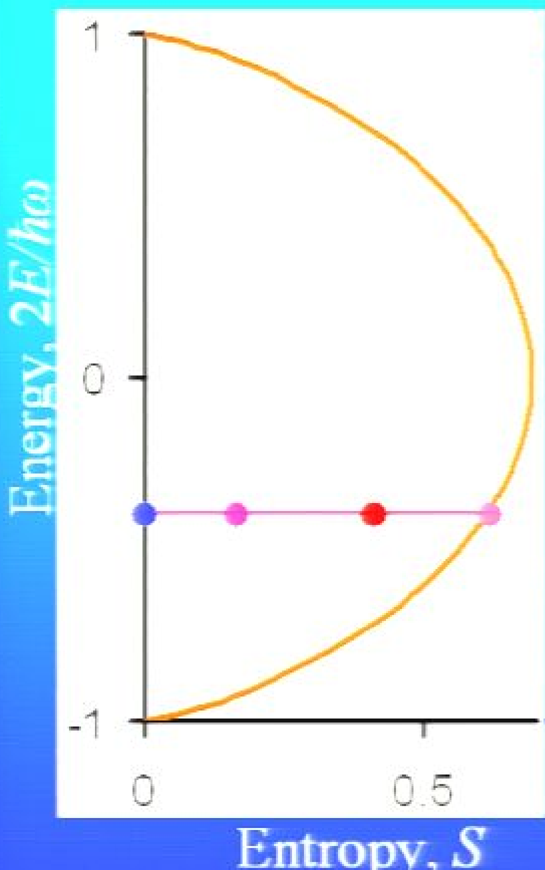
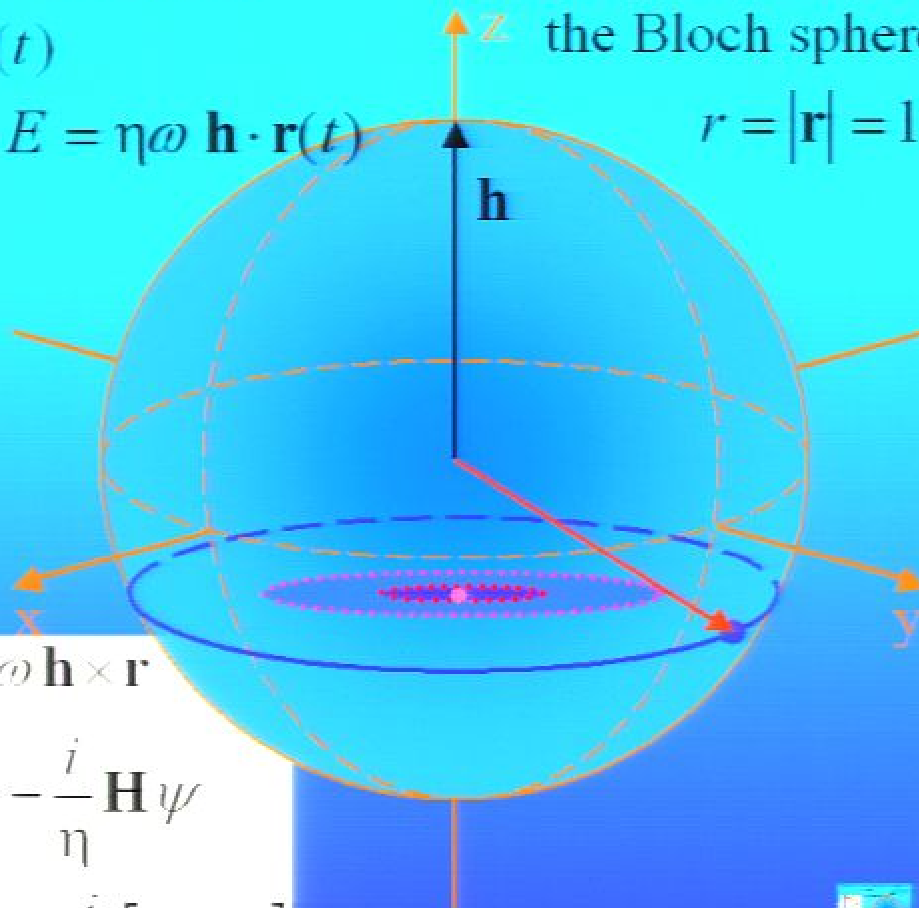
State, $\mathbf{r}(t)$

Energy, $E = \hbar\omega\,\mathbf{h}\cdot\mathbf{r}(t)$

On the surface of the Bloch sphere.

$r = |\mathbf{r}| = 1$





$$\dot{\mathbf{r}} = \omega\,\mathbf{h}\times\mathbf{r}$$

$$\dot{\psi} = -\frac{i}{\hbar}\mathbf{H}\,\psi$$

$$\dot{P}_\psi = -\frac{i}{\hbar}\left[\mathbf{H}, P_\psi\right]$$

$S = 0$

Int.J.Theor.Phys., 24, 119 (1985)

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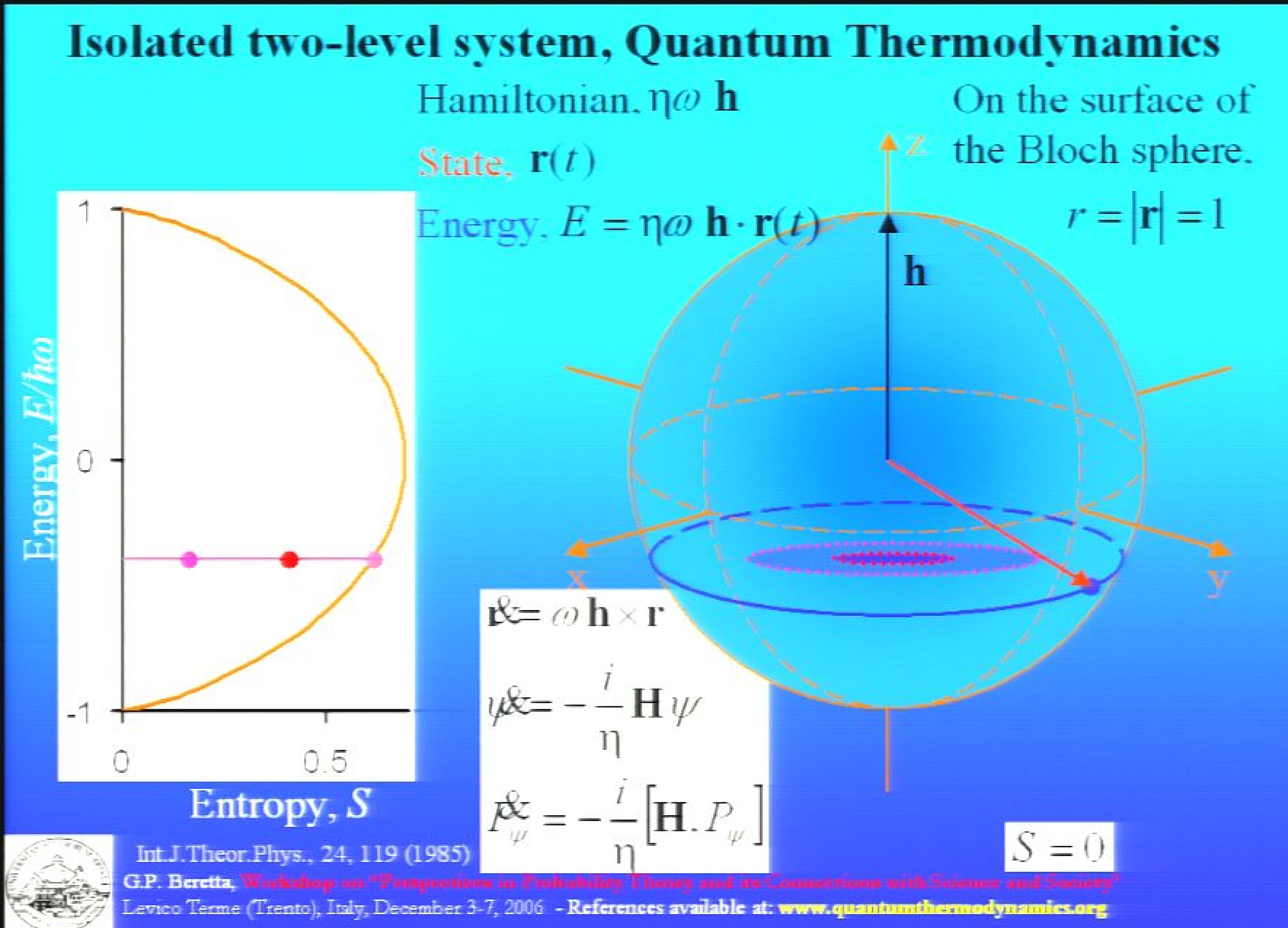
Isolated two-level system, Quantum Thermodynamics
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State, $\mathbf{r}(t)$
Energy, $E = \hbar\omega\,\mathbf{h}\cdot\mathbf{r}(t)$
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$r = |\mathbf{r}| = 1$
z
h
x
y
1
0
-1
0
0.5
Energy, $E/\hbar\omega$
Entropy, S
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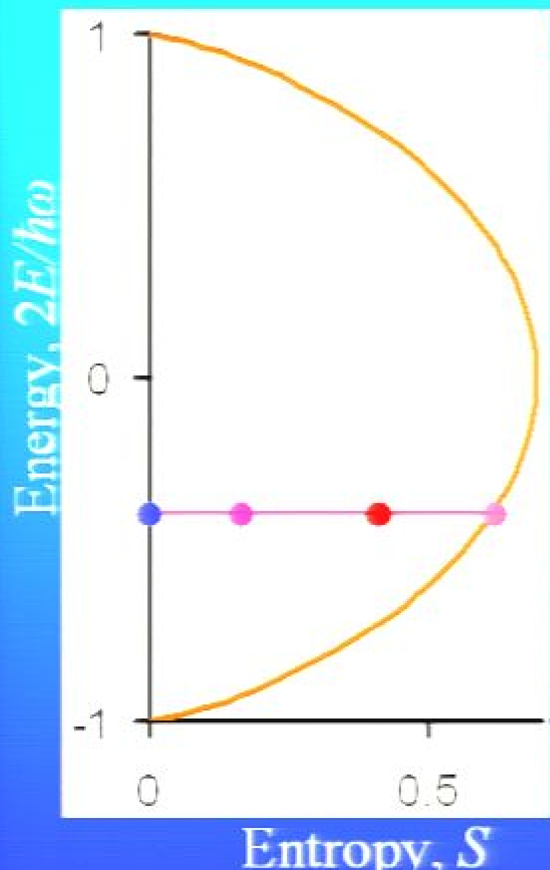

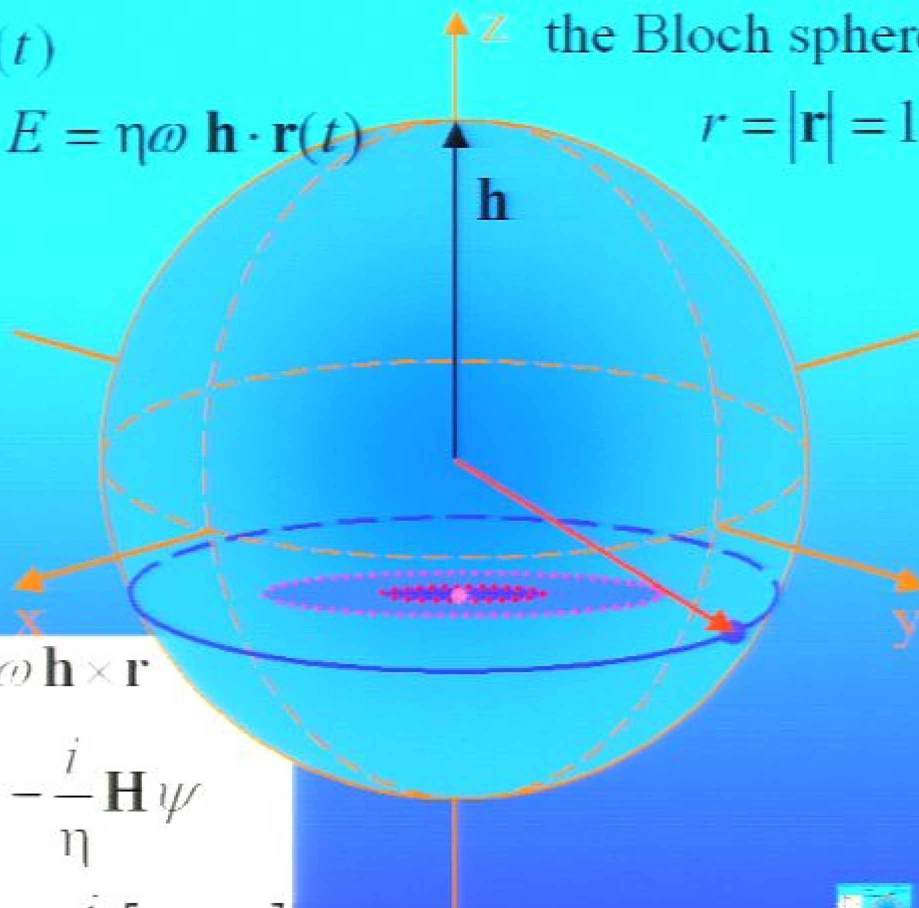

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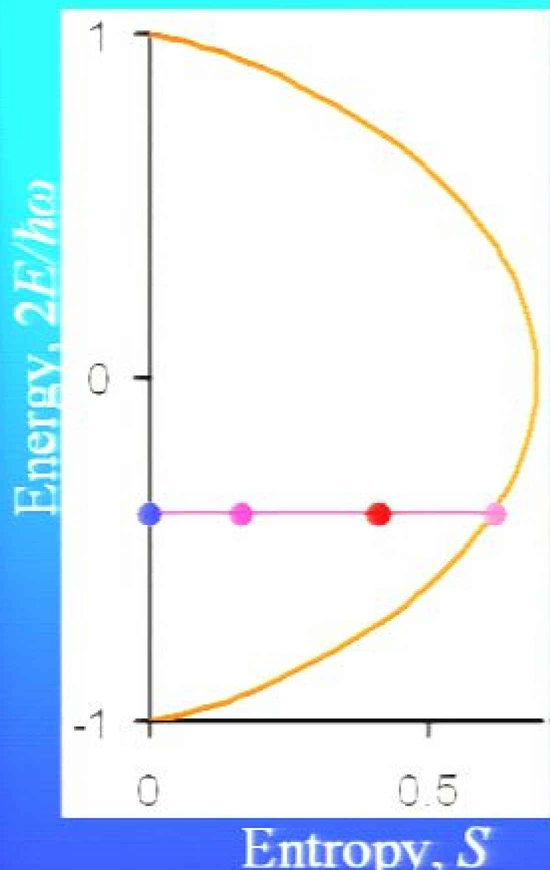

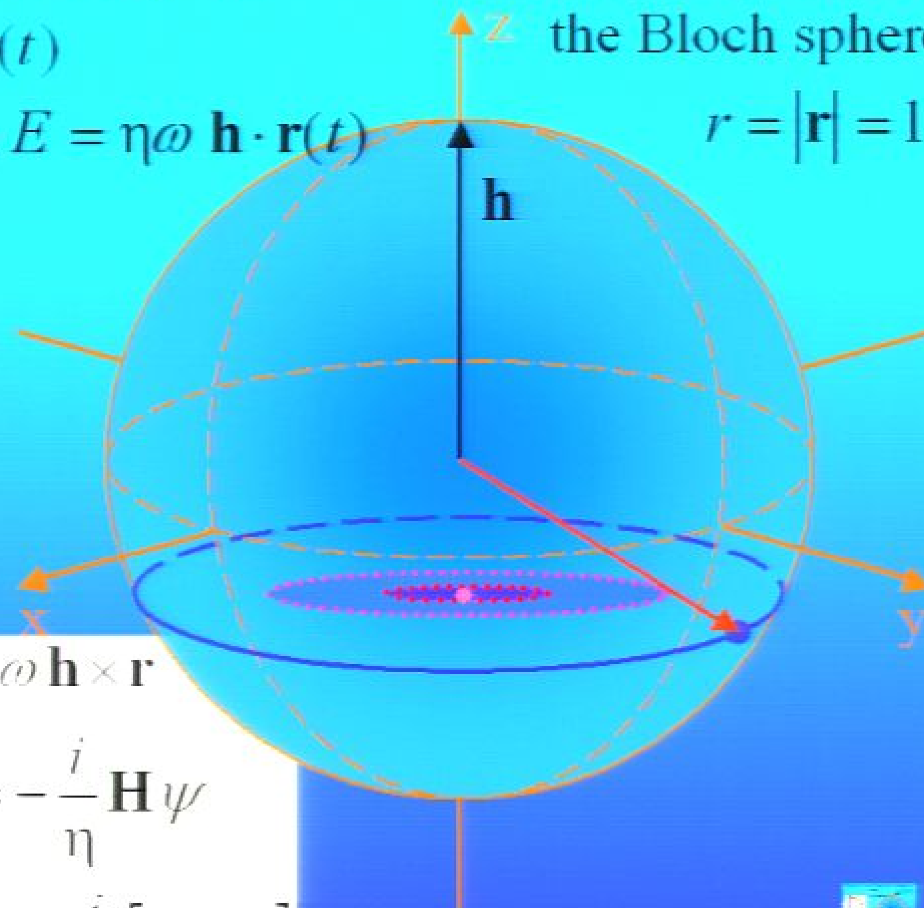

$$\dot{\mathbf{r}} = \omega\,\mathbf{h}\times\mathbf{r}$$

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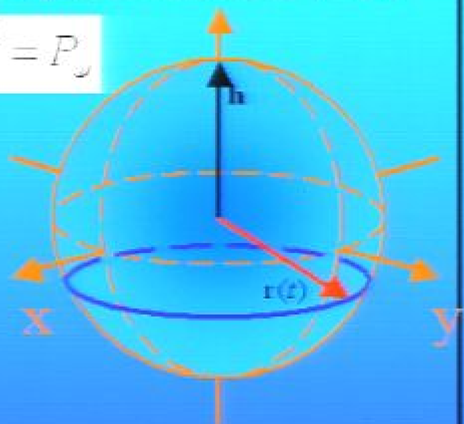
If entropy an intrinsic property of matter…

Isolated and uncorrelated two-level quantum system (quBit)

Quantum Mechanics

Bloch sphere surface, $r = 1$

$\rho = \rho^2 = P_\psi$



Dynamical law (Schrodinger)

$$\dot\psi = -\frac{i}{\hbar} H \psi$$

$$\dot P_\psi = -\frac{i}{\hbar}\left[H, P_\psi\right]$$

$$\dot{\mathbf{r}} = \omega \mathbf{h} \times \mathbf{r}$$

If entropy an intrinsic property of matter…

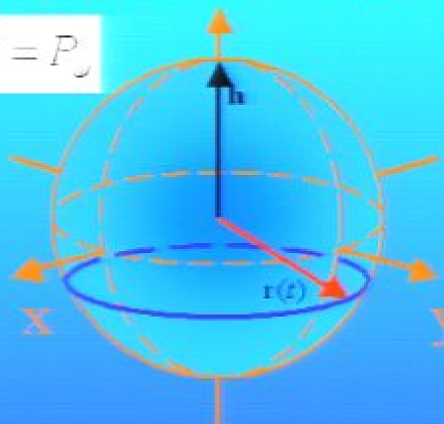
…is irreversibility an intrinsic feature of microscopic dynamics?

Isolated and uncorrelated two-level quantum system (quBit)

Quantum Mechanics

Bloch sphere surface, $r = 1$

$\rho = \rho^2 = P_\psi$



Dynamical law (Schrodinger)

$\dot{\psi} = -\frac{i}{\hbar}\mathrm{H}\psi$

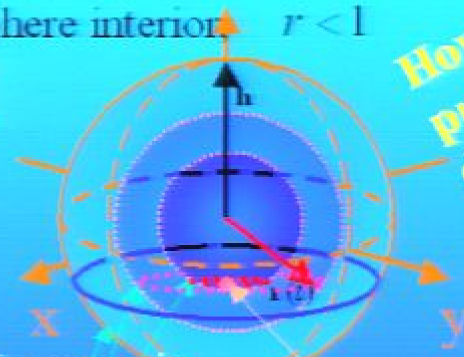
$\dot{P}_\psi = -\frac{i}{\hbar}\left[\mathrm{H}, P_\psi\right]$

$\dot{\mathbf{r}} = \omega \mathbf{h} \times \mathbf{r}$

Quantum Thermodynamics

Hatsopoulos-Gyftopoulos ansatz: also Bloch sphere interior, $r < 1$

$\rho \neq \rho^2$



$S = -k_B\left[\frac{1+r}{2}\ln\frac{1+r}{2} + \frac{1-r}{2}\ln\frac{1-r}{2}\right]$

Dynamical law ?

How can we extend pure state unitary dynamics to include second law?

Irreversibility emerges as an intrinsic feature of microscopic dynamics.

Fundamental ansatz about irreversible steepest-entropy-ascent quantum dynamics

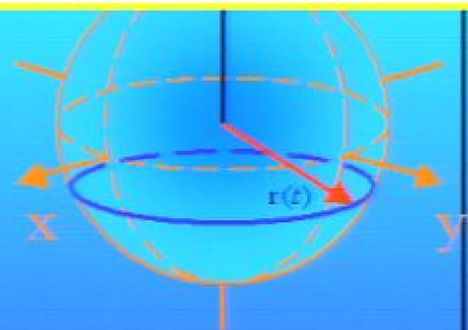
$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] - \frac{1}{\tau}\frac{\begin{vmatrix} \rho\ln\rho & \rho & \frac{1}{2}\{H,\rho\} \\ \mathrm{Tr}\rho\ln\rho & 1 & \mathrm{Tr}\rho H \\ \mathrm{Tr}\rho H\ln\rho & \mathrm{Tr}\rho H & \mathrm{Tr}\rho H^2 \end{vmatrix}}{\mathrm{Tr}\rho H^2 - (\mathrm{Tr}\rho H)^2}$$

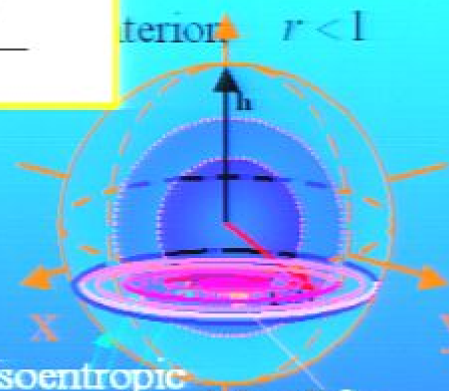
...tem (quBit)

...rmodynamics

...oulos ansatz:

...terior $r<1$





Dynamical law (Schrodinger)

$$\dot{\psi} = -\frac{i}{\hbar}\mathrm{H}\psi$$

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$$\dot{\mathbf{r}} = \omega\mathbf{h}\times\mathbf{r}$$

$$S = -k_B\left[\frac{1+r}{2}\ln\frac{1+r}{2} + \frac{1-r}{2}\ln\frac{1-r}{2}\right]$$

Dynamical law

$$\dot{\mathbf{r}} = \omega\,\mathbf{h}\;\;\mathbf{r} - \frac{f(r)}{\tau}\mathbf{h}\;\;\mathbf{r}\;\;\mathbf{h}$$

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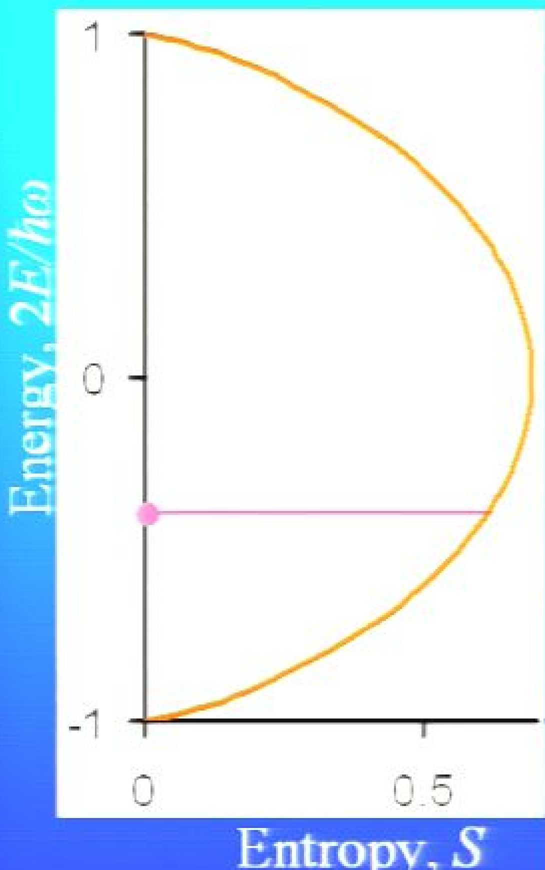
Isolated two-level system, Quantum Thermodynamics

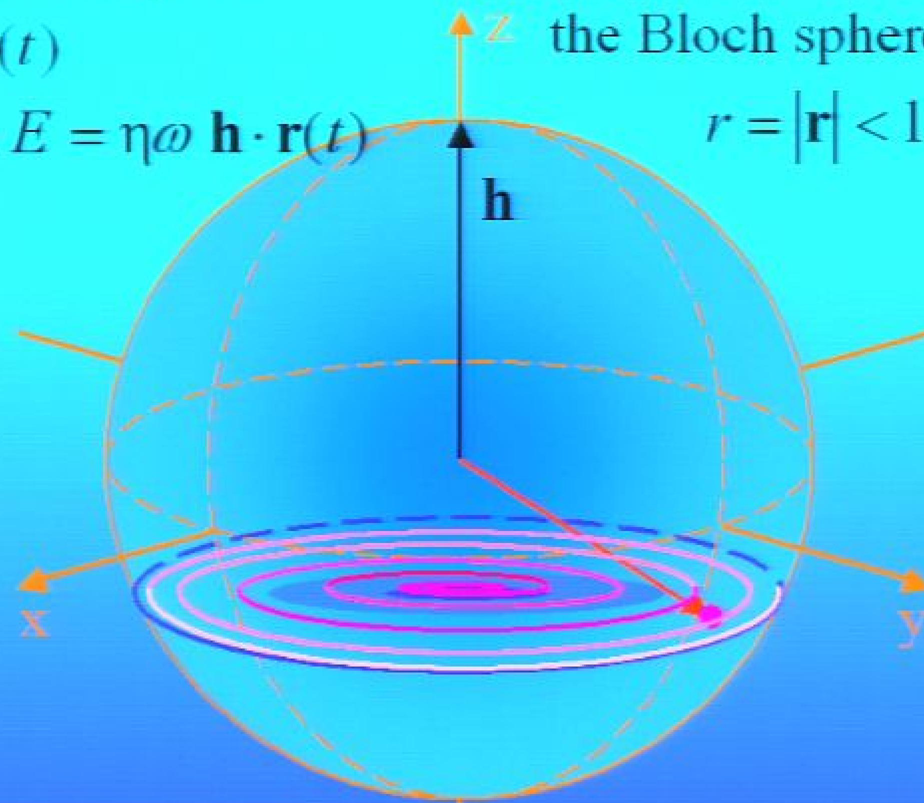
Hamiltonian, $\eta\omega\, \mathbf{h}$

State, $\mathbf{r}(t)$

Energy, $E = \eta\omega\, \mathbf{h}\cdot\mathbf{r}(t)$

Inside the Bloch sphere, $r = |\mathbf{r}| < 1$





$$\dot{\mathbf{r}} = \omega\,\mathbf{h}\times\mathbf{r} - \frac{f(r)}{\tau}\,\mathbf{h}\times\mathbf{r}\times\mathbf{h}$$

$S \neq 0$

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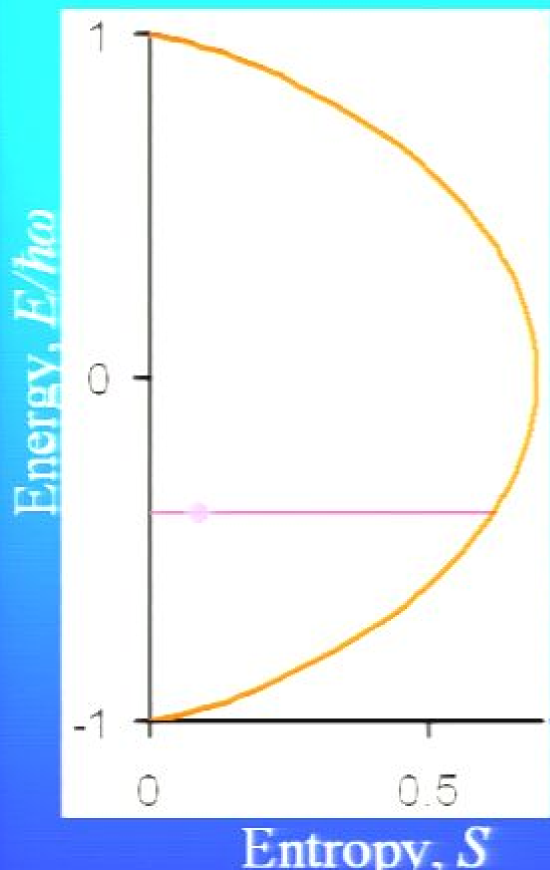

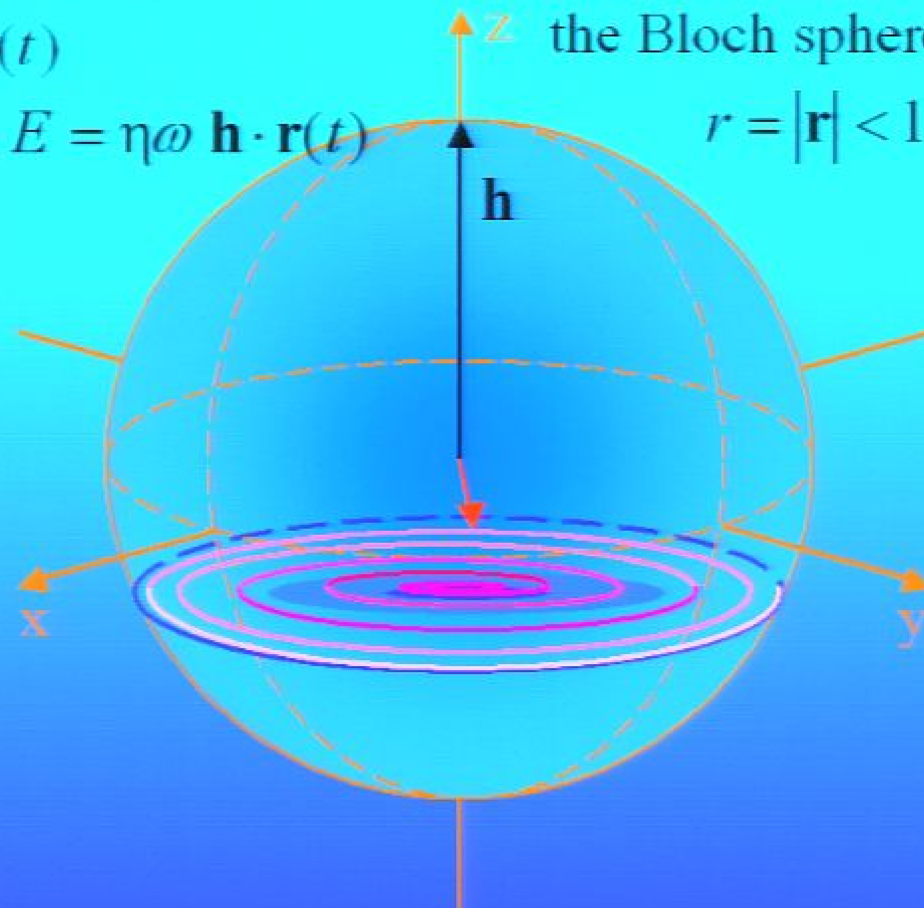

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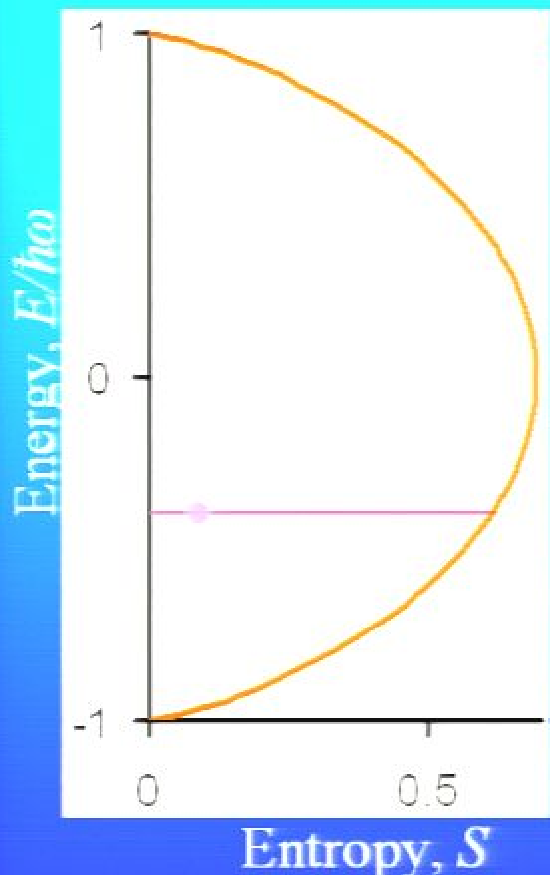

