Title: Analytic, non-perturbative, almost exact QED

Date: Oct 29, 2007 11:00 AM

URL: http://pirsa.org/07100035

Abstract:

Analytic, Non-Perturbative QED – the 2-point functions

- The Goal (of course) is QCD; but one must learn how to walk before one can run.
- This analysis in 4D, Minkowski metric, using a modified Fradkin rep. of a modified Schwingerian functional sol'n for QED.
- This is a work-in-progress presentation...

  ... so far: Σ  , Σ 

- All calculations performed in a special, rel. gauge, using scaling transformations and current conservation; the dressed electron propagator in configuration space, the dressed photon propagator oscillating between config. and momentum space.

- In this (rapid) presentation, suppress mass-renormalization, altho' w.f.r. constants will be obvious; basic ideas in some depth for S_c', and H-W arguments for $\mathcal{D}'_{c,\mu\nu}$.

I. • Begin with modified Schwinger rep. for S_c' :

$$S_c'(x-y) = e^{\mathfrak{D}_A} \cdot G_c(x,y|A) \cdot \frac{e^{L[A]}}{\langle S \rangle} \Big|_{A\to 0} \to e^{\mathfrak{D}_A} G_c(x,y|A) \Big|_{A\to 0} \text{ in quenched approx.}$$

Here: $\mathfrak{D}_A = -\frac{i}{2} \int \frac{\delta}{\delta A_\mu} D_c^{\mu\nu} \frac{\delta}{\delta A_\nu}$, $[m + \gamma \cdot (\partial_x - igA(x))] G_c(x,y|A) = \delta^{(4)}(x-y)$,

$\tilde{D}_c^{\mu\nu}(k) = \frac{1}{k^2 - i\epsilon}\left(\delta_{\mu\nu} - \zeta \frac{k_\mu k_\nu}{k^2 - i\epsilon}\right)$, $L[A] = \mathrm{Tr} \ln [1 - ig\gamma \cdot A\, S_c]$,

$S_c = G_c[A]|_{A\to 0}$, $\langle S \rangle = e^{\mathfrak{D}_A} \cdot e^{L[A]} \Big|_{A\to 0}$.

Introduce Fradkin rep. :

$$G_c(x,y|A) = i\int_0^\infty ds\, e^{-ism_0^2}\, e^{i\int_0^s ds' \frac{\delta^2}{\delta v_\mu(s')^2}}\, \delta\left(x-y+\int_0^s ds' v(s')\right) \cdot \left(m_0 - \gamma_\mu \frac{\delta}{\delta v_\mu(s)}\right)$$

$$\cdot\, e^{-ig\int_0^s ds'\, v_\mu(s')\, A_\mu\left(y-\int_0^{s'} v\right)} \cdot \left(e^{g\int_0^s ds' \sigma_{\mu\nu} F_{\mu\nu}\left(y-\int_0^{s'} v\right)}\right)_+ \Big|_{v_\mu \to 0},$$

Switch from $v_\mu(s')$ to $u_\mu(s') = \int_0^{s'} ds''\, v_\mu(s'')$, perform linkage operations, to obtain, with $u_\mu(0) = 0$, $z = x-y$: $\quad S_c'(z) = $ "$(m-\gamma\cdot\partial)$" $\mathcal{S}_c'(z)$,

$$\mathcal{S}'(z) = i\int_0^\infty ds\, e^{-ism^2} \cdot e^{-\frac{1}{2}\mathrm{Tr}\ln(2ah)} \cdot N'\int d[u]\, e^{\frac{i}{4}\int u(2ah)^{-1}u}\, \delta^{(4)}(z+u(s)) \cdot$$

$$\cdot\, e^{i\frac{g^2}{2}\iint_0^s ds_1 ds_2\, u_\mu'(s_1)\, D_c^{\mu\nu}(u(s_1)-u(s_2))\, u_\nu'(s_2)} \cdot$$

