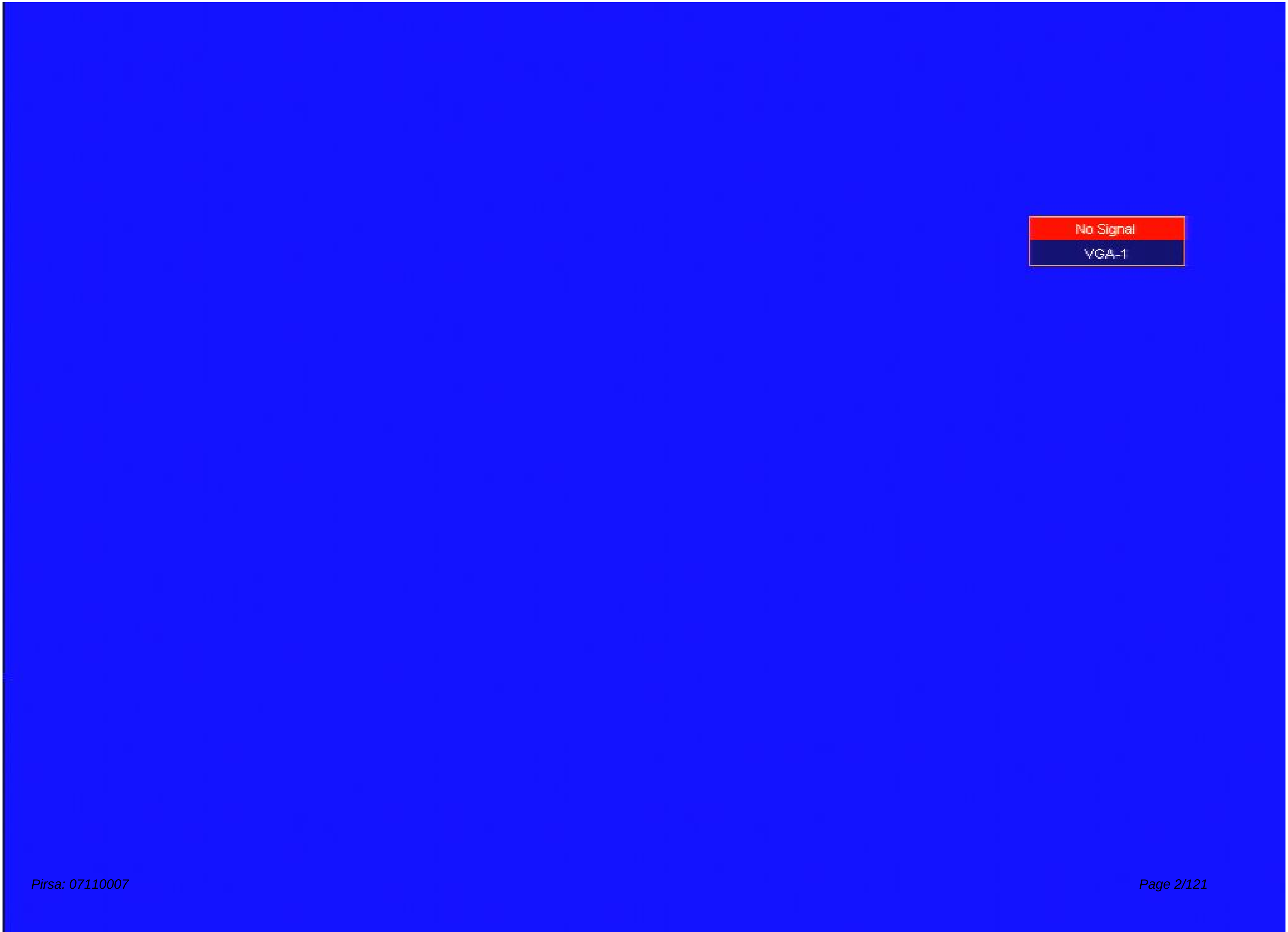
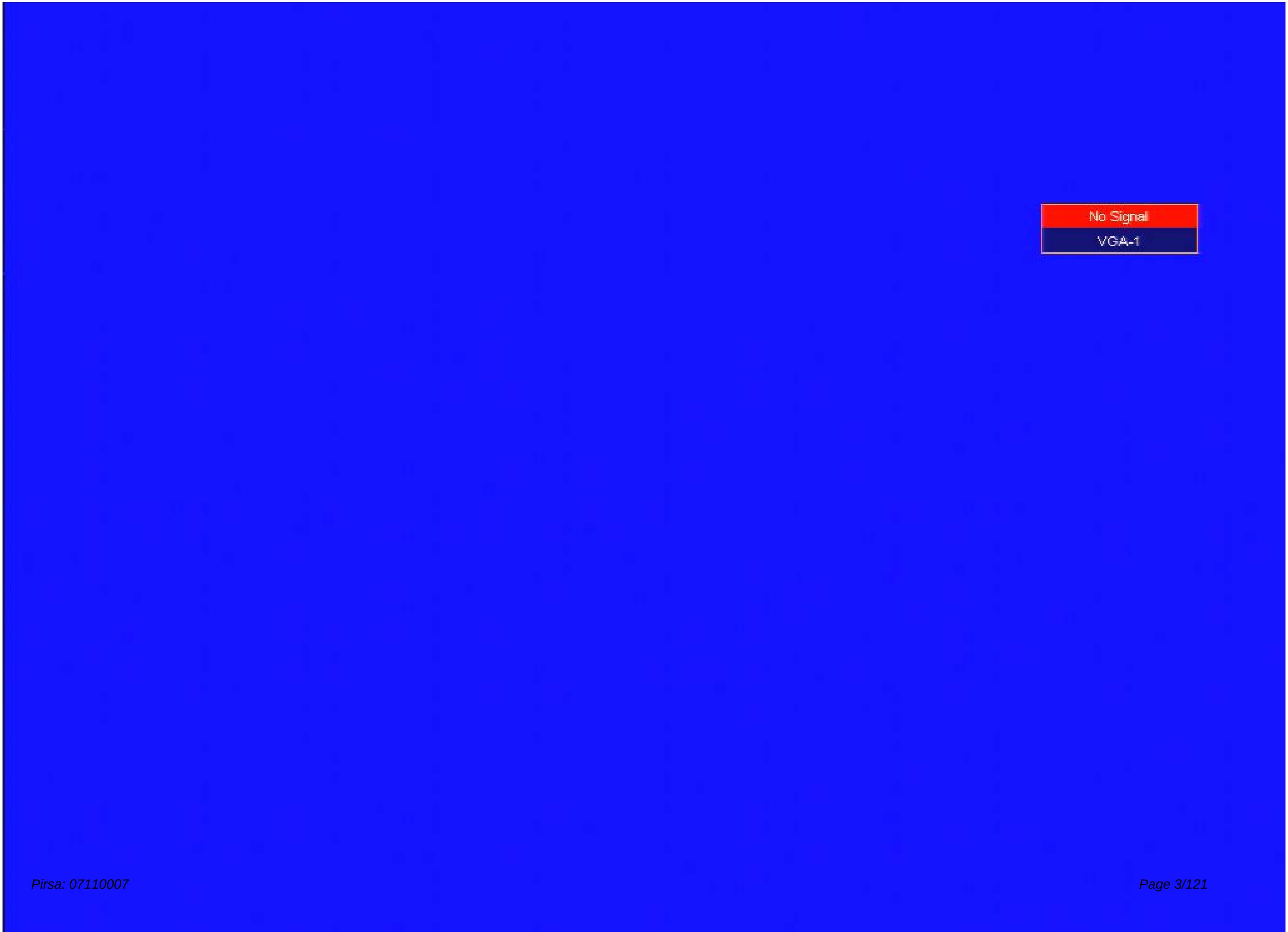
Title: 3d ADS/CFT from a quantum gravity point of view.

Date: Nov 13, 2007 11:00 AM

URL: http://pirsa.org/07110007

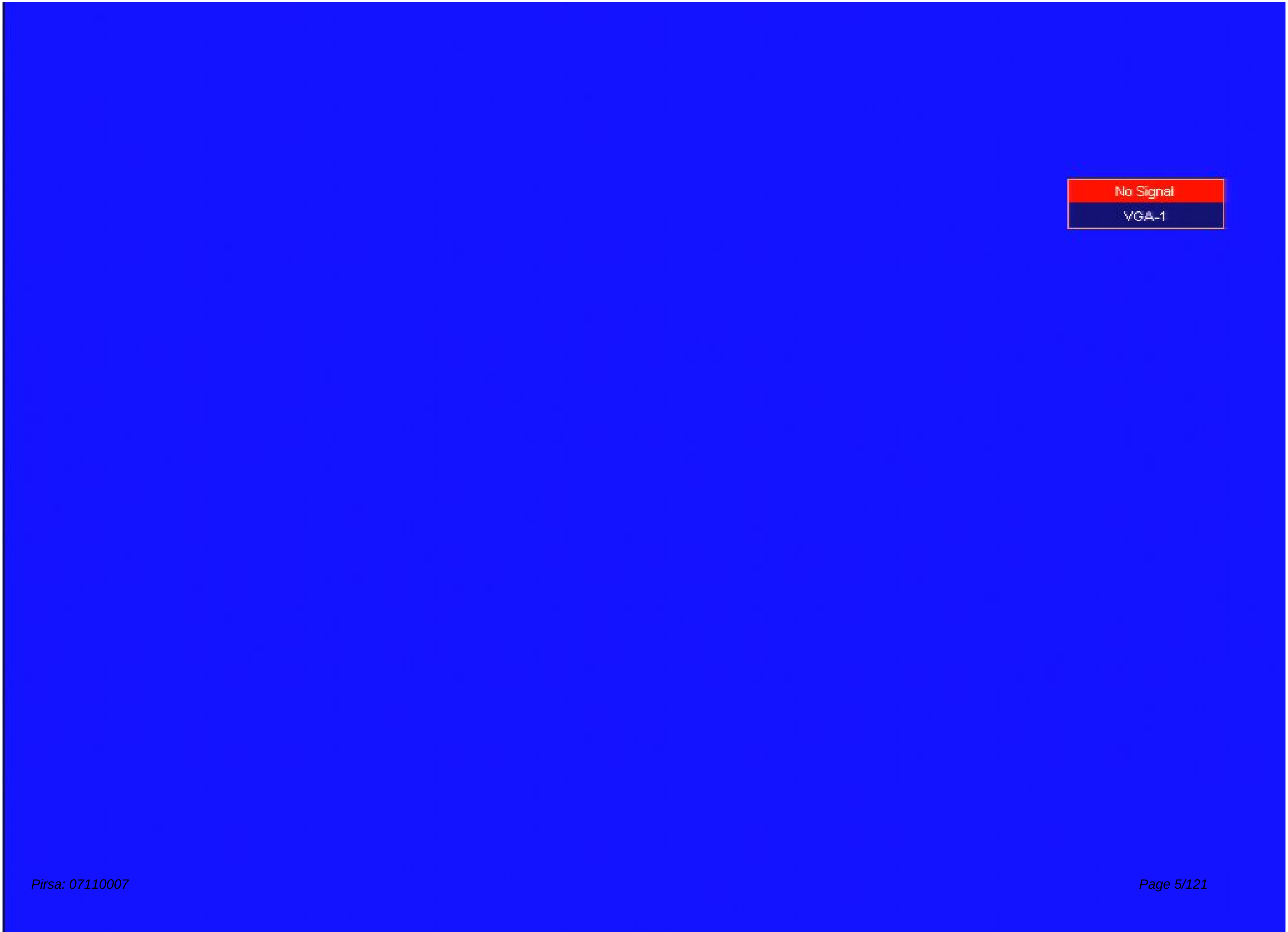
Abstract: TBA

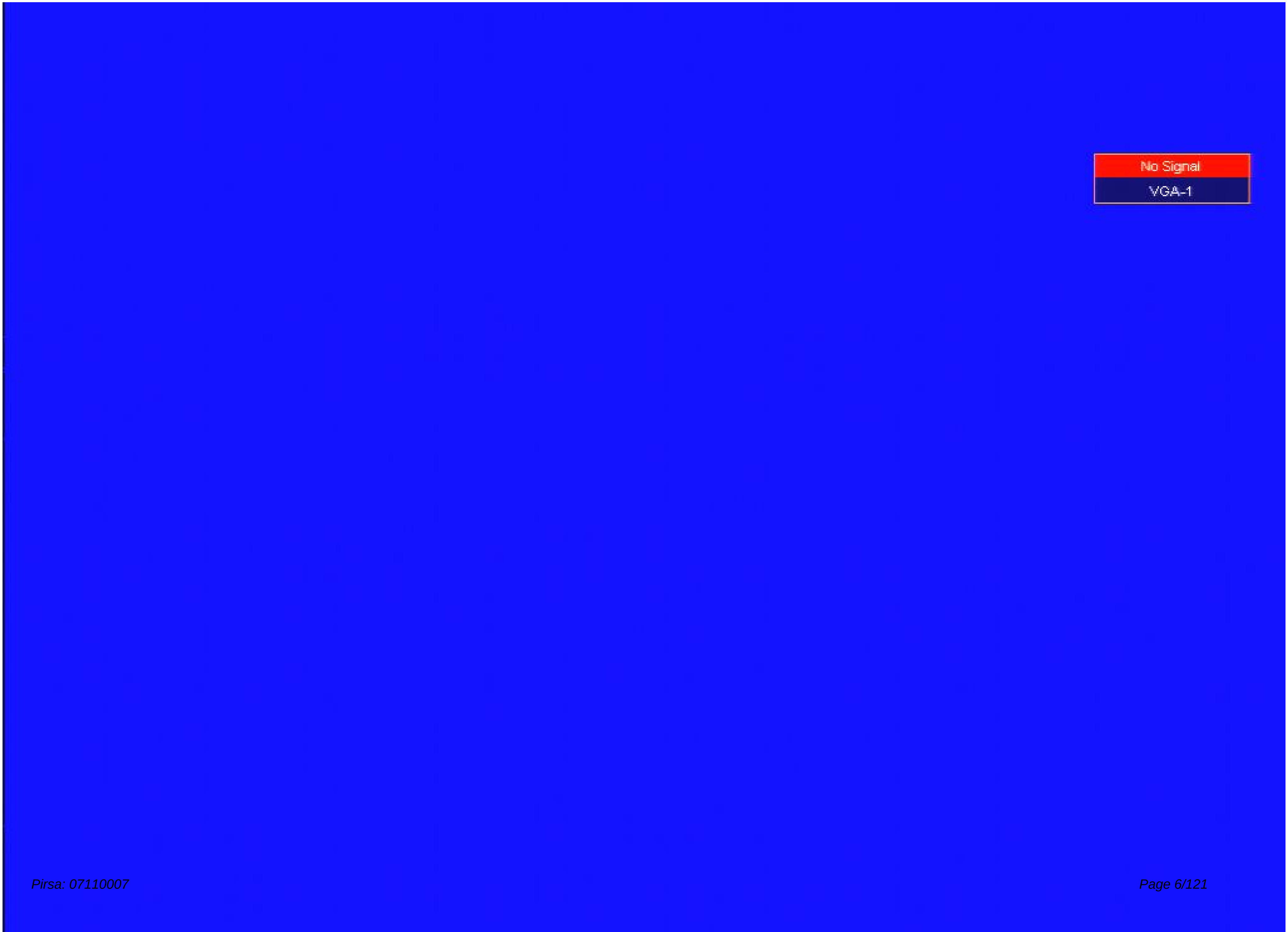

No Signal
VGA-1


No Signal
VGA-1

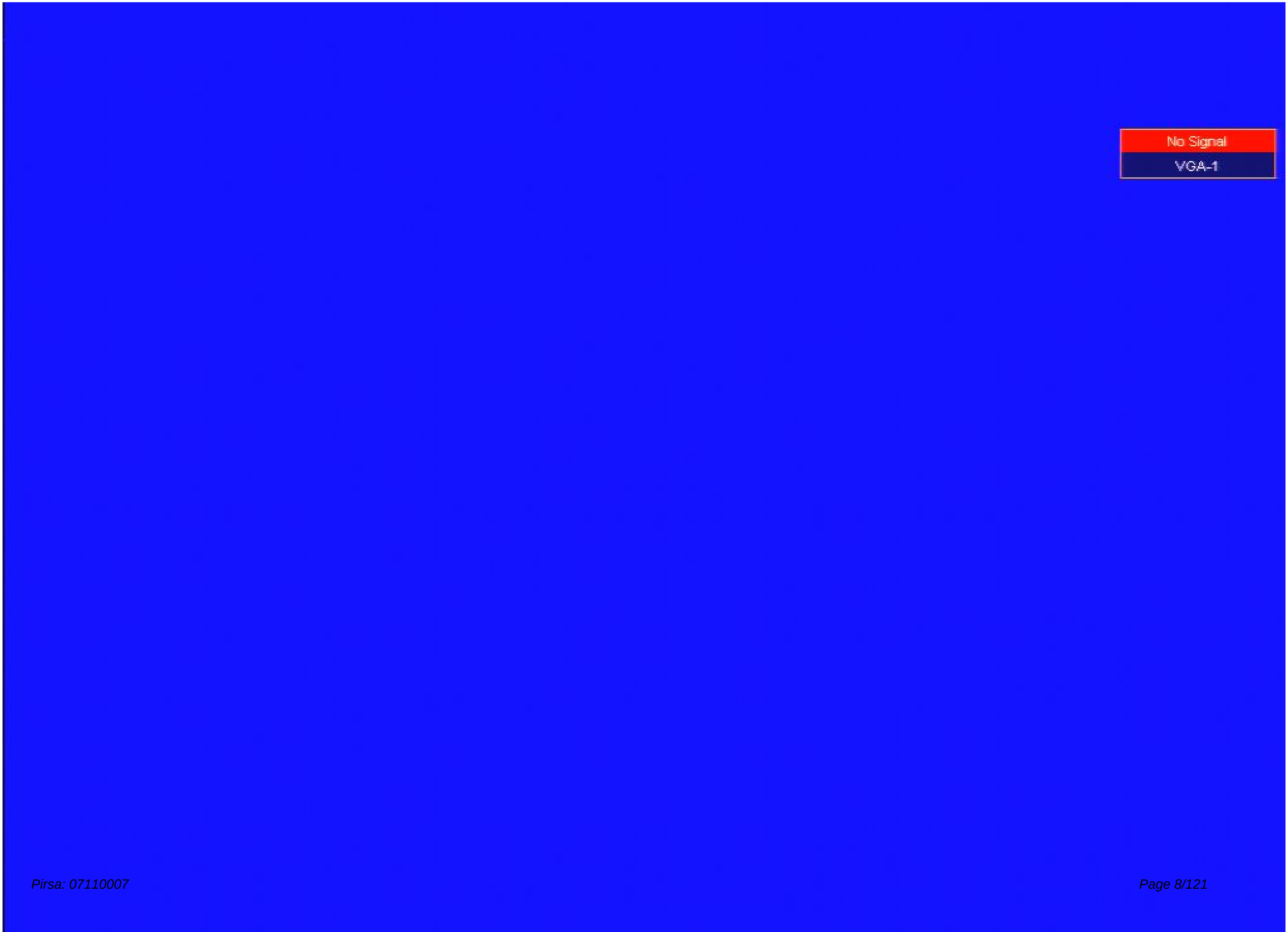
No Signal

VGA-1


No Signal
VGA-1


No Signal
VGA-1

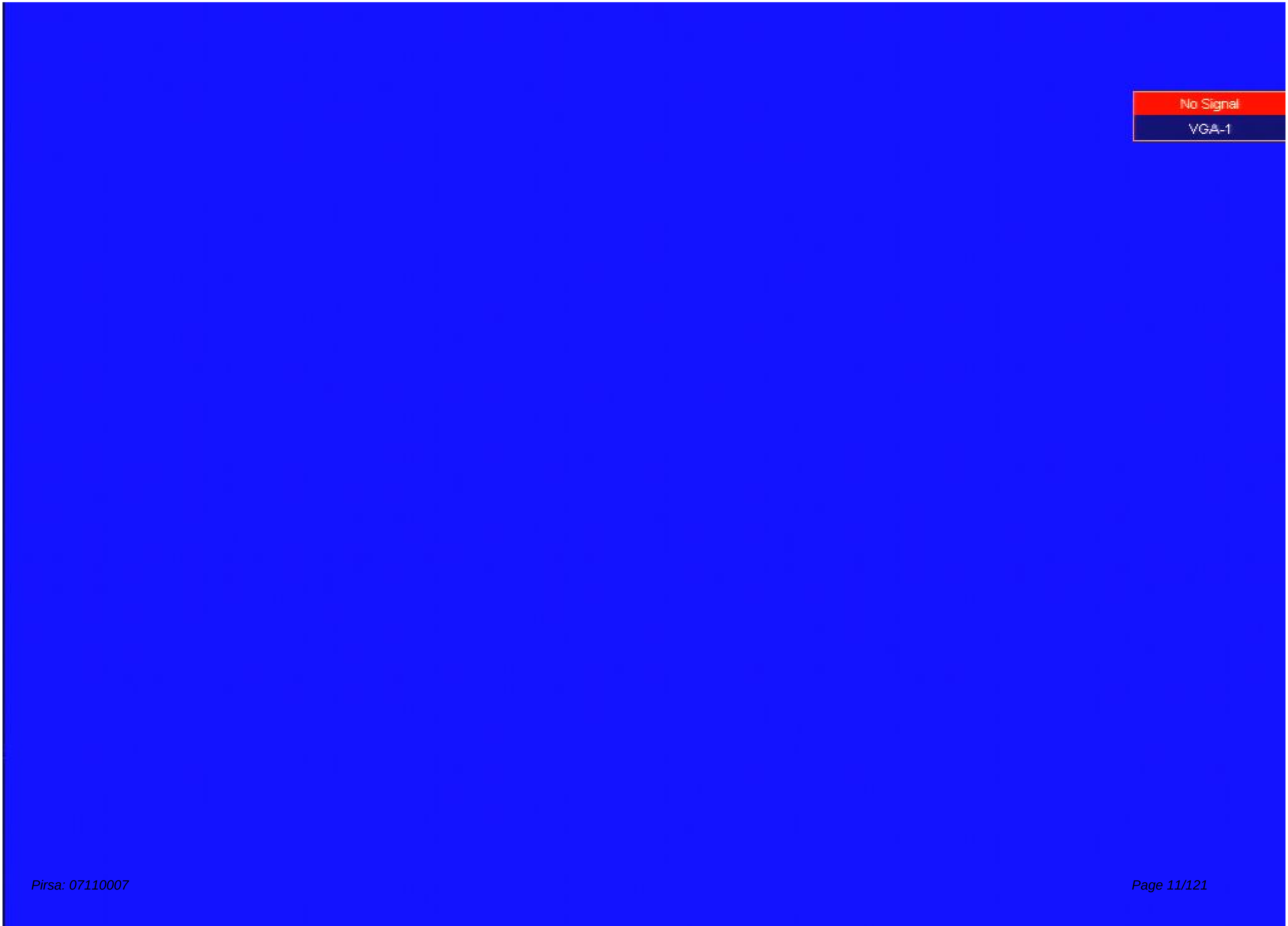

No Signal
VGA-1


No Signal
VGA-1

No Signal
VGA-1

No Signal

VGA-1


No Signal
VGA-1


No Signal
VGA-1

No Signal

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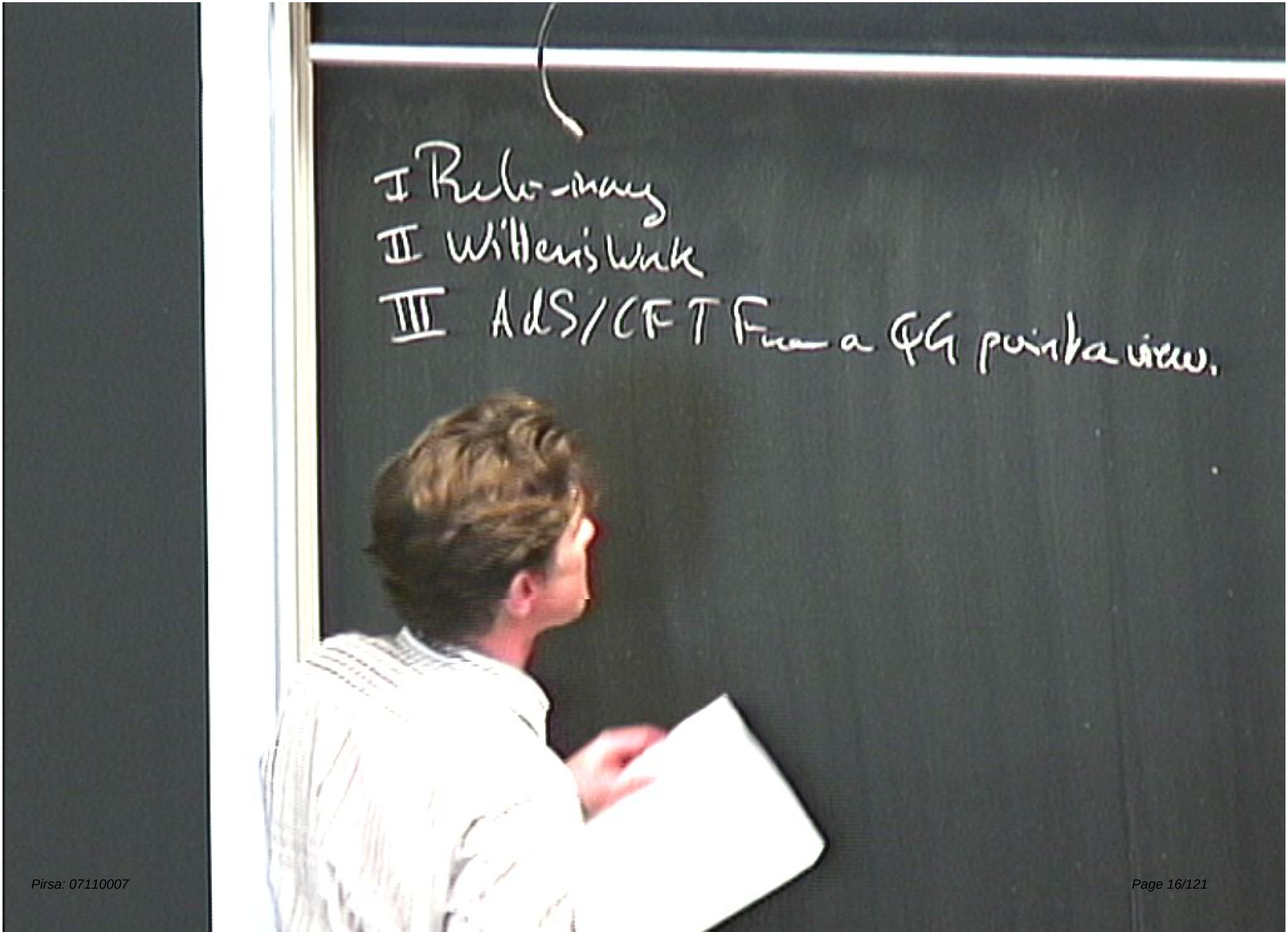
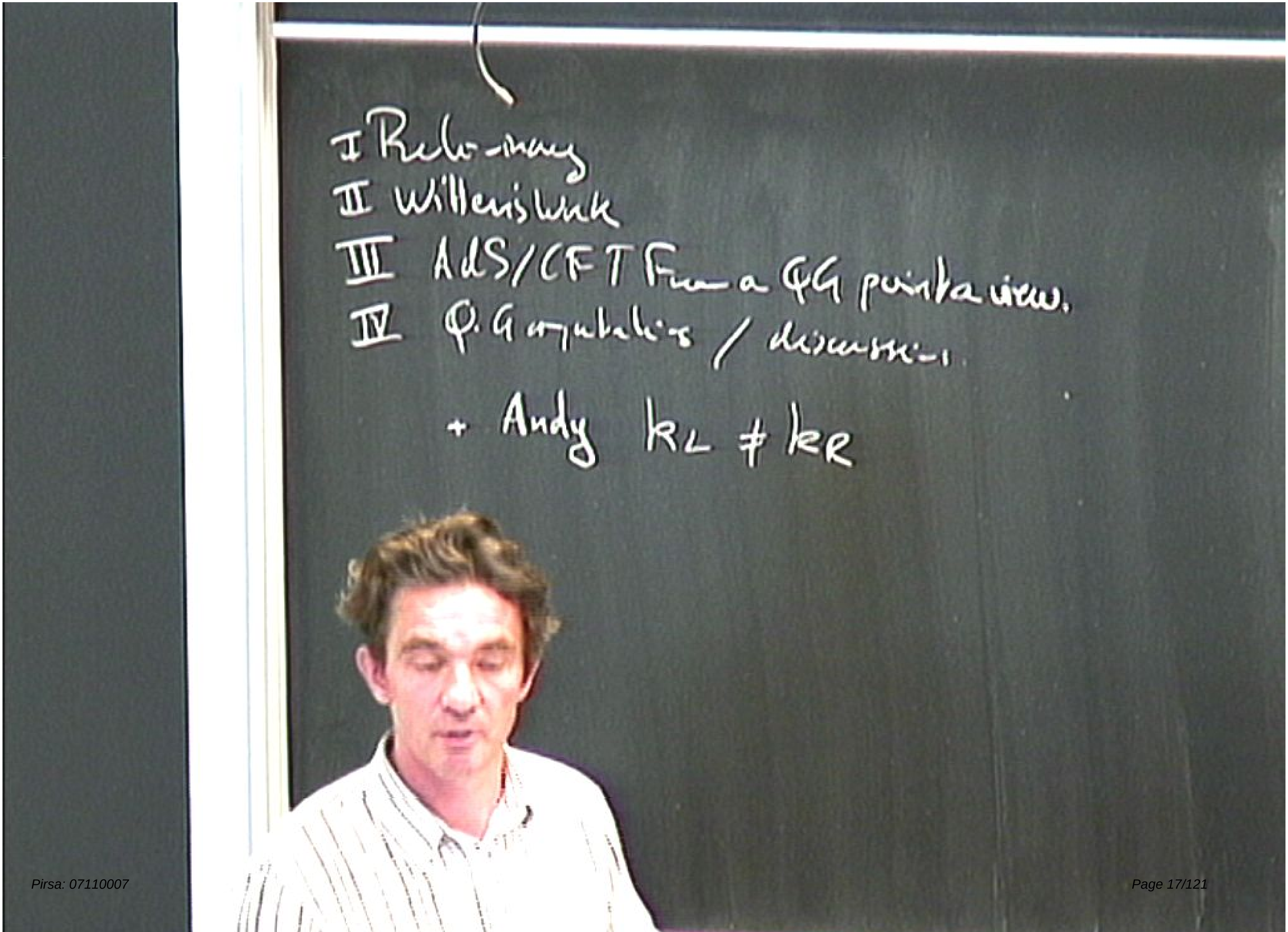
No Signal

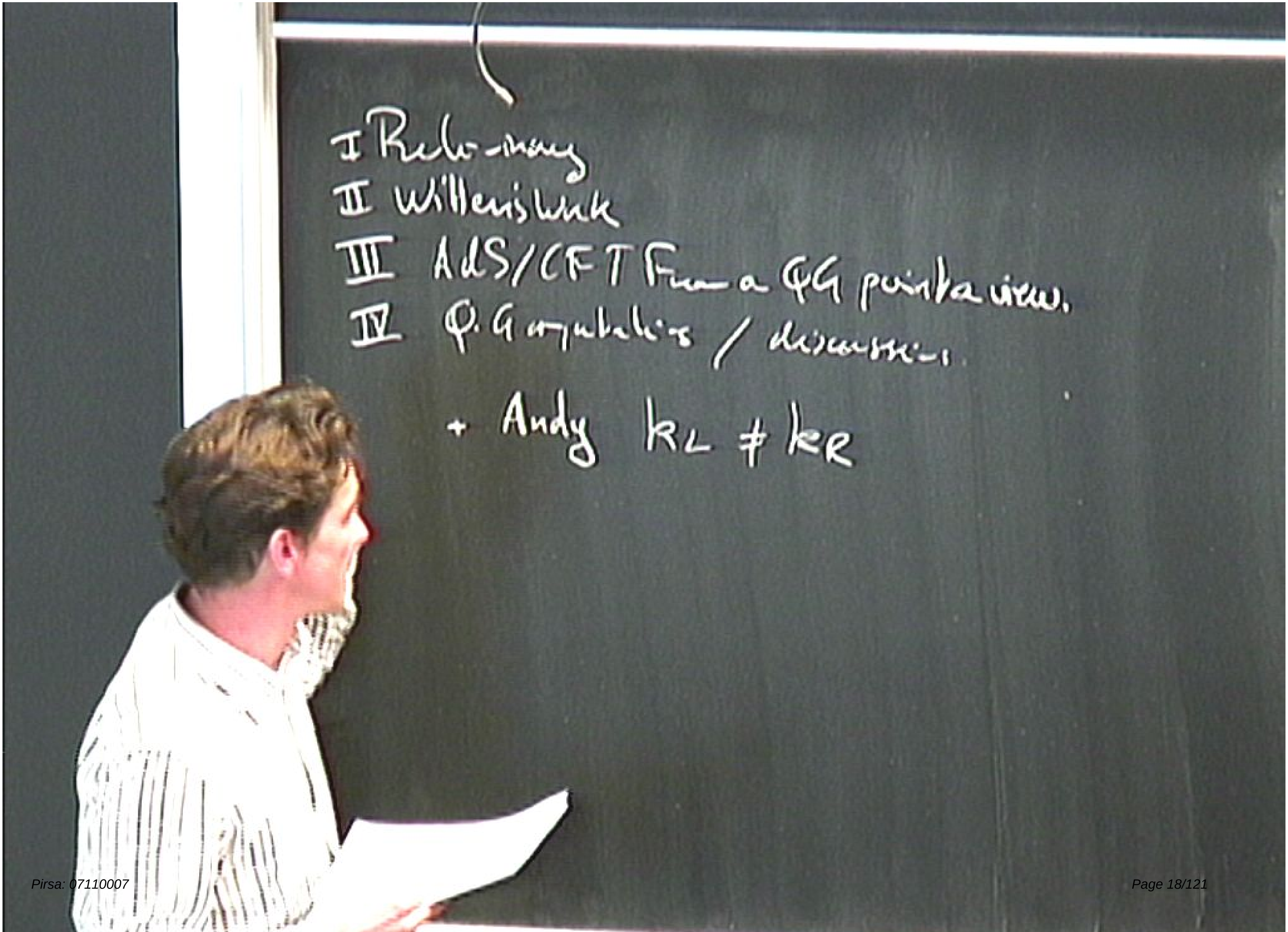
VGA-1

No Signal

VGA-1


I Preliminary
II Witten's Work
III AdS/CFT From a QG point a view.


