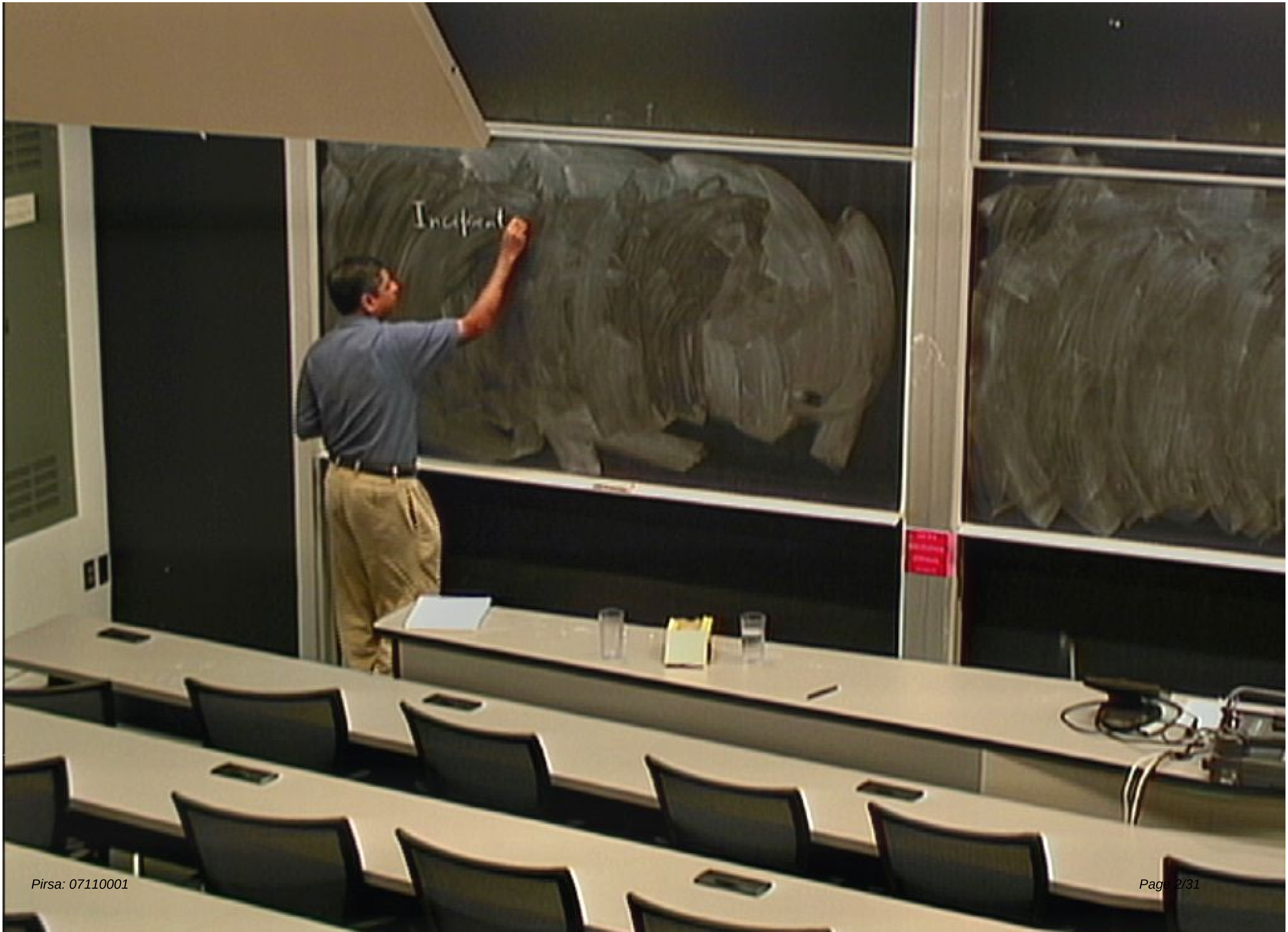
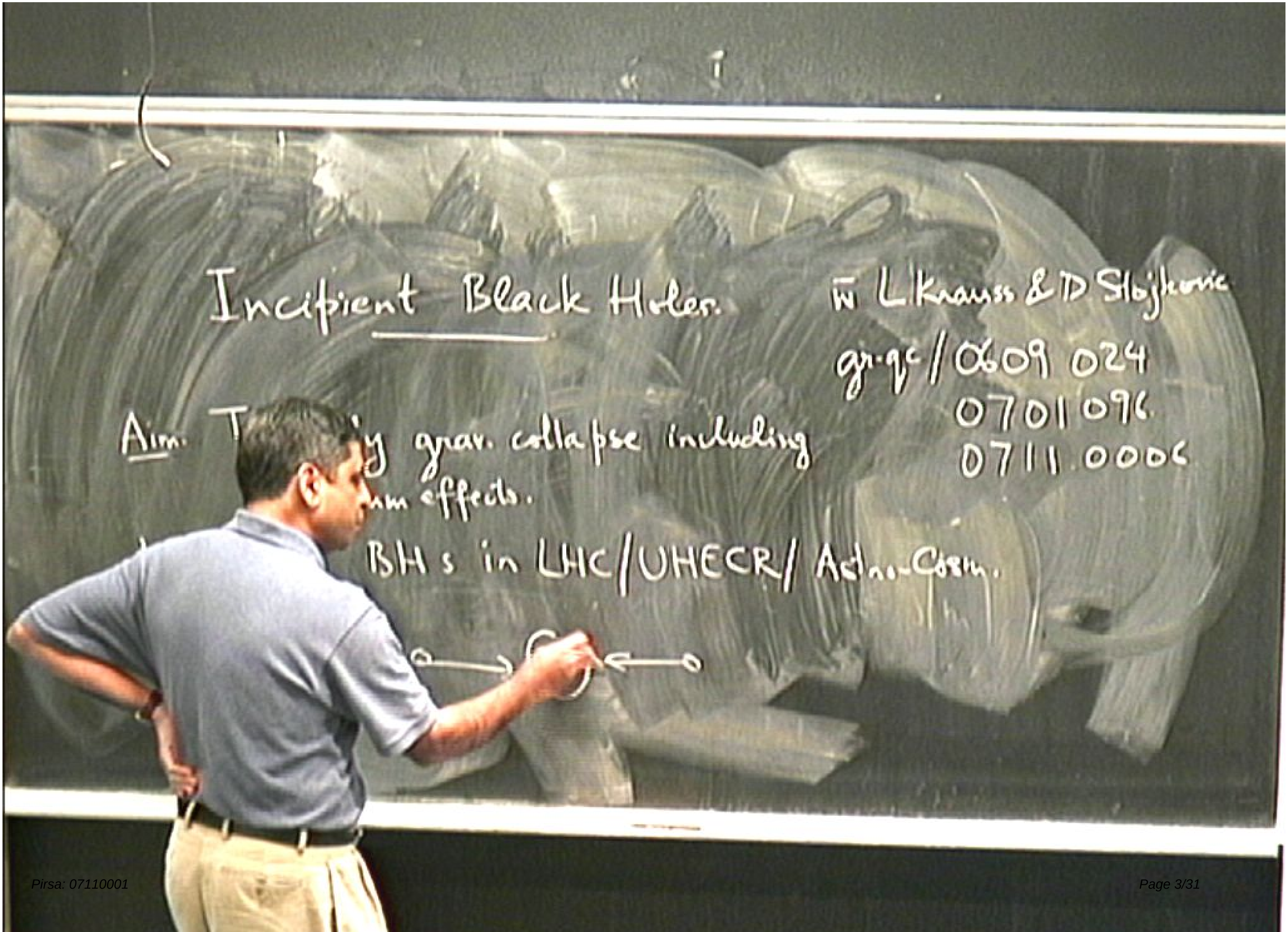
Title: Incipient Black Holes

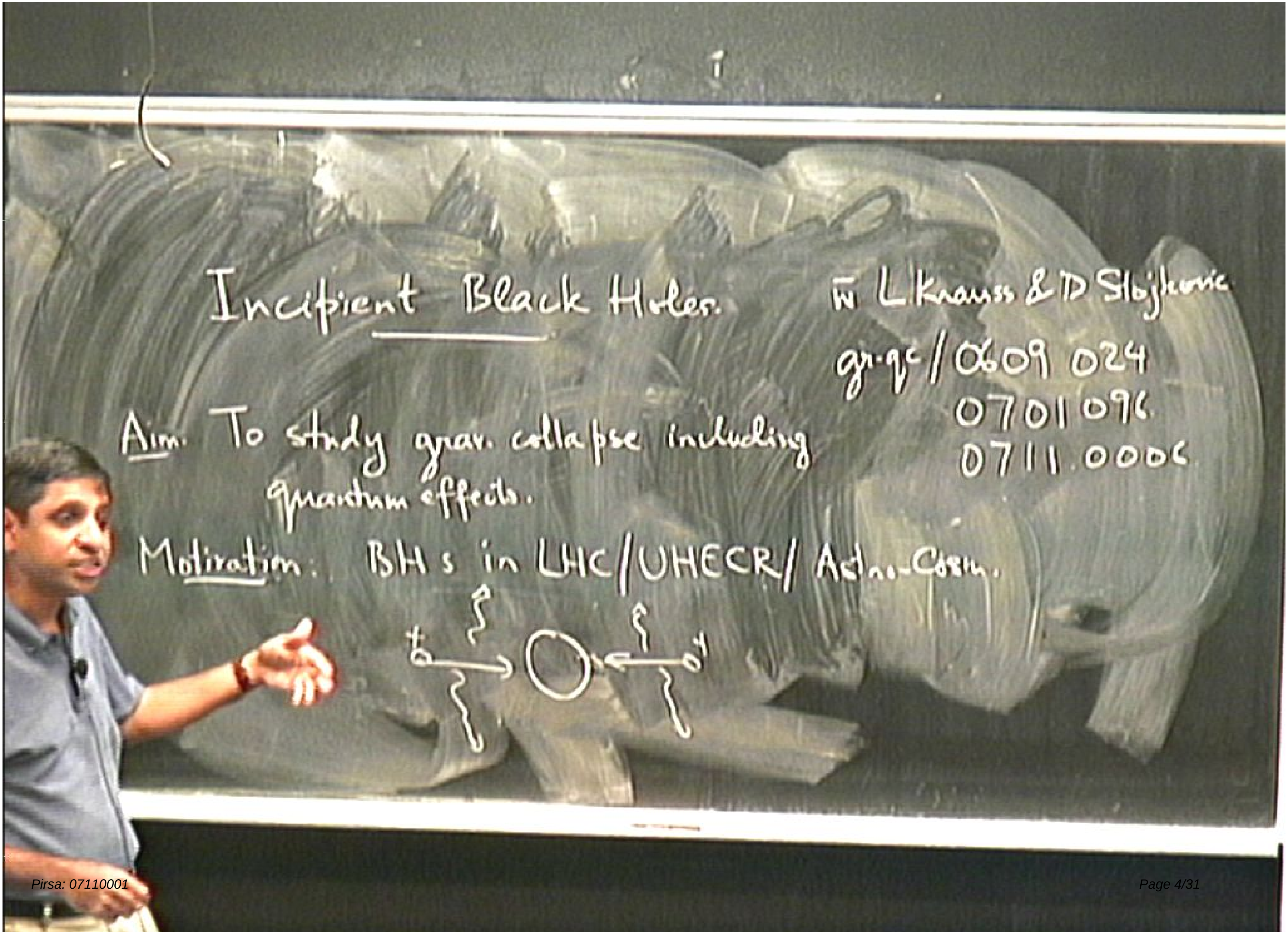
Date: Nov 13, 2007 02:00 PM

URL: http://pirsa.org/07110001

Abstract: TBA




Incipient Black Holes.
w L Krauss & D Stojkovic
gr-qc/0609 024
0701 096
0711.0006
Aim.
grav. collapse including
effects.
BHs in LHC/UHECR/


