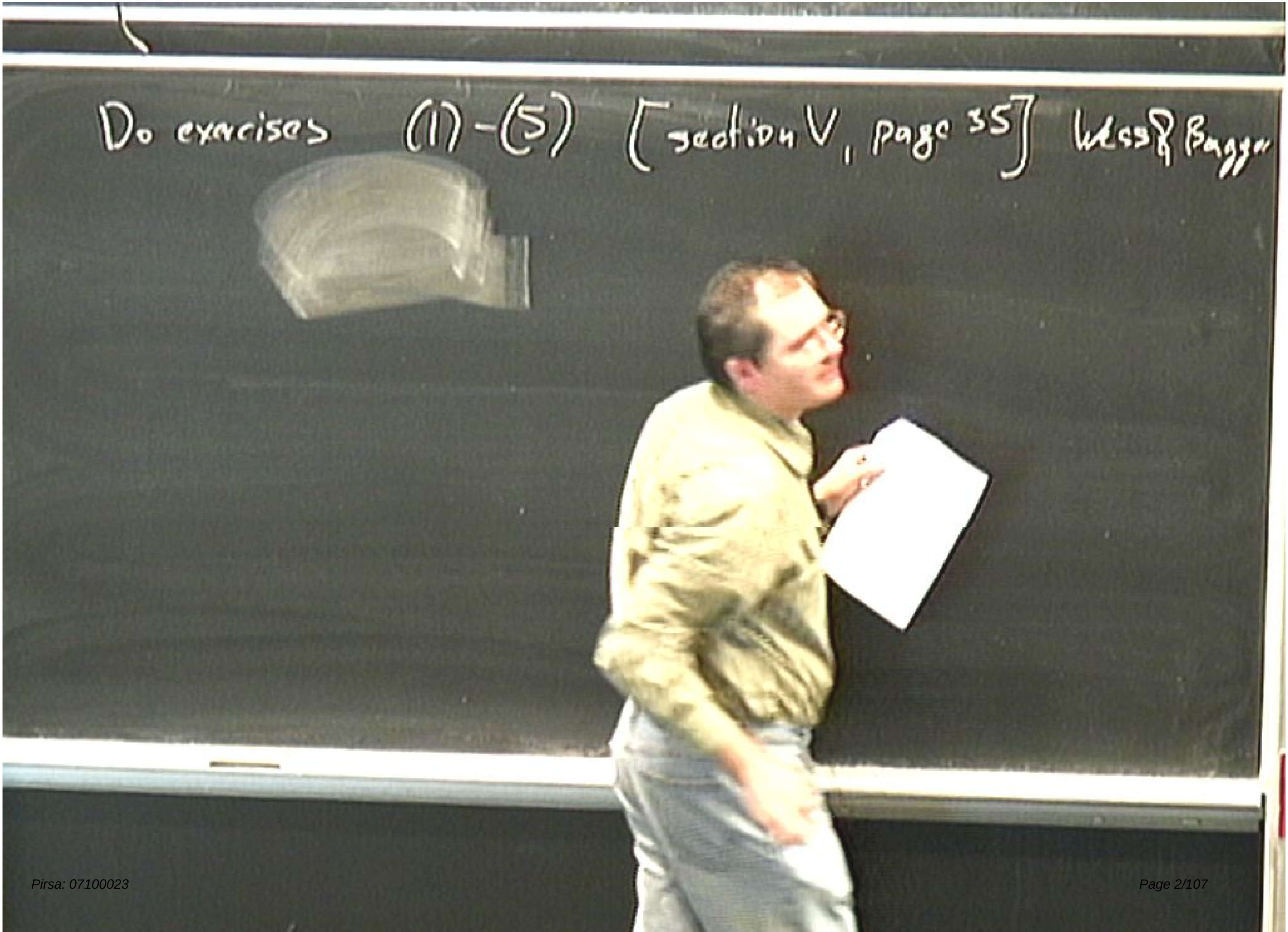
Title: Intro to Supersymmetry 13

Date: Oct 30, 2007 12:30 PM

URL: http://pirsa.org/07100023

Abstract:


Do exercises (1)-(5) [section V, page 35] Wess & Bagg

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Homework #2 (due in 2 week).

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Renormalization and effective field theories

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Renormalization and effective field theories

$$\mathcal{L}_0 = -\frac{1}{2}(\partial\varphi)^2 - \frac{\omega_0^2}{2}\varphi^2 - \frac{\lambda_0}{4!}\varphi^4$$

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$\Rightarrow$  $(-i\lambda_0)$ $+$ 

highe loop corrections

Homework #2 (due in 2 week)

Renormalization and effective field theories

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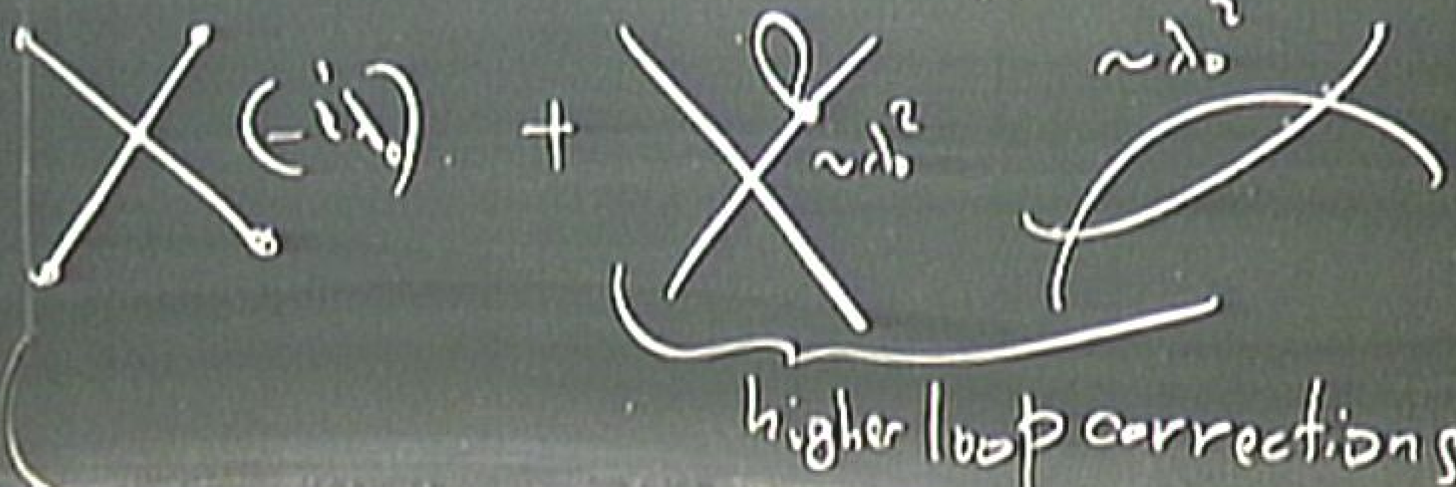

$\Rightarrow$ Direct computat

$\Rightarrow$ Direct computation leads to infinities

$$\mathcal{L}_1 = \mathcal{L}_0$$

renormalized lagrangian at 1-loop

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$$\mathcal{L}_1 = \mathcal{L}_0 - \underbrace{\Delta \mathcal{L}_1}_{\text{counterterms.}}$$

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renormalized lagrangian at 1-loop

Introduce addition vertecies

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$$\mathcal{L}_0 - \underbrace{\Delta \mathcal{L}_1}_{\text{counterterms.}}$$

Introduce addition vertecies

+ tune couplings to get finite results!

$$\mathcal{L}_{\text{physical}} = \mathcal{L}_0 - \underbrace{\sum_{n=1}^{\infty} \Delta \mathcal{L}_{[n]}}$$

In principle can lead to ∞ many additional terms

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$$\mathcal{L}_{physical} = \mathcal{L}_0 - \underbrace{\sum_{n=1}^{\infty} \Delta \mathcal{L}_{[n]}}$$

In principle can lead to ∞ many additional terms

$\Rightarrow$ "renormalizable" QFT

$\hookrightarrow$ counterterms contains operators already present in $\mathcal{L}_0$ (or a finite # of new operators).

