

Title: Intro to Supersymmetry 11

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Abstract:

$$\bar{D}_\alpha \Phi = 0$$

↑
χsf

$$D_\alpha \Psi = 0$$

↑
anti-chiral superfield!

$$\bar{D}_\alpha \Phi = 0$$

↑
χsf

$$D_\alpha \bar{\Phi} = 0$$

↑
anti-chiral superfield

$$K = K(\Phi, \bar{\Phi})$$

↑
Kähler potential

⇒ we can automatically generate SUSY-invariant actions.

$$S = \int d^4\theta \, k(\varphi, \bar{\varphi}) = \int d^4\theta \, b^{\prime\bar{\theta}} \, k(\bar{\varphi}, \bar{\varphi})$$

$$S = \int \int d^4\theta \, k(\varphi, \bar{\varphi}) = \int d^4\theta \, b^2 \bar{\theta} \, k(\bar{\varphi}, \bar{\varphi})$$

$$\delta_{\text{SUSY}} S = \int d^4\psi \int d^4\theta$$

$$S = \int \int d^4\theta \, k(\varphi, \bar{\varphi}) = \int d^4\theta \, b^2 \bar{\theta} \, k(\bar{\varphi}, \varphi)$$

$$S_{\text{cub}} S = \int d^4\gamma \int d^4\theta \, (\mathcal{E} \mathcal{Q} + \bar{\mathcal{E}} \bar{\mathcal{Q}})$$

$$S = \int \int d^4\theta \, k(\varphi, \bar{\varphi}) = \int d^4\theta \, \delta^2\bar{\theta} \, k(\bar{\varphi}, \varphi)$$

$$\delta_{\text{SUSY}} S = \int d^4y \, \underbrace{\int d^4\theta \, (\epsilon Q + \bar{\epsilon} \bar{Q})}_{\delta^2\bar{\theta}} k$$

$$S = \int d^4x \int d^4\theta \, k(\varphi, \bar{\varphi}) = \int d^4\theta \, \delta^4(\theta) \, k(\bar{\varphi}, \varphi)$$

$$\delta_{\text{SUSY}} S = \int d^4x \int d^4\theta \, \underbrace{(\epsilon Q + \bar{\epsilon} \bar{Q})}_{\text{total space-time derivative}} k = \int d^4x \, 2m \left[\text{some} \right]$$

$$S = \int \int d^4\theta \, K(\varphi, \bar{\varphi}) = \int d^4\theta \, \delta^4\theta \, K(\bar{\varphi}, \varphi)$$

$$S_{\text{cov}} = \int d^4x \int d^4\theta \underbrace{(\epsilon \partial + \bar{\epsilon} \bar{\partial})}_{\text{total space-time derivative}} K = \int d^4x \, 2m \left[\text{some} \right]$$

$$K(\varphi, \bar{\varphi}) =$$

$$S = \int d^4x \int d^4\theta \, K(\psi, \bar{\psi}) = \int d^4\theta \, \delta^4(\theta) \, K(\bar{\psi}, \psi)$$

$$\delta_{\text{susy}} S = \int d^4x \int d^4\theta \, \underbrace{(\epsilon Q + \bar{\epsilon} \bar{Q})}_{\text{total space-time derivative}} K = \int d^4x \, 2m \left[\text{some} \right]$$

$$K(\psi, \bar{\psi}) \Rightarrow K(\psi, \bar{\psi}) + F(\psi) + \bar{F}(\bar{\psi})$$

$$S = \int d^4x \int d^4\theta \, K(\varphi, \bar{\varphi}) = \int d^4\theta \, \delta^4(\theta) \, K(\bar{\varphi}, \varphi)$$

$$\delta_{\text{SUSY}} S = \int d^4x \int d^4\theta \, \underbrace{(\epsilon Q + \bar{\epsilon} \bar{Q})}_{\text{total space-time derivative}} K = \int d^4x \, 2m \left[\text{same} \right]$$

$$K(\varphi, \bar{\varphi}) \Rightarrow K(\varphi, \bar{\varphi}) + F(\varphi) + \bar{F}(\bar{\varphi})$$

will lead to the same SUSY inv action

$$K = \varphi \bar{\varphi}$$

\hookrightarrow evaluated $\int d^4\theta \Rightarrow$

$$\mathcal{L}_e =$$

$$\mathbb{I}(\varphi, \theta) = \varphi(\varphi)$$

$$K = \varphi \bar{\varphi}$$

\hookrightarrow evaluated $\int d^4\theta \Rightarrow$

$$\mathcal{L}_\theta =$$

$$\mathcal{L}(\varphi, \theta) = \varphi(\varphi) + \sqrt{2} \theta \psi(\varphi) + \theta^2 F$$

$$\mathbb{I}(y, \theta) = \underbrace{\varphi(y)}_{\text{complex \& data}} + \sqrt{2} \theta \underbrace{\psi(y)}_{\text{weyl f.}} + \theta^2 \underbrace{F(y)}_{\text{auxi. f.}}$$

$$K = \Phi \bar{\Phi}$$

↳ evaluated $\int d^4\theta \Rightarrow$

$$\boxed{\mathcal{L}(\varphi, \theta) = \underbrace{\Phi(\varphi)}_{\text{complex scalar}} + \sqrt{2} \theta \underbrace{\Psi(\varphi)}_{\text{Weyl f.}} + \theta^2 \underbrace{F(\varphi)}_{\text{aux. f.}}$$

$$\mathcal{L}_a = -\partial_\mu \varphi \partial^\mu \bar{\varphi} + i \partial_\mu \bar{\Psi} \bar{\sigma}^{\mu\nu} \Psi + F \bar{F}$$

$$K = \Phi \bar{\Phi}$$

↳ evaluated $\int d^4\theta \Rightarrow$

$$\boxed{\Phi(y, \theta) = \underbrace{\Phi(y)}_{\text{complex scalar}} + \sqrt{2} \theta \underbrace{\Psi(y)}_{\text{Weyl f.}} + \theta^2 \underbrace{F(y)}_{\text{aux. f.}}$$

$$\mathcal{L}_0 = -\partial_\mu \Phi \partial^\mu \bar{\Phi} + i \partial_\mu \bar{\Psi} \bar{\sigma}^{\mu\nu} \Psi + \underbrace{F \bar{F}}_{\text{non-dynamical field}}$$

$$K = \Phi \bar{\Phi}$$

↳ evaluated $\int d^4\theta \Rightarrow$

$$\mathbb{I}(\gamma, \theta) = \underbrace{\Phi(\gamma)}_{\text{complex scalar}} + \sqrt{2} \theta \underbrace{\Psi(\gamma)}_{\text{Weyl f.}} + \theta^2 \underbrace{F(\gamma)}_{\text{aux. f.}}$$

$$\mathcal{L}_d = -\partial_\mu \Phi \partial^\mu \bar{\Phi} + i \partial_\mu \bar{\Psi} \bar{\sigma}^{\mu\nu} \Psi + \underbrace{F \bar{F}}_{\text{non-dynamical field}}$$

↳ when we use

$$K = \Phi \bar{\Phi}$$

↳ evaluated $\int d^4\theta \Rightarrow$

$$\boxed{\mathcal{I}(\gamma, \theta) = \underbrace{\Phi(\gamma)}_{\text{complex scalar}} + \sqrt{2} \theta \underbrace{\Psi(\gamma)}_{\text{w/ff.}} + \theta^2 \underbrace{F(\gamma)}_{\text{aux. f.}}$$

$$\mathcal{L}_a = -\gamma \rho \partial \bar{\rho} + i \partial_m \bar{\Psi} \bar{\sigma}^m \Psi + \underbrace{F \bar{F}}_{\text{non-dynamical field}}$$

↳ when we use $\int d^4\theta K$

$$K = \Phi \bar{\Phi}$$

↳ evaluated $\int d^4\theta \Rightarrow$

$$\boxed{\mathbb{I}(\gamma, \theta) = \underbrace{\Phi(\gamma)}_{\text{complex scalar}} + \sqrt{2} \theta \underbrace{\Psi(\gamma)}_{\text{Weyl f.}} + \theta^2 \underbrace{F(\gamma)}_{\text{aux. f.}}$$

$$\mathcal{L}_d = -\cancel{\partial\phi}^2 \cancel{\partial\bar{\phi}}^2 + i \cancel{\partial}_m \bar{\Psi} \bar{\sigma}^m \Psi + \underbrace{F \bar{F}}_{\text{nondynamical field}}$$