Incipient Black Holes.
w L Krauss & D Stojkovic
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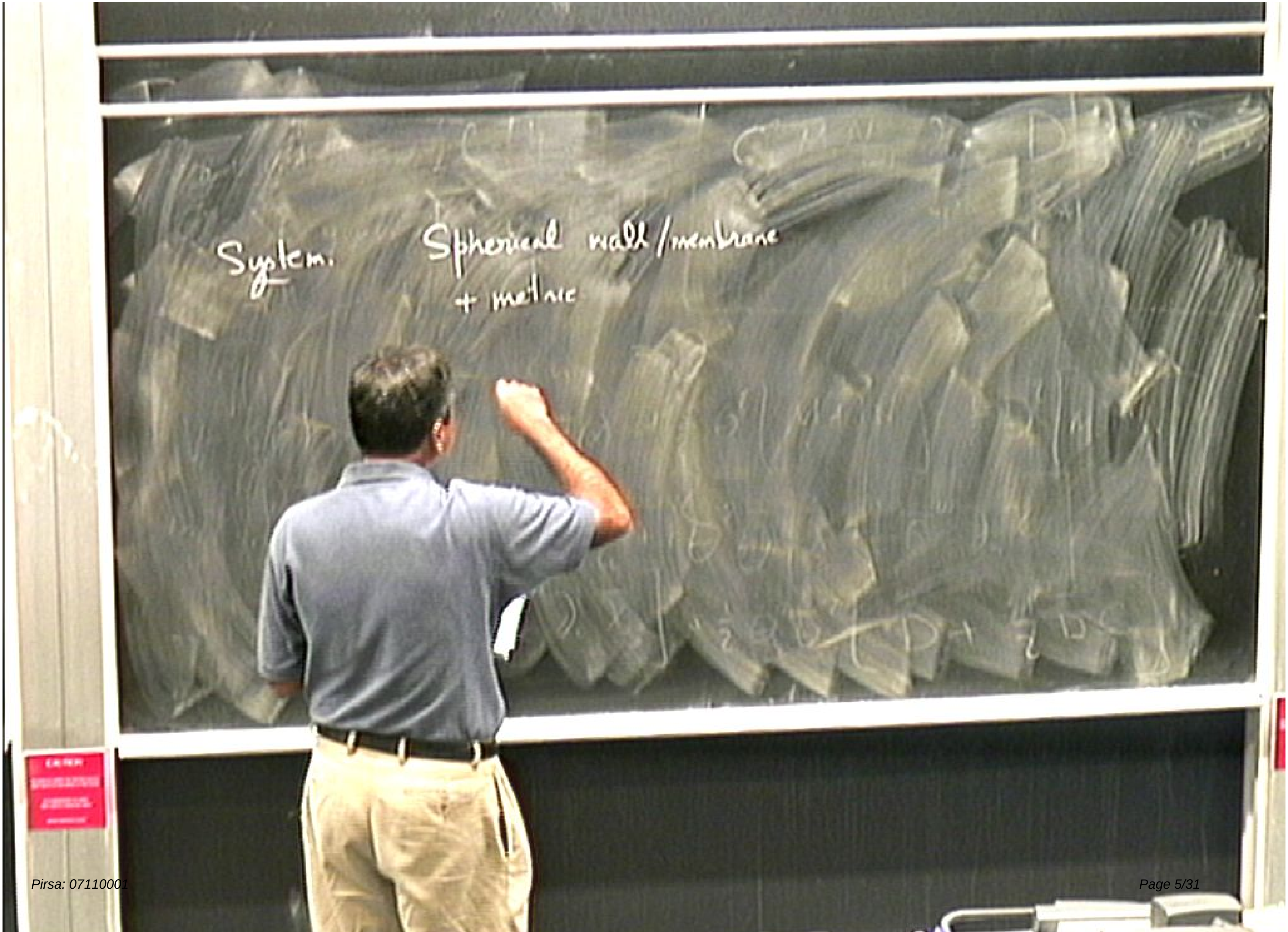
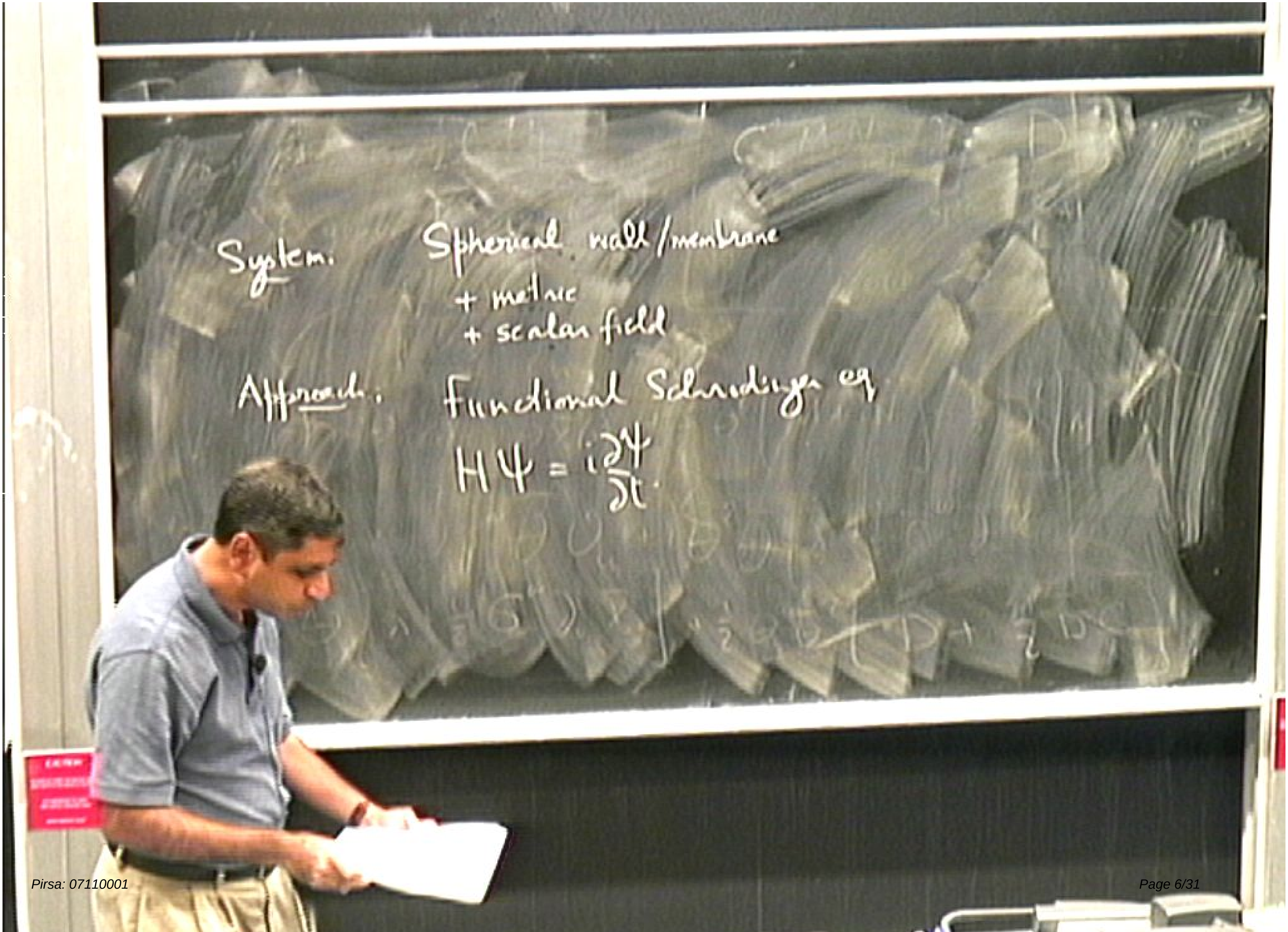
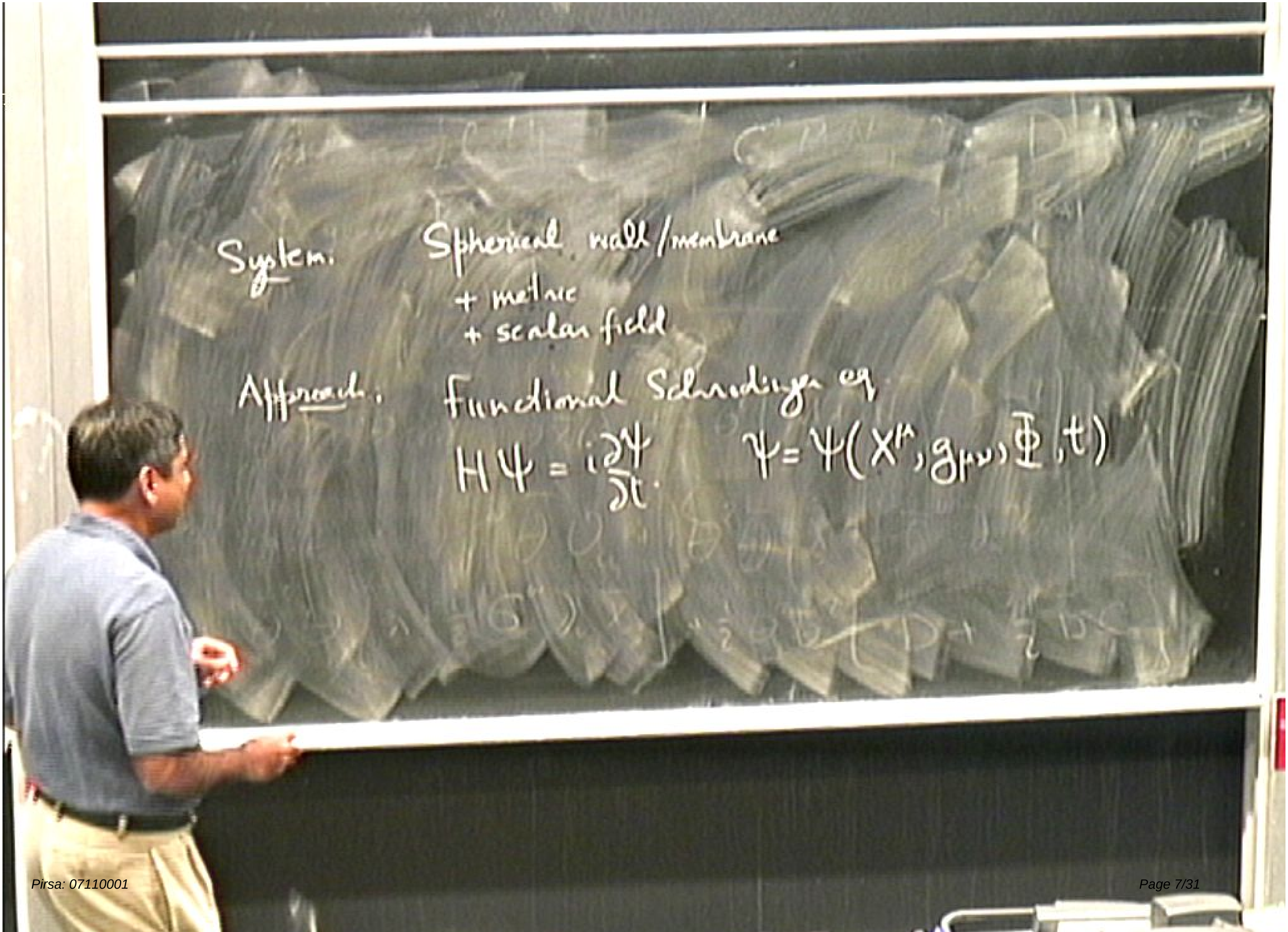
0701 096
0711.0006
Aim. To study grav. collapse including quantum effects.
Motivation: BHs in LHC/UHECR/ Astro-Cosm.