$$\cdot\, e^{\frac{i}{2}g^2\iint_0^s ds_1 ds_2 \frac{\delta}{\delta\chi_{\mu\nu}(s_1)}\overrightarrow{\partial_\nu} D_c(y)\overleftarrow{\partial_{\nu'}} \frac{\delta}{\delta\chi_{\mu'\nu'}(s_2)}} \cdot$$

$$\cdot\, e^{-2ig^2\iint_0^s ds_1 ds_2\, u_\mu'(s_1)\, D_c^{\mu\lambda}(y)\, \overleftarrow{\partial_\nu} \frac{\delta}{\delta\chi_{\nu\lambda}(s_2)}} \cdot \left(e^{\int_0^s ds' \sigma_{\mu\nu}\chi_{\mu\nu}(s')}\right)_+ \Big|_{\chi\to 0} :$$

where : $y_\mu \Rightarrow u_\mu(s_1) - u_\mu(s_2) \equiv \Delta u_\mu(s_1,s_2)$; $h(s_1,s_2) = \frac{1}{2}\left(s_1+s_2-|s_1-s_2|\right)$, and $\underline{a}$ = real, pos. number $\to 1$ at end of the calculation.

- First Observation : Because of Dirac $\sigma_{\mu\nu} = \frac{1}{4}[\gamma_\mu, \gamma_\nu]$ properties, and a-sym. of $\chi_{\mu\nu} = -\chi_{\nu\mu}$,

$$e^{ig^2\int \frac{\delta}{\delta\chi}(\overrightarrow{\partial} D_c \overleftarrow{\partial})\frac{\delta}{\delta\chi}} \left(e^{\int_0^s \sigma\cdot\chi}\right)_+ \Big|_{\chi\to 0} = 1.$$

Surprise! Upon expansion, every term calculated upto $O(g^8) \to 0$; and, now, a simple proof exists for all orders.

- **However**, the $u' \ldots (0\varepsilon)$ linkages do NOT vanish, and generate log. div. terms, in every g^2 order: Q: How to handle these in a non-pert. way?

- **Second Observation**: $\exists$ a special, rel. gauge, in which information can be extracted. A general, rel. gauge is specified by a parameter ρ, in:

$$\tilde{D}_c^{\mu\nu}(k) = \frac{1}{k^2 - i\epsilon}\left(\delta_{\mu\nu} - \rho \frac{k_\mu k_\nu}{k^2 - i\epsilon}\right)$$

ρ	Reason for usage	Name
0	Simplicity	Feynman
1	Landau - Good UV behavior	
-2	Good IR behavior	Yennie
+2	Useful scaling behavior	Special

In config. space, this becomes:

$$D_c^{\mu\nu}(z) = \frac{i}{4\pi^2}\left[\frac{\delta_{\mu\nu}(1-\frac{\rho}{2})}{z^2 + i\epsilon} + \rho \frac{z_\mu z_\nu}{(z^2+i\epsilon)^2}\right],$$

and the special gauge is defined by: $\rho = +2$: $D_c^{\mu\nu}(z) = \frac{i}{2\pi^2} \cdot \frac{z_\mu z_\nu}{(z^2+i\epsilon)^2}$.

Why this gauge? We need evaluate:

$$\exp\left[\tfrac{i}{2} g^2 \iint_0^s ds_1 ds_2\, u'_\mu(s_1)\, D_c^{\mu\nu}(\Delta u)\, u'_\nu(s_2)\right], \quad \text{with}: \Delta u \equiv u(s_1) - u(s_2).$$

$$\hookrightarrow \exp\left[-\frac{g^2}{4\pi^2}\iint_0^s ds_1 ds_2 \cdot \frac{(u'_\mu(s_1)\Delta u_\mu)}{[(\Delta u)^2 + i\epsilon]} \cdot \frac{(\Delta u_\nu \cdot u'_\nu(s_2))}{[(\Delta u)^2 + i\epsilon]}\right]$$

$$\frac{1}{2}\frac{\partial}{\partial s_1} \ln Z \qquad\qquad -\frac{1}{2}\frac{\partial}{\partial s_2}\ln Z,$$

with: $Z = \mu^2((\Delta u)^2 + i\epsilon)$

$$\hookrightarrow \exp\left[\gamma \iint_0^s ds_1 ds_2 \frac{\partial}{\partial s_1}\ln Z \cdot \frac{\partial}{\partial s_2}\ln Z\right], \quad \gamma = \frac{g^2}{16\pi^2}.$$

