Title: Theory of the Nernst effect near quantum phase transitions in condensed matter, and in dyonic black holes

Date: Oct 23, 2007 04:00 PM

URL: http://pirsa.org/07100036

Abstract: We present a general hydrodynamic theory of transport in the vicinity of superfluid-insulator transitions in two spatial dimensions described by ``Lorentz\'\'-invariant quantum critical points. We allow for a weak impurity scattering rate, a magnetic field B, and a deviation in the density, rho, from that of the insulator. We show that the frequency-dependent thermal and electric linear response functions, including the Nernst coefficient, are fully determined by a single transport coefficient (a universal electrical conductivity), the impurity scattering rate, and a few thermodynamic state variables. With reasonable estimates for the parameters, our results predict a magnetic field and temperature dependence of the Nernst signal which resembles measurements in the cuprates, including the overall magnitude. Our theory predicts a ``hydrodynamic cyclotron mode\'\' which could be observable in ultrapure samples. We also present exact results for the zero frequency transport co-efficients of a supersymmetric conformal field theory (CFT), which is solvable by the AdS/CFT correspondence. This correspondence maps the rho and B perturbations of the 2+1 dimensional CFT to electric and magnetic charges of a black hole in the 3+1 dimensional anti-de Sitter space. These exact results are found to be in full agreement with the general predictions of our hydrodynamic analysis in the appropriate limiting regime. The mapping of the hydrodynamic and AdS/CFT results under particle-vortex duality is also described.

Theory of the Nernst effect near quantum phase transitions in condensed matter and in dyonic black holes

Markus Müller

with

Sean Hartnoll (KITP)

Pavel Kovtun (KITP)

Subir Sachdev (Harvard)

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SWISS NATIONAL SCIENCE FOUNDATION

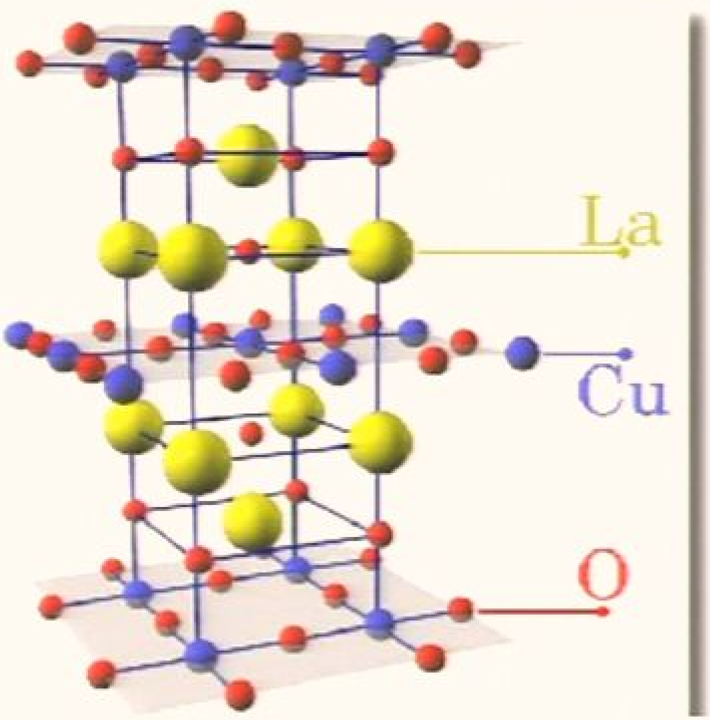
National Science Foundation
WHERE DISCOVERIES BEGIN

Perimeter Institute, Waterloo, 23 October, 2007

Outline

- Nernst experiments in superconductors
- Quantum criticality
  → conformal (relativistic) field theory
- Hydrodynamic analysis of the thermo-electric response functions
- Obtain same results *directly and exactly* via the AdS/CFT correspondence
- Comparison with experiments, predictions

$La_{2-x}Sr_xCuO_4$ (LSCO)



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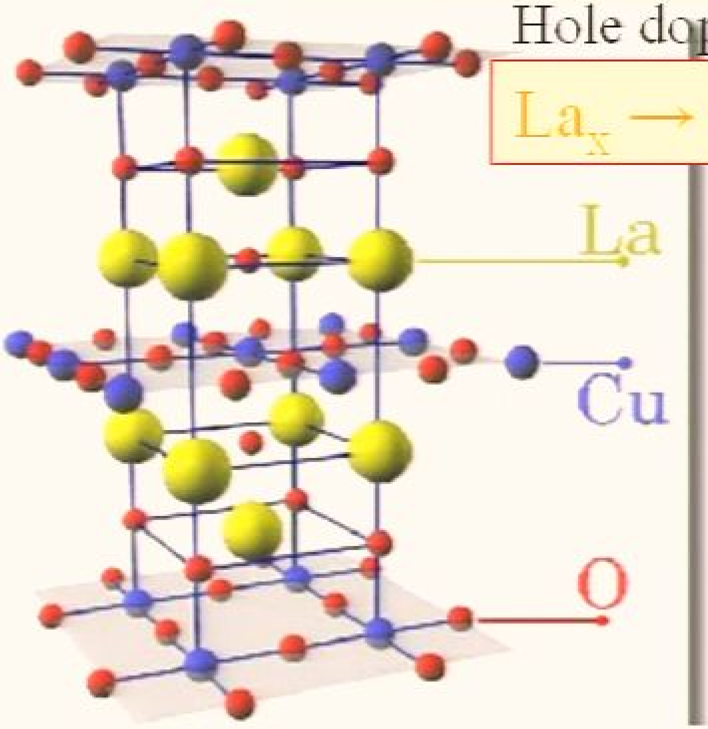
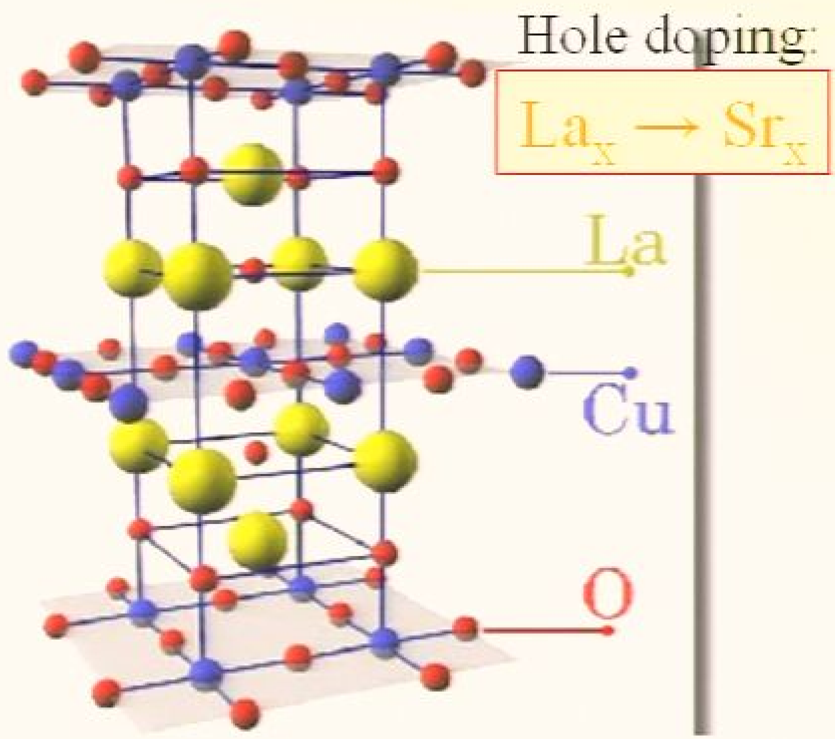
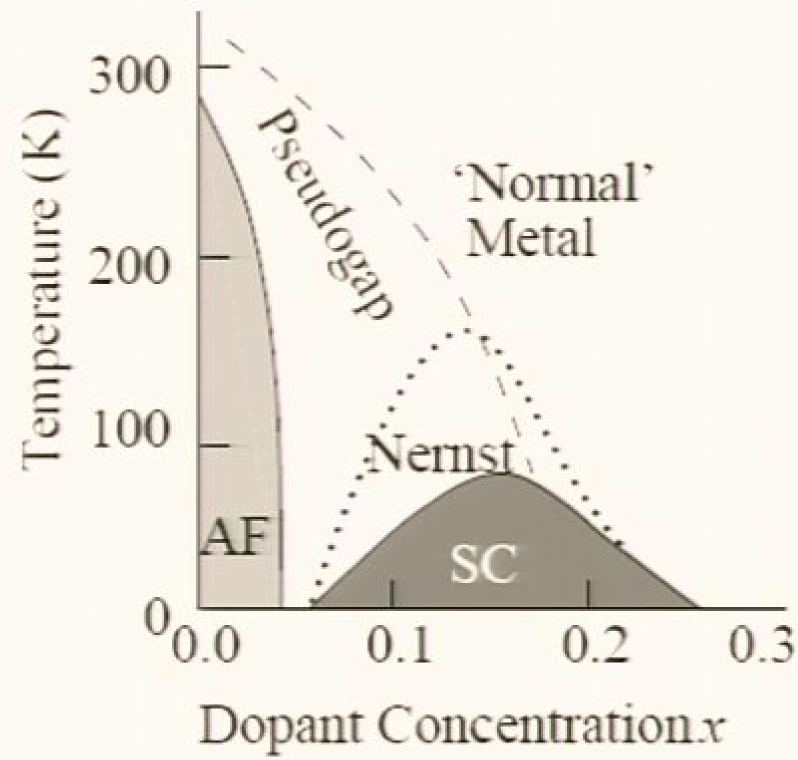


Hole doping:

$La_x \rightarrow Sr_x$

$La_{2-x}Sr_xCuO_4$ (LSCO)



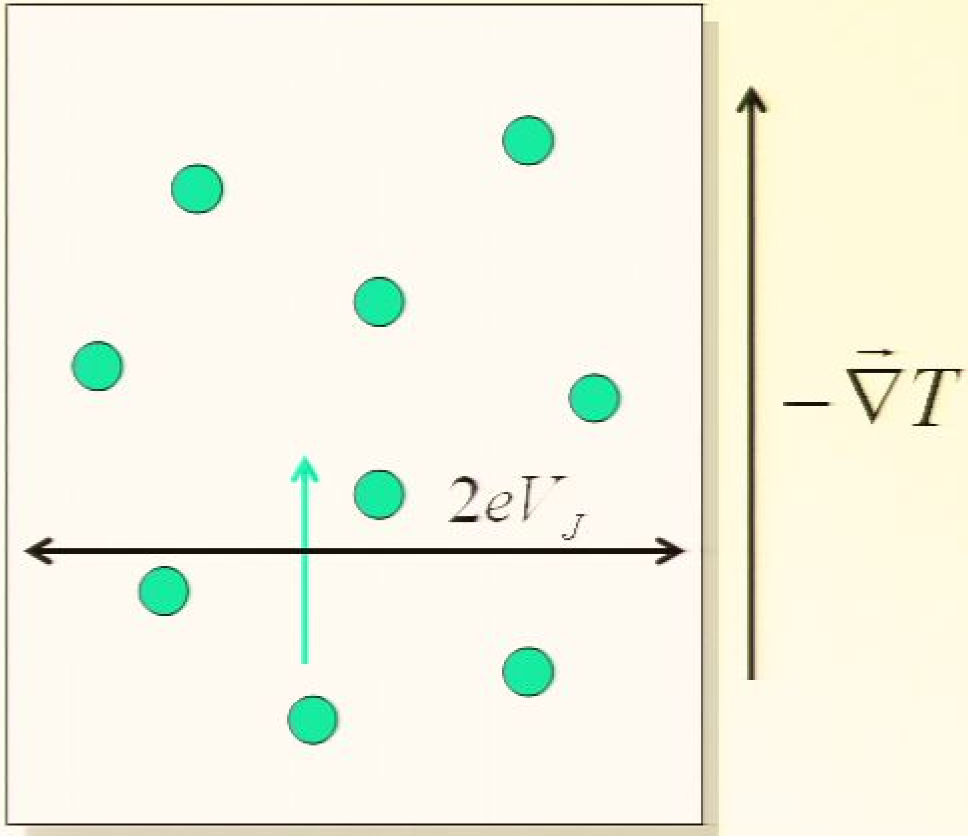


- Undoped $x=0$: antiferromagnetic Mott insulator
- Underdoped-optimally doped $0.05 < x < 0.17$:
  Strong Nernst signal up to $T=(2\text{-}3)T_c$
- Overdoped $0.17 < x$:
  BCS-like transition, very small Nernst signal above T_c

Nernst effect – why?