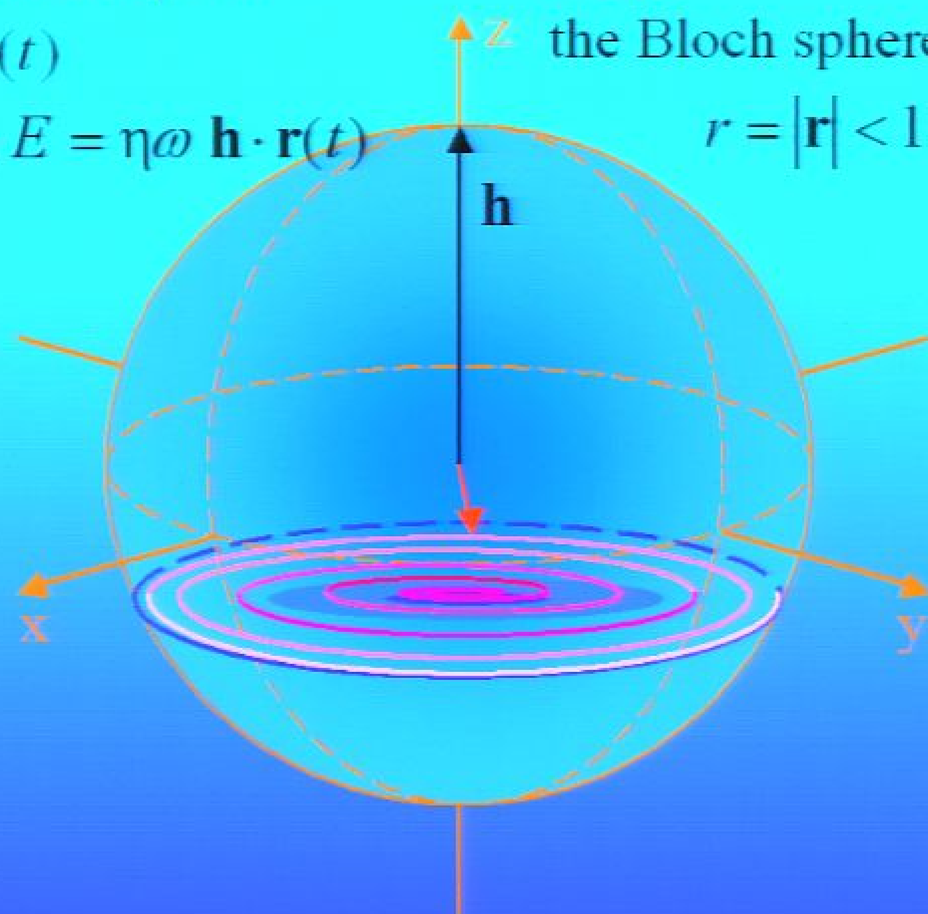

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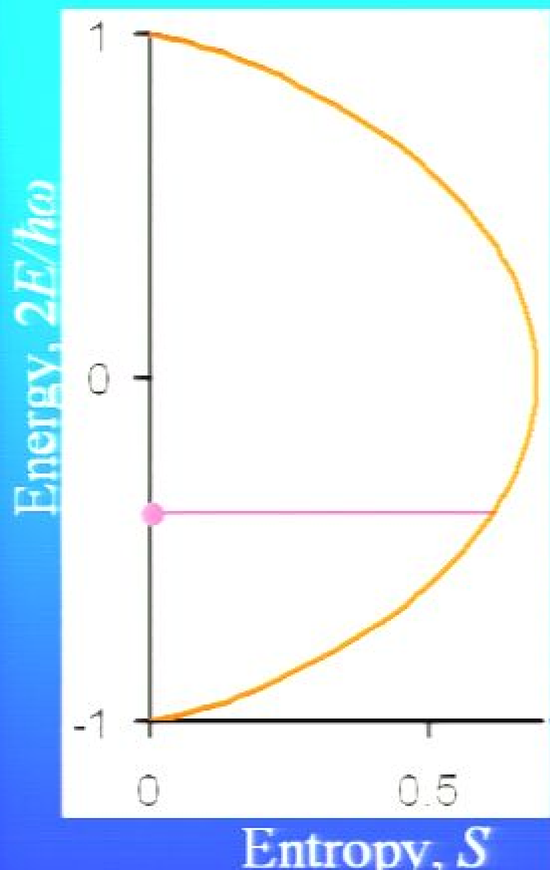

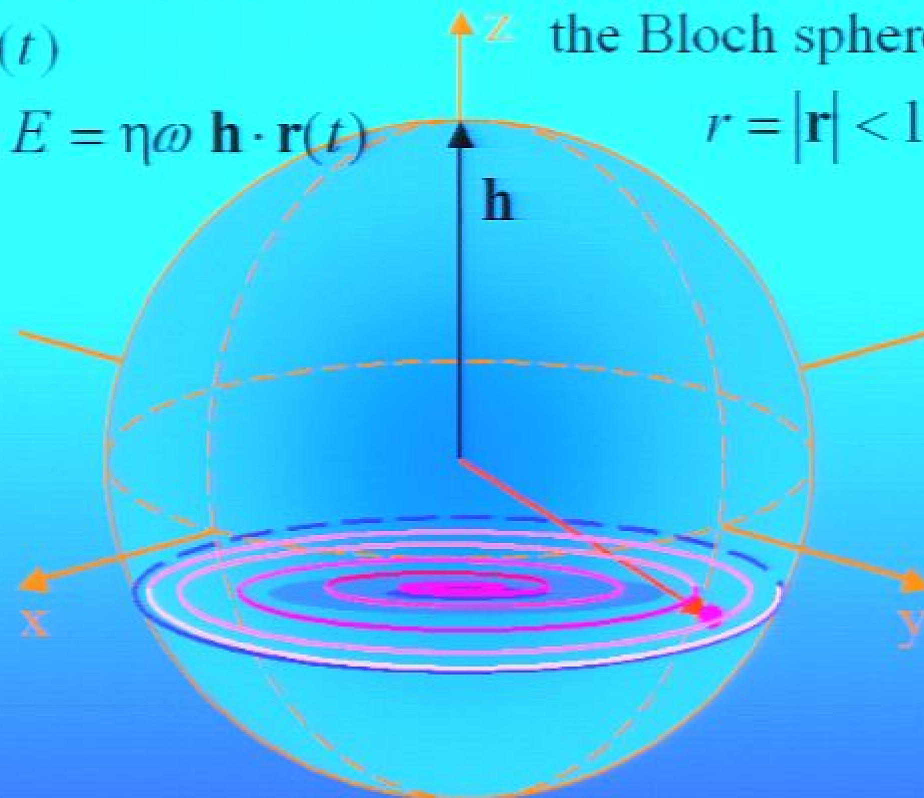

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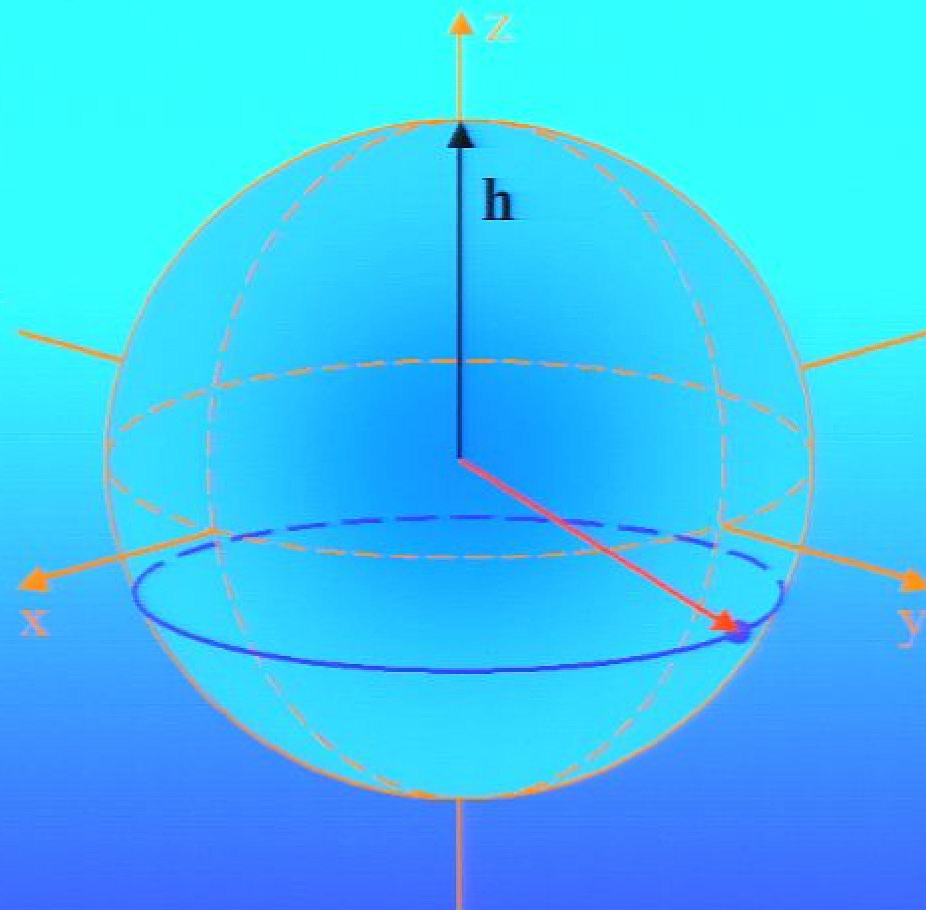
Schrödinger's prescient forecast

Proc. Cambridge Phil. Soc., 32, 446 (1936)

"… in a domain which the present theory (Quantum Mechanics) does not cover, there is room for new assumptions without necessarily contradicting the theory in that region where it is backed by experiment."



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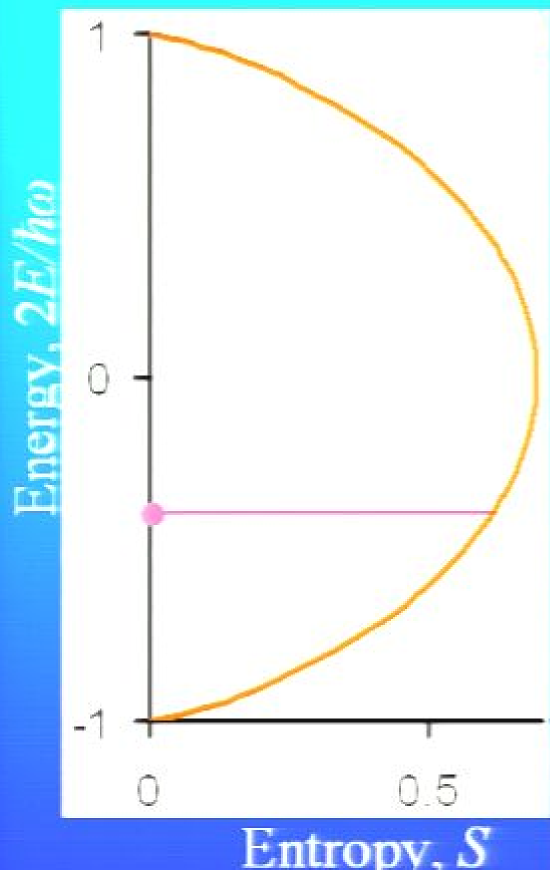

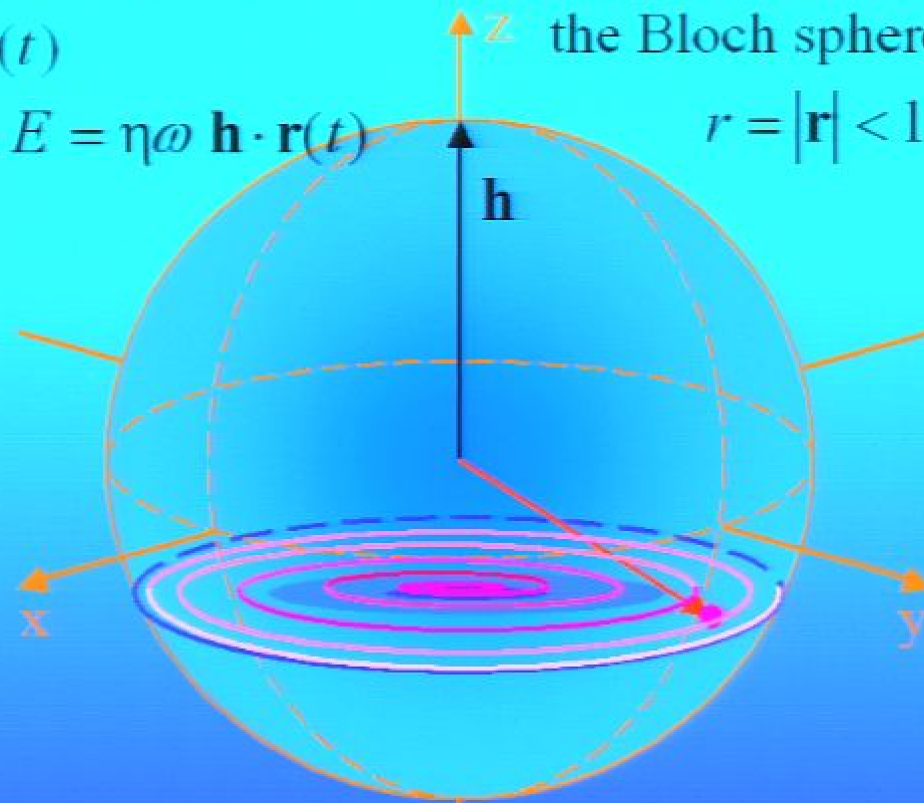

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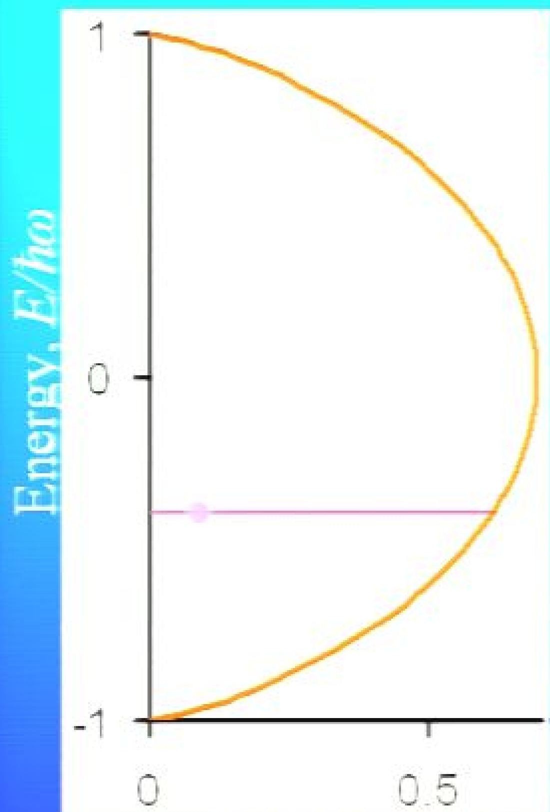

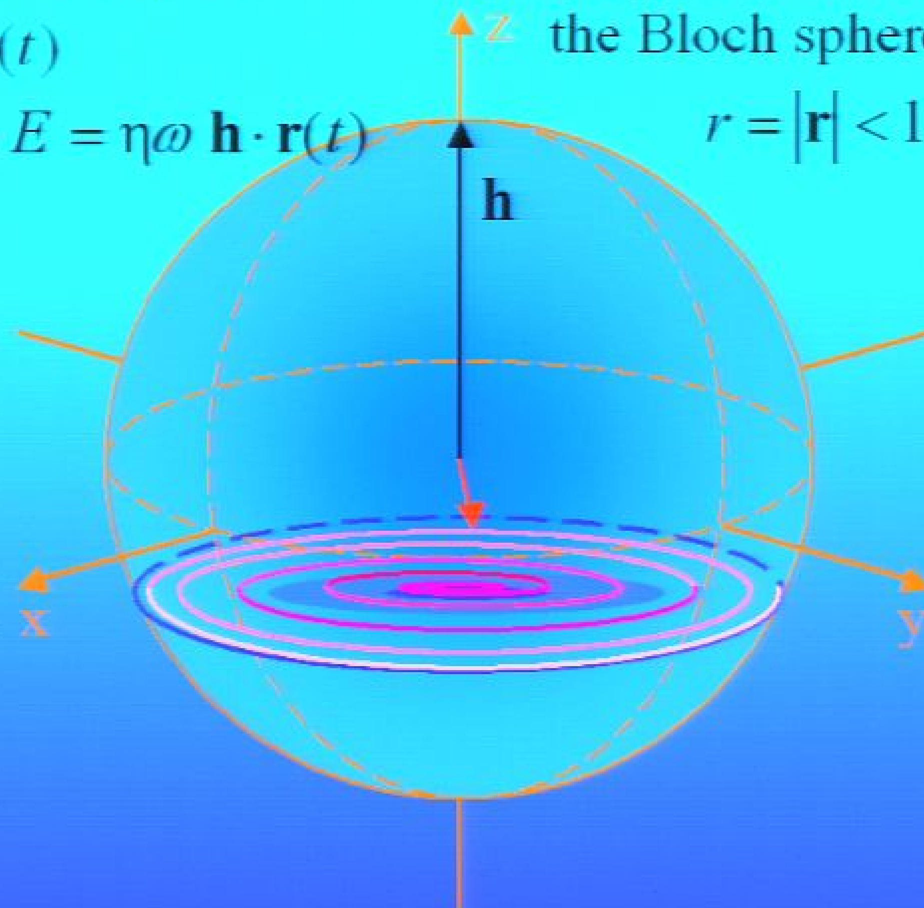

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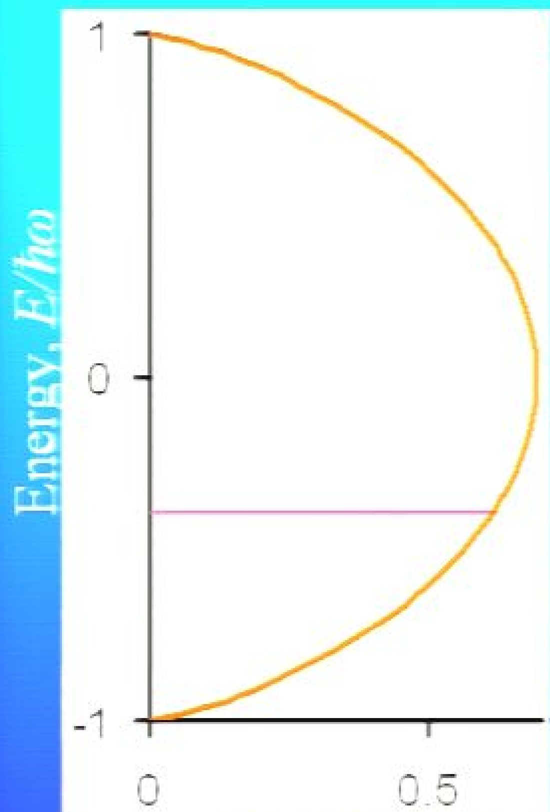
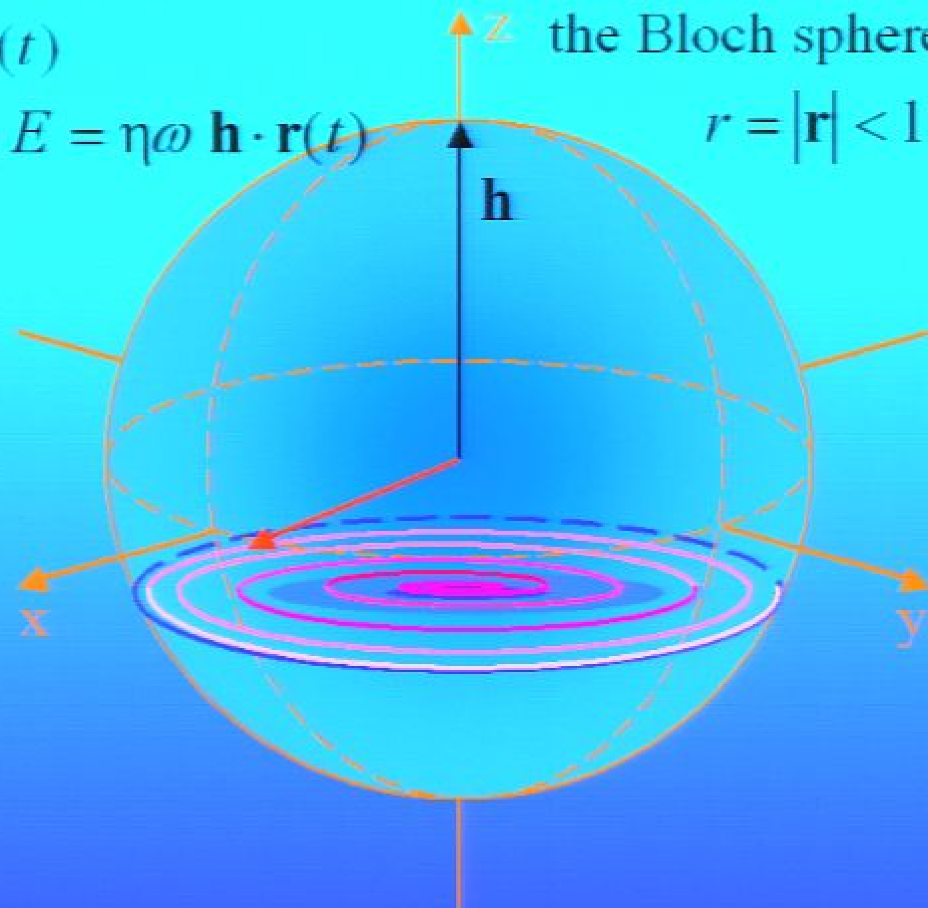


Entropy, S



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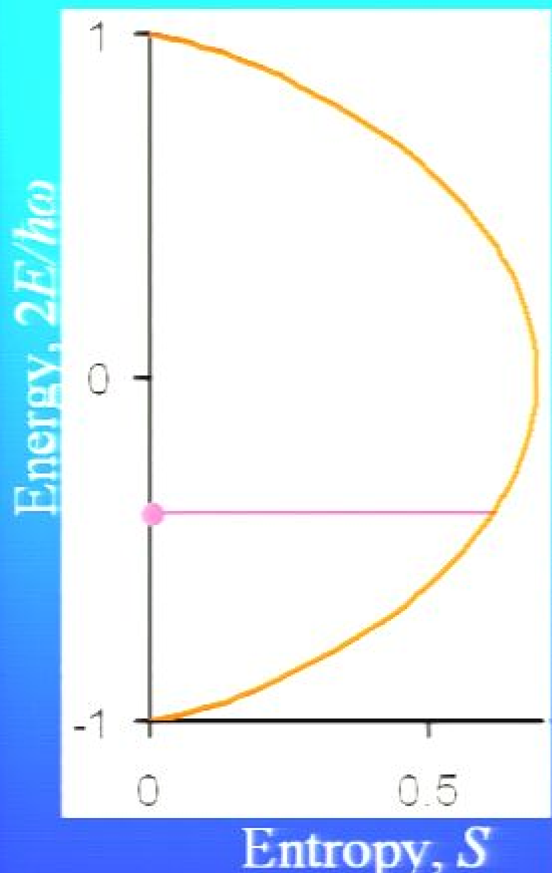

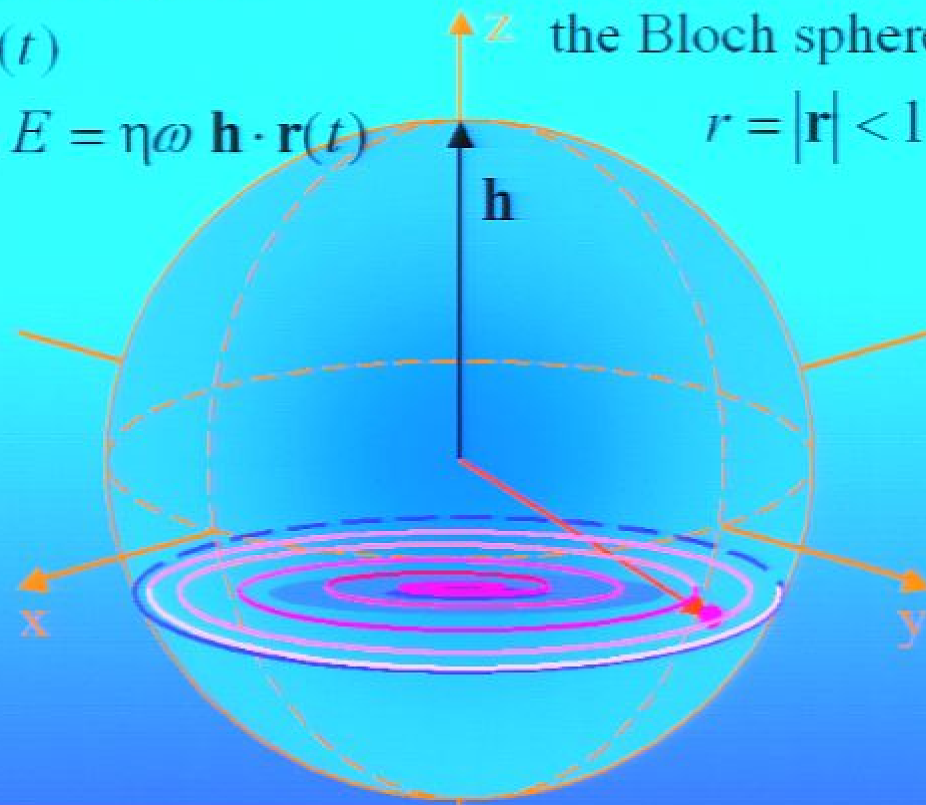

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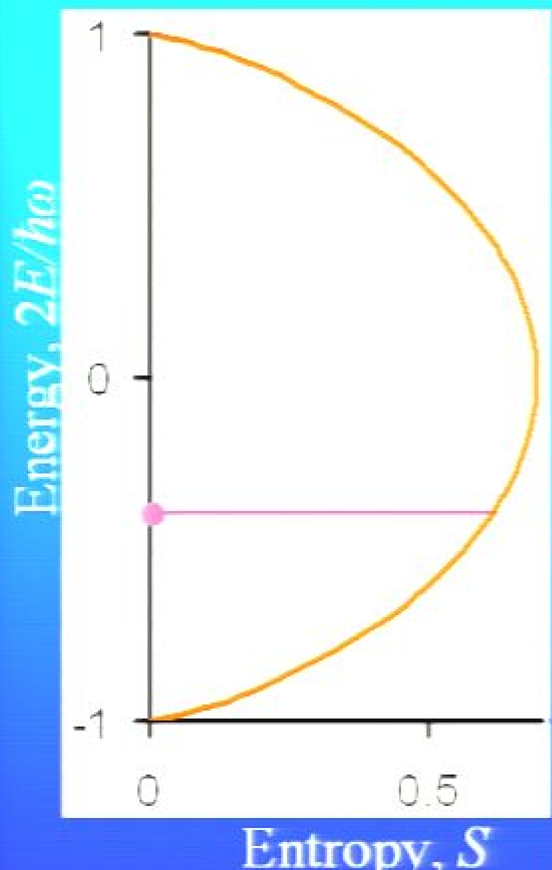

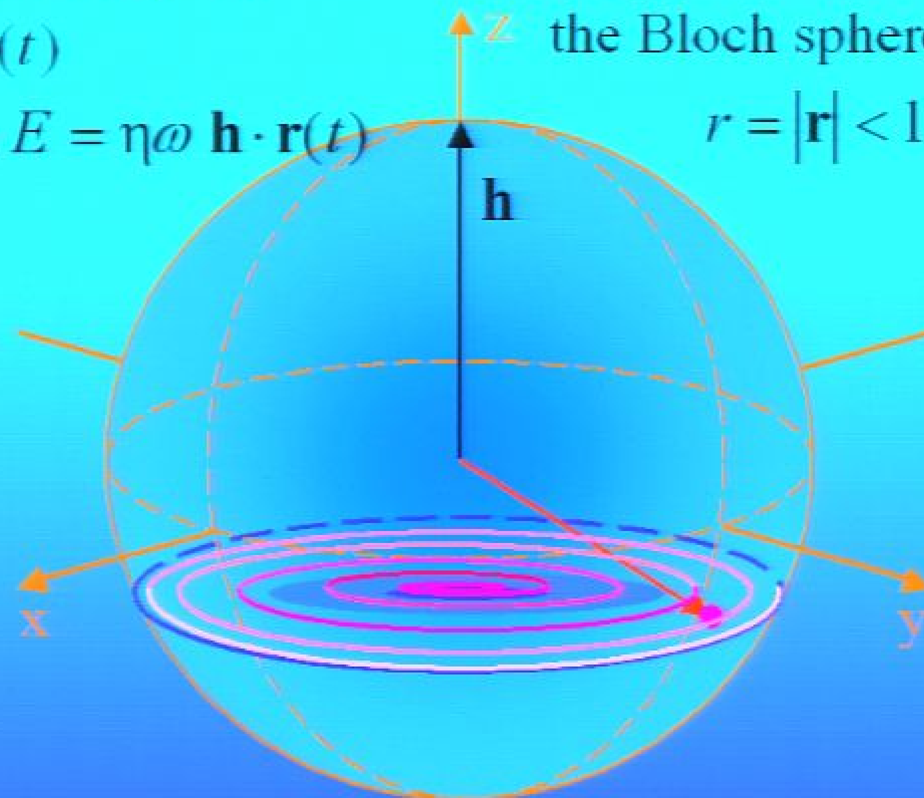

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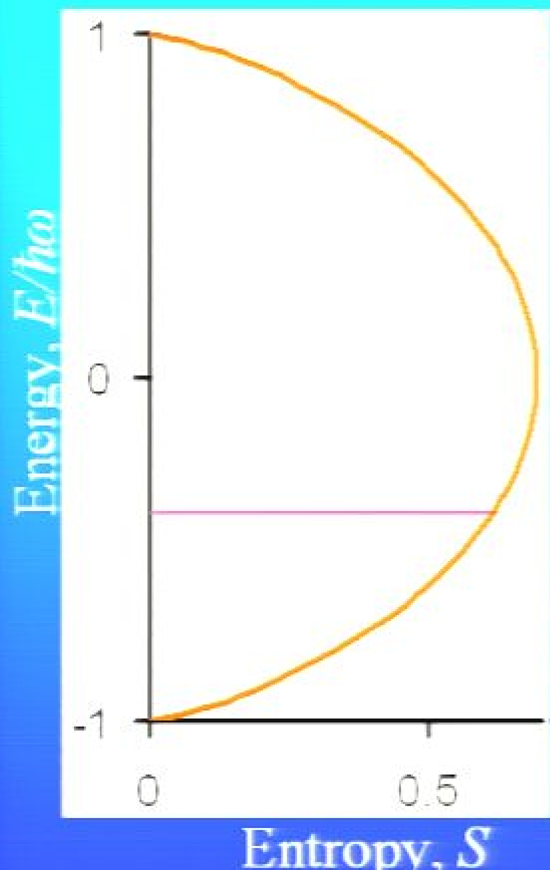

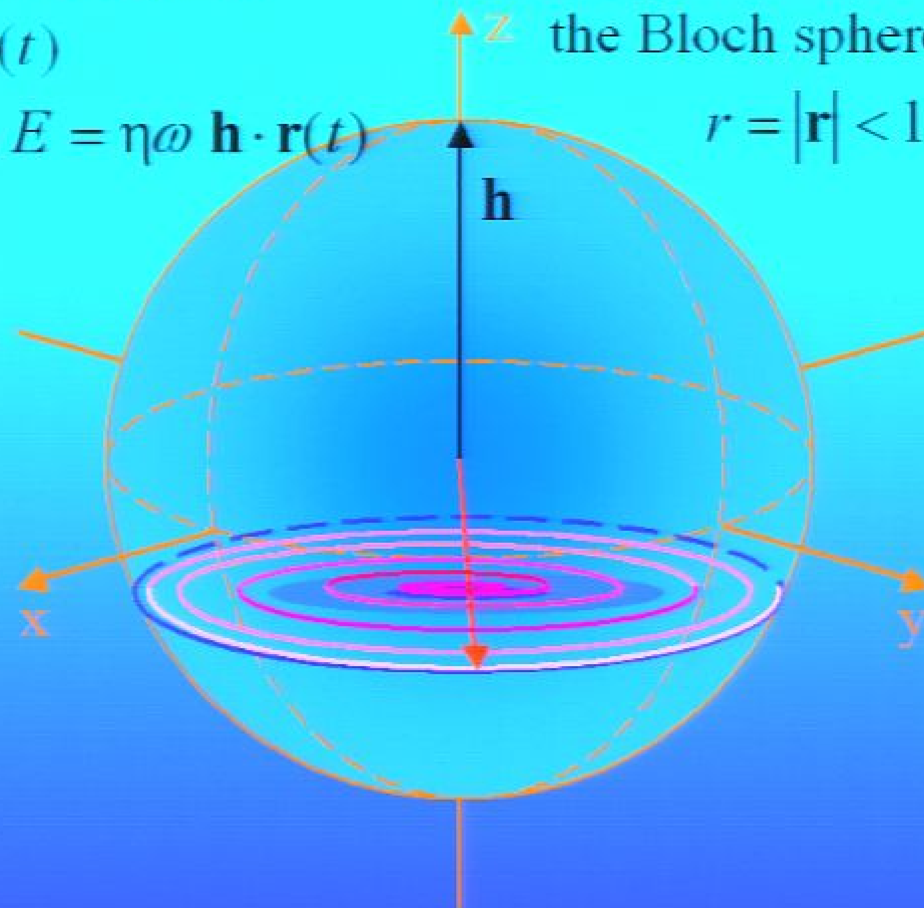

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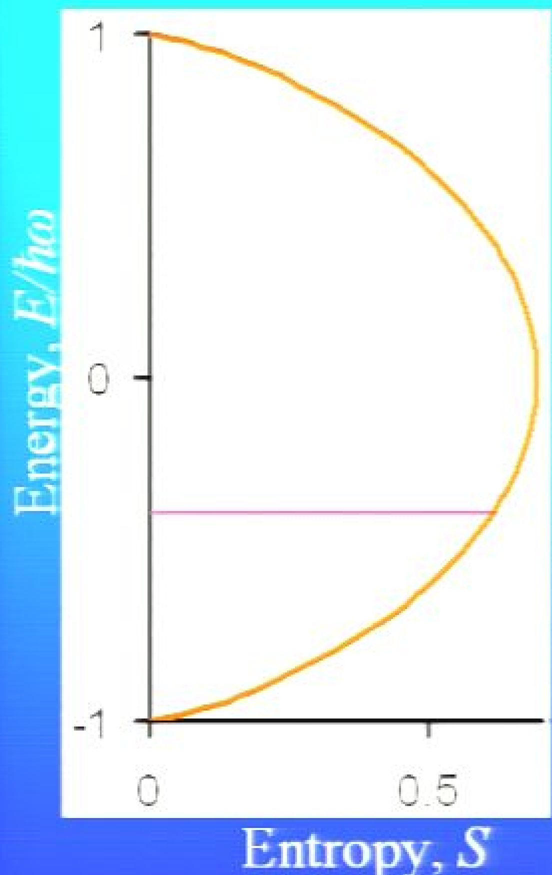