$\left\{ \begin{array}{l} \mu \text{ is arb. mass} \ldots \\ \text{for the moment?} \ldots \end{array} \right.$

This is *almost* a perfect differential, but not quite; we can re-write it as: $\exp\left[\sigma \iint_0^s ds_1 ds_2 \left\{ \frac{1}{2} \frac{\partial}{\partial s_1} \frac{\partial}{\partial s_2} \ln^2 Z - \ln Z \cdot \frac{\partial}{\partial s_1} \frac{\partial}{\partial s_2} \ln Z \right\}\right]$.

- But now, the 1st term is a perfect differential; the $u(s')$ fluctuations don't appear, only the end-pts: $u(0)=0$, $u(s)=-z$.
  ∴ This 1st term contributes:

$$\exp\left[-\sigma \ln^2\left(\frac{z^2+i\epsilon}{i\epsilon}\right) - 2\sigma \ln\left(\frac{z^2+i\epsilon}{i\epsilon}\right) \cdot \ln(i\epsilon M^2)\right].$$

- How to evaluate the 2nd term? $\exp\left[-\sigma \iint_0^s ds_1 ds_2 \ln Z \cdot \frac{\partial^2}{\partial s_1 \partial s_2} \ln Z\right]$

(i) By approximation: Since $\ln Z$ should be "slowly-varying", replace this by:

$$\exp\left[-\sigma \langle \ln Z \rangle \iint_0^s ds_1 ds_2 \frac{\partial^2}{\partial s_1 \partial s_2} \ln Z\right],$$

where $\langle\ \rangle$ is taken over $s_{1,2}$ and u-fluctuations.

- And now, this remaining integral is immediate, because the integrand is a perfect differential,

$$\iint_0^s ds_1 ds_2 \frac{\partial^2}{\partial s_1 \partial s_2} \ln Z \Rightarrow -2 \ln\left(\frac{z^2+i\epsilon}{i\epsilon}\right)$$

- The remaining $\int d(u)$ are trivial, yielding $I_0 = \dfrac{e^{iz^2/4sa}}{16\pi^2 s^2 a^2}$.

Originally, thought this the beginning of a S.C. approx.; but one can do much better:

(ii) Instead of playing with $\langle \ln Z \rangle$, let's add and subtract a term: $-\sigma \ln Q \iint ds_1 ds_2 \frac{\partial^2}{\partial s_1 \partial s_2} \ln Z$, where Q is a real, positive number > 1:

$$\exp\left[-\gamma \iint ds_1 ds_2 \ln Z \cdot \frac{\partial^2}{\partial s_1 \partial s_2} \ln Z\right]$$

$$\equiv \exp\left[-\gamma \ln Q \iint ds_1 ds_2 \frac{\partial^2}{\partial s_1 \partial s_2} \ln Z - \gamma \iint ds_1 ds_2 \ln\left(\frac{Z}{Q}\right) \cdot \frac{\partial^2}{\partial s_1 \partial s_2} \ln\left(\frac{Z}{Q}\right)\right]$$

But: $\ln\left(\frac{Z}{Q}\right) = \ln\left(\mu^2\left[\frac{(\Delta u)^2 + i\epsilon}{Q}\right]\right) \Rightarrow \ln\left(\mu^2\left[\frac{(\Delta u)^2}{Q} + i\epsilon\right]\right)$,

and rescaling : $\bar{u}_\mu(s_i) \equiv \frac{u_\mu(s_i)}{\sqrt{Q}}$, performed consistently, produces for the remaining F.I. ;

