

Title: Summary

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Abstract:

Thanks to the organizers for a great, diverse and provocative conference:

Sabine Hossenfelder, Bianca Dittrich, Tomasz Konopka, and Achim Kempf

- 1) How far have we come?
- 2) Challenges
- 3) DSR from quantum gravity
- 4) Is the dark energy a quantum gravity phenomena?
- 5) Non-locality and quantum gravity
- 6) Other windows into quantum gravity phenomenology

In 1998-2000 there arose the then novel idea that Planck scale phenomena could be observed in high energy astrophysics

These measure the fate of Poincare invariance to order $l_{\text{pl}} E$ or $(l_{\text{pl}} E)^2$:

How far has this idea come?

A key observational question for quantum gravity is:

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Hence, the symmetry of the quantum ground state is a dynamical question.

A key observational question for quantum gravity is:

What is the symmetry of the ground state?

Global Lorentz and Poincare invariance are not symmetries of classical GR, they are only symmetries of the ground state with $\Lambda=0$.

Hence, the symmetry of the quantum ground state is a dynamical question.

Three possibilities

- 1 Poincare invariant
- 2 Broken Lorentz invariance
- 3 Deformed Poincare invariance (DSR)

Deformed or doubly special relativity (DSR)

Principles of deformed special relativity (DSR):

- 1) Relativity of inertial frames
- 2) The constancy of c , a velocity
- 3) The constancy of an energy E_{planck}
- 4) c is the universal speed of photons for $E \ll E_{\text{planck}}$

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Consequences:

- Modified energy-momentum relations
- Momentum space has constant curvature given by E_{planck}
- Energy-momentum conservation becomes non-linear
(Coproduct)

$\hbar m \hbar$

E_p^3

$$\alpha E \sim \sqrt[3]{m^2 E_p}$$

$$\hbar \rightarrow 0$$

$$G \rightarrow 0$$

$$m_p \sim \sqrt{\frac{\hbar}{G}} \rightarrow \text{constant}$$

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Mathematical realizations:

- 1) Deformed poincare algebra is a hopf algebra
Acts on a spacetime geometry which is non-commutative.
- 2) metric becomes scale dependent: $g_{ab}(E)$

Are they different?

What are the differences?

Is there a way to map them to each other?

$$\tanh\left(\frac{m^2 E}{E_p^3}\right)$$

$$E \sim \sqrt[3]{\frac{m^2 E_p}{\hbar}}$$

$$\hbar \rightarrow 0$$

$$G \rightarrow 0$$

$$m_p \sim \sqrt{\frac{\hbar}{G}}$$

$$c \hbar G \rightarrow c(E)$$

$$\hbar m h \left(\frac{1}{E_p} \right)$$

$$\sqrt{m^2 E_p}$$

$$\hbar \rightarrow 0$$

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+

$$c \hbar G \rightarrow c(E)$$

$$\hbar \rightarrow 0$$

$$G \rightarrow 0$$

$$c \hbar$$

$$\hbar \sim \sqrt{\hbar}$$

$$\sqrt{\frac{\hbar}{G}} \rightarrow \text{constant}$$

$$\hbar(E_{\text{Pl}}) \quad \hbar(E_{\text{v}})$$

$$\hbar \rightarrow 0$$

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$$m_p \sim \sqrt{\frac{\hbar}{G}} \rightarrow \text{constant}$$

$$c \hbar \quad G \rightarrow c(E_{\text{Pl}}) \quad \hbar(E_{\text{Pl}})$$

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Models of DSR:

DSR is realized precisely in 2+1 gravity with matter [hep-th/0307084](#)

QFT on kappa-minkowski

Rainbow metric

Energy dependent \hbar and c

How can we experimentally distinguish the three possibilities of exact, broken or deformed Poincare invariance?

There are two basic low energy QG effects:

1) Corrections to energy momentum relations:

$$E^2 = p^2 + m^2 + \alpha l_p E^3 + \beta l_p^2 E^4 + \dots$$
$$v = c(1 + \alpha l_p E + \dots)$$

2) Modifications in the conservation laws.

Some basic consequences:

- Preferred frame allowed processes (photon decay)
- Modifications of thresholds (GZK, TeV photons...)
- Energy dependence of the speed of light, neutrinos ...

Can they be measured to $O(l_{Planck})$?

$$\frac{1}{\hbar m h} \left(\frac{m^2 E}{E_p^3} \right)$$

$$E \sim \sqrt{\hbar m_p}$$

$$\omega_F \sim \sqrt[3]{m^2 E_p}$$

$$\left[\begin{array}{l} \hbar \rightarrow 0 \\ G \rightarrow 0 \end{array} \right. \quad m_p \sim \sqrt{\frac{\hbar}{G}} \rightarrow \text{constant} \quad \left. \begin{array}{l} c \hbar G \rightarrow c(E_{\text{ev}}) \quad \hbar(E_{\text{ev}}) \dots \end{array} \right]$$

Energy dependent speed of light

$$v(E) = c(1 + a l_p E + b l_p^2 E^2 + \dots)$$

- Accumulates for long distances
- Observable in Gamma Ray bursts.
- present limits have $a < 1000$
- *GLAST* will put limits $a < 1$
- **Could be parity even or odd**
- *A parity odd $v(E)$ has been ruled out at $O(l_p)$*
by observations of distant polarized radio galaxies
Also, by polarization observed in Gamma Ray Bursts
Colburn, Boggs, Nature 423, 415–417 (2003). Mitrofanov, Nature, VOL 426 13 Nov 2003
- *GLAST could see $O(l_{pL})$ parity-even $v(E)$*

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Broken lorentz invariance gives modified dispersion relations but unmodified conservation laws



- *GZK threshold moves appreciably*
- *helicity odd energy dependent speed of light*

Deformed lorentz invariance gives both.