$\mathcal{L}_0$ is a bare Lagrangian for a renormalizable QFT.

$\mathcal{L}_0$ is a bare Lagrangian for a renormalizable QF

$$-\sum_{n=1}^{\infty} \mathcal{L}_{[n]} \equiv \frac{(Z_\varphi - 1)}{2}\left[-(\partial\varphi)^2 - m^2\varphi^2\right] - \frac{\lambda}{4!}(Z_\lambda - 1)\varphi^4 + \frac{\delta m^2}{2} Z_\varphi \varphi^2$$

$\mathcal{L}_0$ is a bare Lagrangian for a renormalizable QFT

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Z_φ, Z_λ, $\delta m^2 \rightarrow$ they depend on m_0, λ_0 ...

$\mathcal{L}_0$ is a bare Lagrangian for a renormalizable QFT.

$$-\sum_{n=1}^{\infty} \mathcal{L}_{[n]} \equiv \frac{(Z_\varphi - 1)}{2}\left[-(\partial\varphi)^2 - m^2\varphi^2\right]$$

$$- \frac{\lambda}{4!}(Z_\lambda - 1)\varphi^4 + \frac{\delta m^2}{2} Z_\varphi \varphi^2 = -\Delta\mathcal{L}$$

Z_φ, Z_λ, δm^2 $\rightarrow$ they depend on m_0, λ_0 ...

$$\mathcal{L}_{phys} = \mathcal{L}_0 - \Delta\mathcal{L}$$

$$\mathcal{L}_{phys} + \Delta\mathcal{L} = \mathcal{L}_0$$

$$-\frac{Z_\varphi (\partial\varphi)^2}{2} - \frac{Z_\varphi m^2 \varphi^2}{2} - \frac{\lambda Z_\lambda \varphi^4}{4!} = -\frac{1}{2}(\partial\varphi_0)^2 - \frac{m_0^2\varphi_0^2}{2} - \frac{\lambda_0 \varphi_0^4}{4!}$$

$$\mathcal{L}_{phys} = \mathcal{L}_0 - \Delta\mathcal{L}$$

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$$\underbrace{-\frac{Z_\varphi(\partial\varphi)^2}{2} - \frac{Z_\varphi m^2\varphi^2}{2} - \frac{\lambda Z_\lambda}{4!}\varphi^4}_{\mathcal{L}_{phys} + \Delta\mathcal{L}} = \underbrace{-\frac{1}{2}(\partial\varphi_0)^2 - \frac{m_0^2\varphi_0^2}{2} - \frac{\lambda_0\varphi_0^4}{4!}}_{\text{bare Lagrangian}}$$

$$\lambda\varphi^4 = -\frac{1}{2}(\partial\varphi_0)^2 - \frac{m_0^2\varphi_0^2}{2} - \frac{\lambda_0\varphi_0^4}{4!}$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{\text{bare lagrangian}}$$

$$\mathcal{L}_{phys} = -\frac{1}{2}(\partial\varphi)^2 - \frac{m^2}{2!}\varphi^2 - \frac{\lambda}{4!}\varphi^4$$

$$m^2\varphi^2 \qquad \qquad \qquad$$

$$\underbrace{\qquad \frac{}{2} - \frac{\lambda Z_\lambda \varphi^4}{4!}}_{\mathcal{L}} = \underbrace{-\frac{1}{2}(\partial\varphi_0)^2 - \frac{m_0^2\varphi_0^2}{2} - \frac{\lambda_0\varphi_0^4}{4!}}_{\text{bare lagrangian}}$$

$$\underbrace{-\frac{Z_\varphi(\partial\varphi)^2}{2} - \frac{Z_\varphi m^2\varphi^2}{2} - \frac{\lambda Z_\lambda}{4!}}$$

$$\varphi_0 = Z_\varphi^{1/2}\varphi \qquad \alpha_{phys} + \Delta\alpha$$

$$\varphi_0 = Z_\varphi^{1/2}\,\varphi$$

$$m_0^2 = m^2 - \delta m^2$$

$$\lambda = Z_\lambda^{-1}\, Z_\varphi^2\, \lambda_0$$

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Z_φ is a "wave function" renormalization.

Z_λ is a coupling constant renormalization

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Z_λ is a coupling con[stant] renormali[zation]

δm^2 is a mass ren[ormalization]

How do we know if QFT is renormalizable?

$$V(\varphi_0) =$$

$$\mathcal{L}_{phys} + \Delta\mathcal{L} = \mathcal{L}_0$$

$$\underbrace{-\frac{Z_\varphi (\partial\varphi)^2}{2} - \frac{Z_\varphi (m^2 - \delta m^2)\varphi^2}{2} - \frac{\lambda Z_\lambda}{4!}\varphi^4}_{\mathcal{L}_{phys} + \Delta\mathcal{L}} = -\frac{1}{2}$$

$$\lambda = Z_\lambda^{-1} Z_\varphi^{2} \lambda_0 \Big)$$

renormal

$\delta\mu^2$ is a m

How do we know if QFT is renormalizab

$$V(\varphi_0) = \frac{m_0^2 \varphi_0^2}{2} + \frac{\lambda_0}{4!} \varphi_0^4$$

$\lambda = Z_\lambda \, Z_\varphi \, \lambda_0 \;)$ $\delta\mu^2$ is a mass ren

How do we know if QFT is renormalizable?

$$V(\varphi_0) = \frac{\mu_0^2 \varphi_0^2}{2} + \frac{\lambda_0}{4!} \varphi_0^4$$

couplings: $\frac{\mu_0^2}{2}$ and $\frac{\lambda_0}{4!}$

$$V(\Phi_0) = \sum_{n=1}^{\infty} \underbrace{g_n}_{\text{couplings}} \Phi_0^n$$

That our QFT is renormalizable by power counting if

$$[g_n]$$

$$V(\varphi_0) = \sum_{n=1}^{\infty} \underbrace{g_n}_{\text{couplings}} \varphi_0^n$$

That our QFT is renormalizable by power counting if

$$[g_n] \geq 0$$

mass dimention of couplings

$[m] = +1 \qquad [\partial_\mu] = +1$

$[x] = -1$

Renormalization and

$[m] = +1$ $\quad$ $[\partial_\mu] = +1$

$[x] = -1$ $\quad$ $[S]$

Renormalization an

$[m] = +1$ $[\eta_m] = +1$

$[x] = -1$ $[S] = 0$

Renormalization and effe

$$\mathcal{L}_0 = -\frac{1}{2}(\partial\varphi)^2 - \frac{\omega_0^2}{2}\varphi^2 -$$