$$I(z^2, a) = e^{-\frac{1}{2}\mathrm{Tr}\ln(2ah)} \cdot N' \int d[u]\, e^{\frac{i}{4}\int u (2ah)^{-1} u}\, \delta^{(4)}(z + u(s)) \cdot$$
$$\cdot e^{-\gamma \iint \ln Z \cdot \frac{\partial^2}{\partial s_1 \partial s_2} \ln Z} \cdot e^{-2ig^2 \iint ds_1 ds_2\, u'_\mu(s_1) \Omega^{\mu\lambda} \overleftarrow{\partial}_\nu) \frac{\delta}{\delta \chi_{\nu\lambda}}}$$
$$\cdot \left(e^{\int \sigma \cdot \chi}\right)_+ \Big|_{\chi \to 0}$$

the scaling statement :

$$I(z^2, a) = \frac{e^{2\gamma \ln Q \cdot \ln\left(\frac{z^2 + i\epsilon}{i\epsilon}\right)}}{Q^2} \, I\left(\frac{z^2}{Q}, \frac{a}{Q}; \frac{1}{Q}\right),$$

where : $I\left(\frac{z^2}{Q}, \frac{a}{Q}; \frac{1}{Q}\right) = e^{-\frac{1}{2}\mathrm{Tr}\ln\left(2\frac{a}{Q}h\right)} N' \int d[\bar{u}]\, e^{\frac{i}{4}\int \bar{u}\left(\frac{2a}{Q}h\right)^{-1}\bar{u}} \cdot \delta\left(\bar{u}(s) + \frac{z}{\sqrt{Q}}\right) \cdot$

$$\cdot e^{-\gamma \iint \ln Z(\bar{u}) \cdot \frac{\partial^2}{\partial s_1 \partial s_2} \ln Z(\bar{u})}$$
$$\cdot e^{-\frac{2ig^2}{Q} \iint ds_1 ds_2\, \bar{u}'(s_1) \Omega^{\mu\lambda}(\bar{u}) \overleftarrow{\partial}_\nu \cdot \frac{\delta}{\delta \chi_{\nu\lambda}(\bar{u})}} \left(e^{\int_0 \sigma \cdot \chi}\right)_+ \Big|_{\chi \to 0}$$

Because of the $\frac{1}{Q}$, this is not a useful scaling relation.
But if Q becomes arb. large — larger than any cut-off of the div. $u' \cdots (\Omega)$ terms — then all of these div. (Ω) terms are individually removed, and their sum appears in the exp. factor :

$$e^{2\gamma \ln Q \cdot \ln\left(\frac{z^2 + i\epsilon}{i\epsilon}\right)}$$

And, we now have a useful scaling relation for $I(z^2; a)$:

$$I(z^2;a) = \frac{e^{2\gamma \ln Q \cdot \ln\left(\frac{z^2+i\epsilon}{i\epsilon}\right)}}{Q^2} \cdot I\left(\frac{z^2}{Q}, \frac{a}{Q}\right)$$

NB: The terms from those (OE)'s are gauge-inv; this special gauge provides the framework for their summation.

Now, we can use R.G. methods to provide a Diff. Eq. for I, by considering $Q \to Q + \delta Q$, with Q very large:

Calc.: $0 = Q\frac{\partial}{\partial Q} I(z^2; a)$ generates:

$$0 = \left[2\gamma \ln\left(\frac{z^2+i\epsilon}{i\epsilon}\right) - 2\right] I\left(\frac{z^2}{Q}, \frac{a}{Q}\right) - \left(z^2\frac{\partial}{\partial z^2} + a\frac{\partial}{\partial a}\right) I\left(\frac{z^2}{Q}, \frac{a}{Q}\right).$$

Now, let $z^2 \to z^2 Q$, $a \to aQ$, to obtain:

$$\left[2\gamma \ln Q + 2\gamma \ln\left(\frac{z^2+i\epsilon}{i\epsilon}\right) - 2\right] I(z^2; a) = \left(z^2\frac{\partial}{\partial z^2} + a\frac{\partial}{\partial a}\right) I(z^2; a).$$

- Look for a sol'n of form: $I(z^2;a) = I_0(z^2;a)\, J(z^2;a)$,

where $\mathcal{S}'(z^2)\big|_{g\to 0} = I_0(z^2;a) = \frac{1}{16\pi^2 a^2 s^2} \cdot e^{i z^2/4as}$, as given by $\int d[u] \ldots \big|_{g\to 0}$.

