

Title: Finding out about quantum systems

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Abstract: One of the cool, frustrating things about quantum theory is how the once-innocuous concept of "measurement" gets really complicated. I'd like to understand how we find out about the universe around us, and how to reconcile (a) everyday experience, (b) experiments on quantum systems, and (c) our theory of quantum measurements. In this talk, I'll try to braid three [apparently] separate research projects into the beginnings of an answer. I'll begin from the premise that you make a measurement to find something out, then attack some specific questions: "How do we find out about quantum systems?", "What can we find out about quantum systems?", and finally "What do we actually know, afterward?" I'll give precise statements of these questions, then present [partial] answers.

Finding out about quantum systems

Robin Blume-Kohout
October 2, 2007



Obligatory Reference to Famous Physicist

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I wonder why I wonder.
I wonder why I wonder why
I wonder why I wonder!



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Noisy
communication
channel

I'd like to find out how [stuff works].
I'd like to find out how
I find out how [stuff works].

A general outline

- How does measurement really work in quantum theory?
- Premise: A measurement is an action that tells you something about some thing.
- 1. Environments as Witnesses
or "How I learned to stop worrying and love decoherence."
- 2. The structure of correlations in QM
- 3. Information-preserving structures
in the quantum dynamical processes

I. Environments as Witnesses & Indirect Measurements

- Measurement in QM excludes objective properties.
- Indirect measurements -- through the environment -- let us recover some objectivity
- A necessary condition is **redundant information**.
- But what is that information about?
 - All we know is: it has to be classical

Measurement:

Classical vs. Quantum

- Classical physics: Universe has a state (point in phase space), which evolves.
 - Measurement merely reveals pre-existing objective properties of the state.
- Quantum physics: Measurement is an active part of the theory -- changes the state.
 - Does this leave room for objectivity???

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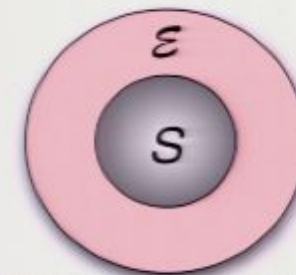
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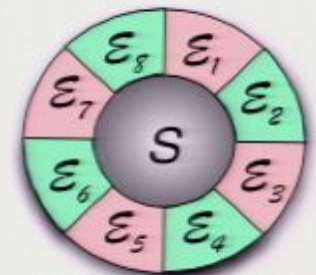
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Environment as a Witness

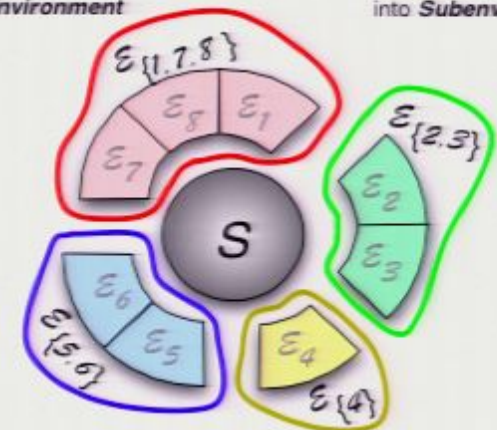
- Measurements are **indirect** -- intercept part of the system's environment.
- Correlations w/environment cause decoherence -- but also enable measurement!
- Observer's resources are limited -- small fragments.



