Title: Topological Quantum Computation: anyons, quantum symmetries and topological order

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Abstract: This will be an introductory talk about Topological Quantum Computation. TQC is attractive because it is intrinsicaly decoherence free. We introduce the basic notions, such as non abelian anyons, quantum symmetries and topological order. A topologically ordered phase is a gapped phase in which the basic degrees of freedom are of a topological nature (denoted as anyons), charactetized by their fusion and braiding properties. If time permits possible implementations based on Quantum Hall systems will be discussed as well.


Topological order, anyons and
quantum computation
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Outline

- Introduction
- Anyons and Topological order
- Topological Quantum Computation
- Implementations TQC
- Topological symmetry breaking
- Conclusions

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The best of two worlds?

1980 Topological phases in gauge theories (FAB)

1982 Anyons (Wilczek)

1983 Conformal Field Theory (BPZ)

1983 Fractional Quantum Hall (Laughlin)
Wave functions

1989 Topological Field Theory (Witten)
Quantum groups WZW-CFT

1991 Chiral Conformal Blocks (Moore and Read)

1992 Anyons in DGT
(FAB, v Driel, De W. Propitius)

1997 (2003) Topological Quantum Computing (Kitaev)

Topological Quantum Computation

- Use non-abelian anyons to store quantum information topologically
  (information corresponds to global characteristics of the state).

- Great advantage: insensitive to local perturbations
  (decoherence exponentially suppressed, scaling properties)

- Need system with topological order <=> Topological field theories
  (I.e. gapped phase with only topological degrees of freedom)

  - Discrete gauge theories
  - Quantum Hall systems
  - Quantum liquid crystals
  - Spin nematics
  - Josephson arrays

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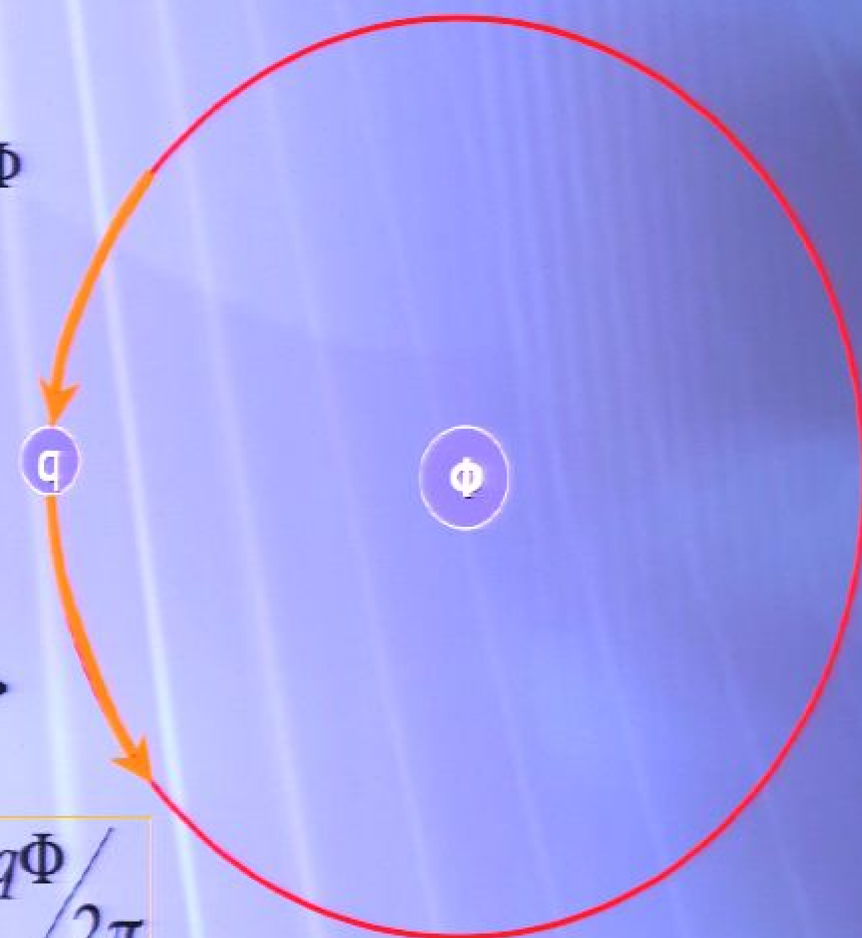
Aharonov Bohm 1959

Charge-flux composites: anyons

Phase change caused by Adiabatic transport of charge along a closed path around flux:

$$W = e^{iq\oint A dx} = e^{iq\Phi}$$

$$|\Phi, q> \Rightarrow$$



$$e^{iq\Phi}|\Phi, q> = e^{2\pi i s}|\Phi, q>$$

Composite has "spin"→ $s = {q\Phi}/{2\pi}$

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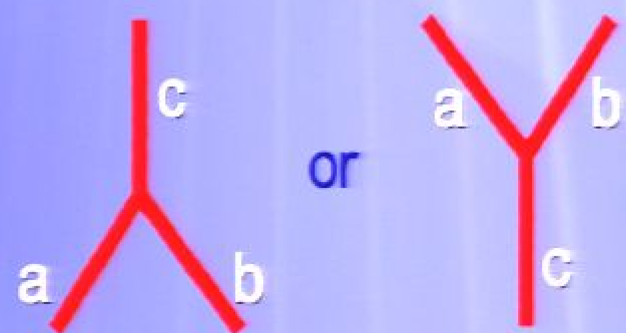
Fusion: composition rules

Basic fusion rules:



$$\Pi_a \otimes \Pi_b = N_{ab}^{c} \Pi_c$$

Anti-particles (conjugate reps):

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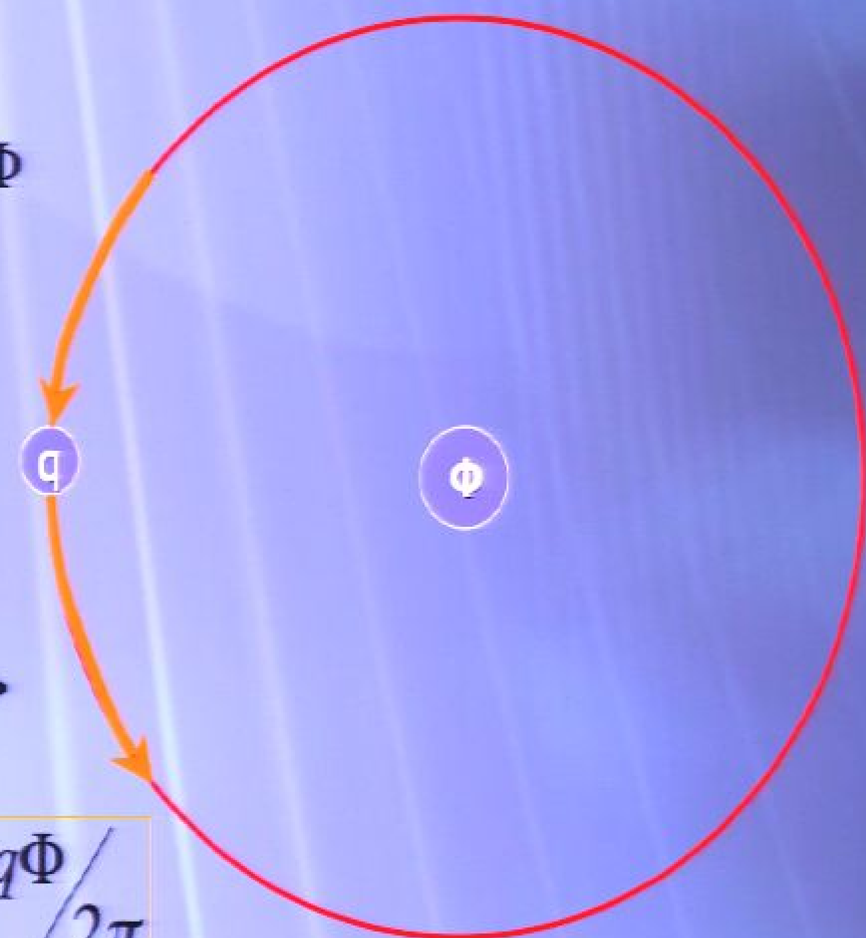

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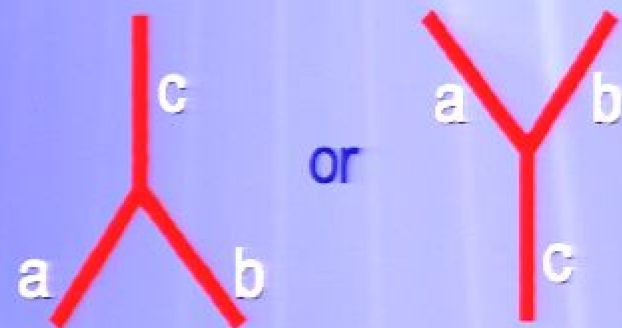

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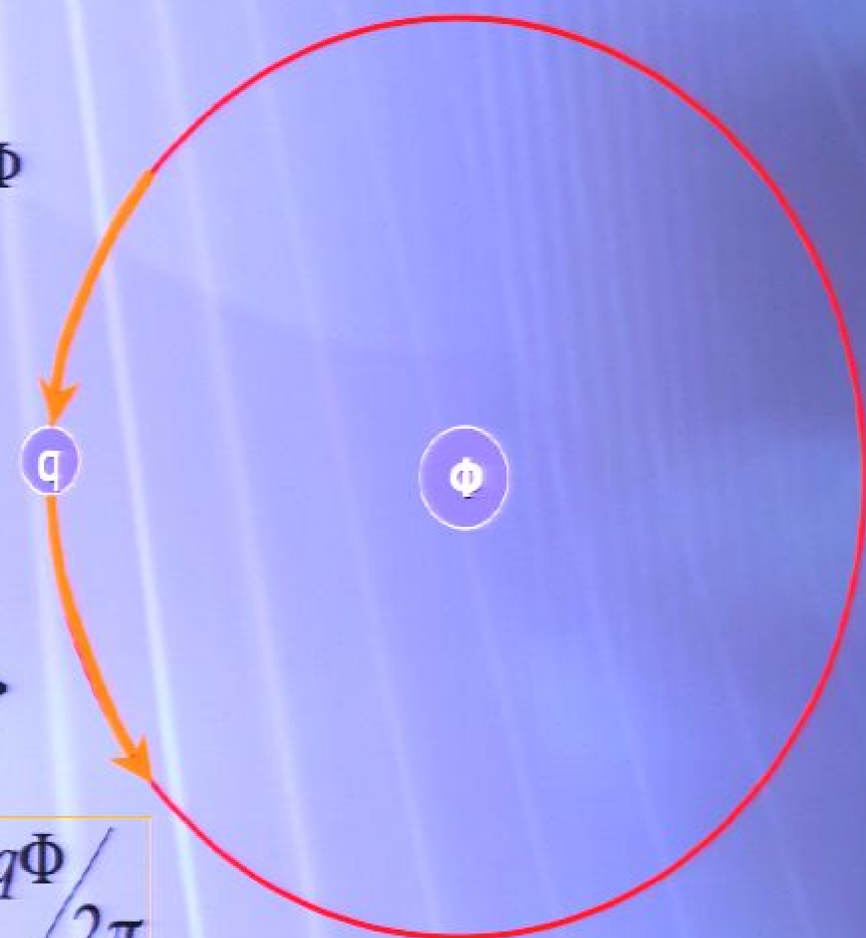

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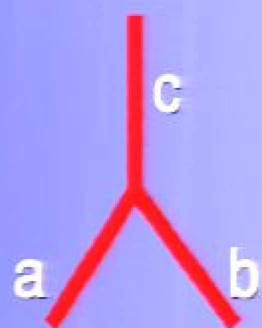
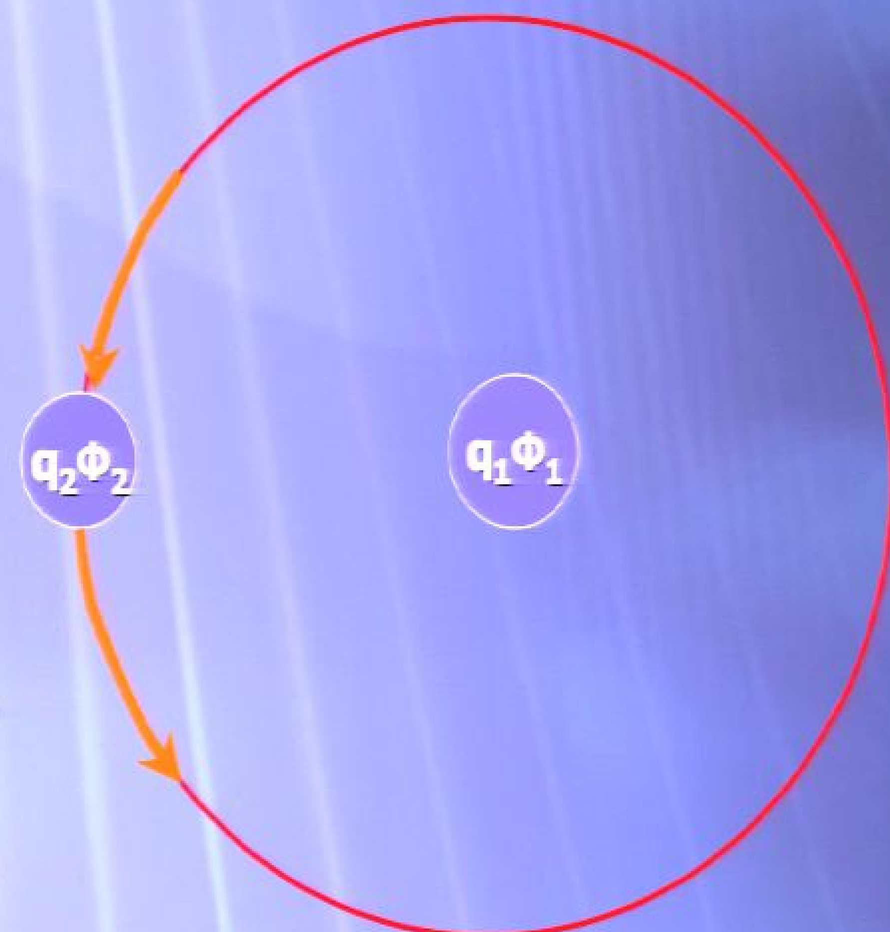


or

$$\Pi_a \otimes \Pi_b = N^c_{ab}\Pi_c$$

Anti-particles (conjugate reps):

Holonomy of two anyons



$$e^{i(q_1\Phi_2+q_2\Phi_1)} = \cdots$$

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Quantum statistics:
Exchange of indistinguishable particles

Leinaas and Myrheim, 1977
Wilczek 1982

$$q_1 = q_2 = q$$

$$\Phi_1 = \Phi_2 = \Phi$$

$$e^{2\pi i S} = e^{iq\Phi} = e^{i\theta}$$

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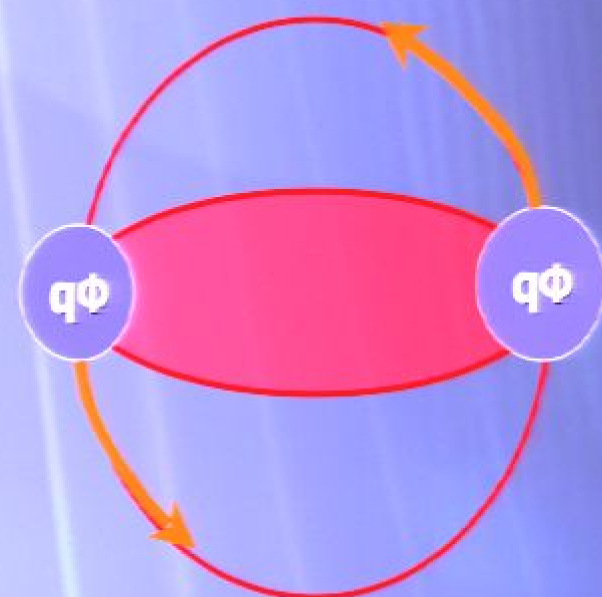
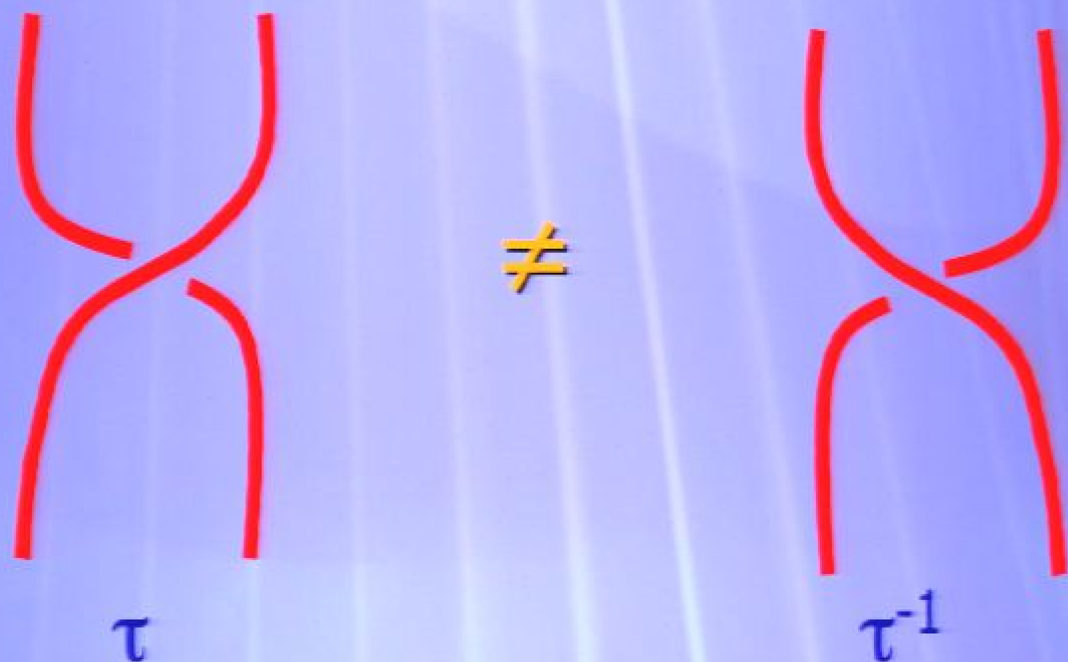
D=3 versus D=2

D= 3: $\tau = \tau^{-1}$



D= 2: $\tau \neq \tau^{-1}$

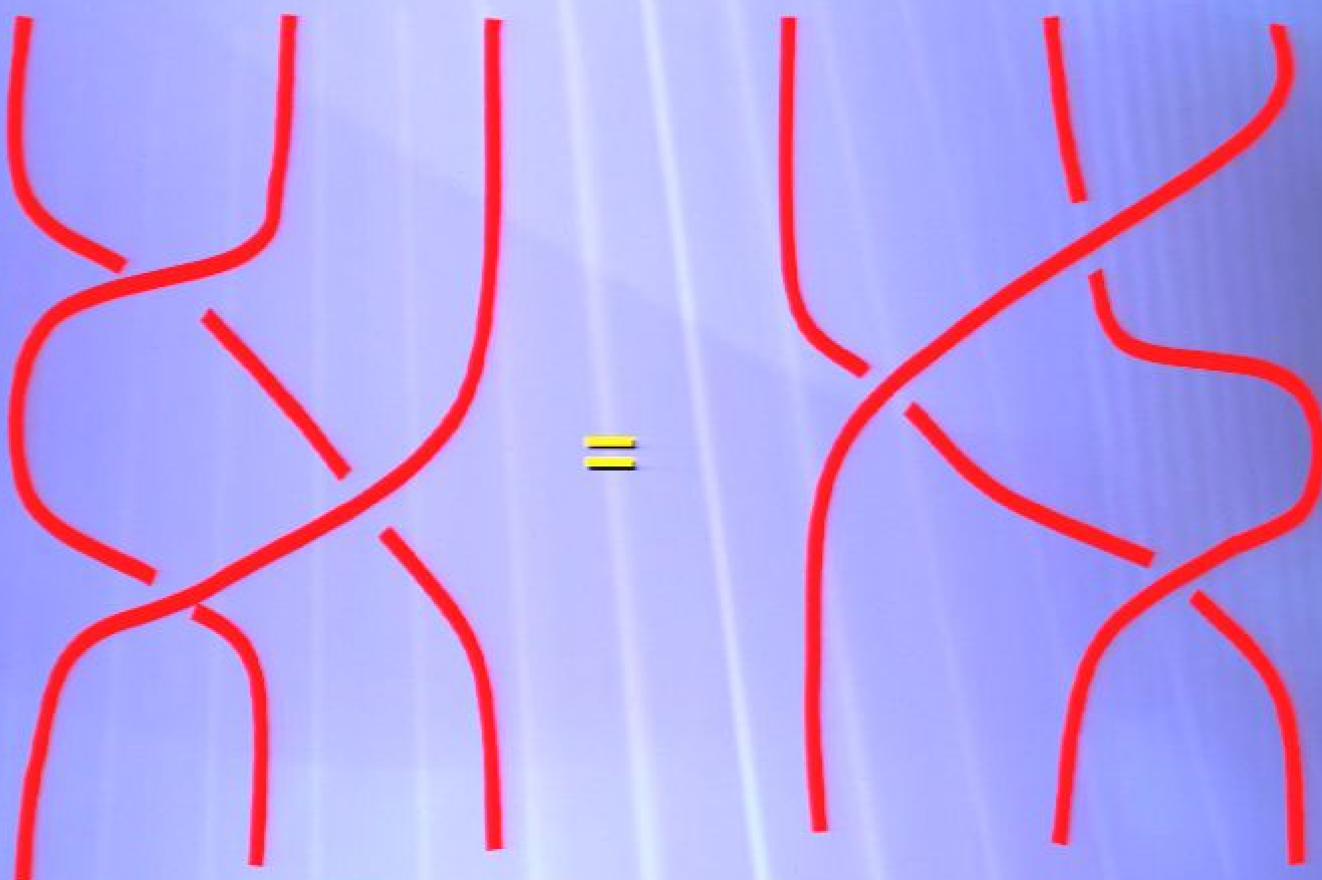
Braiding of world lines --> braid group



Braid group generators

τ_1 τ_2

Relation between generators (Yang Baxter)



$$\tau_1 \tau_2 \tau_1 = \tau_2 \tau_1 \tau_2$$

Defining relations between generators

$D \geq 3$ Permutation group S_n:

$\tau = \tau^{-1} \rightarrow \tau^2 = 1 \rightarrow \tau = \pm 1$ $+1 \rightarrow$ Bosons
$-1 \rightarrow$ Fermions

$D = 2$ Braid group B_n:

$\tau_1 \tau_2 \tau_1 = \tau_2 \tau_1 \tau_2 \rightarrow \tau_j = e^{i\theta}$ Anyons

$(\tau \neq \tau^{-1})$

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$(\tau \neq \tau^{-1})$ $\rightarrow$ Matrices Non-abelian anyons

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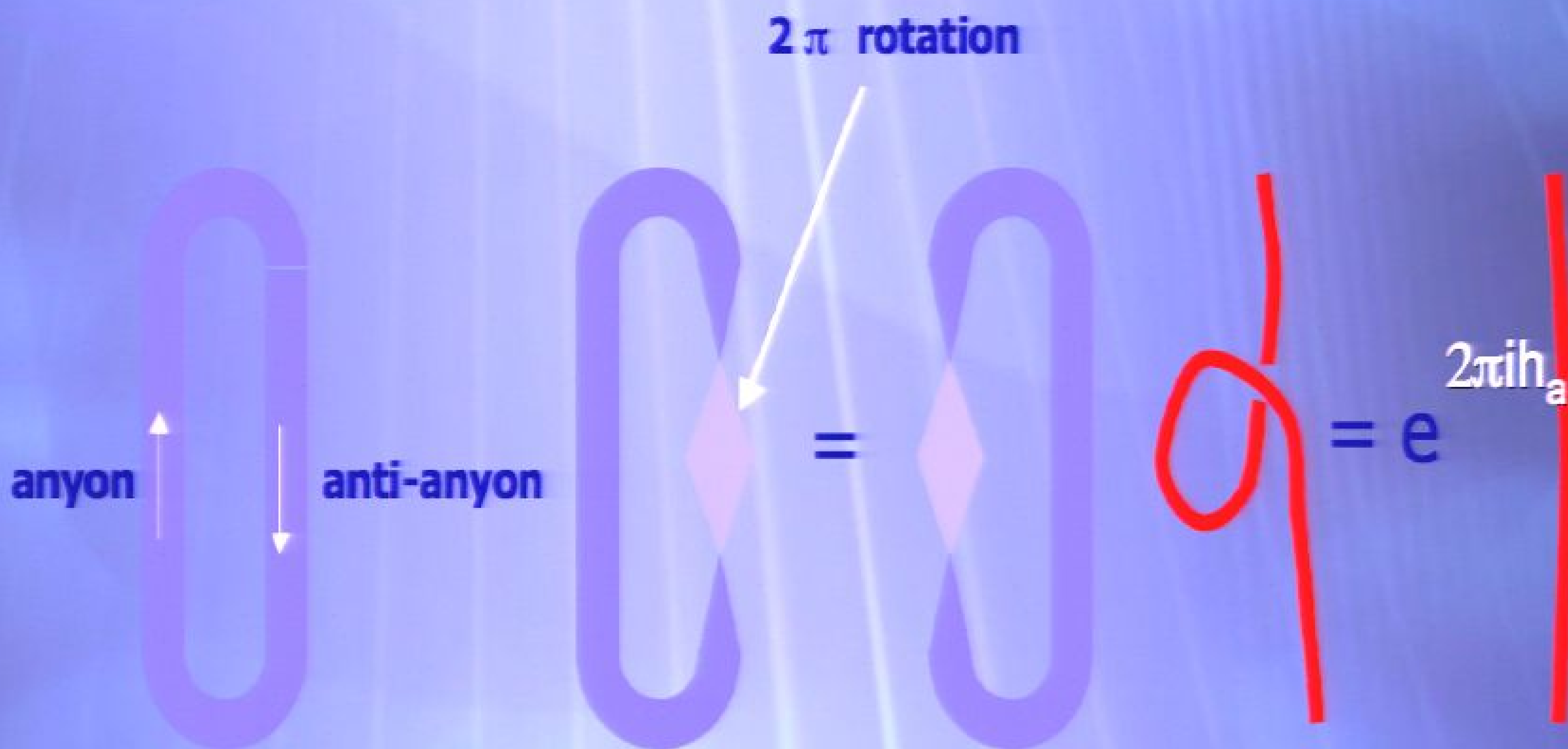
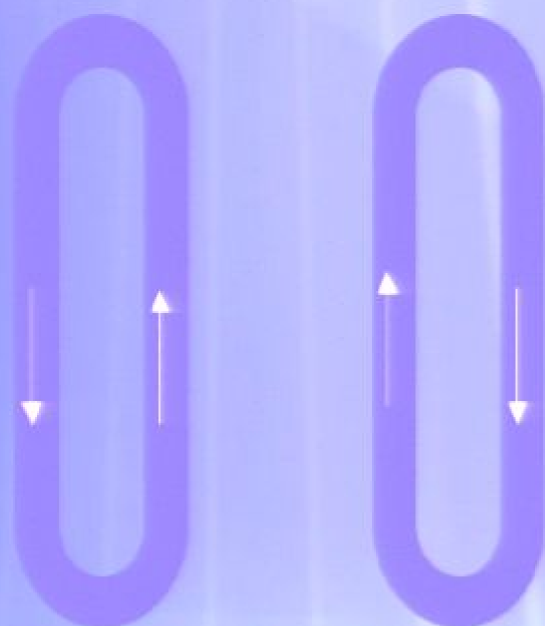
Ribbon algebras

