Title: Intro to Supersymmetry 11

Date: Oct 23, 2007 12:30 PM

URL: http://pirsa.org/07100021

Abstract:

$\bar{D}_{\dot\alpha} \Phi = 0$

$\uparrow$

χsf

$D_\alpha \Phi = 0$

$\uparrow$

anti-chiral superfield

$$\bar{D}_{\dot\alpha}\,\underset{\substack{\uparrow\\ \chi\text{sf}}}{\Phi} = 0 \qquad\qquad D_\alpha \underset{\substack{\uparrow\\ \text{anti-chiral superfield!}}}{\Phi} = 0$$

$$\underset{\substack{\uparrow\\ \text{Kähler potential}}}{K} = K(\Phi, \bar\Phi)$$

⇒ we can automatically generate SUSY-inv actions.

$$S = \int d^4\theta \; K(\Phi, \bar{\Phi}) = \int d^2\theta \, d^2\bar{\theta} \; K(\bar{\Phi}, \bar{\Phi})$$

$$S = \int d^4x \int d^4\theta \; K(\Phi, \bar{\Phi}) = \int d^2\theta \, d^2\bar{\theta} \; K(\bar{\Phi}, \bar{\Phi})$$

$$\delta_{SUSY} S = \int d^4y \int d^4\theta$$

$$S = \int d^4x \int d^4\theta \; K(\Phi, \bar{\Phi}) = \int d^2\theta \, d^2\bar{\theta} \; K(\bar{\Phi}, \bar{\Phi})$$

$$\delta_{SUSY} S = \int d^4x \int d^4\theta \; \left(\xi Q + \bar{\xi} \bar{Q}\right)$$

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$$\delta_{SUSY} S = \int d^4y \int \underbrace{d^4\theta \; {}_{q}(\epsilon Q + \bar{\epsilon} \bar{Q})}_{} K$$

$$S = \int d^4x \int d^4\theta \; K(\Phi, \bar{\Phi}) = \int d^2\theta\, d^2\bar{\theta} \; K(\bar{\Phi}, \bar{\Phi})$$

$$\delta_{susy} S = \int d^4x \int \underbrace{d^4\theta \, (\varepsilon Q + \bar{\varepsilon} \bar{Q})}_{\text{total space-time derivative}} K = \int d^4x \; \partial_m \left[\text{some} \right]$$

$$S = \int d^4x \int d^4\theta \; K(\Phi, \bar{\Phi}) = \int d^2\theta \, d^2\bar{\theta} \; K(\Phi, \bar{\Phi})$$

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$$K(\Phi, \bar{\Phi}) =$$

$$S = \int d^4x \int d^4\theta \; K(\varphi, \bar{\varphi}) = \int d^2\theta \, d^2\bar{\theta} \; K(\bar{\varphi}, \bar{\varphi})$$

$$\delta_{SUSY} S = \int d^4x \int \underbrace{d^4\theta \, (\xi Q + \bar{\xi} \bar{Q})}_{\text{total space-time derivative}} K = \int d^4x \; \partial_m \left[\text{some} \right]$$

$$K(\varphi, \bar{\varphi}) \Rightarrow K(\varphi, \bar{\varphi}) + f(\varphi) + \bar{f}(\bar{\varphi})$$

$$S = \int d^4x \int d^4\theta \; K(\varphi, \bar\varphi) = \int d^2\theta\, d^2\bar\theta \; K(\bar\varphi, \bar\varphi)$$

$$\delta_{SUSY} S = \int d^4x \int \underbrace{d^4\theta \, (\xi Q + \bar\xi \bar Q)}_{\text{total space-time derivative}} K = \int d^4x \; \partial_m \left[\text{some} \right]$$

$$K(\varphi, \bar\varphi) \Rightarrow K(\varphi, \bar\varphi) + f(\varphi) + \bar f(\bar\varphi)$$

will lead to the same SUSY inv action

$K = \Phi \bar{\Phi}$

$\hookrightarrow$ evaluated $\int d^4\theta \Rightarrow$

$\mathcal{L}_{\mu} =$

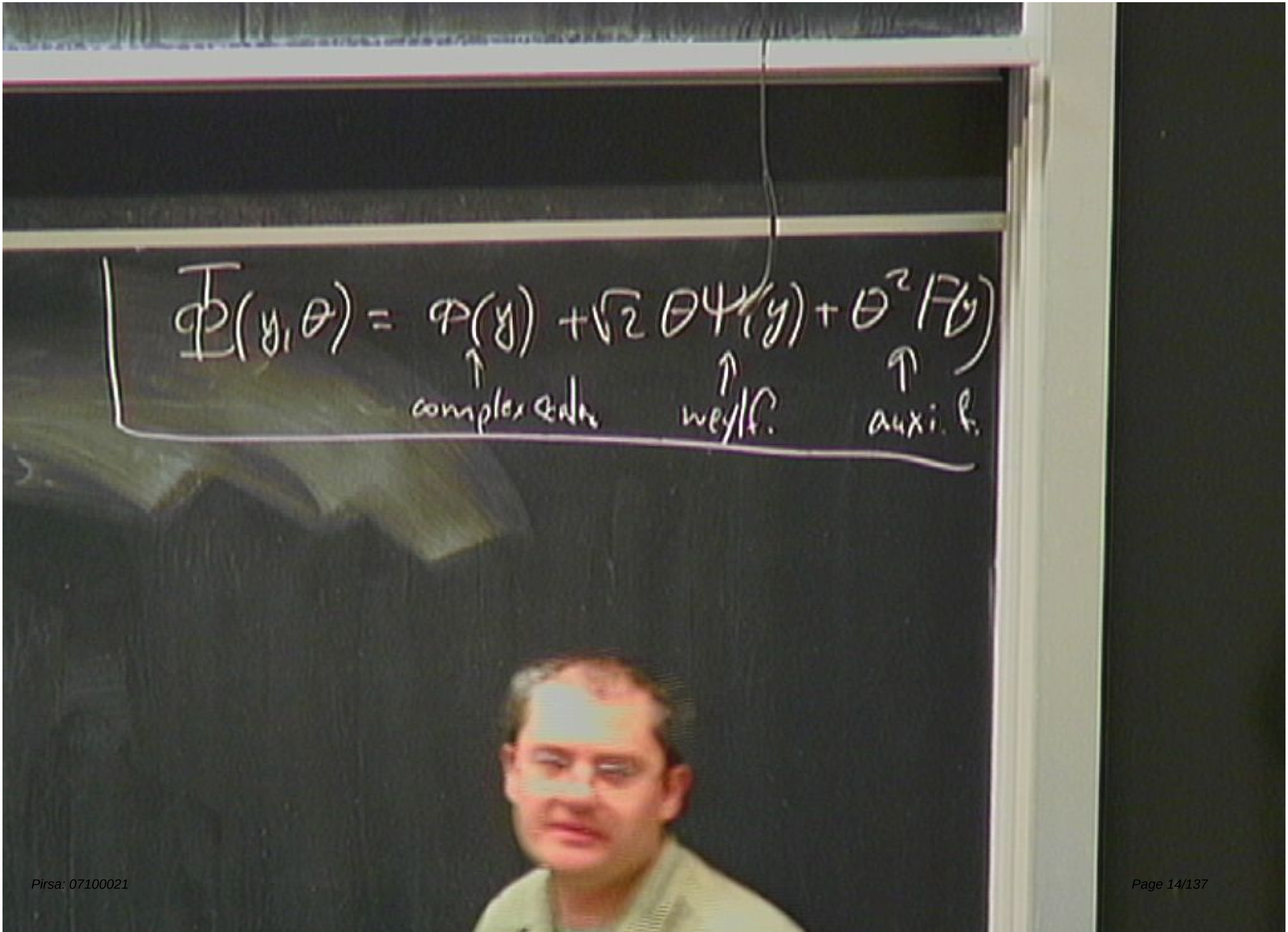
$\Phi(y,\theta) = \varphi(y)$

$$\boxed{\Phi(y,\theta) = \varphi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F}$$

$K = \varphi\bar{\varphi}$

$\hookrightarrow$ evaluated $\int d^4\theta \Rightarrow$

$\mathcal{L}_K =$



$$\Phi(y,\theta) = \underset{\substack{\uparrow \\ \text{complex scalar}}}{\varphi(y)} + \sqrt{2}\,\theta\underset{\substack{\uparrow \\ \text{weyl f.}}}{\psi(y)} + \theta^2 \underset{\substack{\uparrow \\ \text{auxi. f.}}}{F(y)}$$

$$\Phi(y,\theta) = \underset{\text{complex scalar}}{\underset{\uparrow}{\varphi(y)}} + \sqrt{2}\,\underset{\text{weyl f.}}{\underset{\uparrow}{\theta\psi(y)}} + \underset{\text{auxil. f.}}{\underset{\uparrow}{\theta^2 F(y)}}$$

$$K = \varphi\bar{\varphi}$$

$\hookrightarrow$ evaluated $\int d^4\theta \Rightarrow$

$$\mathcal{L}_K = -\partial\varphi\,\partial\bar{\varphi} + i\,\partial_m\bar{\psi}\,\bar{\sigma}^m\psi + F\bar{F}$$

$$K = \Phi\bar{\Phi}$$

↳ evaluated $\int d^4\theta \Rightarrow$

$$\Phi(y,\theta) = \underset{\substack{\uparrow \\ \text{complex scal.}}}{\varphi(y)} + \sqrt{2}\,\theta\underset{\substack{\uparrow \\ \text{weyl f.}}}{\psi(y)} + \theta^2 \underset{\substack{\uparrow \\ \text{auxi. f.}}}{F(y)}$$

$$\mathcal{L}_K = -\partial\varphi\,\partial\bar{\varphi} + i\,\partial_m\bar{\psi}\,\bar{\sigma}^m\psi \underbrace{+ F\bar{F}}_{\text{nondynamical field(s)}}$$

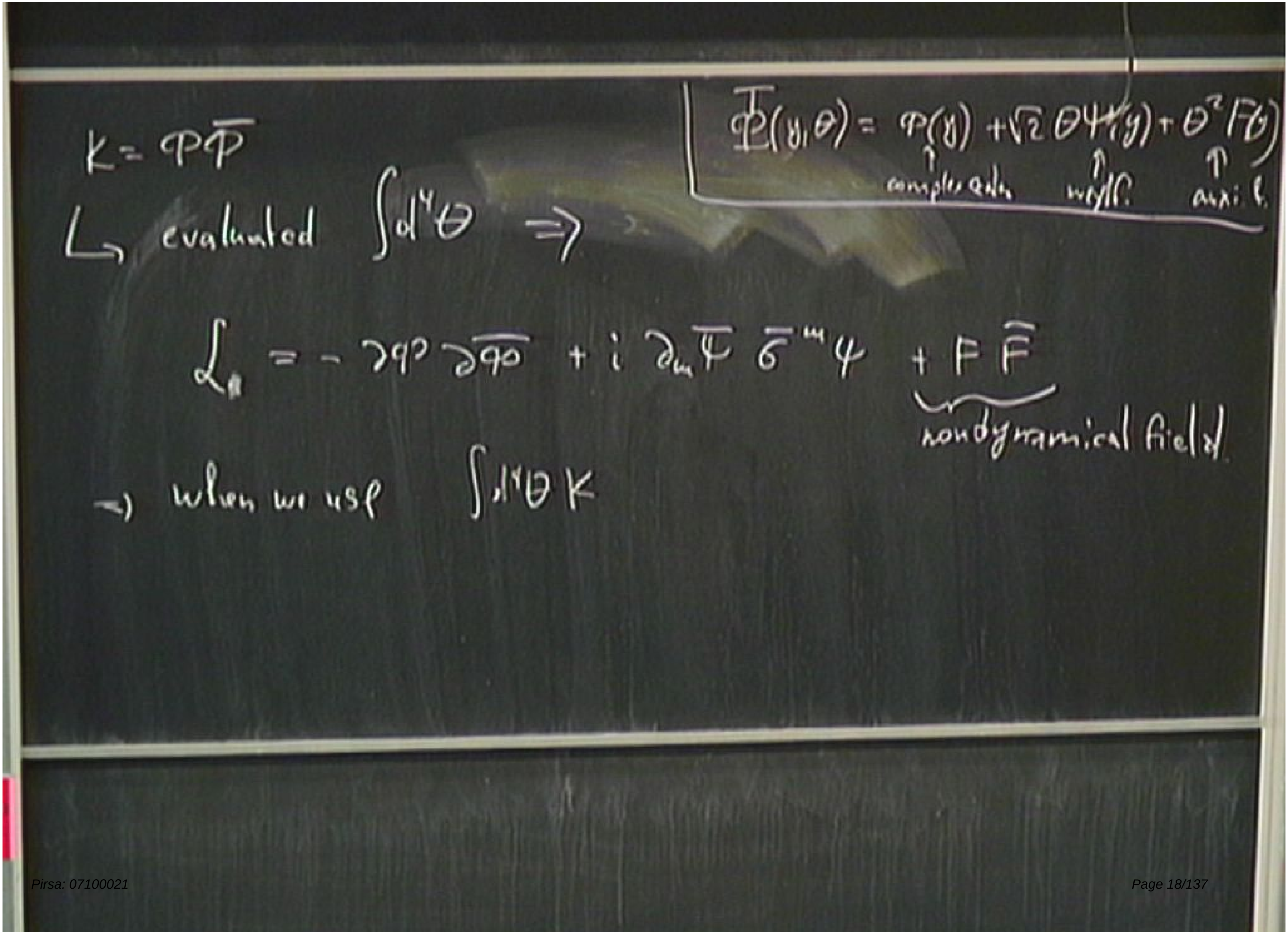
$$\Phi(y,\theta) = \underset{\substack{\uparrow \\ \text{complex scalr}}}{\varphi(y)} + \sqrt{2}\,\theta\underset{\substack{\uparrow \\ \text{weyl f.}}}{\psi(y)} + \theta^2 \underset{\substack{\uparrow \\ \text{auxi. f.}}}{F(y)}$$

$K = \varphi\bar{\varphi}$

↳ evaluated $\int d^4\theta \Rightarrow$

$$\mathcal{L}_{\#} = -\partial\varphi\,\partial\bar{\varphi} + i\,\partial_m\bar{\psi}\,\bar{\sigma}^m\psi \underbrace{+ F\bar{F}}_{\text{nondynamical field}}$$

→ when we use



$$\Phi(y,\theta) = \underset{\substack{\uparrow\\ \text{complex scalar}}}{\varphi(y)} + \sqrt{2}\,\theta\,\underset{\substack{\uparrow\\ \text{weyl f.}}}{\psi(y)} + \theta^2\,\underset{\substack{\uparrow\\ \text{auxi. f.}}}{F(y)}$$

$K = \Phi\bar{\Phi}$

↳ evaluated $\int d^4\theta \Rightarrow$

$$\mathcal{L}_1 = -\partial\varphi\,\partial\bar{\varphi} + i\,\partial_m\bar{\psi}\,\bar{\sigma}^m\psi + \underbrace{F\bar{F}}_{\text{nondynamical field}}$$

→ when we use $\int d^4\theta\, K$

$$\boxed{\Phi(y,\theta) = \underset{\substack{\uparrow \\ \text{complex scalar}}}{\varphi(y)} + \sqrt{2}\,\theta\underset{\substack{\uparrow \\ \text{weyl f.}}}{\psi(y)} + \underset{\substack{\uparrow \\ \text{auxi. f.}}}{\theta^2 F(y)}}$$

$K = \Phi\bar{\Phi}$

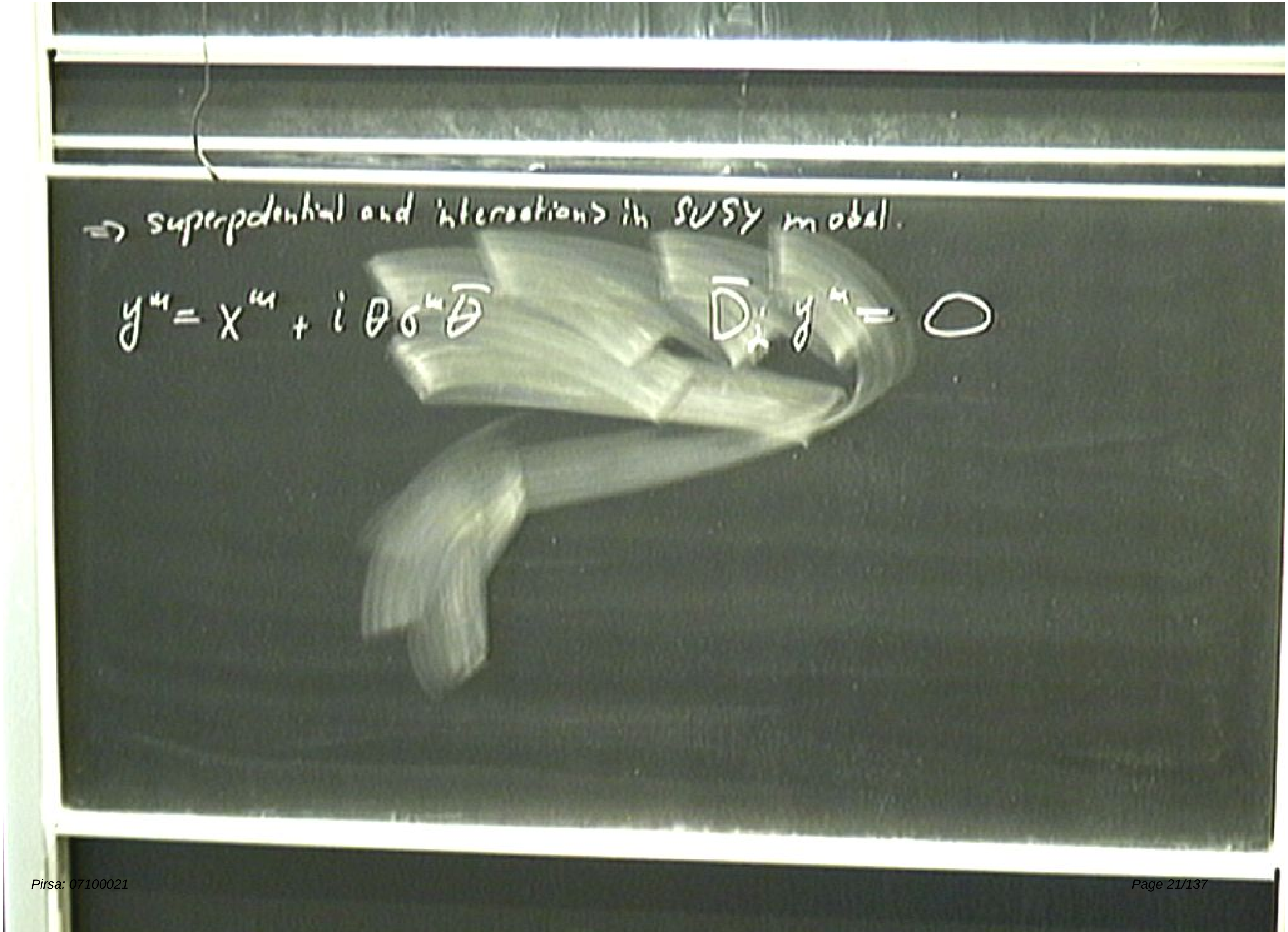
↳ evaluated $\int d^4\theta \Rightarrow$

$$\mathcal{L} = -\partial\varphi\,\partial\bar{\varphi} + i\,\partial_m\bar{\psi}\,\bar{\sigma}^m\psi + \underbrace{F\bar{F}}_{\text{nondynamical field}}$$

⇒ when we use $\int d^4\theta\, K$ we can not get non-derivative interactions

⇒ superpotential and interactions in SUSY model.

$$y^\mu = x^\mu + i\,\theta\sigma^\mu\bar{\theta}$$



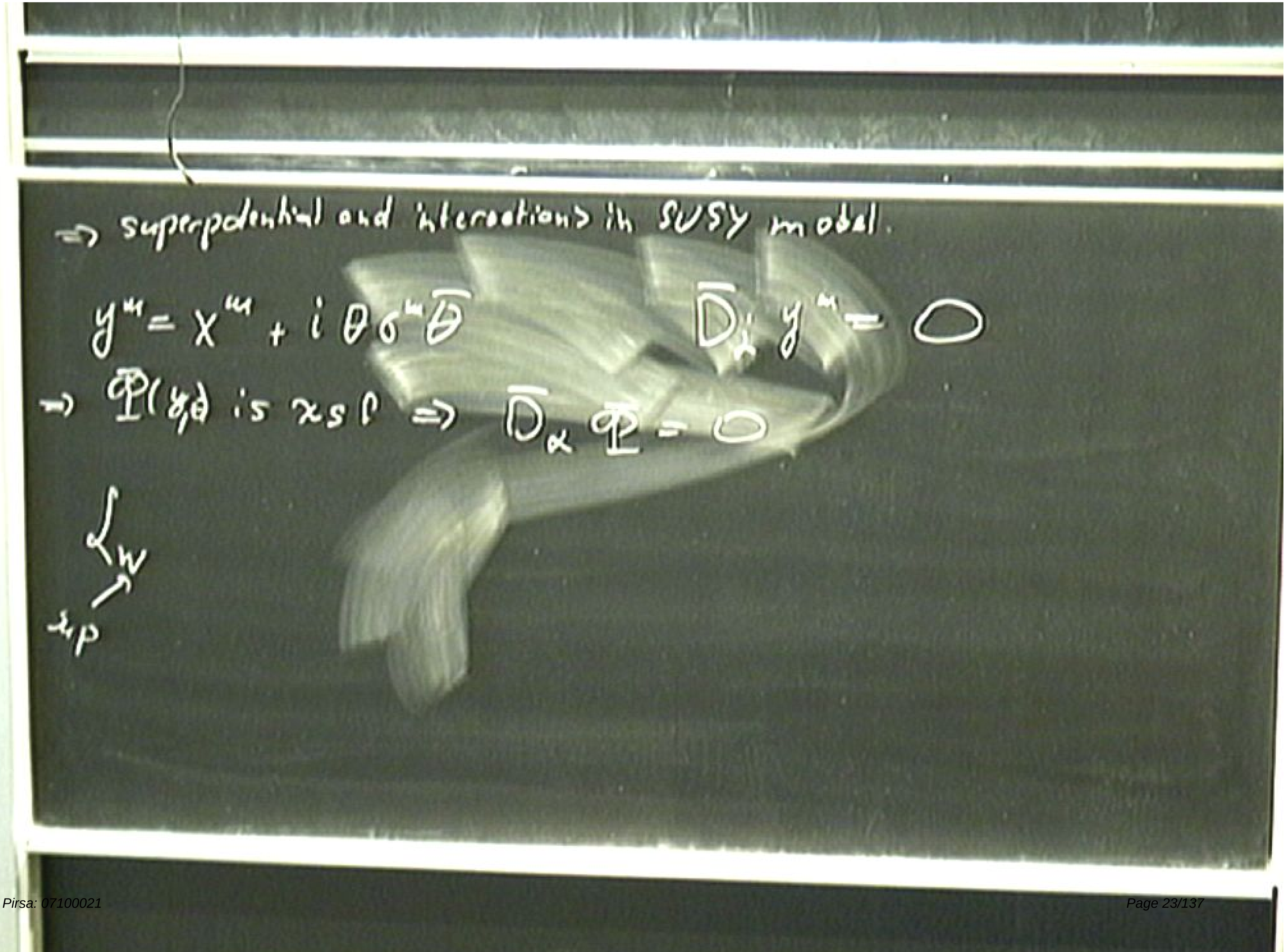
⇒ superpotential and interactions in SUSY model.

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta} \qquad \bar{D}_{\dot\alpha} y^\mu = 0$$

⇒ superpotential and interactions in SUSY model.

$$y^\mu = x^\mu + i\,\theta\sigma^\mu\bar\theta \qquad \bar D_{\dot\alpha}\, y^\mu = 0$$

⇒ $\Phi(y,\theta)$ is χSF ⇒ $\bar D_\alpha \Phi = 0$



⇒ superpotential and interactions in SUSY model.

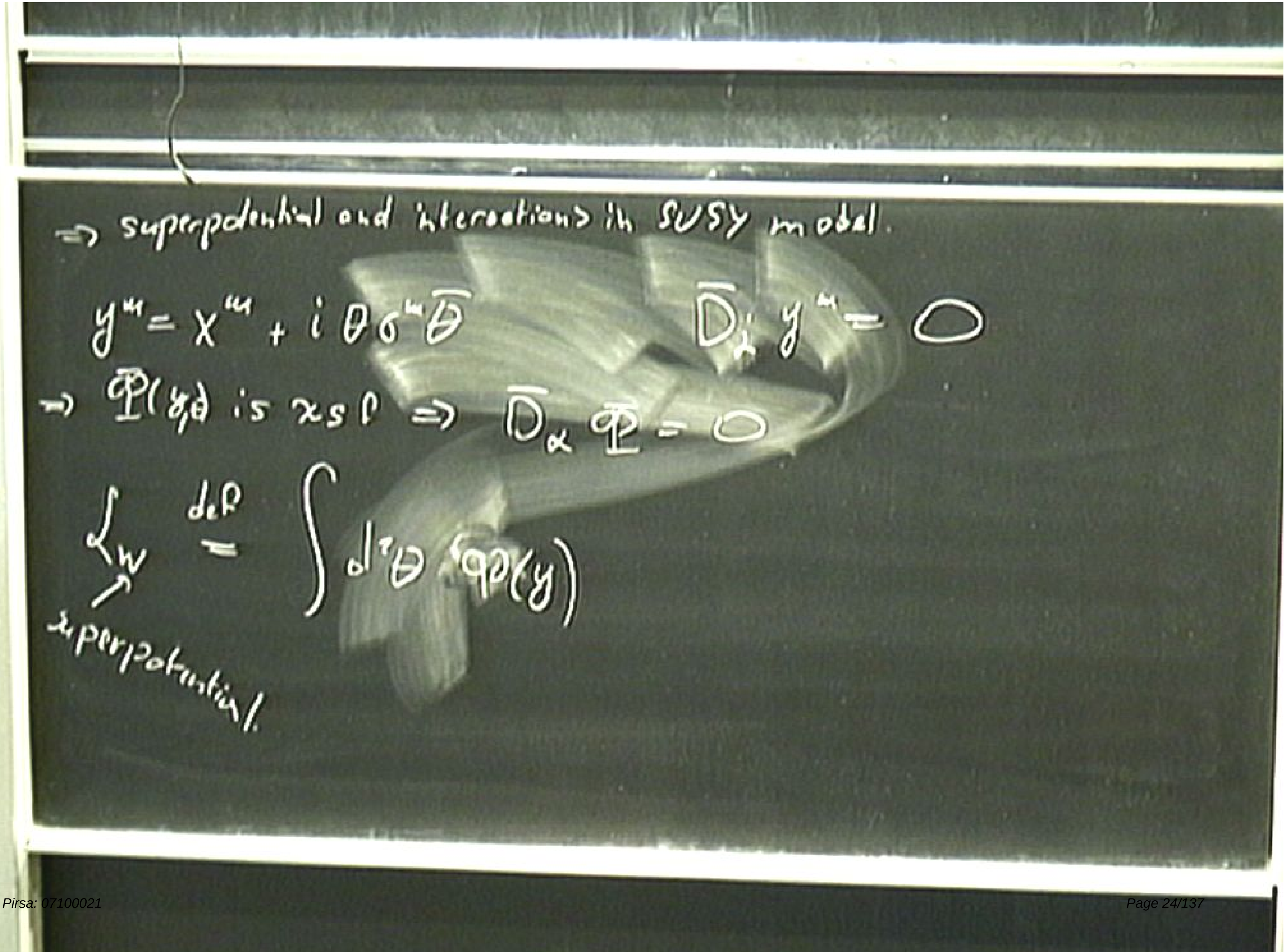
$$y^m = x^m + i\theta\sigma^m\bar{\theta} \qquad \bar{D}_{\dot{\alpha}}\, y^m = 0$$

$\Rightarrow$ $\Phi(y,\theta)$ is xsf $\Rightarrow$ $\bar{D}_{\alpha}\Phi = 0$

$\mathcal{L}_W$

$\nearrow$

x_p



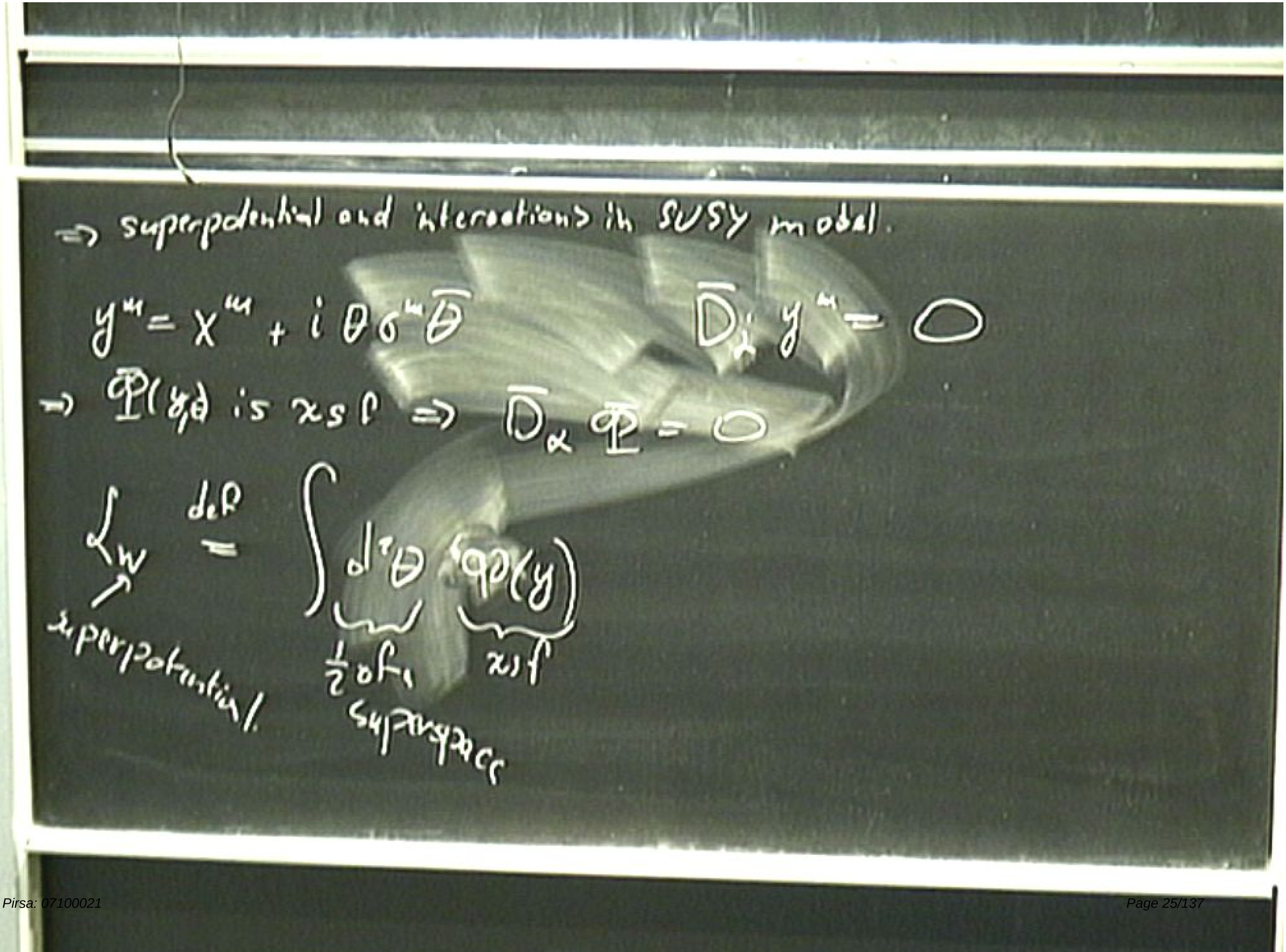
⇒ superpotential and interactions in SUSY model.

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta} \qquad \bar{D}_{\dot\alpha}\, y^\mu = 0$$

⇒ $\Phi(y,\theta)$ is χSF ⇒ $\bar{D}_{\dot\alpha}\Phi = 0$

$$\underset{\nearrow}{\mathcal{L}_W} \overset{\text{def}}{=} \int d^2\theta\, W(\Phi(y))$$

superpotential



⇒ superpotential and interactions in SUSY model.