II Witten's work
III AdS/CFT from a QG point of view.
IV Q.G computations / discussion
+ Andy kL ≠ kR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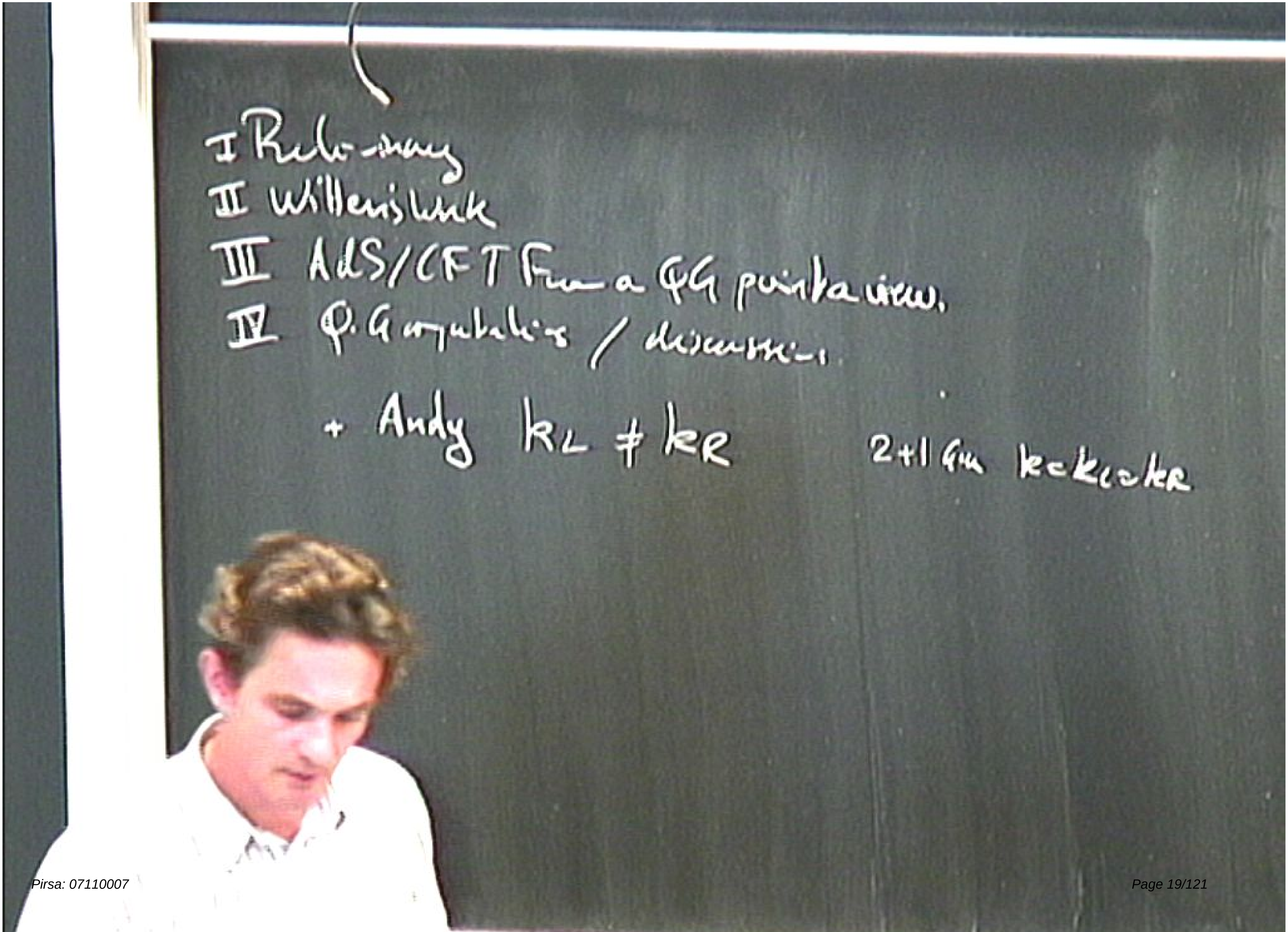
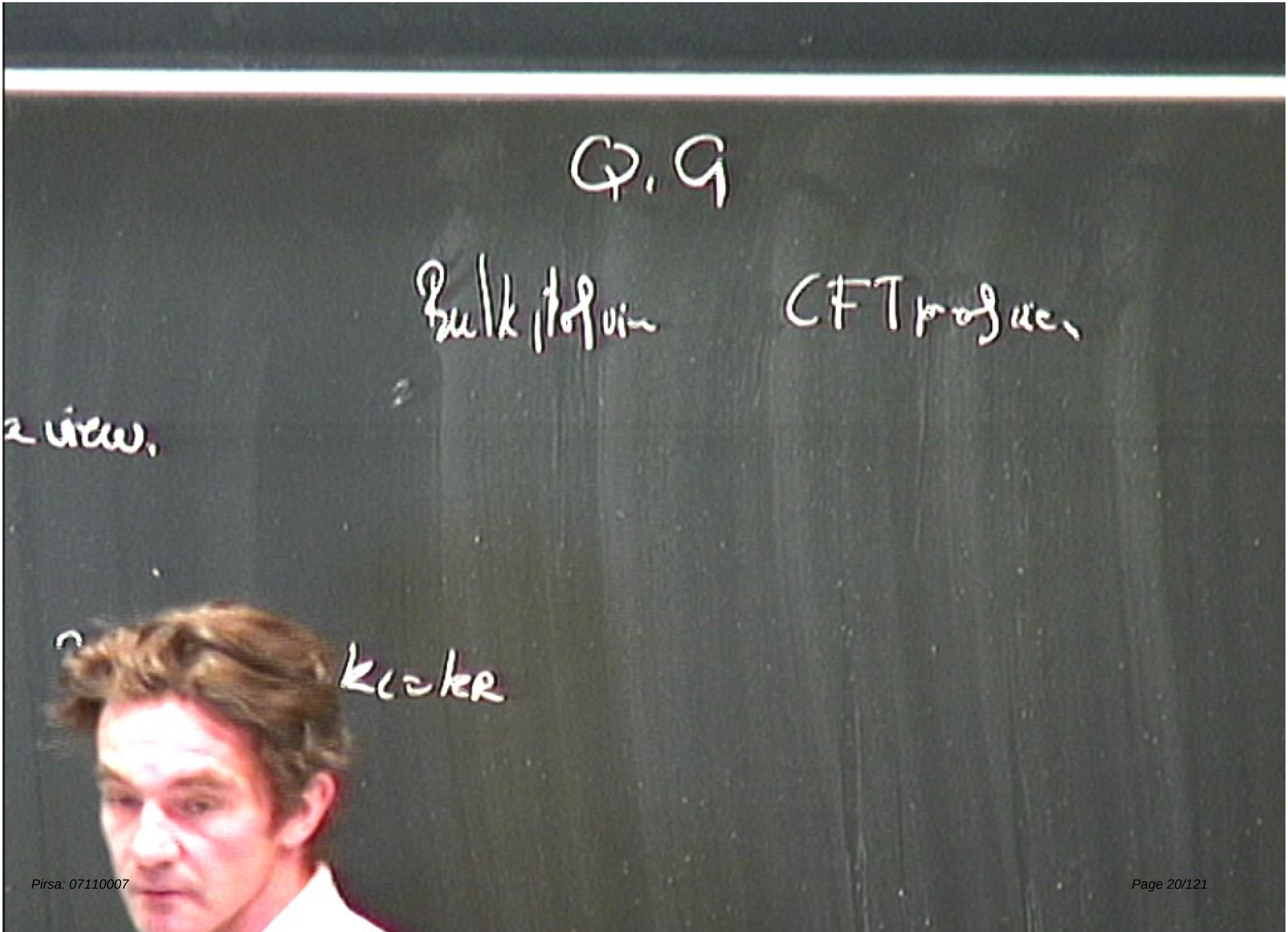
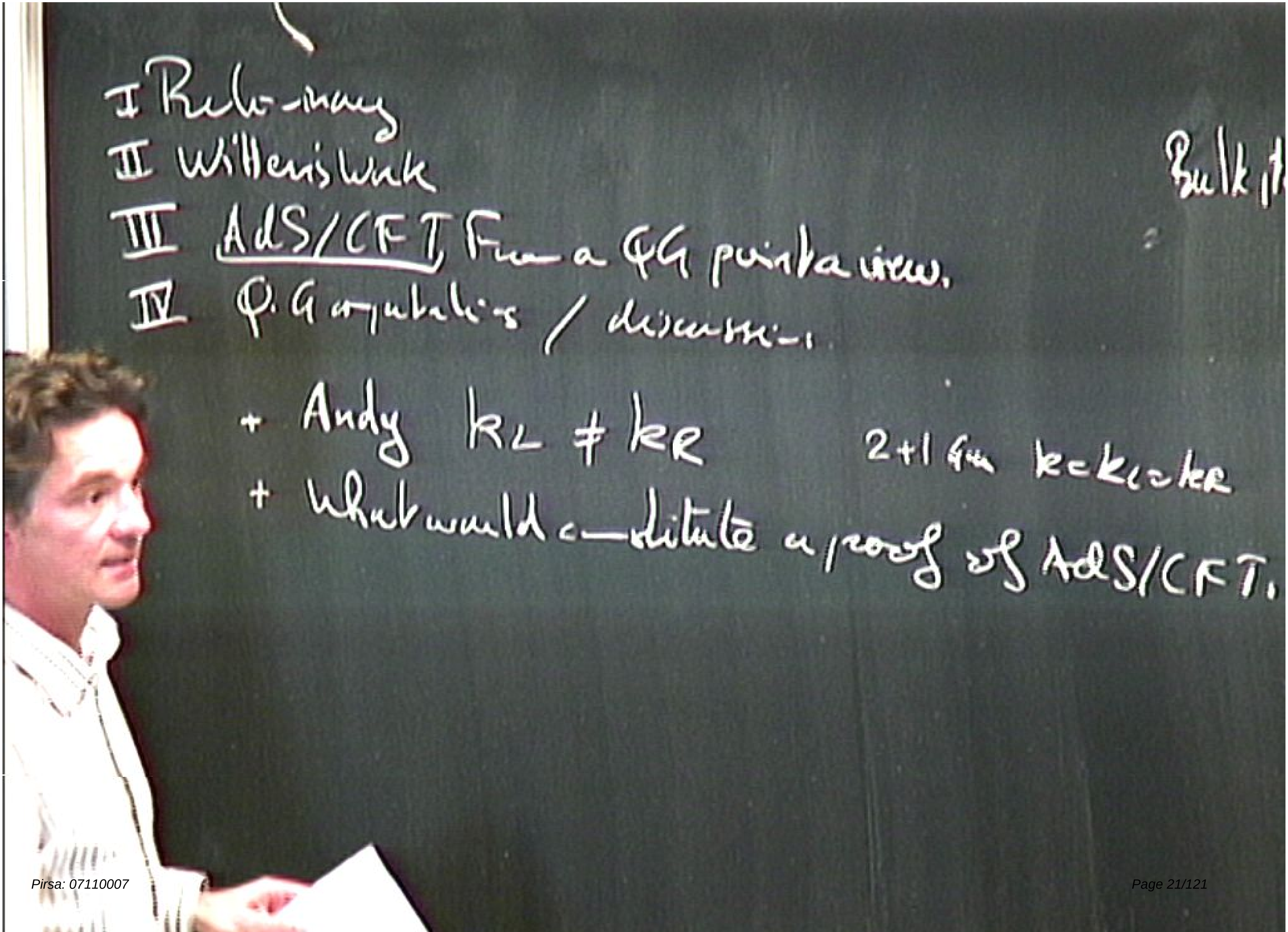
+ Andy $k_L \neq k_R$


III AdS/CFT From a QG point of view.
+ Andy kL ≠ kR
2+1 Grav k=kL=kR



Q.G
view.


I Preliminary
II Witten's Work
III AdS/CFT from a QG point of view.
IV Q.G computations / discussion
+ Andy $k_L \neq k_R$
2+1 grav $k = k_L = k_R$
+ What would constitute a proof of AdS/CFT.
Bulk, th

I Preliminary
II Witten's work
III AdS/CFT From a QG point a view.
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Q.G

Bulk perspective CFT perspective

+ Andy $k_L \neq k_R$ 2+1 grav. $k \neq k$ ...
+ What would constitute a proof of AdS/CFT.
What is exactly the precise dictionary / Lorentzian QG
a CFT

§ AdS/CFT.

dictionary | Lorentzian QG

| a CFT

$e^{-S(g)}$

k=k_L=k_R

AdS/CFT.

Lorentzian QG

$e^{iS(g)}$

a CFT

- a QG point a view.
/ discussion.

kR 2+1 Gm kakicske

c—ditute a proof of AdS/CFT.

exactly the precise dictionnary

$$e^{-S(g)}$$

$$e^{iS(g)}$$

Lorentzian QG

a CFT

Euclidian

$$S = \frac{1}{16\pi G} \int \sqrt{g} \left(R(g) - \frac{2}{\ell^2} \right)$$

AdS

$S(\mathbb{R}^{2}\mathbb{R})$: $A^{\pm} = \omega \pm e$

$$S = \frac{1}{16\pi G}\int \sqrt{g}\left(R(g) - \frac{2}{\ell^2}\right)$$

AdS

$SL(2,\mathbb{R})$: $A^{\pm} = \omega \pm e$ $\qquad S_{CS} = \int \mathrm{Tr}\left(\frac{1}{2} A\, dA + \frac{1}{3} A^3\right)$

$$S = \frac{k}{2\pi}\left(S_{CS}(A^+) - S_{CS}(A^-)\right)$$

$$S = \frac{1}{16\pi G}\int \sqrt{g}\left(R(g) - \frac{2}{\ell^2}\right) + c\, S(\omega)$$

AdS

$SL(2,\mathbb{R})$: $A^{\pm} = \omega \pm e$ $\qquad S_{CS} = \int \mathrm{Tr}\left(\frac{1}{2} A\, dA + \frac{1}{3} A^3\right)$

$$S = \frac{k}{2\pi}\left(S_{cs}(A^+) - S_{cs}(A^-)\right)$$

$e = dx^a e_a^i J_i$

$ds^2 = e_a^i e_b^j \gamma_{ij} = g_{ab}$

w)

A^3)

Brown-Henneaux,
at the classical level
the Poisson alg of
Conserved
~Virasoro

$$c = \frac{3}{2}\frac{\ell}{G}$$

$\frac{2}{\ell^2} \Big) + C.S(\omega)$

$CS = \int \mathrm{Tr} \left(\frac{1}{2} A\, dA + \frac{1}{3} A^3 \right)$

$(A) \Big)$

Brown-Henneaux.
at the classical level
the Poisson alg of
Conserved
~ Virasoro

$c = \frac{3}{2} \frac{\ell}{G}$

$= 6k$

$\frac{2}{\ell^2}\Big) + C.S(\omega)\Big)$

$_{CS} = \int \mathrm{Tr}\left(\frac{1}{2} A\, dA + \frac{1}{3} A^3\right)$

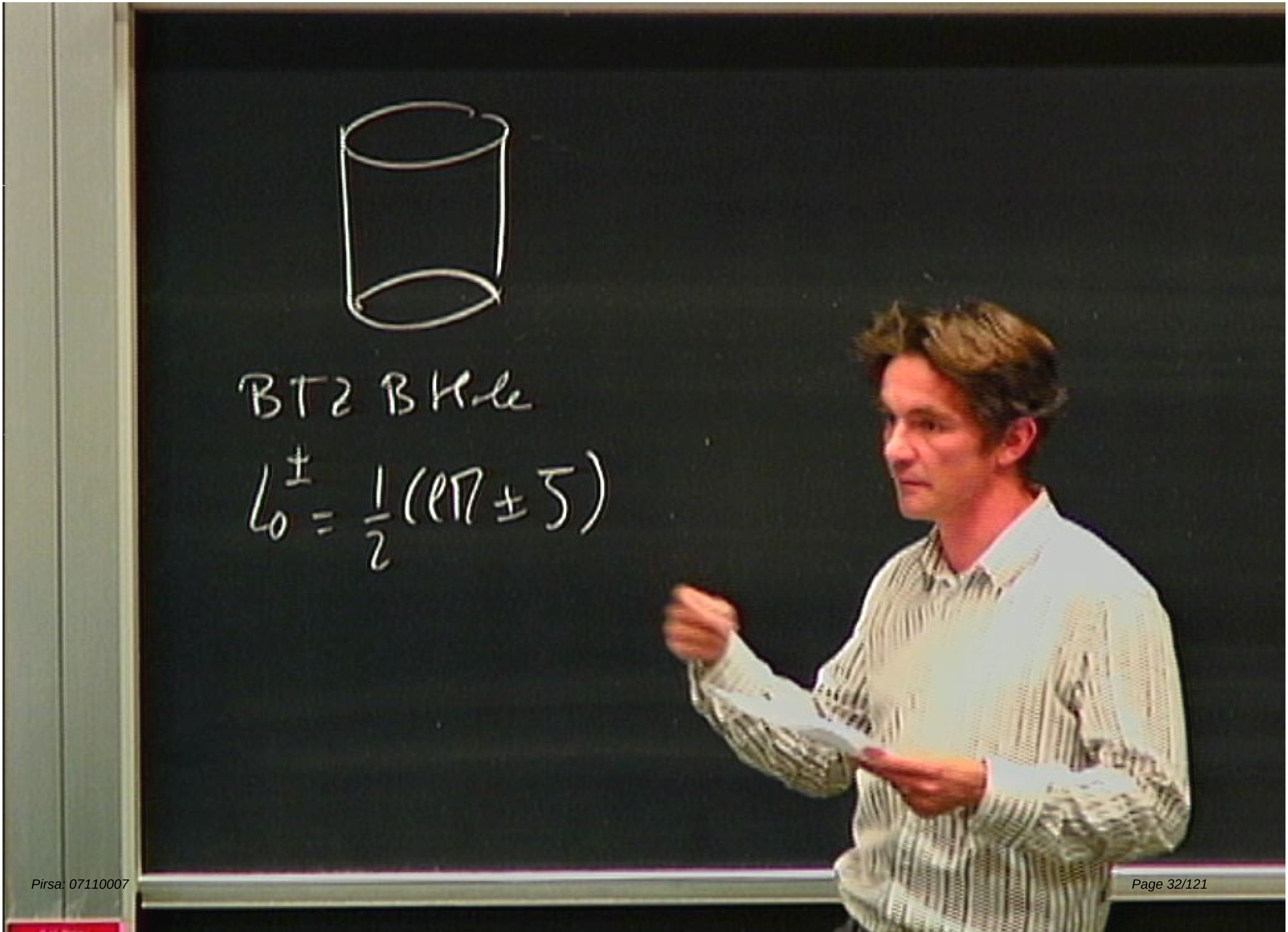
$\frac{}{3}(A^3)\Big)\Big)$

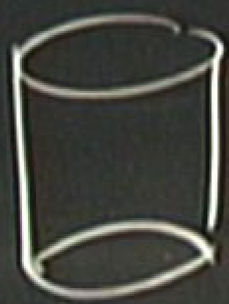
Brown-Henneaux.
at the classical level
the algebra of
Conserved
Virasoro

$c = \frac{3}{2}\frac{\ell}{G}$

$= 6k$

$k_{Hu} = 4 k_{Witt}$


BTZ B Hole
$L_0^{\pm} = \frac{1}{2}(\ell M \pm J)$

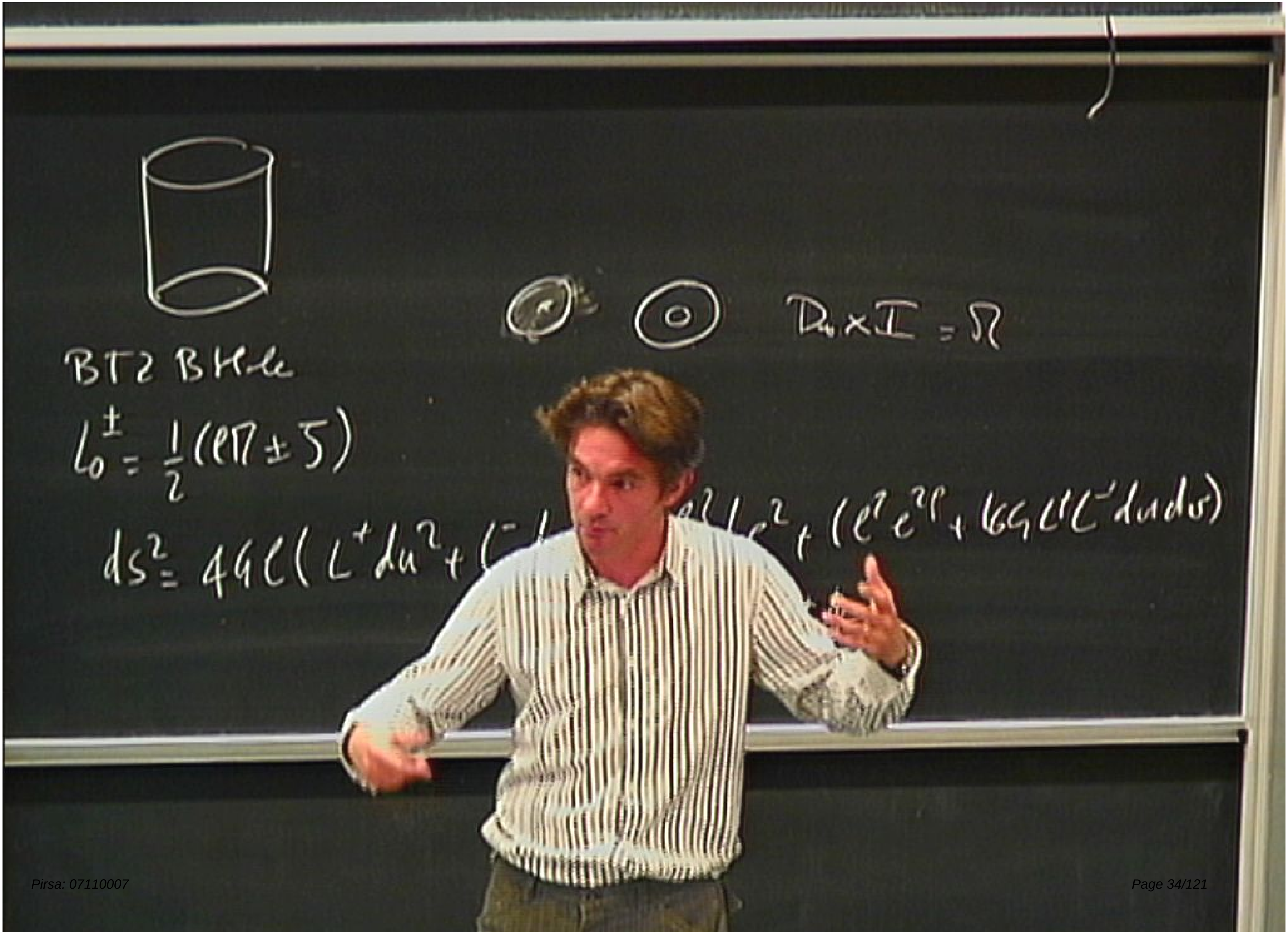


BTZ BHole

$$L_0^\pm = \frac{1}{2}(\ell M \pm J)$$

$$ds^2 = 4G\ell\left(L^+ du^2 + L^- dv^2\right) - \ell^2 d\rho^2 + \left(\ell^2 e^{2\rho} + 16G\,L^+L^- du\,dv\right)$$

$$u, v = t \pm \phi$$


BTZ BHole

$$\mathbb{H}_3 = D_\infty \times I = \Omega$$

$$\ldots + (L^- dv^2) - \ell^2 d\rho^2 + (\ell^2 e^{2\rho} + 16G\, L^+ L^- \, du\, d\ldots$$

$$u, v = t \pm \phi$$

 $M_3 = D_\infty \times I = \Omega$

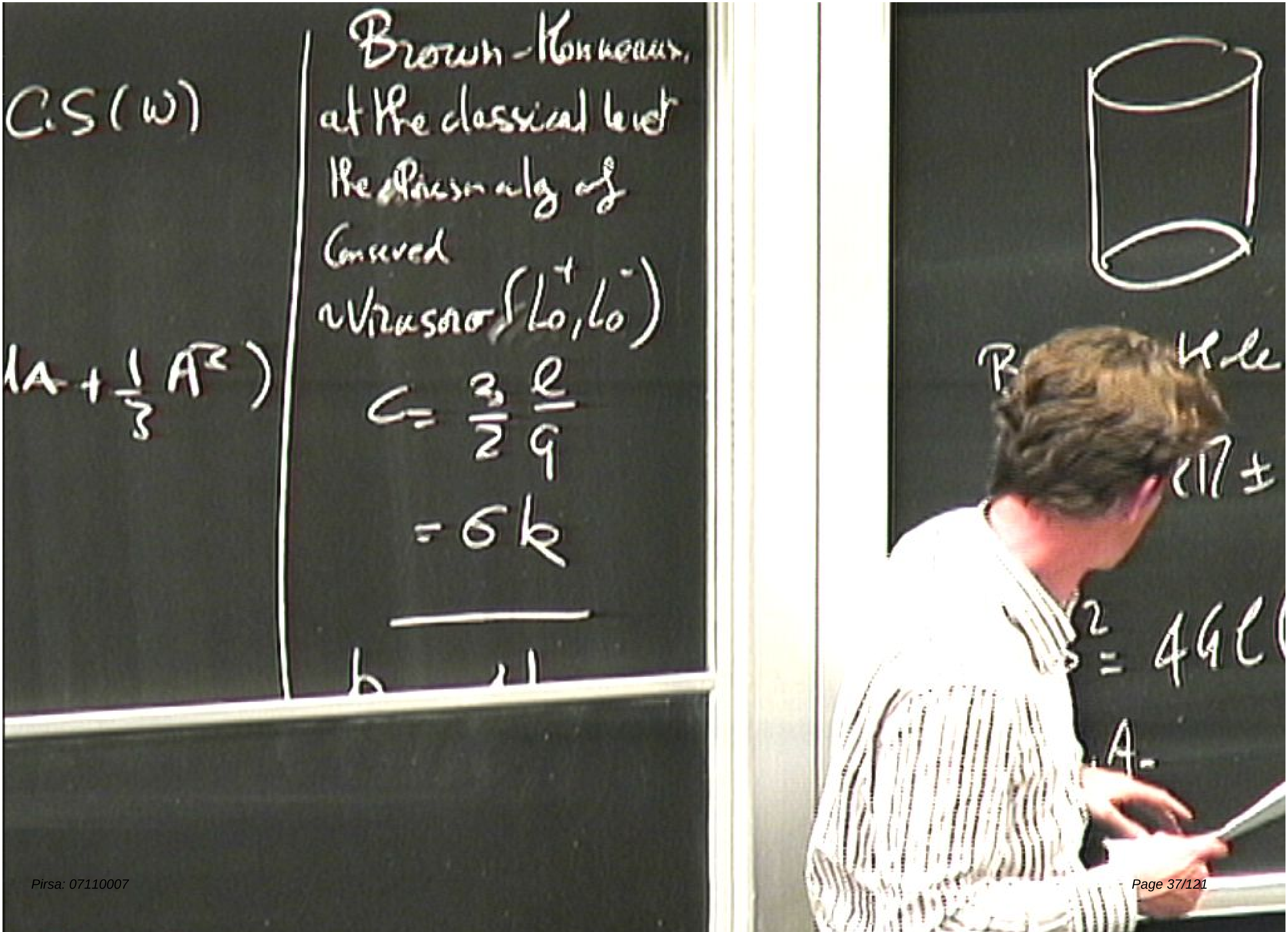
BTZ BHole

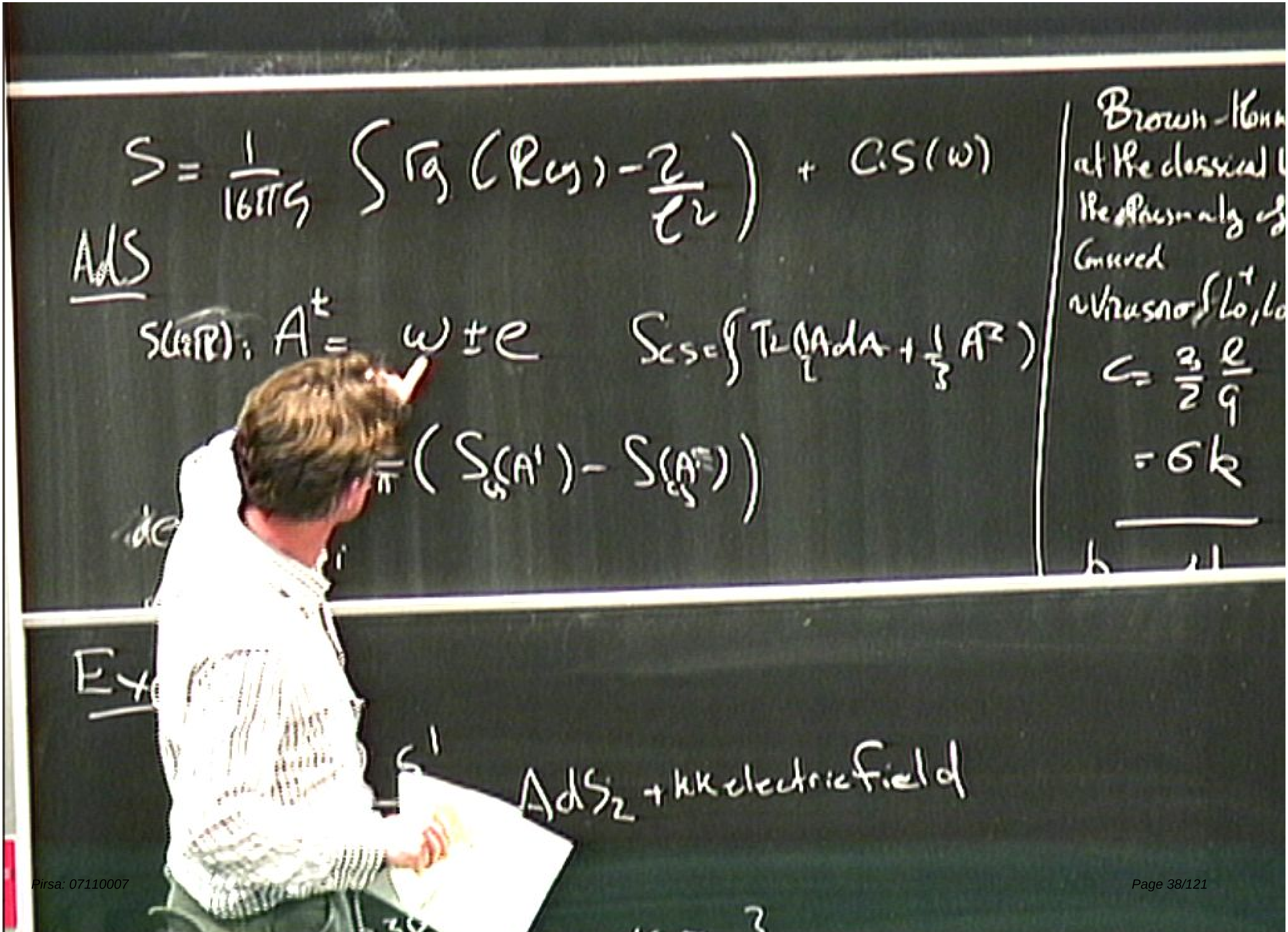
$$L_0^\pm = \frac{1}{2}(\ell M \pm J)$$

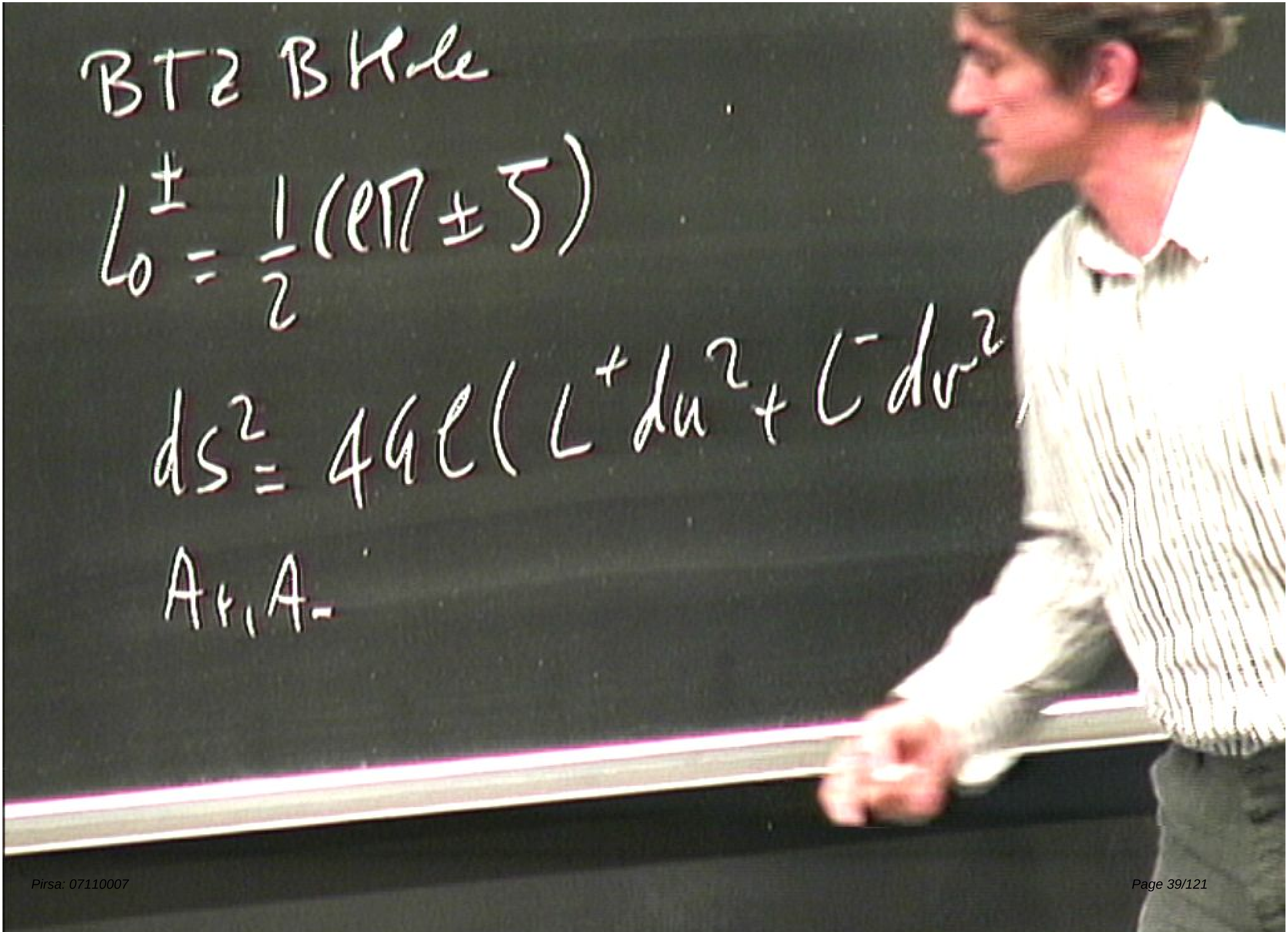
$$ds^2 = 4G\ell(L^+ du^2 + L^- dv^2) - \ell^2 d\rho^2 + (\ell^2 e^{2\rho} + 16G\,L^+L^- du\,dv$$