Ribbon diagrams:
spinfactors, topological twist



Two anyon—anti-anyon pairs



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Anyon interchange

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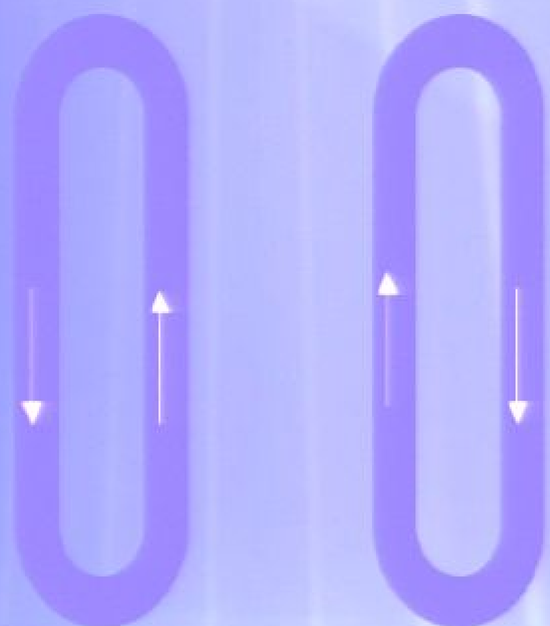
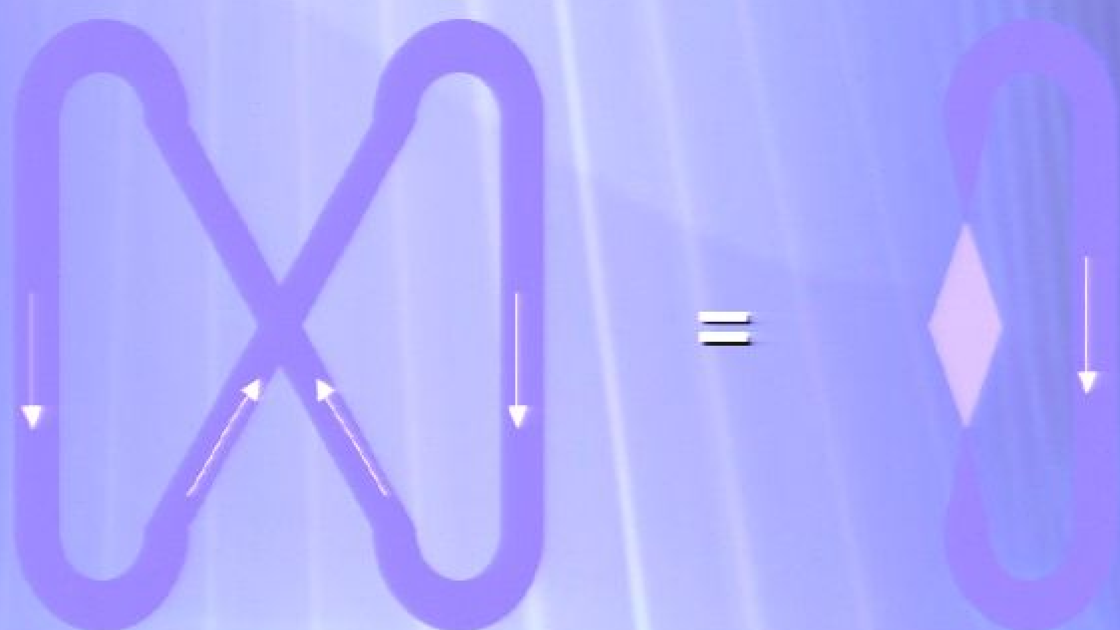
Two anyon—anti-anyon pairs

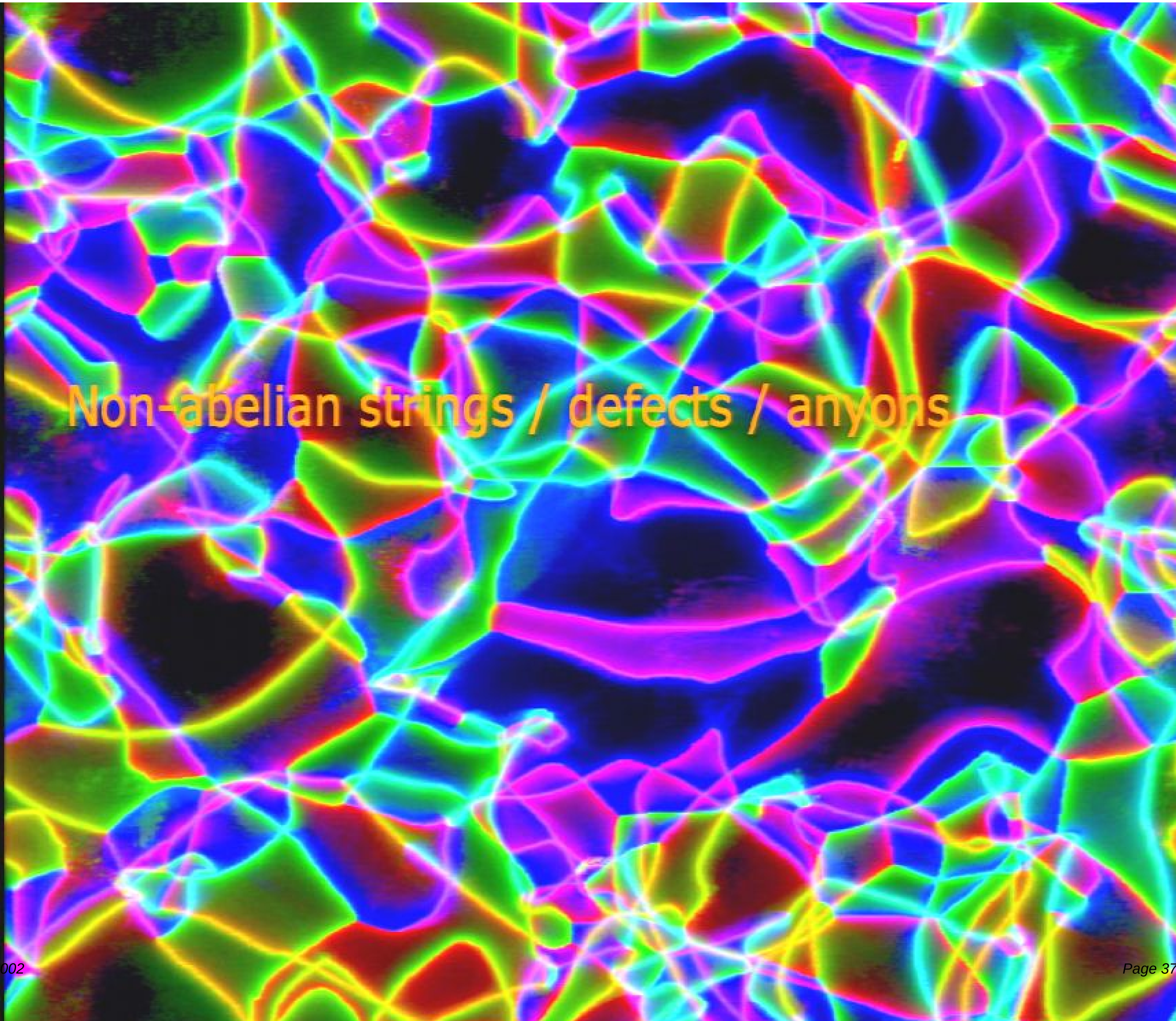


Anyon interchange

Spin-statistics connection
as a topological equivalence



Effect of interchange is equivalent to 2π rotation


Non-abelian strings / defects / anyons

Bais 1980

Non-abelian flux in discrete gauge theories

Setting: continuous group G breaks to discrete non-abelian group H

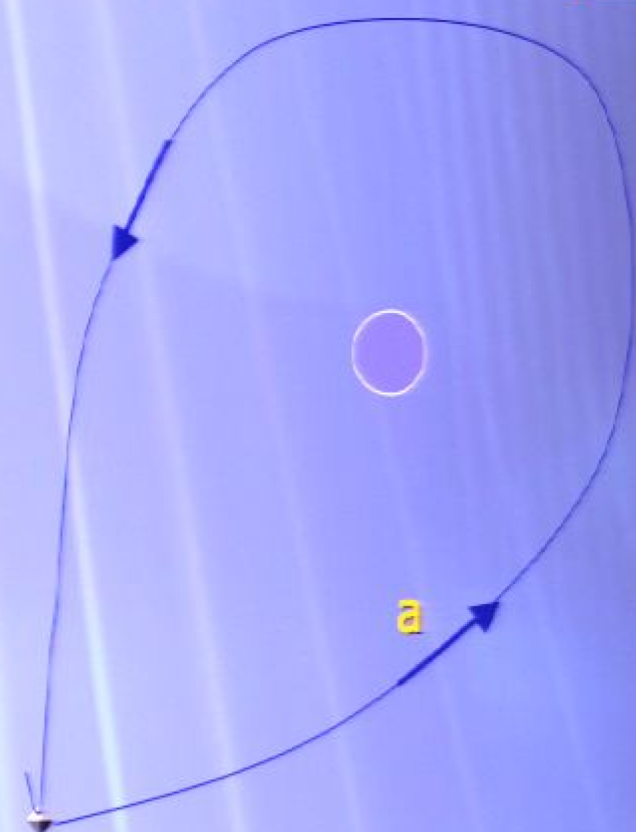
ab ≠ ba

$$W = Pe^{iq\oint A dl}$$

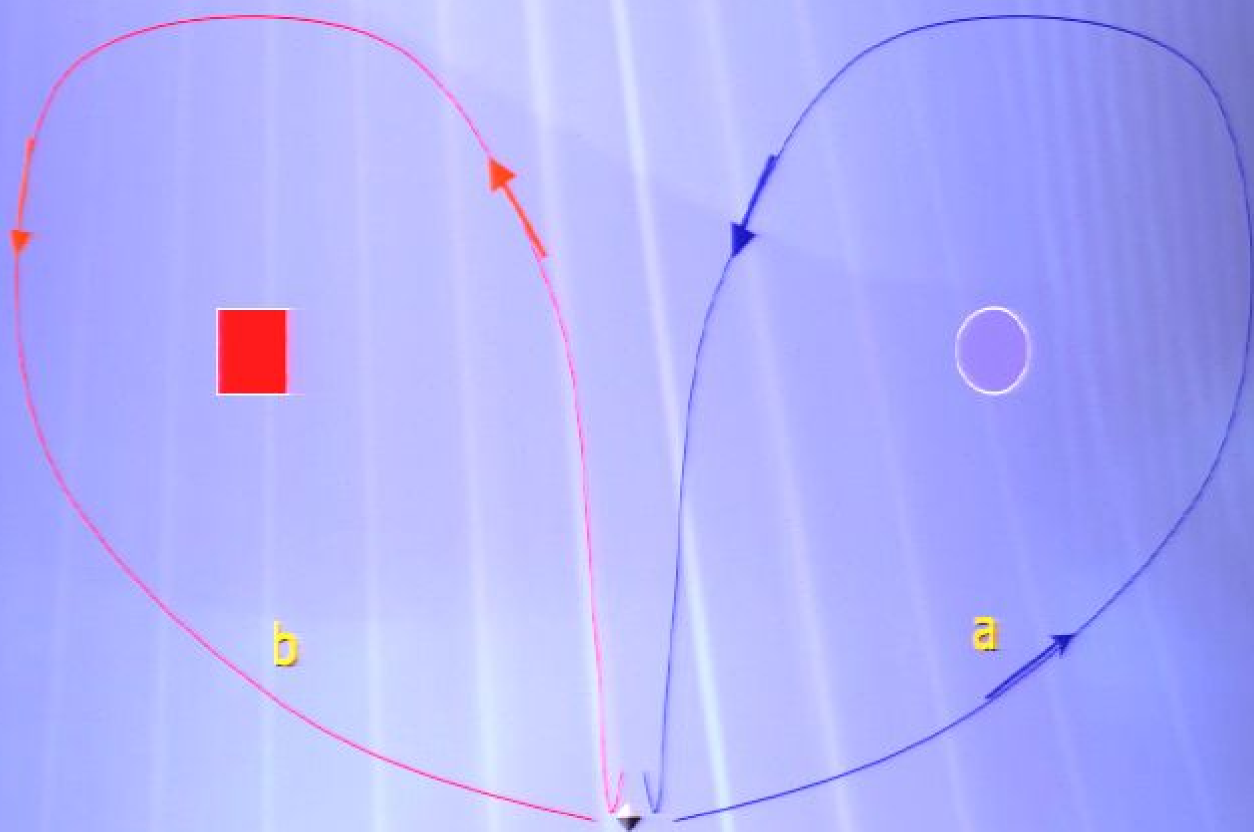
group element of *H*

algebra

$W = a \in H$



Composition rules: Fusion of defects



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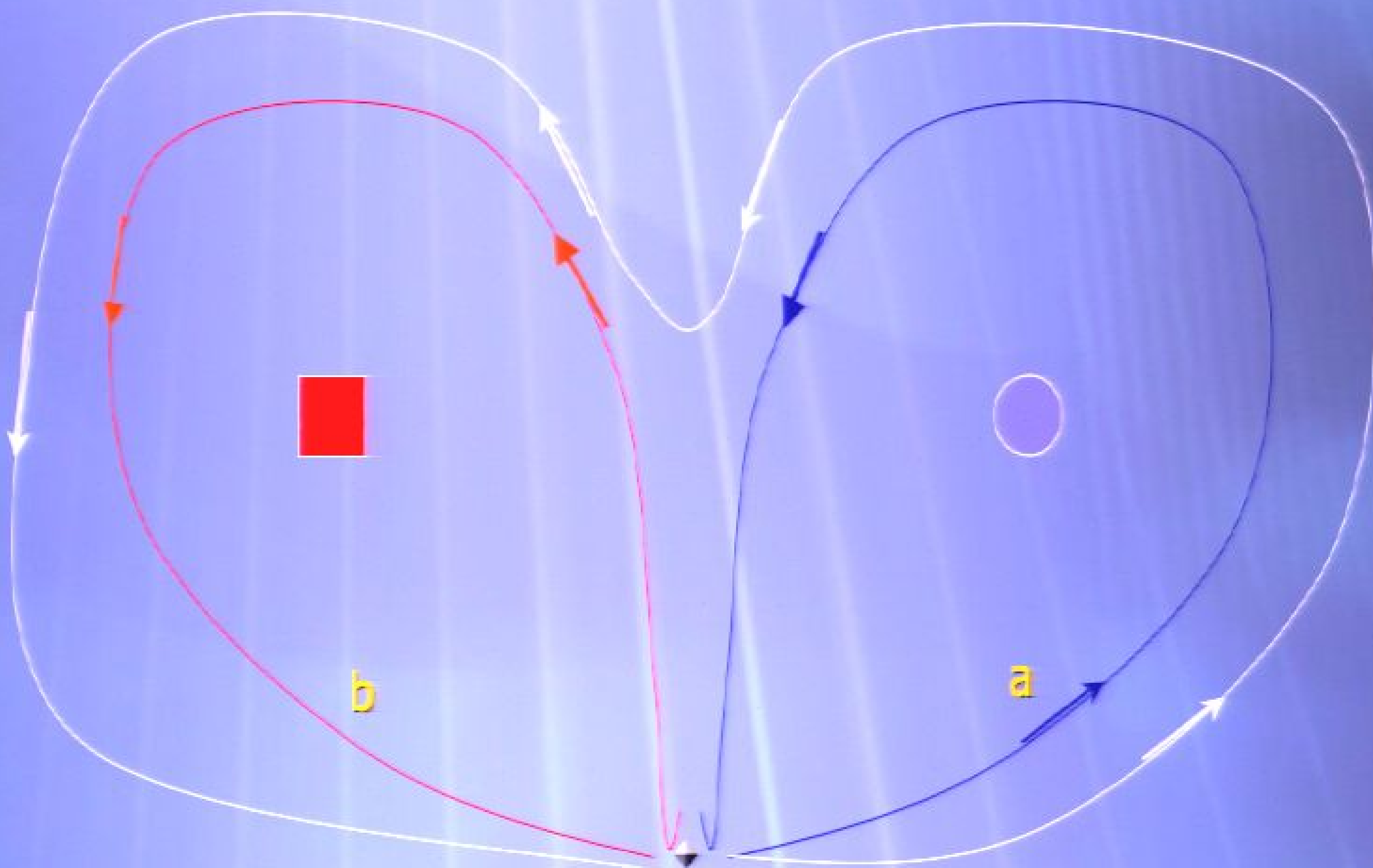

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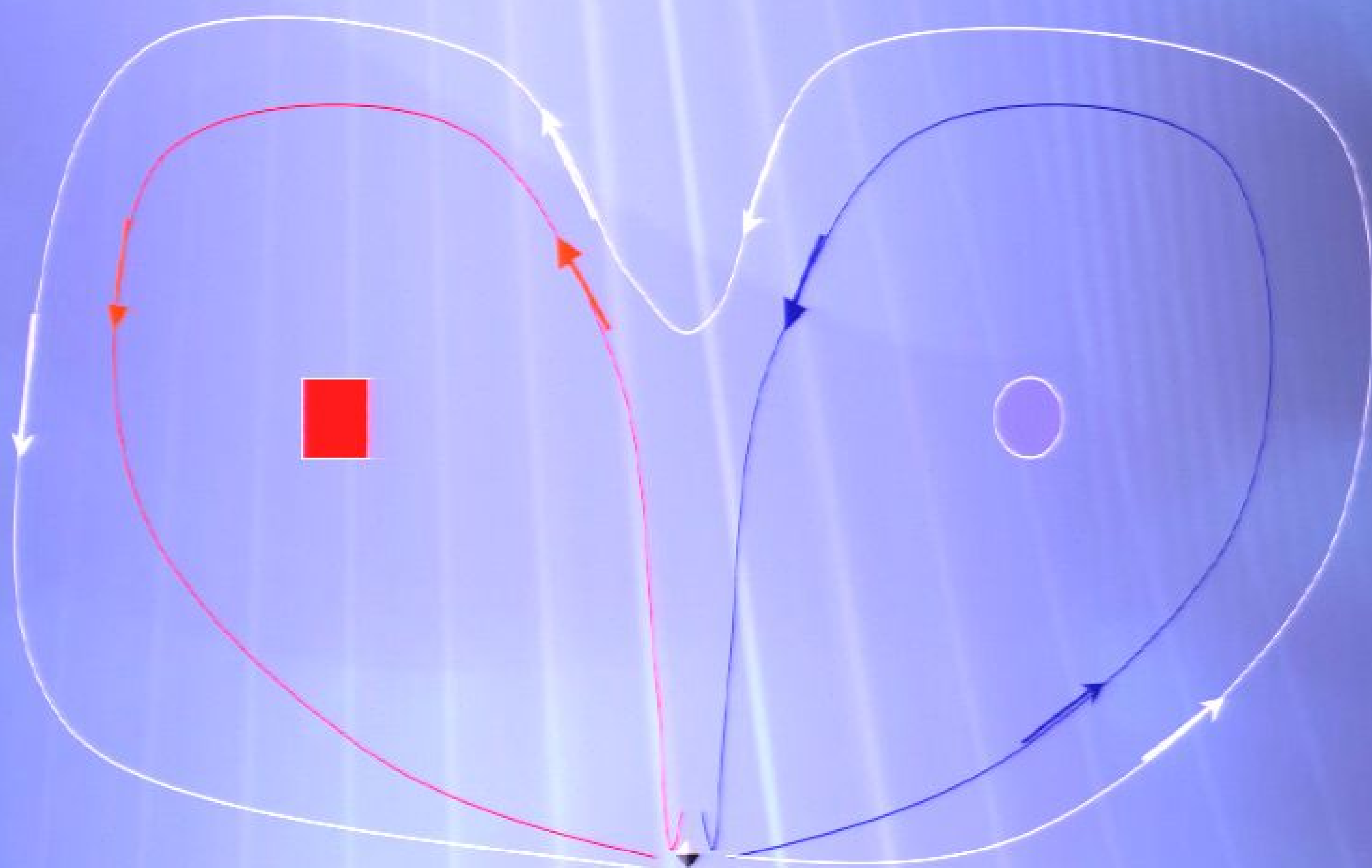

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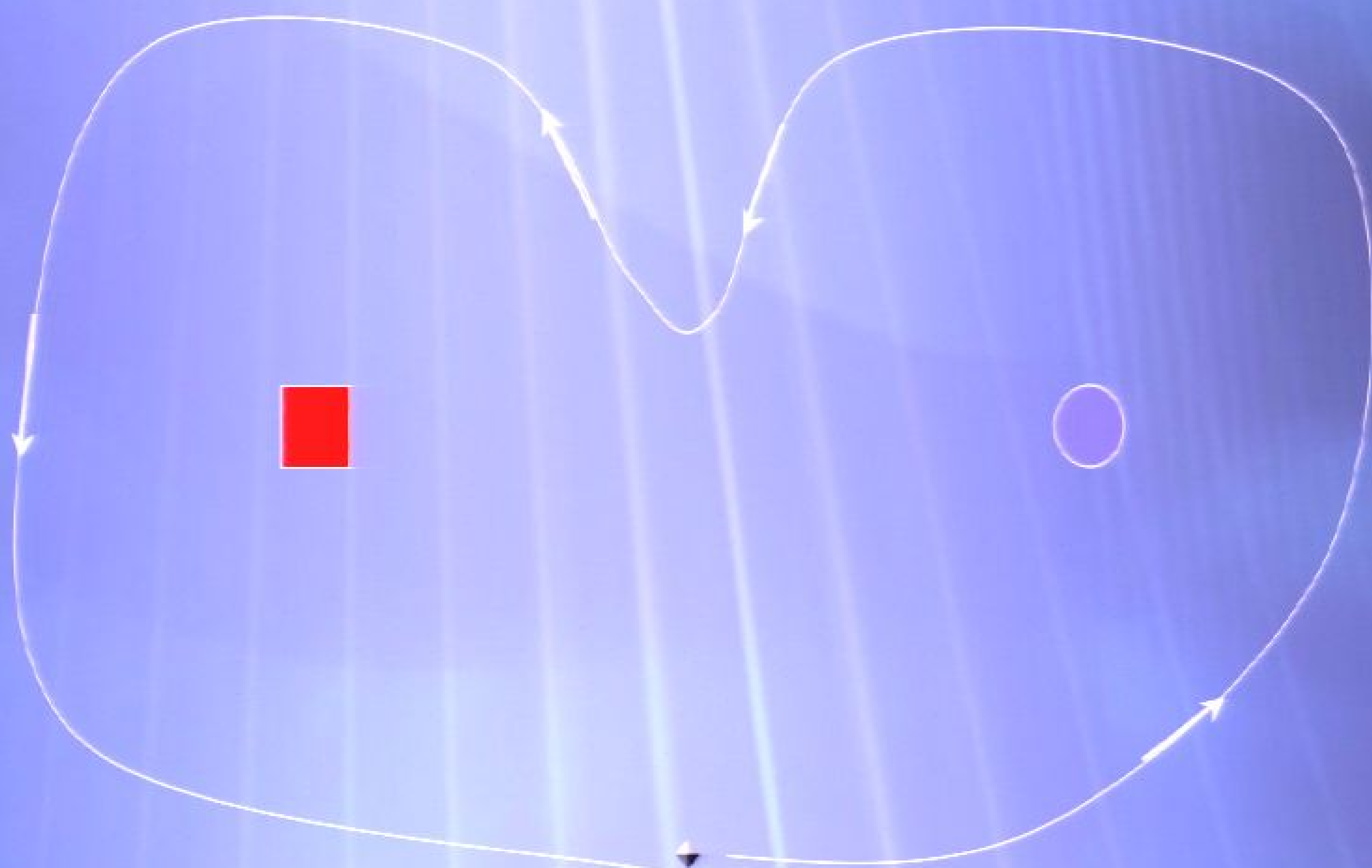

Composition rules: Fusion of defects

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$C_A\, C_B = N_{AB}^{\;C}\, C_C$

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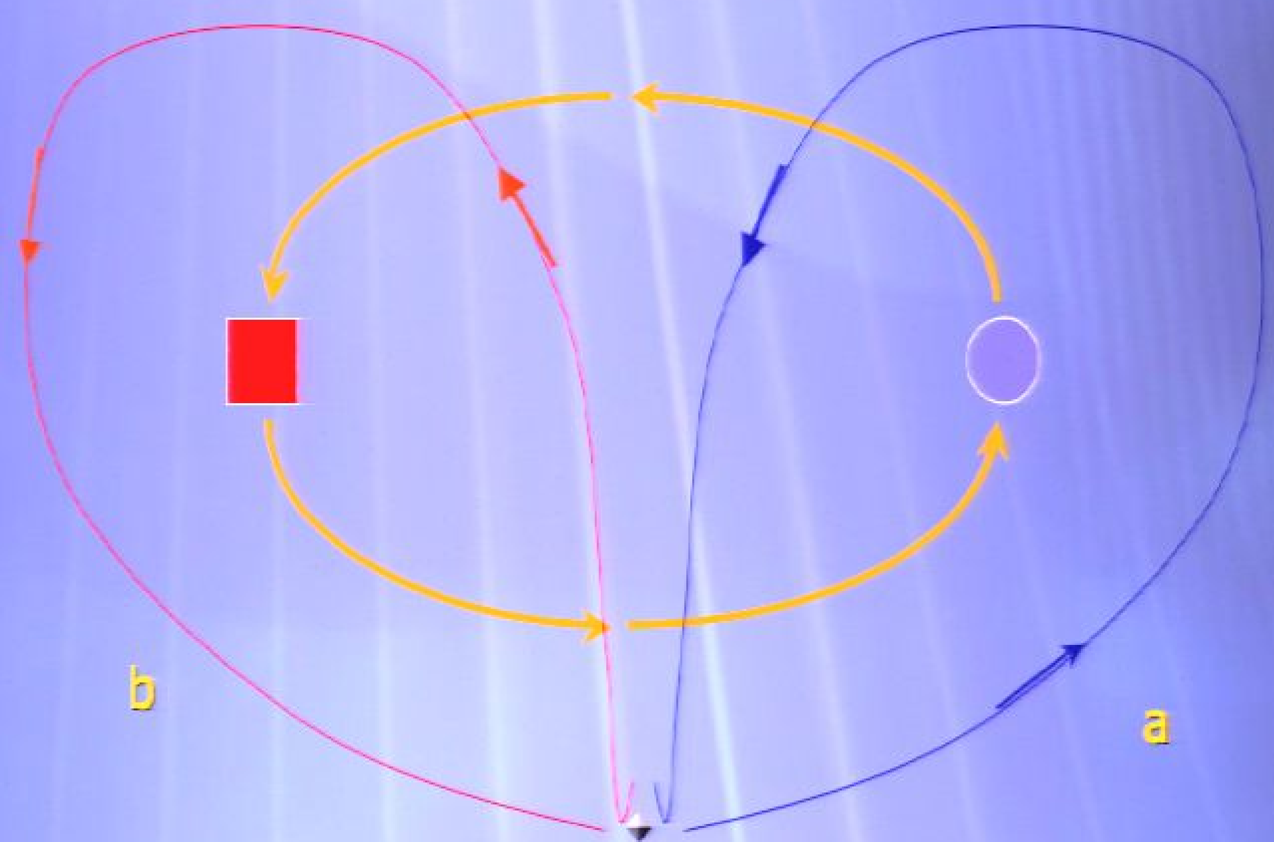
Interchange: braiding of defects