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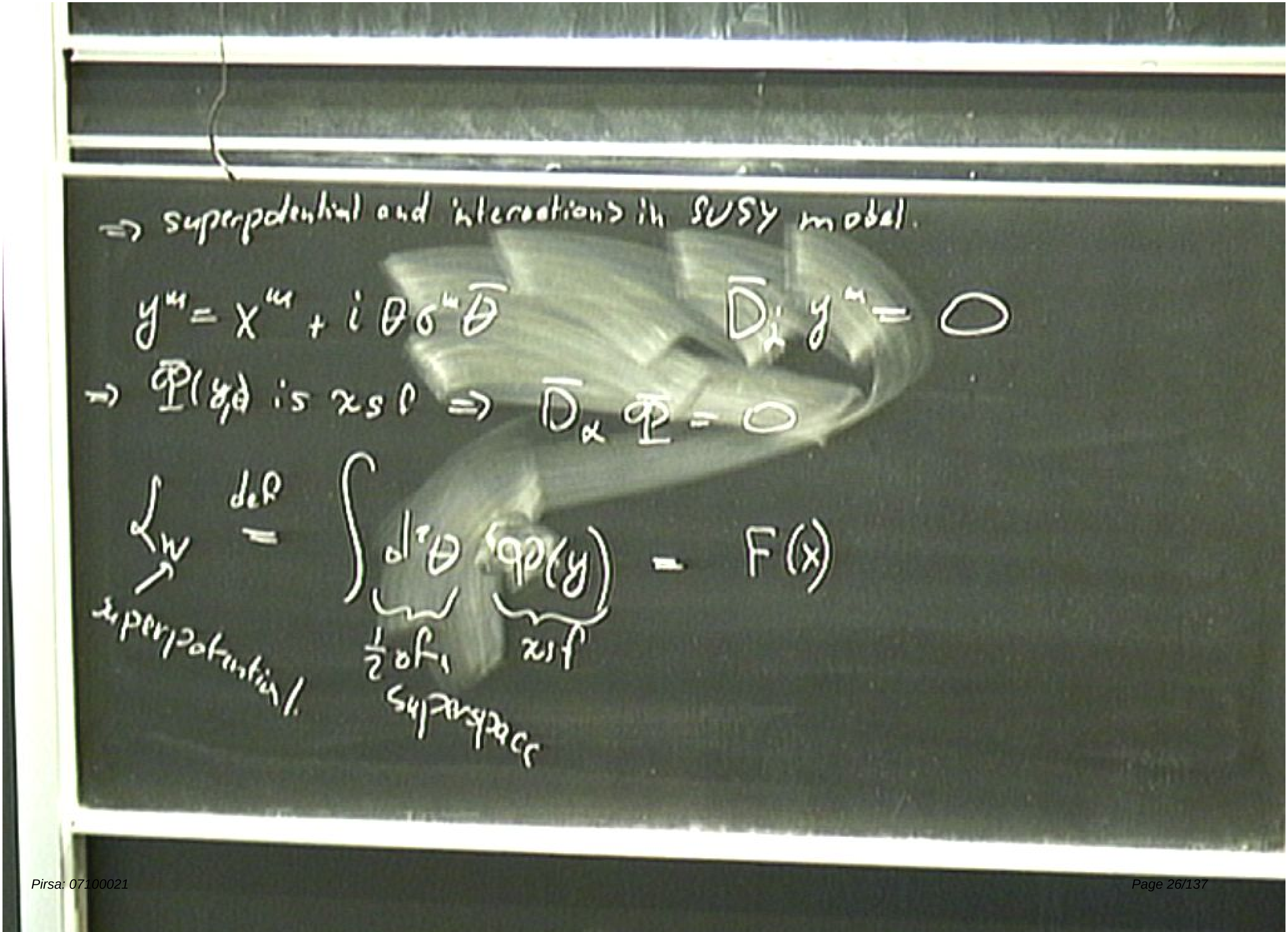
⇒ $\Phi(y,\theta)$ is χsf ⇒ $\bar{D}_{\dot\alpha}\Phi = 0$

$$\mathcal{L}_W \overset{\text{def}}{=} \int \underbrace{d^2\theta}_{\frac{1}{2}\text{ of superspace}} \underbrace{W(\Phi(y))}_{\chi sf}$$

superpotential



⇒ superpotential and interactions in SUSY model.

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar\theta \qquad \bar D_{\dot\alpha}\, y^\mu = 0$$

⇒ $\Phi(y,\theta)$ is χsf ⇒ $\bar D_{\dot\alpha}\Phi = 0$

$$\mathcal{L}_W \overset{\text{def}}{=} \int \underbrace{d^2\theta}_{\frac{1}{2}\text{ of superspace}}\; \underbrace{\mathcal{W}(y)}_{\chi\text{sf}} = F(x)$$

↗ superpotential.

$$\Phi(y,\theta) = \underset{\substack{\uparrow \\ \text{complex scalar}}}{\varphi(y)} + \sqrt{2}\,\theta\underset{\substack{\uparrow \\ \text{weyl f.}}}{\psi}(y) + \boxed{\underset{\substack{\uparrow \\ \text{auxi. f.}}}{\theta^2 F(y)}}$$

$$\bar{\sigma}^m \psi \quad + F\bar{F}$$

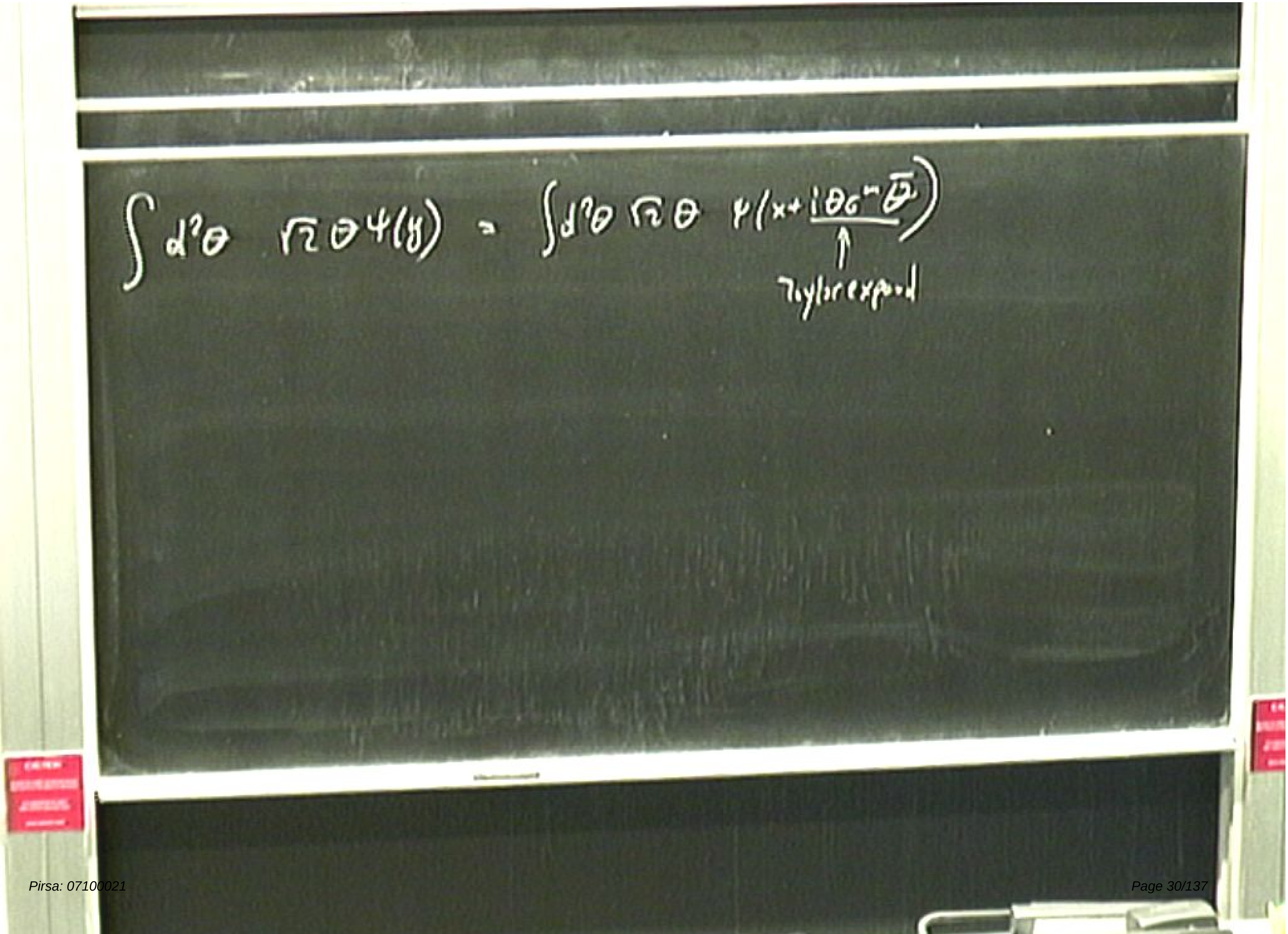
⇒ superpotential and interactions in SUSY model.

$$y^\mu = x^\mu + \underline{i\theta\sigma^\mu\bar{\theta}} \qquad \bar{D}_{\dot{\alpha}}\, y^\mu = 0$$

$$\Rightarrow \Phi(y,\theta) \text{ is xsf} \Rightarrow \bar{D}_{\dot{\alpha}}\,\Phi = 0$$

$$\underset{\nearrow \atop \text{superpotential}}{\mathcal{L}_W} \overset{\text{def}}{=} \int \underbrace{d^2\theta}_{\frac{1}{2}\text{ of superspace}} \underbrace{\Phi(y)}_{\text{xsf}} = F(x) + \underline{\text{nothing else}}$$

$$\int d^2\theta \; \sqrt{2}\,\theta\,\psi(y) = \int d^2\theta \, \sqrt{2}\,\theta \; \psi(x + i\theta\sigma^m\bar{\theta})$$



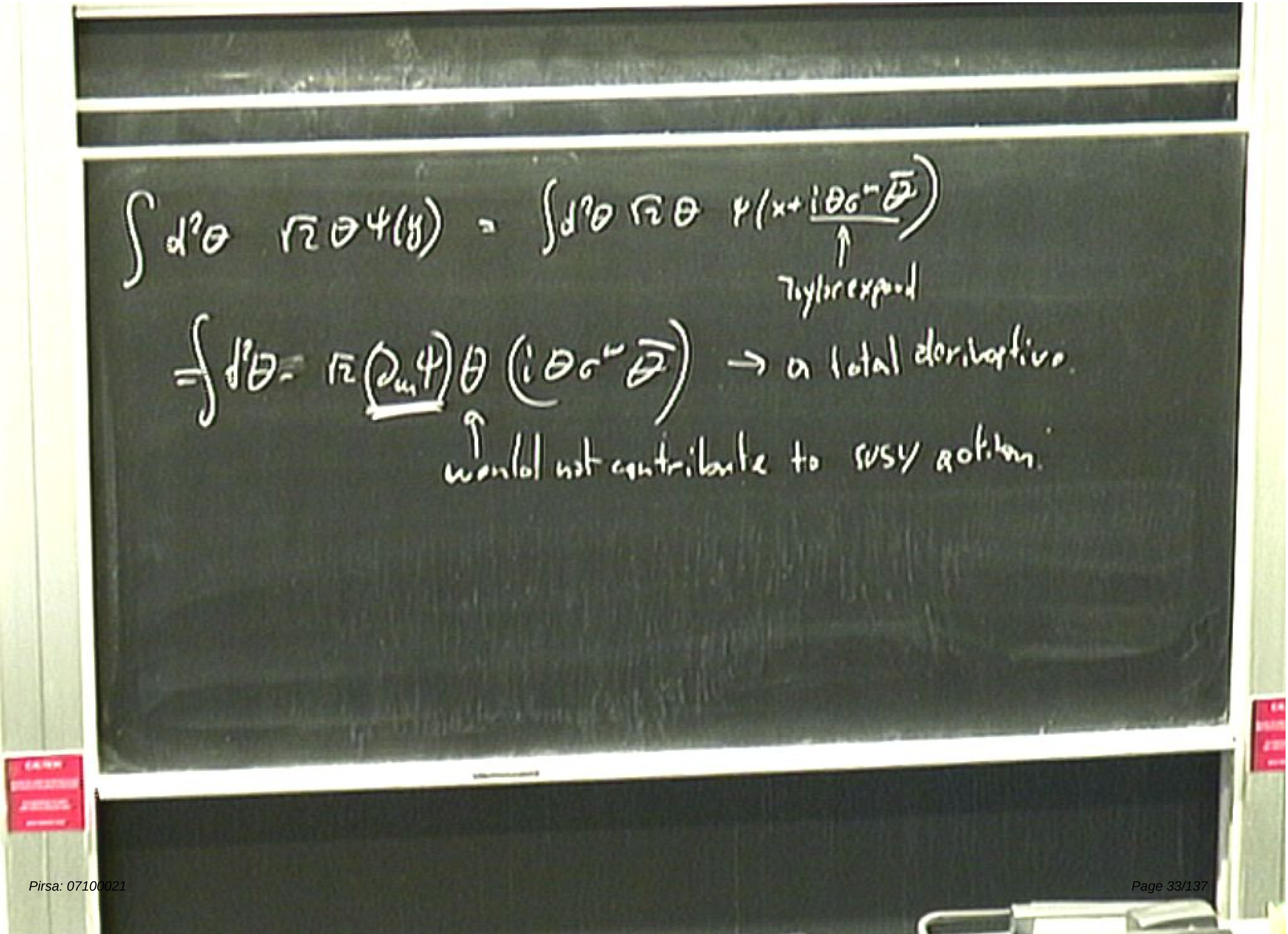
$$\int d^2\theta \ \sqrt{2}\,\theta\psi(y) = \int d^2\theta\ \sqrt{2}\,\theta\ \psi(x+\underbrace{i\theta\sigma^m\bar{\theta}}_{\substack{\uparrow \\ \text{Taylor expand}}})$$

$$\int d^2\theta \; \sqrt{2}\,\theta\psi(y) = \int d^2\theta \, \sqrt{2}\,\theta \; \psi\Big(x+\underset{\overset{\uparrow}{\text{Taylor expand}}}{\underline{i\theta\sigma^m\bar{\theta}}}\Big)$$

$$= \int d^2\theta \; \sqrt{2}\,(\partial_m\psi)\,\theta\,\big(i\theta\sigma^m\bar{\theta}\big)$$

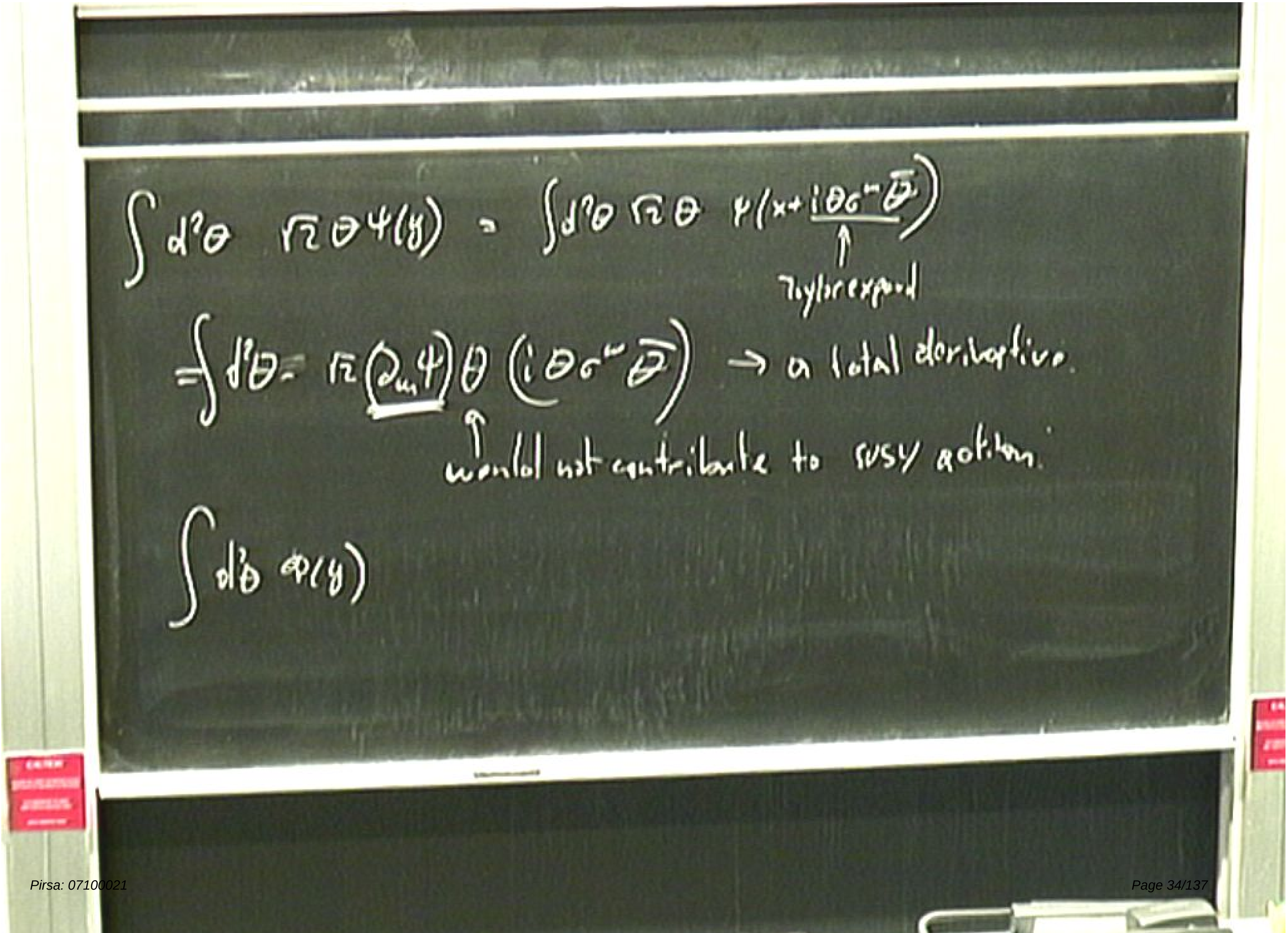
$$\int d^2\theta \; \sqrt{2}\,\theta\psi(y) = \int d^2\theta\,\sqrt{2}\,\theta\;\psi\Big(x+\underset{\substack{\uparrow\\ \text{Taylor expand}}}{\underline{i\theta\sigma^m\bar{\theta}}}\Big)$$

$$= \int d^2\theta \; \sqrt{2}\,\underline{(\partial_m\psi)}\,\theta\,\big(i\theta\sigma^m\bar{\theta}\big) \rightarrow \text{a total derivative}$$



$$\int d^2\theta \;\; \sqrt{2}\,\theta\psi(y) = \int d^2\theta\, \sqrt{2}\,\theta \;\; \psi(x+\underset{\substack{\uparrow \\ \text{Taylor expand}}}{\underline{i\theta\sigma^m\bar{\theta}}})$$

$$= \int d^2\theta \;\; \sqrt{2}\,\underline{(\partial_m\psi)}\,\underset{\substack{\uparrow \\ \text{would not contribute to SUSY action}}}{\theta}\,(i\theta\sigma^m\bar{\theta})$$ → a total derivative



$$\int d^2\theta \; \sqrt{2}\,\theta\psi(y) = \int d^2\theta\, \sqrt{2}\,\theta \; \psi(x + \underset{\substack{\uparrow \\ \text{Taylor expand}}}{\underline{i\theta\sigma^m\bar{\theta}}})$$

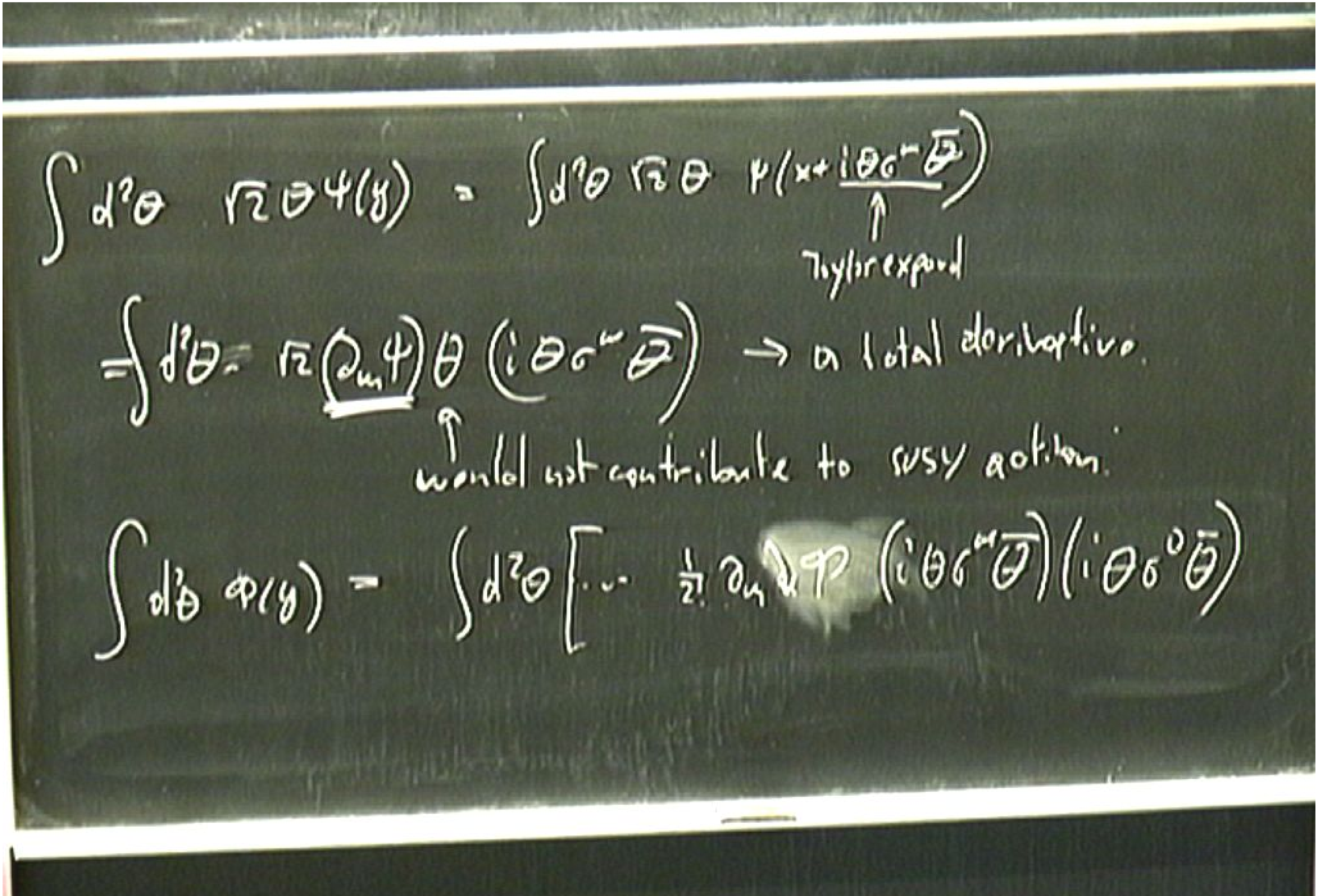
$$= \int d^2\theta \; \sqrt{2}\,\underline{(\partial_m\psi)}\,\underset{\substack{\uparrow \\ \text{would not contribute to SUSY action}}}{\theta}\,\big(i\theta\sigma^m\bar{\theta}\big) \rightarrow \text{a total derivative.}$$

$$\int d^2\theta \; \Phi(y)$$

$$\int d^2\theta \;\; \sqrt{2}\,\theta\psi(y) = \int d^2\theta\, \sqrt{2}\,\theta \;\; \psi(x + \underset{\substack{\uparrow \\ \text{Taylor expand}}}{\underline{i\theta\sigma^\mu\bar\theta}})$$

$$= \int d^2\theta \; \sqrt{2}\,\underline{(\partial_\mu\psi)}\,\underset{\substack{\uparrow \\ \text{would not contribute to susy action.}}}{\theta}\,(i\theta\sigma^\mu\bar\theta) \rightarrow \text{a total derivative.}$$

$$\int d^2\theta \;\; \phi(y) = \int d^2\theta \Big[\cdots \;\; \frac{1}{2!}\partial_\mu\partial_\nu\phi \;\; (i\theta\sigma^\mu\bar\theta)(i\theta\sigma^\nu\bar\theta)$$



$$\int d^2\theta \; \sqrt{2}\,\theta\psi(y) = \int d^2\theta \, \sqrt{2}\,\theta \; \psi(x + \underset{\substack{\uparrow \\ \text{Taylor expand}}}{\underline{i\theta\sigma^m\bar\theta}})$$

$$= \int d^2\theta \; \sqrt{2}\,\underset{\substack{\uparrow \\ \text{would not contribute to susy action.}}}{\underline{(\partial_m\psi)}}\,\theta\,(i\theta\sigma^m\bar\theta) \rightarrow \text{a total derivative.}$$

$$\int d^2\theta \; \phi(y) = \int d^2\theta \left[\cdots \; \frac{1}{2!}\partial_m\partial_n\phi \; (i\theta\sigma^m\bar\theta)(i\theta\sigma^n\bar\theta) \right.$$

$$\int d^2\theta \ \sqrt{2}\,\theta\psi(y) = \int d^2\theta \sqrt{2}\,\theta \ \psi(x + \underset{\substack{\uparrow \\ \text{Taylor expand}}}{\underline{i\theta\sigma^\mu\bar\theta}})$$

$$= \int d^2\theta \ \sqrt{2}\,\underline{(\partial_\mu\psi)}\,\underset{\substack{\uparrow \\ \text{would not contribute to susy action.}}}{\theta}\,(i\theta\sigma^\mu\bar\theta) \rightarrow \text{a total derivative.}$$

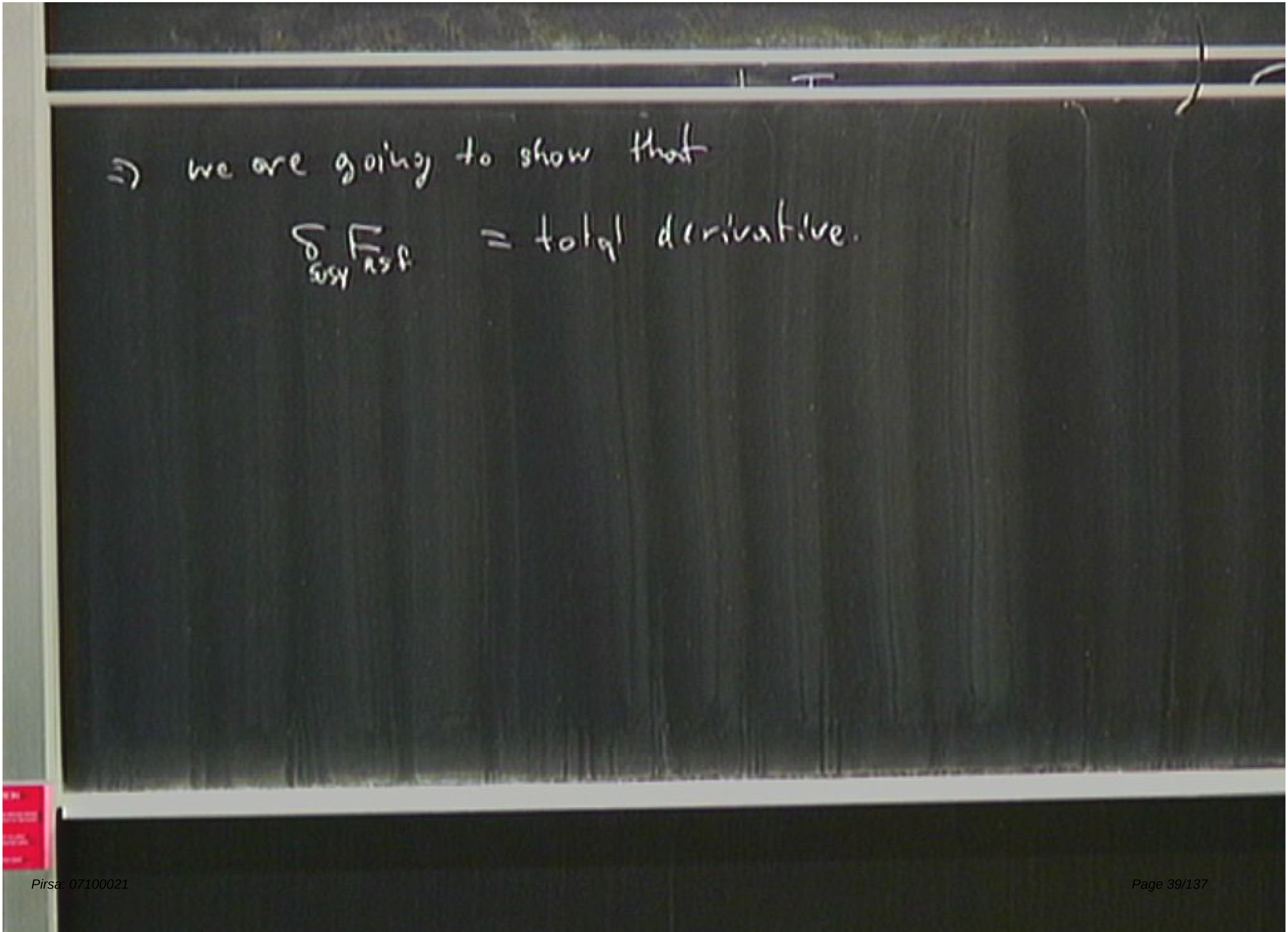
$$\int d^2\theta \ \Phi(y) = \int \underline{d^2\theta}\left[\cdots \ \frac{1}{2!}\partial_\mu\partial_\nu\Phi \ (i\underline{\theta\sigma^\mu}\bar\theta)(i\underline{\theta\sigma^\nu}\bar\theta)\right]$$

$$\int d^2\theta \ \sqrt{2}\,\theta\psi(y) = \int d^2\theta\,\sqrt{2}\,\theta\ \psi(x+\underset{\substack{\uparrow\\ \text{Taylor expand}}}{\underline{i\theta\sigma^m\bar{\theta}}})$$

$$= \int d^2\theta = \sqrt{2}\,\underline{(\partial_m\psi)}\,\underset{\substack{\uparrow\\ \text{would not contribute to SUSY action}}}{\theta}\,(i\theta\sigma^m\bar{\theta}) \to \text{a total derivative}$$

$$\int d^2\theta\ \varphi(y) = \left[\int \underline{d^2\theta}\left[\cdots\ \frac{1}{2!}\underset{\substack{\uparrow\\ \text{a total derivative}}}{\partial_m}\partial\varphi\ (i\underline{\theta\sigma^m}\bar{\theta})(i\underline{\theta\sigma^n}\bar{\theta})\right.\right]$$

