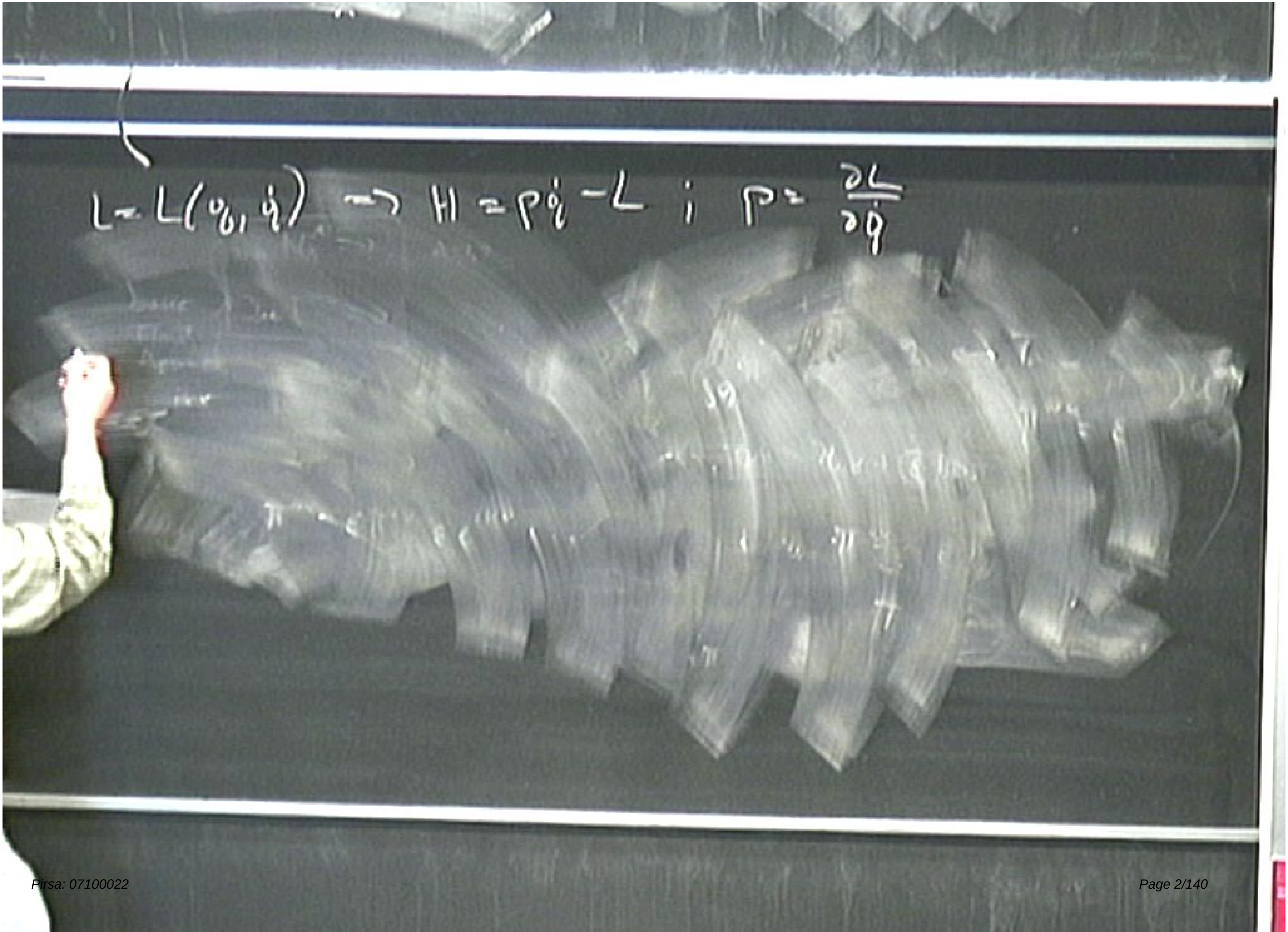
Title: Intro to Supersymmetry 12

Date: Oct 25, 2007 12:30 PM

URL: http://pirsa.org/07100022

Abstract:


$L = L(q, \dot{q}) \rightarrow H = p\dot{q} - L \; ; \; p = \frac{\partial L}{\partial \dot{q}}$

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$$\Rightarrow \quad q(t_0),\; q'(t_0) \longrightarrow q(t)$$

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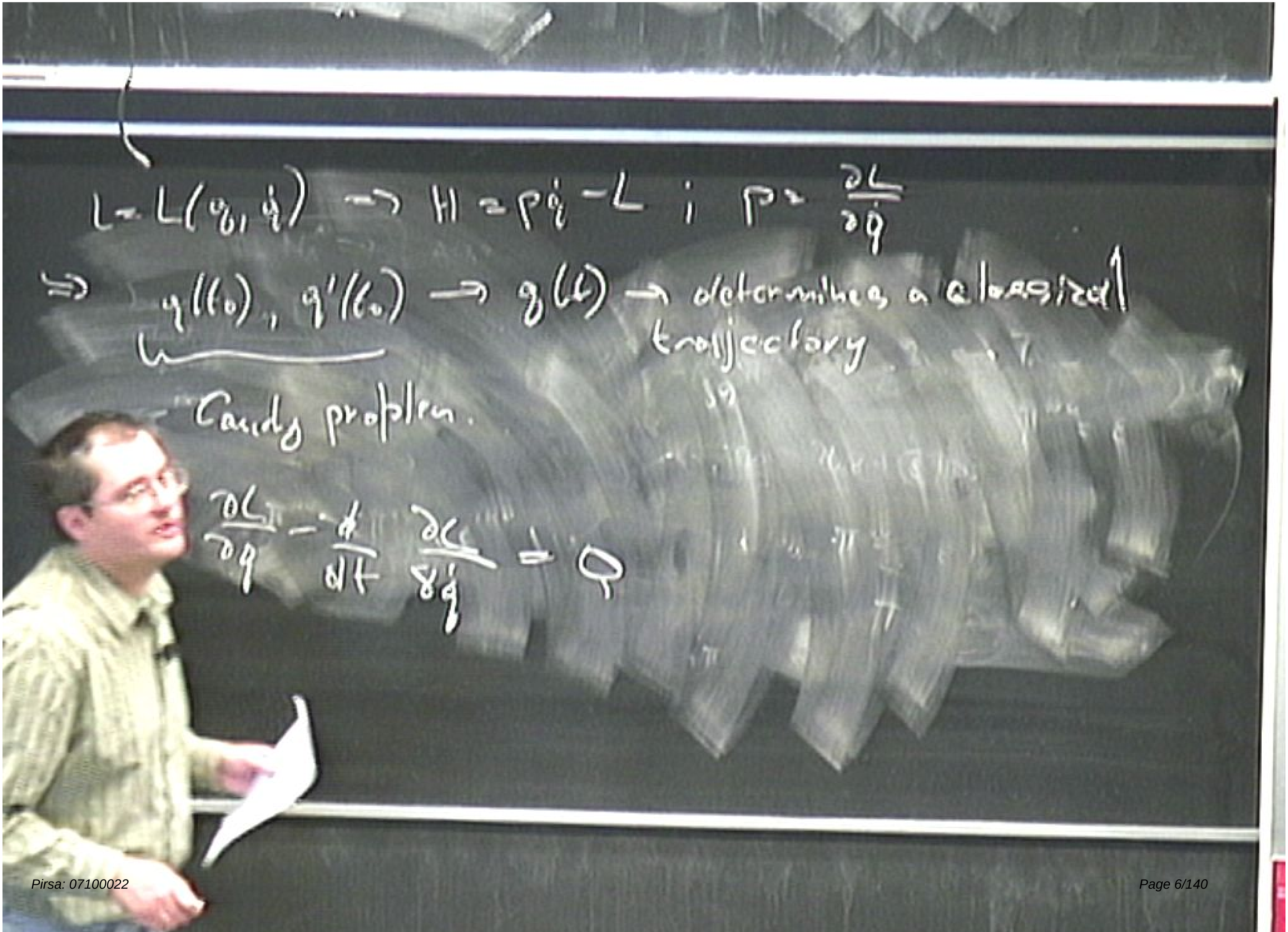
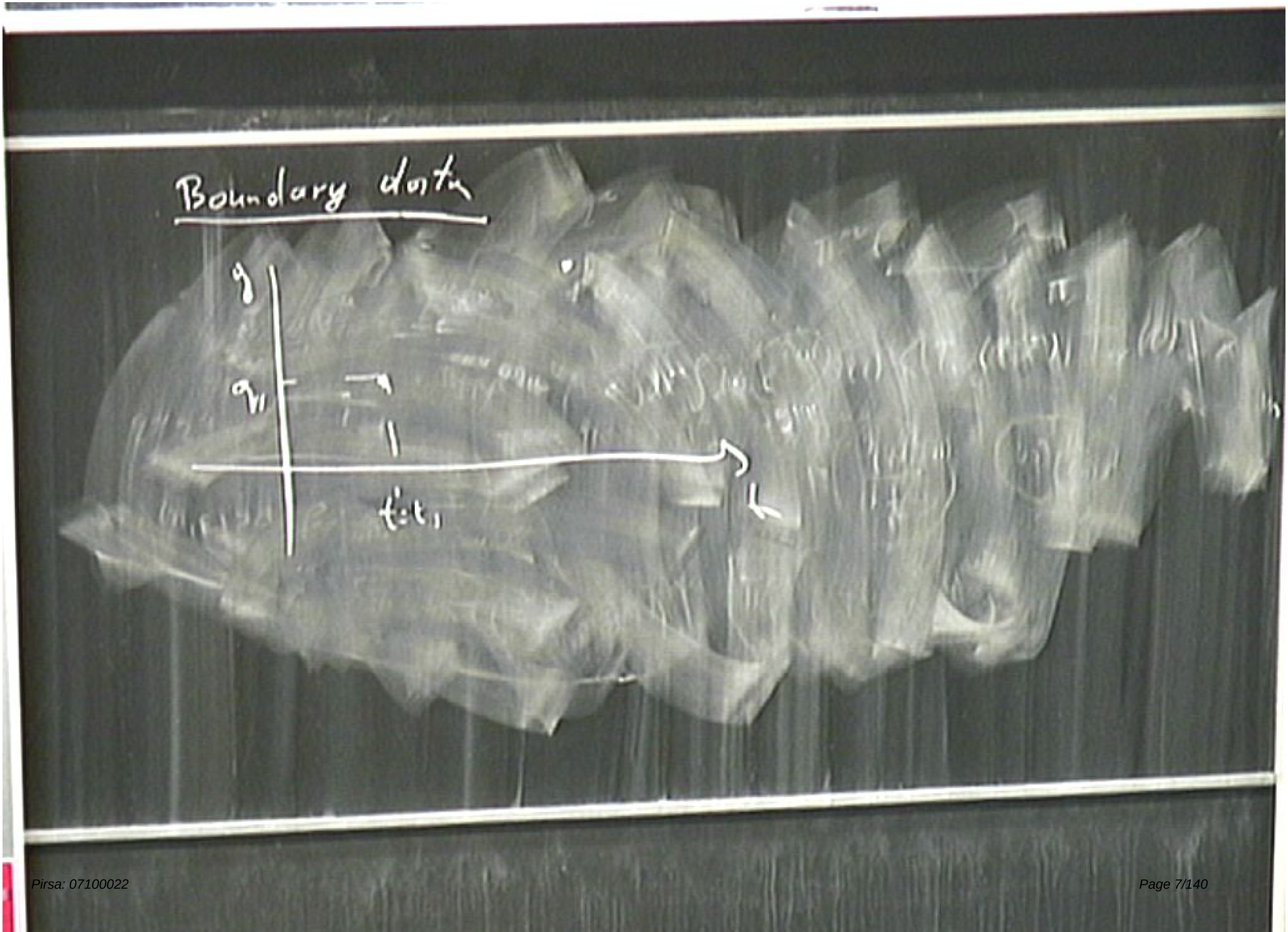
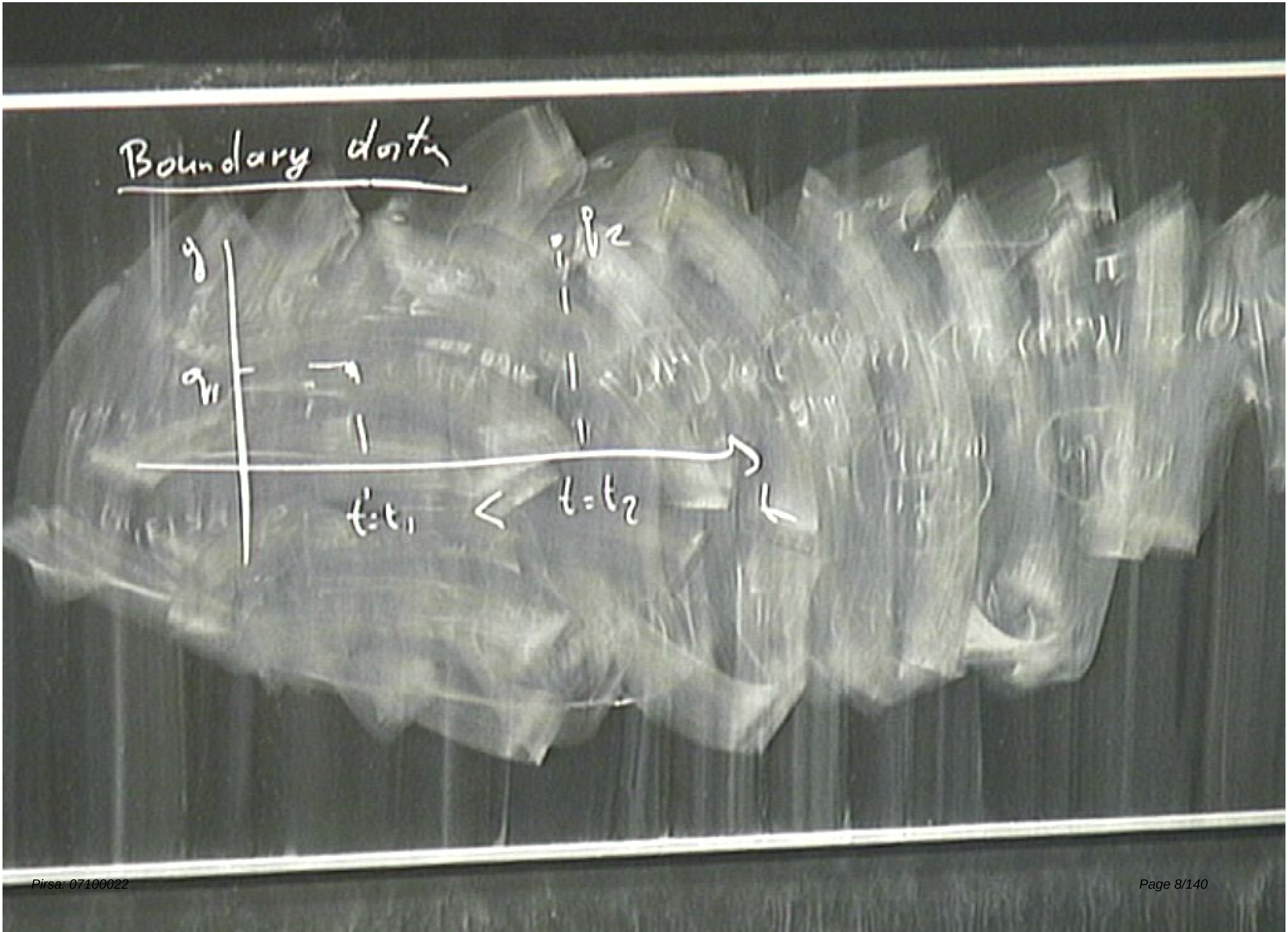
$\Rightarrow \quad q(t_0), \; q'(t_0) \longrightarrow q(t) \rightarrow$ determines a classical trajectory

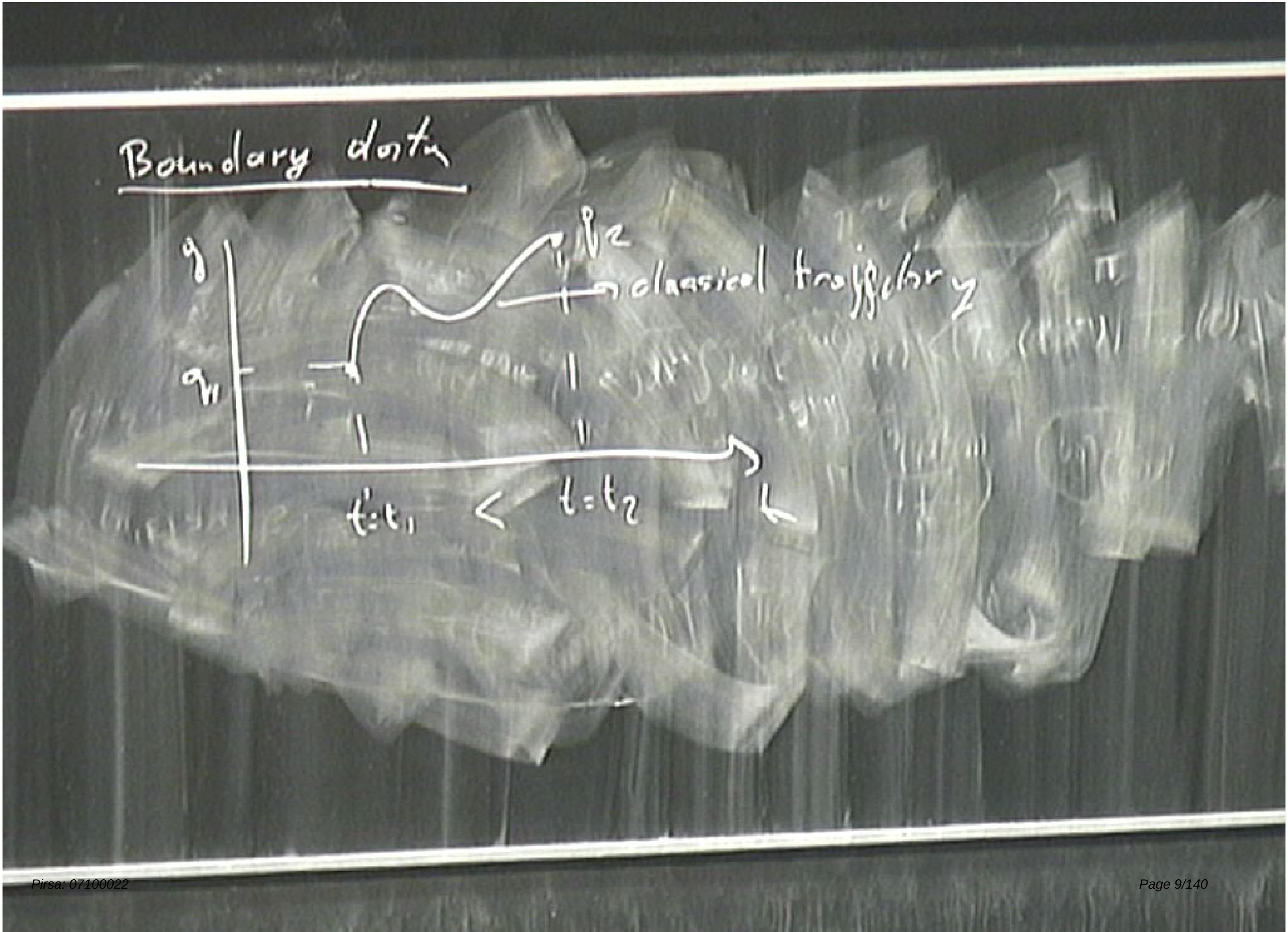
$L = L(q, \dot{q}) \rightarrow H = p\dot{q} - L \;;\; p = \frac{\partial L}{\partial \dot{q}}$

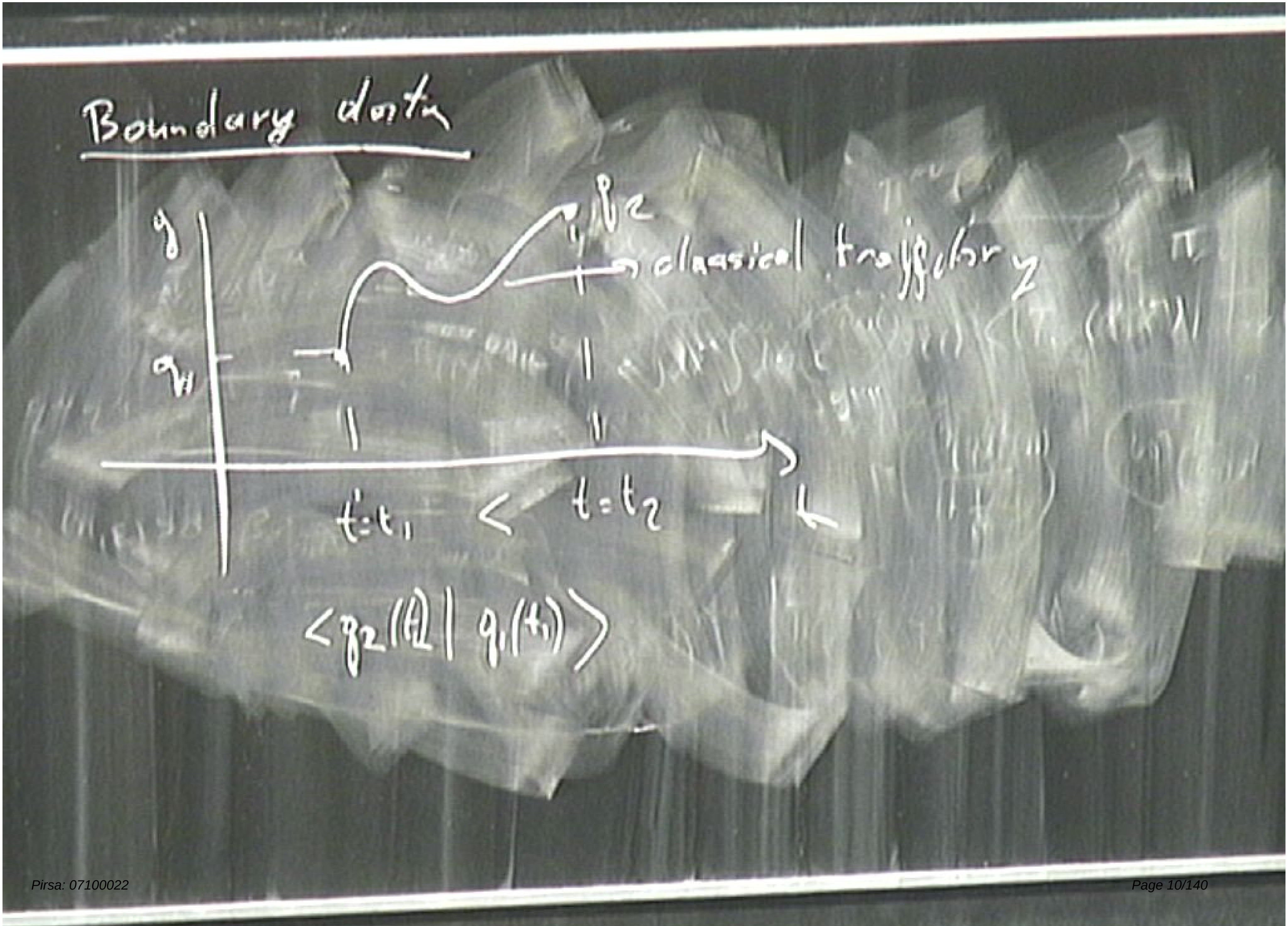
$\Rightarrow \underbrace{q(t_0),\ q'(t_0)}_{\text{Cauchy problem.}} \rightarrow q(t) \rightarrow$ determines a classical trajectory

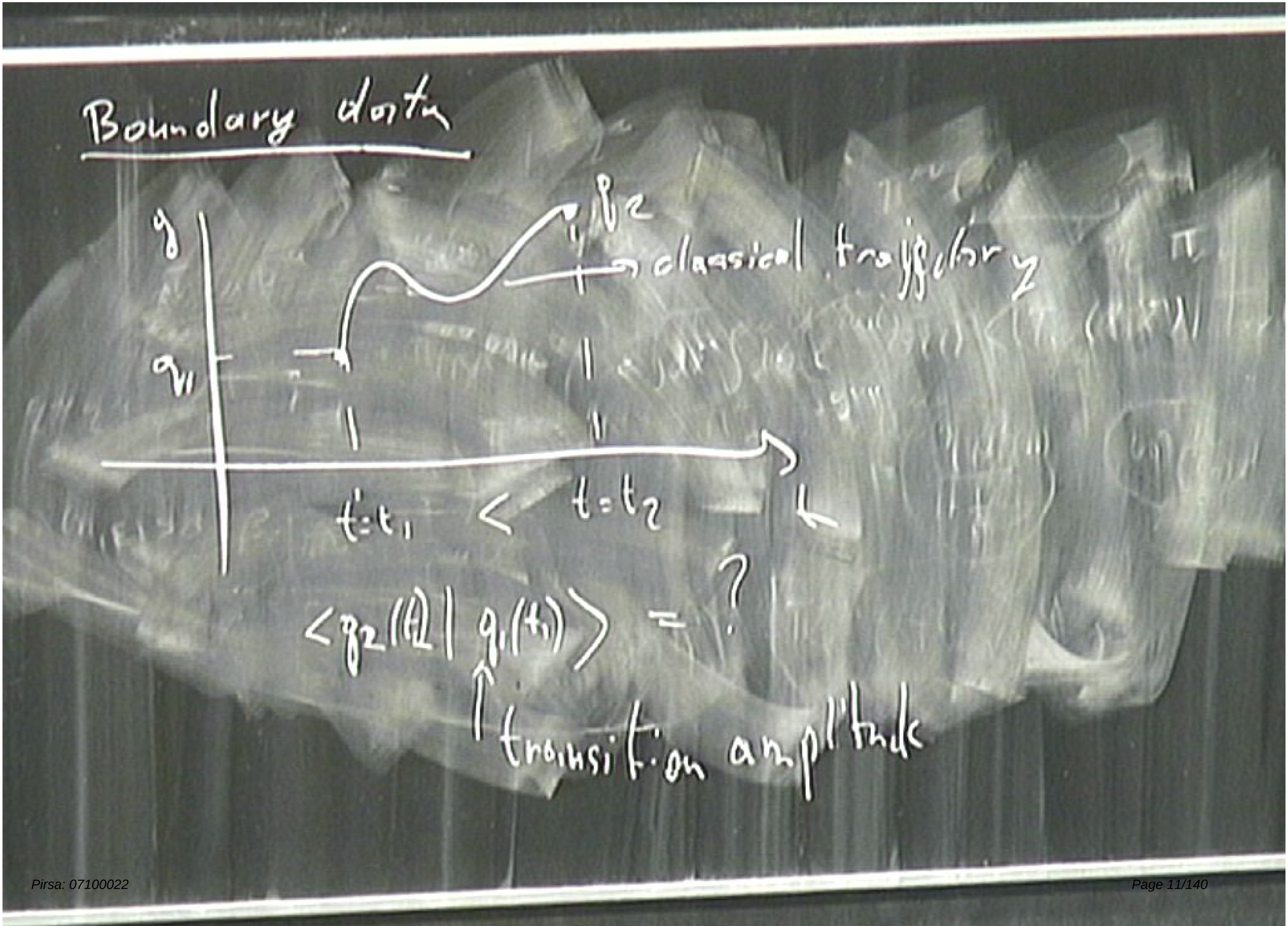

L = L(q, q̇) → H = pq̇ − L ; p = ∂L/∂q̇
⇒ q(t₀), q'(t₀) → q(t) → determines a classical trajectory
Cauchy problem.
∂L/∂q − d/dt ∂L/∂q̇ = Q