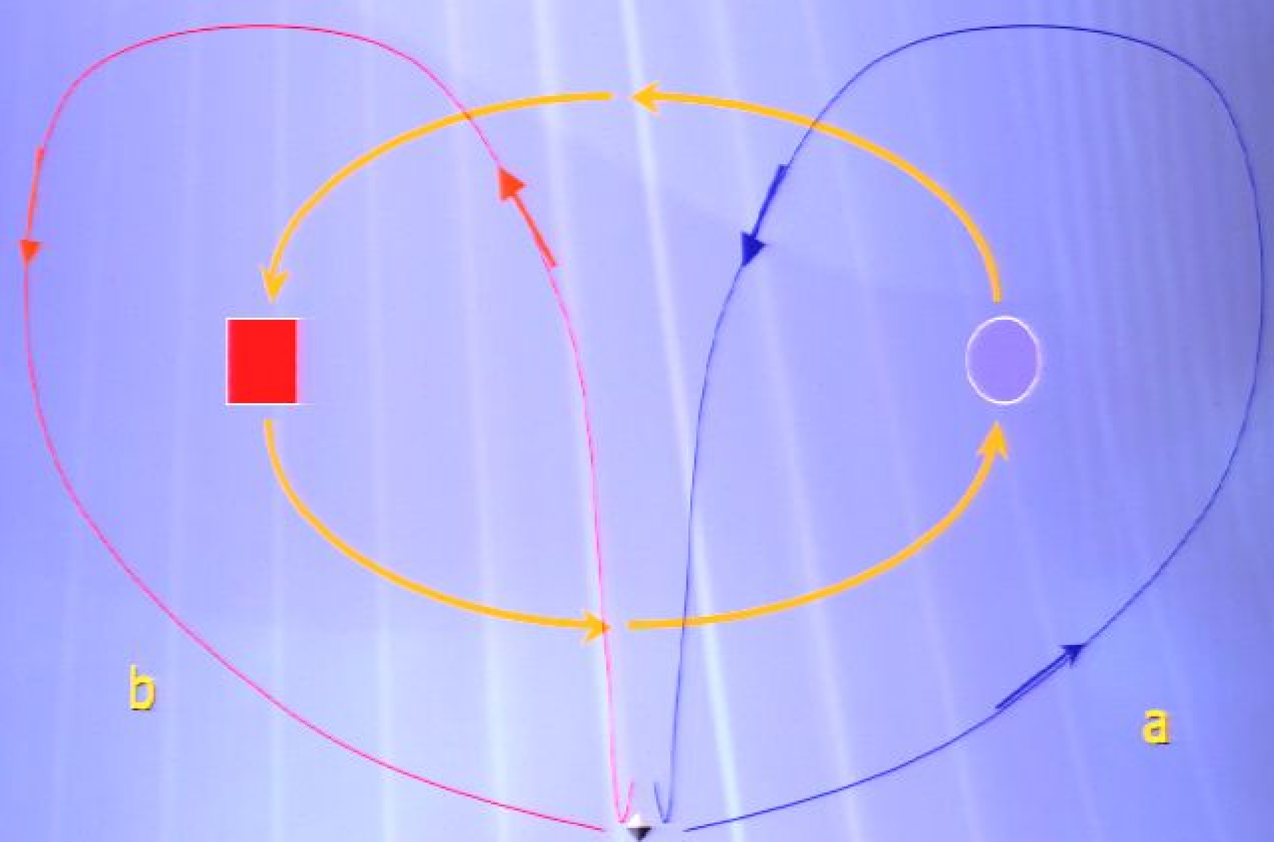


Composition rules: Fusion of defects

$c = ba$

$$C_A\, C_B = N_{AB}{}^C\, C_C$$

Interchange: braiding of defects




Interchange: braiding of defects
R(ba) = a′b′
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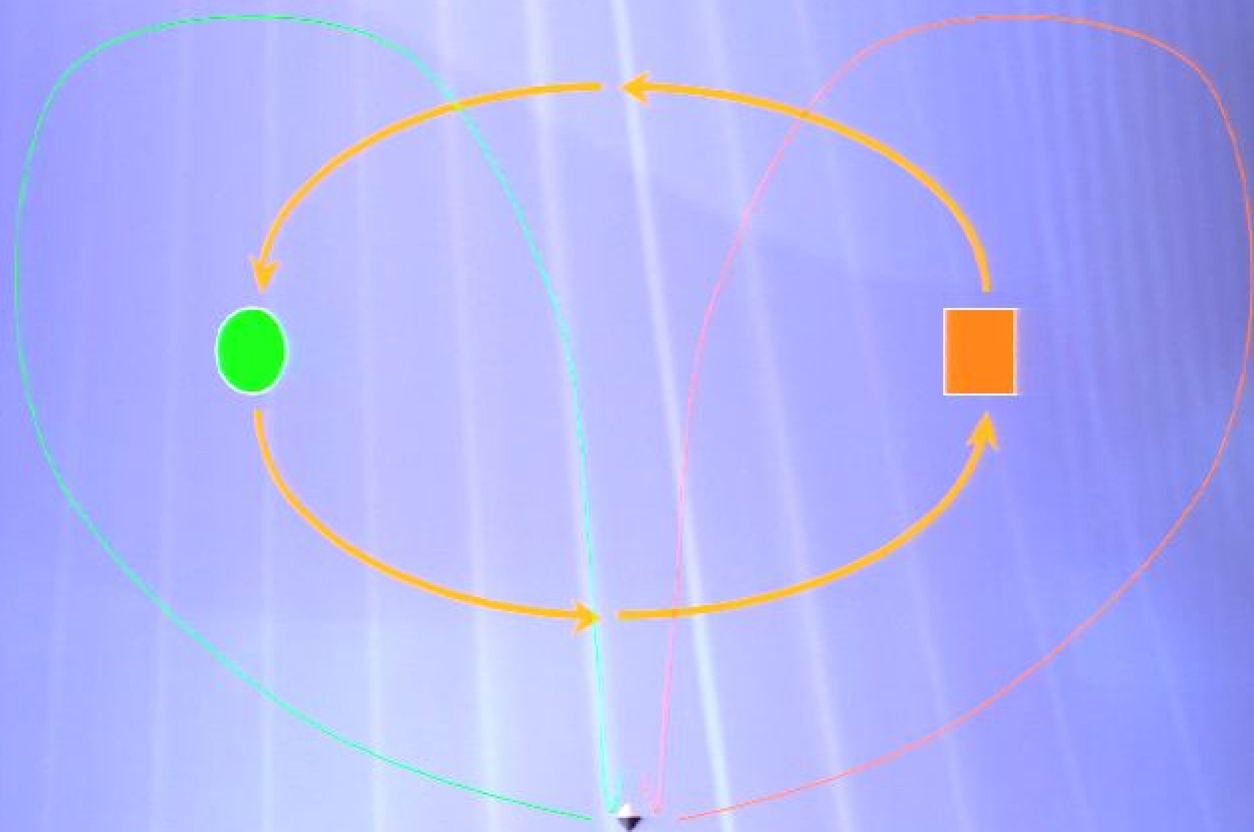
Interchange: braiding of defects



Algebraic argument

- $R(ba) = a'b'$
- in fact $ba = a'b'$ $(c=c)$
- we note that $b'=b$
- → $ba = a'b$
- → $a'=bab^{-1}$

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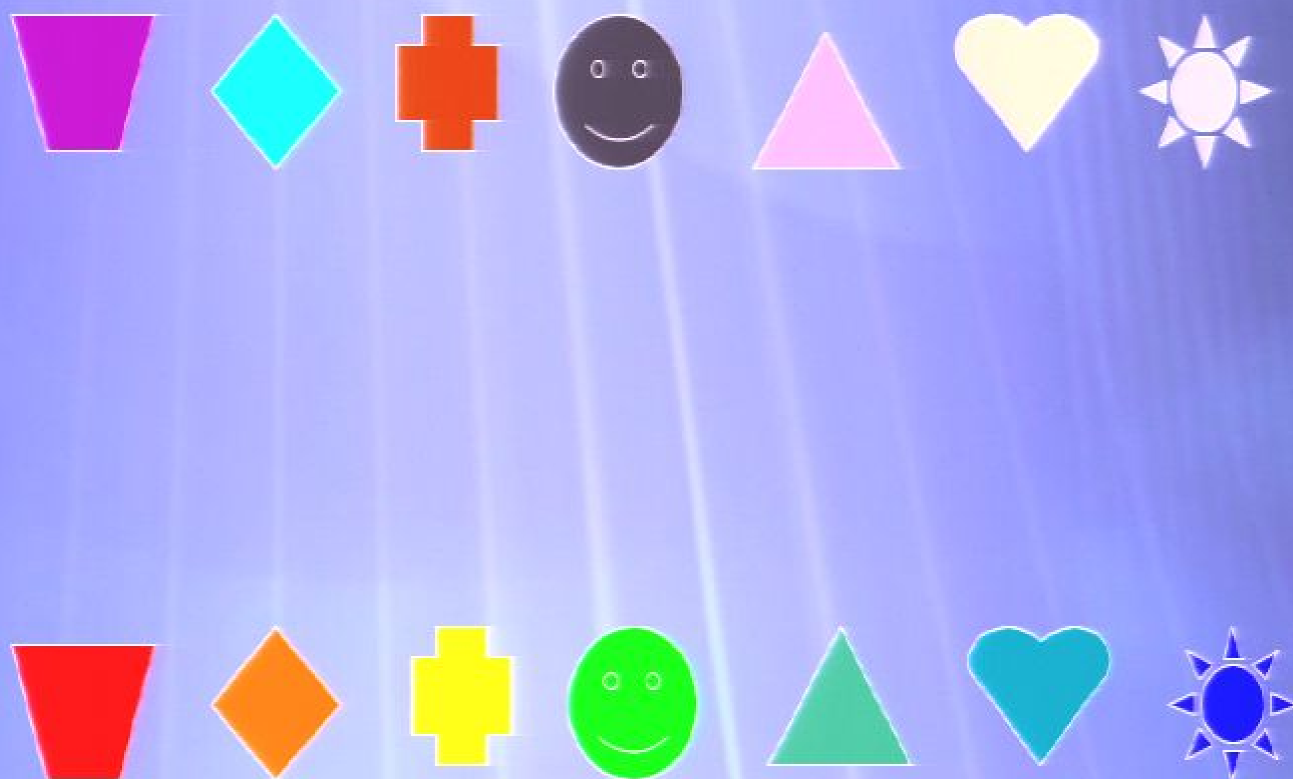
Multiparticle braid relations



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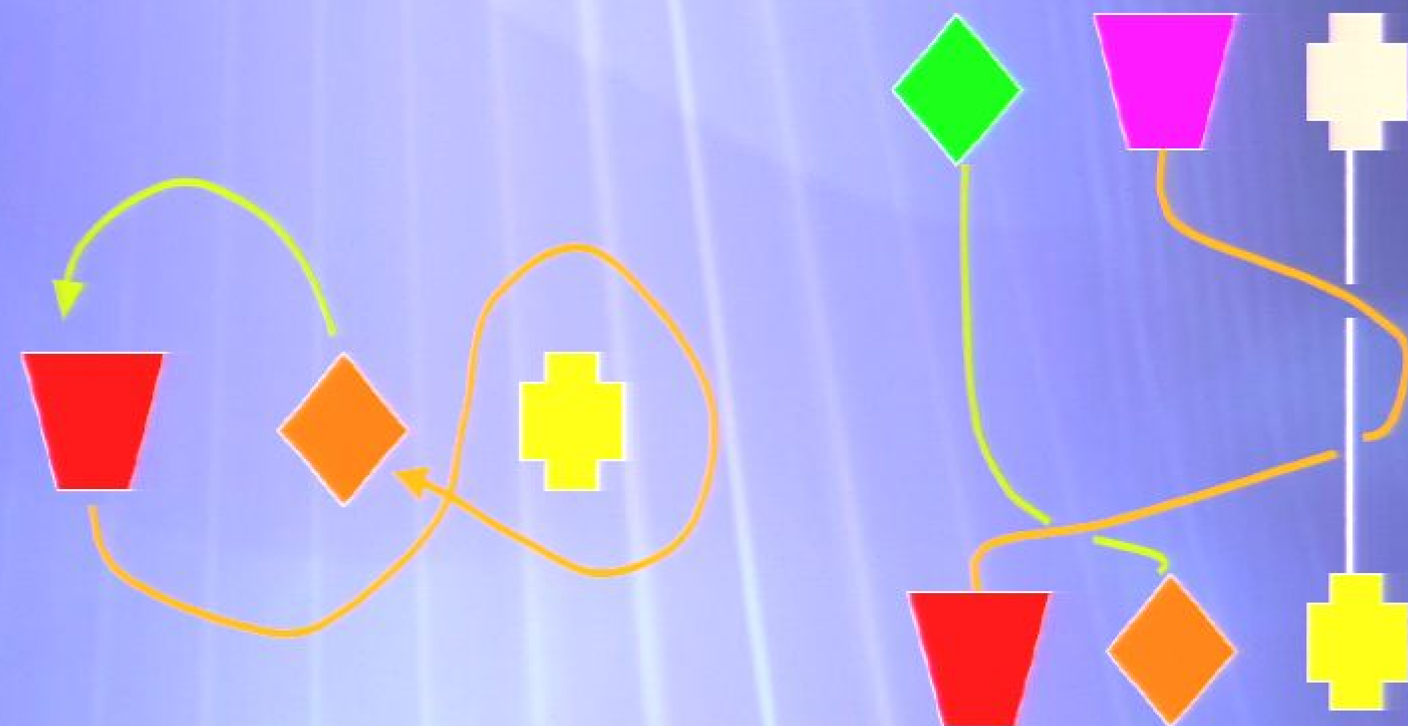
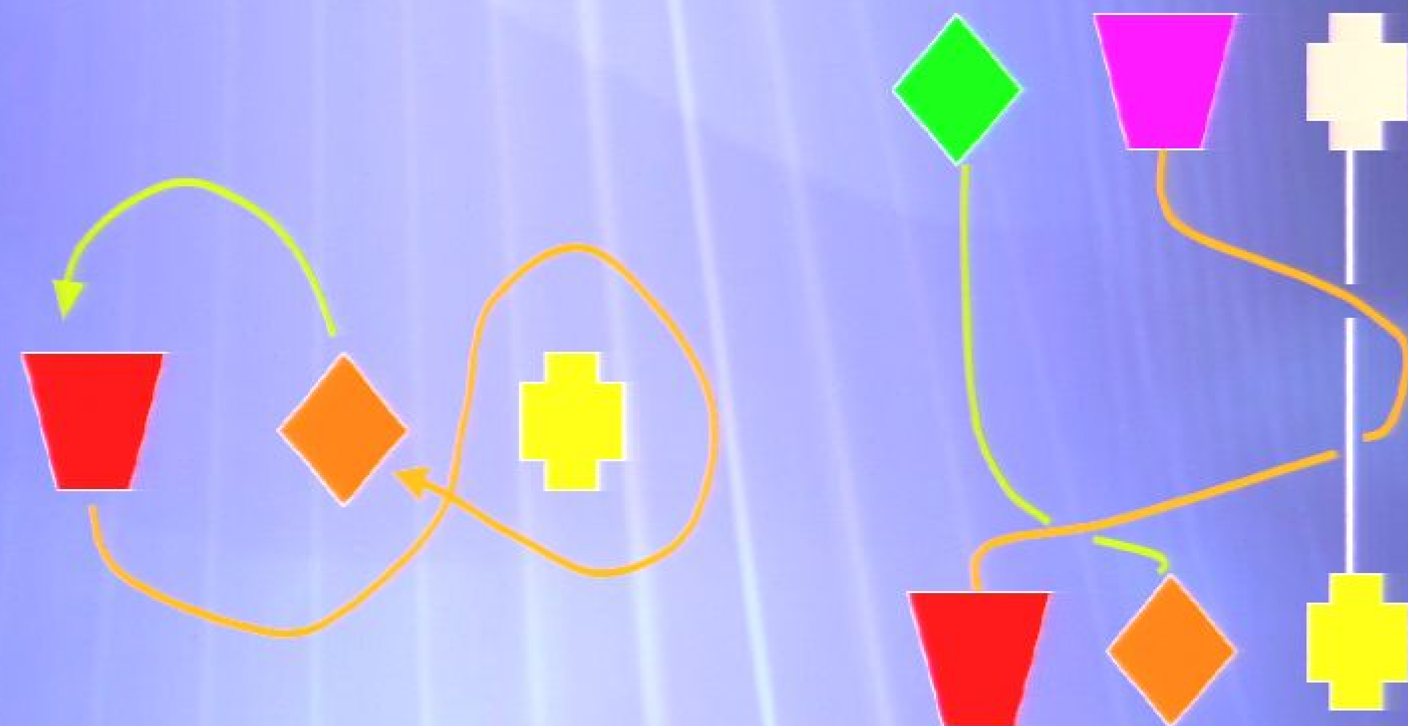
Multiparticle braid relations

Braid group B_n on n strands



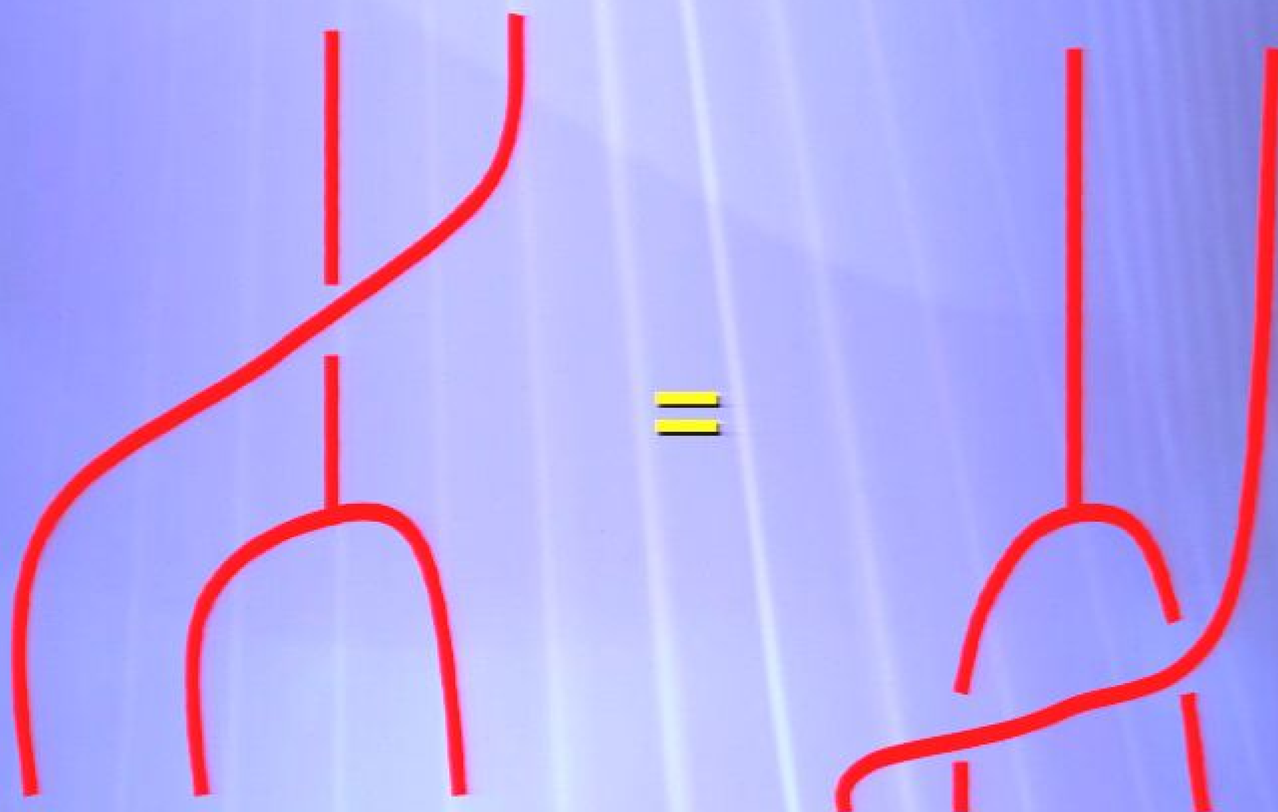
Braids & Knots

Braid group B_n on n strands



Braids & Knots

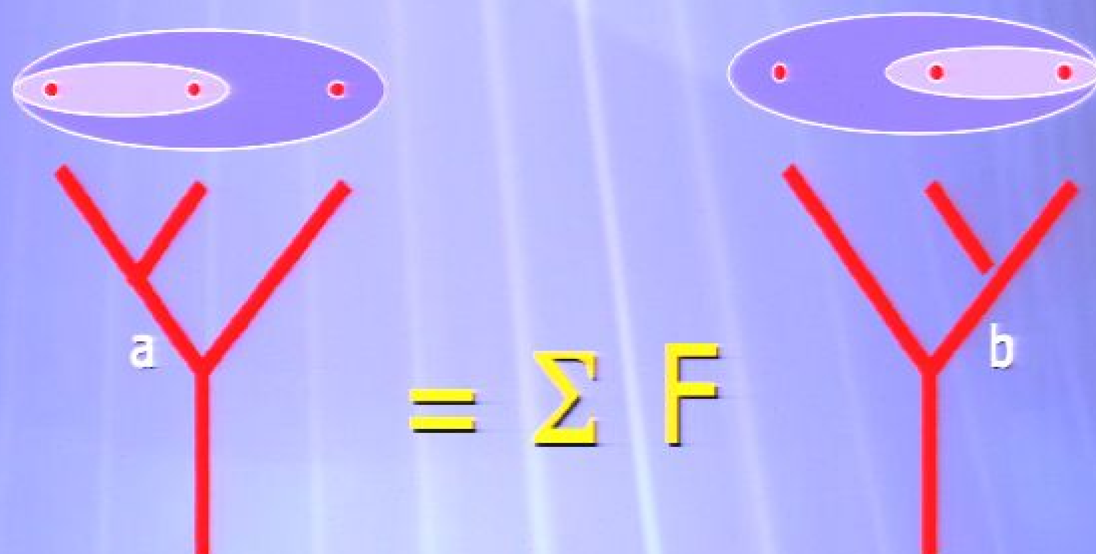
Consistency of braiding and fusion



Fusion rules and their associativety

Basic fusion rules:

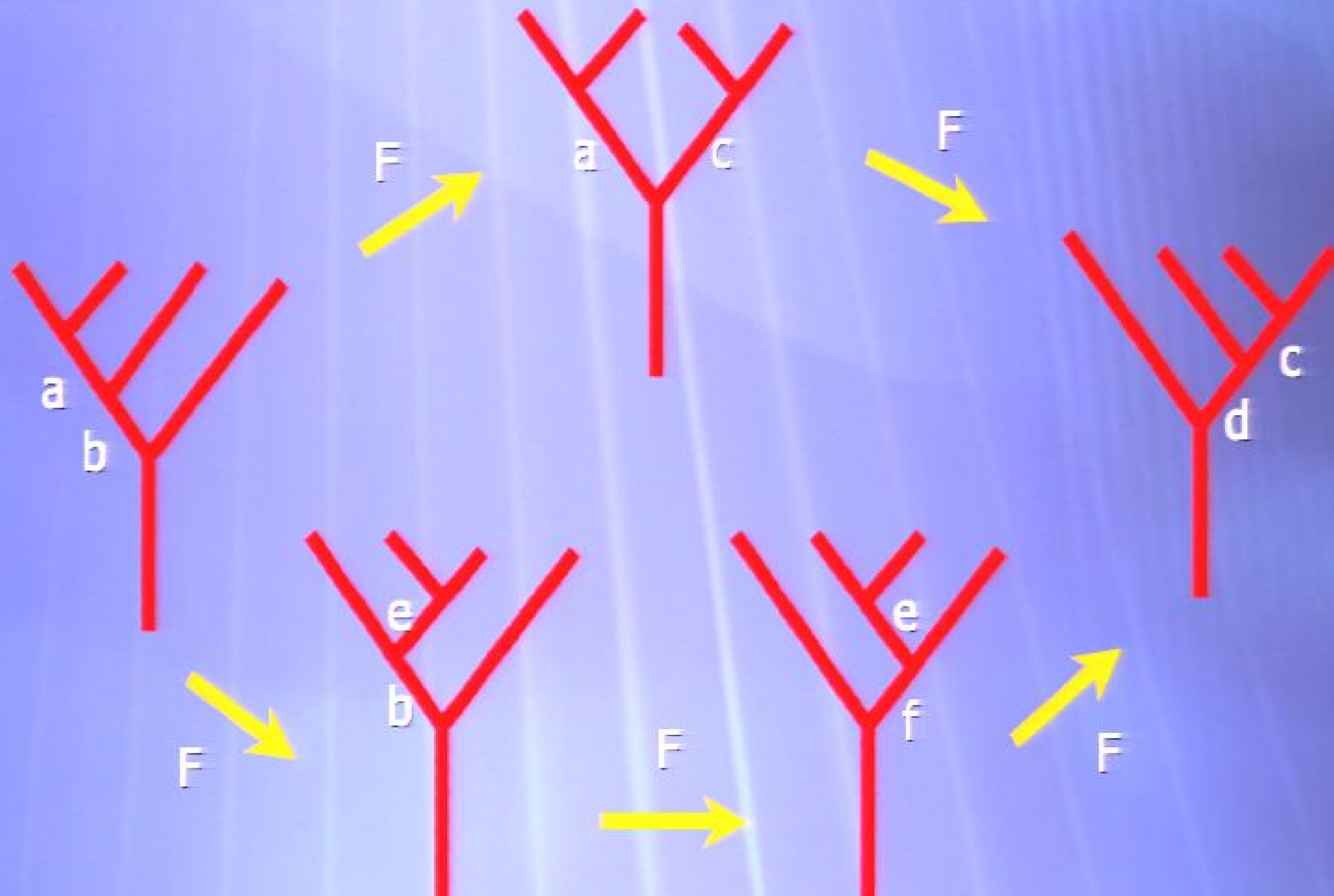
$$\Pi_a \otimes \Pi_b = N_{ab}^{c} \Pi_c$$



