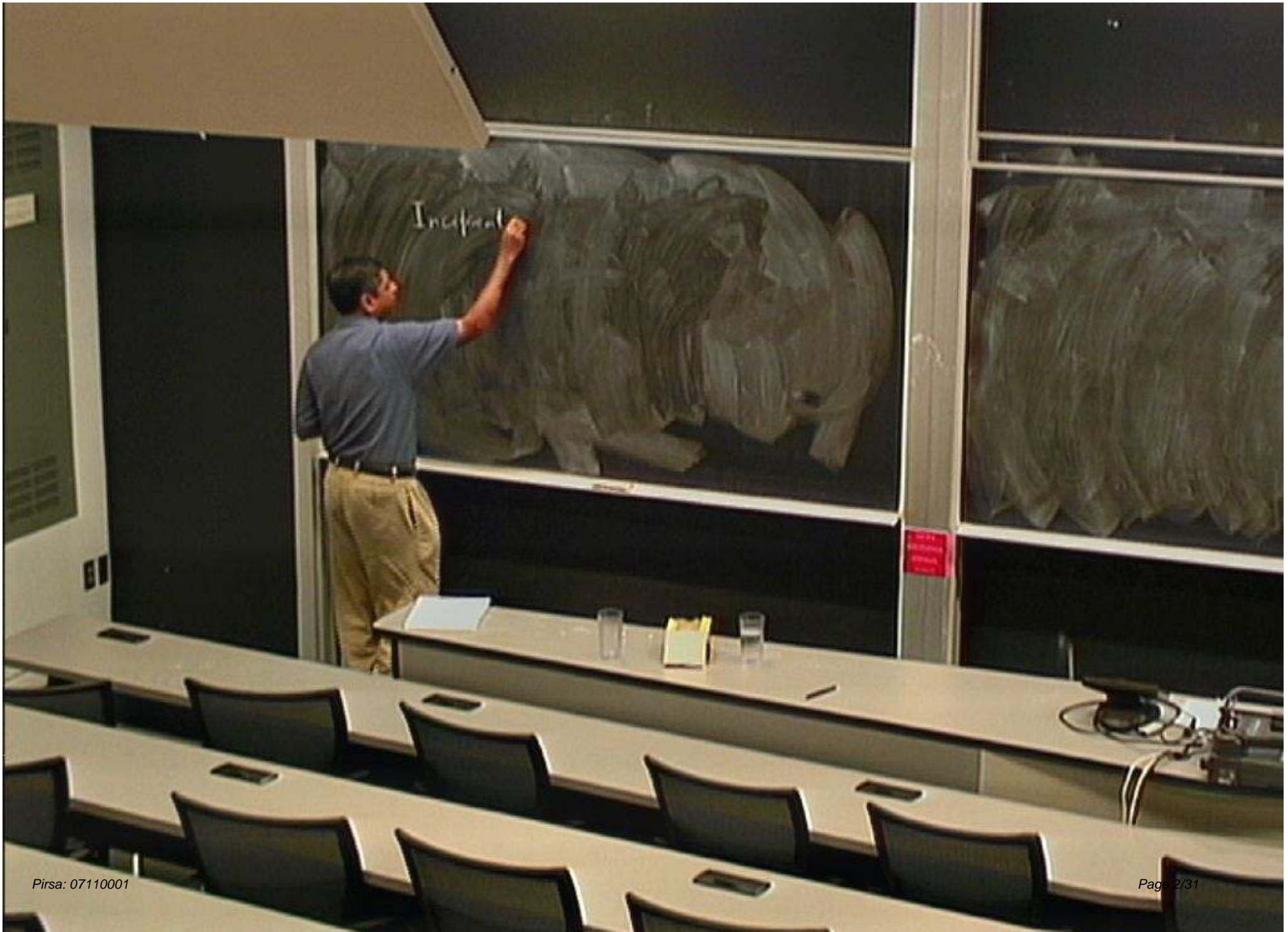


Title: Incipient Black Holes

Date: Nov 13, 2007 02:00 PM

URL: <http://pirsa.org/07110001>

Abstract: TBA



# Incipient Black Holes.

by L Krauss & D Stojkovic

gr-qc / 0609 024

0701096

07110006

Aim.  $T$  grav. collapse including  
quantum effects.

BHs in LHC / UHECR / Act. Cosm.