System. Spherical wall / membrane
+ metric


System: Spherical wall/membrane
+ metric
+ scalar field
Approach: Functional Schrodinger eq
$H\psi = i\frac{\partial\psi}{\partial t}$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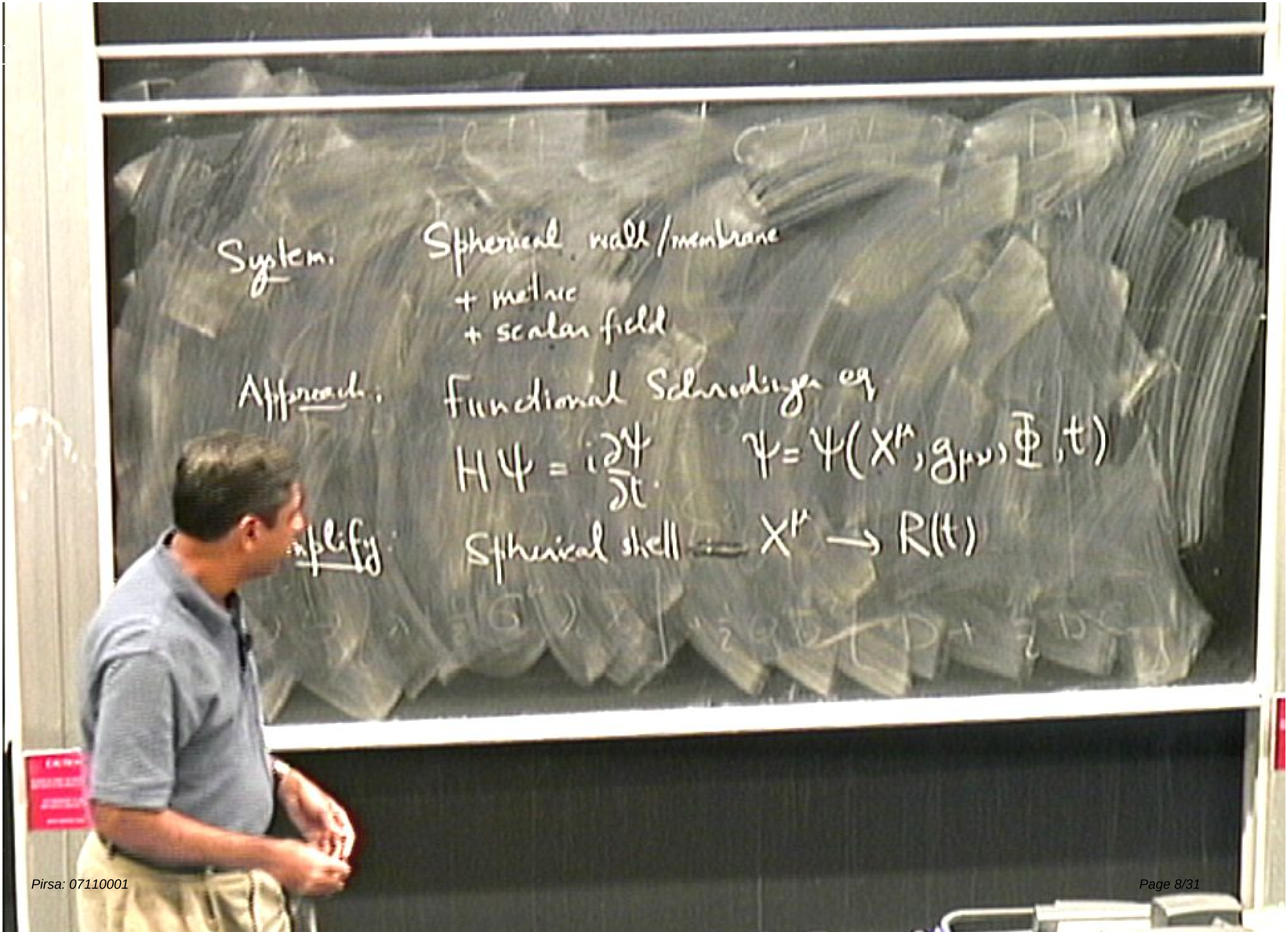
$H\Psi = i\frac{\partial\Psi}{\partial t}$
$\Psi = \Psi(X^\mu, g_{\mu\nu}, \Phi, t)$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System: Spherical wall/membrane