Pentagon relation

(Associativity of Fusion rules =>

Consistency condition on the F symbols)



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Hexagon relation

(Condition for consistent braiding)

Hopf-algebra's: quantum double

Hopf algebra $\mathcal{A}$ Ex: Group algebra $\mathbb{C}H$ Basis: $\{h_i\}$ $h_i \in H$	Dual Hopf algebra $\mathcal{A}^*$ Functions on the group $F(H)$ $\{f_i\}$ $f_i = f_{h_i} = P_{h_i}$ $f_i(x) = \delta_{h_i,x}$

	Algebra		Dual algebra	
product	$\cdot$	$h_1 \cdot h_2 = h_1 h_2$	$\star$	$f_1 \star f_2(x) = f_1 \otimes f_2 \Delta(x)$
unit	e	$eh = he = h$	e^*	$e^* f(x) = \varepsilon(x) = 1$
	Co-algebra		**Dual co-algebra**	
co-product	Δ	$\Delta(h) = h \otimes h$	Δ^*	$\Delta^*(f)(x, y) = f(x \cdot y)$
co-unit	ε	$\varepsilon(h) = 1$	ε^*	$\varepsilon^*(f) = f(e)$
antipode	S	$S(h) = h^{-1}$	S^*	$S^*(f)(x) = f(S(x)) = f(x^{-1})$

The Quantum Double D(H) (Drinfeld, DPR)

Double algebra $D = \mathcal{A}^* \times \mathcal{A}$
Ex: Hopf double algebra $D(H) = F(H) \times \mathbb{C}H$
Basis: $\{f_i \times h\}$ $h \in H$

	Algebra	
product	$\cdot$	$(f_1 \times h_1) \cdot (f_2 \times h_2)(x) = f_1(x) f_2(h_1 x h_1^{-1}) \times h_1 h_2$
unit	e	$(1 \times e)(x) = e$
	Co-algebra	
co-product	Δ	$\Delta(f \times h)(x,y) = f(xy) h \otimes h$
co-unit	ε	$\varepsilon(f \times h)(x) = f(e)$
antipode	S	$S(f \times h)(x) = f(h^{-1} x^{-1} h) h^{-1}$
Central (ribbon) element	c	$c = \sum_h (f_h \times h)$
R-element $R \in D \otimes D$	R	$R = \sum_h (f_h \times e) \otimes (1 \times h)$

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	Co-algebra		**Dual co-algebra**	
co-product co-unit antipode	Δ ε S	$\Delta(h) = h \otimes h$ $\varepsilon(h) = 1$ $S(h) = h^{-1}$	Δ^* ε^* S^*	$\Delta^*(f)(x,y) = f(x \cdot y)$ $\varepsilon^*(f) = f(e)$ $S^*(f)(x) = f(S(x)) = f(x^{-1})$

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Representation Theory

Representations Π_α^A of $D(H) = F(H) \times CH$		
representation	Π_α^A A α	$A \sim$ defect/magnetic label, $\alpha \sim$ ordinary/electric label $C_A \sim$ Conjugacy class (orbit of representative element h_A). $\alpha \sim$ is a representation of the normalizer N_A of h_A in H.
carrier space	V_α^A	$\lvert v >: H \rightarrow V_\alpha \ \{\lvert v(x) > \mid \lvert v(xn) >= \alpha(n^{-1}) \lvert v(x) >, n \in N_A\}$
action of $D(H)$ on V_α^A		$\pi_\alpha^A(f \times h)\lvert v(x) >= f(xhx^{-1}) \lvert v(h^{-1}x) >$
central element spin factor	c s_α^A	$\Pi_\alpha^A(c)\lvert v(x) >= \alpha(h_A^{-1})\lvert v(x) >$ $s_\alpha^A \equiv \alpha(h_A^{-1})$
tensor products	$\Pi_\alpha^A \otimes \Pi_\beta^B$	$\Pi_\alpha^A \otimes \Pi_\beta^B(f \times h) V \otimes W \equiv \Pi_\alpha^A \otimes \Pi_\beta^B \Delta(f \times h) V \otimes W$ Clebsch Gordon series: $\Pi_\alpha^A \otimes \Pi_\beta^B = \sum_{C,\gamma} N_{\alpha\beta C}^{AB\gamma} \Pi_\gamma^C$

Quantum group (Hopf algebra) G_q

- Particles (anyons) <-->Representations

  Π^a <--> $|\phi^a>_i$
- Rotations (fractional spin) <--> s^a (Casimir op)

  $S(2\pi)\ |\phi^a>_i = s^a|\phi^a>_i$
- Fusion <--> tensor product (Clebsch Gordan series)

  $\Pi^a \times \Pi^b = N^{ab}{}_c\ \Pi^c$
- Braiding <--> (Non)abelian Quantum Statistics

  $R\ (|\phi^a>x|\phi^b>) = r^{ab}{}_c(|\phi^b>x\ |\phi^a>)$

- Multiparticle states decompose under $G_q \otimes B_n$

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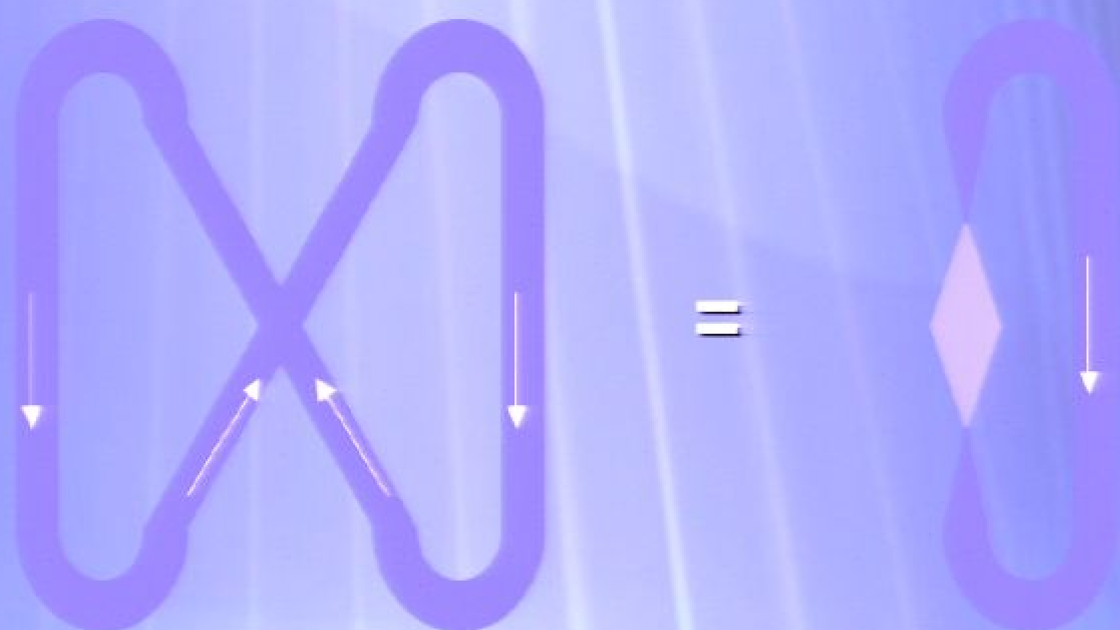
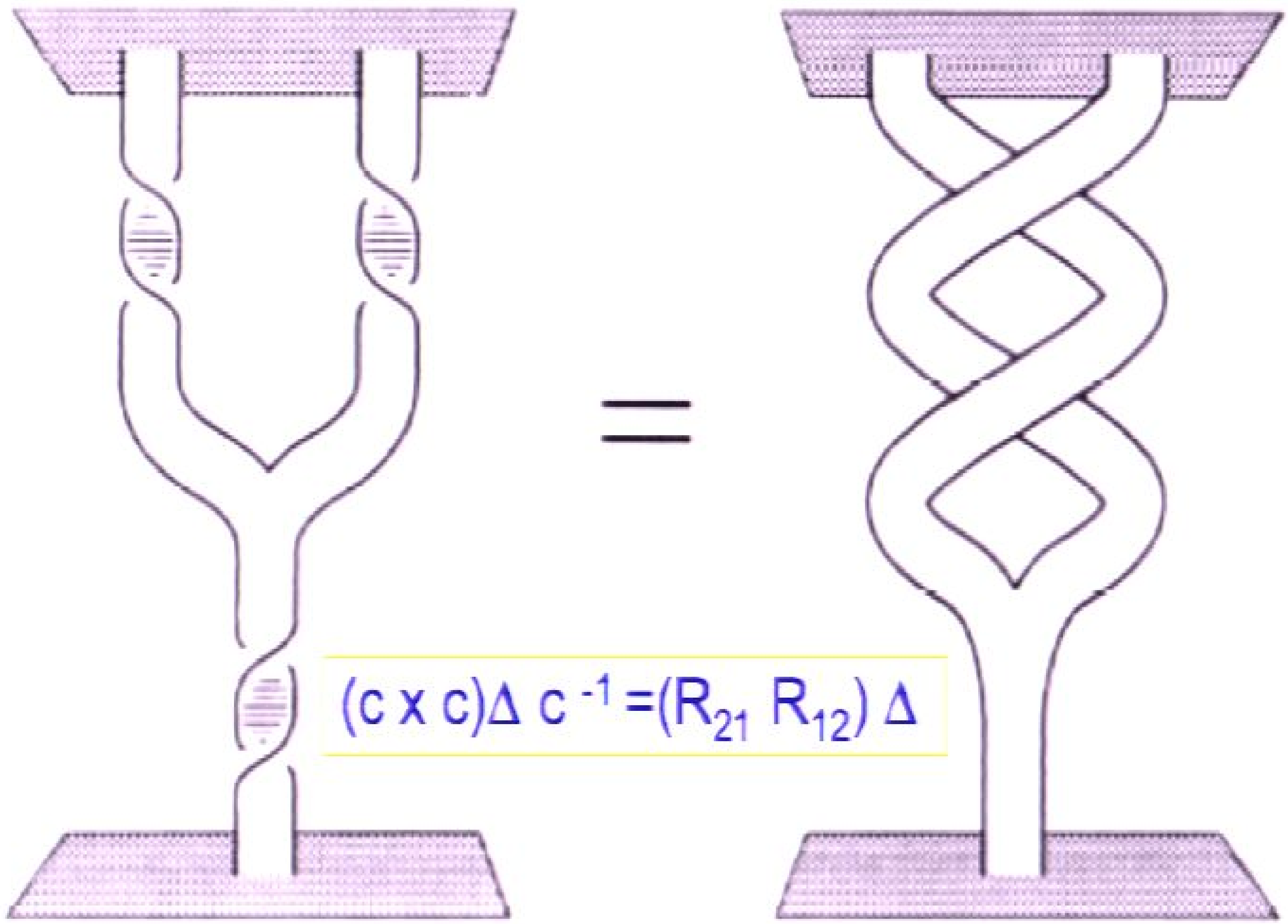
Spin-statistics connection as a topological equivalence



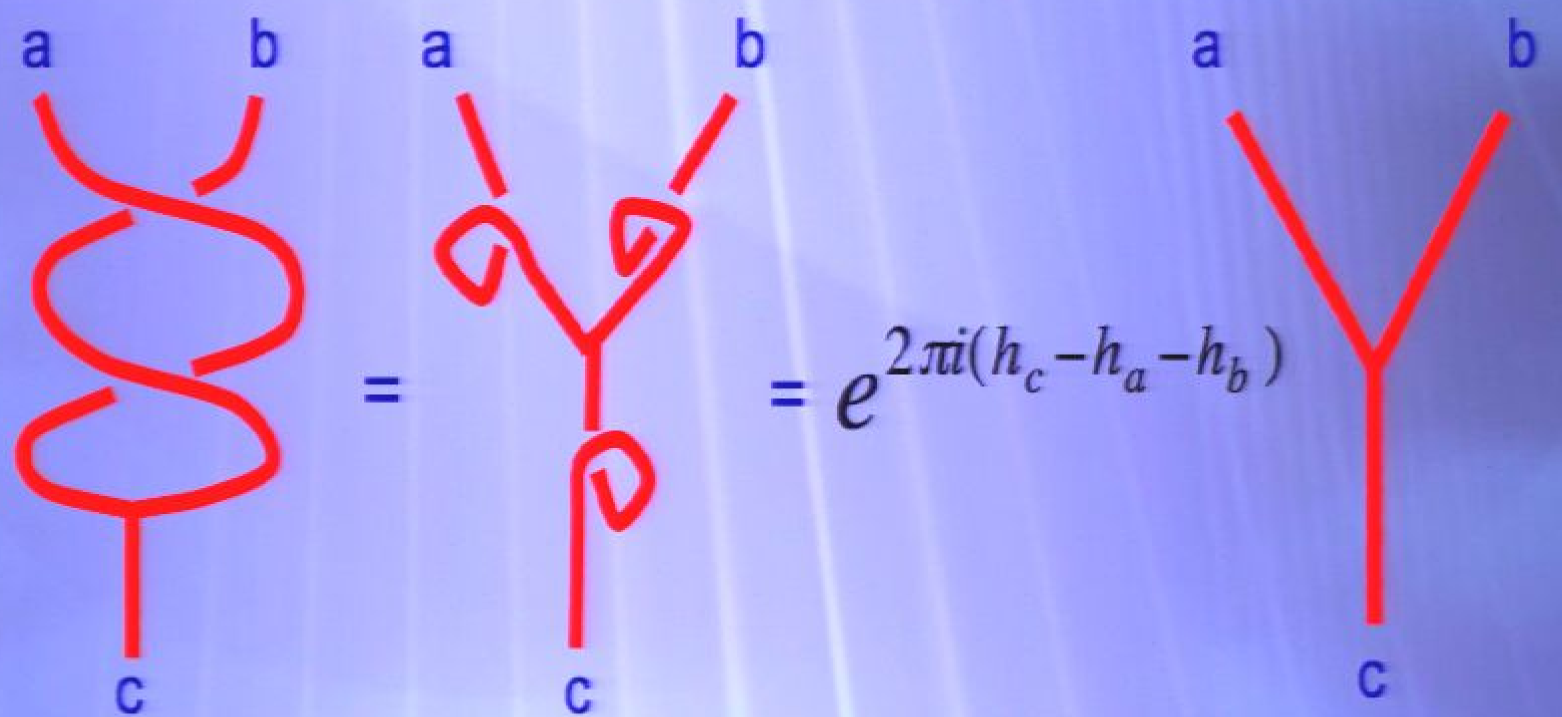
Effect of interchange is equivalent to 2π rotation

Suspenders diagram

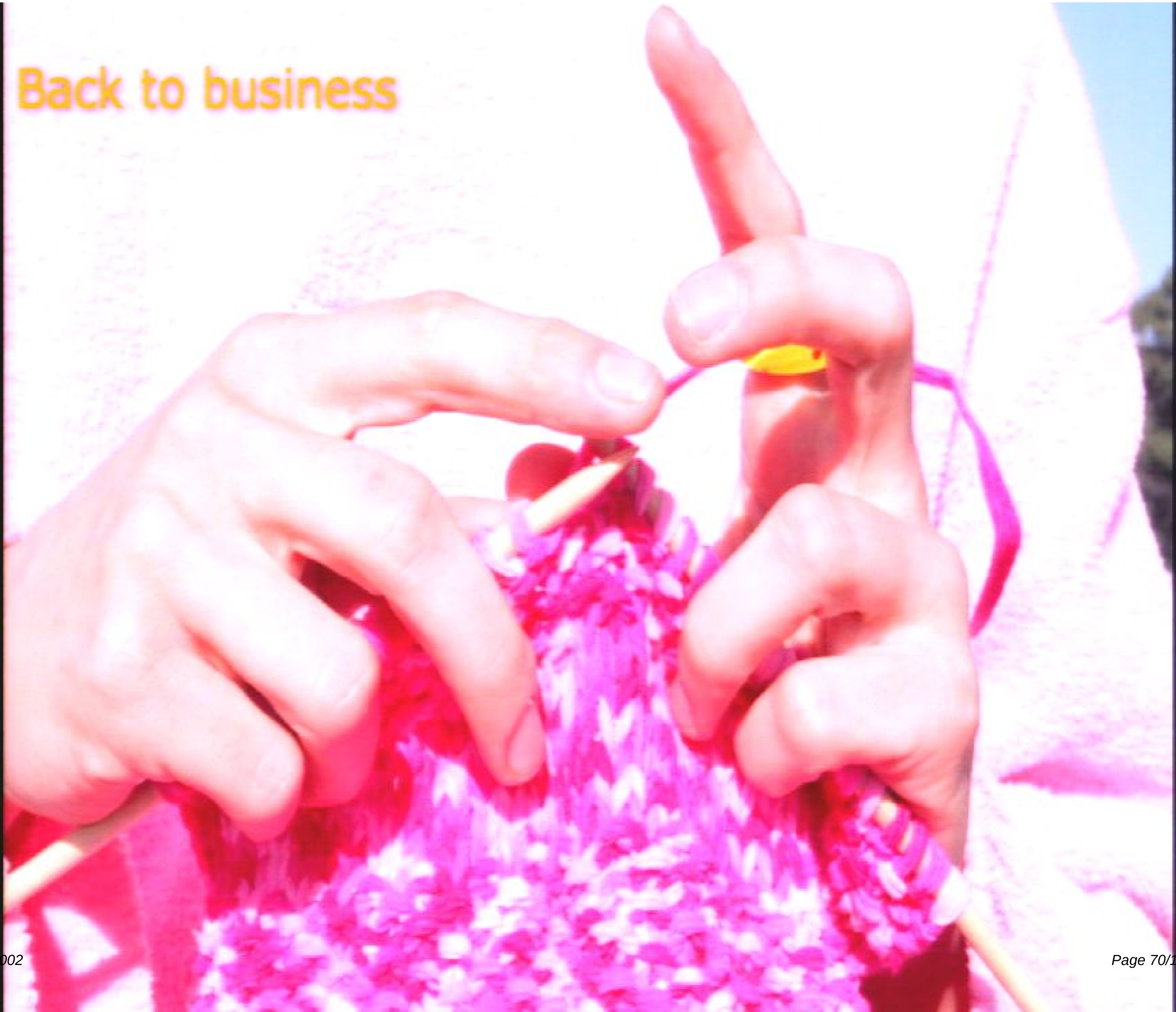


$$(c \times c)\Delta\, c^{-1} = (R_{21}\, R_{12})\, \Delta$$

Suspenders diagram



Back to business

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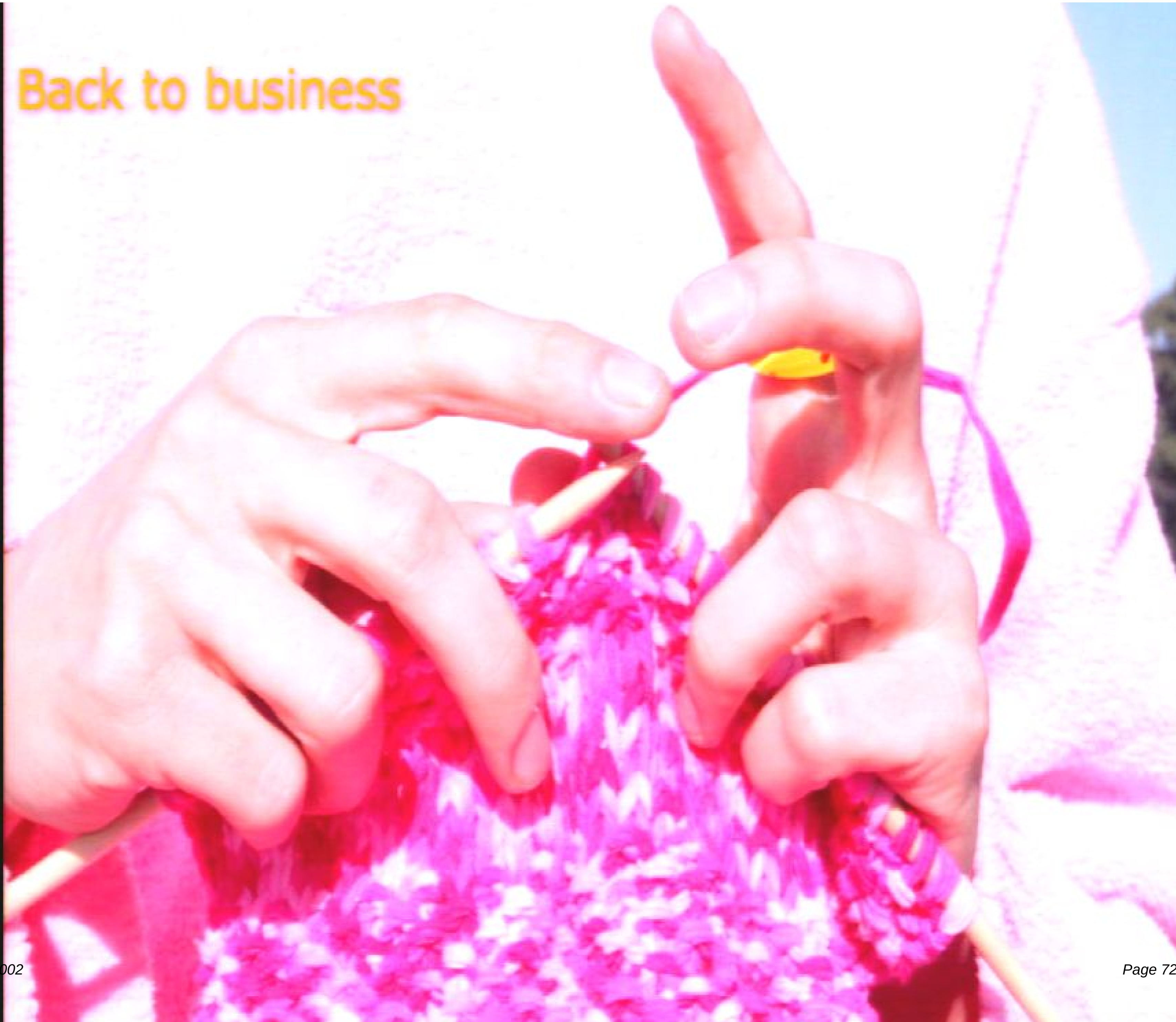
Teaching, Entertaining, Inspiring.
The Art Of
KNITTING
STITCHES ○ COLORS ○ FASHION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PI 20

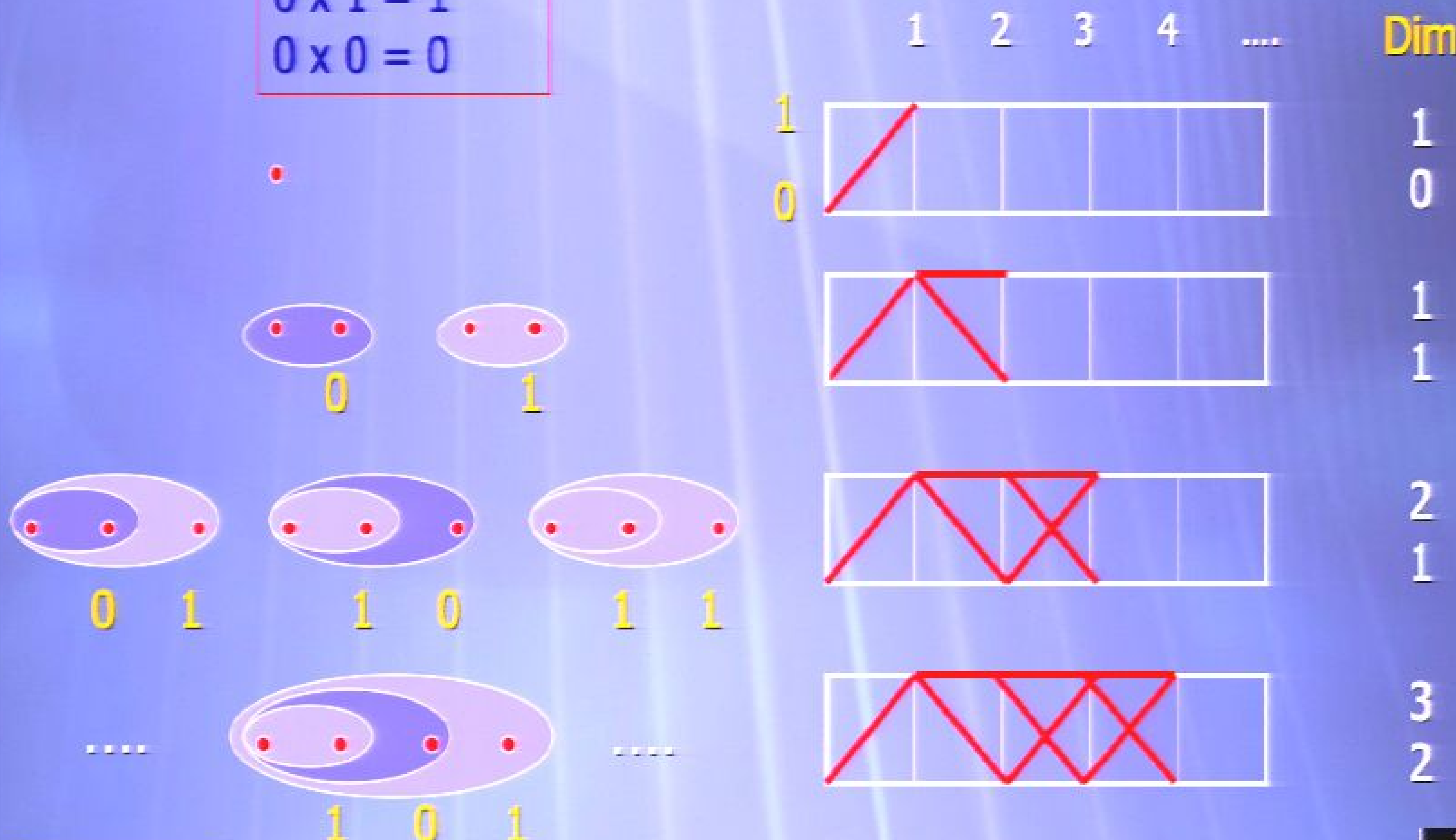
Fusion rules (multi-anyon states)