↳ when we use $\int d^4\theta K$ we cannot get non-derivative interactions

⇒ superpotential and interactions in SUSY model.

$$y^m = X^m + i \theta \sigma^m \bar{\theta}$$

⇒ superpotential and interactions in SUSY model.

$$y^m = X^m + i\theta\sigma^m\bar{\theta} \quad \bar{D}_i y^m = 0$$

⇒ superpotential and interactions in SUSY model.

$$y^m = \chi^m + i\theta\sigma^m\bar{\theta} \quad \bar{D}_i y^m = 0$$

⇒ $\mathcal{P}(y, \theta)$ is χ SP $\Rightarrow \bar{D}_\alpha \mathcal{P} = 0$

⇒ superpotential and interactions in SUSY model.

$$y^m = \chi^m + i\theta\sigma^m\bar{\theta} \quad \bar{D}_i y^m = 0$$

→ $\mathcal{P}(y, \theta)$ is χ s ρ ⇒ $\bar{D}_\alpha \mathcal{P} = 0$

$\begin{matrix} \swarrow \\ \searrow \\ \nearrow \\ \nwarrow \end{matrix}$
 x, p

⇒ superpotential and interactions in SUSY model.

$$y^m = x^m + i \theta \sigma^m \bar{\theta} \quad \bar{D}_i y^m = 0$$

→ $\mathcal{P}(y, \theta)$ is $\chi_{SP} \Rightarrow \bar{D}_\alpha \mathcal{P} = 0$

$$\int_{SP} \mathcal{P} = \int d^2\theta \mathcal{P}(y)$$

superpotential.

⇒ superpotential and interactions in SUSY model.

$$y^m = x^m + i \theta \sigma^m \bar{\theta} \quad \bar{D}_i y^m = 0$$

→ $\mathcal{P}(y, \theta)$ is $\chi_{SP} \Rightarrow \bar{D}_\alpha \mathcal{P} = 0$

$$\int_{\text{superpotential}} d^4\theta = \int_{\frac{1}{2} \text{ of } \theta} d^2\theta \int_{\chi_{SP}} \mathcal{P}(y)$$

⇒ superpotential and interactions in SUSY model.

$$y^m = x^m + i \theta \sigma^m \bar{\theta} \quad \bar{D}_i y^m = 0$$

→ $\mathcal{P}(y, \theta)$ is χ spin $\Rightarrow \bar{D}_\alpha \mathcal{P} = 0$

$$\int_{\text{superspace}} d^2\theta \int_{\text{sp}} d^4p \mathcal{P}(y) = F(x)$$

sp
fix
fix

1/2 of
1/2

superpotential
W

$$\mathbb{I}(y, \theta) = \underbrace{\varphi(y)}_{\text{complex scalar}} + \sqrt{2} \theta \underbrace{\psi(y)}_{\text{weyl f.}} + \underbrace{\theta^2 F(y)}_{\text{auxi. f.}}$$

⇒ superpotential and interactions in SUSY model.

$$y^m = x^m + i \theta \sigma^m \bar{\theta} \quad \bar{D}_i y^m = 0$$

→ $\mathcal{P}(y, \theta)$ is a scalar $\Rightarrow \bar{D}_\alpha \mathcal{P} = 0$

$$\int d^4x \int d^2\theta \int d^2\bar{\theta} \mathcal{P}(y) = F(x) + \text{nothing else}$$

$\int d^2\theta$ is $\frac{1}{2}$ of superspace
 $\int d^2\bar{\theta}$ is $\frac{1}{2}$ of superspace
 $\mathcal{P}(y)$ is a scalar

superpotential

$$\int d^2\theta \sqrt{2\theta} \psi(x) = \int d^2\theta \sqrt{2\theta} \psi(x + i\theta c^{-1} \bar{\theta})$$

$$\int d^2\theta \sqrt{2} \theta^4(y) = \int d^2\theta \sqrt{2} \theta^4 \left(\psi(x + \frac{i\theta\sigma^{\mu\nu}\bar{\theta}}{2}) \right)$$

↑
Taylor expand

$$\int d^2\theta \sqrt{2} \theta^4(x) = \int d^2\theta \sqrt{2} \theta^4 \left(x + \frac{i\theta\theta^c \bar{\theta}}{2} \right)$$

↑
Taylor expand

$$= \int d^2\theta \sqrt{2} (\theta^4(x) + \dots)$$

$$\int d^2\theta \sqrt{2} \psi(y) = \int d^2\theta \sqrt{2} \theta \psi(x + \frac{i\theta\sigma^{\mu\nu}\bar{\theta}}{2})$$

↑
Taylor expand

$$= \int d^2\theta \sqrt{2} \psi(y) \theta (i\theta\sigma^{\mu\nu}\bar{\theta}) \rightarrow \text{a total derivative}$$

$$\int d^2\theta \sqrt{2} \theta^2 \psi(x) = \int d^2\theta \sqrt{2} \theta^2 \psi(x + \frac{i\theta\sigma^{\mu\nu}\bar{\theta}}{2})$$

↑
Taylor expand

$$= \int d^2\theta \sqrt{2} (\psi(x) + \dots) \theta^2 (i\theta\sigma^{\mu\nu}\bar{\theta}) \rightarrow \text{a total derivative.}$$

↑
would not contribute to susy action.

$$\int d^2\theta \sqrt{2} \theta^2 \phi(y) = \int d^2\theta \sqrt{2} \theta^2 \phi\left(x + \frac{i\theta\sigma^{\mu\nu}\bar{\theta}}{2}\right)$$

↑
Taylor expand

$$= \int d^2\theta \sqrt{2} \left(\frac{\partial_\mu \phi}{\partial x^\mu} \right) \theta^2 (i\theta\sigma^{\mu\nu}\bar{\theta}) \rightarrow \text{a total derivative.}$$

↑
would not contribute to susy action.

$$\int d^2\theta \phi(y)$$

$$\int d^2\theta \sqrt{2} \theta \psi(y) = \int d^2\theta \sqrt{2} \theta \psi(x + \frac{i\theta\sigma^{\mu\nu}\bar{\theta}}{2})$$

↑
Taylor expand

$$= \int d^2\theta \sqrt{2} (\partial_{\mu}\psi) \theta (i\theta\sigma^{\mu\nu}\bar{\theta}) \rightarrow \text{a total derivative.}$$

↑
would not contribute to susy action.

$$\int d^2\theta \varphi(y) = \int d^2\theta \left[\dots - \frac{1}{2!} \partial_{\mu}\partial_{\nu}\varphi (i\theta\sigma^{\mu\nu}\bar{\theta})(i\theta\sigma^{\rho\sigma}\bar{\theta}) \right]$$

$$\int d^2\theta \sqrt{2}\theta\psi(y) = \int d^2\theta \sqrt{2}\theta \psi\left(x + \frac{i\theta\sigma^{\mu\nu}\bar{\theta}}{2}\right)$$

↑
Taylor expand

$$= \int d^2\theta \sqrt{2} \left(\partial_{\mu}\psi\right)\theta (i\theta\sigma^{\mu\nu}\bar{\theta}) \rightarrow \text{a total derivative.}$$

↑
would not contribute to susy action.

$$\int d^2\theta \psi(y) = \int d^2\theta \left[\dots - \frac{1}{2!} \partial_{\mu}\psi (i\theta\sigma^{\mu\nu}\bar{\theta})(i\theta\sigma^{\rho\sigma}\bar{\theta}) \right]$$

$$\int d^2\theta \sqrt{2}\theta^4(y) = \int d^2\theta \sqrt{2}\theta^4 \left(\underbrace{\varphi(x + \frac{i\theta\sigma^0\bar{\theta}}{2})}_{\text{Taylor expand}} \right)$$

$$= \int d^2\theta \sqrt{2} \left(\underbrace{\varphi(x)}_{\substack{\uparrow \\ \text{would not contribute to susy action}}} \right) \theta^4 (i\theta\sigma^0\bar{\theta}) \rightarrow \text{a total derivative.}$$

$$\int d^2\theta \varphi(y) = \int d^2\theta \left[\dots - \frac{1}{2!} \partial_\mu \partial_\nu \varphi \left(\underline{i\theta\sigma^0\bar{\theta}} \right) \left(\underline{i\theta\sigma^0\bar{\theta}} \right) \right]$$

$$\int d^2\theta \sqrt{2}\theta\psi(y) = \int d^2\theta \sqrt{2}\theta \psi\left(x + \frac{i\theta\sigma^{\mu\nu}\bar{\theta}}{2}\right)$$