- Why this form? Out of the "landscape" of possible sol'ns to Sch. eqs., this is the one desired.

- Also, we'll demand $J(z^2;a)\big|_{a\to 1,\, z^2\to 0} \Rightarrow 1$, so that ETARs are the same for dressed and free fields, as originally assumed.

With: $J = e^{\Omega}$, $\left(z^2\frac{\partial}{\partial z^2} + a\frac{\partial}{\partial a}\right)\Omega = 2\gamma\left[\ln Q + \ln\left(\frac{z^2+i\epsilon}{i\epsilon}\right)\right]$.

At the end of the calc, $a \to 1$; and there's no a-dep. on RHS.
∴ All a-dep. → const, and that const. chosen below.

∴ Look for sol's for $\Omega \to \Omega(z^2) \to \Omega(z^2+i\epsilon)$: $y \equiv \ln[(z^2+i\epsilon)M^2]$,

and: $\frac{\partial}{\partial y}\Omega = 2\gamma\left[y + \ln\left(\frac{Q}{i\epsilon M^2}\right)\right]$.

∴ $\Omega(y) = \gamma y^2 + 2\gamma y \ln\left(\frac{Q}{i\epsilon M^2}\right) + x$, $\quad x = \text{const.}$

or: $J \to \exp\left[\gamma(z^2+i\epsilon) + 2\gamma(z^2+i\epsilon)\cdot\ln\left(\frac{Q}{i\epsilon M^2}\right) + x\right]$.

Adding in the previous, perfect-differential term,

$$\frac{\gamma}{2}\iint_0^1 dz_1 dz_2 \frac{\partial^2}{\partial z_1 \partial z_2}\ln^2 \Xi = -\gamma \ln^2\left(\frac{z^2+i\epsilon}{i\epsilon}\right) - 2\gamma \ln\left(\frac{z^2+i\epsilon}{i\epsilon}\right)\cdot\ln(i\epsilon M^2),$$

the entire answer becomes:

$$I_0 \cdot \exp\left\{2\gamma \ln\left(\frac{z^2+i\epsilon}{i\epsilon}\right)\cdot\ln\left(\frac{Q}{i\epsilon M^2}\right) + 2\gamma \ln Q \cdot \ln(i\epsilon M^2) - \gamma \ln^2(i\epsilon M^2) + x\right\}$$

and if the $\{\ \} \to 0$ when $z^2 \to 0$, $x \to \gamma \ln^2(i\epsilon M^2) - 2\gamma \ln Q \cdot \ln(i\epsilon M^2) -$
$- is \cdot \delta m^2$

∴ The entire result becomes (with $m_0^2 \to m^2$):

$$I_0 \cdot \exp\left[2\gamma \cdot \ln\left(\frac{z^2+i\epsilon}{i\epsilon}\right)\cdot\ln\left(\frac{Q}{i\epsilon M^2}\right)\right],$$ so that:

$\Delta_c'(z^2) \to \Delta_c^{(0)}(z^2)\cdot\exp[\ \text{"}\ ]$, a multiplicative, div., z^2-dependent

Let's try to guess Z_2, the electron's w.f.r. constant, without performing the Fourier transform integral.

Going to the mass-shell in mom. space $\Leftrightarrow$ $\vec{z}^2 - z_0^2 \to +\infty$ in config. space. $\therefore$ set $z^2 \to +\frac{1}{\mu^2}$ and imagine μ very small. (the mom. space IR div.) Also, $\epsilon \to \Lambda^{-2}$, Λ = UV. cutoff.

then: $\{\ \} \to 2\gamma\left[-\left(\frac{\pi}{2}\right)^2 + \ln\left(\frac{\Lambda^2}{\mu^2}\right)\cdot\ln\left(\frac{Q\Lambda^2}{M^2}\right)\right]$

$$- i\frac{\pi}{2}\cdot\ln\left(\frac{\Lambda^2}{\mu^2}\cdot\frac{\Lambda^2 Q}{M^2}\right)$$