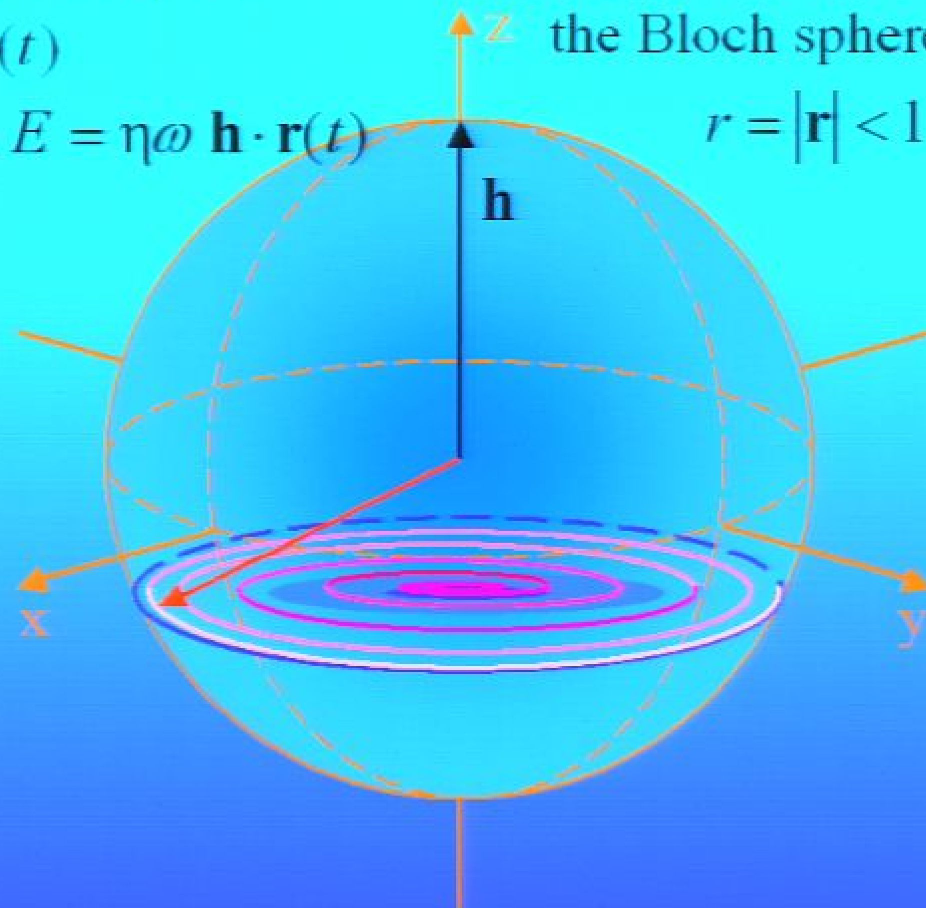

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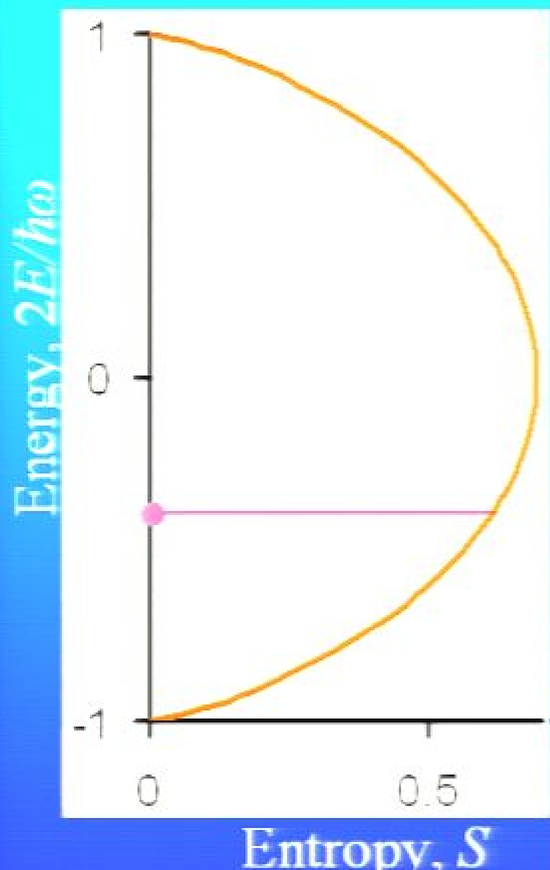

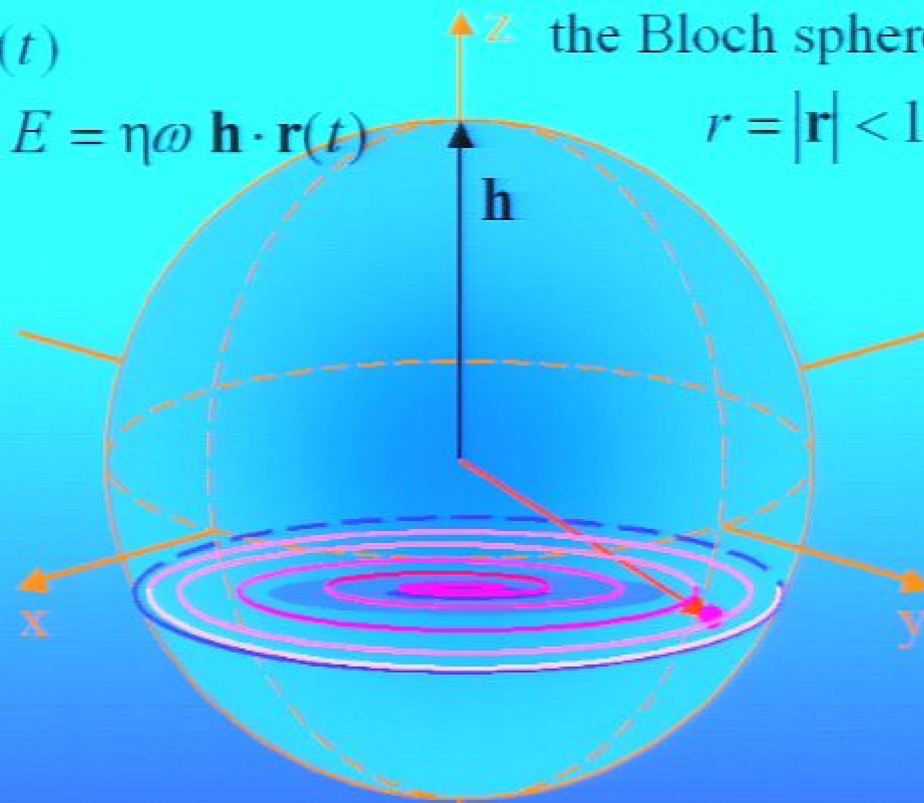

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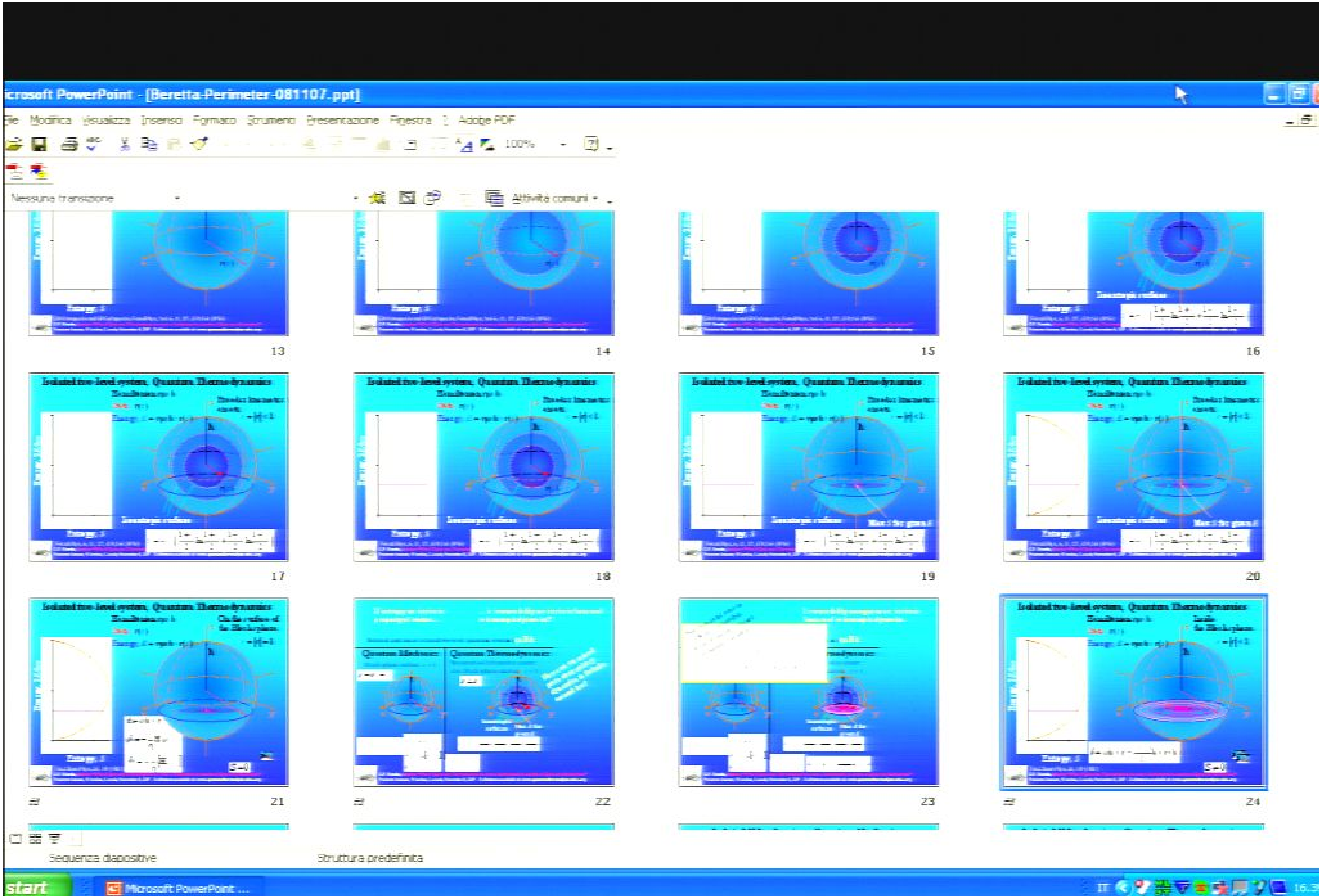
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Isolated two-level system, Quantum Thermodynamics
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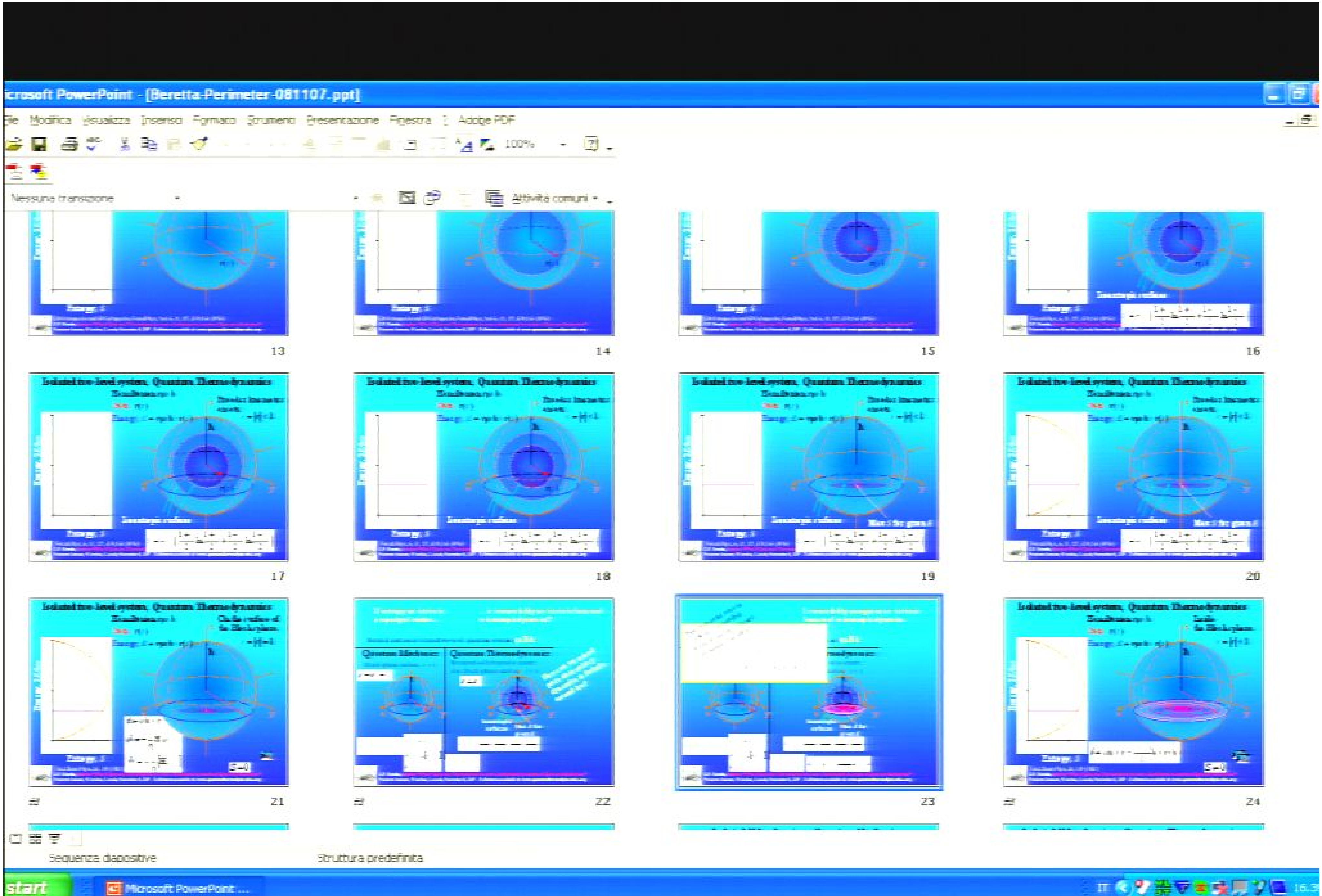
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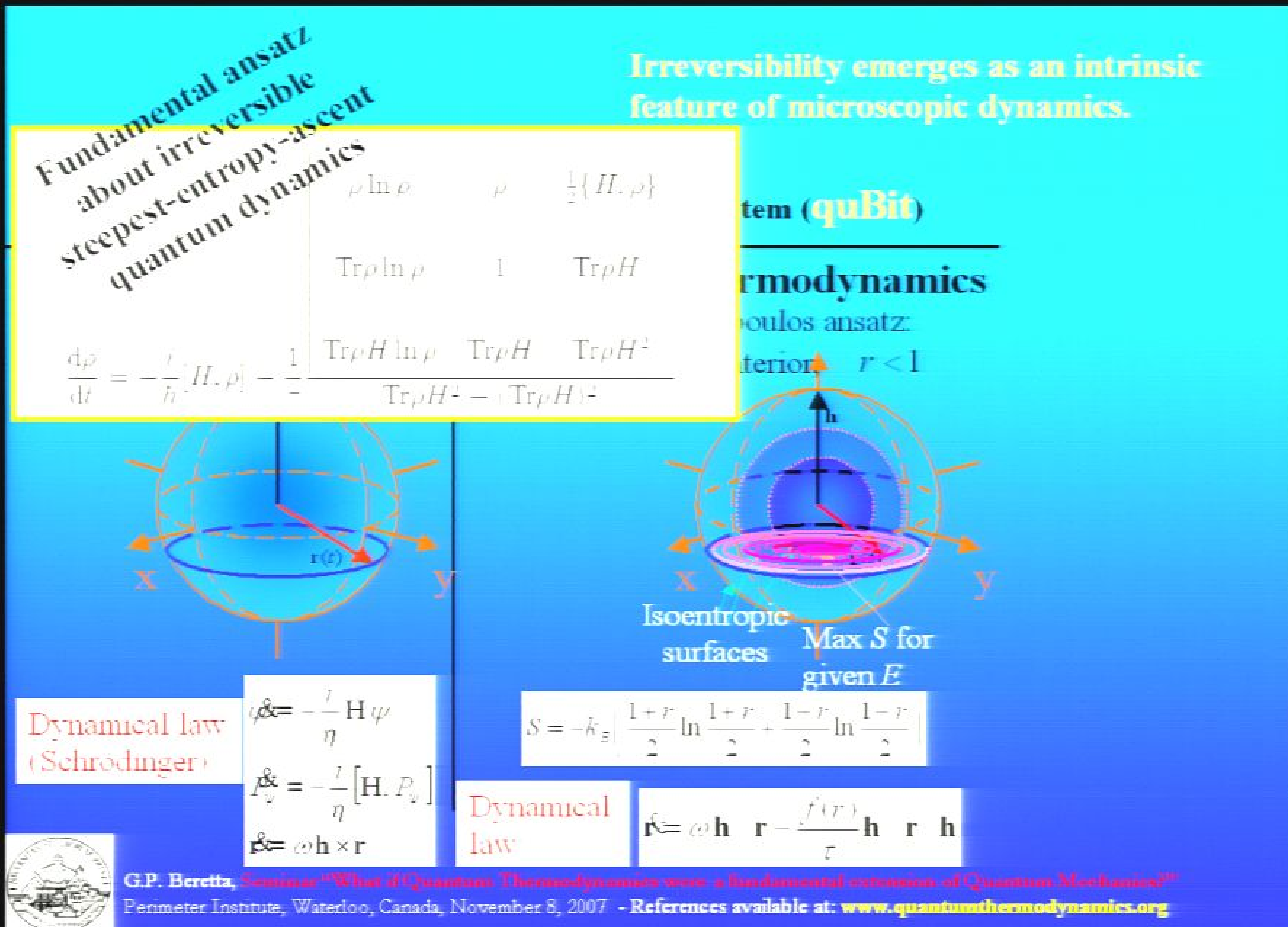
Irreversibility emerges as an intrinsic feature of microscopic dynamics.
Fundamental ansatz about irreversible steepest-entropy-ascent quantum dynamics
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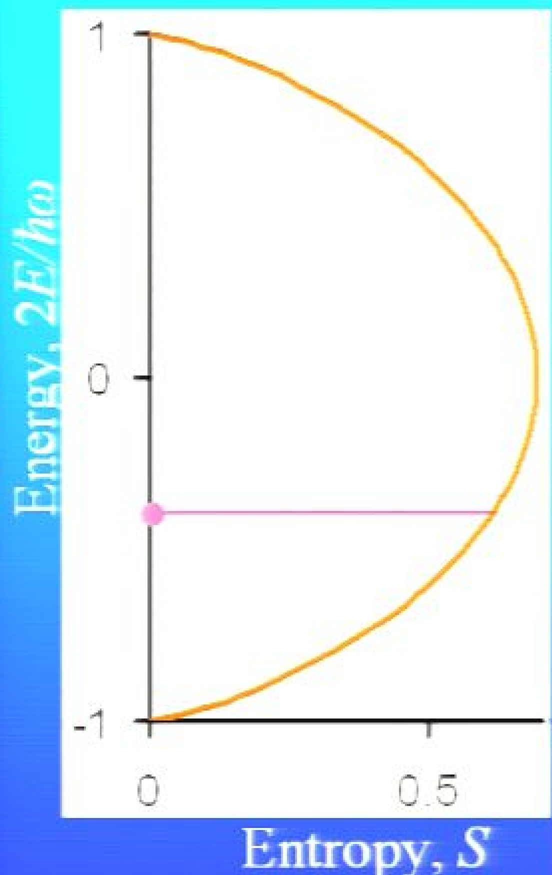

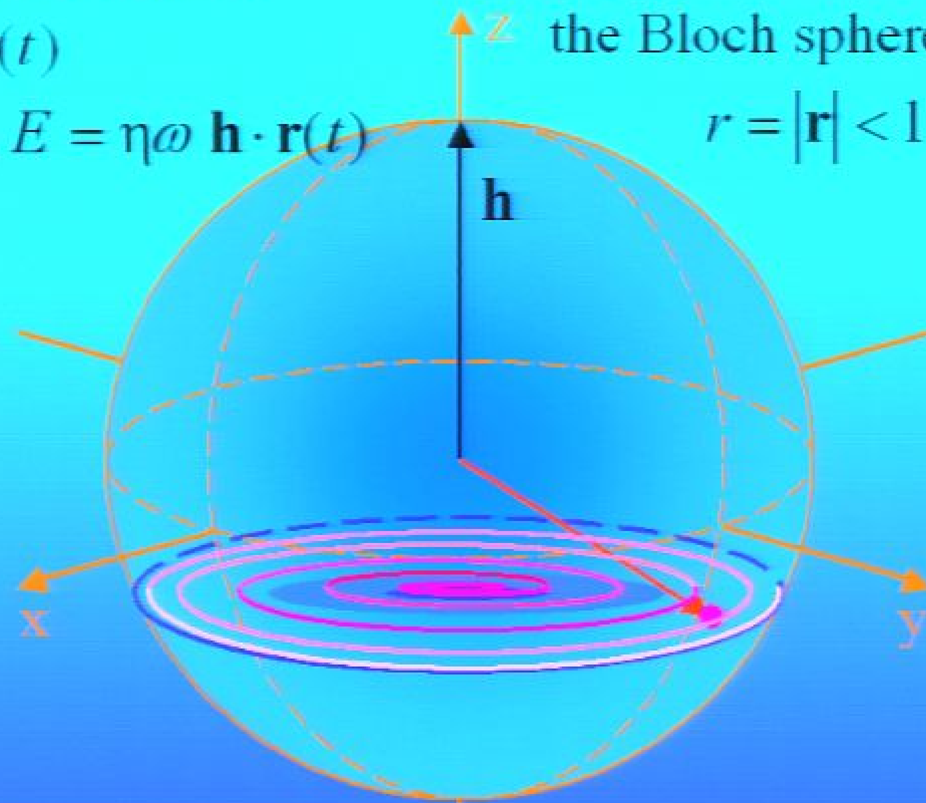

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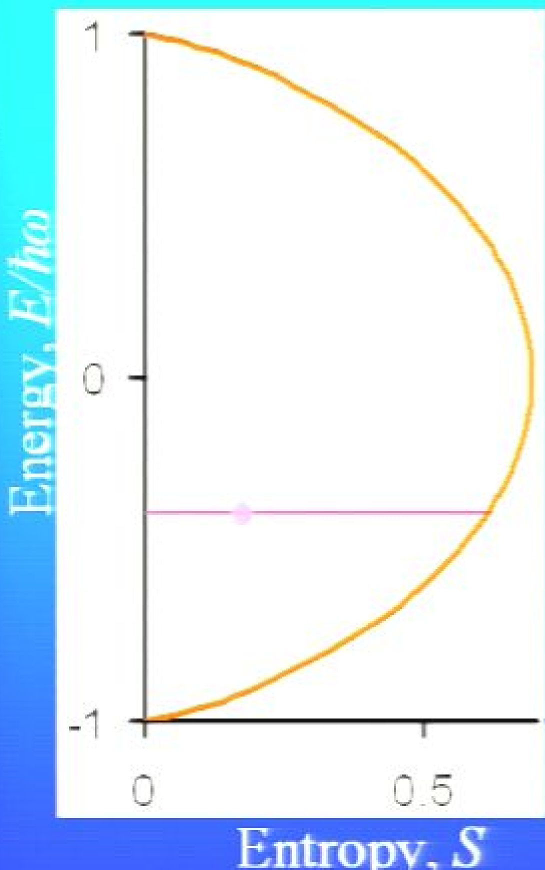

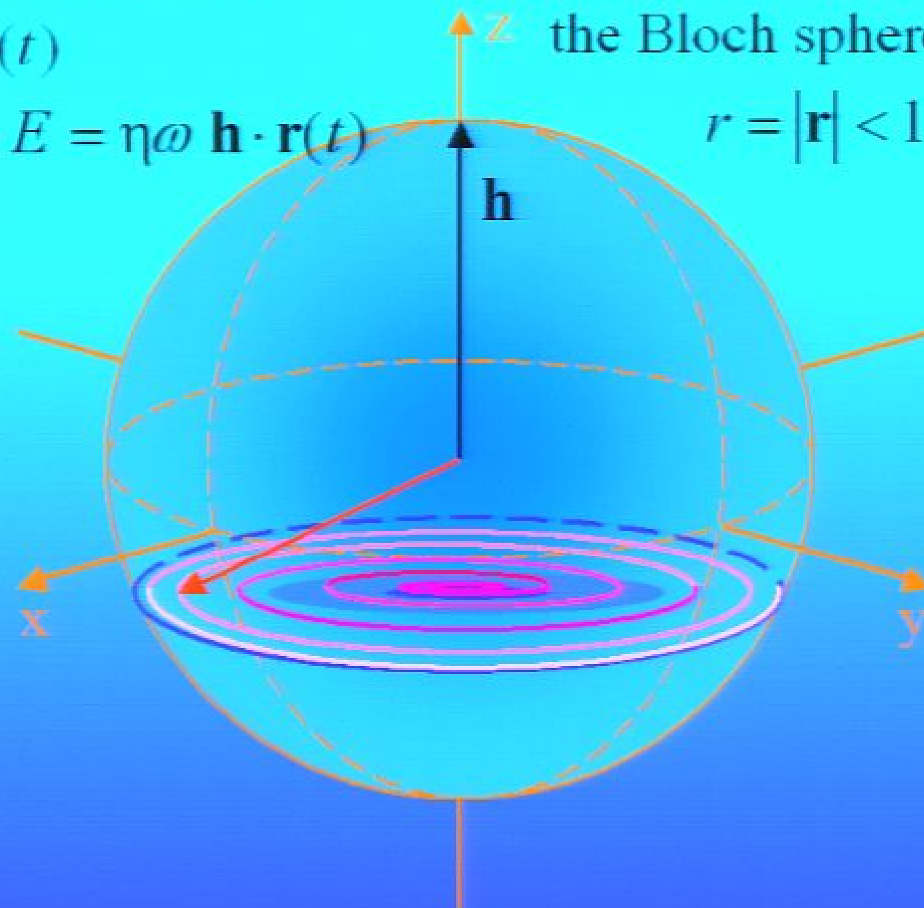

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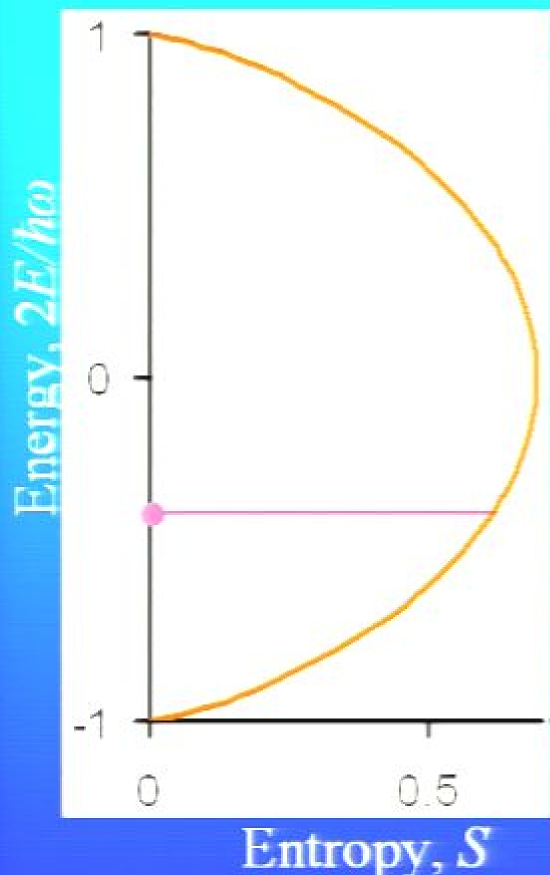

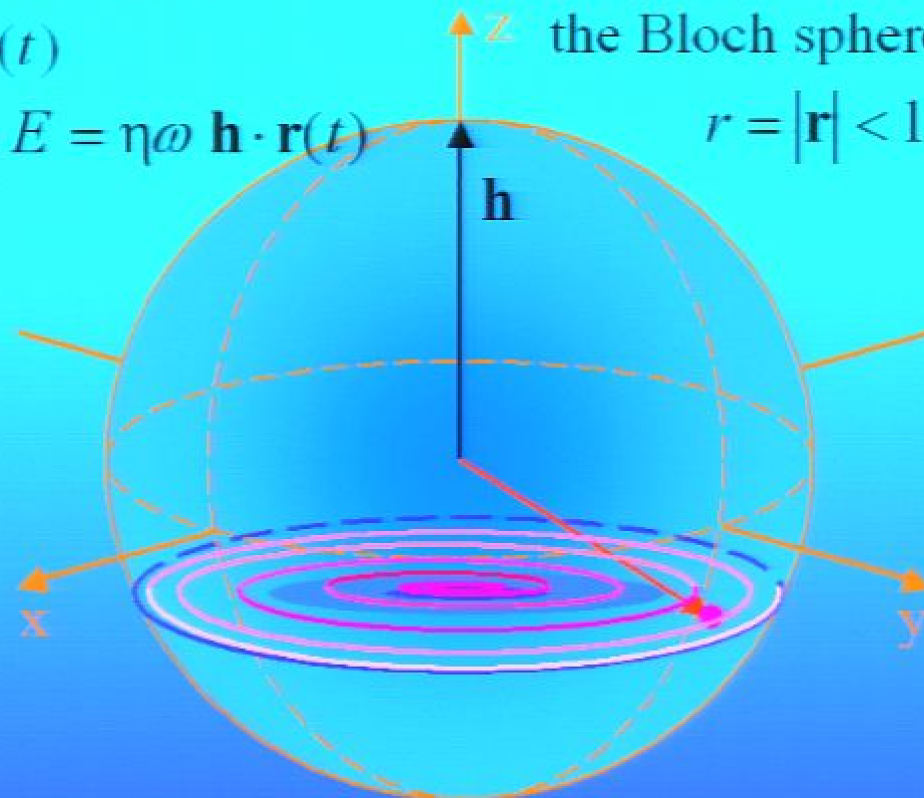

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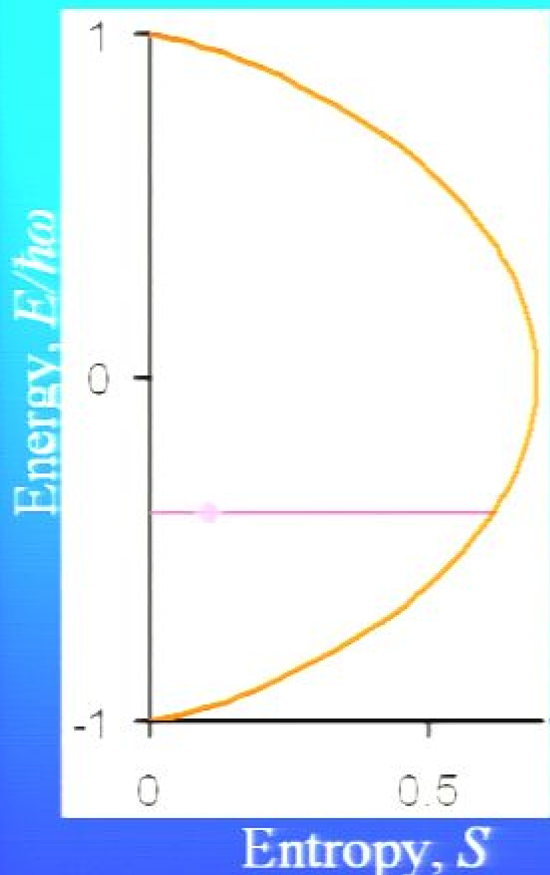

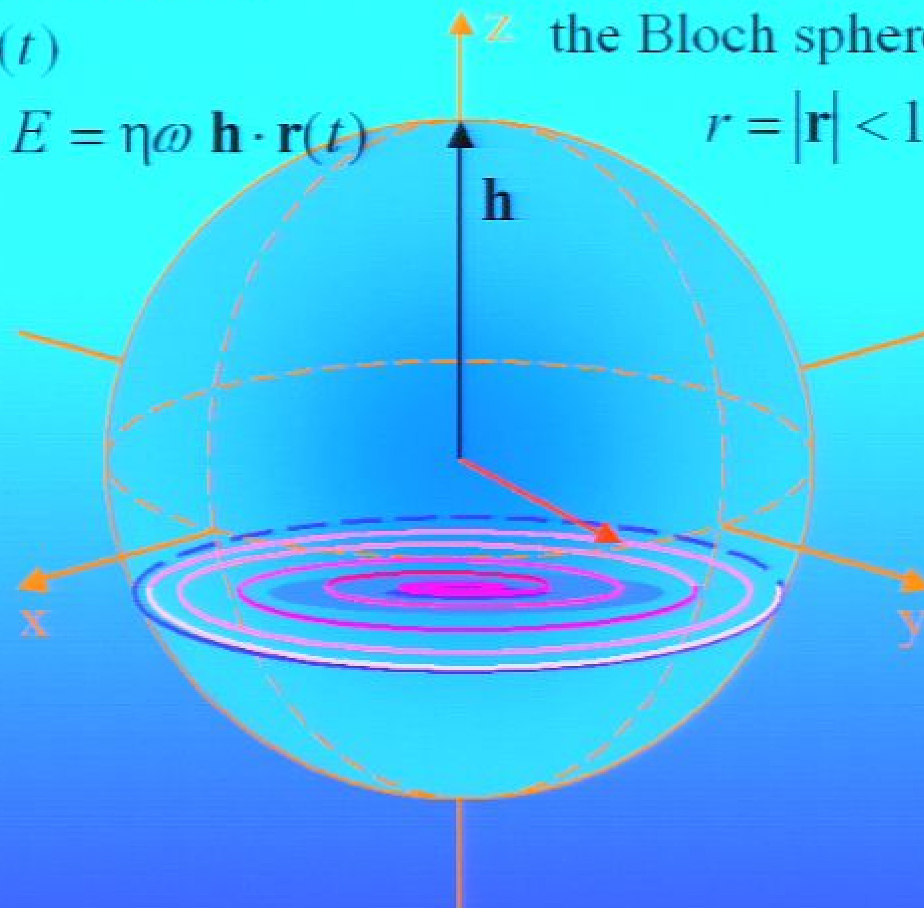

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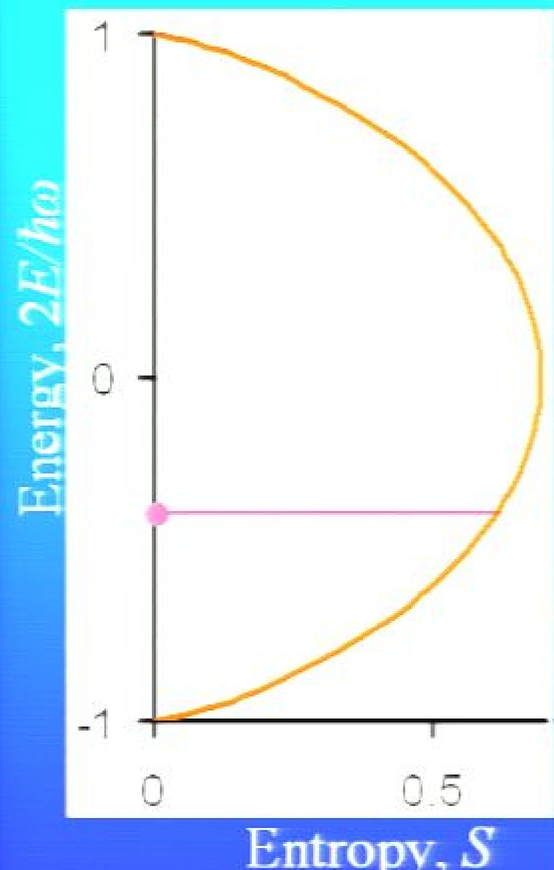