↑
Taylor expand

$$= \int d^2\theta \sqrt{2} \left(\partial_{\mu}\psi\right)\theta (i\theta\sigma^{\mu\nu}\bar{\theta}) \rightarrow \text{a total derivative}$$

↑
would not contribute to SUSY action

$$\int d^2\theta \psi(y) = \int d^2\theta \left[\dots \frac{1}{2!} \partial_{\mu}\psi (i\theta\sigma^{\mu\nu}\bar{\theta})(i\theta\sigma^{\rho\sigma}\bar{\theta}) \right]$$

↑
a total derivative

⇒ we are going to show that

$$\sum_{\text{all } x} F_{x,y} = \text{total derivative.}$$

\Rightarrow we are going to show that

$$\sum_{i=1}^n F_{i,2} F_i = \text{total derivative.}$$

$\Rightarrow W(q_i)$ an arbitrary function of q_i

\Rightarrow we are going to show that

$$\sum_{\text{susy}} F_{\alpha\beta} = \text{total derivative.}$$

\Rightarrow $W(\varphi_i)$ an arbitrary function of φ_i

new susy actions

$$\Rightarrow \int d^2\theta \ W_{\alpha\beta}(\varphi_i)$$

\Rightarrow we are going to show that

$$\delta_{\text{SUSY}} F_{\text{NSR}} = \text{total derivative.}$$

$\Rightarrow W(\varphi_i)$ an arbitrary function of φ_i

new susy actions

$$\Rightarrow \int d^2\theta \ W_{\text{NSR}}(\varphi_i) \equiv F_W$$

⇒ we are going to show that

$$\sum_{\text{susy}} F_{\text{NSF}} = \text{total derivative.}$$

⇒ $W(\varphi_i)$ an arbitrary function of φ_i

new susy actions

$$\Rightarrow \int d^2\theta \ W_{\text{NSF}}(\varphi_i) \equiv F_W$$

$\delta F_W = \text{total derivative}$

⇒ we are going to show that

$$\delta_{\text{susy}} F_{\text{NSL}} = \text{total derivative.}$$

→ superpotential

⇒ $W(\varphi_i)$ an arbitrary function of φ_i

new susy actions

⇒

$$\int d^2\theta W_{\text{NSL}}(\varphi_i) \equiv F_W$$

\uparrow
NSL

$d_W =$

$\delta F_W = \text{total derivative}$

⇒ we are going to show that

$$\delta_{\text{SUSY}} F_{\text{SUSY}} = \text{total derivative.}$$

↘ superpotential

⇒ $W(\varphi_i)$ an arbitrary function of φ_i

new susy actions

⇒

$$\int d^2\theta \, W(\varphi_i) \equiv F_W$$

↑
SUSY

$$L_W = \int d^2\theta$$

$$\delta F_W = \text{total derivative}$$

when we use $\int d^2\theta$ we cannot get
non-derivative interactions

⇒ we are going to show that

$$\delta_{\text{SUSY}} F_{\text{SUSY}} = \text{total derivative.}$$

↘ superpotential

⇒ $W(\varphi_i)$ an arbitrary function of φ_i

new susy actions

⇒

$$\int d^2\theta W(\varphi_i) \equiv F_W$$

$\delta F_W = \text{total derivative}$

$$\mathcal{L}_W = \int d^2\theta W(\varphi_i) + \text{h.c.}$$

→ when we use $\int d^2\theta$ we cannot get
non-derivative interactions

$$\Phi(y, \theta) \rightarrow \chi^2 L.$$

S_{SUSY}

$$\mathcal{K}(y, \theta) \rightarrow \text{real}$$

$$\begin{aligned} \mathcal{S}_{\text{susy}} \mathcal{K} &= (\epsilon Q + \bar{d} \bar{Q}) \mathcal{K} \\ &= (\epsilon^{\dot{\alpha}} Q_{\dot{\alpha}} - \bar{d}^{\dot{\alpha}} \bar{Q}_{\dot{\alpha}}) \mathcal{K} \end{aligned}$$

$$\Phi(\omega) \rightarrow \text{real}$$

$$\begin{aligned} \sum_{k=0}^{\infty} \Phi &= (\epsilon Q + d Q) \Phi \\ &= (\epsilon Q - \epsilon^* Q^*) \Phi \\ &= \epsilon^2 \left[\frac{2}{\omega^2} - i \epsilon^* \theta^* \right] \end{aligned}$$

$$\Phi(\psi, \theta) \rightarrow \chi \in L$$

$$\mathcal{L}_{\text{susy}} \Phi = (\epsilon Q + \bar{\epsilon} \bar{Q}) \Phi$$

$$= \left(\epsilon^\alpha Q_\alpha - \bar{\epsilon}_{\dot{\alpha}} \bar{Q}_{\dot{\alpha}} \right) \Phi$$

$$= \left\{ \epsilon^\alpha \left[\frac{\partial}{\partial \theta^\alpha} - i \sigma_{\alpha\dot{\beta}}^m \bar{\theta}^{\dot{\beta}} \partial_m \right] + \bar{\epsilon}_{\dot{\alpha}} \left[\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \sigma_{\dot{\alpha}\alpha}^m \theta^\alpha \partial_m \right] \right\} \Phi$$

$$\mathcal{L}(\psi, \theta) = P(x) + i\theta c^w \bar{\theta} \gamma_n \psi + \frac{1}{4} \theta^2 \bar{\theta}^2 \gamma_n^2 \psi^2$$

$$+ \sqrt{2} \theta \psi - \frac{i}{\sqrt{2}} \theta^2 \gamma_n \psi c^w \bar{\theta} + \theta^2 F$$

$$\Phi(\psi, \theta) \rightarrow \text{sol}$$

$$\mathcal{S}_{\text{susy}} \Phi = (\epsilon Q + \bar{\epsilon} \bar{Q}) \Phi$$

$$= (\epsilon^{\alpha} Q_{\alpha} - \bar{\epsilon}^{\dot{\alpha}} \bar{Q}_{\dot{\alpha}}) \Phi$$

$$= \left\{ \epsilon^{\alpha} \left[\frac{\partial}{\partial \theta^{\alpha}} - i \sigma_{\alpha\dot{\beta}}^{\mu} \bar{\theta}^{\dot{\beta}} \partial_{\mu} \right] + \bar{\epsilon}^{\dot{\alpha}} \left[\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \sigma_{\dot{\alpha}\beta}^{\mu} \theta^{\beta} \partial_{\mu} \right] \right\} \Phi$$

$$\Phi(\psi, \theta) \rightarrow \text{real}$$

$$\delta_{\text{susy}} \Phi = (\epsilon Q + \bar{\epsilon} \bar{Q}) \Phi$$

$$= \left(\epsilon^\alpha Q_\alpha - \bar{\epsilon}_{\dot{\alpha}} \bar{Q}_{\dot{\alpha}} \right) \Phi$$

$$= \left\{ \epsilon^\alpha \left[\frac{\partial}{\partial \theta^\alpha} - i \sigma_{\alpha\dot{\beta}}^\mu \bar{\theta}^{\dot{\beta}} \partial_\mu \right] + \bar{\epsilon}_{\dot{\alpha}} \left[\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \sigma_{\dot{\alpha}\alpha}^\mu \theta^\alpha \partial_\mu \right] \right\} \Phi$$

$$\begin{aligned} \mathcal{L}(\psi, \bar{\psi}) = & \mathcal{L}(x) + i\theta \sigma^{\mu\nu} \bar{\theta} \lambda_{\mu} \psi + \frac{1}{4} \theta^2 \bar{\theta}^2 \mathcal{L}(x) \\ & + \underline{\mathcal{L}(x)} - \underline{\frac{1}{2} \theta^2 \lambda_{\mu} \psi \sigma^{\mu\nu} \bar{\theta}} + \theta^2 F \end{aligned}$$