- *GZK threshold as in ordinary special relativity*
- *possible helicity even energy dependent speed of light*

To distinguish the three possibilities we need three experiments:

- *AUGER tests GZK*
- *MAGIC, GLAST tests energy dependence of photons*
- *Detection of polarized photons from distant sources tests helicity dependence*

$$E^2 = p^2 + m^2 + E^3 + E^4$$

$$\frac{E_{cr}}{E_{pl}} \sim \frac{E_x}{m_{\pi}} \tanh \left(\frac{m^2 E}{E_p^3} \right)$$

$$\hbar \rightarrow 0$$

$$E^2 = p^2 + m^2 + E^3 + E^4$$

$$\frac{E_{cr}}{E_{pl}} \sim \frac{E_x}{m_{\pi}} + \ln h \left(\frac{m^2 E}{E_p^3} \right) \quad E$$

$$\frac{E_{cm}}{E_{pl}} \quad \left| \begin{array}{l} h \rightarrow 0 \\ G \rightarrow 0 \end{array} \right. \quad m_p \sim \sqrt{\frac{h}{G}} \rightarrow 0$$

Broken lorentz invariance gives modified dispersion relations but unmodified conservation laws



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The GZK threshold:

Cosmic ray protons scattering off the microwave background.

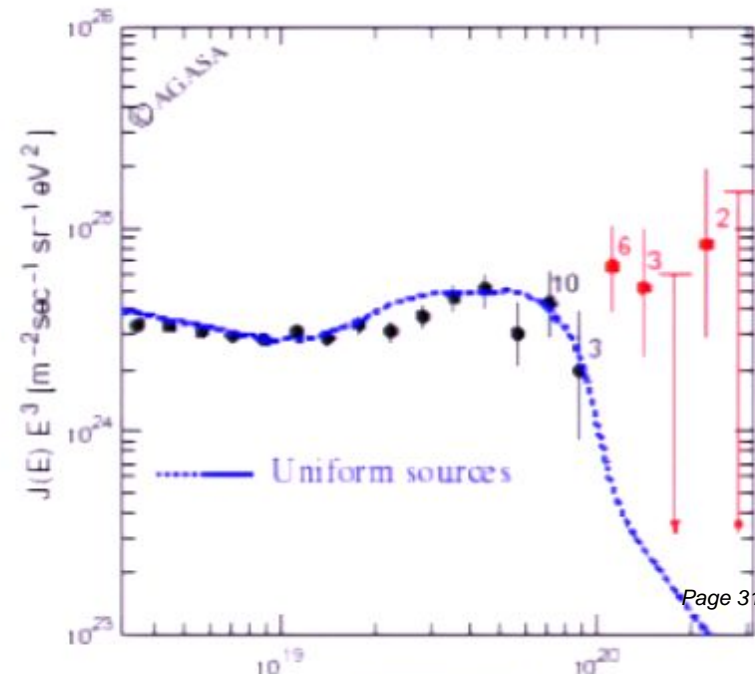
Special relativity predicts a threshold at $3 \cdot 10^{19}$ eV.

An effect of a $O(l_{\text{Planck}})$ modified E-p relation is to move it $O(1)$

$$E^2 = p^2 + m^2 + \alpha l_p E^3 + \beta l_p^2 E^4 + \dots$$

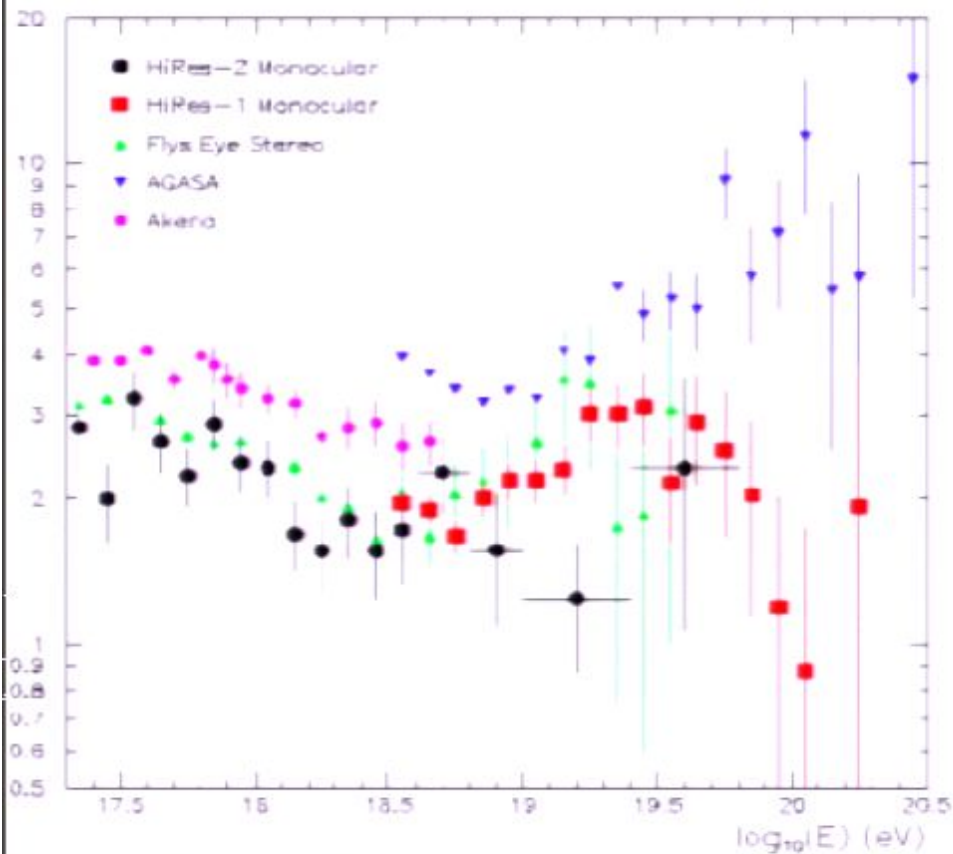
Prediction from Lorentz Inv
+ uniform sources