Boundary data
q
t

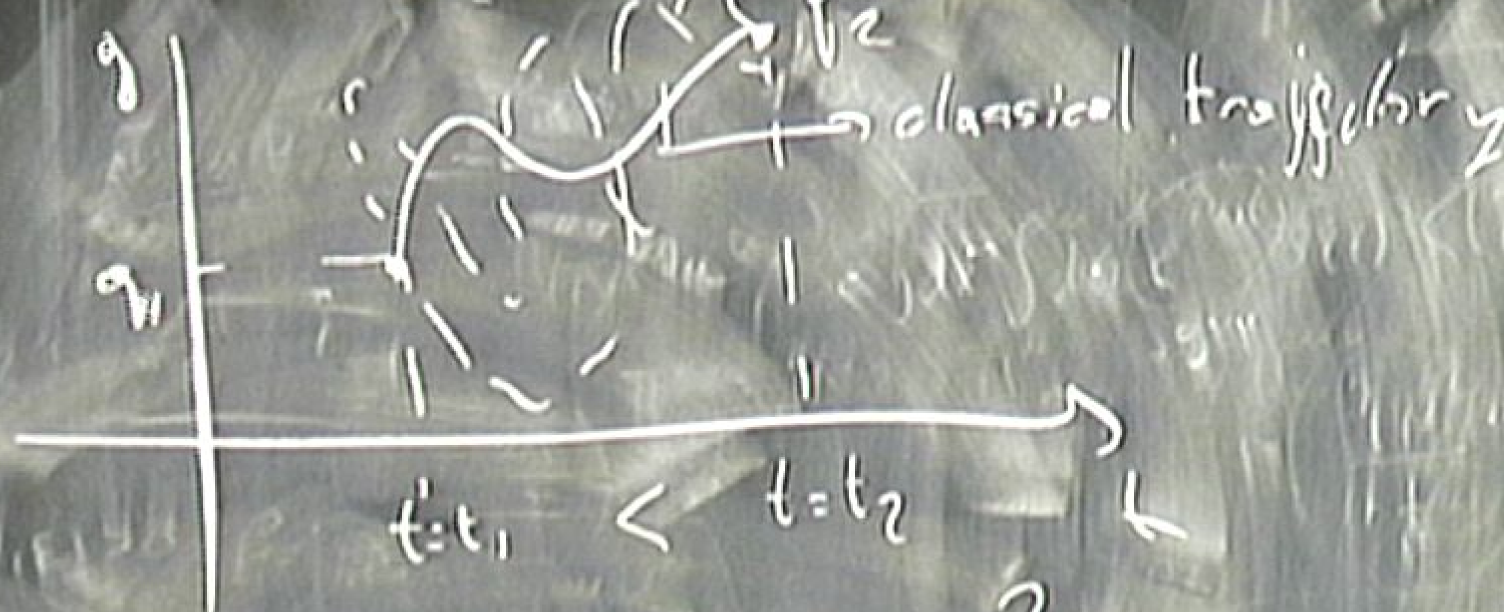

Boundary data
q
q_1
p_2
$t=t_1$
<
$t=t_2$
t


Boundary data
q
q2
classical trajectory
q1
t=t1 < t=t2
t


Boundary data
q
q_2
classical trajectory
q_1
$t = t_1$
<
$t = t_2$
t
$\langle q_2(t_2) | q_1(t_1) \rangle$

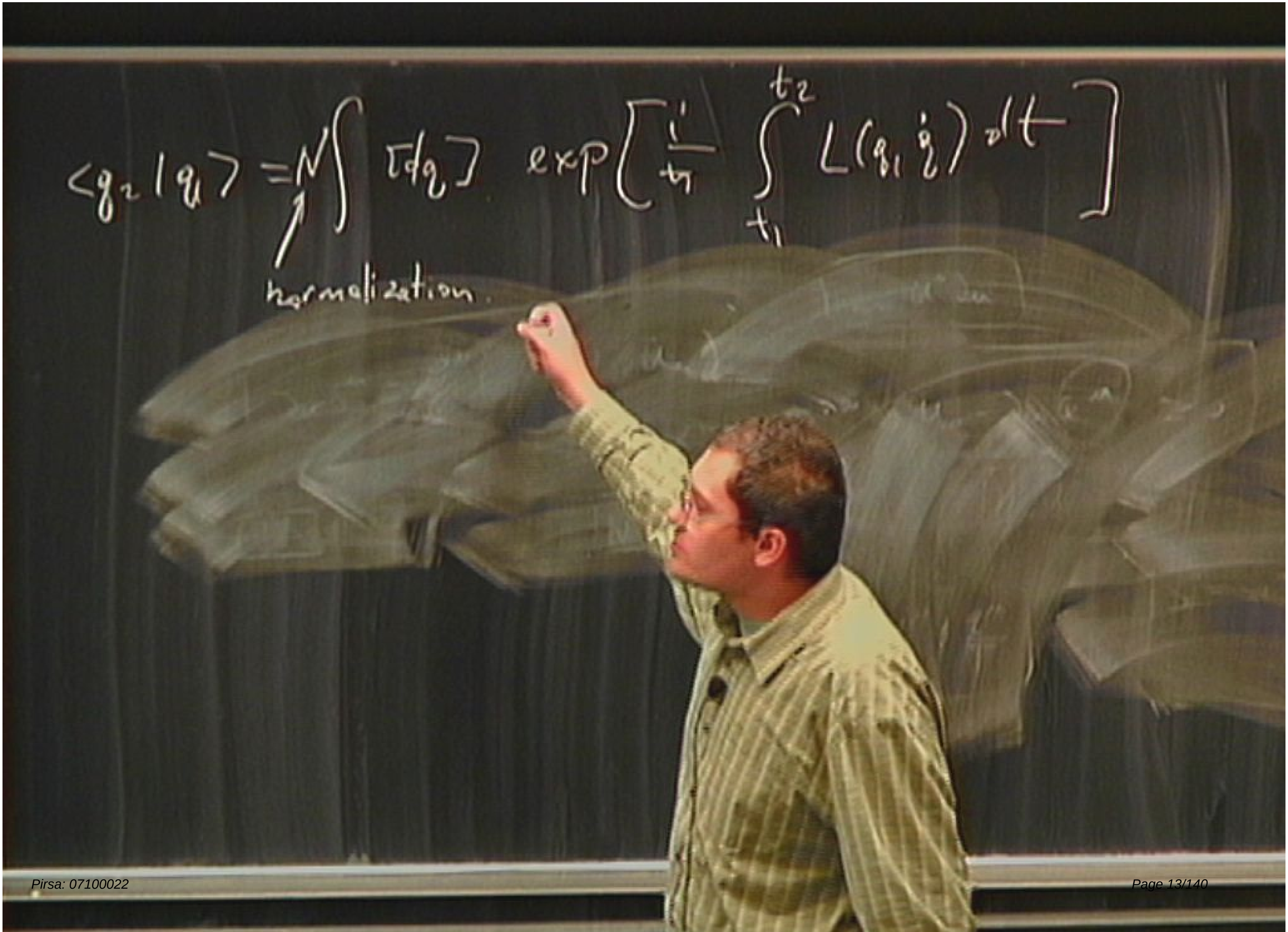

Boundary data
q
q2
classical trajectory
q1
t=t1 < t=t2
t
<q2(t2) | q1(t1)> = ?
transition amplitude

Boundary data



$$\langle q_2(t_2) | q_1(t_1) \rangle = ?$$

↑ transition amplitude


$\langle q_2 | q_1 \rangle = N \int [dq] \exp\left[\frac{i}{\hbar} \int_{t_1}^{t_2} L(q, \dot{q})\, dt \right]$
normalization

$$\langle q_2 | q_1 \rangle = N \int [dq] \, \exp\left[\frac{i}{\hbar} \int_{t_1}^{t_2} L(q, \dot{q}) \, dt \right]$$

↑ normalization

<u>BC:</u>

$$\langle q_2 | q_1 \rangle = N \int [dq] \, \exp\left[\frac{i}{\hbar} \int_{t_1}^{t_2} L(q, \dot{q}) \, dt \right]$$

↑ normalization.

BC: $q(t_1) = q_1$

$q(t_2) = q_2$

$$\langle q_2 | q_1 \rangle = N \int [dq] \exp\left[\frac{i}{\hbar} \int_{t_1}^{t_2} L(q, \dot{q})\, dt \right]$$

↑ normalization

BC. $q(t_1) = q_1$

$q(t_2) = q_2$

⟹ stationary phase approximation

$$\langle q_2 | q_1 \rangle = N \int [dq] \exp\left[\frac{i}{\hbar} \int_{t_1}^{t_2} L(q, \dot{q})\, dt \right] = \int [dq]\, e^{\frac{i}{\hbar} S}$$

↑ normalization

<u>BC.</u> $q(t_1) = q_1$

$q(t_2) = q_2$

⇒ stationary phase approximation

→ dominant contribution would come from configurations

$$\langle q_2 | q_1 \rangle = N \int [dq] \exp\left[\frac{i}{\hbar} \int_{t_1}^{t_2} L(q, \dot{q})\, dt \right] = \int [dq] e^{\frac{i}{\hbar} S}$$

↑ normalization

B.C. $q(t_1) = q_1$

$q(t_2) = q_2$

⇒ stationary phase approximation

→ dominant contribution would come from configurations where phase almost does not chang..

$$\frac{\delta S}{\delta q(t)} = 0 \rightarrow \text{Identify configurations that dominate in } \langle q_2 | q_1 \rangle$$

$\Downarrow$

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$\Downarrow$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0$$

$\frac{\delta S}{\delta q(t)} = 0 \rightarrow$ Identify configurations that dominate in $\langle q_2 | q_1 \rangle$

$\Downarrow$

$$\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = 0 \qquad q(t_1) = q_1 \;;\; q(t_2) = q_2$$

$\rightarrow$ will determine a classical trajectory

$$\langle q_2 | q_1 \rangle \sim \exp\left[\frac{i}{\hbar} S_{\text{classical}}\right] = \exp\left[\frac{i}{\hbar}\int_{t_1}^{t_2} dt\, L(q_{cl}, \dot{q}_{cl})\right]$$

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$\Rightarrow$ the same is in QFT.

$$\langle q_2 | q_1 \rangle \sim \exp\left[\frac{i}{\hbar} S_{classical}\right] = \exp\left(\frac{i}{\hbar}\int_{t_1}^{t_2} dt\, L(q_{cl}, \dot{q}_{cl})\right]$$

$\Rightarrow$ the same is in QFT.

$$L = L(\varphi, \partial\varphi) = \frac{1}{2}(\partial\varphi)^2 - V(\varphi)$$

$$V(\varphi) = -\frac{m^2\varphi^2}{2!} + \frac{\lambda\varphi^4}{4!}$$

correlations functions (observables in QFT)

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$$\langle 0|0\rangle = N \int [d\varphi]$$

↑ vacuum-to-vacuum transition amplitude.

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↑ vacuum-to-vacuum transition amplitude.

correlations functions (observables in QFT).

$$\langle 0|0\rangle = N\int[d\varphi]\, e^{\frac{iS}{\hbar}} = Z$$

$\langle 0|0\rangle$ ↑ vacuum-to-vacuum transition amplitude.

Z ↑ partition function

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In Stat mech.

$$Z = \mathrm{Tr}\, e^{-\beta H}$$

$$\beta = \frac{1}{T}$$

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In Stat mech.

$$Z = \mathrm{Tr}\, e^{-\beta H} = e^{-\beta F}$$

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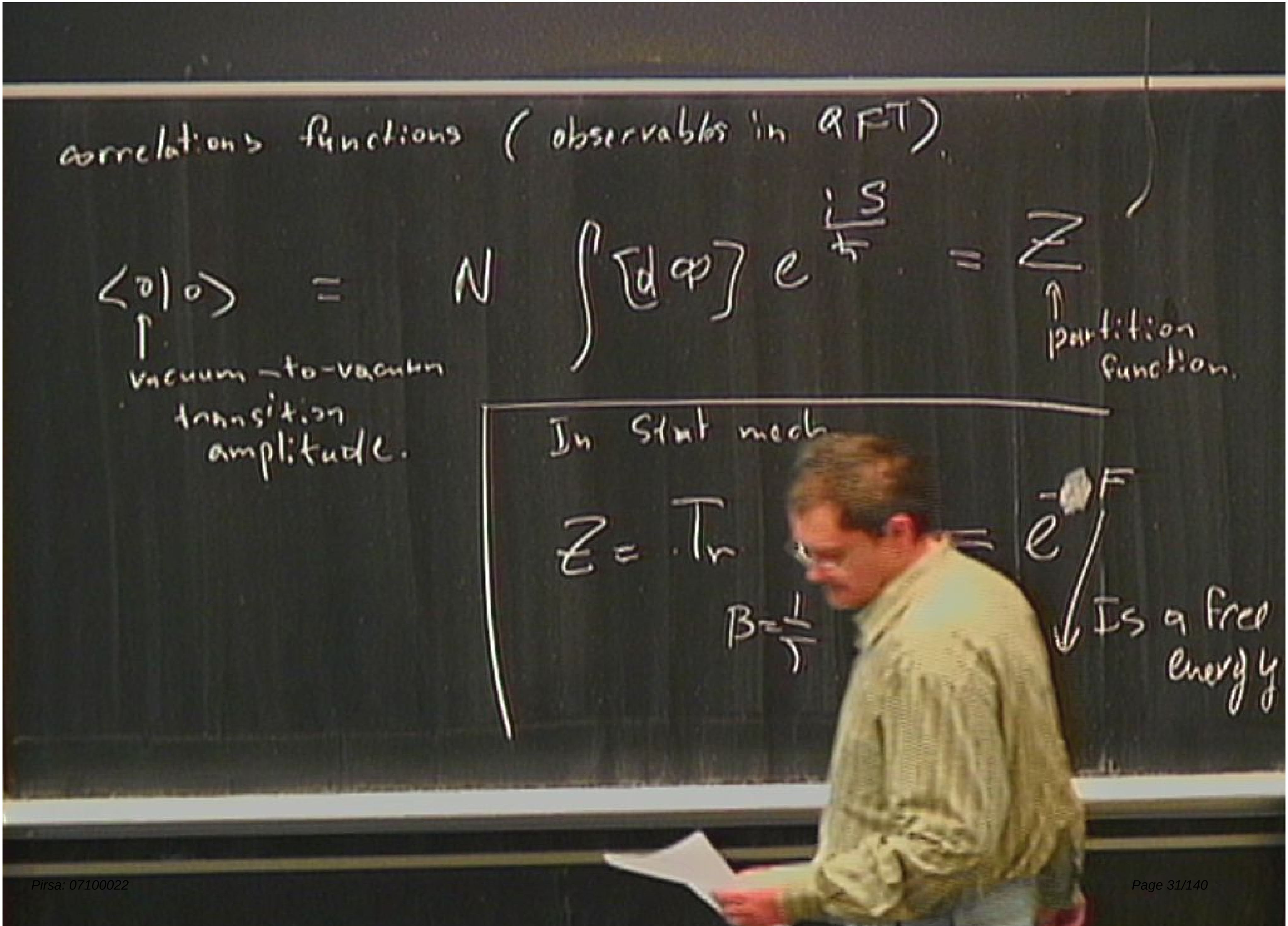
correlations functions (observables in QFT)
<0|0> = N ∫[dφ] e^{iS/ħ} = Z
vacuum-to-vacuum transition amplitude.
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In Stat mech
Z = Tr
= e^{-βF}
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Is a free energy

correlations functions (observables in QFT)

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$$\boxed{F = -\ln Z}$$

In Stat mech.

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$\beta = \frac{1}{T}$

F Is a free energy

$\hat{\phi}(x_1), \ldots \hat{\phi}(x_n)$

$$G^{(n)}(x_1, \ldots x_n) = \langle 0 |$$

$\hat{\phi}(x_1), \ldots \hat{\phi}(x_n)$

$$G^{(n)}(x_1, \ldots x_n) = \langle 0 | T\left(\hat{\phi}_1(x_1) \cdots \hat{\phi}(x_n)\right) | 0 \rangle$$

$\hat{\varphi}(x_1), \dots \hat{\varphi}(x_n)$

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↑ correlation functions

$$T(\hat{O}(t_1)\, \hat{O}(t_2)) = \{$$

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↑ correlation functions

$$T(\hat{O}(t_1)\, \hat{O}(t_2)) = \begin{cases} \hat{O}(t_1)\, \hat{O}(t_2), & t_1 > t_2 \\ \hat{O}(t_2)\, \hat{O}(t_1), & t_2 > t_1 \end{cases}$$

$$\int \prod_{i=1}^{n} d^4x_i \; e^{-i p_i x_i} \, G^{(n)}(x_1, \ldots, x_n) = \exp\left(\frac{i}{\hbar}\int_{t_1}^{t_2} dt \, L(q, \dot q)\right)$$

(Invariance under space-time translations)

$$G^{(n)}(x_i^\mu + a^\mu) = G^{(n)}(x_i)$$

$$\int \prod_{i=1}^{n} d^4x_i \; e^{-ip_i x_i}\, G^{(n)}(x_1, \ldots, x_n) = (2\pi)^4 \,\delta^4(p_1 + \ldots + p_n) \ldots$$

...variance under space-time translations

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Simplest 2-point correlation function is

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$$i\Delta(p) = \int d^4x \, e^{-ipx} \, \mathrm{T}\left(\hat{\varphi}(x)\,\hat{\varphi}(0)\right)$$

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compute for $\lambda = 0$ exactly using canonical quantization.

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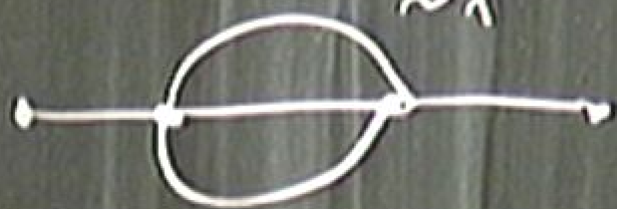
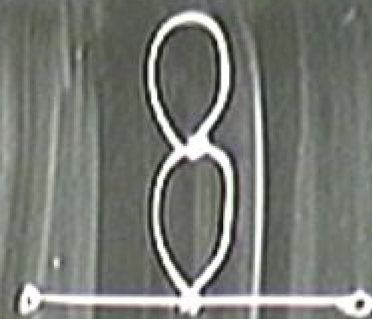
$$i\Delta(p) = \frac{i}{p^2 - m^2}$$ → a "leading order result in λ"

compute for $\lambda = 0$ exactly using canonical quantization.

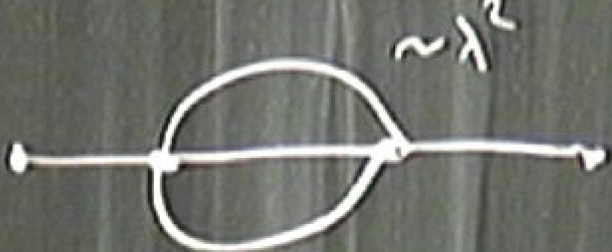
In our $\frac{\lambda\varphi^4}{4!}$ theory

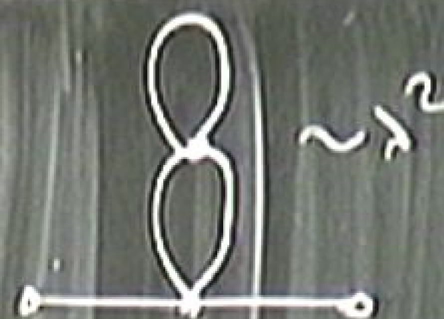
$\sim\lambda^2$



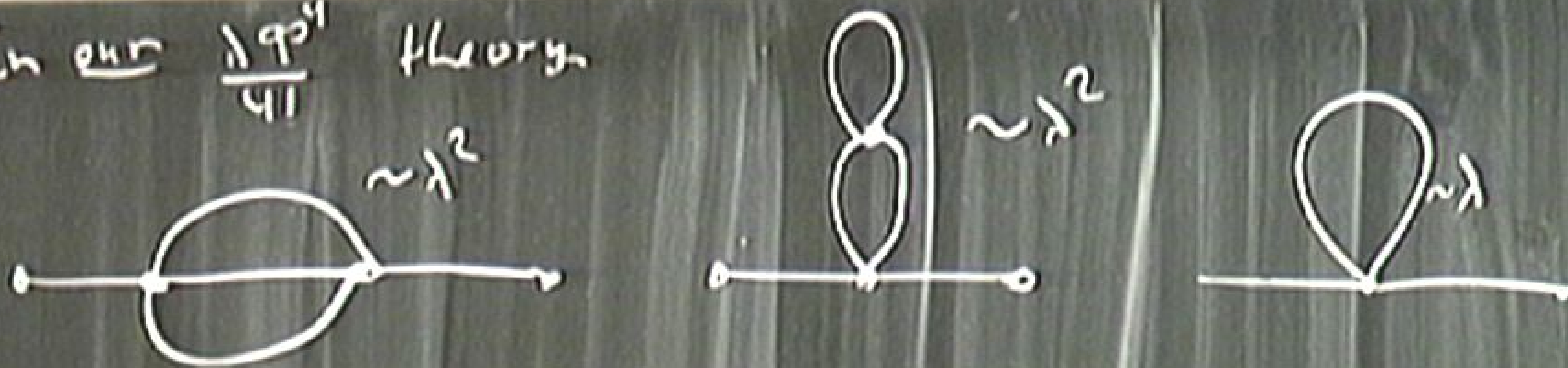


In our $\frac{\lambda \varphi^4}{4!}$ theory



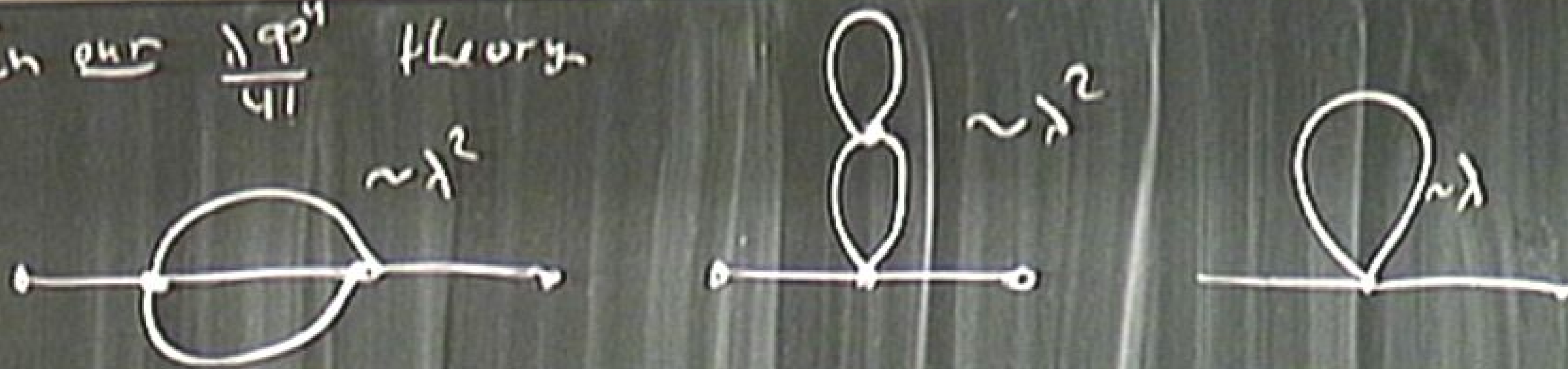


In our $\frac{\lambda \varphi^4}{4!}$ theory

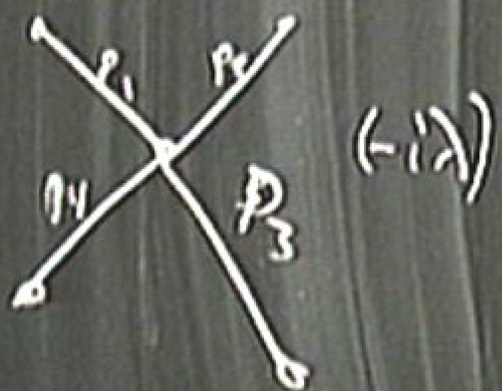


$\Rightarrow$ Interactions are encoded in higher point corr. functions

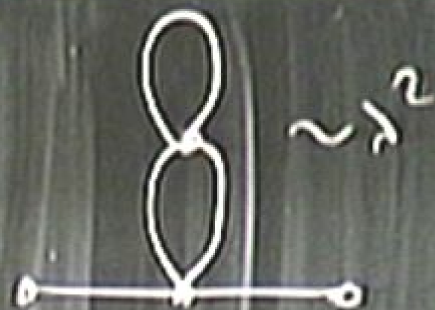
In our $\frac{\lambda\varphi^4}{4!}$ theory

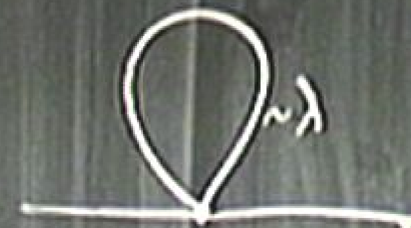


$\Rightarrow$ Interactions are encoded in higher point corr. functn.