lead to the same SUSY inv action



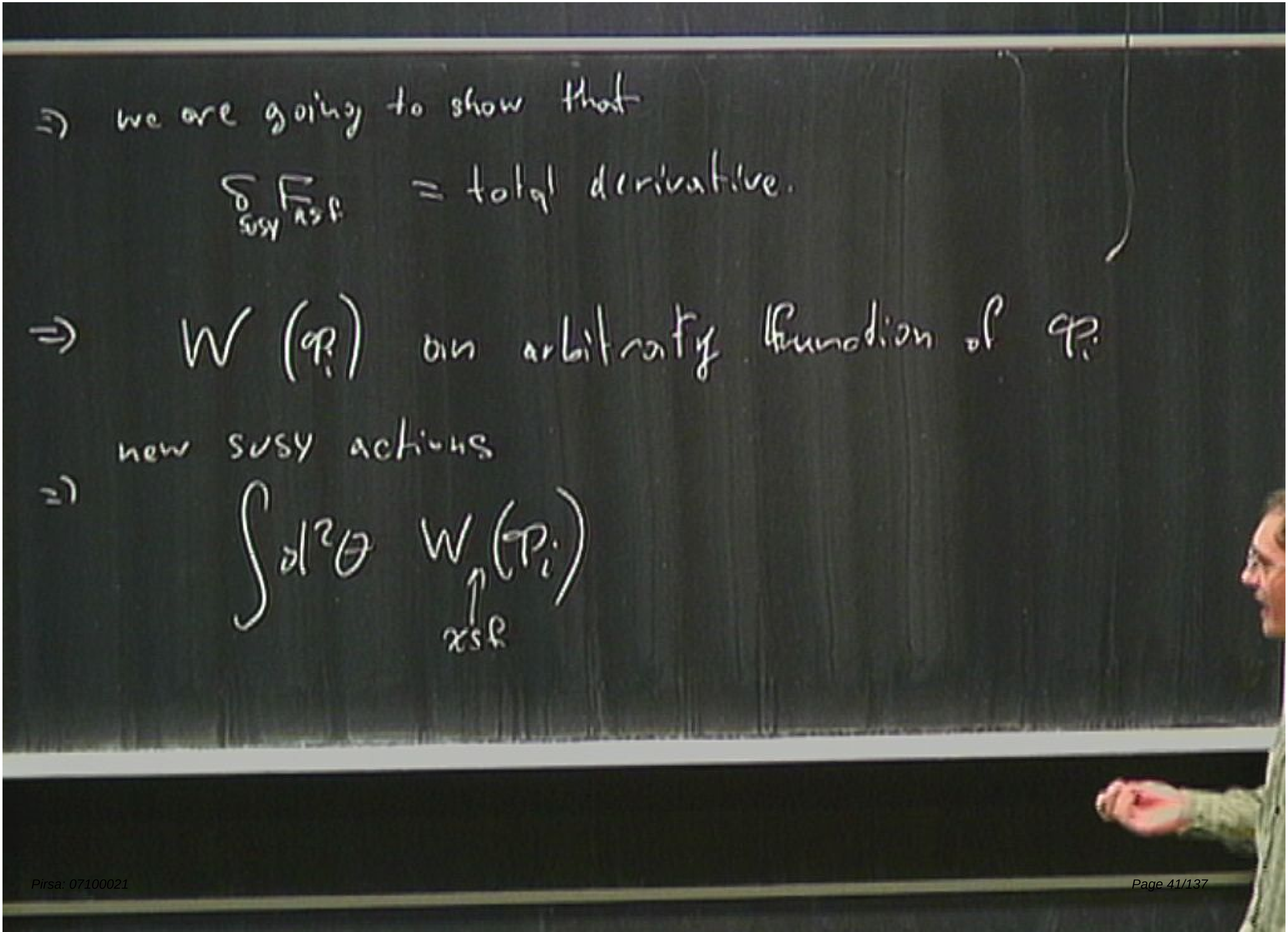
⇒ we are going to show that

$$\delta_{\mathrm{SUSY}} F_{\mathrm{NSR}} = \text{total derivative.}$$

⇒ we are going to show that

$$\delta_{susy} F_{n.s.f.} = \text{total derivative.}$$

⇒ $W(\varphi_i)$ an arbitrary function of φ_i



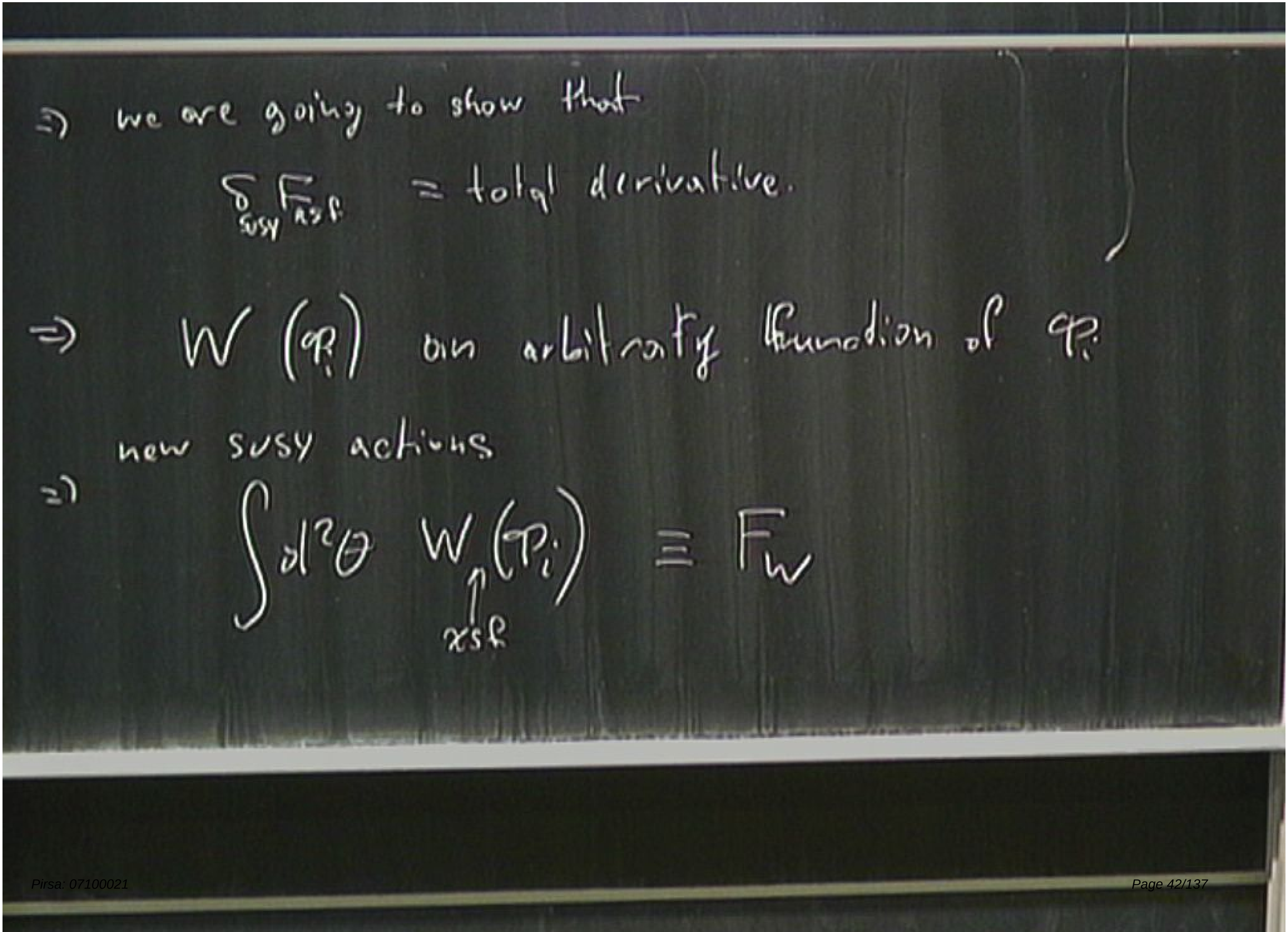
⇒ we are going to show that

$$\delta_{susy} F_{xsf} = \text{total derivative.}$$

⇒ $W(\varphi_i)$ an arbitrary function of φ_i

new susy actions

⇒ $$\int d^2\theta \; \underset{\substack{\uparrow \\ xsf}}{W}(\varphi_i)$$



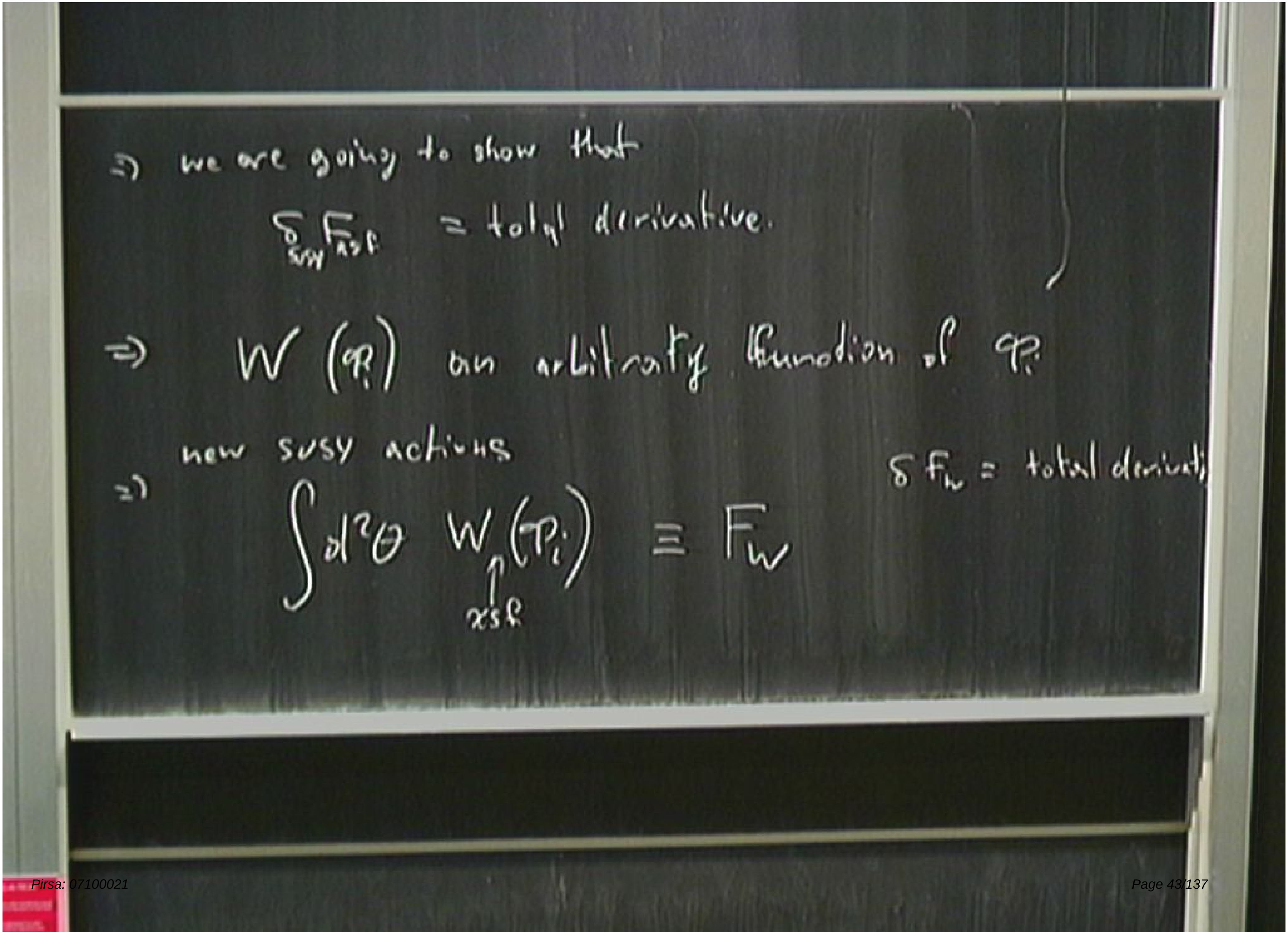
⇒ we are going to show that

$$\delta_{susy} F_{xsf} = \text{total derivative.}$$

⇒ $W(\varphi_i)$ an arbitrary function of φ_i

new susy actions

⇒
$$\int d^2\theta \; \underset{\substack{\uparrow \\ xsf}}{W}(\varphi_i) \equiv F_W$$



⇒ we are going to show that

$$\delta_{susy} F_{xsf} = \text{total derivative.}$$

⇒ $W(\Phi_i)$ an arbitrary function of Φ_i

⇒ new susy actions

$$\int d^2\theta \; \underset{\overset{\uparrow}{xsf}}{W}(\Phi_i) \equiv F_W$$

δF_W = total derivati

⇒ we are going to show that

$$\delta_{susy} F_{xsf} = \text{total derivative.}$$

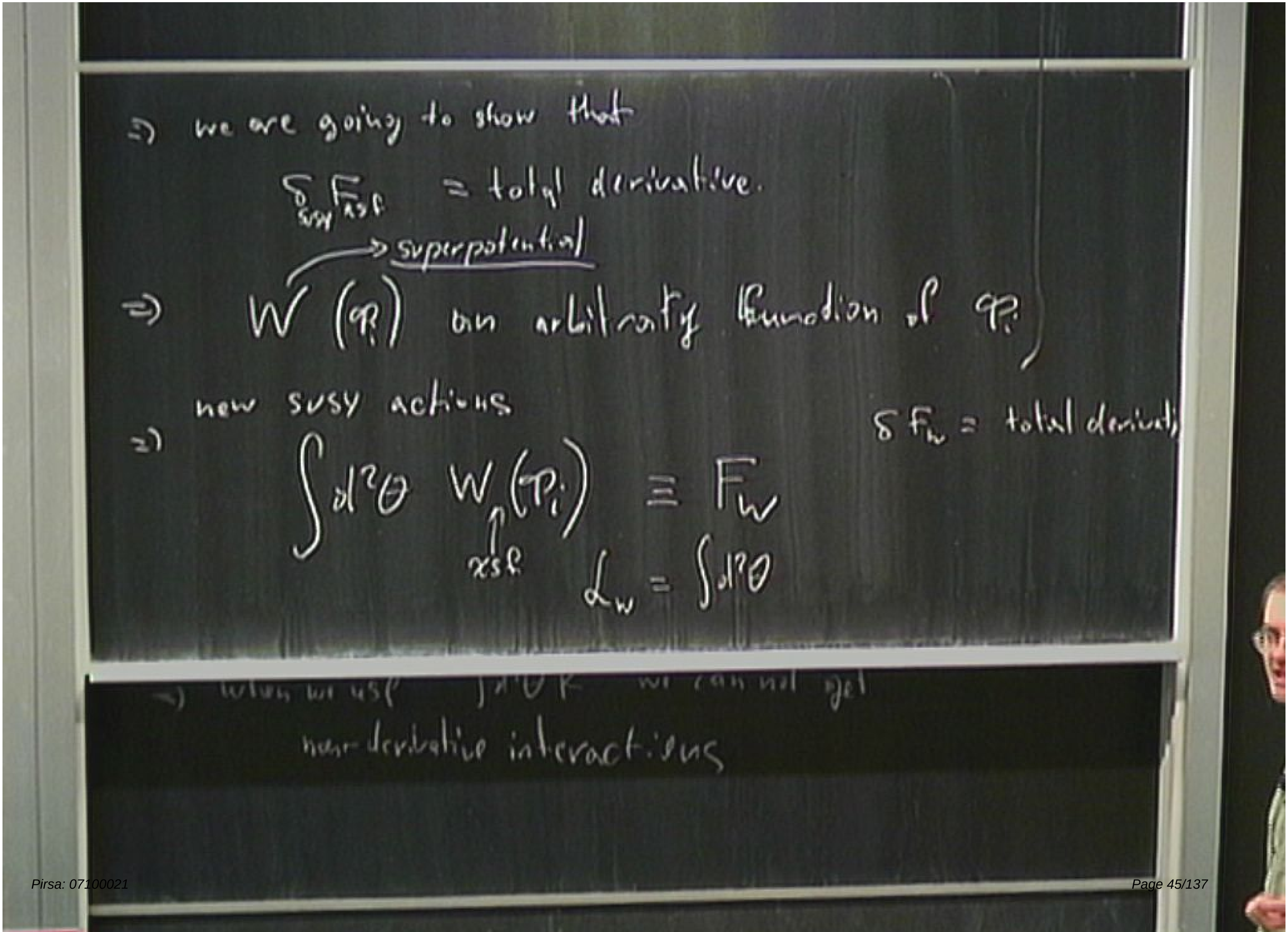
⇒ $\overset{\nearrow\ \underline{\text{superpotential}}}{W}(\varphi_i)$ an arbitrary function of φ_i

new susy actions

⇒ $$\int d^2\theta\ \underset{\overset{\uparrow}{xsf}}{W}(\varphi_i) \equiv F_W$$

δF_W = total derivati

$$\mathcal{L}_W =$$



⇒ we are going to show that

$$\delta_{susy} F_{x.s.f.} = \text{total derivative.}$$

$\longrightarrow$ <u>superpotential</u>

⇒ $W(\varphi_i)$ an arbitrary function of φ_i

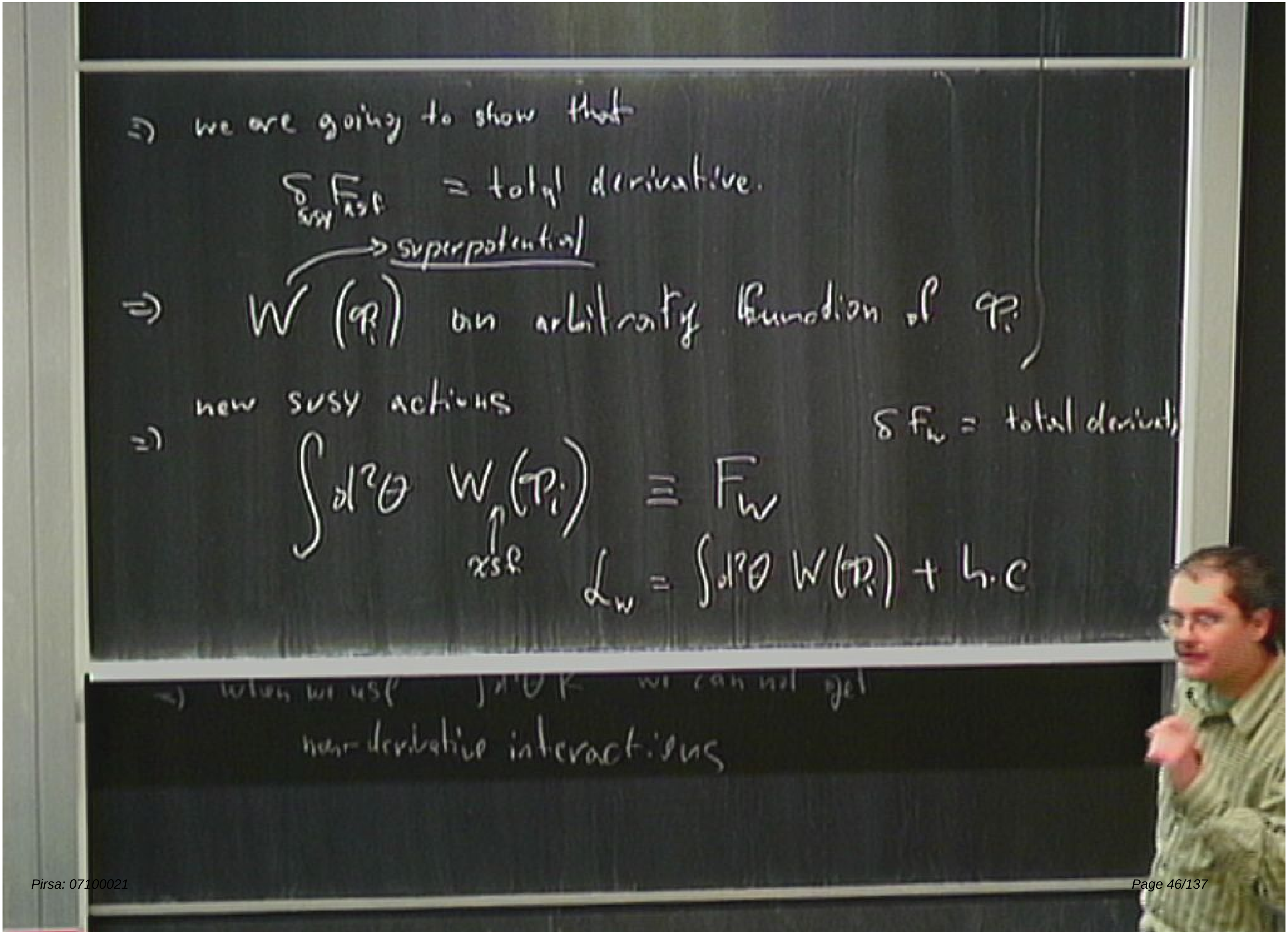
new susy actions

⇒ $$\int d^2\theta \; \underset{\substack{\uparrow \\ x.s.f.}}{W}(\varphi_i) \equiv F_W$$

δF_W = total derivati

$$\mathcal{L}_W = \int d^2\theta$$

⇒ when we use $\int d^4\theta\, K$ we can not get non-derivative interactions



⇒ we are going to show that

$$\delta_{susy} F_{x.s.f.} = \text{total derivative.}$$

$$\overset{\text{superpotential}}{W}(\varphi_i)$$

⇒ $W(\varphi_i)$ an arbitrary function of φ_i

new susy actions

⇒ $$\int d^2\theta \; \underset{\substack{\uparrow \\ x.s.f.}}{W}(\varphi_i) \equiv F_W$$

δF_W = total derivati

$$\mathcal{L}_W = \int d^2\theta \; W(\varphi_i) + h.c$$

⇒ when we use $\int d^4\theta \, K$ we can not get non-derivative interactions

$\Phi(y, \theta) \to x \in L$

S_{susy}

$$\Phi(y,\theta) \rightarrow x \in L$$

$$\delta_{susy}\Phi = \left(\epsilon Q + \bar{\epsilon}\bar{Q}\right)\Phi$$

$$= \left(\xi^{\alpha} Q_{\alpha} - \bar{\xi}^{\dot{\alpha}} \bar{Q}_{\dot{\alpha}}\right)\Phi$$

$\Phi(x,\theta) \to x \in \mathbb{C}$

$$\delta_{susy}\Phi = \left(\varepsilon Q + \bar{\varepsilon}\bar{Q}\right)\Phi$$

$$= \left(\xi^{\alpha} Q_{\alpha} - \bar{\xi}^{\dot{\alpha}} \bar{Q}_{\dot{\alpha}}\right)\Phi$$

$$= \xi^{\alpha}\Big[\frac{\partial}{\partial\theta^{\alpha}} - i\sigma^{m}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}$$

$$\Phi(x,\theta) \rightarrow x \varepsilon L$$

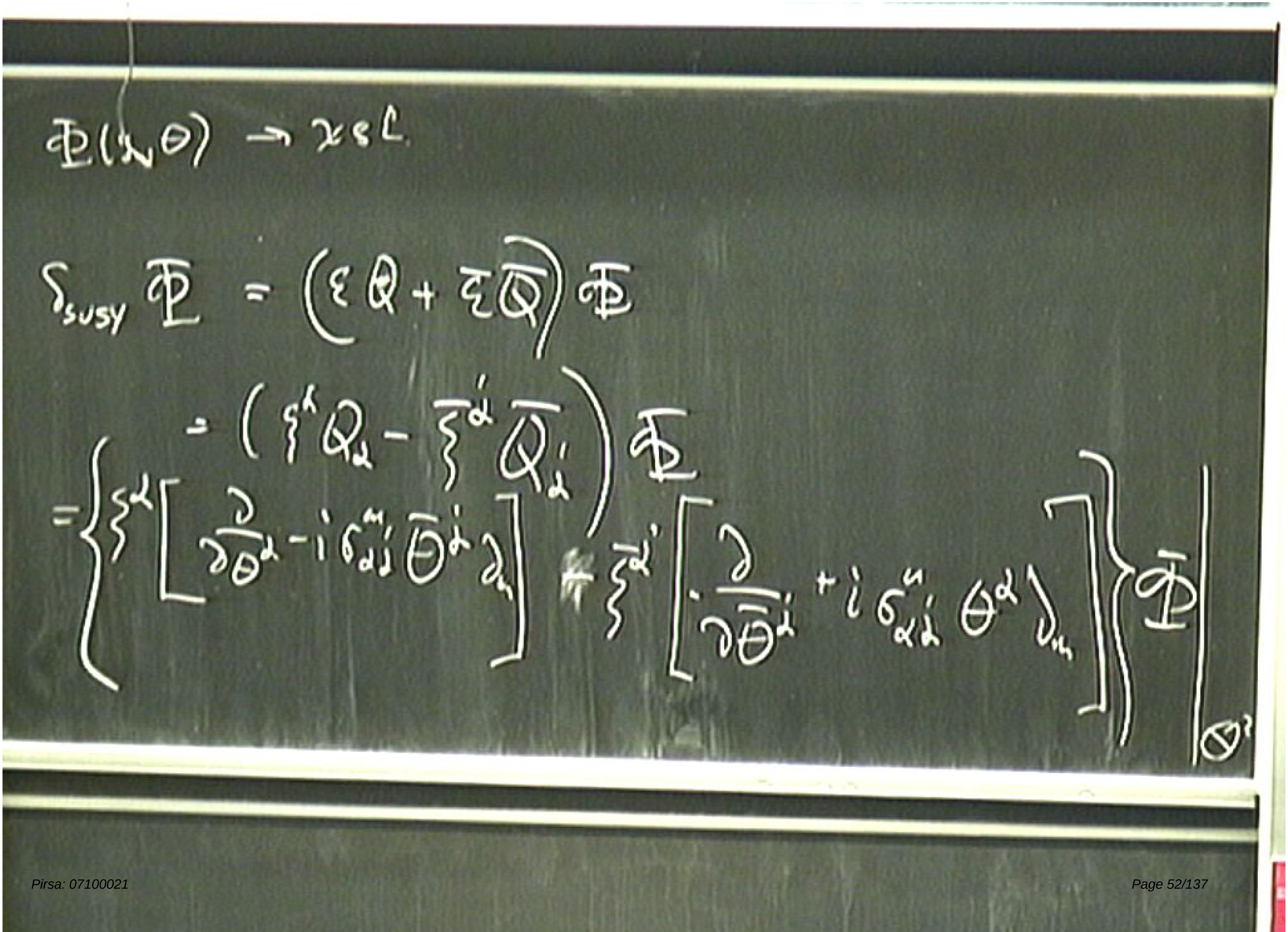
$$\delta_{susy}\Phi = \left(\xi Q + \bar{\xi}\bar{Q}\right)\Phi$$

$$= \left(\xi^{\alpha} Q_{\alpha} - \bar{\xi}^{\dot{\alpha}} \bar{Q}_{\dot{\alpha}}\right)\Phi$$

$$= \left\{ \xi^{\alpha}\left[\frac{\partial}{\partial\theta^{\alpha}} - i\sigma^{m}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_{m}\right] + \bar{\xi}^{\dot{\alpha}}\left[-\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\sigma^{m}_{\alpha\dot{\alpha}}\theta^{\alpha}\partial_{m}\right]\right\}\Phi$$

$$\Phi(y,\theta) = \varphi(x) + i\theta\sigma^{\mu}\bar{\theta}\,\partial_{\mu}\varphi + \frac{1}{4}\theta^{2}\bar{\theta}^{2}\partial^{2}\varphi$$

$$+\sqrt{2}\,\theta\psi - \frac{i}{\sqrt{2}}\theta^{2}\,\partial_{\mu}\psi\sigma^{\mu}\bar{\theta} + \theta^{2}F$$



$\Phi(x,\theta) \rightarrow x \in C$

$$\delta_{susy}\Phi = (\xi Q + \bar{\xi}\bar{Q})\Phi$$

$$= (\xi^{\alpha} Q_{\alpha} - \xi^{\dot{\alpha}} \bar{Q}_{\dot{\alpha}})\Phi$$

$$= \left\{ \xi^{\alpha}\left[\frac{\partial}{\partial\theta^{\alpha}} - i\sigma^{m}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_{m}\right] + \bar{\xi}^{\dot{\alpha}}\left[-\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\sigma^{m}_{\alpha\dot{\alpha}}\theta^{\alpha}\partial_{m}\right]\right\}\Phi\Big|_{\theta^{2}}$$

$$\bar\xi\bar Q\Big)\Phi$$

$$\bar\xi^{\dot\alpha}\bar Q_{\dot\alpha}\Big)\Phi$$

$$\dot\alpha\,\partial_\mu\Big] + \bar\xi^{\dot\alpha}\left[-\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \underbrace{i\sigma^\mu_{\alpha\dot\alpha}\theta^\alpha\partial_\mu}_{}\right]\Big\}\bar\Phi\Big|_{\bar\theta^2}$$

+

$\Phi(x,\theta) \to x \in L$

$$\delta_{susy}\Phi = \left(\epsilon Q + \bar\epsilon\bar Q\right)\Phi$$

$$= \left(\xi^\alpha Q_\alpha - \bar\xi^{\dot\alpha}\bar Q_{\dot\alpha}\right)\Phi$$

$$= \left\{\xi^\alpha\left[\frac{\partial}{\partial\theta^\alpha} - i\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m\right] + \xi^{\dot\alpha}\left[\underline{\underline{\frac{\partial}{\partial\bar\theta^{\dot\alpha}}}} + \underbrace{i\sigma^m_{\alpha\dot\alpha}\theta^\alpha\partial_m}\right]\right\}\Phi\Bigg|_{\bar\theta^\beta}$$

$$\Phi(y,\theta) = \phi(x) + i\theta\sigma^{\mu}\bar{\theta}\,\partial_{\mu}\phi + \frac{1}{4}\theta^{2}\bar{\theta}^{2}\partial^{2}\phi$$

$$+\underline{\sqrt{2}\,\theta\psi} - \frac{i}{\sqrt{2}}\underline{\underline{\theta^{2}\,\partial_{\mu}\psi\sigma^{\mu}\bar{\theta}}} + \boxed{\theta^{2}F}$$

$$\Phi(y,\theta) = \phi(x) + i\theta\sigma^m\bar{\theta}\,\partial_m\phi + \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\phi$$

$$+\underline{\sqrt{2}\theta\psi} - \frac{i}{\sqrt{2}}\underline{\underline{\theta^2\,\partial_m\psi\sigma^m\bar{\theta}}} + \boxed{\theta^2 F}$$

$$-\bar{\xi}^{\dot{\alpha}}\, i\, \sigma^m_{\alpha\dot{\alpha}}\theta^\alpha\theta^\beta\sqrt{2}\,\partial_m\psi_\beta$$

$$\Phi(y,\theta) = \phi(x) + i\theta\sigma^m\bar\theta\,\partial_m\phi + \frac{1}{4}\theta^2\bar\theta^2\,\partial^2\phi$$

$y = x^m + i\theta\sigma^m\bar\theta$

$$+\underline{\sqrt{2}\,\theta\psi} - \underline{\underline{\frac{i}{\sqrt{2}}\theta^2\,\partial_m\psi\,\sigma^m\bar\theta}} + \boxed{\theta^2 F}$$

$$-\bar\xi^{\dot\alpha}\, i\,\sigma^m_{\alpha\dot\alpha}\,\theta^\alpha\bar\theta^\beta\,\sqrt{2}\,\partial_m\psi_\beta + \bar\xi^{\dot\alpha}\,\frac{i}{\sqrt{2}}\theta^2\,\partial_m\psi^\alpha\sigma^m_{\alpha\dot\beta}\,\frac{\partial\bar\theta^{\dot\beta}}{\partial\bar\theta^{\dot\alpha}}$$

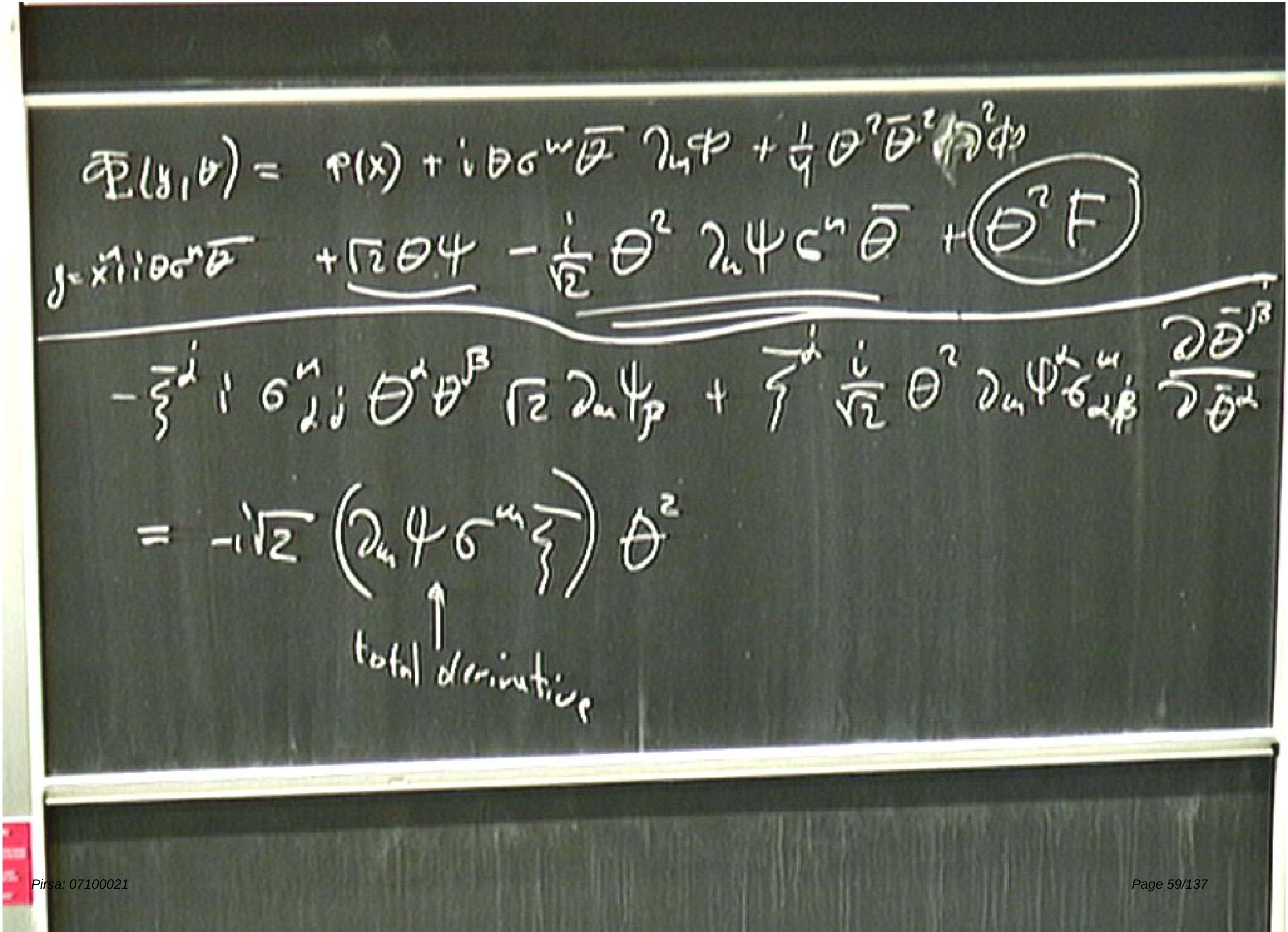
$$\Phi(y,\theta) = \varphi(x) + i\theta\sigma^{\mu}\bar{\theta}\,\partial_{\mu}\varphi + \frac{1}{4}\theta^{2}\bar{\theta}^{2}\partial^{2}\varphi$$