Transverse voltage due to vortices moving to lower T (causing phase slips)

$$2eV_J = \hbar \partial_t \varphi$$
$$= 2\pi\, \hbar \partial_t n_V$$



Nernst signal: $$e_N \equiv N = \frac{E_y}{-\vec{\nabla}_x T}$$

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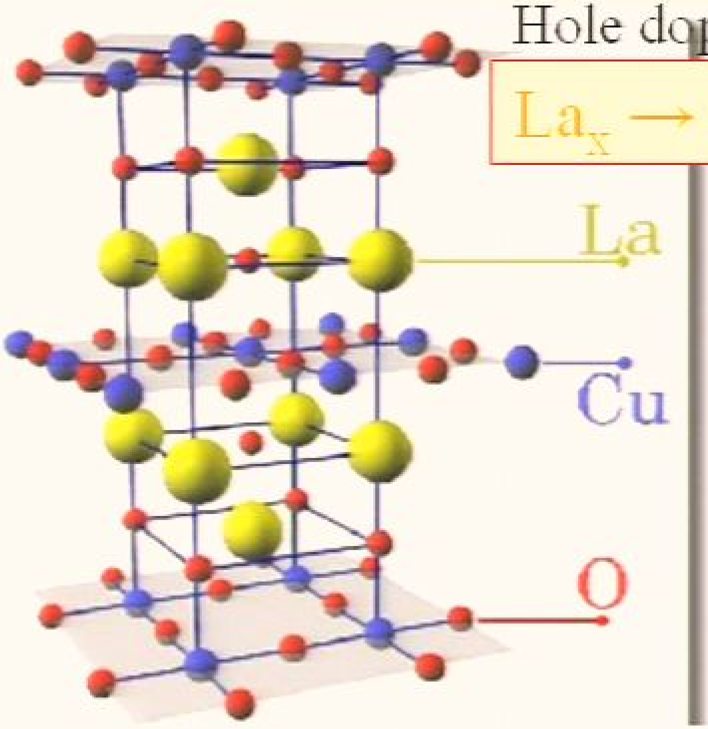
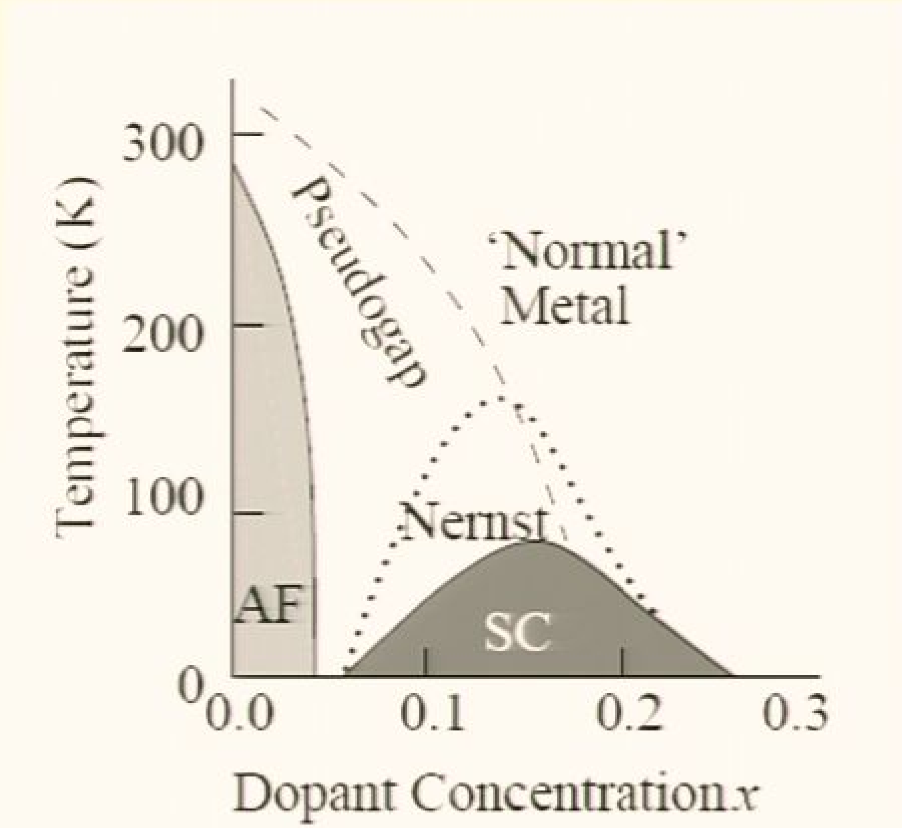


$La_x \rightarrow Sr_x$



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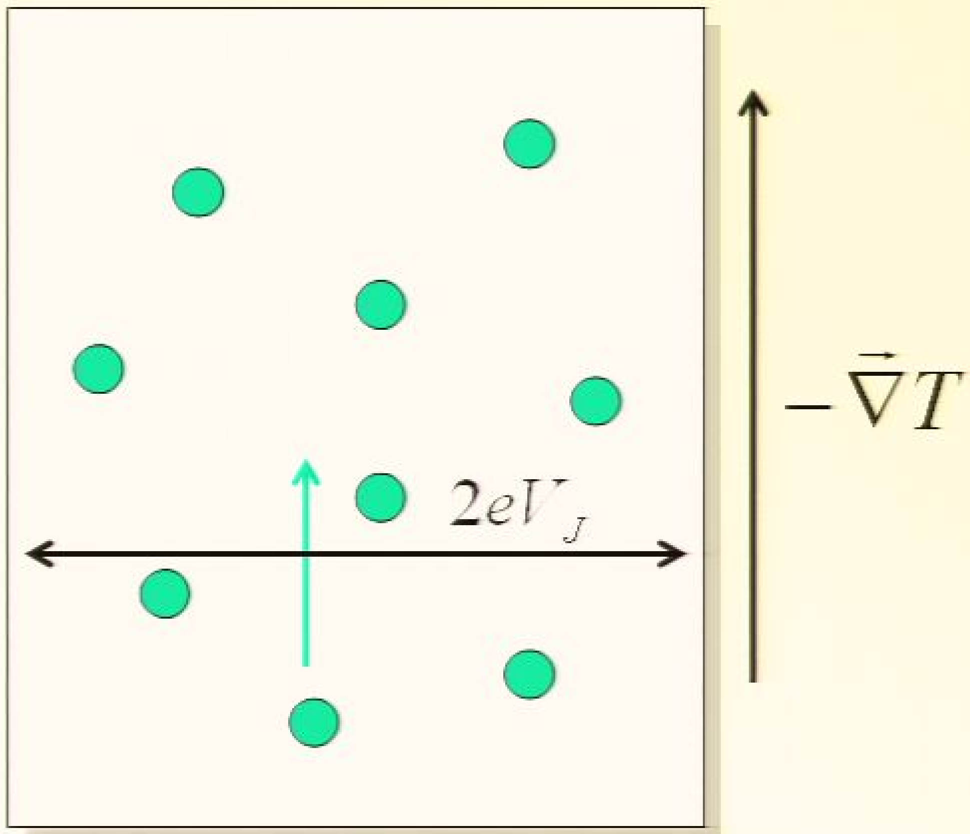

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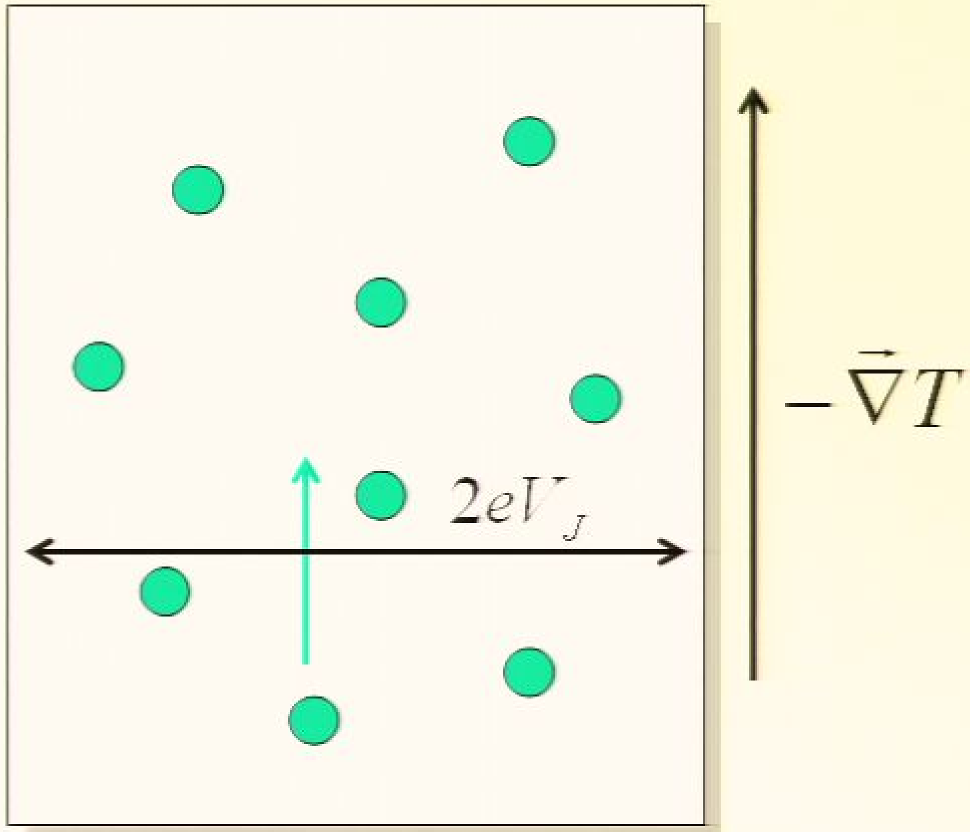

Nernst signal: $$e_N \equiv N = \frac{E_y}{-\vec{\nabla}_x T}$$

In Fermi liquids: usually very small (and with opposite sign) → Big Nernst signal above T_c evidence for a vortex liquid?

Vortex liquid?

Two scenarii for superconducting transition:

$$\Psi = |\Psi| e^{i\varphi}$$

1) BCS-type: Amplitude vanishes at T_c

$$\langle |\Psi|^2 \rangle \to 0$$

2) Phase fluctuations kill long range order: (Kosterlitz-Thouless)

$$\langle e^{i\varphi} \rangle \to 0$$

while a "vortex liquid" with local pairing amplitude $|\Psi|^2 > 0$ survives.

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Probe with Nernst

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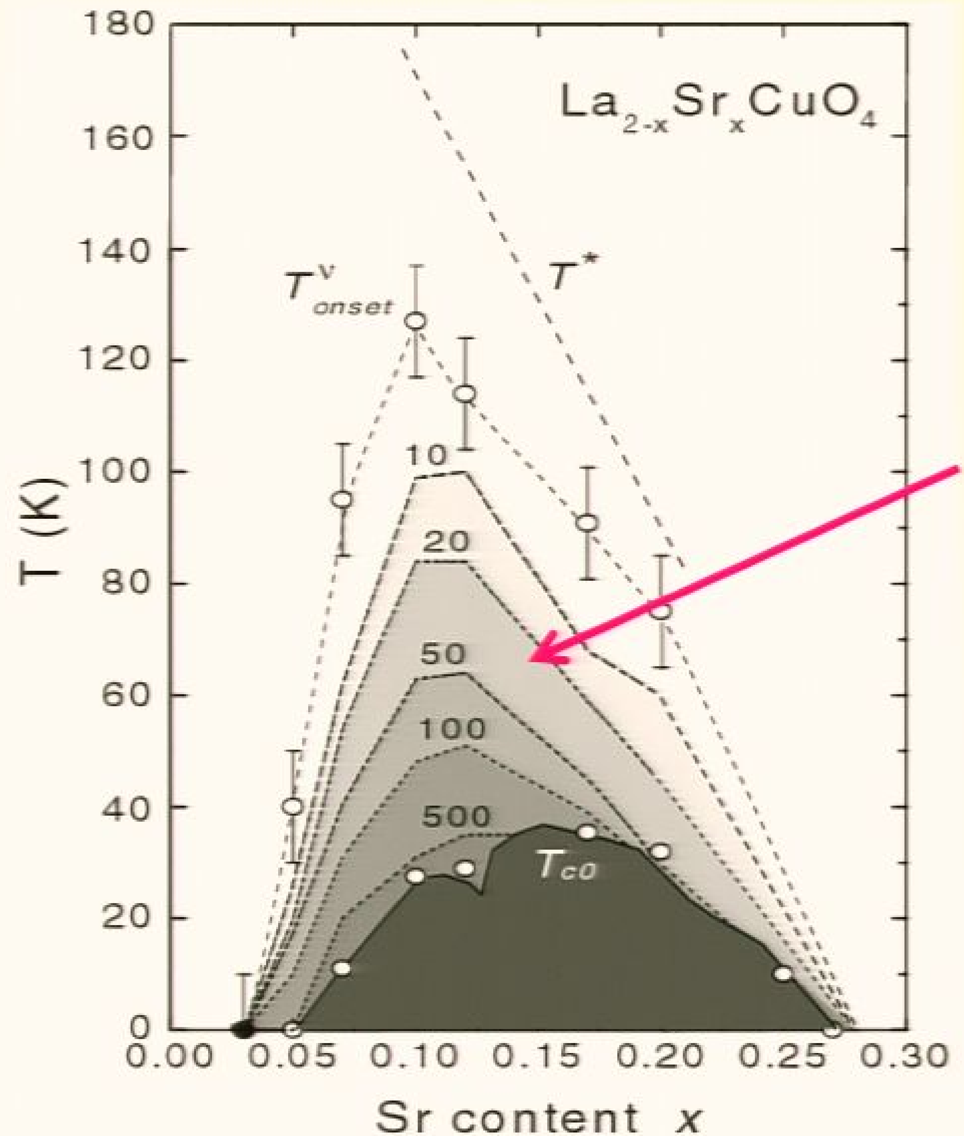
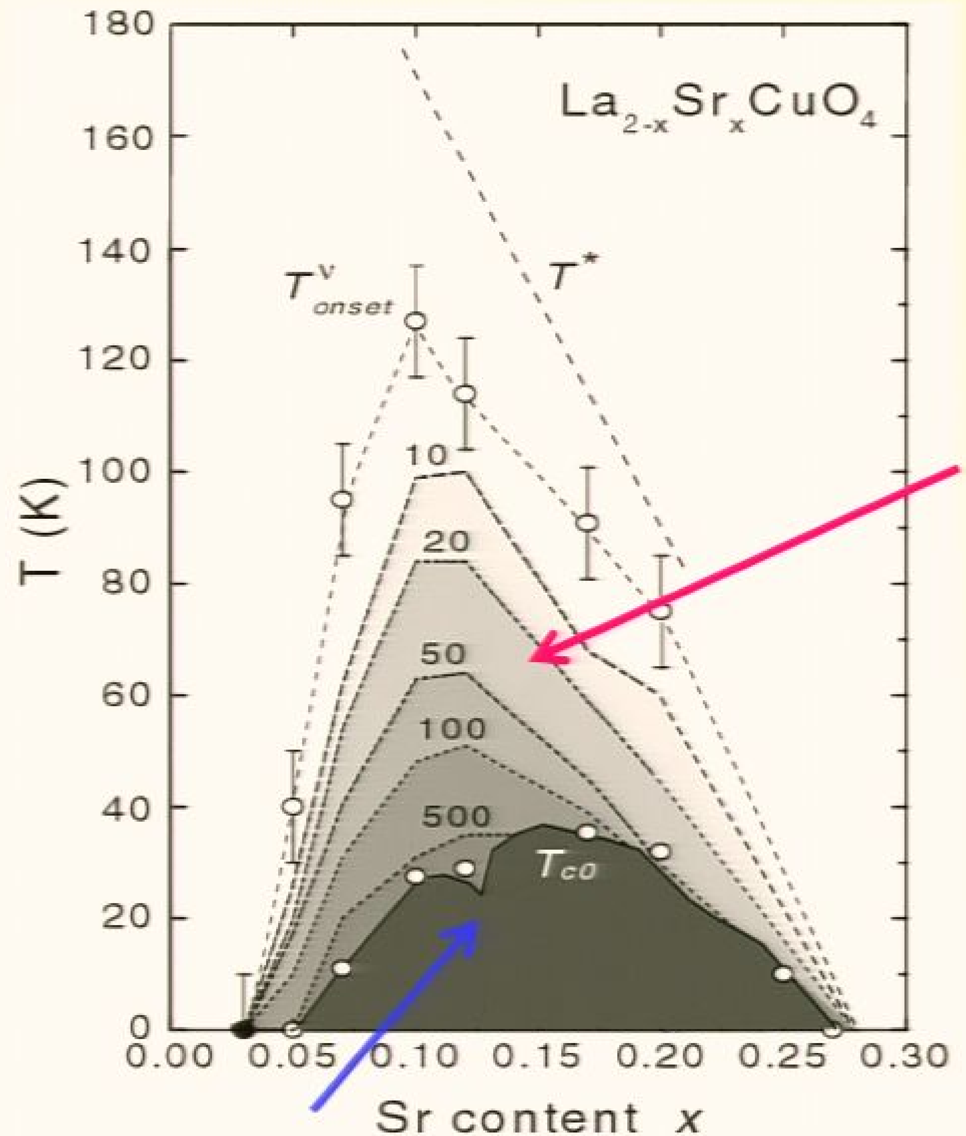
LSCO Phase diagram