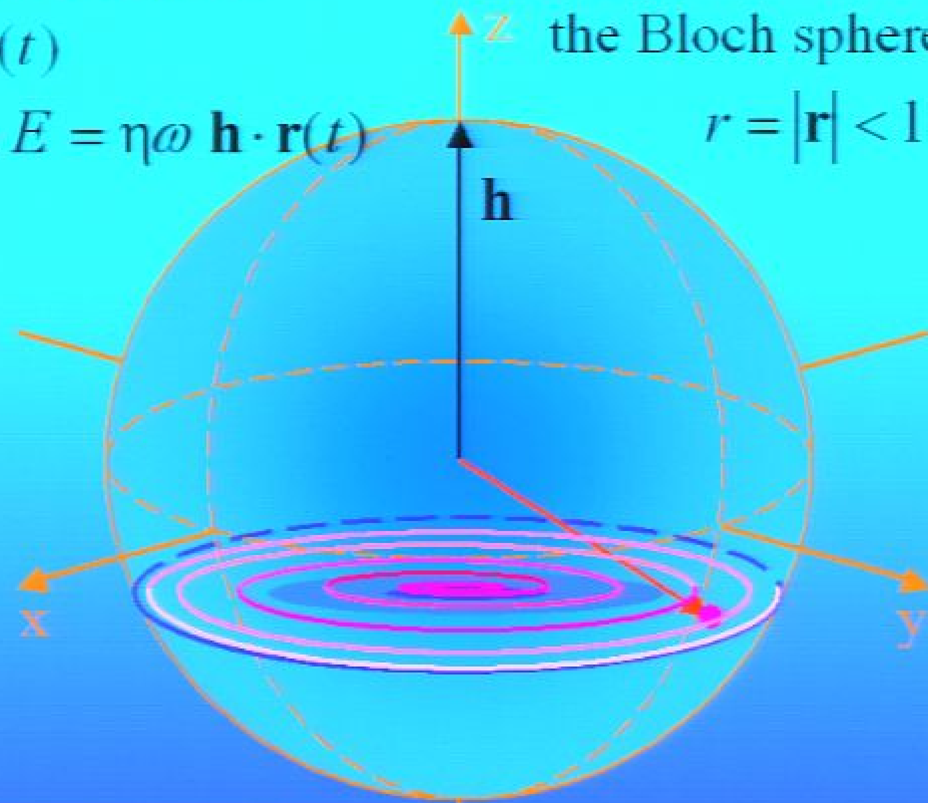

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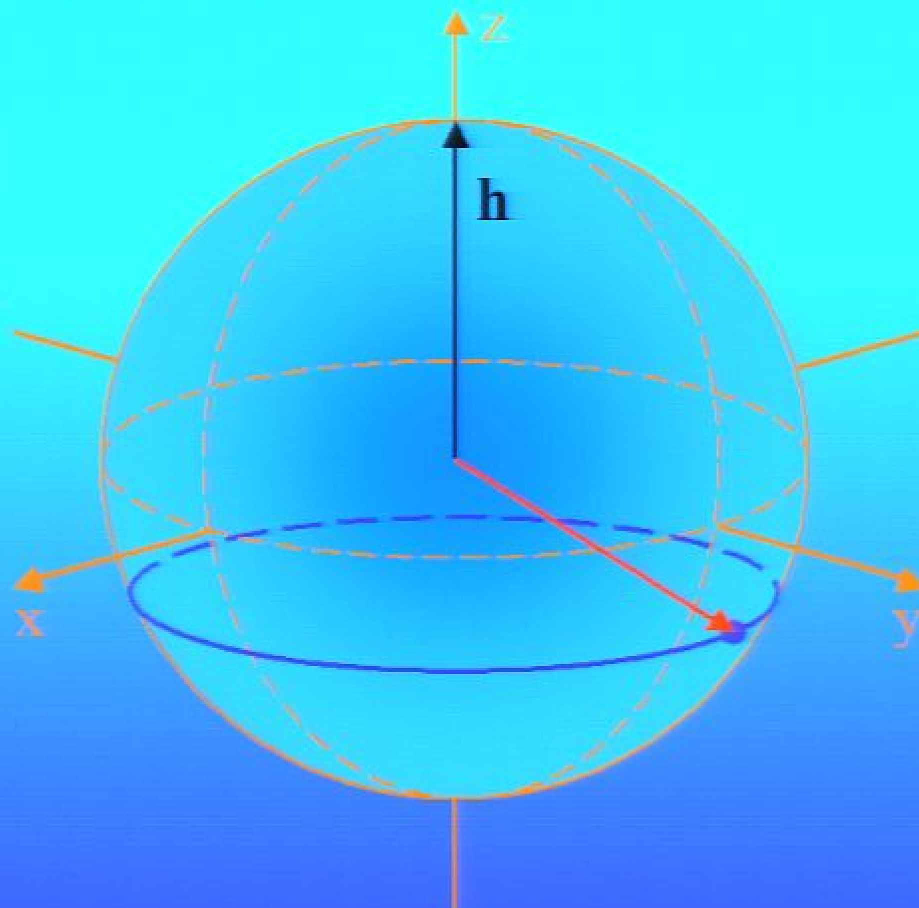

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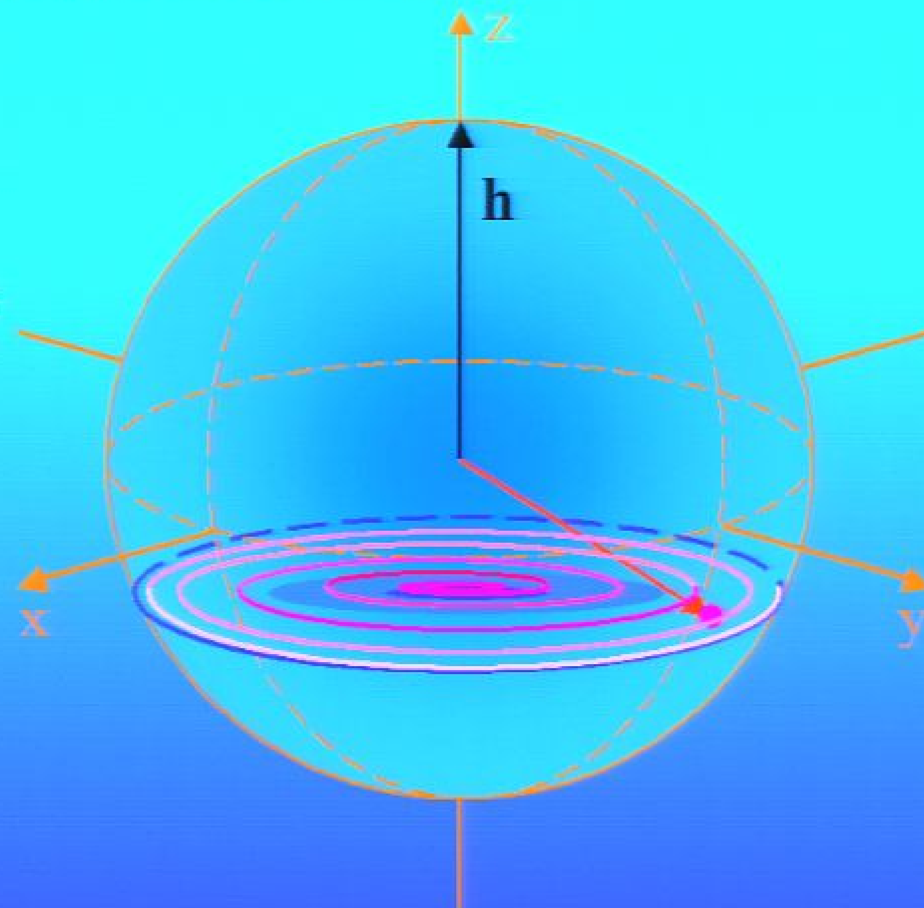
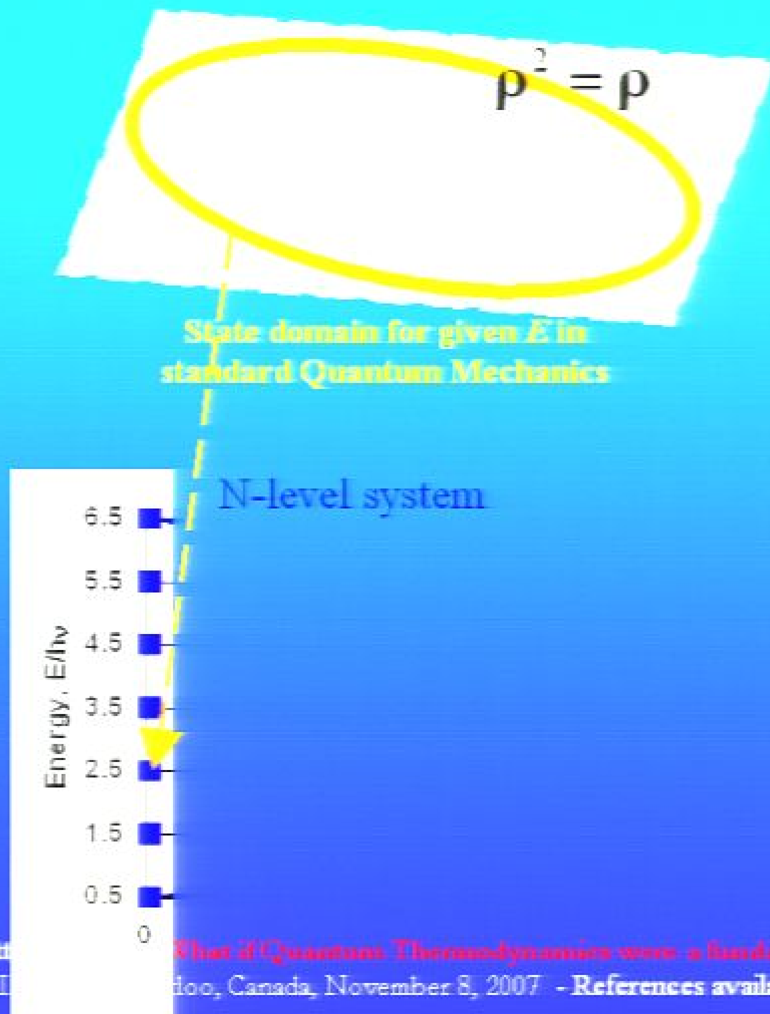


Isolated N-level system, Quantum Mechanics



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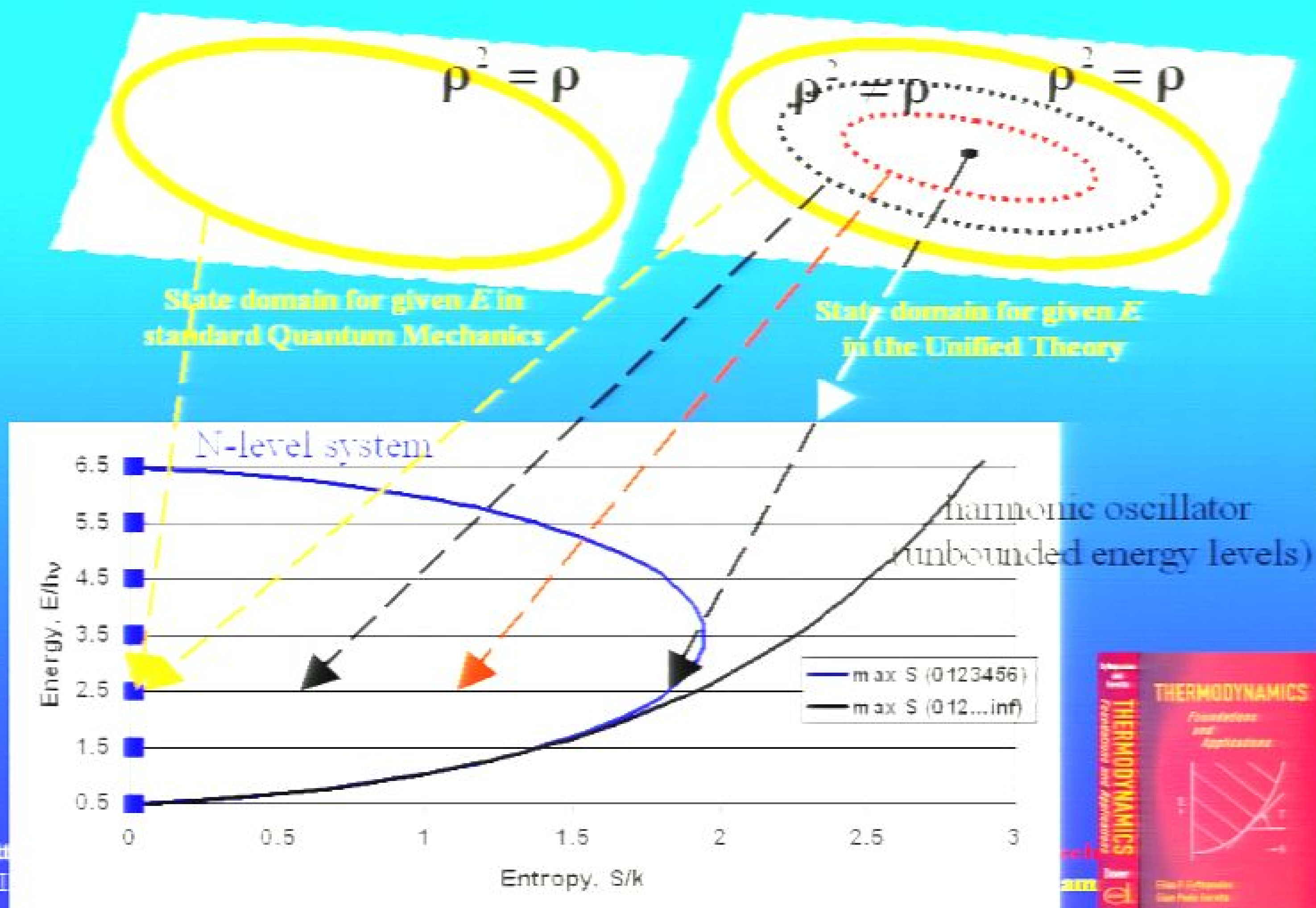

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Perimeter I

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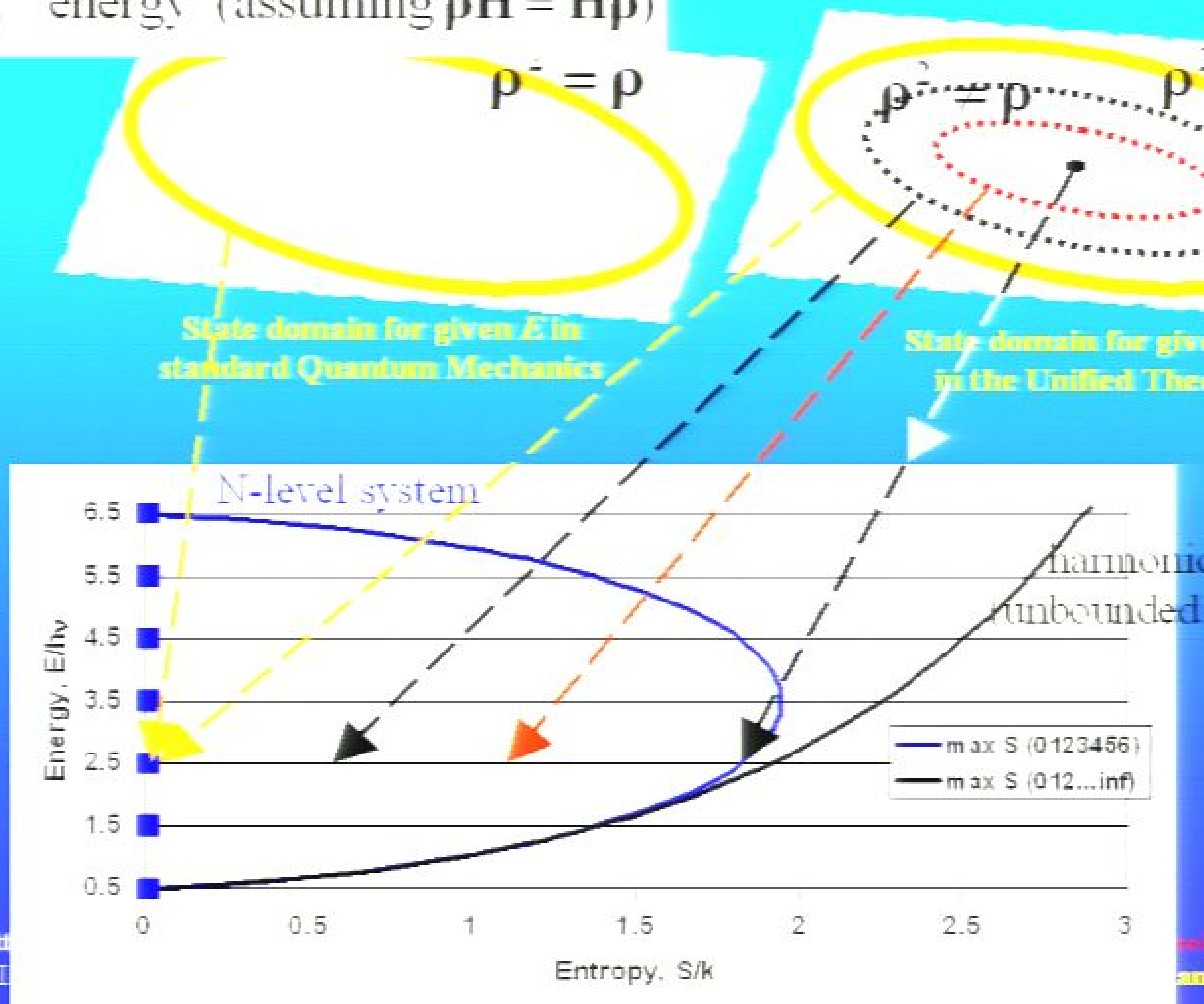

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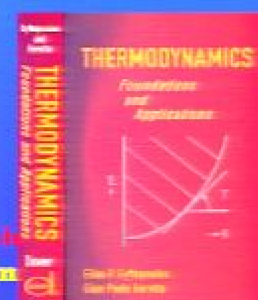

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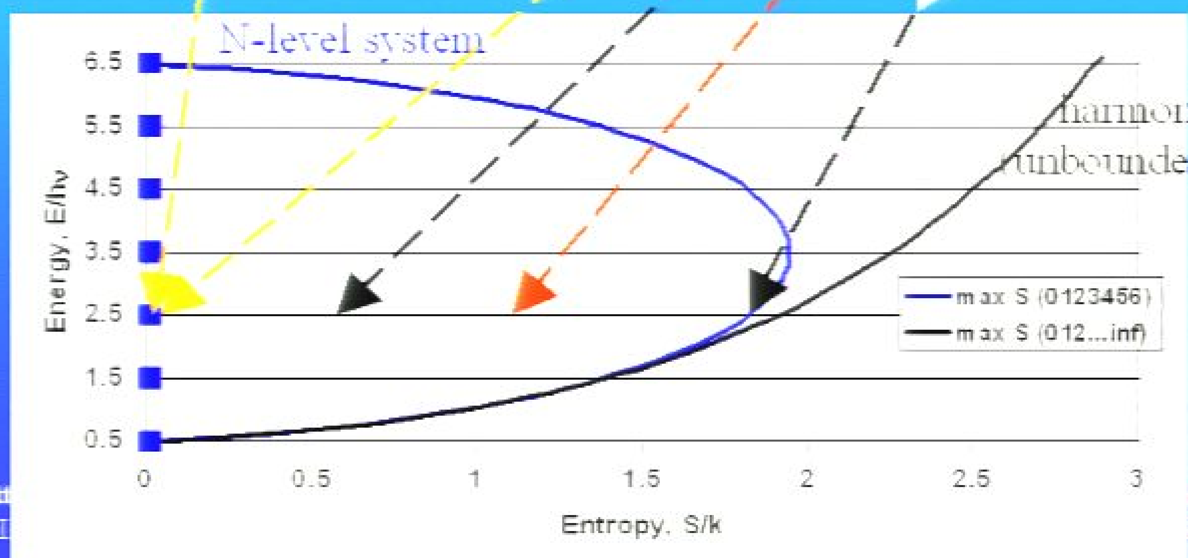
State domain for given E in standard Quantum Mechanics

State domain for given E in the Unified Theory



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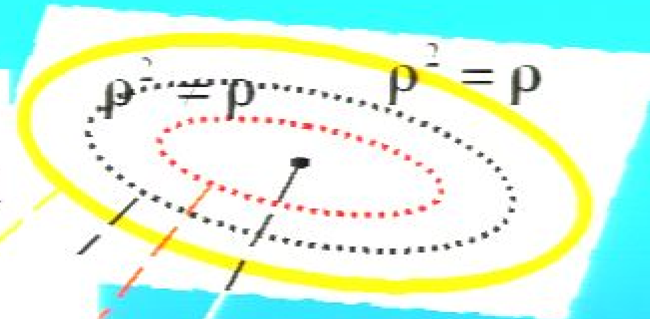

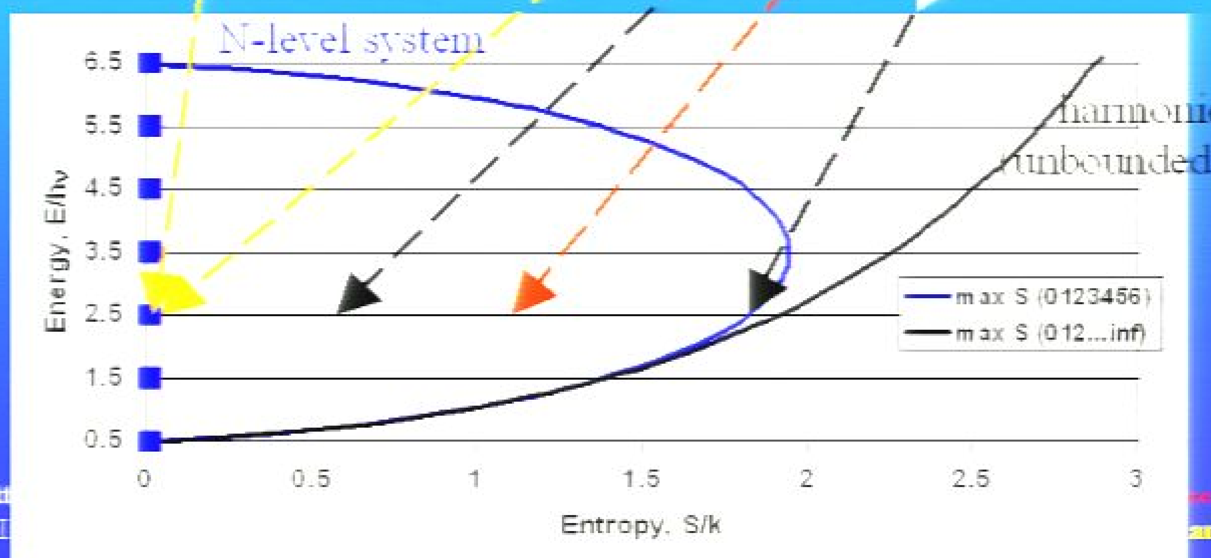

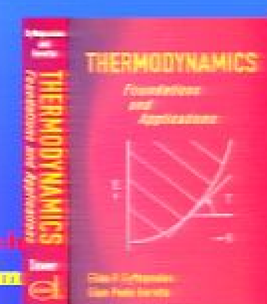

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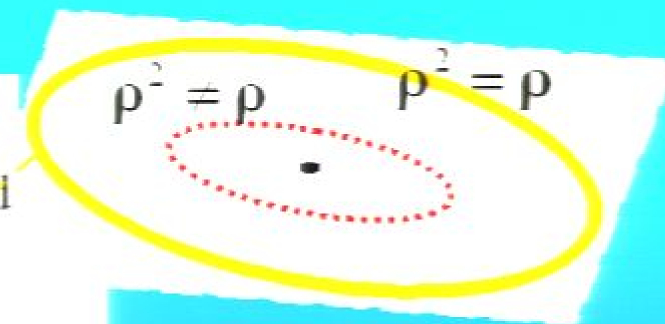

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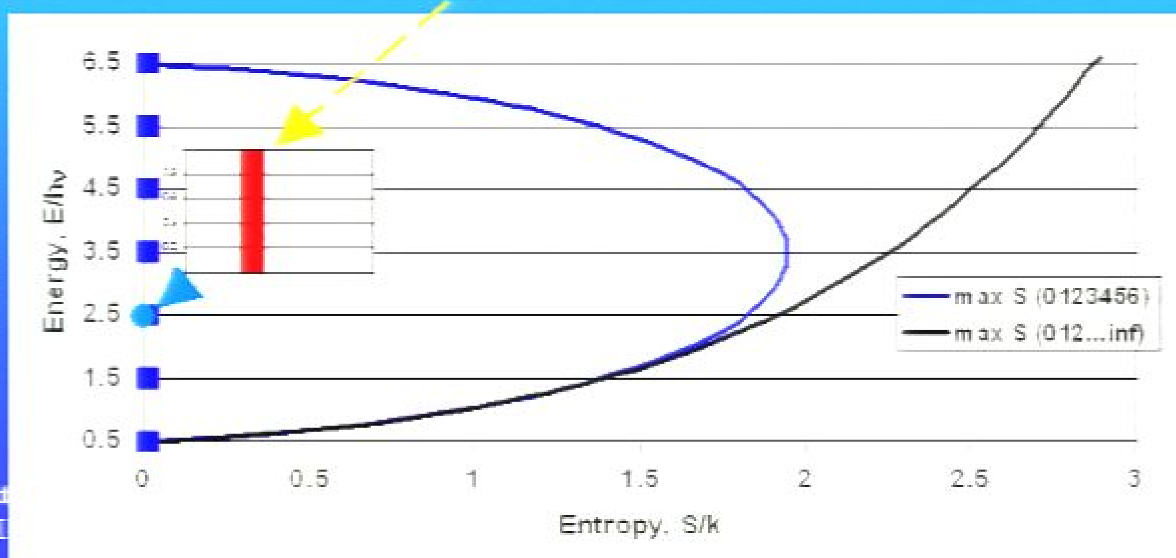

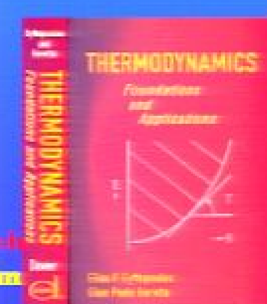

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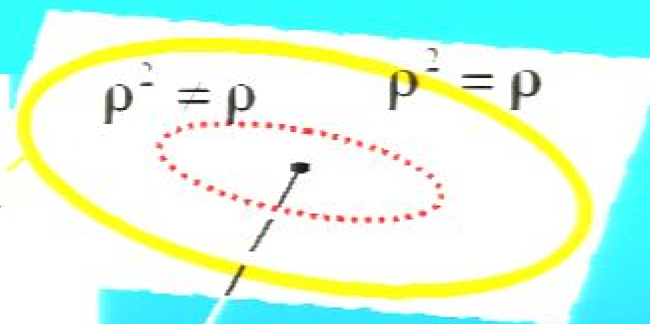

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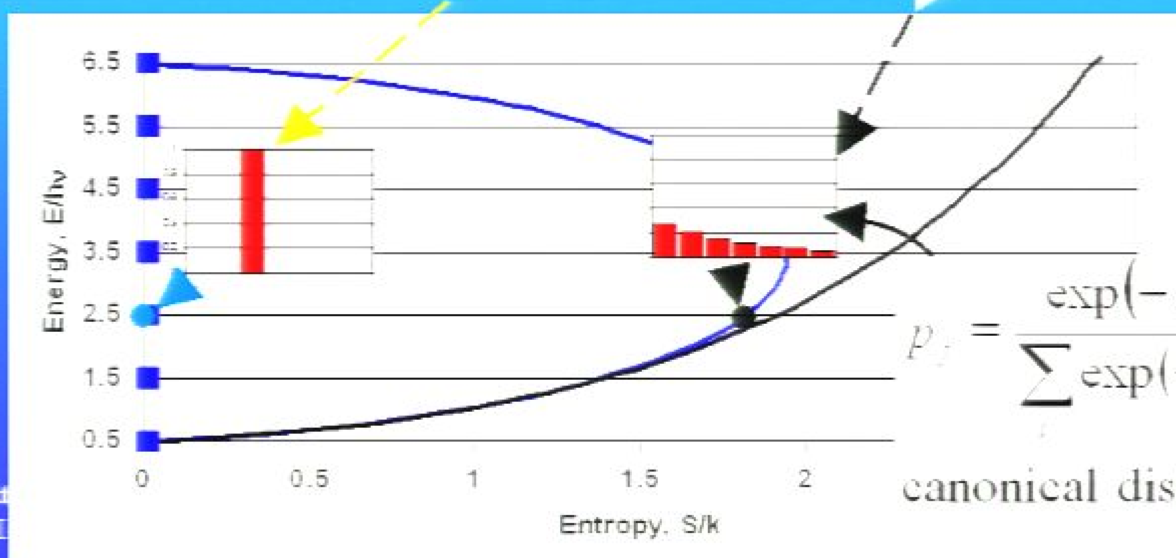


$$p_j = \frac{\exp(-e_j/k_B T)}{\sum_i \exp(-e_i/k_B T)}$$

canonical distribution

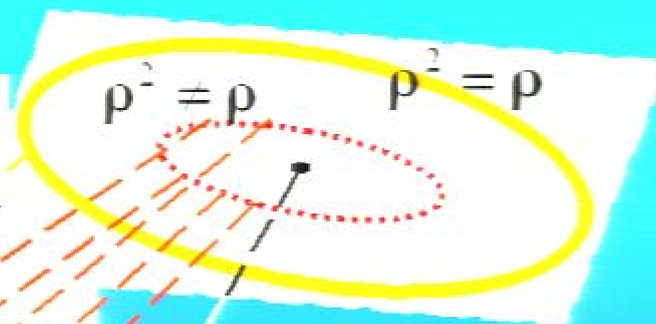
Isolated N-level system, Quantum Thermodynamics

energy levels e_j $\quad j = 1,2,\ldots,N$

$E = \sum_j p_j e_j$ energy (assuming $\rho H = H\rho$)

probability p_j measures the degree of involvement of level e_j in sharing the energy load

$S = -k_B \sum_j p_j \ln p_j$ entropy measures the global degree of sharing the energy load among levels



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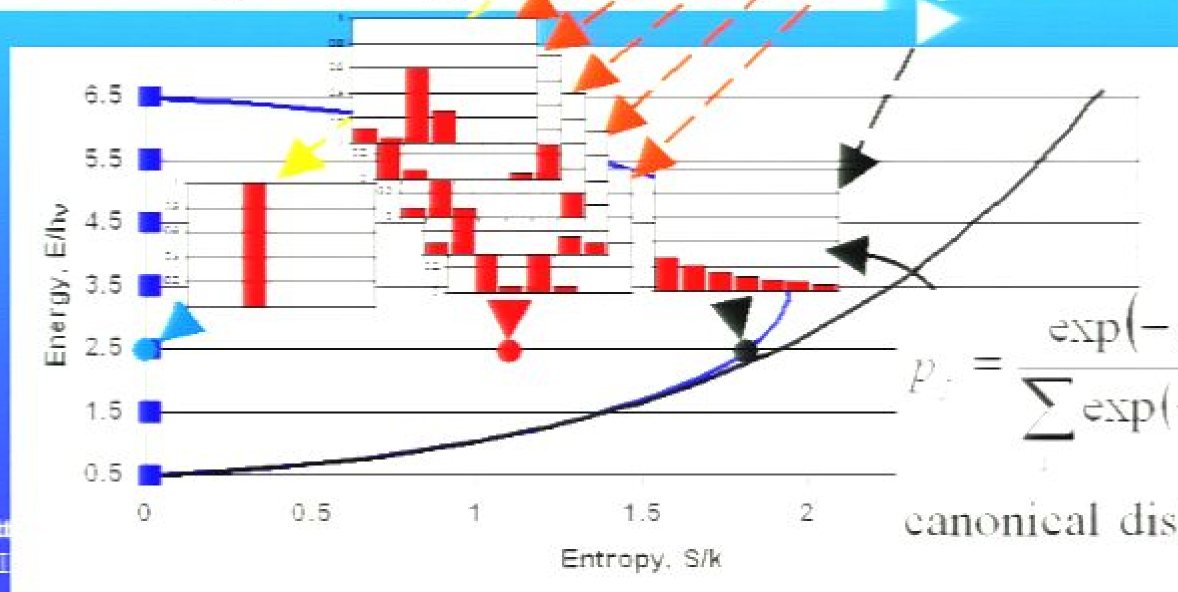