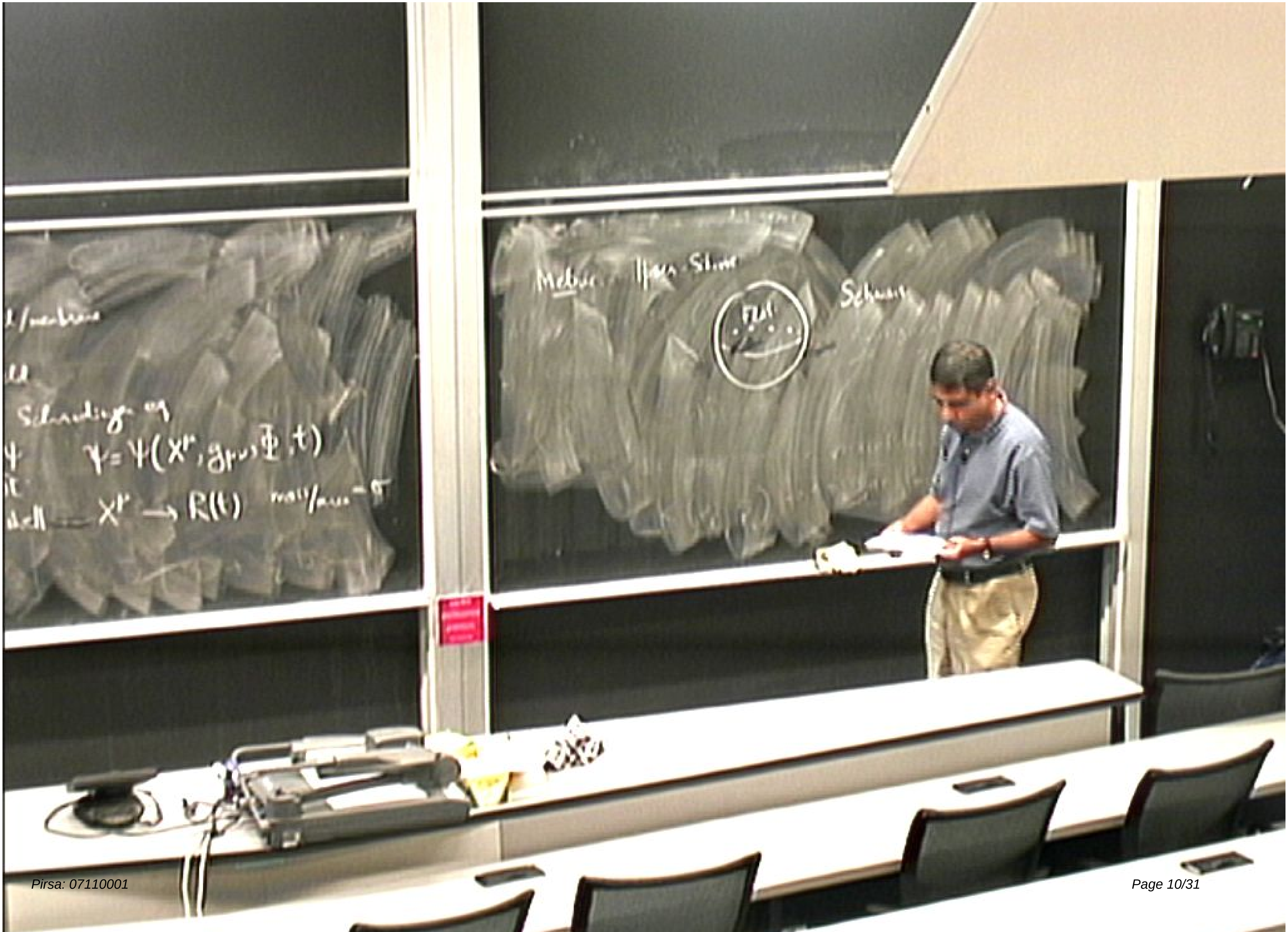
+ metric

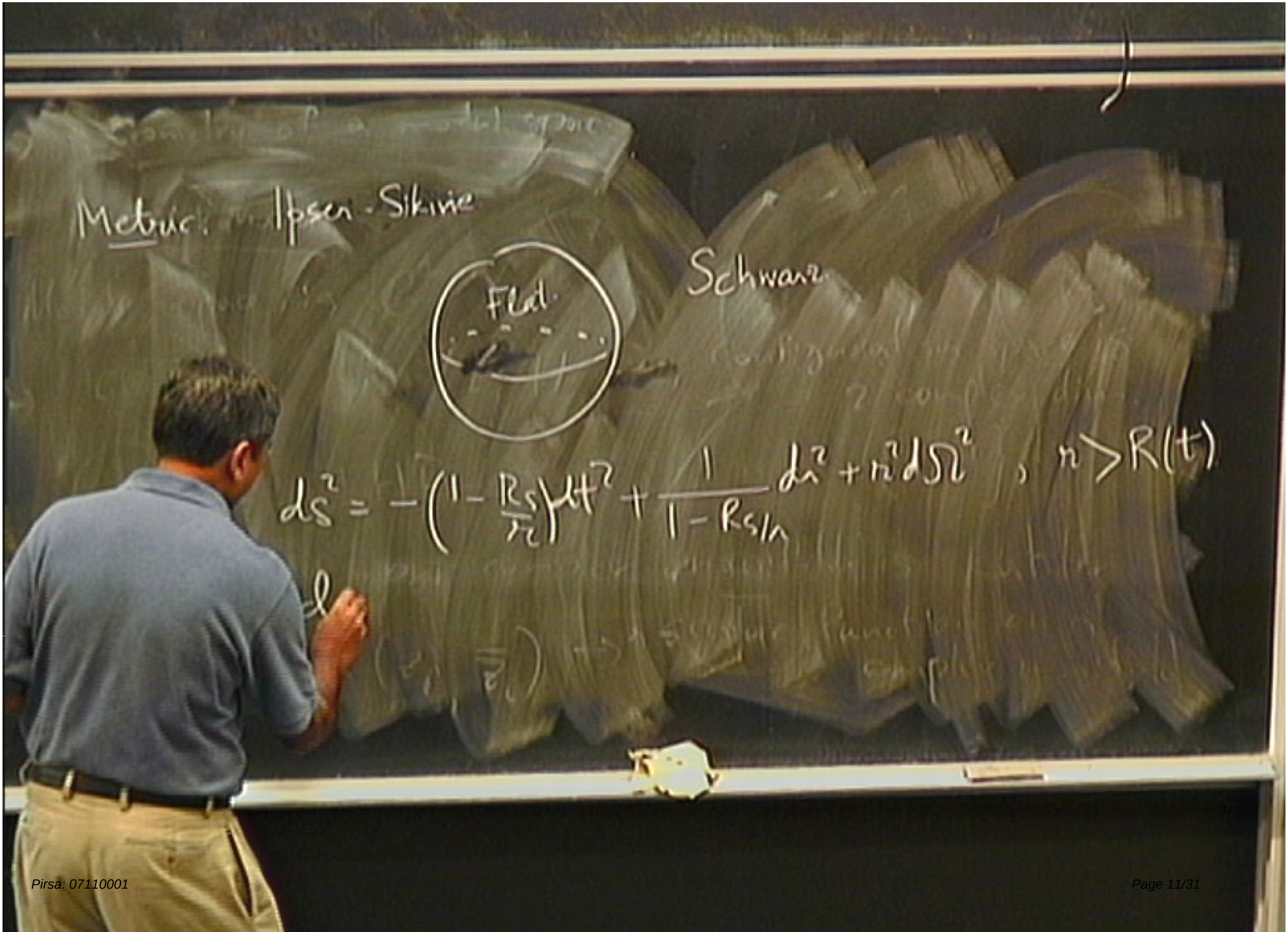
+ scalar field

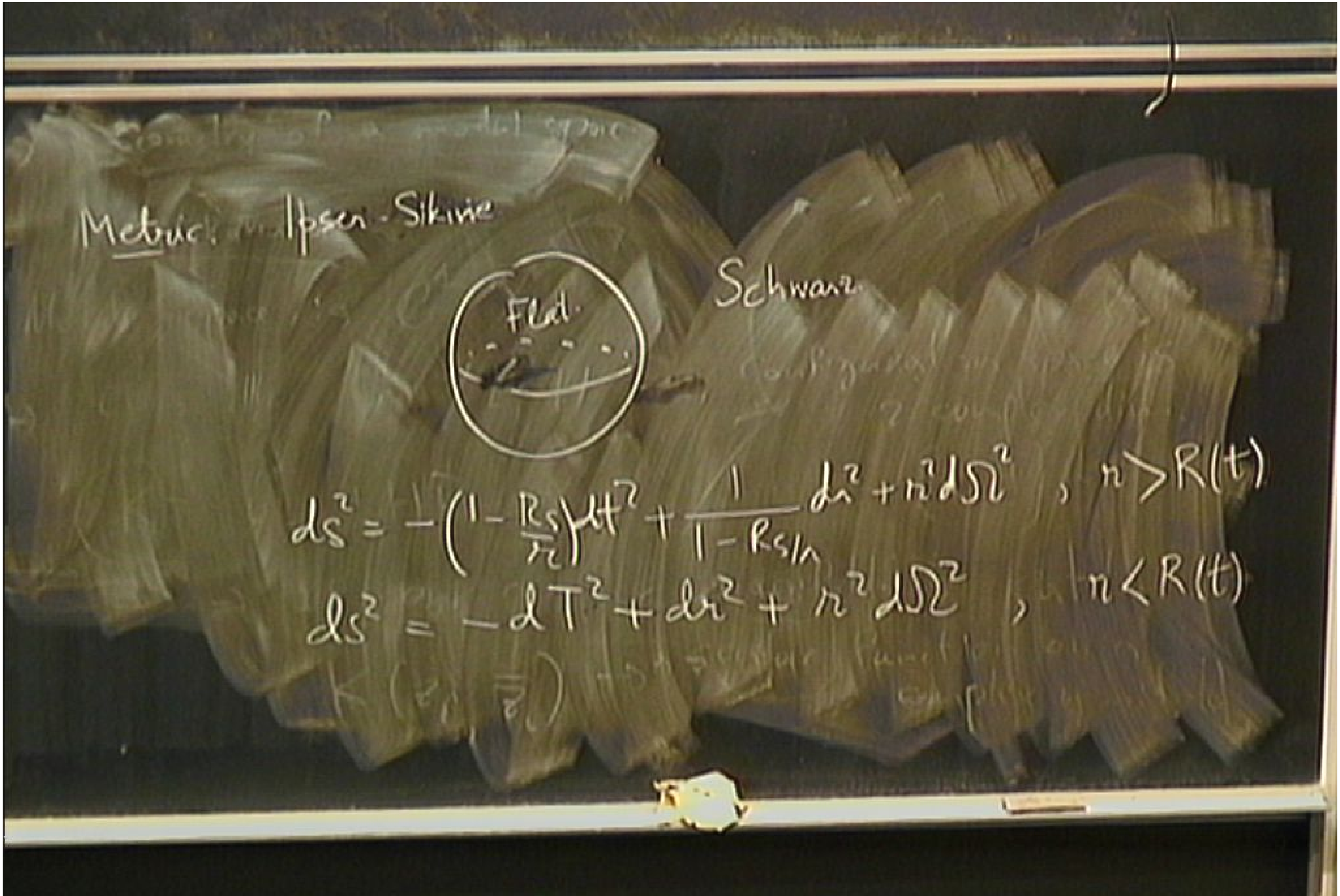
Approach: Functional Schrodinger eq

$$H\Psi = i\frac{\partial \Psi}{\partial t} \qquad \Psi = \Psi(X^\mu, g_{\mu\nu}, \Phi, t)$$

Simplify: Spherical shell — $X^\mu \to R(t)$ mass/area $= \sigma$




Metric: Ipser-Sikivie
Flat
Schwarz
$ds^2 = -\left(1 - \frac{R_s}{r}\right)dt^2 + \frac{1}{1 - R_s/r}\,dr^2 + r^2 d\Omega^2 \quad , \quad r > R(t)$



Metric: Ipser-Sikivie
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$ds^2 = -\left(1-\frac{R_s}{r}\right)dt^2 + \frac{1}{1-R_s/r}dr^2 + r^2 d\Omega^2 \, , \quad r > R(t)$
$ds^2 = -dT^2 + dr^2 + r^2 d\Omega^2 \, , \quad r < R(t)$

Metric: Ipser-Sikivie

$$\frac{dT}{d\tau} = \left[1 + \left(\frac{dR}{d\tau}\right)^2\right]^{1/2}$$

$$\frac{dt}{d\tau} = \frac{1}{B}\left[B + \left(\frac{dR}{d\tau}\right)^2\right]^{1/2}$$

Flat

Schwarz

$$ds^2 = -\left(1 - \frac{R_s}{r}\right)dt^2 + \frac{1}{1 - R_s/r}\,dr^2 + r^2 d\Omega^2 \;,\quad r > R(t)$$

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Metric ... Ipser-Sikivie

$$\boxed{\begin{aligned}\frac{dT}{d\tau} &= \left[1+\left(\frac{dR}{d\tau}\right)^2\right]^{1/2}\\ \frac{dt}{d\tau} &= \frac{1}{B}\left[B+\left(\frac{dR}{d\tau}\right)^2\right]^{1/2}\end{aligned}}$$

Flat

$$\boxed{B = 1 - \frac{R_s}{R}}$$

Schwarz

$$ds^2 = -\left(1-\frac{R_s}{r}\right)dt^2 + \frac{1}{1-R_s/r}\,dr^2 + r^2 d\Omega^2 \,, \quad r > R(t)$$