(Preskill,Freedman, Bonesteel)

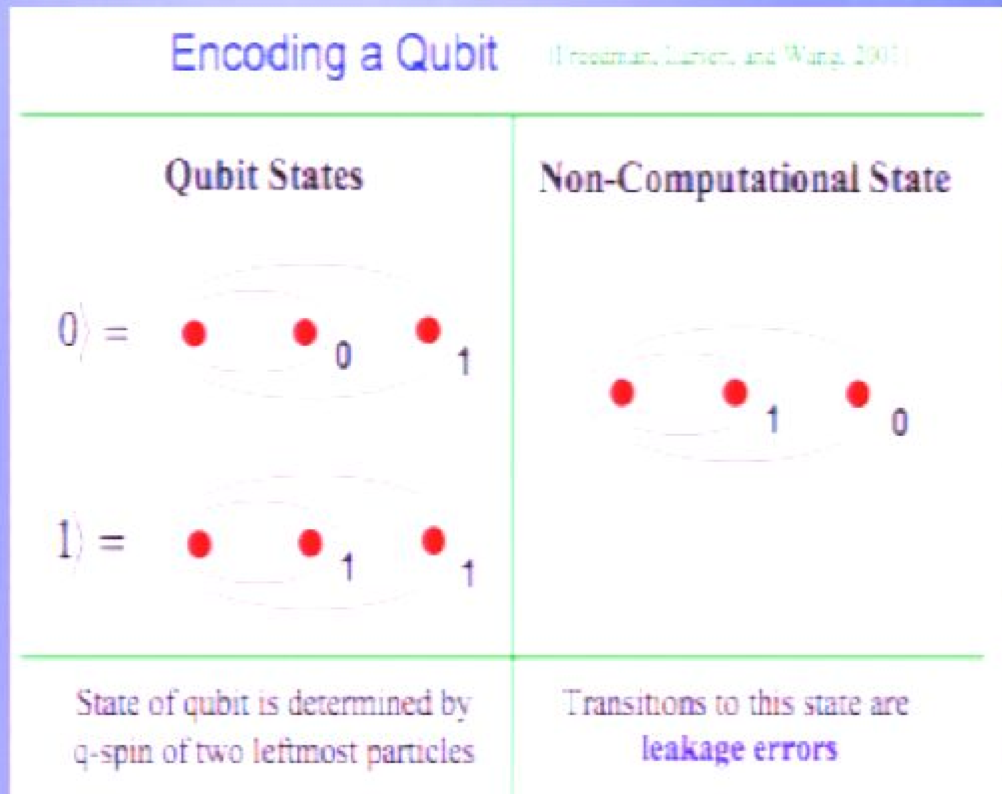
Fibonacci anyons

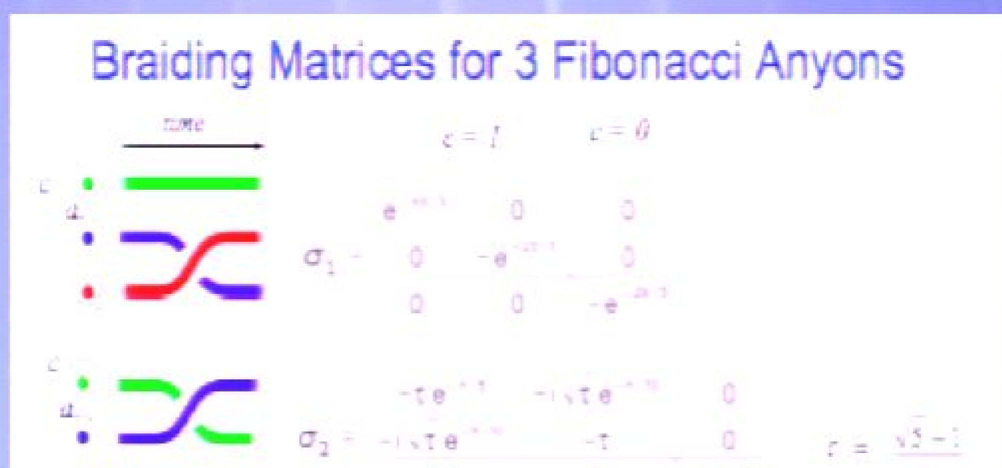
$1 \times 1 = 0 + 1$
$0 \times 1 = 1$
$0 \times 0 = 0$

Dim = # paths



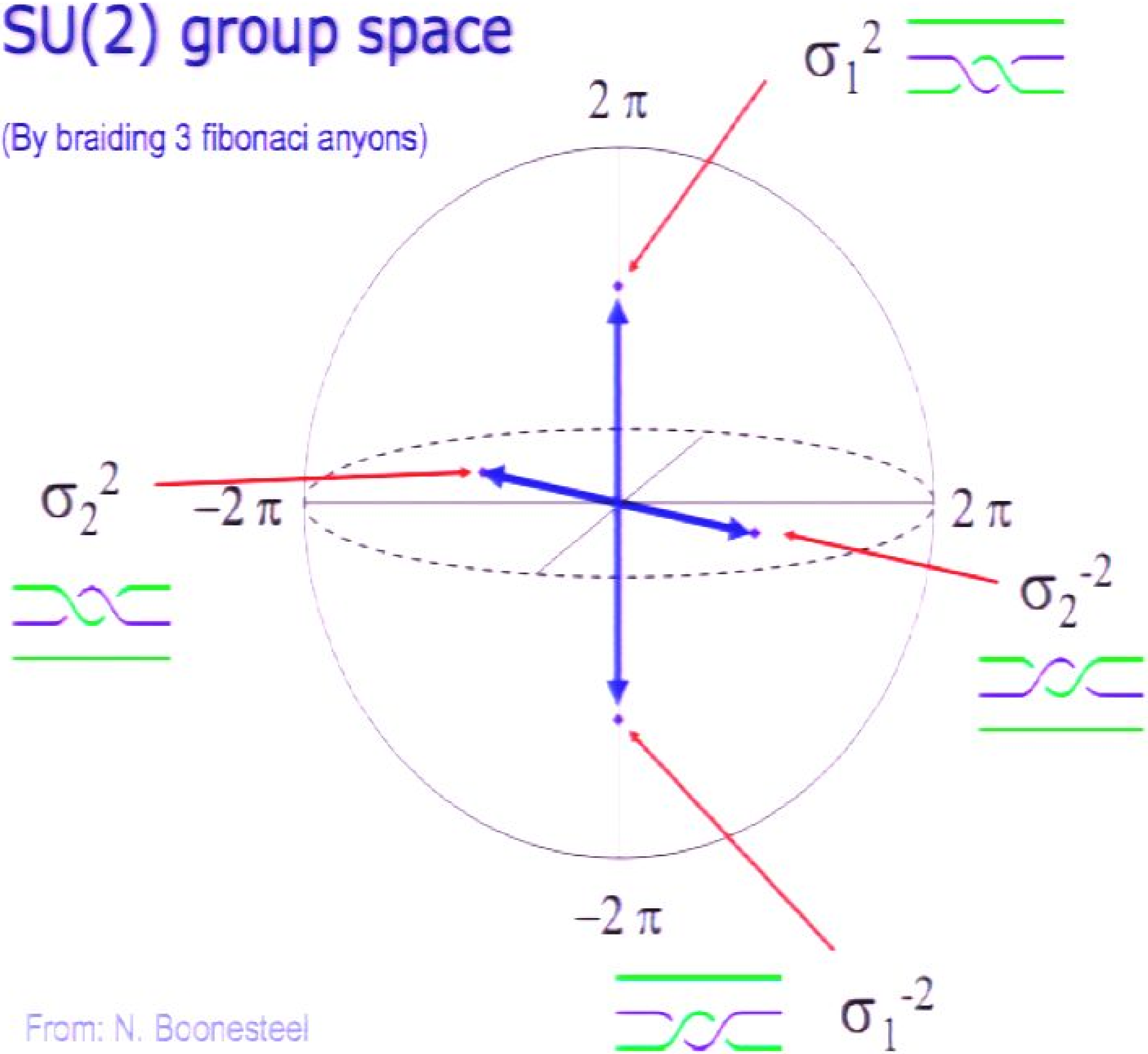
Fibonacci anyons encoding 1 qbit





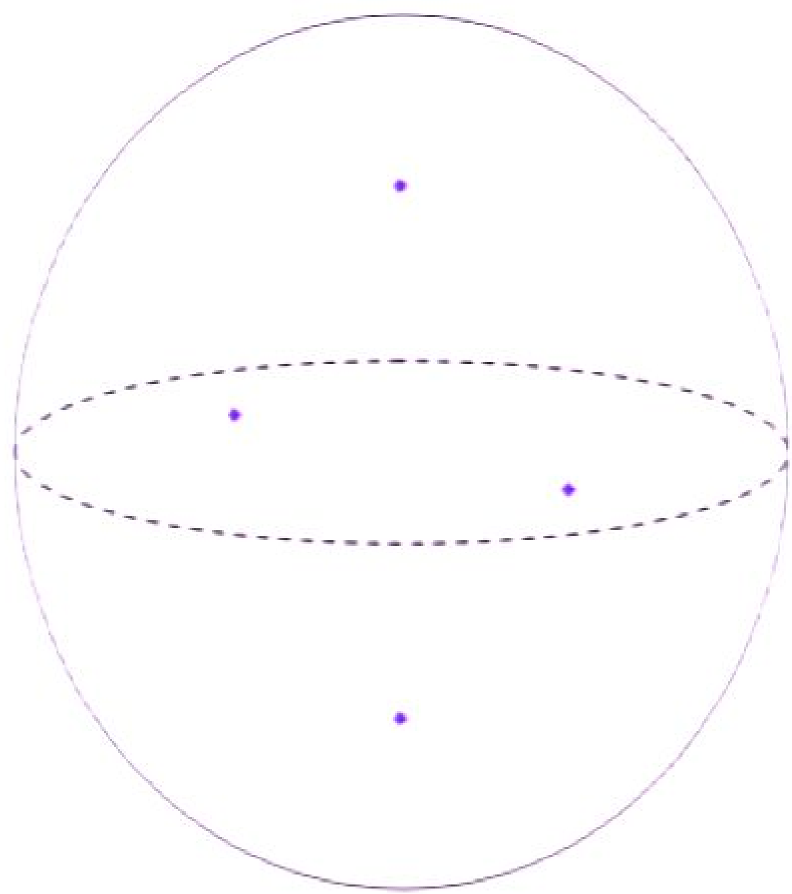
SU(2) group space

(By braiding 3 fibonaci anyons)