Nernst region
e_N [nV/KT]

LSCO Phase diagram



Nernst region
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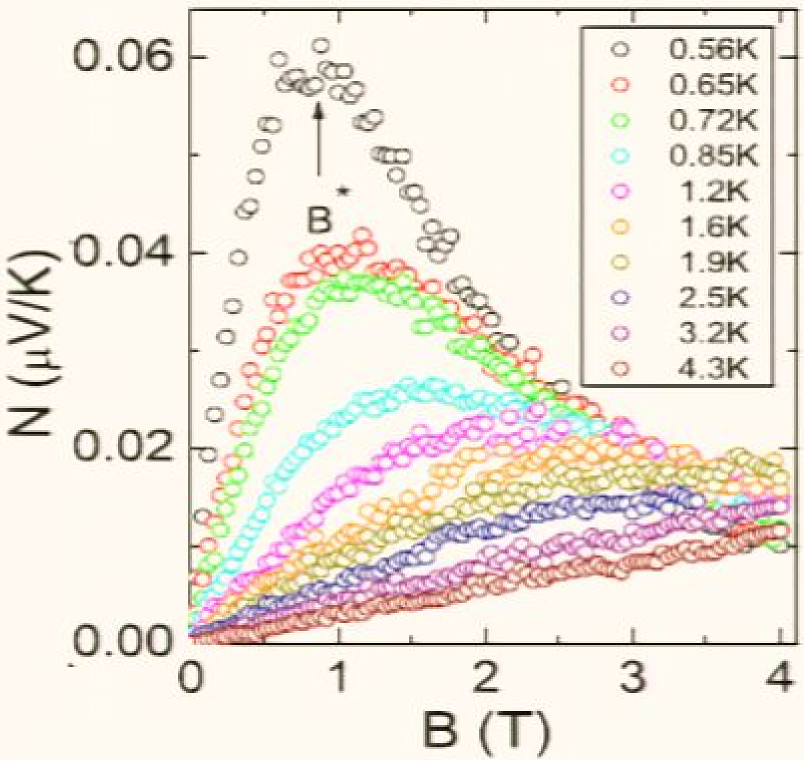
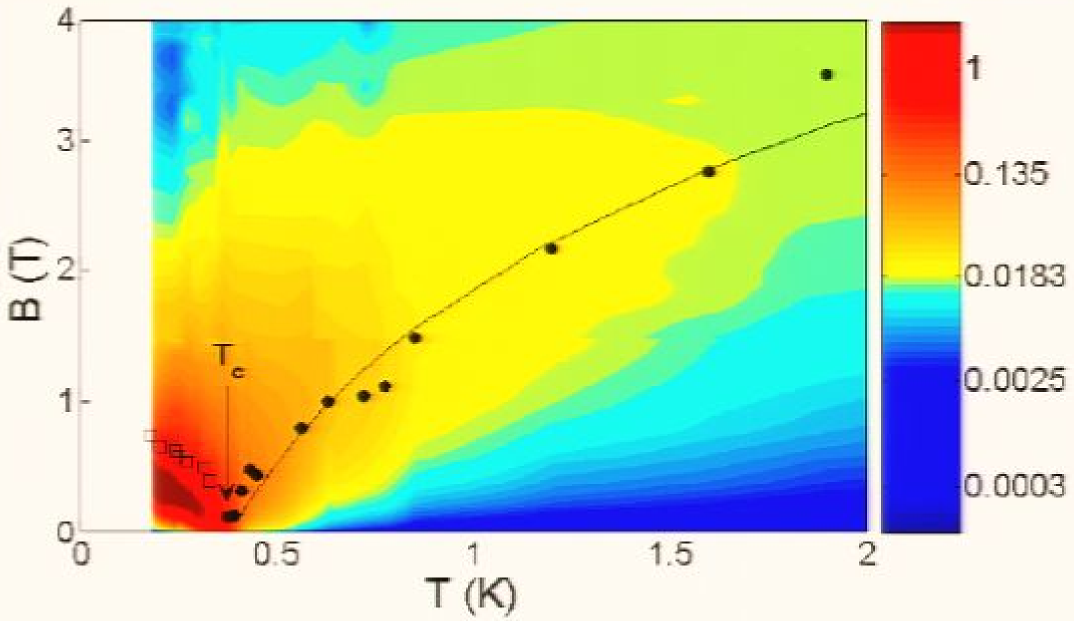
Dip in T_c near x=1/8 indicates proximity of insulator

Nernst effect in $Nb_{0.15}Si_{0.18}$





(A. Pourret, H. Aubin, J. Lesueur, C. A. Marrache-Kikuchi, L. Bergé, L. Dumoulin, K. Behnia, arxiv:0701376 (2007))

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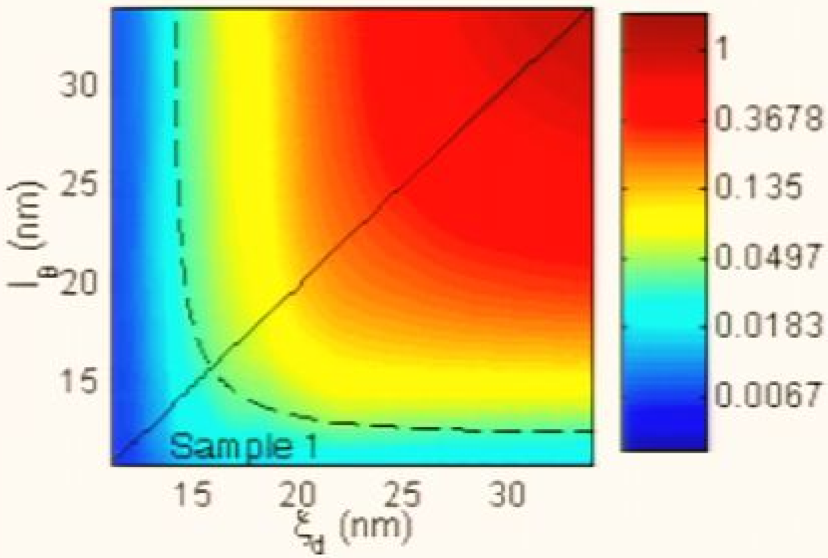

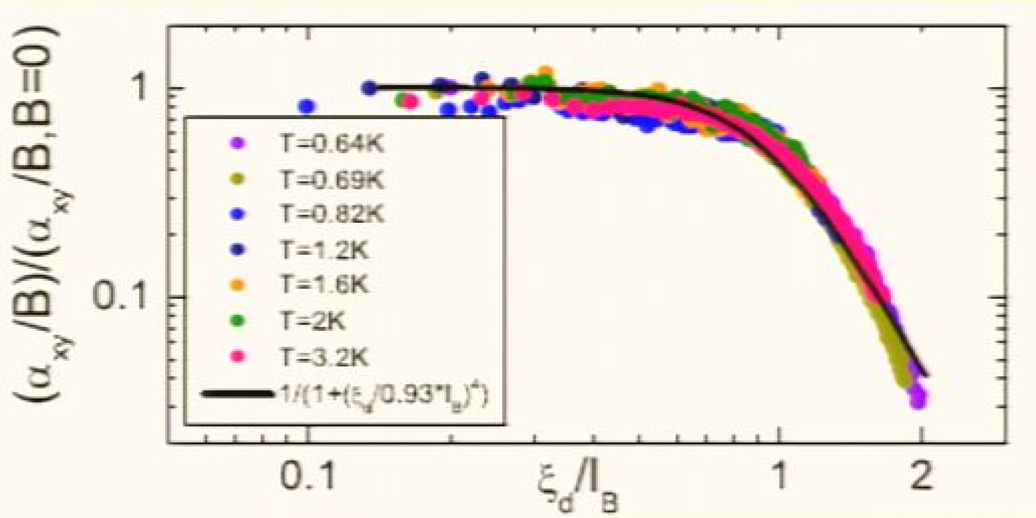

$$\frac{\alpha_{xy}}{B} = \frac{C}{1+\left(\xi_d/\ell_B\right)^4} = \frac{C}{1+\left(B/B_0\right)^2}$$

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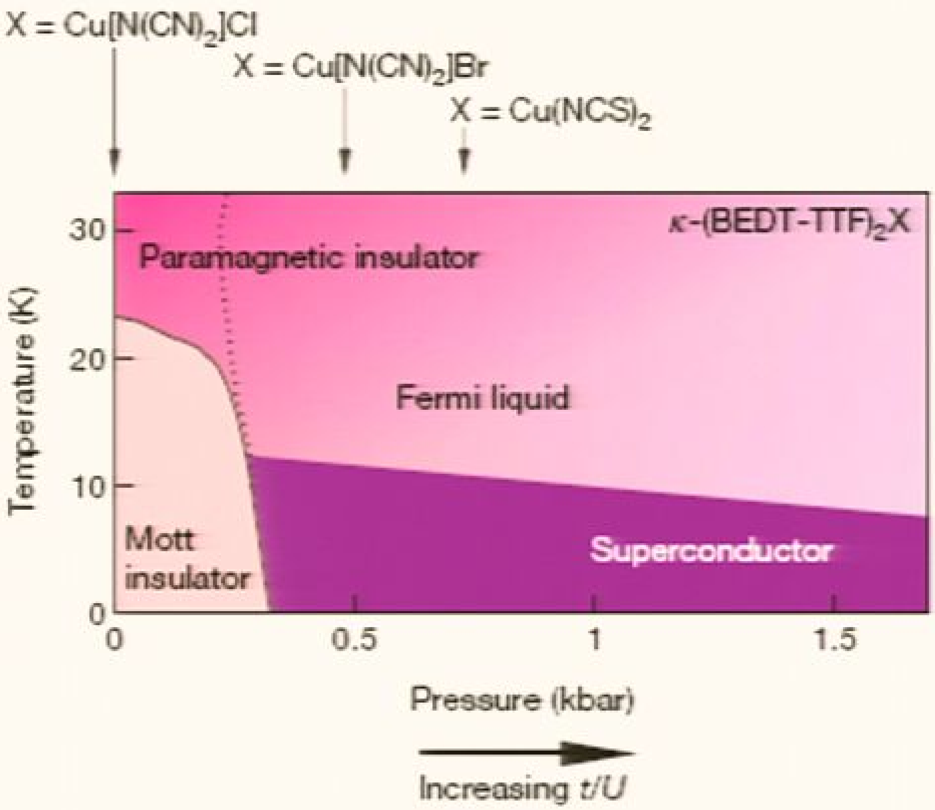
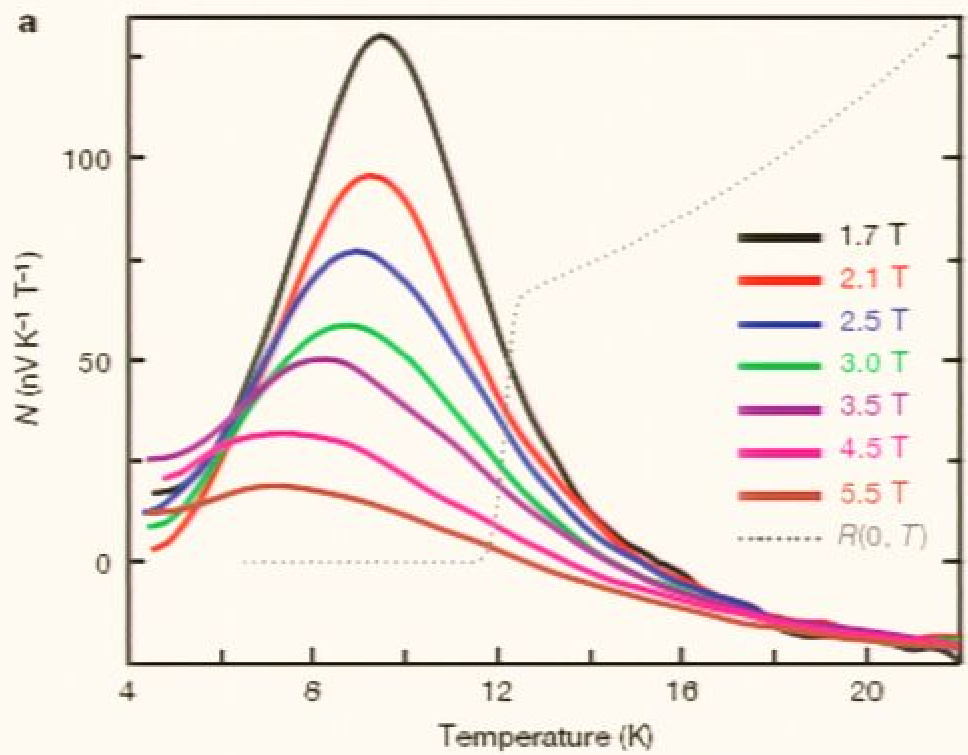
Organic superconductors