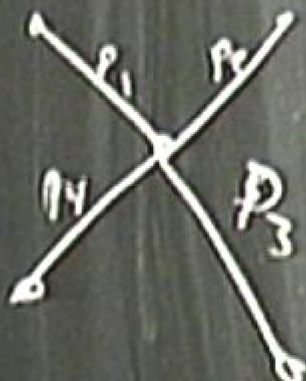


In our $\frac{\lambda \varphi^4}{4!}$ theory



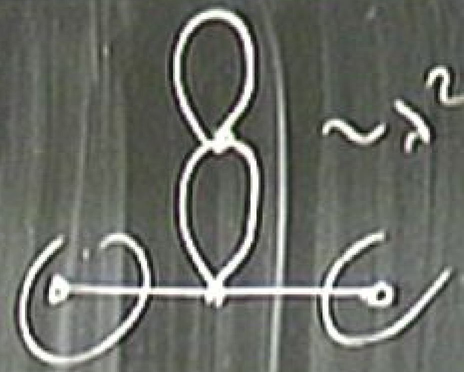


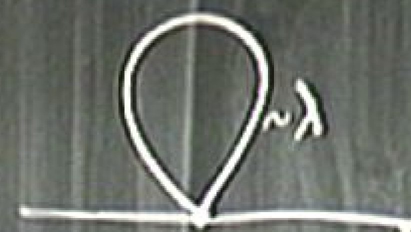
$\Rightarrow$ Interactions are encoded in higher point corr. function.



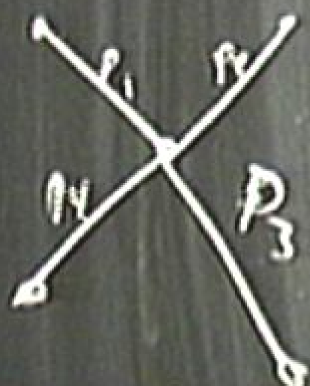
$$(-i\lambda) = G^{(4)}_{amp}(p_1, p_2, p_3, p_4)$$

In our $\frac{\lambda \varphi^4}{4!}$ theory



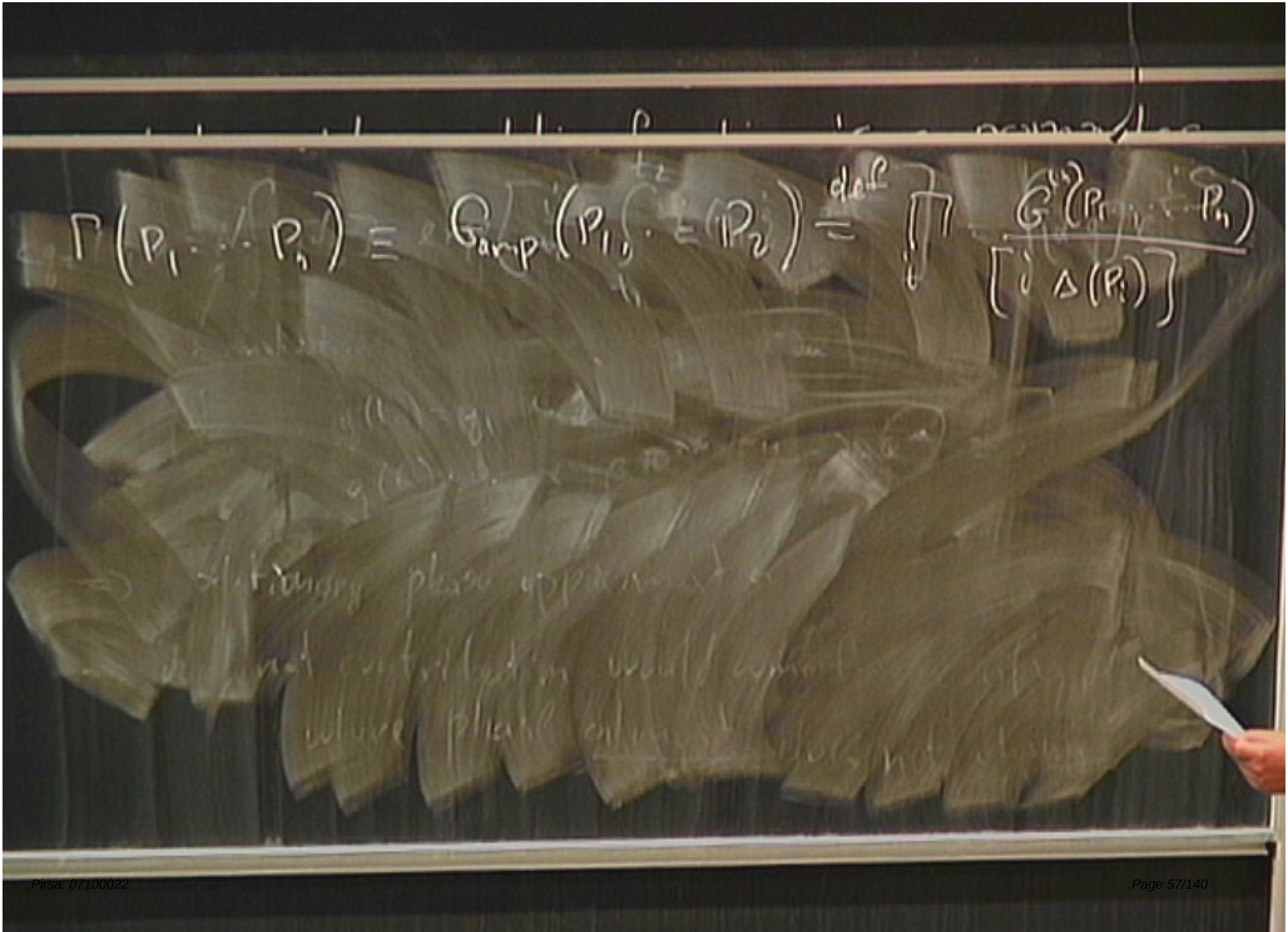


$\Rightarrow$ Interactions are encoded in higher point corr. functn.



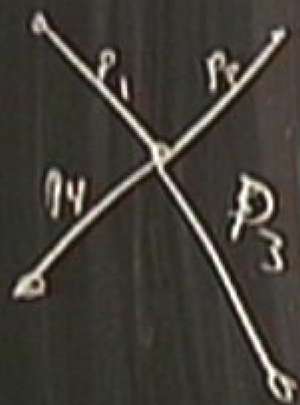
$$(-i\lambda) = G^{(4)}_{amp}(p_1, p_2, p_3, p_4)$$

$$\Gamma(p_1 \ldots p_n) \equiv G_{amp}\left(p_1, \int_{t_1}^{t_2} \ldots p_2\right) \stackrel{\text{def}}{=} \prod_i G^{(i)}(p_1 \ldots p_n)$$


Γ(p1 ... pn) ≡ Gamp(p1, ... pn) def= ∏i G(n)(p1, ... pn) / [i Δ(pi)]

$$\Gamma(p_1 \ldots p_n) \equiv G_{amp}(p_1, \ldots p_n) \stackrel{def}{=} \left(\prod_i \frac{1}{[\,i\,\Delta(p_i)\,]} \right) G(p_1 \ldots p_n).$$

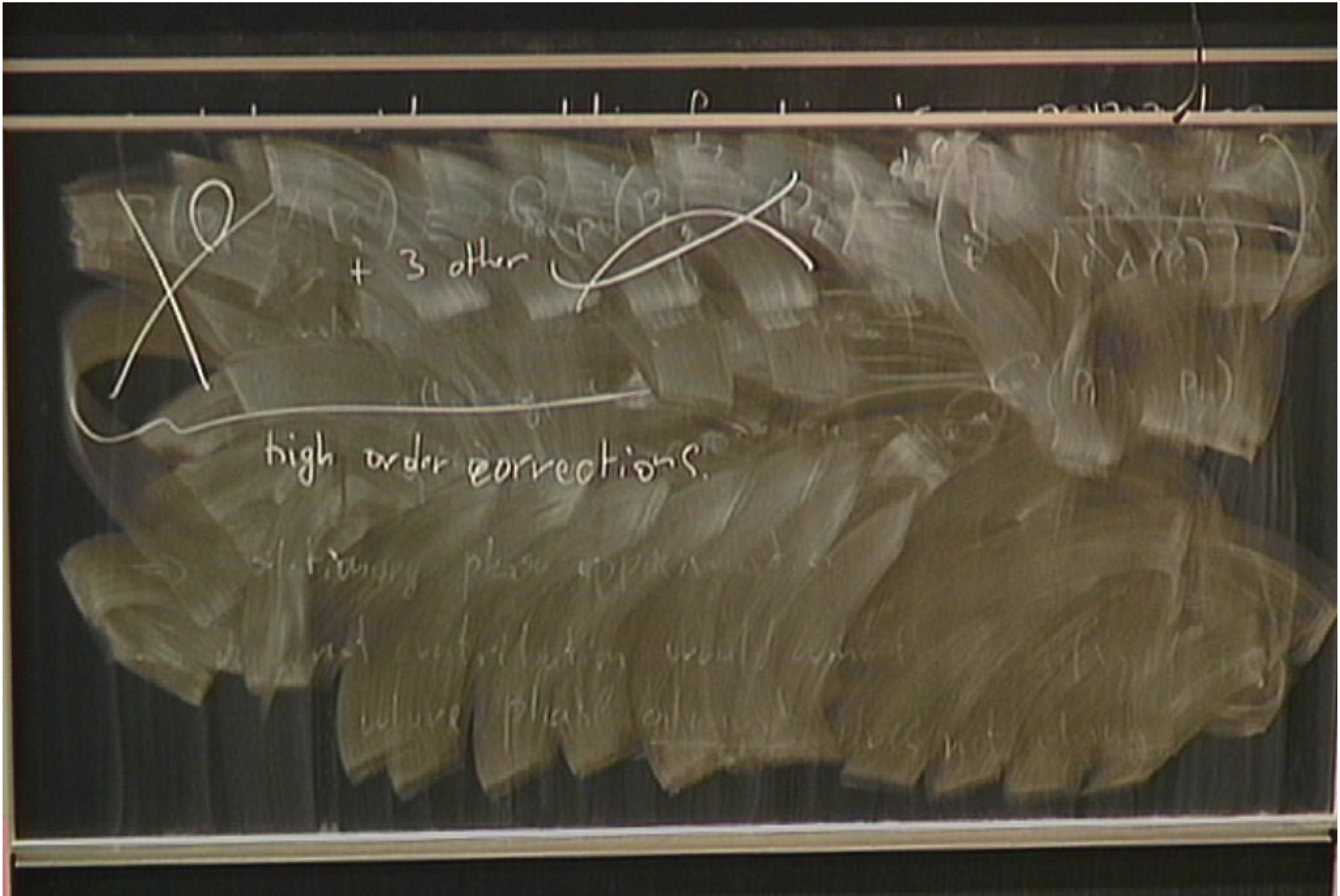
⇒ Interactions are encoded in higher point corr. fun

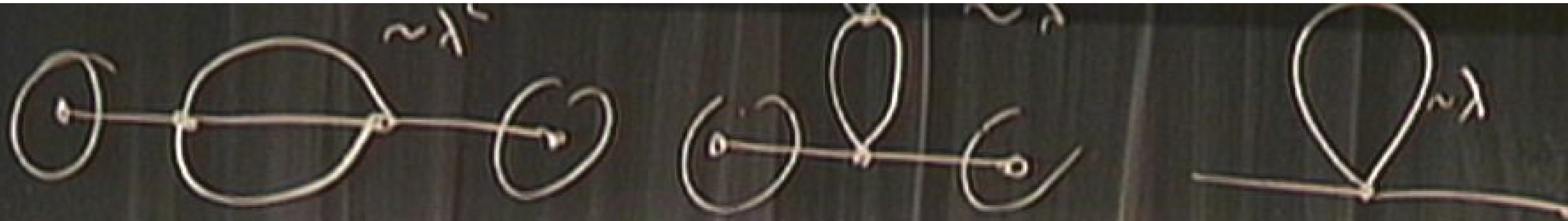


$$(-i\lambda) = G^{(4)}_{amp}(p_1, p_2, p_3, p_4)$$

to leading order $\Gamma^{(4)}(p_1; p_2, p_3, p_4) = -i\lambda$

transition amplitude


+ 3 other
high order corrections.

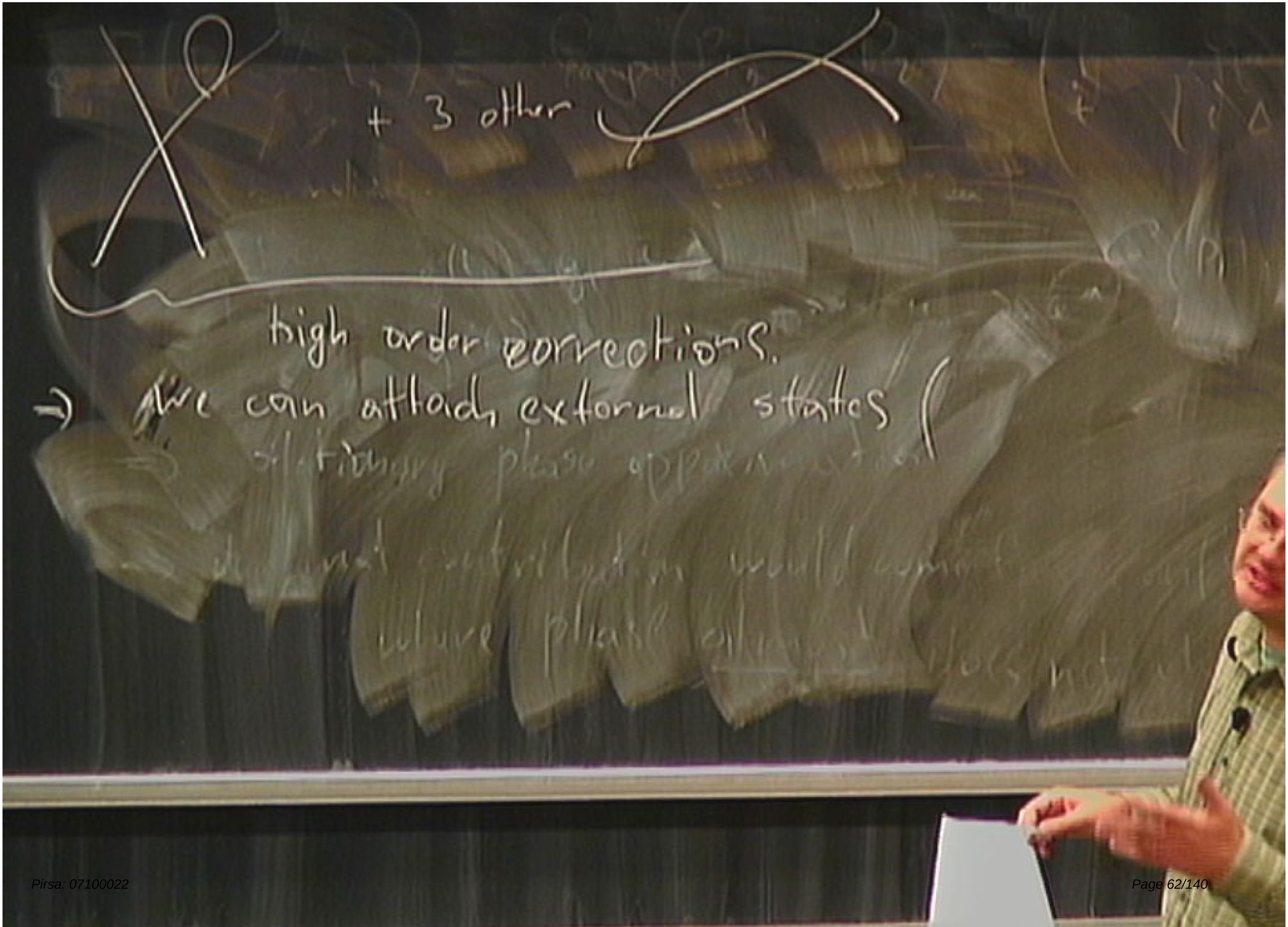


⇒ Interactions are encoded in higher point corr. f...

$$(-i\lambda) = G^{(4)}_{amp}(p_1, p_2, p_3, p_4)$$

to leading order $\Gamma^{(4)}(p_1; p_2, p_3, p_4) = -i\lambda$

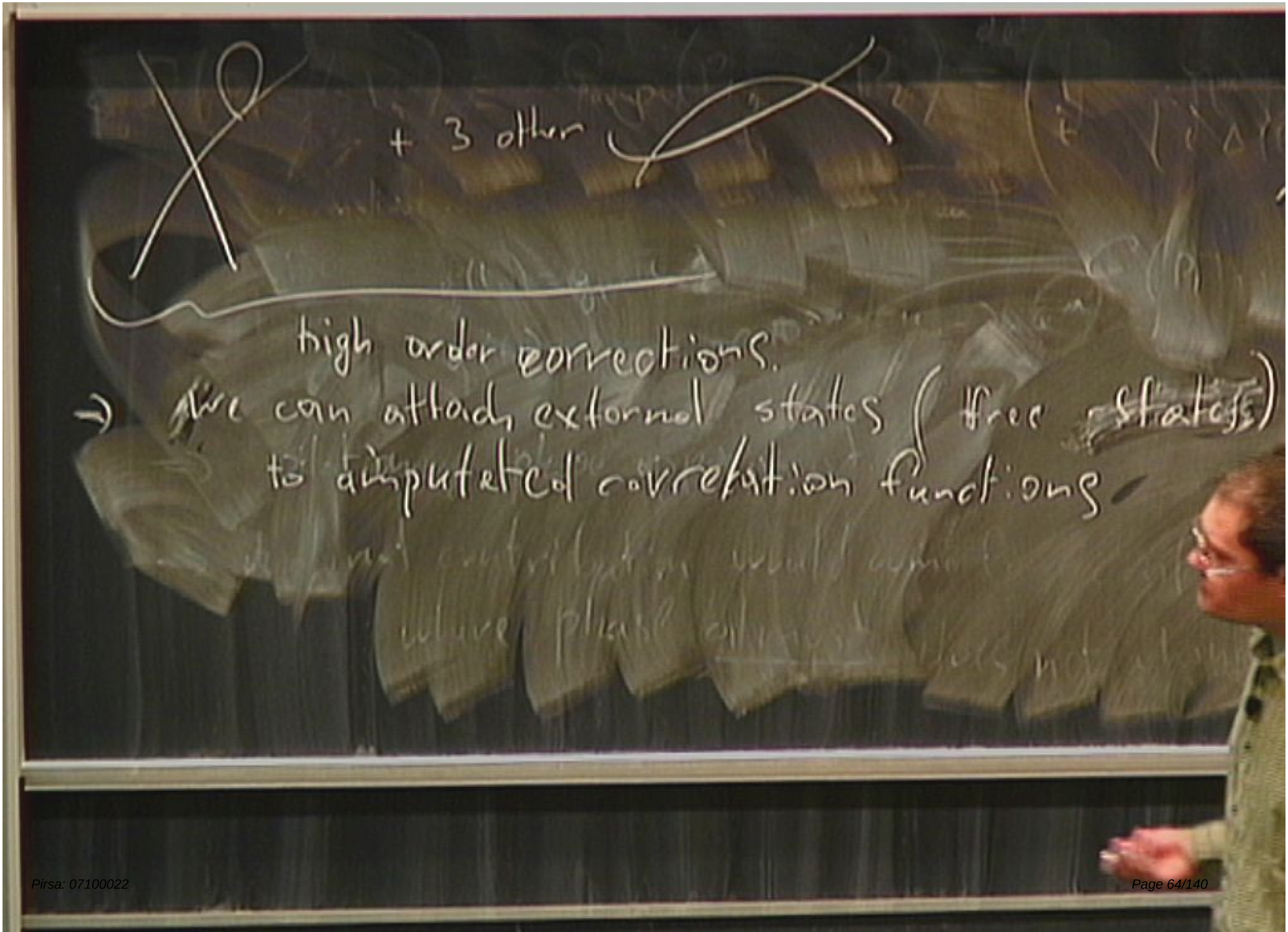
transition amplitude


+ 3 other
high order corrections.
we can attach external states

+ 3 other

high order corrections.

⇒ we can attach external states (free states)


+ 3 other
high order corrections.
⇒ we can attach external states (free states)
to amputated correlation functions

+ 3 other

high order corrections.

⇒ we can attach external states (free states)
to amputated correlation functions
to get scattering amplitude ⇒

+ 3 other

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+ 3 other

high order corrections:

⇒ we can attach external states (free states) to amputated correlation functions

to get scattering amplitude ⇒ Feynman diagrams

$$\delta V(\varphi) = \frac{m^2}{2!}\varphi^2 + \frac{\lambda}{4!}\varphi^4$$ for configurations ...

$\delta q(t)$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot q} = 0$$

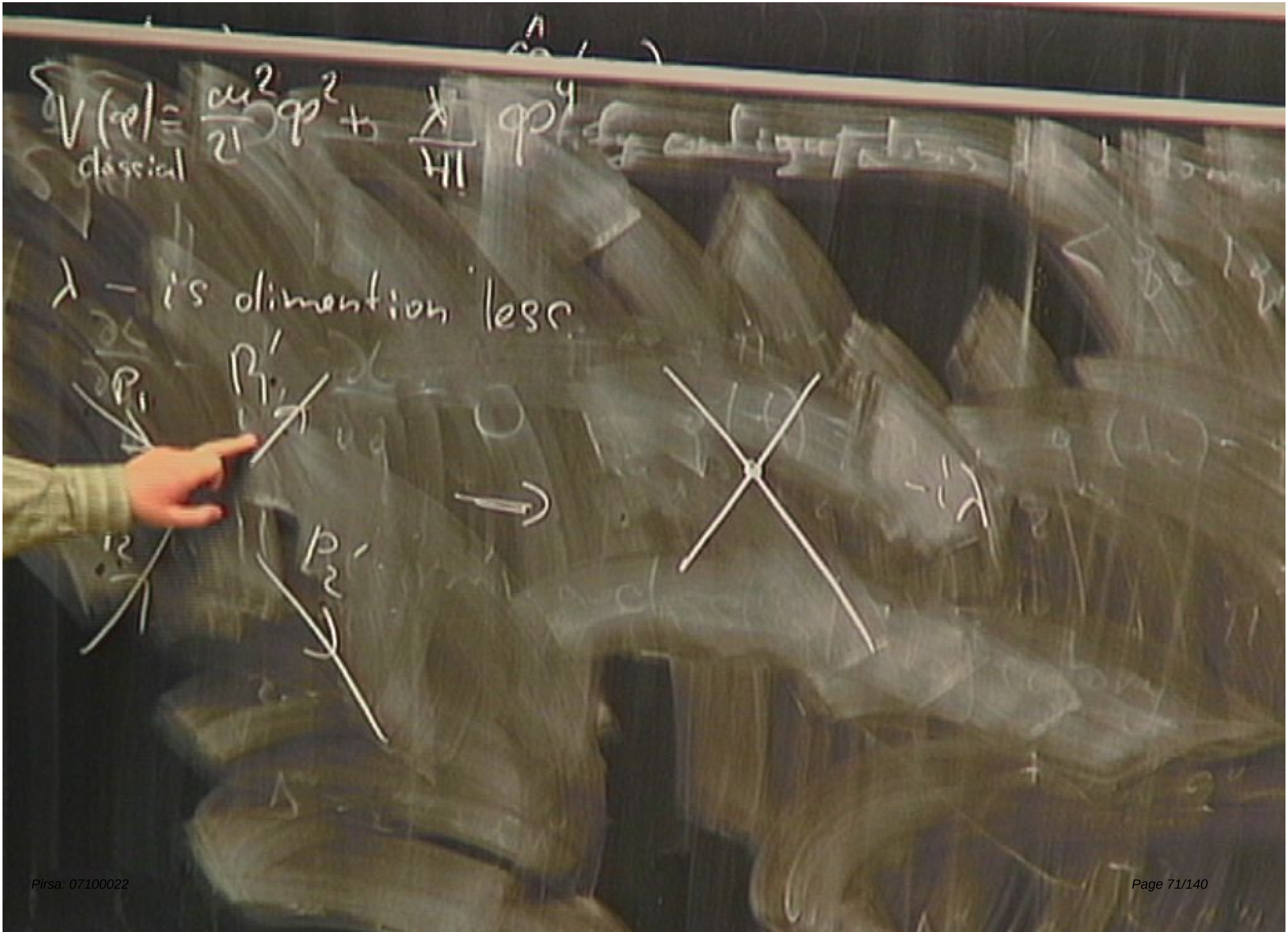
$q(-T) = q_1$ $\quad q(+T) = q_2$

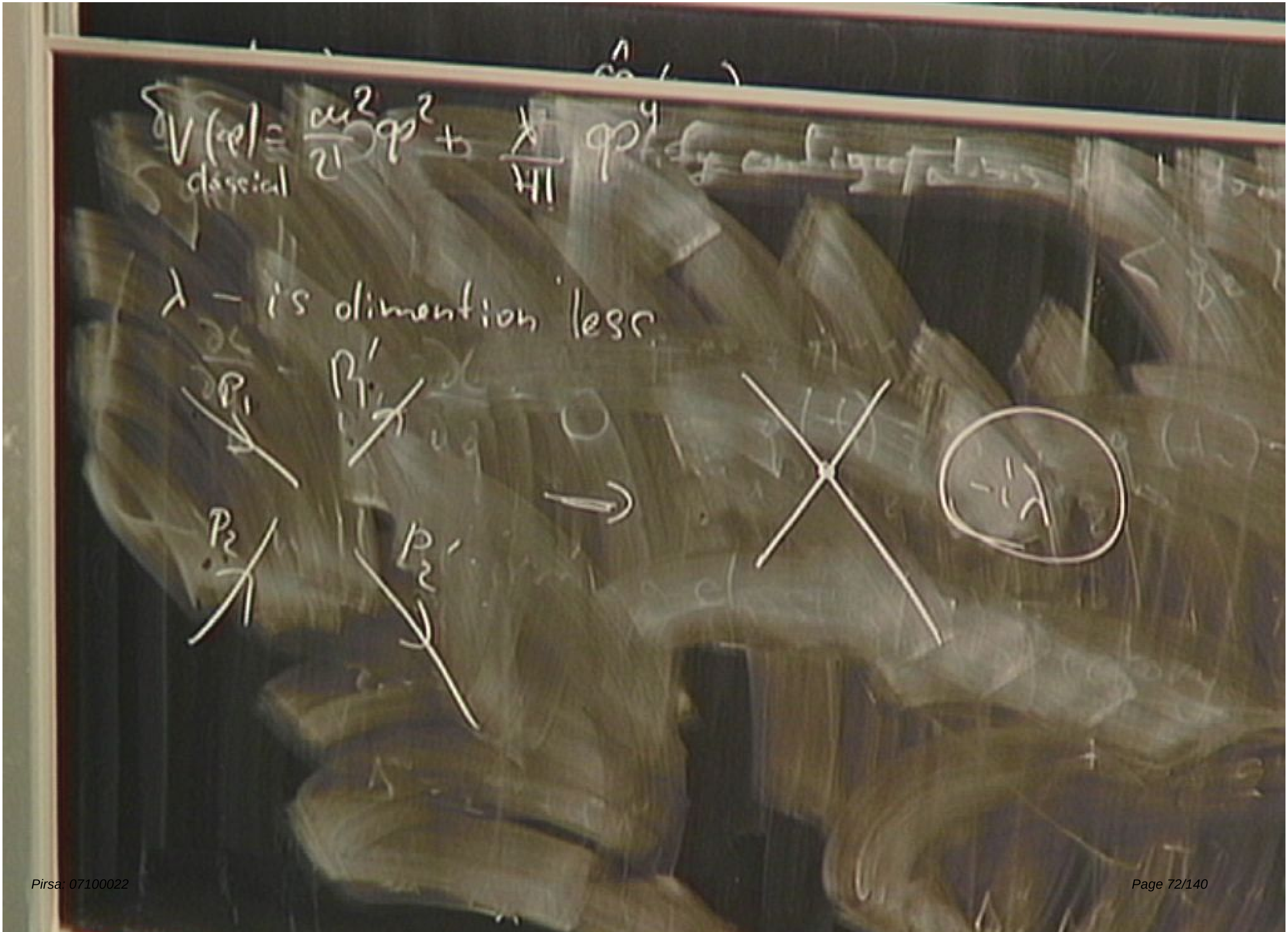
$$\delta V_{\text{classical}}(\varphi) = \frac{m^2}{2!}\varphi^2 + \frac{\lambda}{4!}\varphi^4$$