$y = x^{\mu} + i\theta\sigma^{\mu}\bar{\theta}$

$$+\underline{\sqrt{2}\,\theta\psi} - \underline{\underline{\frac{i}{\sqrt{2}}\theta^{2}\,\partial_{\mu}\psi\sigma^{\mu}\bar{\theta}}} + \boxed{\theta^{2}F}$$

$$-\bar{\xi}^{\dot{\alpha}}\, i\,\sigma^{\mu}_{\alpha\dot{\alpha}}\,\theta^{\alpha}\theta^{\beta}\sqrt{2}\,\partial_{\mu}\psi_{\beta} + \bar{\xi}^{\dot{\alpha}}\frac{i}{\sqrt{2}}\theta^{2}\,\partial_{\mu}\psi^{\alpha}\sigma^{\mu}_{\alpha\dot{\beta}}\frac{\partial\bar{\theta}^{\dot{\beta}}}{\partial\bar{\theta}^{\dot{\alpha}}}$$

$$= -i\sqrt{2}\left(\partial_{\mu}\psi\sigma^{\mu}\bar{\xi}\right)\theta^{2}$$

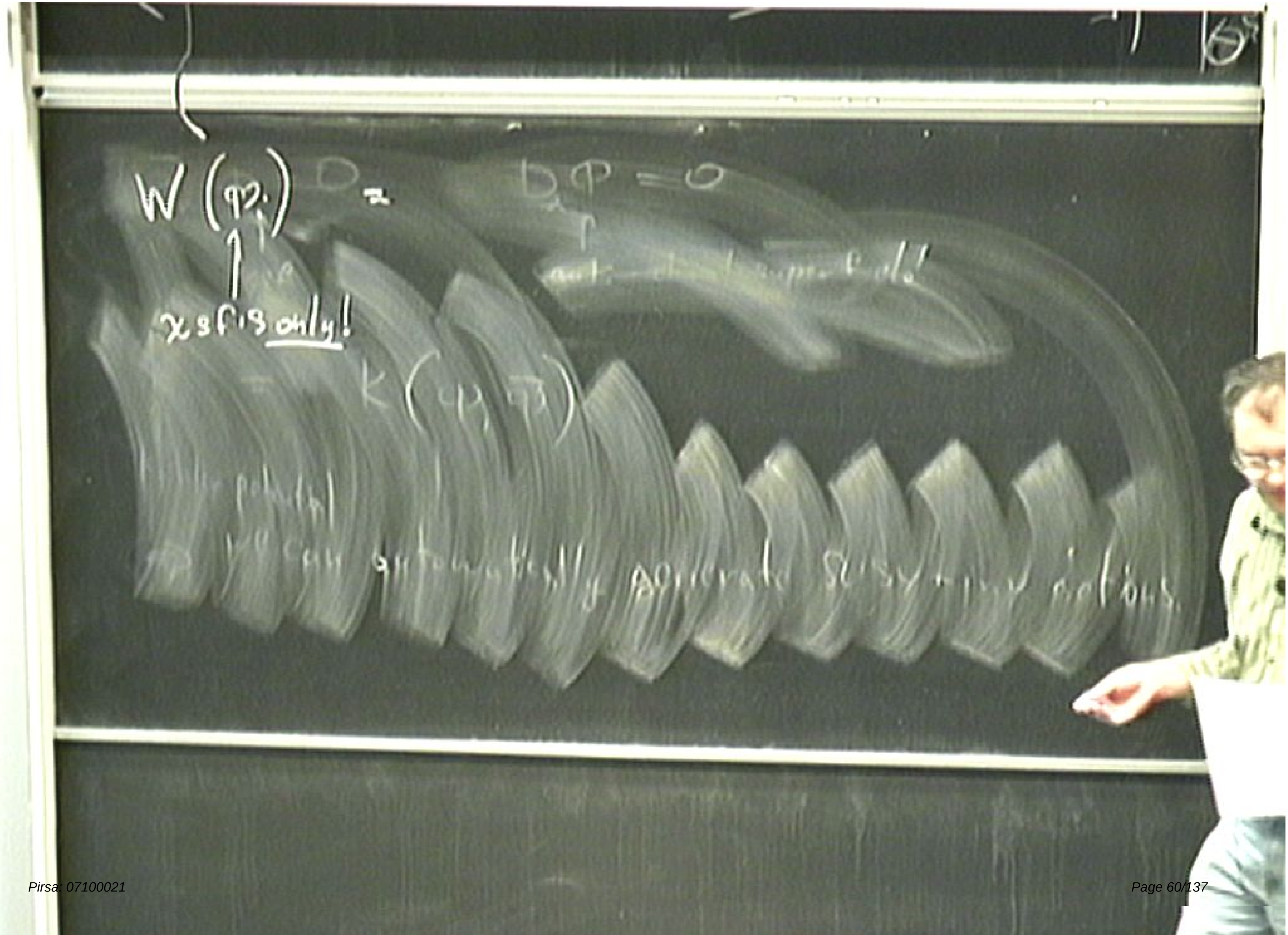


$$\Phi(y,\theta) = \phi(x) + i\theta\sigma^\mu\bar\theta\,\partial_\mu\phi + \frac{1}{4}\theta^2\bar\theta^2\partial^2\phi$$

$$y = x^\mu + i\theta\sigma^\mu\bar\theta \qquad + \underline{\sqrt{2}\,\theta\psi} - \underline{\underline{\frac{i}{\sqrt{2}}\theta^2\,\partial_\mu\psi\sigma^\mu\bar\theta}} + \boxed{\theta^2 F}$$

$$-\bar\xi^{\dot\alpha}\, i\, \sigma^\mu_{\alpha\dot\alpha}\,\theta^\alpha\theta^\beta\sqrt{2}\,\partial_\mu\psi_\beta + \bar\xi^{\dot\alpha}\,\frac{i}{\sqrt{2}}\,\theta^2\,\partial_\mu\psi^\alpha\sigma^\mu_{\alpha\dot\beta}\,\frac{\partial\bar\theta^{\dot\beta}}{\partial\bar\theta^{\dot\alpha}}$$

$$= -i\sqrt{2}\left(\partial_\mu \underset{\substack{\uparrow \\ \text{total derivative}}}{\psi}\sigma^\mu\bar\xi\right)\theta^2$$



$W(\varphi_i)$ D_α

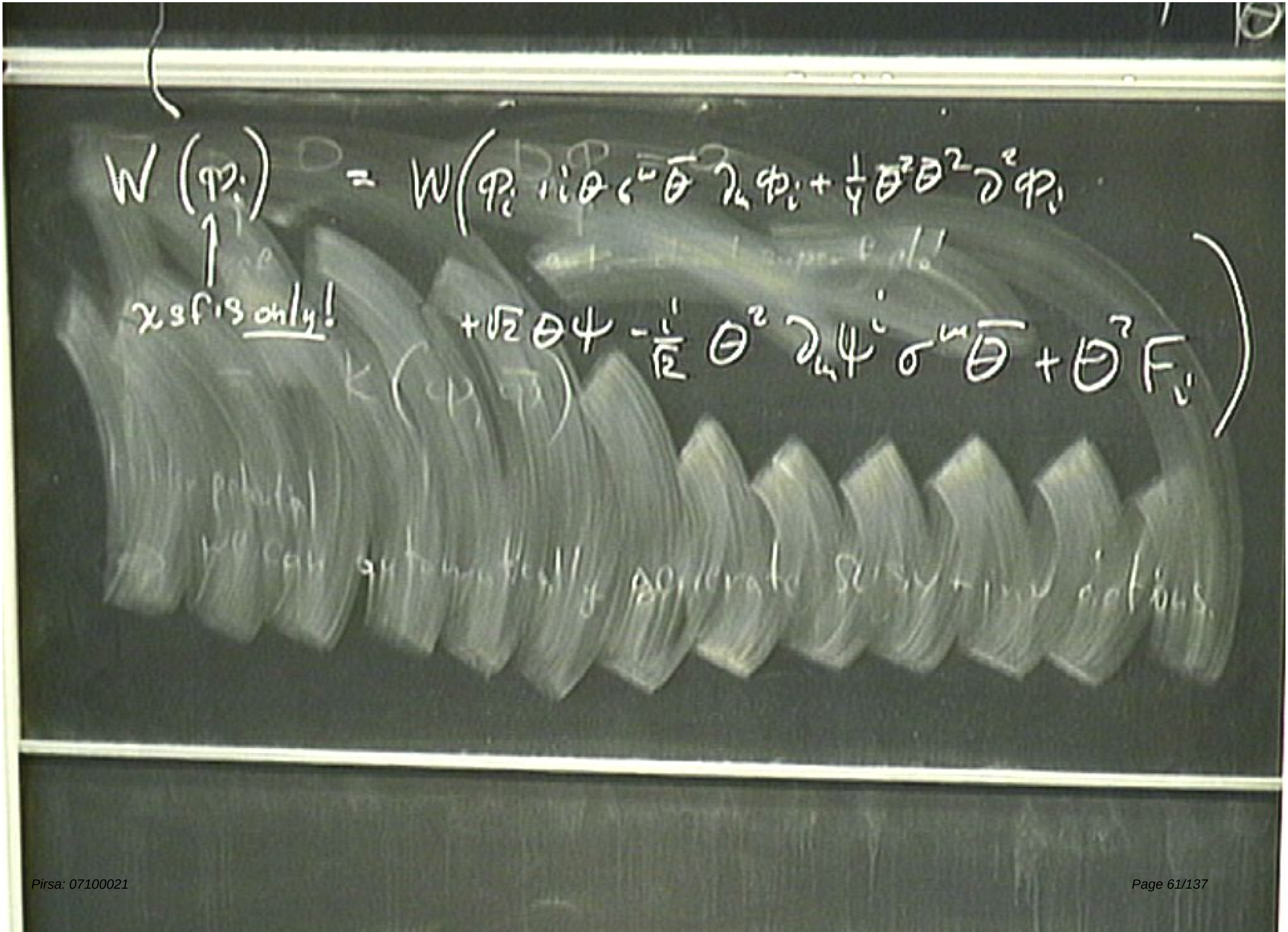
$D_{\dot\alpha}\Phi = 0$

$\uparrow$

x's + θ's only!

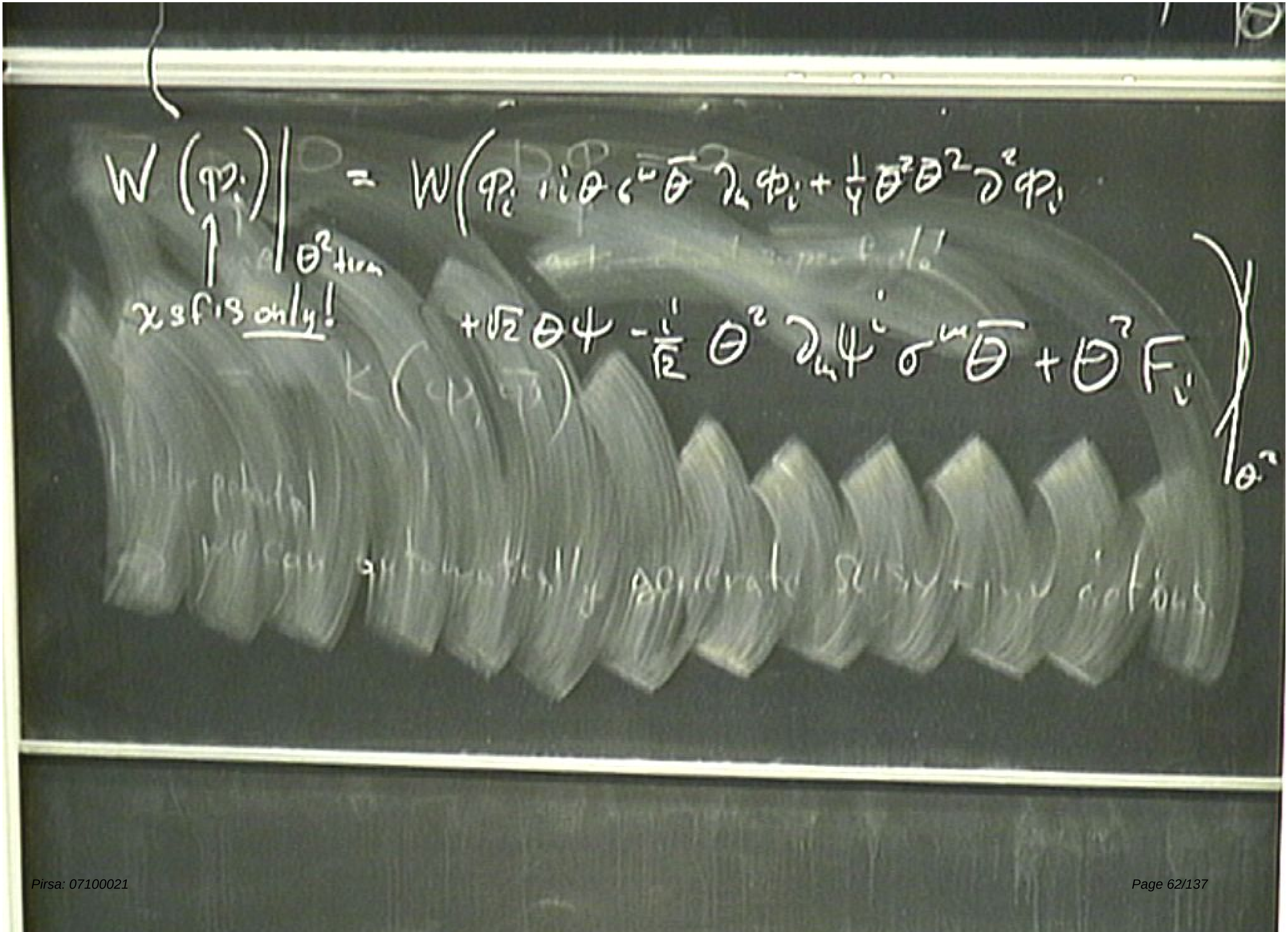
$K(\varphi_i, \bar\varphi_i)$

we can automatically generate supersymmetric actions.



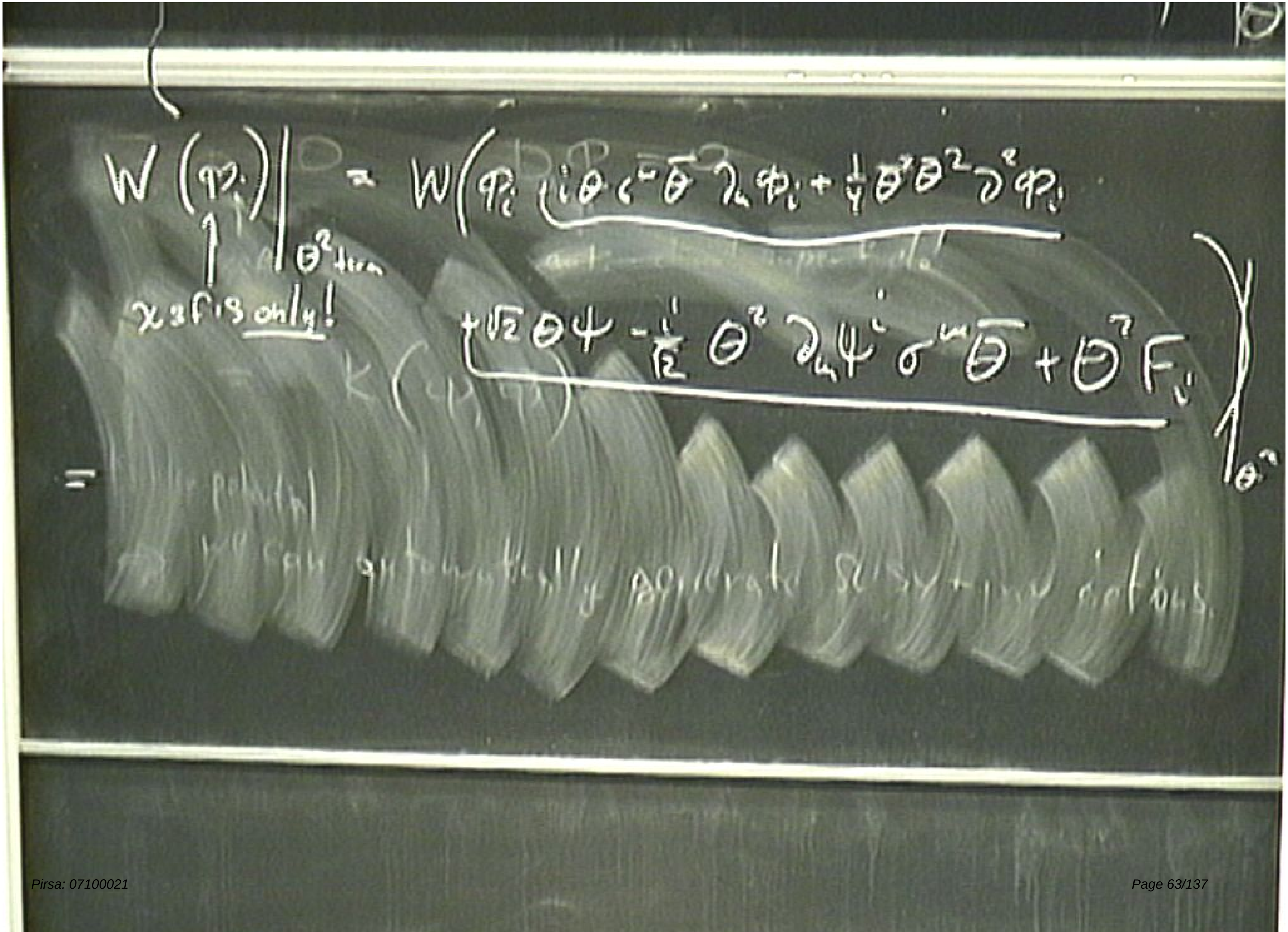
$$W(\varphi_i) = W\Big(\varphi_i + i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi_i + \frac{1}{4}\bar\theta^2\theta^2\partial^2\varphi_i$$
$$+\sqrt{2}\,\theta\psi^i - \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi^i\sigma^\mu\bar\theta + \theta^2 F_i\Big)$$

$\uparrow$ χ sf's only!



$$W(\varphi_i)\Big|_{\theta^2\text{ term}} = W\Big(\varphi_i + i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi_i + \tfrac{1}{4}\bar\theta^2\theta^2\partial^2\varphi_i + \sqrt{2}\,\theta\psi_i - \tfrac{i}{\sqrt{2}}\,\theta^2\,\partial_\mu\psi_i\,\sigma^\mu\bar\theta + \theta^2 F_i\Big)\Big|_{\theta^2}$$

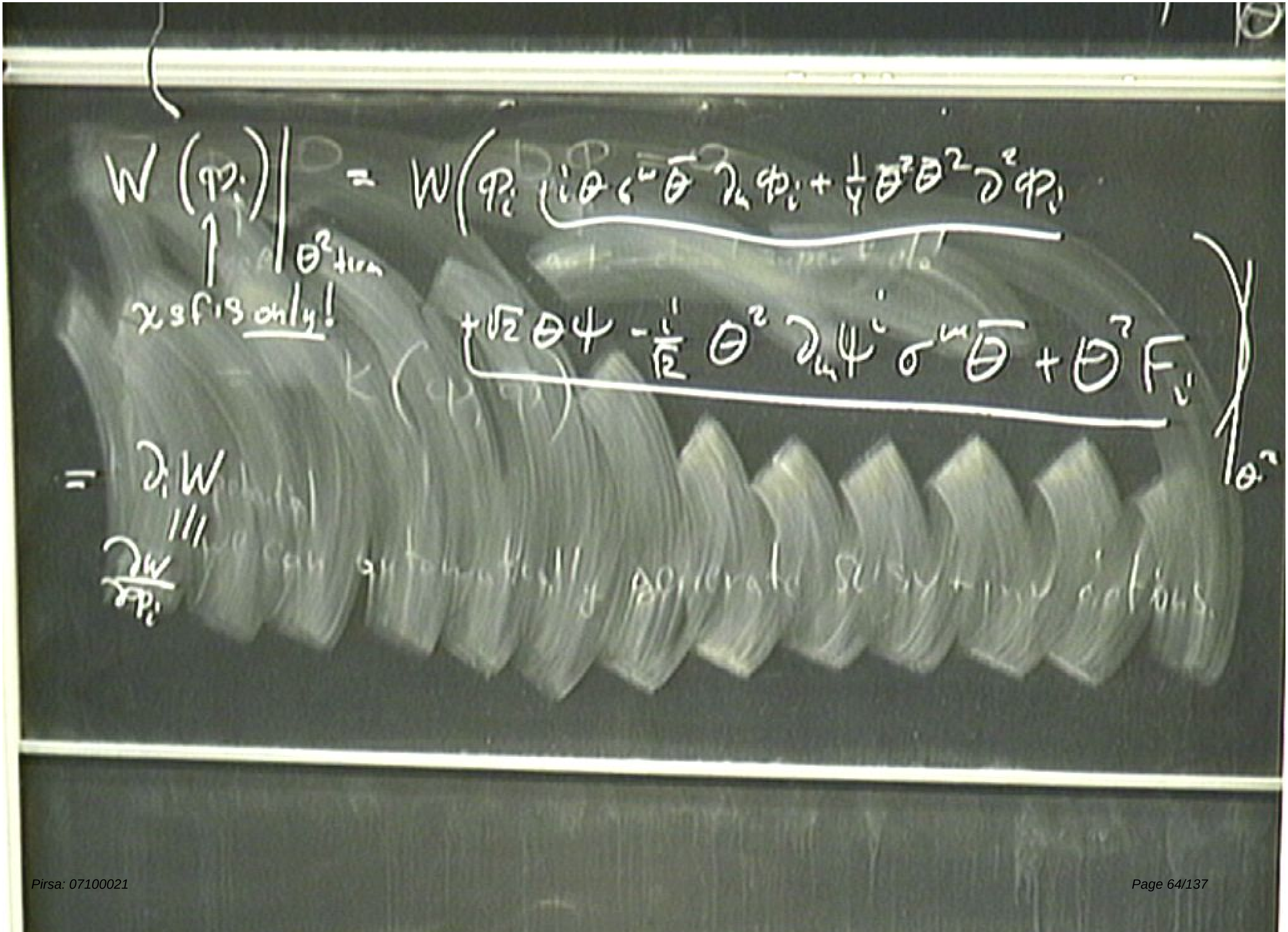
$\uparrow$ χsf's <u>only</u>!



$$W(\varphi_i)\Big|_{\theta^2\text{ term}} = W\Big(\varphi_i \underbrace{+\, i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi_i + \tfrac{1}{4}\theta^2\bar\theta^2\partial^2\varphi_i}_{} \underbrace{+\sqrt{2}\,\theta\psi - \tfrac{i}{\sqrt{2}}\theta^2\,\partial_\mu\psi^i\sigma^\mu\bar\theta + \theta^2 F_i}_{}\Big)\Big|_{\theta^2}$$

$\uparrow$ x's only!

$=$



$$W(\varphi_i)\Big|_{\theta^2 \text{ term}} = W\Big(\varphi_i \underline{+ i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi_i + \tfrac14\bar\theta^2\theta^2\partial^2\varphi_i} \; \underline{+\sqrt2\,\theta\psi - \tfrac{i}{\sqrt2}\,\theta^2\,\partial_\mu\psi^i\sigma^\mu\bar\theta + \theta^2 F_i}\Big)\Big|_{\theta^2}$$

$\uparrow$ x's f'ns only!

$$= \partial_i W \quad \underset{\displaystyle \frac{\partial W}{\partial\varphi_i}}{|||}$$

$$W(\varphi_i)\Big|_{\theta^2\text{term}} = W\Big(\varphi_i \underline{+ i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi_i + \frac{1}{4}\bar\theta^2\theta^2\partial^2\varphi_i} \underline{+\sqrt{2}\,\theta\psi - \frac{i}{\sqrt{2}}\,\theta^2\,\partial_\mu\psi^i\sigma^\mu\bar\theta + \theta^2F_i}\Big)\Big|_{\theta^2}$$

x's fix only!

$$= \partial_i W(x)$$

$$\parallel$$

$$\frac{\partial W}{\partial \varphi_i}$$

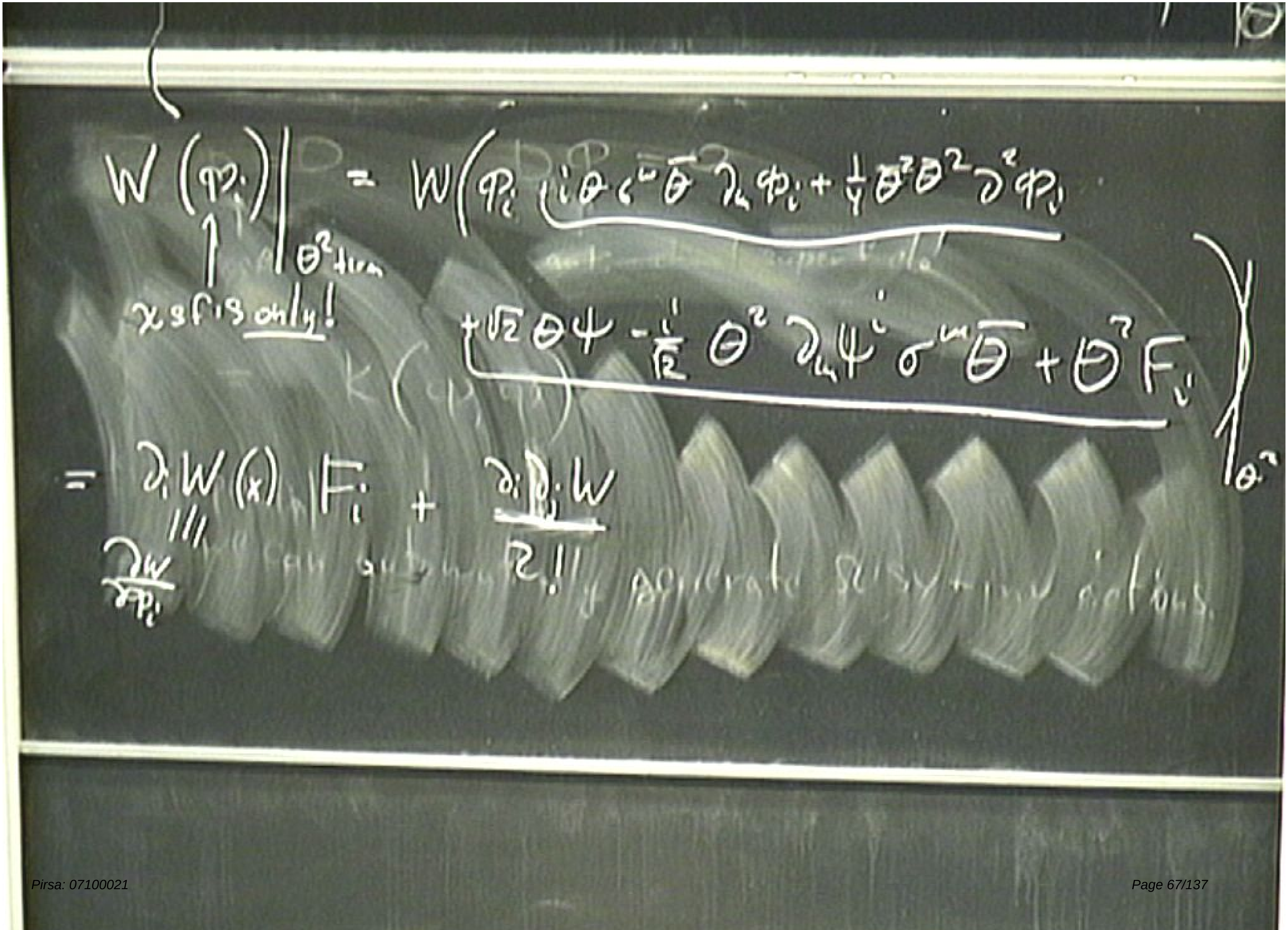
$$W(\varphi_i)\Big|_{\theta^2\,\text{term}} = W\Big(\varphi_i \underbrace{+ i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi_i + \tfrac{1}{4}\bar\theta^2\theta^2\partial^2\varphi_i}$$

$$\underbrace{+\sqrt{2}\,\theta\psi - \tfrac{i}{\sqrt{2}}\,\theta^2\,\partial_\mu\psi^i\sigma^\mu\bar\theta + \theta^2 F_i}\Big)\Big|_{\theta^2}$$

$\uparrow$ x's fns only!

$$= \partial_i W(x)\, F_i$$

$$\partial_i W \equiv \frac{\partial W}{\partial\varphi_i}$$



$$W(\varphi_i)\Big|_{\theta^2\text{ term}} = W\Big(\varphi_i \underbrace{+ i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi_i + \frac{1}{4}\bar\theta^2\theta^2\partial^2\varphi_i}_{} \underbrace{+ \sqrt{2}\,\theta\psi - \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi^i\sigma^\mu\bar\theta + \theta^2 F_i}_{}\Big)\Big|_{\theta^2}$$

$\uparrow$ x's fns only!

$$= \underset{\frac{\partial W}{\partial\varphi_i}}{\underset{|||}{\partial_i W(x)}}\, F_i + \frac{\partial_i\partial_j W}{2!}$$

$$W(\varphi_i)\Big|_{\theta^2 \text{ term}} = W\Big(\varphi_i + \underline{i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi_i + \tfrac{1}{4}\bar\theta^2\theta^2\,\partial^2\varphi_i}$$

$$\underline{+\sqrt{2}\,\theta\psi_i - \tfrac{i}{\sqrt{2}}\,\theta^2\,\partial_\mu\psi^i\sigma^\mu\bar\theta + \theta^2 F_i}\Big)\Big|_{\theta^2}$$

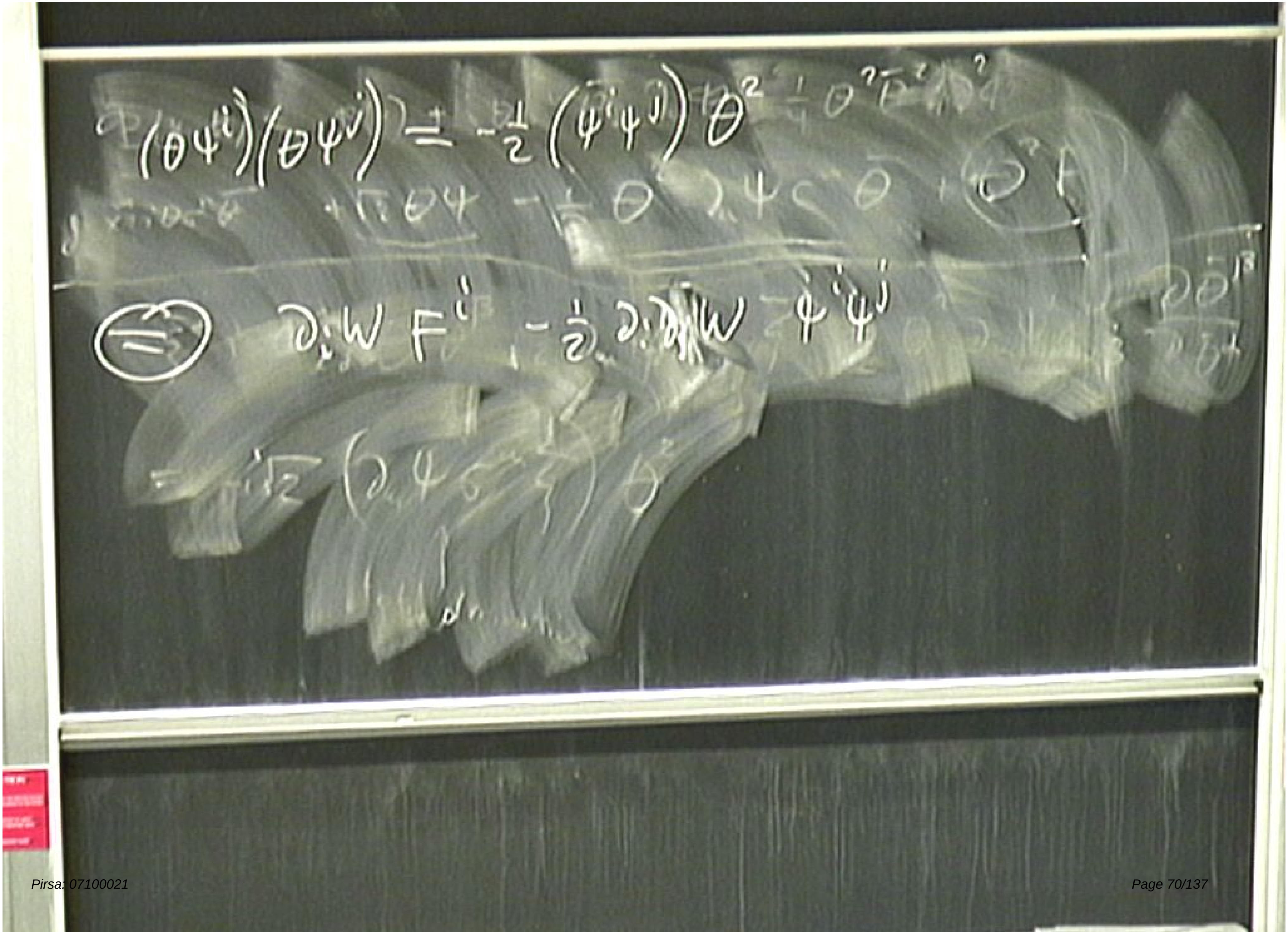
$\uparrow$ x's f's only!

$$= \underset{\substack{|||\\ \frac{\partial W}{\partial\varphi_i}}}{\partial_i W(x)}\, F_i + \frac{\partial_i\partial_j W}{2!}$$

$$W\left(\underset{\substack{\uparrow \\ x\text{'s fns only!}}}{\varphi_i}\right)\Big|_{\theta^2\,\text{term}} = W\Big(\varphi_i \underline{+ i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi_i + \tfrac{1}{4}\bar\theta^2\theta^2\partial^2\varphi_i}$$

$$\underline{\boxed{+\sqrt{2}\,\theta\psi} - \tfrac{i}{\sqrt{2}}\,\theta^2\,\partial_\mu\psi^i\sigma^\mu\bar\theta + \theta^2 F_i}\Big)\Big|_{\theta^2}$$

$$= \underset{\substack{||| \\ \frac{\partial W}{\partial\varphi_i}}}{\partial_i W(x)}\, F_i + \frac{\partial_i\partial_j W}{2!}\, 2\,(\theta\psi^i)(\theta\psi^j)\Big|_{\theta^2},$$