$$\begin{aligned}
 \mathcal{L}(y, \psi) = & p(x) + i \theta \sigma^{\omega \bar{\theta}} \lambda_n \psi + \frac{1}{4} \theta^2 \bar{\theta}^2 \lambda_n^2 \psi \\
 & + \sqrt{2} \theta \psi - \frac{i}{\sqrt{2}} \theta^2 \lambda_n \psi \sigma^{\omega \bar{\theta}} + \theta^2 F \\
 & - \bar{\xi}^i \sigma_{ij}^{\omega} \theta^j \theta^{\beta} \sqrt{2} \lambda_n \psi
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{L}(y, \theta) &= P(x) + i\theta \sigma^{\mu\nu} \bar{\psi} \gamma_{\mu} \psi + \frac{1}{4} \theta^2 \bar{\psi}^2 \psi^2 \\
 &= \sum_{i=1}^n x_i i \theta \sigma^{\mu\nu} \bar{\psi} + \sqrt{2} \theta \psi + \frac{i}{\sqrt{2}} \theta^2 \gamma_{\mu} \psi \sigma^{\mu\nu} \bar{\psi} + \underbrace{\theta^2 F}_{\text{circled}} \\
 &\quad - \sum_{i=1}^n i \sigma_{\alpha\beta}^{\mu} \theta^{\alpha} \bar{\psi}^{\beta} \sqrt{2} \gamma_{\mu} \psi + \sum_{i=1}^n \frac{i}{\sqrt{2}} \theta^2 \gamma_{\mu} \psi \sigma_{\alpha\beta}^{\mu} \frac{\partial \bar{\psi}^{\beta}}{\partial \theta^{\alpha}}
 \end{aligned}$$

$$\begin{aligned}
 \Phi(y, \theta) &= \Psi(x) + i\theta \sigma^{\mu\nu} \bar{\psi} \gamma_{\mu} \psi + \frac{1}{4} \theta^2 \bar{\psi}^2 \psi^2 \\
 &= \underbrace{\Psi(x) + \sqrt{2} \theta \psi}_{\text{linear}} - \frac{i}{\sqrt{2}} \theta^2 \gamma_{\mu} \psi \sigma^{\mu\nu} \bar{\psi} + \underbrace{\theta^2 F}_{\text{quadratic}} \\
 &= -\frac{1}{\sqrt{2}} i \sigma^{\mu\nu} \xi^{\alpha} \theta^{\alpha} \theta^{\beta} \sqrt{2} \gamma_{\mu} \psi_{\beta} + \frac{1}{\sqrt{2}} i \theta^2 \gamma_{\mu} \psi^{\alpha} \sigma_{\alpha\beta} \frac{\partial \bar{\psi}^{\beta}}{\partial \theta^{\alpha}} \\
 &= -i\sqrt{2} \left(\gamma_{\mu} \psi \sigma^{\mu\nu} \xi \right) \theta^2
 \end{aligned}$$

$$\Phi(y, \theta) = P(x) + i\theta \sigma^{\mu\nu} \bar{\theta} \lambda_{\mu} \psi_{\nu} + \frac{1}{4} \theta^2 \bar{\theta}^2 \lambda^{\mu} \psi_{\mu}$$

$$= x^{\mu} \lambda_{\mu} \sigma^{\mu\nu} \bar{\theta} + \sqrt{2} \theta^{\alpha} \psi_{\alpha} - \frac{i}{\sqrt{2}} \theta^2 \lambda_{\mu} \psi^{\mu} \bar{\theta} + \theta^2 F$$

$$- \sum_i^i \lambda_{\mu} \sigma^{\mu\nu} \theta^{\alpha} \bar{\theta}^{\beta} \sqrt{2} \partial_{\alpha} \psi_{\nu} + \sum_i^i \frac{i}{\sqrt{2}} \theta^2 \partial_{\mu} \psi^{\mu} \sigma_{\alpha\beta} \frac{\partial \bar{\theta}^{\beta}}{\partial \theta^{\alpha}}$$

$$= -i\sqrt{2} \left(\partial_{\mu} \psi^{\mu} \sigma^{\mu\nu} \zeta \right) \theta^2$$

total derivative

$$W(\varphi_2)$$

χ s f is only!

$$D_{\alpha} P = 0$$

$$K(\varphi_2, \varphi_3)$$

you can automatically generate susy-type variations.

$$W(\varphi_i) = W\left(\varphi_i + i\theta \epsilon^{\mu\nu} \bar{\theta} \gamma_{\mu} \varphi_i + \frac{1}{4} \theta^2 \theta^2 \partial^2 \varphi_i + \sqrt{2} \theta \psi - \frac{1}{\sqrt{2}} \theta^2 \gamma_{\mu} \psi^i \sigma^{\mu} \bar{\theta} + \theta^2 F_i\right)$$

ant. - chiral superfield!

$\chi, \psi, \bar{\psi}$ only!

$K(\varphi, \bar{\varphi})$

we can automatically generate SUSY + grav actions.

$$W(\varphi_i) \Big|_0 = W(\varphi_i + i\theta \epsilon^{\mu\nu} \bar{\theta} \gamma_\mu \varphi_i + \frac{1}{4} \theta^2 \theta^2 \partial^2 \varphi_i$$

act on chiral superfield!

$$+ \sqrt{2} \theta \psi - \frac{i}{\sqrt{2}} \theta^2 \gamma_\mu \psi^i \sigma^\mu \bar{\theta} + \theta^2 F_i)$$

$K(\varphi, \bar{\varphi})$

\uparrow θ^2 term
χ s f's only!

we can automatically generate SUSY + grav actions

$$W(\varphi_i) \Big|_0 = W\left(\varphi_i \left(i\theta \sigma^{\mu\nu} \partial_\mu \varphi_i + \frac{1}{4} \theta^2 \partial^2 \varphi_i \right. \right.$$

↑ θ^2 term
χ, ψ only!

$$\left. + \sqrt{2} \theta \psi - \frac{i}{2} \theta^2 \partial_\mu \psi^i \sigma^{\mu\nu} \bar{\theta} + \theta^2 F_i \right)$$

= ...
 you can automatically generate SUSY + grav actions.

$$W(\varphi_i) \Big|_0 = W(\varphi_i \underbrace{[i\theta \epsilon^\mu \bar{\theta} \gamma_\mu \varphi_i + \frac{1}{4} \theta^2 \theta^2 \partial^2 \varphi_i]}_{\text{and: chiral superfield!}})$$

↑ θ^2 term
χ, ψ only!

$$\left. \begin{aligned} &+ \sqrt{2} \theta \psi - \frac{i}{\sqrt{2}} \theta^2 \gamma_\mu \psi^i \sigma^\mu \bar{\theta} + \theta^2 F_i \end{aligned} \right\} \theta^2$$

$$= \frac{\partial W}{\partial \varphi_i}$$

we can automatically generate susy + mv actions.

$$W(\varphi_i) \Big|_0 = W(\varphi_i \left(i\theta \epsilon^\mu \bar{\theta} \partial_\mu \varphi_i + \frac{1}{4} \theta^2 \bar{\theta}^2 \partial^2 \varphi_i \right.$$

↑ $\theta^2 \epsilon^\mu \epsilon_\mu$
χ, ψ is only!

$$\left. + \sqrt{2} \theta \psi - \frac{i}{\sqrt{2}} \theta^2 \partial_\mu \psi^i \sigma^\mu \bar{\theta} + \theta^2 F_i \right)$$

$$= \partial_i W(x)$$

/// $\frac{\partial W}{\partial \varphi_i}$

we can automatically generate SUSY + grav actions.

$$W(\varphi_i) \Big|_0 = W(\varphi_i \underbrace{(i\theta \epsilon^\mu \bar{\theta} \gamma_\mu \varphi_i + \frac{1}{4} \theta^2 \theta^2 \partial^2 \varphi_i)}_{\text{act. - chiral superfield!}})$$

\uparrow θ^2 term
χ, ψ is only!

$$\left. \begin{aligned} &+ \sqrt{2} \theta \psi - \frac{i}{\sqrt{2}} \theta^2 \gamma_\mu \psi^i \sigma^\mu \bar{\theta} + \theta^2 F_i \end{aligned} \right\} \theta^2$$

$$= \underbrace{\partial_i W(x)}_{\text{the}} F_i$$

can substantially generate SUSY + grav actions.

$$W(\varphi_i) \Big|_{\theta^2 \text{ terms}} = W(\varphi_i) \left(i\theta \sigma^\mu \bar{\theta} \partial_\mu \varphi_i + \frac{1}{4} \theta^2 \theta^2 \partial^2 \varphi_i \right)$$

\uparrow θ^2 terms only!

$$+ \sqrt{2} \theta \psi - \frac{i}{2} \theta^2 \partial_\mu \psi^i \sigma^\mu \bar{\theta} + \theta^2 F_i$$

$$= \underbrace{\partial_\mu W(x)}_{\frac{\partial W}{\partial \varphi_i}} F_i + \frac{\partial W}{\partial \psi_i} \psi$$

can substitute ψ and generate susy + mv actions.