λ – is dimention less

$$\delta V(\varphi)_{\text{classical}} = \frac{m^2}{2!}\varphi^2 + \frac{\lambda}{4!}\varphi^4$$

λ – is dimention less


V(φ) = m²/2! φ² + λ/4! φ⁴
classical
λ – is dimention less
p₁'
p₂'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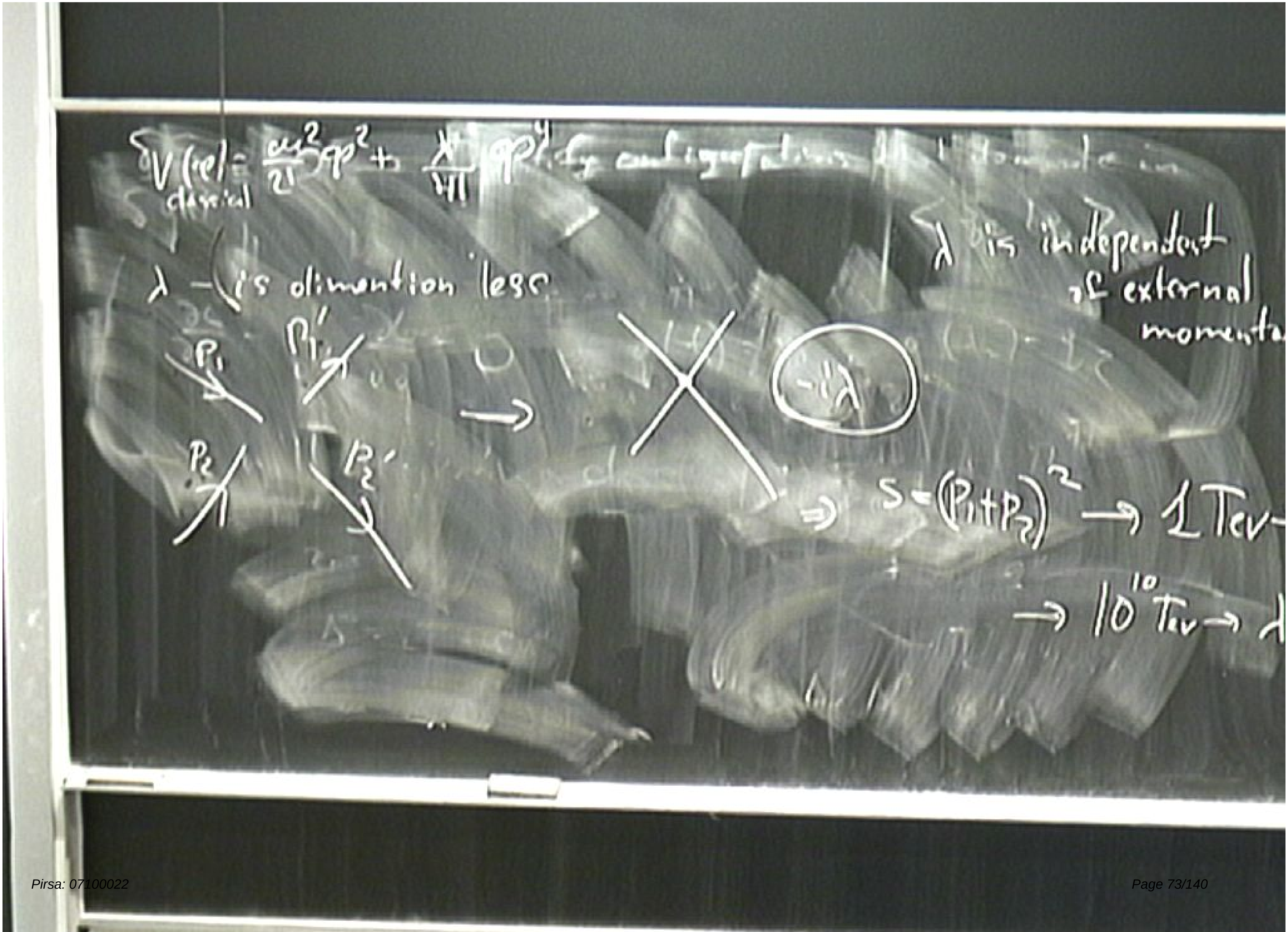
classical
λ – is dimention less
p₁ p₁'
p₂ p₂'
−iλ


V(φ) classical = m²/2! φ² + λ/4! φ⁴
λ – is dimention less
λ is independent
of external
momenta
P1 P1'
P2 P2'
-iλ
S = (P1+P2)² → 1 Tev
→ 10¹⁰ Tev → λ

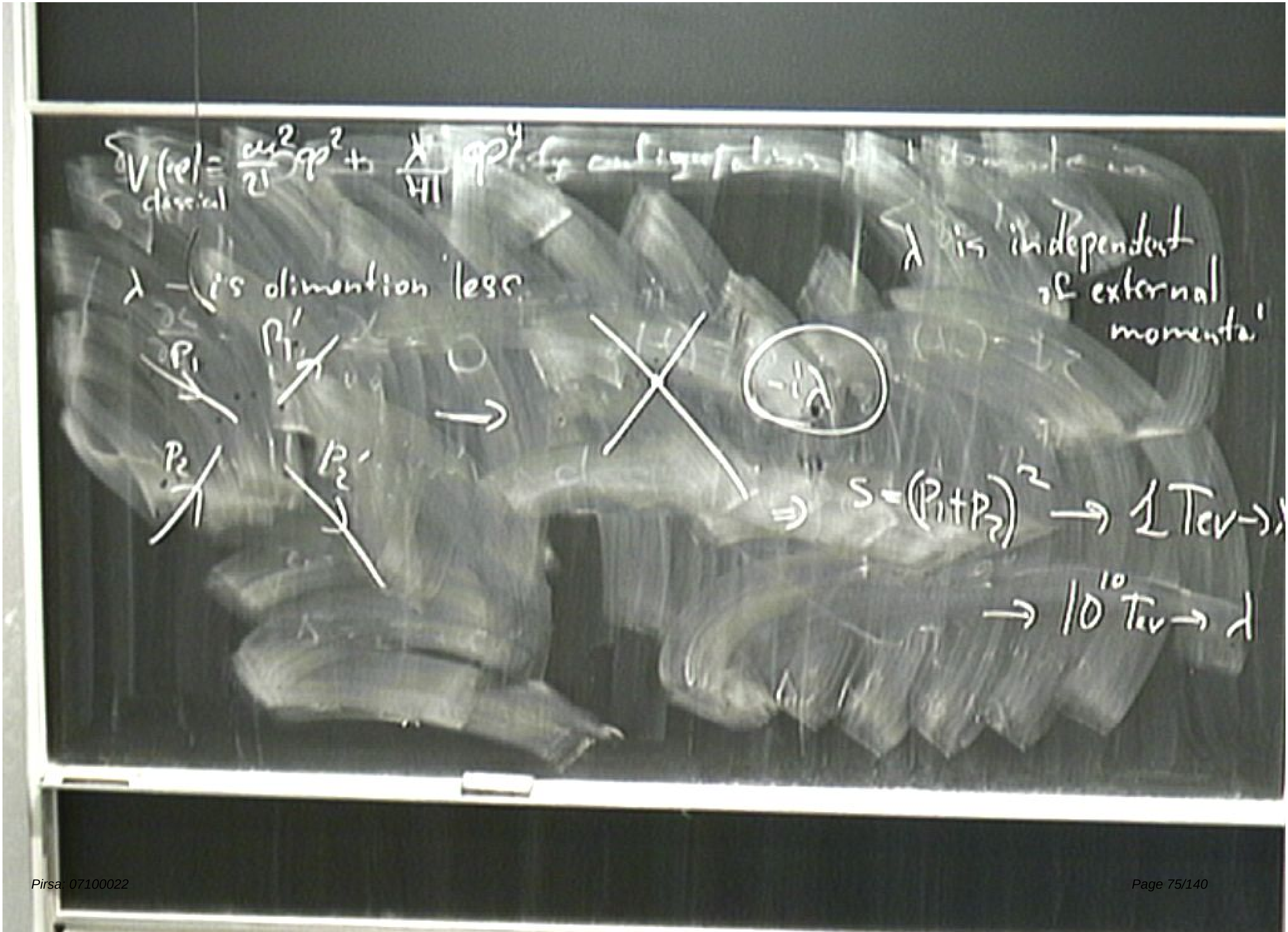
$V_{classical}(\varphi) = \frac{m^2}{2!}\varphi^2 + \frac{\lambda}{4!}\varphi^4$

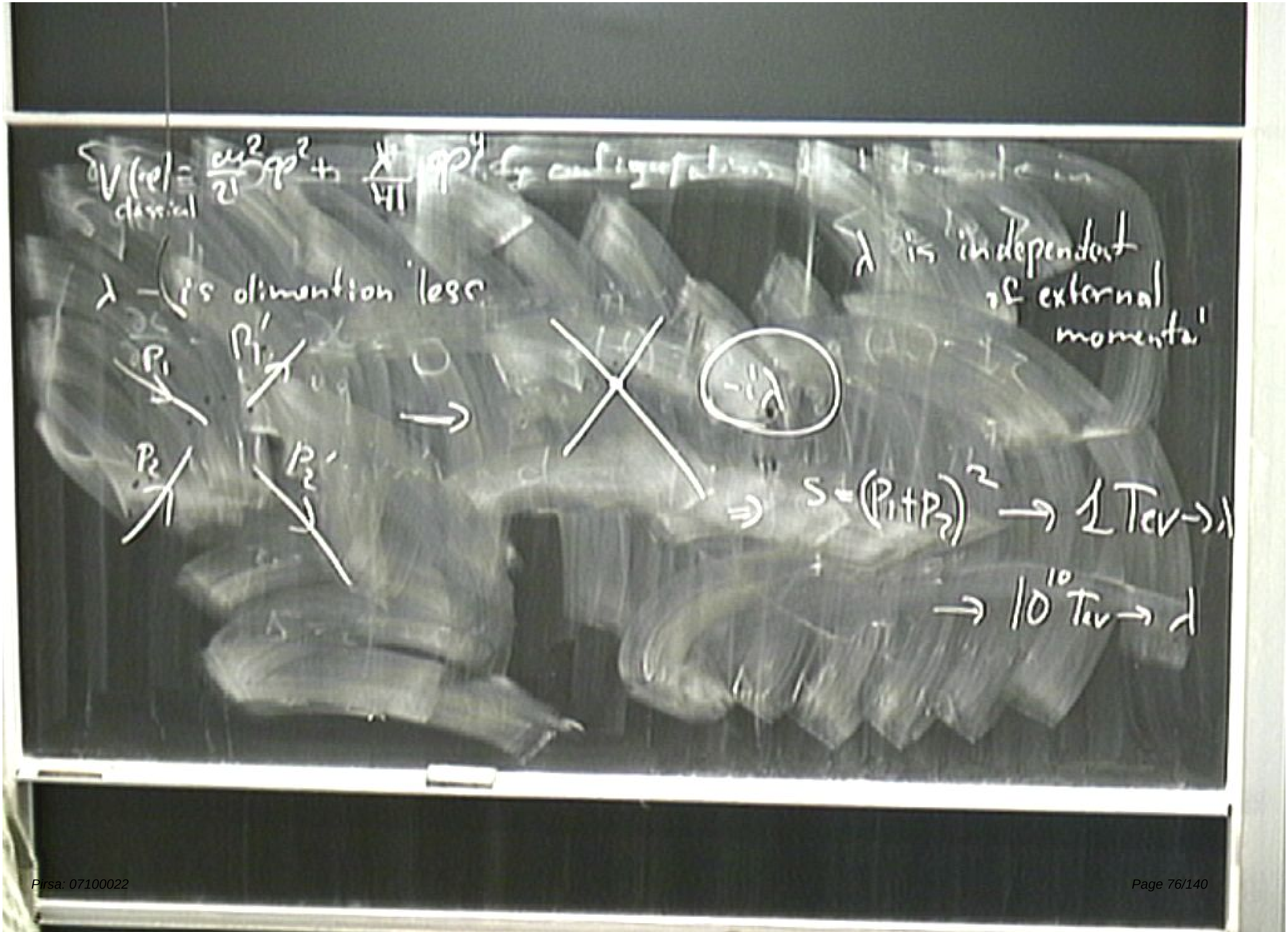
λ — is dimention less

λ is independent of external momenta

$\Rightarrow s = (p_1+p_2)^2 \to 1\ \text{Tev} \to \lambda$

$\to 10^{10}\ \text{Tev} \to \lambda$


V(φ) classical = m²/2! φ² + λ/4! φ⁴
λ — is dimention less
λ is independent of external momenta
P1
P1'
P2
P2'
-iλ
⇒ S = (P1+P2)² → 1 TeV → λ
→ 10¹⁰ TeV → λ


V(φ) = m²/2! φ² + λ/4! φ⁴ for configurations that dominate in
classical
λ – is dimention less
λ is independent of external momenta
P₁ P₁′ P₂ P₂′
−iλ
⇒ s = (P₁+P₂)² → 1 Tev → λ
→ 10¹⁰ Tev → λ

p_1 p_1'

p_2 p_2' $\longrightarrow$ $\times$ $= (-i\lambda)$

$\Rightarrow s = (p_1+p_2)^2 \longrightarrow$

$\Rightarrow$ That λ is not constant $\lambda = \lambda(s) \longrightarrow 10^{10}$ TeV

$V(\varphi)_{\text{classical}} = \frac{m^2}{2!}\varphi^2 + \frac{\lambda}{4!}\varphi^4$ for configurations ... dominate in

λ – is dimention less

λ is independent of external momenta

$\Rightarrow s = (p_1 + p_2)^2 \rightarrow 1\,\text{Tev} \rightarrow \lambda$

$\rightarrow 10^{10}\,\text{Tev} \rightarrow \lambda$

$\Rightarrow$ That λ is not constant $\quad \lambda = \lambda(s)$

$[\lambda] = 0 \Rightarrow \lambda = \lambda\left(\frac{s}{m}\right) \quad [s] = 2$ (in units of mass)

$V_{classical}(\varphi) = \frac{m^2}{2!}\varphi^2 + \frac{\lambda}{4!}\varphi^4$ for configurations ...

λ is 'independent of external momenta'

λ - is dimention less

$\Rightarrow s = (p_1 + p_2)^2 \to 1\ \text{Tev} \to \lambda$

$\to 10^{10}\ \text{Tev} \to \lambda$

$\Rightarrow$ That **λ is not constant** $\quad \lambda = \lambda(s)$

$[\lambda] = 0 \Rightarrow \lambda = \lambda\left(\frac{s}{\mu}\right)$ $\quad [s] = 2$ (in units of mass)

$\to \mu$ is some finite scale

$T(\hat{\varphi}(t_1)\,\hat{O}(t_2)) = \begin{cases} \hat{O}(t_2)\,\hat{O}(t_1), & t_2 > t_1 \end{cases}$

$V_{classical}(\varphi) = \frac{m^2}{2!}\varphi^2 + \frac{\lambda}{4!}\varphi^4$

λ – is dimention less.

λ is independent of external momenta

$\longrightarrow$

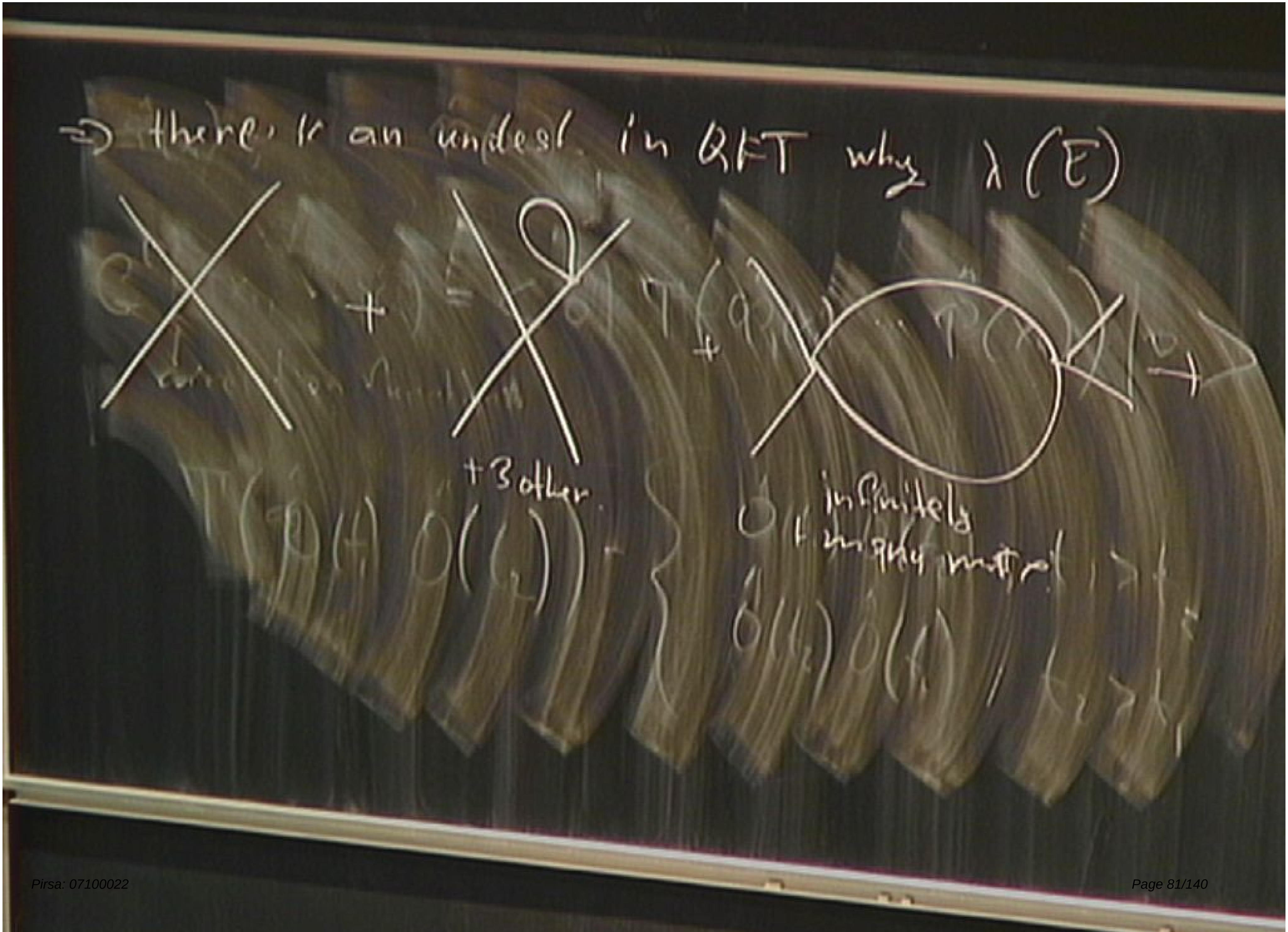
$\Rightarrow s = (p_1+p_2)^2 \longrightarrow 1\ \text{TeV} \rightarrow \lambda$

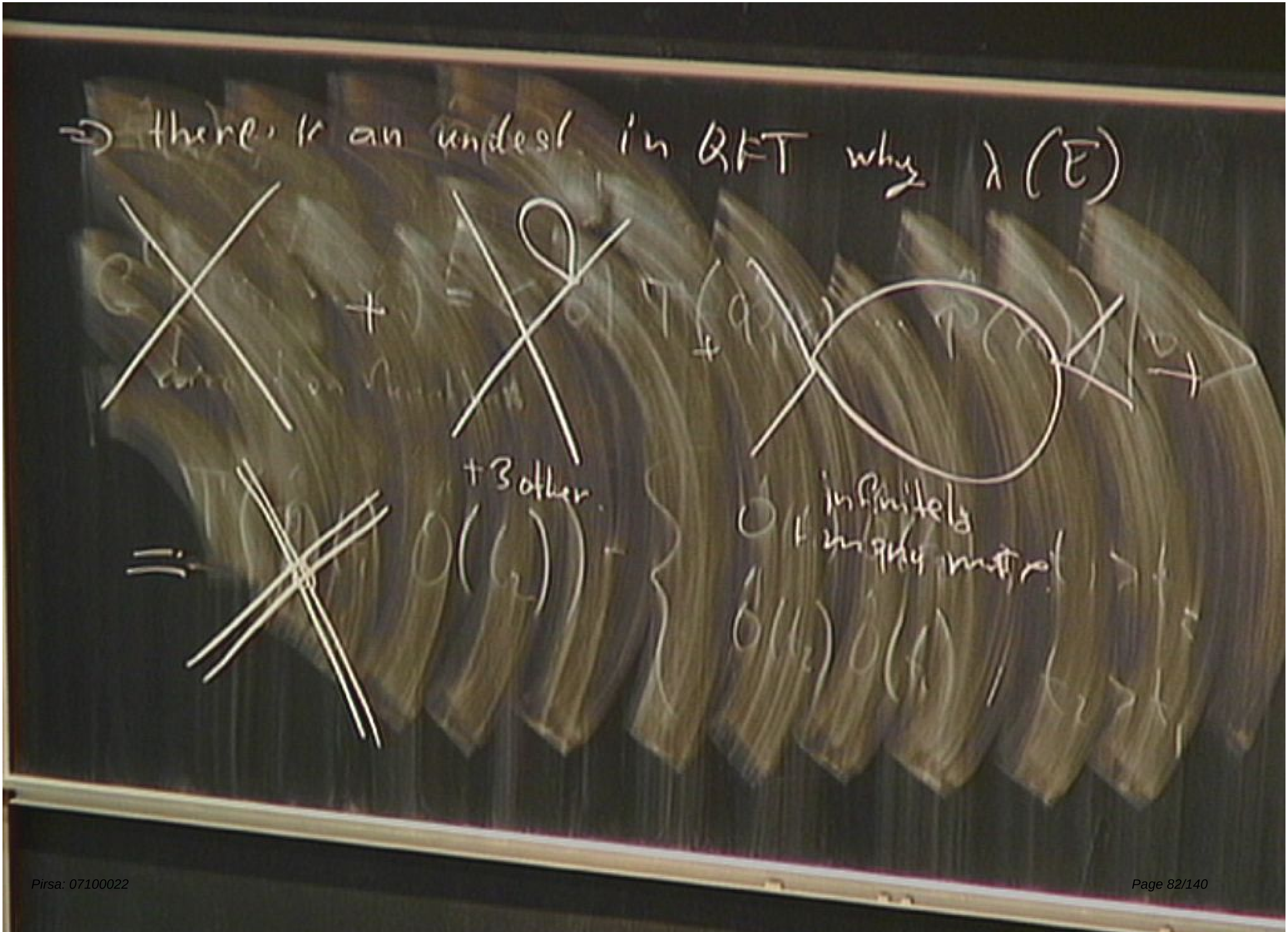
$\longrightarrow 10^{10}\ \text{TeV} \rightarrow \lambda$