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by L Krauss & D Stojkovic

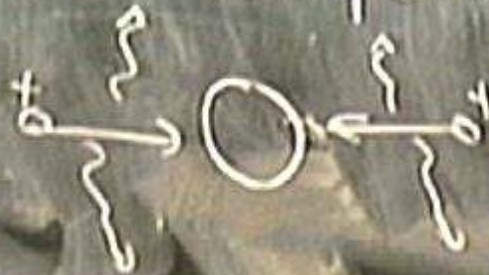
gr-qc / 0609 024

0701 096

0711.0006

Aim. To study grav. collapse including quantum effects.

Motivation: BHs in LHC/UHECR/Actino-Cosm.



System.

Spherical wall/membrane  
+ metric

CAUTION  
OBSTRUCTIVE  
EQUIPMENT  
REMOVED

System:

Spherical wall/membrane  
+ metric  
+ scalar field

Approach:

Functional Schrödinger eq  
$$H\Psi = i\frac{\partial\Psi}{\partial t}$$

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simplify

$$\text{Spherical shell} \Rightarrow X^\mu \rightarrow R(t)$$

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Approach:

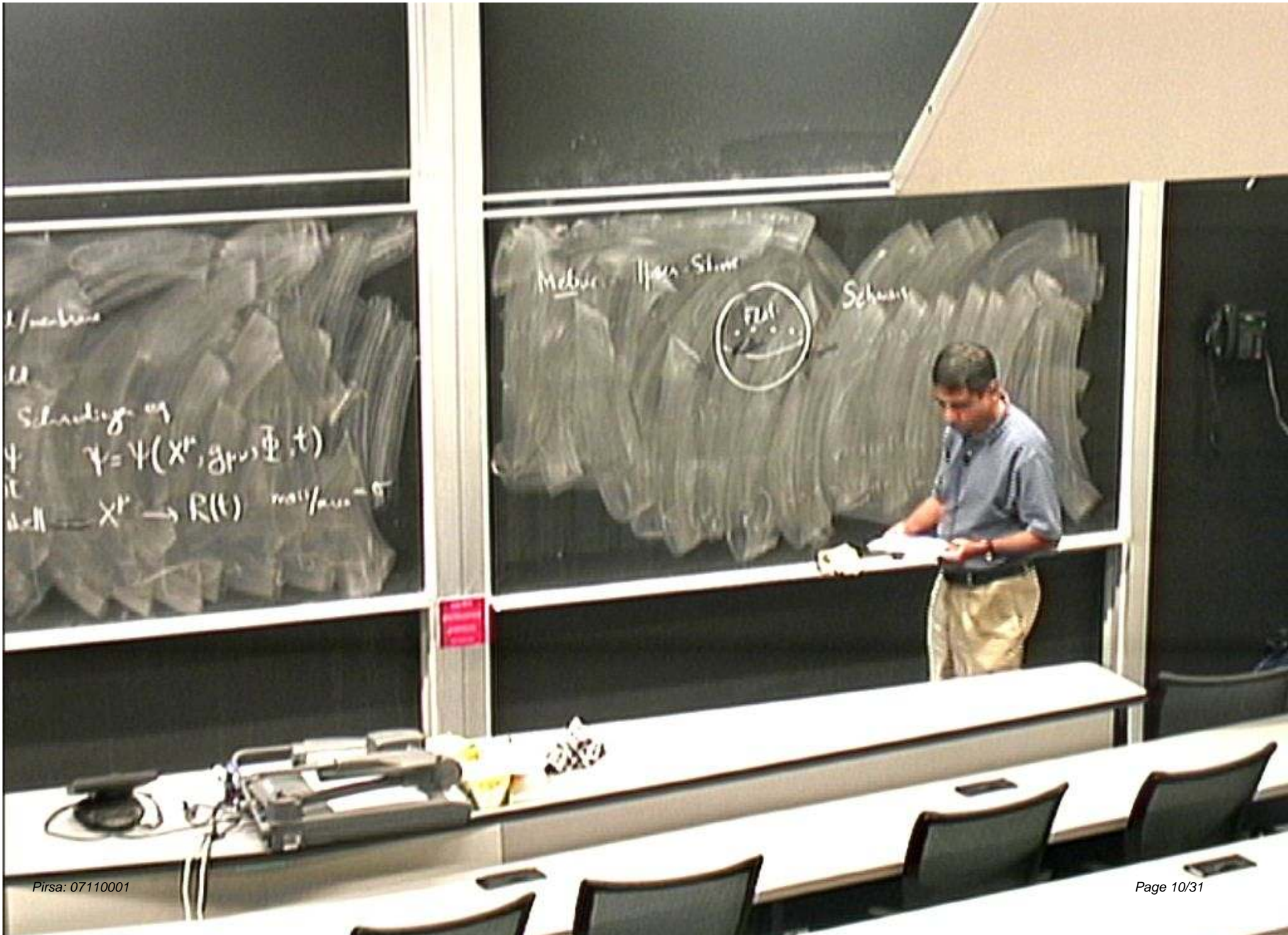
Functional Schrödinger eq

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Simplify

Spherical shell  $\rightarrow X^\mu \rightarrow R(t)$  mass/area  $= \sigma$



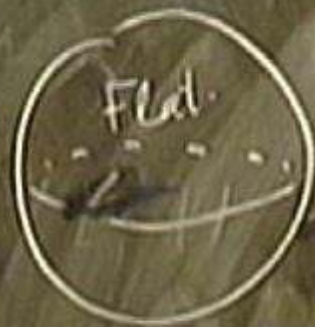
Metric: Ipser-Sikwie



Schwarz.

$$ds^2 = -\left(1 - \frac{R_s}{r}\right) dt^2 + \frac{1}{1 - R_s/r} dr^2 + r^2 d\Omega^2, \quad r > R(t)$$

Metric: ulpsen - Sikwie



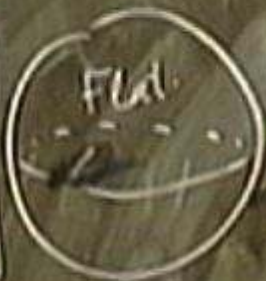
Schwarz

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$$ds^2 = -dT^2 + dr^2 + r^2 d\Omega^2, \quad r < R(t)$$

Metric: Ipsen-Skine

$$\frac{dT}{dt} = \left[ 1 + \left( \frac{dR}{dt} \right)^2 \right]^{1/2}$$