(a) Decoherence Paradigm:
Universe is divided into
System & Environment



(b) Redundancy Paradigm:
Environment is divided
into **Subenvironments**



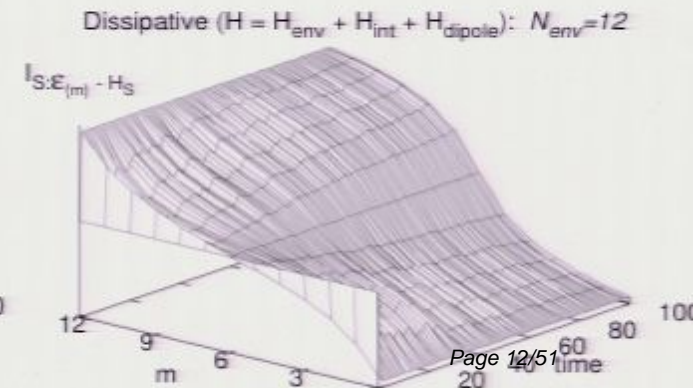
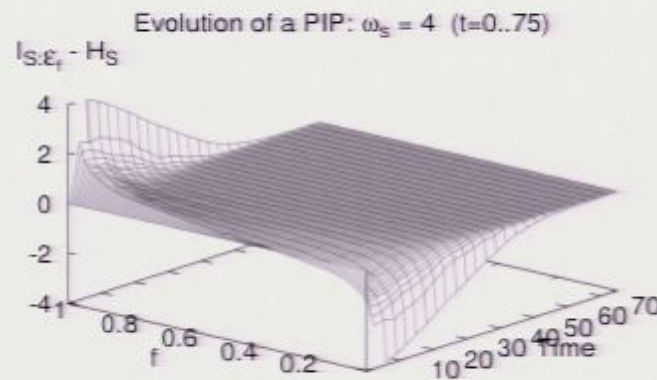
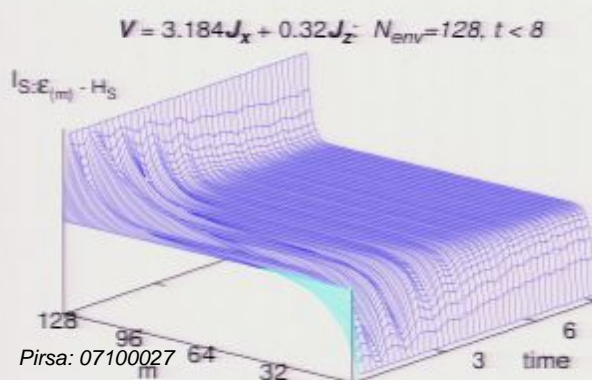
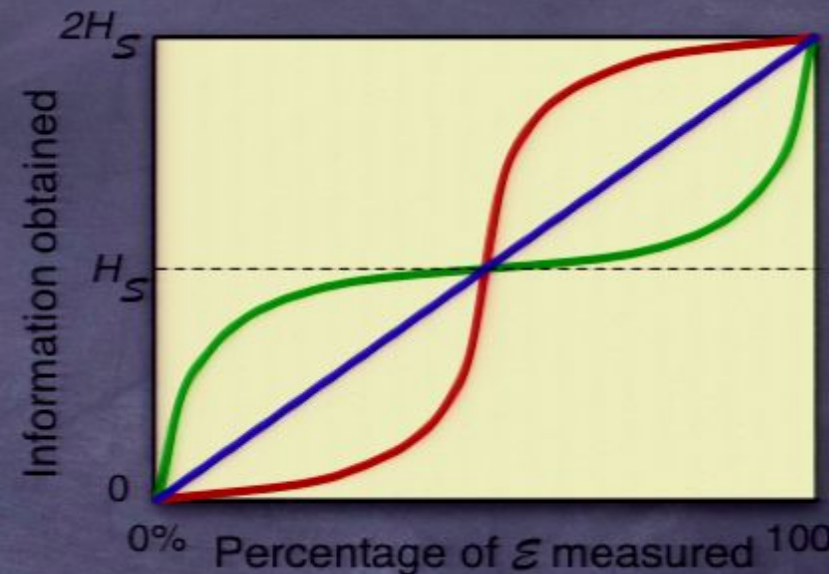
(c) **Subenvironments** are combined
into **Fragments** that each have
nearly-complete information.

Redundant Information

- Redundancy: small fragments provide a lot of information.
- Analyze joint state of Sys-Env using *Quantum Mutual Information*:

$$I_{S\mathcal{E}} = H_S + H_{\mathcal{E}} - H_{S\mathcal{E}}$$

where $H \equiv -\text{Tr}(\rho \ln \rho)$



Information about What?

- There are a lot of small fragments, each of which provides a lot of information... so **something** must be redundant. But what?
- At most, a single classical observable
 - Redundant quantum info would violate no-cloning (or no-broadcasting) theorems.
- How do we identify the observable... or even whether it **is** an observable...

Information about What?

- Information = a resource that can be used to decrease uncertainty [about the system].
= ability to effectively make a measurement [on the system].
- So, what properties can we measure indirectly, using a generic small fragment?
- Those properties are **objective**.
 - * Deep: Do they describe our world?
 - * Less deep: How do we identify them?