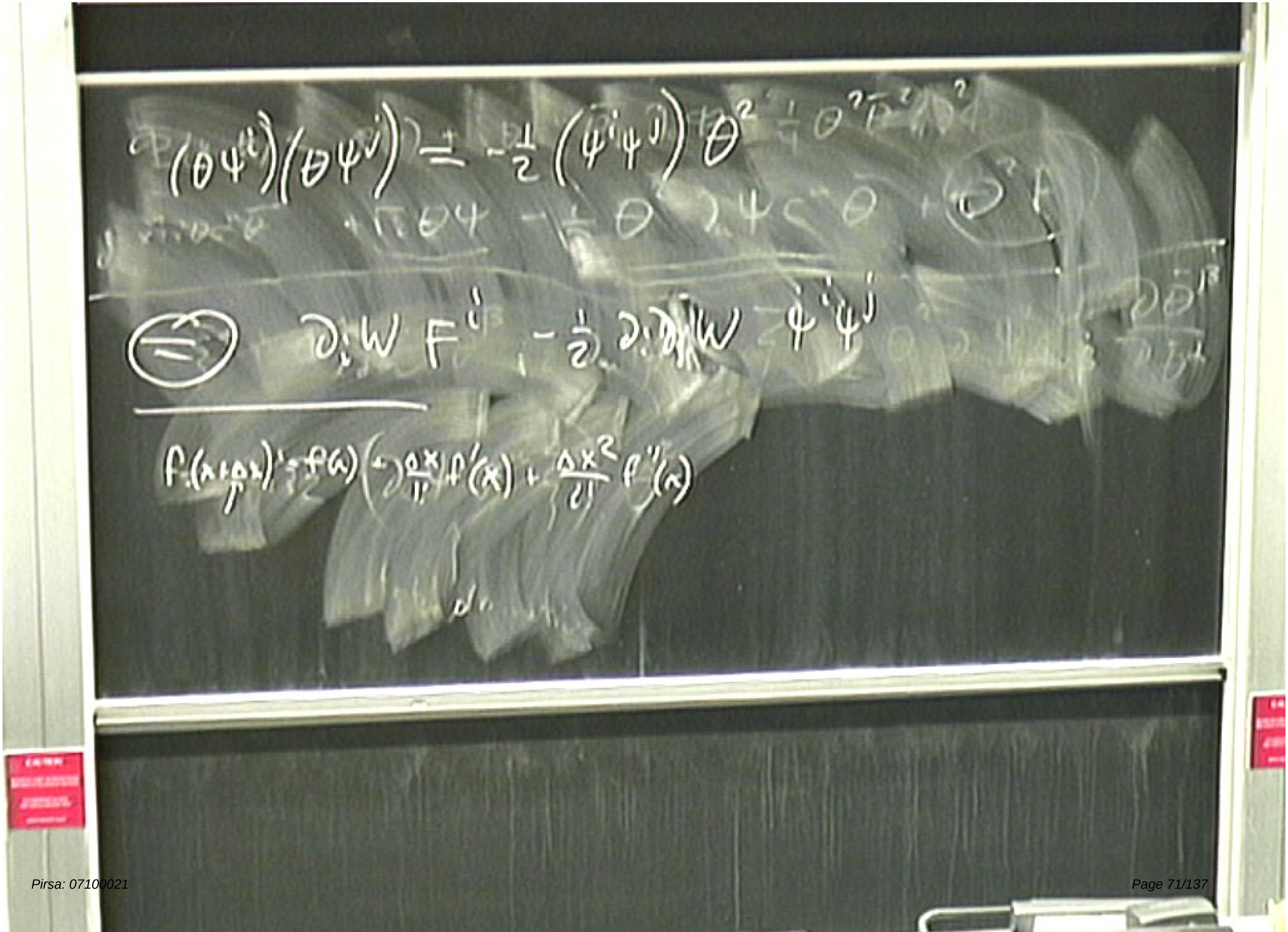


$$(\theta\psi^i)(\theta\psi^j) = -\frac{1}{2}(\psi^i\psi^j)\theta^2$$

$$+\sqrt{2}\theta\psi - \frac{i}{2}\theta\,\partial_\mu\psi\sigma^\mu\bar\theta + \theta^2 F$$

$$= \partial_i W F^i - \frac{1}{2}\partial_i\partial_j W\,\psi^i\psi^j$$

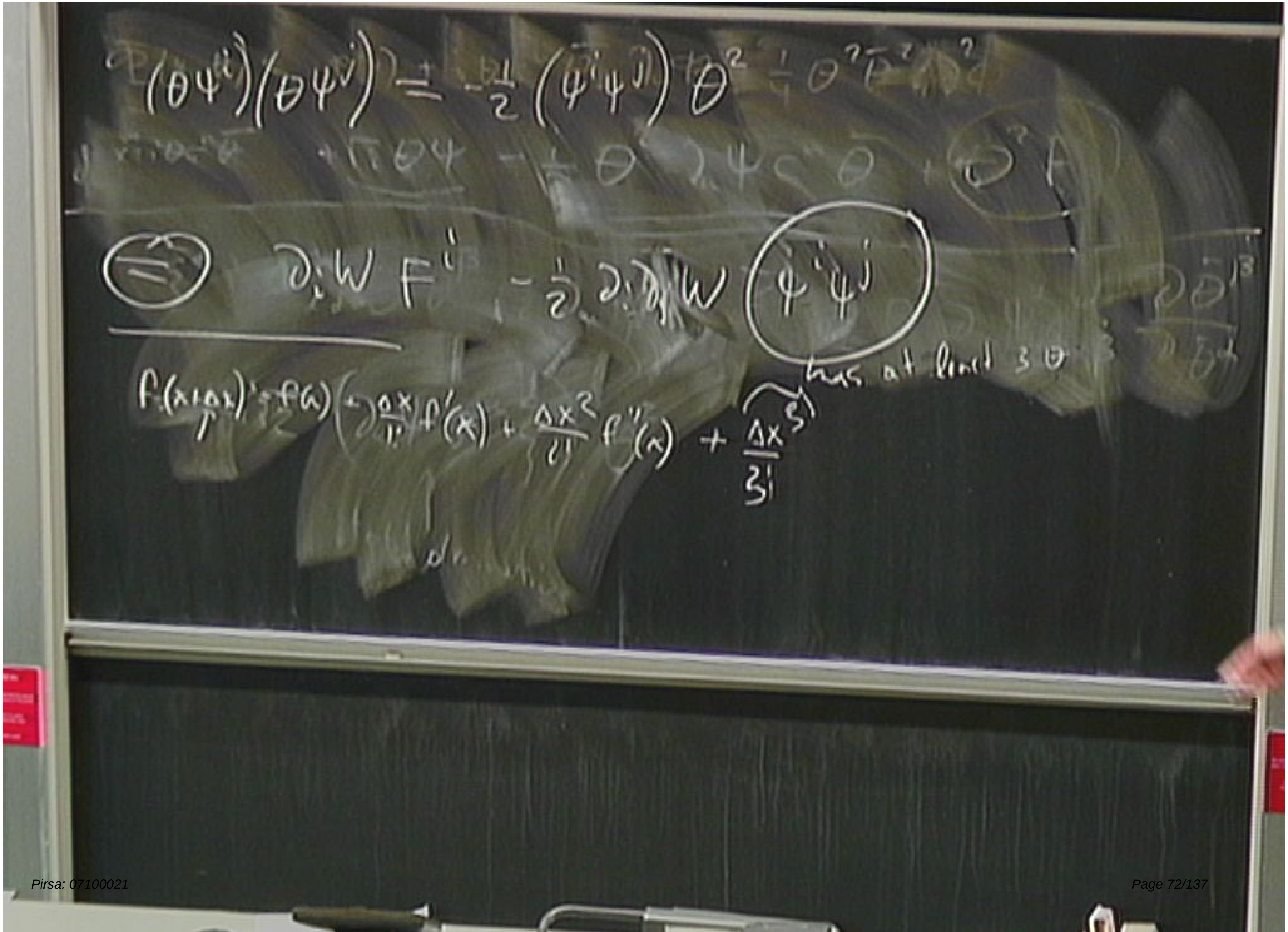
$$-\frac{i}{\sqrt{2}}(\partial_\mu\psi\sigma^\mu\bar\theta)\theta^2$$



$$(\theta\psi^i)(\theta\psi^j) = -\frac{1}{2}(\psi^i\psi^j)\theta^2$$

$$\textcircled{=}\ \partial_i W F^i - \frac{1}{2}\partial_i\partial_j W\,\psi^i\psi^j$$

$$f(x+\Delta x) = f(x) + \frac{\Delta x}{1!}f'(x) + \frac{\Delta x^2}{2!}f''(x)$$

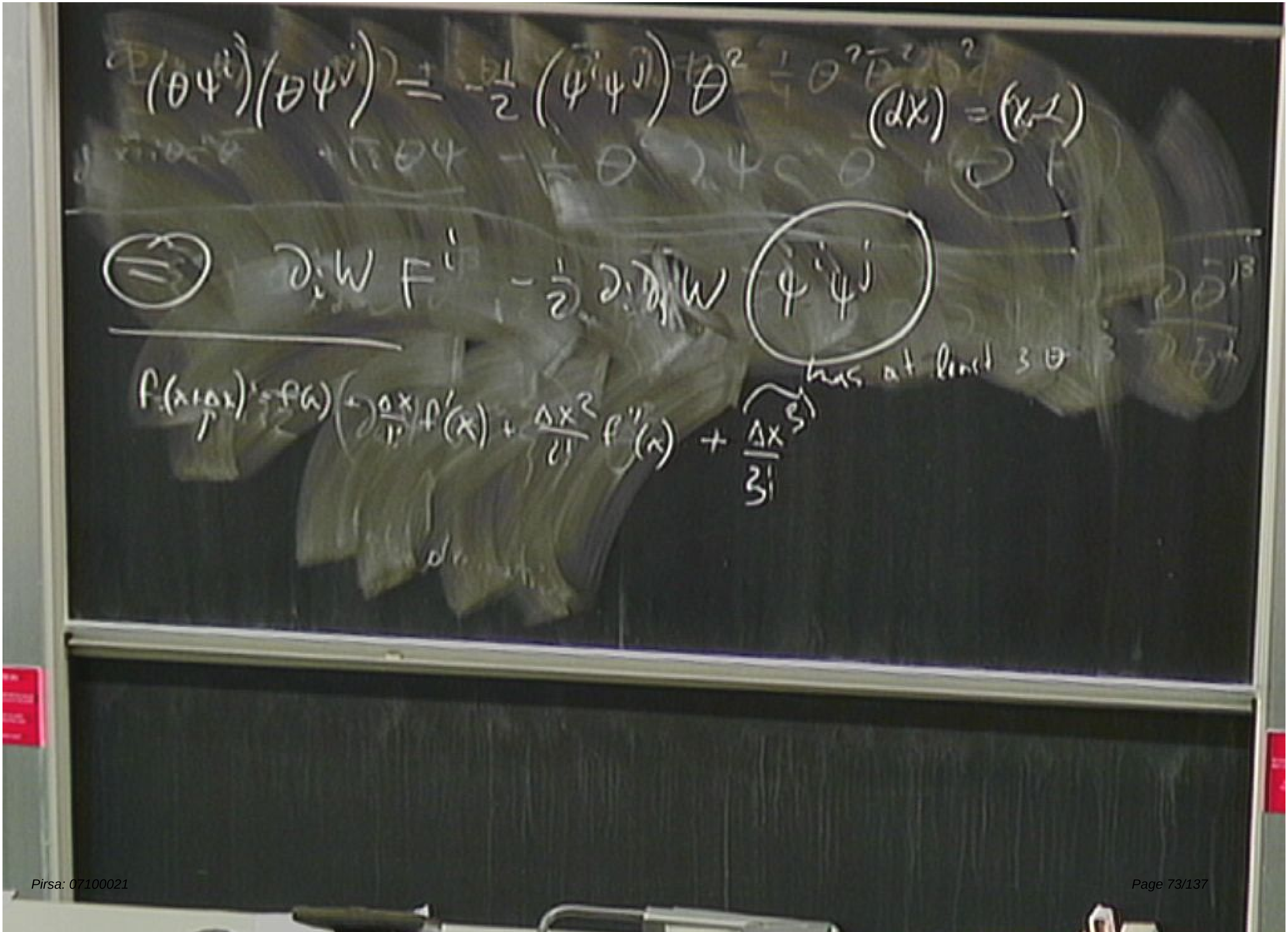


$$(\theta\psi^i)(\theta\psi^j) = -\frac{1}{2}(\psi^i\psi^j)\,\theta^2$$

$$\partial_i W F^i - \frac{1}{2}\partial_i\partial_j W \, \boxed{\psi^i\psi^j}$$

has at least 3 θ

$$f(x+\Delta x) = f(x) + \frac{\Delta x}{1!} f'(x) + \frac{\Delta x^2}{2!} f''(x) + \frac{\Delta x^3}{3!}$$



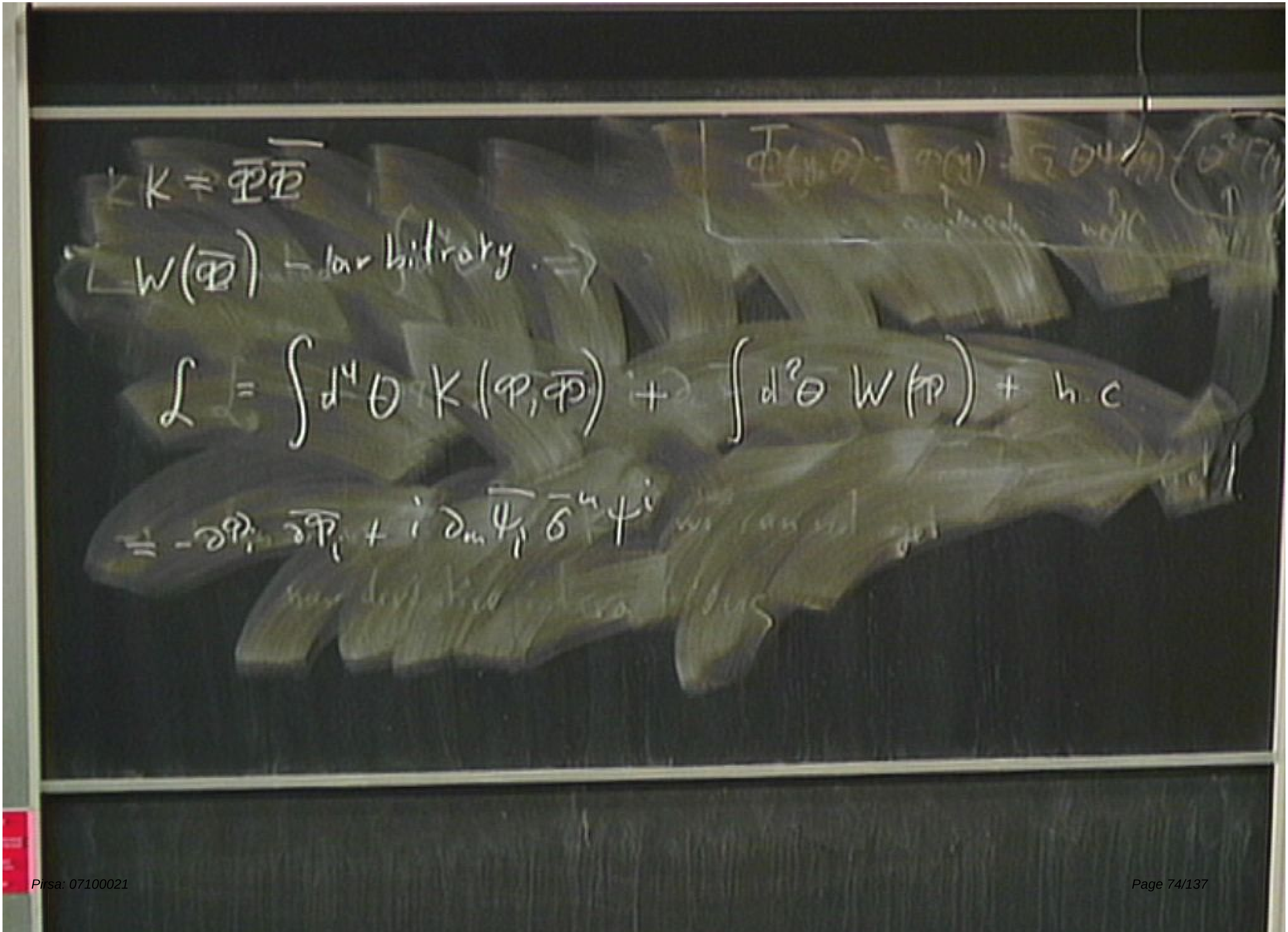
$$(\theta\psi^i)(\theta\psi^j) = -\frac{1}{2}(\psi^i\psi^j)\theta^2$$

$$(dX) = (X-1)$$

$$\boxed{=}\ \partial_i W F^i - \frac{1}{2}\partial_i\partial_j W \boxed{\psi^i\psi^j}$$

has at least 3 θ

$$f(x+\Delta x) = f(x) + \frac{\Delta x}{1!}f'(x) + \frac{\Delta x^2}{2!}f''(x) + \frac{\Delta x^3}{3!}$$



$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$$

complex scalar

weyl

$$K = \Phi\bar{\Phi}$$

$W(\Phi)$ – arbitrary $\Rightarrow$

$$\mathcal{L} = \int d^4\theta\, K(\Phi,\bar{\Phi}) + \int d^2\theta\, W(\Phi) + h.c.$$

$$= -\partial\phi_i\,\partial\bar{\phi}_i + i\,\partial_m\bar{\psi}_i\,\bar{\sigma}^m\psi^i$$

$$\Phi(y,\theta) = \varphi(y) + \sqrt{2}\,\theta\psi$$

complex scalar

weyl

$K = \bar{\Phi}_i \Phi_i$

$W(\Phi_i)$ arbitrary $\Rightarrow$

$$\mathcal{L} = \int d^4\theta \; K(\Phi_i, \bar{\Phi}_i) + \int d^2\theta \; W(\Phi_i) + h.c$$

$$= -\partial\varphi_i \, \partial\bar{\varphi}_i + i \, \partial_m \bar{\psi}_i \, \bar{\sigma}^m \psi^i$$

$K = \Phi_i \bar{\Phi}_i$

$W(\Phi_i)$ — arbitrary $\Rightarrow$

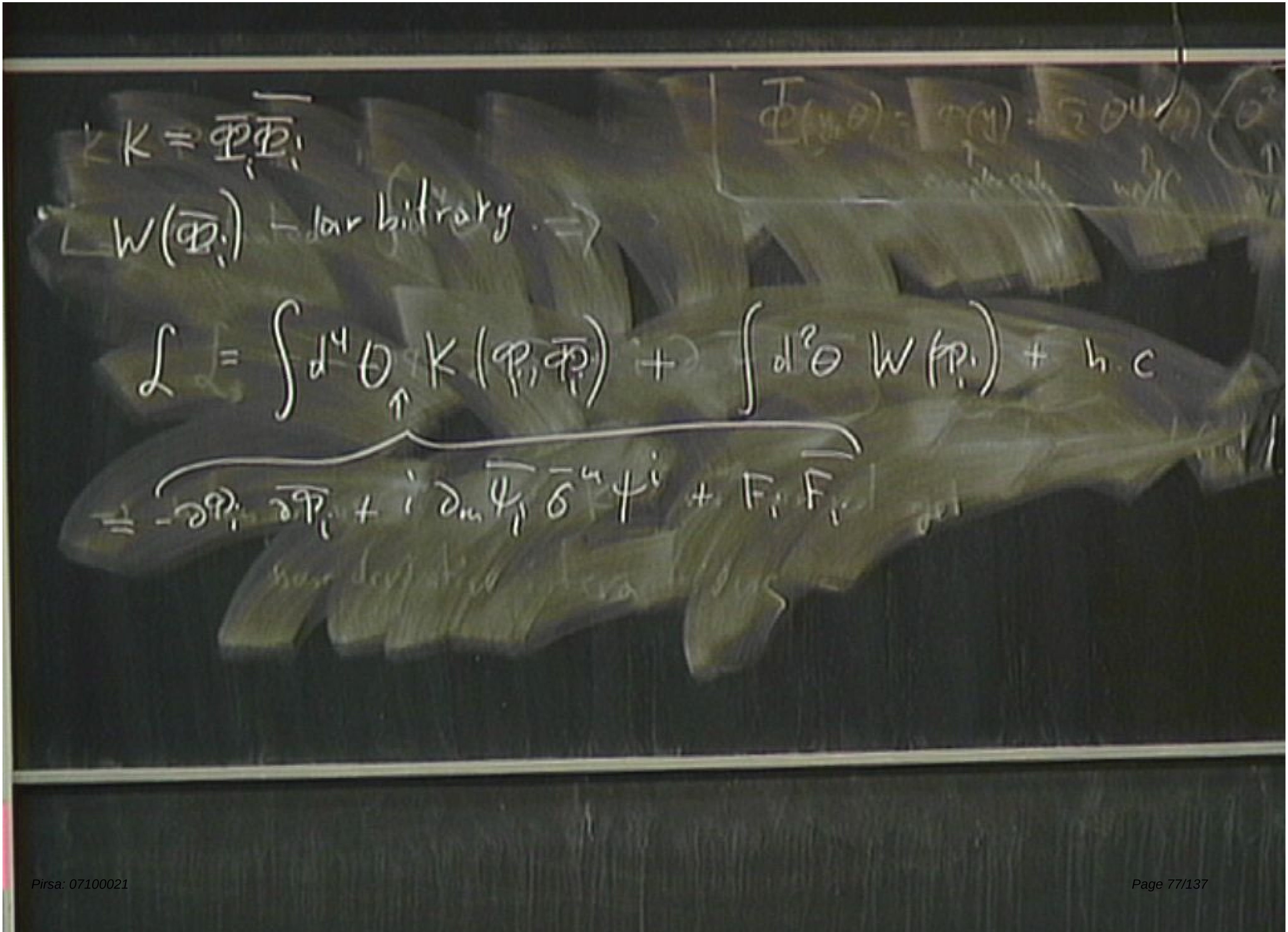
$\Phi(y,\theta) = \varphi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2$

complex scalar

weyl

$$\mathcal{L} = \int d^4\theta\, K(\Phi_i, \bar{\Phi}_i) + \int d^2\theta\, W(\Phi_i) + h.c$$

$$= -\partial\varphi_i\, \partial\bar{\varphi}_i + i\,\partial_m \bar{\psi}_i\, \bar{\sigma}^m \psi^i + F_i \bar{F}_i$$



$K = \Phi_i \overline{\Phi}_i$

$W(\Phi_i)$ arbitrary $\Rightarrow$

$$\mathcal{L} = \int d^4\theta \, K(\Phi_i, \overline{\Phi}_i) + \int d^2\theta \, W(\Phi_i) + h.c.$$

$$= -\partial\varphi_i \, \partial\overline{\varphi}_i + i \, \partial_m \overline{\psi}_i \, \overline{\sigma}^m \psi^i + F_i \overline{F}_i$$

$\Phi(y,\theta) = \varphi(y) + \sqrt{2}\,\theta\psi(y)$

$K = \Phi_i \overline{\Phi}_i$

$W(\Phi_i)$ — arbitrary $\Rightarrow$

$$\mathcal{L} = \int d^4\underset{\uparrow}{\theta}\, K(\varphi_i, \overline{\varphi}_i) + \int d^2\theta\, W(\varphi_i) + h.c$$

$$\overbrace{= -\partial\varphi_i\,\partial\overline{\varphi}_i + i\,\partial_m\overline{\psi}_i\,\overline{\sigma}^m\psi^i + F_i\overline{F}_i}\ +$$

$+$

$K = \Phi_i \bar{\Phi}_i$

$W(\Phi_i)$ — arbitrary $\Rightarrow$

$$\mathcal{L} = \int d^4 \underset{\uparrow}{\theta}\, K(\Phi_i, \bar{\Phi}_i) + \int d^2\theta\, W(\Phi_i) + h.c.$$

$$= -\partial\varphi_i\, \partial\bar{\varphi}_i + i\,\partial_m \bar{\psi}_i\, \bar{\sigma}^m \psi^i + F_i \bar{F}_i +$$

$$+ \partial_i W F^i + \overline{\partial_i W F^i}$$

$\Phi(y,\theta) = \varphi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$

complex scalar — Weyl — aux

$K = \bar{\Phi}_i \bar{\Phi}_i$

$W(\Phi_i)$ — arbitrary $\Rightarrow$

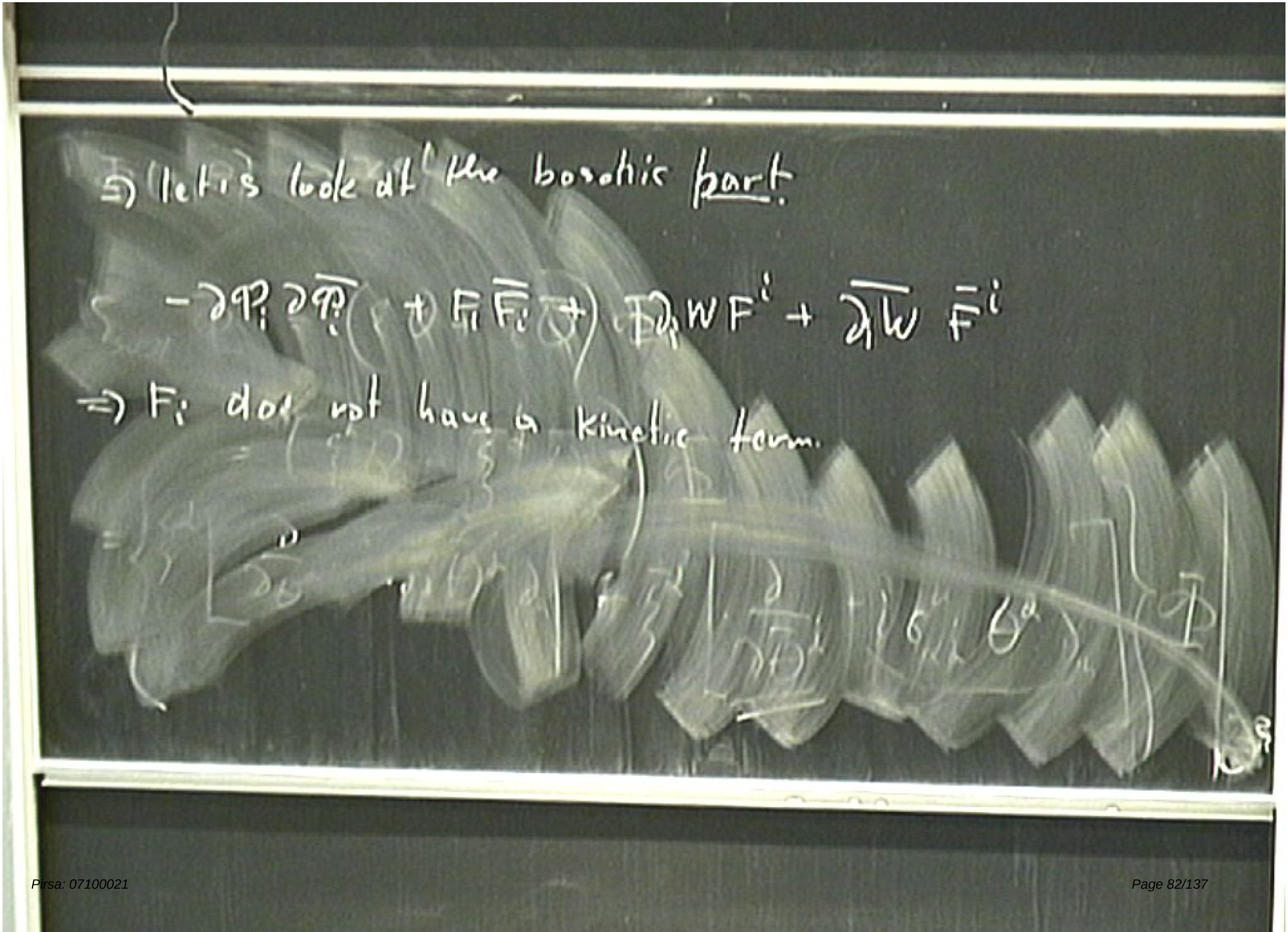
$$\mathcal{L} = \int d^4\theta\, K(\Phi_i, \bar{\Phi}_i) + \int d^2\theta\, W(\Phi_i) + h.c.$$

$$= -\partial\varphi_i\, \partial\bar{\varphi}_i + i\,\partial_m \bar{\psi}_i \bar{\sigma}^m \psi^i + F_i \bar{F}_i +$$

$$+ \partial_i W F^i + \overline{\partial_i W F^i} - \frac{1}{2}\partial_i\partial_j W\, \psi^i\psi^j - \frac{1}{2}\overline{\partial_i\partial_j W}\; \overline{\psi^i\psi^j}$$

⇒ let's look at the bosonic part.

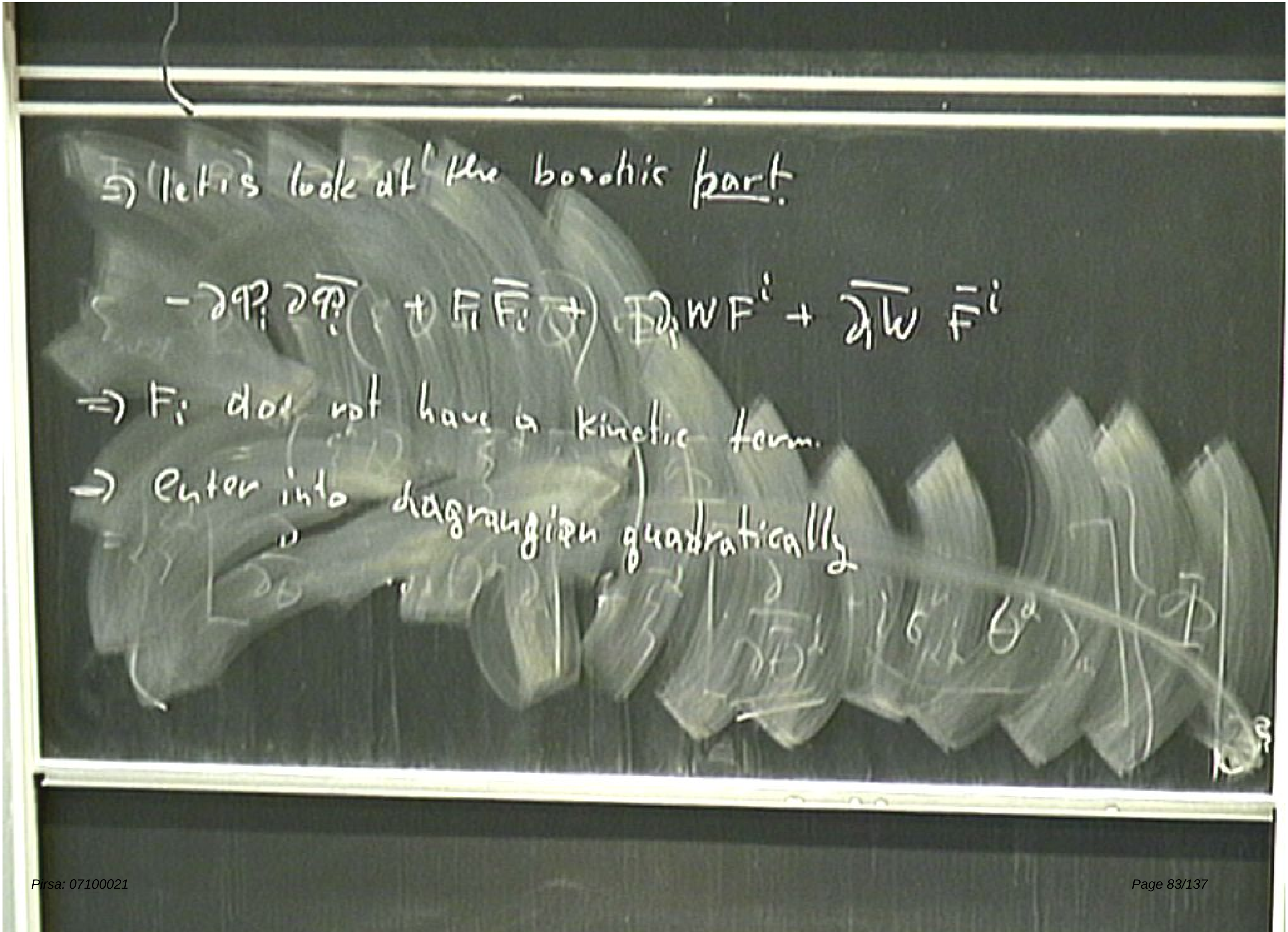
$$-\partial\varphi_i\,\partial\bar{\varphi}^i + F_i\bar{F}^i + \partial_i W F^i + \overline{\partial W}\,\bar{F}^i$$



⇒ let's look at the bosonic part.

$$-\partial\varphi_i\,\partial\bar{\varphi}^i + F_i\bar{F}_i + \partial_i W F^i + \overline{\partial_i W}\,\bar{F}^i$$

⇒ F_i does not have a kinetic term.