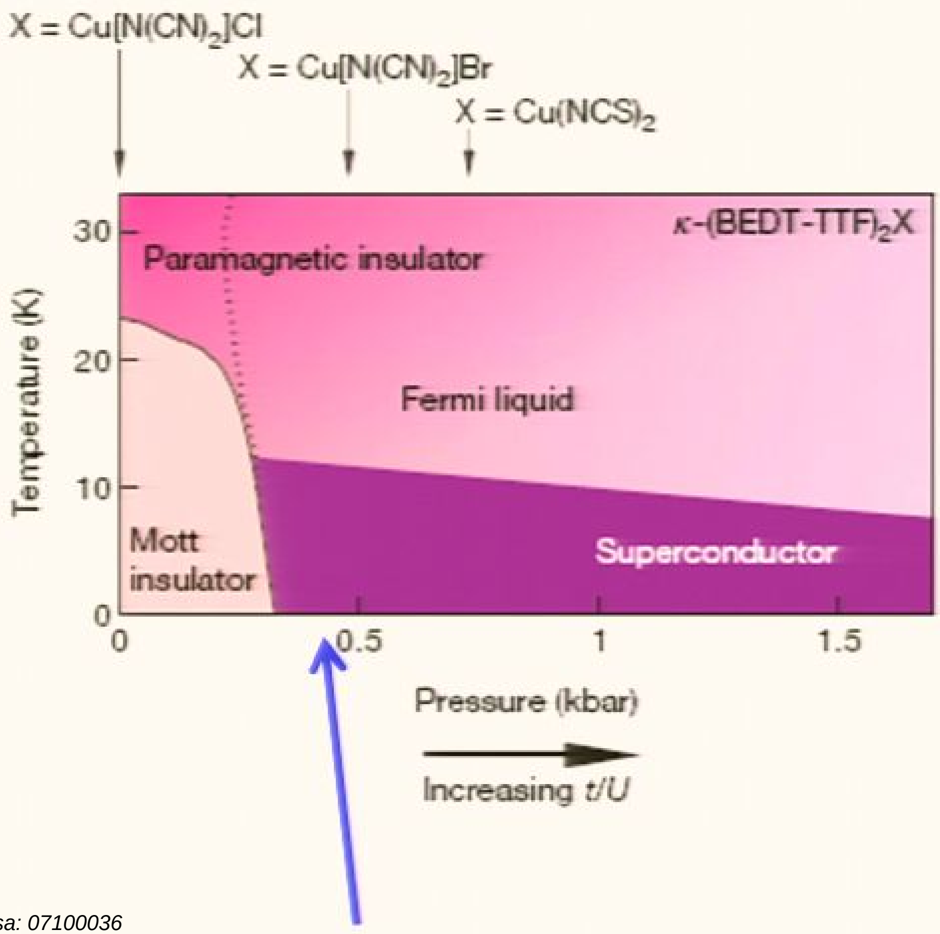


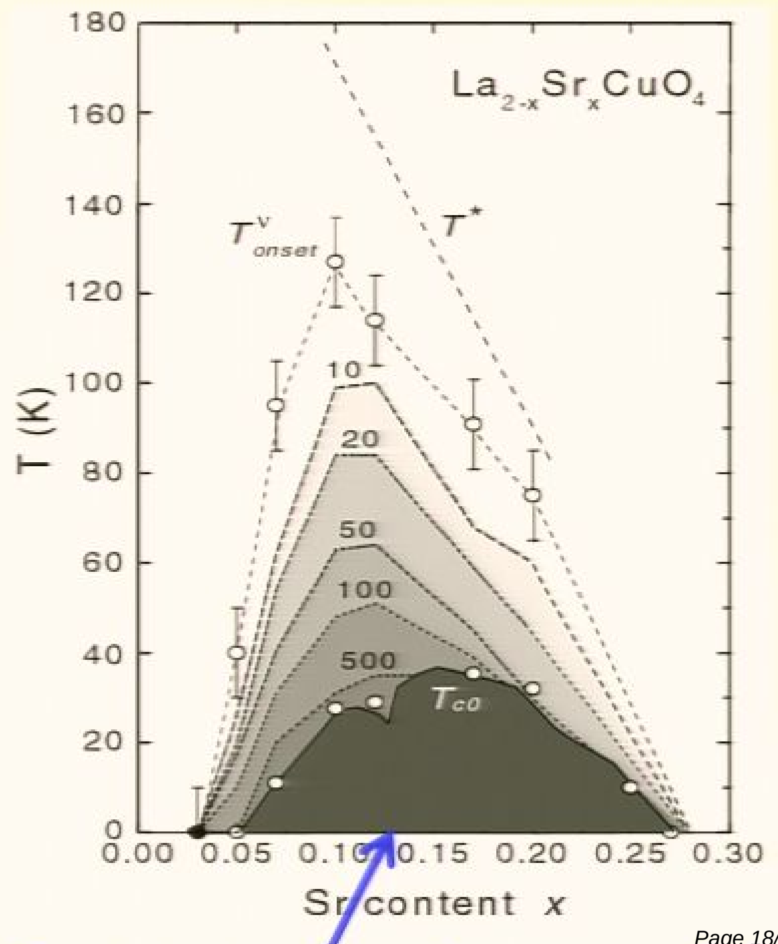


M. Nam, A. Ardavan, S. J. Blundell, and J. A. Schlueter, Nature ***449****, 584 (2007).*

Quantum criticality

Proximity to transition: Superconductor ↔ Mott insulator





Proximity to an insulator near x=1/8

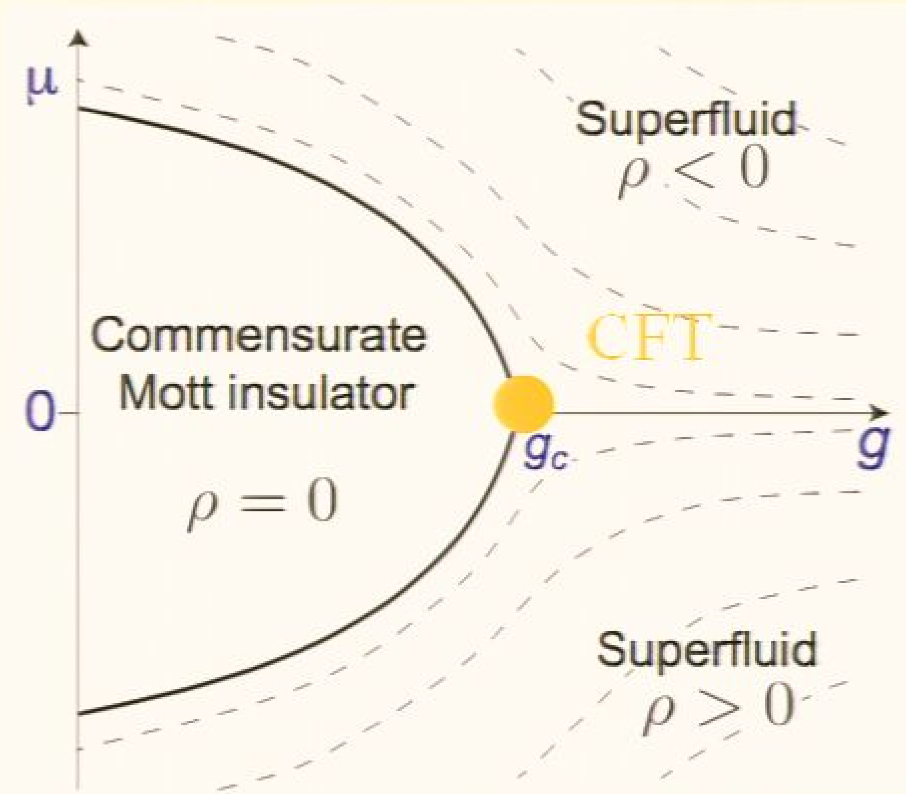
SI-transition: Bose Hubbard model

Bose-Hubbard model

$$H = -t\sum_{\langle ij\rangle} b_j^+ b_i + U\sum_i n_i^2 - \mu\sum_i n_i$$

Coupling

$g \equiv \frac{t}{U}$ tunes the SI-transition



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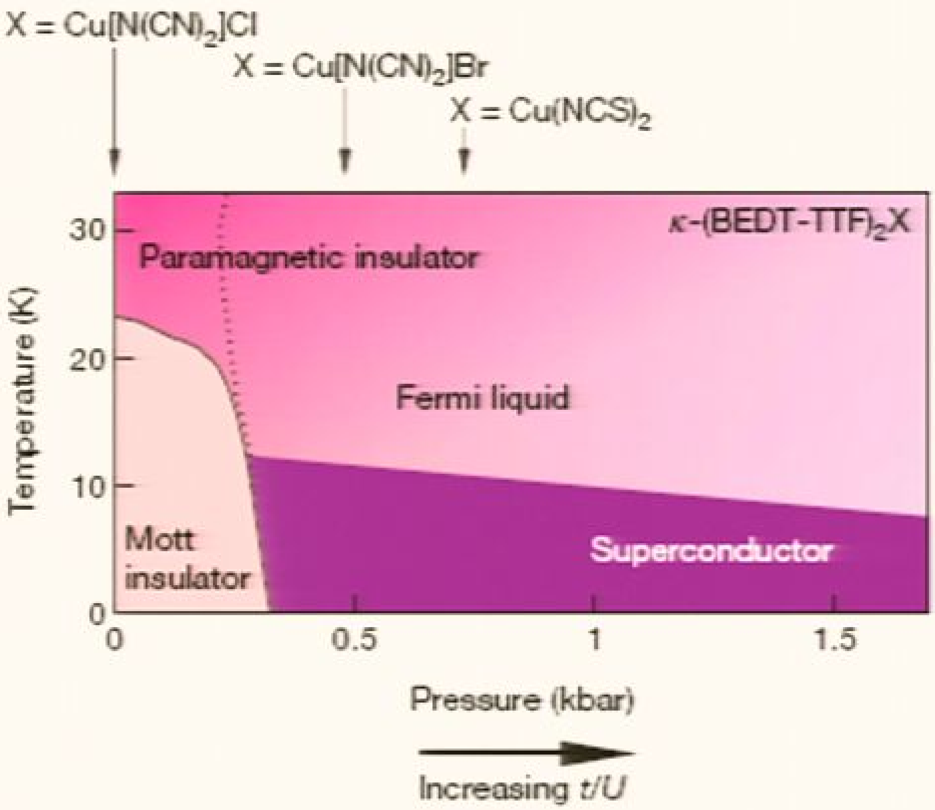

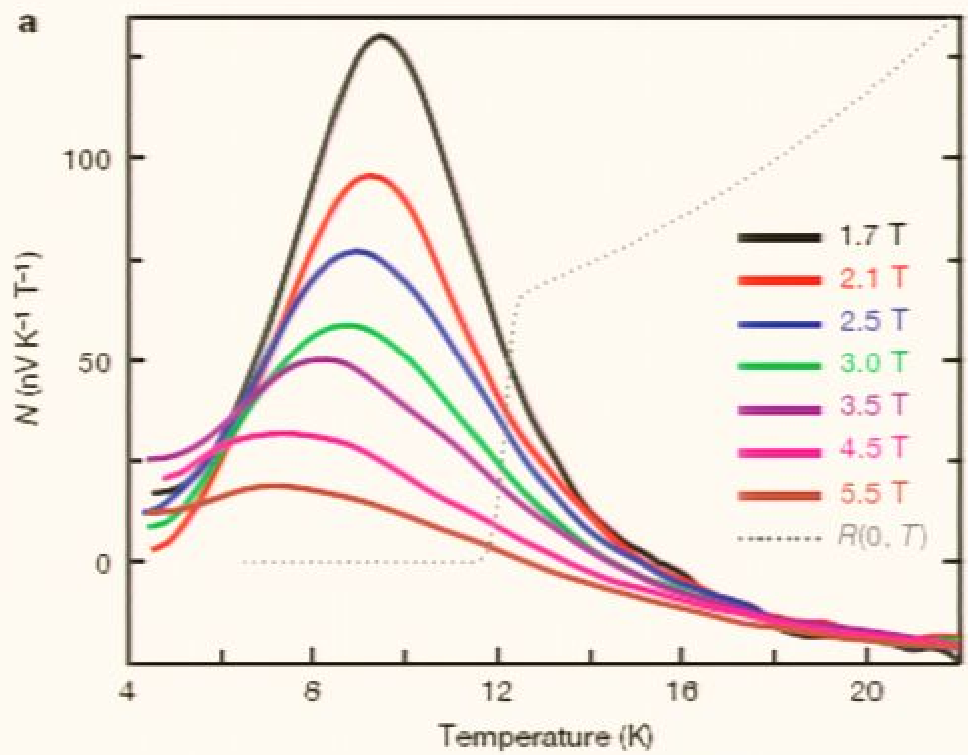

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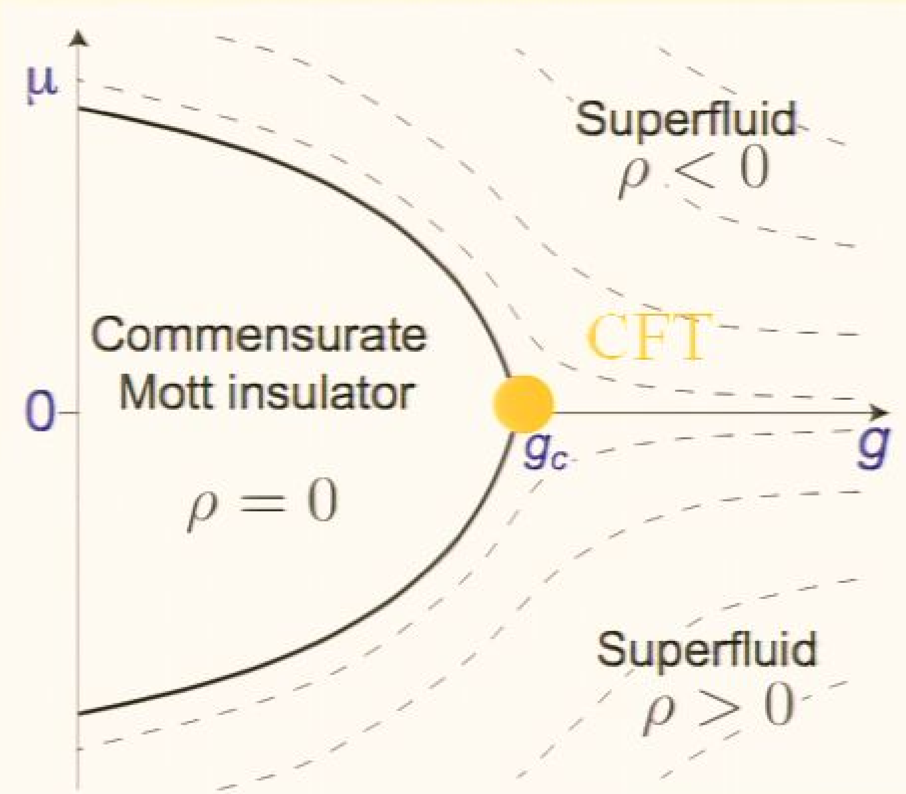