II. Structure of Correlations

What can system **B** “know” about system **A**?

- Measurements can **retrodict** and/or **predict**.
- Can analyze both via **quantum channels**.
- Perfect information has **algebraic structure**.
- Information is typically conditional:
 - **B** can “know” multiple **conditional properties**...
 - ...but only one **unconditional property** of **A**.
- We can efficiently identify this -- i.e.:

What **B** “knows” unconditionally about **A**.

Retrodiction vs. Prediction

(What does system **B** “know” about system **A**?)

- Classical: measurement reveals current state
 - ⇒ implies both past & future (symmetrically)
- Quantum: measurement collapses the state
 - ⇒ sharp prediction (“X will definitely occur”)
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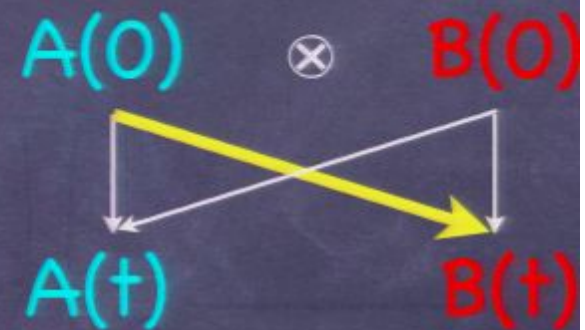


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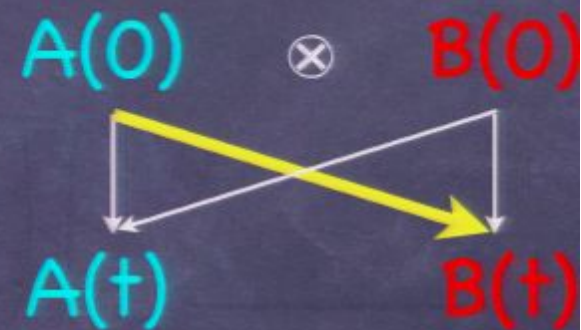