$[m] = +1 \qquad [\partial_\mu] = +1$

$[x] = -1 \qquad [S] = 0 \Rightarrow [\mathcal{L}] = +4$

$\Rightarrow$ kinetic term

$$[(\partial\varphi)^2] = 2 + 2[\varphi] = 4 \Rightarrow [\varphi] = 1$$

Renormalization and effective field theories

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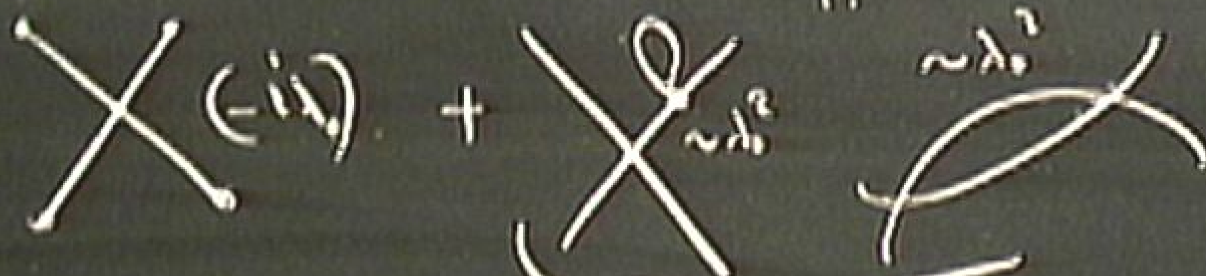

higher loop corrections

$[m] = +1$ $\quad [\hbar_m] = +1$

$[x] = -1$ $\quad [S] = 0 \Rightarrow [\mathcal{L}] = +4$

$\Rightarrow$ kinetic term

$[(\partial\varphi)^2] = 2 + 2[\varphi] = 4 \Rightarrow [\varphi] = 1$ $\qquad [m_0] = +1$

$[\lambda_0] = 0$

Renormalization and effective field theories

$$\mathcal{L}_0 = -\frac{1}{2}(\partial\varphi)^2 - \frac{m_0^2}{2}\varphi^2 - \frac{\lambda_0}{4!}\varphi^4$$

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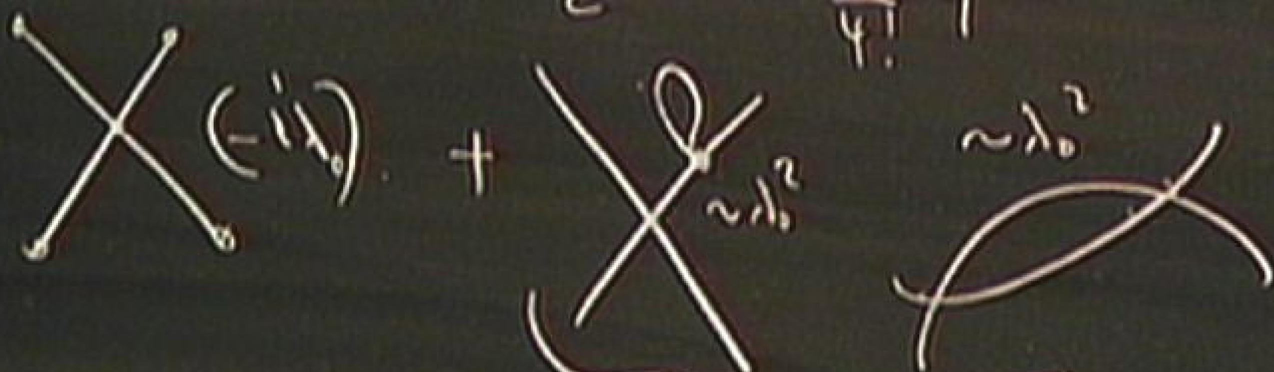

higher loop corrections

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$$[(\partial\varphi)^2] = 2 + 2[\varphi] = 4 \Rightarrow [\varphi] = 1$$

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$\Rightarrow$ We use (canonical kinetic term) to assign scaling dimension for φ

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$$- \quad \varphi^2 (\partial\varphi)^2 - \quad V(\varphi)$$

higher loop corrections

$\Rightarrow$ We use (canonical kinetic term) to assign scaling dimention for φ

$$-\underbrace{\varphi^2(\partial\varphi)^2}_{} - V(\varphi)$$

Not canonical kinetic term.

higher loop corrections

$[(\partial\varphi)^2] = 2 + 2[\varphi] = 4 \Rightarrow [\varphi] = 1$

$[\lambda_0]$

$\Rightarrow$ We use (canonical kinetic term) to assign scaling dimension for φ

$$- \underbrace{\varphi^2 (\partial\varphi)^2}_{} - V(\varphi) \qquad \partial\hat{\varphi} = \varphi\, \partial\varphi$$

Not canonical kinetic term.

$\Rightarrow$ First to ~~re~~define $\varphi \to \hat{\varphi}$

higher loop corrections

$\mathcal{L}_{phys} + \Delta\mathcal{L}$

bo

$[(\partial\varphi)^2] = 2 + 2[\varphi] = 4 \Rightarrow [\varphi] = 1$

$[\lambda_0] =$

$\Rightarrow$ We use (canonical kinetic term) to assign scaling dimension for φ

$$- \underbrace{\varphi^2(\partial\varphi)^2}_{\text{Not canonical kinetic term.}} - V(\varphi)$$

$\partial\hat{\varphi} = \varphi\,\partial\varphi$

$\partial\hat{\varphi} = \partial\,\frac{1}{2}\varphi^2$

$\Rightarrow$ First to redefine $\varphi \to \hat{\varphi}$

Higher loop corrections

$\mathcal{L}_{phys} + \Delta\mathcal{L}$

bare

$\Rightarrow$ We use (canonical kinetic term) to assign scaling dimention for φ

$$-\underbrace{\varphi^2(\partial\varphi)^2}_{} - V(\varphi)$$

Not canonical kinetic term.