$$W(\varphi_i) \Big|_0 = W(\varphi_i) \left(i\theta \sigma^{\mu\nu} \bar{\theta} \partial_\mu \varphi_i + \frac{1}{4} \theta^2 \bar{\theta}^2 \partial^2 \varphi_i \right)$$

↑ θ^2 term
χ, ψ only!

$$\left(+\sqrt{2} \theta \psi - \frac{1}{\sqrt{2}} \theta^2 \partial_\mu \psi^i \sigma^{\mu\nu} \bar{\theta} + \theta^2 F_i \right)$$

$$= \underbrace{\partial_\mu W(x)}_{\frac{\partial W}{\partial \varphi_i}} F_i + \frac{\partial W}{\partial \psi^i} \psi^i$$

can substitute ψ^i and generate SUSY + grav actions.

$$W(\varphi_i) \Big|_{\theta^2 \text{ terms}} = W\left(\varphi_i \left(i\theta \sigma^{\mu\nu} \gamma_\mu \varphi_i + \frac{1}{4} \theta^2 \theta^2 \gamma^2 \varphi_i \right) \right)$$

\uparrow
 θ^2 terms
xsfs only!

$$\left(+\sqrt{2}\theta\psi - \frac{i}{2} \theta^2 \gamma_\mu \psi^i \sigma^{\mu\nu} \bar{\theta} + \theta^2 F_i \right)$$

$$= \underbrace{\gamma_i W(x)}_{\frac{\partial W}{\partial \varphi_i}} F_i + \frac{\partial W}{\partial \varphi_i} \left(\frac{\partial W}{\partial \varphi_i} \right) \Big|_{\theta^2}$$

$$(\partial\psi^i)(\partial\psi^j) = -\frac{1}{2}(\psi^i\psi^j)\partial^2 + \dots$$

$$\textcircled{=} \partial_i W F^i = -\frac{1}{2} \partial_i \partial_j W \psi^i \psi^j$$

$$= -\frac{1}{2} (\partial_i \psi^i \psi^j) \partial^2$$

$$\frac{\partial (\theta \psi^i)}{\partial \psi^j} = -\frac{1}{2} (\psi^i \psi^j) \theta^2 + \theta^2 \delta^{ij}$$

$$\textcircled{=} \quad \partial_i W F^i = -\frac{1}{2} \partial_i \partial_j W \psi^i \psi^j$$

$$f(x+\Delta x) = f(x) + \frac{\Delta x}{1!} f'(x) + \frac{\Delta x^2}{2!} f''(x)$$

$$(\theta \psi^i)(\theta \psi^j) = -\frac{1}{2} (\psi^i \psi^j) \theta^2 + \theta^2 \psi^i \psi^j$$

$$\textcircled{=} \quad \partial_i W F^i = -\frac{1}{2} \partial_i \partial_j W \textcircled{\psi^i \psi^j}$$

has at least 3 θ

$$f(x+\Delta x) = f(x) + \frac{\Delta x}{1!} f'(x) + \frac{\Delta x^2}{2!} f''(x) + \frac{\Delta x^3}{3!} f'''(x) + \dots$$

$$(\theta \psi^i)(\theta \psi^j) = -\frac{1}{2} (\psi^i \psi^j) \theta^2 + \dots \quad (dx) = (x, 1)$$

$$\textcircled{=} \quad \partial_i W F^i = -\frac{1}{2} \partial_i \partial_j W \textcircled{\psi^i \psi^j}$$

has at least 3 θ

$$f(x+\Delta x) = f(x) + \frac{\Delta x}{1!} f'(x) + \frac{\Delta x^2}{2!} f''(x) + \frac{\Delta x^3}{3!} f'''(x) + \dots$$

$$K = \overline{\Phi \Phi}$$

$W(\Phi)$ - arbitrary \rightarrow

$$\mathcal{L} = \int d^4\theta K(\Phi, \overline{\Phi}) + \int d^2\theta W(\Phi) + \text{h.c.}$$

$$= -\partial_\mu \Phi_i \partial^\mu \overline{\Phi}_i + i \partial_m \overline{\Psi}_i \bar{\sigma}^m \Psi_i$$

we can not get
new derivative
so this

$\mathcal{L}(\Psi, \theta) = \mathcal{L}(\Psi) + \int d^4x \mathcal{L}(\Psi)$
 $\mathcal{L}(\Psi) = \mathcal{L}(\Psi) + \int d^4x \mathcal{L}(\Psi)$

$$K = \overline{\Phi}_i \Phi_i$$

$W(\overline{\Phi}_i)$ - arbitrary \Rightarrow

$$\mathcal{L} = \int d^4\theta K(\Phi_i, \overline{\Phi}_i) + \int d^2\theta W(\Phi_i) + \text{h.c.}$$

$$= -\partial_\mu \Phi_i \partial^\mu \overline{\Phi}_i + i \partial_\mu \overline{\Psi}_i \bar{\sigma}^{\mu\nu} \Psi_i$$

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$$K = \overline{\Phi}_i \Phi_i$$

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$$= -\partial_\mu \Phi_i \partial^\mu \overline{\Phi}_i + i \partial_\mu \overline{\Psi}_i \bar{\sigma}^{\mu\nu} \Psi_i + F_i \overline{F}_i$$

kinetic terms

$$K = \overline{\Phi} \Phi$$

$W(\Phi_i)$ - arbitrary \Rightarrow

$$\mathcal{L} = \int d^4\theta \left[K(\Phi_i, \overline{\Phi}_i) + \int d^2\theta W(\Phi_i) + \text{h.c.} \right]$$

$$= -\partial_\mu \Phi_i \partial^\mu \overline{\Phi}_i + i \partial_\mu \overline{\Psi}_i \bar{\sigma}^{\mu\nu} \Psi_i + F_i \overline{F}_i$$

$$K = \overline{\Phi} \Phi$$

$W(\Phi)$ - arbitrary \Rightarrow

$$\mathcal{L} = \int d^4\theta K(\Phi, \overline{\Phi}) + \int d^2\theta W(\Phi) + \text{h.c.}$$

$$= \underbrace{-\partial_\mu \Phi \partial^\mu \overline{\Phi} + i \partial_\mu \overline{\Psi} \sigma^\mu \Psi + F \overline{F}}_{\text{kinetic terms}} + \dots$$

+

$$K = \overline{\Phi}_i \Phi_i$$

$W(\Phi_i)$ - arbitrary \rightarrow

$$\mathcal{L} = \int d^4\theta \left[K(\Phi_i, \overline{\Phi}_i) + \int d^2\theta W(\Phi_i) + \text{h.c.} \right]$$

$$= \left[-\partial_\mu \Phi_i \partial^\mu \overline{\Phi}_i + i \partial_\mu \overline{\Psi}_i \bar{\sigma}^\mu \Psi^i + F_i \overline{F}_i \right] +$$

$$+ \lambda W F^i + \overline{\lambda W F^i}$$

$$K = \overline{\Phi_i} \Phi_i$$

$W(\overline{\Phi_i})$ - arbitrary \rightarrow

$$\mathcal{L} = \int d^4\theta \uparrow K(\Phi_i, \overline{\Phi_i}) + \int d^2\theta W(\Phi_i) + h.c$$

$$= \underbrace{-\partial\Phi_i \partial\overline{\Phi_i} + i \partial_m \overline{\Psi_i} \delta^m \Psi_i + F_i \overline{F_i}}_{\text{herm. conj.}} + \lambda W F^i + \overline{\lambda W F^i} - \frac{1}{2} \lambda_i \partial_j W \Phi^i \Psi^j - \frac{1}{2} \overline{\lambda_i \partial_j W} \overline{\Psi^i \Psi^j}$$

5) Let's look at the bosonic part.

$$-\sum_{\mathbf{r}} \partial_{\mathbf{r}} \bar{\psi} (\psi + \bar{\psi} \bar{\psi}) \partial_{\mathbf{r}} \psi + \bar{\psi} \psi$$

⇒ let's look at the bosonic part.

$$-\partial\varphi_i \partial\varphi_i + F_i \bar{F}_i + \mathcal{D}_\mu W F^i + \overline{\lambda} W F^i$$

⇒ F_i does not have a kinetic term.

5) let's look at the bosonic part.

$$-\partial\varphi_i \partial\bar{\varphi}_i + F_i \bar{F}_i + \mathcal{D}_\mu W F^i + \overline{\mathcal{D}_\mu W} \bar{F}^i$$

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→ enter into Lagrangian quadratically

5) let's look at the bosonic part.