**AGASA reported events
over the GZK threshold!**



GZK: AGASA, Sugar saw
anomalous events

HIRES didn't



AUGER Astro-ph/0608136

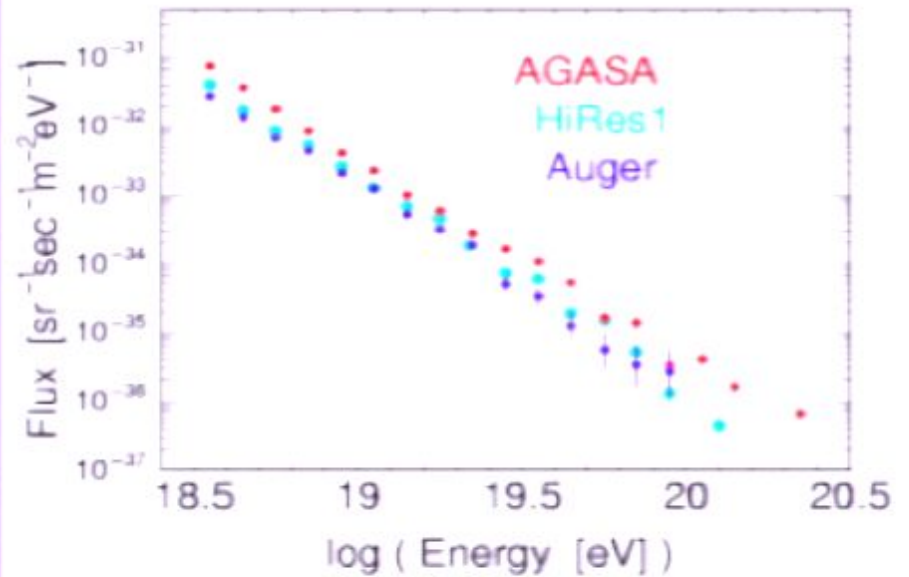
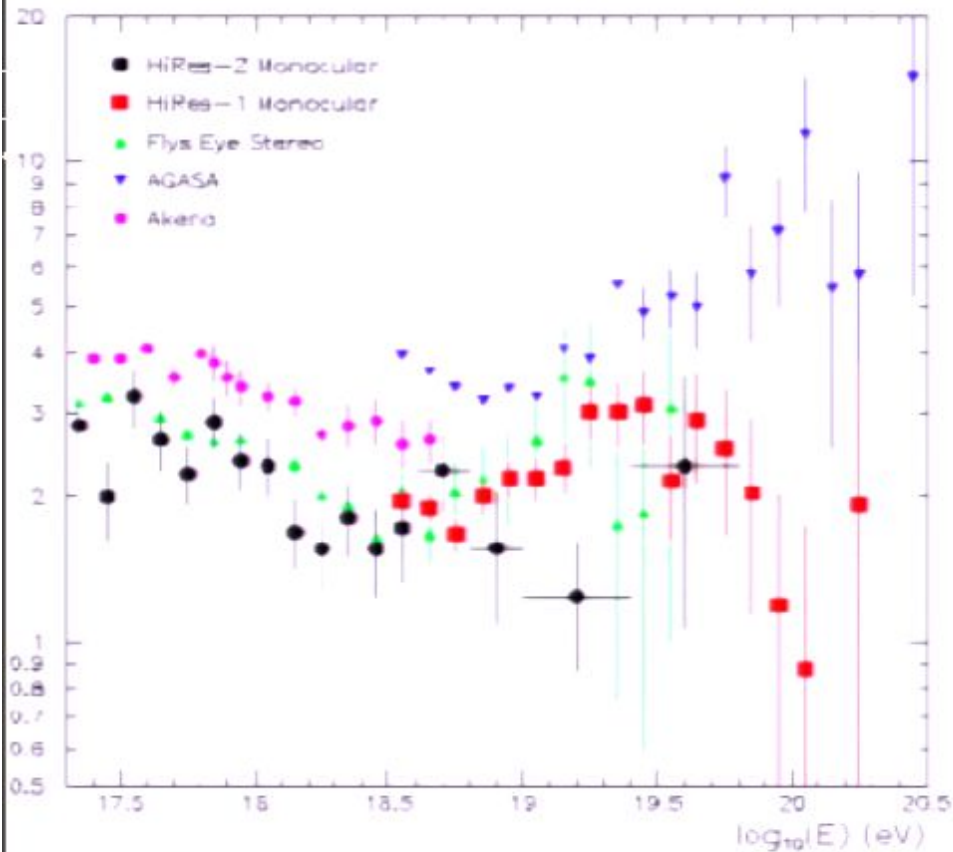
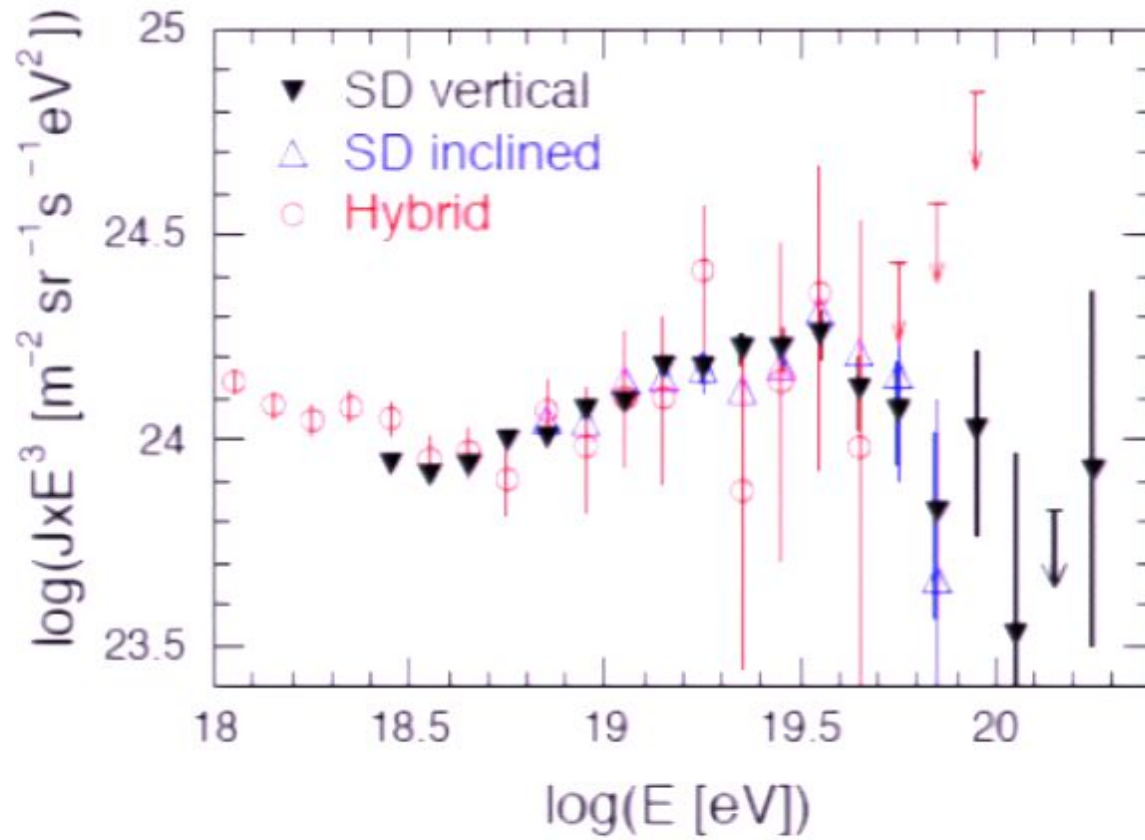