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Desiderata for a general dynamical law (isolated system)

Mod.Phys.Lett.A 20, 977 (2005)

Energy conservation, $\frac{d}{dt}\left(\sum_j p_j e_j\right) = 0$

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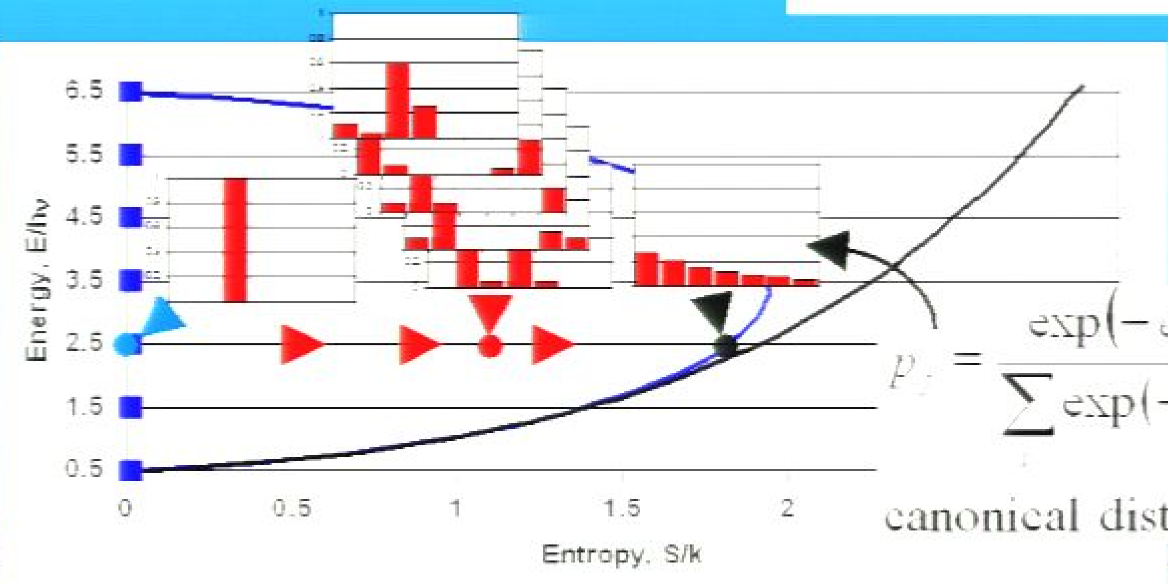

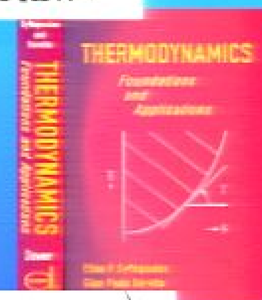

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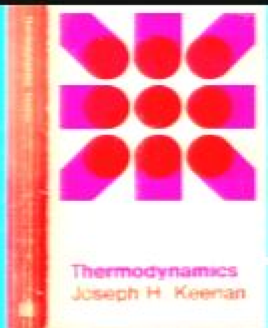
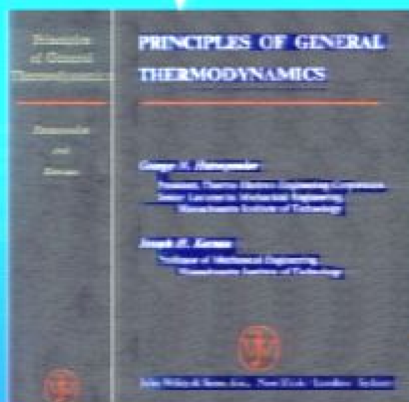
The Hatsopoulos-Keenan statement of the Second Law and the role of stability at thermodynamic equilibrium

Books by members of the "Keenan school of thermodynamics"

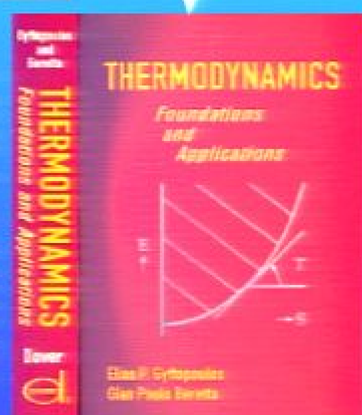
1941



1965



1991 (2005)



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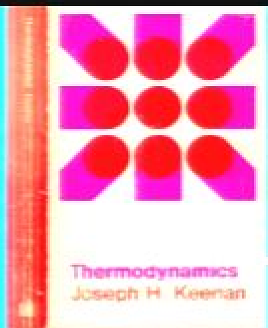

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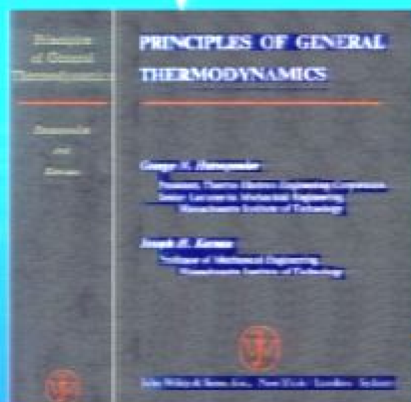

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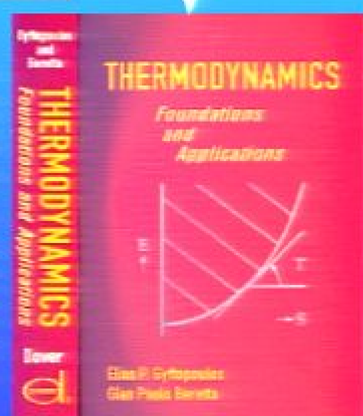

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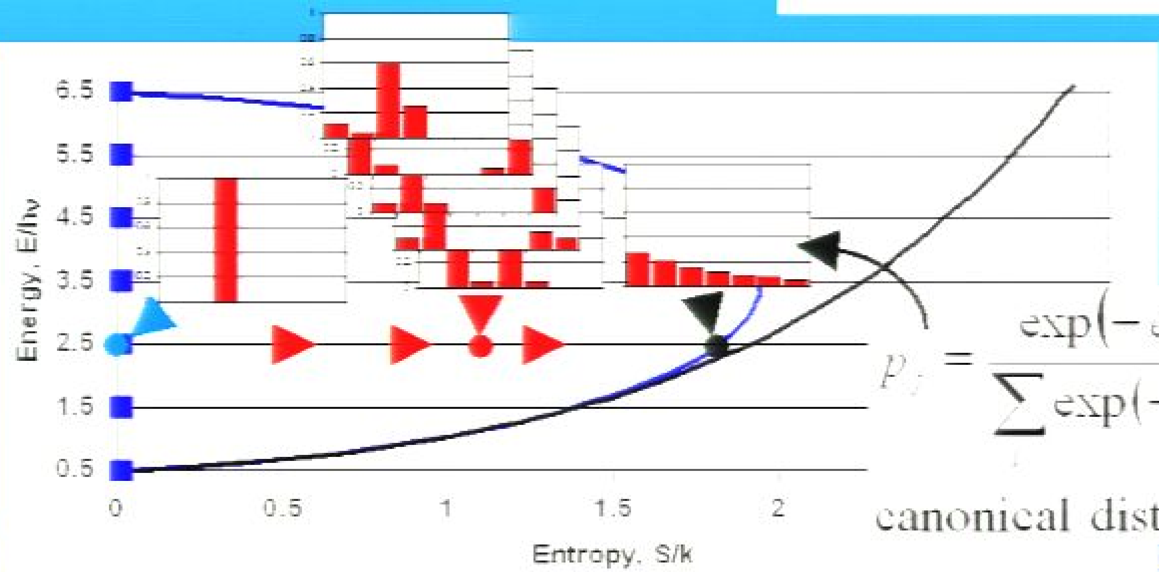

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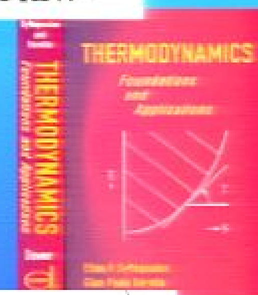

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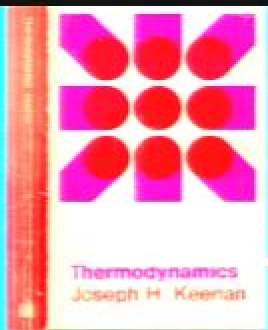

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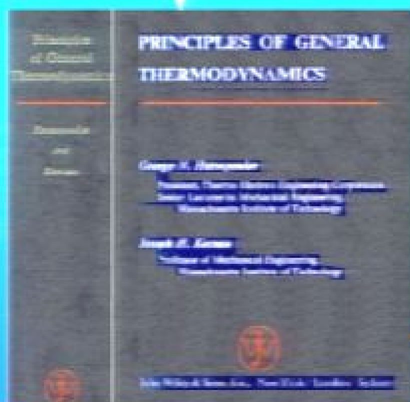

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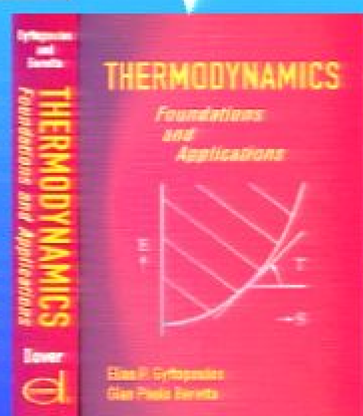

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quant-ph-0511091

We assume a dynamical equation of the form

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\frac{i}{\hbar}[H, \rho] + \frac{1}{2k_{\mathrm{B}}\tau(\rho)}\{\Delta M(\rho), \rho\}$$

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G.P. Beretta
Perimeter In

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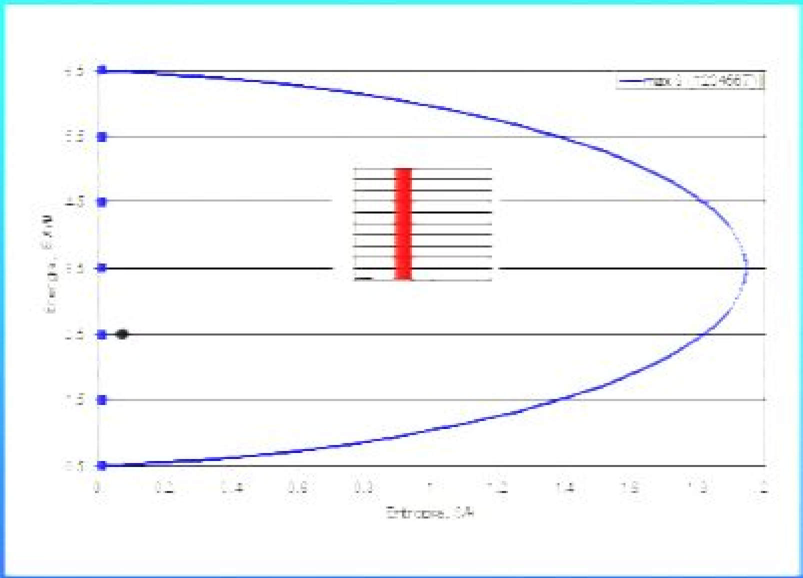

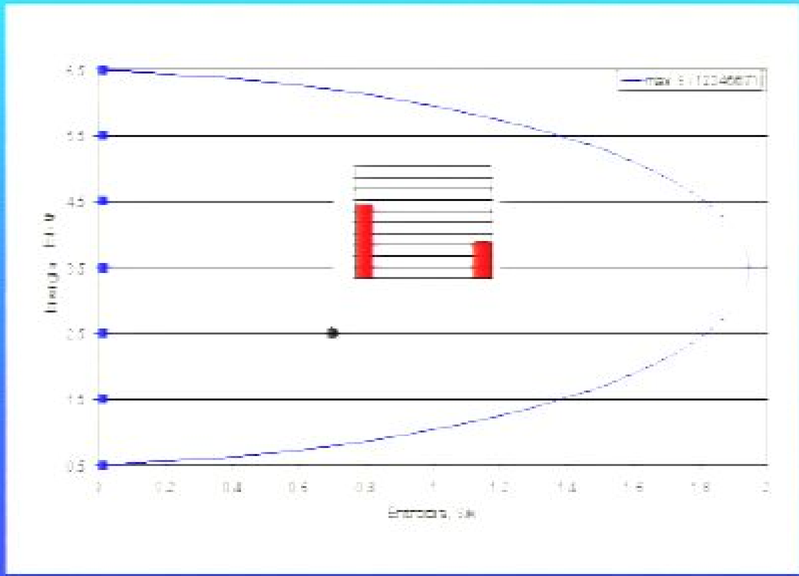

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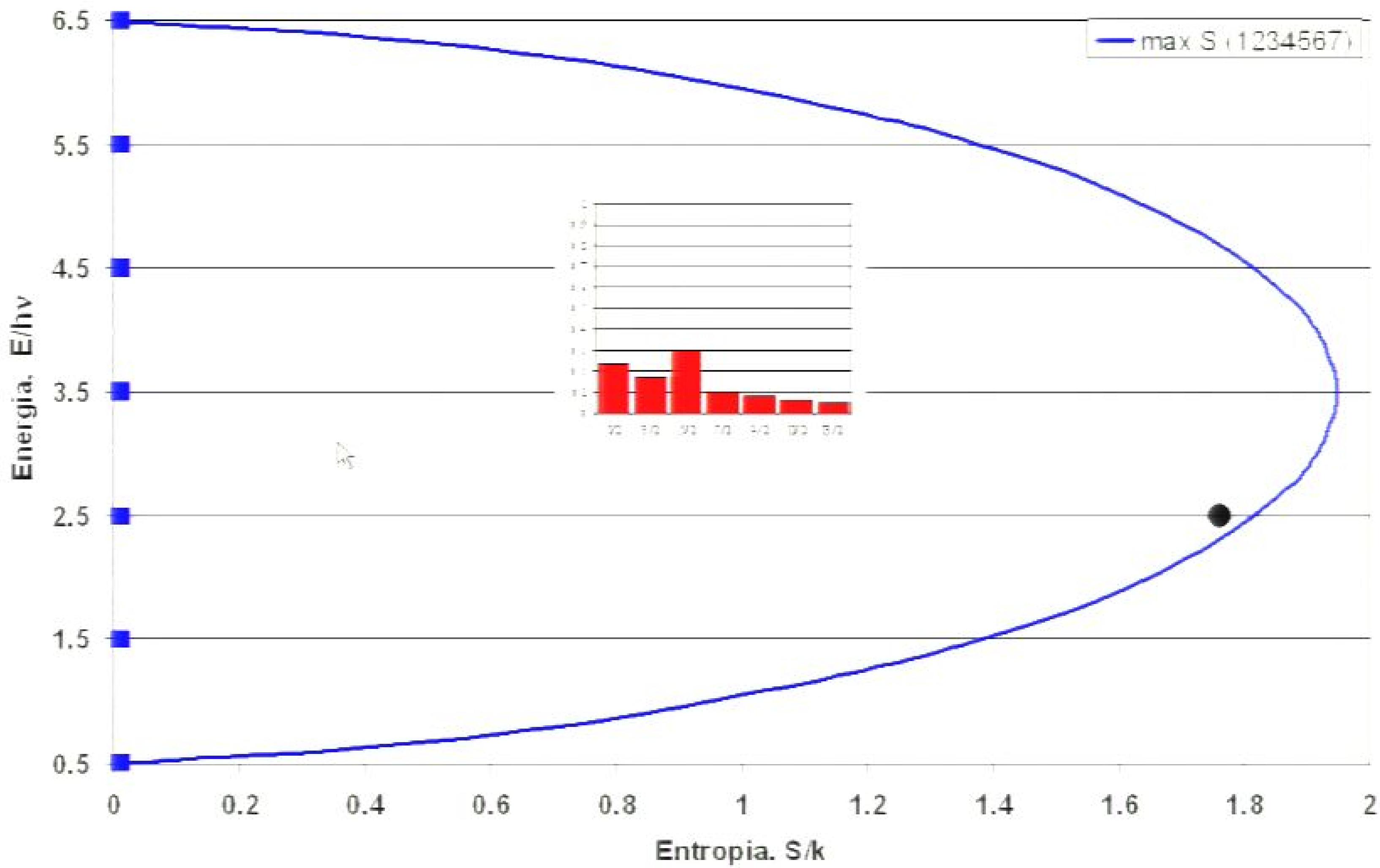

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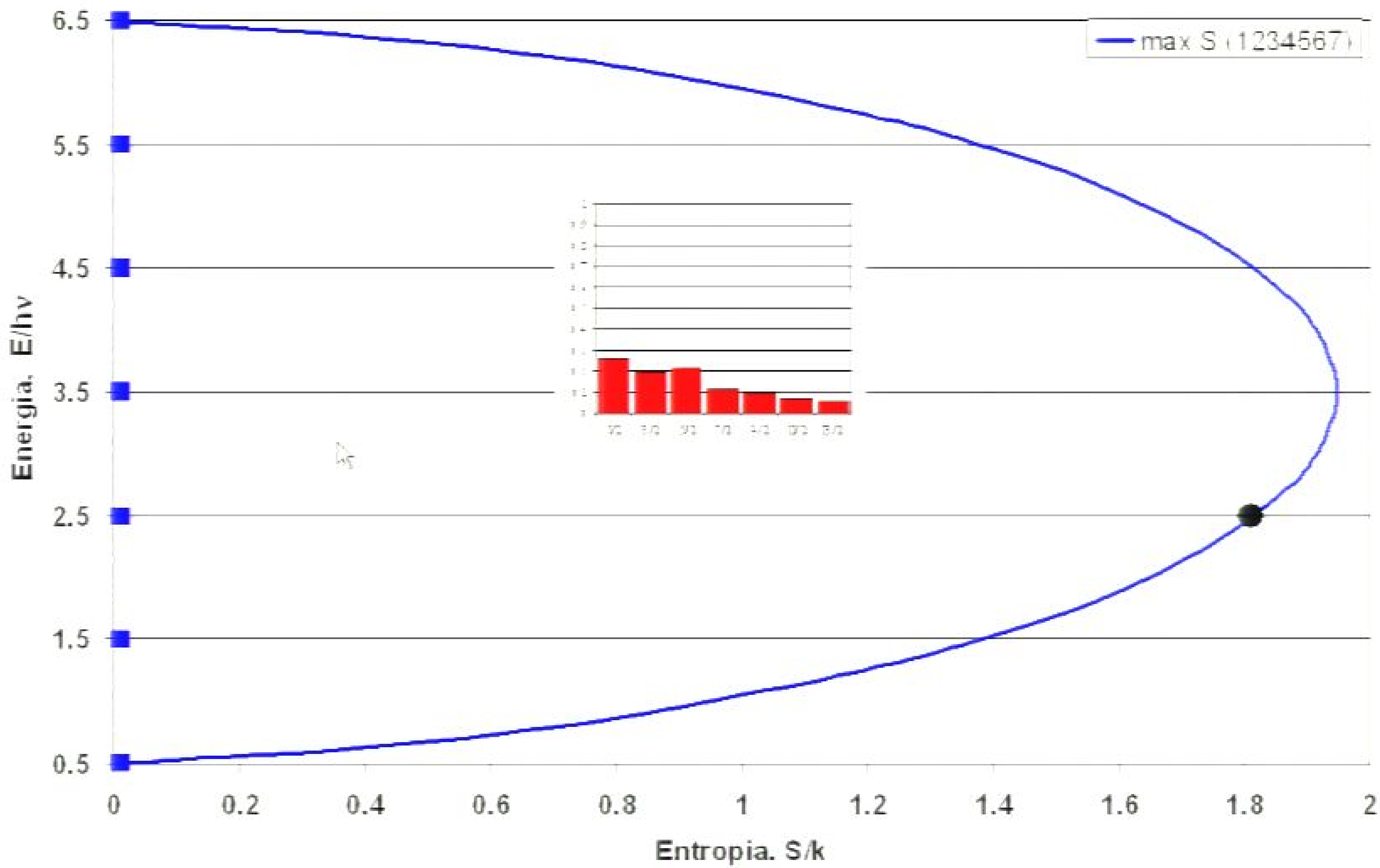

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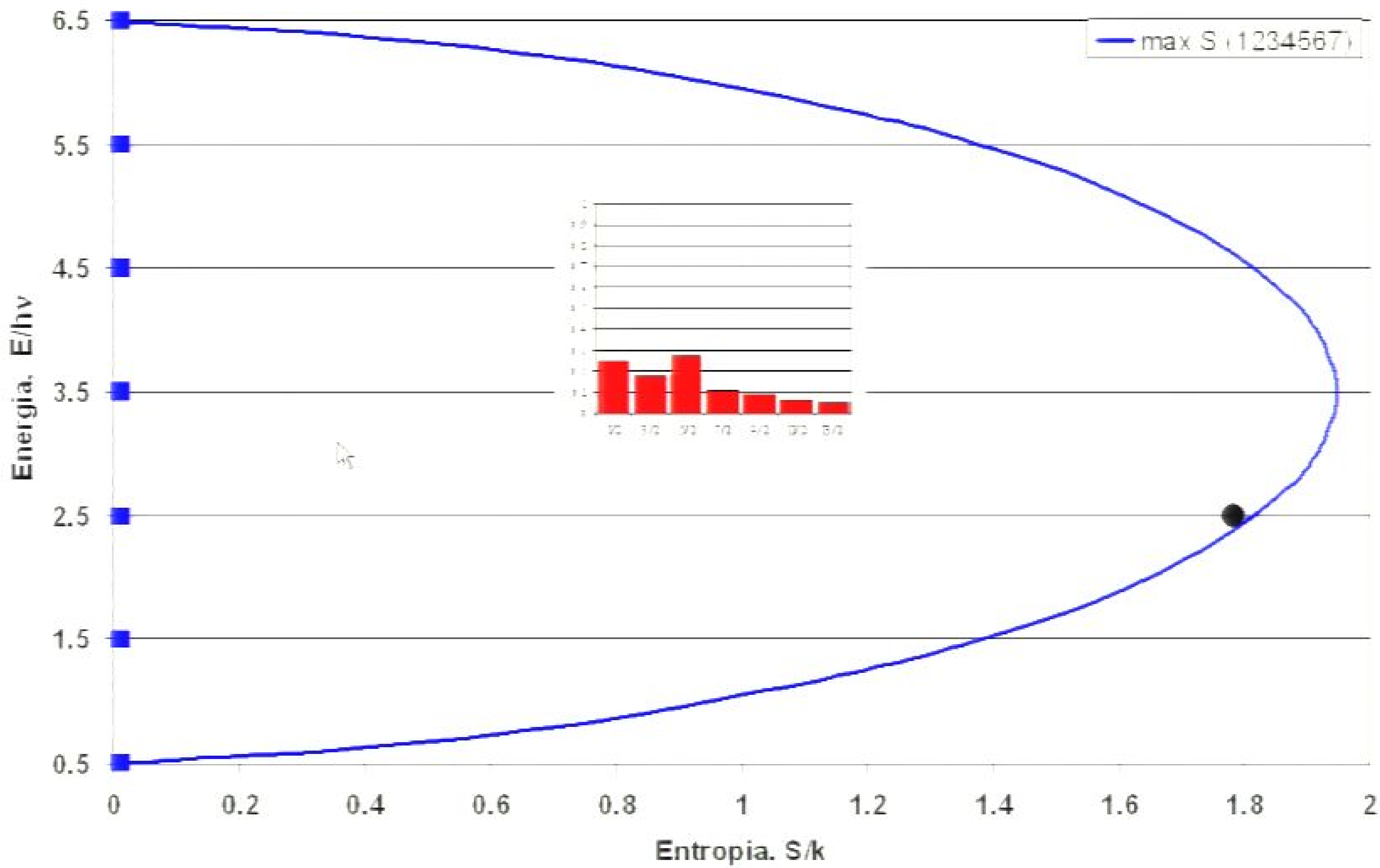
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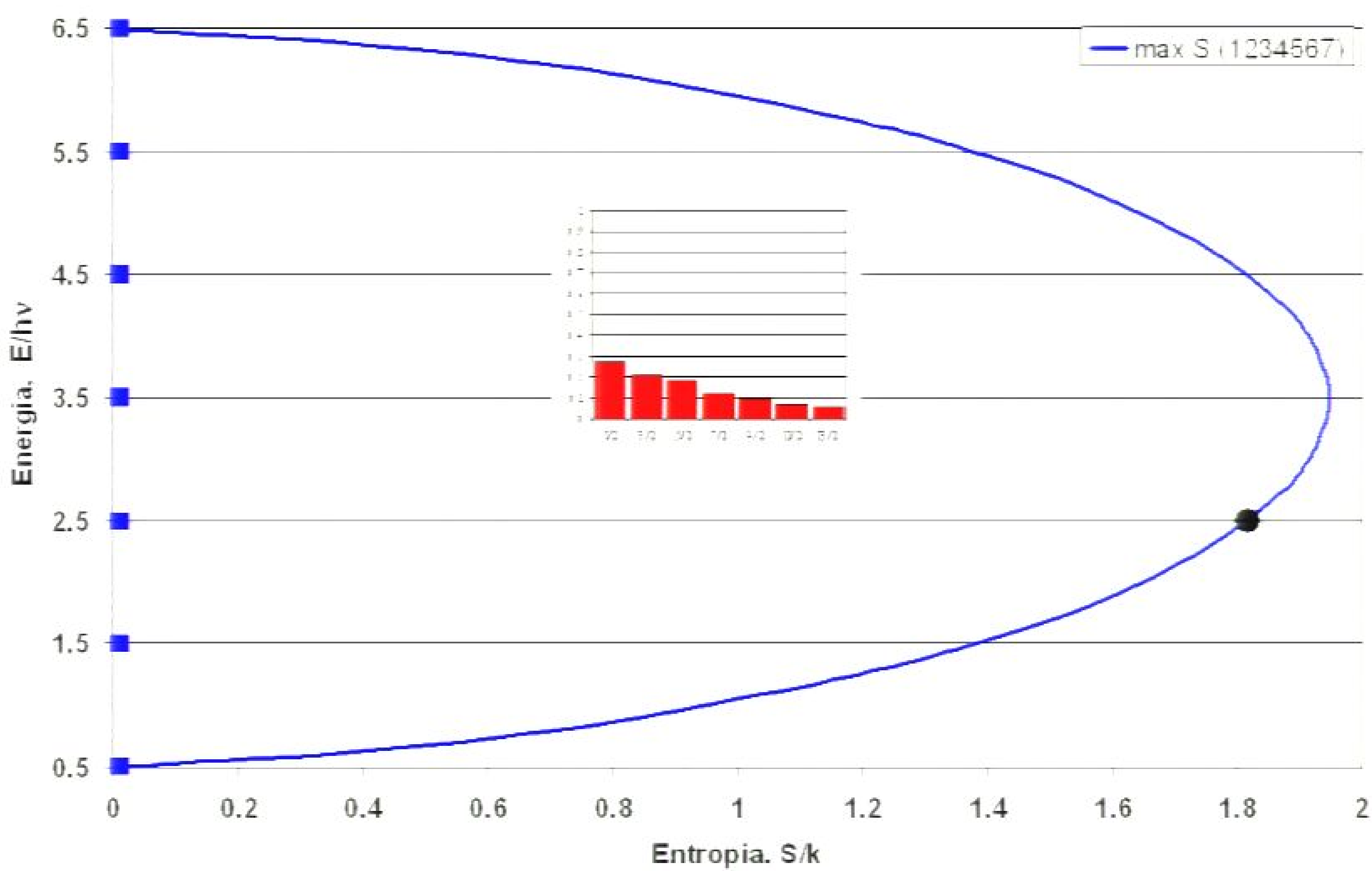
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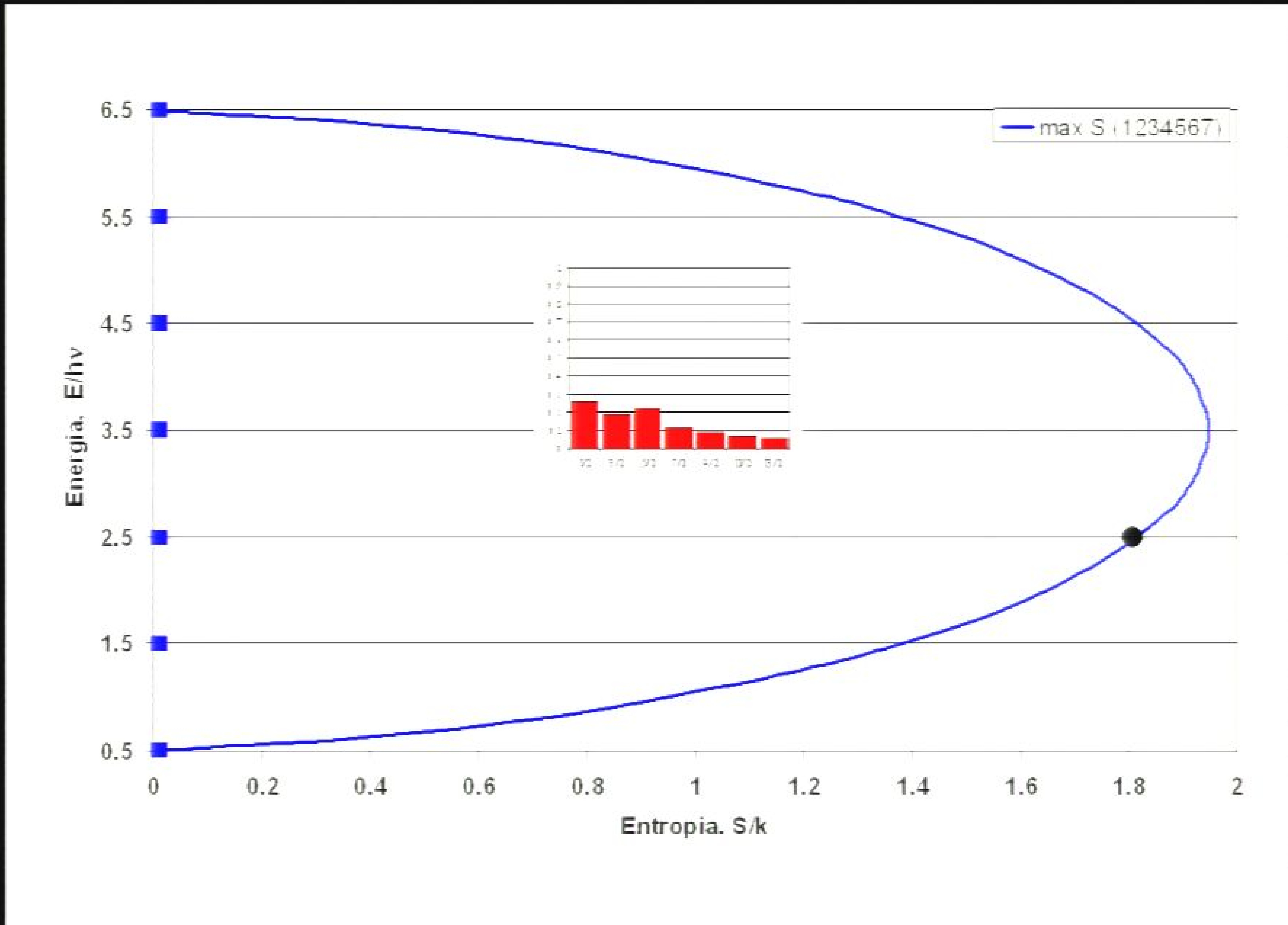