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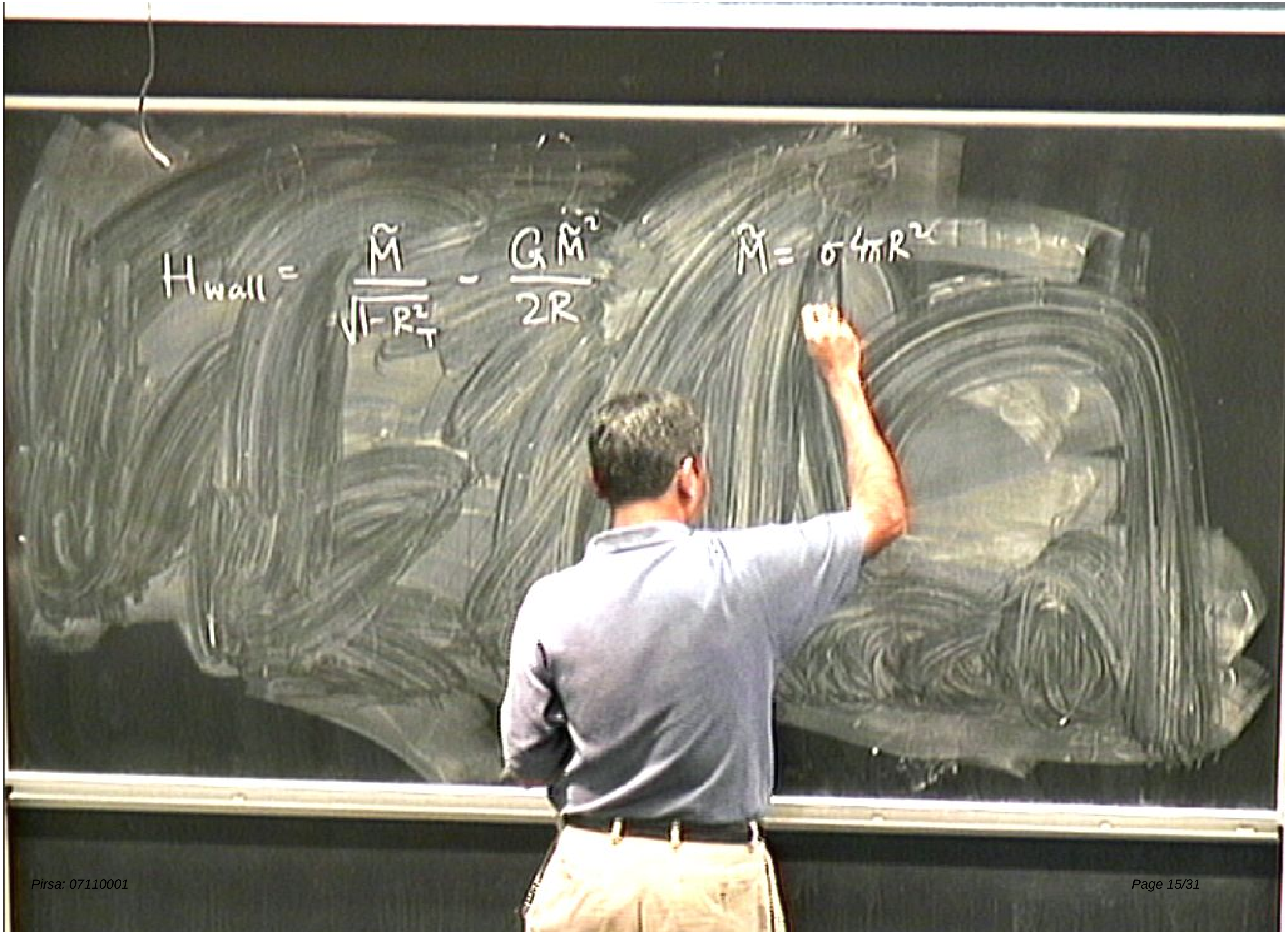
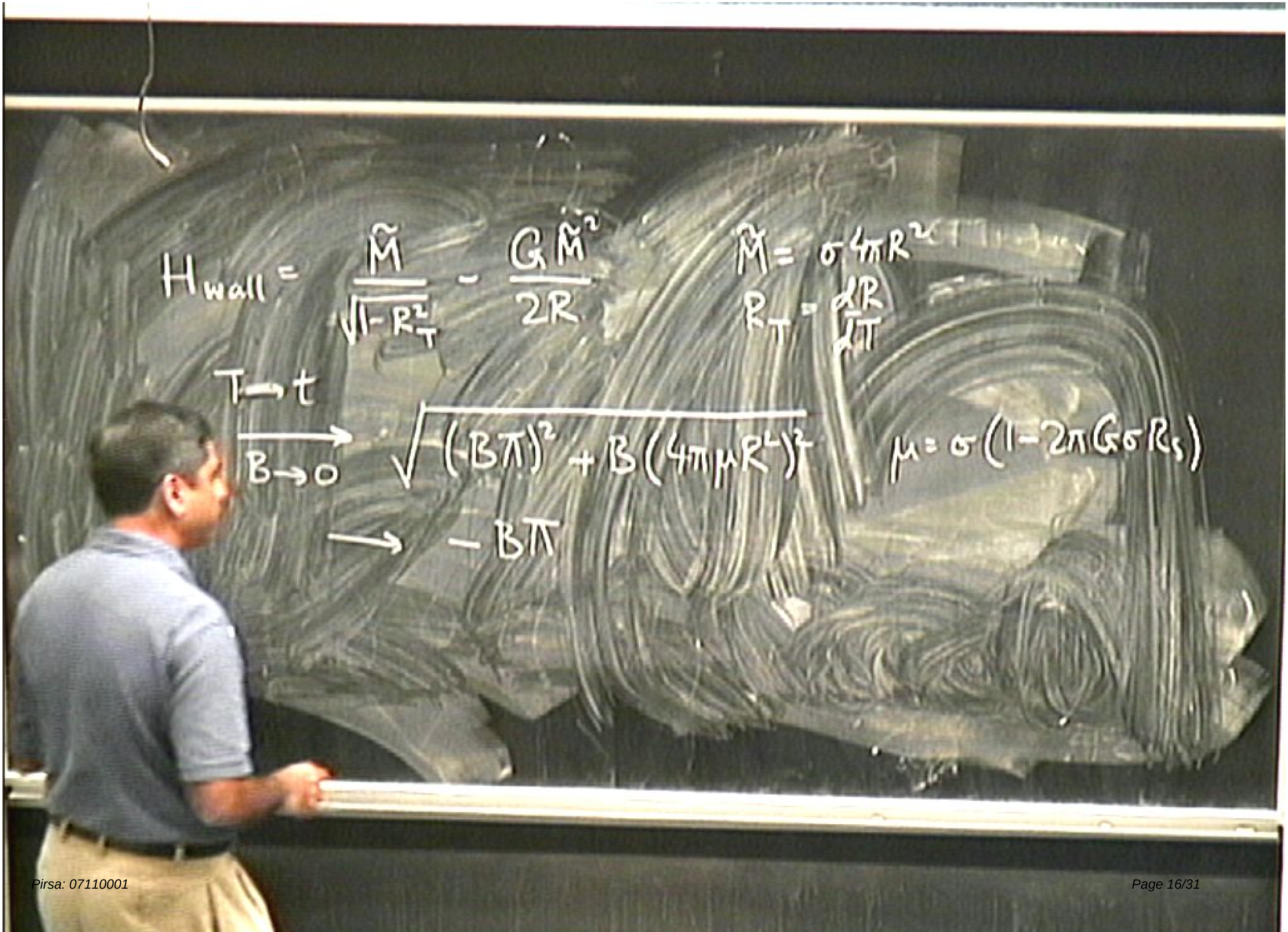

$H_{wall} = \frac{\tilde{M}}{\sqrt{1-R_T^2}} - \frac{G\tilde{M}^2}{2R}$
$\tilde{M} = \sigma 4\pi R^2$


$H_{wall} = \frac{\tilde{M}}{\sqrt{1-R_T^2}} - \frac{G\tilde{M}^2}{2R}$
$\tilde{M} = \sigma 4\pi R^2$
$R_T = \frac{dR}{dT}$
$T \to t$
$B \to 0$
$\to \sqrt{(B\pi)^2 + B(4\pi\mu R^2)^2}$
$\mu = \sigma(1 - 2\pi G\sigma R_s)$
$\to -B\pi$

$$H_{wall} = \frac{\tilde{M}}{\sqrt{1-R_T^2}} - \frac{G\tilde{M}^2}{2R} \qquad \tilde{M} = \sigma 4\pi R^2$$

$$R_T = \frac{dR}{dT}$$

$$T \to t$$

$$\xrightarrow[B\to 0]{} \sqrt{(B\pi)^2 + B(4\pi\mu R^2)^2} \qquad \mu = \sigma(1 - 2\pi G\sigma R_s)$$

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$$H_{wall} = \frac{\tilde{M}}{\sqrt{1-R_T^2}} - \frac{G\tilde{M}^2}{2R} \qquad \tilde{M} = \sigma 4\pi R^2 \qquad R_T = \frac{dR}{dT}$$

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$$\xrightarrow[B\to 0]{} \sqrt{(B\Pi)^2 + B(4\pi\mu R^2)^2} \qquad \mu = \sigma(1 - 2\pi G\sigma R_s)$$

$$\longrightarrow \underbrace{-B\Pi}_{\text{ultra-relativistic particle}}$$

$$\Pi = \frac{4\pi\mu R^2 \dot{R}}{\sqrt{B}\sqrt{B^2 - \dot{R}^2}}$$

$$H_{wall} = \frac{\tilde{M}}{\sqrt{1-R_T^2}} - \frac{G\tilde{M}^2}{2R}$$

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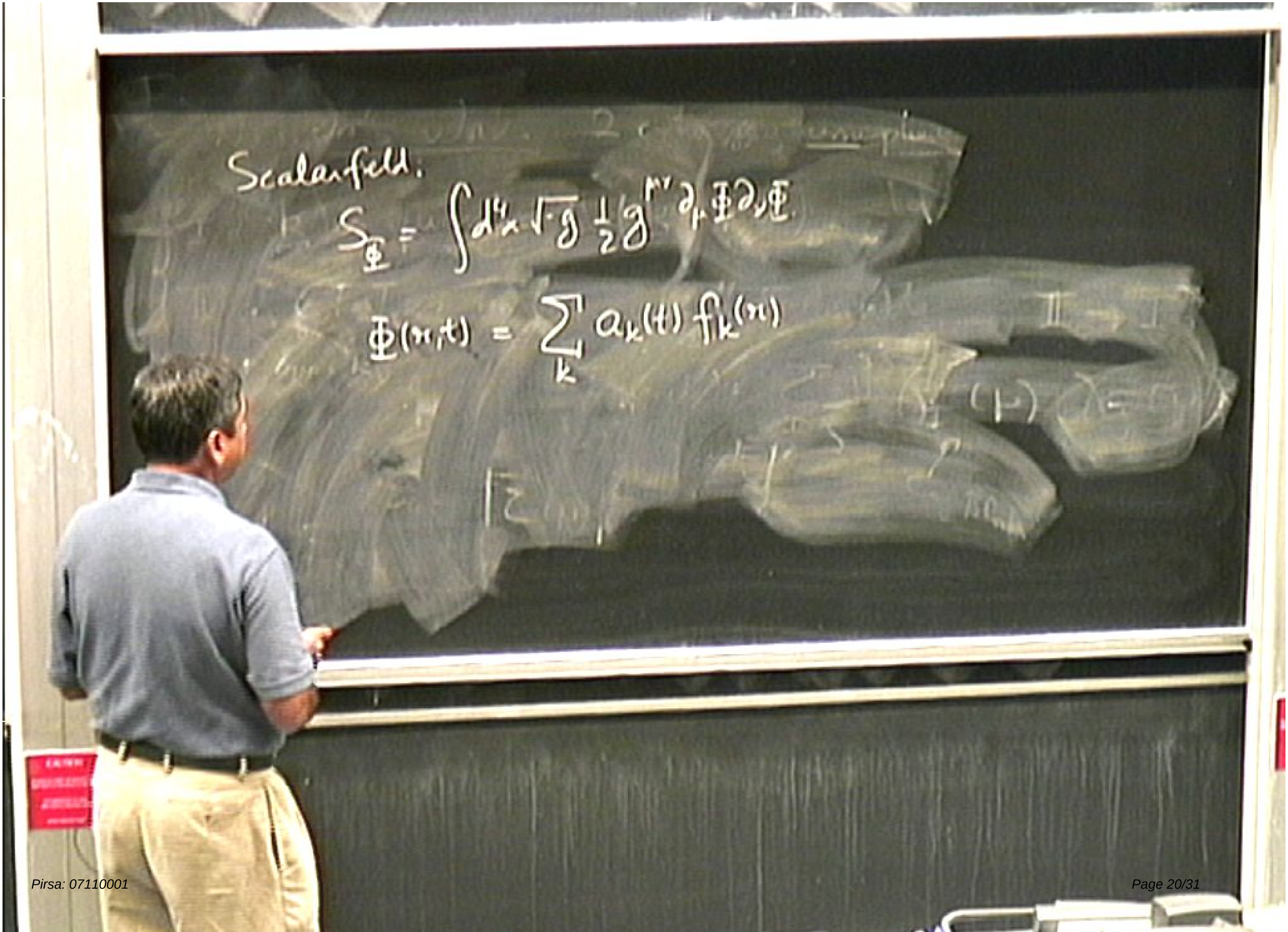
$$T \to t$$