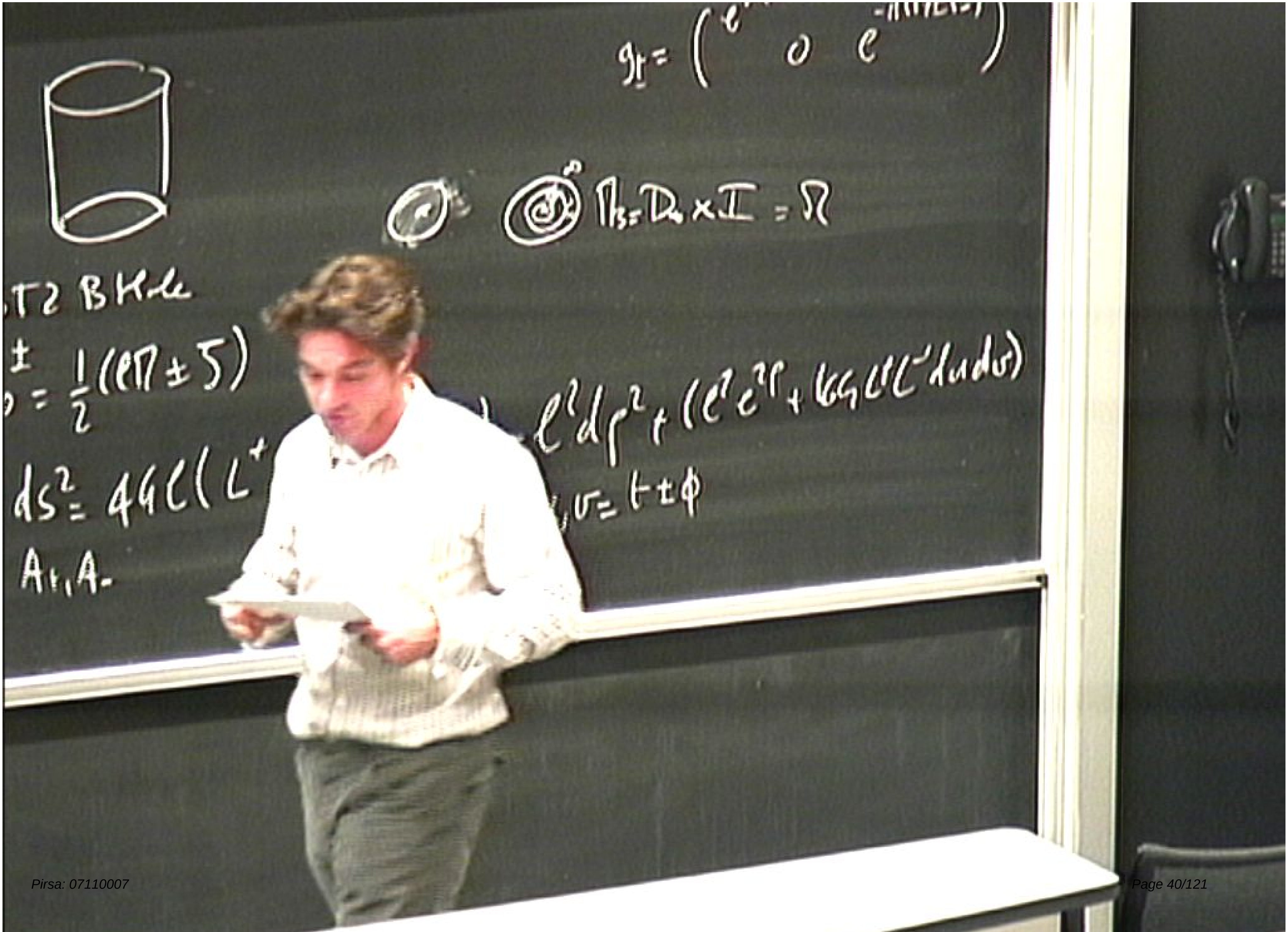
A_+, A_-

$u, v = t \pm \phi$


C.S(ω)
Brown-Henneaux
at the classical level
the Virasoro alg of
Conserved
Virasoro (L_0^+, L_0^-)
$c = \frac{3}{2}\frac{\ell}{G}$
$= 6k$

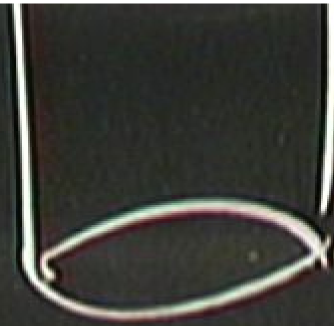

S = 1/16πG ∫ √g (R(g) − 2/ℓ²) + C.S(ω)
AdS
A± = ω ± e
S_CS = ∫ Tr(½ A dA + ⅓ A³)
(S_CS(A⁺) − S_CS(A⁻))
Ex
AdS₂ + KK electric field
c = 3/2 ℓ/G
= 6k


BTZ BHole
$L_0^{\pm} = \frac{1}{2}(\ell\Pi \pm J)$
$ds^2 = 4G\ell(L^+ du^2 + L^- dv^2$
A_+, A_-



$$g_{\pm} = \begin{pmatrix} e^{\pi(r_+ \pm r_-)} & 0 \\ 0 & e^{-\pi(r_+ \pm r_-)} \end{pmatrix}$$

$$M_3 = D_{\infty} \times I = \Omega$$



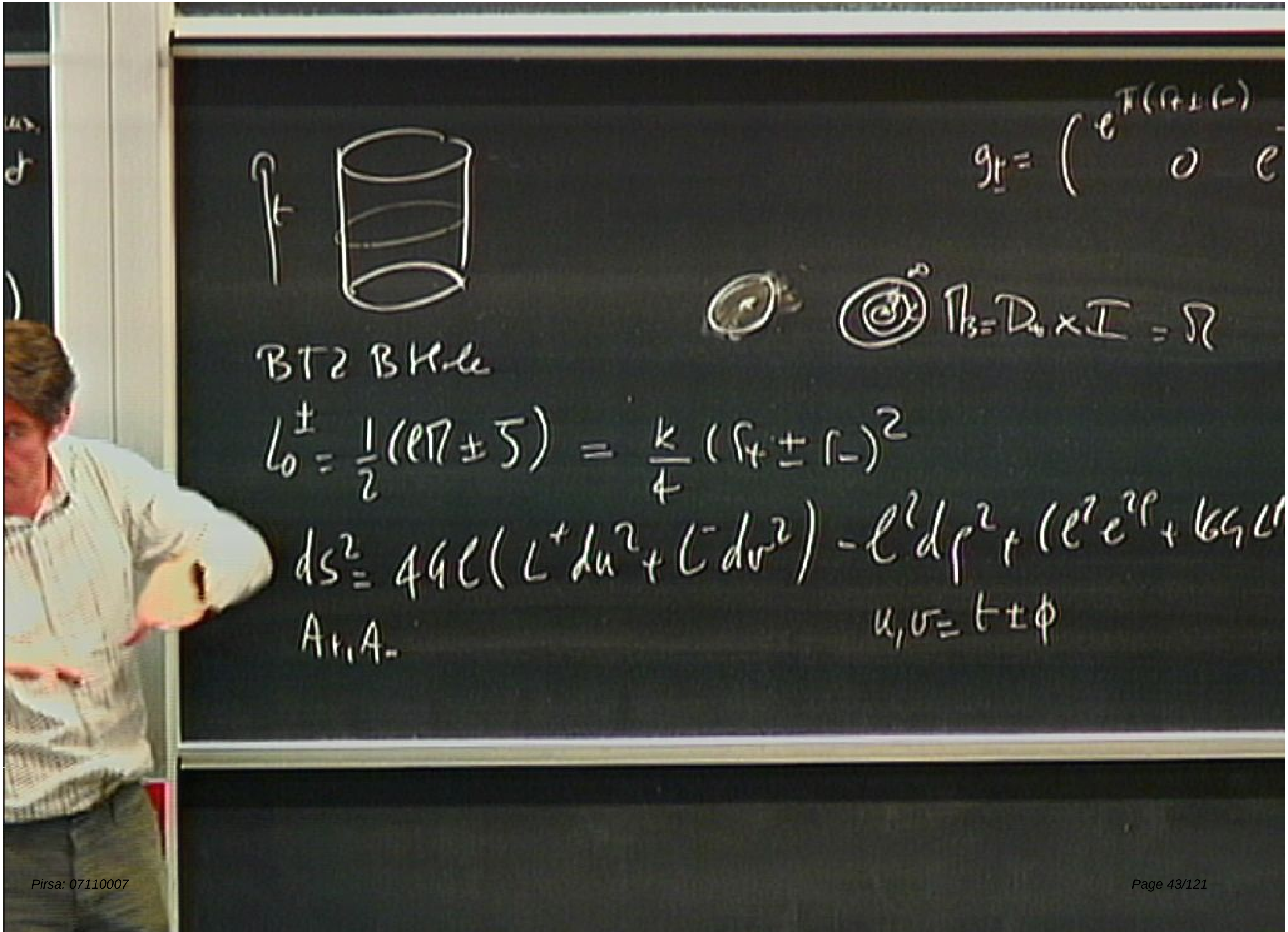
BTZ BHole

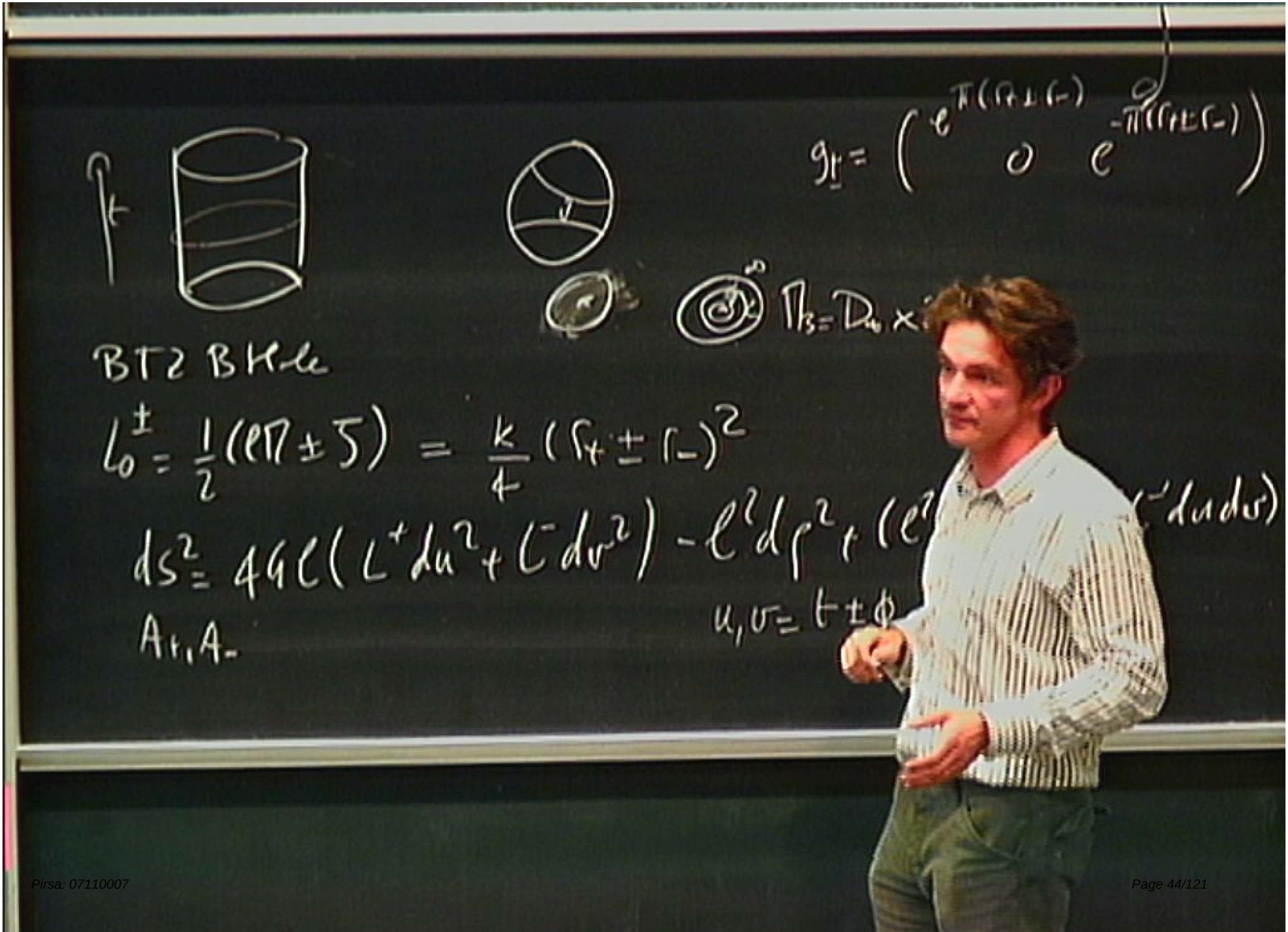
$$L_0^{\pm} = \frac{1}{2}(\ell M \pm J) = \frac{k}{4}(r_+ \pm r_-)^2$$

$$ds^2 = 4G\ell(L^+ du^2 + L^- dv^2) - \ell^2 d$$

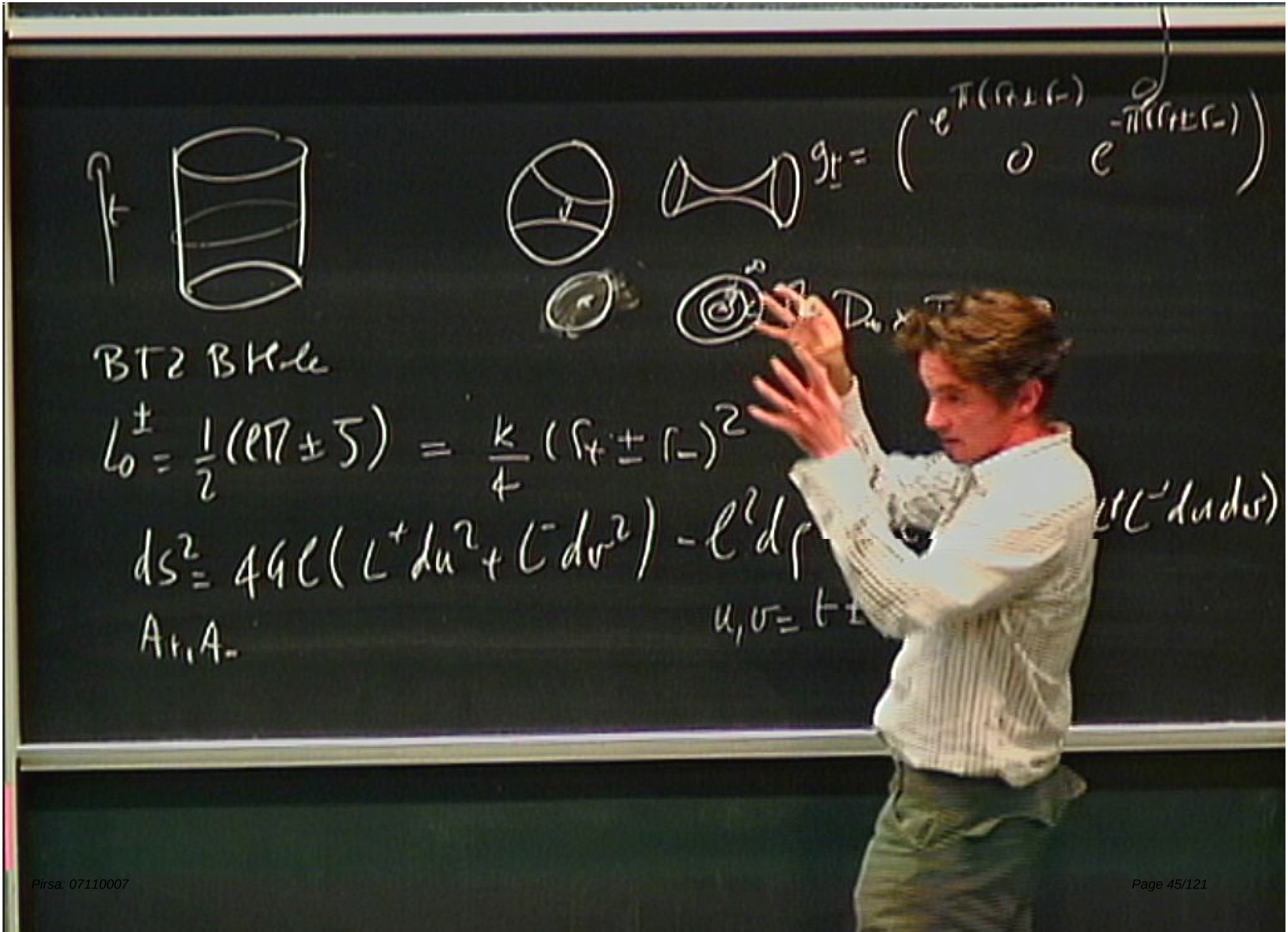
A_+, A_-

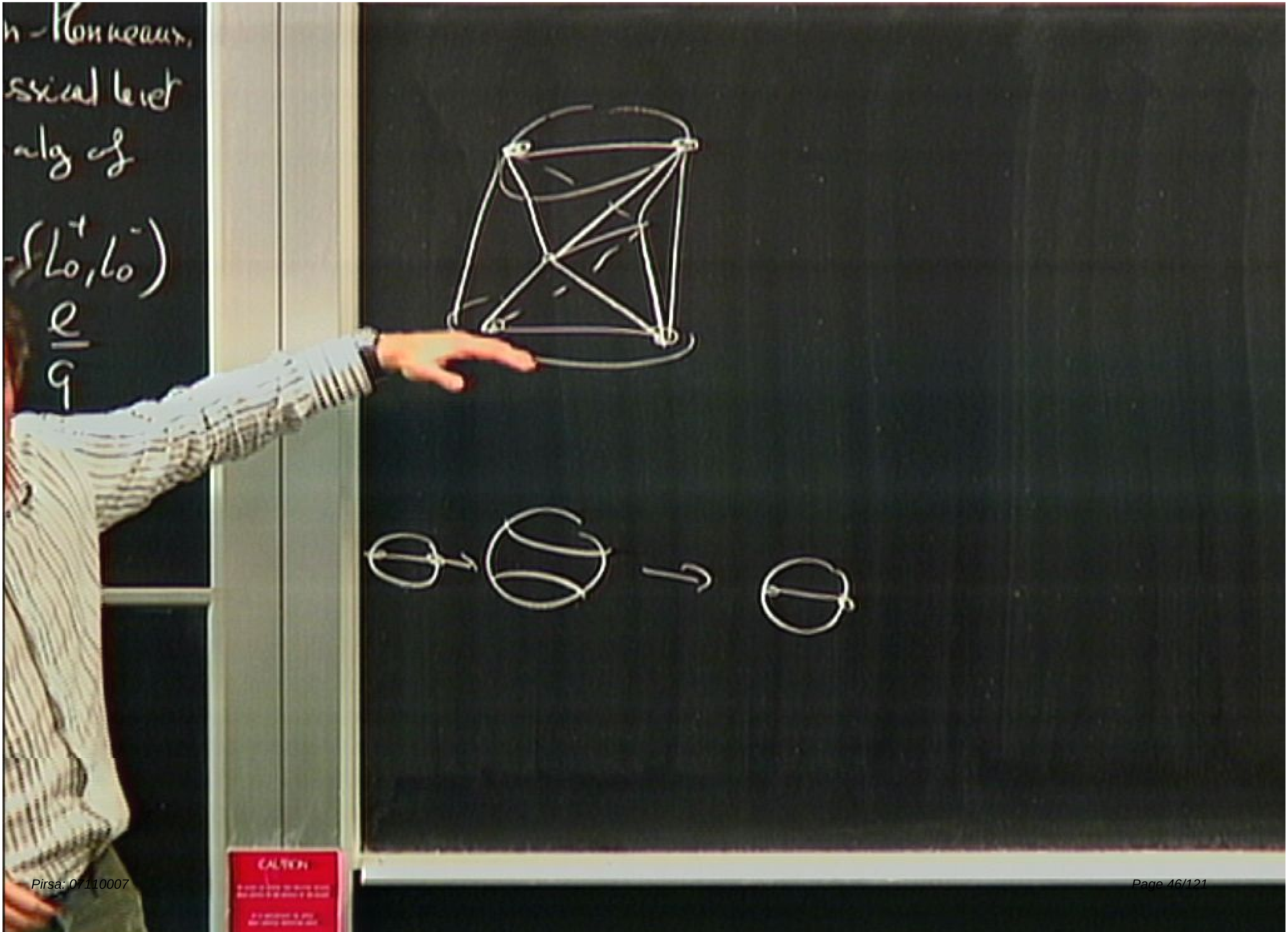
$u, v =$

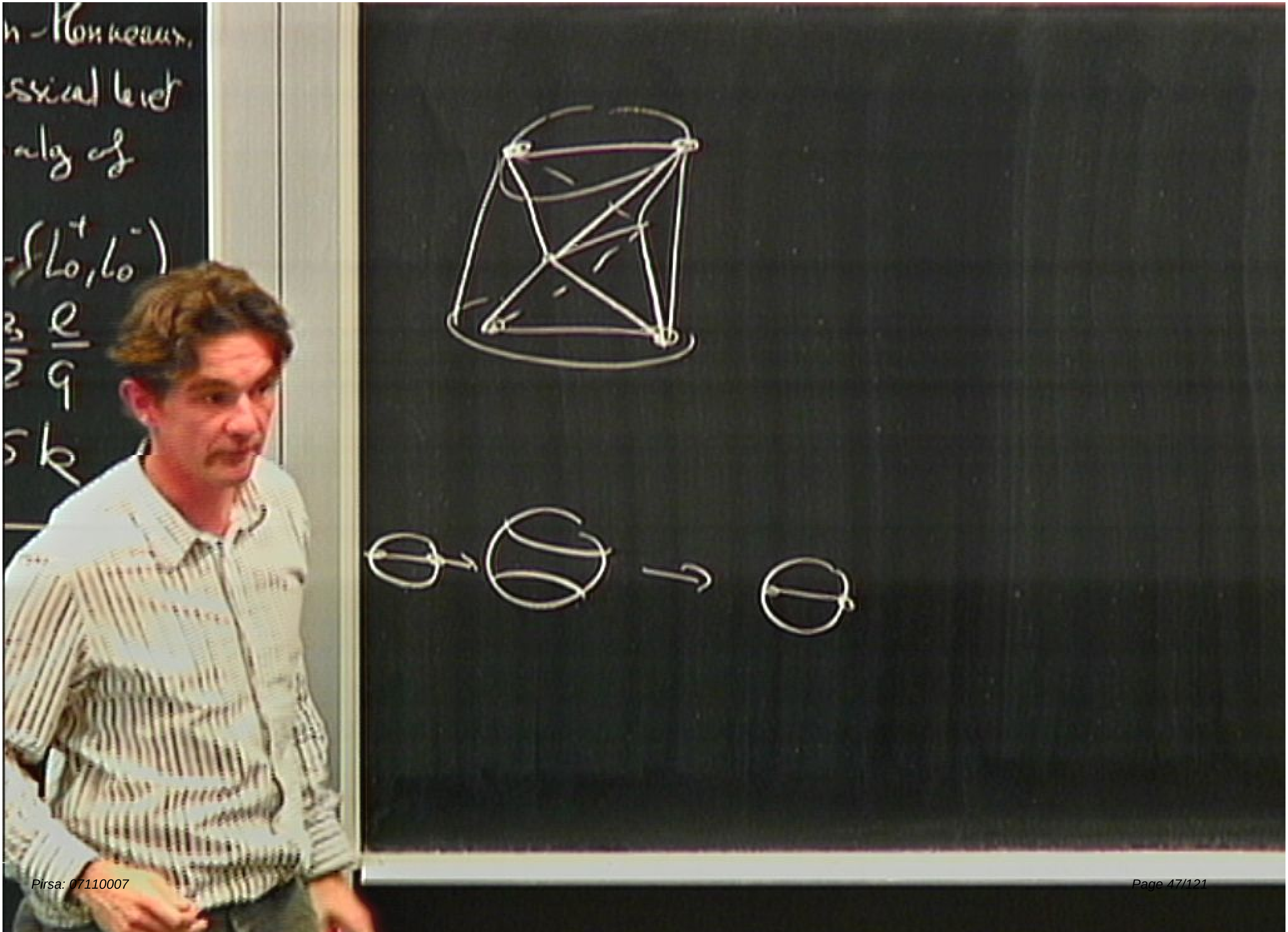

BTZ B Hole
L_0^± = 1/2 (ℓM ± J) = k/4 (r+ ± r-)^2
ds^2 = 4Gℓ(L^+ du^2 + L^- dv^2) - ℓ^2 dρ^2 + (ℓ^2 e^{2ρ} + 16G
A+, A-
u, v = t ± φ

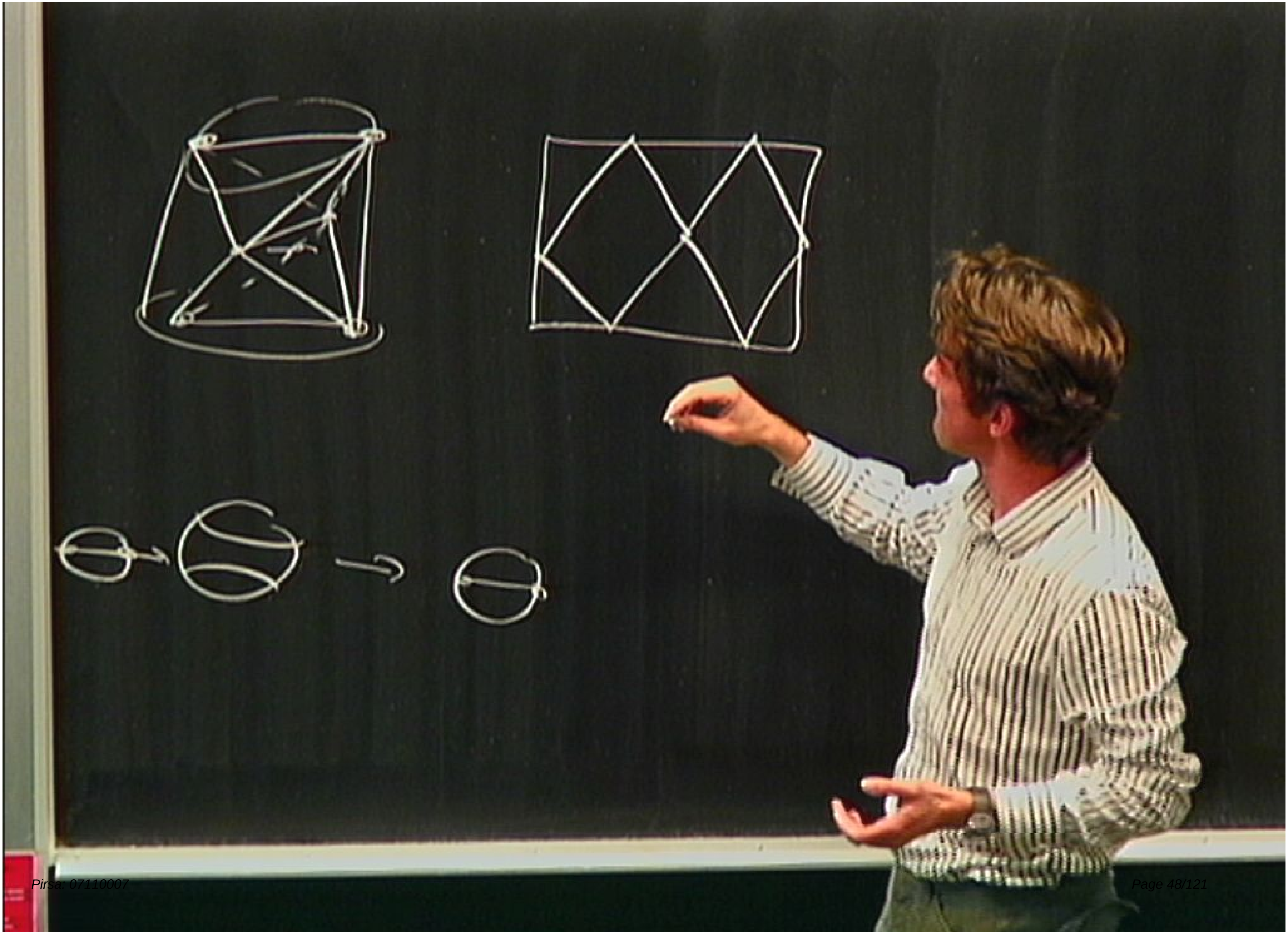


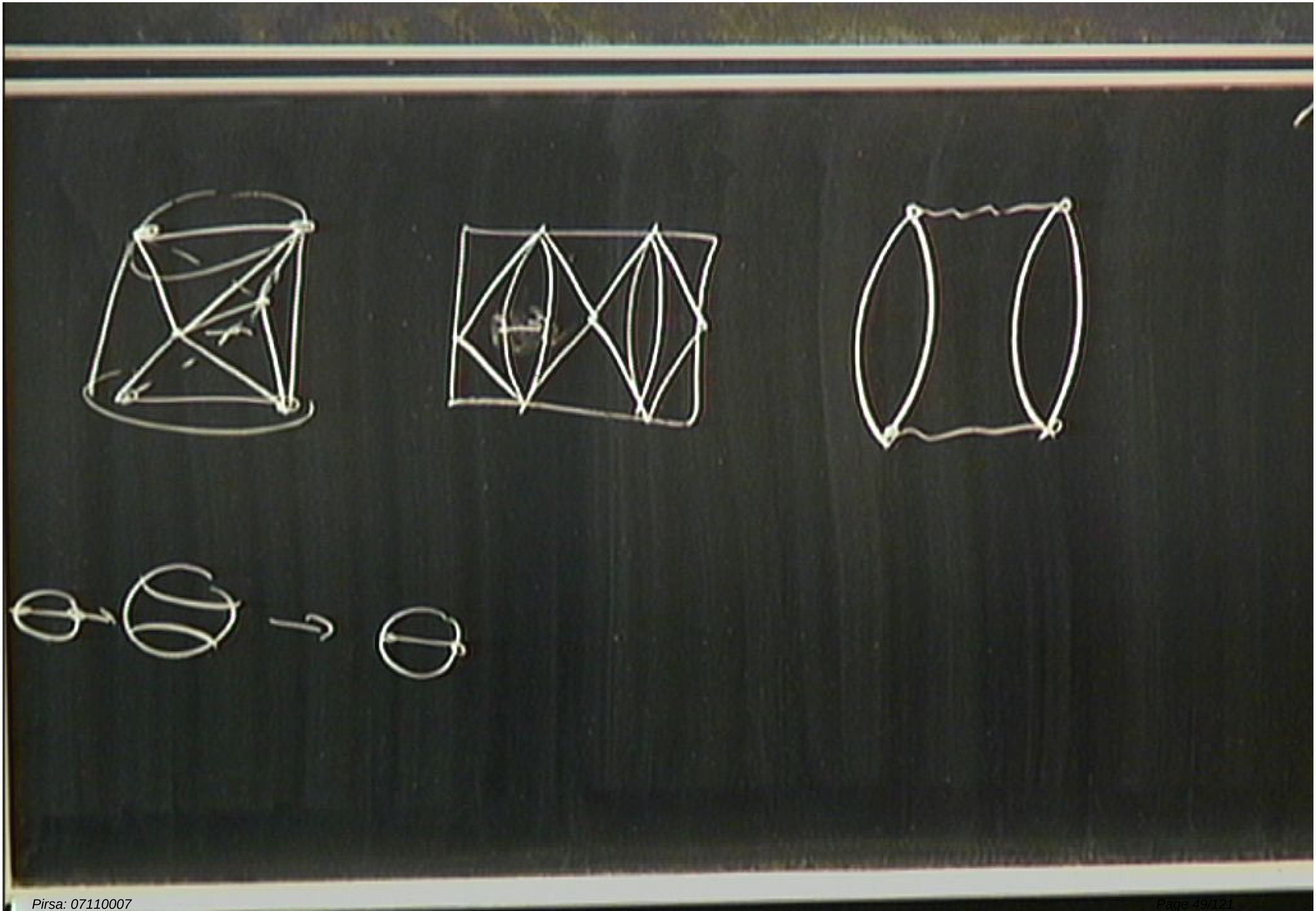
g± = (e^{π(r+ ± r-)} 0 ; 0 e^{-π(r+ ± r-)})
M3 = D∞ ×
BTZ BHole
L0± = 1/2 (ℓM ± J) = k/4 (r+ ± r-)²
ds² = 4Gℓ(L⁺du² + L⁻dv²) − ℓ²dρ² + (ℓ²
dudv)
u, v = t ± φ
A+, A-

