

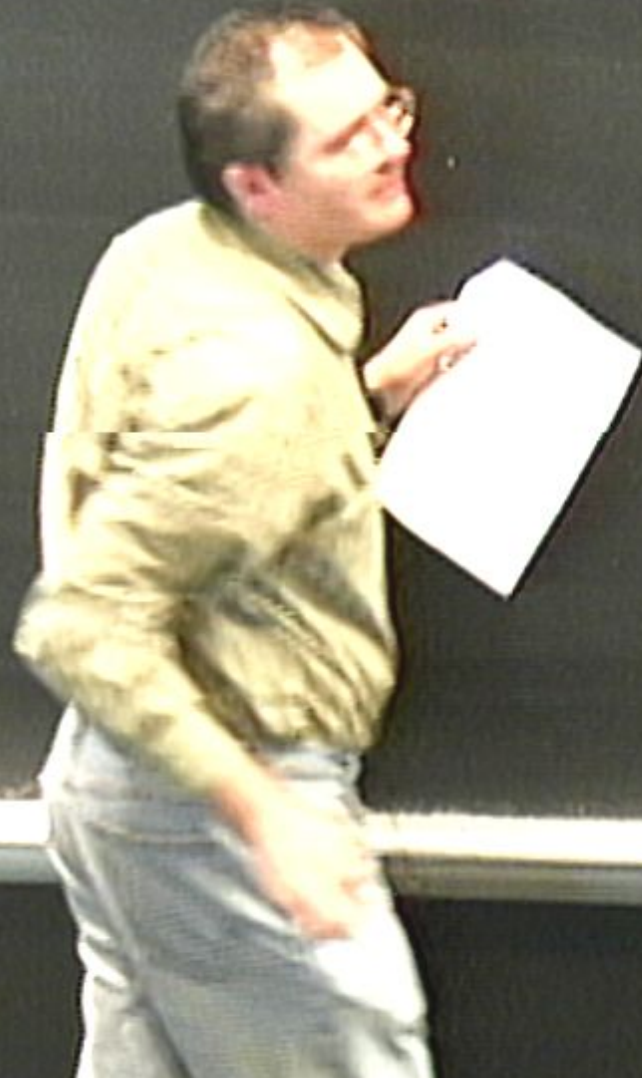
Title: Intro to Supersymmetry 13

Date: Oct 30, 2007 12:30 PM

URL: <http://pirsa.org/07100023>

Abstract:

Do exercises (1)-(5) [section V, page 35] Mess & Baggen



Do exercises (1)-(5) [section V, page 35] Weiss & Bagger

↑
Homework #2 (due in 2 weeks)

Homework #2 (due in 2 week).

Renormalization and effective field theories

homework #1 (due in 2 weeks)

Renormalization and effective field theories

$$\mathcal{L}_0 = -\frac{1}{2} (\partial\phi)^2 - \frac{\omega_0^2}{2} \phi^2 - \frac{\lambda_0}{4!} \phi^4$$

homework #1 (due in 2 weeks)

Renormalization and effective field theories

$$\mathcal{L}_0 = -\frac{1}{2} (\partial\phi)^2 - \frac{\omega_0^2}{2} \phi^2 - \frac{\lambda_0}{4!} \phi^4$$

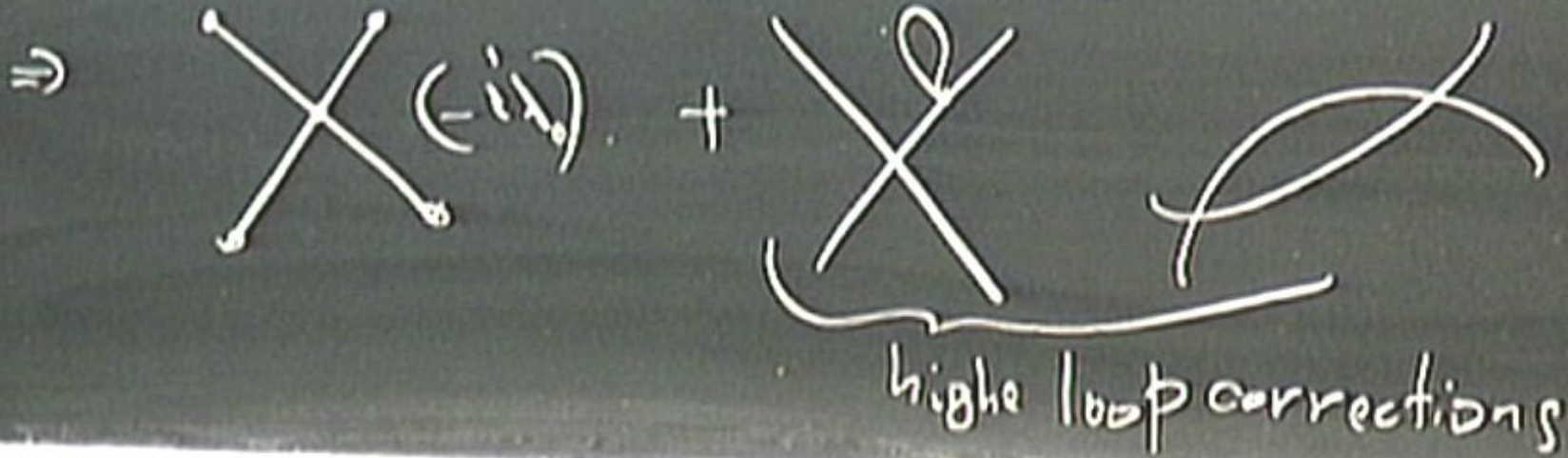
\Rightarrow



Homework #2 (due in 2 week)