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⇒ F_i do not have a kinetic term.

→ enter into Lagrangian quadratically

⇒ integrate F_i out (exact QM)

$$\frac{\partial L}{\partial F_i} = F_i + \lambda_i W = 0 \Rightarrow F_i = -\lambda_i W$$

$$\frac{\partial L}{\partial \lambda_i} = F_i + \lambda_i W = 0 \Rightarrow F_i = -\lambda_i W$$

5) let's look at the bosonic part

$$-\partial\varphi_i \partial\bar{\varphi}_i + \underbrace{F_i \bar{F}_i + F_i W F^i + \bar{W} \bar{F}_i}_{-V(\varphi_i)}$$

$\Rightarrow F_i$ does not have a kinetic term.

\rightarrow enter into Lagrangian quadratically

\Rightarrow integrate F_i out (exact QM)

$$\frac{\partial \mathcal{L}}{\partial F_i} = F_i + \lambda_i W = 0 \Rightarrow F_i = -\lambda_i W$$

$$\frac{\partial \mathcal{L}}{\partial F_i} = F_i + \lambda_i W = 0 \Rightarrow F_i = -\lambda_i W$$

$$-v(\mathbf{w}_i) = \|\lambda_i W\|^2 - \|\partial_i W\|^2$$

5) let's look at the bosonic part.

$$-\partial\varphi_i \partial\bar{\varphi}_i + \underbrace{F_i \bar{F}_i}_{\textcircled{1}} + \underbrace{F_i W F^i}_{\textcircled{2}} + \underbrace{\lambda \bar{W} F^i}_{\textcircled{3}}$$

$\Rightarrow F_i$ does not have a kinetic term.

\rightarrow enter into Lagrangian quadratically

\Rightarrow integrate F_i out (exact QM)

$$\frac{\partial \mathcal{L}}{\partial F_i} = F_i + \lambda_i W = 0 \Rightarrow F_i = -\lambda_i W$$

$$\frac{\partial \mathcal{L}}{\partial F_i} = F_i + \lambda_i W = 0 \Rightarrow F_i = -\lambda_i W$$

$$-v(\lambda_i) = \overset{\textcircled{1}}{|W|^2} - \overset{\textcircled{2}}{|\lambda_i W|^2} - \overset{\textcircled{3}}{|\lambda_i W|^2}$$

$$\frac{\delta \mathcal{L}}{\delta F_i} = F_i + \gamma_i W = 0 \Rightarrow F_i = -\gamma_i W$$

$$\frac{\delta \mathcal{L}}{\delta F_i} = F_i + \gamma_i \overline{W} = 0 \Rightarrow F_i = -\gamma_i \overline{W}$$

$$-V(\varphi_i) = \overset{\textcircled{1}}{|W|^2} - \overset{\textcircled{2}}{|\gamma_i W|^2} + \overset{\textcircled{3}}{|\gamma_i \overline{W}|^2}$$

$$-V(\varphi_i) = -|\gamma_i W|^2 \quad V(\varphi_i) =$$

$$\frac{\partial \mathcal{L}}{\partial F_i} = F_i + \lambda_i W = 0 \Rightarrow F_i = -\lambda_i W$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_i} = F_i + \lambda_i W = 0 \Rightarrow F_i = -\lambda_i W$$

$$-V(p_i) = \overset{\textcircled{1}}{|W|^2} - \overset{\textcircled{2}}{|W|^2} - \overset{\textcircled{3}}{|W|^2}$$

$$- |W|^2 \quad \boxed{V(p_i) = |W|^2}$$

$$\partial_i W = 0 \Rightarrow F_i = -\partial_i W$$

$$\overline{\partial_i W} = 0 \Rightarrow F_i = -\partial_i \overline{W}$$

$$|\partial_i W|^2 - |\partial_i \overline{W}|^2$$

$$|\partial_i W|^2$$

$$V(\varphi_i) = |\partial_i W|^2$$

$$K = \mathcal{L}(\varphi_i)$$

$$\mathcal{L} = \int d^4 \theta_i K$$

$$= -\partial_{\mu} \varphi_i \partial^{\mu} \varphi_i + i \partial_{\mu} \varphi_i \partial^{\mu} \overline{\varphi}_i + \partial_i W$$

• $v > 0 \Rightarrow$ If sys is unbroken: $v = 0$

• $V \geq 0 \Rightarrow$ If susy is unbroken: $V|_{\text{vacuum}} = 0$

- $V \geq 0 \Rightarrow$ If susy is unbroken: $V|_{\text{vacuum}} = 0$
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• $V \geq 0 \Rightarrow$ If susy is unbroken: $V|_{\text{vacuum}} = 0$

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• In order to have (a perturbative SUSY vacuum)
 $F_i = 0$

• $V \geq 0 \Rightarrow$ If susy is unbroken: $V|_{\text{vacuum}} = 0$

$\partial_i W = 0$ for all i

• In order to have (a perturbative SUSY vacuum)
 $F_i = 0$ (F-term condition).

• $V \geq 0 \Rightarrow$ If SUSY is unbroken: $V|_{\text{vacuum}} = 0$

$\lambda_i W = 0$ for all i

• In order to have (a perturbative SUSY vacuum)
 $F_i(\phi_i) = 0$ (F-term condition).

• $V \geq 0 \Rightarrow$ If susy is unbroken: $V|_{\text{vacuum}} = 0$

$\lambda_i W = 0$ for all i

• In order to have (a perturbative SUSY vacuum)

$$F_i(\phi_i) = 0 \quad (\text{F-term condition}).$$

some \nearrow times have multiple solutions.

• $V \geq 0 \Rightarrow$ If susy is unbroken: $V|_{\text{vacuum}} = 0$

$\lambda_i W = 0$ for all i

• In order to have (a perturbative SUSY vacuum)

$F_i(\phi_i) = 0$ (F-term condition).

some times have multiple solutions.

\Rightarrow a space of solution

• $V \geq 0 \Rightarrow$ If susy is unbroken: $V|_{\text{vacuum}} = 0$

$\lambda_i W = 0$ for all i

• In order to have (a perturbative SUSY vacuum)

$F_i(\phi_i) = 0$ (F-term condition).

some \nearrow times have multiple solutions.

\Rightarrow a space of solution = a moduli space of a model

$$W \underbrace{\partial_i \partial_j W}_{\text{a scalar}} \psi^i \psi^j \rightarrow W(\eta_{ij} \theta^i \theta^j + \frac{1}{4} \theta^i \theta^2 \theta^j \theta^k)$$

$$\chi^i \psi^j \rightarrow \frac{1}{2} \theta^i \theta^j \theta^k \theta^l + \theta^i \theta^j \theta^k + \theta^i \theta^j \theta^k$$

$$\psi^i \psi^j \rightarrow \theta^i \theta^j \theta^k \theta^l + \theta^i \theta^j \theta^k + \theta^i \theta^j \theta^k$$

$\underbrace{\delta_i \delta_j W}_{\text{a scalar}} \psi^i \psi^j \rightarrow$ mass term, Yukawa couplings

$\chi^2 F^2$

$(\psi^i \psi^j) \left(\frac{1}{2} \theta^2 \psi^k \psi^l + \theta^2 F \right)$

$(\psi^i \psi^j) \left(\frac{1}{2} \theta^2 \psi^k \psi^l + \theta^2 F \right)$

$(\psi^i \psi^j) \left(\frac{1}{2} \theta^2 \psi^k \psi^l + \theta^2 F \right)$

$\underbrace{\partial_i \partial_j W}_{\text{a scalar}} \psi^i \psi^j \rightarrow$ mass term, Yukawa couplings

nlsm
nonlinear sigma model

$\int d^4\theta W(\varphi, \psi)$ $\psi^i \psi^j$ \rightarrow mass term, Yukawa couplings
 $W(\varphi, \psi) = \varphi^i \psi^j \psi^k$

a scalar

SUSY \hookrightarrow nlsm
 nonlinear sigma model

$$L = \int d^4\theta K(\varphi, \bar{\varphi}) + \int d^2\theta W(\varphi) + h.c.$$

$\int d^4\theta W(\varphi, \psi) \rightarrow$ mass term, Yukawa couplings
a scalar

SUSY nlsm
 non-linear sigma model

K, W are arbitrary

$$L = \int d^4\theta K(\varphi, \bar{\varphi}) + \int d^2\theta W(\varphi) + h.c.$$