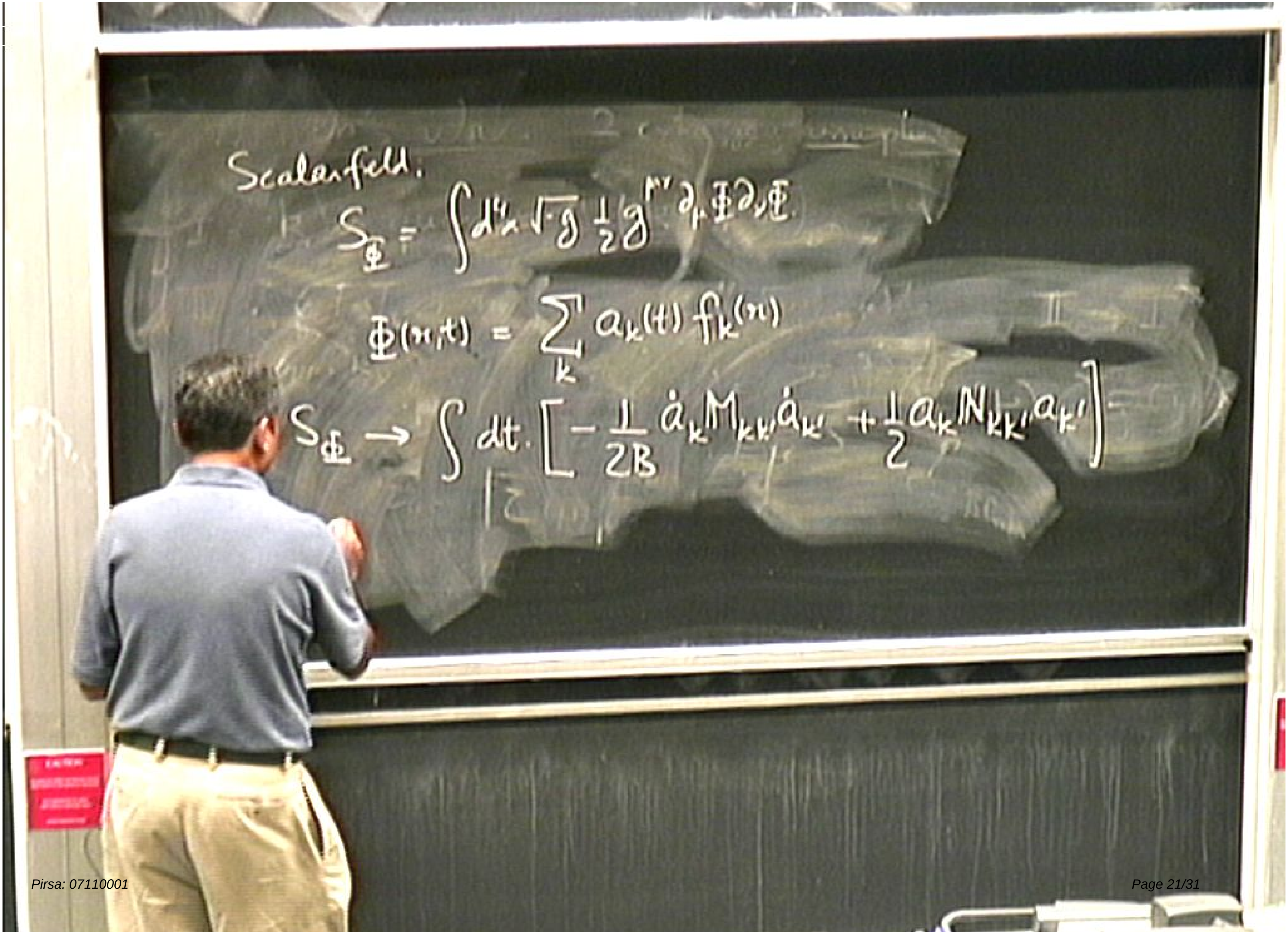
$$\xrightarrow[B\to 0]{} \sqrt{(B\Pi)^2 + B(4\pi\mu R^2)^2}$$

$$\mu = \sigma(1 - 2\pi G\sigma R_s)$$

$$\dot{R} \simeq B\left(1 - \frac{1}{2}\frac{BR^4}{h^2}\right) \longrightarrow \underbrace{-B\Pi}_{\text{ultra-relativistic particle}}$$

$$\Pi = \frac{4\pi\mu R^2 \dot{R}}{\sqrt{B}\sqrt{B^2 - \dot{R}^2}}$$


Scalar field.
$S_\Phi = \int d^4x \sqrt{-g}\, \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi$
$\Phi(x,t) = \sum_k a_k(t) f_k(x)$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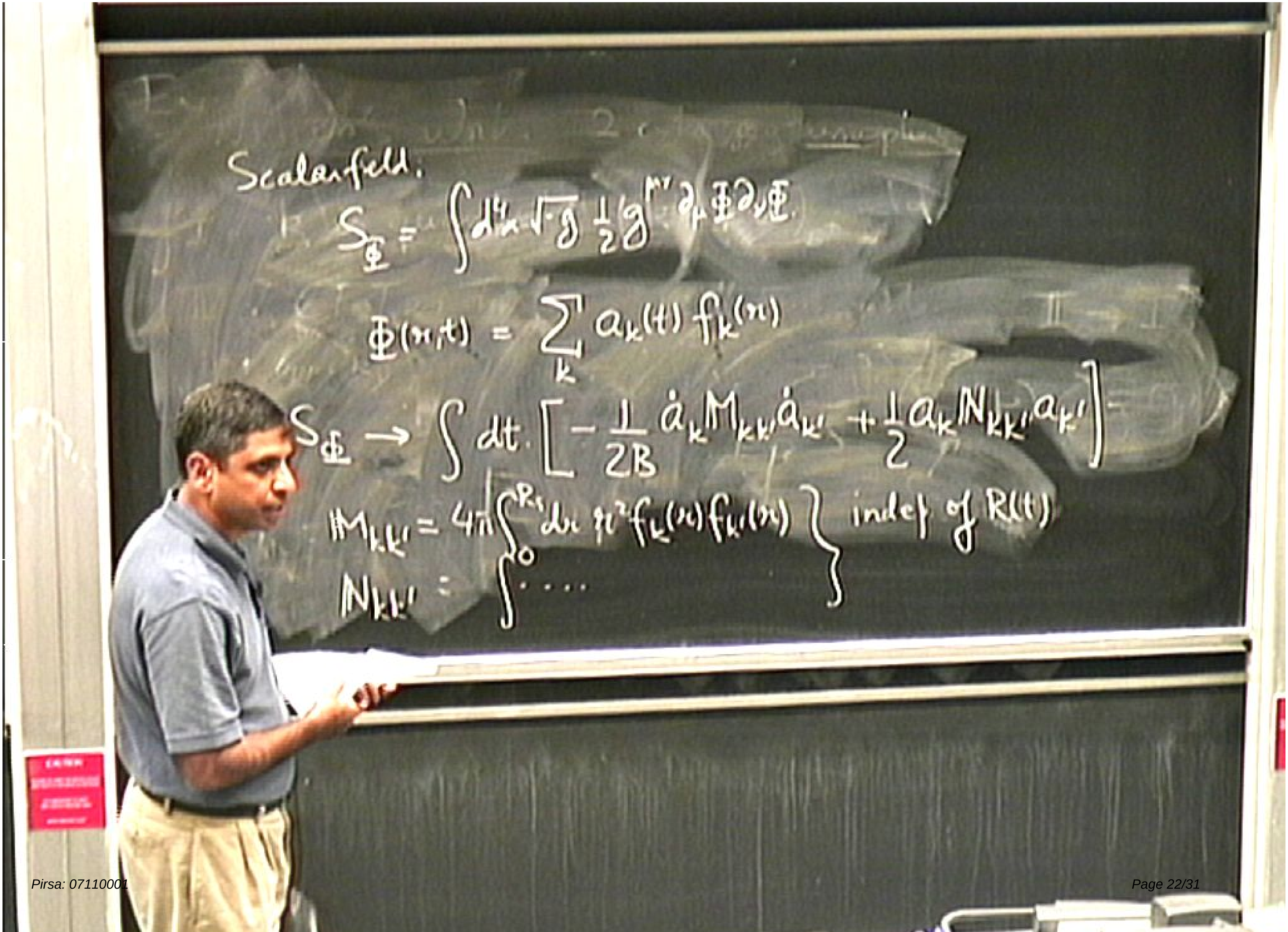
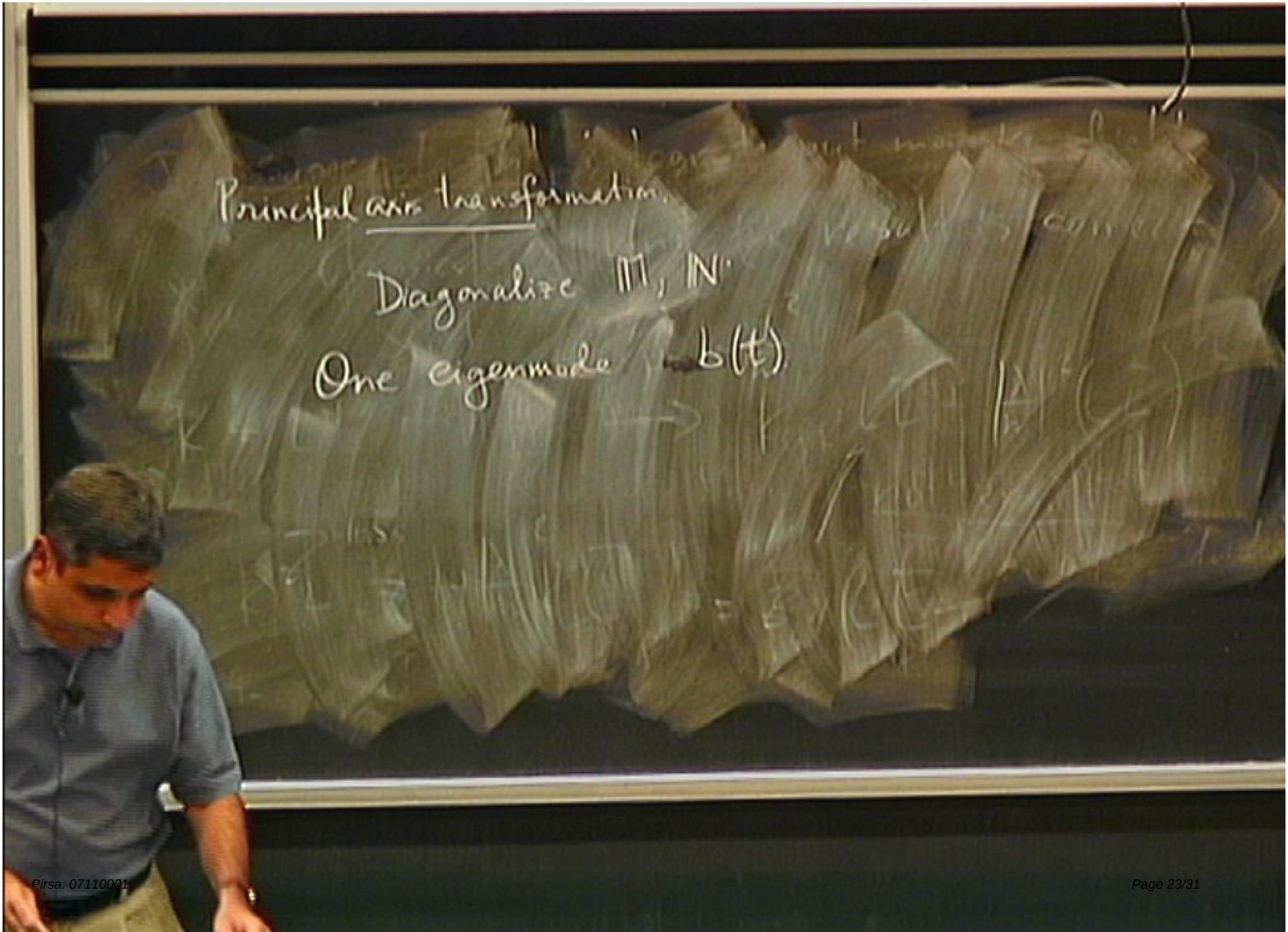
$\Phi(x,t) = \sum_k a_k(t) f_k(x)$
$S_\Phi \rightarrow \int dt. \left[-\frac{1}{2B} \dot{a}_k M_{kk'} \dot{a}_{k'} + \frac{1}{2} a_k N_{kk'} a_{k'} \right]$