In order for Z_2 to be real, $\frac{\Lambda^4 Q}{\mu^2 M^2} = 1$, or $M^2 = \frac{Q\Lambda^4}{\mu^2}$ (really large!)

then, $\ln\left(\frac{Q\Lambda^2}{M^2}\right) = \ln\left(\frac{\mu^2}{\Lambda^2}\right) = -\ln\left(\frac{\Lambda^2}{\mu^2}\right)$,

and: $Z_2 \Rightarrow \exp\left[-2\gamma\left(\left(\frac{\pi}{2}\right)^2 + \ln^2\left(\frac{\Lambda^2}{\mu^2}\right)\right)\right]$,

so that: $0 < Z_2 < 1$, which satisfies a formal property of the theory.

$\to$ "Physical", not "Mathematical" Uniqueness —

II. The Photon Propagator (in one-fermion-loop approximation):

$K_{\mu\nu}(z) = \sum_G$  $+ \cdots$

$$K_{\mu\nu}(z) \sim g^2 e^{\mathfrak{D}_A}\, \mathrm{tr}\left[\gamma_\mu G_c(x,y|A)\,\gamma_\nu\, G_c(y,x|A)\right]\Big|_{A\to 0}\,,$$

and must satisfy: $\partial_\mu K_{\mu\nu}(z) = 0$.

Let's try to guess Z_2, the electron's w.f.r. constant, without performing the Fourier transform integral.

Going to the mass-shell in mom. space $\leftrightarrow$ $z^2 - z_0^2 \to +\infty$ in config. space. $\therefore$ Set $z^2 \to + \frac{1}{\mu^2}$ and imagine μ very small. (the mom. space IR div.) Also, $\epsilon \to \Lambda^{-2}$, Λ = UV. cutoff.

$$\text{then}: \{ \ \} \to 2r\left[-\left(\tfrac{\pi}{2}\right)^2 + \ln\left(\frac{\Lambda^2}{\mu^2}\right)\cdot \ln\left(\frac{Q\Lambda^2}{M^2}\right)\right]$$
$$- i\frac{\pi}{2}\cdot \ln\left(\frac{\Lambda^2}{\mu^2}\cdot\frac{\Lambda^2 Q}{M^2}\right)$$

In order for Z_2 to be real, $\frac{\Lambda^4 Q}{\mu^2 M^2} = 1$, or $M^2 = \frac{Q\Lambda^4}{\mu^2}$. (really large!)

$$\text{then},\ \ln\left(\frac{Q\Lambda^2}{M^2}\right) = \ln\left(\frac{\mu^2}{\Lambda^2}\right) = -\ln\left(\frac{\Lambda^2}{\mu^2}\right),$$

$$\text{and}: Z_2 \Rightarrow \exp\left[-2r\left(\left(\tfrac{\pi}{2}\right)^2 + \ln^2\left(\frac{\Lambda^2}{\mu^2}\right)\right)\right],$$

so that : $0 < Z_2 < 1$, which satisfies a formal property of the theory.

→ "Physical", not "Mathematical" Uniqueness —

II. The Photon Propagator (in one-fermion-loop approximation):

$K_{\mu\nu}(z) = \sum_G$ 

$$K_{\mu\nu}(z) \sim g^2 e^{\mathfrak{D}_A}\, \mathrm{tr}\left[\gamma_\mu G_c(x,y|A)\,\gamma_\nu\, G_c(y,x|A)\right]\Big|_{A\to 0}.$$

and must satisfy: $\partial_\mu K_{\mu\nu}(z) = 0$.

Procedure: 1) Insert $\int \overset{(s)}{d[u]} \cdots \int \overset{(t)}{d[v]} \cdots$ with $i\int_0^\infty ds\, e^{-ism_0^2} \cdot i\int_0^\infty dt\, e^{-im_0^2 t}$,

choose special gauge; note that all $e^{g^2 \int \frac{\delta}{\delta x}(\partial\cdot\partial)\frac{\delta}{\delta x}} (e^{i(\cdots)})_+ \Big|_{x\to 0} \to 1$

2) Perform mass-renorm. in both F.I.s, $m_0 \to m$.