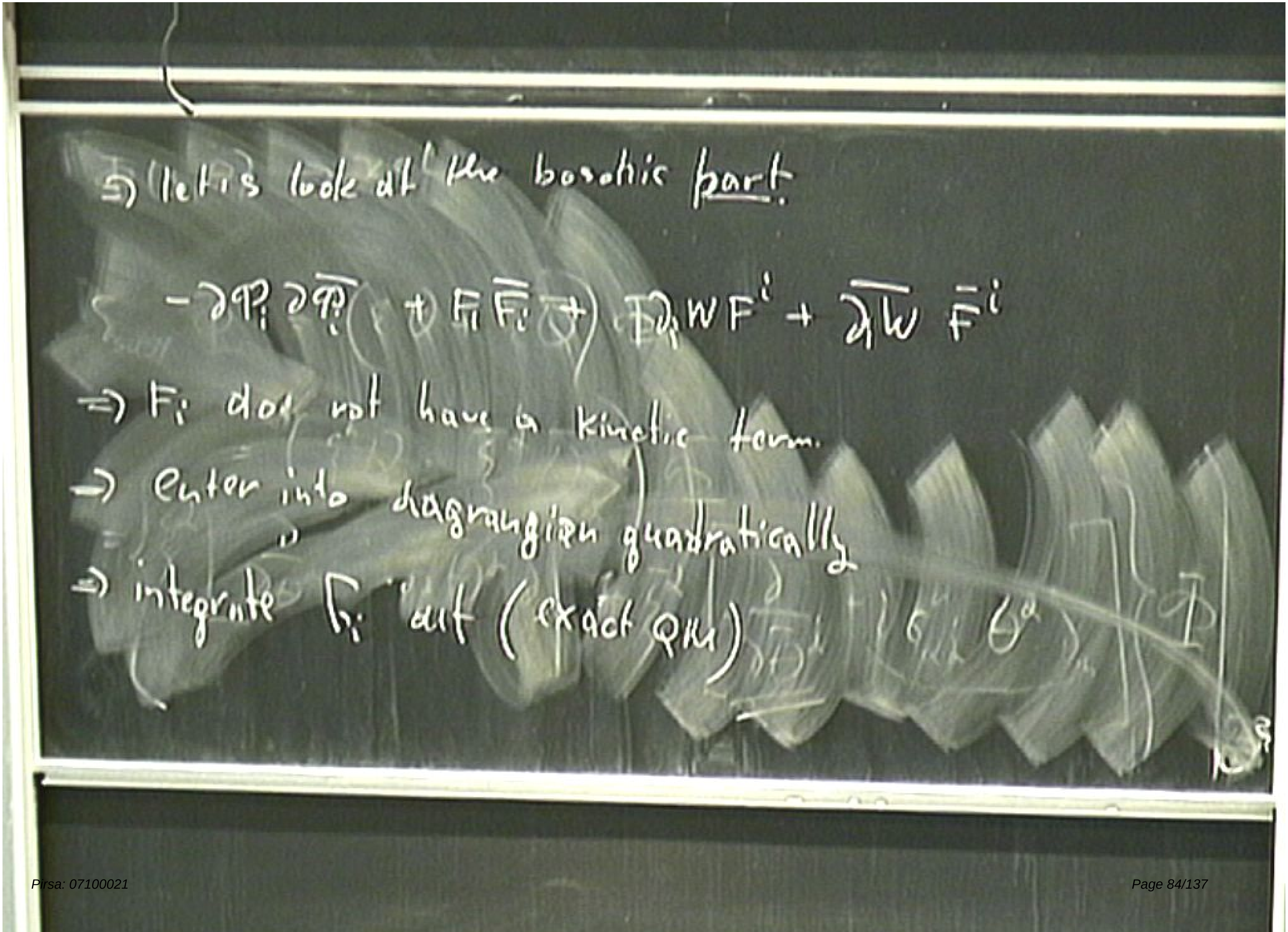


⇒ let's look at the bosonic part

$$-\partial\varphi_i\,\partial\bar{\varphi}_i + F_i\bar{F}_i + \partial_i W F^i + \overline{\partial W}\,\bar{F}^i$$

⇒ F_i does not have a kinetic term.

⇒ enter into Lagrangian quadratically



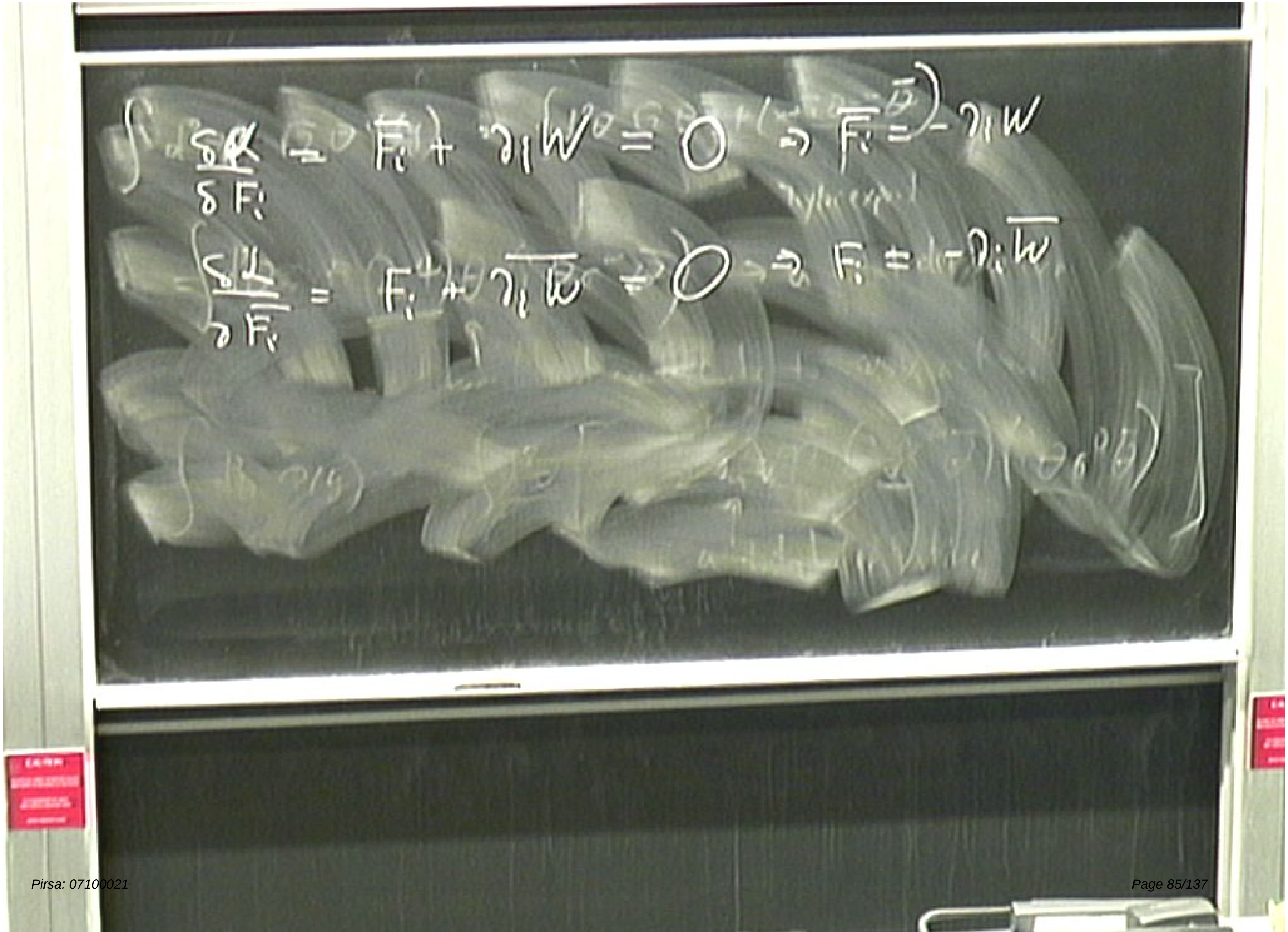
⇒ let's look at the bosonic part

$$-\partial\varphi_i \partial\bar{\varphi}_i + F_i \bar{F}_i + \partial_i W F^i + \overline{\partial_i W}\, \bar{F}^i$$

⇒ F_i does not have a kinetic term.

⇒ enter into Lagrangian quadratically

⇒ integrate F_i out (exact QM)



$$\frac{\delta \mathcal{L}}{\delta F_i} = \overline{F}_i + \partial_i W = 0 \Rightarrow \overline{F}_i = -\partial_i W$$

$$\frac{\delta \mathcal{L}}{\delta \overline{F}_i} = F_i + \partial_i \overline{W} = 0 \Rightarrow F_i = -\partial_i \overline{W}$$

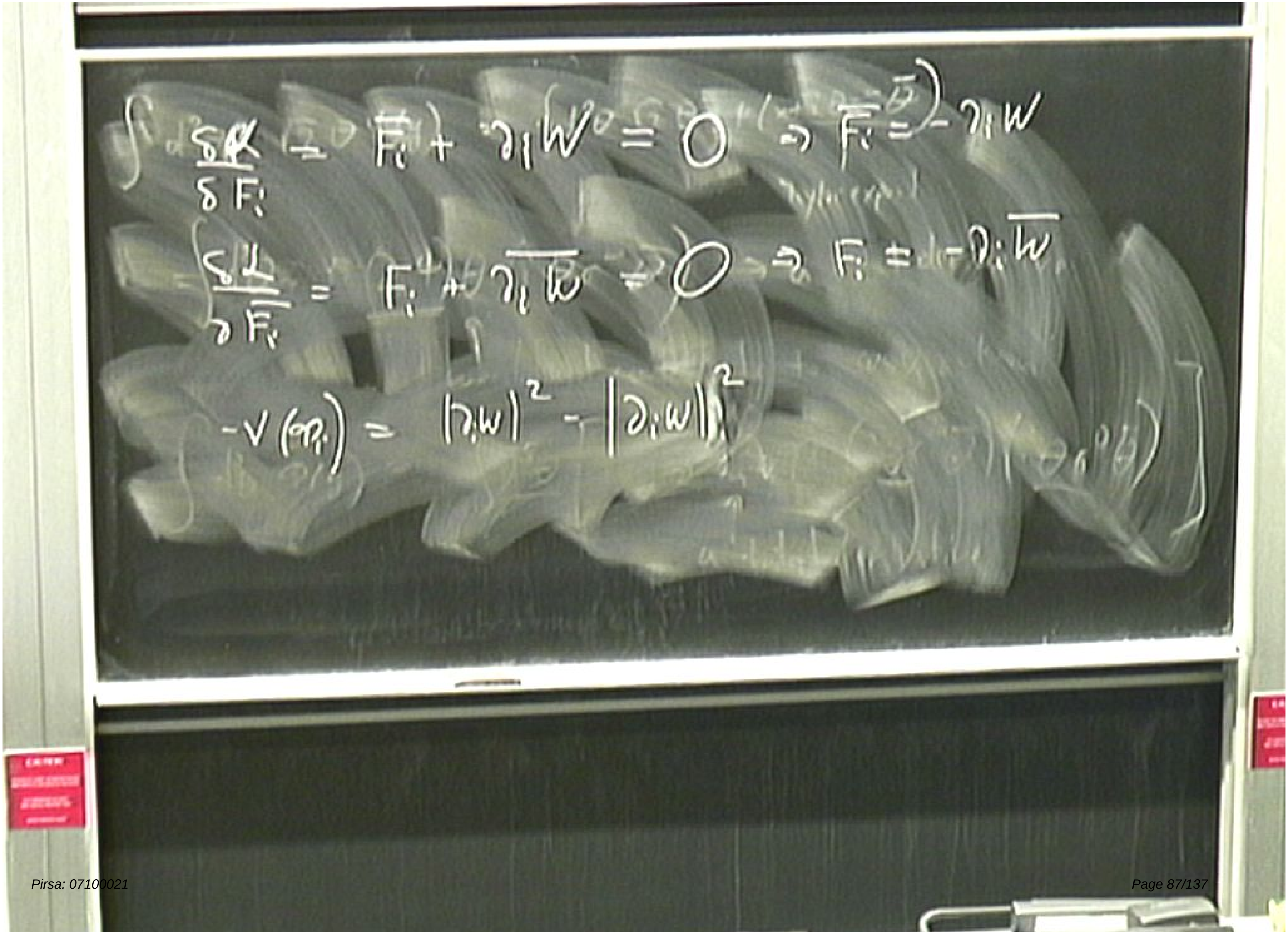
$\Rightarrow$ let's look at the bosonic part.

$$-\partial\varphi_i \partial\bar{\varphi}_i + \underbrace{F_i \bar{F}_i + \partial_i W F^i + \overline{\partial_i W}\, \bar{F}^i}_{-V(\varphi_i)}$$

$\Rightarrow$ F_i does not have a kinetic term.

$\Rightarrow$ enter into Lagrangian quadratically

$\Rightarrow$ integrate F_i out (exact QM)



$$\frac{\delta \mathcal{L}}{\delta F_i} = \bar{F}_i + \partial_i W = 0 \Rightarrow \bar{F}_i = -\partial_i W$$

$$\frac{\delta \mathcal{L}}{\delta \bar{F}_i} = F_i + \overline{\partial_i W} = 0 \Rightarrow F_i = -\partial_i \overline{W}$$

$$-V(\varphi_i) = |\partial_i W|^2 - |\partial_i W|^2$$

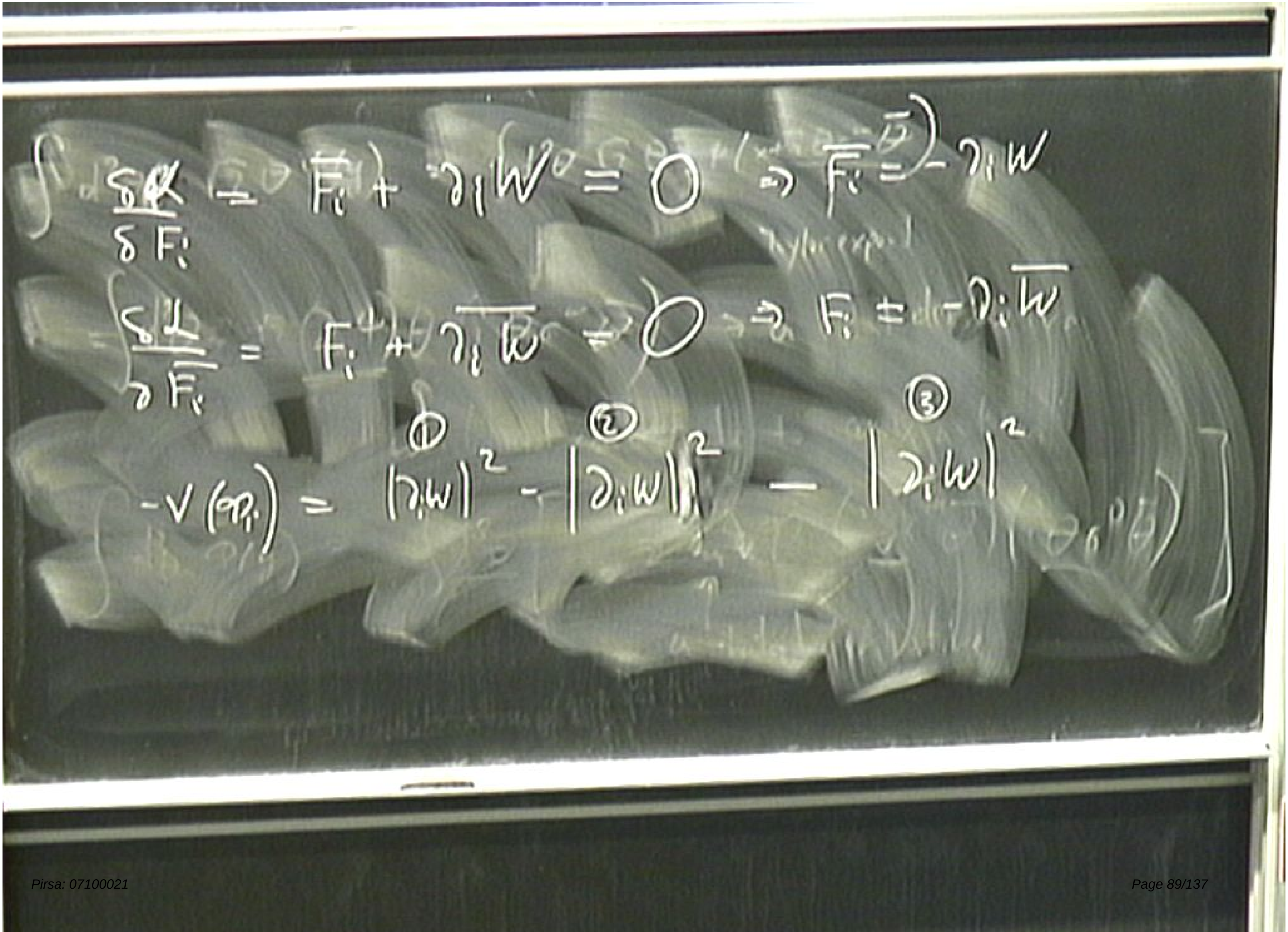
⇒ let's look at the bosonic part:

$$-\partial\varphi_i\,\partial\bar{\varphi}_i + \underbrace{\overset{①}{F_i\bar{F}_i} + \overset{②}{\partial_i W F^i} + \overset{③}{\overline{\partial_i W}\,\bar{F}^i}}_{-V(\varphi_i)}$$

⇒ F_i does not have a kinetic term.

⇒ enter into Lagrangian quadratically

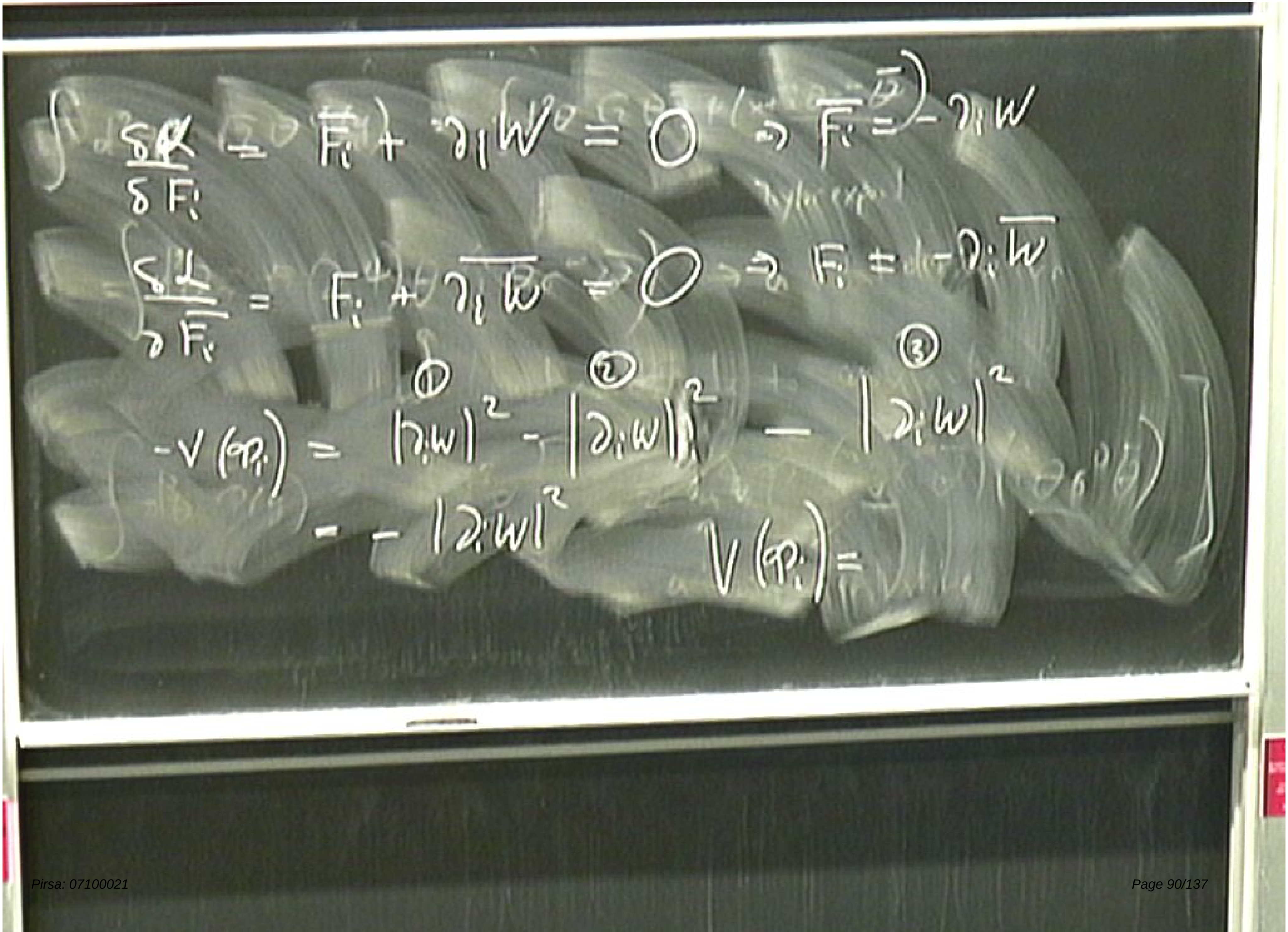
⇒ integrate F_i out (exact QM)



$$\frac{\delta \mathcal{L}}{\delta F_i} = F_i + \partial_i W = 0 \Rightarrow \overline{F_i} = -\partial_i W$$

$$\frac{\delta \mathcal{L}}{\delta \overline{F_i}} = F_i + \partial_i \overline{W} = 0 \Rightarrow F_i = -\partial_i \overline{W}$$

$$-V(\varphi_i) = \overset{①}{|\partial_i W|^2} - \overset{②}{|\partial_i W|^2} - \overset{③}{|\partial_i W|^2}$$



$$\frac{\delta \mathcal{L}}{\delta F_i} = \bar{F}_i + \partial_i W = 0 \Rightarrow \bar{F}_i = -\partial_i W$$

$$\frac{\delta \mathcal{L}}{\delta \bar{F}_i} = F_i + \overline{\partial_i W} = 0 \Rightarrow F_i = -\overline{\partial_i W}$$

$$-V(\varphi_i) = \overset{①}{|\partial_i W|^2} - \overset{②}{|\partial_i W|^2} - \overset{③}{|\partial_i W|^2}$$

$$= -|\partial_i W|^2$$

$$V(\varphi_i) =$$

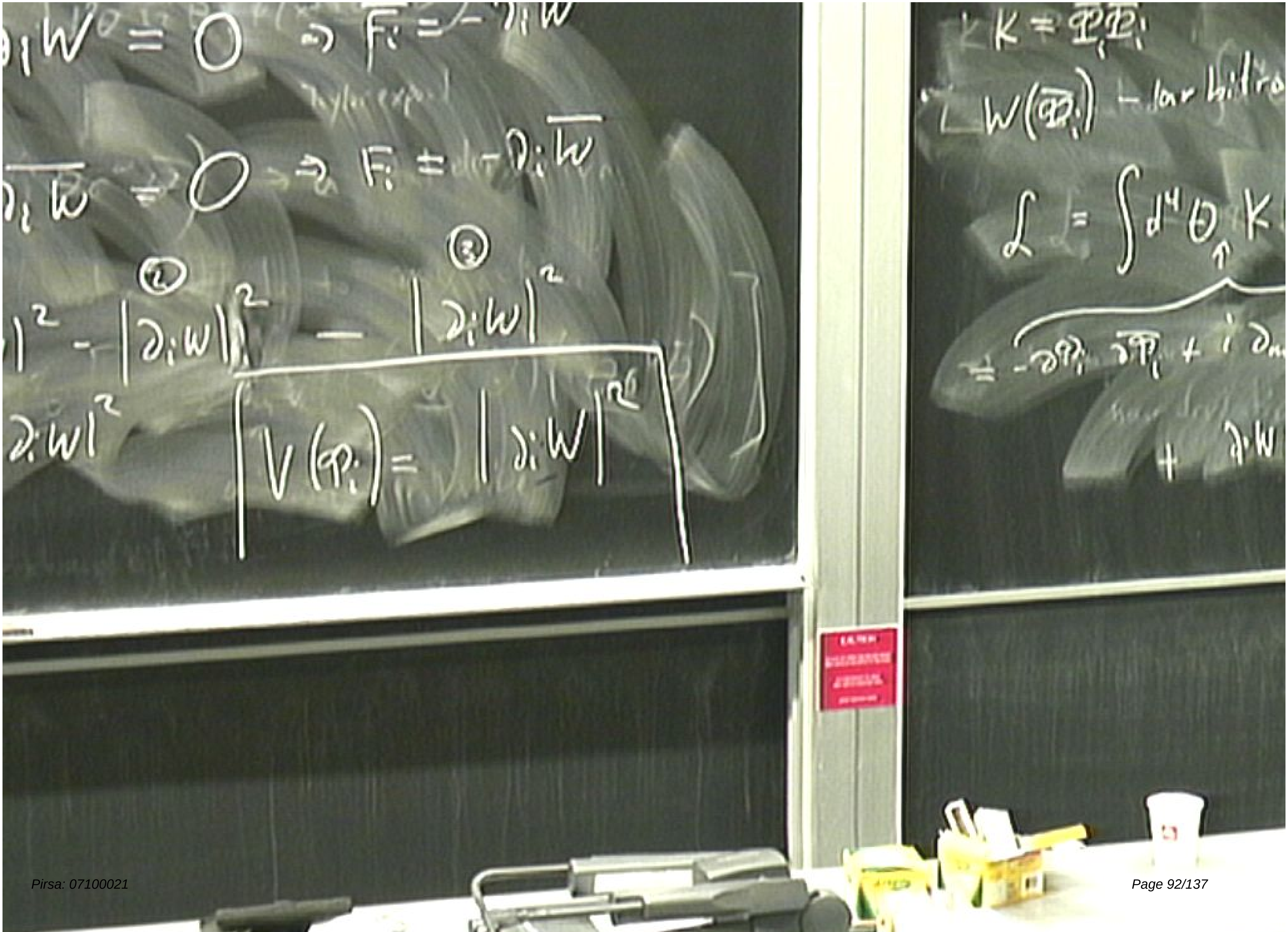
$$\frac{\delta \mathcal{L}}{\delta F_i} = F_i + \partial_i W = 0 \Rightarrow \overline{F_i} = -\partial_i W$$

$$\frac{\delta \mathcal{L}}{\delta \overline{F_i}} = F_i + \partial_i \overline{W} = 0 \Rightarrow F_i = -\partial_i \overline{W}$$

$$-V(\varphi_i) = \overset{①}{|\partial_i W|^2} - \overset{②}{|\partial_i W|^2} - \overset{③}{|\partial_i W|^2}$$

$$= -|\partial_i W|^2$$

$$\boxed{V(\varphi_i) = |\partial_i W|^2}$$



$\partial_i W = 0 \Rightarrow F_i = -\partial_i W$

$\overline{\partial_i W} = 0 \Rightarrow F_i = -\partial_i \overline{W}$

② ③

$|^2 - |\partial_i W|^2 - |\partial_i W|^2$

$\partial_i W|^2$

$$\boxed{V(\Phi_i) = |\partial_i W|^2}$$

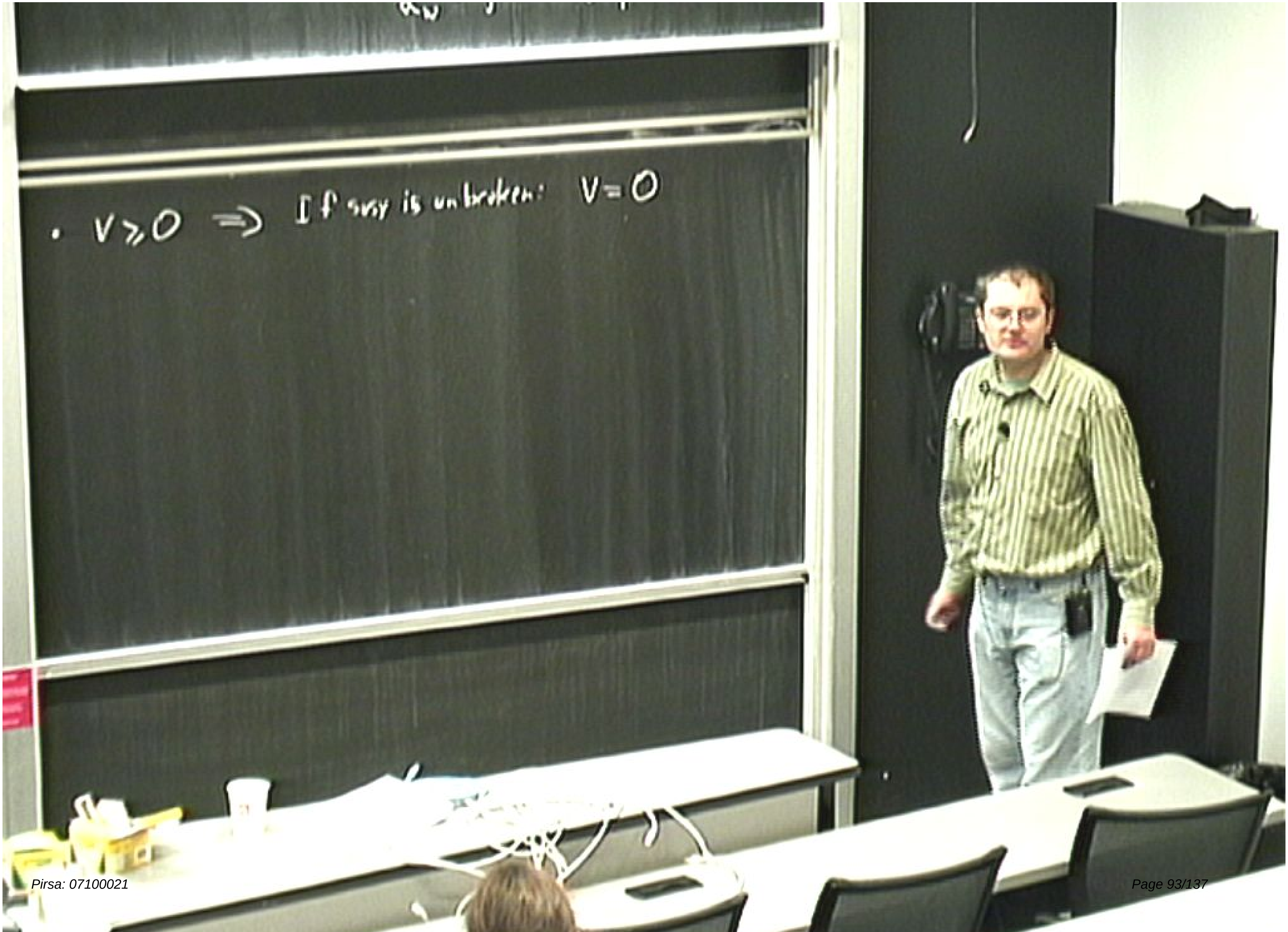
$K = \Phi_i \bar{\Phi}_i$

$W(\Phi_i)$ - arbitrary

$\mathcal{L} = \int d^4\theta\, K$

$= -\partial\Phi_i\, \partial\bar{\Phi}_i + i\,\partial_\mu$

$+ \partial_i W$



- $V \geq 0 \Rightarrow$ If susy is unbroken: $V = 0$

- $V \geq 0 \Longrightarrow$ If susy is unbroken: $V\big|_{\text{vacuum}} = 0$

- $V \geq 0 \Longrightarrow$ If susy is unbroken: $V\big|_{\text{vacuum}} = 0$

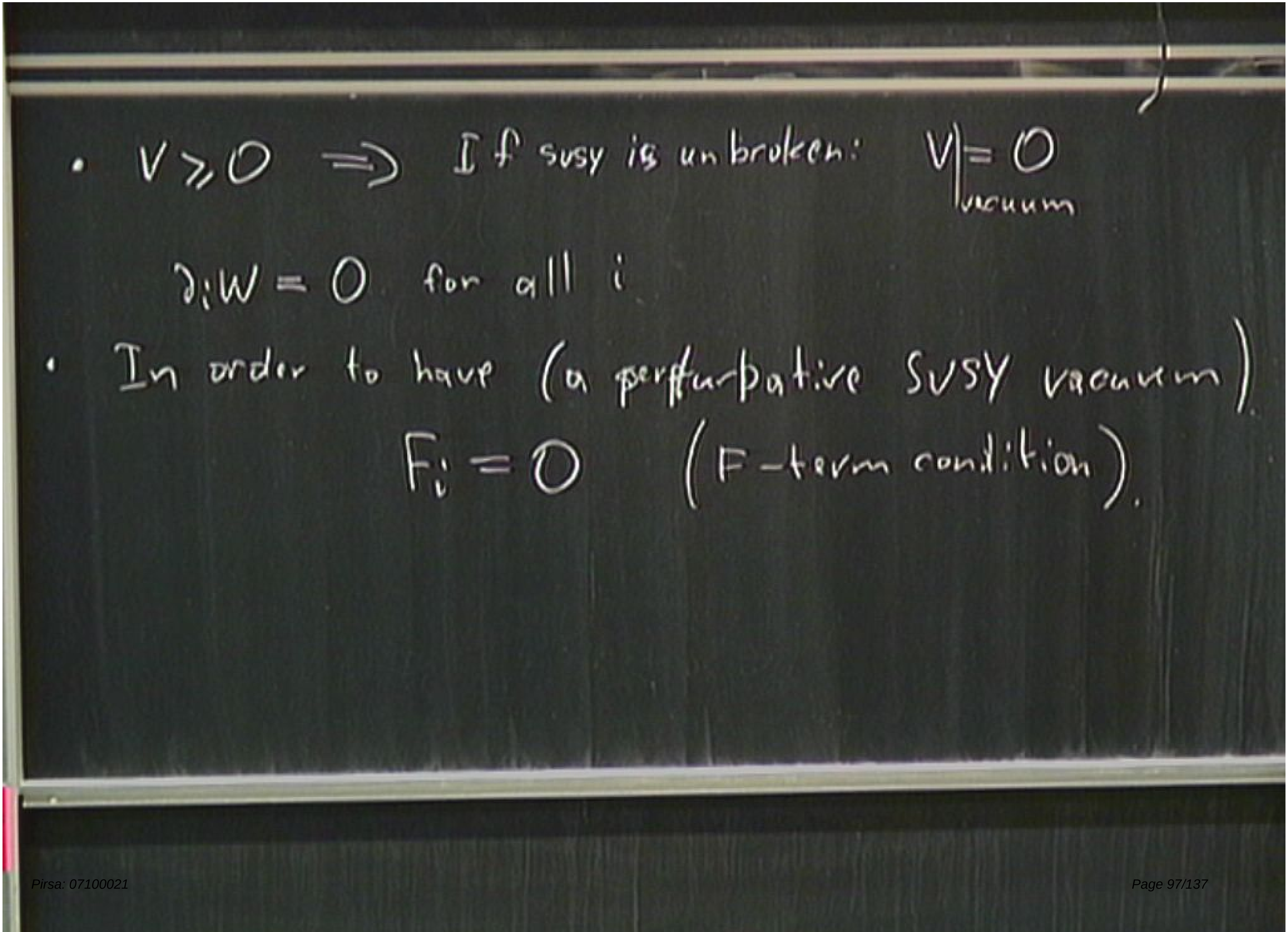
$\partial_i W = 0$ for all i

- $V \geq 0 \implies$ If susy is unbroken: $V\big|_{\text{vacuum}} = 0$

  $\partial_i W = 0$ for all i

- In order to have (a perturbative SUSY vacuum)