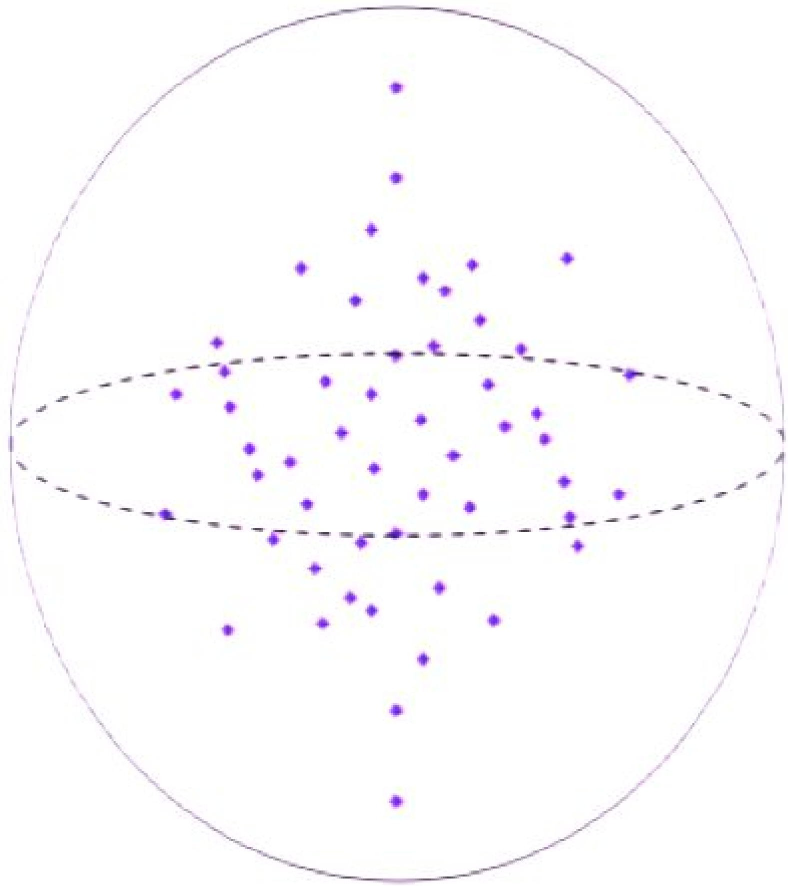


From: N. Boonesteel

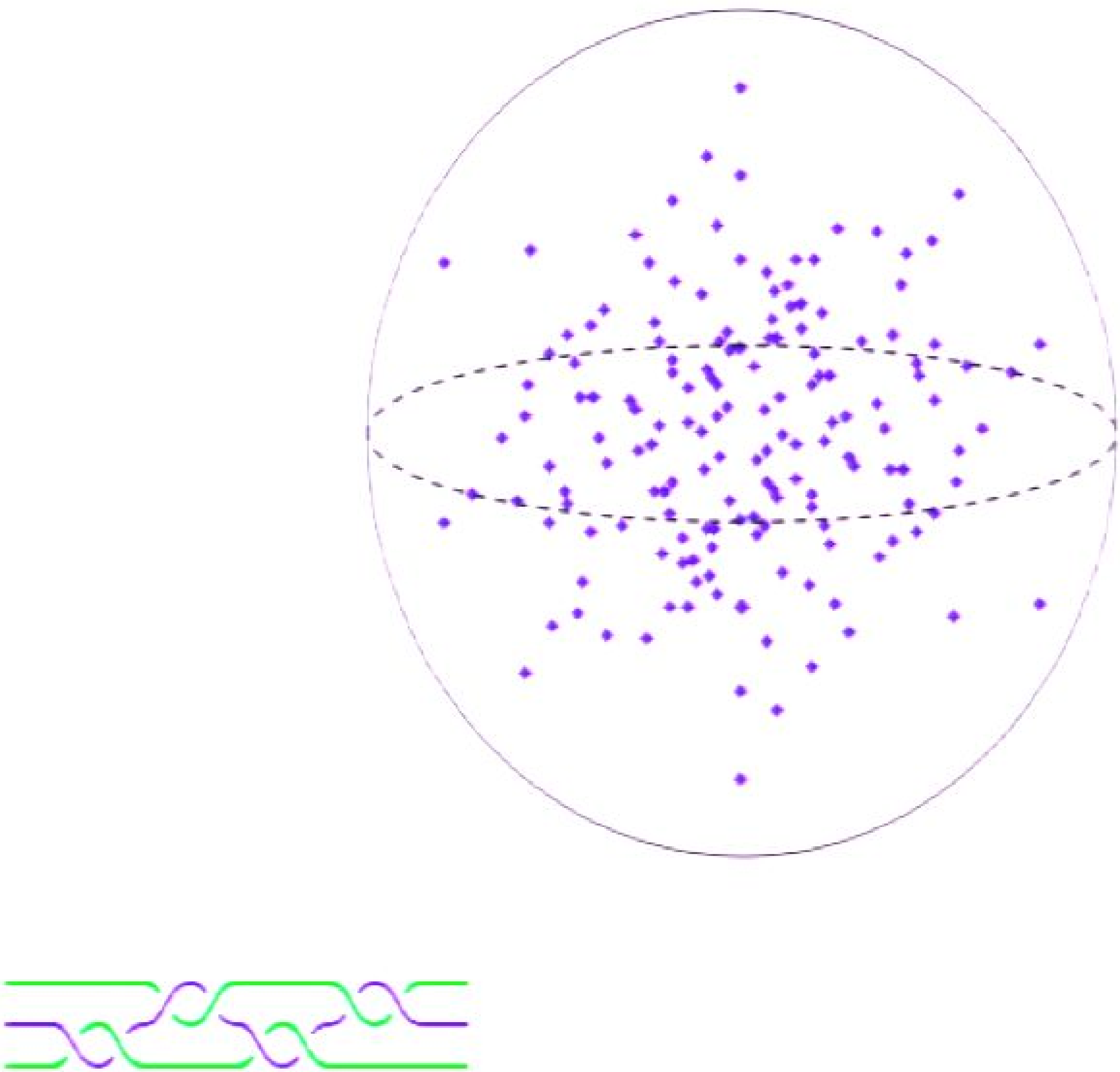
N = 1



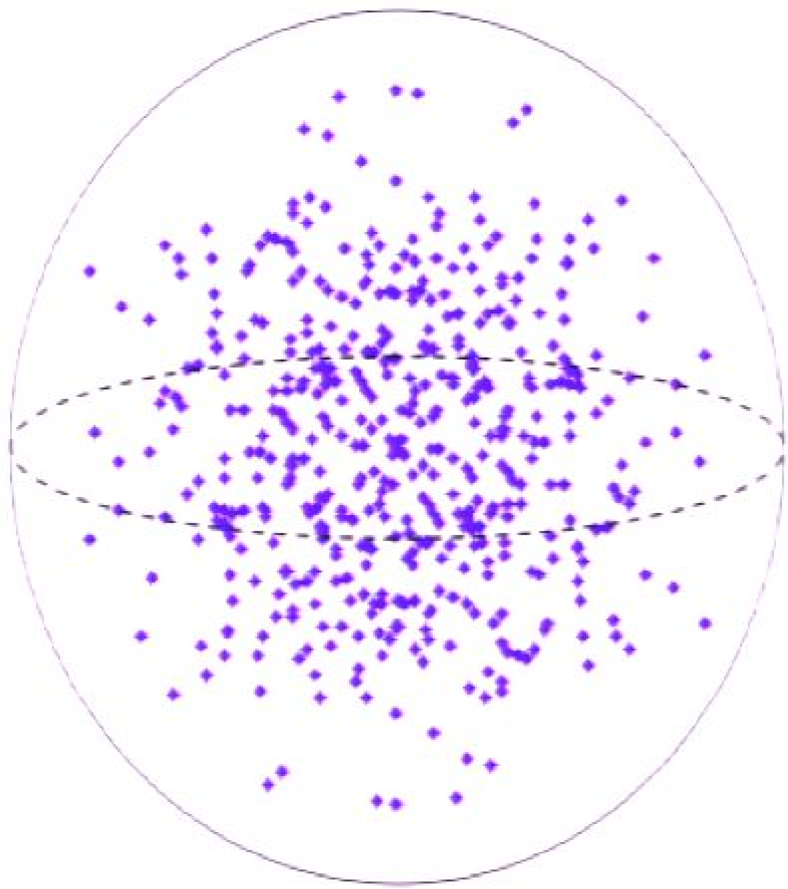
N = 3

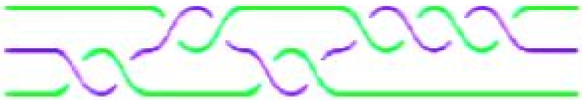


N = 4

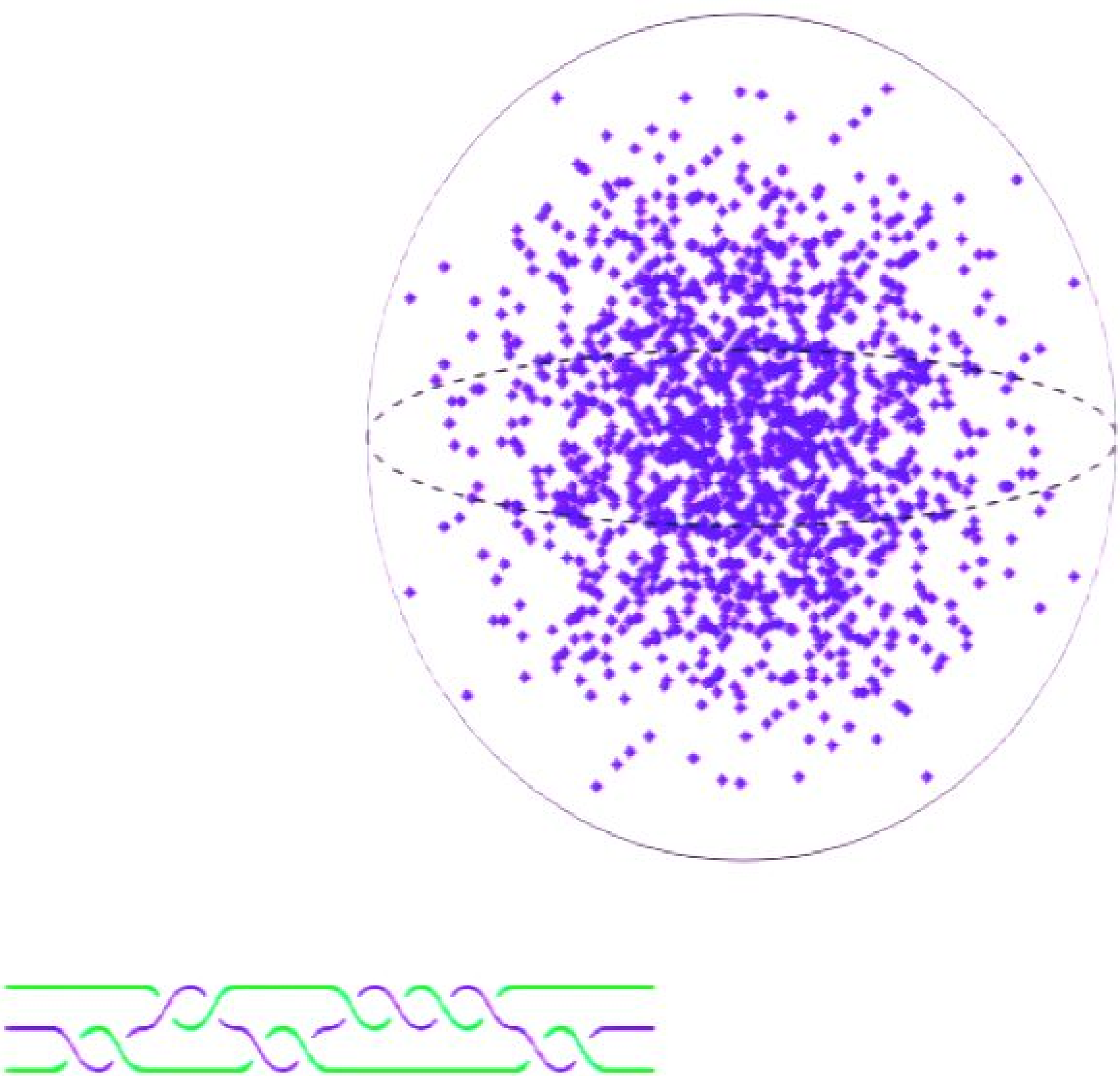


N = 5

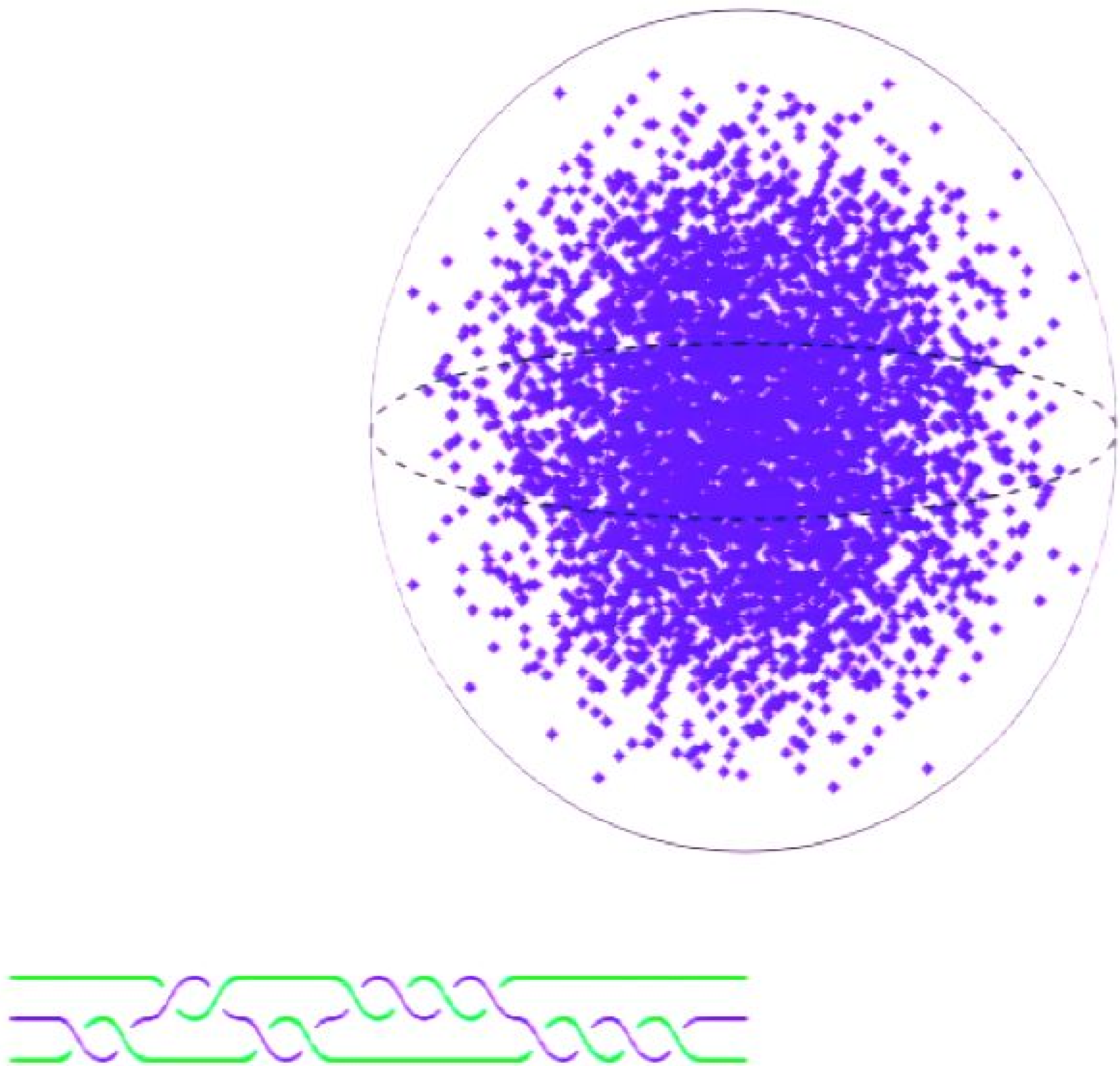




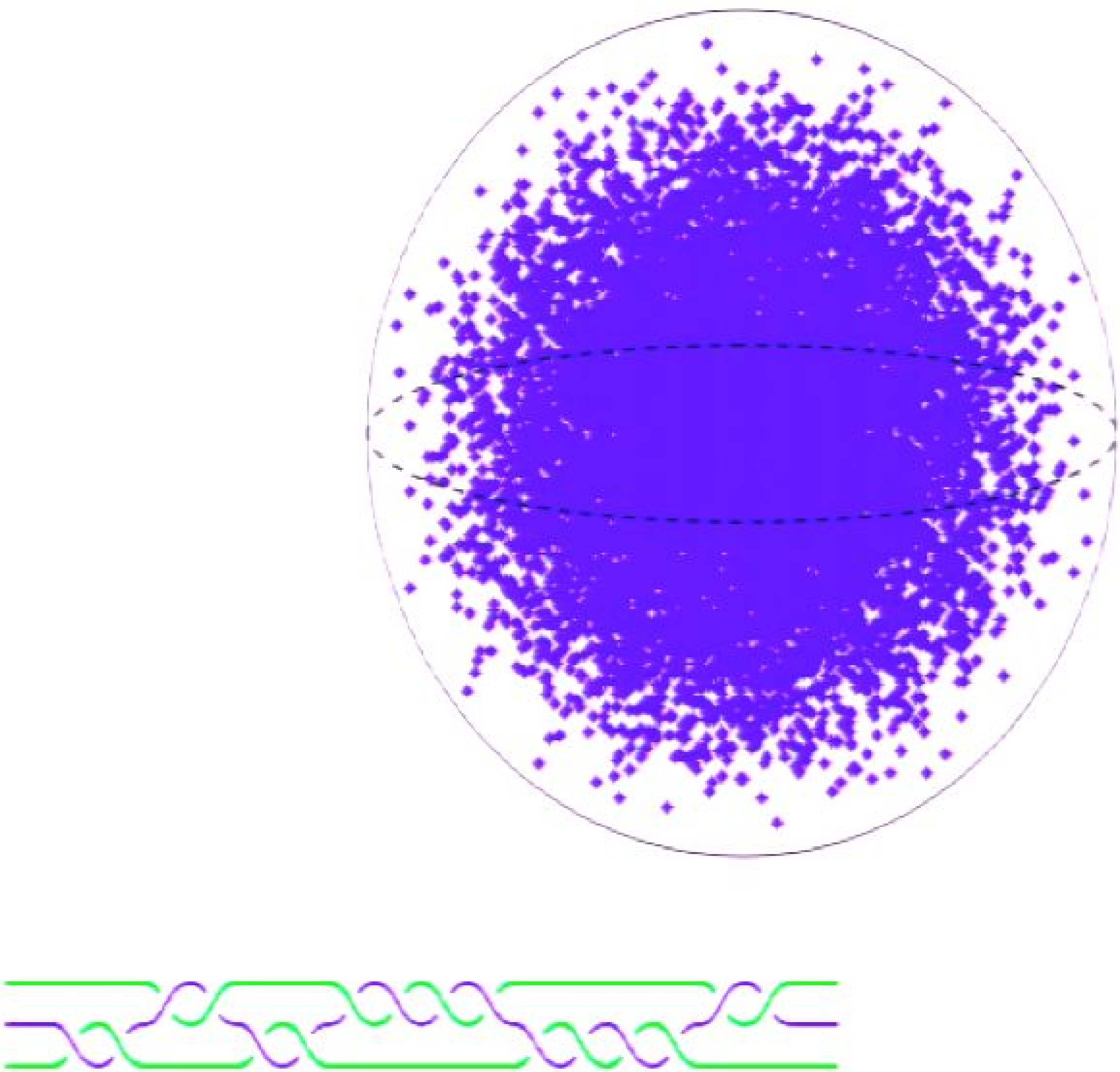
N = 6

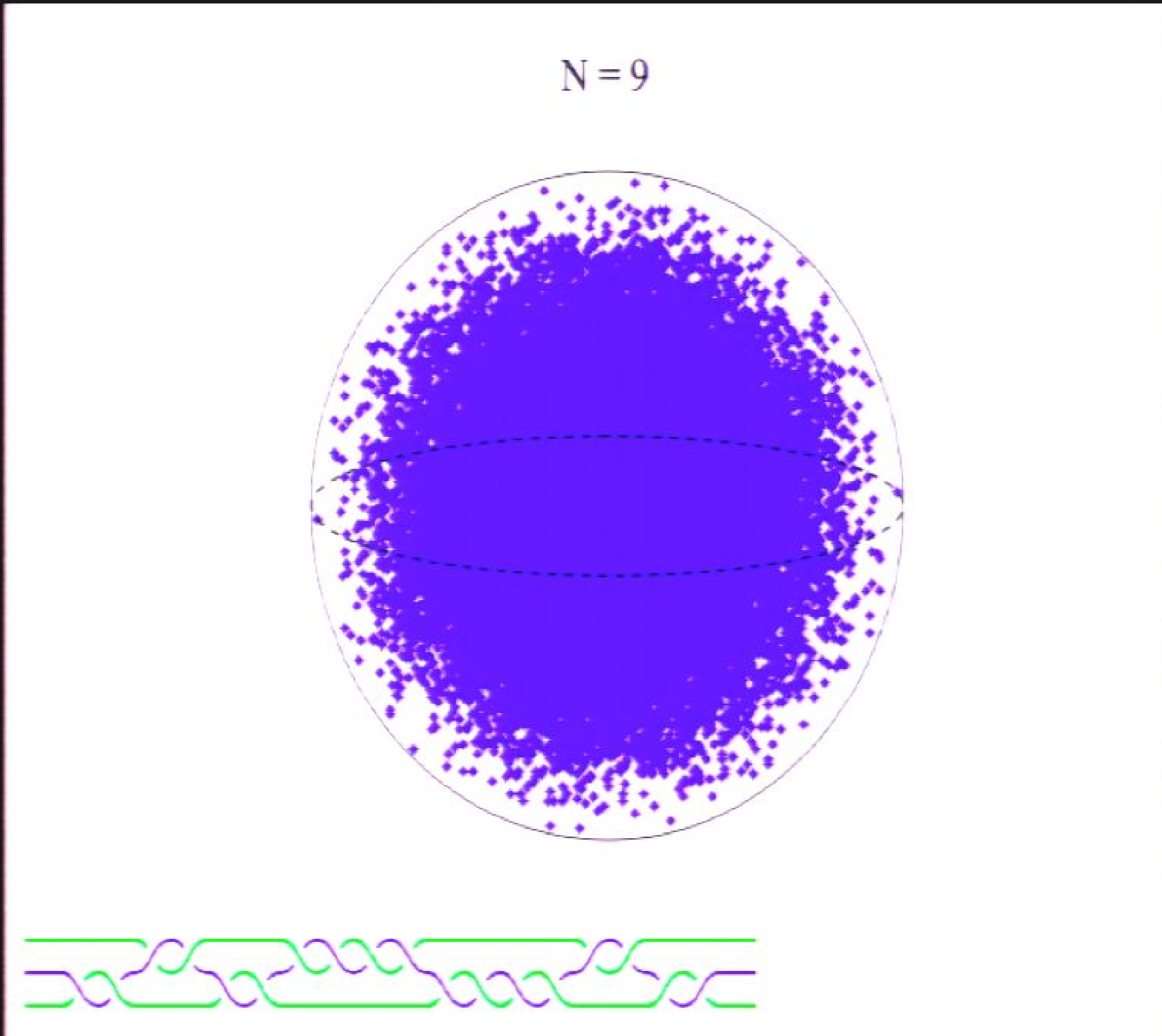


N = 7

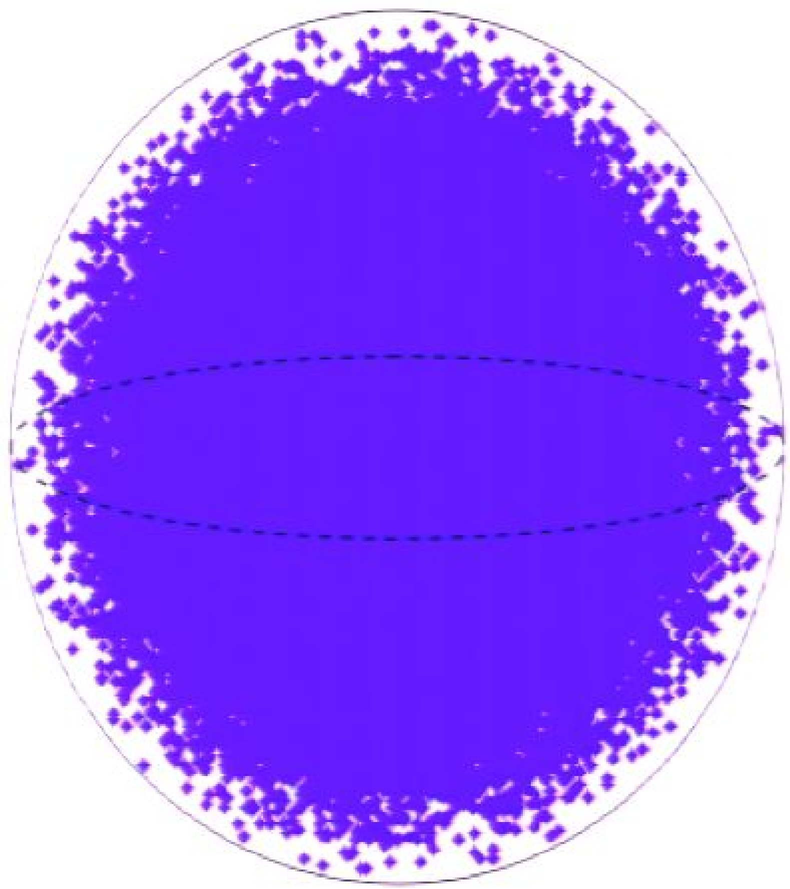


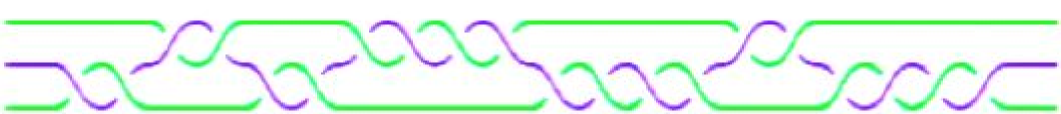
N = 8



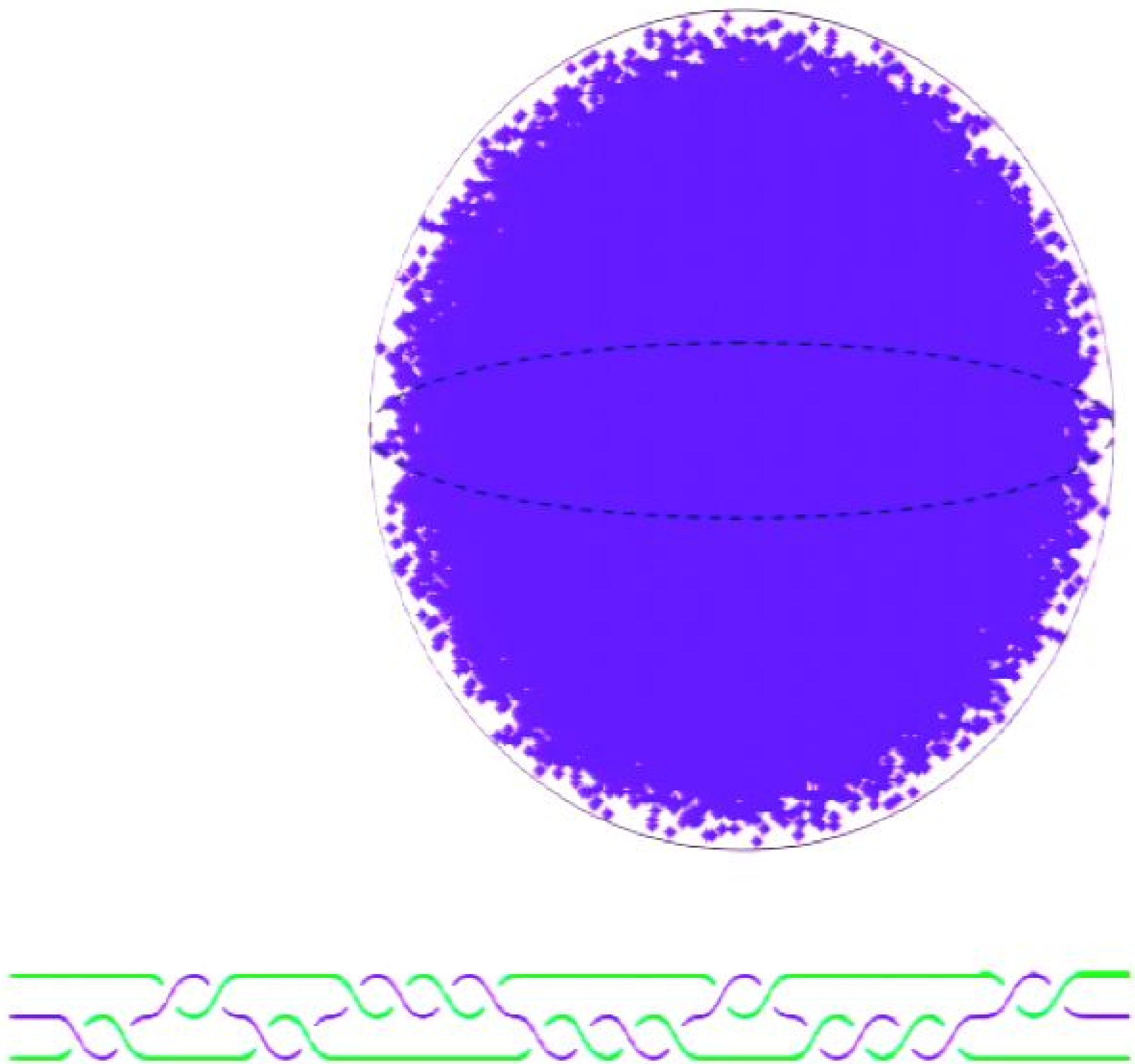

N = 9

N = 10





N = 11



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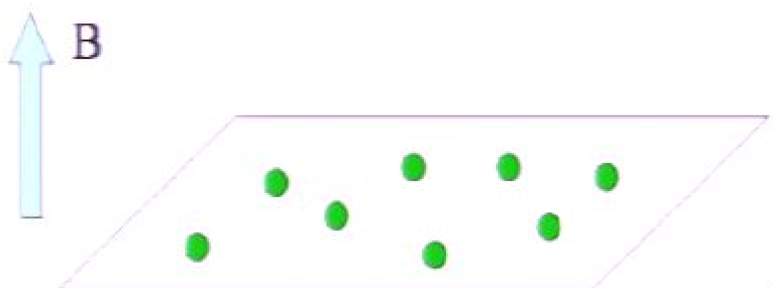
The Fractional Quantum Hall Effect



Occurs when a **two-dimensional electron gas** is placed in a magnetic field

An **incompressible quantum liquid** can form when the Landau level filling fraction $\nu = n_{elec}(hc/eB)$ is a rational fraction.

Quasiparticle excitations can have **fractional charge**.

PI

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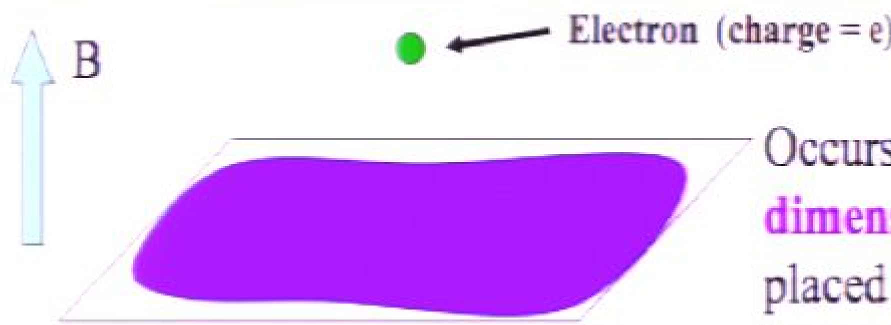

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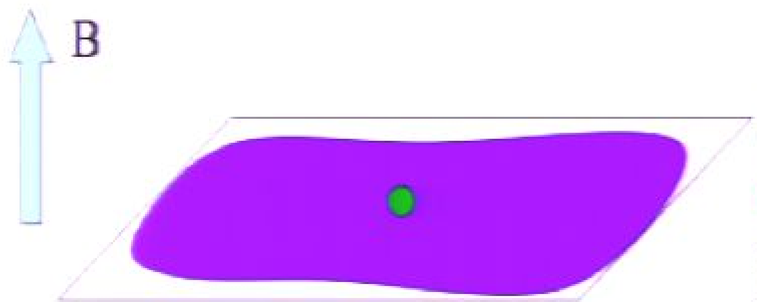

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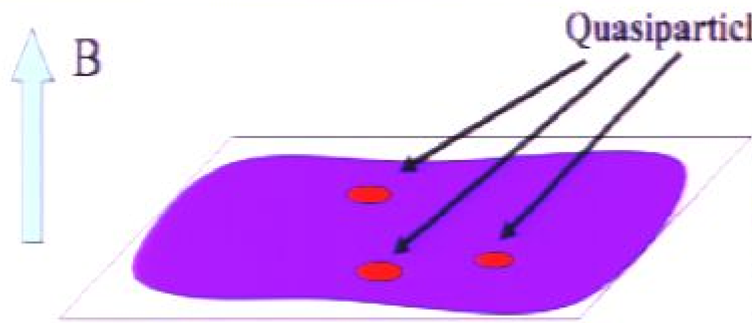

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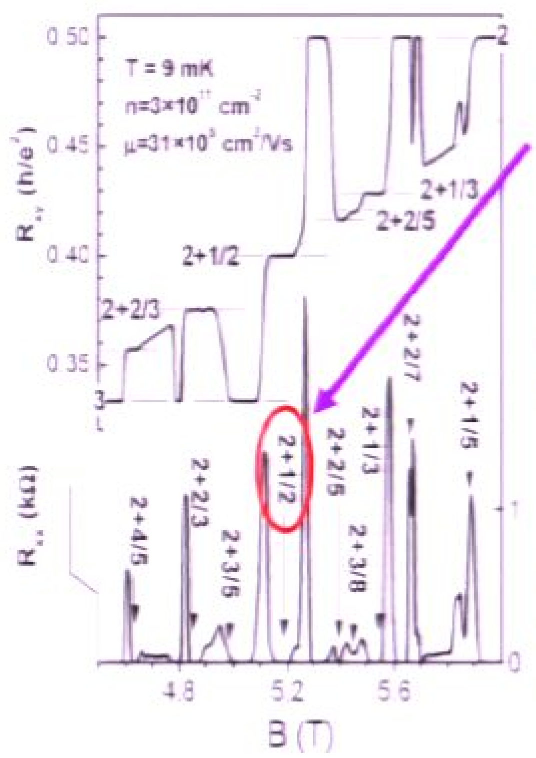


J.S. Xia et al., PRL (2004).

$\nu = 5/2$

Very likely a Moore-Read "Pfaffian" state.

Moore and Read, 1991
Morf, 1998

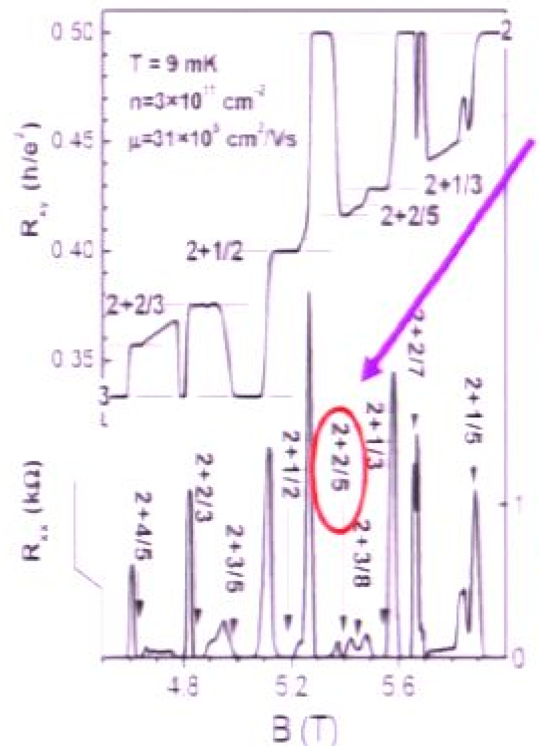
Charge *e/4* quasiparticles with braiding properties described by $SU(2)_2$ Chern-Simons Theory.

Nayak and Wilczek, 1996

Not sufficiently "rich" nonabelian statistics to do universal quantum computation.

But see, S. Bravyi, quant-ph/0511178 and M. Freedman, C. Nayak and K. Walker, cond-mat/0512066.

The Fractional Quantum Hall Effect



J.S. Xia et al., PRL (2004).

$\nu = 12/5$

Possibly a Read-Rezayi k = 3 "Parafermion" state.

Read and Rezyai, 1999

Charge *e/5* quasiparticles with braiding properties described by $SU(2)_3$ Chern-Simons Theory.

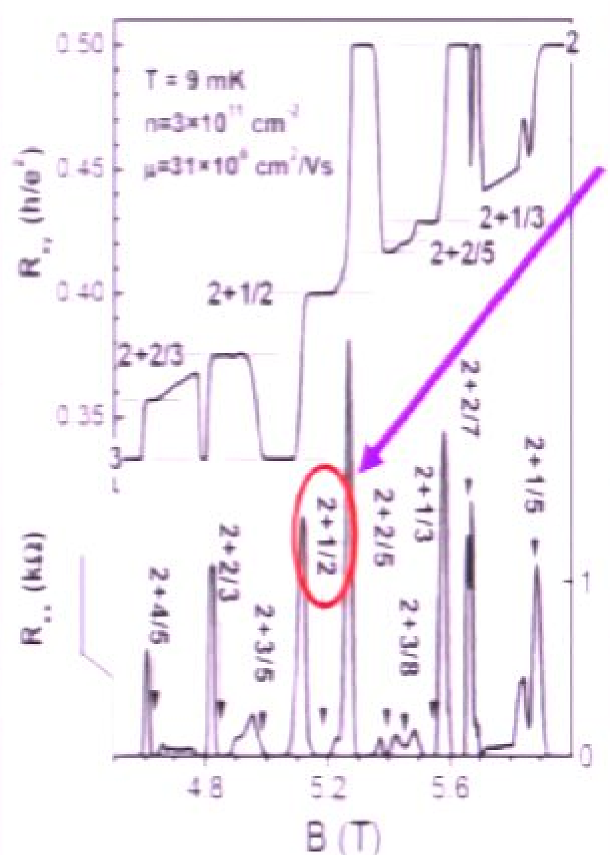
Slingerland and Bais, 2001

$SU(2)_3$ ***is*** sufficiently "rich" to do universal quantum computation.

Freedman, Larsen, and Wang, 2001

PI

The Fractional Quantum Hall Effect



J.S. Xia et al., PRL (2004).