3) Insert: $\pm \Big(\ln Q \iint ds_1 ds_2 \frac{\partial^2}{\partial s_1 \partial s_2} \ln \Sigma(u(s)) + \ln Q \cdot \iint dt_1 dt_2 \frac{\partial^2}{\partial t_1 \partial t_2} \ln \Sigma(v(t)) +$

$+ \ln Q \iint ds_1 dt_1 \frac{\partial^2}{\partial s_1 \partial t_1} \ln \Sigma(u(s'), v(t')) \Big)$

4) Rescale, and take $Q \ggg 1$.

5) In remainder, all exponential factors cancel; and one is left with:

$$K_{\mu\nu}(z|s,t) = \frac{g^2}{Q^4}\int d[\bar{u}(s')] \int d[\bar{v}(t')] \cdot \left[4m^2 \delta_{\mu\nu} + (u'(s), v'(t))_{\mu\nu} + g^2 [z]_{\mu\nu} \right] \cdot \delta[\bar{u}(s) + \tfrac{z}{\sqrt{Q}}] \cdot \delta[\bar{v}(t) - \tfrac{z}{\sqrt{Q}}] \cdot e^{\frac{ig^2}{2}\iint \bar{u}' \cdot D_c \cdot \bar{u}' + \frac{ig^2}{2}\iint \bar{v}' \cdot D_c \cdot \bar{v}' + ig^2 \iint \bar{u}' \cdot D_c \cdot \bar{v}'},$$

with: $[z]_{\mu\nu} = \frac{2i}{\pi^2}\left[\frac{\delta_{\mu\nu}}{(z^2+i\epsilon)} - 2\frac{z_\mu z_\nu}{(z^2+i\epsilon)^2} \right]$.

Alternately:

$$K_{\mu\nu}(z|s,t) = g^2 \left[4m^2 \delta_{\mu\nu} + (u', v')_{\mu\nu} + g^2 [z]_{\mu\nu} \right] C\left(\frac{z^2}{a}, \frac{a}{Q}\right) \cdot \frac{1}{Q^4};$$

and $K_{\mu\nu}(z|s,t)$ is indep. of Q. For $Q \ggg 1$, $Q \cdot \frac{\partial}{\partial Q} K_{\mu\nu}(z|s,t) = 0$,

which gives: $4C + \left(z^2 \frac{\partial}{\partial z^2} + a \frac{\partial}{\partial a}\right) C\left(\frac{z^2}{Q}, \frac{a}{Q}\right) = 0$.

How is this possible? Consider: $C(\frac{z^2}{Q}, \frac{a}{Q})|_{g^2=0} = C_0$,

and by direct f-integration: $C_0(z^2, a) \sim \iint \frac{ds}{s^2}\frac{dt}{t^2}\frac{1}{a^4} e^{-im^2(s+t) + i\frac{z^2}{4}(\frac{1}{sa}+\frac{1}{ta})}$,

i.e., $C_0(z^2, a) = \frac{1}{a^4} e^{i\frac{z^2}{4a\bar{s}}}$, $\frac{1}{\bar{s}} = \frac{1}{s} + \frac{1}{t}$,

so: $C_0(\frac{z^2}{Q}, \frac{a}{Q})\frac{1}{Q^4}$ is indep. of Q.

$\therefore$ We now assume sol's of form: $C(z^2, a)$ has a "normalization" factor: $C \to \sum_{n=0}^{\infty} C_{(2n)}\, \phi \cdot \exp[i z^2/4a\bar{s}]$, $\phi \sim \int \frac{ds}{s^2}\frac{dt}{t^2} e^{-im^2(s+t)} \cdot \frac{1}{a^4}$,

So that: $C_{(2n)}(\frac{z^2}{Q}, \frac{a}{Q}) \cdot \frac{1}{Q^4}$ is indep. of Q.