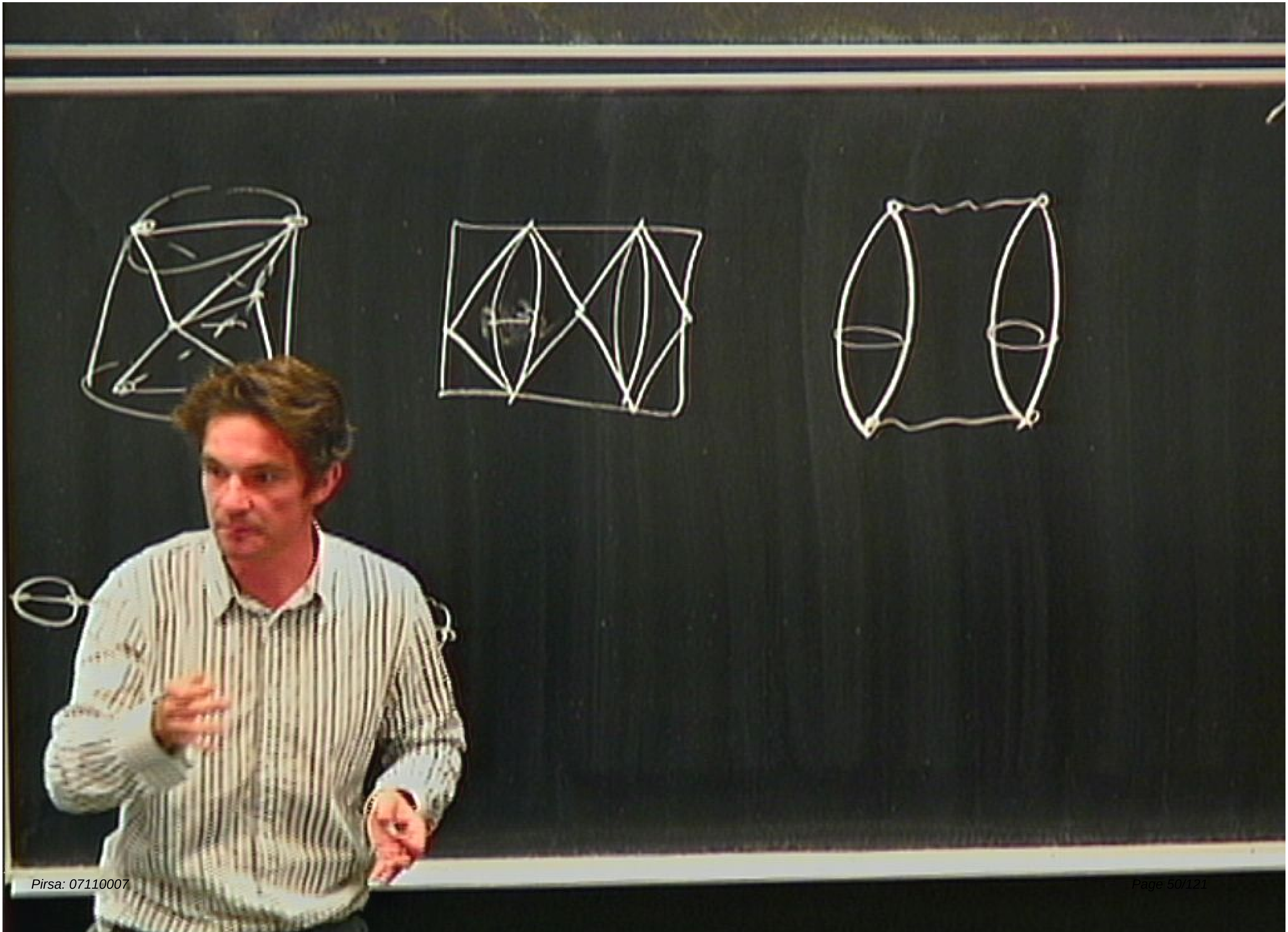

$g_\pm = \begin{pmatrix} e^{\pi(r_+ \pm r_-)} & 0 \\ 0 & e^{-\pi(r_+ \pm r_-)} \end{pmatrix}$
$D_\infty \times T$
BTZ BHole
$L_0^\pm = \frac{1}{2}(\ell\Pi \pm J) = \frac{k}{4}(r_+ \pm r_-)^2$
$ds^2 = 4G\ell(L^+ du^2 + L^- dv^2) - \ell^2 d\rho$
$L^- du dv)$
$u, v = t \pm$
A_+, A_-

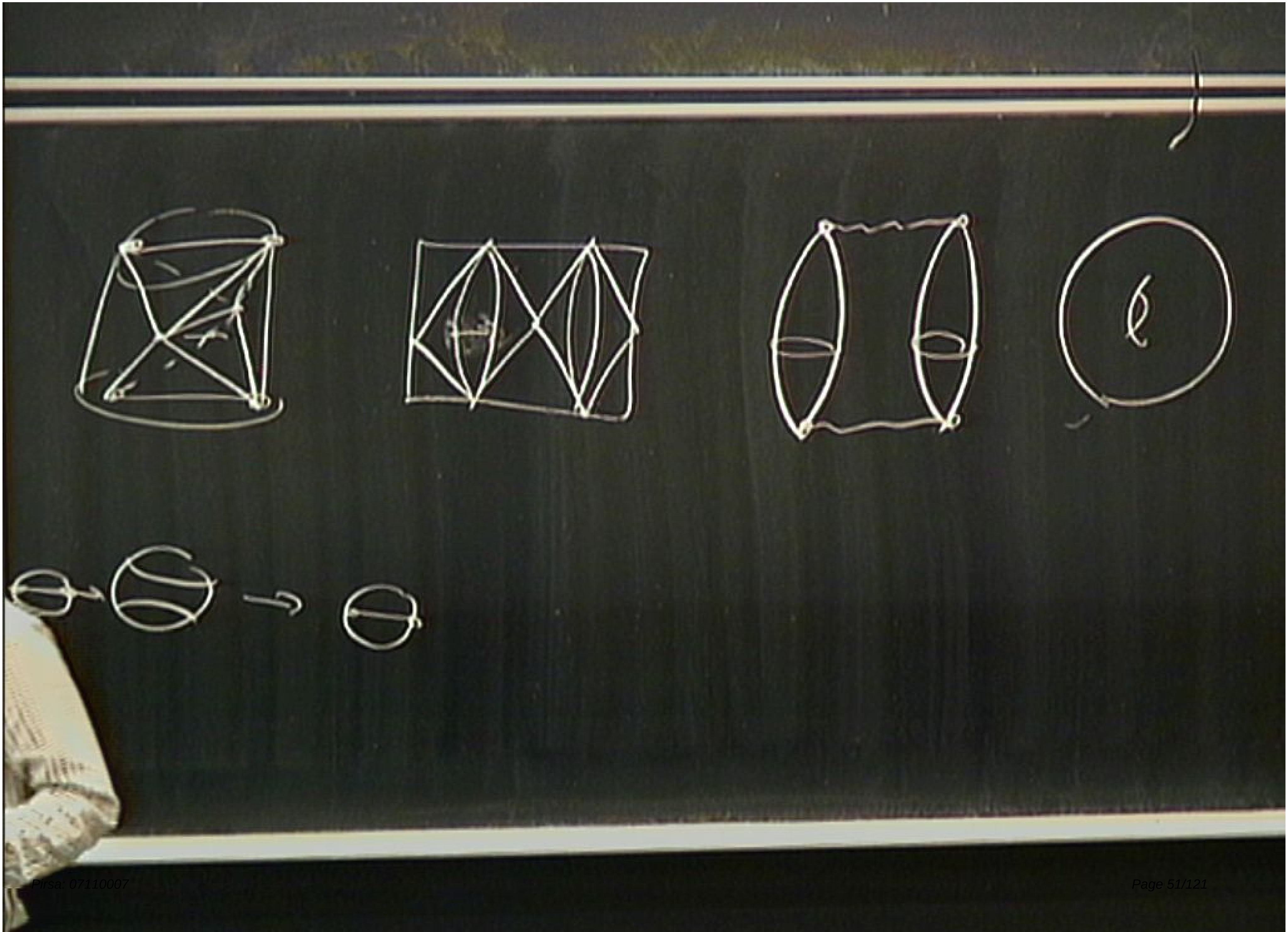

CAUTION

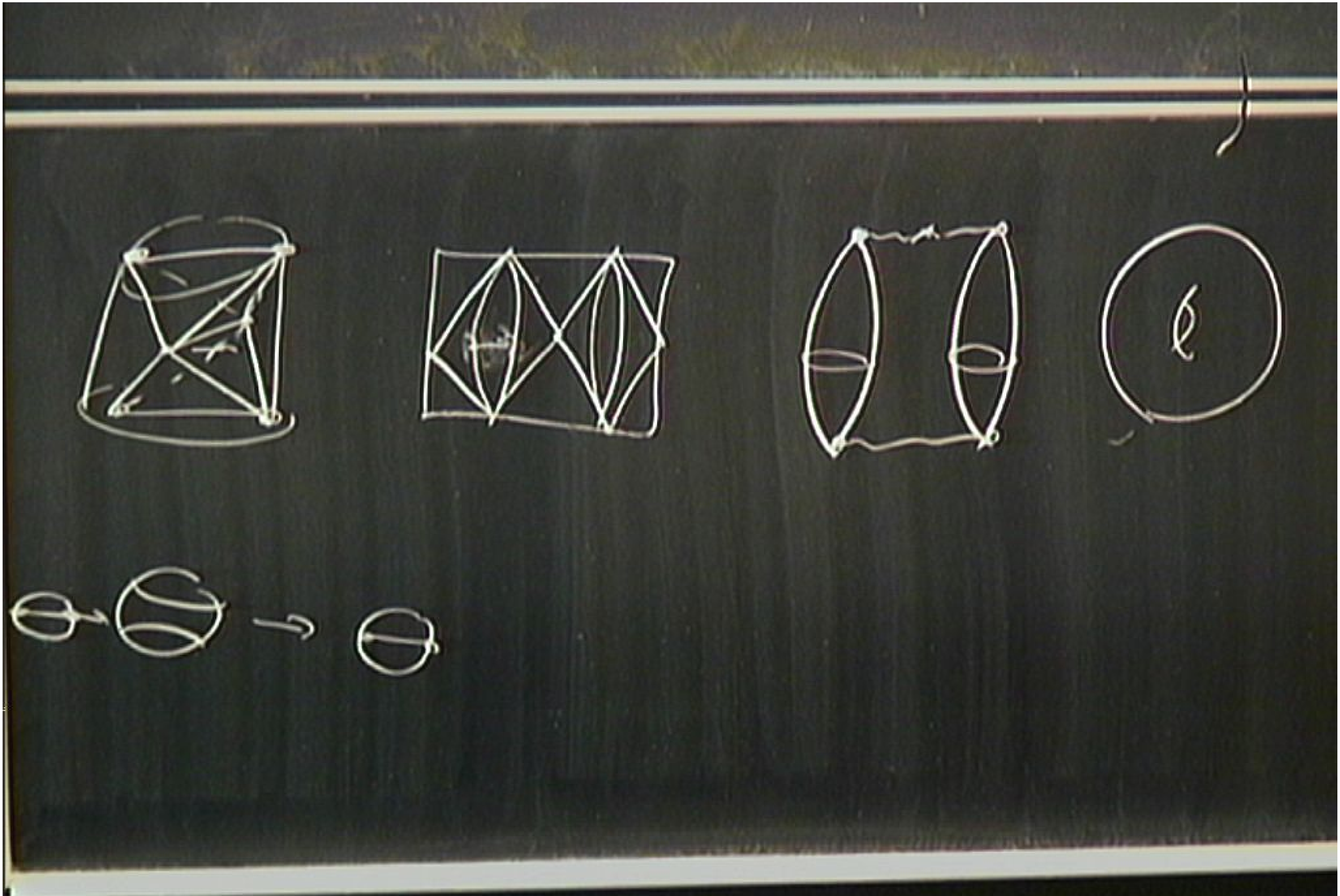


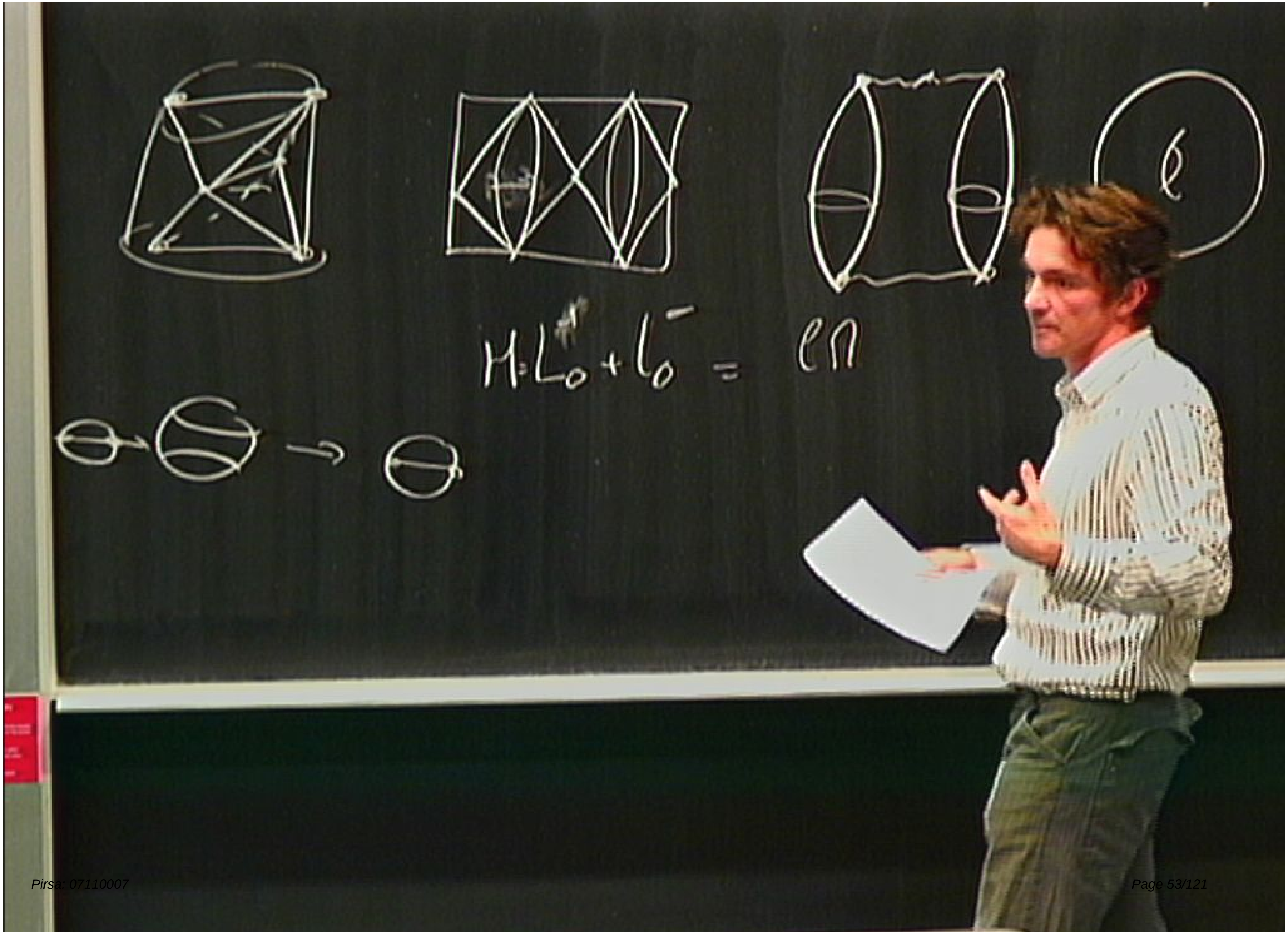


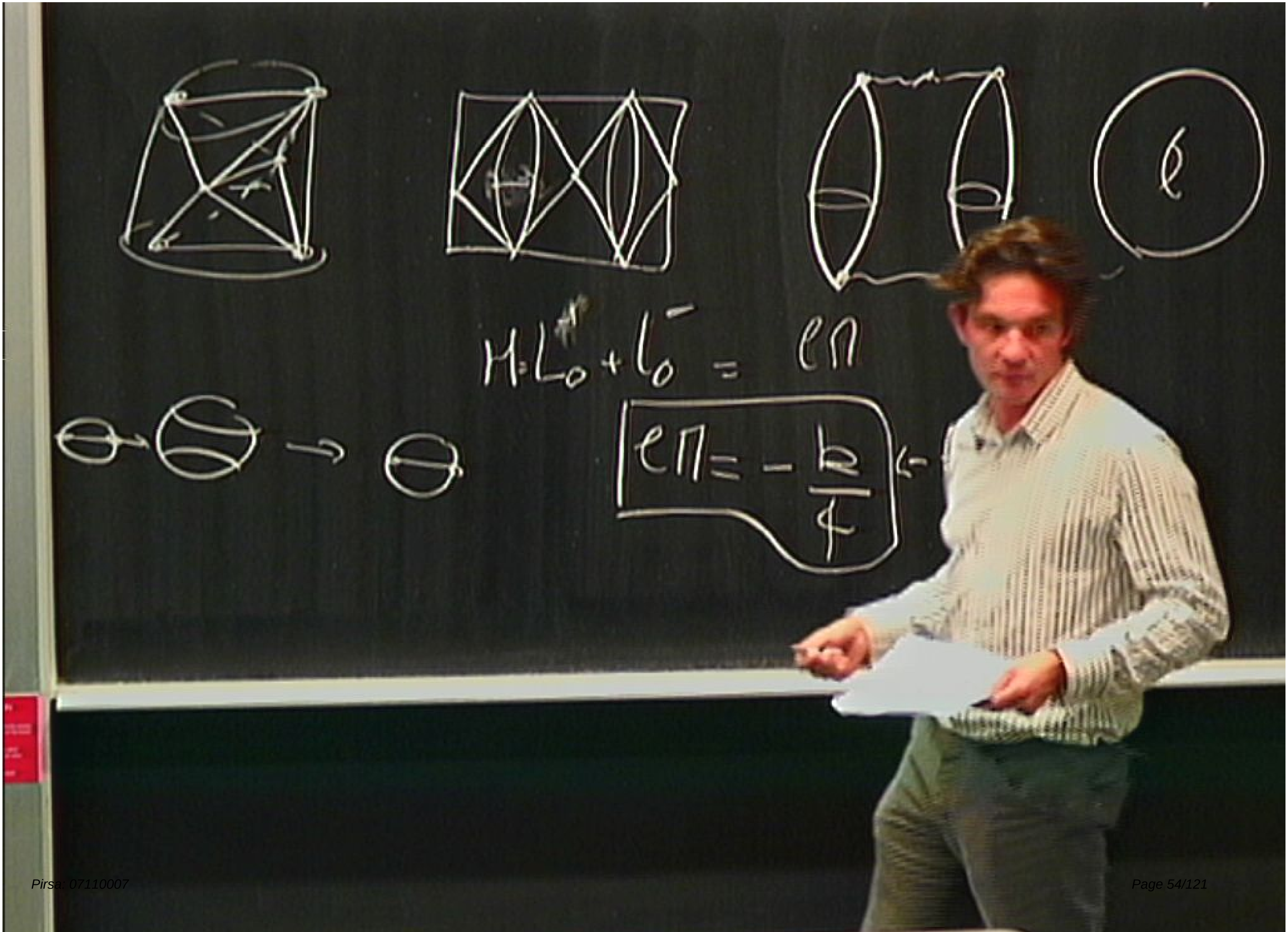


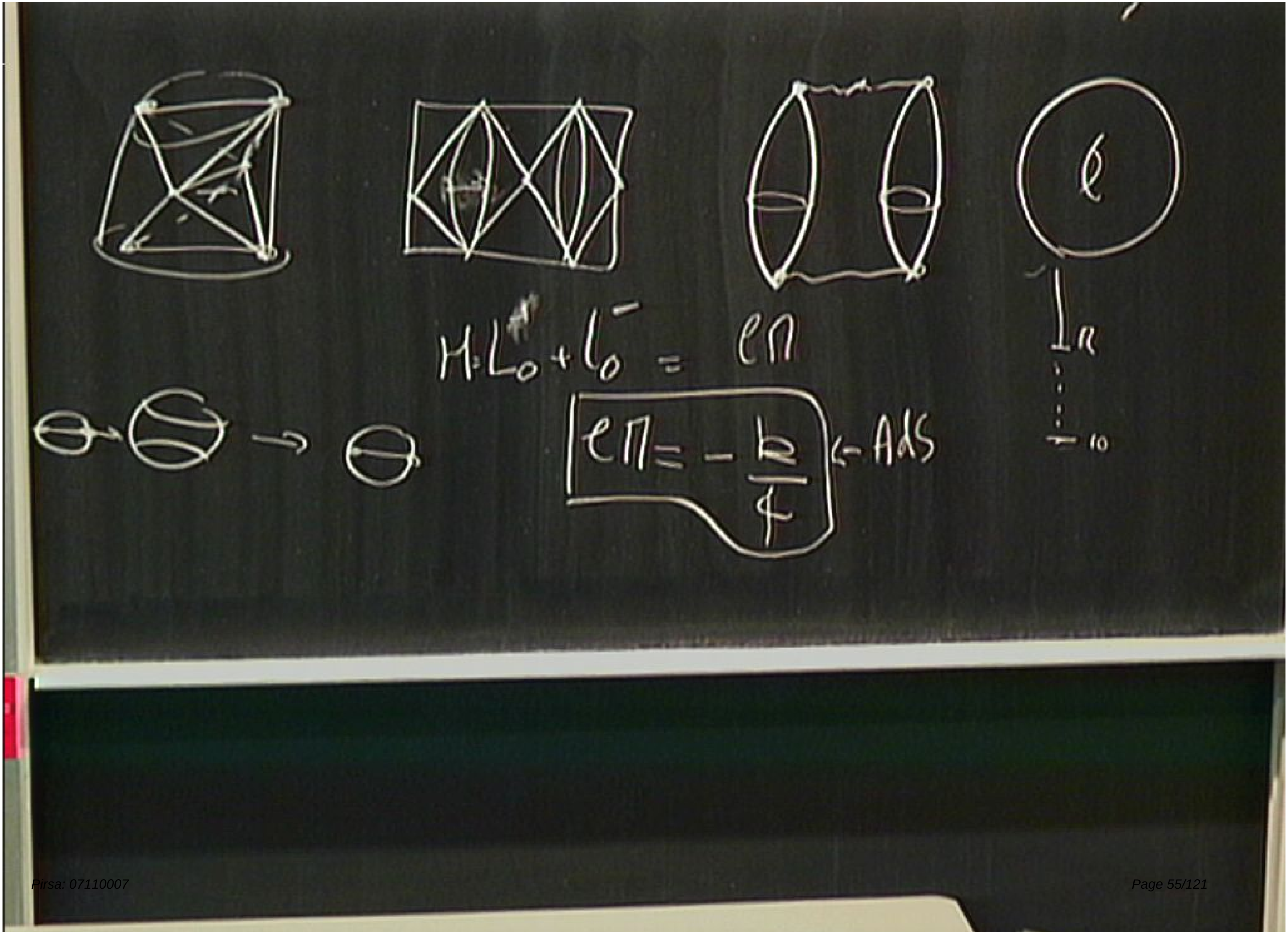


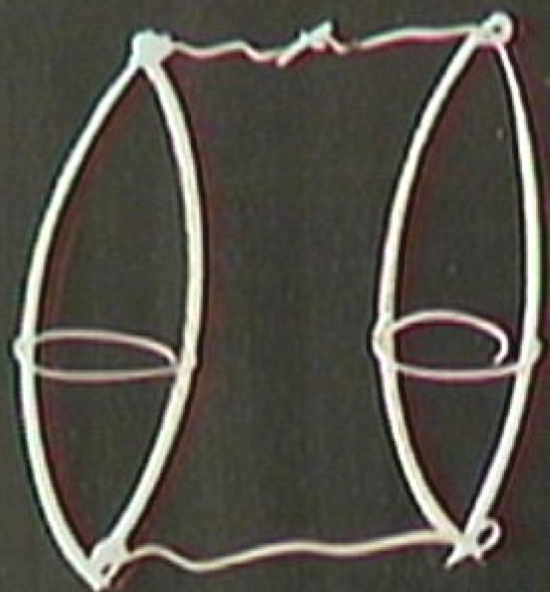


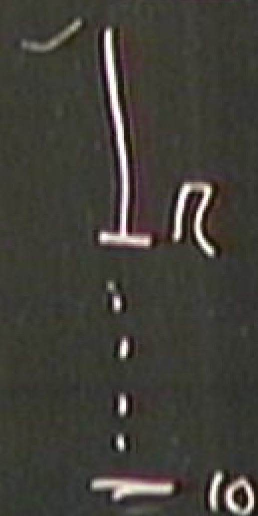



H=L_0^+ + L_0^- = ℓn




AdS

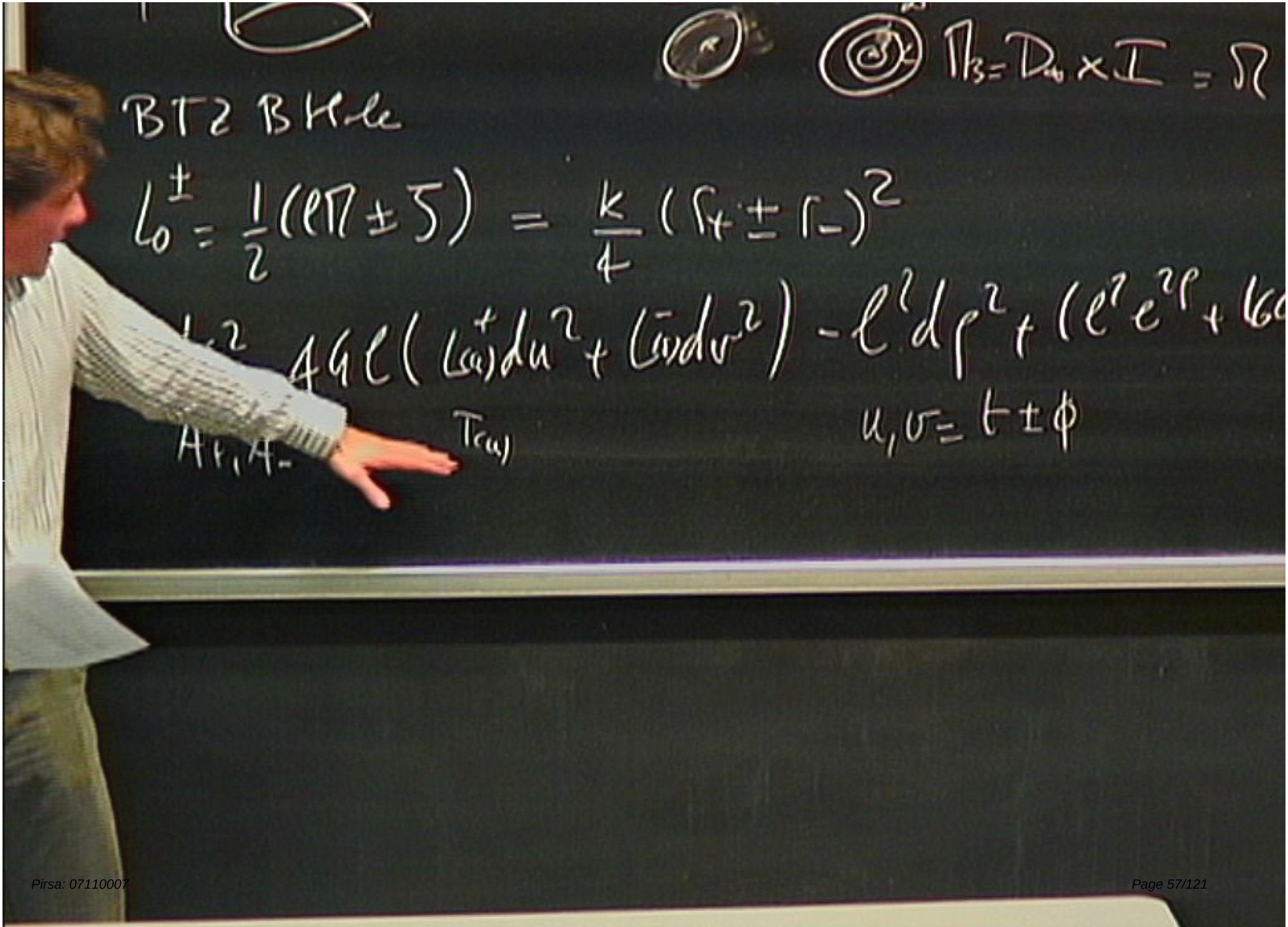




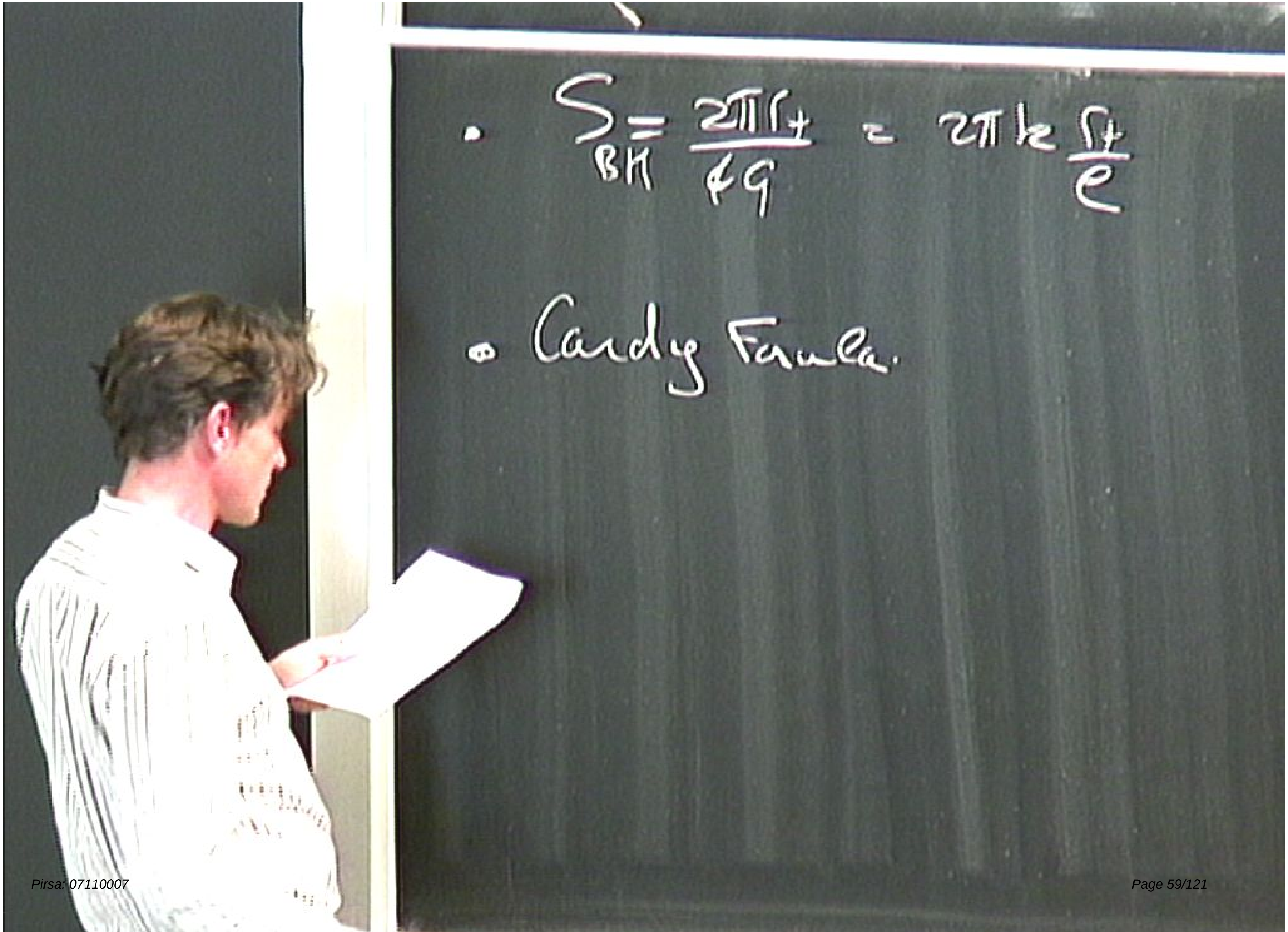
ℓn

$=$

$$\boxed{= -\frac{k}{4}} \leftarrow \text{AdS}$$


BTZ BHole
$L_0^{\pm} = \frac{1}{2}(\ell M \pm J) = \frac{k}{4}(r_+ \pm r_-)^2$
$u, v = t \pm \phi$
$T(u)$
A_+, A_-