Renormalization and effective field theories

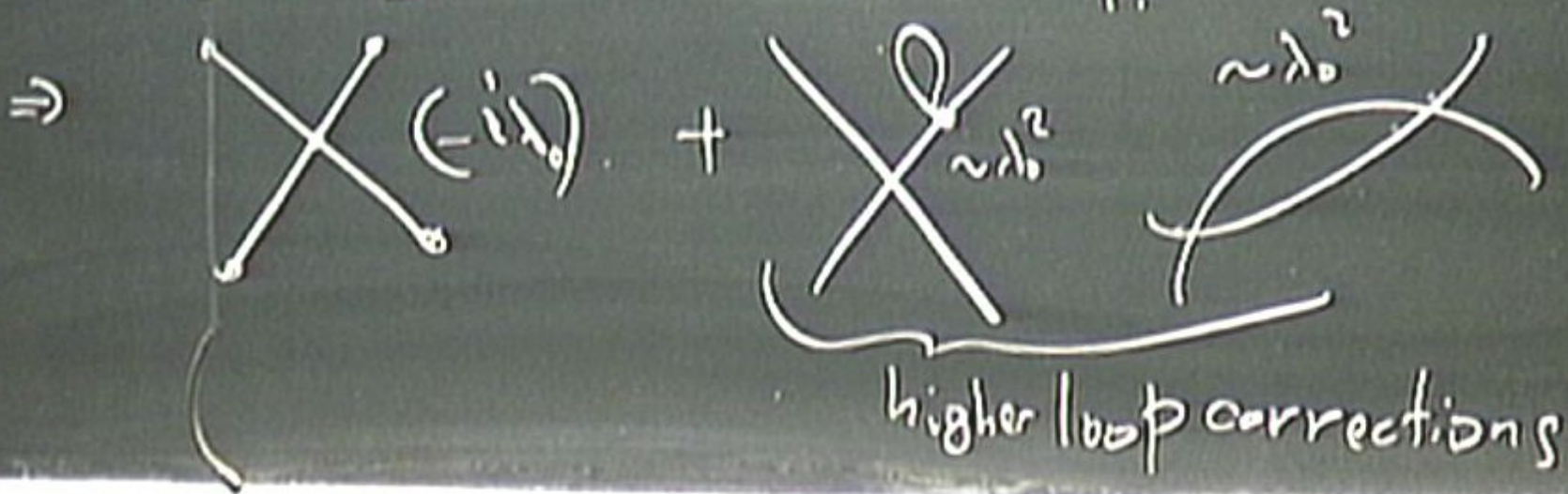
$$\mathcal{L}_0 = -\frac{1}{2} (\partial\phi)^2 - \frac{m^2}{2} \phi^2 - \frac{\lambda_0}{4!} \phi^4$$



Homework #2 (due in 2 week)

Renormalization and effective field theories

$$\mathcal{L}_0 = -\frac{1}{2} (\partial\phi)^2 - \frac{\omega_0^2}{2} \phi^2 - \frac{\lambda_0}{4!} \phi^4$$



\Rightarrow Direct computation

\Rightarrow Direct computation leads to infinities

$$\mathcal{L}_1 = \mathcal{L}_0$$

renormalized
lagrangian
at 1-loop

\Rightarrow Direct computation leads to infinities

$$\mathcal{L}_1 = \mathcal{L}_0 - \underbrace{\Delta \mathcal{L}_1}_{\text{counterterms}}$$

renormalized
lagrangian
at 1-loop

\Rightarrow Direct computation leads to infinities

$$\mathcal{L}_2 = \mathcal{L}_0 - \underbrace{\Delta \mathcal{L}_1}_{\text{counterterms}}$$

renormalized
lagrangian
at Δ -loop

Introduce additional
vertices

ation leads to infinities

L_0

$$- \underbrace{\Delta L_1}_{\text{counterterms}}$$

counterterms.

↑
Introduce additional
vertices

+ tune couplings to get finite results.

$$L_{\text{physical}} = L_0 - \underbrace{\sum_{n=1}^{\infty} \Delta L(n)}$$

In principle can lead to ∞ many additional terms

$$\mathcal{L}_{\text{physical}} = \mathcal{L}_0 - \underbrace{\sum_{n=1}^{\infty} \Delta \mathcal{L}(n)}$$

In principle can lead to ∞ many
additional terms

\Rightarrow "renormalizable" QFT

$$\mathcal{L}_{\text{physical}} = \mathcal{L}_0 - \underbrace{\sum_{n \geq 1} \Delta \mathcal{L}(n)}$$

In principle can lead to ∞ many
additional terms

\Rightarrow "renormalizable" QFT

$$\mathcal{L}_{\text{physical}} = \mathcal{L}_0 - \underbrace{\sum_{n \geq 1} \delta \mathcal{L}_n}_{\text{additional terms}}$$

In principle can lead to ∞ many
additional terms
 \Rightarrow "renormalizable" QFT

\hookrightarrow counterterms contains operators already
present in \mathcal{L}_0 (or a finite
of new operators).

\mathcal{L}_0 is a bare Lagrangian for a renormalizable QFT.