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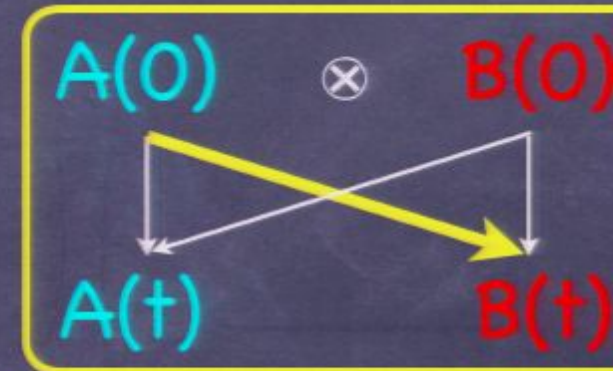
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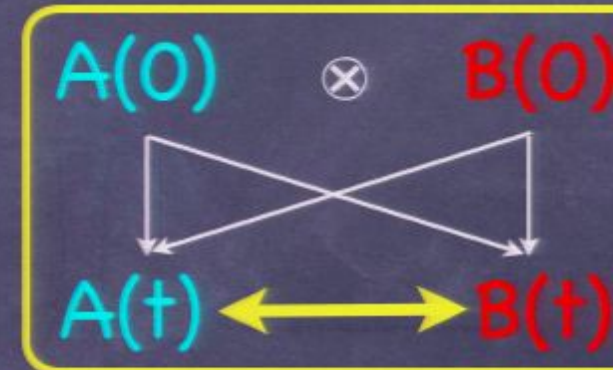
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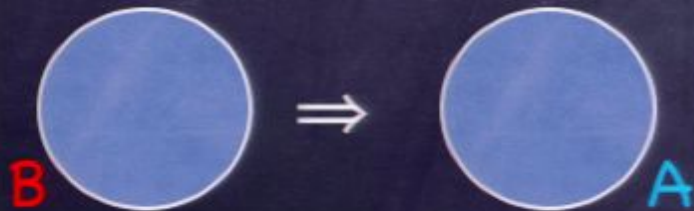
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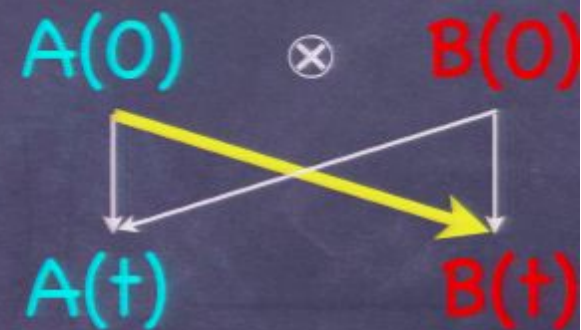
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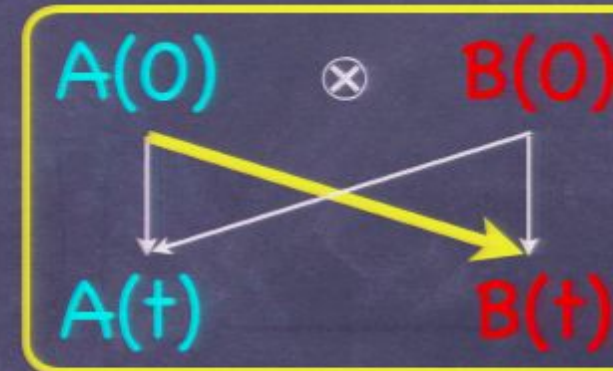
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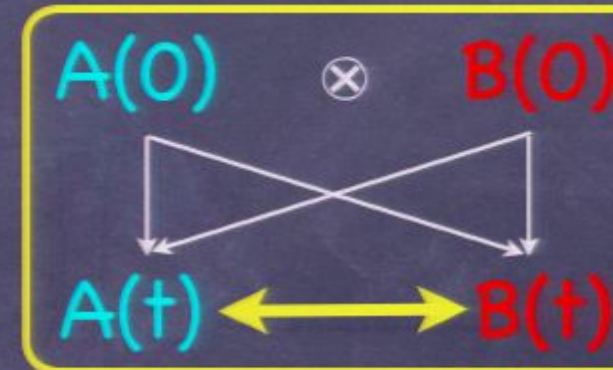
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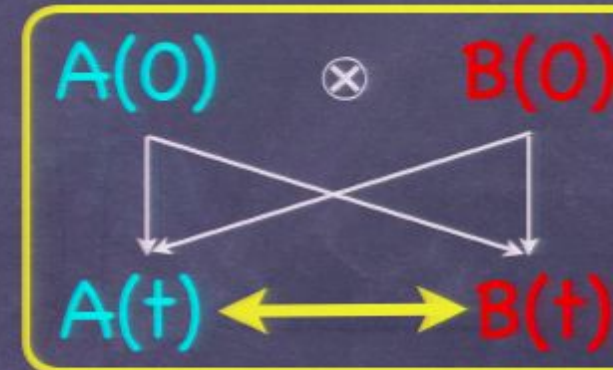
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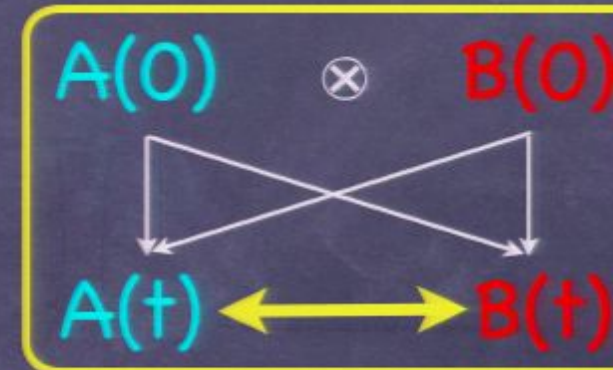
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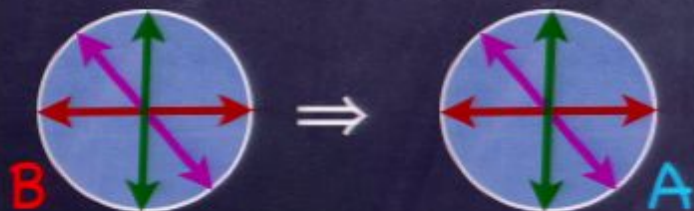
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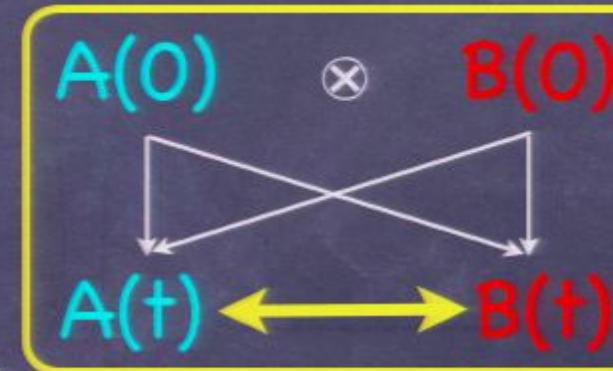
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- B can "know" about non-orthogonal states of A