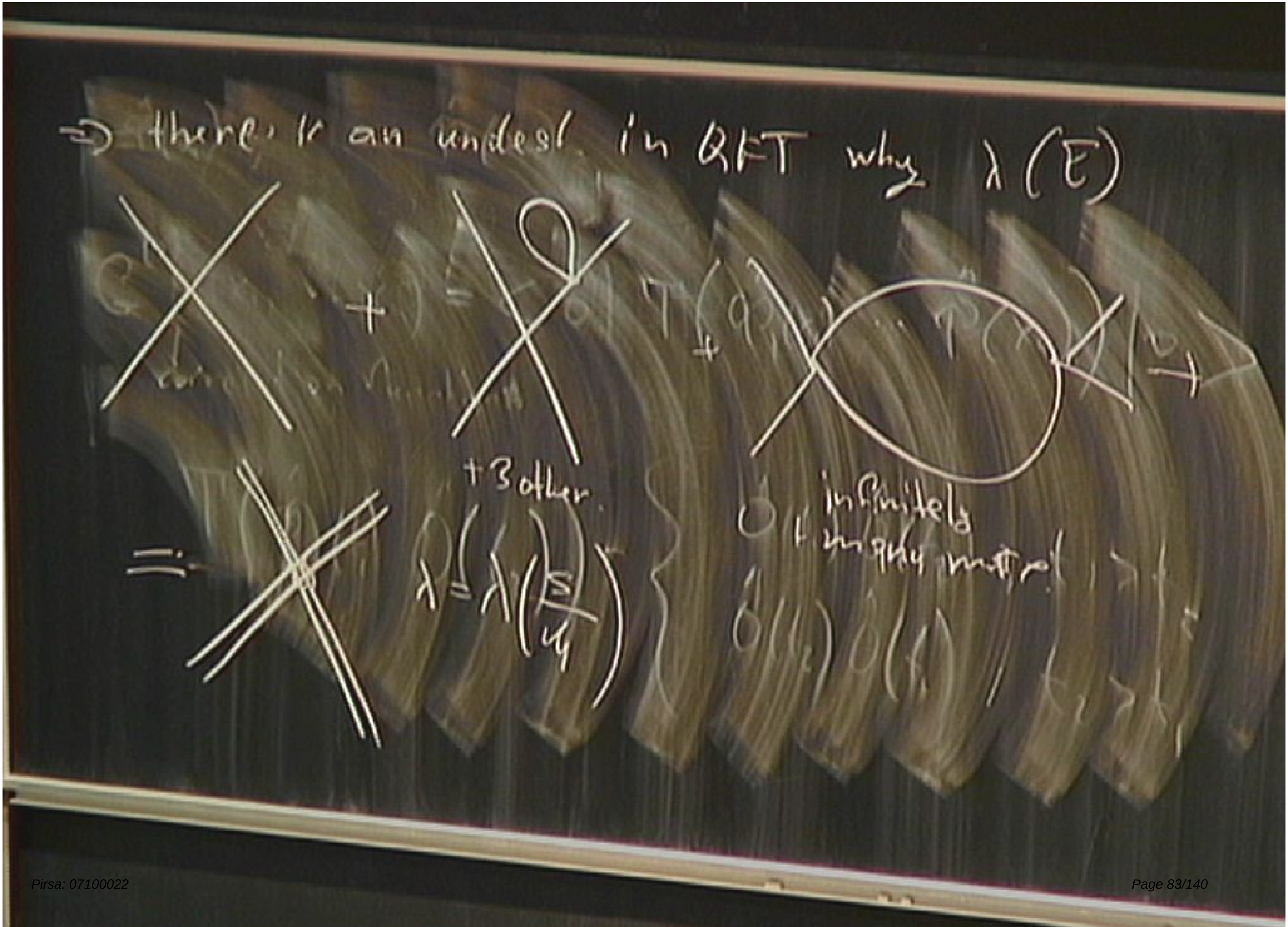
$\Rightarrow$ That <u>λ is not constant</u> $\quad \lambda = \lambda(s)$

$[\lambda] = 0 \Rightarrow \lambda = \lambda\left(\frac{s}{\mu^2}\right) \longrightarrow \mu$ is some finite scale $\quad [s] = 2$ (in units of mass)

$T\left(\hat{O}(t_1)\,\hat{O}(t_2)\right) = \begin{cases} \hat{O}(t_2)\,\hat{O}(t_1), & t_2 > t_1 \end{cases}$


⇒ there is an undest. in QFT why λ(E)
+3 other
infinitely


in QFT why λ(E)
+3 other
infinitely


in QFT why λ (E)
+3 other
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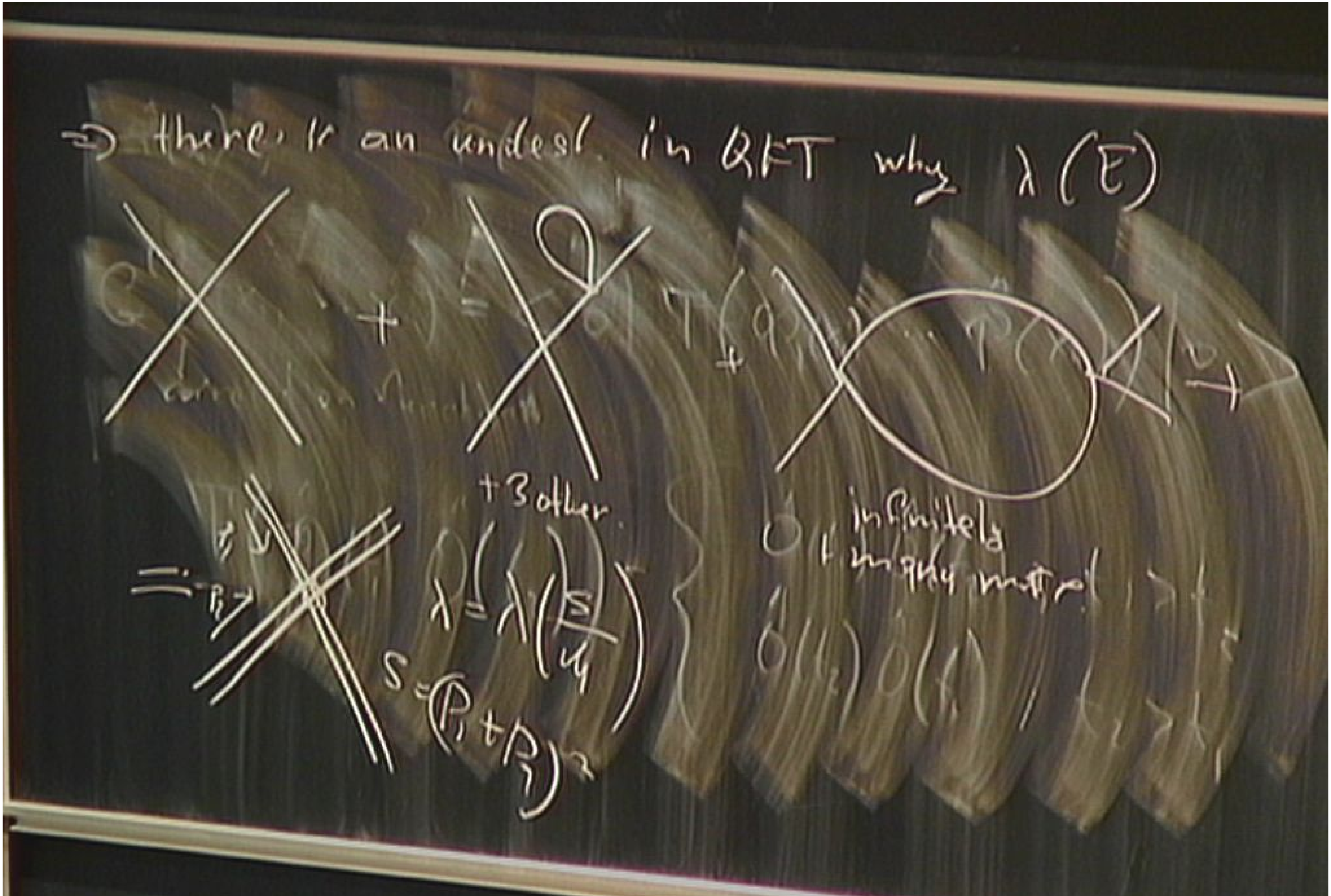
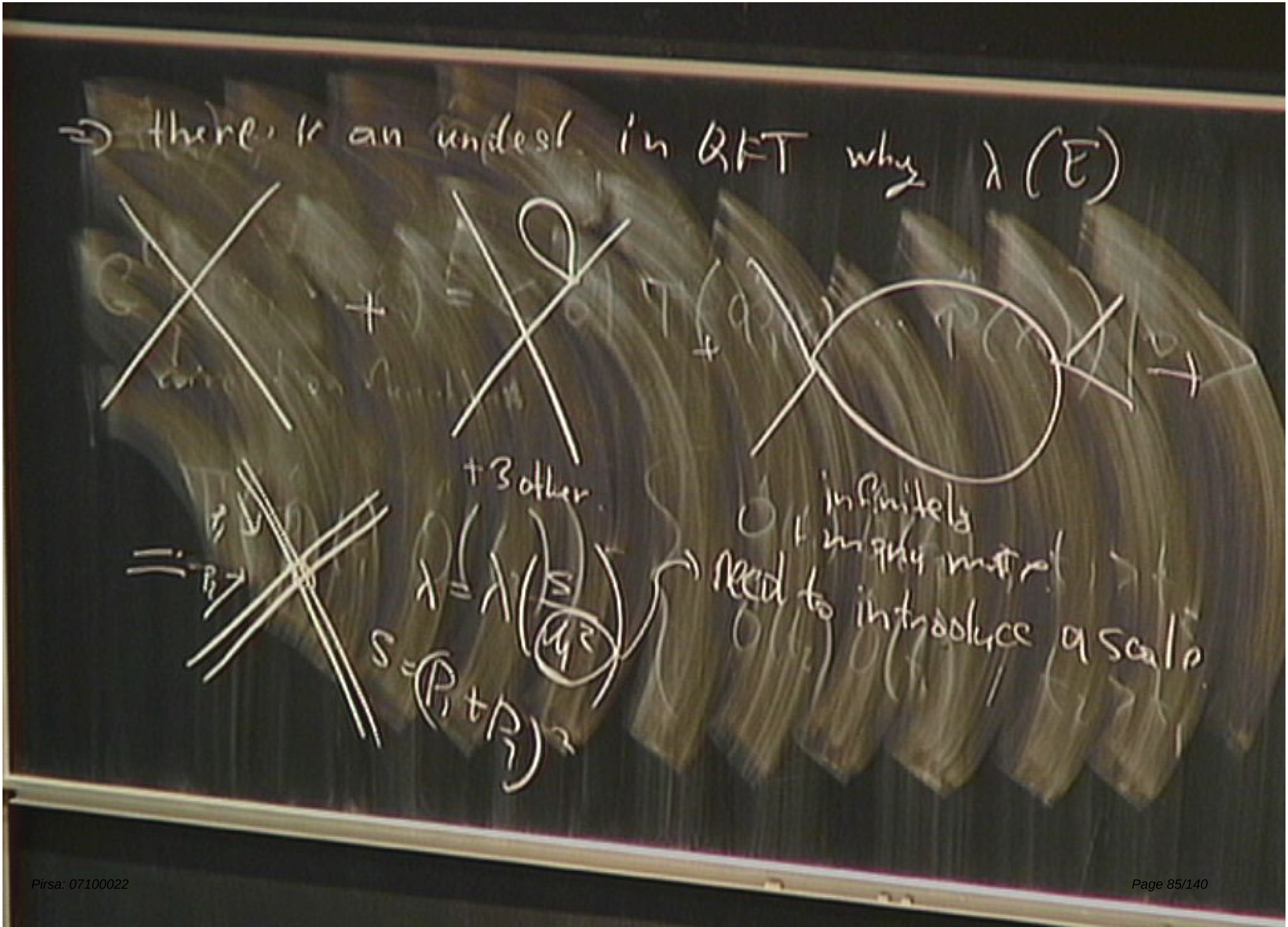
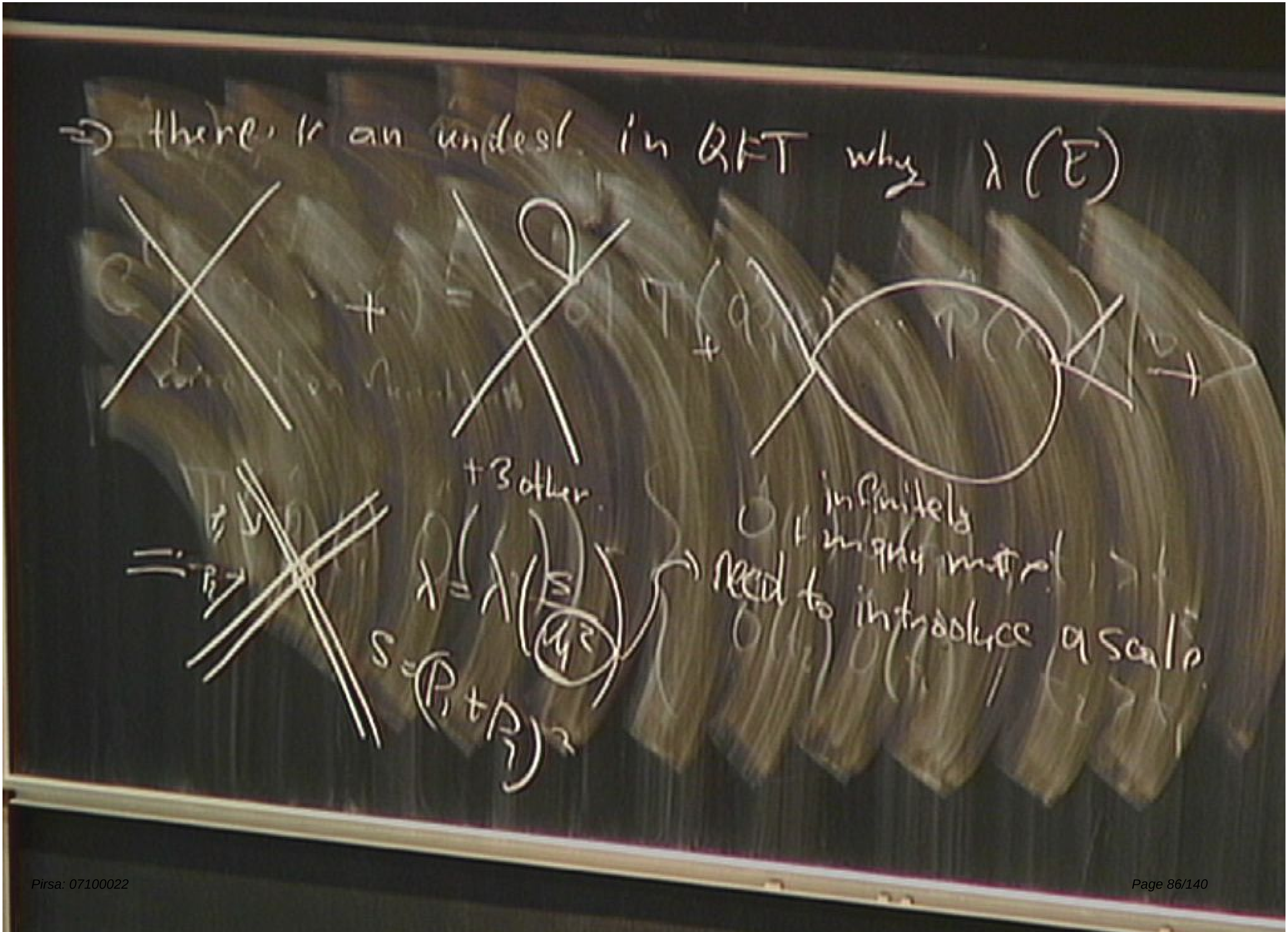

⇒ there is an undest. in QFT why λ(E)
+3 other
infinitely
λ = λ(s/μ4)
s = (p1+p2)2


⇒ there is an undest. in QFT why λ(E)
+ 3 other
infinitely
need to introduce a scale
λ = λ(s/μ²)
s = (p₁+p₂)²


⇒ there is an undest. in QFT why λ(E)
+3 other
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λ = λ(s/μ²)
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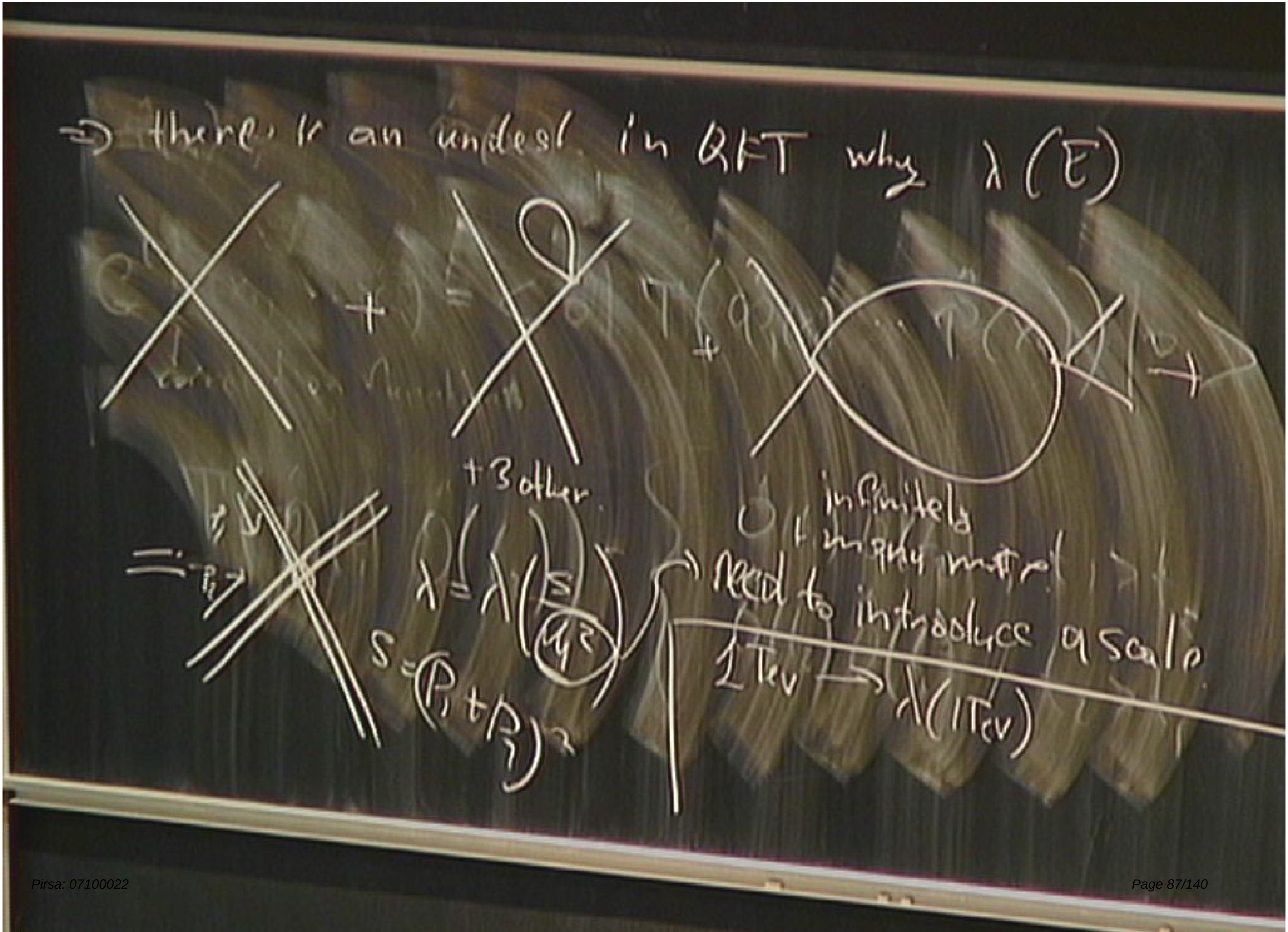
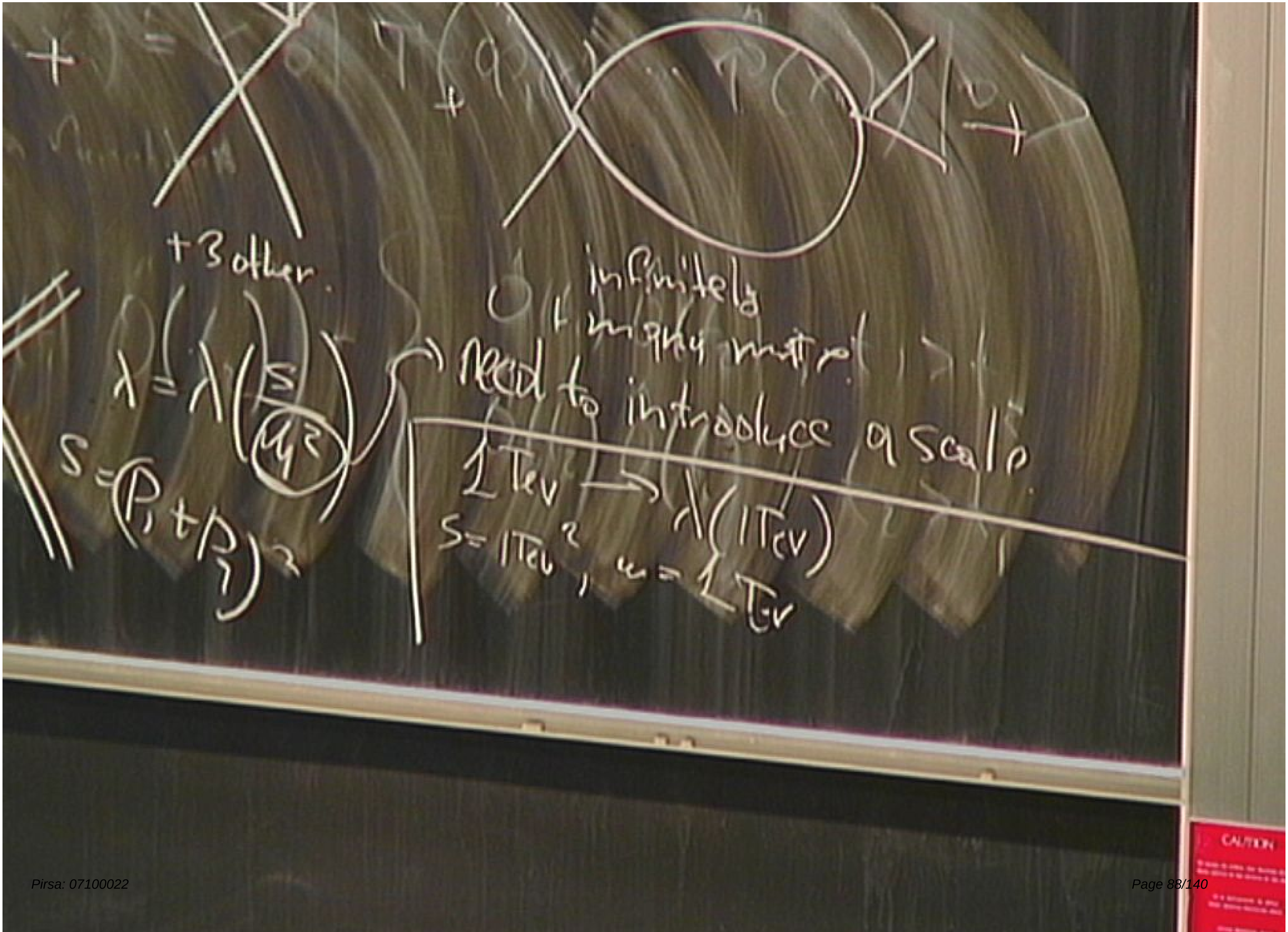
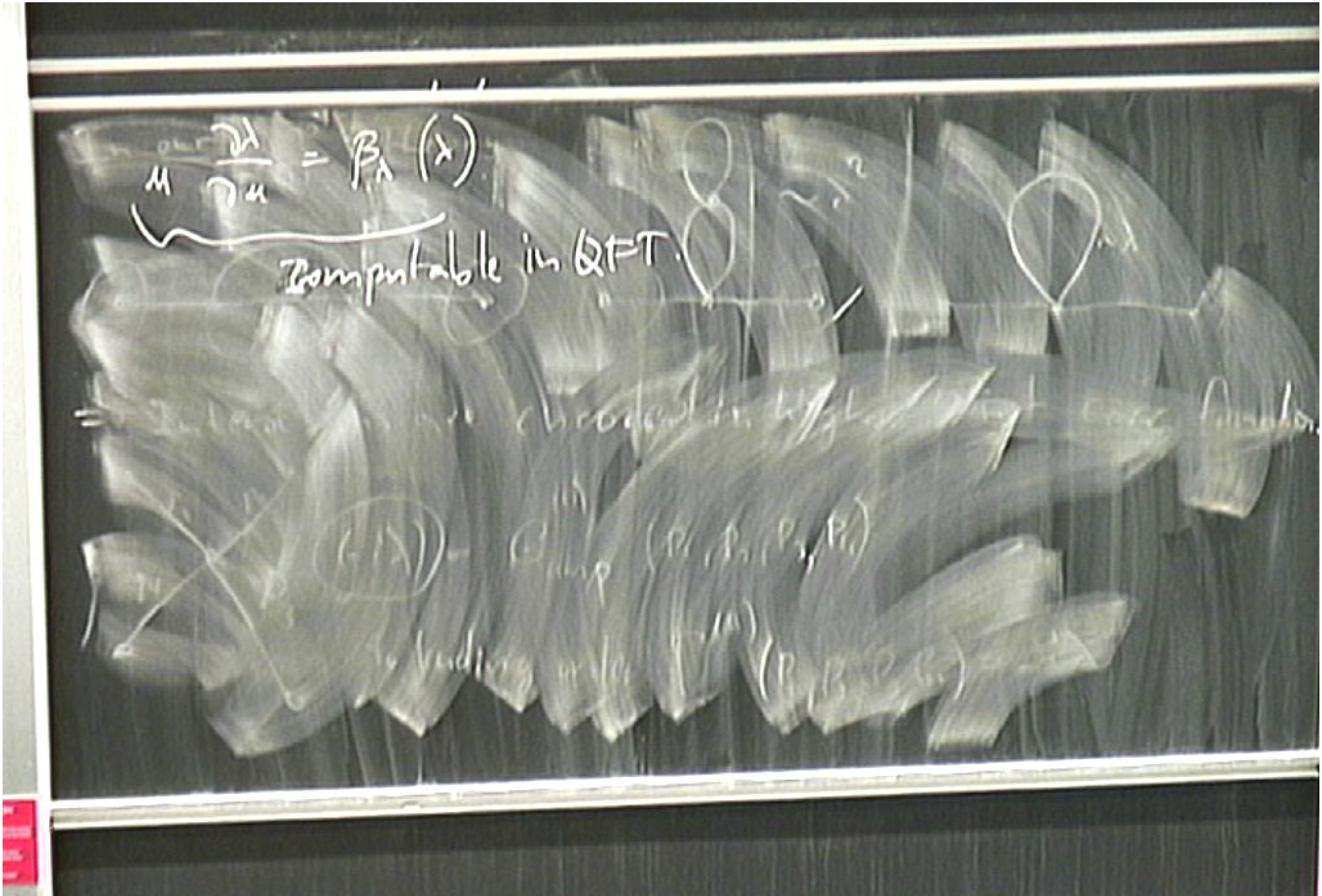

⇒ there is an undest. in QFT why λ(E)
+ 3 other
λ = λ(s, μ^2)
s = $(p_1+p_2)^2$
infinitely
need to introduce a scale
1 TeV ⟶ λ(1 TeV)


+3 other
infinitely
need to introduce a scale
1 TeV
λ(1 TeV)
CAUTION


$\mu \frac{\partial \lambda}{\partial \mu} = \beta_\lambda(\lambda)$
computable in QFT.

$$\mu \frac{\partial \lambda}{\partial \mu} = \beta_\lambda(\lambda)$$

Computable in QFT.

Your experiment @ 1 TeV $\rightarrow$ IC for $\lambda(1.5)$

$$\mu \frac{\partial \lambda}{\partial \mu} = \beta_\lambda(\lambda)$$

Computable in QFT.