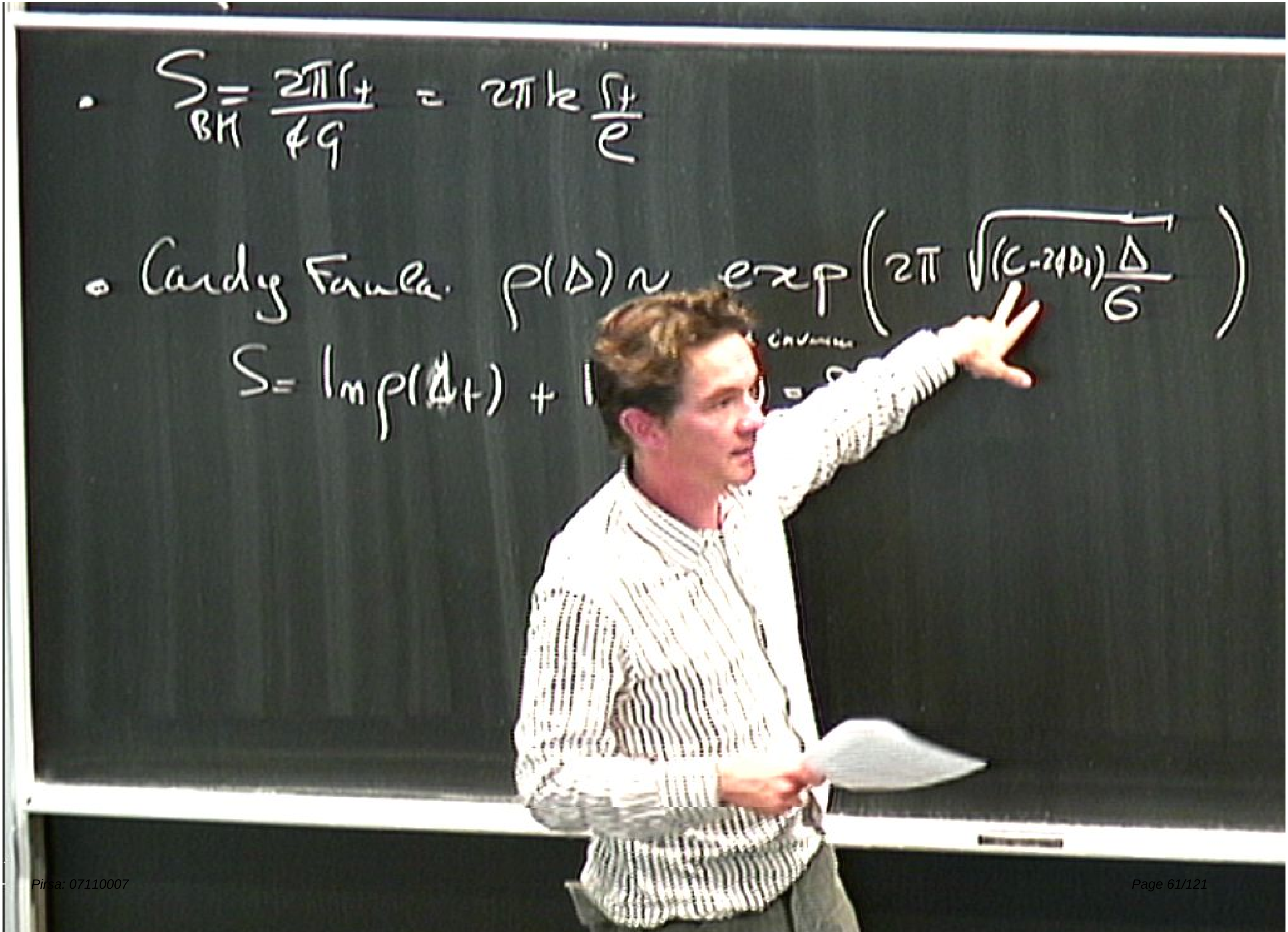
- $S_{BH} = \frac{2\pi r_+}{4G} = 2\pi k \frac{r_+}{\ell}$
-


$S_{BH} = \frac{2\pi r_+}{4G} = 2\pi k \frac{r_+}{\ell}$
Cardy Formula.

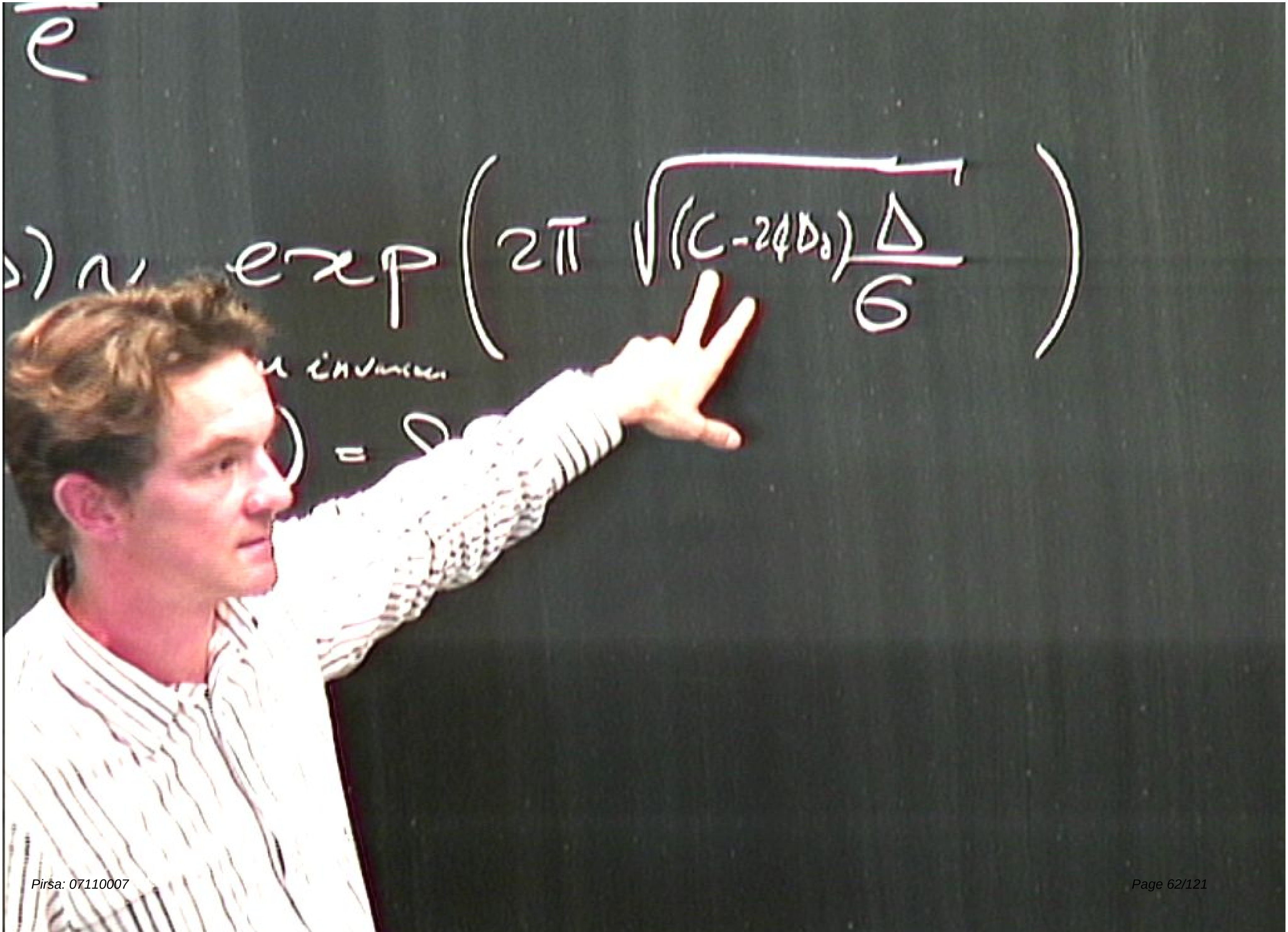
- $$S_{BH} = \frac{2\pi r_+}{4G} = 2\pi k \frac{r_+}{\ell}$$

- Cardy Formula: $\rho(\Delta) \sim$

$$S = \ln\rho(\Delta_+) + \ln\rho(\Delta_-) = S_{BH}$$



S_BH = 2πr+/4G = 2πk r+/ℓ
Cardy Formula ρ(Δ) ~ exp(2π √((c−24|D|)Δ/6))
S = ln ρ(Δ+) +

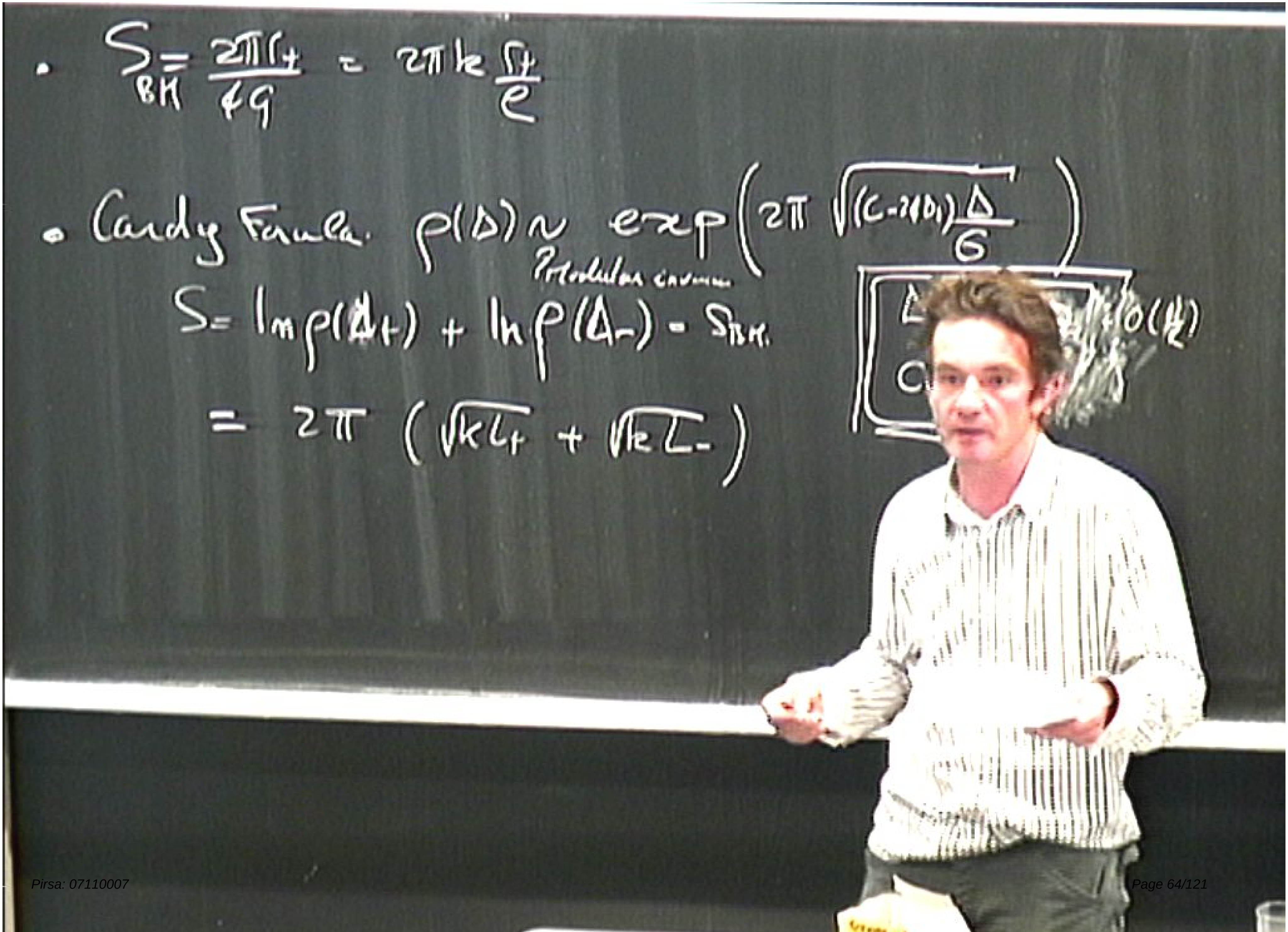

$\exp\left(2\pi\sqrt{(c-24D_0)\frac{\Delta}{6}}\right)$

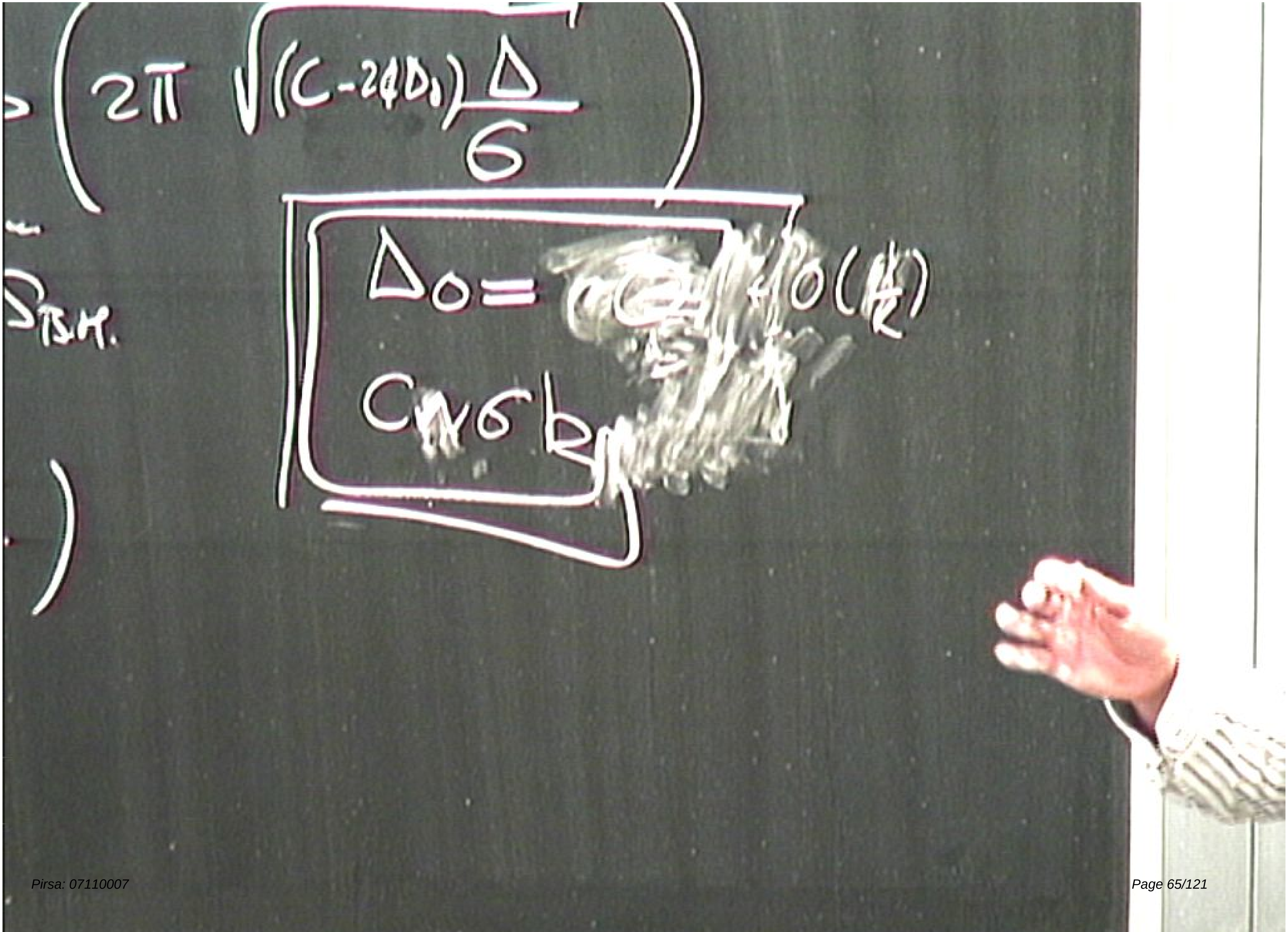
$$) \sim \exp\left(2\pi\sqrt{(c-24\Delta_0)\frac{\Delta}{6}}\right)$$

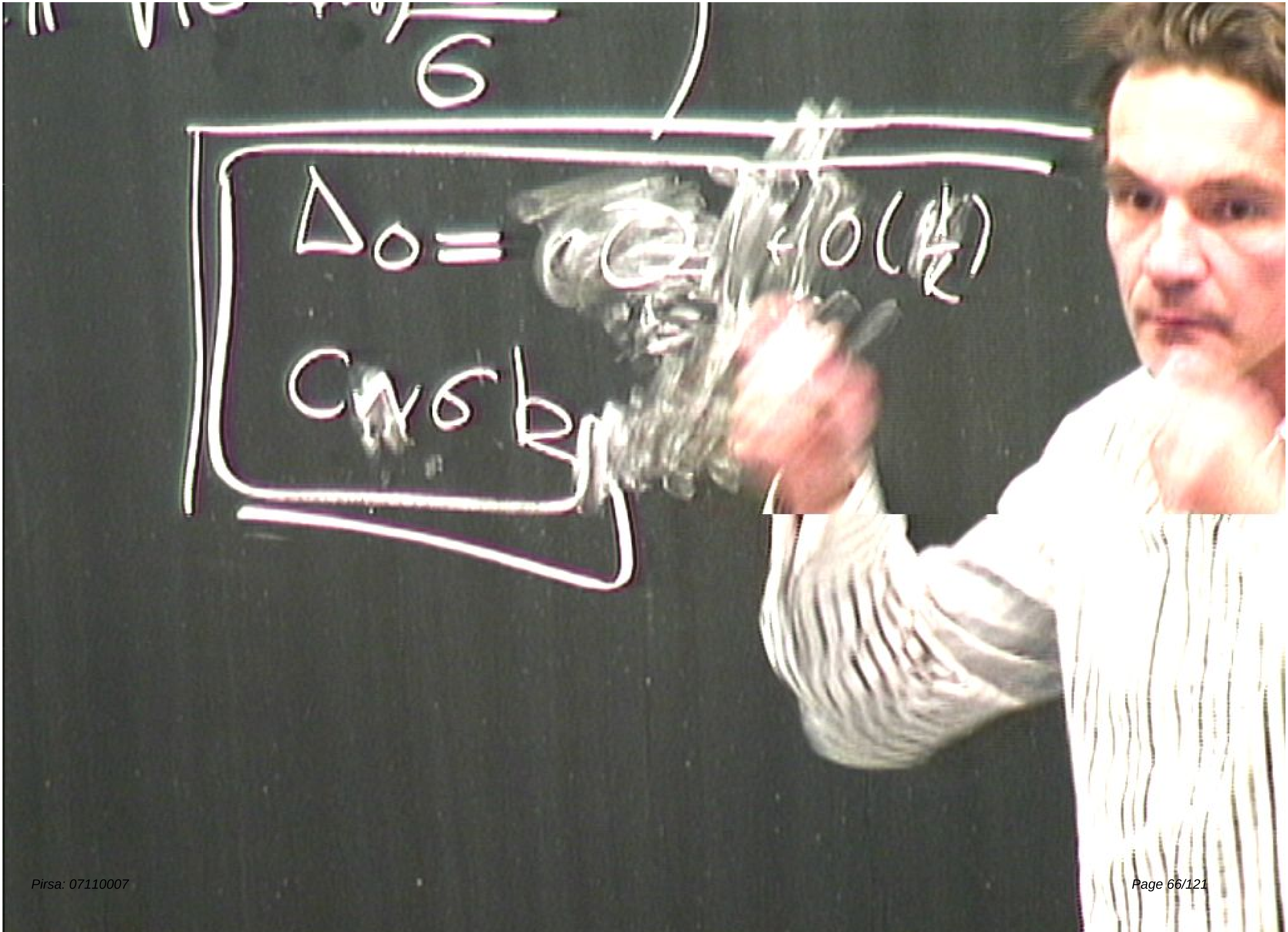
Modular invariance

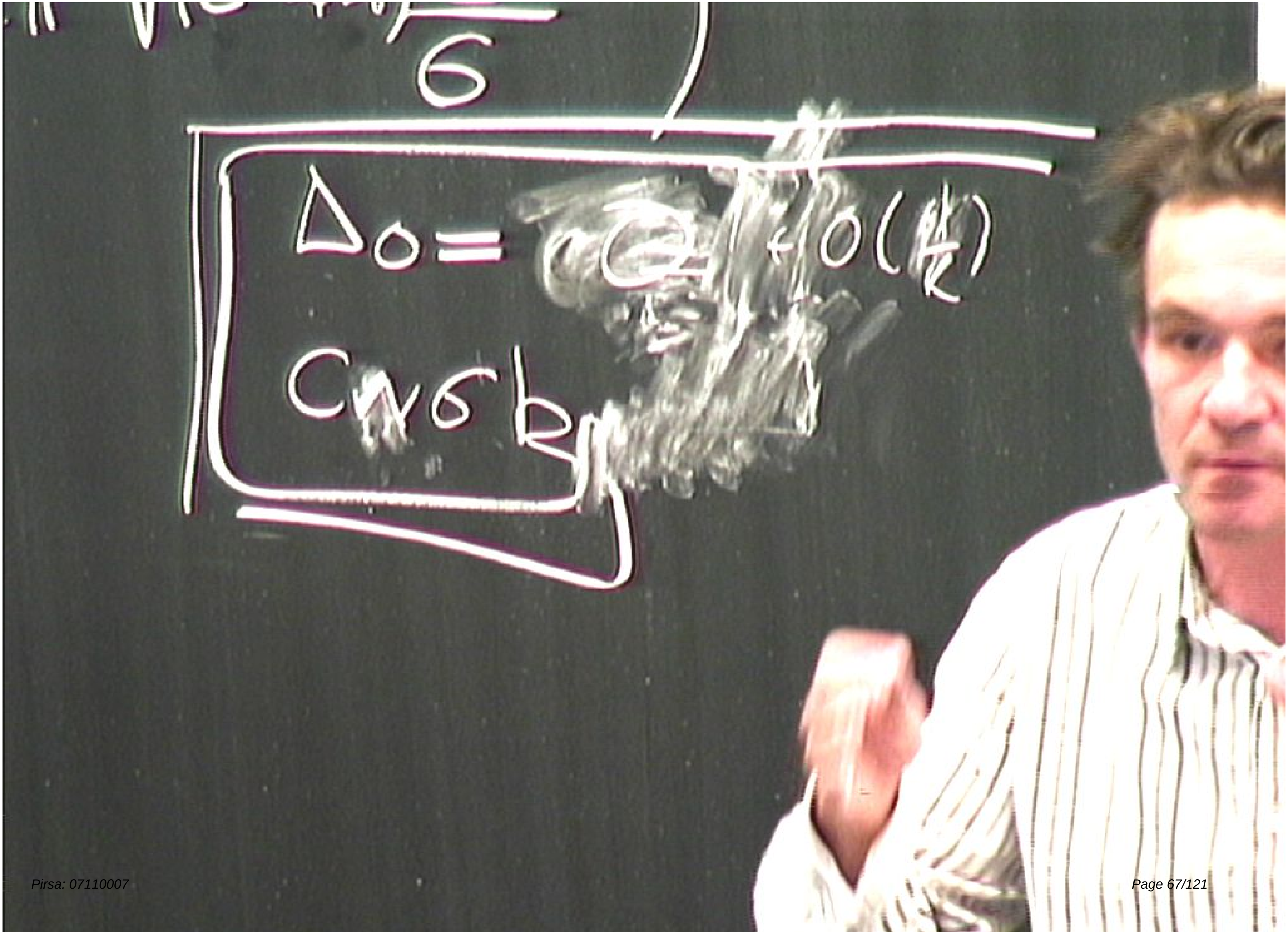
$$\ln\rho(\Delta_-) = S_{B.H.}$$

$$\boxed{\begin{array}{l}\Delta_0 = \text{[illegible]} \\ c = 6k\end{array}}$$


$S_{BH} = \frac{2\pi r_+}{4G} = 2\pi k \frac{r_+}{\ell}$
Cardy Formula $\rho(\Delta) \sim \exp\left(2\pi\sqrt{(c-24|D_0|)\frac{\Delta}{6}}\right)$
$S = \ln\rho(\Delta_+) + \ln\rho(\Delta_-) = S_{BH}$
$= 2\pi\left(\sqrt{kL_+} + \sqrt{kL_-}\right)$
$O(\frac{1}{k})$


2π √((C-24D₀) Δ/6)
S_BH
Δ₀ =
C~6b





$$\rho(\Delta) \sim \exp\left(2\pi \sqrt{(c - 24\Delta_0)\frac{\Delta}{6}}\right)$$

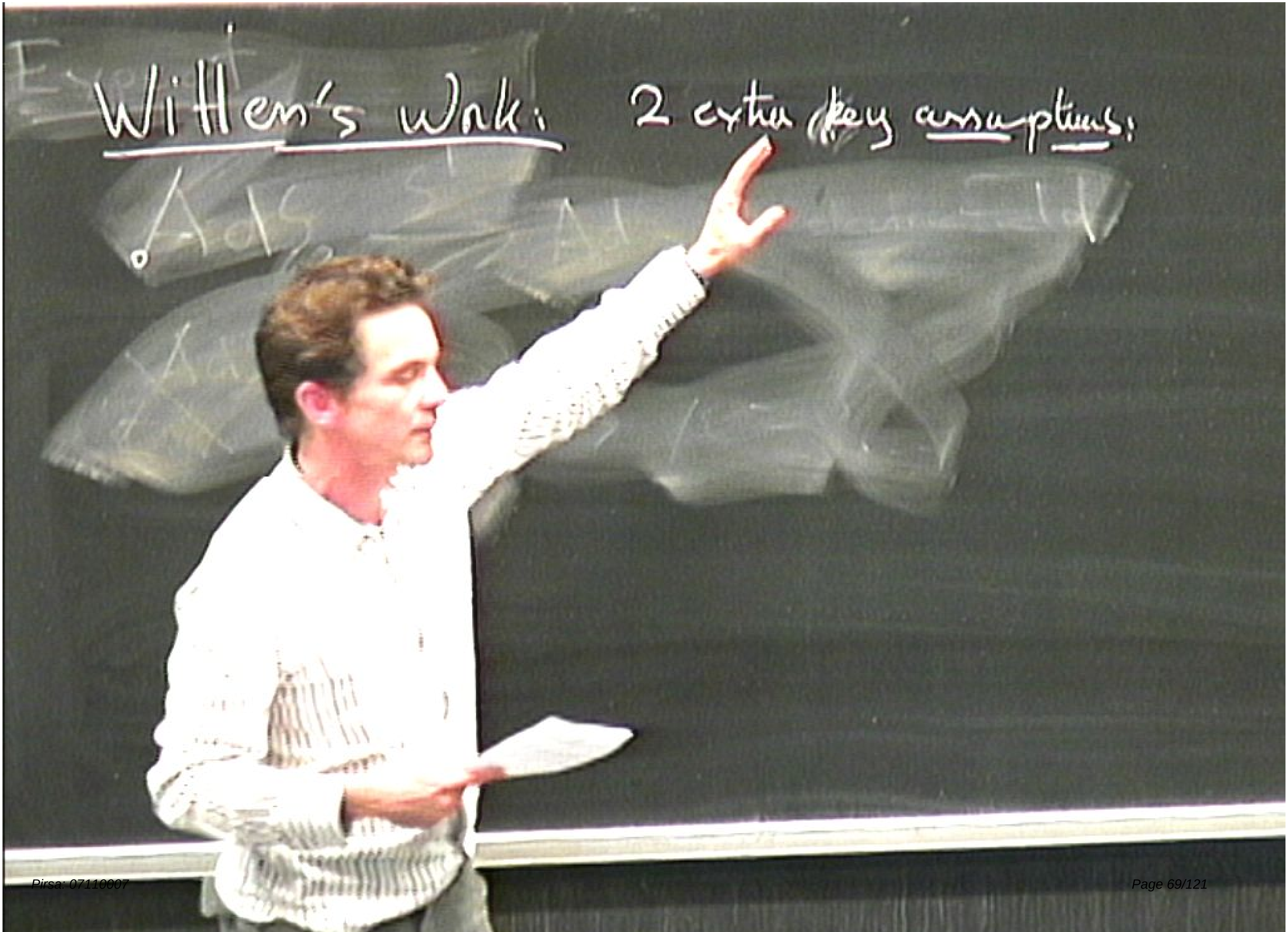
Potentiator invariance

$$) + \ln\rho(\Delta_-) = S_{BH}.$$

$$\left(\sqrt{kL_+} + \sqrt{kL_-}\right)$$

$$\boxed{\begin{aligned} \Delta_0 &= \ldots + O(\tfrac{1}{k}) \\ c &\sim 6k + O(\tfrac{1}{k}) \end{aligned}}$$

$$k = \left(\frac{\ell}{\ell_p}\right)$$


Witten's Work: 2 extra key assumptions:

Witten's Work: 2 extra key assumptions:

I. k is quantised

II.

Witten's Work: 2 extra key assumptions:

I. k is quantised

II. CFT is holomorphically factorisable.

xtra key assumptions:

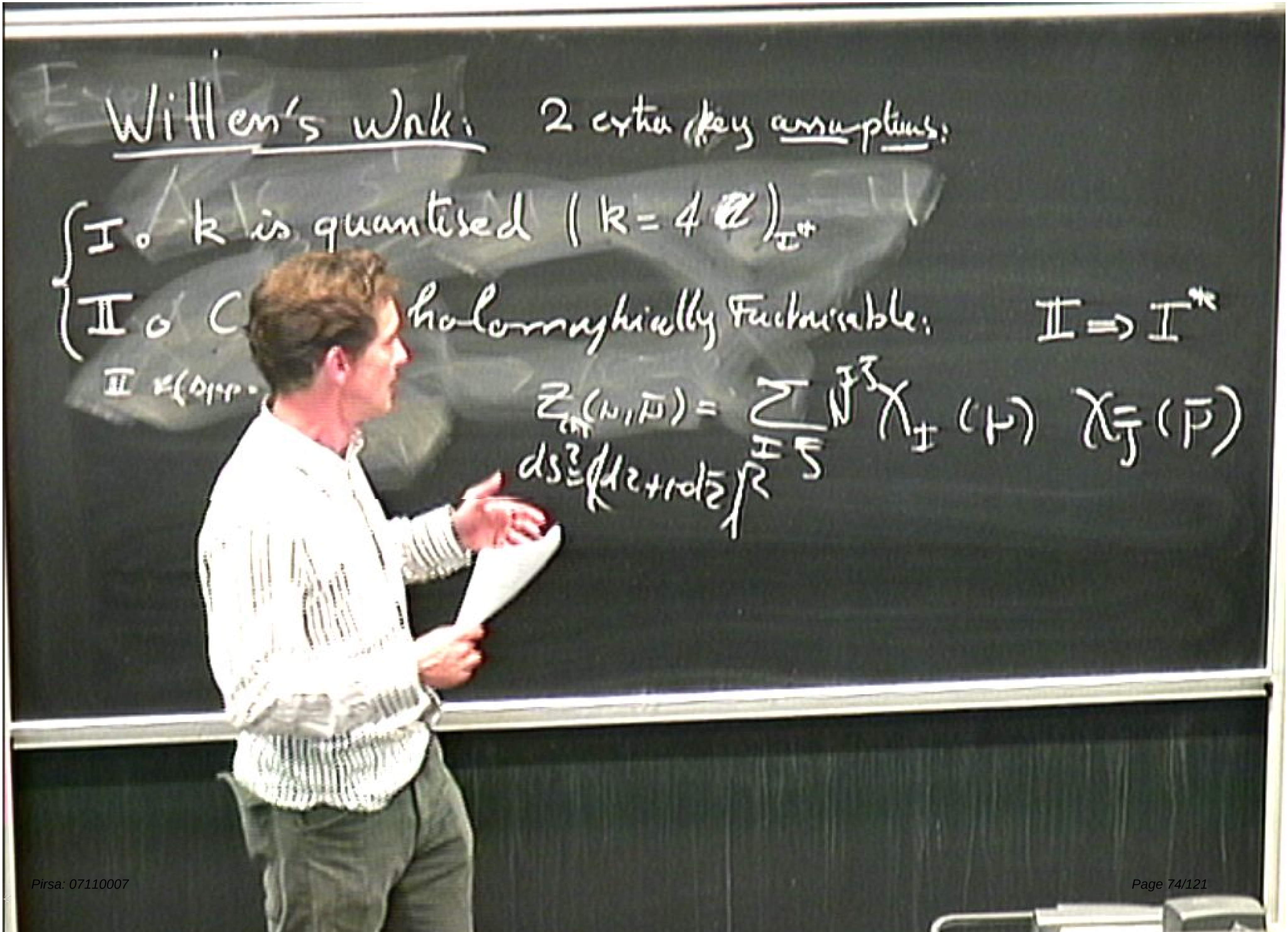
hically Factorisable: $\mathrm{II} \Rightarrow \mathrm{I}$

Witten's Work: 2 extra key assumptions:

I. k is quantised $(k = 4\mathbb{Z})_{I^*}$

II. CFT is holomorphically factorisable.

$\mathrm{II} \Rightarrow$


Witten's Work: 2 extra key assumptions:
I. k is quantised (k = 4
II. C
holomorphically Factorisable.
II ⇒ I*

ed $\left(k = 4 \cancel{2}\right)_{\mathbb{I}^4}$

lomorphically Factorizable: $\quad \mathbb{I} \Rightarrow \mathbb{I}^*$

$$Z_{CFT}(\mu,\bar{\mu}) = \sum_{I,\bar{J}} N^{I\bar{J}} \chi_I(\mu)\, \chi_{\bar{J}}(\bar{\mu})$$

$ds^2 = \left|dz + \tau d\bar{z}\right|^2$

$\ldots(\mu)|^2$

Conf Bl Block.