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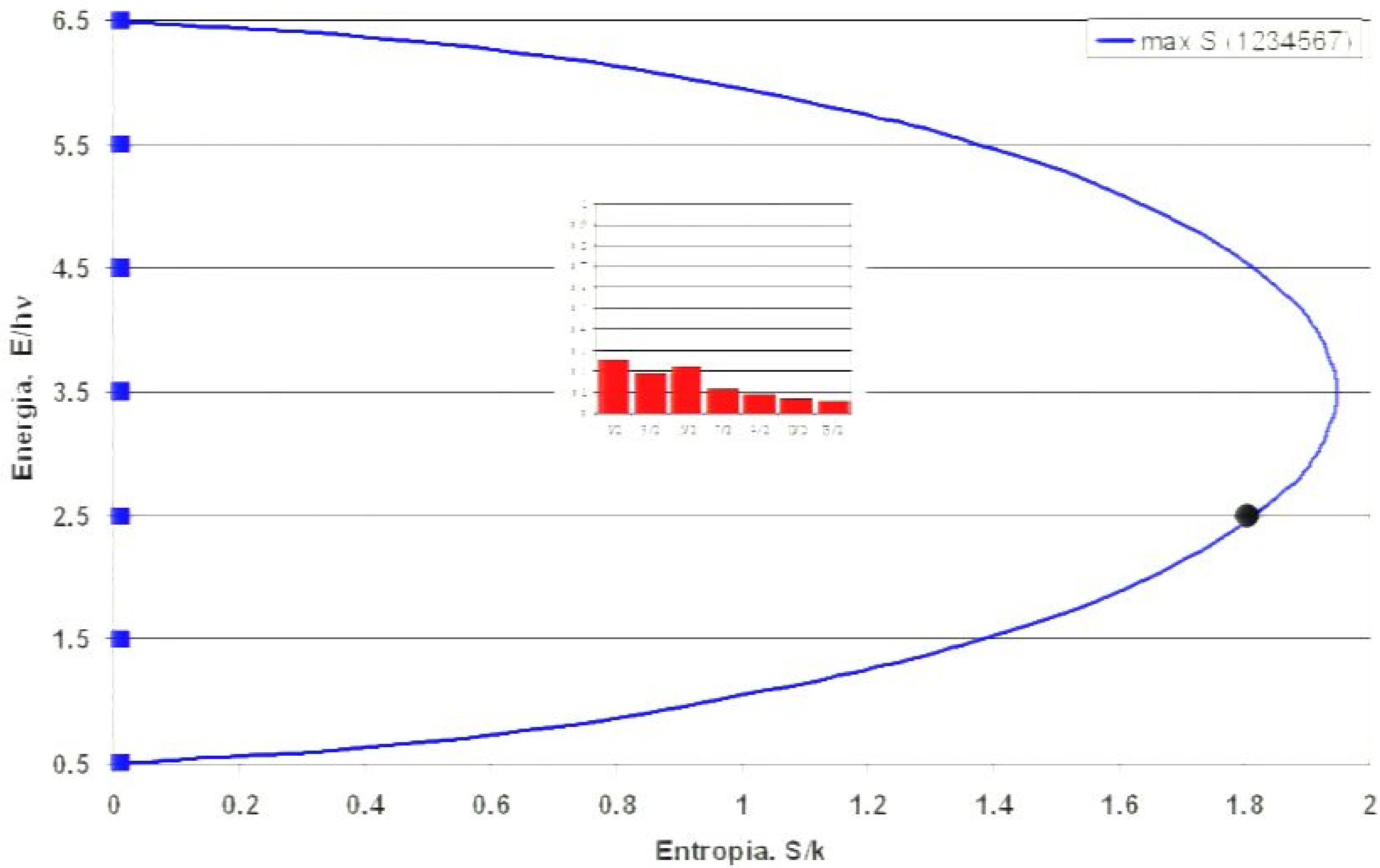
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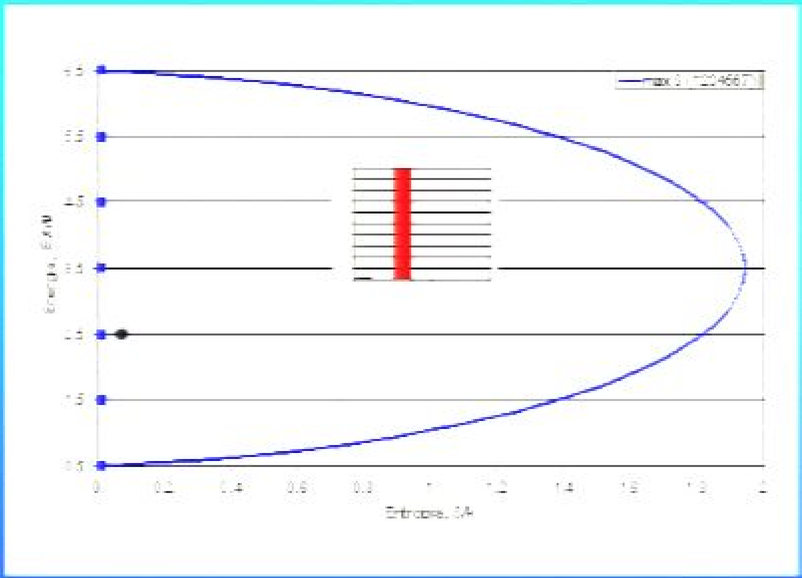

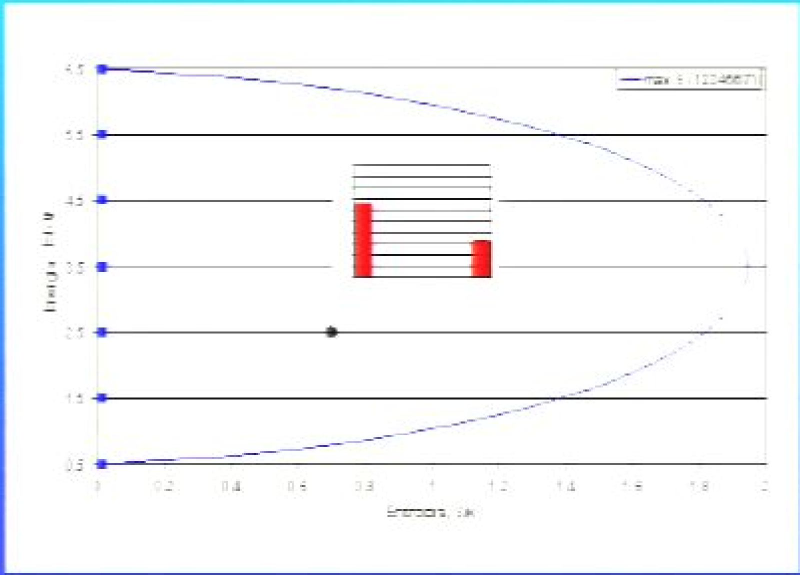

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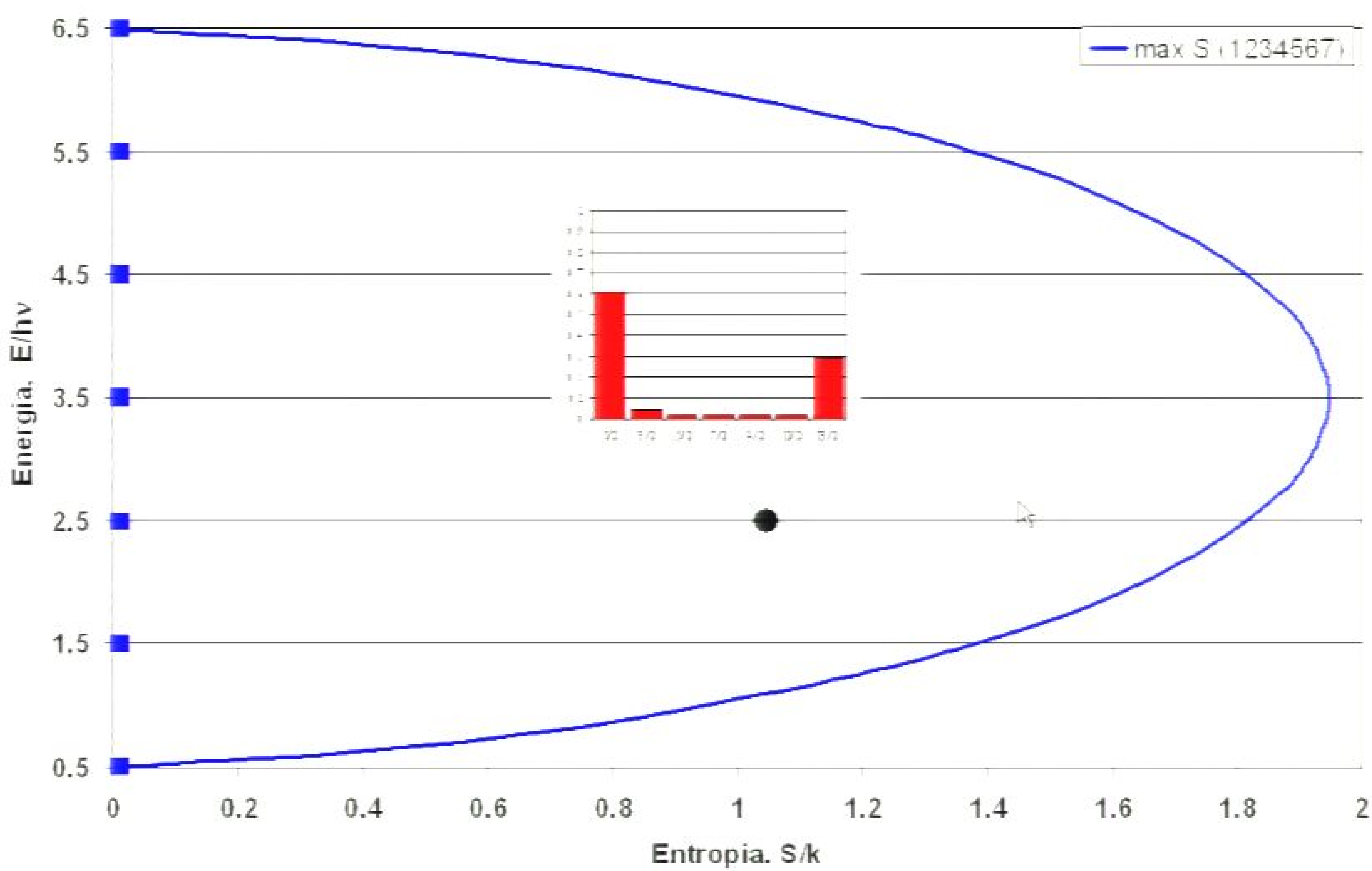
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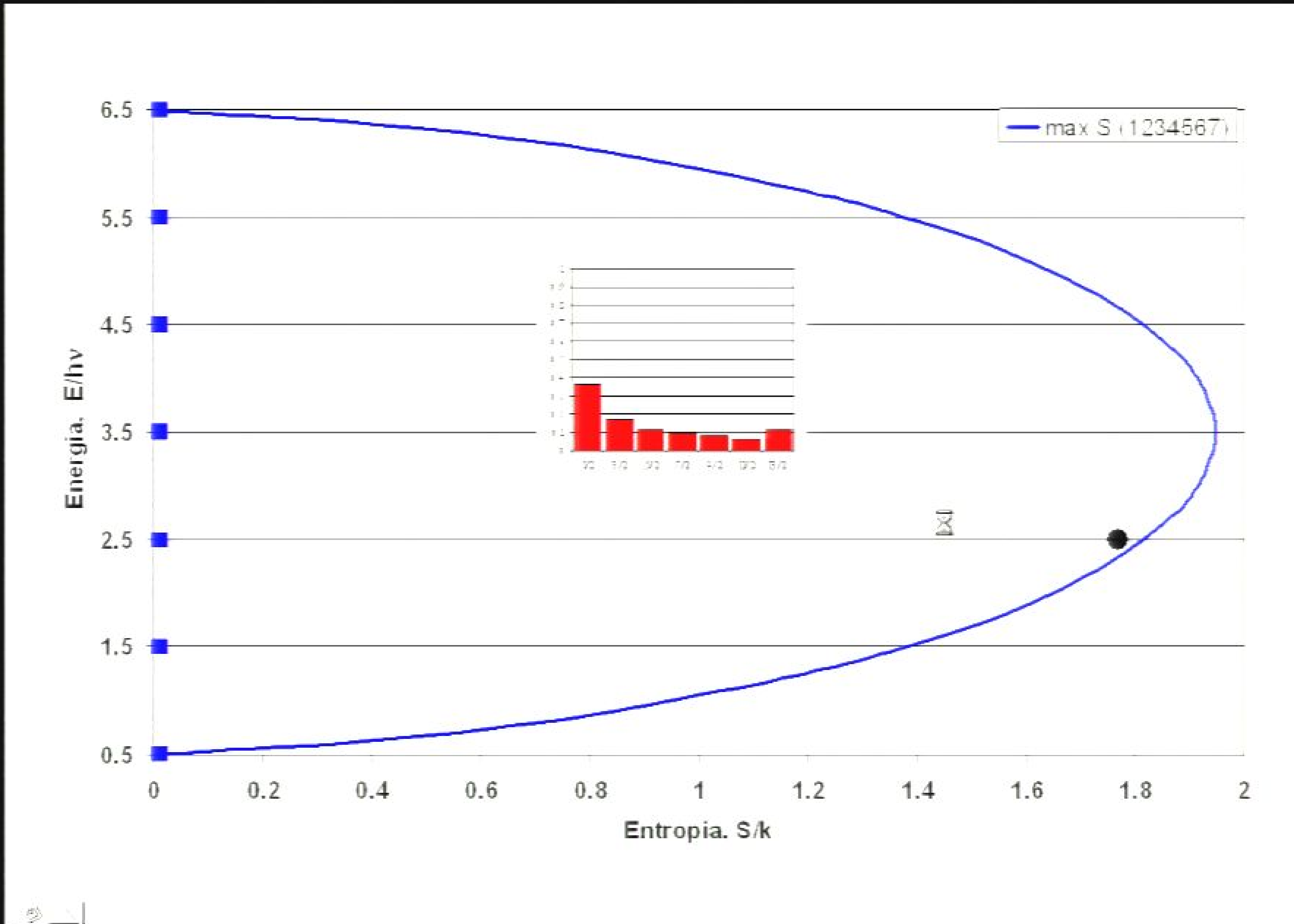
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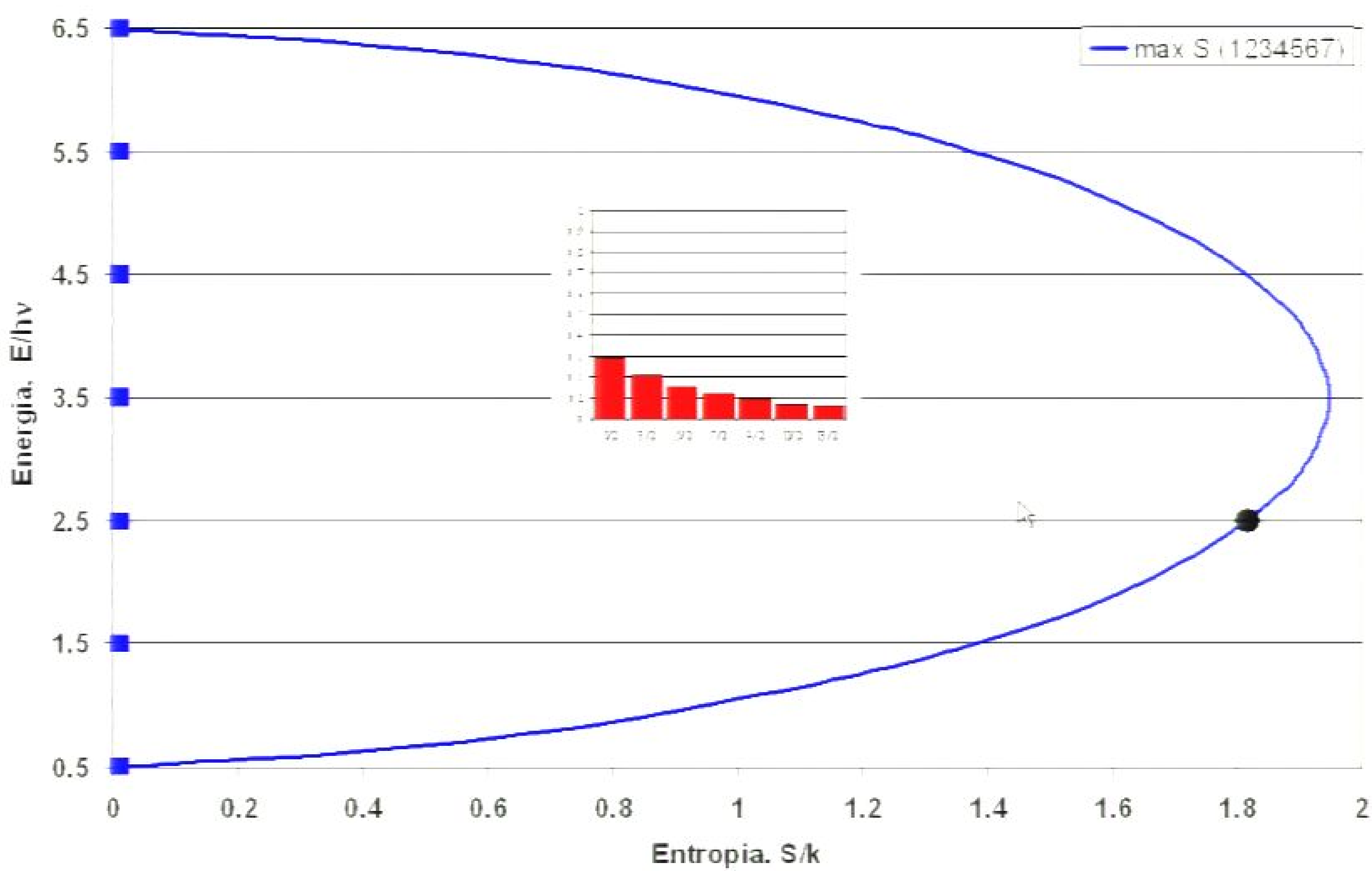

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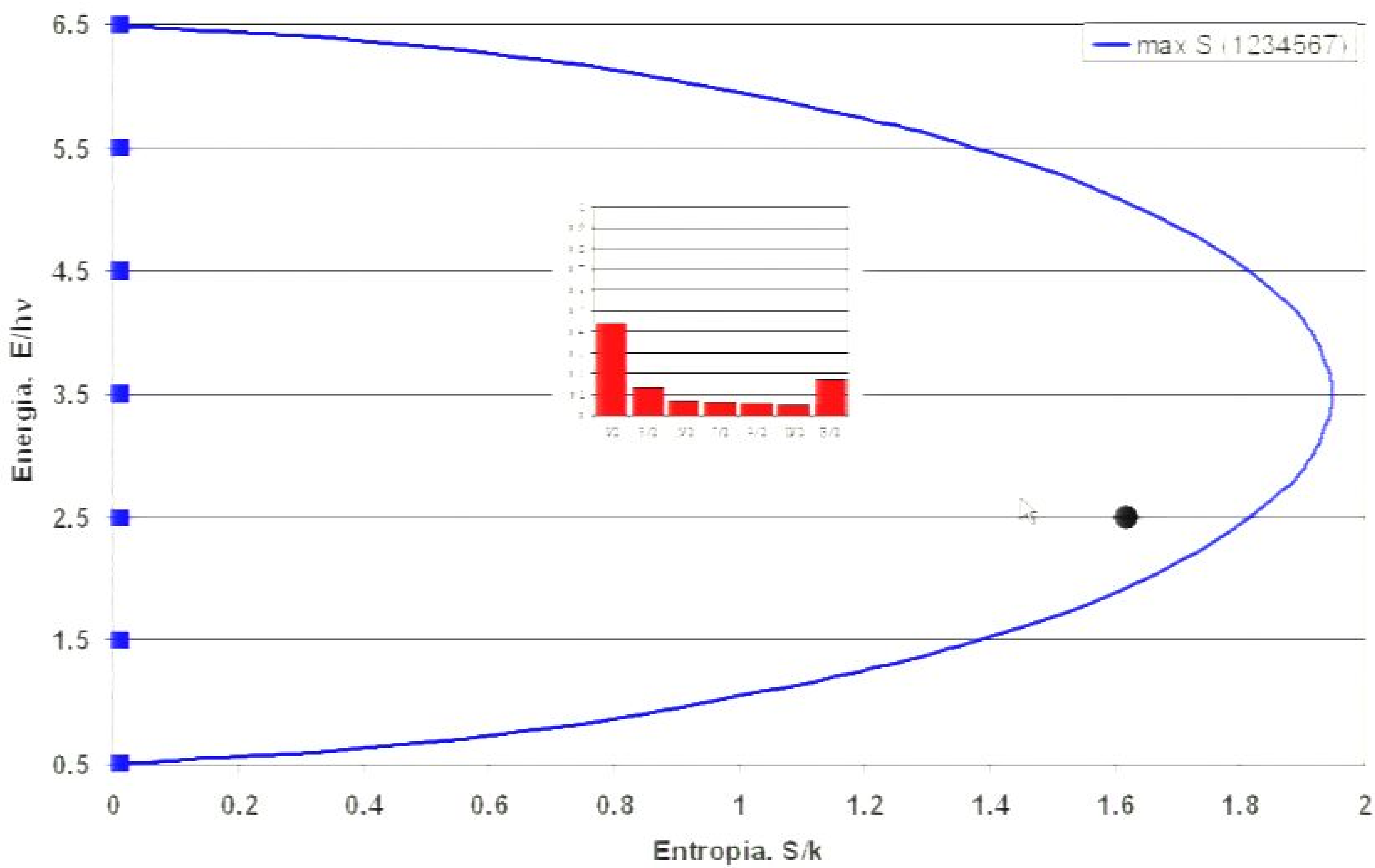

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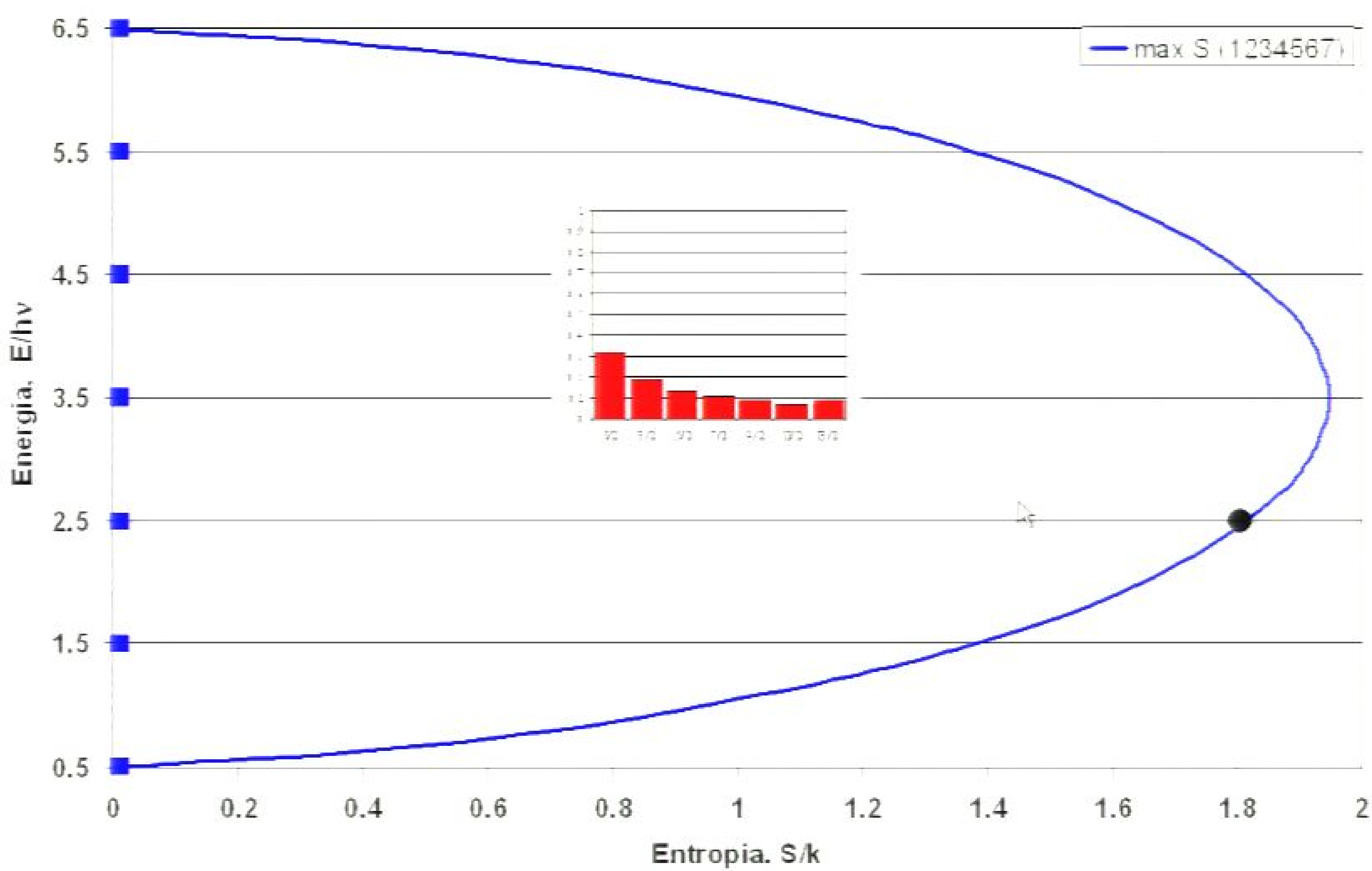
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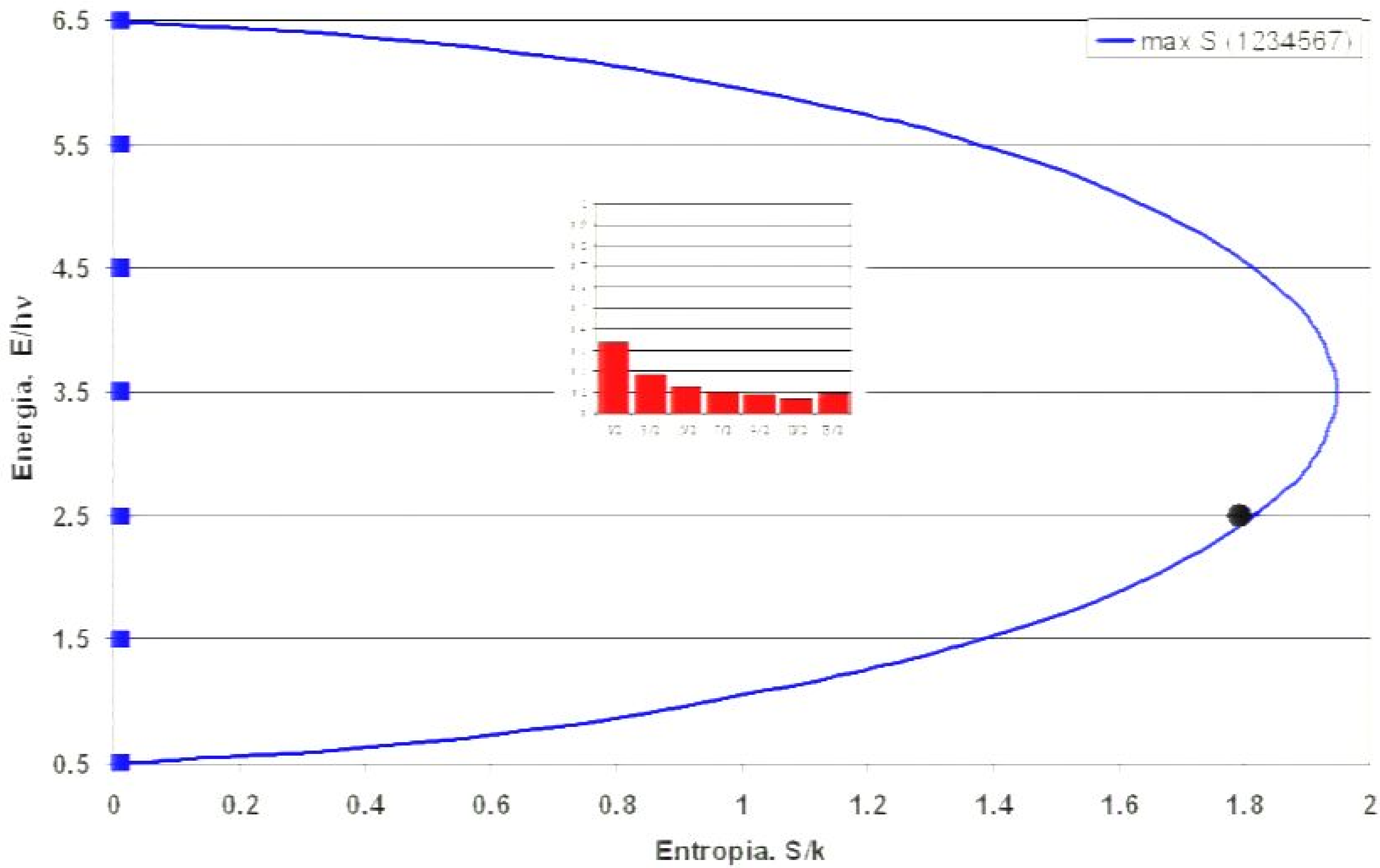

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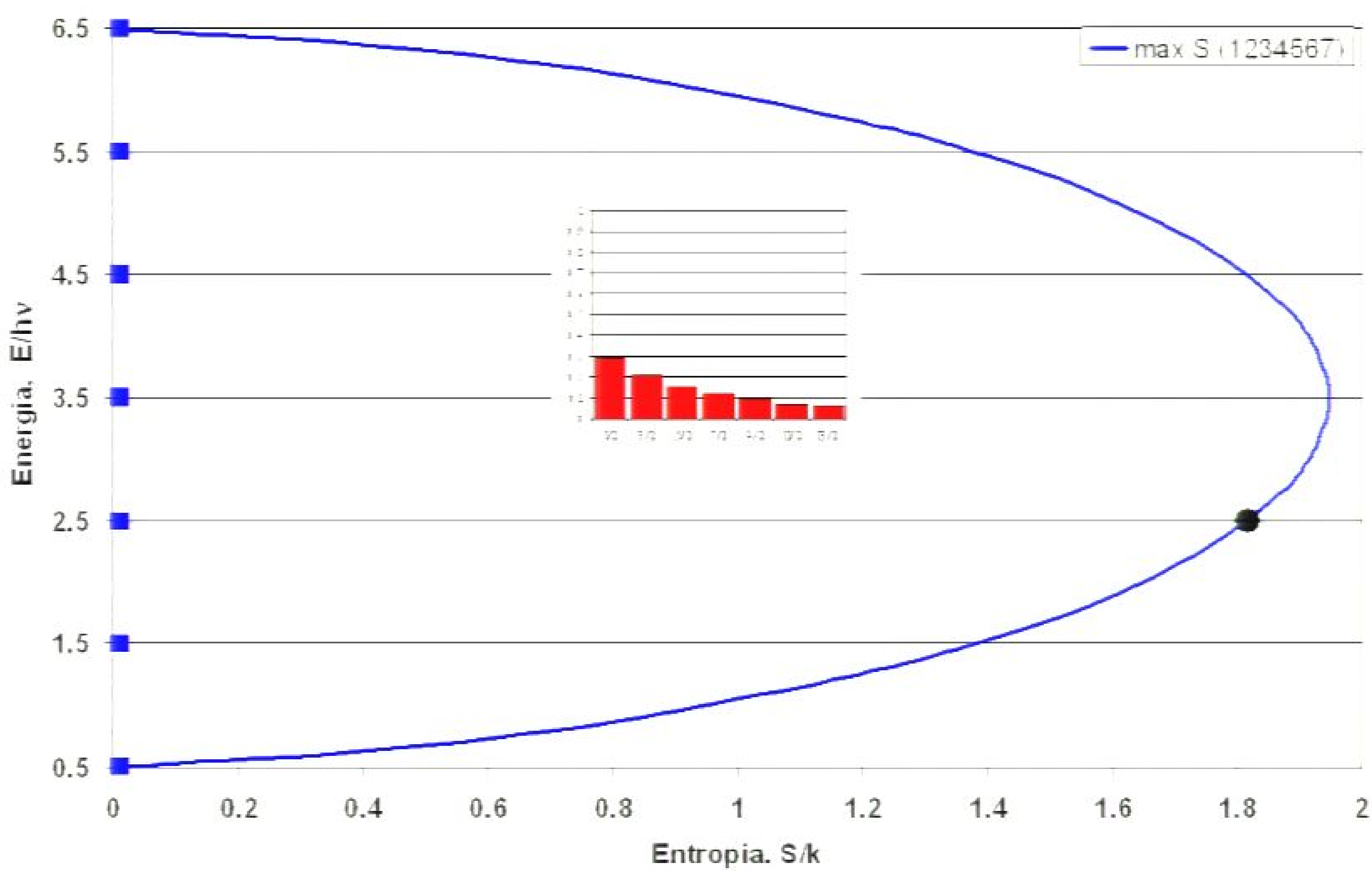
2.5
1.5
0.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



6.5
5.5
4.5
3.5
2.5
1.5
0.5
Energia. E/hv
max S (1234567)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Entropia. S/k

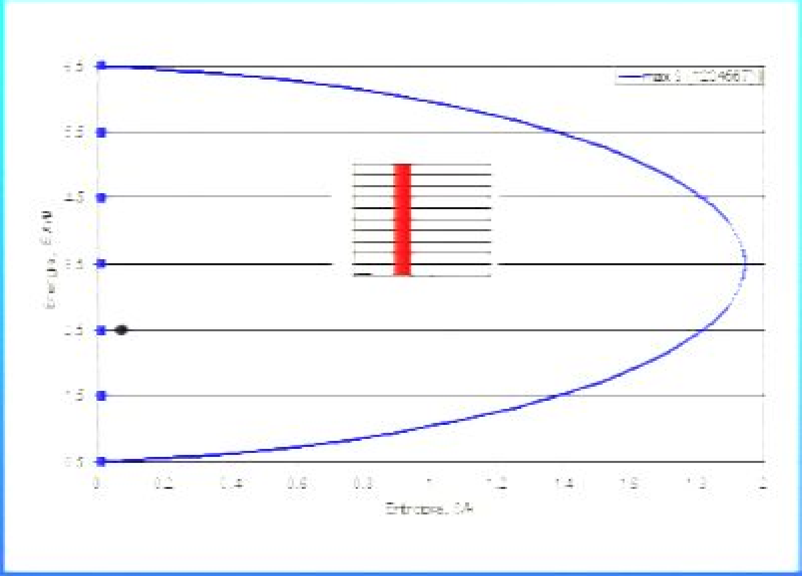
Steepest-entropy-ascent dynamical ansatz (isolated system)

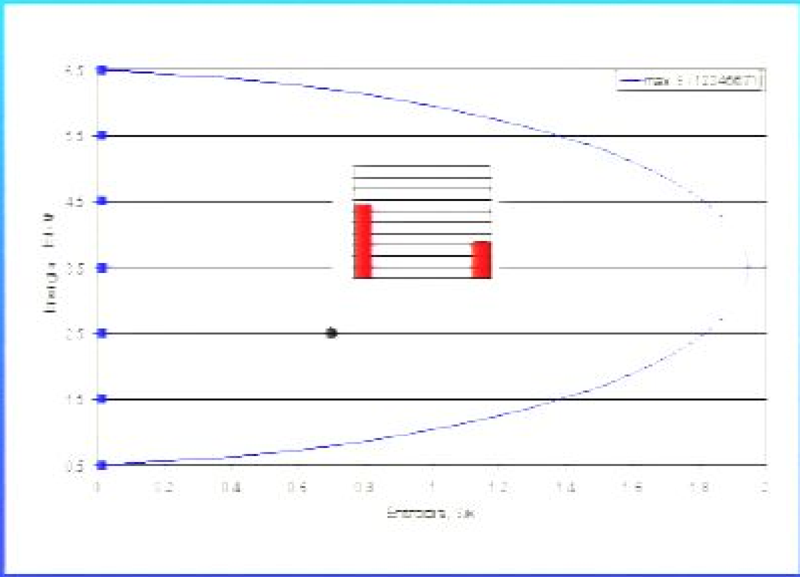
Ph.D.thesis, MIT (1981), quant-ph/0509116