Your experiment @ 1 TeV → ΣC for $\lambda(s)$

$\beta_\lambda| = 0$ → can happen sometimes.

$$\mu \frac{\partial \lambda}{\partial \mu} = \beta_\lambda(\lambda)$$

Computable in QFT.

Your experiment @ 1 TeV → IC for $\lambda(s)$

$\beta_\lambda(\lambda) = 0$ → can happen sometime.

$\mu \frac{\partial \lambda}{\partial \mu} = 0 \rightarrow \lambda = \text{const}$

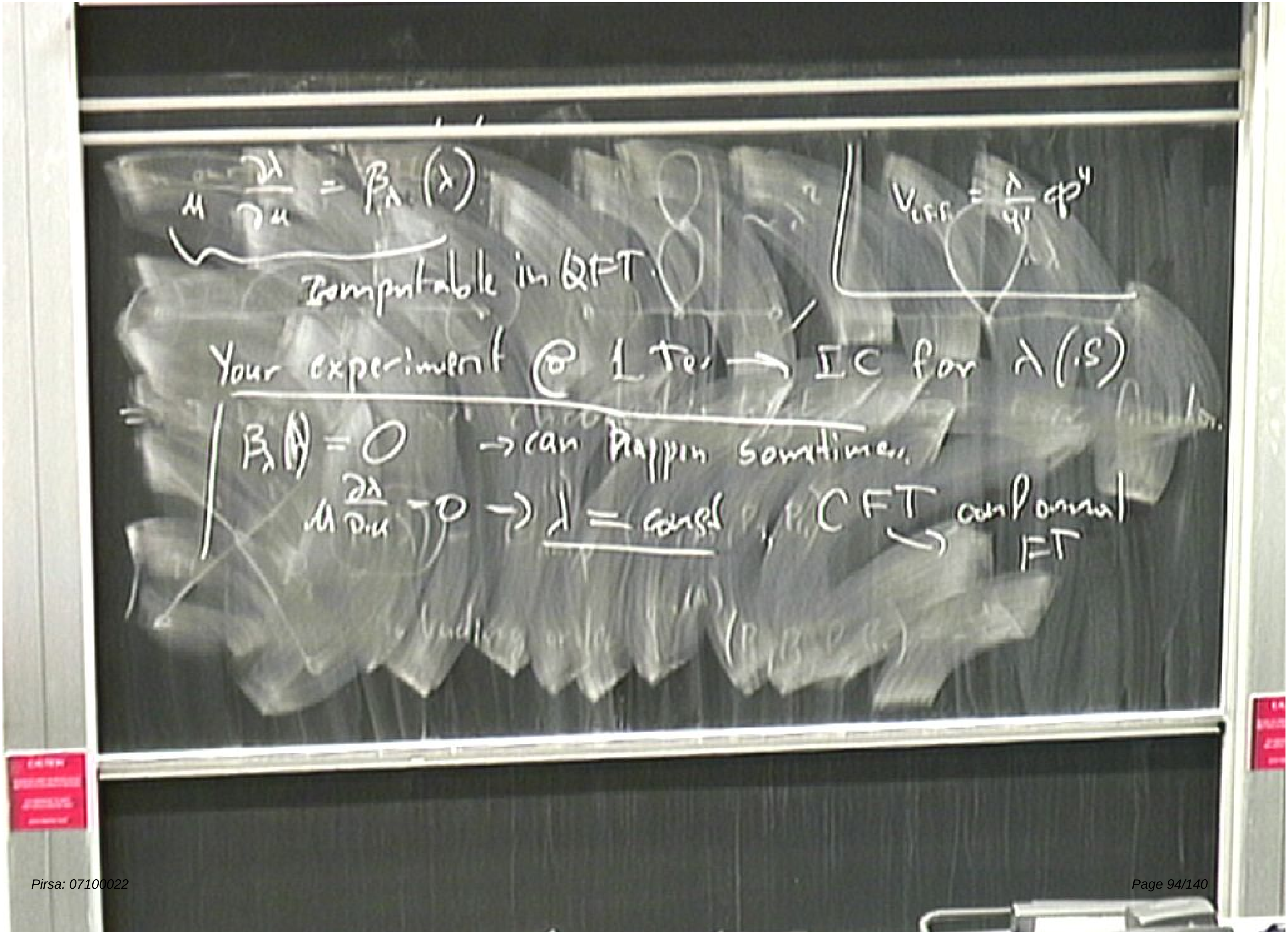
$$\mu \frac{\partial \lambda}{\partial \mu} = \beta_\lambda(\lambda)$$

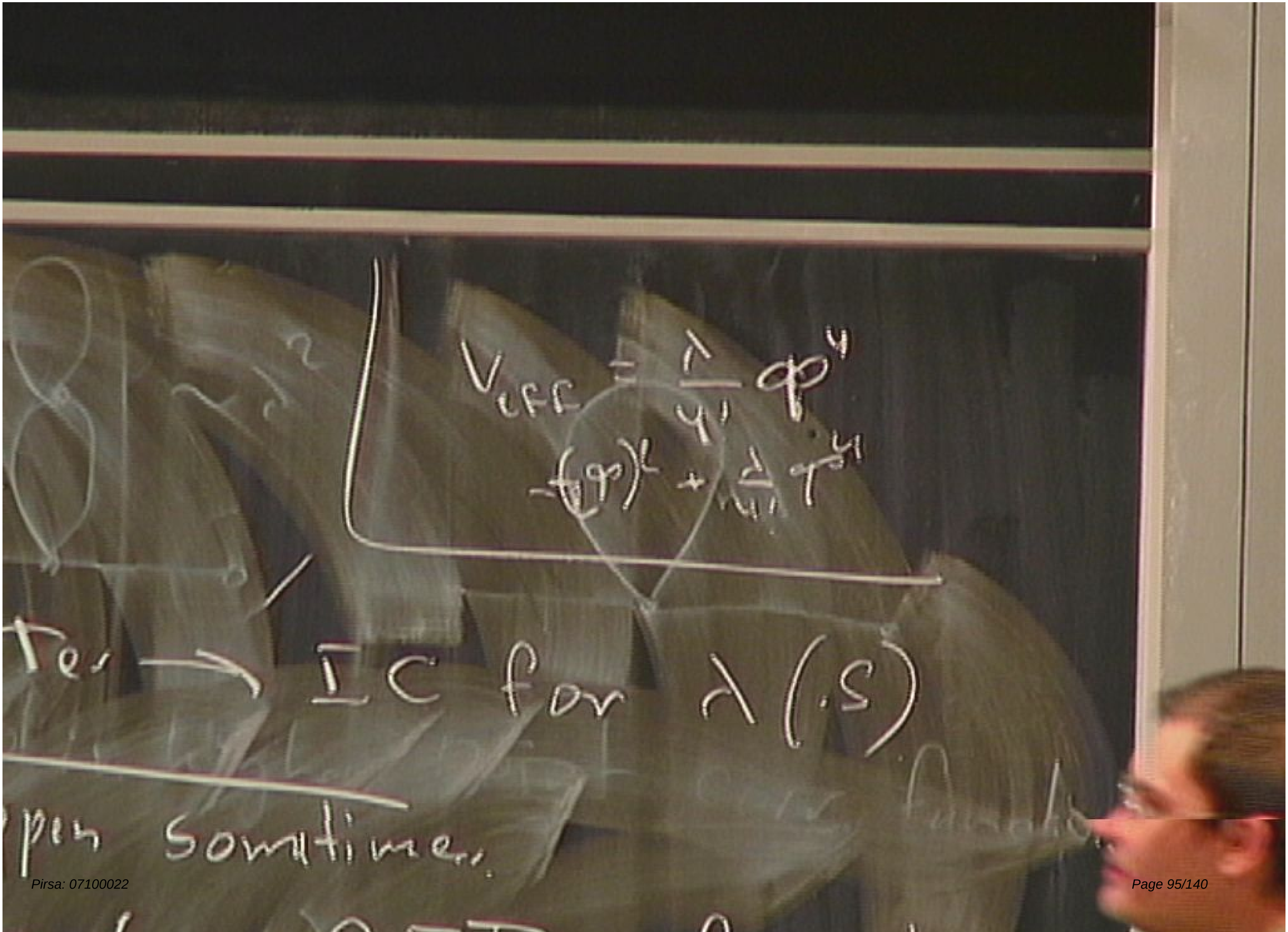
computable in QFT.

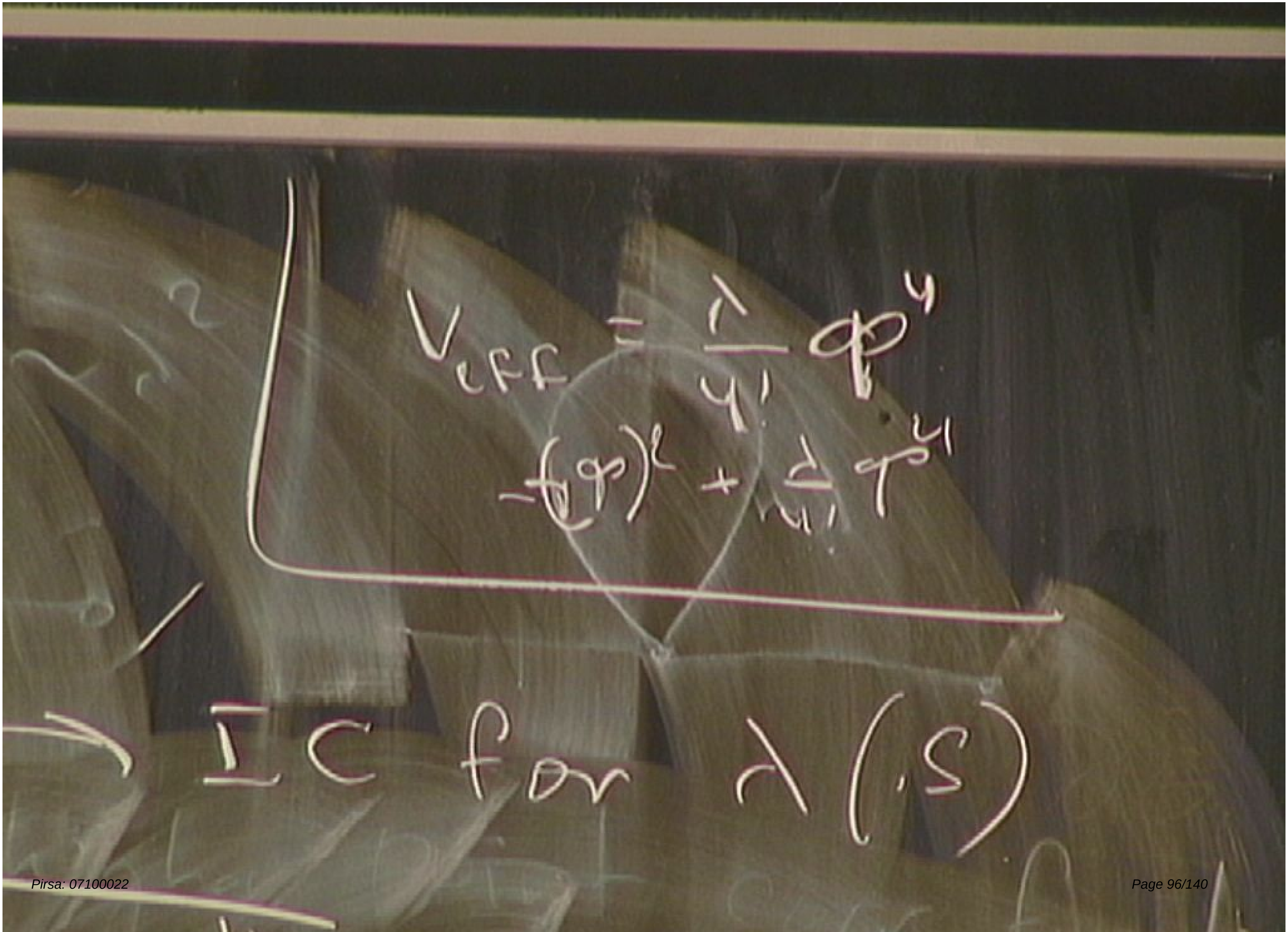
Your experiment @ 1 TeV → IC for $\lambda(s)$

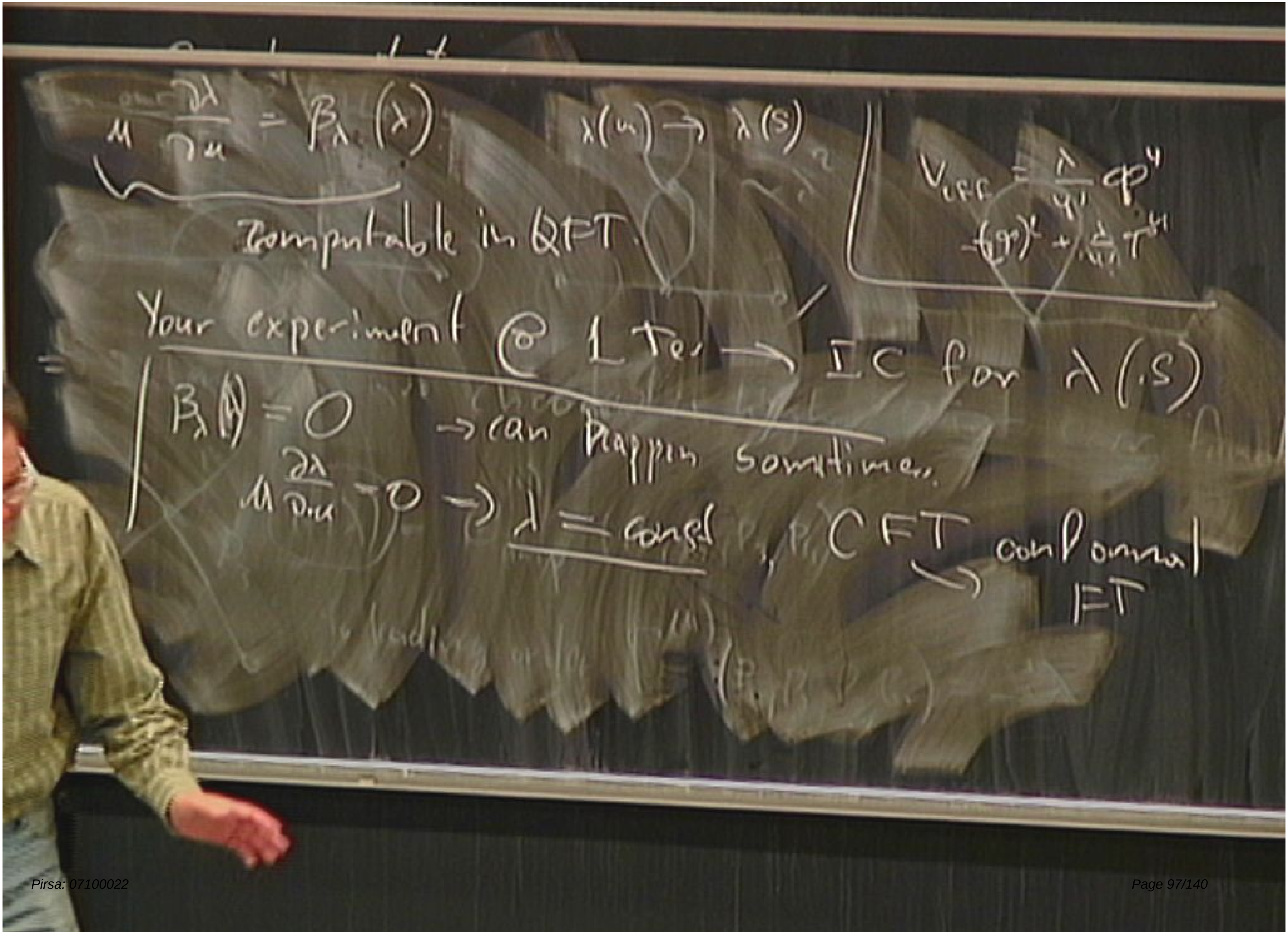
$\beta_\lambda(\lambda) = 0$ → can happen sometimes.

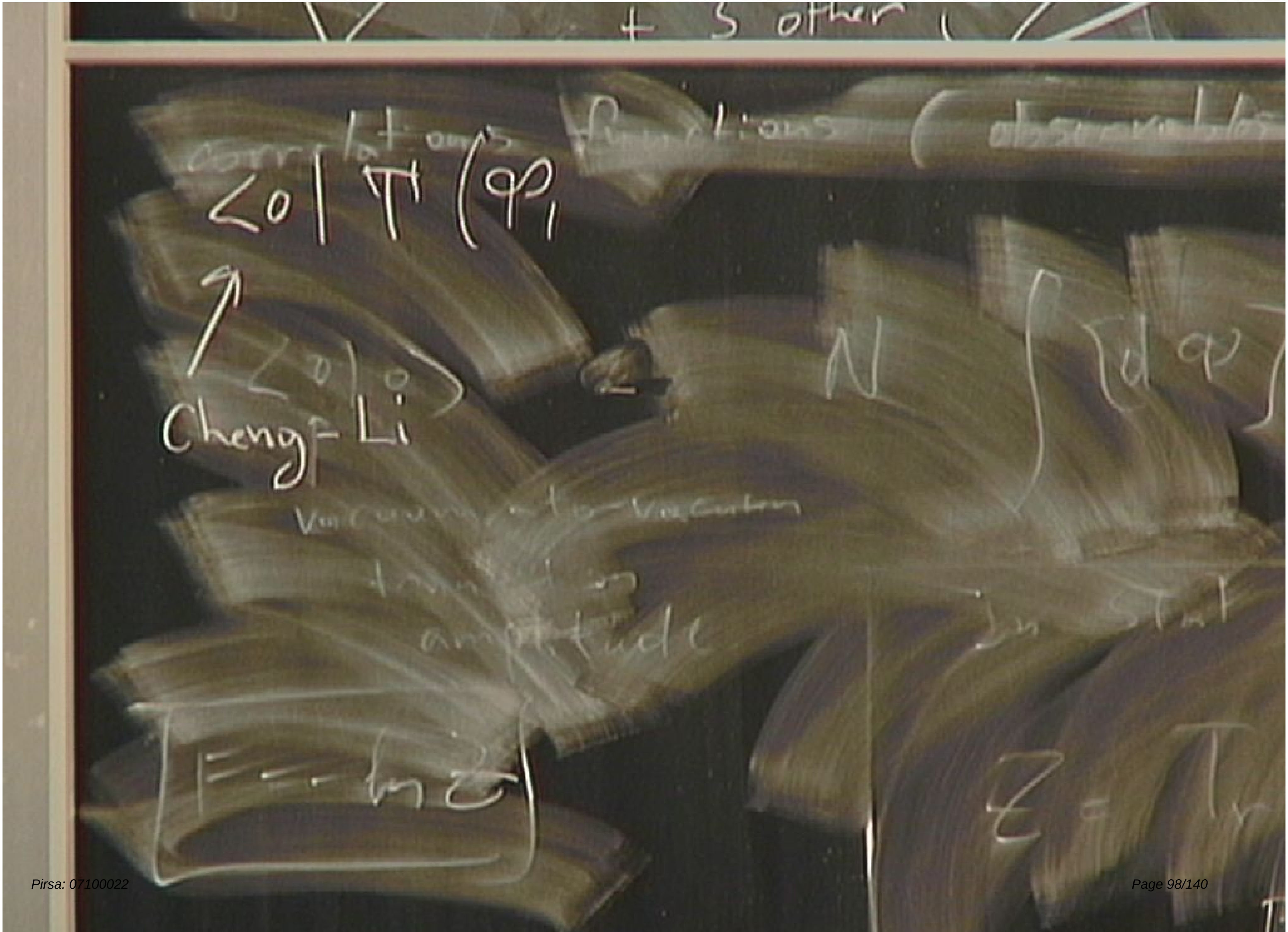
$\mu \frac{\partial \lambda}{\partial \mu} = 0$ → $\lambda = \text{const}$, CFT → conformal FT


μ ∂λ/∂μ = β_λ(λ)
computable in QFT
V_eff = λ/4! φ^4
Your experiment @ 1 TeV → IC for λ(.5)
β_λ(λ) = 0 → can happen sometimes
μ ∂λ/∂μ = 0 → λ = const
CFT → conformal FT


V_eff = λ/4! φ^4
IC for λ(s)
sometimes


$V_{eff} = \frac{\lambda}{4!}\varphi^4$
$-(\partial\varphi)^2 + \frac{\lambda}{4!}\varphi^4$
IC for λ(.5)

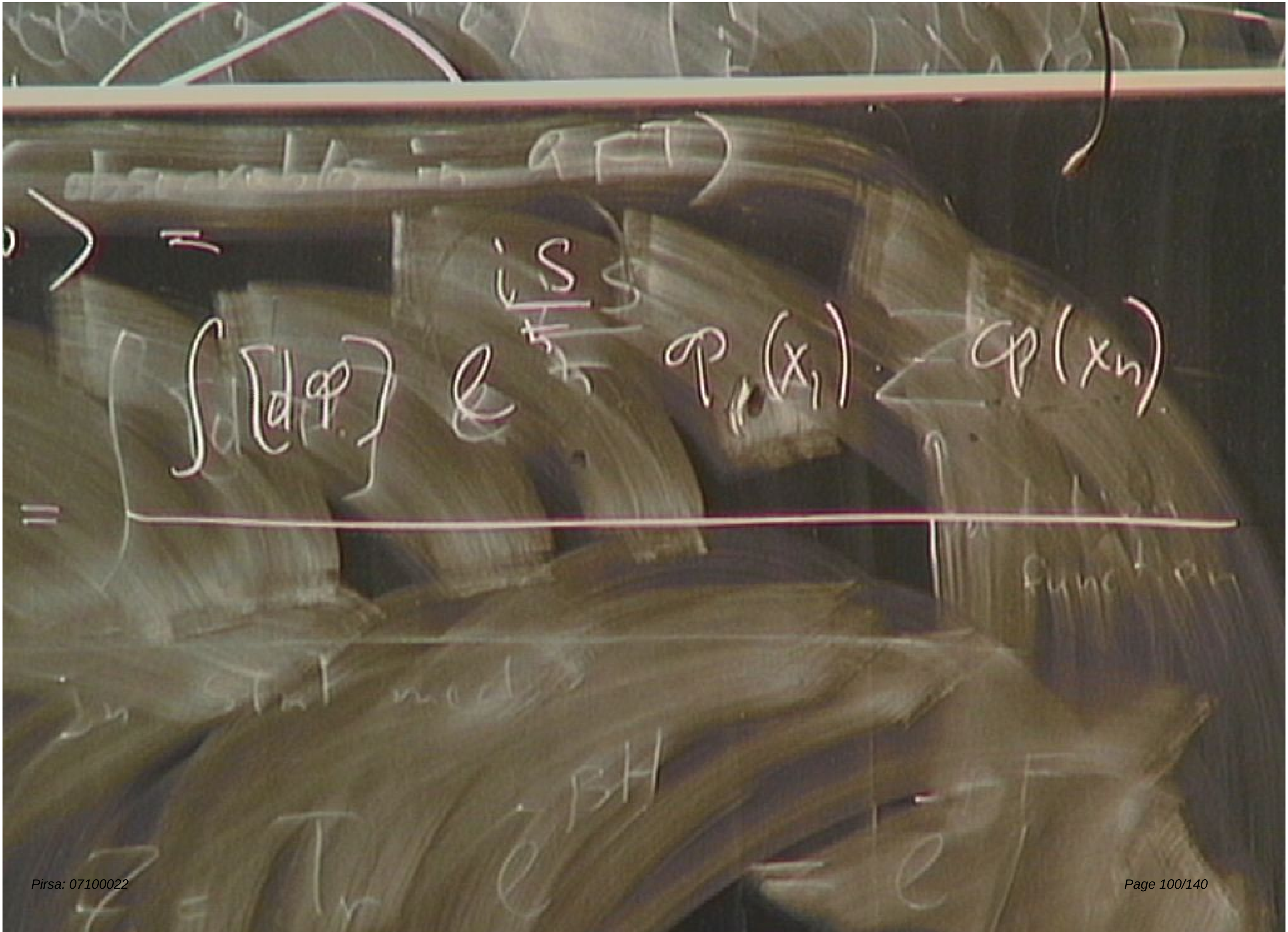

μ ∂λ/∂μ = β_λ(λ)
Computable in QFT.
λ(μ) → λ(s)
V_eff = λ/4! φ^4
Your experiment @ 1 TeV → IC for λ(s)
β_λ(λ) = 0 → can happen sometimes.
μ ∂λ/∂μ = 0 → λ = const , CFT
conformal FT


+ 5 other
correlations functions
<0| T (φ
<0|0>
Cheng-Li
vacuum-to-vacuum
transition amplitude
F = - ln Z
N ∫ [dφ
Z = Tr

correlation functions (observables in QFT)

$$\langle 0 | T(\varphi_1, \ldots \varphi_n) | 0 \rangle = N \int [d\varphi]\, e^{\frac{iS}{\hbar}}\, \varphi_1(x_1) \ldots \varphi(x_n)$$

Cheng-Li


observables in QFT
$= \int [d\varphi]\, e^{\frac{iS}{\hbar}}\, \varphi_1(x_1) \cdots \varphi(x_n)$
function

$$\ldots 0\rangle = \frac{\int [d\varphi]\, e^{\frac{iS}{\hbar}}\, \varphi_1(x_1) \cdots \varphi(x_n)}{\int [d\varphi]\, e^{\frac{iS}{\hbar}}}$$

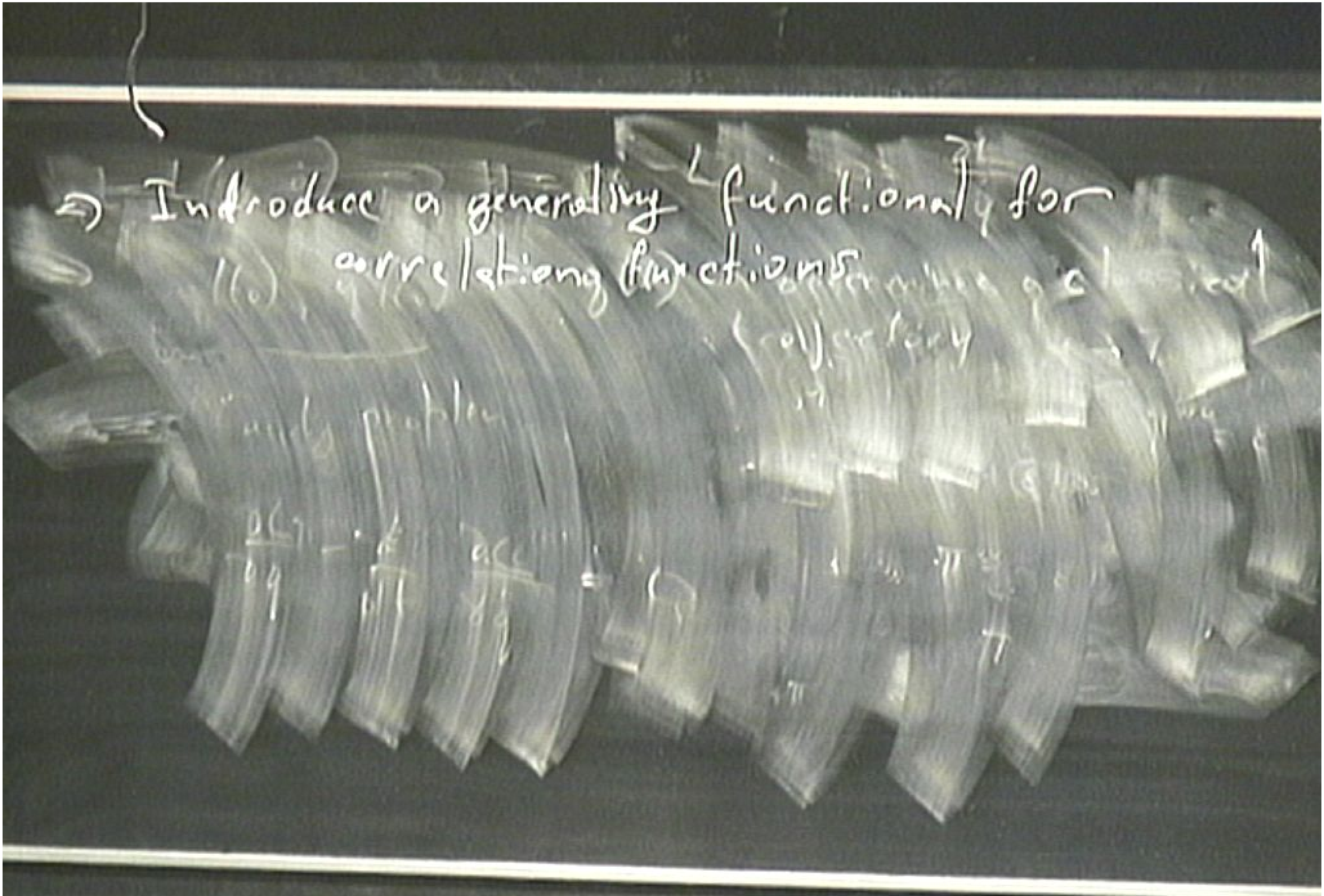
(observables in QFT)

correlation function

$$\langle 0 | T(\varphi_1 \cdots \varphi_n) | 0 \rangle = N \frac{\int [d\varphi]\, e^{\frac{iS}{\hbar}}\, \varphi_1(x_1) \cdots \varphi(x_n)}{\int [d\varphi]\, e^{\frac{iS}{\hbar}}}$$

Cheng-Li

$$F = -\ln Z$$


⇒ Introduce a generating functional for
correlation functions

⇒ Introduce a generating functional for correlation functions.