$$\frac{dt}{d\tau} = \frac{1}{B} \left[ B + \left( \frac{dR}{d\tau} \right)^2 \right]^{1/2}$$

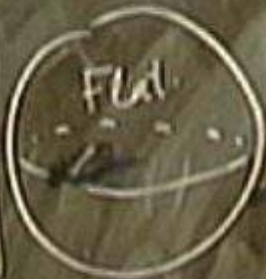


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Schwarz

$$B = 1 - \frac{R_S}{R}$$

$$ds^2 = - \left( 1 - \frac{R_S}{r} \right) dt^2 + \frac{1}{1 - R_S/r} dr^2 + r^2 d\Omega^2, \quad r > R(t)$$
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$$\tilde{M} = \sigma 4\pi R^2$$

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$T \rightarrow t$

$B \rightarrow 0$

$$\sqrt{(-B\pi)^2 + B(4\pi R^2)^2}$$

$\rightarrow -B\pi$

$$\mu = \sigma(1 - 2\pi G\sigma R_s)$$

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$$R_T = \frac{2R}{\lambda T}$$

$$T \rightarrow t$$

$$B \rightarrow 0$$

$$\sqrt{(-BT)^2 + B(4\pi\mu R^2)^2}$$

$$\mu = \sigma(1 - 2\pi G\sigma R_s)$$

$$\rightarrow$$

$$-BT$$

ultra-relativistic particle

$$T = \frac{4\pi\mu R^2 \dot{R}}{\sqrt{B} \sqrt{B^2 - R^2}}$$

$$H_{\text{wall}} = \frac{\tilde{M}}{\sqrt{1-R_T^2}} - \frac{G\tilde{M}^2}{2R}$$

$$\tilde{M} = \sigma 4\pi R^2$$

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$T \rightarrow t$

$B \rightarrow 0$

$$\sqrt{(-B\lambda)^2 + B(4\pi\mu R^2)^2}$$

$$\mu = \sigma(1 - 2\pi G\sigma R_s)$$

$$\dot{R} \approx B \left( 1 - \frac{1}{2} \frac{BR^4}{h^2} \right)$$

$\rightarrow$

$$-B\lambda$$

ultra-relativistic particle

$$\lambda = \frac{4\pi\mu R^2 \dot{R}}{\sqrt{B} \sqrt{B^2 - R^2}}$$

Scalar field.