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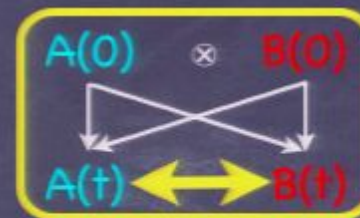
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 - Write ρ_{AB} as $\rho_{AB} = (\mathbb{1}_A \otimes \mathcal{E}_B) [|\Psi_{AB}\rangle\langle\Psi_{AB}|]$
 - Note: (1) nothing happened to A ; (2) no interaction
 - \Rightarrow " $B(t)$ knows X about $A(t)$ " iff " $B(t)$ knows X about $A(0)$ ".
 - and $B(0)$ has perfect [distorted] correlations with $A(0)$...
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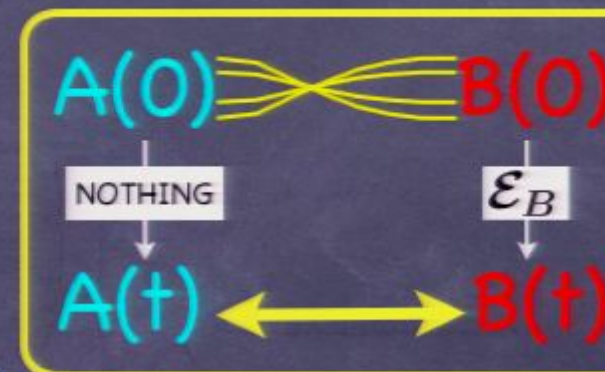


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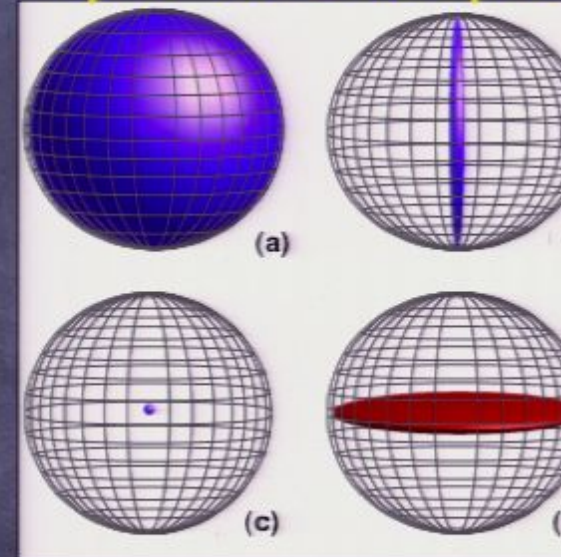
III. Information preserved by a quantum process

• **Question:** What information is preserved by a quantum channel \mathcal{E} ?

• **Answer:**

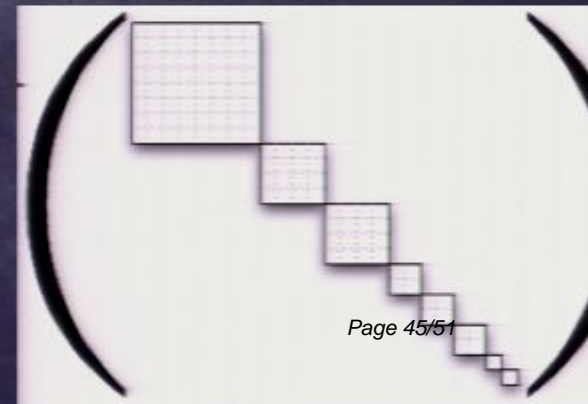
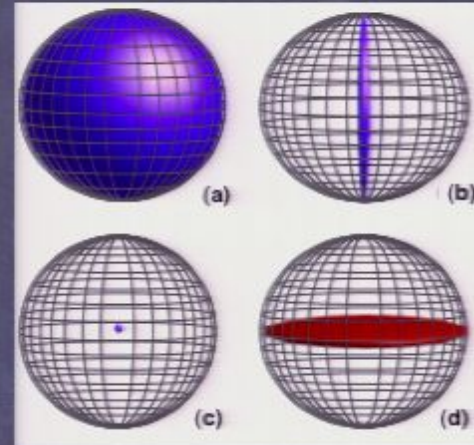
- Represent information by a set of states -- a **code**.
- A code is preserved by \mathcal{E} if its states retain their distinguishability.
- A preserved code must be **isometric** (geometrically equivalent) **to an algebra**.
- Equivalent codes (= same information) share an **Information-preserving structure (IPS)** = algebra