\mathcal{L}_0 is a bare Lagrangian for a renormalizable QFT

$$-\sum_{n=1}^{\infty} \mathcal{L}(\varphi) \equiv \frac{(Z_\varphi - 1)}{2} \left[-(\partial\varphi)^2 - m^2 \varphi^2 \right] \\ - \frac{\lambda}{4!} (Z_\lambda - 1) \varphi^4 + \frac{\delta m^2}{2} Z_\varphi \varphi^2$$

\mathcal{L}_0 is a bare Lagrangian for a renormalizable QFT

$$-\sum_{n=1}^{\infty} \mathcal{L}_n \equiv \frac{(Z_\varphi - 1)}{2} \left[-(\partial\varphi)^2 - m^2 \varphi^2 \right] \\ - \frac{\lambda}{4!} (Z_\lambda - 1) \varphi^4 + \frac{\delta m^2}{2} Z_\varphi \varphi^2$$

$Z_\varphi, Z_\lambda, \delta m^2 \rightarrow$ they depend on μ, \mathcal{L}_0 .

\mathcal{L}_0 is a bare Lagrangian for a renormalizable QFT.

$$-\sum_{n=1}^{\infty} \alpha_n \mathcal{L}_n \equiv \frac{(Z_\phi - 1)}{2} \left[-(\partial\phi)^2 - m^2 \phi^2 \right] \\ - \frac{\lambda}{4!} (Z_\lambda - 1) \phi^4 + \frac{\delta m^2}{2} Z_\phi \phi^2 = -\Delta \mathcal{L}$$

$Z_\phi, Z_\lambda, \delta m^2 \rightarrow$ they depend on m, λ, α_0 .

$$d_{\text{phys}} = d_0 - \Delta L$$

$$d_{\text{phys}} + \Delta L = d_0$$

$$\frac{-Z_{\text{op}} (\partial \varphi)^2}{2} - \frac{Z_{\text{op}} m^2 \varphi^2}{2} - \frac{\lambda Z_{\text{op}} \varphi^4}{4!} = -\frac{1}{2} (\partial \varphi_0)^2 - \frac{m^2 \varphi_0^2}{2} - \frac{\lambda \varphi_0^4}{4!}$$

$$d_{\text{phys}} = d_0 - \Delta L$$

$$L_{\text{phys}} + \Delta L = L_0$$

$$\underbrace{\frac{-Z_0 (\partial \varphi)^2}{2} - \frac{Z_0 m^2 \varphi^2}{2} - \frac{\lambda Z_\lambda \varphi^4}{4!}}_{L_{\text{phys}} + \Delta L} = \underbrace{-\frac{1}{2} (\partial \varphi_0)^2 - \frac{m^2 \varphi_0^2}{2} - \frac{\lambda \varphi_0^4}{4!}}_{\text{bare lagrangian}}$$

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi_0)^2 - \frac{\mu_0^2 \varphi_0^2}{2} - \frac{1}{4!}\varphi_0^4$$

bare lagrangian

$$d_{\text{phys}} = -\frac{1}{2}(\partial\varphi)^2 - \frac{\mu^2}{2!}\varphi^2 - \frac{\lambda}{4!}\varphi^4$$

$$\frac{\mu^2\varphi^2}{2} - \frac{\lambda\varphi^4}{4!} = \underbrace{-\frac{1}{2}(\partial\varphi)^2 - \frac{\mu^2\varphi^2}{2} - \frac{\lambda\varphi^4}{4!}}_{\text{bare lagrangian}}$$

$$\underbrace{\frac{-Z_{\text{eff}}(\theta)}{2} - \frac{Z_{\text{eff}} m \phi}{2} - \frac{\lambda Z_{\text{eff}}}{4!}}$$

$$\phi_0 = Z_{\text{eff}}^{1/2} \phi_{\text{phys}} + \Delta\phi$$

$$P_0 = Z_{\varphi}^{1/2} \varphi$$

$$m_0^e = m^2 - \delta m^2$$

$$\lambda = Z_{\lambda}^{-1} Z_{\varphi}^2 \lambda_0$$

$$\varphi_0 = Z_\varphi^{1/2} \varphi$$

$$m_0^2 = m^2 - \delta m^2$$

$$\lambda = Z_\lambda^{-1} Z_\varphi^2 \lambda_0$$

} Z_φ is a "wave function" renormalization.

Z_λ is a coupling constant renormalization

~~Z_m~~ is a mass renormalization.

Z_A is a "wave function" renormalization.

Z_λ is a coupling constant renormalization

~~Z_M~~ is a mass renormalization

$$m_0^2 = m^2 - \delta m^2$$

$$\lambda = Z_\lambda^{-1} Z_\phi^2 \lambda_0$$

Z_λ is a coupling constant renormalization

~~δm^2~~ is a mass renormalization

How do we know if QFT is renormalizable?

$$V(\phi_0) =$$

$$\alpha_{\text{phys}} + \Delta\alpha = \alpha_0$$

$$\frac{-Z_{\text{eff}}(\alpha) \alpha^2}{2} - \frac{Z_{\text{eff}} \alpha^2}{2} - \frac{\lambda Z_{\lambda} \alpha^4}{4!} = -\frac{1}{2}$$

$$\alpha_{\text{phys}} + \Delta\alpha$$

$$\lambda = Z_\lambda^{-1} Z_\varphi^2 \lambda_0$$

renormal

~~is a~~ is a m

How do we know if QFT is renormalizable?

$$V(\varphi_0) = \frac{m_0^2 \varphi_0^2}{2} + \frac{\lambda_0}{4!} \varphi_0^4$$

$\lambda = \sum_{\varphi} \lambda_{\varphi} \lambda_0$) ~~SM~~? is a mass ren

How do we know if QFT is renormalizable?