Nuovo Cimento B, 82, 169 (1984); 87, 77 (1985)

NATO-ASI Lecture Notes, 278, 441 (1986)

Complete review at quant-ph/0112046





G.P. Beretta, Seminar "What if Quantum Thermodynamics were a fundamental extension of Quantum Mechanics?"
Perimeter Institute, Waterloo, Canada, November 8, 2007 - References available at: www.quantumthermodynamics.org

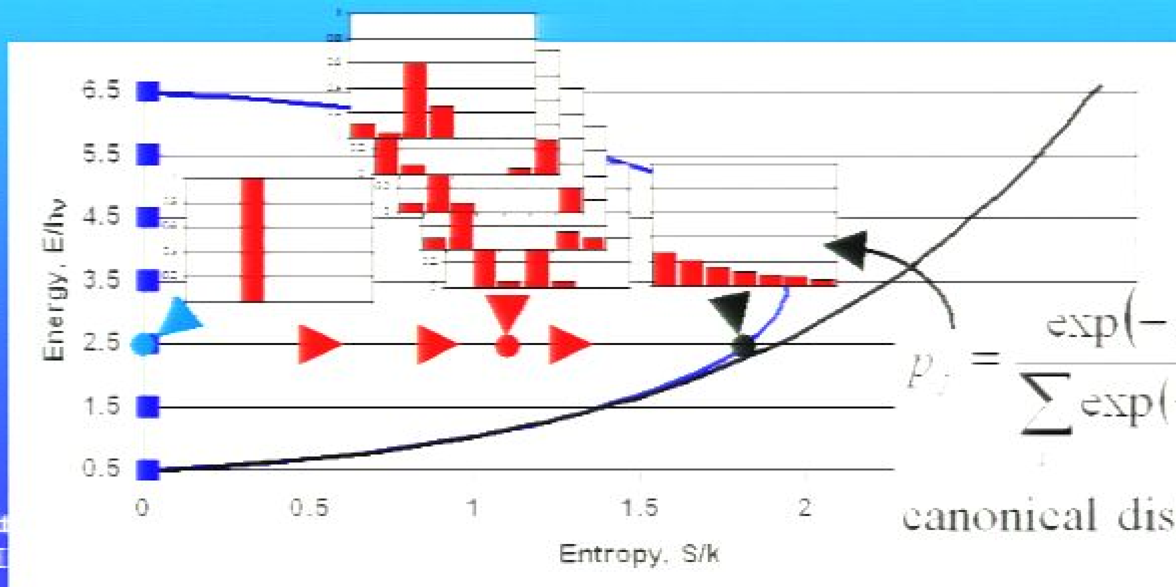
Desiderata for a general dynamical law (isolated system)

Mod.Phys.Lett.A 20, 977 (2005)

Energy conservation, $\frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_j p_j e_j\right)=0 \Rightarrow$

Entropy nondecrease, $\frac{\mathrm{d}}{\mathrm{dt}}\left(-k_B \sum_j p_j \ln p_j\right)\ge 0 \Rightarrow$

Probability conservation, $p_j \ge 0$, $\frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_j p_j\right)=0 \Rightarrow$



$$p_j = \frac{\exp(-e_j/k_B T)}{\sum_i \exp(-e_i/k_B T)}$$

canonical distribution

G.P. Berett
Perimeter I

Desiderata for a general dynamical law (isolated system)

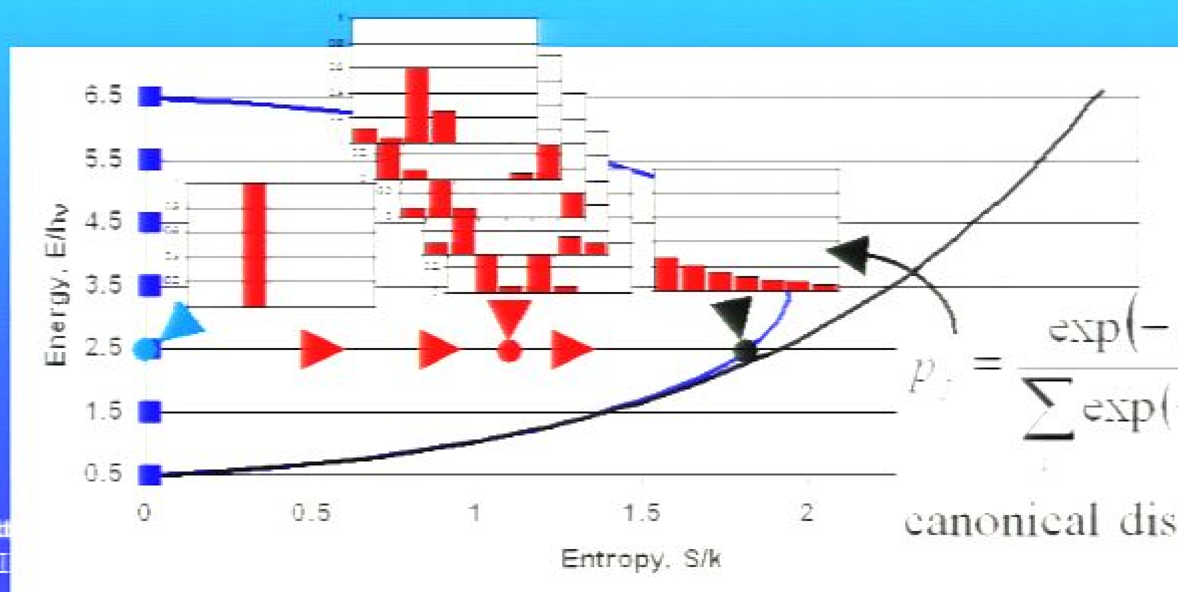
Mod.Phys.Lett.A. 20, 977 (2005)

Reformulate in terms of $r_j = \sqrt{p_j}$

Energy conservation: $\frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_j p_j e_j\right) = 0$ $\qquad \frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_j r_j^2 e_j\right) = 0$

Entropy nondecrease: $\frac{\mathrm{d}}{\mathrm{dt}}\left(-k_B \sum_j p_j \ln p_j\right) \geq 0$ $\qquad \frac{\mathrm{d}}{\mathrm{dt}}\left(-k_B \sum_j r_j^2 \ln r_j^2\right) \geq 0$

Probability conservation: $p_j \geq 0,\ \frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_j p_j\right) = 0$ $\qquad p_j = r_j^2 \geq 0,\ \frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_j r_j^2\right) = 0$



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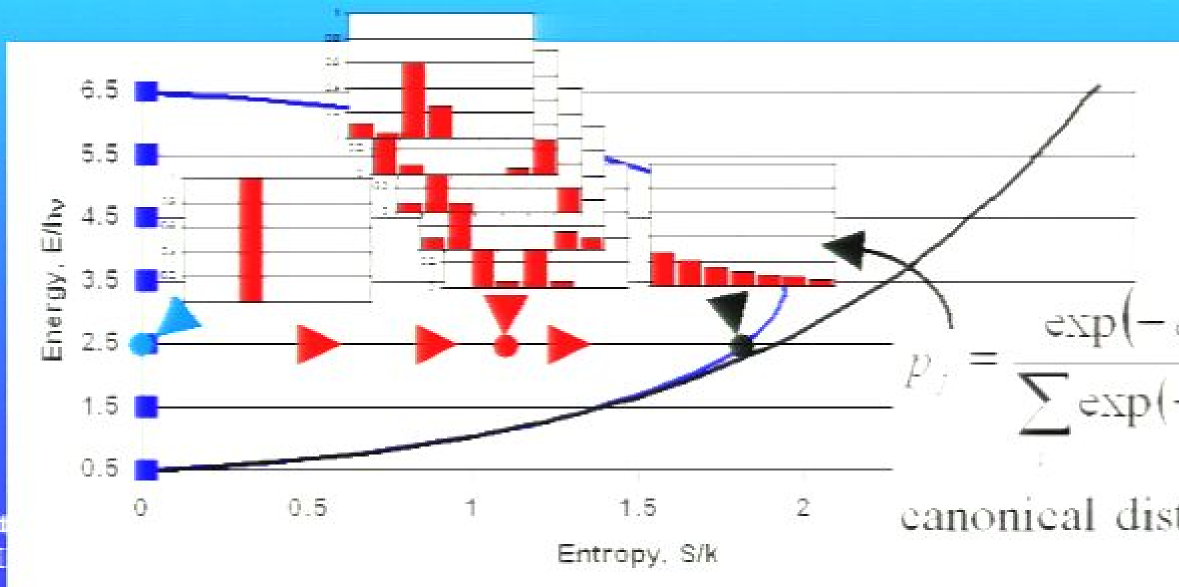

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Perimeter I

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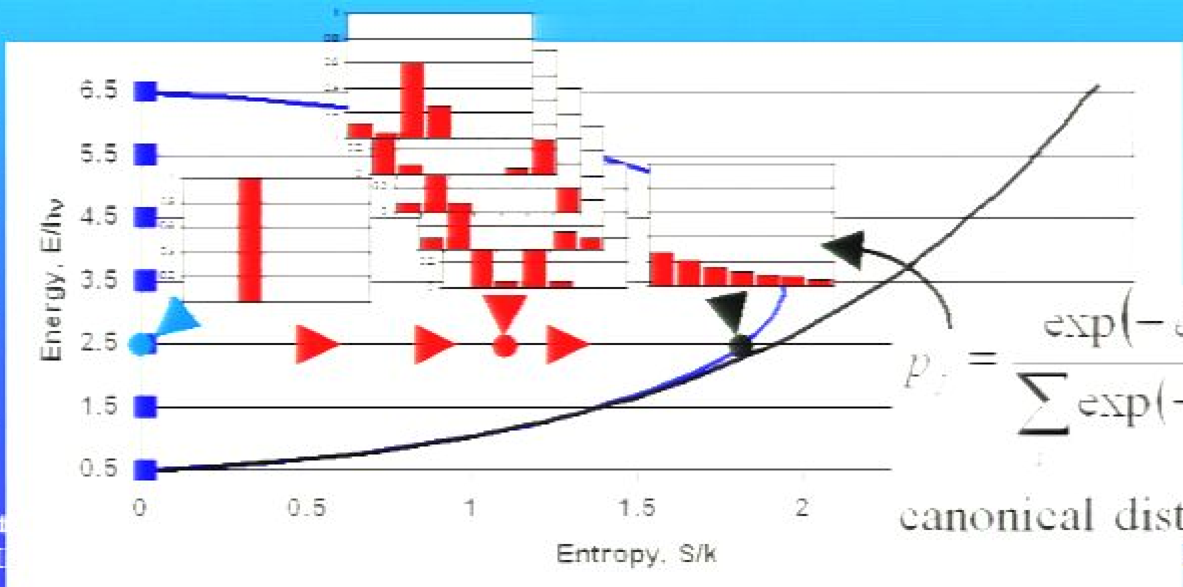
$$p_j = r_j^2 \geq 0, \quad \frac{d}{dt}\left(\sum_j r_j^2\right) = 0$$



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G.P. Berett

Perimeter I

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Probabi...

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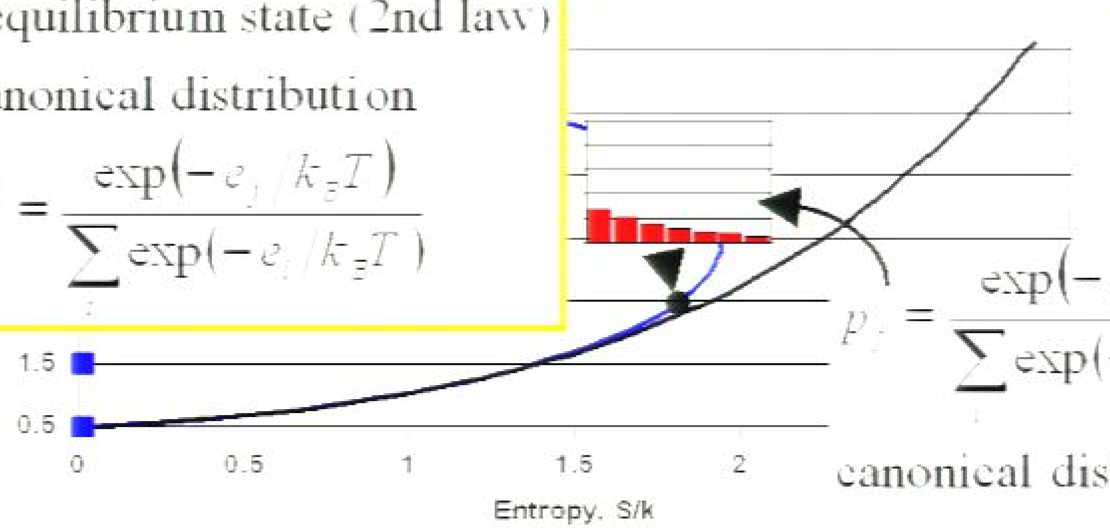
$$p_j = r_j^2 \geq 0, \quad \frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_j r_j^2\right) = 0$$

For each value of $E = \sum_j p_j e_j$ there must be one and only one stable equilibrium state (2nd law) with canonical distribution

$$p_j = r_j^2 = \frac{\exp(-e_j / k_B T)}{\sum_i \exp(-e_i / k_B T)}$$



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canonical distribution

G.P. Berett...
Perimeter I...

Equivalent variational formulation

Gheorghiu-Svirschevski, Phys.Rev.A. 63, 054102 (2001)

quant-ph/0112046

In terms of $r_j = \sqrt{p_j}$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_j r_j^2 e_j\right) = 0$$

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steepest - entropy - ascent dynamical ansatz :

$$\frac{\mathrm{d}r_j}{\mathrm{dt}} \text{ in the direction of max } \frac{\mathrm{d}S}{\mathrm{dt}} \text{ subject to } \frac{\mathrm{d}E}{\mathrm{dt}} = 0$$

$$\text{and } \frac{\mathrm{d}r^2}{\mathrm{dt}} = \frac{1}{\tau^2} \text{ where } \tau > 0 \text{ depends on } r$$

Equivalent variational formulation

Gheorghiu-Svirschevski, Phys.Rev.A 63, 054102 (2001)

quant-ph/0112046

Variational formulation :

$$\max \frac{\mathrm{d}}{\mathrm{dt}}\left(-k_B \sum_j r_j^2 \ln r_j^2\right) \text{ subject to}$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_j r_j^2\right)=0,\ \frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_j r_j^2 e_j\right)=0,\ \sum_j \dot{r}_j^2 = 1 \quad \tau^2(r)$$

In terms of $r_j = \sqrt{p_j}$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_j r_j^2 e_j\right)=0$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(-k_B \sum_j r_j^2 \ln r_j^2\right)\geq 0$$

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In terms of $r_j = \sqrt{p_j}$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_j r_j^2 e_j\right)=0$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(-k_B \sum_j r_j^2 \ln r_j^2\right)\geq 0$$

$$p_j = r_j^2 \geq 0,\ \frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_j r_j^2\right)=0$$

$$\frac{\partial}{\partial \&}\left[\frac{\mathrm{d}}{\mathrm{dt}}\left(-k_B \sum \ \ln \ \right) - \lambda \frac{\mathrm{d}}{\mathrm{dt}}\left(\sum \ \right) - \lambda \frac{\mathrm{d}}{\mathrm{dt}}\left(\sum \ \right) - 2\tau \sum \ \&\right] = 0$$

$$\frac{\partial}{\partial \&}\left[-2k_B \sum (\ - \ \ln \ )\& - 2 \ \sum \ \& - 2 \ \sum \ \& - 2\tau \sum \ \&\right] = 0$$

$$-k_B(\ - \ \ln \ ) - \ - \ - 2\tau\& = 0$$

$$\tau \frac{\mathrm{d}p_i}{\mathrm{dt}} = \tau \frac{\mathrm{d}r_i^2}{\mathrm{dt}} = 2\tau r_i \& = -k_B p_i \ln p_i - (k_B + \lambda_1) p_i - \lambda_E p_i e_i$$

Steepest-entropy-ascent dynamical ansatz (isolated system)

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] - \frac{1}{\tau}\frac{\begin{vmatrix} \rho\ln\rho & \rho & \frac{1}{2}\{H,\rho\} \\ \mathrm{Tr}\rho\ln\rho & 1 & \mathrm{Tr}\rho H \\ \mathrm{Tr}\rho H\ln\rho & \mathrm{Tr}\rho H & \mathrm{Tr}\rho H^2 \end{vmatrix}}{\mathrm{Tr}\rho H^2 - (\mathrm{Tr}\rho H)^2}$$

Ph.D.thesis, MIT (1981), quant-ph/0509116

Nuovo Cimento B, 82, 169 (1984); 87, 77 (1985)

J.Maddox, Nature, 316, 11 (1985)

NATO-ASI Lecture Notes, 278, 441 (1986)

Complete review at quant-ph/0112046

Phys.Rev.E, 73, 026113 (2006)

When $[\mathbf{H},\boldsymbol{\rho}] = 0$

$$\frac{dp_i}{dt} = -\frac{1}{\tau}\frac{\begin{vmatrix} p_i\ln p_i & p_i & e_i p_i \\ \sum p_i\ln p_i & 1 & \sum e_i p_i \\ \sum e_i p_i\ln p_i & \sum e_i p_i & \sum e_i^2 p_i \end{vmatrix}}{\begin{vmatrix} 1 & \sum e_i p_i \\ \sum e_i p_i & \sum e_i^2 p_i \end{vmatrix}}$$

$$\frac{dS}{dt} = \frac{k}{\tau}\frac{\begin{vmatrix} \sum p_i(\ln p_i)^2 & \sum p_i\ln p_i & \sum e_i p_i\ln p_i \\ \sum p_i\ln p_i & 1 & \sum e_i p_i \\ \sum e_i p_i\ln p_i & \sum e_i p_i & \sum e_i^2 p_i \end{vmatrix}}{\begin{vmatrix} 1 & \sum e_i p_i \\ \sum e_i p_i & \sum e_i^2 p_i \end{vmatrix}} \geq 0$$

J.Math.Phys. 27, 305 (1986), review in quant-ph-0112046

Stability of equilibrium states and second law

Indeed, Equation (2) preserves the rank and nullity of ρ (2) or, as said in Ref. 12, preserves the cardinality of the set of nonzero eigenvalues of ρ. Because the equation of motion attracts the state operator towards the highest entropy state or limit cycle compatible with the initial mean values of the generators of the motion and the number of zero eigenvalues, a minor perturbation of the state that changes an initially zero eigenvalue to an arbitrarily small nonzero value would cause an irreversible departure of the state towards a different (higher entropy) equilibrium state or limit cycle. Hence, as long as there are zero eigenvalues of ρ, i.e., unless $B = I$, all equilibrium states and limit cycles are unstable. Without this conclusion, we could not claim that the equation of motion entails the second law of thermodynamics (see Appendixes A and B, and the original papers, for further discussion of this important point).

The two ansatzs of "Quantum Thermodynamics"

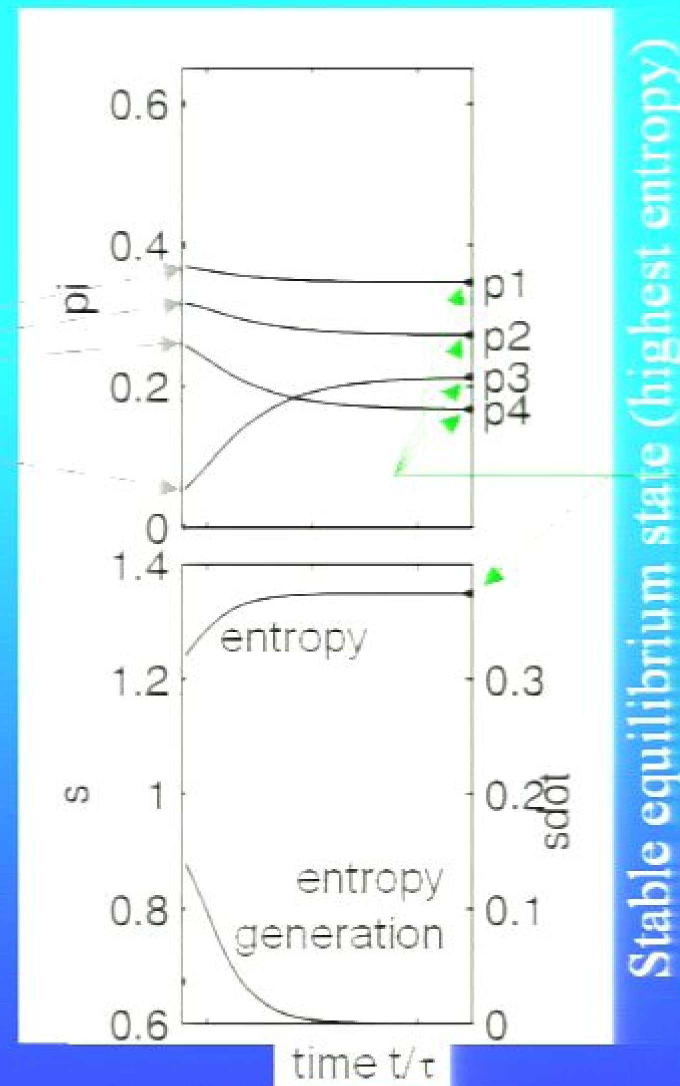
- The second law is forced directly into the microscopic laws and emerges as a theorem of the generalized dynamical law (stability of equilibrium states)

- Entropy emerges from the microscopic level as a measure of the degree of "load sharing" among the energy levels. The higher the sharing the lower the energy available for conversion to work.

- Irreversibility emerges as a manifestation of spontaneous internal load redistribution among the initially occupied energy levels.
- Experimentally verifiable? Phenomenological or fundamental?
- If proved not fundamental, the nonlinear dynamical law remains a valuable phenomenological nonlinear master equation, that guarantees all compatibility requirements with thermodynamics.

Time dependence of energy-level occupation probabilities

An arbitrary initial distribution (state)



Phys.Rev.E, 73, 026113 (2006)

G.P. Beretta, Seminar "What if Quantum Thermodynamics were a fundamental extension of Quantum Mechanics?"
Perimeter Institute, Waterloo, Canada, November 8, 2007 - **References available at: www.quantumthermodynamics.org**

Time dependence of energy-level occupation probabilities

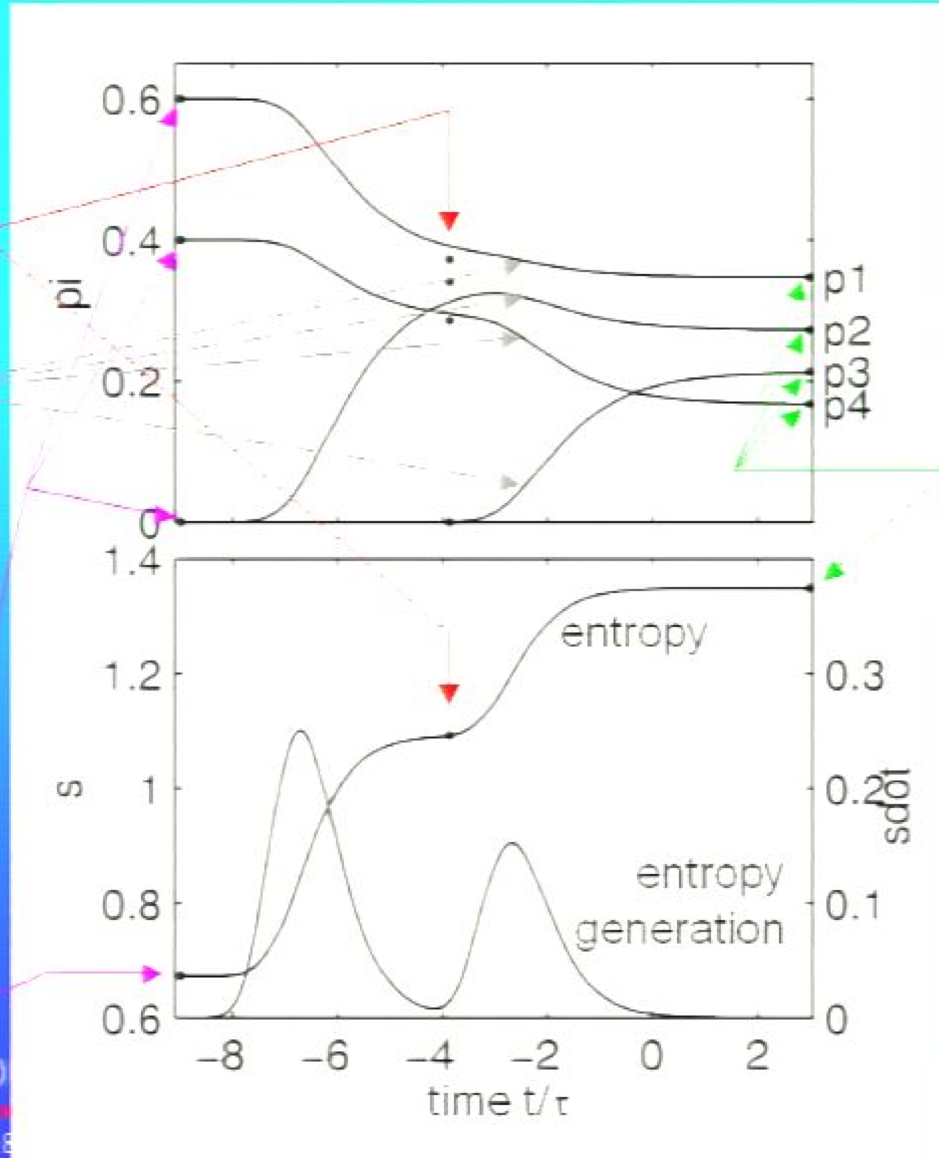
The trajectory passes very close to an unstable equilibrium state

An arbitrary initial distribution (state)

Strong causality:

given any initial state the trajectory is unique and defined for $-\infty < t < +\infty$

We can trace back the lowest entropy 'ancestral' state



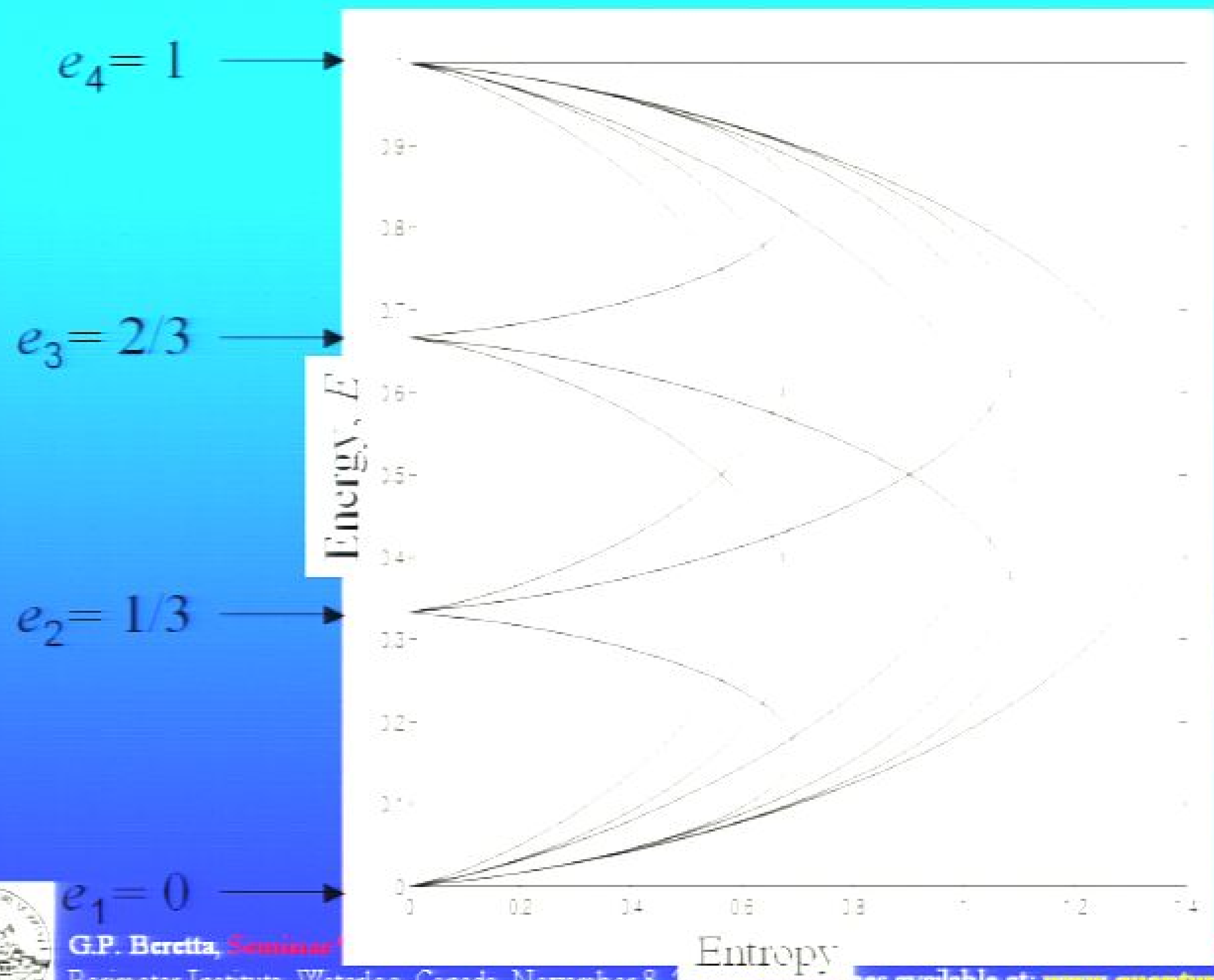
Stable equilibrium state (highest entropy)

Phys.Rev.E, 73, 026113 (2006)

G.P. Beretta, Seminar "What if Quantum Therm
Perimeter Institute, Waterloo, Canada, November

Internal dynamical structure for a 4-level system

Phys.Rev.E. 73, 026113 (2006)



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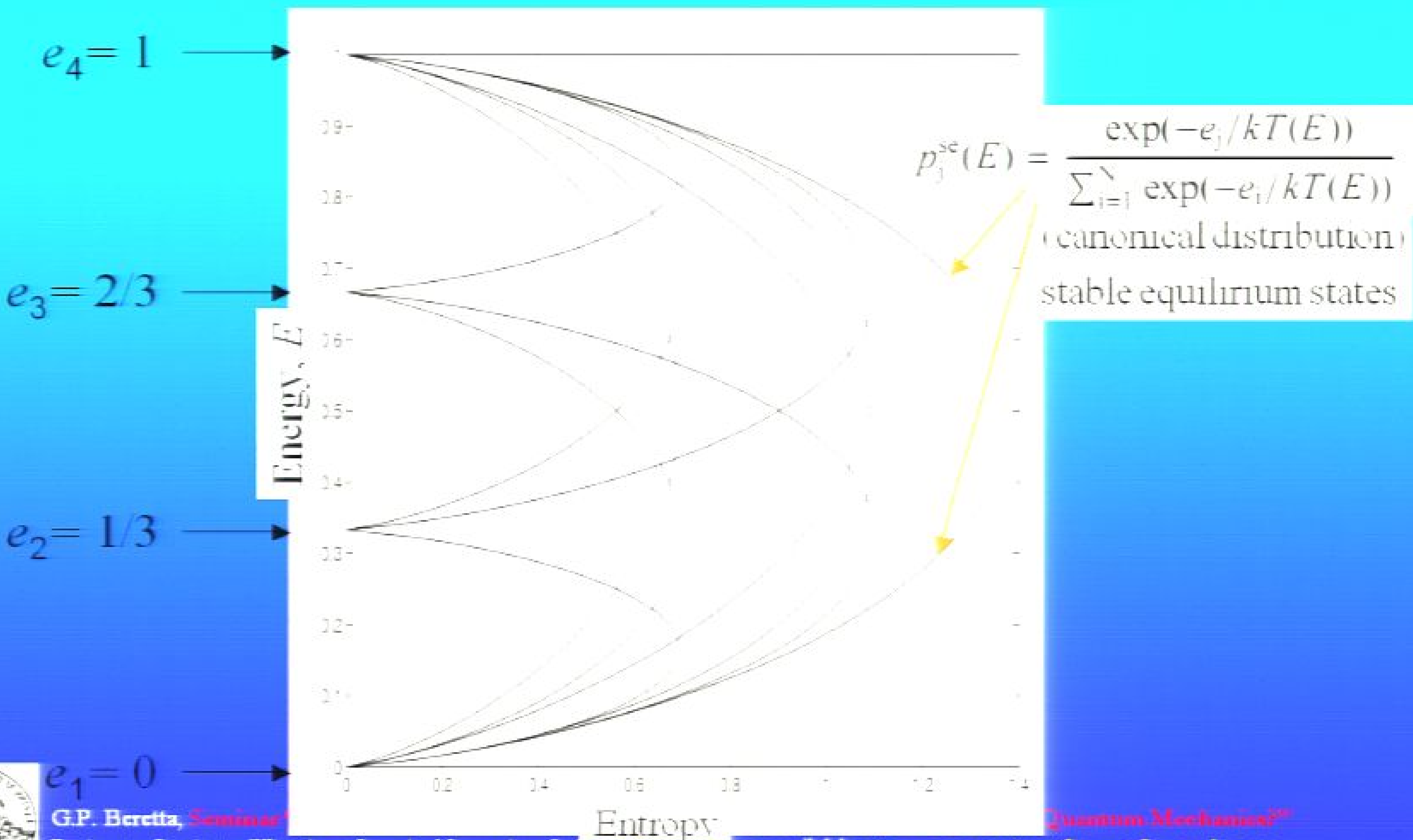
Phys.Rev.E. 73, 026113 (2006)



Internal dynamical structure for a 4-level system

Phys.Rev.E. 73, 026113 (2006)

$$p_j^{pe}(E,\boldsymbol{\delta}) = \frac{\delta_j \exp(-\beta^{pe}(E,\boldsymbol{\delta})\, e_j)}{\sum_{i=1}^{N} \delta_i \exp(-\beta^{pe}(E,\boldsymbol{\delta})\, e_i)}$$

lowest entropy and unstable equilibrium states

$$p_j^{se}(E) = \frac{\exp(-e_j/kT(E))}{\sum_{i=1}^{N} \exp(-e_i/kT(E))}$$

(canonical distribution)