  $$F_i = 0$$

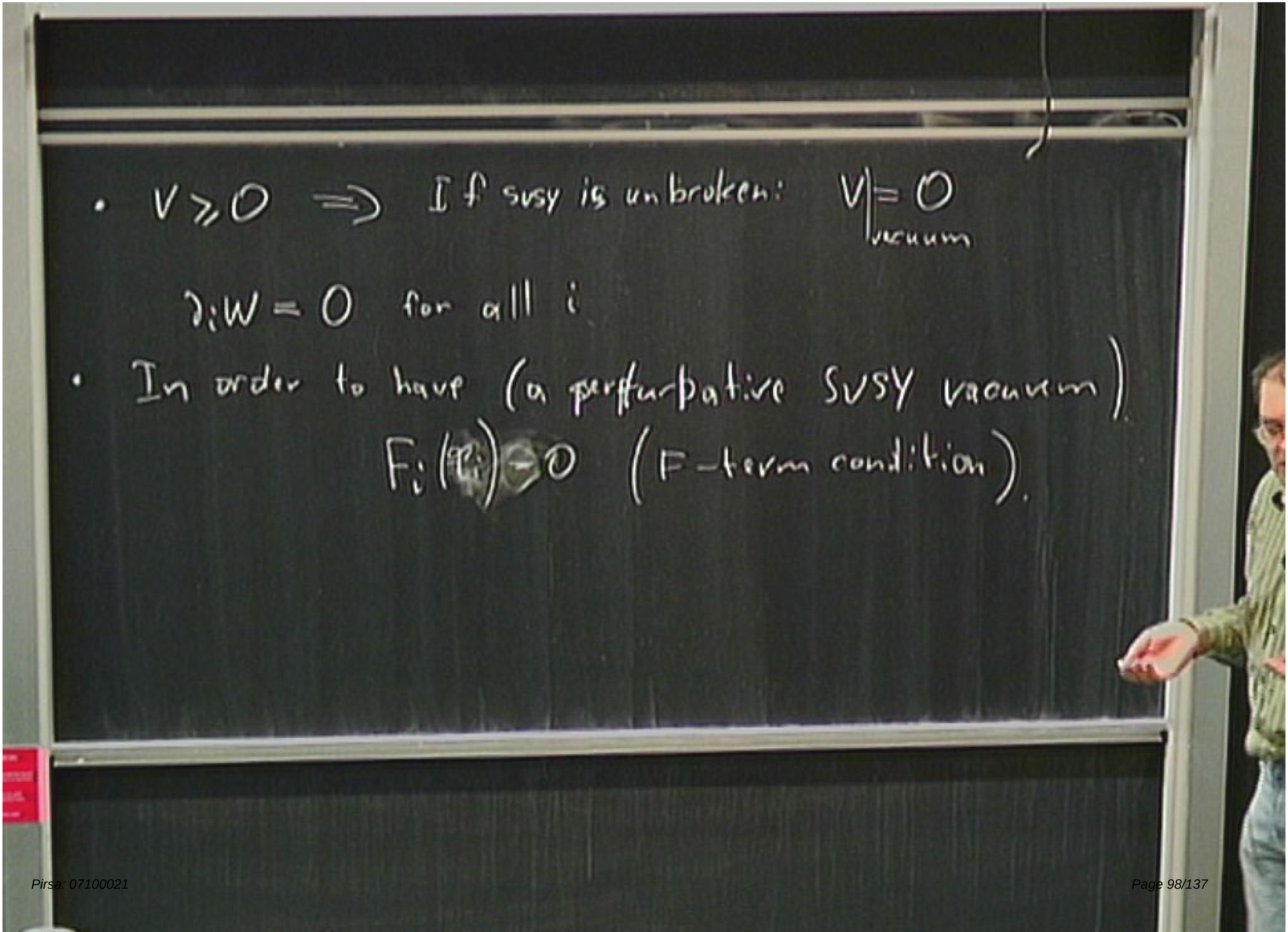


- $V \geqslant 0 \Rightarrow$ If susy is unbroken: $V\big|_{\text{vacuum}} = 0$

  $\partial_i W = 0$ for all i

- In order to have (a perturbative SUSY vacuum)

  $F_i = 0$ (F-term condition).



- $V \geq 0 \Longrightarrow$ If susy is unbroken: $V\big|_{\text{vacuum}} = 0$

  $\partial_i W = 0$ for all i

- In order to have (a perturbative SUSY vacuum)

  $F_i(\varphi_i) = 0$ (F-term condition).

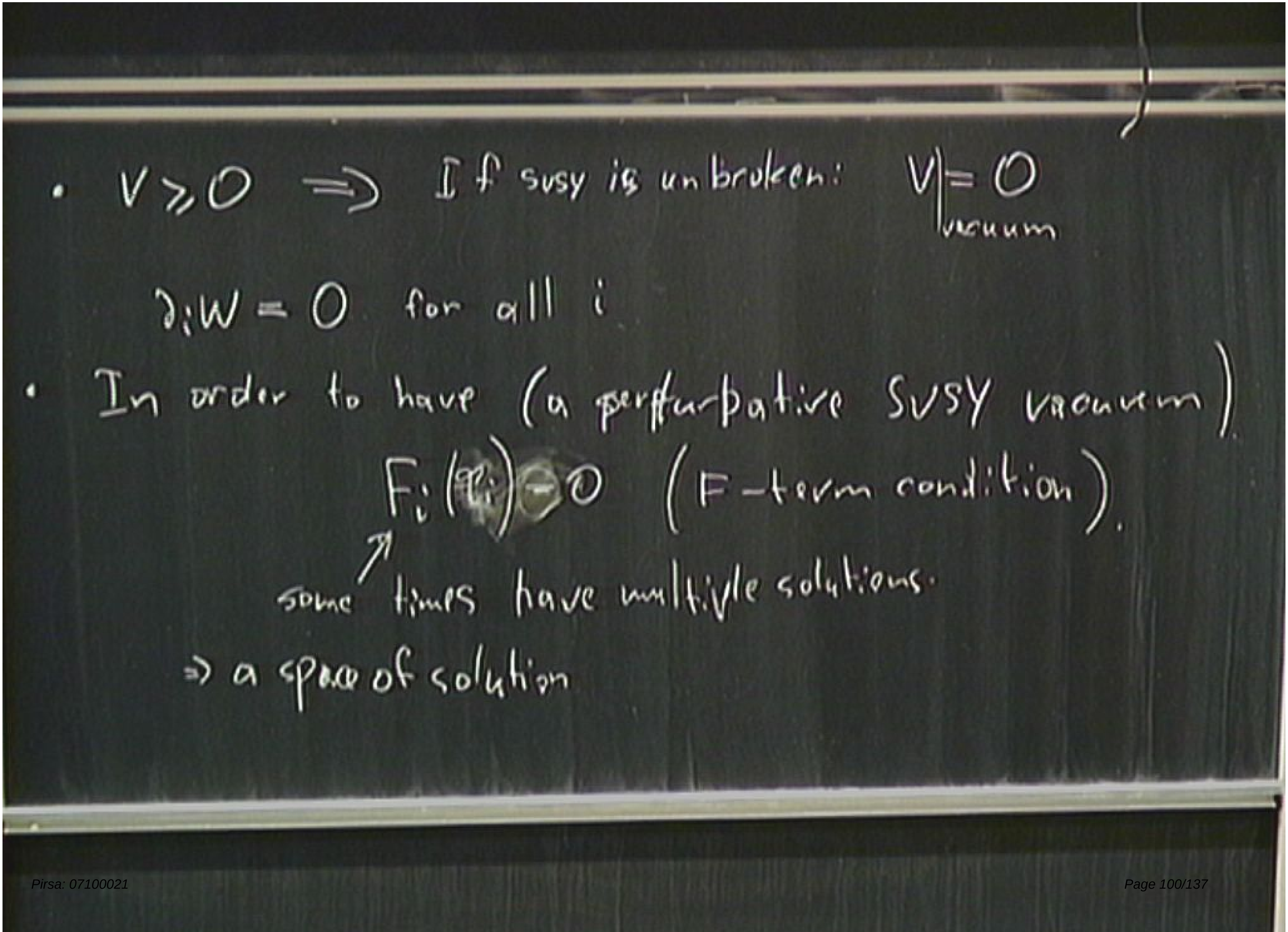
- $V \geq 0 \Rightarrow$ If susy is unbroken: $V\big|_{\text{vacuum}} = 0$

  $\partial_i W = 0$ for all i

- In order to have (a perturbative SUSY vacuum)

  $F_i(\varphi_i) = 0$ (F-term condition).

  ↗ sometimes have multiple solutions.



- $V \geq 0 \Rightarrow$ If susy is unbroken: $V|_{\text{vacuum}} = 0$

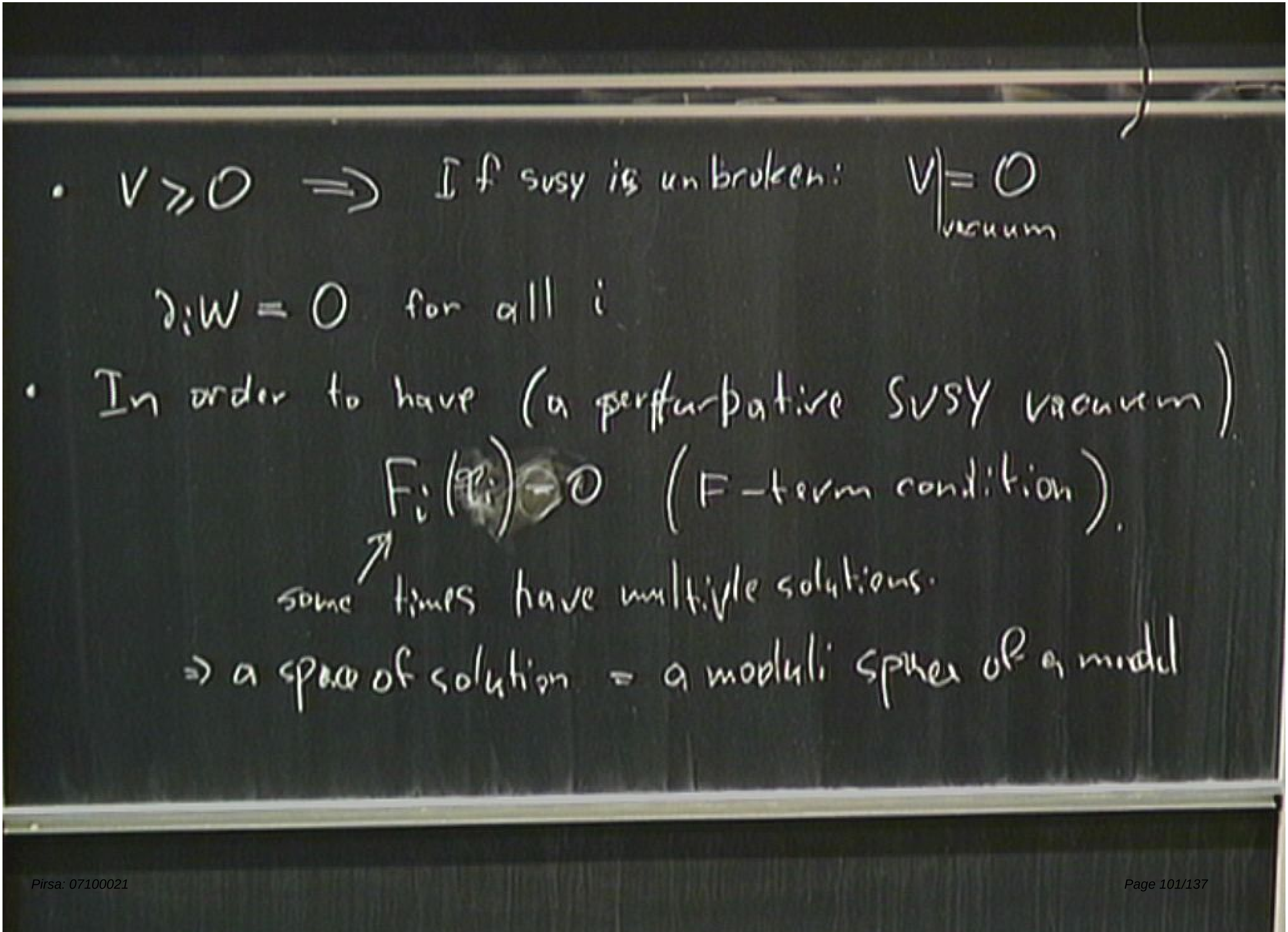
  $\partial_i W = 0$ for all i

- In order to have (a perturbative SUSY vacuum)

$$F_i(\varphi_i) = 0 \quad (\text{F-term condition}).$$

$\nearrow$ sometimes have multiple solutions.

$\Rightarrow$ a space of solution



- $V \geq 0 \Longrightarrow$ If susy is unbroken: $V\big|_{\text{vacuum}} = 0$

  $\partial_i W = 0$ for all i

- In order to have (a perturbative SUSY vacuum)

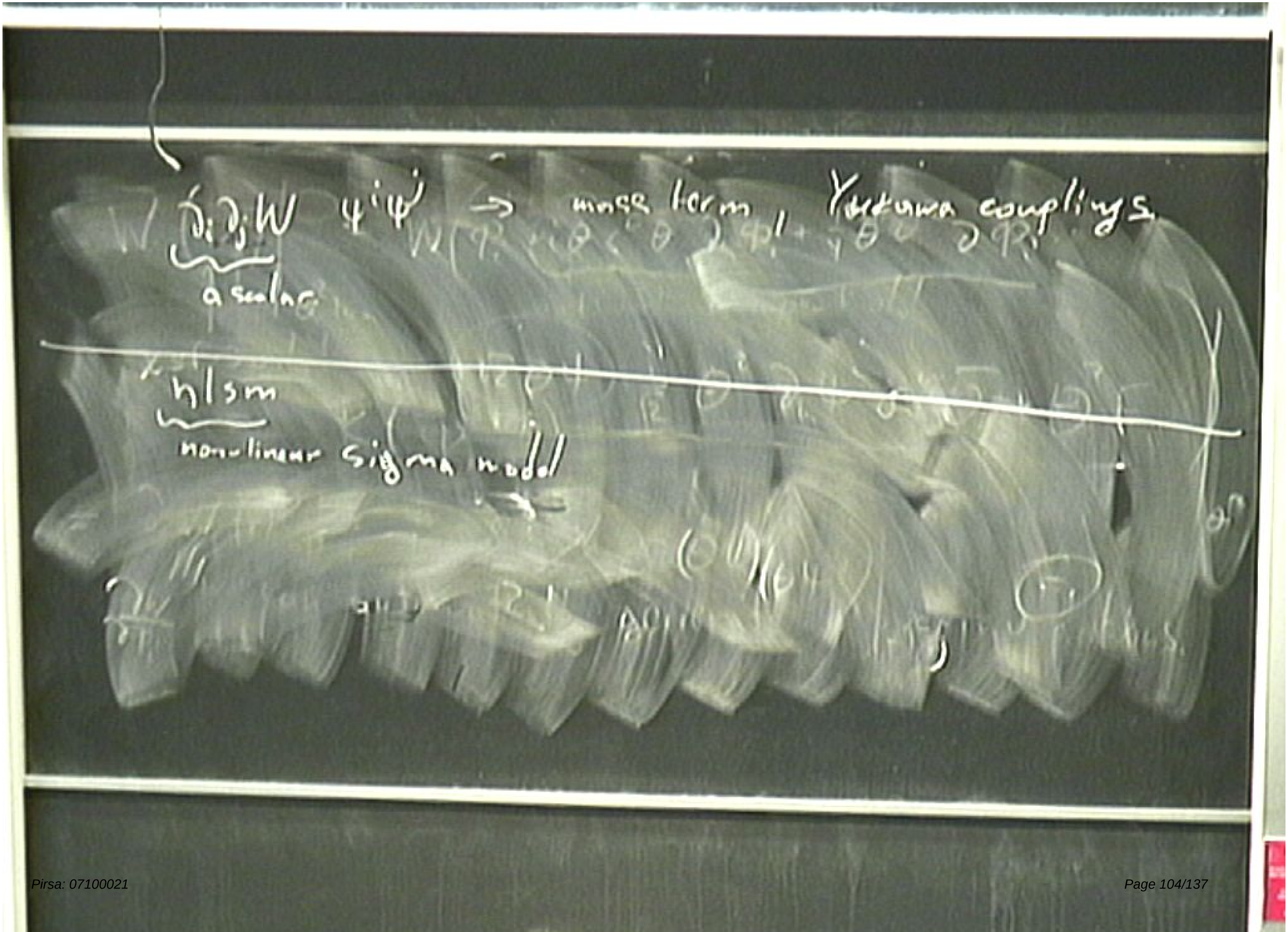
  $F_i(\varphi_i) = 0$ (F-term condition).

  $\nearrow$ some times have multiple solutions.

  $\Rightarrow$ a space of solution = a moduli space of a model

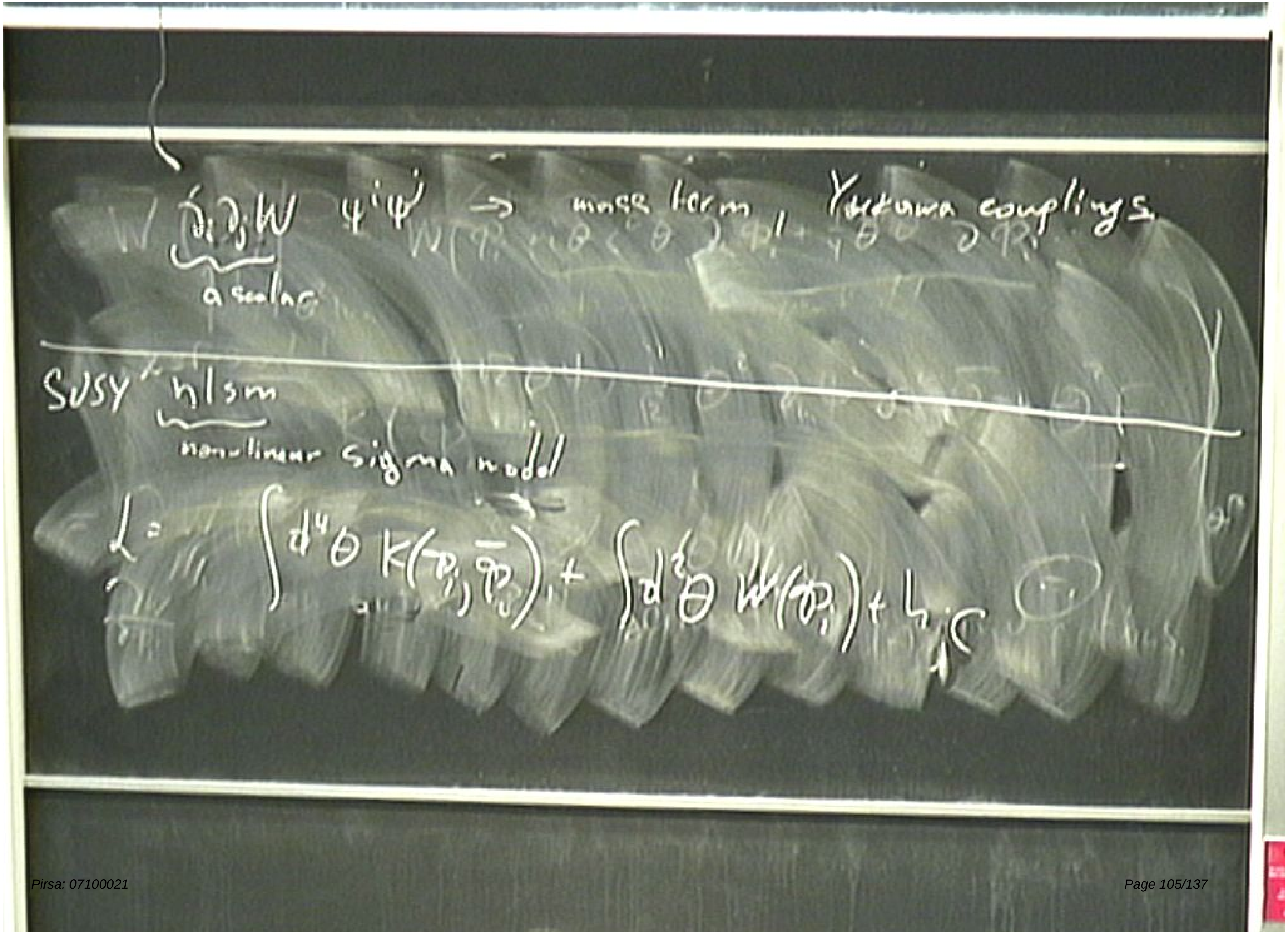
$$\underbrace{\partial_i \partial_j W}_{\text{a scalar}} \ \psi^i \psi^j \rightarrow W(\phi_i + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi_i + \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\phi_i$$

$\underbrace{\partial_i \partial_j W}_{\text{a scalar}} \psi^i \psi^j \rightarrow$ mass term, Yukawa couplings.



$\underbrace{\partial_i \partial_j W}_{\text{a scalar}} \psi^i \psi^j \rightarrow$ mass term, Yukawa couplings.

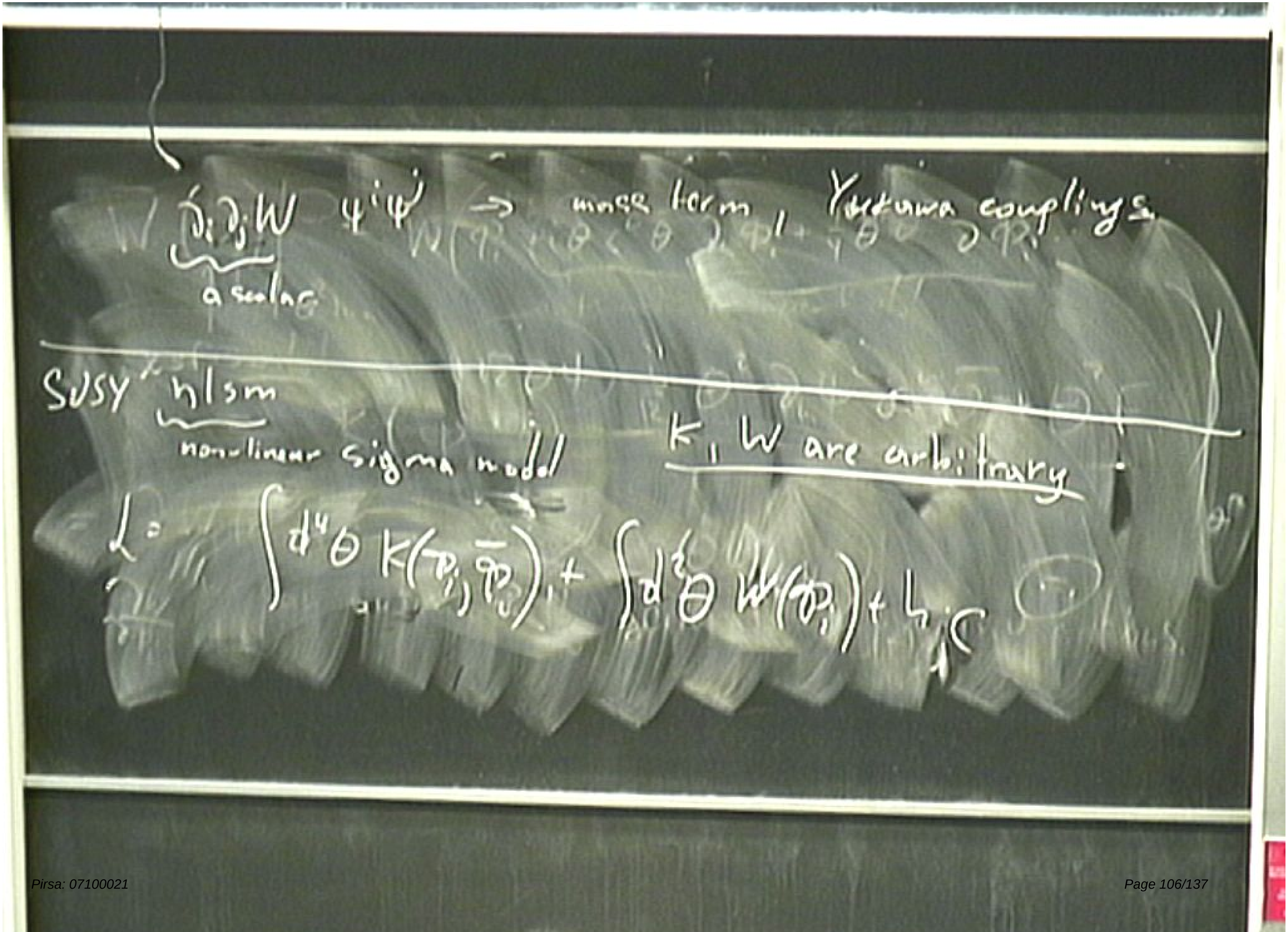
$\underbrace{\text{nlsm}}_{\text{nonlinear sigma model}}$



$\underbrace{\partial_i \partial_j W}_{\text{a scalar}} \psi^i \psi^j \rightarrow$ mass term, Yukawa couplings.

SUSY $\underbrace{\text{nlsm}}_{\text{non-linear sigma model}}$

$$\mathcal{L} = \int d^4\theta \, K(\Phi_i, \bar{\Phi}_i) + \int d^2\theta \, W(\Phi_i) + h.c.$$



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SUSY $\underbrace{\text{nlsm}}_{\text{non-linear sigma model}}$

$\underline{K, W \text{ are arbitrary}}$.

$$\mathcal{L} = \int d^4\theta \, K(\Phi_i, \bar{\Phi}_i) + \int d^2\theta \, W(\Phi_i) + \text{h.c.}$$

$\underbrace{\partial_i \partial_j W}_{\text{a scalar}} \ \psi^i \psi^j \rightarrow$ mass term, Yukawa couplings.

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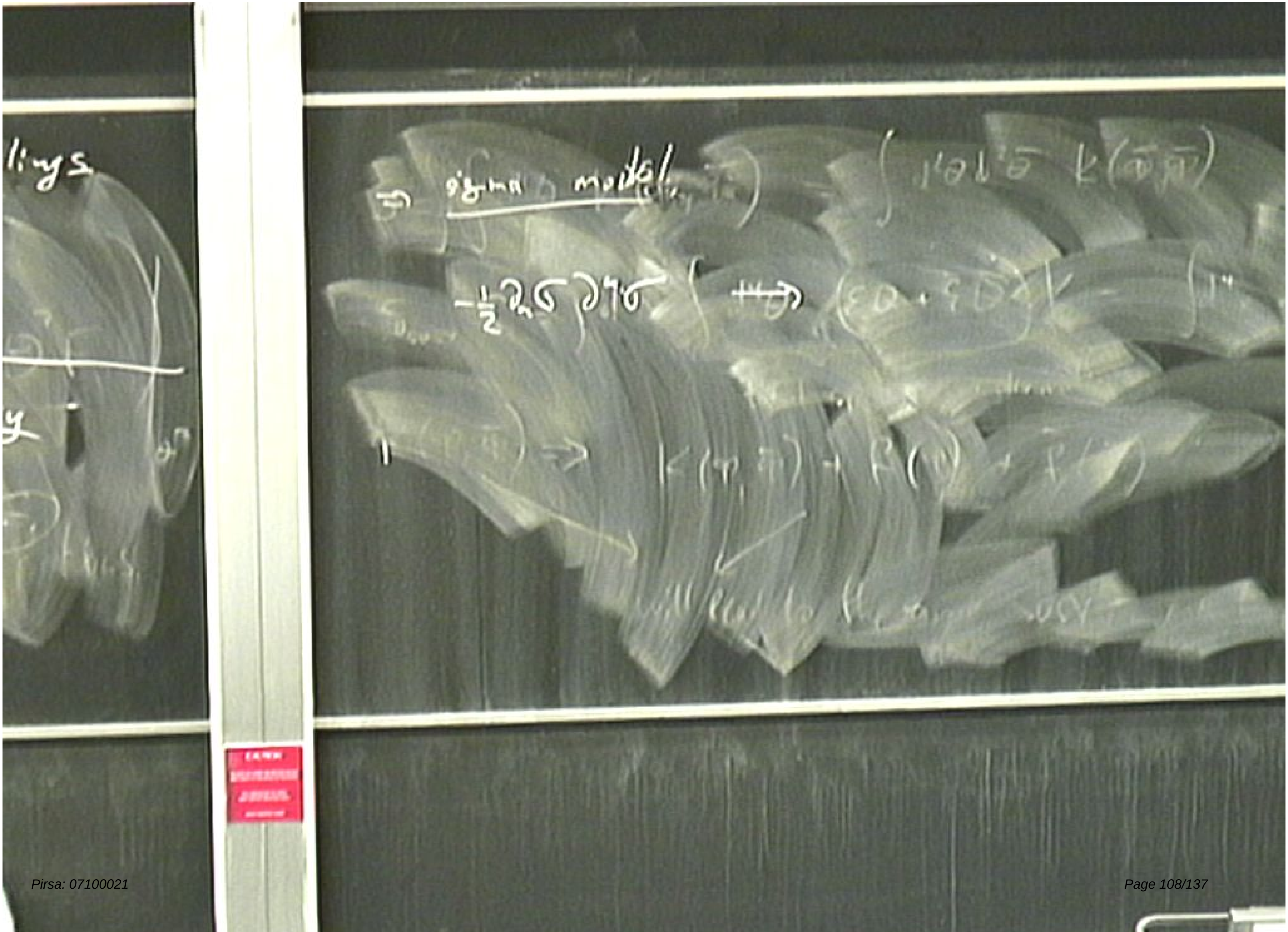
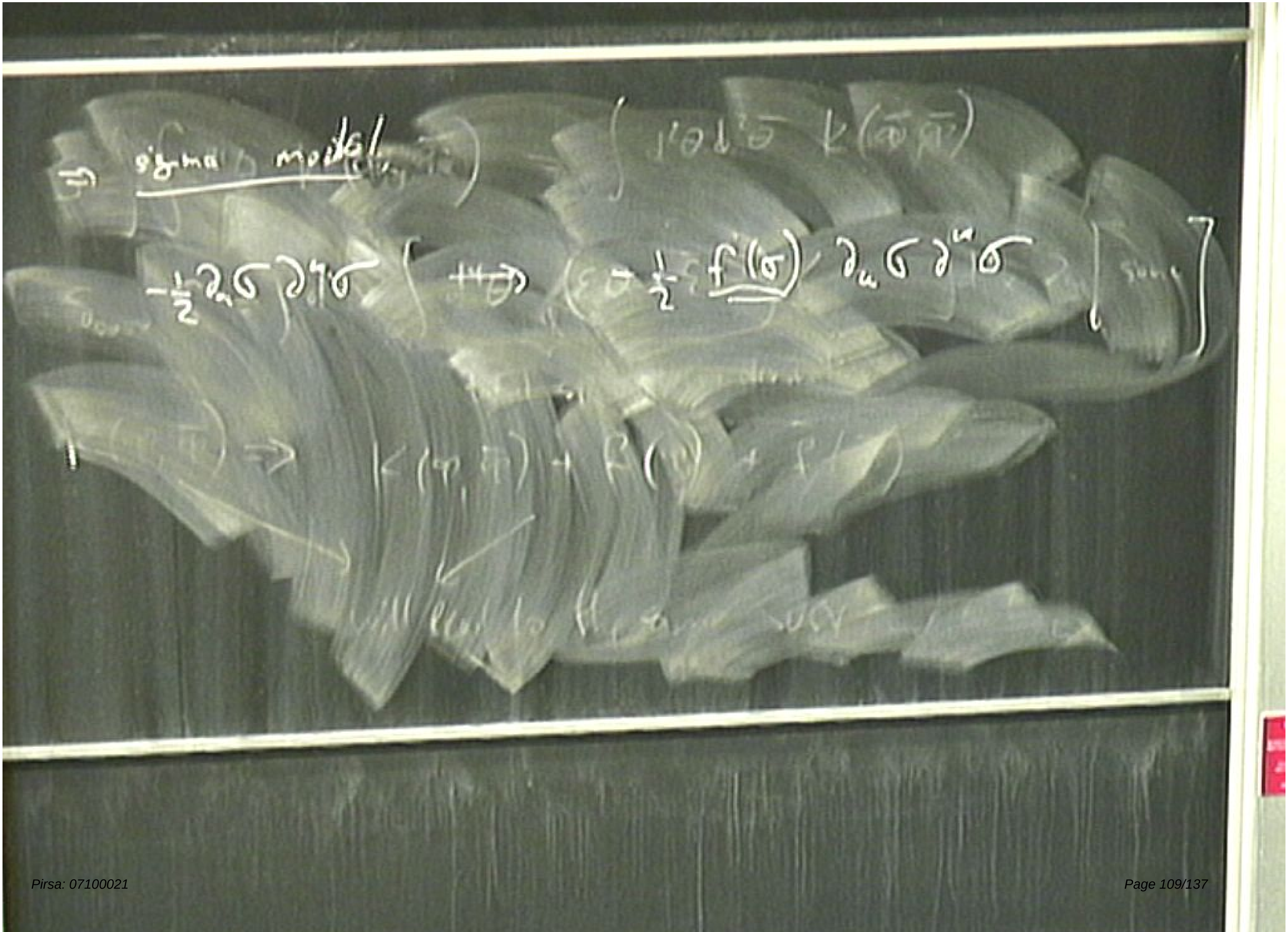
$$K = \sum_i \Phi_i \bar{\Phi}_i$$


lings
⇒ sigma model
$-\frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma$
$\int d^2\theta\, d^2\bar{\theta}\; K(\bar{\Phi},\Phi)$