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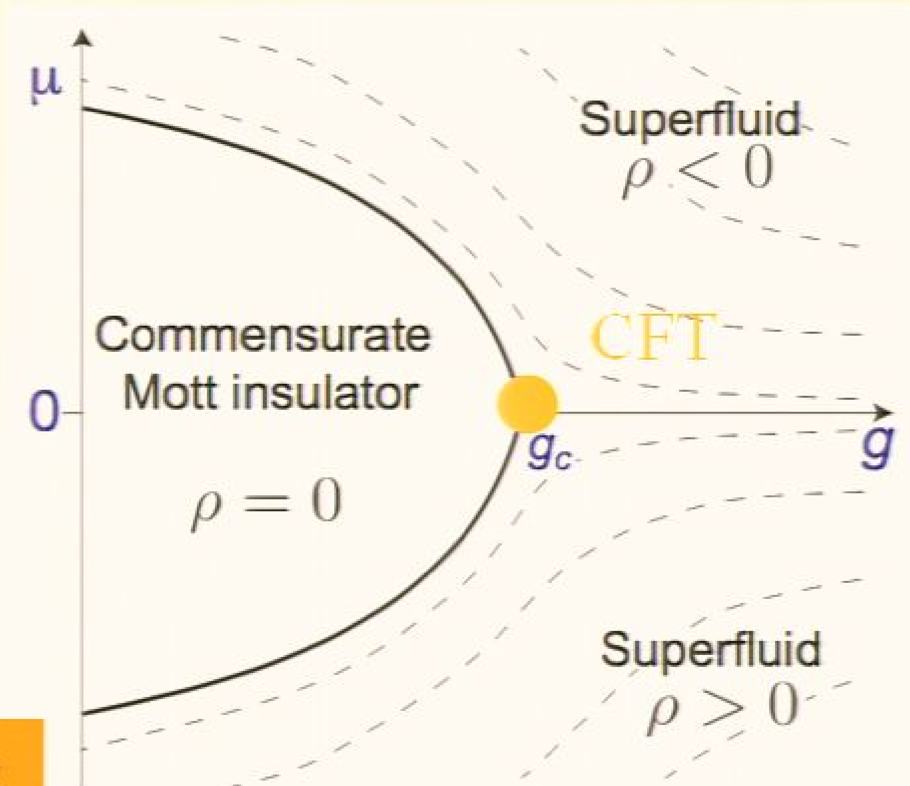
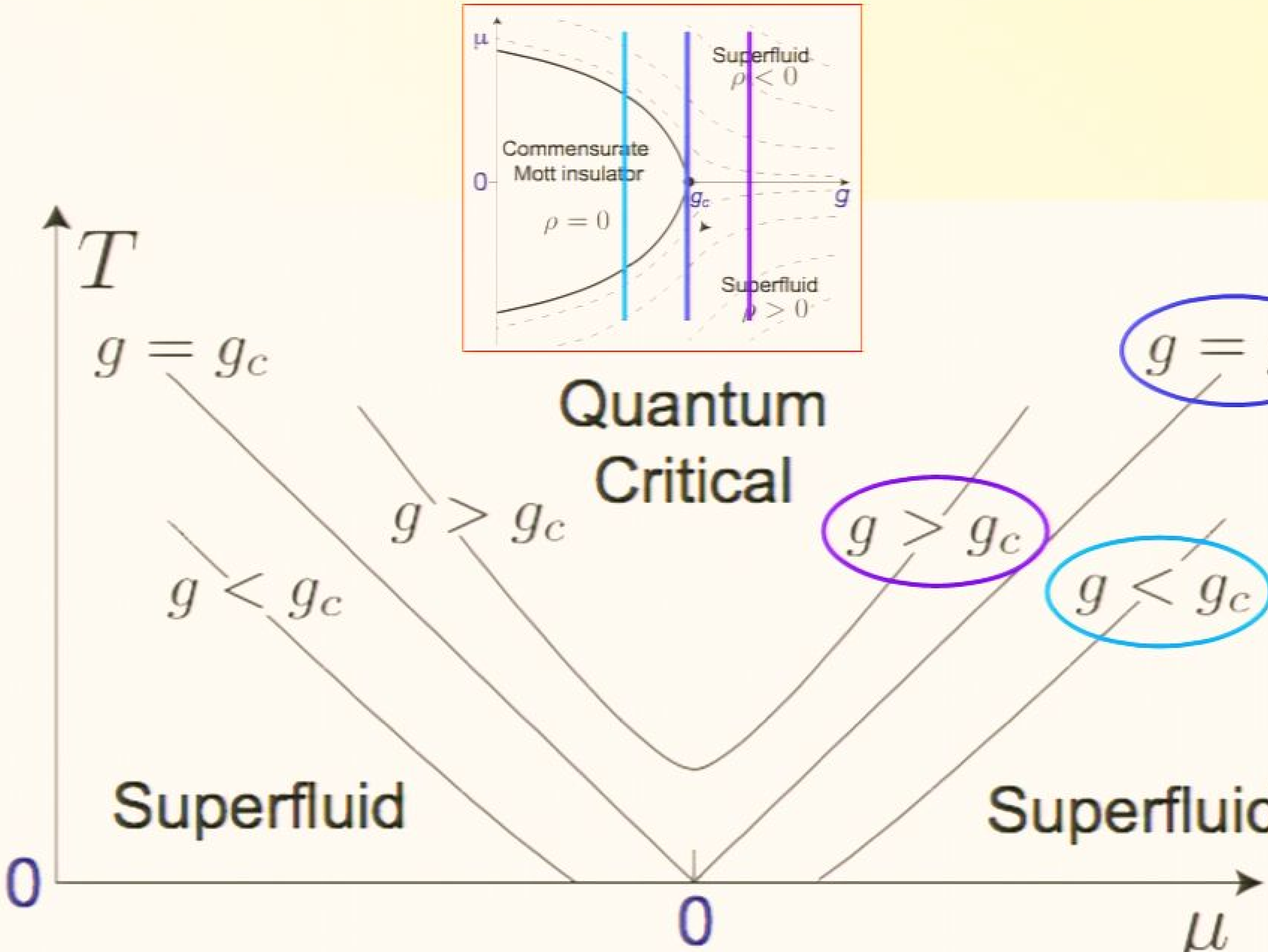


Effective action around g_c ($\mu = 0$):

$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 \left|\vec{\nabla}\psi\right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

→ Relativistic (conformal) CFT in d=2+1



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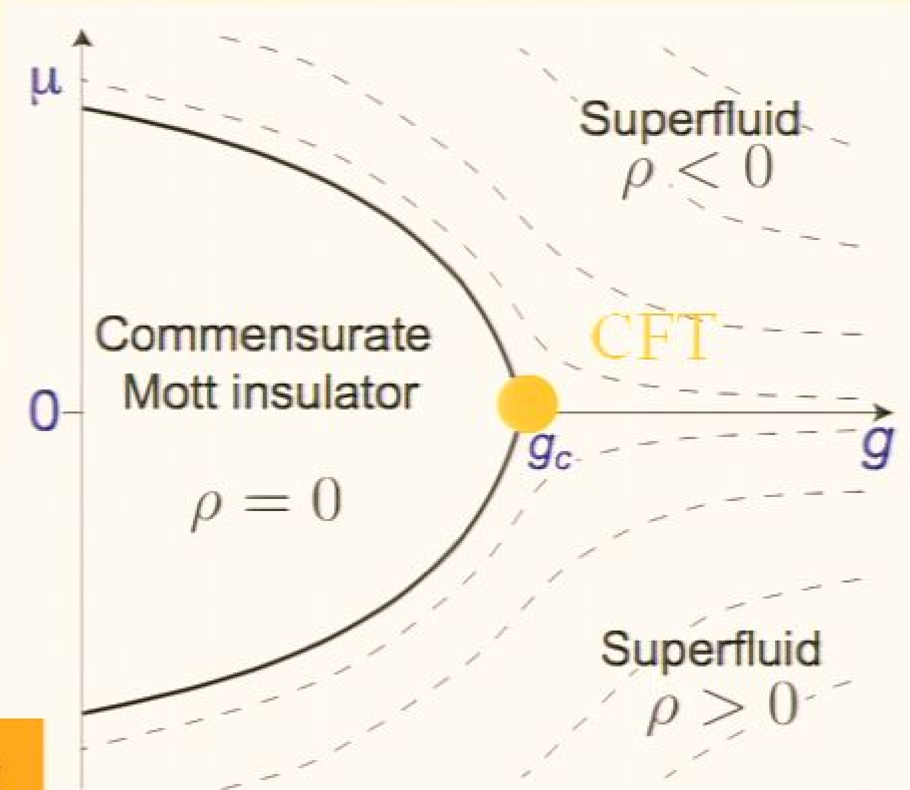

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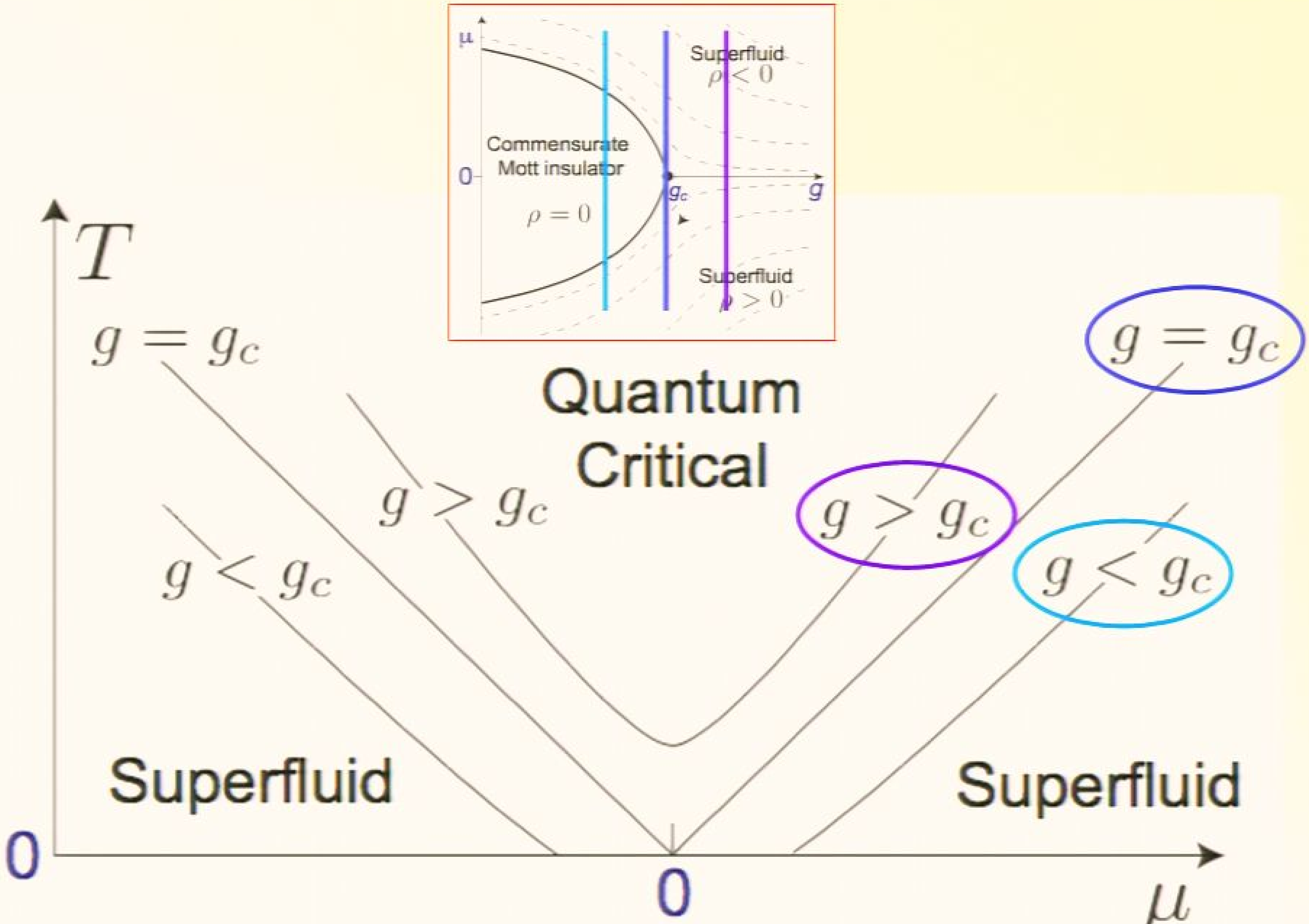