Witten's Work: 2 extra (key) assumptions:

$$\begin{cases} \mathrm{I}.\ k \text{ is quantised } (k = 4\mathbb{Z})_{\mathrm{I}^*} \\ \mathrm{II}.\ \text{CFT is holomorphically Factorisable:} \end{cases} \qquad \mathrm{II} \Rightarrow \mathrm{I}^*$$

$$Z_{CFT}(\mu,\bar{\mu}) = \sum_{I,\bar{J}} N^{I\bar{J}} \chi_I(\mu)\ \chi_{\bar{J}}(\bar{\mu})$$

$ds^2 = |dz + \mu d\bar{z}|^2$

Conf. str. Param.

$$Z_{CFT} = |Z(\mu)|^2$$

Witten's Work: 2 extra key assumptions:

I. k is quantised $(k = 4\mathbb{Z})_{c.c.}$

II. CFT is holomorphically factorisable. $\quad \mathrm{II} \Rightarrow \mathrm{I}^*$

II #(Ops ~ k_ω)

$$Z_M(\mu,\bar\mu) = \sum_{I\bar J} N^{I\bar J}\,\chi_I(\mu)\,\chi_{\bar J}(\bar\mu)$$

$ds^2 = |dz + \mu\, d\bar z|^2$

$Z_{CFT} = |Z(\mu)|^2$

Witten's Work: 2 extra (key) assumptions:

I. k is quantised $(k = 4\mathbb{Z})_{I^*}$

II. CFT is holomorphically factorisable: $\quad II \Rightarrow I^*$

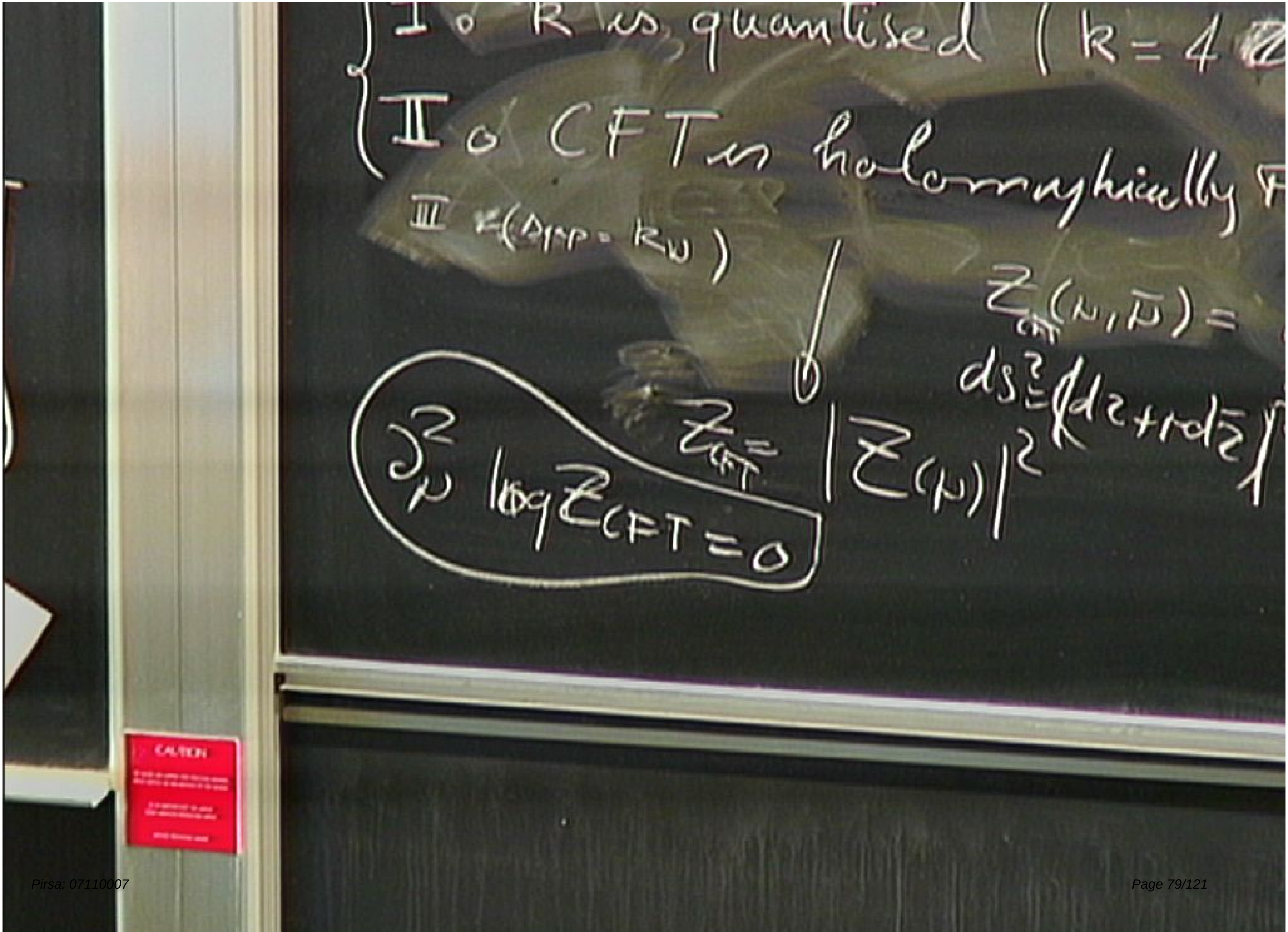
II $k(\ldots k_0)$

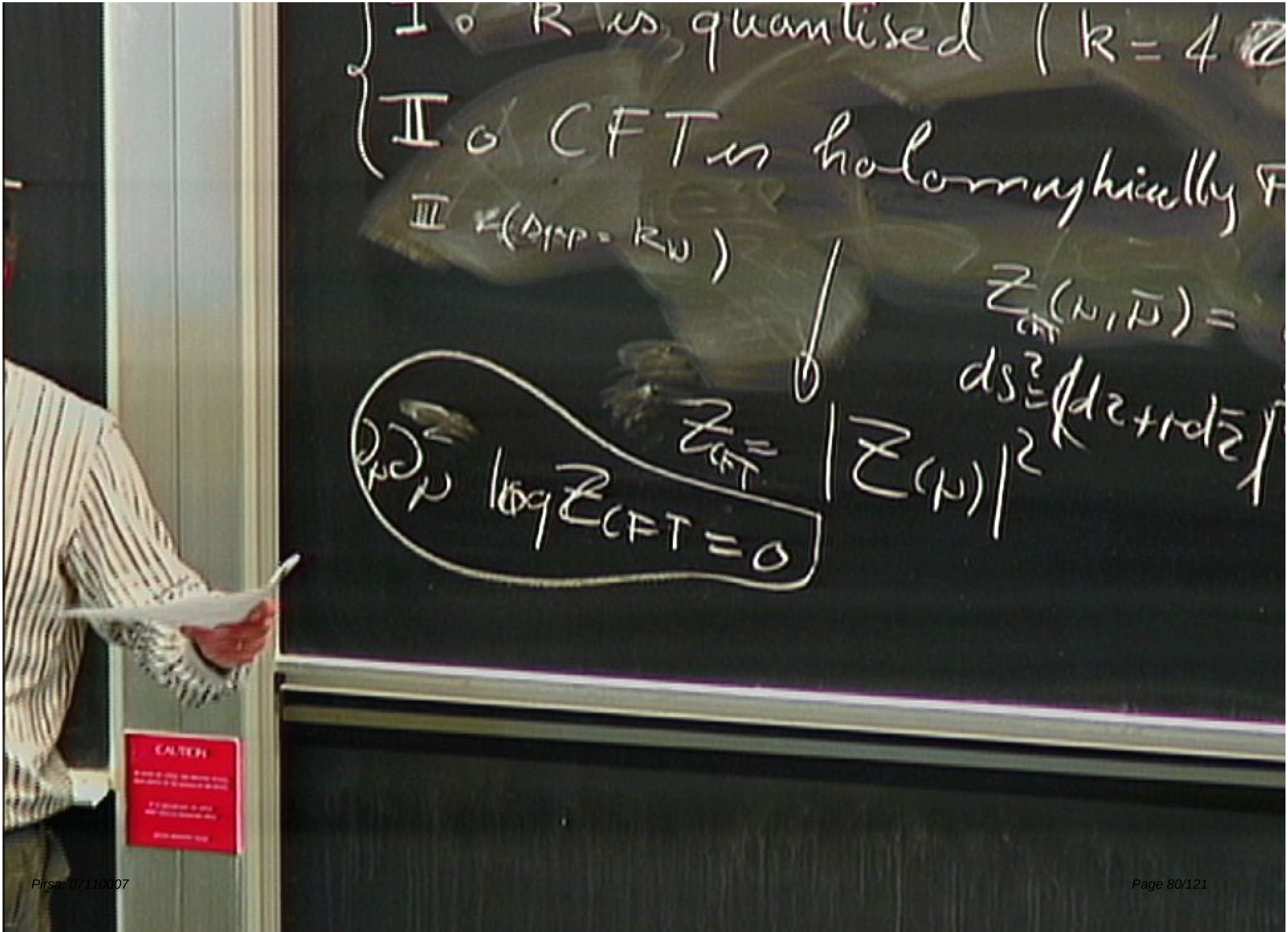
$$Z_{CFT}(\mu, \bar{\mu}) = \sum_{I, J} N^{IJ} \chi_I(\mu) \, \chi_{\bar{J}}(\bar{\mu})$$

$ds^2 = |dz + \mu d\bar{z}|^2$

Conf. TS Bundle.

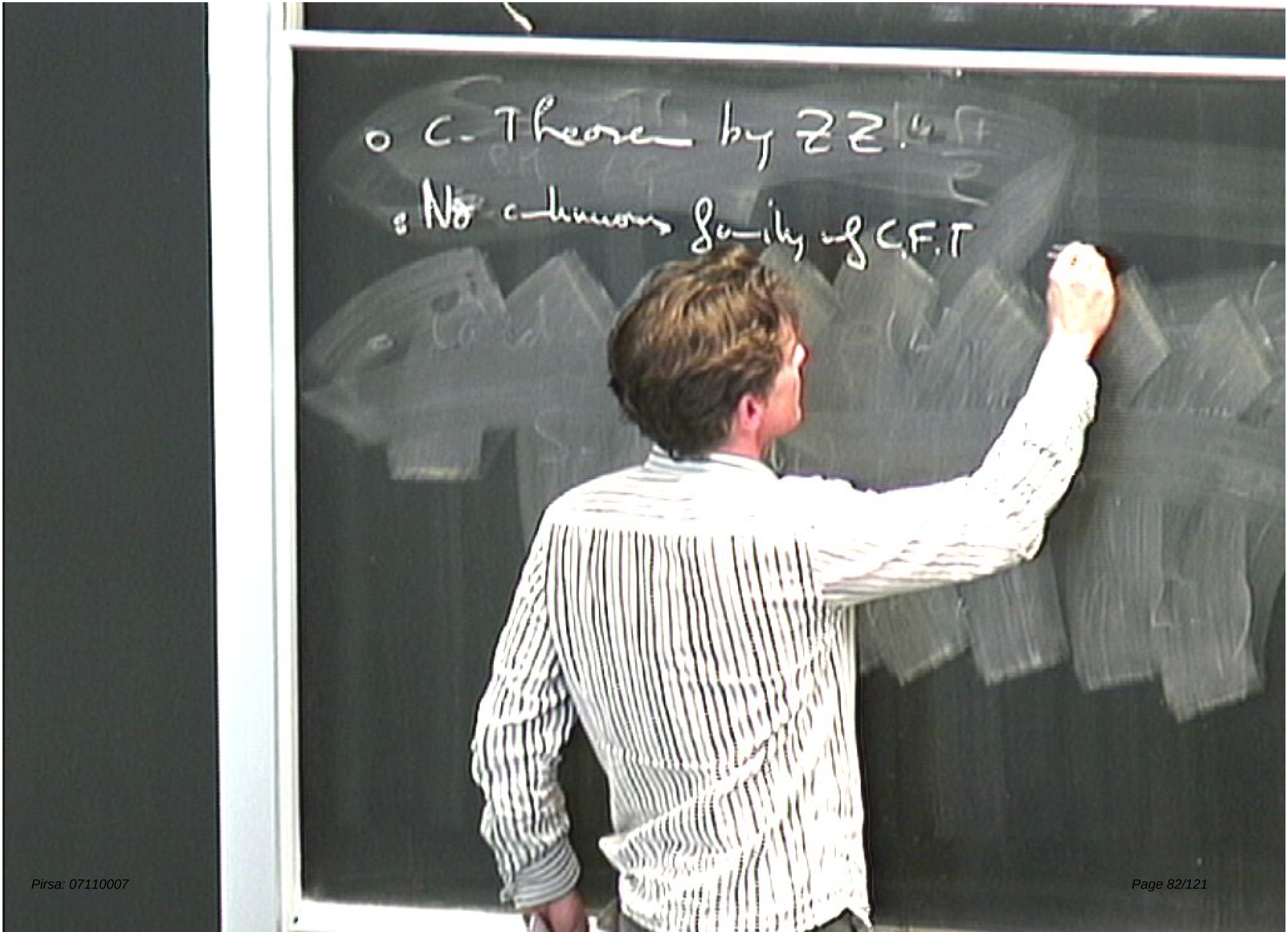
$$Z_{CFT} = |Z(\mu)|^2$$

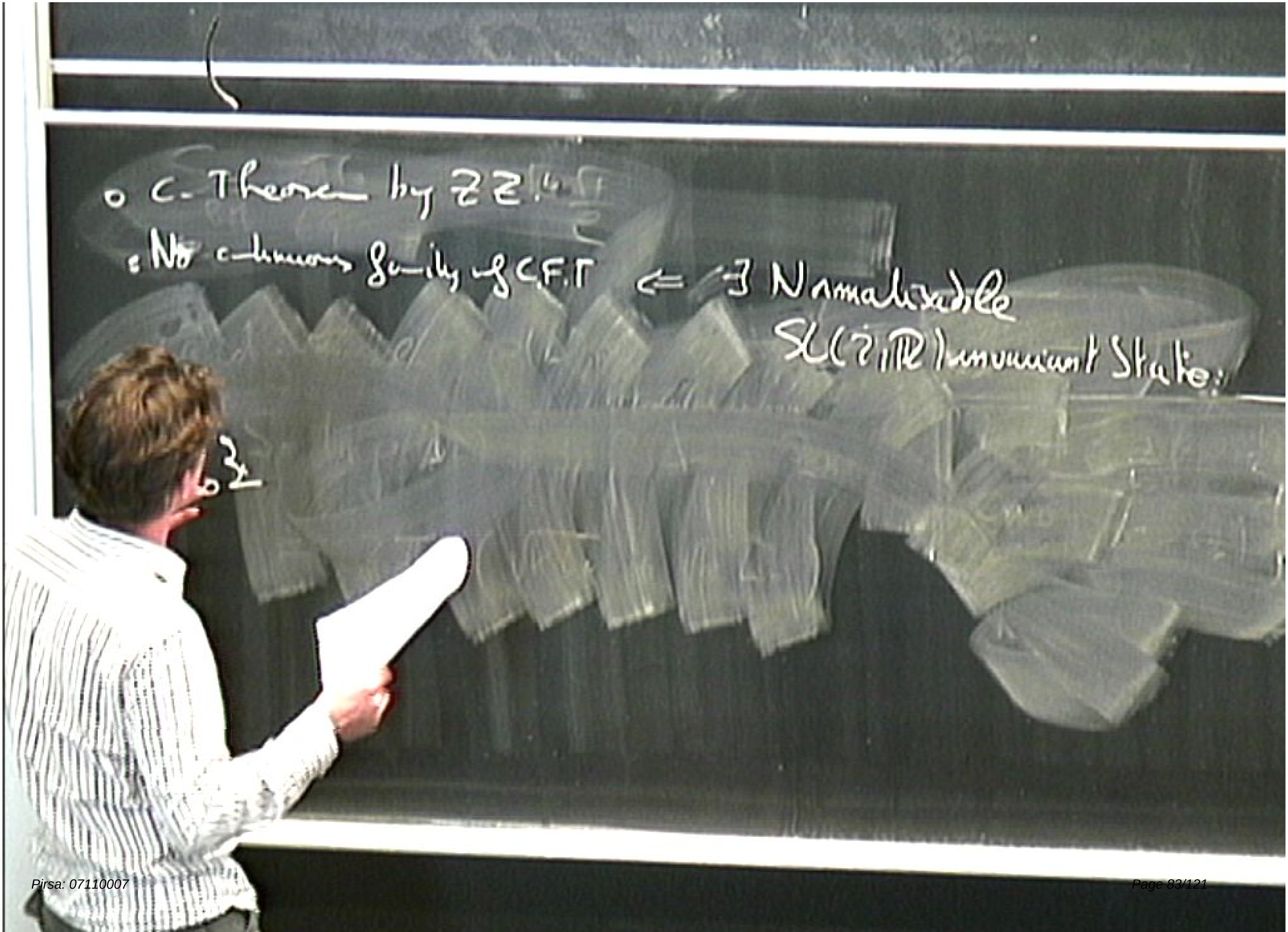

I o k is quantised (k = 4
II o CFT is holomorphically f
Z(τ, τ̄) =
ds² = (dz + τ dz̄)
Z_CFT = |Z(τ)|²
∂²_τ̄ log Z_CFT = 0
CAUTION

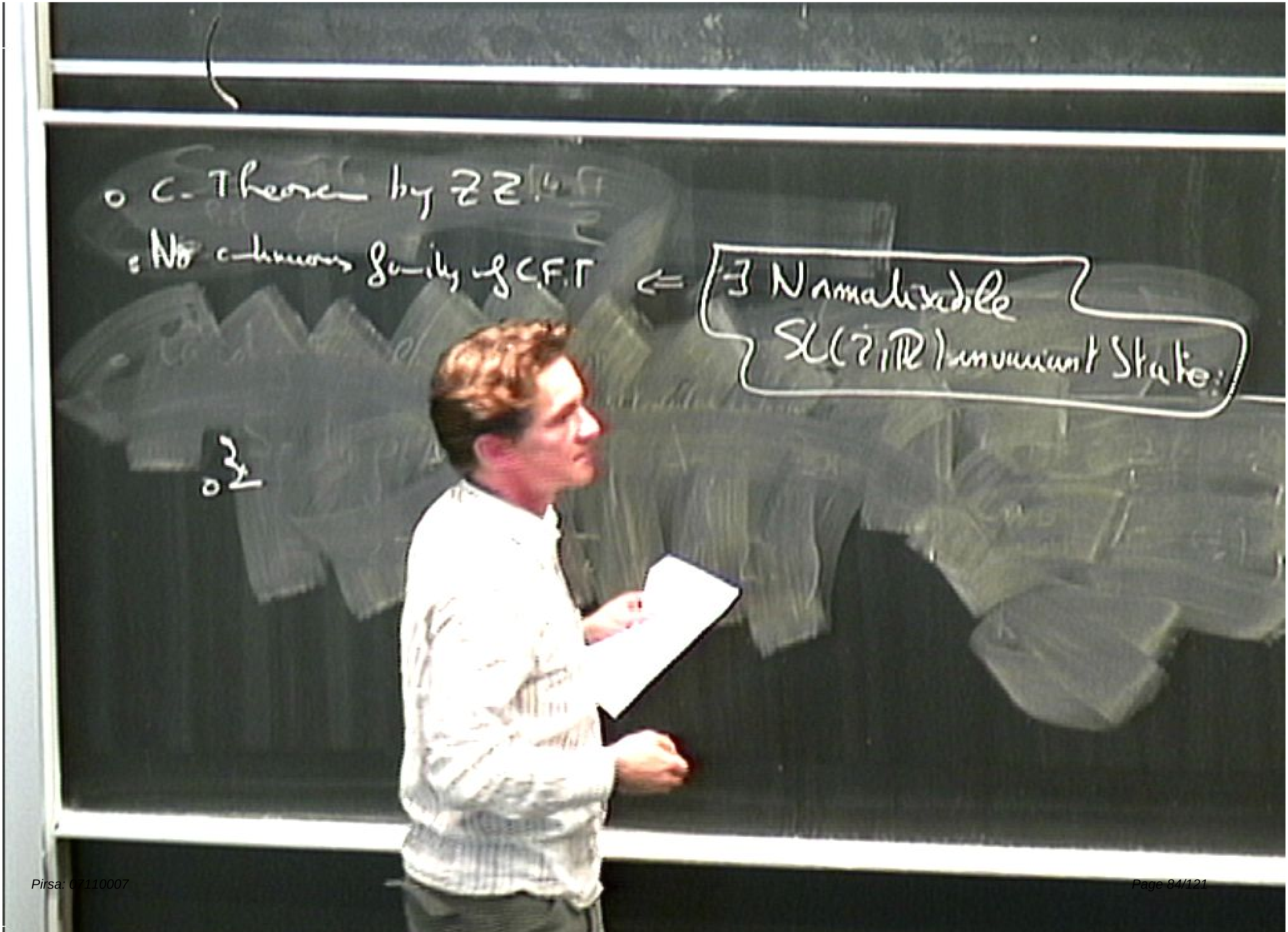


I o k is quantised (k = 4
II o CFT is holomorphically
Z_CFT = |Z(μ)|^2
log Z_CFT = 0
CAUTION

- C-Theorem by ZZ


o C-Theorem by ZZ!
o No continuous family of C.F.T


∘ c-Theorem by ZZ!
∘ No continuous family of CFT ⇐ ∃ Normalizable
SL(2,ℝ) invariant State:


c-Theorem by ZZ
No continuous family of C.F.T
∃ Normalizable
SL(2,R) invariant State

∘ C-Theorem by ZZ!

∘ No continuous family of C.F.T $\Leftarrow$ $\exists$ Normalizable $SL(2,\mathbb{R})$ Invariant State.

∘ $\frac{3}{2}$

$$\boxed{k = \frac{p}{q}}$$

normalisable

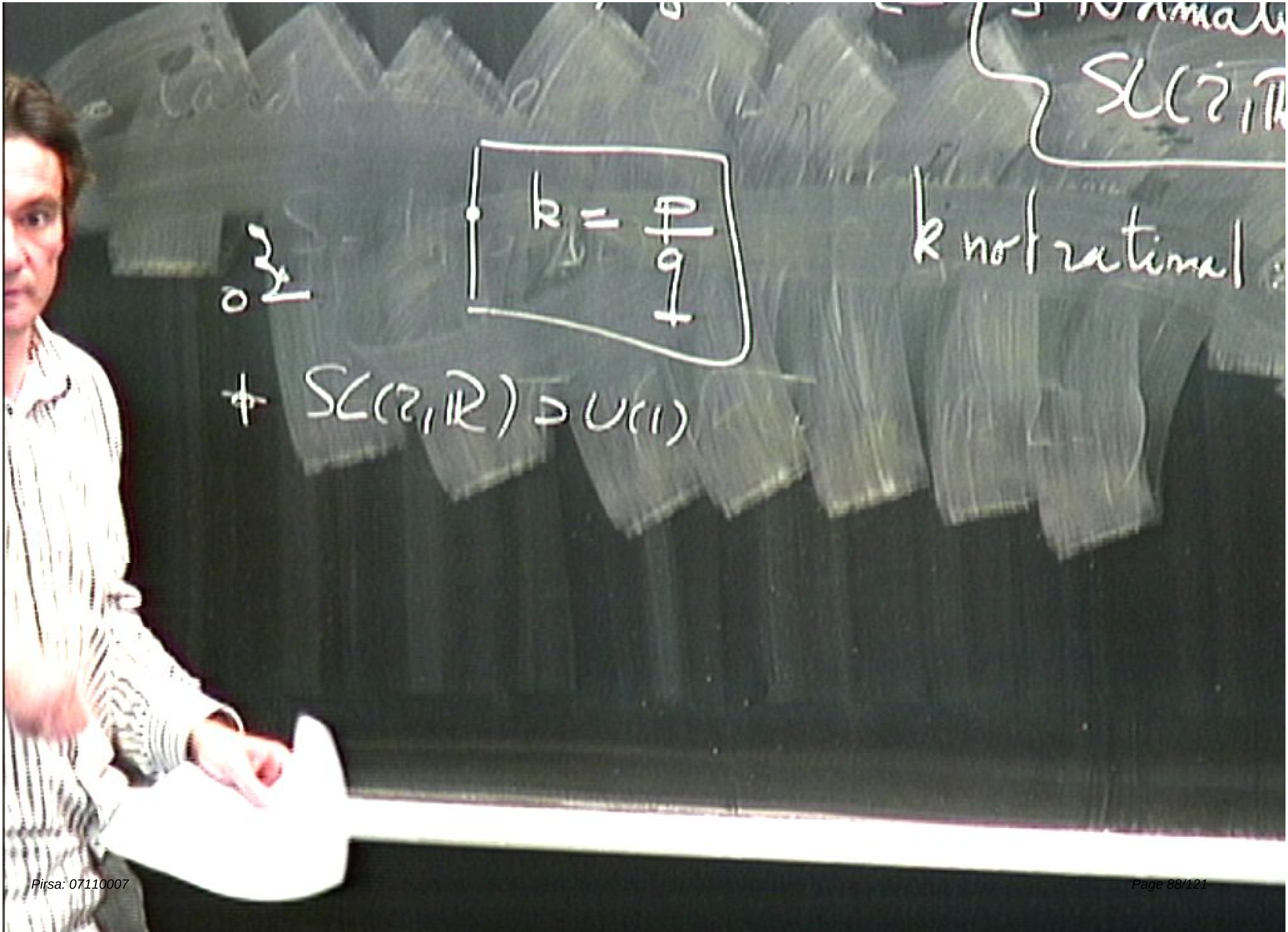
$SL(2,\mathbb{R})$ invarian

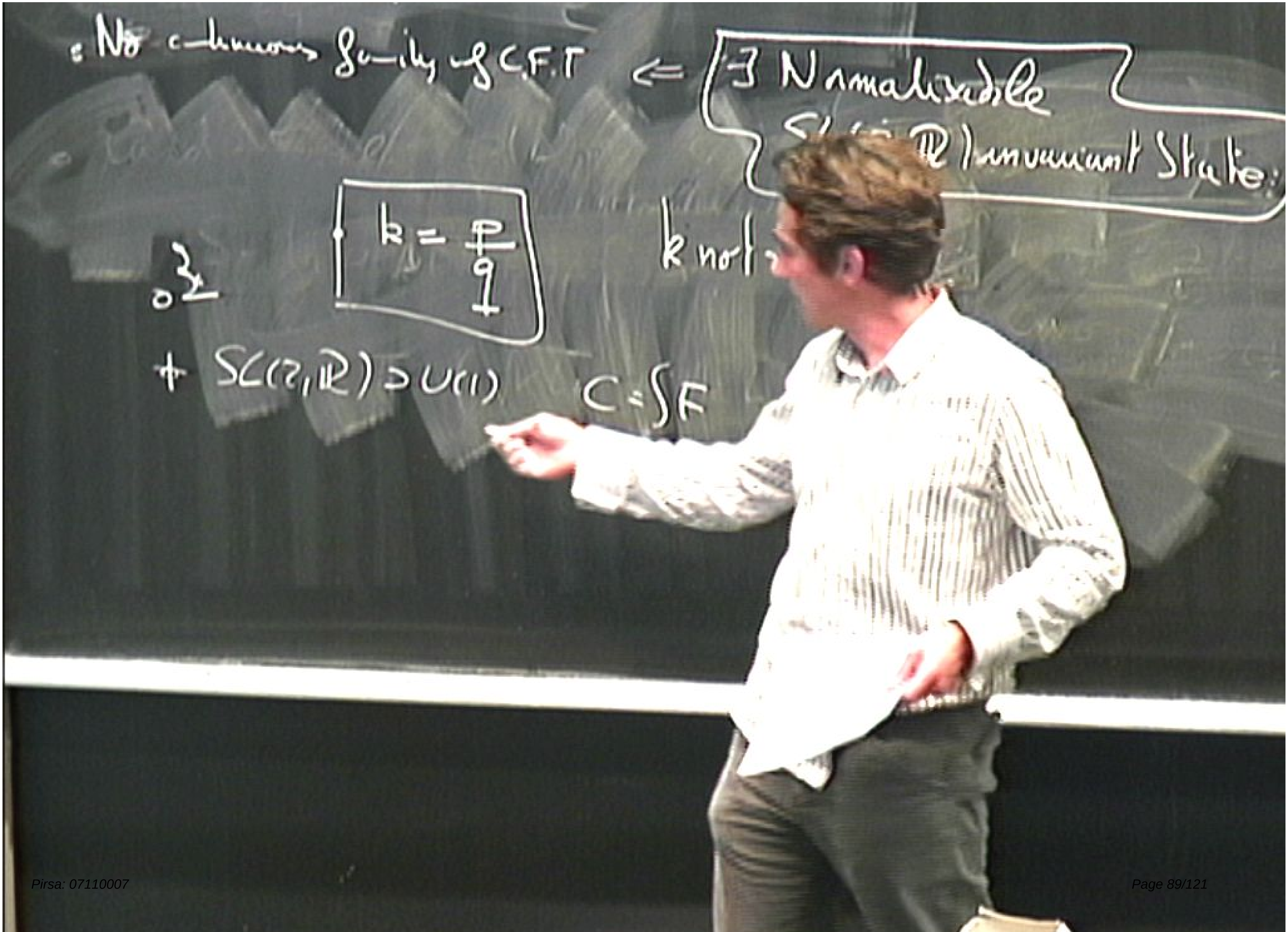
$$\boxed{k = \frac{p}{q}}$$

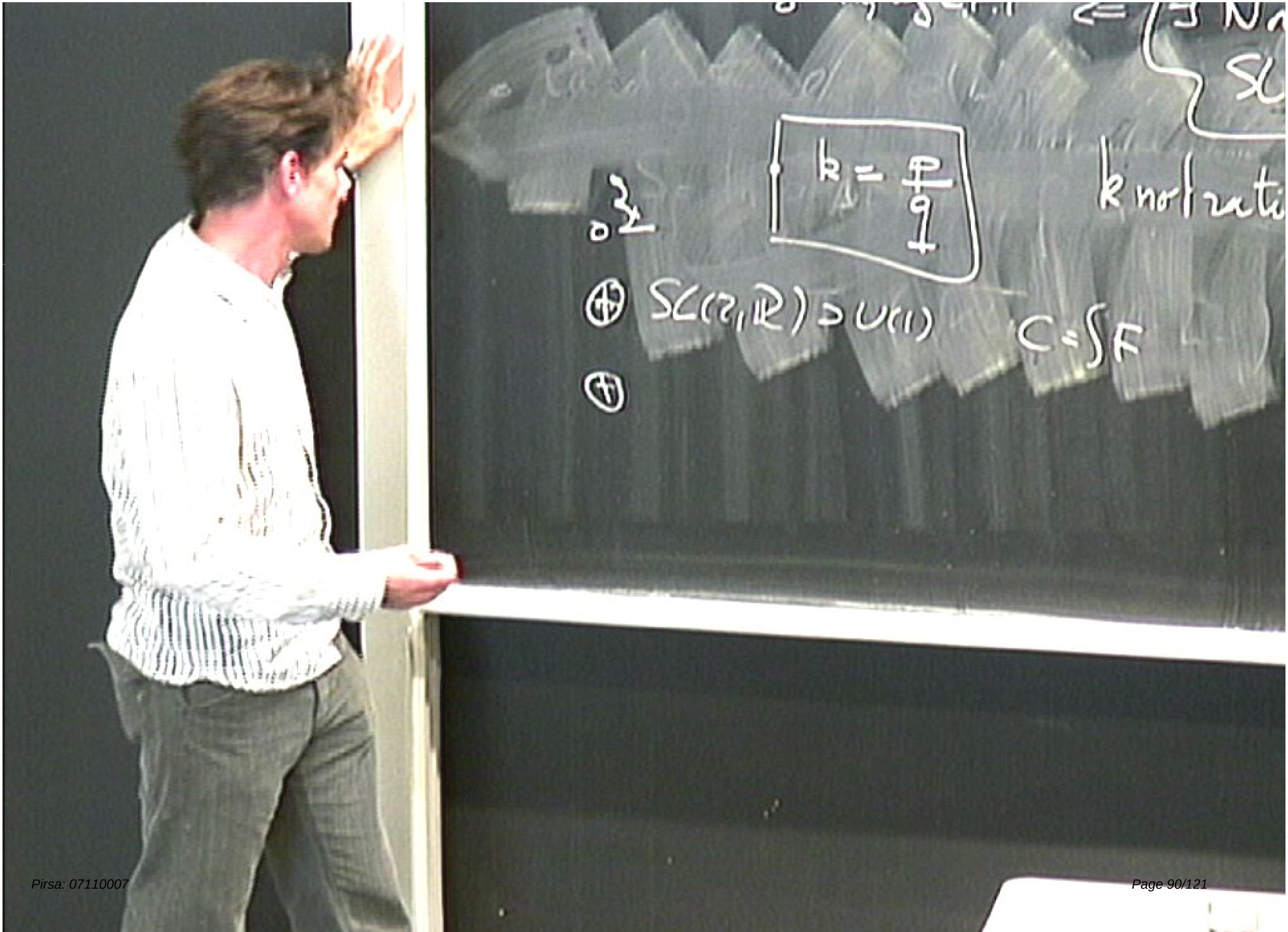
k not rational

$\Leftarrow$ $\exists$ Normalizable $SL(2,\mathbb{R})$ invariant State:

k not rational


k = p/q
k not rational
SL(2,R) ⊃ U(1)