Fig. 6. The energy spectrum of EHECRs measured by the Pierre Auger experiment compared with the AGASA ³⁸ and HiRes-I ³⁹ results.

AUGER July 07



Correlation of the Highest-Energy Cosmic Rays with Nearby Extragalactic Objects

The Pierre Auger Collaboration*

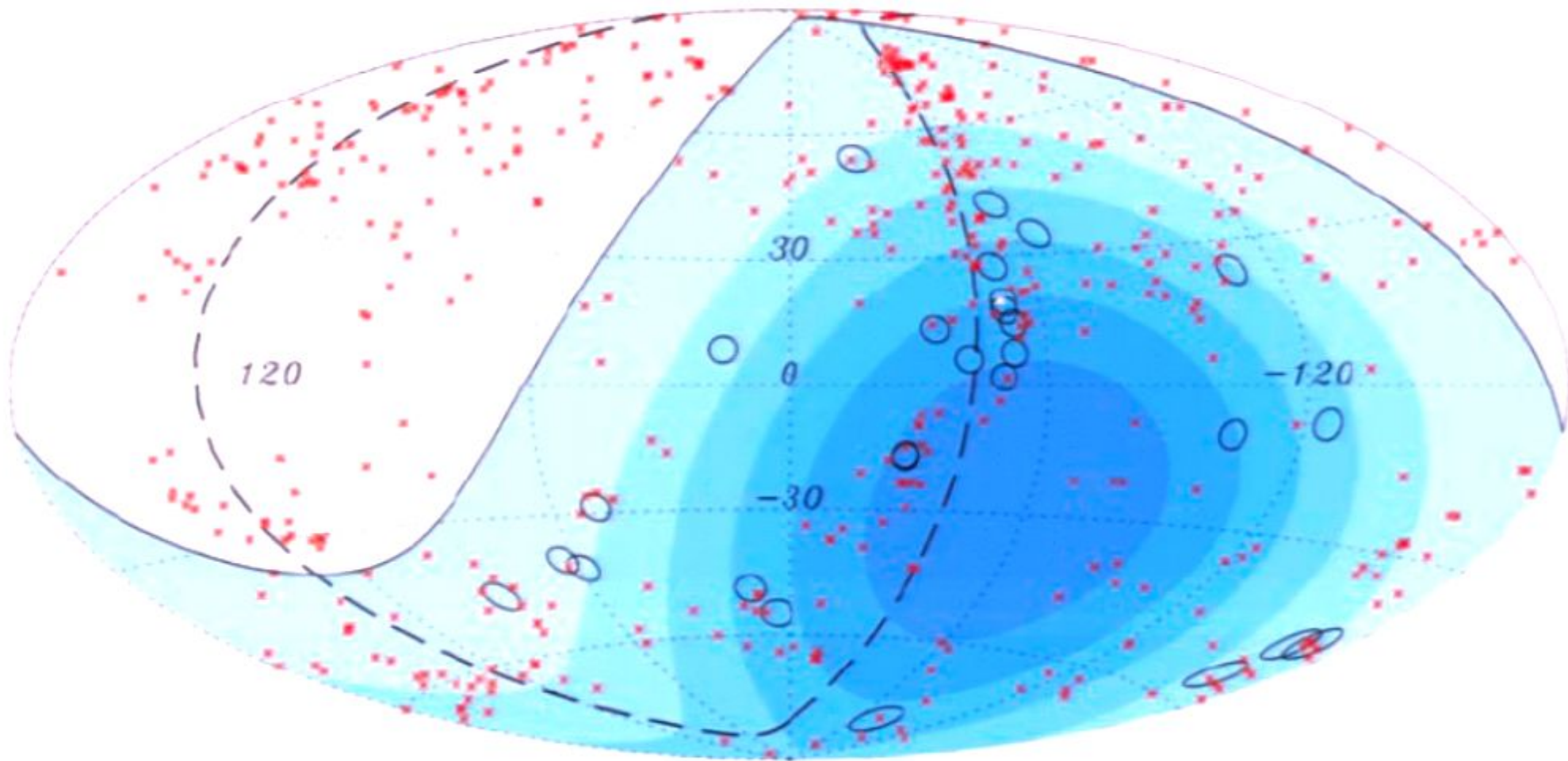


Fig. 2. Aitoff projection of the celestial sphere in galactic coordinates with circles of radius 3.1° centered at the arrival directions of the 27 cosmic rays with highest energy detected by the Pierre Auger Observatory. The positions of the 472 AGN (318 in the field of view of the Observatory) with redshift $z \leq 0.018$ ($D < 75$ Mpc) from the 12th edition of the catalog of quasars and active nuclei (12) are indicated by red asterisks. The solid line represents the border of the field of view (zenith angles smaller than 60°). Darker color indicates larger relative exposure. Each colored band has equal integrated exposure. The dashed line is the supergalactic plane. Centaurus A, one of our closest AGN, is marked in white.

Tentative but pretty compelling conclusions:

- **There is a GZK cutoff**
- **Lorentz symmetry breaking is dead, at least at first order**
- **DSR and good old fashioned SR are fine.**

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1 sec dilates to 10,000 years!!

1 cm contracts to 100 fermi!!

Hubble scale contracts to light days!!

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Probing Quantum Gravity using Photons from a Mkn 501 Flare Observed by MAGIC

We use the timing of photons observed by the MAGIC gamma-ray telescope during a flare of the active galaxy Markarian 501 to probe a vacuum refractive index $\simeq 1 - (E/M_{\text{QC}n})^n$, $n = 1, 2$, that might be induced by quantum gravity. The peaking of the flare is found to maximize for quantum-gravity mass scales $M_{\text{QC}1} \sim 0.4 \times 10^{18}$ GeV or $M_{\text{QC}2} \sim 0.6 \times 10^{11}$ GeV, and we establish lower limits $M_{\text{QC}1} > 0.26 \times 10^{18}$ GeV or $M_{\text{QC}2} > 0.39 \times 10^{11}$ GeV at the 95% C.L. Monte Carlo studies confirm the MAGIC sensitivity to propagation effects at these levels. Thermal plasma effects in the source are negligible, but we cannot exclude the importance of some other source effect.