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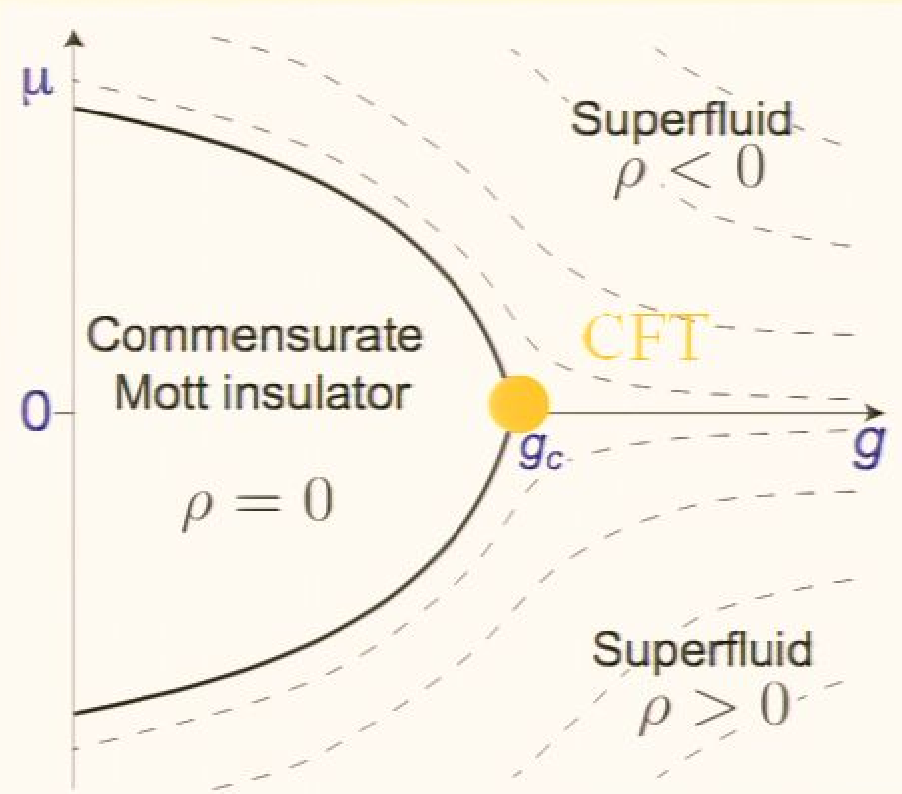

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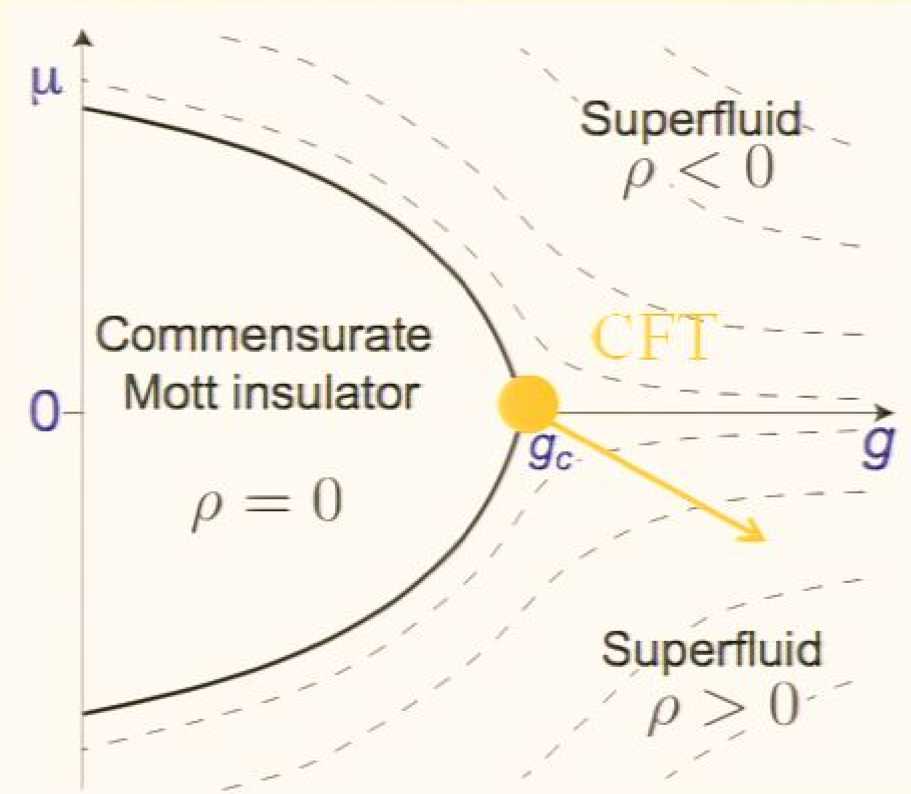
Perturb the CFT with

- a chemical potential μ
- a magnetic field B



$$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left|(\vec{\nabla} - i\vec{A})\psi\right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B$$

Hydrodynamics

Relativistic Hydrodynamics

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B 76, 144502 (2007).

Energy-momentum tensor $$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

Current 3-vector $$J^\mu = \rho u^\mu + \nu^\mu$$

u^μ : Energy velocity: $u^\mu = (1,0,0) \rightarrow$ No energy current

ν^μ : Dissipative current

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Thermodynamic relations

$$\varepsilon + P = Ts + \mu\rho, \quad d\varepsilon = Tds + \mu d\rho,$$

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Conservation laws (equations of motion):

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Q: **How to determine the dissipative terms ν^μ, $\tau^{\mu\nu}$?**

(Landau-Lifschitz)

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A: Heat current $Q^{\mu} = (\varepsilon + P) u^{\mu} - \mu J^{\mu}$ $\rightarrow$ Entropy current Q^{μ}/T

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Positivity of entropy production: → $\partial_\mu \left(\dfrac{Q^\mu}{T} \right) = a_\mu \left(\partial^\mu T, \partial^\mu \mu, F^{\mu\nu} u_\nu \right) + b_{\mu\nu} \partial^\mu u^\nu \geq 0$

Relativistic Hydrodynamics

S. Hartnoll, P. Kovton, MM. and S. Sachdev, Phys. Rev. B **76**, 144502 (2007).

$$J^\mu = \rho u^\mu + \nu^\mu \qquad T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

Conservation laws (equations of motion):

$\partial_\mu J^\mu = 0$ Charge conservation

$\partial_\nu T^{\mu\nu} = F^{\mu\nu} J_\nu$. Energy/momentum conservation

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix}$$

$\partial_\nu T^{\mu\nu} = F^{\mu\nu} J_\nu + \frac{1}{\tau_{\text{imp}}}(\delta^\mu_\nu + u^\mu u_\nu) T^{\nu\gamma} u_\gamma$. Momentum relaxation

Positivity of entropy production: →

$$\nu^\mu = \sigma_Q (g^{\mu\nu} + u^\mu u^\nu)\left[(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T}\right]$$

$$\tau^{\mu\nu} = -(g^{\mu\lambda} + u^\mu u^\lambda)[\eta(\partial_\lambda u^\nu + \partial^\nu u_\lambda) + (\zeta - \eta)\delta^\nu_\lambda \partial_\alpha u^\alpha]$$

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Irrelevant for response at $k \to 0$

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Irrelevant for response at $k \to 0$

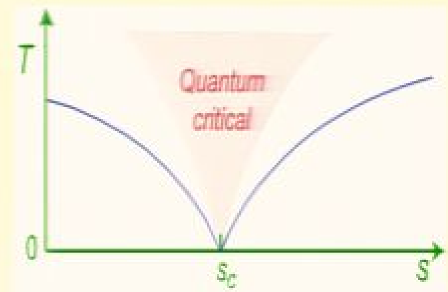
One single transport coefficient!

Universal conductivity σ_Q

Quantum critical, relativistic regime

- Quantum criticality:
  Relaxation time set by temperature alone

$$\tau_{rel} \approx \frac{\hbar}{k_B T}$$



- Relativistic regime: (Charge) current can relax via pair creation/annihilation without violating momentum conservation.
  This is possible because $\vec{J}, \vec{P} = \vec{J}^E$ are not proportional, and thus $\vec{J}$ is not conserved!

(particle)

(hole)

$\vec{J} = 0, \ \vec{J}^E \neq 0$ $\qquad$ $\vec{J} \neq 0, \ \vec{J}^E = 0$

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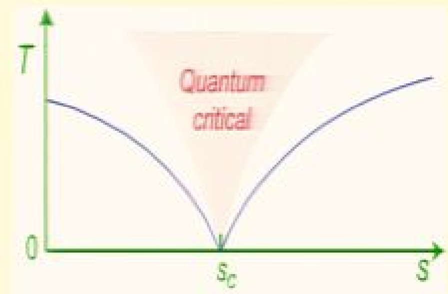

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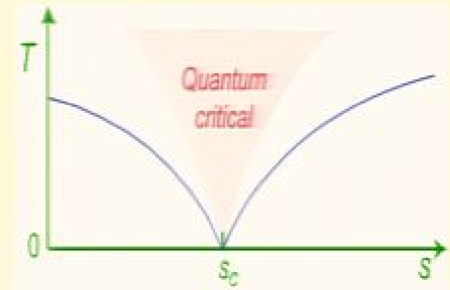

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Universal conductivity

$$\sigma_{Drude} = \frac{e}{m}\rho\tau \rightarrow \sigma_Q \sim \frac{1}{\varepsilon/\rho}\rho\tau_{rel} \sim \frac{e^2}{h}\frac{1}{T}T^2\frac{1}{T} \sim \frac{e^2}{h}$$

Energy vs. charge current

$$J^\mu = \rho u^\mu + \nu^\mu$$
$$J^{E\mu} = (\varepsilon + P) u^\mu$$

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$$\vec{J}^E = \frac{\varepsilon + P}{\rho} \left[\vec{J} - \sigma_Q \left(-\vec{\nabla}\mu + \vec{v} \times \vec{B} + \mu \frac{\vec{\nabla} T}{T} \right) \right]$$

$$= \frac{\varepsilon + P}{\rho} \vec{J} + \frac{(\varepsilon + P)^2}{T\rho^2} \sigma_Q \vec{\nabla} T + \frac{\varepsilon + P}{\rho^2} \sigma_Q (-\vec{\nabla} P + \vec{J} \times \vec{B})$$

Energy current due to matter flow

Heat current due to a thermal gradient (with $\kappa \propto \sigma_Q$!)

Purely relativistic contribution to the heat current proportional to the acceleration!

Thermoelectric response

S. Hartnoll, P. Kovtun, MM. and S. Sachdev, Phys. Rev. B 76, 144502 (2007).

Charge and heat current: $J^\mu = \rho u^\mu + \nu^\mu$ $\quad Q^\mu = (\varepsilon + P)u^\mu - \mu J^\mu$

Thermo-electric response in the particle picture

$$\begin{pmatrix} \vec{J} \\ \vec{Q} \end{pmatrix} = \begin{pmatrix} \hat{\sigma} & \hat{\alpha} \\ T\hat{\alpha} & \hat{\bar{\kappa}} \end{pmatrix} \begin{pmatrix} \vec{E} \\ -\vec{\nabla} T \end{pmatrix} \qquad \hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix} \quad \text{etc.}$$

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$$\begin{pmatrix} \vec{E} \\ \vec{Q} \end{pmatrix} = \begin{pmatrix} \hat{\rho} & \hat{\vartheta} \\ T\hat{\vartheta} & \hat{\kappa} \end{pmatrix} \begin{pmatrix} \vec{J} \\ -\vec{\nabla} T \end{pmatrix}$$

Nernst signal $\quad e_N \equiv \vartheta_{yx}$

Nernst coefficient $\quad \nu = e_N / B$

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Nernst signal $e_N \equiv \vartheta_{yx}$

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Task: i) Solve linearized hydrodynamic equations;
ii) Read off the response functions (Kadanoff & Martin 1960)

Cyclotron resonance

Poles in the response $\omega = \pm\omega_c^{rel} - i\gamma$ $\left(\tau_{imp}^{-1} = 0\right)$

Cyclotron frequency

$$\omega_c^{rel} = \frac{v^2}{c^2}\frac{2e\,B}{(\varepsilon + P)/\rho c} \leftrightarrow \omega_c^{nonrel} = \frac{2e\,B}{mc}$$

Damping: particle picture ($\hat{\sigma}, \hat{\alpha}, \hat{\bar{\kappa}}$) $\gamma = \sigma_Q \frac{v^2}{c^2}\frac{B^2}{\varepsilon + P}$

Damping: vortex picture ($\hat{\rho}, \hat{\vartheta}, \hat{\kappa}$) $\gamma_v = \frac{\omega_c^2}{\gamma} = \frac{4e^2}{\sigma_Q} v^2 \frac{\rho^2}{\varepsilon + P}$

Particle vortex duality!

Response functions

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Longitudinal conductivity

$$\sigma_{xx} = \sigma_Q \left[\frac{(\omega + i/\tau_{\text{imp}})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\text{imp}})}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2} \right] .$$

$$= \sigma_Q + \frac{4e^2\rho^2 v^2}{(\varepsilon + P)} \frac{1}{(-i\omega + 1/\tau_{\text{imp}})} \quad \text{as } B \to 0$$

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$$= \sigma_Q + \frac{2en}{m} \frac{1}{(-i\omega + 1/\tau_{\rm imp})} \quad \text{as } B \to 0$$

(non-relativistic limit)

Response functions

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Thermal conductivity

$$\begin{aligned} \kappa_{xx} &= \sigma_Q \left(\frac{k_B^2 T}{4e^2}\right) \left(\frac{\varepsilon + P}{k_B T \rho}\right)^2 \left[\frac{(\omega_c^2/\gamma)(\omega_c^2/\gamma + 1/\tau_{\rm imp})}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right] \\ &= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B}\right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\rm imp})}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right] \end{aligned}$$

Response functions

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Thermal conductivity

$$\kappa_{xx} = \sigma_Q \left(\frac{k_B^2 T}{4e^2}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)^2 \quad \boxed{\rightarrow 1 \text{ as } B \rightarrow 0}$$

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$\rightarrow 1$ as $\rho \rightarrow 0$

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Wiedemann-Frantz-like relations!