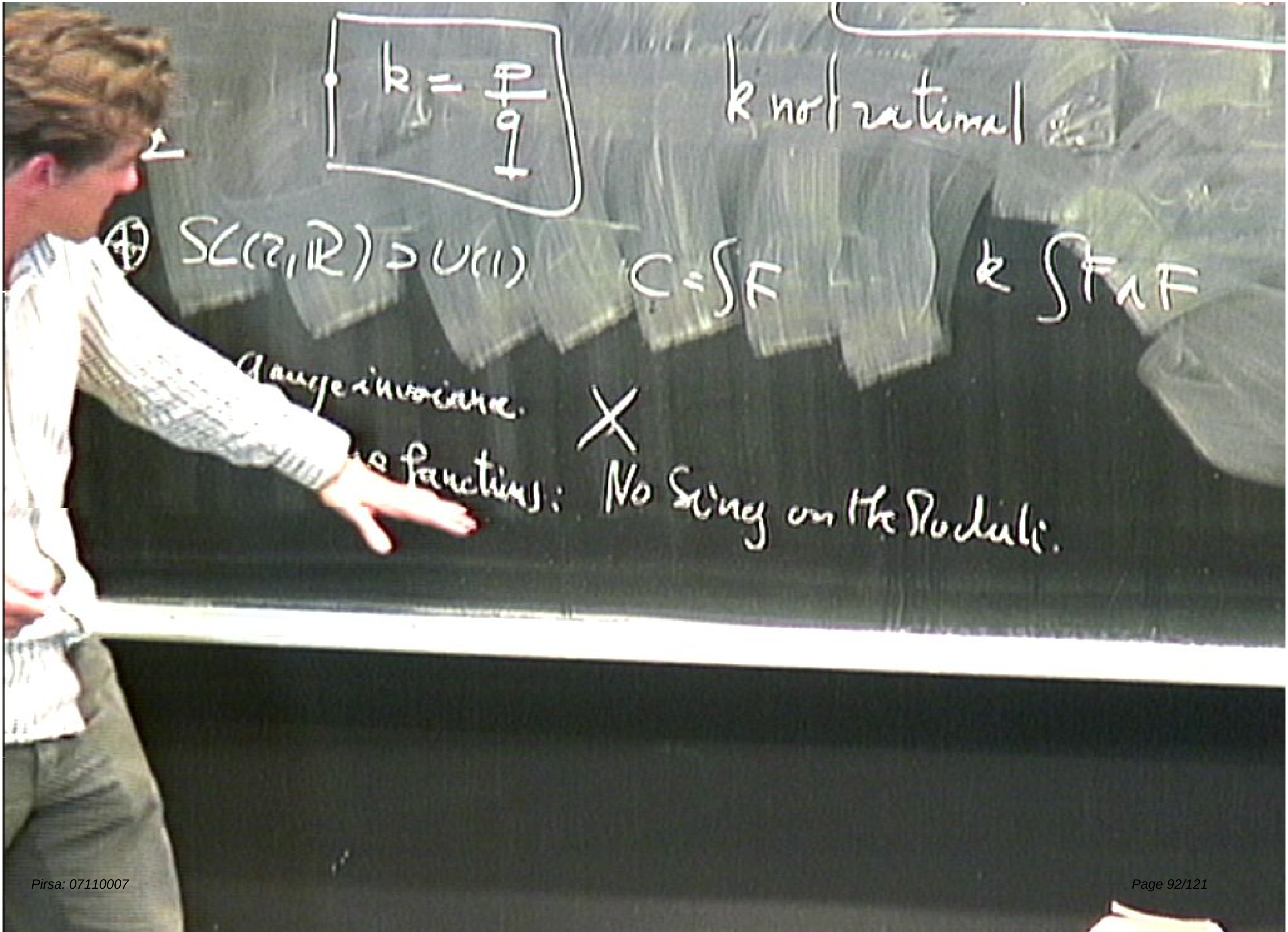

No continuous family of C.F.T ⇐ ∃ Normalisable SL(2,R) invariant State
k = p/q
k not
+ SL(2,R) ⊃ U(1)
C = ∫F

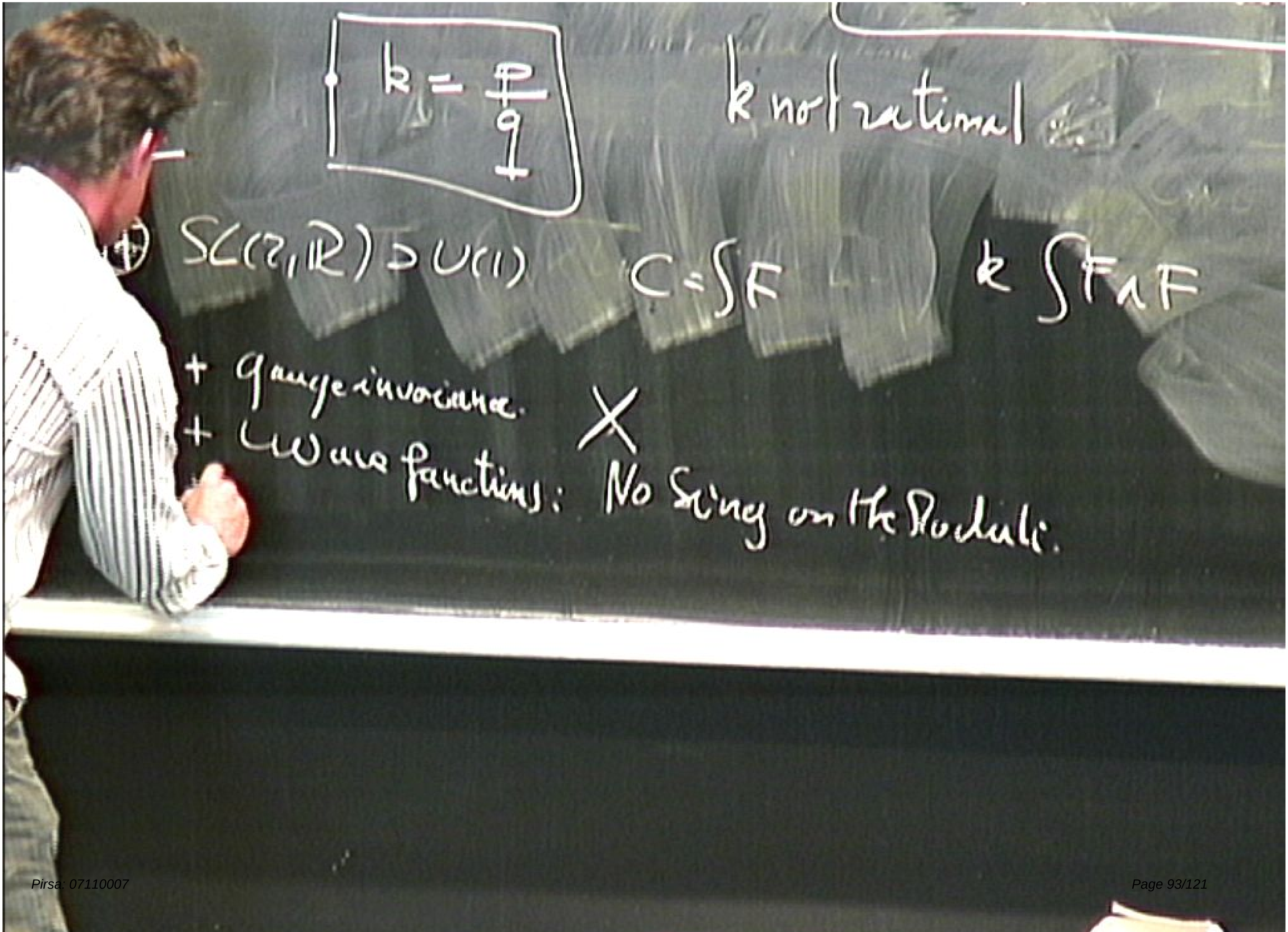

k = p/q
SL(2,R) ⊃ U(1)
C = ∫F

$k = \frac{p}{q}$ k not rational

⊕ $SL(2,\mathbb{R}) \supset U(1)$ $c = \int F$ $k \int$

⊕ + gauge invariance ×
+ L^2 wave functions:


k = p/q
k not rational
SL(2,R) ⊃ U(1)
C = ∫F
k ∫F∧F
gauge invariance.
X
functions: No Swing on the Moduli.



$\boxed{k = \frac{p}{q}}$
k not rational
$SL(2,\mathbb{R}) \supset U(1)$
$C = \int F$
$k \int F \wedge F$
+ gauge invariance.
+ Wave functions:
on the Moduli.

q

k not rational

$\oplus$ $SL(2,\mathbb{R}) \supset U(1)$ $\quad C = \int F \quad k \int F \wedge F$

$\oplus$
+ gauge invariance. $\times$
+ L^2 wave functions: No String on the Moduli.
+

⊕ $SL(2,\mathbb{R}) \supset U(1)$ $C = \int F$ k

↳ does not simply connected.

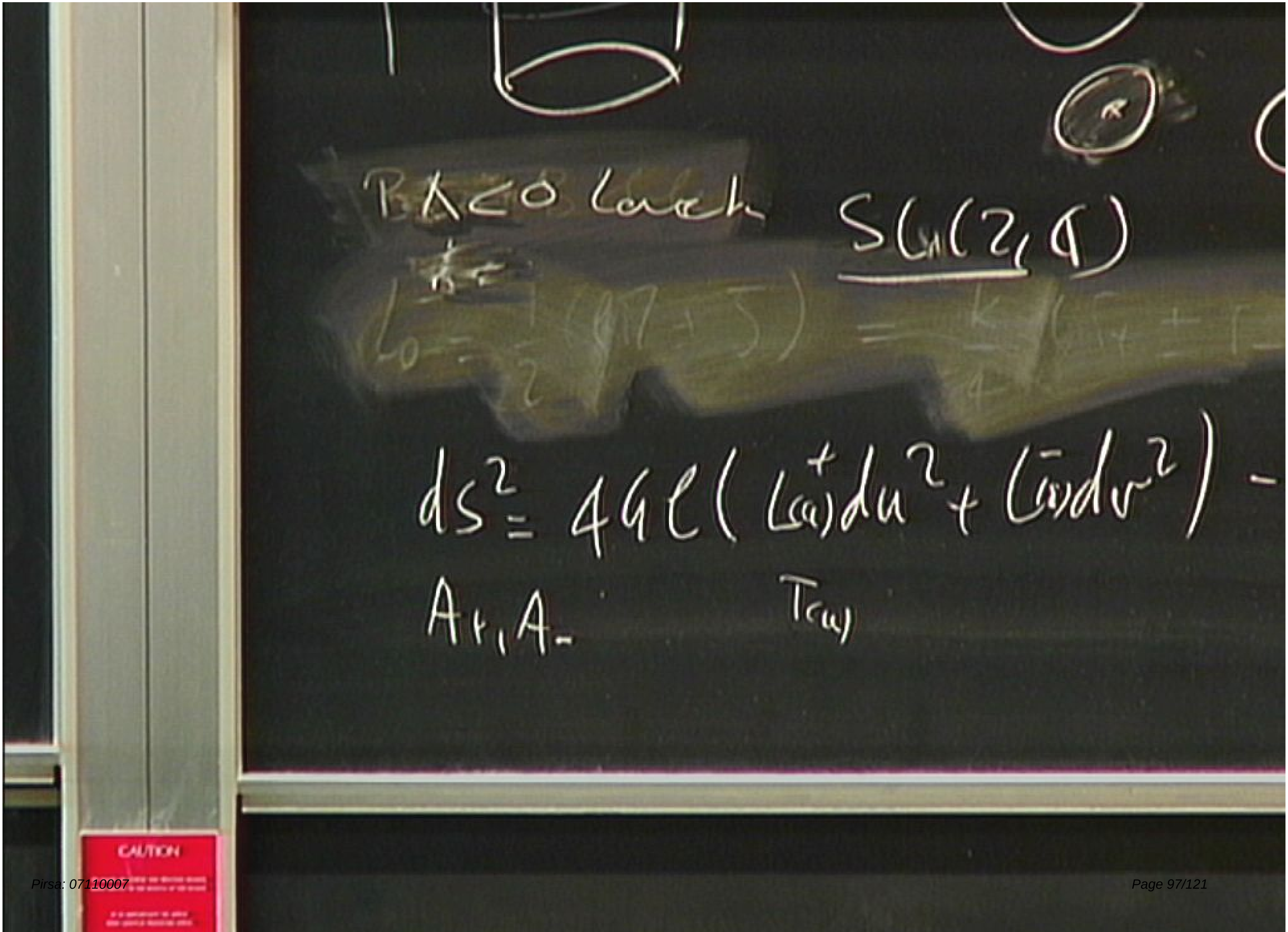
⊕ + gauge invariance. X

+ LW ave functions: No Strings on the Moduli

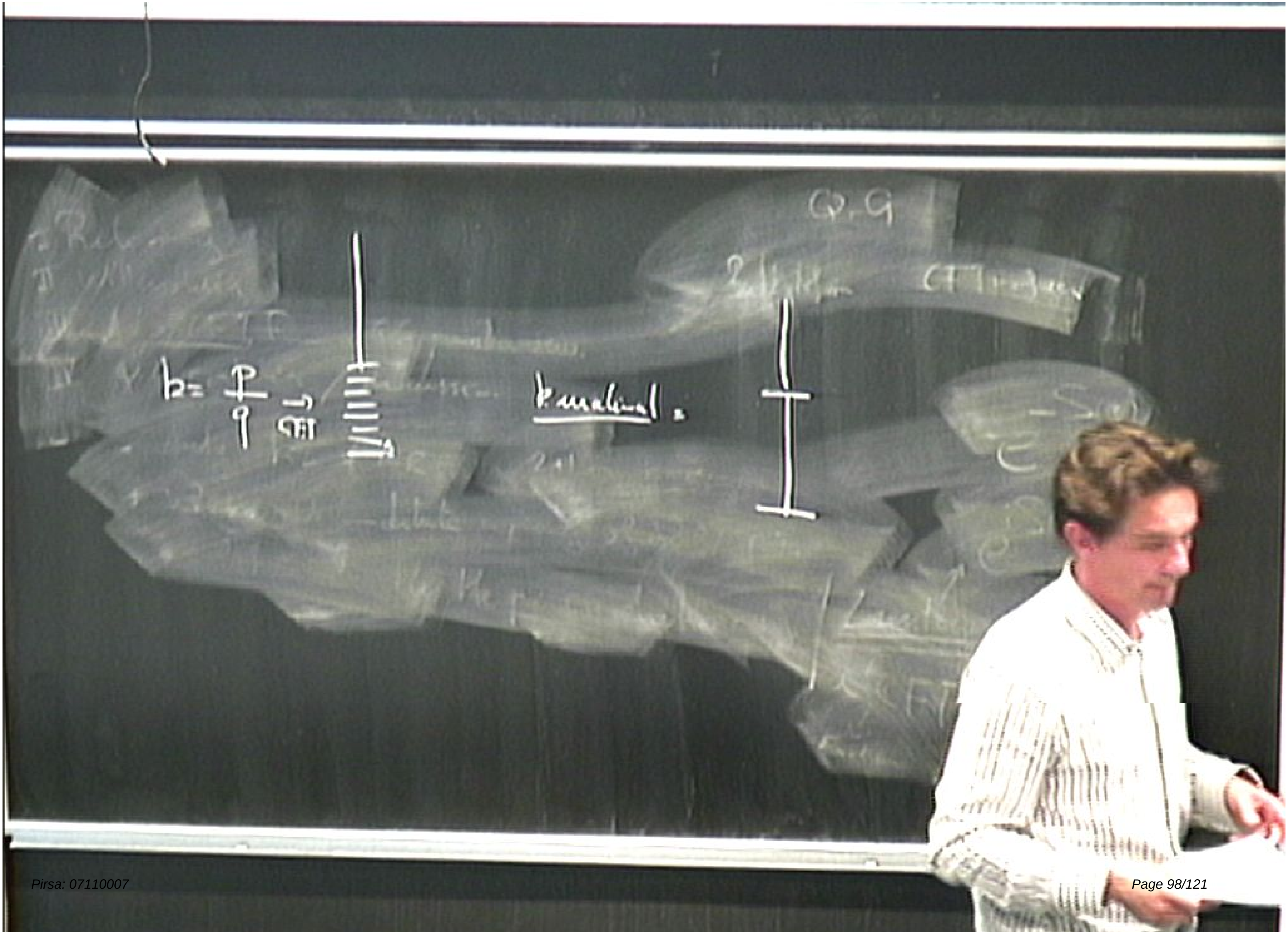
+ SL_n

⊕ $SL(2,\mathbb{R}) \supset U(1)$ $C = \int F$ &
does not simply connected.

⊕ + gauge invariance. X
+ LW wave functions: No String on the Moduli.
+ SL
$q \to \frac{p}{T}$



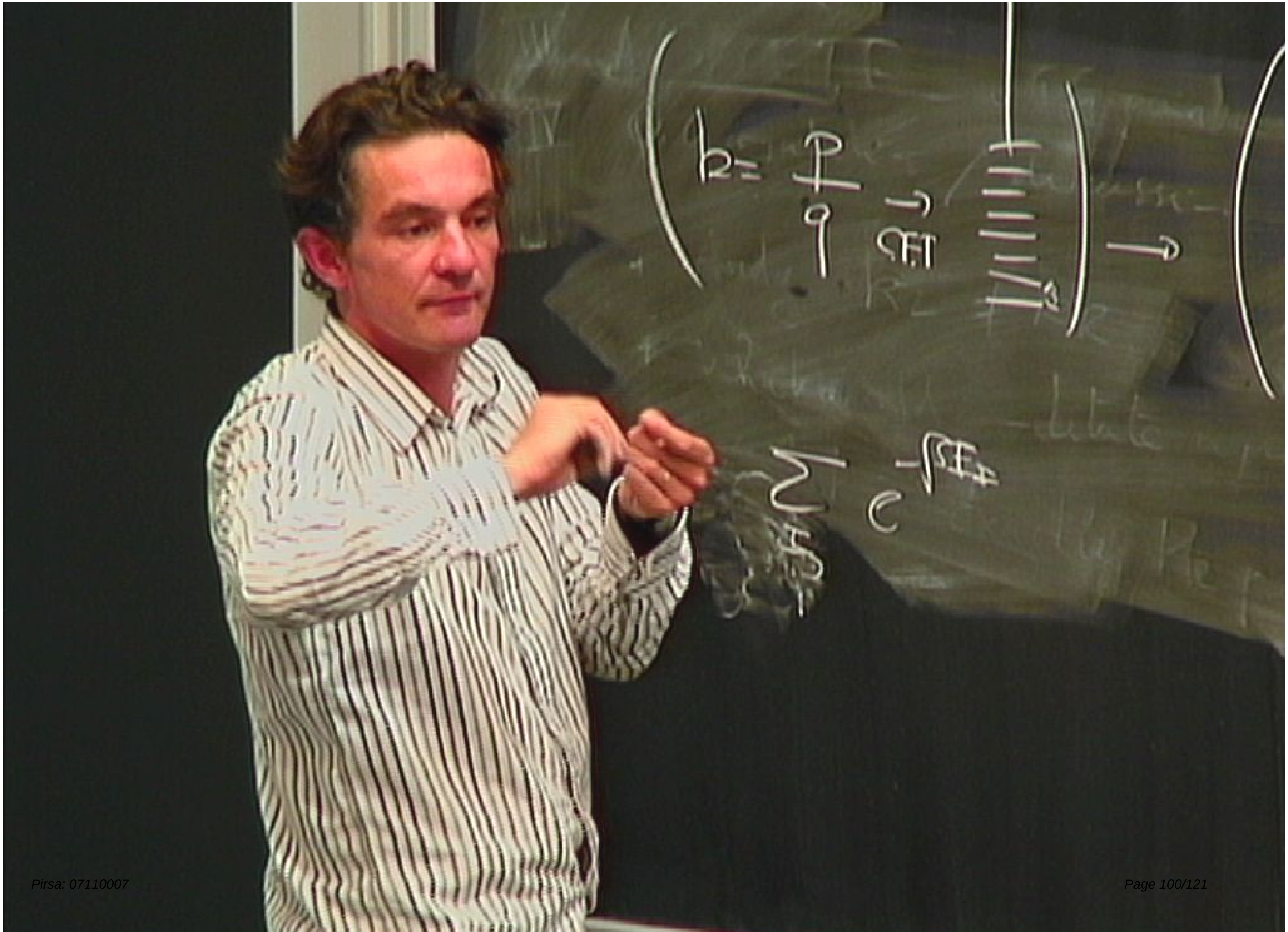
SL(2,R)
ds^2 = 4Gl(L+(u)du^2 + L-(v)dv^2) -
A+, A-
T(u)

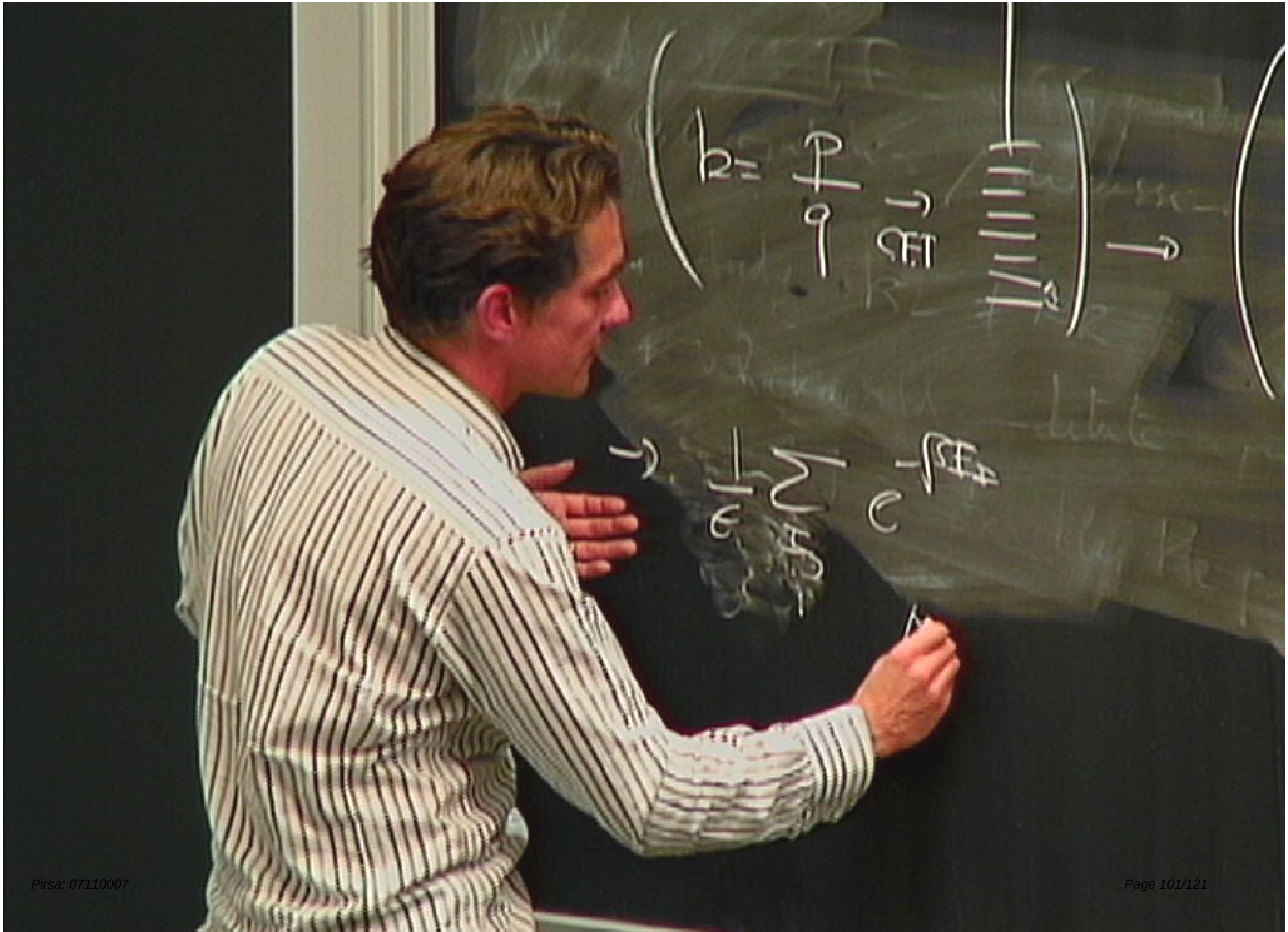


$k = \frac{p}{q} \to \frac{\vec{\ }}{q\pi}$

Kunnabinal =

No Normalisable $SL(2,\mathbb{R})$ inv State



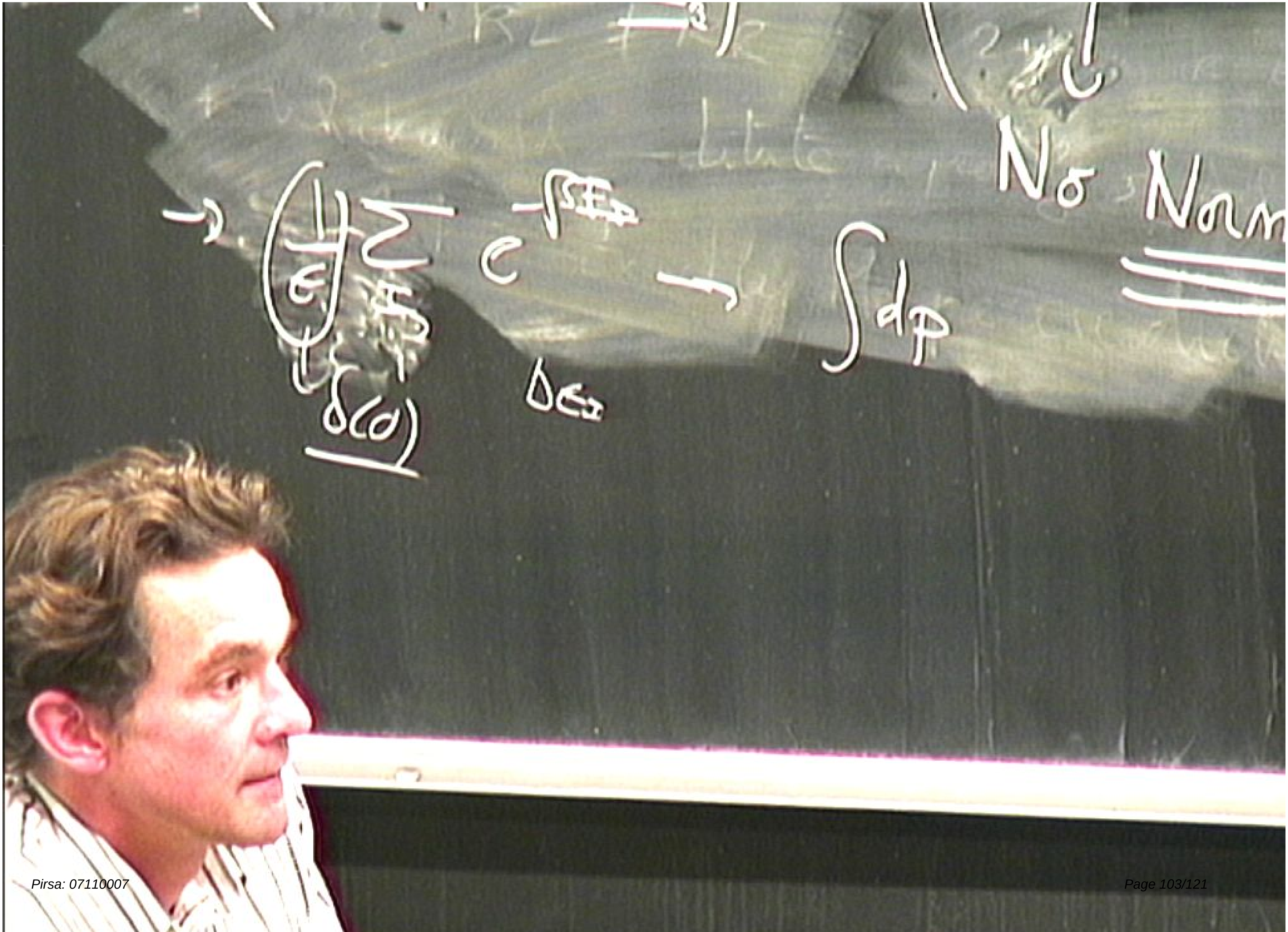


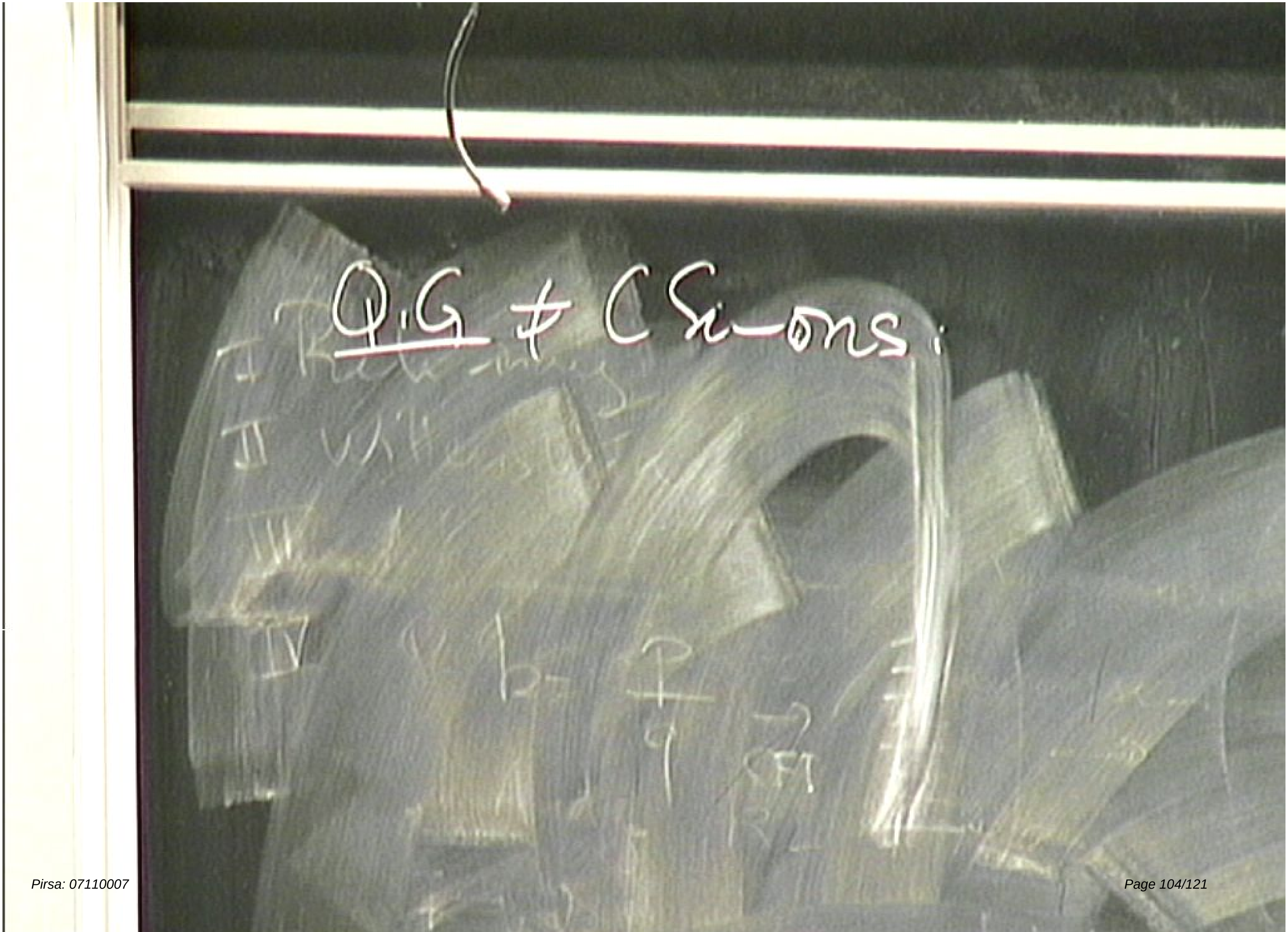
$k = \frac{p}{q} \to q_{ET}$ $\longrightarrow$ $k_{\text{analytical}} =$

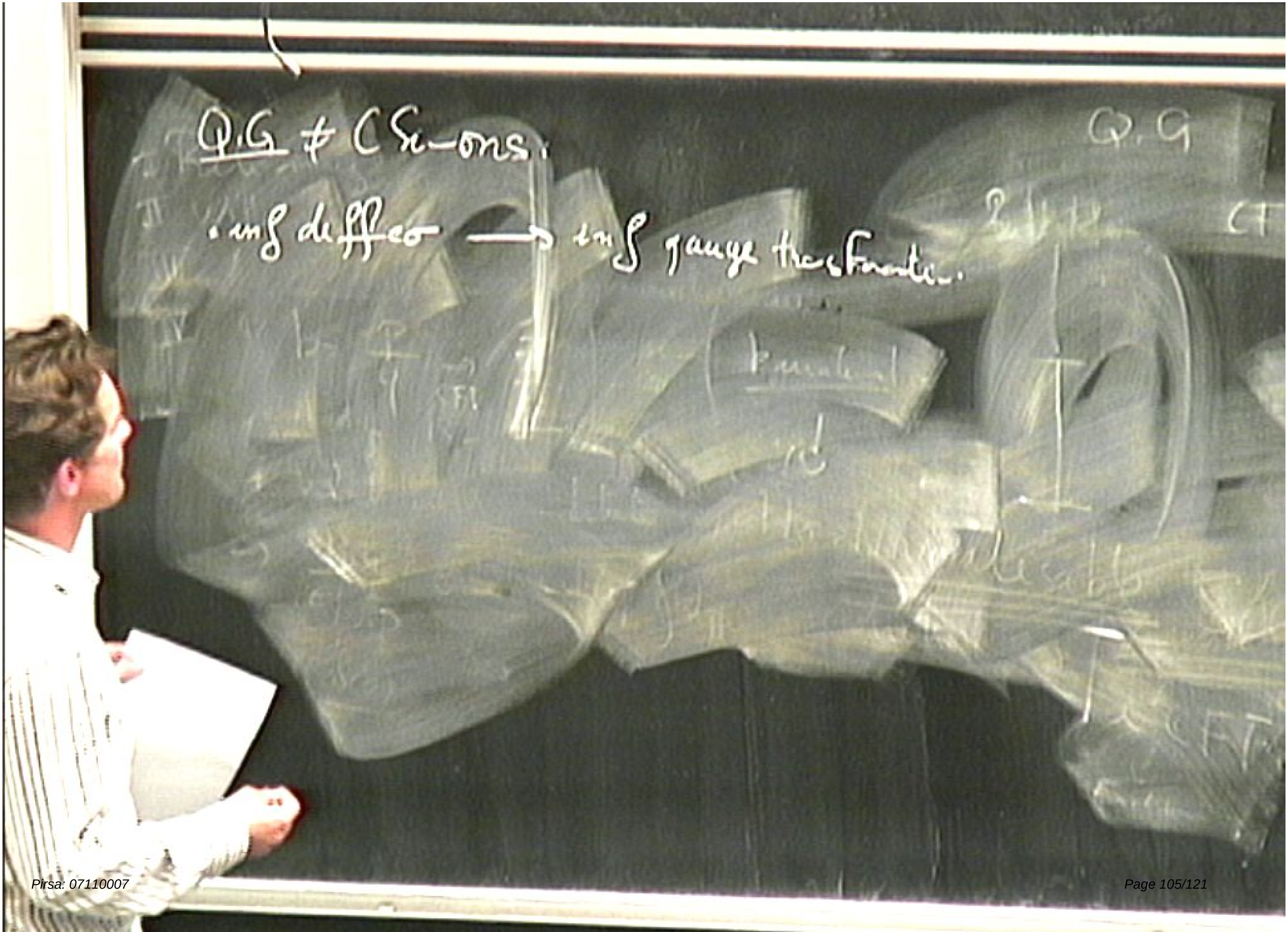
$\to \left(\frac{1}{\epsilon}\right) \sum e^{-\int S E_x} \longrightarrow \int dp$

$\delta(0)$ $\quad$ δE_x

No Norm




Q.G ≠


Q.G ≠ C Si-ons.
• inf diffeo ⟶ inf gauge transformation.
Q.G

Q.G $\neq$ C Si-ons:

· inf diffeo $\longrightarrow$ inf gauge transformation.