$Z[J(x)]$

J — an arbitrary classical source

⇒ Introduce a generating functional for correlations functions

$$Z[J(x)] = N \int [d\phi]\, e^{\frac{iS}{\hbar} + \int d^4x\, J(x)\phi(x)}$$

$J(x)$: an arbitrary classical source

⇒ Introduce a generating functional for correlation functions.

$$Z[J(x)] = N \int [d\phi]\, e^{\frac{iS}{\hbar} + i\int d^4x\, J(x)\phi(x)}$$

J: an arbitrary classical source

$Z\big|_{J=0}$

⇒ Introduce a generating functional for correlation functions

$$Z[J(x)] = N \int [d\phi]\, e^{\frac{iS}{\hbar} + i\int d^4x\, J(x)\phi(x)}$$

$J(x)$: an arbitrary classical source

$$Z\big|_{J=0} = \langle 0|0\rangle$$

Boundary

$$\frac{\delta Z}{\delta J(x_1)}$$

$$\frac{\delta Z}{\delta J(x_1)}$$

$$\frac{\delta}{\delta\varphi(x)} \int J(x)\varphi(x)\, dx = \int J(x)$$

Def $\frac{\delta\varphi(x)}{\delta\varphi(x_1)} \stackrel{def}{=} \delta(x-x_1)$?

$$\frac{\delta Z}{\delta J(x_1)}$$

$$\frac{\delta}{\delta \varphi(x)} \int J(x)\varphi(x)\, dx = \int J(x)\, \delta(x - x_1)\, dx$$

Def $\quad \dfrac{\delta \varphi(x)}{\delta \varphi(x_1)} \stackrel{\text{def}}{=} \delta(x - x_1)$?

$$\frac{\delta Z}{\delta J(x_1)}$$

$$\frac{\delta}{\delta\varphi(x)}\int J(x)\varphi(x)\,dx = \int J(x)\,\delta(x-x_1)\,dx = J(x_1)$$

Def $\frac{\delta\varphi(x)}{\delta\varphi(x_1)} \overset{def}{=} \delta(x-x_1)$?

$$\frac{\delta Z}{\delta J(x_1)}$$

$$\frac{\delta}{\delta \varphi(x)} \int J(x)\varphi(x)\, dx = \int J(x)\, \delta(x-x_1)\, dx = J(x_1)$$

Def $\frac{\delta \varphi(x)}{\delta \varphi(x_1)} \stackrel{\text{def}}{=} \delta(x-x_1)$

$$\frac{\delta Z}{\delta J(x_1)} =$$

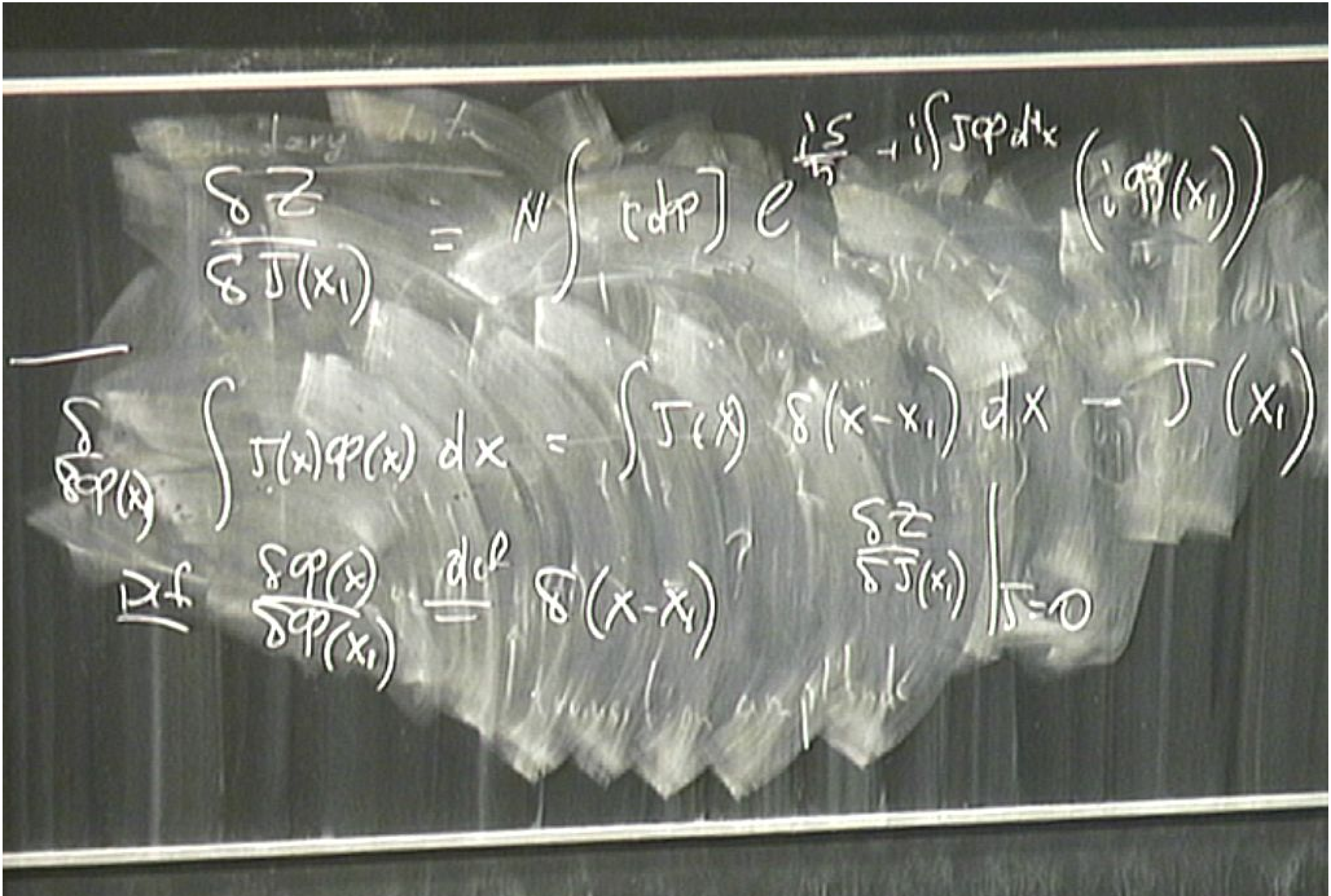
$$\frac{\delta}{\delta \varphi(x)} \int J(x)\varphi(x)\, dx = \int J(x)\, \delta(x-x_1)\, dx = J(x_1)$$

Def $\quad \frac{\delta \varphi(x)}{\delta \varphi(x_1)} \overset{\text{def}}{=} \delta(x-x_1)$

$$\frac{\delta Z}{\delta J(x_1)} = N \int [d\varphi]\, e^{\frac{iS}{\hbar} + i\int J\varphi\, d^4x} \left(i\,\varphi(x_1) \right)$$

$$\frac{\delta}{\delta\varphi(x)} \int J(x)\varphi(x)\, dx = \int J(x)\, \delta(x-x_1)\, dx = J(x_1)$$

Def $\frac{\delta\varphi(x)}{\delta\varphi(x_1)} \stackrel{def}{=} \delta(x-x_1)$?


$\frac{\delta Z}{\delta J(x_1)} = N \int [d\varphi]\, e^{\frac{iS}{\hbar} + i\int J\varphi\, d^4x} \left(i\varphi(x_1) \right)$
$\frac{\delta}{\delta\varphi(x)} \int J(x)\varphi(x)\, dx = \int J(x)\, \delta(x - x_1)\, dx = J(x_1)$
Def $\frac{\delta\varphi(x)}{\delta\varphi(x_1)} \overset{def}{=} \delta(x - x_1)$?
$\frac{\delta Z}{\delta J(x_1)}\Big|_{J=0}$

$$\frac{\delta Z}{\delta J(x_1)} = N \int [d\varphi]\, e^{\frac{iS}{\hbar} + i\int J\varphi\, d^4x} \left(i\varphi(x_1) \right)$$

$$\frac{\delta}{\delta\varphi(x)} \int J(x)\varphi(x)\, dx = \int J(x)\, \delta(x - x_1)\, dx = J(x_1)$$

$$\frac{\delta Z}{\delta J(x_1)}\bigg|_{J=0} = i \langle 0|T\varphi_1(x_1)|0\rangle$$

Def $\frac{\delta\varphi(x)}{\delta\varphi(x_1)} \overset{def}{=} \delta(x - x_1)$

$$\left.\frac{\delta^{(n)} Z}{\delta J(x_1)\cdots J(x_n)}\right|_{J=0} = \left(\frac{i}{i}\right)^n \cdot G^{(n)}(x_1, \ldots, x_n)$$

$$= (i)^n \langle 0 | T(\varphi(x_1) \ldots \varphi(x_n)) | 0 \rangle$$

$$\left.\frac{\delta^{(n)} Z}{\delta J(x_1)\cdots J(x_n)}\right|_{J=0} = \left(\frac{1}{i}\right)^n G^{(n)}(x_1,\dots,x_n)$$

$$= \left(\frac{1}{i}\right)^n \langle 0|T(\varphi(x_1)\cdots\varphi(x_n))|0\rangle$$

Divergences

$$\mathcal{L}_0 = -\frac{1}{2}(\partial\varphi)^2 - \frac{m_0^2}{2}\varphi^2 - \frac{\lambda_0}{4!}\varphi^4$$

"bare" Lagrangian

$$\frac{\delta^{(n)} Z}{\delta J(x_1) \cdots J(x_n)}\Bigg|_{J=0} = \left(\frac{1}{i}\right)^n G^{(n)}(x_1 \ldots x_n)$$

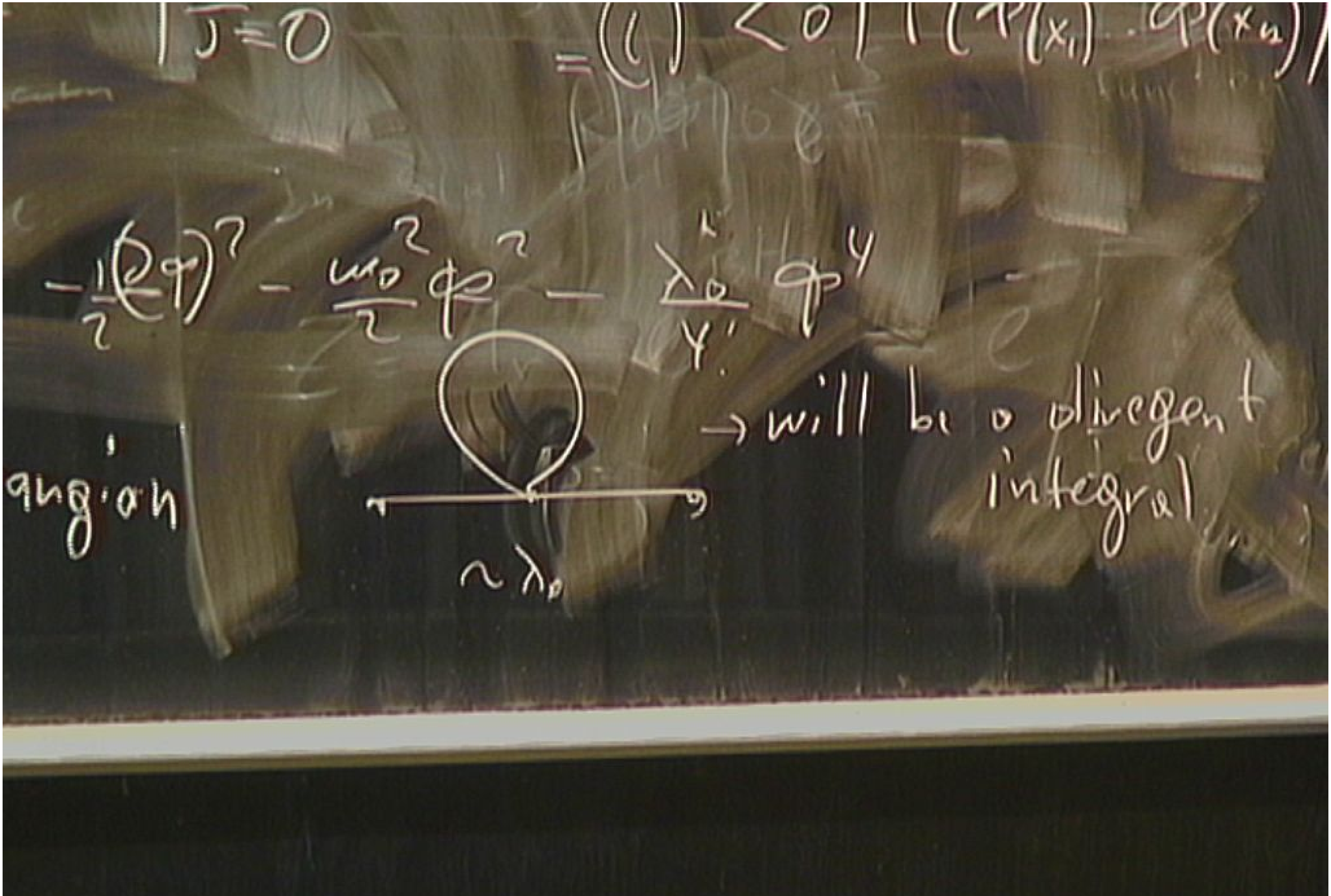
$$= \left(\frac{1}{i}\right)^n \langle 0 | T(\varphi(x_1) \ldots \varphi(x_n)) | 0 \rangle$$

Divergences

$$\mathcal{L}_0 = -\frac{1}{2}(\partial\varphi)^2 - \frac{m_0^2}{2}\varphi^2 - \frac{\lambda_0}{4!}\varphi^4$$

"bare" Lagrangian

$\sim \lambda_0$


J=0
= (i) <0|T(φ(x1) . φ(xn))
-1/2(∂φ)^2 - m0^2/2 φ^2 - λ0/4! φ^4
angian
→ will be a divergent integral
~λ0

$\Big|_{J=0} = (i)\,\langle 0|T(\varphi(x_1)\ldots\varphi(x_n))$

$-\frac{1}{2}(\partial\varphi)^2 - \frac{m_0^2}{2}\varphi^2 - \frac{\lambda_0}{4!}\varphi^4$

angian

$\sim \lambda_0$

→ will be a divergent integral.

$$\frac{\delta^{(n)} Z}{\delta J(x_1)\cdots J(x_n)}\Big|_{J=0} = (i)^n \cdot G^{(n)}(x_1,\dots,x_n) = (i)^n \langle 0|T(\varphi(x_1)\cdots\varphi(x_n))|0\rangle$$

Divergences

$\Rightarrow$

$$\mathcal{L}_0 = -\frac{1}{2}(\partial\varphi)^2 - \frac{m_0^2}{2}\varphi^2 - \frac{\lambda_0}{4!}\varphi^4$$

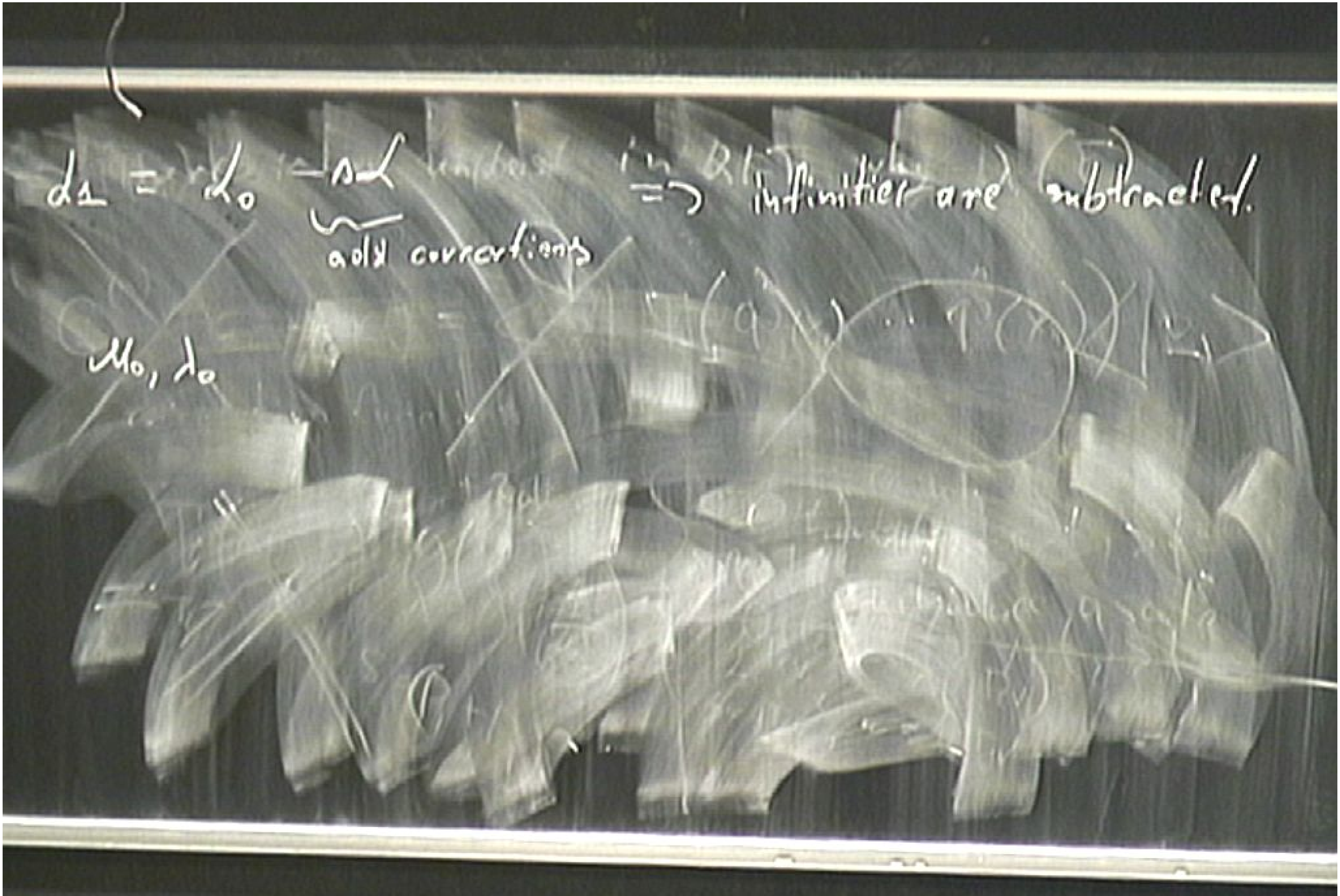
"bare" Lagrangian

→ will be a divergent integral

$\sim \lambda_0 \Rightarrow$ divergence from loop integration (momentum)

$\alpha_1 = \alpha_0 - \underbrace{\Delta\alpha}_{ad}$ in $B(\cdot)$ why $\lambda(\xi)$

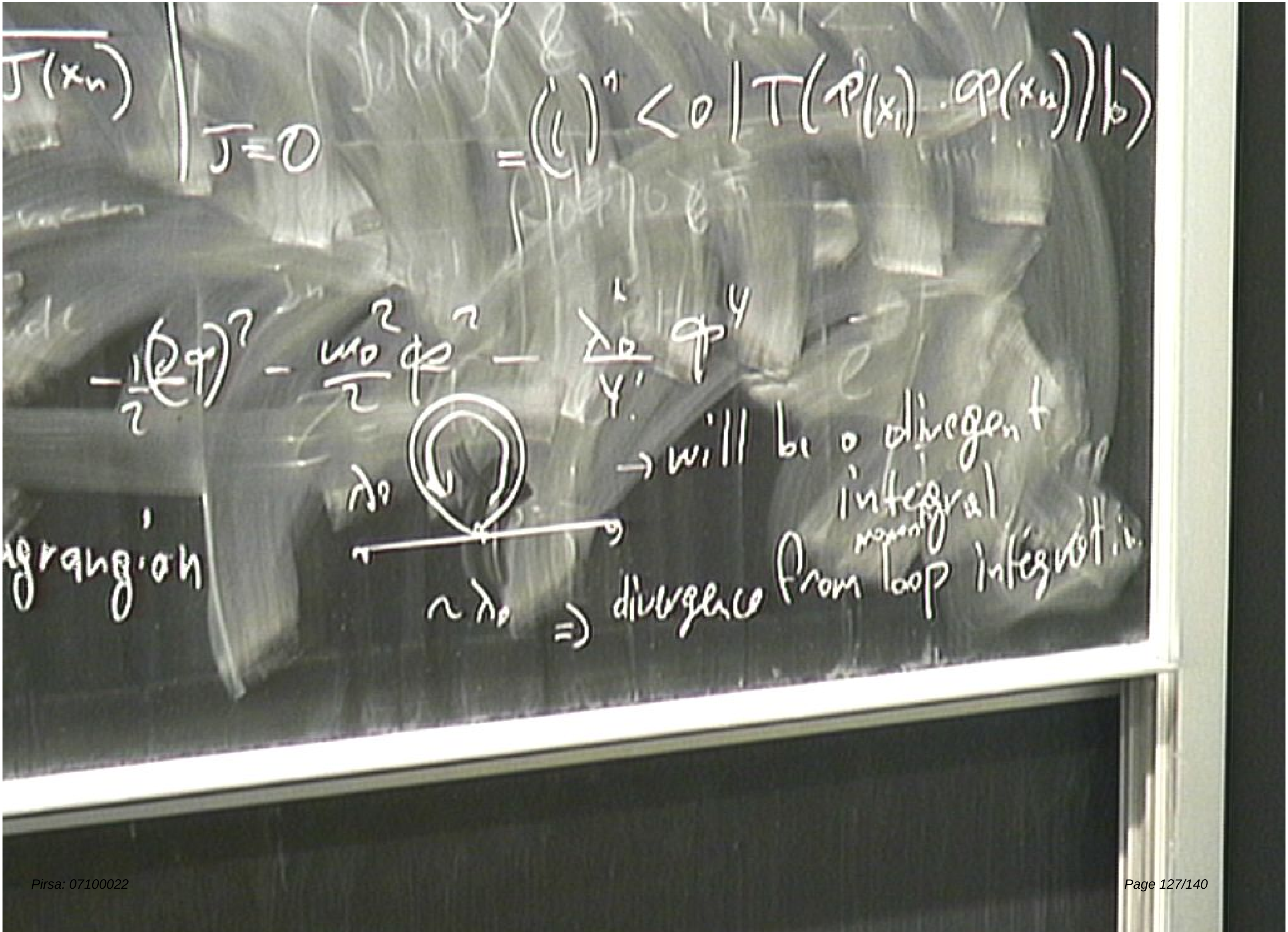
$\alpha_1 = \alpha_0 \quad \underbrace{\delta\alpha}_{\text{add corrections}} \Rightarrow$ infinities are subtracted.


λ1 = λ0 + δλ ⟹ infinities are subtracted.
add corrections
m0, λ0

$\lambda_R = \lambda_0 - \delta\lambda$ $\Rightarrow$ infinities are subtracted.