$\underbrace{W(\phi, \psi)}_{\text{a scalar}} \psi^i \psi^j \rightarrow$ mass term, Yukawa couplings
 $W(\phi, \psi) = \psi^\dagger \psi + \psi \psi^\dagger$

SUSY $\underbrace{\text{nlsm}}_{\text{non-linear sigma model}}$

K, W are arbitrary

$$\mathcal{L} = \int d^4\theta K(\phi, \bar{\phi}) + \int d^2\theta W(\phi) + \text{h.c.}$$

$K = \sum_i p_i \bar{\phi}_i$

lings

y
 x
lines

\Rightarrow sigma model

$\{ |\theta| \leq k(\theta) \}$

$\frac{1}{2} \sigma^2 \rho^2 \Rightarrow \{ \theta \in \mathbb{R}^n \}$

$\rightarrow \langle \rho, \rho \rangle + \rho(\theta) + \rho(\theta)$
all related to the same

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sigma model \rightarrow $\int d^4x \sqrt{-g} \mathcal{L}(\phi, \partial\phi)$

$$-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi \rightarrow \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} f(\phi) \partial_\mu \phi \partial^\mu \phi \right]$$

$$\mathcal{L}(\phi) \rightarrow \mathcal{L}(\phi, \partial\phi) + R(\phi) + f(\phi)$$

sigma model $\left\{ \begin{array}{l} \text{1st part } K(\varphi, \bar{\varphi}) \\ \text{2nd part } f(\varphi) \end{array} \right.$

$$-\frac{1}{2} g_{\mu\nu} \partial_\mu \varphi^\alpha \partial_\nu \varphi^\beta \rightarrow -\frac{1}{2} f(\varphi) g_{\mu\nu} \partial_\mu \varphi^\alpha \partial_\nu \varphi^\beta$$

$$\frac{1}{2} g_{\mu\nu} \partial_\mu \varphi^\alpha \partial_\nu \varphi^\beta + \frac{1}{2} g_{\mu\nu} \partial_\mu \varphi^\alpha \partial_\nu \varphi^\beta$$

$$\rightarrow K(\varphi, \bar{\varphi}) + f(\varphi) + f(\bar{\varphi})$$

will lead to Higgs and SUSY

$\int d^4\theta W(\Phi, \bar{\Phi}) \psi^i \psi^j \rightarrow$ mass term, Yukawa couplings
a scalar

SUSY nlsm
 non-linear sigma model

K, W are arbitrary

$$\mathcal{L} = \int d^4\theta K(\Phi, \bar{\Phi}) + \int d^2\theta W(\Phi) + h.c.$$

$K = \sum_i \Phi_i \bar{\Phi}_i$

$$\int \mathcal{L} \Theta K = g_{i\bar{i}} F^i \bar{F}^{\bar{i}} - \frac{1}{2} F^i g_{i\bar{i}} \Gamma_{j\bar{k}}^{\bar{i}} \bar{\Psi}^{\bar{j}} \bar{\Psi}^{\bar{k}} + \dots$$

[The following text is heavily obscured by white chalk scribbles and is largely illegible.]

$$\int \mathcal{L} \Theta K = g_{:i} F^i \bar{F}^{\bar{i}} - \frac{1}{2} F^i g_{:i\bar{i}} \Gamma_{\bar{j}\bar{k}}^{\bar{i}} \bar{\Psi}^{\bar{j}} \bar{\Psi}^{\bar{k}} + \dots$$

→ h.c

$$\int \mathcal{L} \theta K = g_{i\bar{i}} F^i \bar{F}^{\bar{i}} - \frac{1}{2} F^i g_{i\bar{i}} \Gamma_{j\bar{k}}^{\bar{i}} \bar{\Psi}^{\bar{j}} \bar{\Psi}^{\bar{k}} + \dots$$

$g_{i\bar{i}} = \partial_i \partial_{\bar{i}} K$ (a metric on a Kähler manifold ^{the} parameterized by $\{\varphi_i, \bar{\varphi}_{\bar{i}}\}$)

$$\int M \Theta K = g_{i\bar{i}} F^i \bar{F}^{\bar{i}} - \frac{1}{2} F^i g_{i\bar{i}} \Gamma_{j\bar{k}}^{\bar{i}} \bar{\Psi}^{\bar{j}} \bar{\Psi}^{\bar{k}} + \dots$$

$g_{i\bar{i}} = \partial_i \partial_{\bar{i}} K(\varphi_i, \bar{\varphi}_{\bar{i}})$ (a metric on a Kähler manifold ^{to be} parameterized by $\{\varphi_i, \bar{\varphi}_{\bar{i}}\}$)

$$\Gamma_{j\bar{k}}^{\bar{i}} = g^{i\bar{i}} g_{i\bar{j}}, \bar{k}$$

$$\int \mathcal{L}_W(\mathbb{D}) = g_{i\bar{i}} F^i \bar{F}^{\bar{i}} - \frac{1}{2} F^i g_{i\bar{i}} \Gamma_{j\bar{k}}^{\bar{i}} \bar{\Psi}^{\bar{j}} \bar{\Psi}^{\bar{k}} + \dots$$

$g_{i\bar{i}} = \partial_i \partial_{\bar{i}} K(\varphi_i, \bar{\varphi}_{\bar{i}})$ (a metric on a Kähler manifold ^{the} parameterized by $\{\varphi_i, \bar{\varphi}_{\bar{i}}\}$)

$$\Gamma_{j\bar{k}}^{\bar{i}} = g_{i\bar{i}} g_{i\bar{j}}, \bar{k}$$

$$\int \mathcal{L} \Theta K = g_{i\bar{i}} \frac{F^i \bar{F}^{\bar{i}}}{\int} - \frac{1}{2} F^i g_{i\bar{i}} \Gamma_{j\bar{k}}^{\bar{i}} \bar{\Psi}^{\bar{j}} \bar{\Psi}^{\bar{k}} + \dots$$

Integrate F^i

$g_{i\bar{i}} = \partial_i \partial_{\bar{i}} K(\varphi_i, \bar{\varphi}_{\bar{i}})$ (a metric on a Kähler manifold parameterized by $\{\varphi_i, \bar{\varphi}_{\bar{i}}\}$)

$$\Gamma_{j\bar{k}}^{\bar{i}} = g^{i\bar{i}} g_{i\bar{j}, \bar{k}}$$

$$\mathcal{L} = -g_{i\bar{j}} \partial_\mu \Phi^i \partial^\mu \bar{\Phi}^{\bar{j}} - i g_{i\bar{j}} \bar{\Psi}^{\bar{i}} \gamma^\mu D_\mu \Psi^i$$

$$g_{i\bar{j}} = g_{i\bar{j}}(\tau_i, \bar{\tau}_{\bar{j}})$$

$$+ \frac{1}{4} R_{i\bar{j}k\bar{l}} \bar{\tau}_{\bar{j}} \tau_{\bar{l}}$$

$$\mathcal{L} = - \underbrace{g_{i\bar{j}} \partial_\mu \Phi^i \partial^\mu \bar{\Phi}^{\bar{j}}}_{\text{scalar}} - i g_{i\bar{j}} \bar{\Psi}^{\bar{i}} \gamma^\mu D_\mu \Psi^i$$

$$g_{i\bar{j}} = g_{i\bar{j}}(\tau_i, \bar{\tau}_{\bar{j}})$$

$$+ \frac{1}{4} R_{i\bar{j}k\bar{l}} \Psi^i \Psi^{\bar{k}} \bar{\Psi}^{\bar{j}} \bar{\Psi}^{\bar{l}}$$

$$\mathcal{L} = - \underbrace{g_{i\bar{j}} \partial_\mu \Phi^i \partial^\mu \bar{\Phi}^{\bar{j}}}_{\text{kinetic}} - i g_{i\bar{j}} \bar{\Psi}^{\bar{i}} \bar{\sigma}^\mu D_\mu \Psi^i$$

$$g_{i\bar{j}} = g_{i\bar{j}}(\mathcal{P}_i, \bar{\mathcal{P}}_{\bar{j}})$$

$$+ \frac{1}{4} R_{i\bar{j}k\bar{l}} \Psi^i \Psi^k \bar{\Psi}^{\bar{j}} \bar{\Psi}^{\bar{l}}$$

$$D_\mu = \partial_\mu \Psi^i + \Gamma_{j\bar{k}}^i \partial_\mu \mathcal{P}^{\bar{k}} \Psi^j$$