$\Rightarrow$ First to [illegible] $\varphi \to \hat{\varphi}$

$\partial\hat{\varphi} = \varphi\,\partial\varphi$

$\partial\hat{\varphi} = \partial\frac{1}{2}\varphi^2$

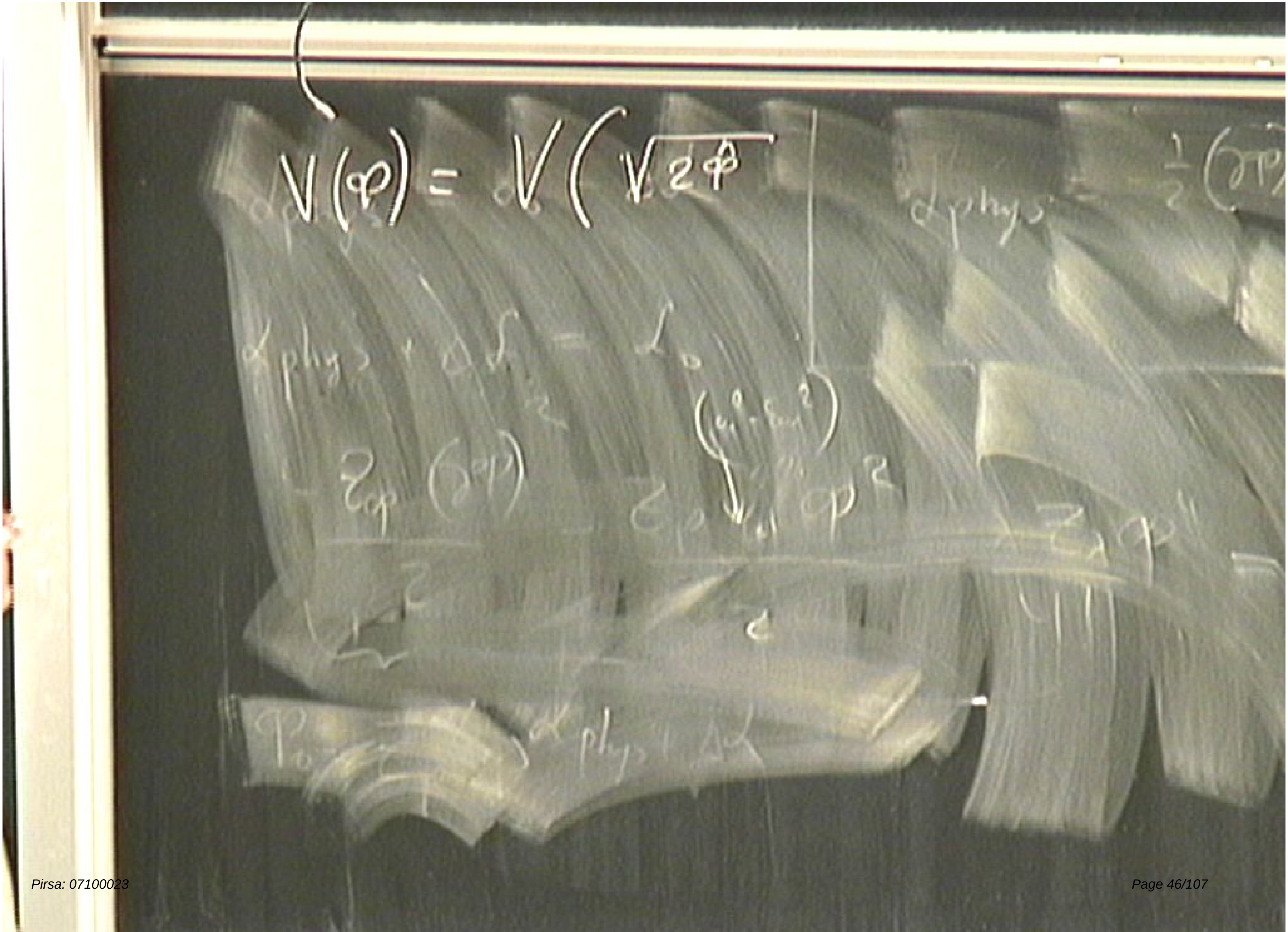
$\hat{\varphi} = \frac{1}{2}\varphi^2$

higher loop

$2 \quad 2 \quad 4!$

$\mathcal{L}_{phys} + \Delta\mathcal{L}$

bare


V(φ) = V(√2φ

$$V(\varphi) = V\left(\sqrt{2\varphi}\right)$$

$$\hookrightarrow \frac{\lambda_8}{8!}\varphi^8$$

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$$-\varphi^2(\partial\varphi)^2$$

$$\Longrightarrow \frac{\lambda_8}{8!} R^2 \hat{\varphi}^4$$

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$$\hookrightarrow \underbrace{\frac{\lambda_8}{8!}\varphi^8}_{\text{would be non-renormalizable with } -(\partial\varphi)^2}$$

$$\Longrightarrow \underbrace{\frac{\lambda_8}{8!} R^2 \hat{\varphi}^4}_{\text{Is renormalizable.}}$$

$$-\varphi^2(\partial\varphi)^2$$

$$\Longrightarrow \quad \underbrace{\frac{\lambda_8}{8!}}\ \mathcal{R}\ \varphi^{\dots}$$

$$-\varphi^2(\partial\varphi)^2$$

Is renormalizable.

Cheng & Li

"Gauge theories of Particle Physics"

$$V(\varphi) = V(\sqrt{2\hat{\varphi}})$$

$$\hookrightarrow \underbrace{\frac{\lambda_8}{8!}\varphi^8}_{\text{would}} \Longrightarrow \underbrace{\frac{\lambda_8}{8!}\hat{\varphi}^2\,\hat{\varphi}^4}$$

would be non-renormalizable with $-(\partial\varphi)^2$

$-\varphi^2(\partial\varphi)^2$

Is renormalizable.

<u>Cheng & Li</u>

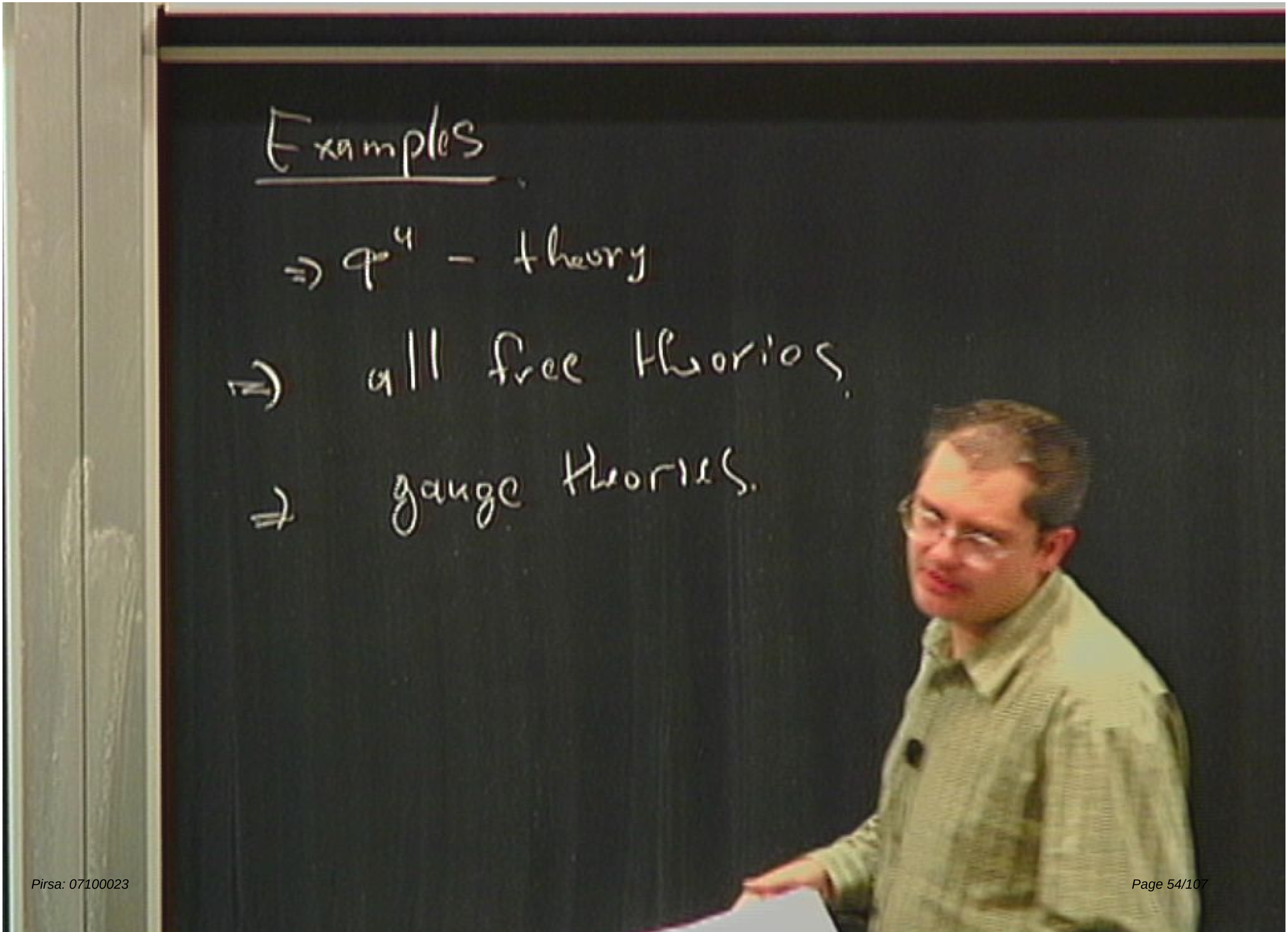
"Gauge theories of Particle physics"