$$V(\varphi_0) = \frac{m_0^2 \varphi_0^2}{2} + \frac{\lambda_0}{4!} \varphi_0^4$$

couplings: $\frac{m_0^2}{2}$ and $\frac{\lambda_0}{4!}$

$$V(\phi_0) = \sum_{n=1}^{\infty} \underbrace{g_n}_{\text{couplings}} \phi_0^n$$

That our QFT is renormalizable
by power counting if

$$[g_n]$$

$$V(\phi_0) = \sum_{n=1}^{\infty} \underbrace{g_n}_{\text{couplings}} \phi_0^n$$

That our QFT is renormalizable
by power counting if

$$[g_n] \geq 0$$

mass dimension
of couplings

$$[m] = +1$$
$$[x] = -1$$

$$[m] = +1$$

Renormalization and

$$[m] = +1$$
$$[x] = -1$$

$$[m] = +1$$
$$[S]$$

Renormalization and

$$[m] = +1$$
$$[x] = -1$$

$$[2m] = +1$$
$$[S] = 0$$

Renormalization and effe

$$\mathcal{L}_0 = -\frac{1}{2} (\partial\varphi)^2 - \frac{\omega_0^2}{2} \varphi^2 -$$

$$[\eta] = +1$$

$$[x] = -1$$

$$[\lambda] = +1$$

$$[S] = 0$$

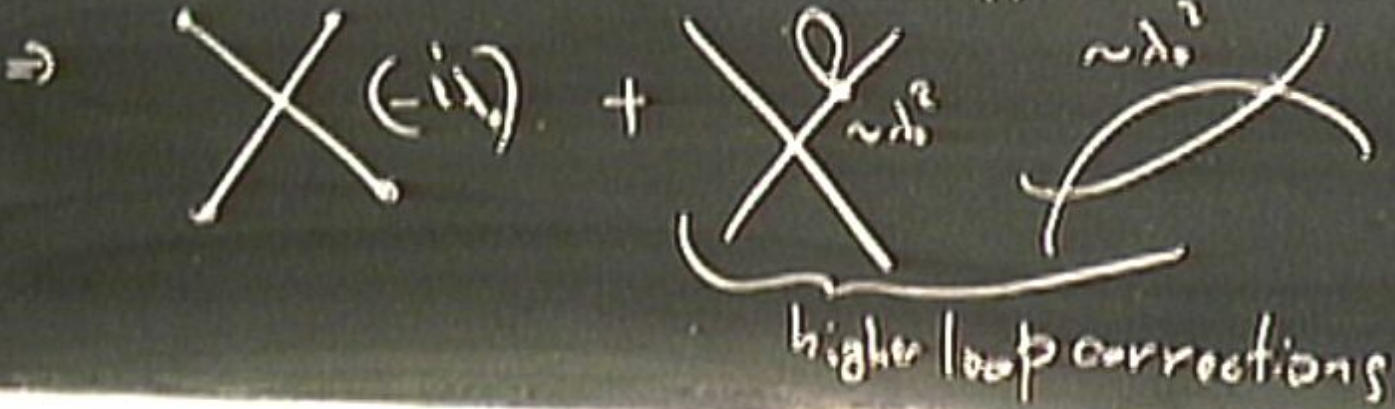
$$\Rightarrow [L] = +4$$

kinetic term

$$[(\partial\phi)^2] = 2 + 2[\phi] = 4 \Rightarrow [\phi] = 1$$

Renormalization and effective field theories

$$d_0 = -\frac{1}{2} (\partial\phi)^2 - \frac{\omega_0^2}{2} \phi^2 - \frac{\lambda_0}{4!} \phi^4$$



$$[m] = +1$$

$$[x] = -1$$

$$[\mu] = +1$$

$$[S] = 0$$

(5)

$$[L] = +4$$

$$[(\partial\phi)^2] = 4$$

⇒ kinetic term

$$[(\partial\phi)^2]$$

$$2 + 2[\phi] = 4 \Rightarrow [\phi] = 1$$

$$[\mu] = +1$$

$$[\lambda_0] = 0$$

Renormalization and effective field theories

$$d_0 = -\frac{1}{2} (\partial\phi)^2 - \frac{m^2}{2} \phi^2 - \frac{\lambda_0}{4!} \phi^4$$

⇒

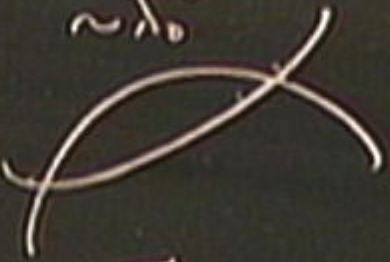


$(-i\lambda_0)$

+



$\sim \lambda_0^2$



$\sim \lambda_0^2$

higher loop corrections

\Rightarrow kinetic term $[(\partial\phi)^2] + 2 + 2[\phi] = 4 \Rightarrow [\phi] = 1$ $[\mu_0] = +1$
 $[\lambda_0] = 0$

\Rightarrow We use (canonical kinetic term) to assign scaling dimension for ϕ

higher loop corrections

→ We use (canonical kinetic term) to assign scaling dimension for ϕ

$$- \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi)$$

higher loop corrections

→ We use (canonical kinetic term) to assign scaling dimension for ϕ

$$- \underbrace{\phi^2 (\partial\phi)^2} - V(\phi)$$

Not canonical kinetic term.

higher loop corrections

$$\frac{(\partial\phi)^2}{2} + 2[\phi] = 4 \Rightarrow [\phi] = 2$$

⇒ We use (canonical kinetic term) to assign scaling dimension for ϕ

$$-\underbrace{\phi^2 (\partial\phi)^2}_{\text{Not canonical kinetic term}} - \underbrace{V(\phi)}$$

$$\gamma_{\phi^2} = \phi, 2\phi$$

Not canonical kinetic term.