Scalar field.
$S_\Phi = \int d^4x \sqrt{-g}\, \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi$
$\Phi(r,t) = \sum_k a_k(t) f_k(r)$
$S_\Phi \to \int dt \left[-\frac{1}{2B} \dot{a}_k M_{kk'} \dot{a}_{k'} + \frac{1}{2} a_k N_{kk'} a_{k'} \right]$
$M_{kk'} = 4\pi \int_0^{R_s} dr\, r^2 f_k(r) f_{k'}(r)$ indep of $R(t)$
$N_{kk'} = \int \ldots$

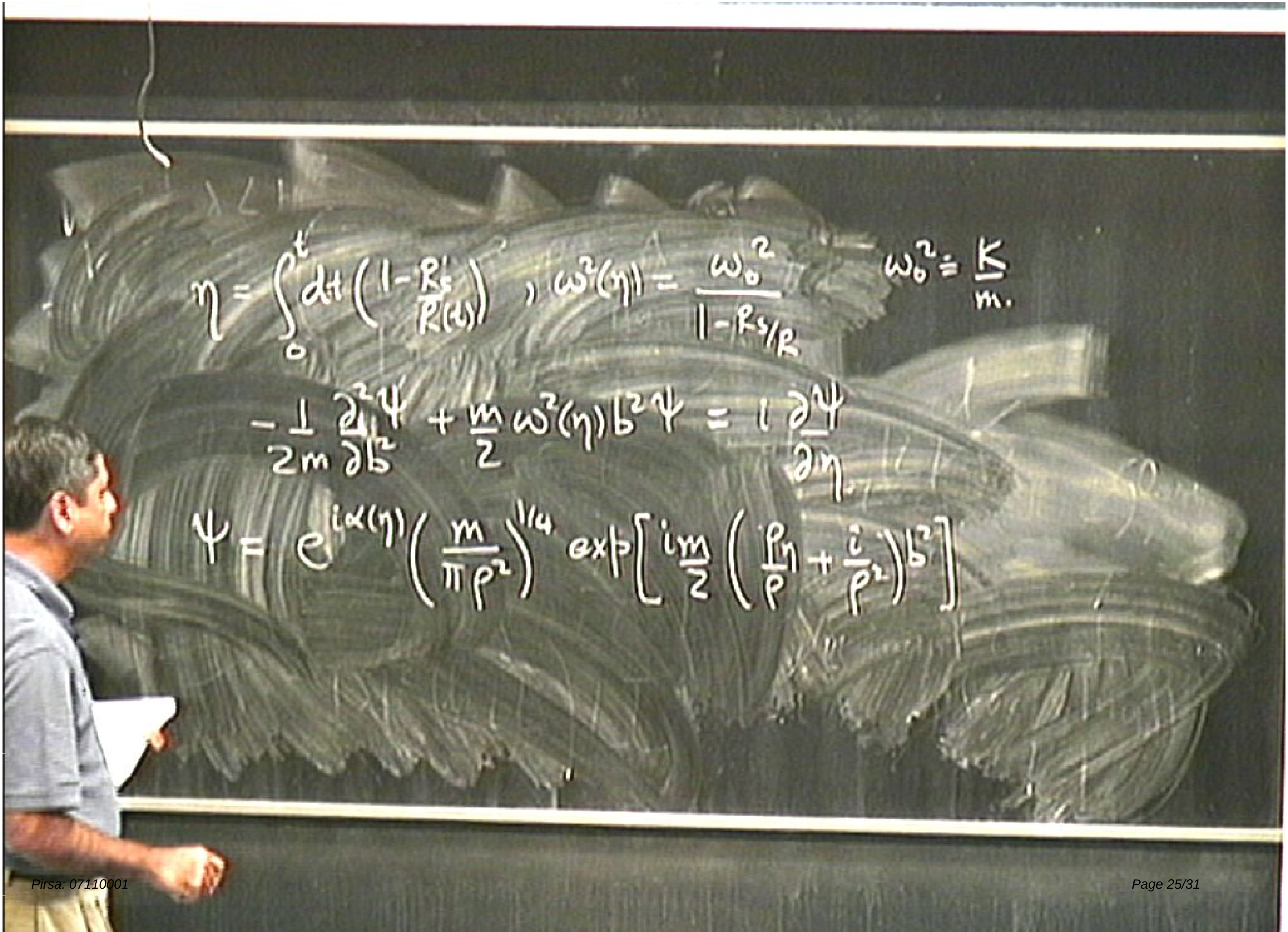

Principal axis transformation
Diagonalize M, N
One eigenmode b(t)

Principal axis transformation.

Diagonalize $\mathbb{M}$, $\mathbb{N}$

One eigenmode $b(t)$

$$H_b = -\left(1-\frac{R_s}{R}\right)\frac{1}{2m}\frac{\partial^2}{\partial b^2} + \frac{1}{2}K b^2$$


$\eta = \int_0^t dt \left(1 - \frac{R_s}{R(t)}\right)$, $\omega^2(\eta) = \frac{\omega_0^2}{1 - R_s/R}$
$\omega_0^2 = \frac{K}{m}$.
$-\frac{1}{2m}\frac{\partial^2\psi}{\partial b^2} + \frac{m}{2}\omega^2(\eta) b^2 \psi = i\frac{\partial\psi}{\partial\eta}$.
$\psi = e^{i\alpha(\eta)}\left(\frac{m}{\pi\rho^2}\right)^{1/4} \exp\left[\frac{im}{2}\left(\frac{\rho_\eta}{\rho} + \frac{i}{\rho^2}\right)b^2\right]$

$$\eta = \int_0^t dt \left(1 - \frac{R_s}{R(t)}\right), \quad \omega^2(\eta) = \frac{\omega_0^2}{1 - R_s/R} \qquad \omega_0^2 = \frac{K}{m}.$$

$$-\frac{1}{2m}\frac{\partial^2 \psi}{\partial b^2} + \frac{m}{2}\omega^2(\eta) b^2 \psi = i \frac{\partial \psi}{\partial \eta}$$

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$$\rho_{\eta\eta} + \omega^2(\eta)\rho = 1/\rho^3$$

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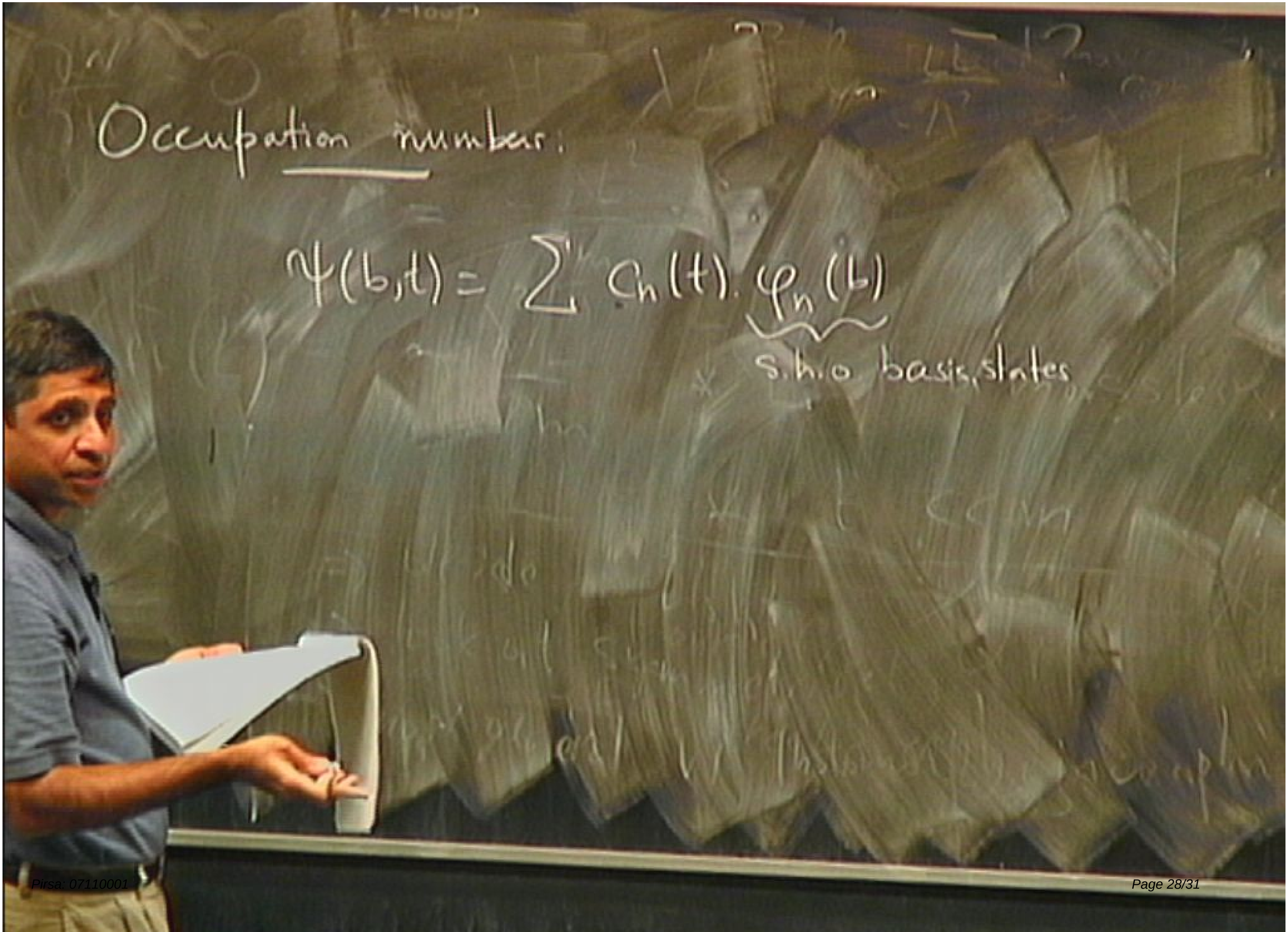
$$\rho_{\eta\eta} + \omega^2(\eta)\rho = 1/\rho^3 \qquad \alpha(\eta) = -\frac{1}{2}\int_0^\eta \frac{d\eta'}{\rho^2(\eta')}$$