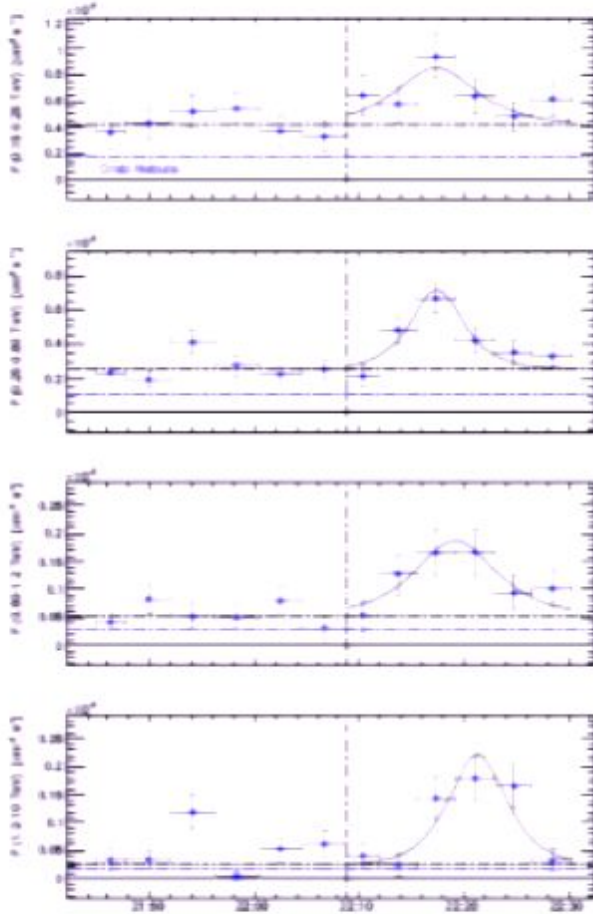


FIG. 1.— LC for the night July 9 with a time binning of 4 minutes, and separated in different energy bands, from the top to the bottom, 0.15–0.25 TeV, 0.25–0.6 TeV, 0.6–1.2 TeV, 1.2–3 TeV. The vertical bars denote 1σ statistical uncertainties. For comparison, the Crab emission is also shown as a lilac dashed horizontal line. The vertical dot-dashed line divides the data into ‘stable’ (i.e., pre-burst) and ‘variable’ (i.e., in-burst) emission. The horizontal black dashed line represents the average of the ‘stable’ emission. The ‘variable’ (in-burst) of all energy ranges were fit with a flare model described by equation 2, where $\tau_r = \tau_f$ (rise=fall time). All parameters were left free in the fit. All light curves were considered simultaneously in the fit (combined fit). The resulting parameters from this combined fit are reported in table 4.

Pirsa: 07100039

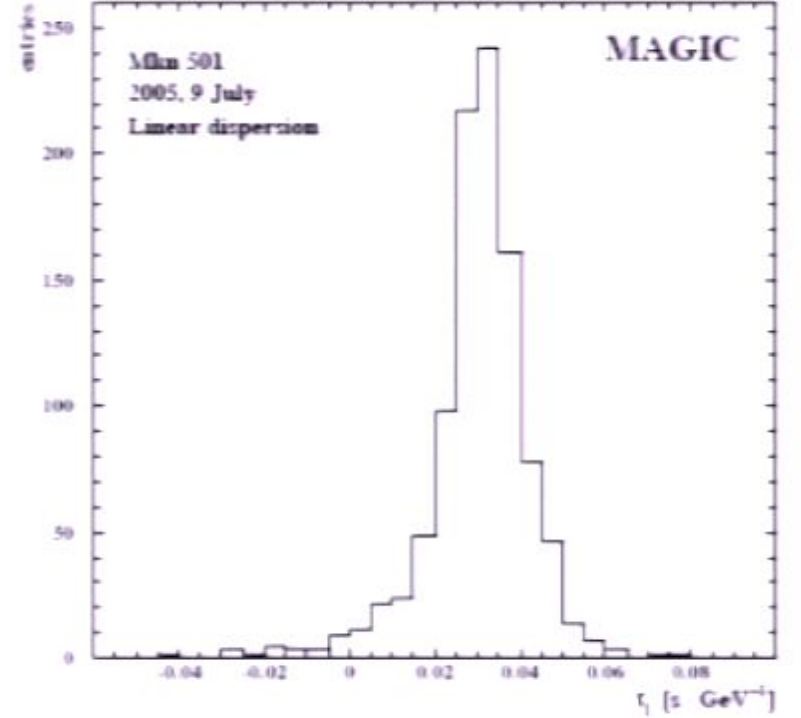


FIG. 2.— The τ_1 distribution from fits to the ECFs of 1000 realizations of the July 9 flare with photon energies smeared by Monte Carlo.

More flares??

GLAST launches in “May”

Too tempting not to say:

If both experiments are correct we have

- GZK cutoff
- Parity even energy dependent speed of light.
- No parity odd variation in c

These are the signatures of DSR!!

Some questions for Lorentz violators:

EFT+ birefringence results imply no first order in l_{Pl} violation.

- *If the MAGIC results are a real variation in the speed of light, how can this then be lorentz violation?*

With respect to EFT, saying it isn't so doesn't make it not so.

- *Are there really broken lorentz violating theories that EFT does not apply to? Why doesn't EFT apply also to open quantum systems?*

- *Are there interesting theories with Lorentz violation at second order and not at first order?*

- *If there really is a preferred frame, what about dimension 1,2,4 operators? Why should Einstein be so wrong but SR work so well?*

Some questions for DSRers:

- *Shouldn't there be a version of EFT appropriate to DSR or deformed Poincare symmetry?*
- *Is DSR a more general category of theories than deformed Poincare symmetry?*
- *Is it so hard to write a full interacting QFT with DSR?*
- *Are there really different versions of DSR? Are the deformed Poincare, energy dependent metric, energy dependent \hbar and c different theories or different representations of one class of theories?*
- *Is there a universal version of DSR, with parameters to represent different versions?*

A question for everyone:

Effective field theory versus non-locality?

Questions for quantum gravityists:

Can DSR be derived from some version of quantum gravity?

Would the result be generic, or theory dependent?