Read Rezayi states

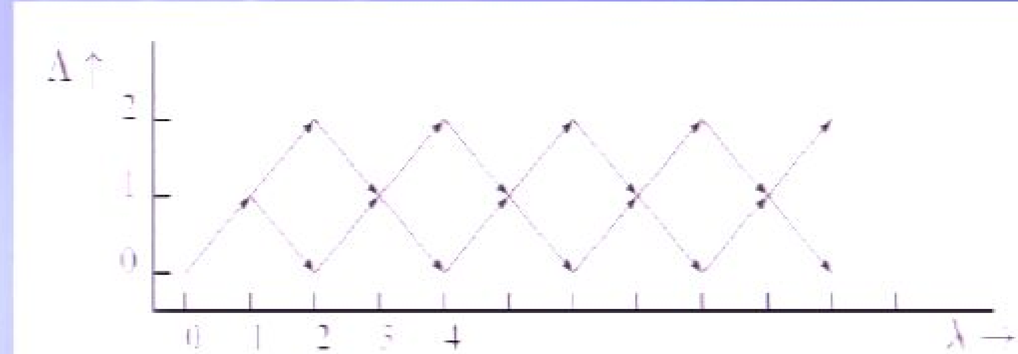
$$SU(2)_k / U(1)_k \otimes \mathit{Chiral\ boson}$$

Filling fraction: $\nu = \dfrac{k}{kM+2}$

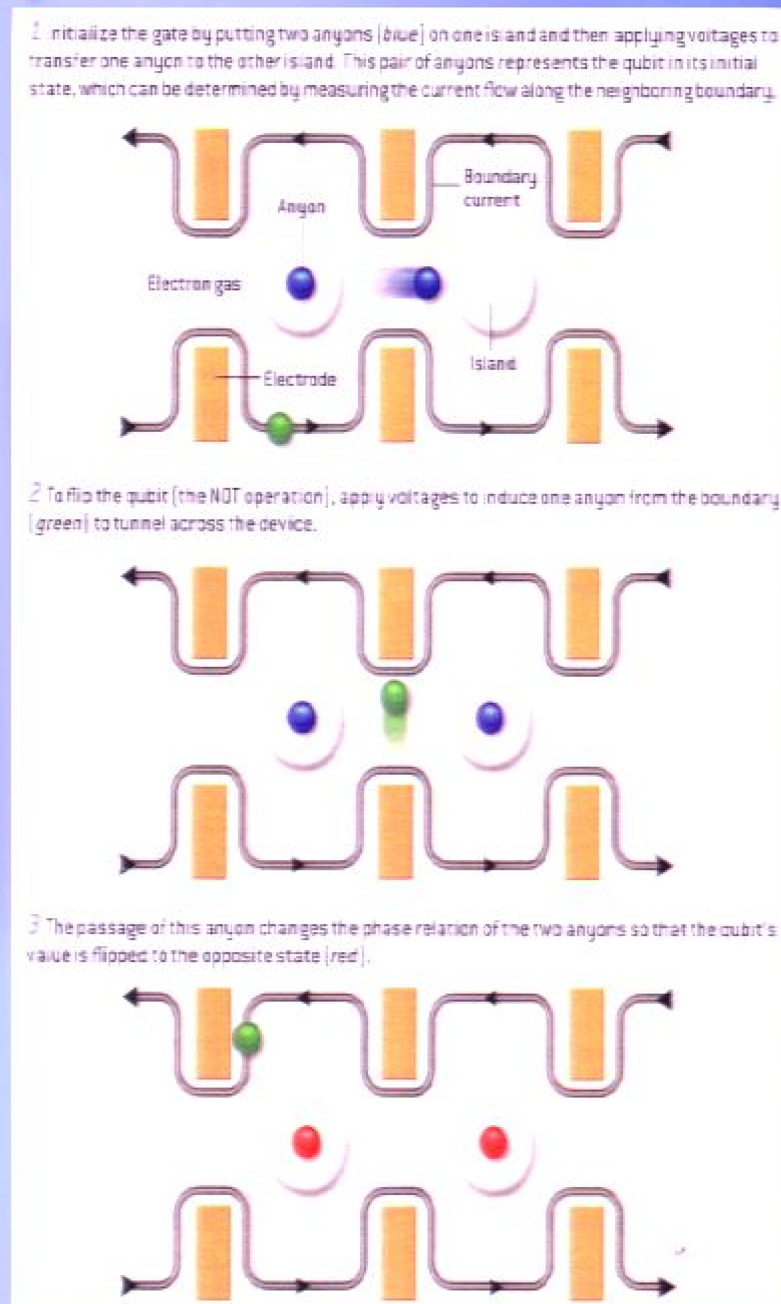
Quantum group:

$$A_{q_1,q_2} := U_{q_1}(sl(2)) \otimes CZ_{4k,q_2}$$

with $q_1 = e^{2\pi i/k+2}$ and $q_2 = e^{-\pi i/k}$

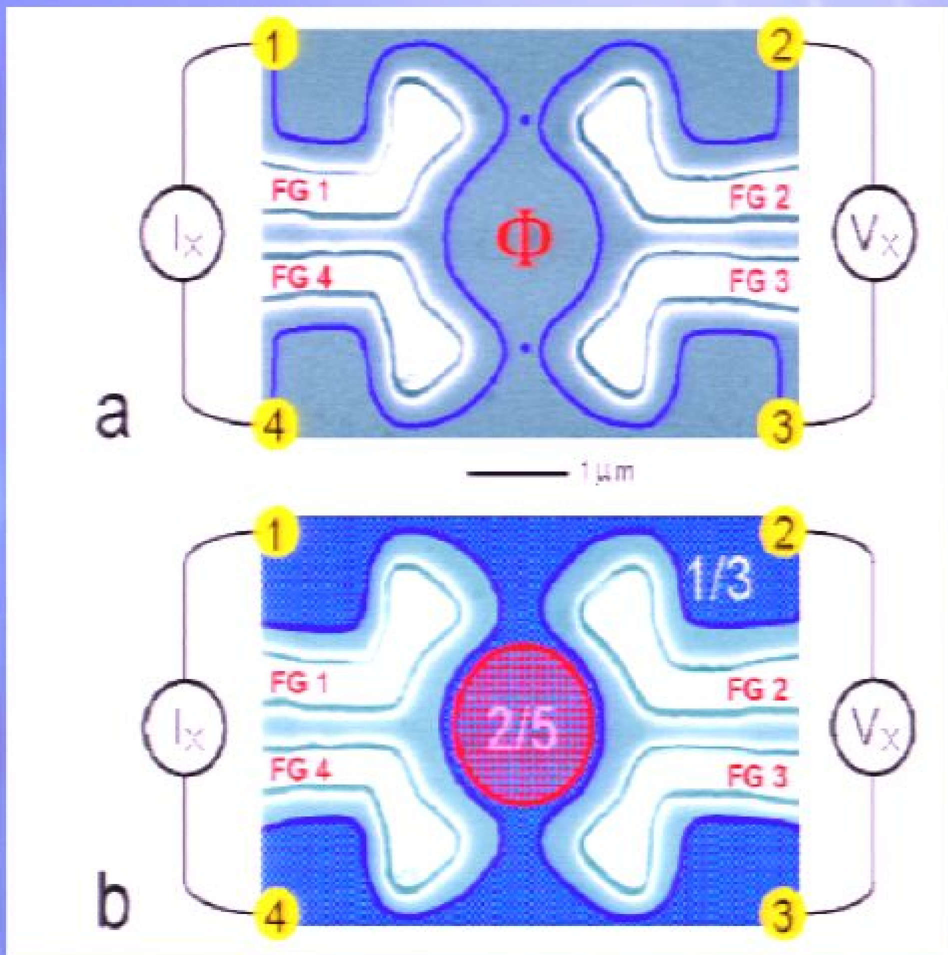


Manipulation of anyons (Das Sarma, Freedman Nayak)



Quasiparticle interferometers

(Goldman et al.)

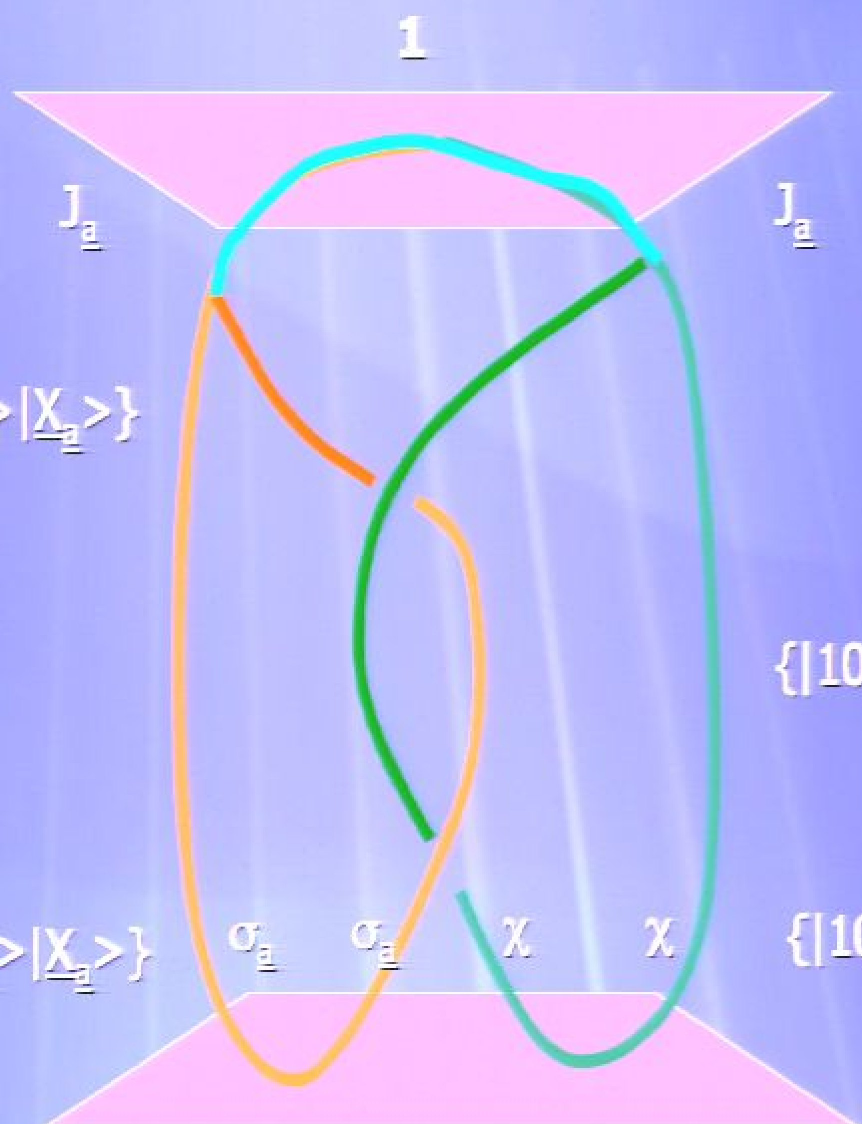


Theory:

E. Verlinde (1991)
Overbosch, Bais (2001)
Das Sarma, Freedman (2005)
Stern, Halperin (2006)
Bonderson, Kitaev, Shtengel, Slingerland ... (2006, 2007)

Entanglements (FAB, De Wild Propitius (1994))

$\{|\underline{X}_{\underline{a}}\rangle|X_{\underline{a}}\rangle-|X_{\underline{a}}\rangle|\underline{X}_{\underline{a}}\rangle\}$



$\{|10\rangle|01\rangle+|01\rangle|10\rangle\}$

$\{|\underline{X}_{\underline{a}}\rangle|X_{\underline{a}}\rangle+|X_{\underline{a}}\rangle|\underline{X}_{\underline{a}}\rangle\}$

$\{|10\rangle|01\rangle-|01\rangle|10\rangle\}$

1

1

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PI

Breaking quantum symmetries

On breaking:

- FAB, Schroers, Slingerland. PRL 89:18601, 2002
- FAB, Schroers, Slingerland. JHEP 05:068, 2003
- FAB, Mathy, Phys. Rev. B 73, 224120 (2006) , cond-mat/0602109 (Ann. of Phys (in press)), cond-mat/0602115 (Annals of Phys (in press))
- Beekman, Zaanen (in progress)
- FAB, Slingerland, (in preparation)
- FAB, Romers (in preparation)

Topological Symmetry Breaking and Bose Condensation (FAB, Schroers, Slingerland)

- Can describe topological order by extended "symmetry" concepts: TQFTs, Tensor Categories, Hopf Algebras, Quantum Groups

Particle types	⟷	Irreducible representations
Fusion	⟷	Tensor Product
Braiding	⟷	R-matrix
Twist	⟷	Ribbon Element

- IDEA:

  Relate topological phases by "Symmetry Breaking"

- Mechanism: Order parameter <--> Bose Condensate.

  Break the Quantum Group to the "Stabiliser" of the (dis)order parameter

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Criteria on condensate and residual symmetry

(BSS)

0. We assume a macroscopic number of particles to condense in a state $|\phi_0>$ in some representation Π_ϕ
1. We require the condensate to be bosonic.
2. We require that $|\phi_0>$ braids trivially with itself.

$$R\,(|\phi_0> \times |\phi_0>) = (|\phi_0> \times |\phi_0>)$$

3. The residual symmetry algebra T consists of the elements *a* that leave the vacuum state "invariant":

$$\Pi_\phi\,(a)\,|\phi_0> = \varepsilon\,(a)\,|\phi_0>$$

Note: T need not necessarily be a Hopf algebra

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What is the physics of breaking?

- Construct representations Ω_j of $T \leq D(H)$
- Decompose Π^{A}_{α} into $\{\Omega_j\}$
- Look at braid relations of $|\phi_r>$ and states in other T reps. (!?)
- If $|\phi_r>$ and $|w> \varepsilon\ \Omega_j$ have nontrivial braiding (R^2) then:
  → $|\phi_r>$ cannot be single valued around $|w>$
  → Ω_j particles will have a string (domain wall) attached
  → Ω_j particle will be confined!
  Note: this is a property of the representation.
- Particles with trivial braiding survive.
- Tensor products $\Omega_i \times \Omega_j = N_{ijk}\, \Omega_k$

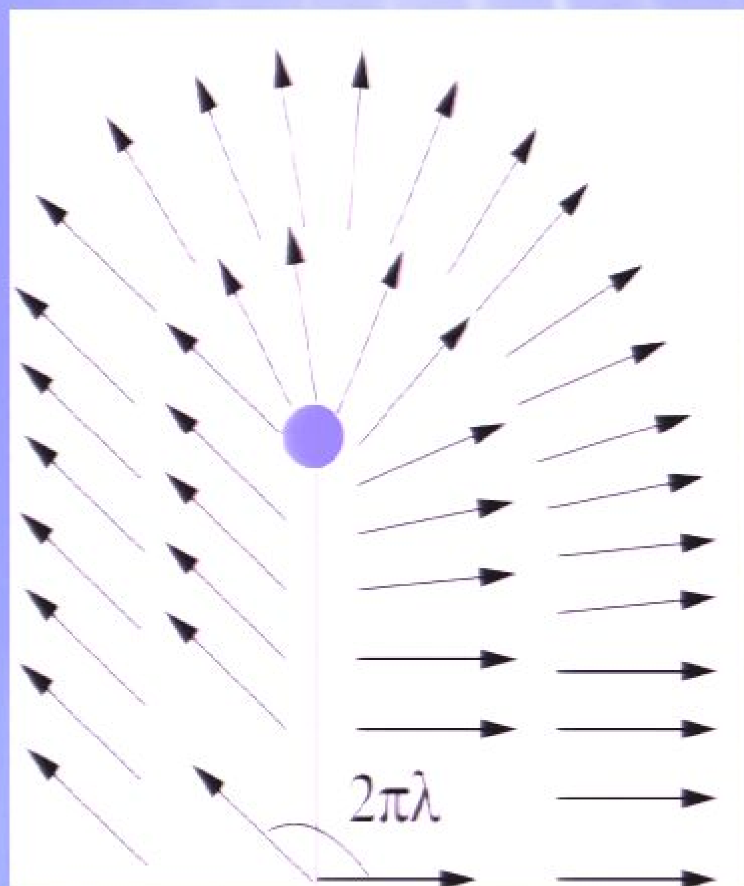
PI

Confined defect

$|\phi_r>$

PI

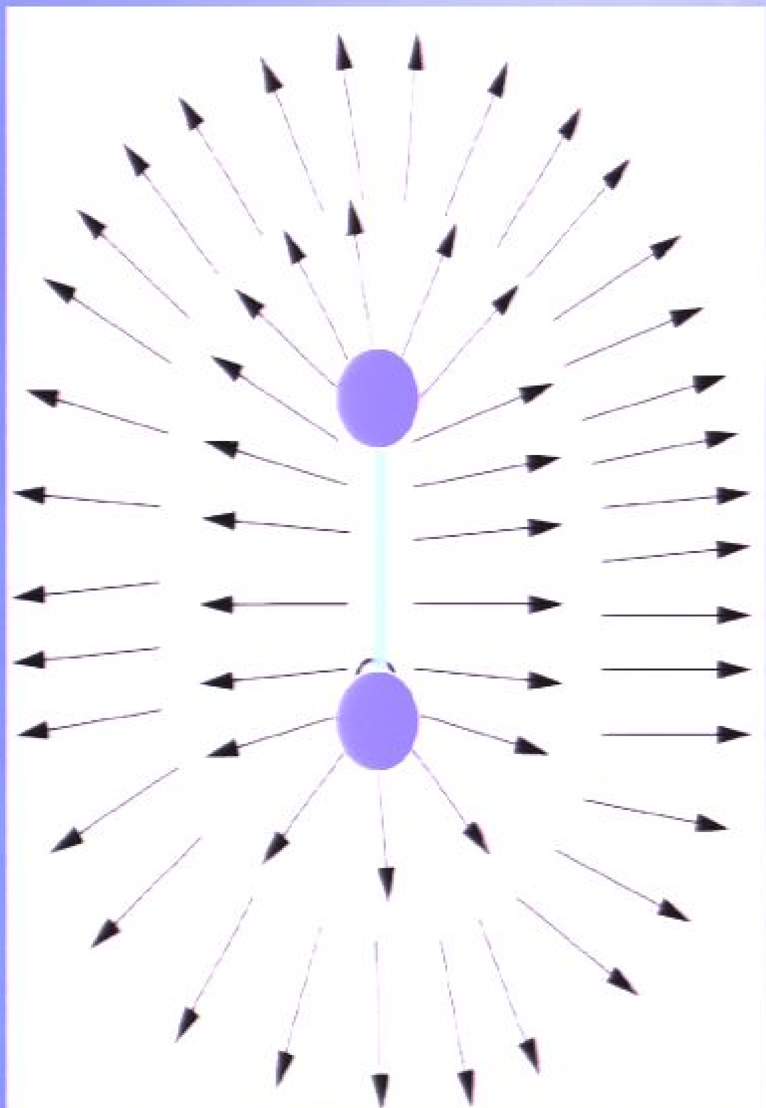
Confined defect



$|\phi_r>$

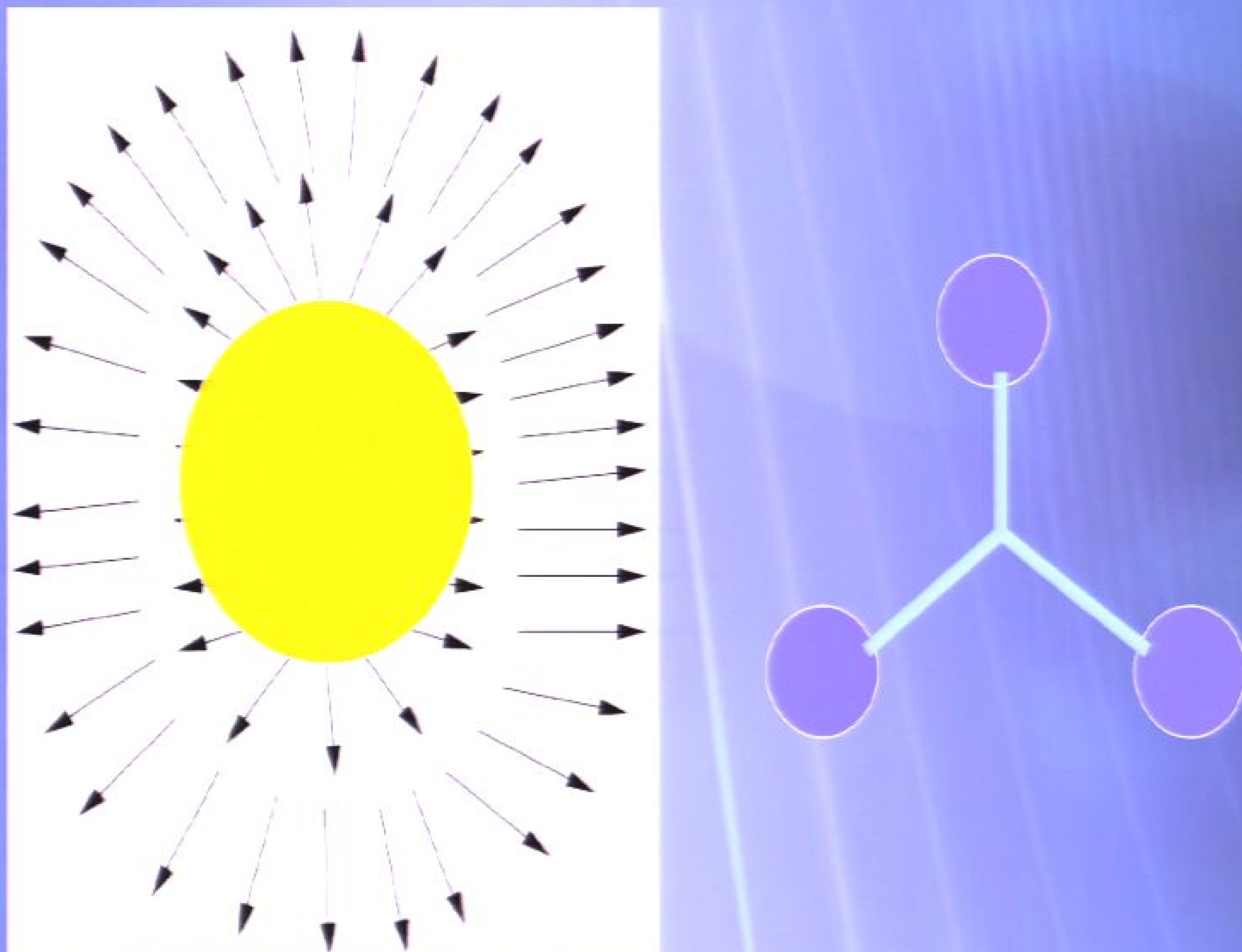
Hadrons (composites) in uniaxial nematic

Hadrons (composites) in uniaxial nematic



PI

Hadrons (composites) in uniaxial nematic



Example: Z_N Gauge Theory / Toric Code

Particles are:

- Electric charges, labelled by representations of the gauge group.
  Under gauge transformations:

$$(\alpha \in Z_N) : |q\rangle \mapsto e^{\frac{2\pi i q \alpha}{N}} |q\rangle$$

- Magnetic fluxes, labelled by monodromies (Wilson loop) 0,1,…N-1
  Can think of these as carriers of representations of a dual group (also Z_N)
- Dyons, with flux and charge (transform under Z_N x Z_N).

Topological interactions:

- Fusion (described by tensor product of Z_N x Z_N reps)
- Monodromies, Aharonov-Bohm effect, phase factors are:

$$\exp\left(\frac{2\pi i(q_1 p_2 + p_1 q_2)}{N}\right)$$

Condensation described by breaking of the full symmetry group (incl dual).
On top of that, have confinement, from AB-interactions

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Residual Hopf symmetry

- The non-confined representations of T form a closed set under tensor product of T
- This set can be viewed as the representations of yet another Hopf algebra U
- There is a surjective map $\Gamma : T \rightarrow U$
- Walls are characterized by reps of *Ker* Γ

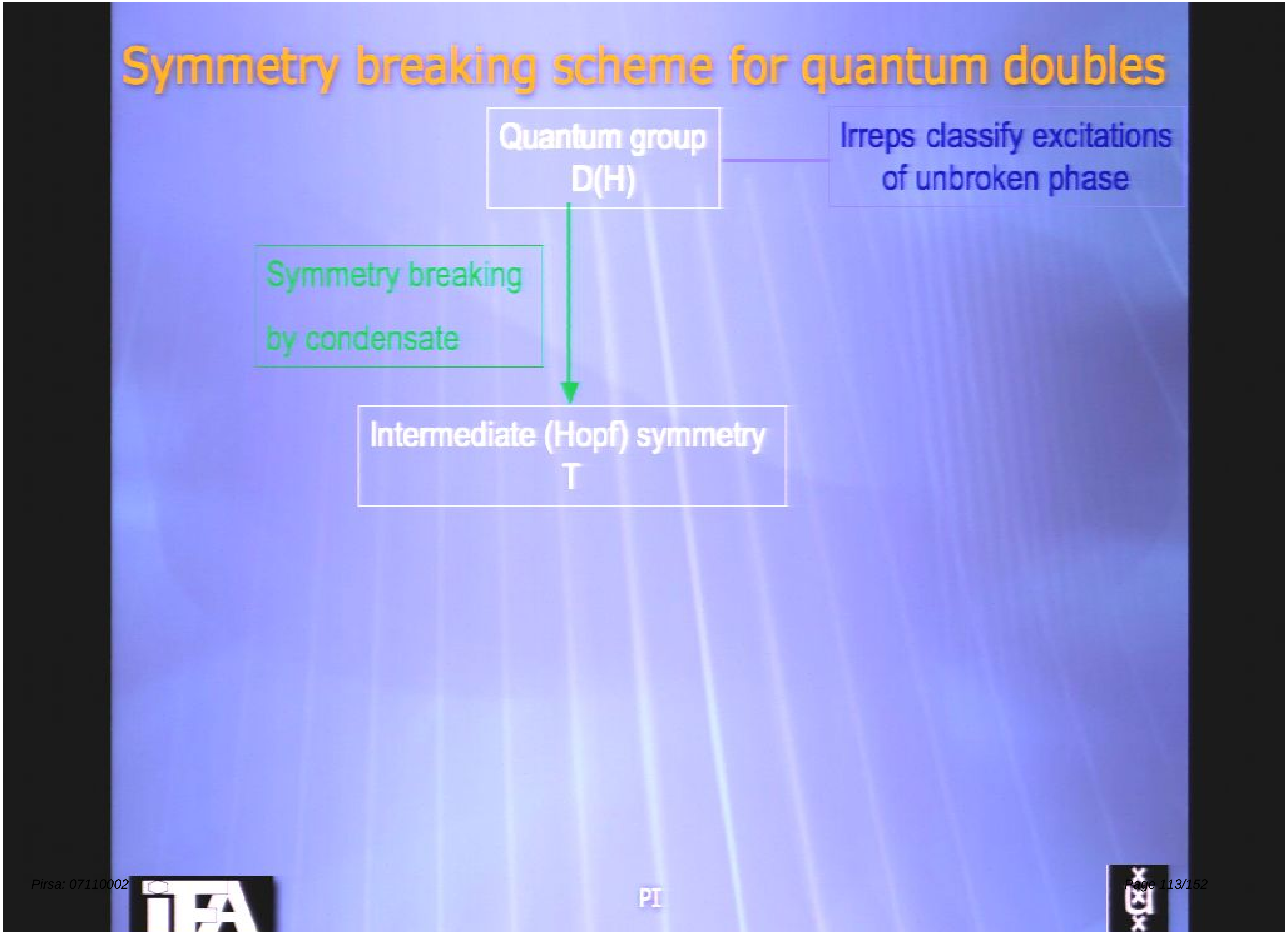
PI

Symmetry breaking scheme for quantum doubles

Quantum group
D(H)

Irreps classify excitations
of unbroken phase

PI


Symmetry breaking scheme for quantum doubles
Quantum group
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Irreps classify excitations
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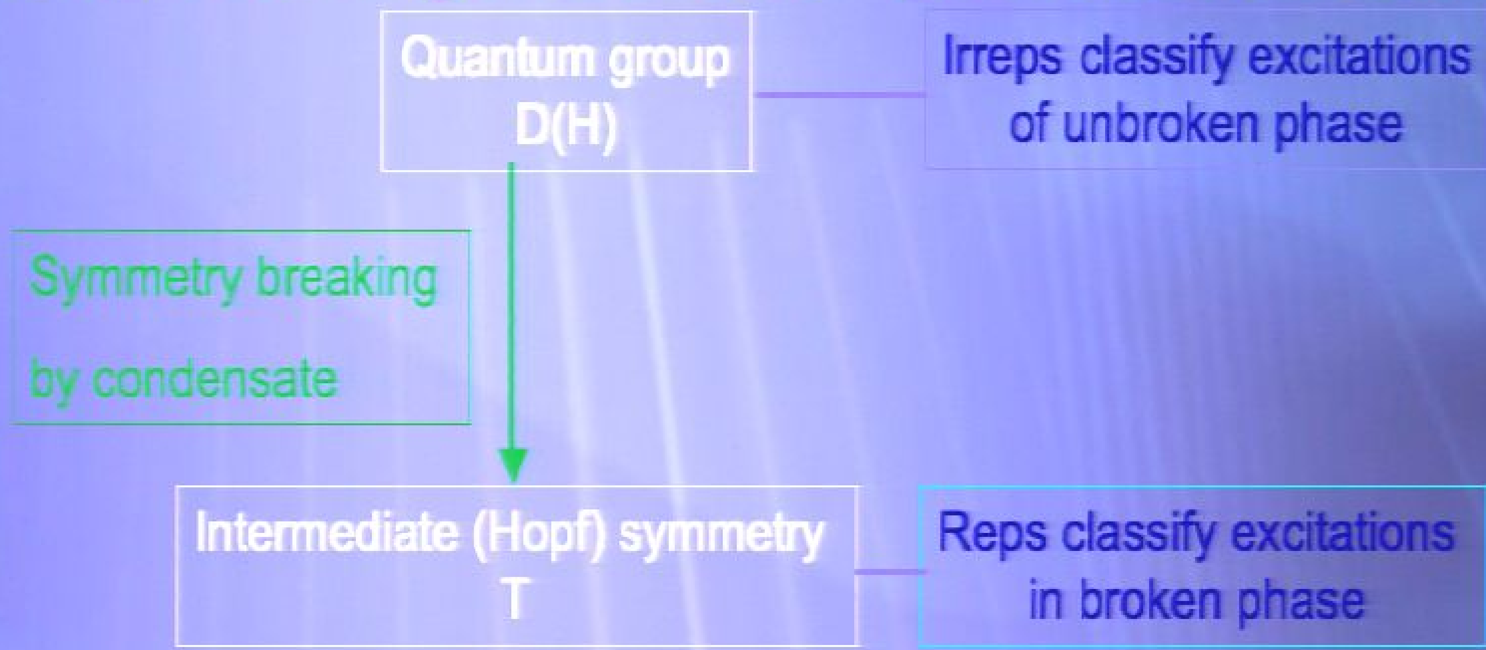
Symmetry breaking
by condensate
Intermediate (Hopf) symmetry
T
PI