⇒ First to ~~cancel~~ define $\phi \rightarrow \phi^1$

higher loop corrections

$\alpha_{\text{phys}} + \Delta\alpha$

$$\left(\frac{\partial \phi}{\partial x} \right)^2 + 2 + 2[\phi] = 4 \Rightarrow [\phi] = 2 \quad [\lambda_0] =$$

→ We use (canonical kinetic term) to assign scaling dimension for ϕ

$$-\underbrace{\phi^2 \left(\frac{\partial \phi}{\partial x} \right)^2}_{\text{Not canonical kinetic term}} - \underbrace{V(\phi)}_{\text{potential}}$$

$$\frac{\partial \phi}{\partial x} = \phi \cdot \frac{\partial \phi}{\partial x}$$

⇒ First to ~~cancel~~ define $\phi \rightarrow \phi^{\frac{1}{2}}$ higher loop corrections

$\alpha_{\text{phys}} + \Delta\alpha$

bare

$$V(\varphi) = V(\sqrt{2}\phi)$$

$$\Delta_{phys} = \frac{1}{2}(\Delta\varphi)$$

$$\Delta_{phys} + \Delta\varphi = L_0$$

$$Z_{op}(\Delta\varphi)$$

$$Z_{op}(\Delta\varphi) \propto \int \mathcal{D}\varphi e^{-S[\varphi]}$$

$$Z_{op}(\Delta\varphi)$$

$$P_0 = \int \mathcal{D}\varphi e^{-S[\varphi]}$$

$$V(\varphi) = V(\sqrt{2}\varphi)$$



$$\frac{1}{8} \varphi^8$$

$\Delta_{phys} = \frac{1}{2} (\partial \varphi)^2$

φ_0 $\Delta_{phys} = \Delta$

$$V(\varphi) = V(\sqrt{2}\varphi)$$

$$\rightarrow \frac{\lambda_8 \varphi^8}{8!}$$

would
be non-renormalizable
with $-(\partial\varphi)^2$

phys = $\frac{1}{2}(\partial\varphi)^2$

$$V(\varphi) = V(\sqrt{2}\phi)$$

$$\hookrightarrow \frac{\lambda_8 \varphi^8}{8!}$$

would
be non-renormalizable
with $-(\partial\phi)^2$

$$\Rightarrow \frac{\lambda_8}{8!} \mathbb{R}^2 \varphi^4$$

$$\frac{\lambda_8}{8!}$$

$$\mathbb{R}^2 \varphi^4$$

$$V(\varphi) = V(\sqrt{2\phi})$$

$$\hookrightarrow \frac{\lambda_6}{81} \phi^8$$

would
be non-renormalizable
with $-\phi^2$

$$\Rightarrow -\phi^2 (\partial\phi)^2$$

$$\frac{\lambda_8}{81} \phi^2 \partial^4 \phi$$

Is renormalizable.

$$\rightarrow \frac{18}{81} \mathbb{R}^p$$

$$-\varphi^2 (\partial\varphi)^2$$

Is renormalizable.

Cheng & Li

"Gauge theories of Particle Physics"

$$V(\phi) = V(\sqrt{2}\phi)$$

$$\rightarrow \frac{\lambda_6}{81} \phi^8$$

would
be non-renormalizable
with $-(\partial\phi)^2$



$$\frac{\lambda_8}{81} \phi^2 (\partial\phi)^2$$

Is renormalizable.

Cheng & Li

"Gauge theories of Particle Physics"

Examples

\Rightarrow \mathcal{Q}^4 - theory

\Rightarrow all free theories

\Rightarrow

Examples

\Rightarrow q^4 - theory

\Rightarrow all free theories.

\Rightarrow gauge theories.

Examples

$\Rightarrow \varphi^4$ - theory

\Rightarrow all free theories

\Rightarrow gauge theories



Infrared free



asymptotically free

Examples

$\Rightarrow \varphi^4$ - theory

\Rightarrow all free theories

\Rightarrow gauge theories



$$(-i\lambda)$$



$$\lambda = \lambda(E)$$



Infrared free

asymptotically free

Examples

\Rightarrow Q^4 - theory

\Rightarrow all free theories

\Rightarrow gauge theories



$$(-i\lambda)$$



$$\lambda = \lambda(E)$$

$\lambda(E) \rightarrow$ because small at $E \rightarrow$

Infrared free

asymptotically free

Examples

$\Rightarrow q^4$ - theory

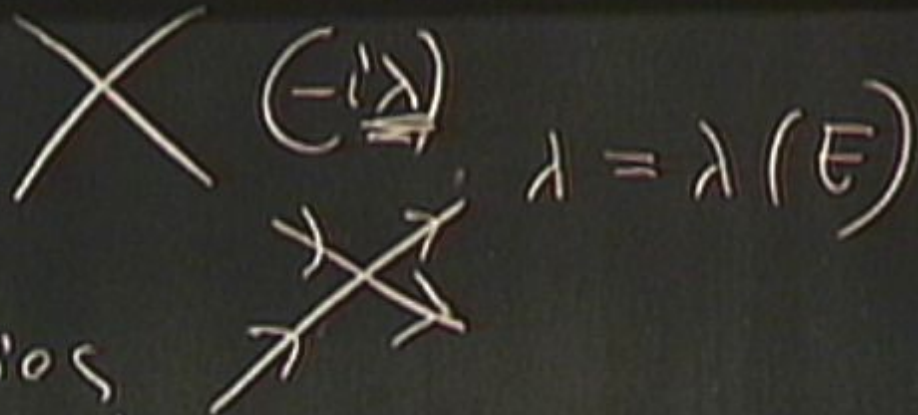
\Rightarrow all free theories

gauge theories.