Energy dependence of the metric is a consequence of quantum gravity.

hep-th/05010901

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Assumptions:

•Ashtekar variables

$$\{\mathcal{A}_a^i(x), \tilde{E}_j^b(y)\} = \rho \delta_a^b \delta_i^j \delta^d(x, y)$$

$[\rho] = \text{length}^2$

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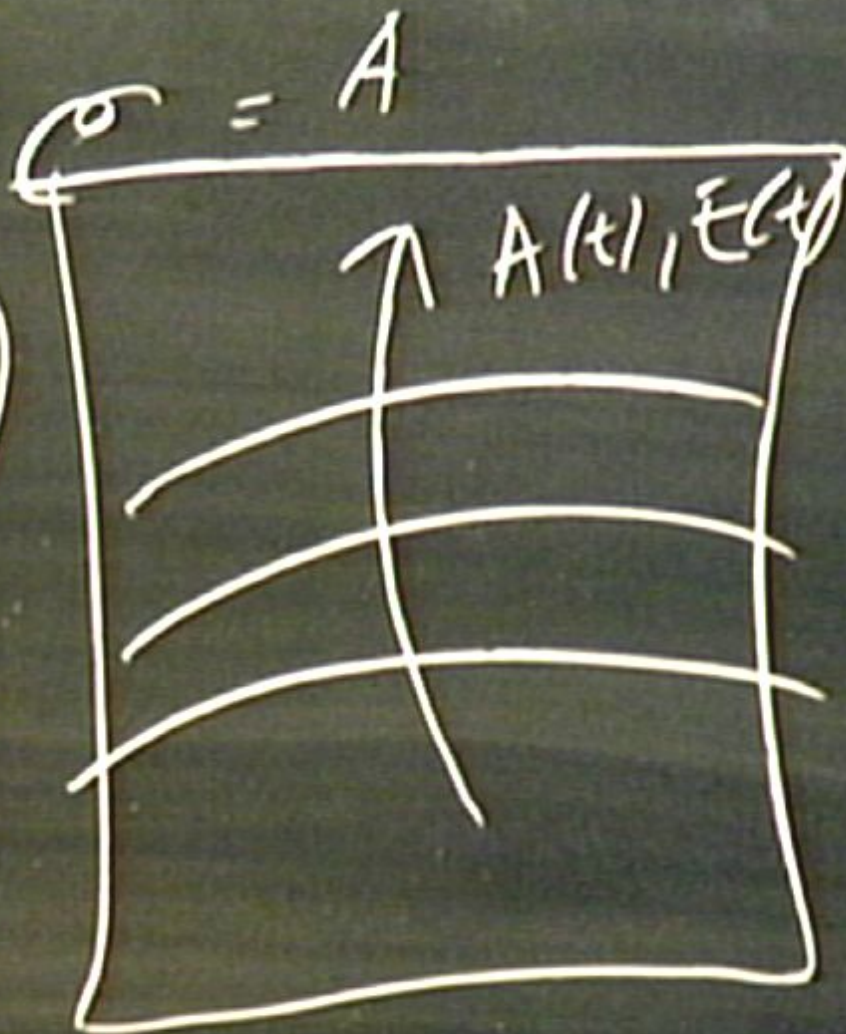
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- Connection rep: $\tilde{E}_i^a(x) = -i\hbar\rho \frac{\delta}{\delta \mathcal{A}_a^i(x)}$ $\langle A | \Psi \rangle = \Psi(A)$
- Semiclassical states: $\Psi_0[\mathcal{A}] = e^{i \frac{S[\mathcal{A}]}{\hbar}}$ $S(A)$ =Hamilton-Jacobi function

$$\Phi\left(A, \frac{\delta S}{\delta A} = E\right) = 0$$



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•Matter fields:

$$\Psi[\mathcal{A}, \phi] = \Psi_0[\mathcal{A}] \chi[\mathcal{A}, \phi]$$

Consider a solution to the Hamilton-Jacobi equations for GR: $S[\mathcal{A}]$

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Gradient flow along $S(\mathcal{A})$ gives a classical solution:

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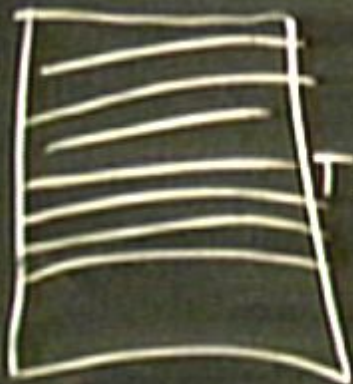
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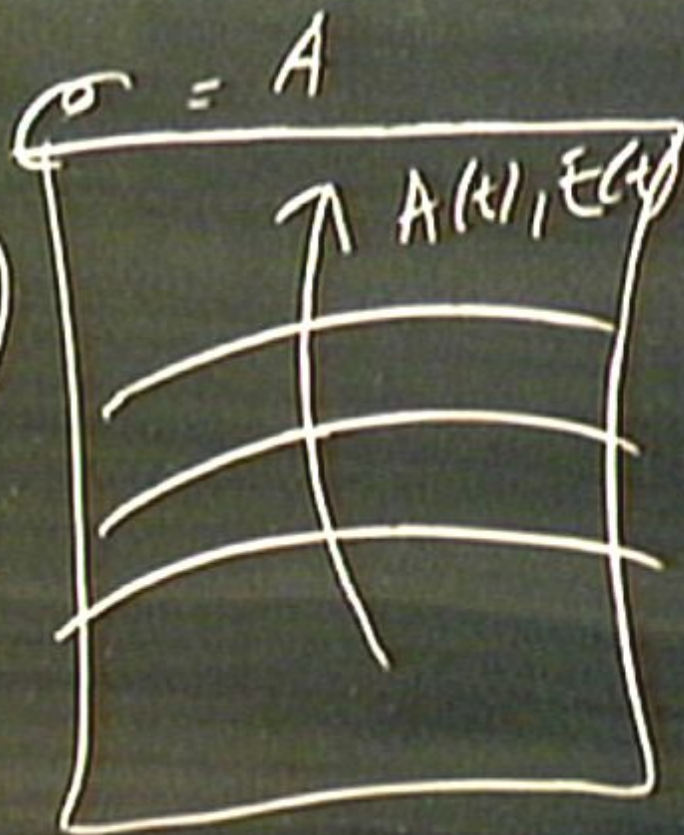
On the classical solution there is a time coordinate T proportional to $S(\mathcal{A})$