Examples

⇒ φ^4 - theory

⇒ all free theories

⇒


Examples
⇒ φ^4 - theory
⇒ all free theories
⇒ gauge theories.

Examples

$\Rightarrow \varphi^4$ - theory

$\Rightarrow$ all free theories

$\Rightarrow$ gauge theories

$\rightarrow$ Infrared free

$\rightarrow$ asymptotically free

Examples

 $(-i\lambda)$

$\lambda = \lambda(E)$

$\Rightarrow \varphi^4$ - theory

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Examples

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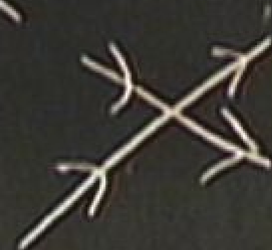

$\Rightarrow$ φ^4 - theory

$\Rightarrow$ all free theories

$\Rightarrow$ gauge theories

$\lambda(E) \longrightarrow$ because small at $E \rightarrow$

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Examples

$(-i\lambda)$

$\lambda = \lambda(E)$

$\Rightarrow$ φ^4 - theory

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Examples

$(-i\lambda)$

$\lambda = \lambda(E)$

$\Rightarrow \varphi^4$ - theory

$\Rightarrow$ all free theories

$\Rightarrow$ gauge theories

$g(E) \rightarrow$ because small at $E \rightarrow$

$\rightarrow$ Infrared free

asymptotically free

$g(E)$

Examples

 $(-i\lambda)$ $\quad \lambda = \lambda(E)$

$\Rightarrow$ φ^4 - theory

$\Rightarrow$ all free theories

$\Rightarrow$ gauge theories. $\quad g(E) \rightarrow$ because small at $E \rightarrow$

$\rightarrow$ Infrared free

$\rightarrow$ asymptotically free

$g(E) \rightarrow 0 \qquad E \rightarrow +\infty$

...eories.

...ies.

$g_{YM}(E) \rightarrow$ because small at $E \rightarrow$

$g_{YM}(E) \rightarrow +\infty$

$\rightarrow$ Infrared free

$E \rightarrow E_{crit}$

asymptotically free

$\rightarrow$

$g_{YM}(E) \rightarrow 0 \qquad E \rightarrow +\infty$

Nonrenormalizable QFT (In $d=4$)

$\Rightarrow \varphi^n \quad n>4$

$\Rightarrow$ Nonrenormalizable QFT (In $d=4$)

$\Rightarrow$ φ^n $\quad n>4$

$\Rightarrow$ gravity: $S = \frac{1}{16\pi G_N}\int d^4x \sqrt{g}\, R$

$8\pi G_N = m_{Pl}^{-2}$

⇒ Nonrenormalizable QFT (In $d=4$)

⇒ $\varphi^n \quad n > 4$

⇒ gravity: $S = \dfrac{1}{16\pi G_N}\displaystyle\int d^4x \, \underbrace{\sqrt{g}\, R}_{\text{Einstein gravity.}}$

$8\pi G_N = m_{Pl}^{-2}$

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$$g_{\mu\nu} = \underbrace{\eta_{\mu\nu}}_{\text{Minkowsky}}$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

Einstein gravi

with $-(\partial\phi)^2$

"Gauge theorie

$\Rightarrow$ gravity: $S = \frac{1}{16\pi G_N}\int d^4x \underbrace{\sqrt{-g}\, R}_{\text{Einstein gravity.}}$

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$g_{\mu\nu} = \underbrace{\eta_{\mu\nu}}_{\text{Minkowsky}} + \underbrace{h_{\mu\nu}}_{\text{small fluctuation}}$ is analog of φ

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$\Rightarrow \varphi^n \qquad n > 4$

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$8\pi G_N = m_{Pl}^{-2}$

$g_{\mu\nu} = \eta_{\mu\nu} + \underbrace{h_{\mu\nu}}_{\text{small fluctuation}}$ is analog of φ

$\eta_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$ Minkowsky

$\Rightarrow$ h will have a kinetic term!

$\Rightarrow$ will not be canonicall!

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$\Rightarrow$ There will be an ∞ # of interaction verticies.

⇒ h will have a kinetic term!

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⇒ there will be an ∞ # of interaction vertecies.

$\sqrt{g}\, R$ is nonlinear

$\Rightarrow$ h will have a kinetic term!

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$\sqrt{g}\,R$ is nonlinear $\quad hhhh$

$\Rightarrow$

$\Rightarrow$ h will have a kinetic term.

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$\Rightarrow$ There will be an ∞ # of interaction vertecies.

$[R] = +2$

$\sqrt{g}\, R$ is nonlinear $hhhh, \ldots$

$\Rightarrow \sqrt{g}\left[R + \frac{R^2}{m_{pl}^2}\right.$

$\Rightarrow$ h will have a kinetic term.