$\lambda(E) \rightarrow$ because small at $E \rightarrow \infty$

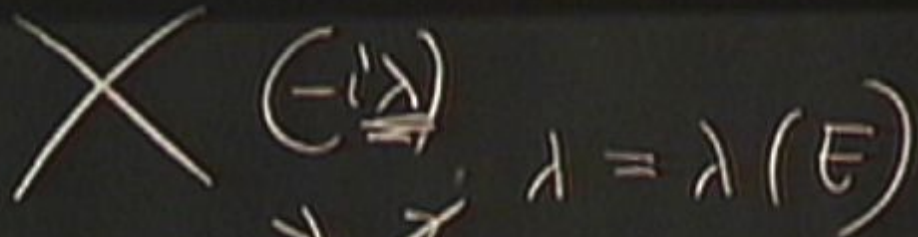
Infrared free

asymptotically free



Examples

\Rightarrow q^4 - theory



\Rightarrow all free theories



\Rightarrow gauge theories. $g_{eff}(E) \rightarrow$ because small at $E \rightarrow \infty$

Infrared free

asymptotically free

$g(E)$

Examples

$\Rightarrow q^4$ - theory

$$\lambda = \lambda(E)$$

\Rightarrow all free theories



\Rightarrow gauge theories. $g_{eff}(E) \rightarrow$ because small at $E \rightarrow \infty$

Infrared free

asymptotically free

$$g_{eff}(E) \rightarrow 0 \quad E \rightarrow +\infty$$

scenarios

cases

$\mathcal{L}_{\text{YM}}(F) \rightarrow$ because small at $F \rightarrow$

→

Infrared free

$\mathcal{L}_{\text{YM}}(F) \rightarrow +\infty$
 $F \rightarrow F_{\text{canc}}$

asymptotically free

→

$\mathcal{L}_{\text{YM}}(F) \rightarrow 0$ $F \rightarrow +\infty$

Nonrenormalizable QFT (to dim 4)

\Rightarrow q^m
 $n > 4$

renormalized
masses
of Δ loop

infrared
divergence
of Δ loop
in $n > 4$
dimensions
is not
removed
by
renormalization
of
masses
and
couplings
to
get
finite
results

= Nonrenormalizable QFTs (to infinity) (In $d=4$)

$\Rightarrow \varphi^n$ $n > 4$

\Rightarrow gravity: $S = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} R$

$$8\pi G_N = m_{Pl}^{-2}$$

introduce additional vertices
+ new couplings to get finite

Nonrenormalizable QFTs (to infinity)
(In $d=4$)

$\Rightarrow \varphi^n$ $n > 4$

\Rightarrow gravity: $S = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} R$

$8\pi G_N = m_{Pl}^{-2}$

Einstein gravity.

$$8\pi G_N = m_{\text{Pl}}^{-2}$$

$$g_{\mu\nu} = \eta_{\mu\nu}$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \text{Minkowski}$$

Einstein gravi-

+ Yang couplings

with $-(\partial\phi)^2$

"Gauge theory

→ gravity: $\int \sqrt{-g} \mathcal{L} d^4x$ $\int \mathcal{L} d^4x$

$$8\pi G_N = m_{\text{Pl}}^{-2}$$

Einstein gravity

$$g_{\mu\nu} = \underbrace{\eta_{\mu\nu}}_{\text{Minkowski}} + \underbrace{h_{\mu\nu}}_{\text{small fluctuation}}$$

$\eta_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$ Minkowski is analog of φ

\Rightarrow gravity: $S = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} R$

$8\pi G_N = m_{Pl}^{-2}$

Einstein gravity.

$g_{\mu\nu} = \underbrace{\eta_{\mu\nu}}_{\text{Minkowski}} + h_{\mu\nu}$

$h_{\mu\nu}$ is small fluctuation
 is analog of φ

$\eta_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$ Minkowski

$P_0 = \hbar$ will have a kinetic term.

\Rightarrow will not be canonical.

M_0

\mathcal{L}_1 is a coupling
 \mathcal{L}_2 is a mass

How do we know

$$V(\phi) = \frac{m^2 \phi^2}{2} + \frac{\lambda \phi^4}{4!}$$

$\times F$
couplings

$P_0 = \hbar^2 \nabla^2$ will have a kinetic term.

\Rightarrow will not be canonically.

\Rightarrow there will be an ∞ # of interaction vertices.

"wave function" renormalization

g is a coupling constant renormalization

m is a mass renormalization

How do we know?

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

couplings

$P_0 = \hbar^2 \nabla^2 \psi$ will have a kinetic term.

\Rightarrow will not be canonically

\Rightarrow there will be an ∞ # of interaction vertices.

$\sqrt{g} R$ is nonlinear

"wave function" renormalization

g is a coupling constant renormalization

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couplings

$P_0 = \hbar^2 \nabla^2 \psi$ will have a kinetic term.

\Rightarrow will not be canonically.

\Rightarrow there will be an ∞ # of interaction vertices.

$\sqrt{g} \psi^2$ is nonlinear

hhhh

\rightarrow

couplings

"wave function" renormalization

z_2 is a coupling constant renormalization

m is a mass renormalization

$P_0 = \hbar^2 \nabla^2 \psi$ will have a kinetic term.

\Rightarrow will not be canonically

\Rightarrow there will be an ∞ # of interaction vertices.