$$S_{\Phi} = \int d^4x \sqrt{g} \frac{1}{2} g^{\mu\nu} \partial_{\mu} \Phi \partial_{\nu} \Phi$$

$$\Phi(x, t) = \sum_k a_k(t) f_k(x)$$

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$$M_{kk'} = 4\pi \int_0^{R_1} dx x^2 f_k(x) f_{k'}(x) \quad \left. \vphantom{M_{kk'}} \right\} \text{indep of } R(t)$$

$$N_{kk'} = \int \dots$$

Principal axis transformation

Diagonalize  $M, N$

One eigenmode  $b(t)$

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Diagonalize  $M, N$ .

One eigenmode  $b(t)$

$$H_b = -\left(\frac{1-R_s}{R}\right) \frac{1}{2m} \dot{b}^2 + \frac{1}{2} K b^2$$

$$\eta = \int_0^t dt \left(1 - \frac{R'_s}{R(t)}\right), \quad \omega^2(\eta) = \frac{\omega_0^2}{1 - R'_s/R}, \quad \omega_0^2 = \frac{K}{m}$$

$$-\frac{1}{2m} \frac{\partial^2 \psi}{\partial b^2} + \frac{m}{2} \omega^2(\eta) b^2 \psi = i \frac{\partial \psi}{\partial \eta}$$

$$\psi = e^{i\alpha(\eta)} \left(\frac{m}{\pi \rho^2}\right)^{1/4} \exp\left[\frac{im}{2} \left(\frac{\rho}{\rho'} + \frac{i}{\rho^2}\right) b^2\right]$$

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$$\rho \eta + \omega^2(\eta) \rho = 1/\rho^3$$

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$$\rho_{\eta\eta} + \omega^2(\eta)\rho = 1/\rho^3$$

$$\rho(0) = \frac{1}{\sqrt{\omega_0}}, \quad \rho_{\eta}(0) = 0$$

$$\alpha(\eta) = -\frac{1}{2} \int_0^{\eta} \frac{d\eta'}{\rho^2(\eta')}$$

Occupation numbers:

$$\Psi(b,t) = \sum c_n(t) \underbrace{\varphi_n(b)}_{\text{S.H.O basis states}}$$

S.H.O basis states

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s.h.o basis states

$$N(t, \bar{\omega}) = \sum_n n |c_n|^2$$

$$= \frac{\bar{\omega}}{\sqrt{2}} p^2 \left[ \left( 1 - \frac{1}{\bar{\omega} p^2} \right)^2 + \left( \frac{p_n}{\bar{\omega} p} \right)^2 \right]$$

$$\psi = e^{i\alpha(\eta)} \left( \frac{m}{\pi \rho^2} \right)^{1/4} \exp \left[ i \frac{m}{2} \left( \frac{p}{\rho} + \dots \right) \right]$$

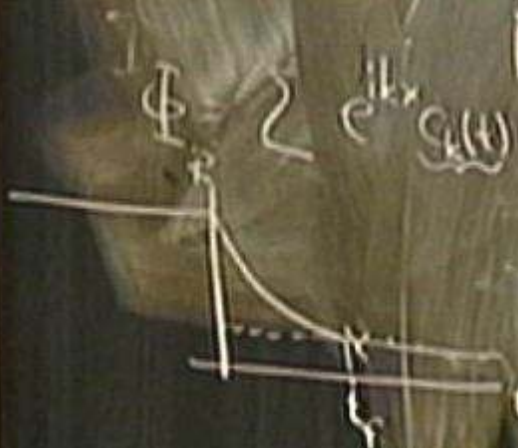
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$$\Psi(b,t) = \sum c_n(t) \underbrace{\varphi_n(b)}_{\text{st. basis states}}$$

$$H_n(b) e^{i E_n t / \hbar}$$



$$N(t, \bar{\omega}) = \sum n$$

$$= \left[ \frac{1}{\bar{\omega} P^2} \right]^2 + \left[ \left( \frac{P_n}{\bar{\omega} P} \right)^2 \right]$$

$$= P_0 + \dots$$

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SANTA BARBARA