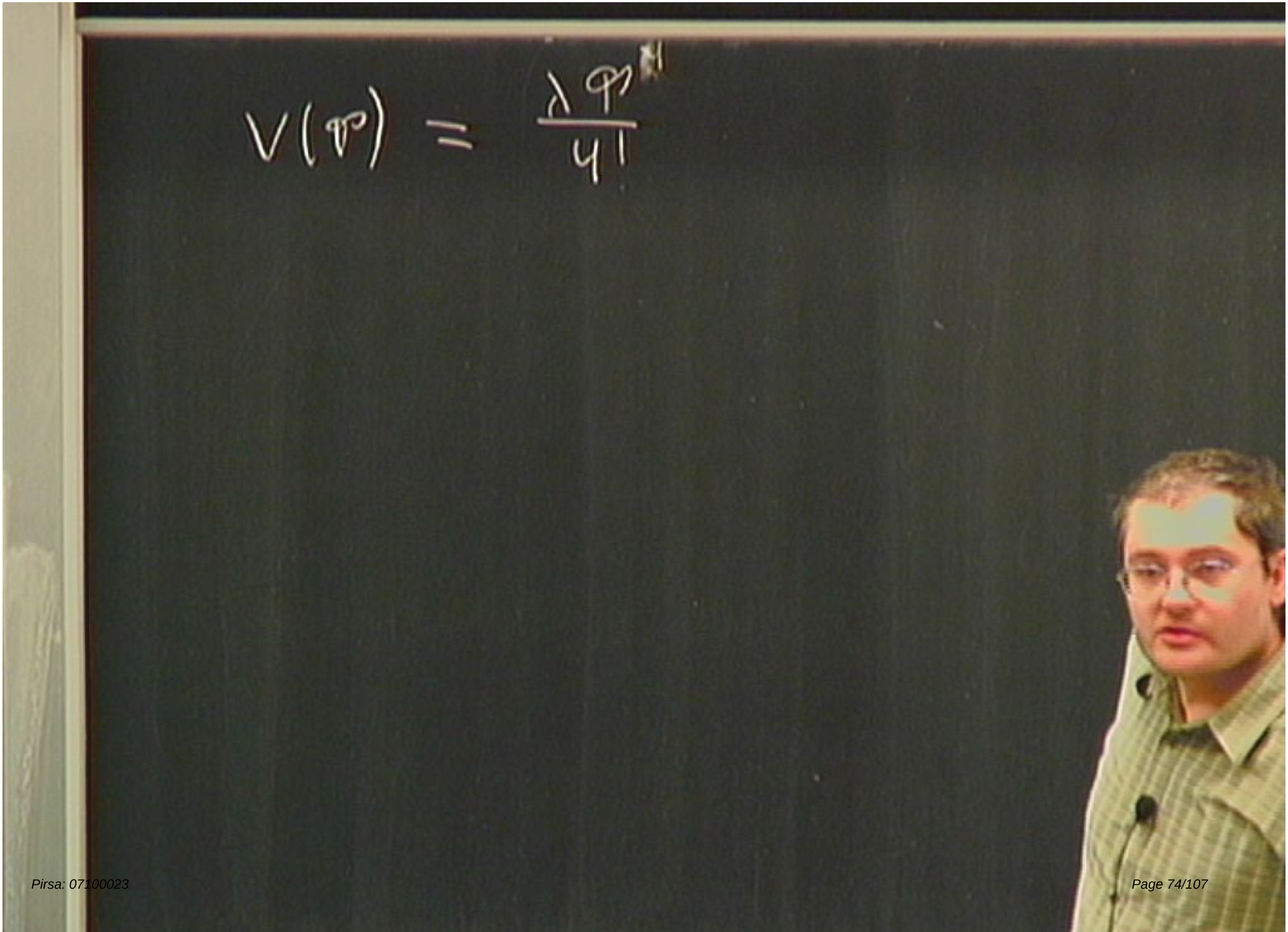
$\Rightarrow$ will not be canonicall

$\Rightarrow$ there will be an ∞ # of interaction vertecies.

$[R] \neq +2$

$\sqrt{g}\, R$ is nonlinear $hhhh$

$$\Rightarrow \sqrt{g}\left[R + \# \frac{R^2}{m_{pl}^2} + \# \frac{R^3}{m_p^4} + \# \frac{R^4}{m_p^6} + \cdots\right]$$


$V(\varphi) = \frac{\lambda \varphi^4}{4!}$

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$\Rightarrow$ mass term will be generated

$$V(\varphi) = \frac{\lambda \varphi^{M}}{4!}$$

$\Rightarrow$ mass term will be generated

Q: How do we live with non-ren QFT?

$\Rightarrow$ Effective field theory

$\Rightarrow$ Back to renormalizable theories.

$Z[J]$

$\Rightarrow$ Back to renormalizable theories.

$$\underbrace{Z[J]}_{\text{generating functional for corr functions}}$$

$\Rightarrow$ Back to renormalizable theories.

$$\underbrace{Z[J]}_{\text{generating functional for corr functions}} = \int [d\varphi]$$

generating functional for corr functions

$\Rightarrow$ Back to renormalizable theories.

$$\underbrace{Z[J]}_{\text{generating functional for corr functions}} = \int [d\varphi]\, e^{iS + i\int J(x)\varphi(x)\, d^4x} = e^{iW[J]}$$

$\Rightarrow$ Back to renormalizable theories.

$$Z[J] = \int [d\varphi]\, e^{iS + i\int J(x)\,\varphi(x)\, d^4x}$$

$$\underbrace{Z[J]}_{\substack{\text{generating} \\ \text{functional} \\ \text{for corr} \\ \text{functions}}} = e^{i\underbrace{W[J]}}$$

W is some functional

$\Rightarrow$ Back to renormalizable theories.

$$\underbrace{Z[J]}_{\substack{\text{generating}\\ \text{functional}\\ \text{for corr}\\ \text{functions}}} = \int [d\varphi]\, e^{iS + i\int J(x)\varphi(x)\, d^4x}$$

$$= e^{i\underbrace{W[J]}}$$

W is some functional

$$iW[J] \stackrel{\text{def}}{\equiv} \ln Z[J]$$

⇒ path integral is dominated in a "stationary phase approximation" by a classical trajectory

$$\frac{\delta}{\delta\varphi_{cl}(x)}\left[S+\int J(x)\varphi(x)\right]\Bigg|_{\varphi(x)=\varphi_{cl}(x)}=0$$

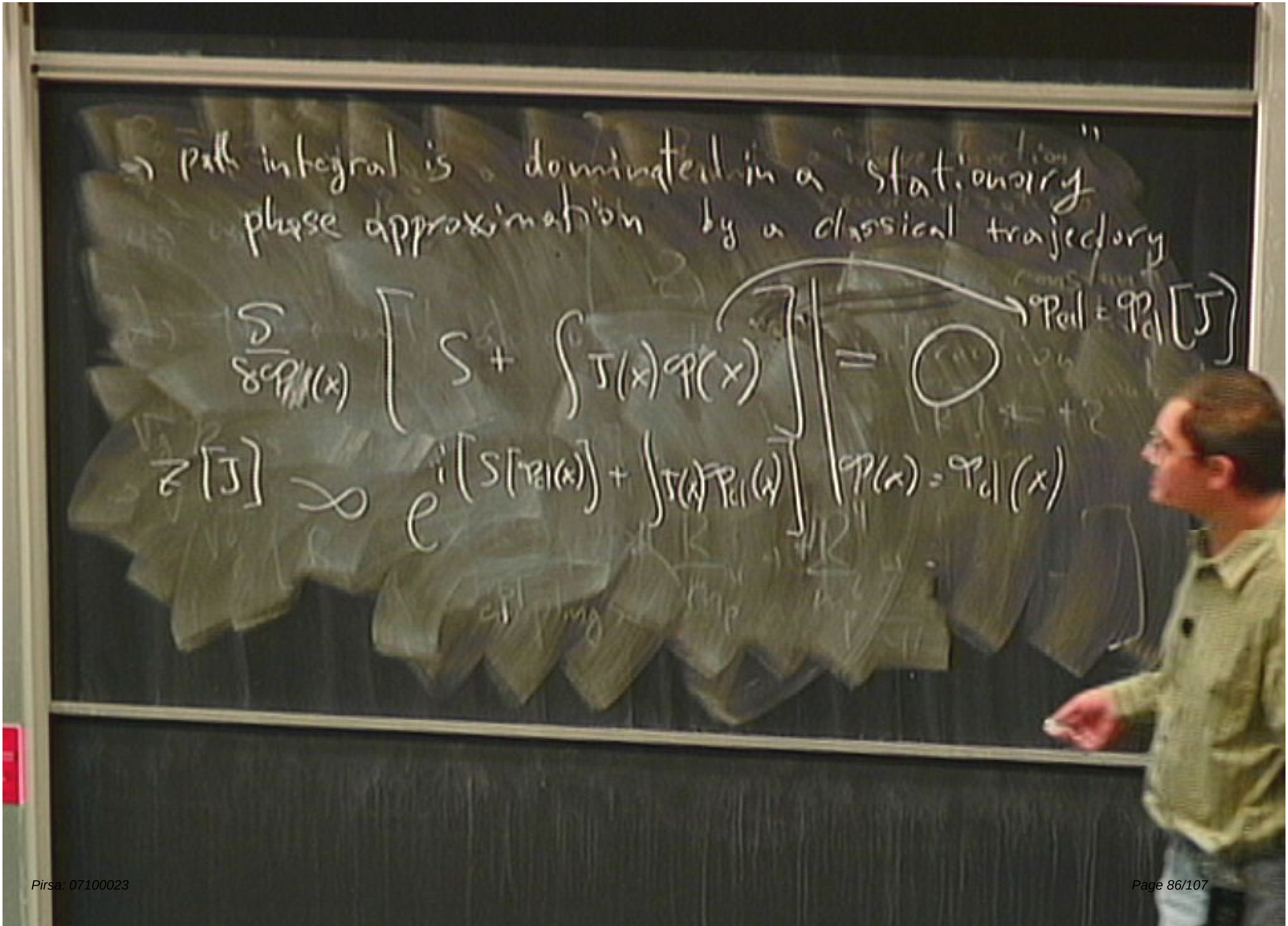
$\Rightarrow$ path integral is dominated in a stationary phase approximation by a classical trajectory