$\sqrt{g} R$ is nonlinear $hhhh$

$\rightarrow \sqrt{g} [R + \frac{m^2}{2\Lambda^2} R^2]$ plings

"Wave function" renormalization

λ is a coupling constant renormalization

$$[R] = +2$$

$P_0 = \hbar^2$ will have a kinetic term. a "wave function" renormalization

\Rightarrow will not be canonically

\Rightarrow there will be an ∞ # of interaction vertices. 2λ is a coupling constant renormalization via $[R] \# + 2$

$\sqrt{g} R$ is nonlinear $hkhkh$

$$\sqrt{g} \left[R + \frac{\# R^2}{m_p^2} + \frac{\# R^3}{m_p^3} + \frac{\# R^4}{m_p^4} + \dots \right]$$

$$V(\varphi) = \frac{\lambda \varphi^4}{4!}$$

$$V(\varphi) = \frac{\lambda \varphi^4}{4!}$$

\Rightarrow mass term will be generated

$$V(\varphi) = \frac{\lambda \varphi^4}{4!}$$

\Rightarrow mass term will be generated

Q: How do we live with non-ren QFT?

\Rightarrow Effecting field theory

⇒ Back to renormalizable theories.

$$Z[J]$$

$$n \geq 4$$

$$S = \int \phi \square \phi$$

$$Z = \int \mathcal{D}\phi e^{-S}$$

Wilson by small fluctuations

\Rightarrow Back to renormalizable theories.

$$Z[\mathcal{J}]$$

generating
functional
for corr
functions

$$S[\phi] + \int \mathcal{J} \phi$$

\Rightarrow Back to renormalizable theories.

$$Z[J] = \int \mathcal{D}\phi$$

generating functional for correlation functions

⇒ Back to renormalizable theories.

$$iS + i \int J(x) \varphi(x) d^4x$$

$$Z[J] = \int [d\varphi] e^{iW[J]}$$

generating functional for correlation functions

$$= e^{iW[J]}$$

⇒ Back to renormalizable theories.

$$iS + i \int J(x) \varphi(x) d^4x$$

$$Z[J]$$

$$= \int [d\varphi] e$$

generating functional for correlation functions

$$= e^{iW[J]}$$

W is some functional

\Rightarrow Back to renormalizable theories.

$$iS + i \int J(x) \varphi(x) d^4x$$

$$Z[J] = \int [d\varphi] e$$

generating
functional
for corr
functions

$$= e^{iW[J]}$$

W is some functional

$$iW[J] \stackrel{\text{def}}{=} \ln Z[J]$$

→ path integral is dominated in a "stationary" phase approximation by a classical trajectory

$$\frac{\delta}{\delta \varphi(x)} \left[S + \int T(x) \varphi(x) \right] = 0$$

$\varphi(x) = \varphi_{cl}(x)$

→ path integral is dominated in a stationary phase approximation by a classical trajectory

$$\frac{\delta}{\delta \varphi(x)} \left[S + \int T(x) \varphi(x) \right] = 0$$

$\varphi(x) = \varphi_d(x)$

→ path integral is dominated in a "stationary" phase approximation by a classical trajectory

$$\frac{\delta}{\delta \varphi(x)} \left[S + \int T(x) \varphi(x) \right] = 0$$

$$Z[J] \propto e^{i \left[S[\varphi_{cl}(x)] + \int T(x) \varphi_{cl}(x) \right]} \quad \varphi(x) = \varphi_{cl}(x)$$

→ path integral is dominated in a stationary phase approximation by a classical trajectory

$$\frac{\delta}{\delta \varphi(x)} \left[S + \int T(x) \varphi(x) \right] = 0 \quad \rightarrow \varphi_{cl} = \varphi_{cl}[J]$$

$$Z[J] \approx e^{i \left(S[\varphi_{cl}(x)] + \int T(x) \varphi_{cl}(x) \right)} \quad \varphi(x) = \varphi_{cl}(x)$$

→ path integral is dominated in a stationary phase approximation by a classical trajectory

$$\frac{\delta}{\delta \varphi(x)} \left[S + \int T(x) \varphi(x) \right] = 0 \quad \rightarrow \varphi_{cl} = \varphi_{cl}[J]$$

$$Z[J] \propto e^{i \left(S[\varphi_{cl}(x)] + \int T(x) \varphi_{cl}(x) \right)} \quad \varphi(x) = \varphi_{cl}(x)$$

In a stationary phase app

$$iW[J] \approx S[\varphi_{cl}[J]] + \int T(x) \varphi_{cl}[J] d^4x$$

→ path integral is dominated in a stationary phase approximation by a classical trajectory

$$\frac{\delta}{\delta \varphi(x)} \left[S + \int T(x) \varphi(x) \right] = 0 \quad \rightarrow \varphi_{cl} = \varphi_{cl}[J]$$

$$Z[J] \propto e^{i \left(S[\varphi_{cl}(x)] + \int T(x) \varphi_{cl}(x) \right)} \quad \varphi(x) = \varphi_{cl}(x)$$

In a stationary phase app

$$W[J] \approx S[\varphi_{cl}[J]] + \int T(x) \varphi_{cl}[J] d^4x$$

$$\frac{\delta}{\delta \rho(x)} \left(S + \int T(x) \rho(x) \right) = 0$$

$$Z[J] \propto e^{i \left(S[\rho_{cl}(x)] + \int T(x) \rho_{cl}(x) \right)} \quad \rho(x) = \rho_{cl}(x)$$

In a stationary phase app

$$W[J] \approx \left(S[\rho_{cl}[J]] + \int T(x) \rho_{cl}[J] dx \right)$$

$$W[J] = \text{Seff}[\varphi_d] + \int d^4x J(x) \varphi_d(x)$$

our
by
exact

mass dimension
of couplings

$$W[J] = \underbrace{S_{\text{eff}}[\varphi_0]}_{\text{renormalizable}} + \int d^4x J(x) \varphi_0(x)$$