$$\mathcal{L} = - \underbrace{g_{i\bar{j}} \partial_\mu \Phi^i \partial^\mu \bar{\Phi}^{\bar{j}}}_{\text{scalar}} - i g_{i\bar{j}} \bar{\Psi}^{\bar{i}} \gamma^\mu D_\mu \Psi^i$$

$$g_{i\bar{j}} = g_{i\bar{j}}(\varphi_i, \bar{\varphi}_{\bar{j}})$$

$$+ \frac{1}{4} R_{i\bar{j}k\bar{l}} \Psi^i \Psi^k \bar{\Psi}^{\bar{j}} \bar{\Psi}^{\bar{l}}$$

$$D_\mu = \partial_\mu \Psi^i + \Gamma_{j\bar{k}}^i \partial_\mu \varphi^k \Psi^{\bar{j}}$$

$$\mathcal{L} = - \underbrace{\partial_{\mu} \bar{\psi}^i \gamma^{\mu} \psi^i}_{\text{Dirac}} - i g_{ij} \bar{\psi}^i \sigma^{\mu\nu} D_{\mu} \psi^j$$

$$g_{ij} = g_{ij}(p_i, p_j) + \frac{1}{4} R_{ij\kappa\epsilon} \psi^i \psi^{\kappa} \bar{\psi}^j \bar{\psi}^{\epsilon}$$

$$k = \bar{p}_i, \bar{q}_j$$

$$g_{ij} = \delta_{ij}$$

$$D_{\mu} \psi^i = \partial_{\mu} \psi^i + \Gamma_{jk}^i \partial_{\mu} p^k \psi^j$$

$$\mathcal{L} = -g_{i\bar{j}} \dot{\varphi}^i \dot{\bar{\varphi}}^{\bar{j}} - ig_{i\bar{j}} \bar{\psi}^i \bar{\sigma}^{\mu} D_{\mu} \psi^j$$

$$g_{i\bar{j}} = g_{i\bar{j}}(\varphi_i, \bar{\varphi}_{\bar{j}}) + \frac{1}{4} R_{i\bar{j}k\bar{l}} \psi^i \psi^k \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{l}}$$

$$k = \bar{i}, \bar{j}$$

$$g_{i\bar{i}} = \delta_{i\bar{i}} \Rightarrow$$

$$D_{\mu} = \partial_{\mu}$$

$$D_{\mu} \psi^i = \partial_{\mu} \psi^i + \Gamma_{jk}^i \partial_{\mu} \varphi^k \psi^j$$

A special case of nkms is WZ model.

$$K_i = \sum_{j=1}^n P_i \overline{P_j} \quad \text{and}$$

In order to have a preparative system

$$F_i(\mathbb{R}) \oplus \mathbb{D} \quad (\text{E-keem on})$$

soln must have small values
→ a span of solutions = a w

A special case of integris is WZ model.

$$K_i = \sum_{j=1}^n p_j \overline{p_j} \quad \text{and} \quad W = v_i p_i +$$

In order to have a property $(+ a_{ij} p_i p_j + \lambda_{ijk} p_i p_j p_k)$

$$F_i(\mathbb{R}) \oplus \mathbb{D} \quad (\text{F-ternary})$$

subalgebras have ...
 \Rightarrow a span of solutions = ...

A special case of integris is WZ model.

$$K_i = \sum_{j=1}^n p_i \overline{p_j} \quad \text{and} \quad W = v_i p_i +$$

In order to have a proper $(\sum_{ij} \alpha_{ij} p_i p_j + \sum_{ijk} \lambda_{ijk} p_i p_j p_k)$
 $v_i, \alpha_{ij}, \lambda_{ijk}$ "are couplings"

other lines have null solutions

⇒ a special solution = a wave

A special case of integrals is WZ model

$$K_i = \sum_{j=1}^n p_i \overline{p_j} \quad \text{and} \quad W = \sum_{i,j} u_{ij} p_i p_j + \sum_{i,j,k} \lambda_{ijk} p_i p_j p_k$$

In order to have the property $\sum_{i,j} u_{ij} p_i p_j + \sum_{i,j,k} \lambda_{ijk} p_i p_j p_k$
 u_{ij}, λ_{ijk} "are couplings"

...
 \Rightarrow a space of solutions

A special case of n-bosons is the WZ model

$$K_i = \sum_{j=1}^n p_i \overline{p_j} \quad \text{and} \quad W = \sum_{i=1}^n p_i + \sum_{i=1}^n \overline{p_i} + \dots$$

In order to have a propagator $\dots + \underbrace{c_{ij} p_i p_j}_{\text{couplings}} + \underbrace{\lambda_{ijk} p_i p_j p_k}_{\text{couplings}} + \dots$

... a space of solutions ...

A special case of integrals is WZ model

$$K_i = \sum_{j=1}^n p_i \overline{p_j} \quad \text{and} \quad W = \sum_{i,j} p_i p_j + \dots$$

non-terminating action

would lead to renormalization (QFT)

In order to have a proper $\int p_i p_j + \lambda_{ijk} p_i p_j p_k + \dots$
 $v_i, \lambda_{ij}, \lambda_{ijk}$ "are couplings"

1.1.1 Interacting ϕ^4 model

$$W(\phi) = \nu \phi + \mu \phi^2 + \lambda \phi^3$$

↳ compute a scalar potential.

$$V(\phi) = |\lambda \phi|^2$$

$\phi(x) = f(x)$ relating also

1.1.1 Interactions, Model

$$W(\varphi) = v\varphi + m\varphi + \lambda\varphi^3$$

↳ compute a scalar potential.

$$V(\varphi) = |\gamma W|^2 = |v + m\varphi + \lambda\varphi^2|^2$$

scalar field interactions, $\lambda \phi^3$ model

$$W(\phi) = v\phi + m\phi + \lambda\phi^3$$

↳ compute a scalar potential.

$$V(\phi) = |W|^2 = |v + m\phi + \lambda\phi^2|^2$$

① $v=0, \lambda=0$

$$V(\phi) = |m|^2 \phi \bar{\phi}$$

massless scalar field interactions

$$W(\phi) = v\phi + m\phi^2 + \lambda\phi^3$$

↳ compute a scalar potential.

$$V(\phi) = |iW|^2 = |v + 2im\phi + 3i\lambda\phi^2|^2$$

① $v=0 \quad \lambda=0$

$$V(\phi) = 4|m|^2 \phi\bar{\phi} \rightarrow \text{mass term.}$$

interactions

$$W(\varphi) = v\varphi + \mu\varphi^2 + \lambda\varphi^3$$

compute a scalar potential

$$V(\varphi) = |W|^2 = |v + 2\mu\varphi + 3\lambda\varphi^2|^2$$

① $v=0, \lambda=0$

$$V(\varphi) = \mu^2 \varphi \bar{\varphi} \rightarrow \text{mass term}$$

② $v=0, \mu=0$

$$V(\varphi) = 9|\lambda|^2 |\varphi|^4$$



$$V(\varphi) = |\gamma W|^2 = |v + 2\mu\varphi|^2$$

$$\textcircled{1} v=0 \quad \mu=0$$

$$V(\varphi) = \frac{1}{2} |\mu|^2 \varphi \bar{\varphi} \rightarrow \text{mass term}$$

$$\textcircled{2} v=0 \quad \mu=0$$

$$V(\varphi) = g |\lambda|^2 |\varphi|^4$$

$$K = \sum_i \varphi_i \bar{\varphi}_i$$

interactions $\lambda \phi^3$ model

$$W(\phi) = \nu \phi + \mu \phi^2 + \lambda \phi^3$$

would typically break SUSY at $\phi=0$

↳ compute a scalar potential.

$$V(\phi) = |\lambda W|^2 = \left| \nu + 2\mu\phi + 3\lambda\phi^2 \right|^2$$

① $\nu=0 \quad \lambda=0$

$$V(\phi) = 4|\mu|^2 \phi \bar{\phi} \rightarrow \text{mass term.}$$

② $\nu=0 \quad \mu=0$

$$V(\phi) = 9|\lambda|^2 |\phi|^4$$

$\Rightarrow V$ can be absorbed by a field redefinition $\phi \rightarrow \phi + \frac{v}{2\mu}$ ($\lambda=0$)

$$W = \frac{1}{2} \dot{\phi}^2 + \mu^2 \phi^2 = \frac{1}{2} \left(\dot{\phi} + \frac{v}{2\mu} \right)^2 - \frac{v^2}{4\mu}$$