$$\frac{\delta}{\delta\varphi(x)}\left[S + \int J(x)\varphi(x) \right]\Bigg|_{\varphi(x)=\varphi_{cl}(x)} = 0$$

$\Rightarrow$ path integral is dominated in a stationary phase approximation by a classical trajectory

$$\frac{\delta}{\delta\varphi(x)}\left[S + \int J(x)\varphi(x)\right]\Bigg|_{\varphi(x)=\varphi_{cl}(x)} = 0$$

$$Z[J] \propto e^{i\left(S[\varphi_{cl}(x)] + \int J(x)\varphi_{cl}(x)\right)}$$


→ path integral is dominated in a "stationary phase approximation" by a classical trajectory
$\frac{\delta}{\delta \varphi(x)} \left[S + \int J(x)\varphi(x) \right] = 0$
$\varphi_{cl} = \varphi_{cl}[J]$
$Z[J] \propto e^{i\left(S[\varphi_{cl}(x)] + \int J(x)\varphi_{cl}(x)\right)} \Big|_{\varphi(x) = \varphi_{cl}(x)}$

⇒ path integral is dominated in a stationary phase approximation by a classical trajectory

$$\frac{\delta}{\delta\varphi(x)}\left[S + \int J(x)\varphi(x)\right]\Bigg|_{\varphi_{cl}} = 0 \qquad \varphi_{cl} = \varphi_{cl}[J]$$

$$Z[J] \propto e^{i\left[S[\varphi_{cl}(x)] + \int J(x)\varphi_{cl}(x)\right]}\Bigg|_{\varphi(x)=\varphi_{cl}(x)}$$

In a stationary phase app

$$iW[J] \approx S[\varphi_{cl}[J]] + \int J(x)\varphi_{cl}[J]\, d^4x$$

⇒ path integral is dominated in a "stationary phase approximation" by a classical trajectory

$$\frac{\delta}{\delta\varphi(x)}\left[S + \int J(x)\varphi(x) \right]\Bigg|_{\varphi_{cl} = \varphi_{cl}[J]} = 0$$

$$Z[J] \propto e^{i\left[S[\varphi_{cl}(x)] + \int J(x)\varphi_{cl}(x) \right]} \Bigg|_{\varphi(x) = \varphi_{cl}(x)}$$

In a stationary phase app

$$W[J] \simeq S[\varphi_{cl}[J]] + \int J(x)\varphi_{cl}[J]\, d^4x$$

$$\frac{\delta}{\delta\varphi(x)}\left[S + \int J(x)\varphi(x)\right]\Bigg|_{\varphi(x)=\varphi_{cl}(x)} = 0$$

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In a stationary phase app

$$W[J] \approx S[\varphi_{cl}[J]] + \int J(x)\varphi_{cl}[J]\, d^4x$$

$$W[J] \underset{\text{exact}}{=} S_{eff}[\varphi_{cl}] + \int d^4x \, J(x)\, \varphi_{cl}(x).$$

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φ_{cl} has nothing to do with φ in path integral

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We assumed

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$\rightarrow$ Invert it

$J(x) = J[\hat{\varphi}(x)]$

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(uniquely defines)

$$S_{eff} = S_{eff}[J] = S_{eff}[J[\hat{\varphi}]]$$

$$\boxed{\frac{\delta W[J]}{\delta J(x)} = \varphi(x)} \qquad J(x) = J[\hat{\varphi}(x)]$$

$$S_{eff} = W[J] - \int d^4x \, J[x] \frac{\delta W}{\delta J}$$

uniquely defined

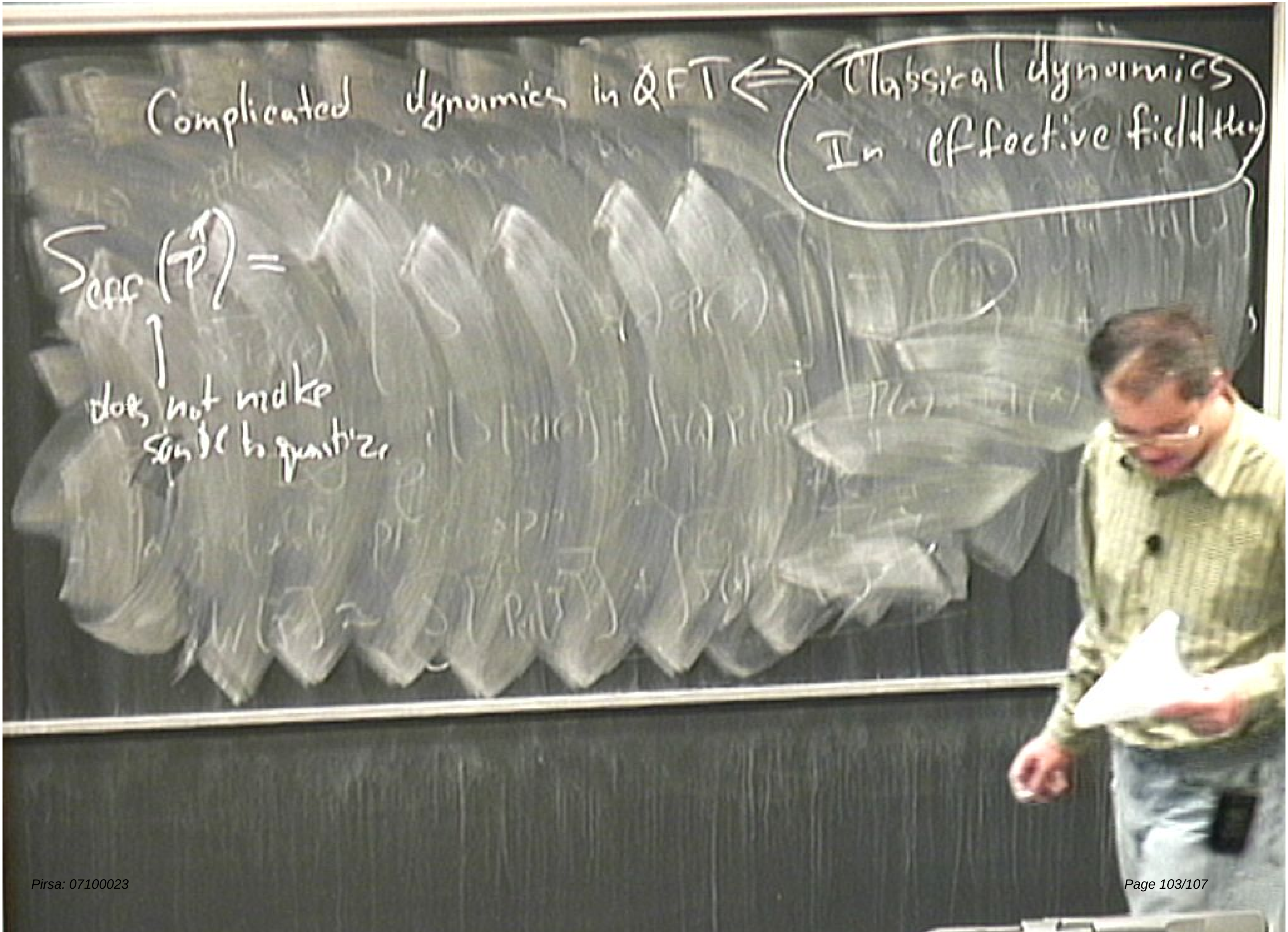
$$S_{eff} = S_{eff}[J] = \underbrace{S_{eff}[J[\hat{\varphi}]]}_{\text{effective action}}$$