$$\frac{d}{dT} = \mu \frac{\delta}{\delta \mathcal{S}[\mathcal{A}]} \quad [\mu] = \text{length}^{-1}$$

$$\Phi(A, \frac{\delta S}{\delta A} = E)$$



$$T = S(H) = 0$$



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E^a_i is a densitized frame field, related to frame 1-form e_i :

The metric g is

$$g = -dT^2 + \sum_i e_i^0 \otimes e_i^0$$

Variations of A are along the solution and orthogonal to it;

$$\frac{\delta}{\delta \mathcal{A}_a^i(x)} = \frac{1}{M} \tilde{E}_{i0}^a \frac{\delta}{\delta \mathcal{S}} + \frac{\delta}{\delta a_a^i} \quad \tilde{E}_0^{ai} \delta a_{ai} = 0$$

$M = \text{length}^2$

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M=length²

We act on the product state: $\Psi[\mathcal{A}, \phi] = \Psi_0[\mathcal{A}] \chi[\mathcal{A}, \phi]$

Since S(A) is a coordinate on C $\chi[\mathcal{A}, \phi] = \chi[\mathcal{S}, a_{ai}, \phi]$

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$$\begin{aligned} \hat{\tilde{E}}_i^a(x) \chi[\mathcal{A}, \phi] &= -i\hbar\rho \frac{\delta \chi[\mathcal{A}, \phi]}{\delta \mathcal{A}_a^i(x)} \\ &= \left(\tilde{E}_i^{0a} \frac{i\hbar\rho}{M} \frac{\delta}{\delta \mathcal{S}(x)} - i\hbar\rho \frac{\delta}{\delta a_{ai}(x)} \right) \chi[\mathcal{S}, a_{ai}, \phi] \end{aligned}$$

Putting it all together, we have $\frac{i\hbar\rho}{M} \frac{\delta}{\delta\mathcal{S}(x)} = \frac{i\hbar\rho}{M\mu} \frac{d}{dT}$

But dimensionally $\frac{\hbar\rho}{M\mu}$ is a time. But there is only one time in the problem, so

$$\frac{\hbar\rho}{M\mu} = \alpha l_{Pl}$$

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$$\frac{\hbar\rho}{M\mu} = \alpha l_{Pl}$$

Hence, on all semiclassical states we find to leading order:

$$\hat{\tilde{E}}_i^a(x) \Psi[\mathcal{A}, \phi] = \Psi_0[\mathcal{A}] \tilde{E}_i^{0a} \left(1 - i\alpha l_{Pl} \frac{d}{dT} \right) \chi[T, a_{ai}, \phi]$$

Putting it all together, we have
$$\frac{i\hbar\rho}{M} \frac{\delta}{\delta\mathcal{S}(x)} = \frac{i\hbar\rho}{M\mu} \frac{d}{dT}$$

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This implies modified dispersion relations

$$m^2 = -g(\omega)^{\mu\nu} k_\mu k_\nu = \omega^2 - \frac{k_i^2}{(1 - \alpha l_P l \omega)}$$

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- *Can the value of α be predicted?*

In principle, in a more detailed treatment.

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Yes, because only the spatial components of the metric are operators:

$$\mathbf{g} = -dT^2 + \sum_i \mathbf{e}_i^0 \otimes \mathbf{e}_i^0$$

$$\text{So: } m^2 = -g(\omega)^{\mu\nu} k_\mu k_\nu = \omega^2 - \frac{k_i^2}{(1 - \alpha l_{Pl} \omega)}$$

Hence, DSR in the form of an energy dependent metric, is a consequence of quantum gravity, in the semiclassical approximation.

Hence, a parity even energy dependent speed of light is a prediction of quantum gravity, in the semiclassical approximation.

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Or, more generally:

$$\hat{E}_i^a(x) \Psi[\mathcal{A}, \phi] = \Psi_0[\mathcal{A}] \tilde{E}_i^{0a} \left(1 - i \alpha l_{Pl} \frac{d}{dT} \right) \chi[T, a_{ai}, \phi]$$

What is the scale of quantum gravity effects?

Are they only at the Planck scale?

The vacuum energy is a quantum effect.

Is the dark energy then a quantum gravity effect?

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$$\not\partial \bar{\Psi} = 0$$

$$= \left(H^{g_{\mu\nu}} + H^{\mu\nu} \left(\frac{\delta}{\delta \phi} \phi \right) \right) \bar{\Psi}_\nu \chi$$

$$\Rightarrow i \frac{d}{dt} - \hat{H}^{\mu\nu}(\vec{E}_0) \chi = 0$$

$$\not\phi \bar{\Psi} = 0$$

$$= \left(H^{g_{\mu\nu}} + H^m (\xi_\phi \phi) \right) \bar{\Psi}_\nu \chi$$

$$\Rightarrow i \frac{1}{1+t} - \left[\hat{H}^m (E^0) \right] \chi = 0$$

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