Spin- $\frac{1}{2}$ Example



Algebras

- An associative algebra is closed under linear combination & multiplication.
- Every matrix algebra is isomorphic to a direct sum: $\mathcal{A} = \bigoplus_k \mathcal{M}_{d_k} \otimes \mathbb{1}_{n_k}, \quad n_k, d_k \in \mathbb{N},$
- This is a "hybrid quantum memory".
Can be described by its shape: $\{d_1, d_2 \dots d_N\}$
- Quantum info is stored in blocks;
classical info is stored across them.



Proof Outline

- If code \mathcal{C} is preserved by \mathcal{E} , then it is **correctable** -- i.e., some \mathcal{R} puts \mathcal{C} back where it came from...
...making it **noiseless** (for $\mathcal{R} \circ \mathcal{E}$).

- An explicit construction is: $\hat{\mathcal{E}}(\rho_{\text{out}}) = \mathcal{E}^\dagger \left(\frac{1}{\sqrt{\mathcal{E}(\mathbb{1})}} \rho_{\text{out}} \frac{1}{\sqrt{\mathcal{E}(\mathbb{1})}} \right)$

- A noiseless code for $\mathcal{R} \circ \mathcal{E}$ is isometric to **fixed points** of $\mathcal{R} \circ \mathcal{E}$.

- An **optimal** noiseless code is isometric to all the states in the fixed-point set.

- The fixed-point set of $\hat{\mathcal{E}} \circ \mathcal{E}$ is an algebra.

Information is Conditional

- A channel can have multiple IPS:
- Example: a map on 3 qubits, $A \times B \times C$
 - (1) Measure $\{|0\rangle, |1\rangle\}$ on A
 - (2) If "0", decohere $B \times C \Rightarrow$ 2 classical bits
 - (3) If "1", ignore B and obliterate $C \Rightarrow$ 1 qubit
 - (4) Force A into $|0\rangle$.
- Each **support** P (subspace of H) has an IPS
(N.B. most will be trivial...)
- P is a **promise** or **precondition** -- "If the state is supported on P , then the IPS is preserved"

Things I Don't Know

- How to find \mathcal{E} 's **biggest** IPS (QMA-complete)
- How to find **all** of \mathcal{E} 's IPSs (even harder)
- The **structure** of all of \mathcal{E} 's IPSs (???)
- How to answer "Does \mathcal{E} have (e.g.) a 1-qubit IPS?" (?)
- How to deal with **post-conditions** (working on it...)

Unconditionally Preserved Information

- Precondition = promise about input
-- useful for communication theory!
- Postcondition = promise about the output
-- will only be true some of the time
- Q: What does B know about A **no matter what?**
- A: \mathcal{E} 's unconditionally preserved IPS
= the IPS with no pre- or post-conditions
= algebra of fixed points of $\hat{\mathcal{E}} \circ \mathcal{E}$ (with no P)
- This is something we can find.

Tying it together

- We have some tools for dissecting measurement:
 - **Environment as a Witness:**
 - permits operational objectivity
 - separates prediction from retrodiction
 - **Information-preserving structures:** let us (in theory) answer the question “What information is perfectly preserved?”
 - **Correlation structures:** extend the domain of IPS to predictive measurements.

Open Problems

- So far, more questions are **raised** than are answered!
- Does the unconditionally-preserved IPS really characterize “What B knows about A?”
- Are other kinds of information allowed by post-conditions?
- What kinds of information can be **approximately** preserved?