· non inf diffeo $\not\longrightarrow$ non trivial gauge transformation.

$$g \xrightarrow{\text{gauge transformation}} g'$$

$$g' \neq g.$$

Q.G $\neq$ C S-ons.

- inf diffeo $\longrightarrow$ inf gauge transformation.
- non inf diffeo $\neq$ non trivial gauge transformation.
- 2 distinct geometries $g \neq g'$ $\quad g \xrightarrow{\text{gauge transform}} g'$ $\quad g' \neq g$.

Q.G $\neq$ C.S–ons:

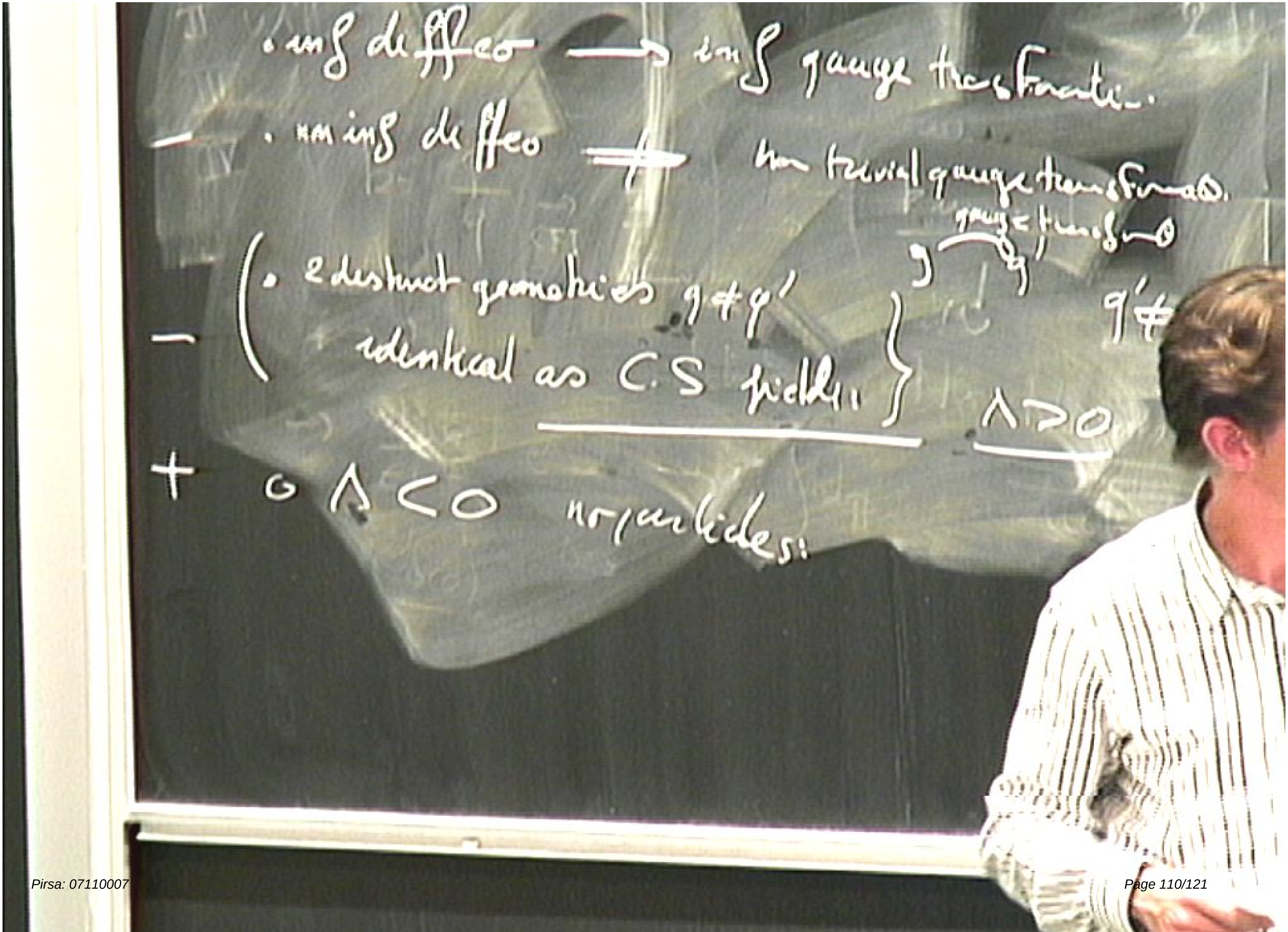
- inf diffeo $\longrightarrow$ inf gauge transformati–.
- non inf diffeo $\neq$ non trivial gauge transformation.
- 2 distinct geometries $g \neq g'$ identical as C.S fields. $\Big\}$ $\quad g \xrightarrow{\text{gauge transformation}} g''$ $\quad g'' \neq g$.

Q.G $\neq$ C S-ons:

- inf diffeo $\longrightarrow$ inf gauge transformation.
- non inf diffeo $\not\Longrightarrow$ non trivial gauge transformation.
- 2 distinct geometries $g \neq g'$ identical as C.S fields. $\Big\}$ $g \xrightarrow{\text{gauge transform}} g'$ $\quad g' \neq g$.

$\Lambda > 0$

G



• inf diffeo → inf gauge transformati..
— • non inf diffeo ⇸ non trivial gauge transformation
gauge transformation
g → g'
g' ≠
— (• 2 distinct geometries g ≠ g' identical as C.S fields }
Λ > 0
+ • Λ < 0 no particles!

− · non inf diffeo $\neq$ non trivial gauge transform.

$$- \left(\begin{array}{l} \cdot \text{ 2 distinct geometries } g \neq g' \\ \quad \text{identical as CS fields} \end{array} \right\} \; g \xrightarrow{\text{gauge transform.}} g' \quad g' \neq g.$$

$\underline{\Lambda > 0}$

+ · $\Lambda < 0$ no particles. $\Sigma_g \times I$

gauge transformations

gauge transf.

$g \to g'$ $\quad g' \neq g$

(• 2 distinct geometries $g \neq g'$
identical as C.S fields }

$\Lambda > 0$

• $\Lambda < 0$ no particles:

$$\Sigma_g \times I \quad \begin{matrix} \nearrow \pi_1\left(\Sigma_g \to SL(2,\mathbb{R})\right) / SL(2,\mathbb{R}) = \bigcup_{|n| \leq 2g-2} C_n \quad n = \text{Chern class (n||)} \\ \searrow \text{Teich} / (S(\Sigma_g)) \end{matrix}$$

$\lambda > 0$

+ o $\Lambda < 0$ no particles:

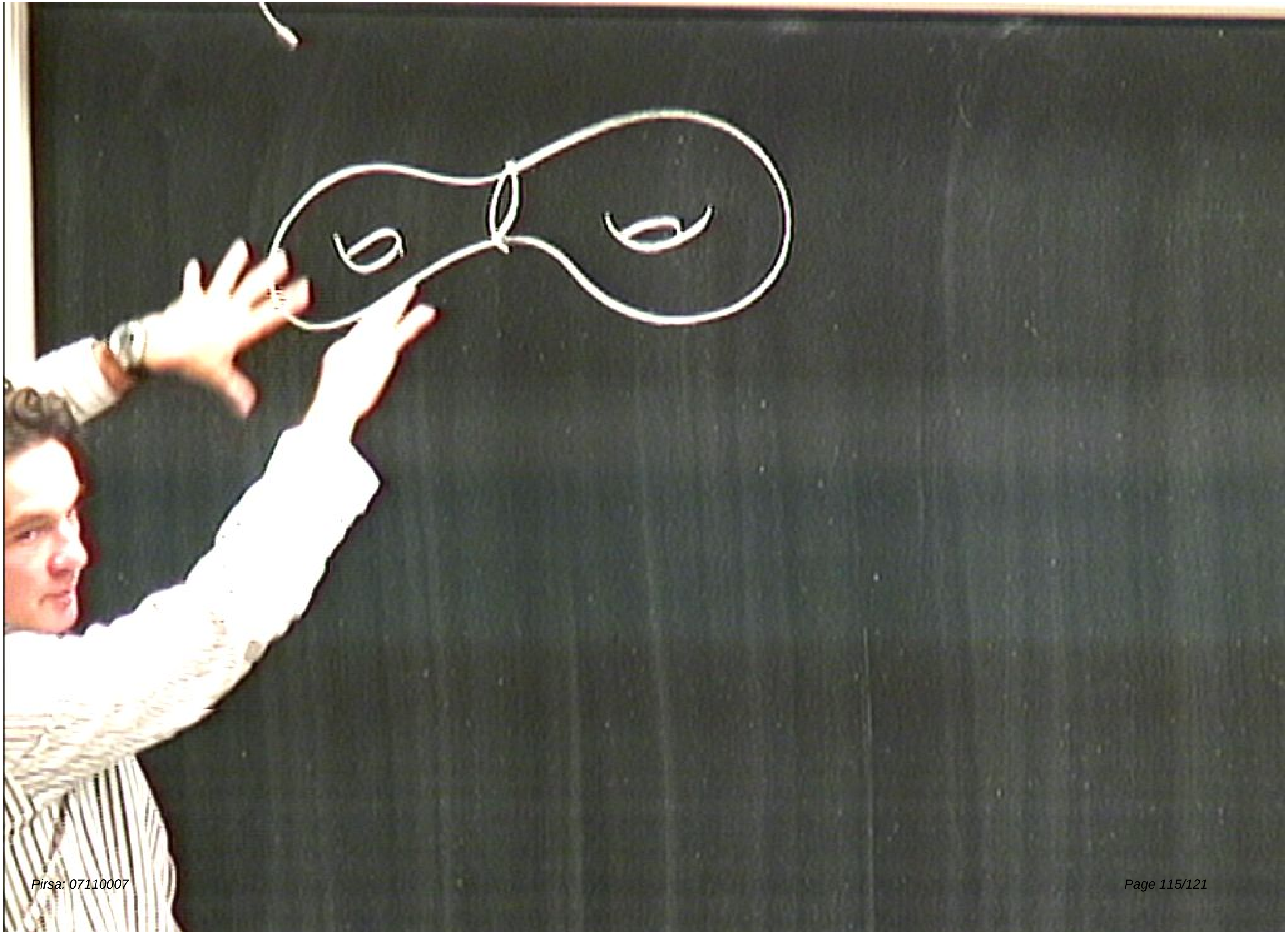
$\Sigma_g \times I$

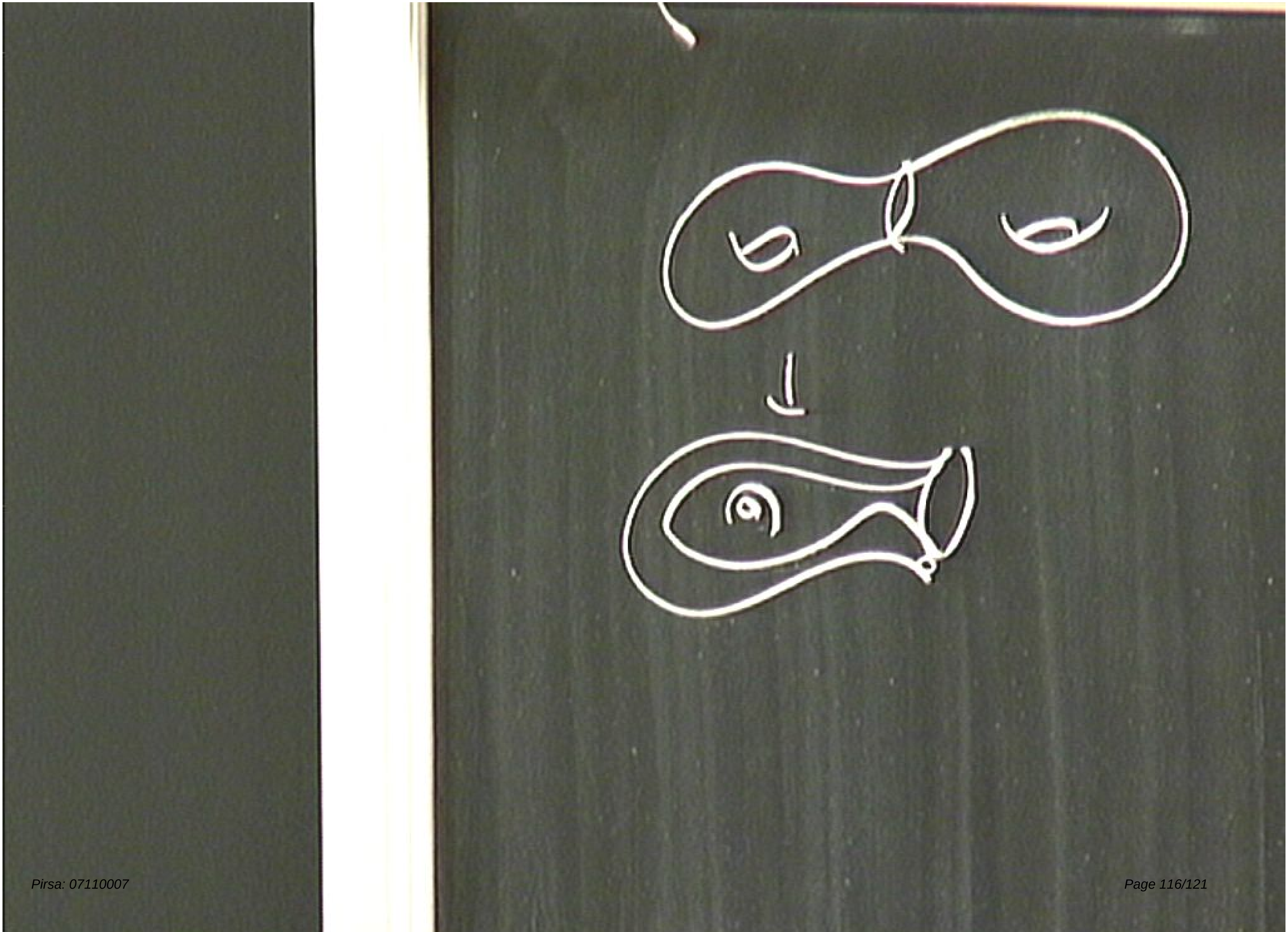
$\lambda > 0$

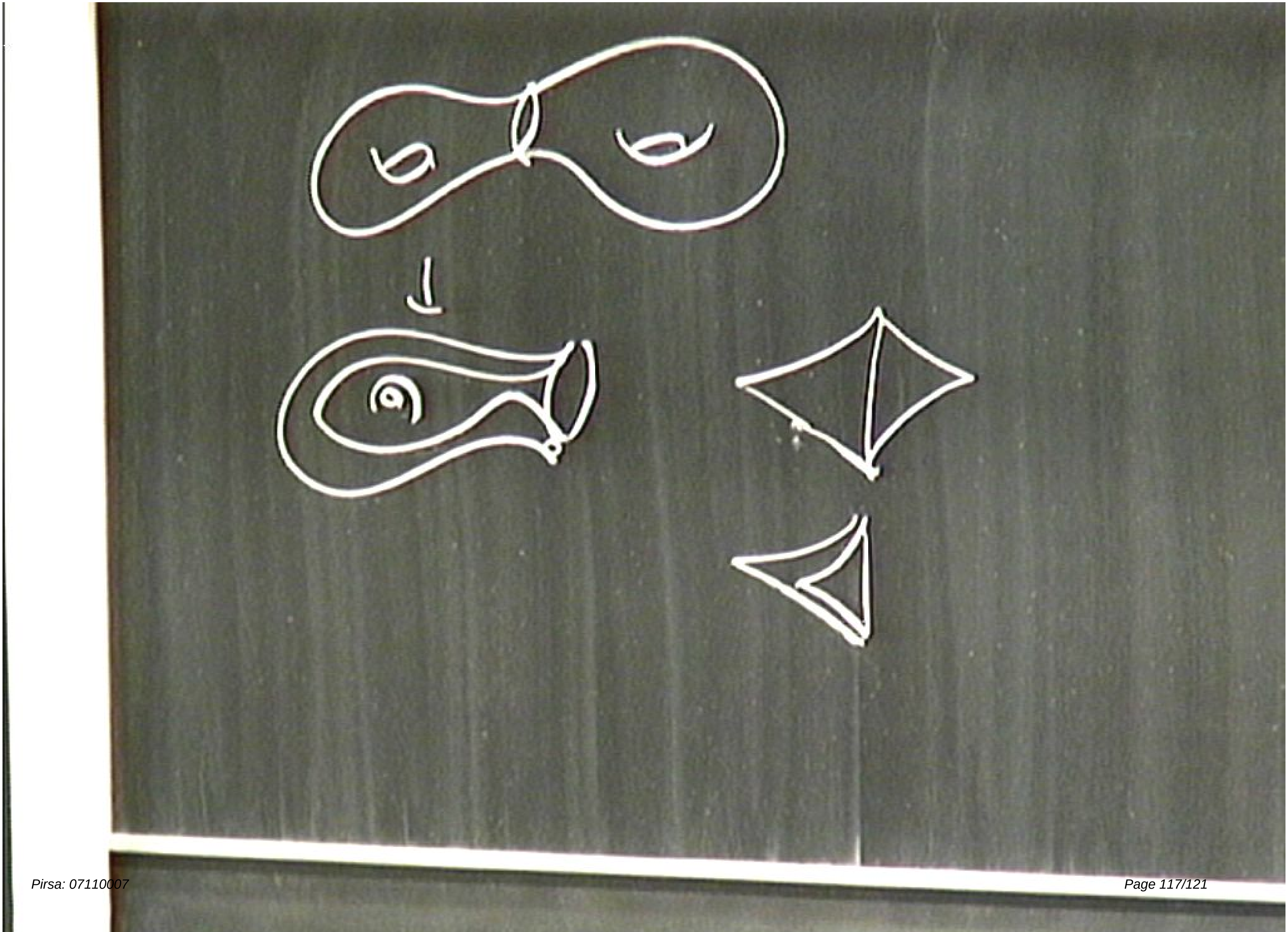
$$\Sigma_g \times I$$

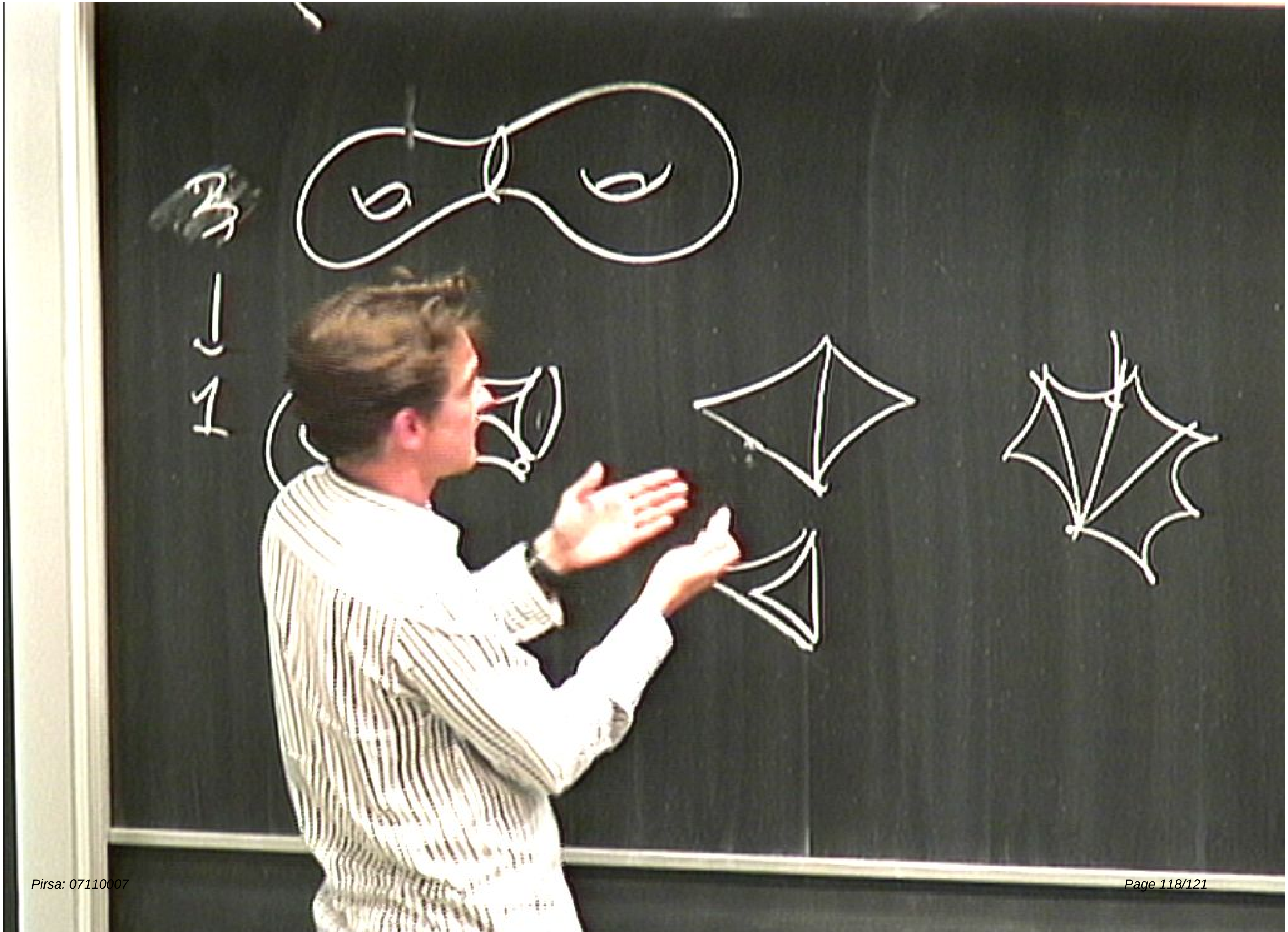
$$\pi_1\left(\Sigma_g \to SL(2,\mathbb{R})\right) / SL(2,\mathbb{R}) = \bigcup_{n=-2g-2} C_n \qquad n = \text{Chern Class}\,(U(1))$$

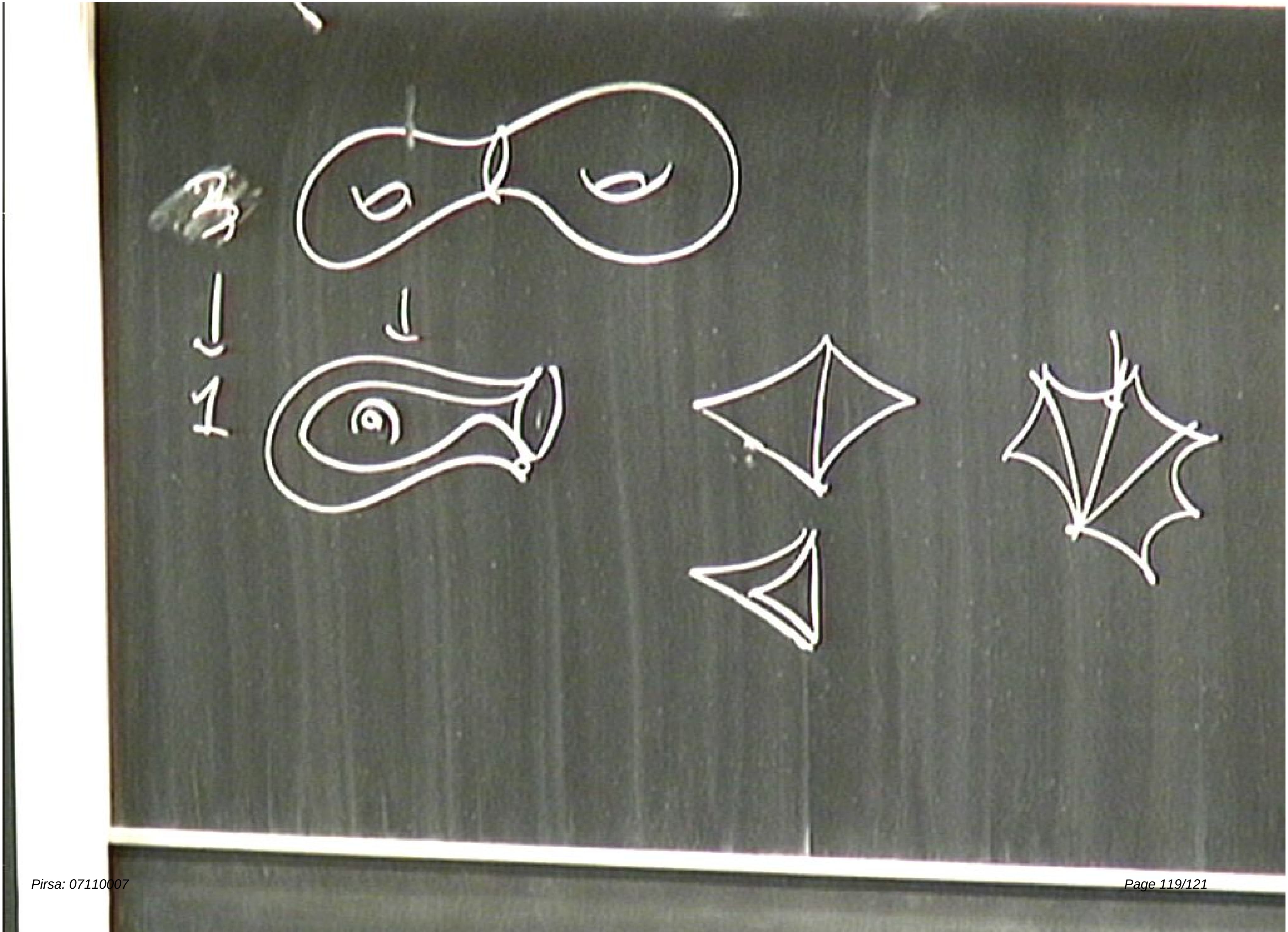
$$\text{Teich} / (SL(2,\mathbb{R}))$$







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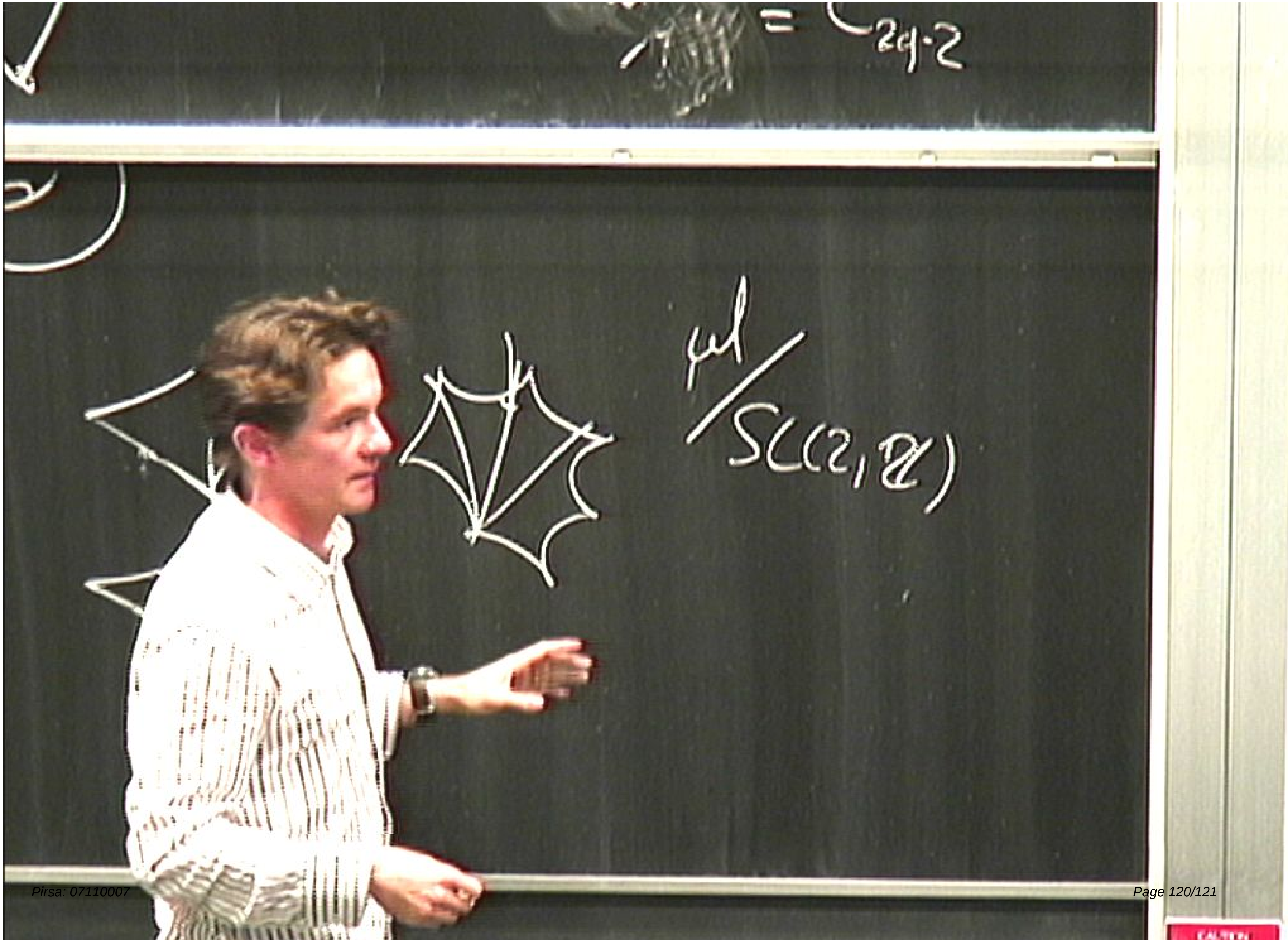
$= C_{2g-2}$
$SL(2,\mathbb{Z})$