($\delta\lambda$: add corrections)

$\mu_0, \lambda_0 \rightarrow$ "bare" mass and "bare" coupling


J(x_n) |_{J=0} = (i)^n <0|T(φ(x_1) ... φ(x_n))|0>
-1/2(∂φ)^2 - m_0^2/2 φ^2 - λ_0/4! φ^4
→ will be a divergent integral
λ_0
grangian
~ λ_0 ⇒ divergence from loop integration

$\lambda_1 = \lambda_0 - \Delta\lambda$ $\Rightarrow$ infinities are subtracted.

add corrections

$m_0, \lambda_0 \rightarrow$ "bare" mass and "bare" coupling

$m, \lambda \rightarrow$ physical (finite couplings)

$\lambda_0 + \# \lambda_0^2 + \# \lambda_0^3 + \cdots$

$\mu_0, \lambda_0 \to$ "bare" mass and "bare" coupling

$\mu, \lambda \to$ physical (finite) couplings

$$\lambda_0 + \# \lambda_0^2 + \# \lambda_0^3 + \cdots \Big|_{\Delta\ell = 0}$$

$\mu_0, \lambda_0 \to$ "bare" mass and "bare" coupling

$\mu, \lambda \to$ physical (finite couplings.)

$$\lambda_0 + \# \lambda_0^2 + \# \lambda_0^3 + \cdots \Big|_{\Delta\mathcal{L}=0}$$

∞

$\lambda_1 = \lambda_0 - \Delta\lambda$ $\Rightarrow$ infinities are subtracted.

($\Delta\lambda$: add corrections)

$m_0, \lambda_0 \rightarrow$ "bare" mass and "bare" coupling

$m, \lambda \rightarrow$ physical (finite couplings)

$\lambda_0 + \#\lambda_0^2 + \#\lambda_0^3 + O(\ ) \Big|_{\Delta\lambda = 0}$

$\# \rightarrow \infty$

$\lambda_1 = \lambda_0 - \Delta\lambda$

add corrections

$\Rightarrow$ infinities are subtracted.

$\mu_0, \lambda_0 \to$ "bare" mass and bare coupling

$\mu, \lambda \to$ physical (finite couplings)

$\lambda_0 + \infty\lambda_0^2 + \infty\lambda_0^3 + \ldots$

$\lambda_0 \to \lambda - \Delta\lambda \quad \infty$

$\Delta\lambda = 0$

$= \lambda + \lambda^2 + \lambda^3 + \ldots$

$\lambda_1 = \lambda_0 - \Delta\lambda \Rightarrow$ infinities are subtracted.

add corrections

$m_0, \lambda_0 \rightarrow$ "bare" mass and "bare" coupling

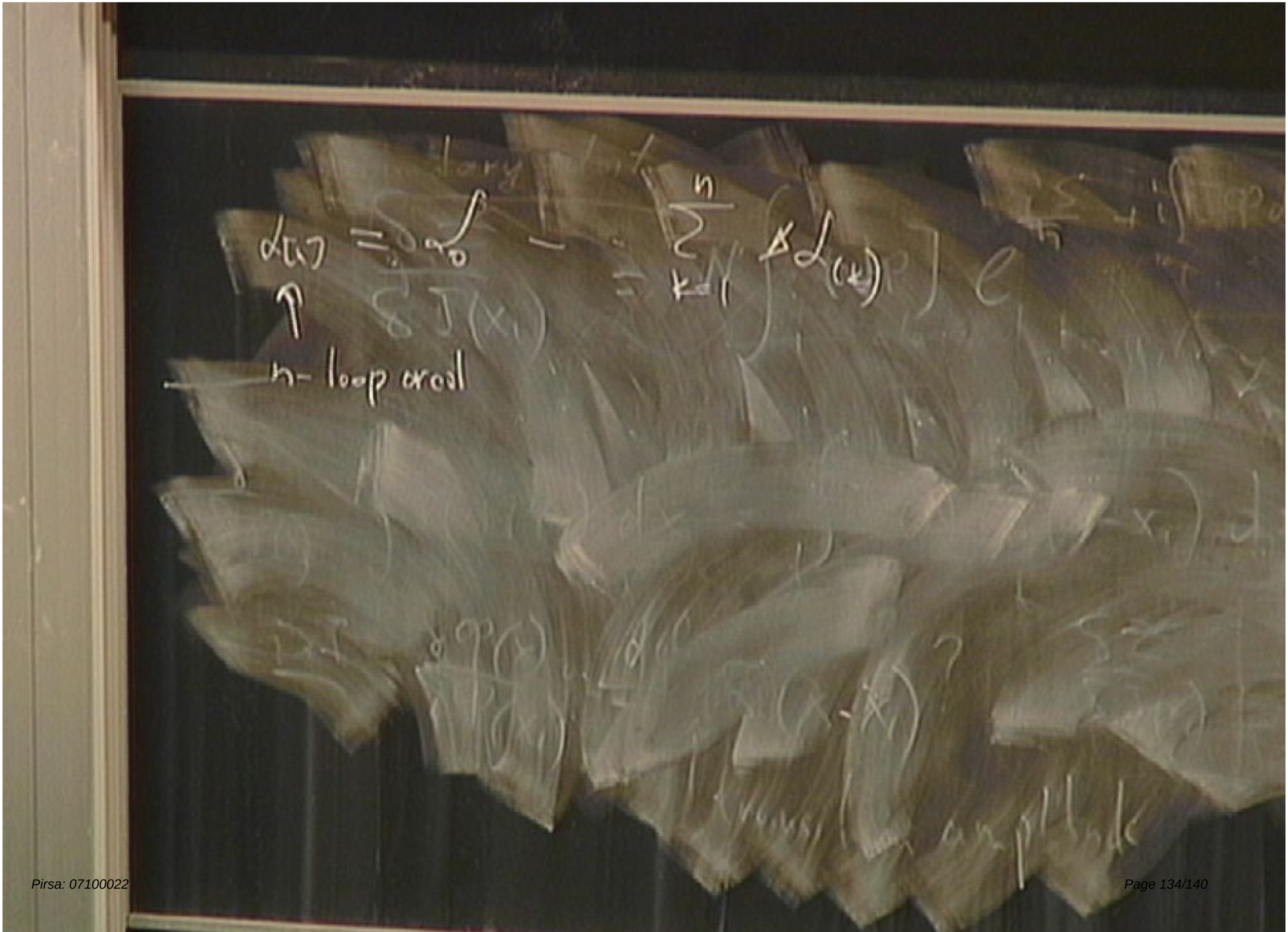
$m, \lambda \rightarrow$ physical (finite couplings)

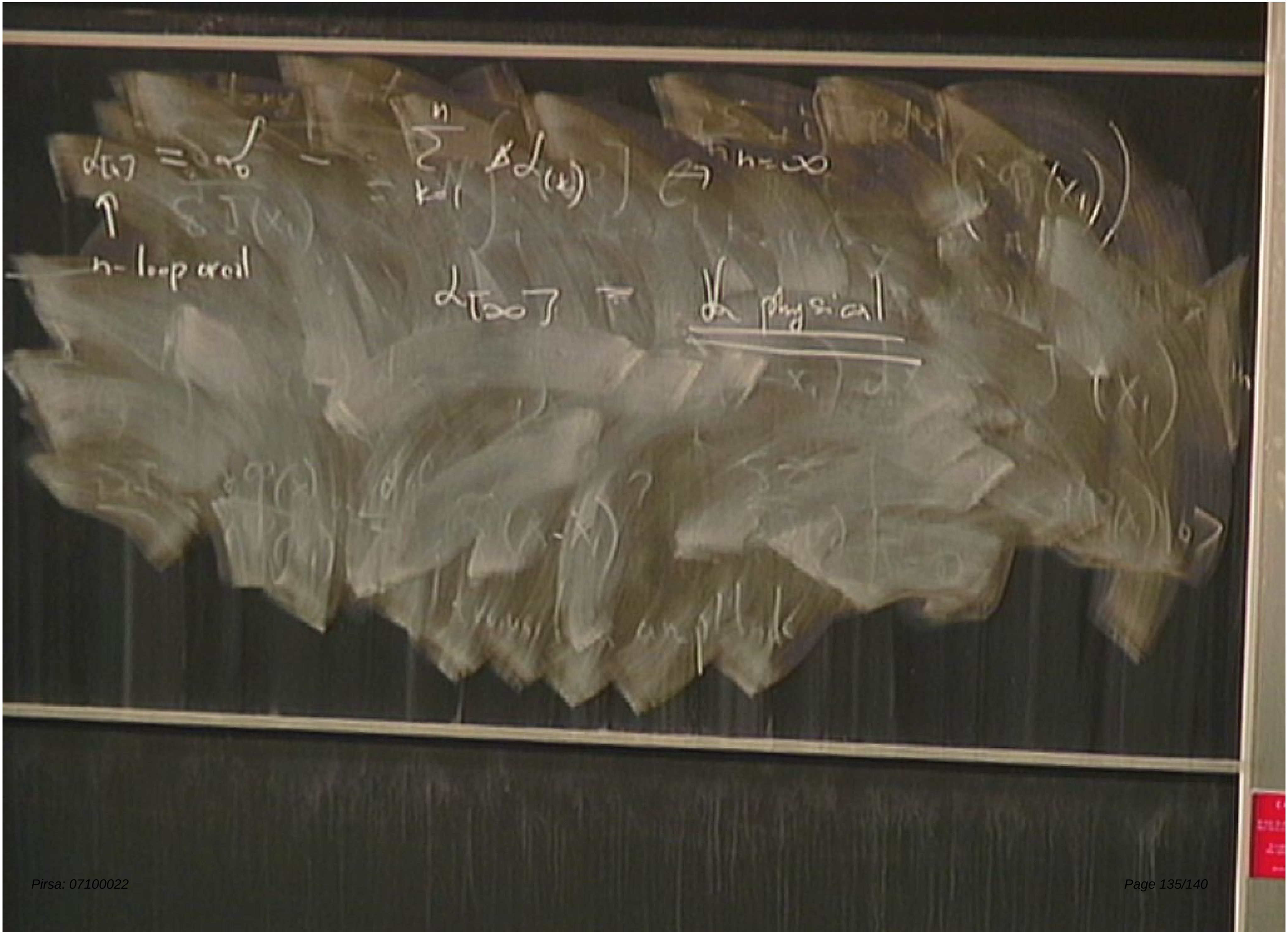
$\lambda_0 + (\#)\lambda_0^2 + (\#)\lambda_0^3 + \mathcal{O}(\ldots)$

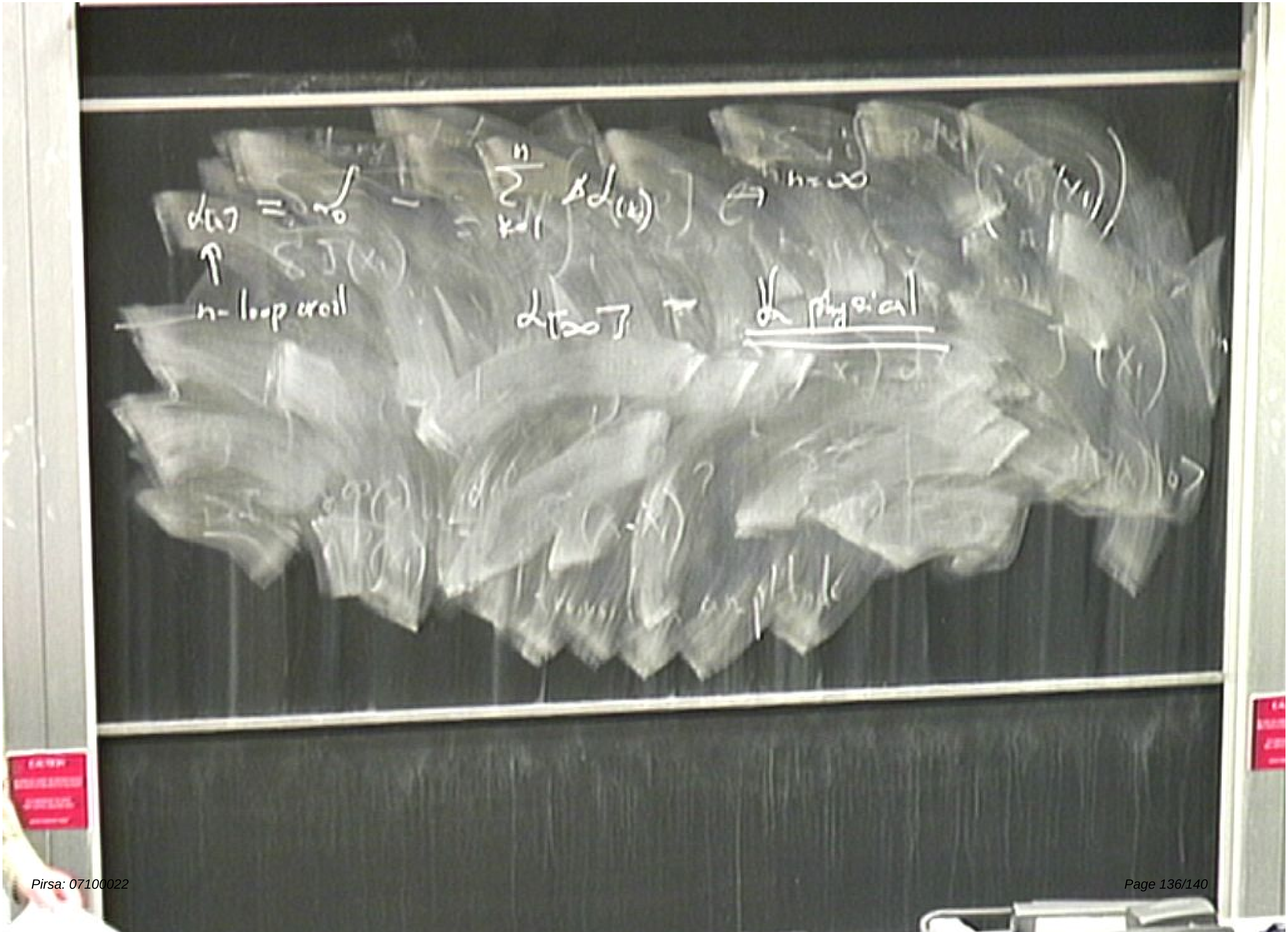
$\lambda_0 \rightarrow \lambda + \Delta\lambda \quad \infty \qquad \Big|_{\Delta\lambda = 0}$

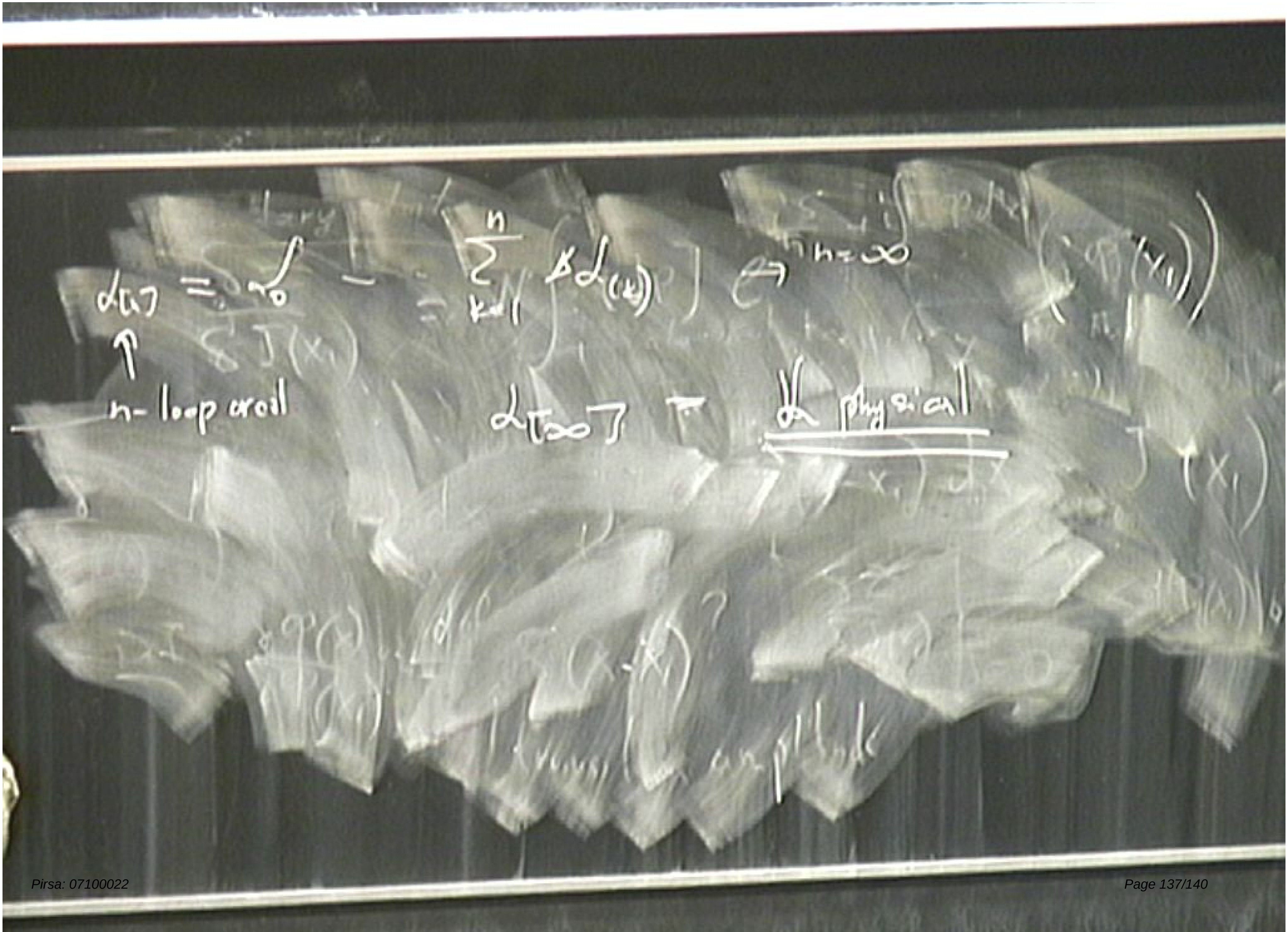
finite

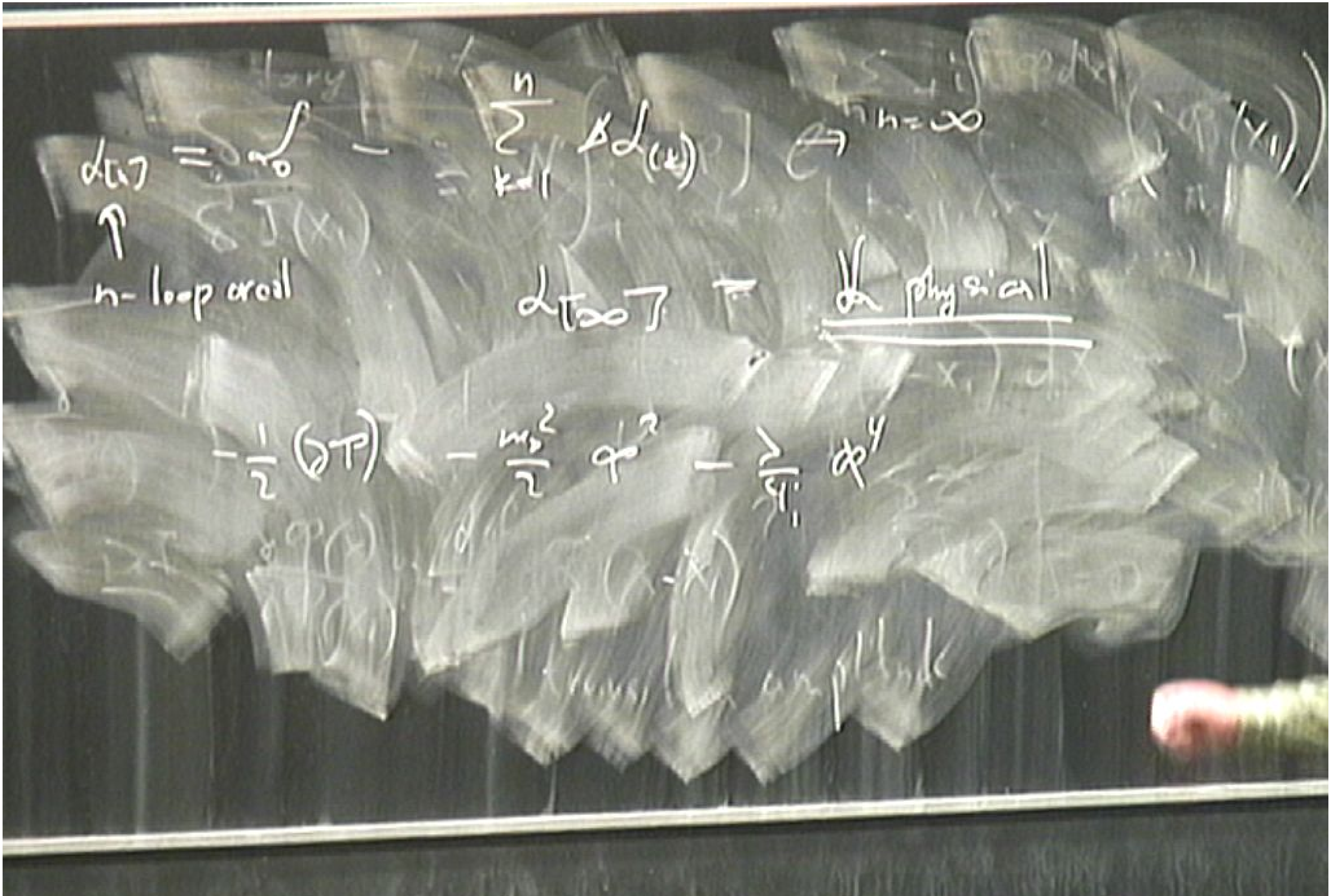
$= \lambda + \#\lambda^2 + \#\lambda^3 + \ldots$


n- loop


n-loop
physical


α[n] = α0 − Σ_{k=1}^{n} δα(k)
n = ∞
n-loop corr
α[∞] = α physical


α[n] = δΓ₀ / δJ(x₁)
= Σ_{k=1}^{n} Δα_{(k)}
→ n=∞
↑ n-loop coeff
α[∞] = α physical


$\mathcal{L}[n] = \mathcal{L}_0 - \sum_{k=1}^{n} \delta\mathcal{L}_{(k)} \rightarrow n=\infty$
n-loop
$\mathcal{L}[\infty] = \mathcal{L}$ physical
$-\frac{1}{2}(\partial\phi)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$

n-loop ord

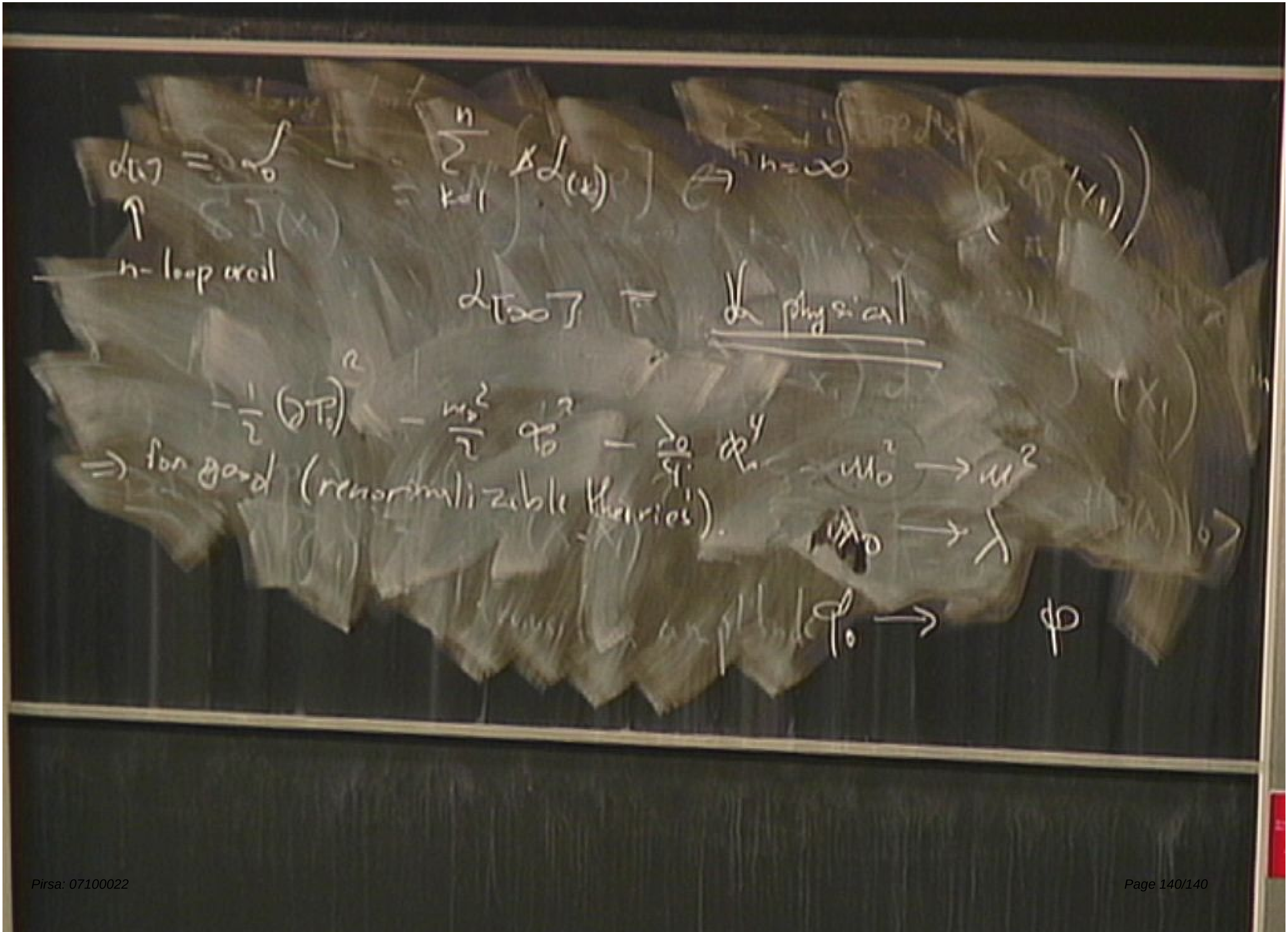
$$\mathcal{L}[\infty] = \mathcal{L}_{\text{physical}}$$

$$-\frac{1}{2}(\partial\phi)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda_0}{4!}\phi^4$$

$\Rightarrow$ for good (renormalizable theories)

$m_0^2 \to m^2$

$\lambda_0 \to \lambda$


$\mathcal{L}_{[\infty]} = \mathcal{L}_{\text{physical}}$
$-\frac{1}{2}(\partial\phi_0)^2 - \frac{m_0^2}{2}\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4$
⇒ for good (renormalizable theories)
$m_0^2 \to m^2$
$\lambda_0 \to \lambda$
$\phi_0 \to \phi$