I.E. Assume sol's of form: $C(z^2, a) \to \frac{1}{a^4} C(\frac{z^2}{a})$, and, as such, are indep. of Q. $\therefore$ all Q-dep. is removed; & we set $Q \to 1$.

How to Proceed? Adopt $\partial_z K_{\mu\nu}(z) \equiv 0$ as a requirement, in every g^2-order.

Starting from:

$$K_{\mu\nu}(z|s,t) = g^2\left[4m^2\delta_{\mu\nu} + \langle(u'(s), w'(t))_{\mu\nu}\rangle + g^2[z]_{\mu\nu}\right] C(z^2|s,t),$$

where $\langle(u'(s), w'(t))_{\mu\nu}\rangle$ is highly singular, and must be defined properly,

$$K^{(2)}_{\mu\nu} = g^2\{4m^2\delta_{\mu\nu} + \langle(u', w')_{\mu\nu}\rangle\} C_0\, \phi\, e^{iz^2/4\bar{s}}$$

, and "proper def." (of ...

If $C = \sum_{n=0}^{\infty} C_{(2n)} = C_0 + C_2 + C_4 + \cdots$, and $C_0 \equiv 1$,

the sum becomes :

$$K^{(2)}_{\mu\nu}(C-1) + g^4 [z]_{\mu\nu} \phi e^{i z^2/4s} C - \sum_{n=2}^{\infty} K^{(2n)}_{\mu\nu} ,$$

or:

$$C \cdot K^{(2)}_{\mu\nu} + g^4 [z]_{\mu\nu} \phi e^{i z^2/4s} C = \sum_{n=1}^{\infty} K^{(2n)}_{\mu\nu} = K_{\mu\nu}(z)$$

Since $\partial_\mu K_{\mu\nu}(z) = 0$, we have a D. Eq. for C':

$$\underbrace{K^{(2)}_{\mu\nu}(z) \cdot 2z_\mu}_{2z_\nu T} C'(z^2) + \underbrace{g^4 [z]_{\mu\nu} \phi e^{i z^2/4s} \cdot 2z_\mu}_{2z_\nu U} C'(z^2) + \Big(\underbrace{\partial_\mu g^4 [z]_{\mu\nu} \phi e^{i z^2/4s}}_{2z_\nu R}\Big) C = 0$$

$\therefore \; (T+U)C' + RC = 0$,

$$\text{and: } C = x\, e^{-\int_{z_0^2}^{z^2} du \left(\frac{R(u)}{U(u)+T(u)} \right)} \quad , \; x \text{ and } z_0^2 \text{ constants}$$

Since $R \sim g^4 + \cdots$, $T \sim g^2 + \cdots$, $U \sim g^4 + \cdots$,

$$C \sim x\, e^{-\int_{z_0^2}^{z^2} du \left[O(g^2) + \cdots \right]}$$

$\therefore \; x \to C_0 = 1$;

and comparing this with C_2, or $K^{(4)}_{\mu\nu}$, determines z_0^2.

Finally, $K_{\mu\nu}(z) = C(z^2) \left(K^{(2)}_{\mu\nu}(z) + g^4 [z]_{\mu\nu} \phi e^{i z^2/4s} \right)$.

<u>NB</u> : T, U, R are built from $K^{(2)}_{\mu\nu}$ and $K^{(4)}_{\mu\nu}$; and the non-perturbative soln has this product form in coordinate space.

To Summarize :

For configuration-space QED :

(i) Write relevant, modified Schwinger–Fradkin Functional Representations of exact n-point functions.

(ii) Introduce the Special Gauge

(iii) Rescale, with Q arbitrarily large

(iv) If $\exists$ an inhomogeneous term to scaled, R.G., F. Integral, the simplest (class of solns) obtainable.

(v) If $\exists$ no inhomogeneous term in scaled, R.G. F. Integrals, require (current conservation) gauge-inv. to all orders in g^2. Then, determine and solve the D.Eq. for the sum of all orders.

(vi) Take Fourier transforms ...

- Work-in-Progress on QED Vertex ...

- Generalization to QCD ...