$$\rho(0) = \frac{1}{\sqrt{\omega(0)}}, \quad \rho_\eta(0) = 0$$


Occupation number:
ψ(b,t) = Σ c_n(t). φ_n(b)
s.h.o basis states

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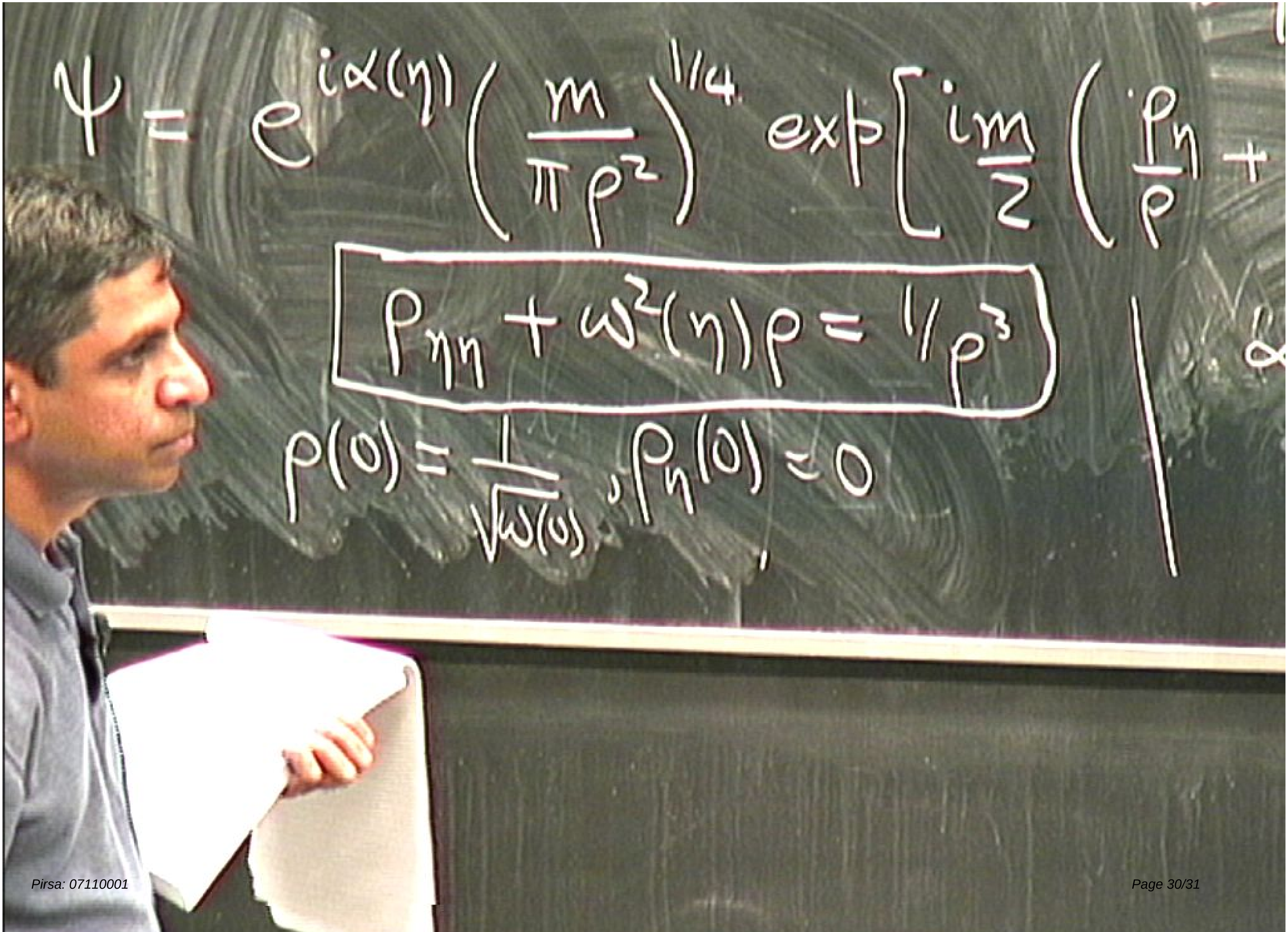
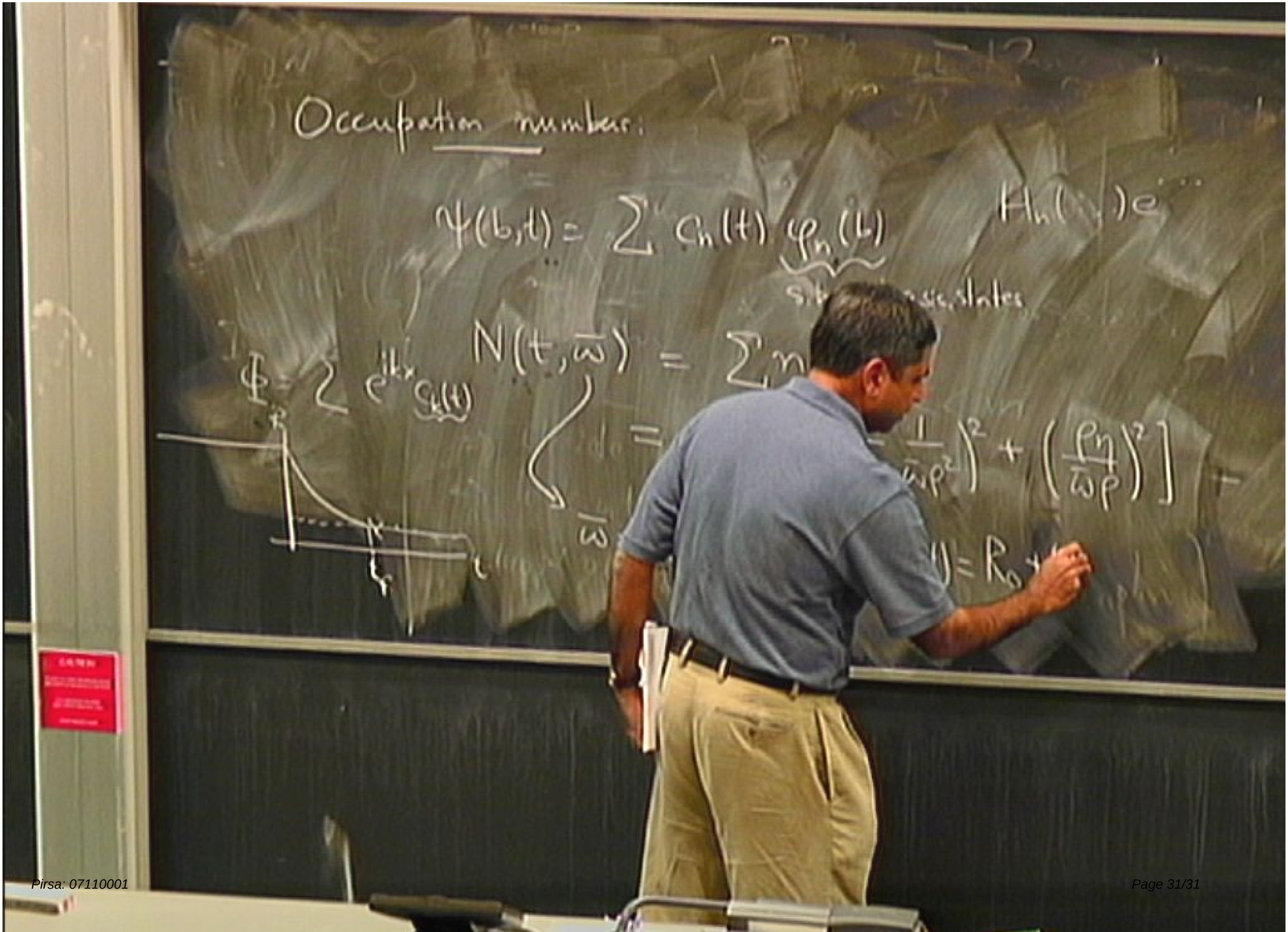
$$\psi(b,t) = \sum c_n(t)\, \underbrace{\varphi_n(b)}_{\text{s.h.o. basis states}}$$

$$N(t,\bar{\omega}) = \sum n |c_n|^2$$

$$= \frac{\bar{\omega}\rho^2}{\sqrt{2}} \left[\left(1 - \frac{1}{\bar{\omega}\rho^2}\right)^2 + \left(\frac{\rho\eta}{\bar{\omega}\rho}\right)^2 \right]$$


$\psi = e^{i\alpha(\eta)} \left(\frac{m}{\pi\rho^2}\right)^{1/4} \exp\left[\frac{im}{2}\left(\frac{\rho_\eta}{\rho} +\right.\right.$
$\boxed{\rho_{\eta\eta} + \omega^2(\eta)\rho = 1/\rho^3}$
$\rho(0) = \frac{1}{\sqrt{\omega(0)}}, \rho_\eta(0) = 0$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