Response functions

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Nernst signal

$$e_N = \left(\frac{k_B}{2e}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)\left[\frac{\omega_c/\tau_{\text{imp}}}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right]$$

Quantum unit for Nernst signal $\frac{k_B}{2e} = 43.086\ \mu\text{V/K}$

AdS/CFT correspondence

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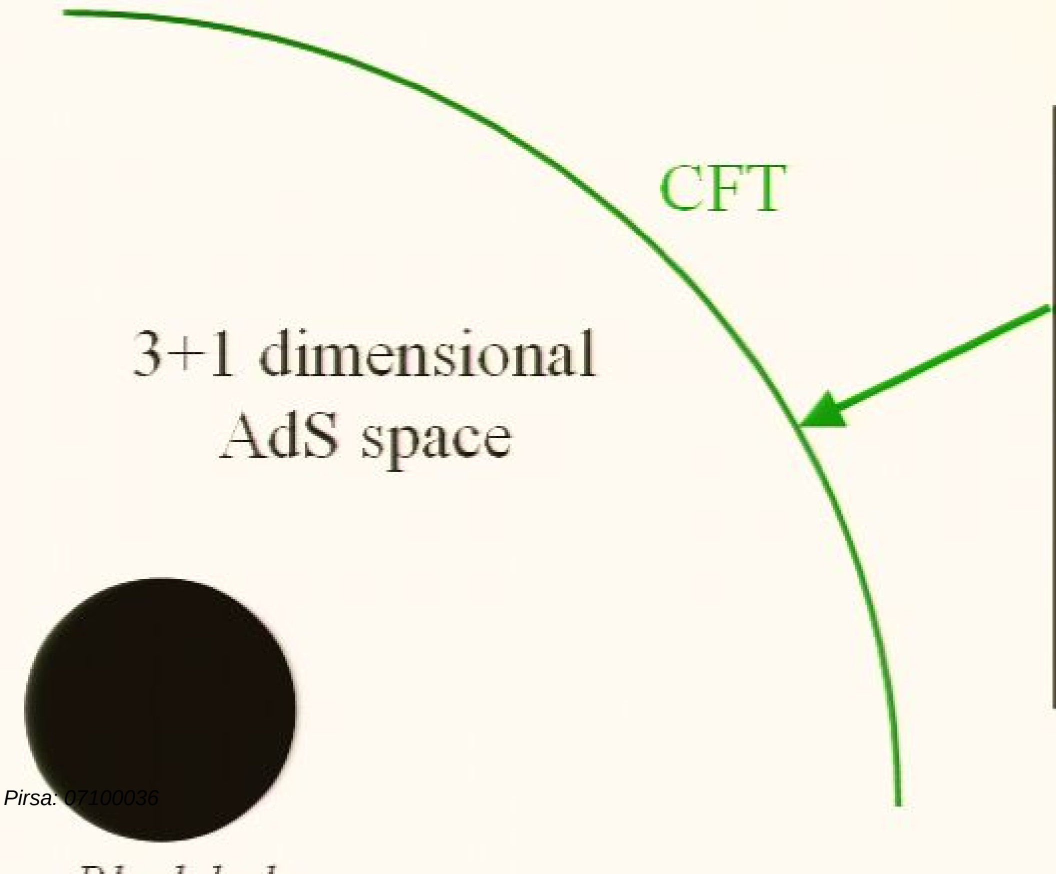
AdS/CFT correspondence

AdS/CFT

The AdS/CFT correspondence (Maldacena, Polyakov) relates CFTs to the quantum gravity theory of a black hole in Anti-de Sitter space.



The black hole physics is holographically represented in a 2+1 dimensional CFT on the boundary of AdS space, at a temperature equal to the Hawking temperature of the black hole.

AdS/CFT

Idea:

- Obtain exact results at the critical point (CFT) for transport coefficients (σ_Q) from mapping to a solvable gravity problem.
- Find precisely the response functions of magnetohydrodynamics, *without* putting in knowledge of dissipative terms nor the principle of positivity of entropy production!
- Go beyond hydrodynamic regime.
- Future: Obtain quantum critical crossover functions exactly?

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Concretely:

To the solvable supersymmetric, Yang-Mills theory CFT, we add

- A chemical potential μ
- A magnetic field B

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

- An electric charge
- A magnetic charge

AdS/CFT

Simplest gravitational dual to CFT_{2+1}: Einstein-Maxwell theory

$$I = \frac{1}{g^2}\int d^4x\sqrt{-g}\left[-\frac{1}{4}R + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{3}{2}\right].$$

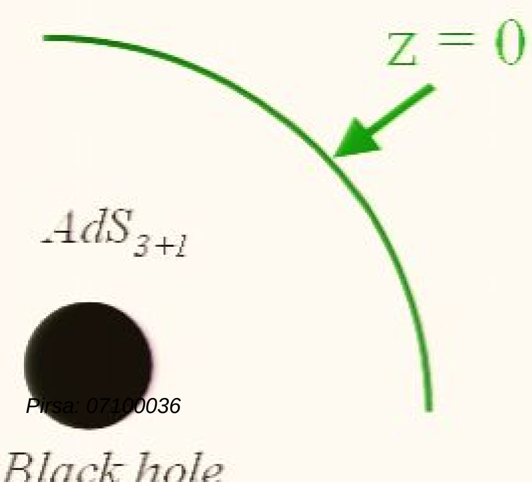
(embedded in M theory as $AdS_4 \times S^7$: $1/g^2 \sim N^{3/2}$)

It has a black hole solution (with electric and magnetic charge):

$$ds^2 = \frac{\alpha^2}{z^2}\left[-f(z)dt^2 + dx^2 + dy^2\right] + \frac{1}{z^2}\frac{dz^2}{f(z)}.$$

$$F = h\alpha^2 dx \wedge dy + q\alpha dz \wedge dt.$$

$$f(z) = 1 + (h^2+q^2)z^4 - (1+h^2+q^2)z^3.$$



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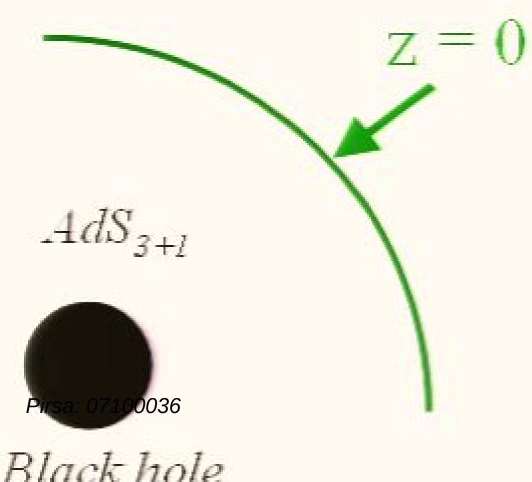

Background $\leftrightarrow$ Equilibrium

Transport $\leftrightarrow$ Perturbations in $g_{tx,ty}, A_{x,y}$.

Response via Kubo formula from $\delta^2 I/\delta(g,A)^2$.

AdS/CFT

Main results

- Precise agreement with MHD, *without* imposing the principle of positivity of entropy production!
- Exact value for σ_Q (in large N).
- Proven potential to go beyond MHD.

Comparison with experiments

AdS/CFT

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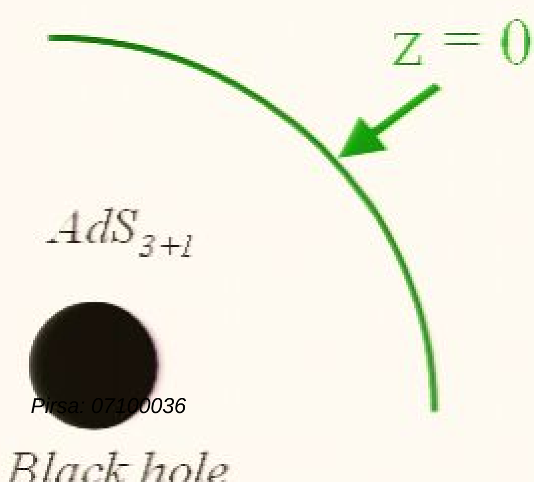

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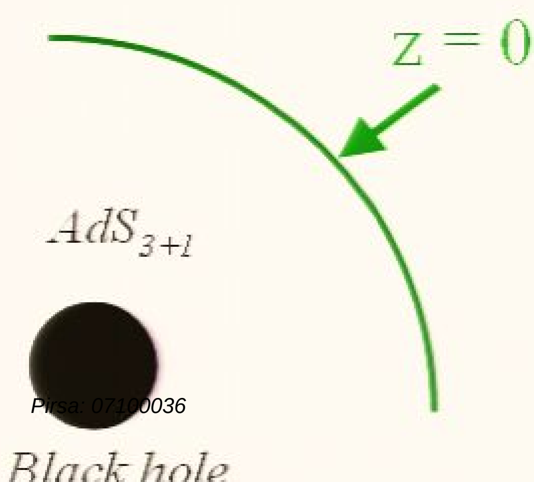

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Comparison with experiment: Peltier coefficient

$$\alpha_{xy} = \left(\frac{2ek_B}{h}\right)\left(\frac{s/k_B}{B/\phi_0}\right)\left[\frac{\gamma^2 + \omega_c^2 + \gamma/\tau_{\rm imp}\{1 - \mu\rho/(Ts)\}}{(\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right]$$

Quantum critical scaling: $\varepsilon, P = \# T^3$; $s = \# T^3$; $\sigma_Q = \#$

$$\alpha_{xy} \propto \frac{BT^2\left(\#\rho^2\tau_{imp} + \#T^3\right)}{T^6 + \#B^2\rho^2\tau_{imp}^2}$$

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$$\alpha_{xy} \propto \frac{BT^2\left(\#\rho^2\tau_{imp} + \#T^3\right)}{T^6 + \# B^2\rho^2\tau_{imp}^2}$$

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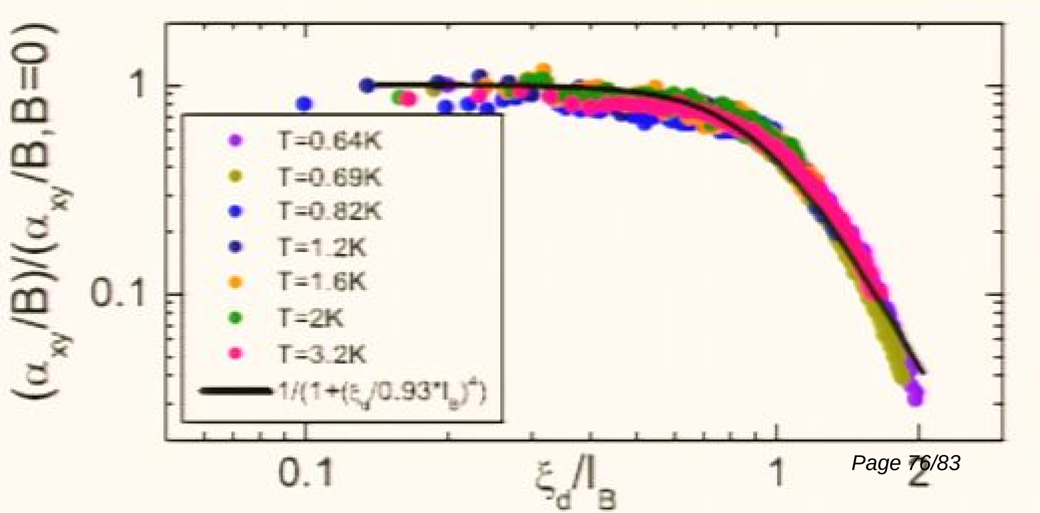
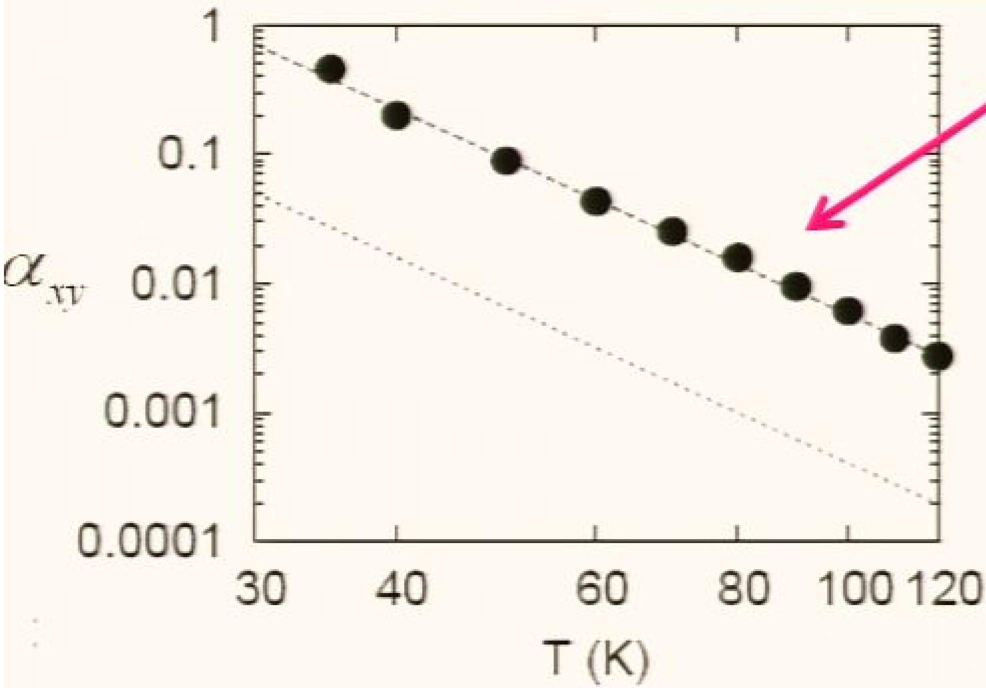