Symmetry breaking scheme for quantum doubles



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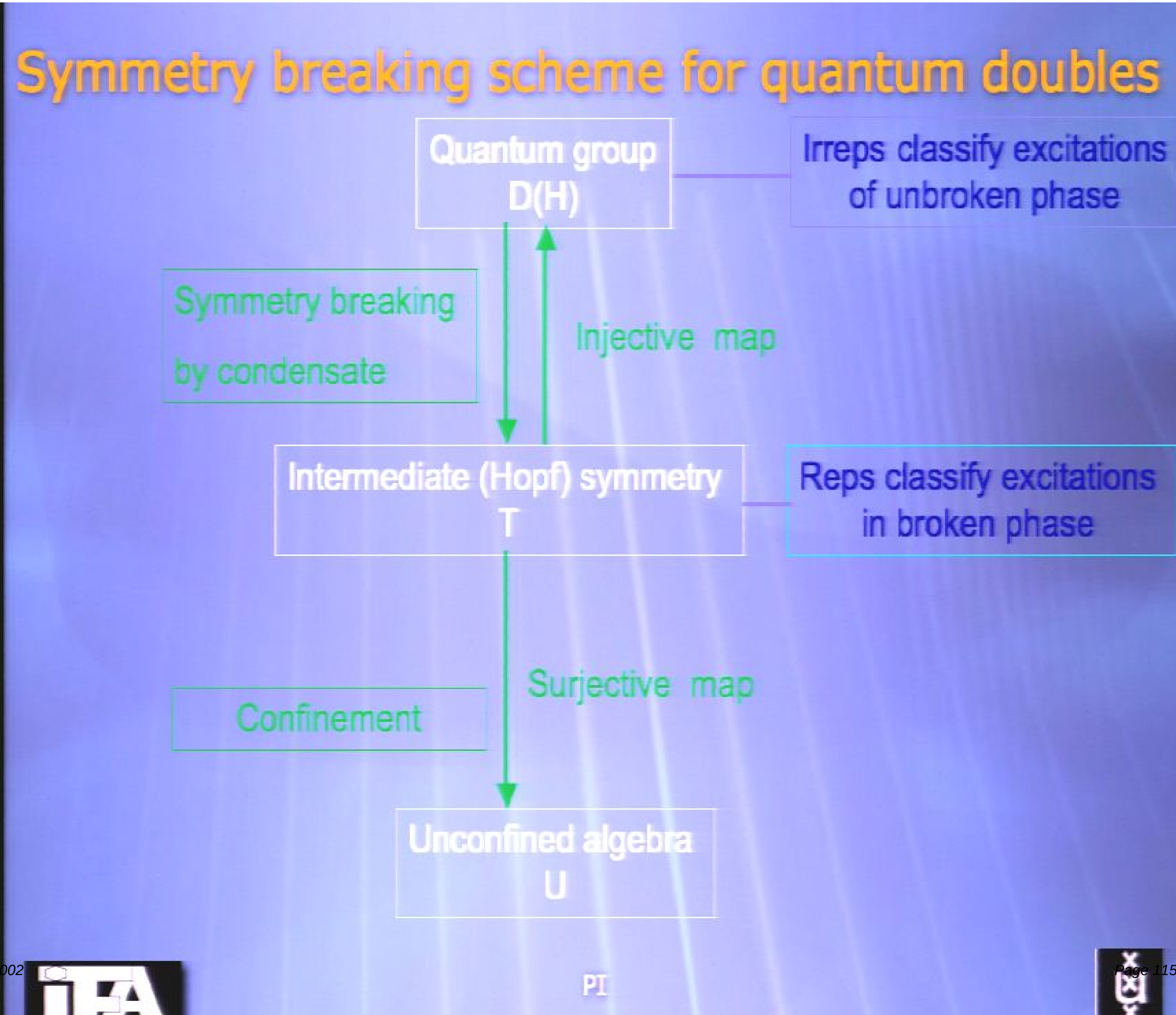
Symmetry breaking scheme for quantum doubles
Quantum group
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T
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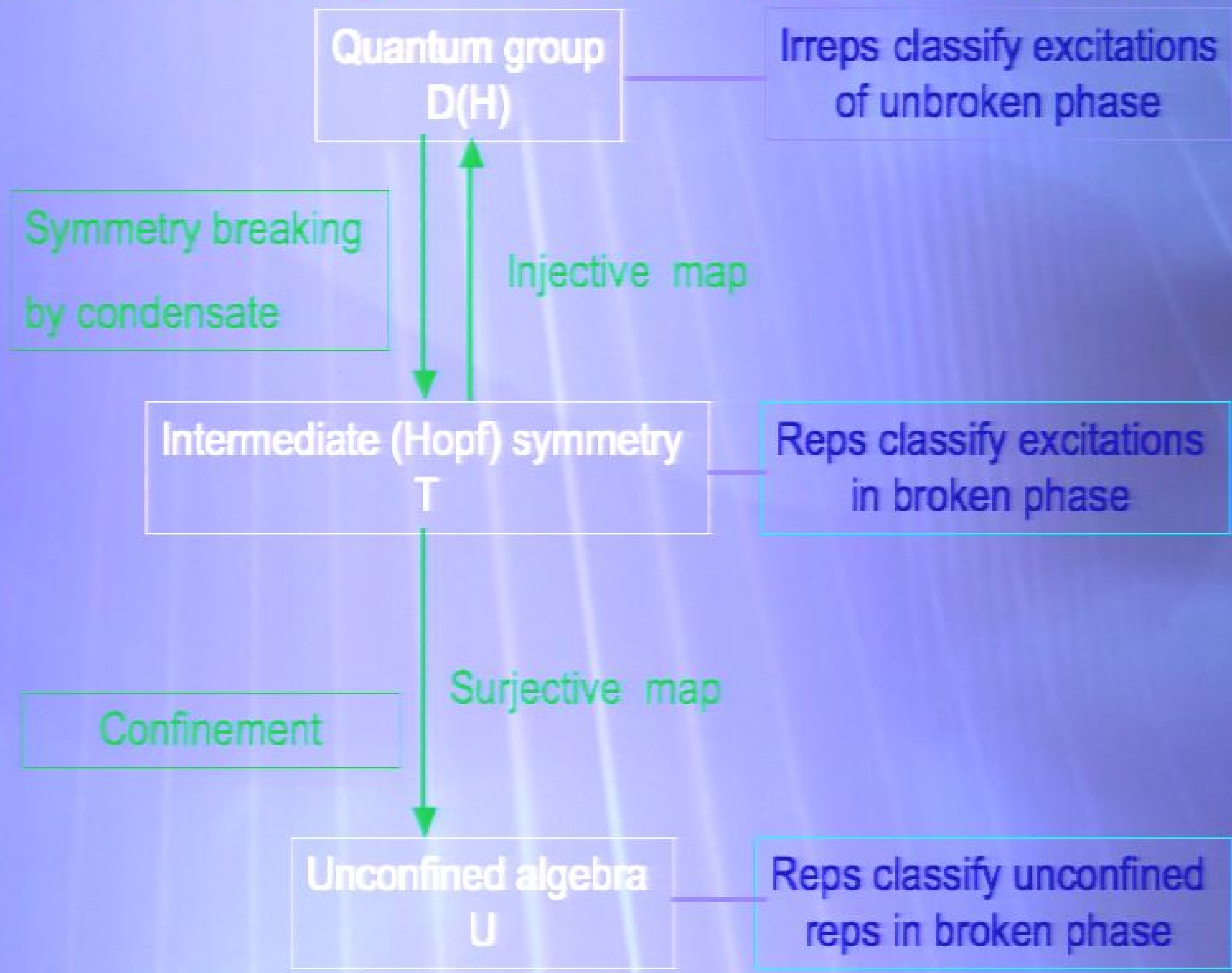
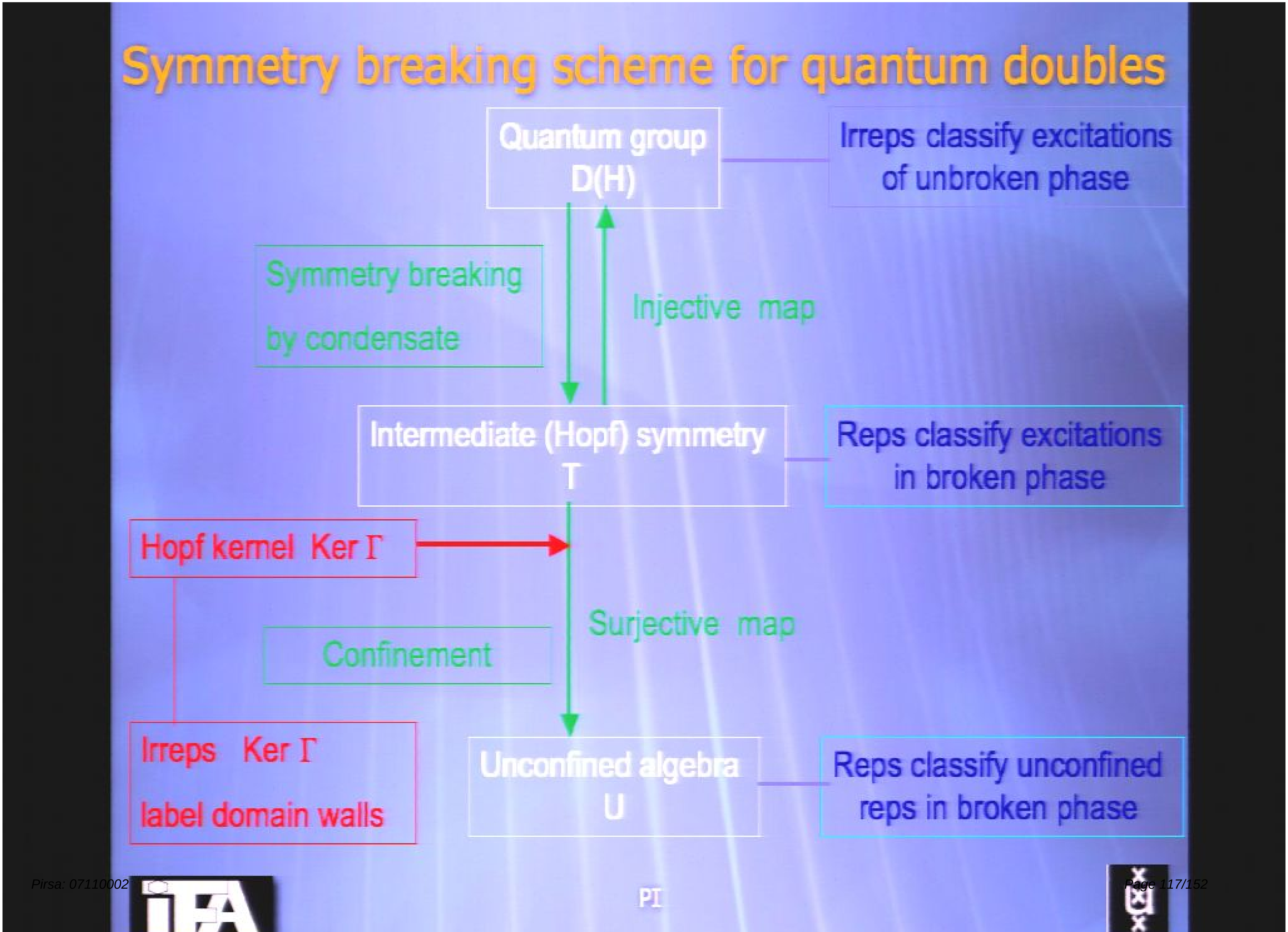
Confinement
Unconfined algebra
U
PI

Symmetry breaking scheme for quantum doubles

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Hopf kernel Ker Γ
Surjective map
Confinement
Irreps Ker Γ
label domain walls
Unconfined algebra
U
Reps classify unconfined
reps in broken phase
PI

Symmetry Breaking using representation theory

(FAB,Slingerland)

Use dual of algebra, Just Labels and Branching:

$$a \to \sum_i n_{a,i} a_i$$

Three requirements:

1. The new labels themselves form a fusion algebra (need associativity, vacuum and charge conjugation)
2. Branching and fusion are compatible,

$$a \otimes b = c \to (\sum_i n_{a,i} a_i) \otimes (\sum_j n_{b,j} b_j) = \sum_k n_{c,k} c_k$$

3. Not more branching/identification than necessary for 1. and 2. (want the full "stabiliser")

PI

Breaking $U_q(su(2))$

$SU(2)_4$

0	$d_0 = 1$	$h_0 = 0$
1	$d_1 = \sqrt{3}$	$h_1 = 1/8$
2	$d_2 = 2$	$h_2 = 1/3$
3	$d_3 = \sqrt{3}$	$h_3 = 5/8$
4	$d_4 = 1$	$h_4 = 1$

$1\times 1 = 0+2$			
$1\times 2 = 1+3$	$2\times 2 = 0+2+4$		
$1\times 3 = 2+4$	$2\times 3 = 1+3$	$3\times 3 = 0$	
$1\times 4 = 3$	$2\times 4 = 2$	$3\times 4 = 1$	$4\times 4 = 0$

Λ $\quad d_\Lambda = [\Lambda+1]_q$ $\quad h_\Lambda = \dfrac{\Lambda(\Lambda+2)}{4(k+2)}$

Condensate, splitting and identification

Assume a bosonic condensate forms in the 4 rep of $SU(2)_4$:

$$2\times 2 = 0+2+4 = 0+2+0$$

$\Rightarrow 2 := 2_1 + 2_2$ possible because $d_2=2$

$$2_1\times 2_1 + 2_1\times 2_2 + 2_2\times 2_1 + 2_2\times 2_2 = 0 + 2_1 + 2_2 + 0$$

$$\Rightarrow 2_1\times 2_2 = 0$$

if $2_1\times 2_1 = 2_1$

then $2_2\times(2_1\times 2_1) = 2_2\times 2_1 = 0$

$(2_2\times 2_1)\times 2_1 = 0\times 2_1 = 2_1$

$\Rightarrow 2_1\times 2_1 = 2_2$ and $2_2\times 2_2 = 2_1$

(follows also from associativity)

$$1\times 1 = 0 + 2_1 + 2_2$$

$$1\times 3 = 0 + 2_1 + 2_2$$

$$\Rightarrow 1 \Leftrightarrow 3$$

0

1

→ $2 := 2_1 + 2_2$

$3 \Leftrightarrow 1$

$4 \Leftrightarrow 0$

$1\times 1 = 0+1$		
$1\times 2_1 = 1$	$2_1\times 2_1 = 2_2$	
$1\times 2_2 = 1$	$2_1\times 2_2 = 1$	$2_2\times 2_2 = 2_2$

Breaking $U_q(su(2))$

$SU(2)_4$

0	$d_0 = 1$	$h_0 = 0$
1	$d_1 = \sqrt{3}$	$h_1 = 1/8$
2	$d_2 = 2$	$h_2 = 1/3$
3	$d_3 = \sqrt{3}$	$h_3 = 5/8$
4	$d_4 = 1$	$h_4 = 1$

$1\times 1 = 0 + 2$			
$1\times 2 = 1 + 3$	$2\times 2 = 0 + 2 + 4$		
$1\times 3 = 2 + 4$	$2\times 3 = 1 + 3$	$3\times 3 = 0$	
$1\times 4 = 3$	$2\times 4 = 2$	$3\times 4 = 1$	$4\times 4 = 0$

$$\Lambda \qquad d_\Lambda = [\Lambda + 1]_q \qquad h_\Lambda = \frac{\Lambda(\Lambda+2)}{4(k+2)}$$

Condensate, splitting and identification

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if $2_1\times 2_1 = 2_1$

then $2_2\times(2_1\times 2_1) = 2_2\times 2_1 = 0$

$(2_2\times 2_1)\times 2_1 = 0\times 2_1 = 2_1$

$\Rightarrow 2_1\times 2_1 = 2_2$ and $2_2\times 2_2 = 2_1$

(follows also from associativity)

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$\Rightarrow 1 \Leftrightarrow 3$

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→ $2 := 2_1 + 2_2$

$3 \Leftrightarrow 1$

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PI

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Breaking $U_q(su(2))$

$SU(2)_4$

0	$d_0 = 1$	$h_0 = 0$
1	$d_1 = \sqrt{3}$	$h_1 = 1/8$
2	$d_2 = 2$	$h_2 = 1/3$
3	$d_3 = \sqrt{3}$	$h_3 = 5/8$
4	$d_4 = 1$	$h_4 = 1$

$1\times 1 = 0+2$			
$1\times 2 = 1+3$	$2\times 2 = 0+2+4$		
$1\times 3 = 2+4$	$2\times 3 = 1+3$	$3\times 3 = 0$	
$1\times 4 = 3$	$2\times 4 = 2$	$3\times 4 = 1$	$4\times 4 = 0$

Λ $\quad d_\Lambda = [\Lambda+1]_q$ $\quad h_\Lambda = \dfrac{\Lambda(\Lambda+2)}{4(k+2)}$

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$$

PI

Relation to Conformal Embedding

Central charges satisfy c(G) = c(H) ==> c(G/H) = 0
with $C(G_k) = k\,\dim G/(k+h)$ h=dual Coxeter number

Coset Virasoro algebra is trivial.
==> Finite branching of inf. Dim. KM representations

Example: $SU(2)_4$ ==> $SU(3)_1$ (c=2)

$SU(3)_1$ Irreps:

1	$d_1 = 1$	$h_1 = 0$
3	$d_3 = 1$	$h_3 = 1/3$
$\bar{3}$	$d_{\bar{3}} = 1$	$h_{\bar{3}} = 1/3$

$3 \times 3 = \bar{3}$
$3 \times \bar{3} = 1$
$\bar{3} \times \bar{3} = 3$

→ branching

$1 \to 0 + 4$
$3 \to 2$
$\bar{3} \to 2$

PI

Topological symmetrybreaking and the coset construction

Consider condensate in G x <u>H:</u>

<u>Example:</u> $G = SU(2)_1 \times SU(2)_1$ $H = SU(2)_2$ $c_{g/h} = 2 - 3/2 = 1/2$

$SU(2)_1$	h	d
0	0	1
1	1/4	1

$SU(2)_2$	h	d
0	0	1
1	3/16	$\sqrt{2}$
2	1/2	1

2x2x3=12 velden

Condensate (boson): (112) = (000)

==> (112)x(112) = (000) (simple current)

PI

Identifications en confinement

(112)x (00;012) = (11;210)

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A physical interpretation of the coset construction!

PI

Identifications en confinement

(112)x (00;012) = (11;210) ⇕ (112)x (11;210) = (00;012)

(112)x (01;012) = (10;210) ⇕ (112)x (10;210) = (01;012)

We are left with 6 fields:

Φ	000	001	002	010	011	012
h	0	13/16	1/2	1/4	1/16	3/4
h′	1	5/16	1/2	3/4	1/16	1/4
Φ'	112	111	110	102	101	100

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We are left with 6 fields:

Φ	000		002		011	
h	0		1/2		1/16	
h' / Φ'	d=1		d=1		$d=\sqrt{2}$	

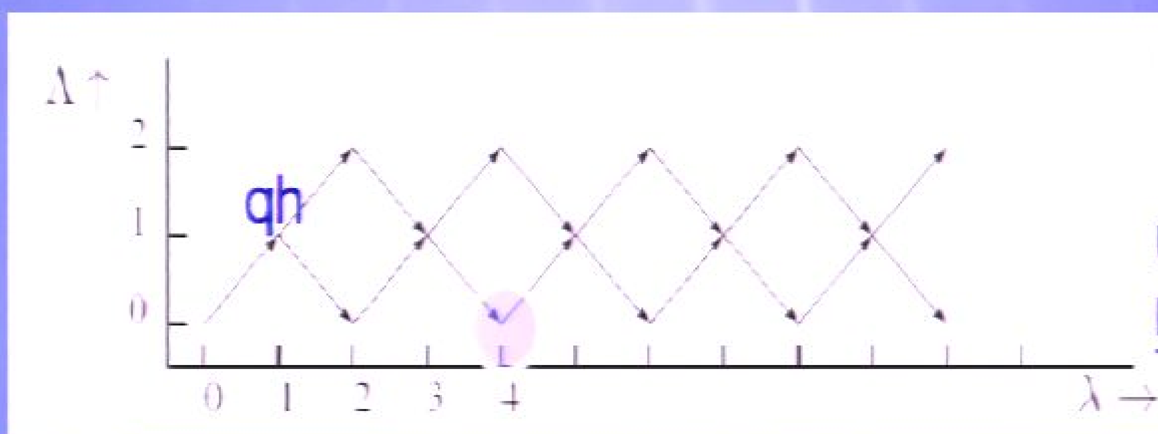
Conclusion: After confinement we are left with the Ising model<==> Coset model.

A physical interpretation of the coset construction!

Bosons in the Read-Rezayi States

- RR states: reps of $A_{q_1,q_2} := U_{q_1}(sl(2)) \otimes CZ_{4k,q_2}$

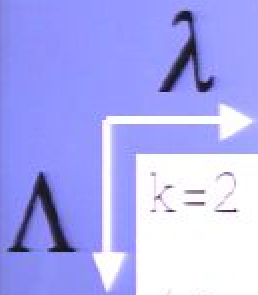
  with $q_1 = e^{2\pi i/k+2}$ and $q_2 = e^{-\pi i/k}$
- What is a boson? A particle with
  - trivial twist factor <==> integer conformal weight
  - trivial self braiding in at least one fusion channel
    <==> at least one of the fusion products also has trivial twist/integer weight
- Have a boson in the Pfaffian state (below)
  and lots of bosons in the higher RR-states (k=4 upwards)



Contains $e^{i\sqrt{2}\varphi}$

U(1) conformal primary with h_b=1, Trivial self-braiding

Another RR-boson



k=2

$$\begin{pmatrix} 0 & X & \frac{3}{4} & X & 0 \\ X & \frac{1}{8} & X & \frac{5}{8} & X \\ \frac{1}{2} & X & \frac{1}{4} & X & \frac{1}{2} \end{pmatrix}$$

$$h_\lambda^\Lambda = \frac{\Lambda(\Lambda+2)-\lambda^2}{4(k+2)} + \text{integer}$$

k=3

$$\begin{pmatrix} 0 & X & \frac{4}{5} & X & \frac{1}{5} & X \\ X & \frac{1}{10} & X & \frac{7}{10} & X & \frac{3}{10} \\ \frac{2}{5} & X & \frac{1}{5} & X & \frac{3}{5} & X \\ X & \frac{?}{10} & X & \frac{3}{10} & X & \frac{1}{2} \end{pmatrix}$$

k=4

$$\begin{pmatrix} 0 & X & \frac{5}{6} & X & \frac{1}{3} & X & \frac{1}{2} \\ X & \frac{1}{12} & X & \frac{3}{4} & X & \frac{1}{12} & X \\ \frac{1}{3} & X & \frac{1}{6} & X & \frac{2}{3} & X & \frac{5}{6} \\ X & \frac{?}{12} & X & \frac{1}{4} & X & \frac{?}{12} & X \\ 0 & X & \frac{5}{6} & X & \frac{1}{3} & X & \frac{1}{2} \end{pmatrix}$$

Note: several fractions in the k=3 and k=4 matrices are very faint; entries marked ? are [illegible].

More RR-Bosons

k=8

$$
\begin{pmatrix}
0 & X & \frac{9}{10} & X & \frac{2}{5} & X & \frac{1}{10} & X & \frac{1}{5} & X & \frac{1}{2} \\
X & \frac{1}{40} & X & \frac{17}{40} & X & \frac{5}{8} & X & \frac{17}{40} & X & \frac{1}{40} & X \\
\frac{1}{5} & X & \frac{1}{10} & X & \frac{4}{5} & X & \frac{3}{10} & X & \frac{2}{5} & X & \frac{7}{10} \\
X & \frac{7}{40} & X & \frac{3}{40} & X & \frac{3}{8} & X & \frac{3}{40} & X & \frac{7}{40} & X \\
\frac{2}{5} & X & \frac{1}{2} & X & \frac{1}{5} & X & \frac{7}{10} & X & 0 & X & \frac{1}{10} \\
X & \frac{17}{40} & X & \frac{13}{40} & X & \frac{1}{8} & X & \frac{13}{40} & X & \frac{17}{40} & X \\
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X & \frac{11}{40} & X & \frac{7}{40} & X & \frac{23}{40} & X & \frac{7}{40} & X & \frac{11}{40} & X \\
0 & X & \frac{9}{10} & X & \frac{2}{5} & X & \frac{1}{10} & X & \frac{1}{5} & X & \frac{1}{2}
\end{pmatrix}
$$

PI

Some results for the Read-Rezayi states

- k=2: Ising x $U(1)_8$ ➔ $SO(3)_2$ x $U(1)_2$ or (Ising/Z_2) x $U(1)_2$
  Interpretation not obvious (superconductor?!).
  Possibly connected to Fradkin-Nayak-Schoutens '98?

- k=4: Pf_4 x $U(1)_{24}$ ➔ ($U(1)_6$ x $U(1)_6$)/Z_2
  Expect a Hall state at filling 2/3 (neutral condensate)
  Get Abelian topological order with right behavior for the quasiholes.
  More precise connection difficult (naïve ground state gives nu=1/3 Laughlin)

- k=6: Much like k=2, but stays nonabelian (contains $SO(3)_6$)

- k=7: Stays nonabelian (contains "twisted" version of Pf_7)

- k=8: Stays nonabelian and contains Fibonacci x Fibonacci (chiral version)

PI

Conclusions and summary

Results

- **Topological ordered phases in condensed matter are important**
- Nonabelian anyons provide a means to implement quantum computation
- Quantum groups, Hopf algebra's etc. naturally appear in 2+1 dim.
- **Topological quantum computation beats decoherence**
- For the moment study FQH systems (ν= 5/2, 12/5,…) in detail
- Physics of ordered phases in quantum (spin) nematics etc.
- Theory: exploit connections to TQFT and CFT.
- **Breaking topological order links topological phases**
- Extended topological symmetry breaking to TQFTs with non-integer quantum dimensions
- Breaking has relations to conformal embeddings, coset construction etc.
- Had a first go at application to nonabelian FQH states
- Breaking in 2+1 dim gravity?