\uparrow
 exact
 by $S_{\text{eff}} \neq S$

\uparrow
 mass dimension
 of couplings

$$W[J] = \underbrace{S_{\text{eff}}[\varphi_{cl}]}_{\text{exact}} + \int d^4x J(x) \varphi_{cl}(x)$$

by $S_{\text{eff}} \neq S$

φ_{cl} has nothing to do with φ in path integral

$$W[J] = \underbrace{S_{\text{eff}}[\hat{\phi}]}_{\text{exact}} + \int d^4x J(x) \hat{\phi}(x)$$

by path integral
 $S_{\text{eff}} \neq S$

$\hat{\phi}$ has nothing to do with ϕ in path integral

All assumed

$$W[J] = i \ln Z[J] \quad (\text{in principle uniquely def. func.})$$

$$\frac{\delta W[J]}{\delta J(x)}$$

\equiv

[The rest of the page is heavily scribbled out with dark grey/black ink, obscuring any text or equations that might have been present.]

All assumed

$$W[J] = i \ln Z[J] \quad (\text{In principle uniquely def. func.})$$

$$\frac{\delta W[J]}{\delta J(x)} \stackrel{!}{=} \hat{\phi}(x)$$

$$\begin{aligned} W[J] &= i \ln Z[J] \\ \frac{\delta W[J]}{\delta J(x)} &= \frac{i}{Z[J]} \frac{\delta Z[J]}{\delta J(x)} \end{aligned}$$

$$\begin{aligned} \frac{\delta Z[J]}{\delta J(x)} &= \int \mathcal{D}\phi \frac{\delta}{\delta J(x)} e^{iS[\phi, J]} \\ &= i \int \mathcal{D}\phi \phi(x) e^{iS[\phi, J]} \end{aligned}$$

All assumed

$$W[J] = i \ln Z[J]$$

(In principle uniquely def. func.)

$$\frac{\delta W[J]}{\delta J(x)} = \hat{\phi}(x)$$

All assumed

$$W[J] = -i \ln Z[J]$$

(In principle uniquely defined)

$$\frac{\delta W[J]}{\delta J(x)} = \hat{\phi}(x)$$

$$S_{\text{eff}} = W[J] - \int d^4x J(x) \frac{\delta W}{\delta J}$$

All assumed

$$W[J] = \ln Z[J]$$

(In principle uniquely defined)

$$\frac{\delta W[J]}{\delta J(x)} = \hat{\phi}(x)$$

$$S_{\text{eff}} = W[J] - \int d^4x J(x) \frac{\delta W}{\delta J}$$

uniquely defined

All assumed

$$W[J] = i \ln Z[J]$$

(In principle uniquely defined)

$$\frac{\delta W[J]}{\delta J(x)} \stackrel{!}{=} \hat{\phi}(x)$$

→ Invert it

$$J(x) = J[\hat{\phi}(x)]$$

$$S_{\text{eff}} = W[J] - \int d^4x J(x) \frac{\delta W}{\delta J}$$

↑
uniquely defined

All assumed

$$W[J] = \int \mathcal{D}\phi \exp(iZ[\phi]) \quad (\text{In principle uniquely defined})$$

$$\left[\frac{\delta W[J]}{\delta J(x)} = \hat{\phi}(x) \right] \rightarrow \text{Invert it}$$
$$J(x) = J[\hat{\phi}(x)]$$

$$S_{\text{eff}} = W[J] - \int d^4x J(x) \frac{\delta W}{\delta J}$$

uniquely defined

$$S_{\text{eff}} = S_{\text{eff}}[J] = S_{\text{eff}}[J[\hat{\phi}]]$$

$$\frac{\delta W(\varphi)}{\delta J(x)} = \varphi(x) \quad \rightarrow \quad J(x) = J[\varphi(x)]$$

$$S_{\text{eff}} = W[J] - \int d^4x \varphi(x) \frac{\delta W}{\delta J}$$

↑
uniquely defined

$$S_{\text{eff}} = S_{\text{eff}}[J] = S_{\text{eff}}[J[\varphi]]$$

effective action

Functional = e^{-W} is some functional
for corr functions

$$W[J] \stackrel{\text{def}}{=} \ln Z[J]$$

Complicated dynamics in QFT \Leftrightarrow Classical dynamics
 In effective field theory

[The following text is heavily obscured by white chalk scribbles and is largely illegible. It appears to contain mathematical expressions and symbols.]

Complicated dynamics in QFT

Classical dynamics
In effective field theory

$$S_{\text{eff}}(\vec{P}) =$$

does not make
sense to quantize

Complicated dynamics in QFT

Classical dynamics
In effective field theory

$$S_{\text{eff}}(\vec{P}) = \int d^4x$$

does not make sense to quantize

Complicated dynamics in QFT

Classical dynamics
In effective field theory

$$S_{\text{eff}}(\vec{P}) = \int d^4x \left[\dots \right]$$

does not make sense to quantize

derivative expansion

Complicated dynamics in QFT

Classical dynamics
In effective field theory

$$S_{\text{eff}}(\vec{\varphi}) = \int d^4x \left[-V_{\text{eff}}(\vec{\varphi}) - \frac{1}{2} (\partial\vec{\varphi})^2 Z(\vec{\varphi}) + \dots \right]$$

does not make sense to quantize

derivative expansion

Complicated dynamics in QFT

Classical dynamics
In effective field theory

$$S_{\text{eff}}(\vec{P}) = \int d^4x \left[-V_{\text{eff}}(\vec{\Phi}) - \frac{1}{2} (\partial\vec{\Phi})^2 Z(\vec{\Phi}) + \dots \right]$$

does not make sense to quantize

derivative expansion