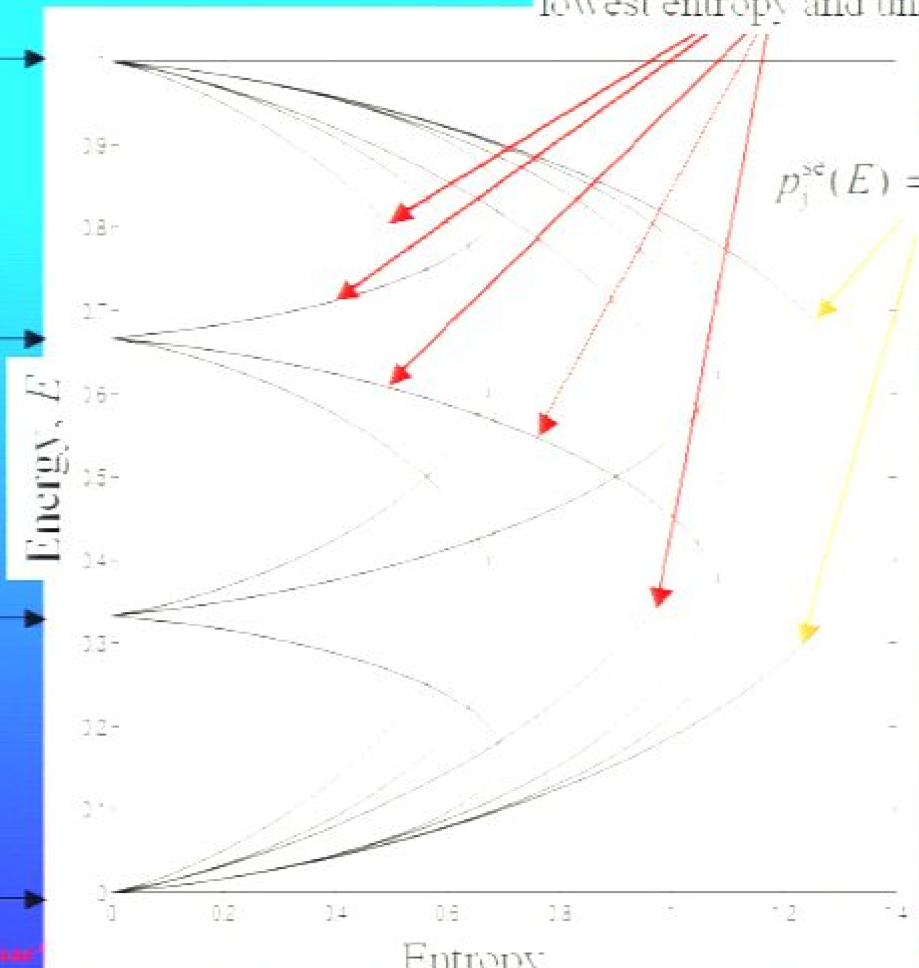
stable equilibrium states

$e_4 = 1$

$e_3 = 2/3$

$e_2 = 1/3$

$e_1 = 0$



Standard Time-Energy Uncertainty Relations

In the Mandelstam-Tamm-Messiah interpretation, for any F the characteristic time

$$\tau_F = \Delta_F / |\mathrm{Tr}(\dot{\rho} F)|$$

represents the time required for the mean value $\langle F \rangle$ to change by an amount equal to the width Δ_F of the distribution. τ_F is just a feature of the dynamics, a measure of how fast (or slow) the mean value $\langle F \rangle$ changes with time.

quant-ph/0511091

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For Hamiltonian (unitary) time evolution,

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\frac{i}{\hbar}[H, \rho] \; .$$

the "internal" rate of change of $\langle F \rangle$ is given by

$$\mathrm{Tr}(\dot{\rho} F) = \text{[illegible]} = \frac{\langle i[F, H]/2 \rangle}{\hbar/2} = -\frac{\eta_{FH}}{\hbar/2}$$

Steepest-entropy-ascent time-energy uncertainty relations

quant-ph-0511091

The internal rate of change of $\langle F \rangle$ is

$$\mathrm{Tr}(\dot{\rho} F) = \frac{\langle i[F,H]/2 \rangle}{\hbar/2} + \frac{\langle \Delta F \Delta M \rangle}{k_{\rm B}\tau} = -\frac{\eta_{FH}}{\hbar/2} + \frac{\sigma_{FM}}{k_{\rm B}\tau} .$$

We obtain the general exact uncertainty relation

$$\tau_F \Delta_H = \frac{\hbar/2}{|c_{FH} + a_\tau r_{FM}|}$$

where a_τ is the nonnegative, dimensionless functional

$$a_\tau = \frac{\hbar/2}{k_{\rm B}} \frac{\Delta_M/\tau}{\Delta_H} .$$

In general, if the dynamics is dissipative ($\tau \neq \infty$, $a_\tau \neq 0$) there are density operators for which $|c_{FH} + a_\tau r_{FM}| > 1$ so that $\tau_F \Delta_H$ takes a value less than $\hbar/2$ and thus, expectedly, the usual time-energy uncertainty relation is violated. The sharpest general time-energy uncertainty relation always satisfied when both Hamiltonian and dissipative dynamics are active is

$$\left[\frac{\hbar/2}{\tau_F \Delta_H}\right]^2 \leq 1 + a_\tau^2 + 2a_\tau c_{MH} .$$

Steepest-entropy-ascent time-energy / time-entropy uncertainty relations

quant-ph-0511091

The rate of entropy generation is bounded, i.e.,

$$\frac{\mathrm{d}\langle S\rangle}{\mathrm{d}t} = -k_{\mathrm{B}}\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Tr}(\rho\ln\rho) = \frac{\sigma_{MM}}{k_{\mathrm{B}}\tau} \leq \frac{\Delta_S\Delta_M}{k_{\mathrm{B}}\tau} \leq \frac{\sigma_{SS}}{k_{\mathrm{B}}\tau}$$

and we have a time-entropy uncertainty relation

$$\frac{k_{\mathrm{B}}\tau(\rho)}{\tau_S\Delta_S} = r_{SM}^2 \leq r_{SM} \leq 1$$

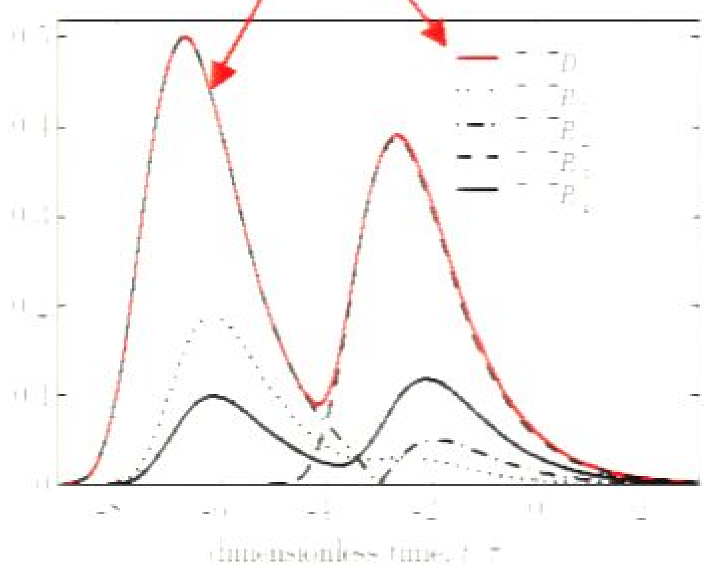
Moreover, we find the time-energy/time-entropy uncertainty relations

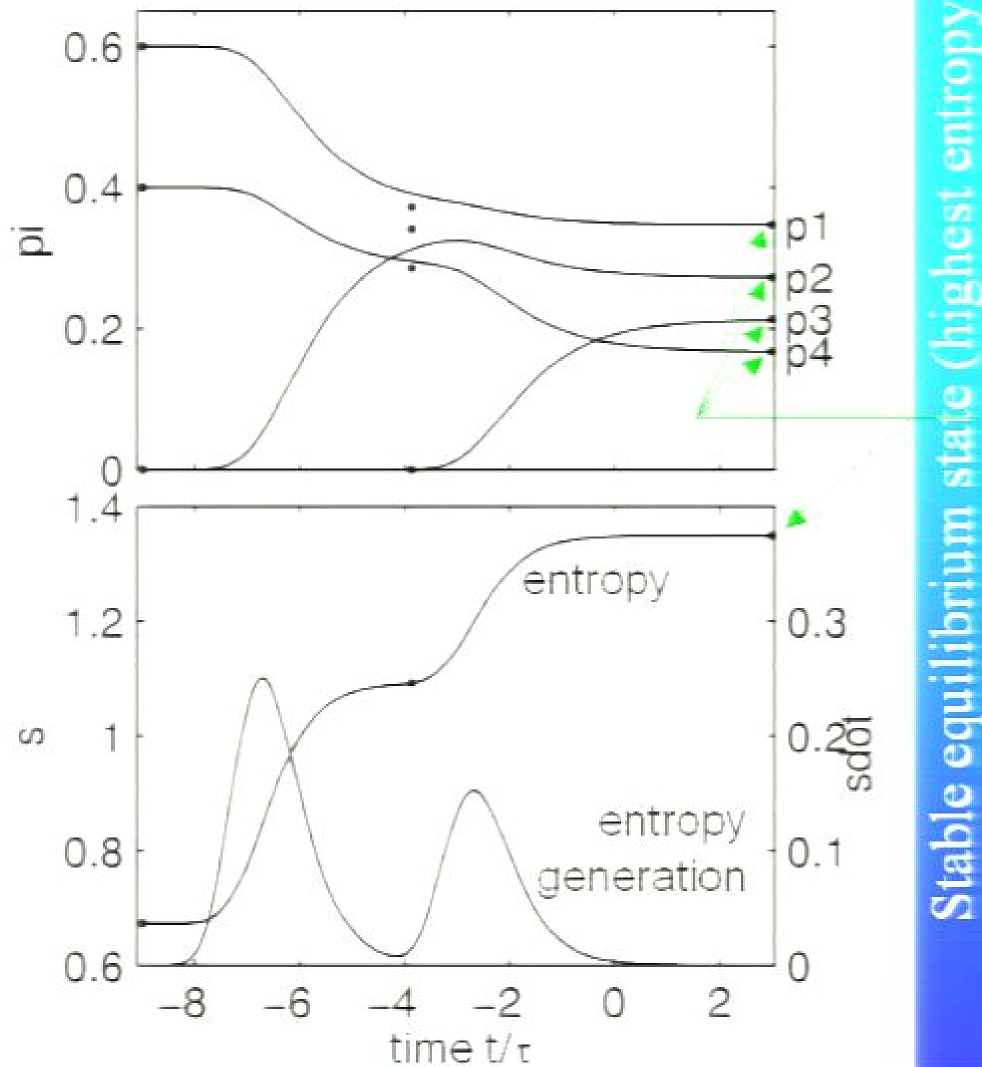
$$\left(\frac{\tau_F\Delta_H}{\hbar/2}\right)^2 + \left(\frac{\tau_F\Delta_S}{k_{\mathrm{B}}\tau(\rho)}\right)^2 \geq 1$$

Time dependence of energy-level occupation probabilities

quant-ph-0511091

time-entropy uncertainty bound





Found.Phys., 17, 365 (1987), review in quant-ph-0112046

Affinities and dissipative rates of change

Any nonequilibrium ρ can be written as [6]

$$\rho = \frac{B\exp(-\sum_j f_j X_j)}{\mathrm{Tr} B\exp(-\sum_j f_j X_j)} \ . \tag{39}$$

where the set $\{f_j, X_j\}$ spans the real space $\mathcal{L}(\mathcal{H})$. Hence,

$$\sqrt{\rho}\ln\rho = -f_0\sqrt{\rho} - \sum_j f_j \sqrt{\rho} X_j \ , \tag{40}$$

$$x_j(\rho) = \mathrm{Tr}(\rho X_j) \ . \tag{41}$$

$$s(\rho) = k_B f_0 + k_B \sum_j f_j \, x_j(\rho) \ . \tag{42}$$

$$\text{where} \quad k_B f_j = \left.\frac{\partial s(\rho)}{\partial x_j(\rho)}\right|_{x_{i\neq j}(\rho)} \tag{43}$$

may be interpreted as a generalized affinity or force, and

$$\frac{\mathrm{D}x_i(\rho)}{\mathrm{D}t} = 2\,(E_D|\sqrt{\rho}X_i) \ . \tag{44}$$

is the dissipative rate of change of property $x_j(\rho)$.

Found.Phys., 17, 365 (1987), review in quant-ph-0112046

Generalized Onsager conductivities and reciprocity

The rate of entropy change may be rewritten as a quadratic form of the generalized affinities.

$$\frac{\mathrm{d}s(\rho)}{\mathrm{d}t} = k_\mathrm{B} \sum_i \sum_j f_i f_j L_{ij}(\rho) \ . \qquad (47)$$

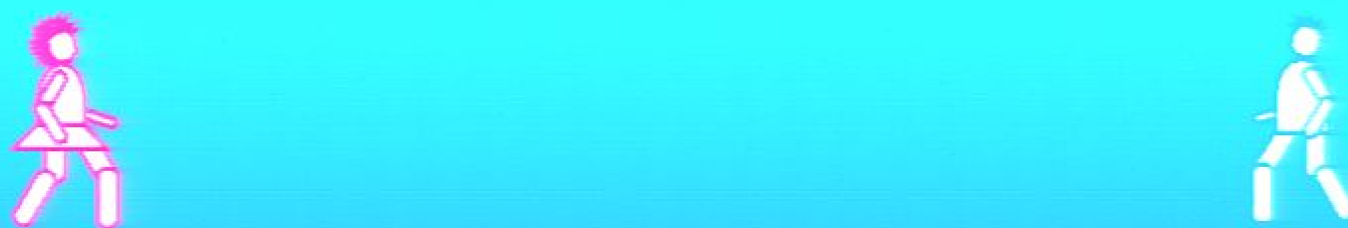
Being a Gram matrix, $[L_{ij}(\rho)]$ has non-negative definite determinant $\det[L_{ij}(\rho)] \geq 0$, strict positive iff all operators [illegible] are linearly independent, in which case Eq. (47) may be solved to yield

$$f_j = \sum_i L^{-1}_{ij}(\rho) \frac{\mathrm{D}x_i(\rho)}{\mathrm{D}t} \qquad (48)$$

and the rate of entropy change can be written as a quadratic form of the dissipative rates

$$\frac{\mathrm{d}s(\rho)}{\mathrm{d}t} = k_\mathrm{B} \sum_i \sum_j L^{-1}_{ij}(\rho) \frac{\mathrm{D}x_i(\rho)}{\mathrm{D}t} \frac{\mathrm{D}x_j(\rho)}{\mathrm{D}t} \ . \qquad (49)$$

STEEPEST-ENTROPY-ASCENT DYNAMICS FOR COMPOSITE SYSTEMS. NO-SIGNALING CONDITION AND LOCAL EFFECTS OF CORRELATIONS



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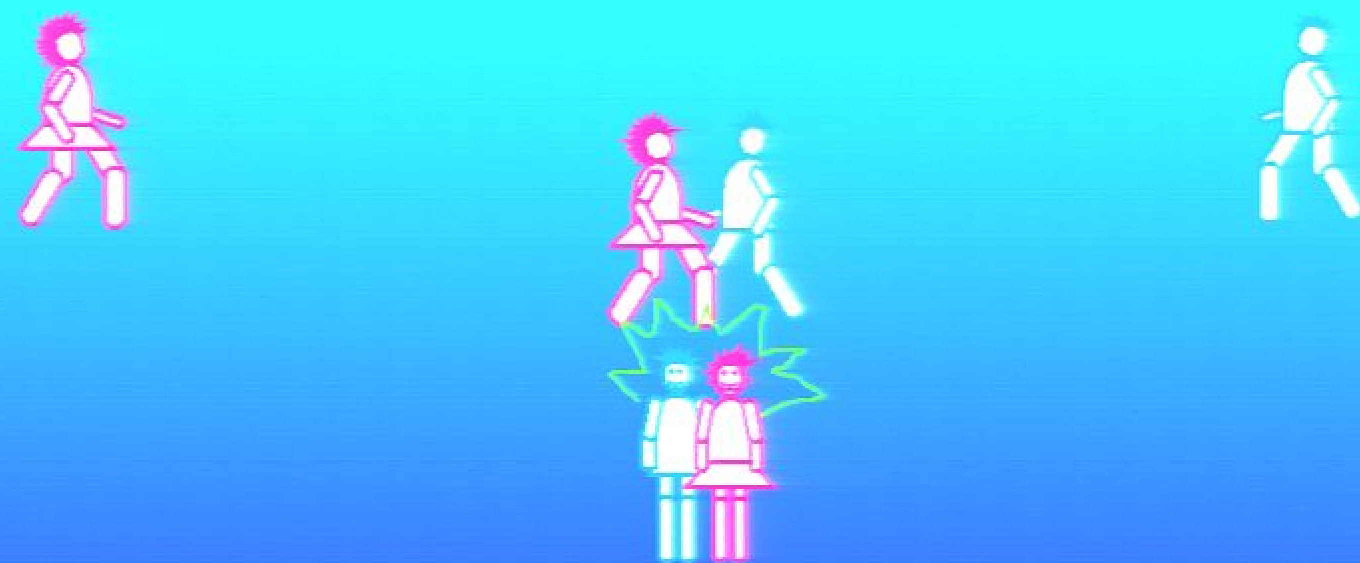
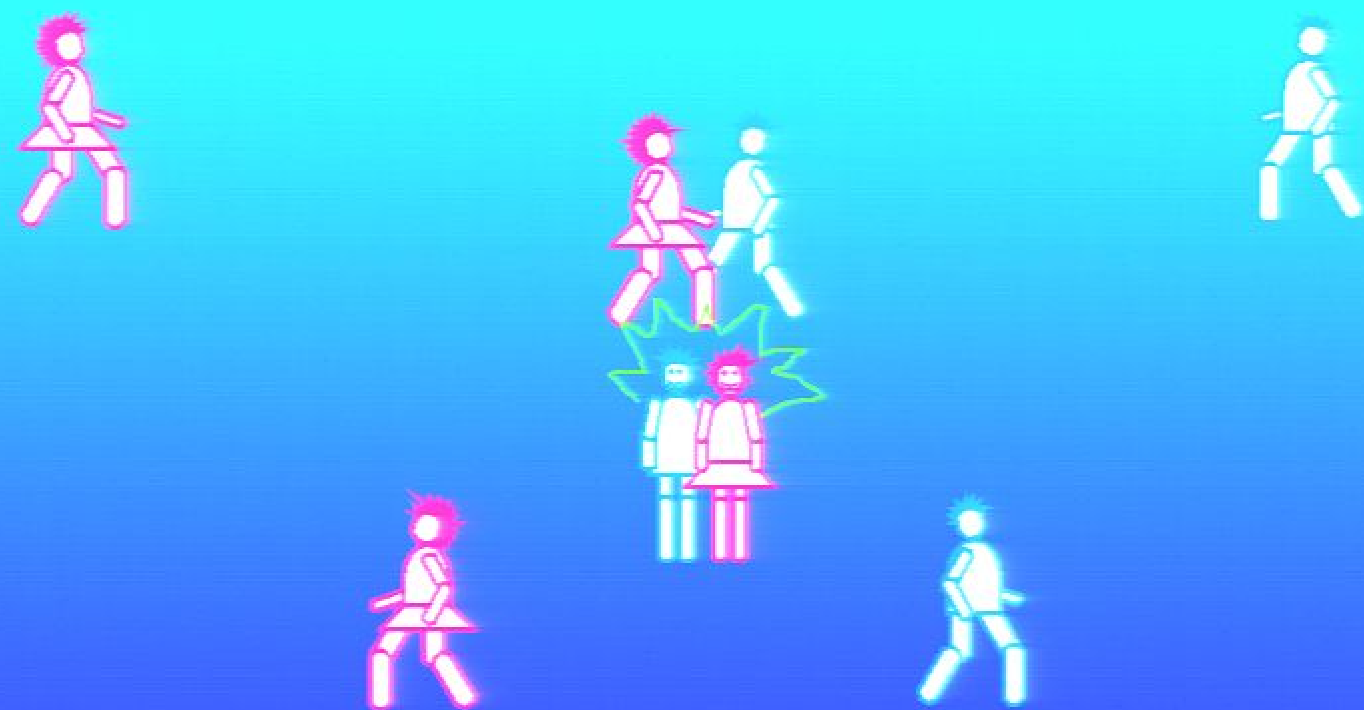


STEEPEST-ENTROPY-ASCENT DYNAMICS FOR COMPOSITE SYSTEMS.

NO-SIGNALING CONDITION AND LOCAL EFFECTS OF CORRELATIONS



STEEPEST-ENTROPY-ASCENT DYNAMICS FOR COMPOSITE SYSTEMS.

NO-SIGNALING CONDITION AND LOCAL EFFECTS OF CORRELATIONS

In general, if A and B are either interacting ($H = H_A \otimes I_B + I_A \otimes H_B + V_{AB}$) or correlated ($S \neq S_A \otimes I_B + I_A \otimes S_B$, where $S = -k_B (P_{\mathrm{Ran}\,\rho}) \ln \rho$), or both:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \frac{1}{2k_B\tau_A}(\sqrt{\rho_A}D_A + D_A^\dagger\sqrt{\rho_A}) \otimes \rho_B + \frac{1}{2k_B\tau_B}\rho_B \otimes (\sqrt{\rho_B}D_B + D_B^\dagger\sqrt{\rho_B})$$

$$D_A = [\sqrt{\rho_A}(S)^A]_{\perp\mathcal{L}(\sqrt{\rho_A},\sqrt{\rho_A}(H)^A)}$$
$$(H)^A = \mathrm{Tr}_B[(I_A \otimes \rho_B)H]$$
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All zero entropy states ($\rho^2 = \rho$), even if A and B are entangled, obey the Schroedinger equation $\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho]$.

G.P. Beretta, Seminar "What if Quantum Thermodynamics were a fundamental extension of Quantum Mechanics?"
Perimeter Institute, Waterloo, Canada, November 8, 2007 - References available at: www.quantumthermodynamics.org

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G.P. B

Perimeter Institute, Waterloo, Canada, November 8, 2007 - **References available at: www.quantumthermodynamics.org**

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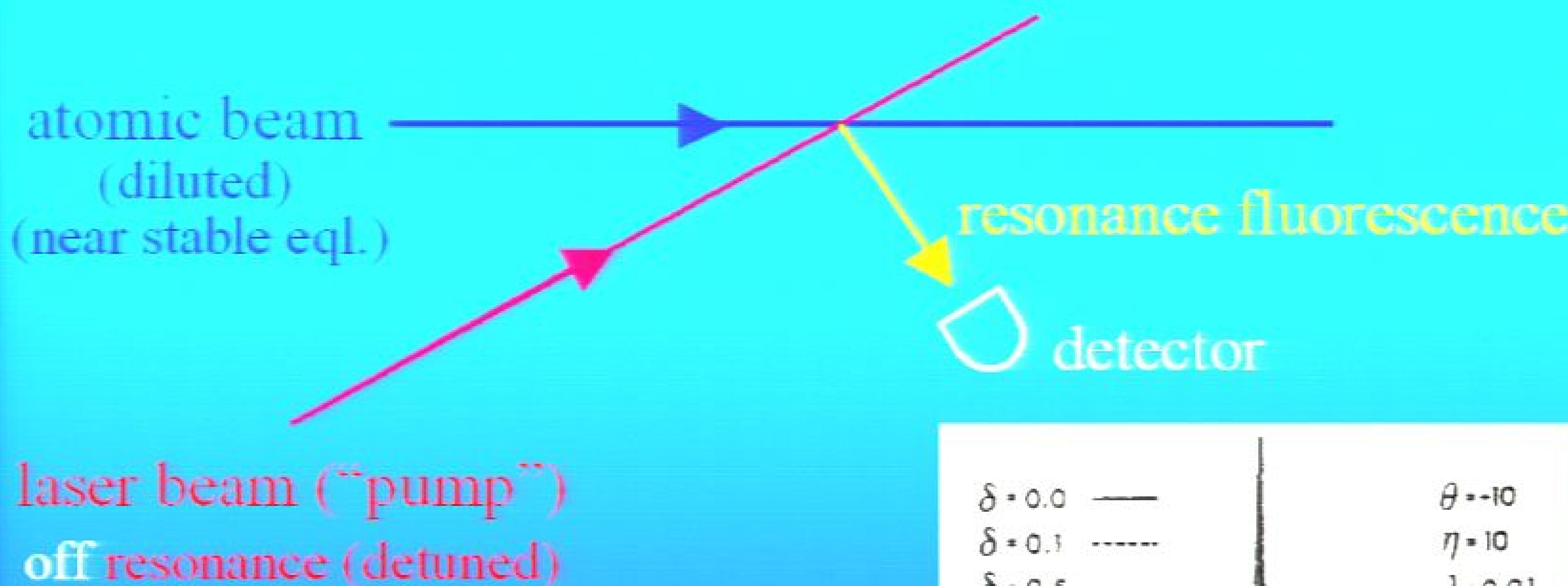
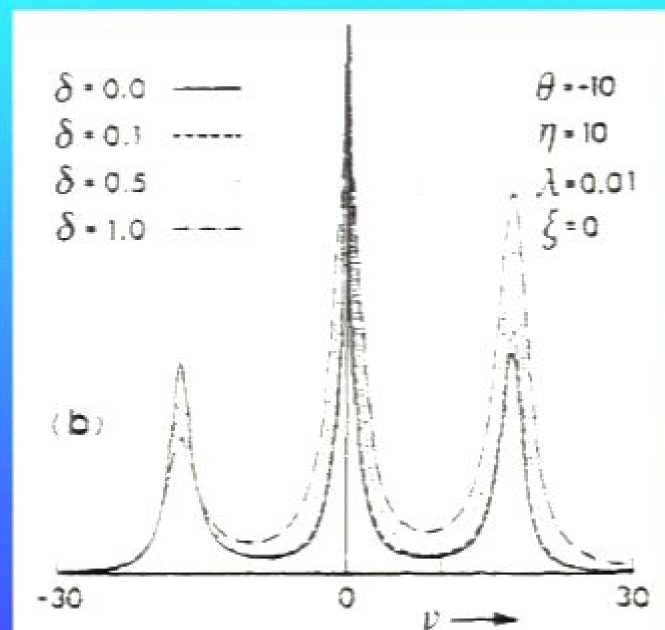
Measurable effects?



Irreversible internal redistribution
implies
asymmetries
in the spectral distribution



Int.J.Theor.Phys., 24, 1233 (1985)

NATURE VOL. 316 4 JULY 1985

Uniting mechanics and statistics

An adventurous scheme which seeks to incorporate thermodynamics into the quantum laws of motion may end arguments about the arrow of time — but only if it works.

None of this implies that the arguments about the reconciliation between microscopic reversibility and macroscopic irreversibility will now be stilled. Indeed, while for as long as the present justification of the basis of statistical mechanics holds water, there will be many who say that what Beretta *et al*. have done is strictly unnecessary. But this is a field in which the proof of the pudding is in the eating.

John Maddox

Nuovo Cimento B, 87, 77 (1985), review in quant-ph-0112046

Separability: structure of the equation of motion for…

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \sqrt{\rho}\,E + E^\dagger\sqrt{\rho}\,, \qquad E_D = \frac{1}{2k_\mathrm{B}\tau(\rho)}\,[\sqrt{\rho}S]_{\perp\mathcal{L}\{\sqrt{\rho}R_i\}}\,.$$

$$E = E_H + E_D\,, \qquad \{\sqrt{\rho}R_i\} = \text{ linear span of } \sqrt{\rho}I,\ \sqrt{\rho}H,\ \sqrt{\rho}N_k$$

$$E_H = i\sqrt{\rho}\,H/\hbar\,, \qquad (F|G) = \mathrm{Tr}(F^\dagger G + G^\dagger F)/2\,.$$

$$S = -k_\mathrm{B} P_{\mathrm{Ran}\,\rho} \ln\rho\,.$$

…an indivisible system

$$\mathcal{H} = \mathcal{H}^1 \otimes \mathcal{H}^2 \otimes \cdots \otimes \mathcal{H}^M = \mathcal{H}^J \otimes \mathcal{H}^{\overline{J}}\,.$$

…a composite system

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \sqrt{\rho}\,E + E^\dagger\sqrt{\rho}\,,$$

$$E = E_H + E_D\,, \qquad \sqrt{\rho}\,E_D = \sum_{J=1}^{M} \sqrt{\rho_J}\,E_{DJ} \otimes \rho_{\overline{J}}\,.$$

$$E_H = i\sqrt{\rho}\,H/\hbar\,,$$

$$S = -k_\mathrm{B} P_{\mathrm{Ran}\,\rho} \ln\rho\,, \qquad E_{DJ} = \frac{1}{2k_\mathrm{B}\tau_J(\rho)}\left[\sqrt{\rho_J}(S)^J\right]_{\perp\mathcal{L}\{\sqrt{\rho_J}(R_{iJ})^J\}}\,.$$

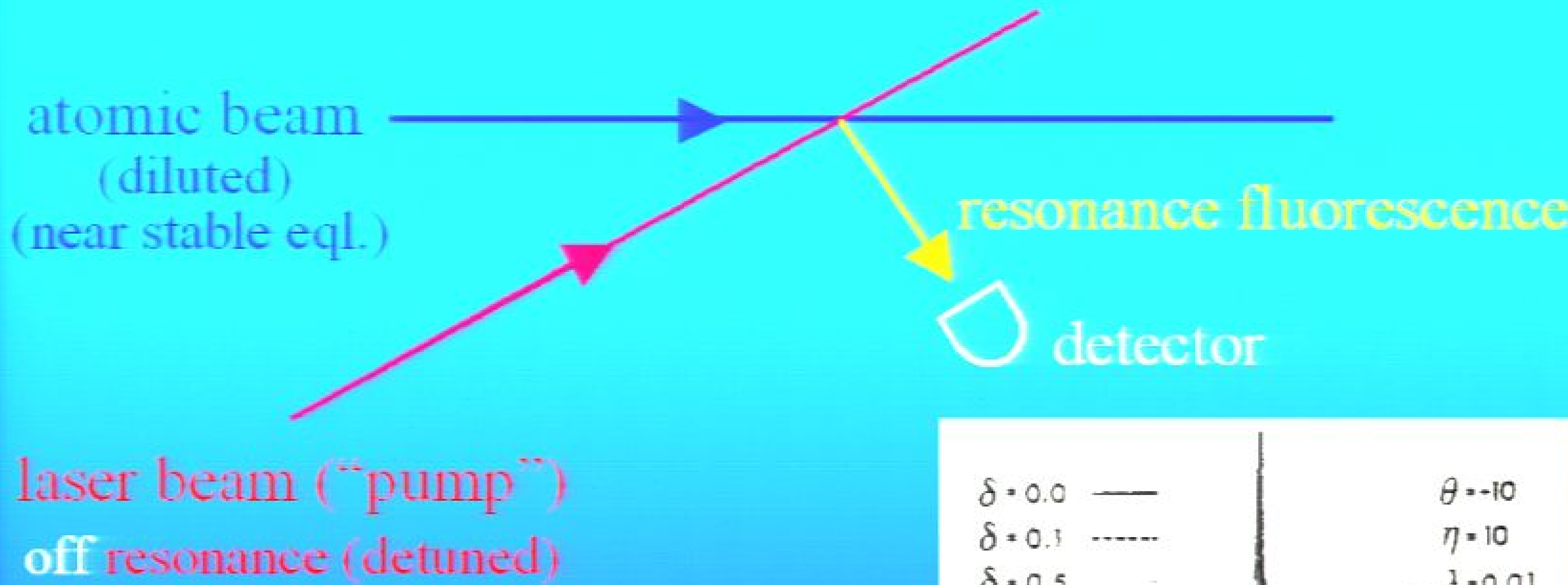
$$\mathcal{L}\{\sqrt{\rho_J}(R_{iJ})^J\} = \text{ lin. span of } \sqrt{\rho_J}I_J,\ \sqrt{\rho_J}(H_J)^J,\ \sqrt{\rho_J}(N_{kJ})^J$$

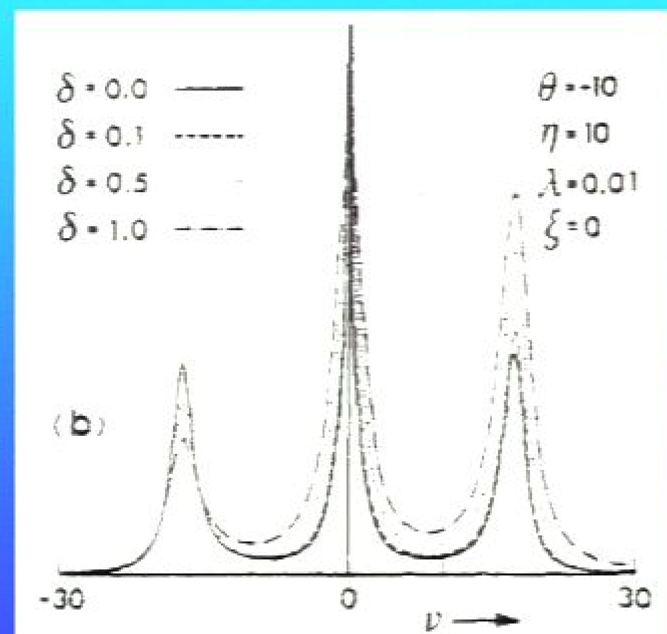
$$(F_J|G_J)_J = \mathrm{Tr}_J(F_J^\dagger G_J + G_J^\dagger F_J)/2\,.$$

$$(R_{iJ})^J = \mathrm{Tr}_{\overline{J}}[(I_J \otimes \rho_{\overline{J}})R_{iJ}]\,.$$

$$(S)^J = \mathrm{Tr}_{\overline{J}}[(I_J \otimes \rho_{\overline{J}})S]\,.$$

Measurable effects?





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