Functional for corr functions $= e$ W is some functional

$$iW[J] \stackrel{def}{\equiv} \ln Z[J]$$

Complicated dynamics in QFT $\Longleftrightarrow$ Classical dynamics

In effective field thy


Complicated dynamics in QFT ⟺ Classical dynamics In effective field th
$S_{eff}(\vec{\varphi}) =$
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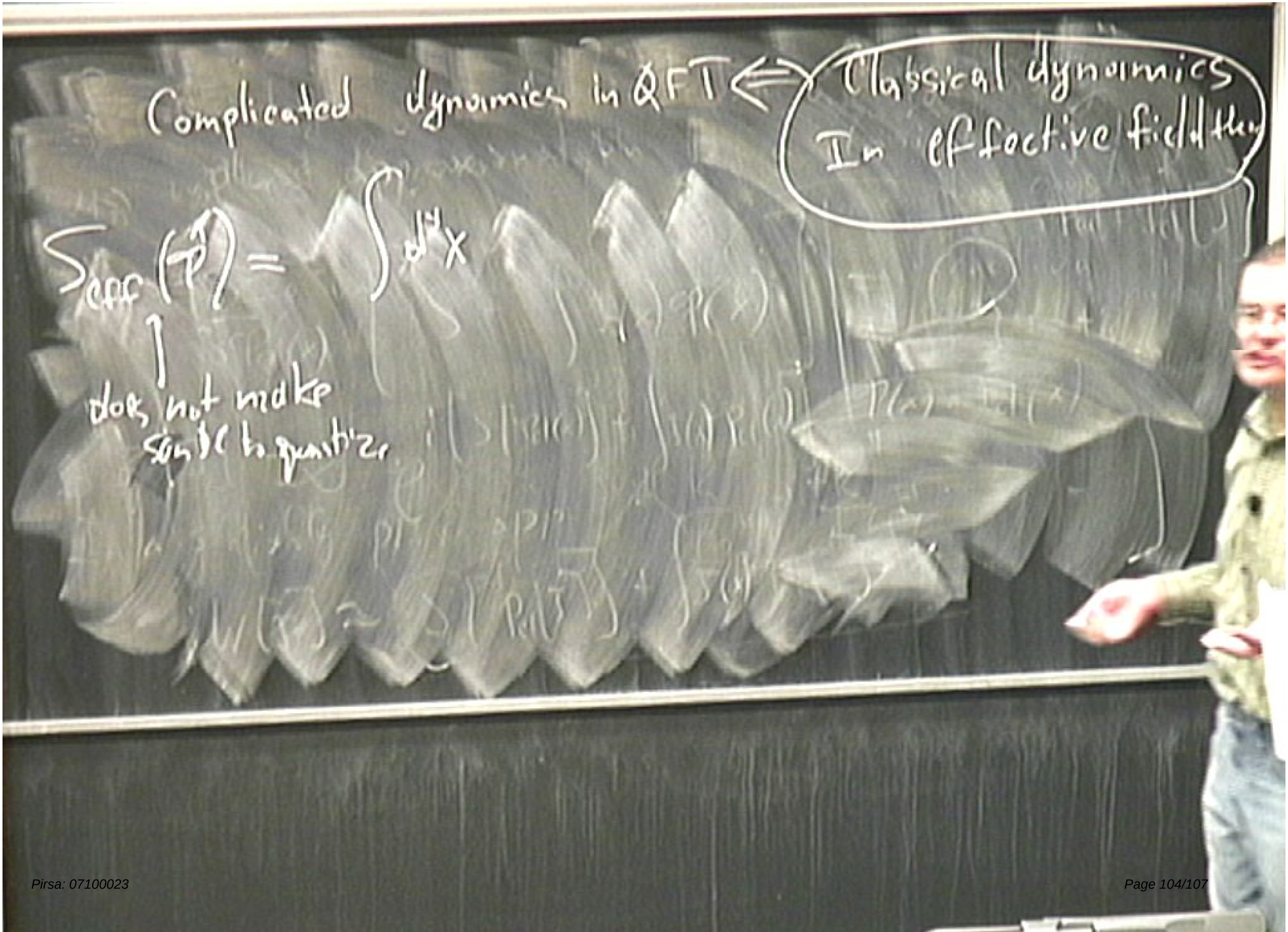
Complicated dynamics in QFT $\Longleftrightarrow$ Classical dynamics
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Complicated dynamics in QFT $\Longleftrightarrow$ Classical dynamics In effective field th[eory]

$$S_{eff}(\vec{\varphi}) = \int d^4x \, \Big[\qquad \Big]$$

does not make sense to quantize

derivative expansion

Complicated dynamics in QFT $\Longleftrightarrow$ Classical dynamics In effective field theory

$$S_{eff}(\vec{\varphi}) = \int d^4x \left[-V_{eff}(\vec{\varphi}) - \frac{1}{2}(\partial\vec{\varphi})^2 Z(\vec{\varphi}) + \text{higher derivs} + \text{so on} \right],$$

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Complicated dynamics in QFT $\Longleftrightarrow$ Classical dynamics In effective field th[eory]

$$S_{\text{eff}}(\hat{\varphi}) = \int d^4x \left[-V_{\text{eff}}(\hat{\varphi}) - \frac{1}{2}(\partial\hat{\varphi})^2 Z(\hat{\varphi}) + \text{[illegible] + so on} \right],$$

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