LSCO Experiments

Measurement of $\alpha_{xy} \approx \sigma_{xx} e_N$



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(T small)

$$\frac{\alpha_{xy}}{B}(B \to 0) \approx \left(\frac{2ek_B}{h\phi_0}\right)\frac{\Phi_s}{\Phi^2_{\varepsilon+P}}\left(\frac{2\pi\tau_{\text{imp}}}{\hbar}\right)^2\frac{\rho^2(\hbar v)^6}{(k_B T)^4}$$

Y. Wang et al., Phys. Rev. B **73**, *024510 (2006).*

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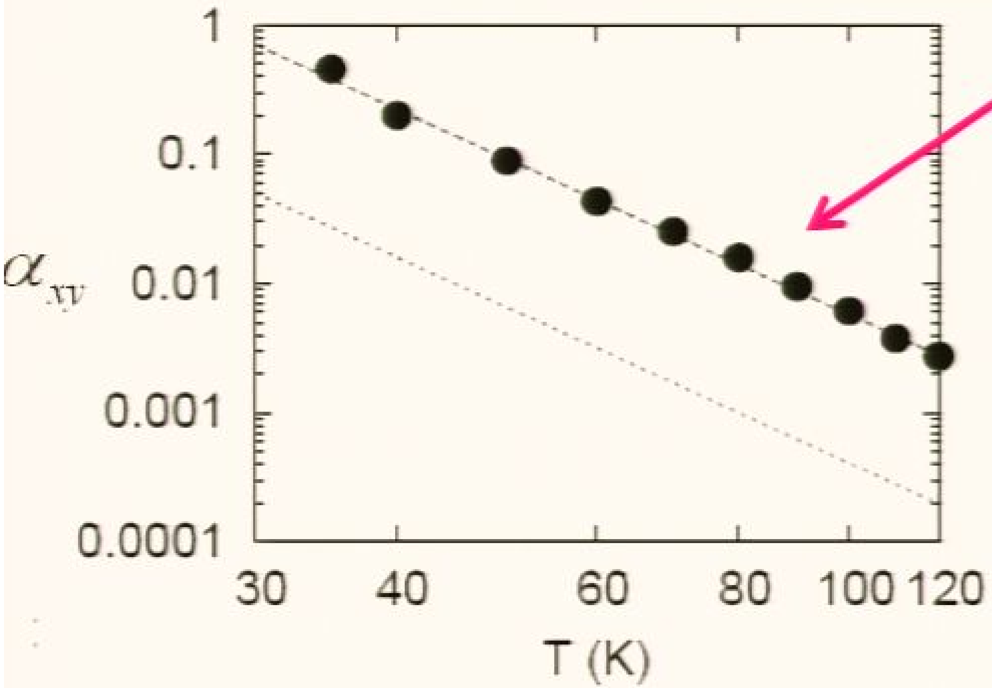

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$$\hbar v \approx 47\mathrm{meV}\,\mathring{A} \qquad \tau_{imp} \approx 10^{-12}\,\mathrm{s}$$

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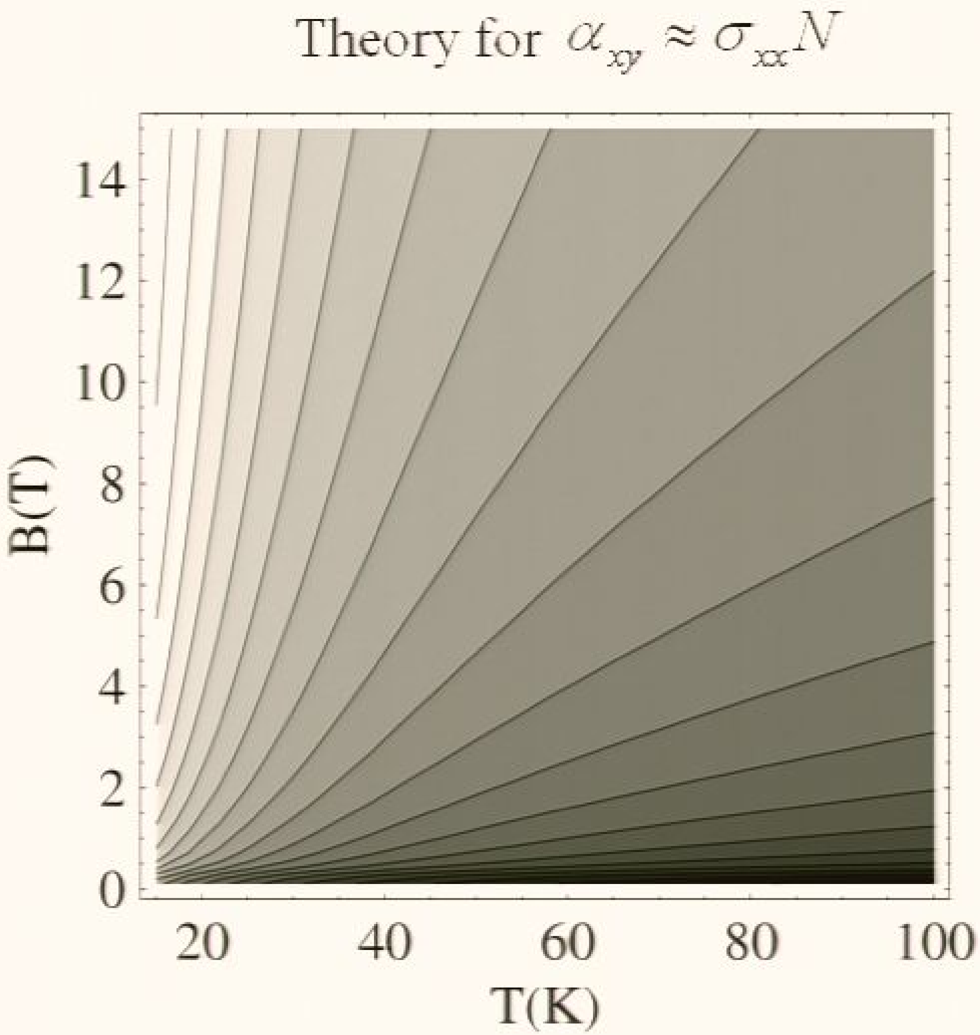
→ Prediction for ω_c:

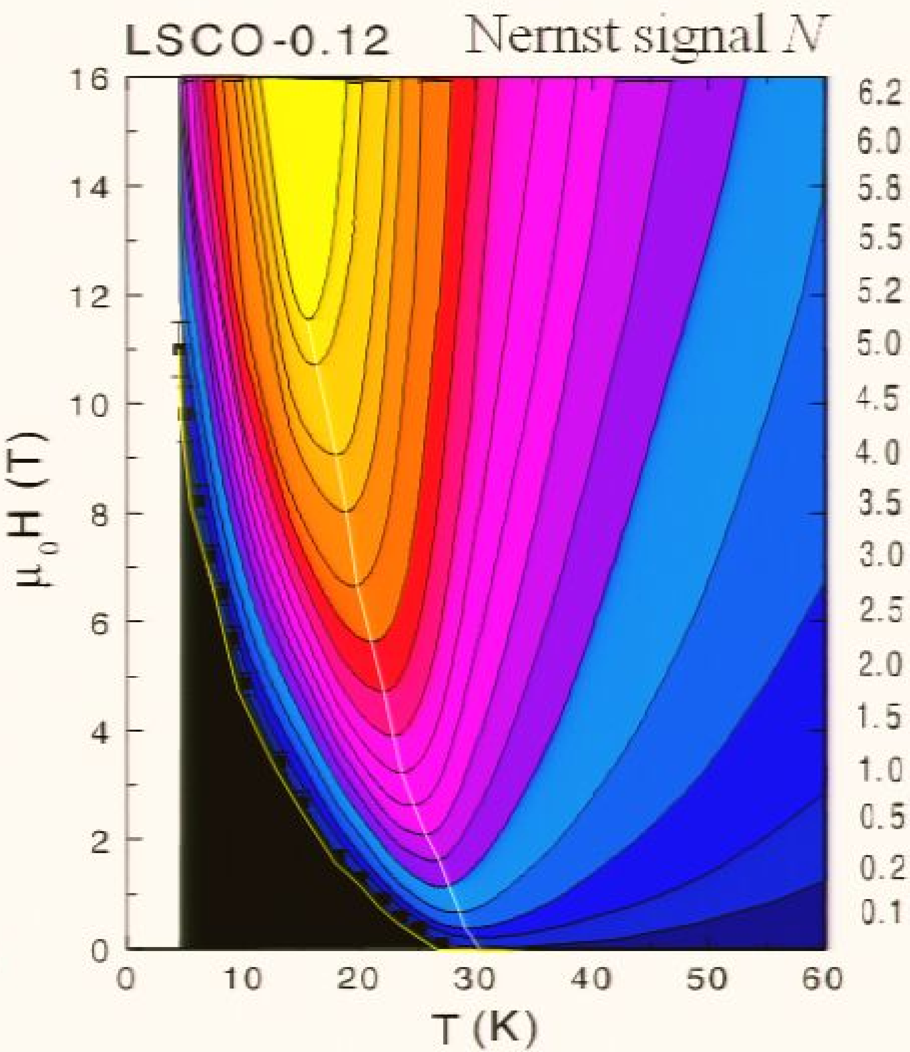
$$\omega_c = 6.2\ \mathrm{GHz}\frac{B}{1\ \mathrm{T}}\left(\frac{35\ \mathrm{K}}{T}\right)^3$$

- T-dependent cyclotron frequency!
- 0.035 times smaller than the cyclotron frequency of free electrons (at T=35 K)
- Only observable in ultra-pure samples where $\tau_{imp}^{-1} < \omega_c$

LSCO Experiments

B,T -dependence





Y. Wang, L. Li, and N. P. Ong, Phys. Rev. B **73**, 024510 (2006).

Conclusions

Conclusions

- General theory of transport in a weakly disordered "vortex liquid" state.
- "Relativistic" magnetohydrodynamics offers a transparent way to disentangling energy and charge transport
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Simplest model reproduces many trends of the Nernst measurements in cuprates.

Comparison with experiment: Peltier coefficient

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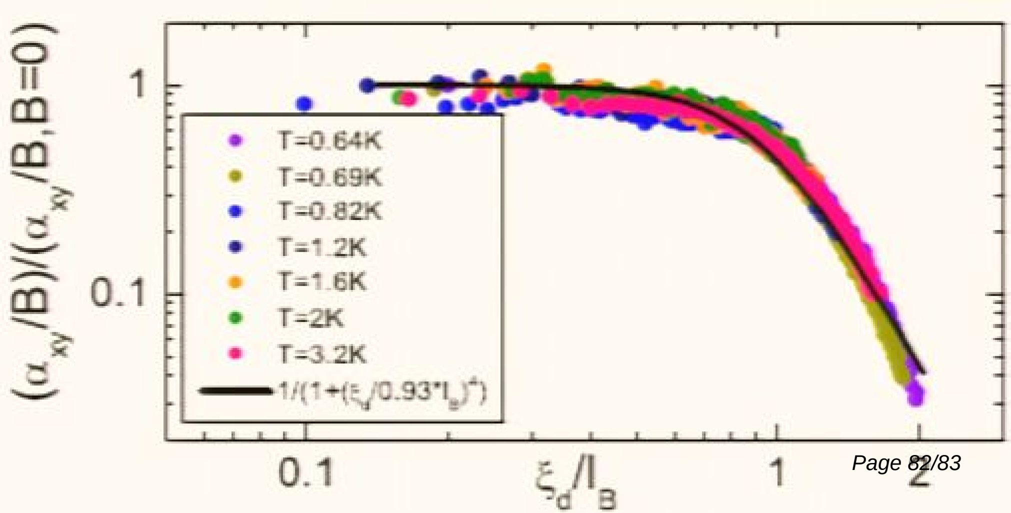

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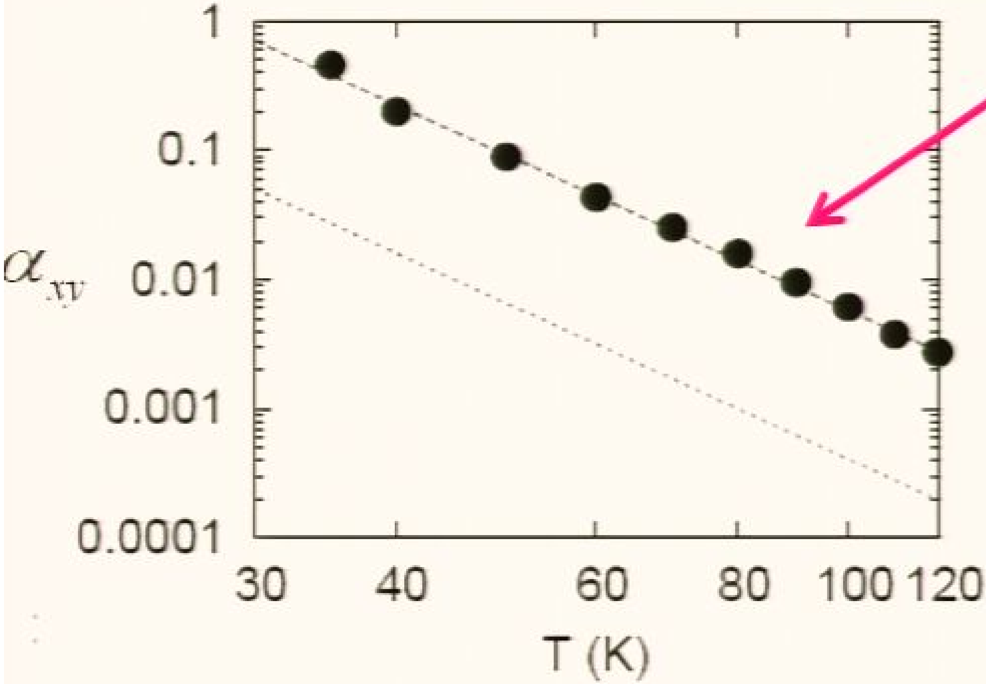

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