⇒ sigma model

$\int d^4\theta\, d^4\bar\theta\; K(\sigma, \bar\sigma)$

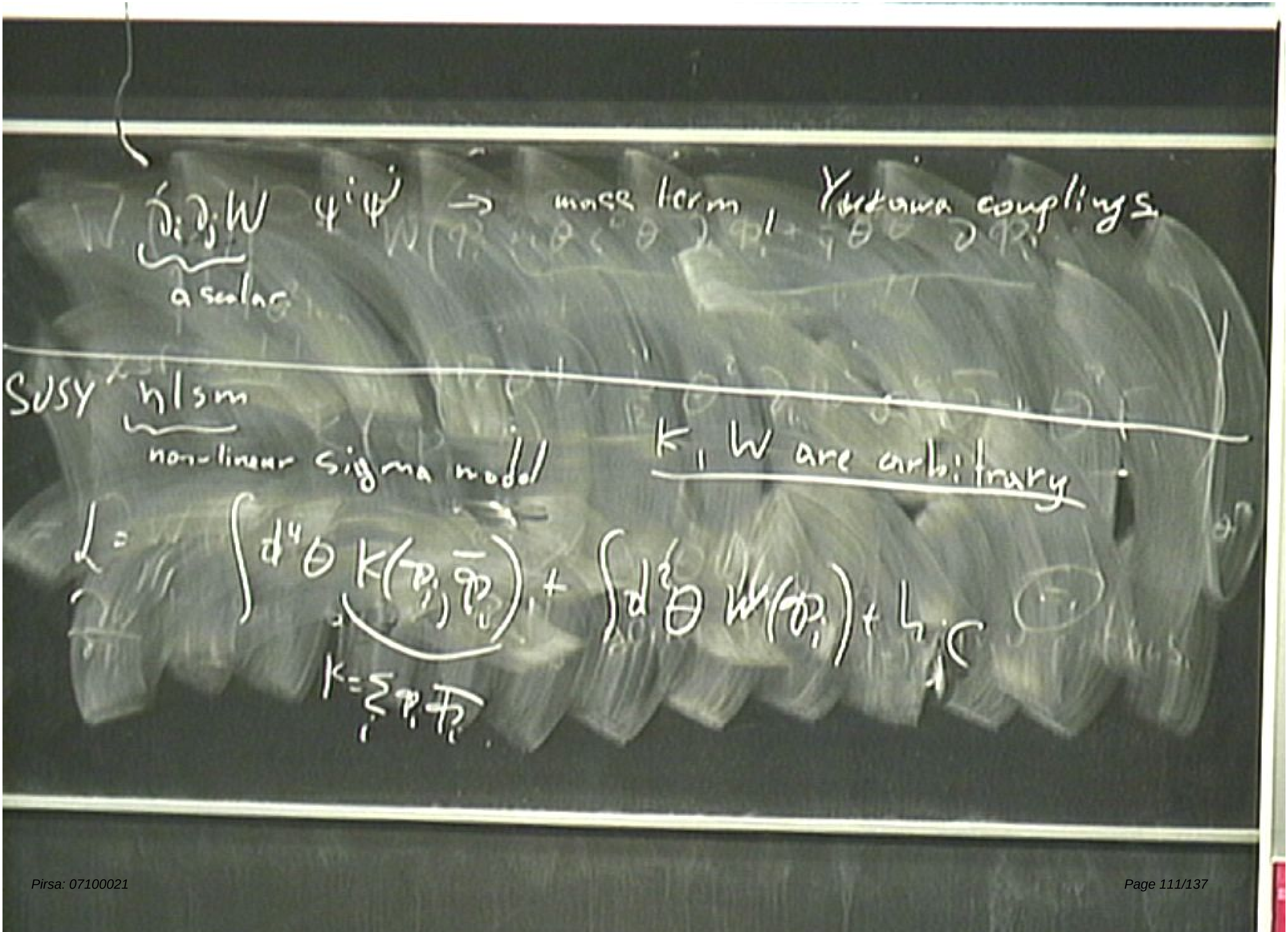
$-\frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma \mapsto -\frac{1}{2} f(\sigma)\,\partial_\mu\sigma\,\partial^\mu\sigma$ [sum]

$K(\sigma,\bar\sigma) \to K(\sigma,\bar\sigma) + F(\sigma) + \bar{F}$

⇒ sigma model

$$-\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma \Longrightarrow -\frac{1}{2}\underline{f(\sigma)}\,\partial_\mu\sigma\partial^\mu\sigma$$

$$-\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \underline{\sigma\,\partial_\mu\sigma\partial^\mu\sigma}$$



$\underbrace{\partial_i \partial_j W}_{\text{a scalar}} \ \psi^i \psi^j \rightarrow$ mass term, Yukawa couplings.

SUSY $\underbrace{\text{nlsm}}_{\text{non-linear sigma model}}$

K, W are arbitrary!

$$\mathcal{L} = \int d^4\theta \, \underbrace{K(\Phi_i, \bar{\Phi}_i)}_{K = \sum_i \Phi_i \bar{\Phi}_i} + \int d^2\theta \, W(\Phi_i) + h.c.$$

$$\int d^4\theta K = g_{i\bar{i}} \, F^i \bar{F}^{\bar{j}} - \frac{1}{2} F^i g_{i\bar{i}} \Gamma^{\bar{i}}_{\bar{j}\bar{k}} \bar{\psi}_1^{\bar{j}} \bar{\psi}^{\bar{k}} + \cdots$$

$$\int d^4\theta K = g_{i\bar{i}} F^i \bar{F}^{\bar{i}} \underbrace{- \frac{1}{2} F^i g_{i\bar{i}} \Gamma^{\bar{i}}_{\bar{j}\bar{k}} \bar{\psi}_1^{\bar{j}} \bar{\psi}^{\bar{k}}}_{+h.c} + \cdots$$

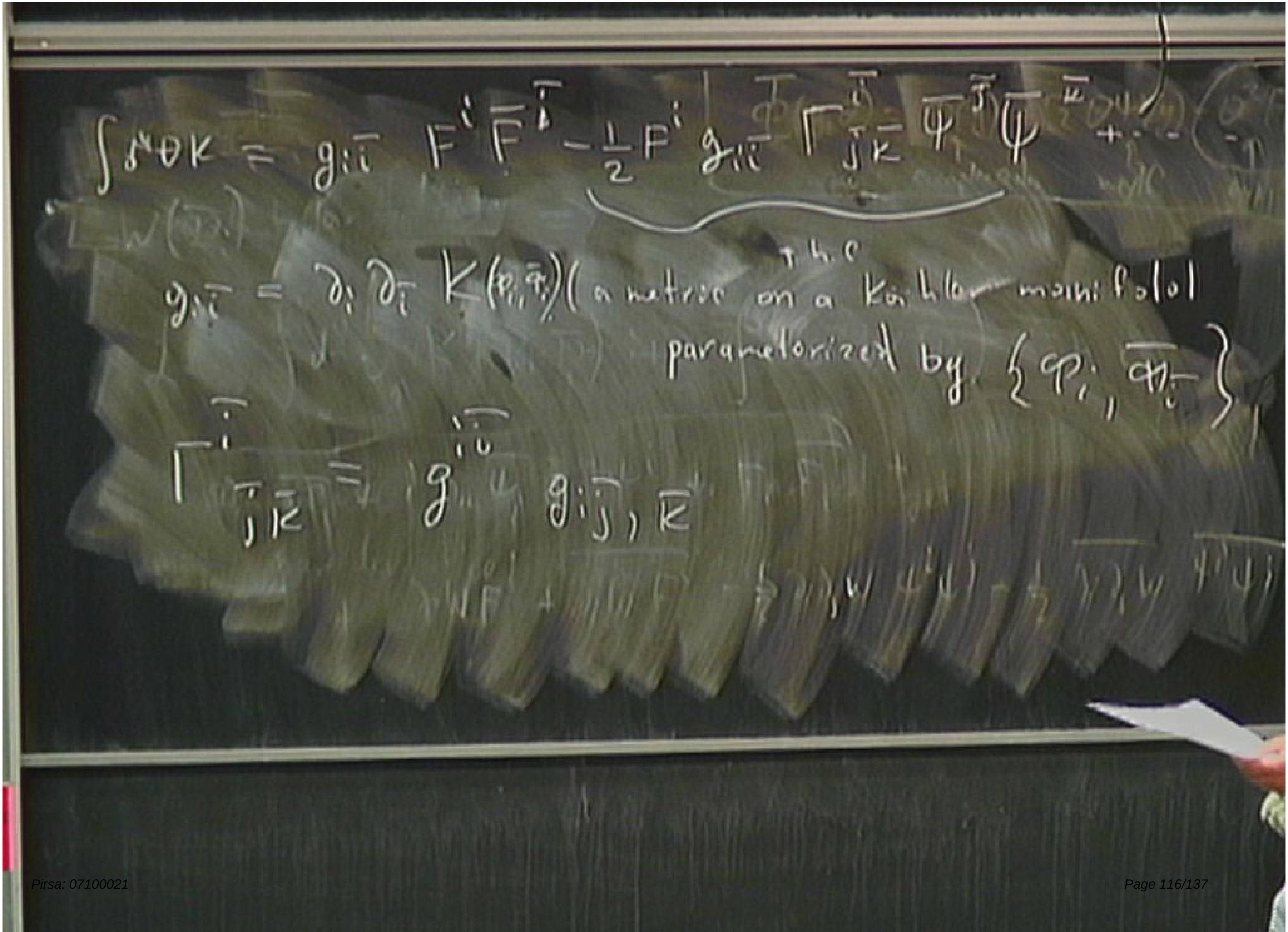
$$\int d^4\theta K = g_{i\bar{i}} F^i \bar{F}^{\bar{i}} \underbrace{- \frac{1}{2} F^i g_{i\bar{i}} \Gamma^{\bar{i}}_{\bar{j}\bar{k}} \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{k}}}_{+\text{h.c}} + \cdots$$

$$g_{i\bar{i}} = \partial_i \partial_{\bar{i}} K$$ (a metric on a Kähler manifold parametorized by $\{\varphi_i, \bar{\varphi}_{\bar{i}}\}$)

$$\int d^4\theta K = g_{i\bar{i}}\, F^i \bar{F}^{\bar{i}} \underbrace{- \frac{1}{2} F^i g_{i\bar{i}} \Gamma^{\bar{i}}_{\bar{j}\bar{k}} \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{k}}}_{+\text{h.c}} + \cdots$$

$g_{i\bar{i}} = \partial_i \partial_{\bar{i}} K(\varphi_i, \bar{\varphi}_{\bar{i}})$ (a metric on a Kähler manifold parameterized by $\{\varphi_i, \bar{\varphi}_{\bar{i}}\}$)

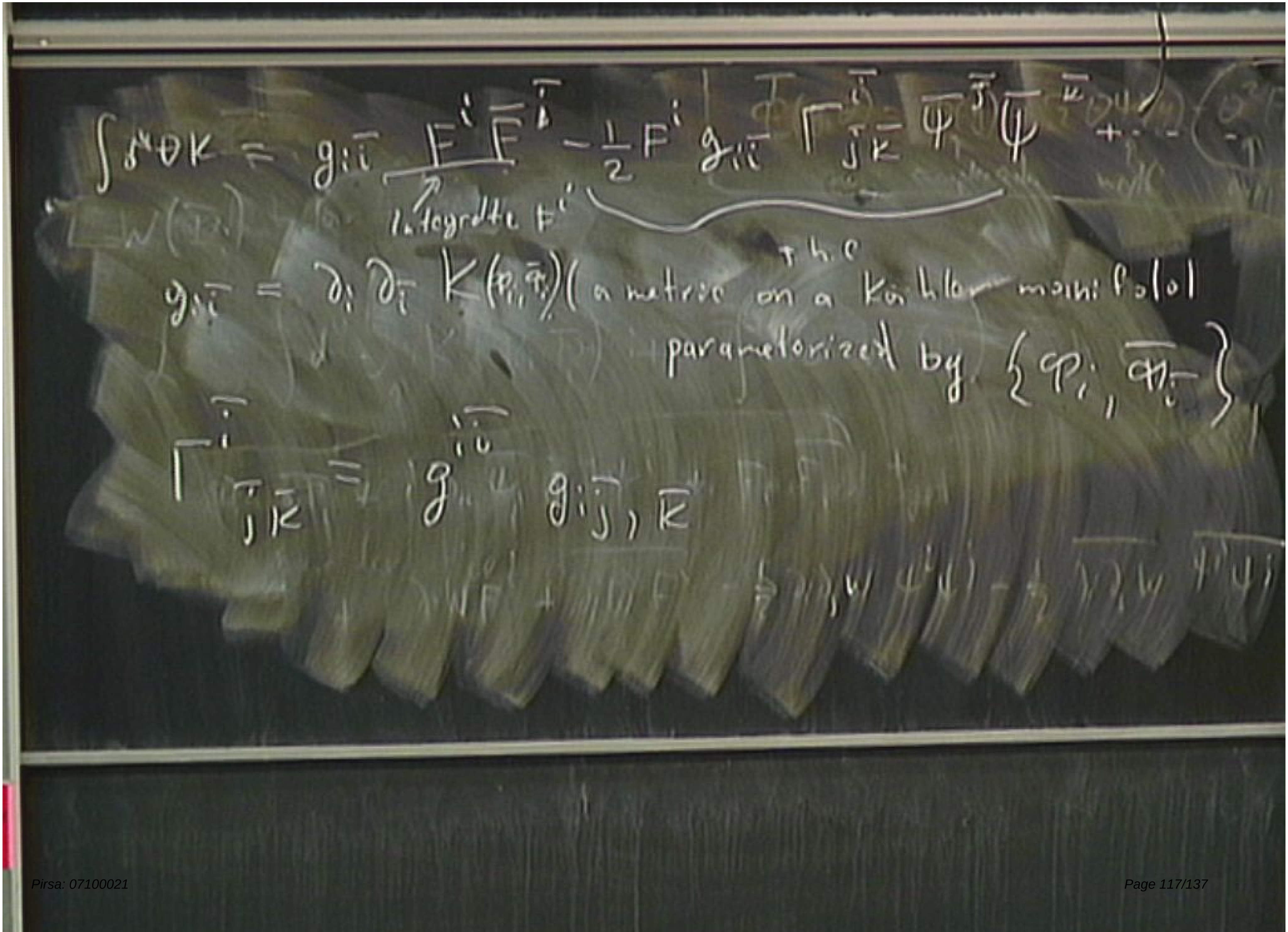
$$\Gamma^{\bar{i}}_{\bar{j}\bar{k}} = g^{i\bar{i}}\, g_{i\bar{j},\bar{k}}$$



$$\int d^4\theta\, K = g_{i\bar i}\, F^i \bar F^{\bar i} \underbrace{- \frac{1}{2} F^i g_{i\bar i}\, \Gamma^{\bar i}_{\bar j \bar k}\, \bar\psi_1^{\bar j}\, \bar\psi^{\bar k}} + \cdots$$

$g_{i\bar i} = \partial_i \partial_{\bar i} K(\varphi_i, \bar\varphi_{\bar i})$ (a metric on $\overset{\text{the}}{\text{a}}$ Kähler manifold parametorized by $\{\varphi_i, \bar\varphi_{\bar i}\}$

$$\Gamma^{\bar i}_{\bar j \bar k} = g^{\bar i i}\, g_{i\bar j, \bar k}$$



$$\int d^4\theta\, K = g_{i\bar i}\, \underset{\uparrow}{F^i \bar F^{\bar i}} \underbrace{- \frac{1}{2} F^i g_{i\bar i}\, \Gamma^{\bar i}_{\bar j \bar k}\, \bar\psi^{\bar j} \bar\psi^{\bar k}} + \cdots$$

integrate F^i

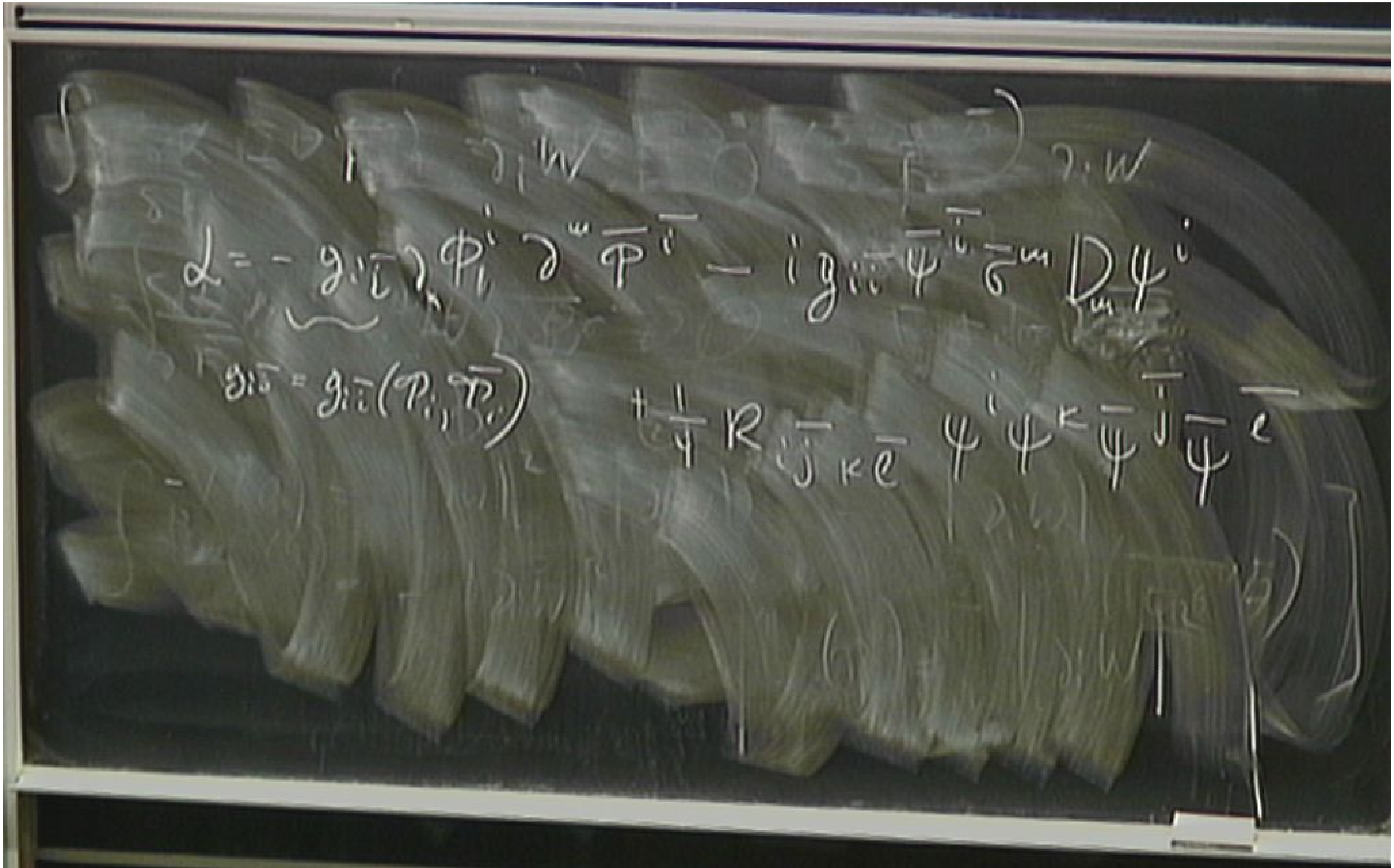
$g_{i\bar i} = \partial_i \partial_{\bar i} K(\varphi_i, \bar\varphi_{\bar i})$ (a metric on the a Kahler manifold parameterized by $\{\varphi_i, \bar\varphi_{\bar i}\}$

$$\Gamma^{\bar i}_{\bar j \bar k} = g^{i \bar i}\, g_{i \bar j, \bar k}$$

$\int \ldots \partial_i W \ldots) \; \partial_i W$

$$\mathcal{L} = - g_{i\bar i}\, \partial_m \varphi^i \partial^m \bar\varphi^{\bar i} - i g_{i\bar i} \bar\psi^{\bar i} \bar\sigma^m D_m \psi^i$$

$$g_{i\bar i} = g_{i\bar i}(\varphi_i, \bar\varphi_i) \qquad + \frac{1}{4} R_{i\bar j k \bar l}$$



$\partial_i W \qquad) \; \partial_i W$

$$\mathcal{L} = - \underbrace{g_{i\bar{i}}}\, \partial_m \varphi^i \partial^m \bar{\varphi}^{\bar{i}} - i g_{i\bar{i}} \bar{\psi}^{\bar{i}} \bar{\sigma}^m D_m \psi^i$$

$$g_{i\bar{i}} = g_{i\bar{i}}(\varphi_i, \bar{\varphi}_i) \qquad + \frac{1}{4} R_{i\bar{j}k\bar{l}}\, \psi^i \psi^k \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{l}}$$

$$\int \qquad \partial_i W \qquad\qquad \partial_i W$$

$$\mathcal{L} = - \underbrace{g_{i\bar{i}}} \partial_m \varphi^i \partial^m \bar{\varphi}^{\bar{i}} - i g_{i\bar{i}} \bar{\psi}^{\bar{i}} \bar{\sigma}^m D_m \psi^i$$

$$\boxed{g_{i\bar{i}} = g_{i\bar{i}}(\varphi_i, \bar{\varphi}_i)} \qquad + \frac{1}{4} R_{i\bar{j}k\bar{l}} \psi^i \psi^k \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{l}}$$

$$D_m = \partial_m \psi^i + \Gamma^i_{jk} \partial_m \varphi^k \psi^j$$

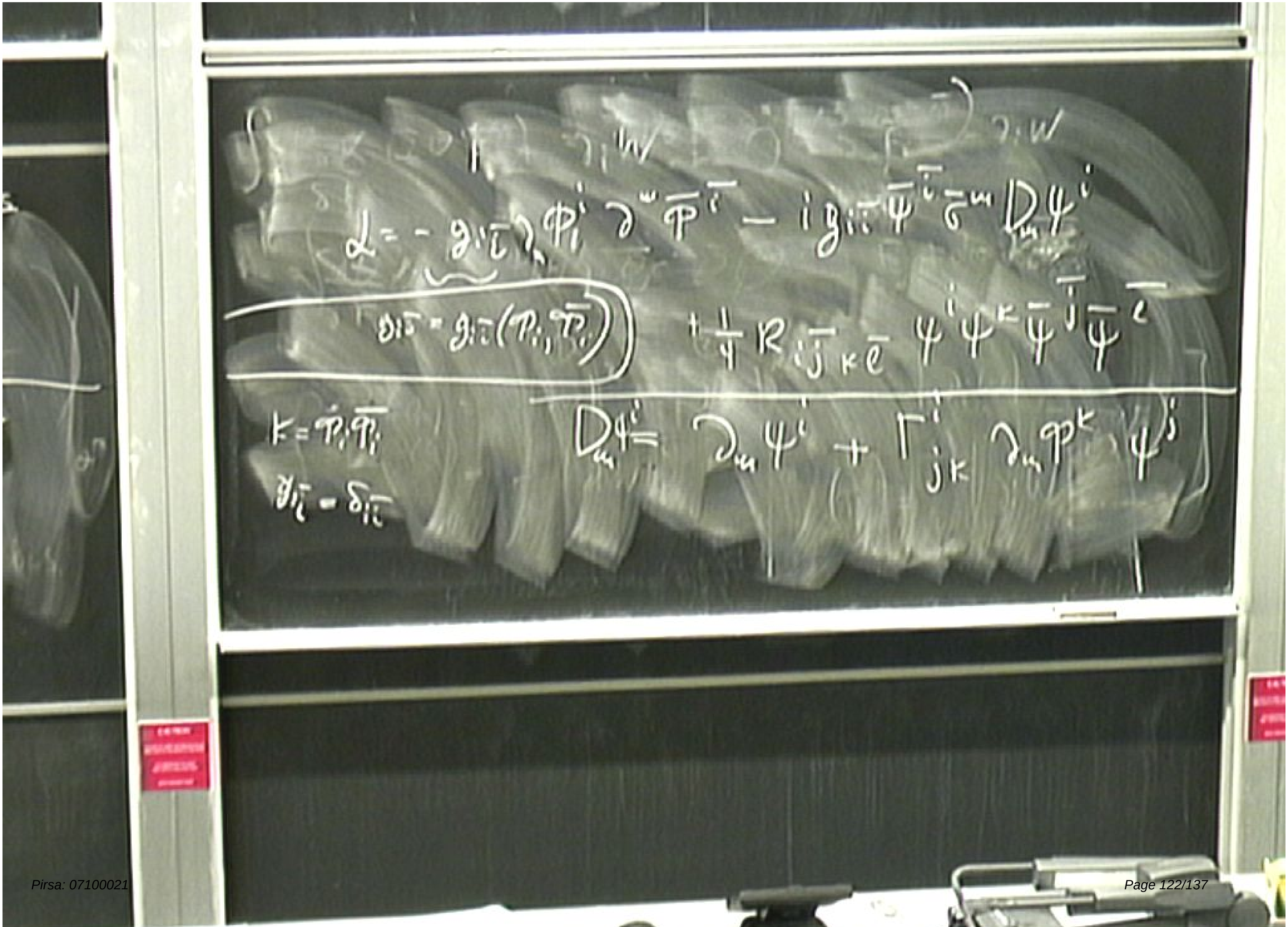
$\partial_i W \qquad) \, \partial_i W$

$$\mathcal{L} = - \underbrace{g_{i\bar{i}}} \partial_m \varphi^i \partial^m \bar{\varphi}^{\bar{i}} - i g_{i\bar{i}} \bar{\psi}^{\bar{i}} \bar{\sigma}^m D_m \psi^i$$

$$\boxed{g_{i\bar{i}} = g_{i\bar{i}}(\varphi_i, \bar{\varphi}_{\bar{i}})}$$

$$+ \frac{1}{4} R_{i\bar{j}k\bar{l}} \psi^i \psi^k \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{l}}$$

$$D_m = \partial_m \psi^i + \Gamma^i_{jk} \partial_m \varphi^k \psi^j$$



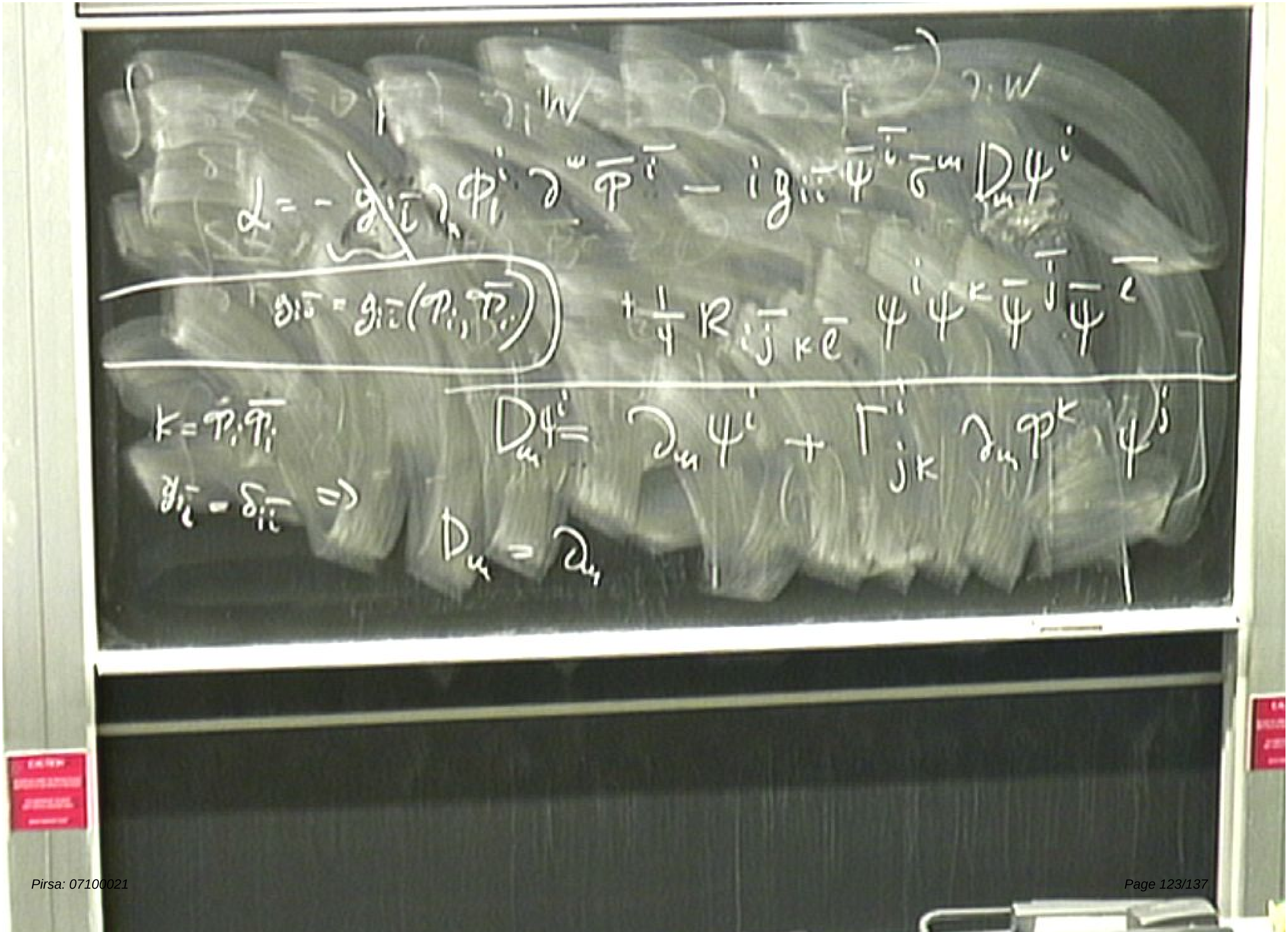
$$\mathcal{L} = -g_{i\bar{i}}\,\partial_m \varphi^i \,\partial^m \bar{\varphi}^{\bar{i}} - i g_{i\bar{i}}\, \bar{\psi}^{\bar{i}} \bar{\sigma}^m D_m \psi^i$$

$$\boxed{g_{i\bar{i}} = g_{i\bar{i}}(\varphi_i, \bar{\varphi}_i)} \qquad + \frac{1}{4} R_{i\bar{j}k\bar{l}}\, \psi^i \psi^k \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{l}}$$

$K = \varphi_i \bar{\varphi}_i$

$g_{i\bar{i}} = \delta_{i\bar{i}}$

$$D_m\psi^i = \partial_m \psi^i + \Gamma^i_{jk}\, \partial_m \varphi^k \,\psi^j$$



$$\mathcal{L} = -\cancel{g_{i\bar{i}}}\,\partial_\mu \varphi^i \partial^\mu \bar{\varphi}^{\bar{i}} - i g_{i\bar{i}} \bar{\psi}^{\bar{i}} \bar{\sigma}^\mu D_\mu \psi^i$$

$$+ \frac{1}{4} R_{i\bar{j}k\bar{l}}\, \psi^i \psi^k \bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{l}}$$

$$\boxed{g_{i\bar{i}} = g_{i\bar{i}}(\varphi_i, \bar{\varphi}_{\bar{i}})}$$

$$K = \varphi_i \bar{\varphi}_i$$

$$D_\mu \psi^i = \partial_\mu \psi^i + \Gamma^i_{jk}\, \partial_\mu \varphi^k\, \psi^j$$

$$g_{i\bar{i}} = \delta_{i\bar{i}} \Rightarrow$$

$$D_\mu = \partial_\mu$$

A special case of nlsms is WZ model

$K = \sum_{i=1}^{n} \Phi_i \overline{\Phi}_i$ and

In order to have (a perturbative SUSY breaking)

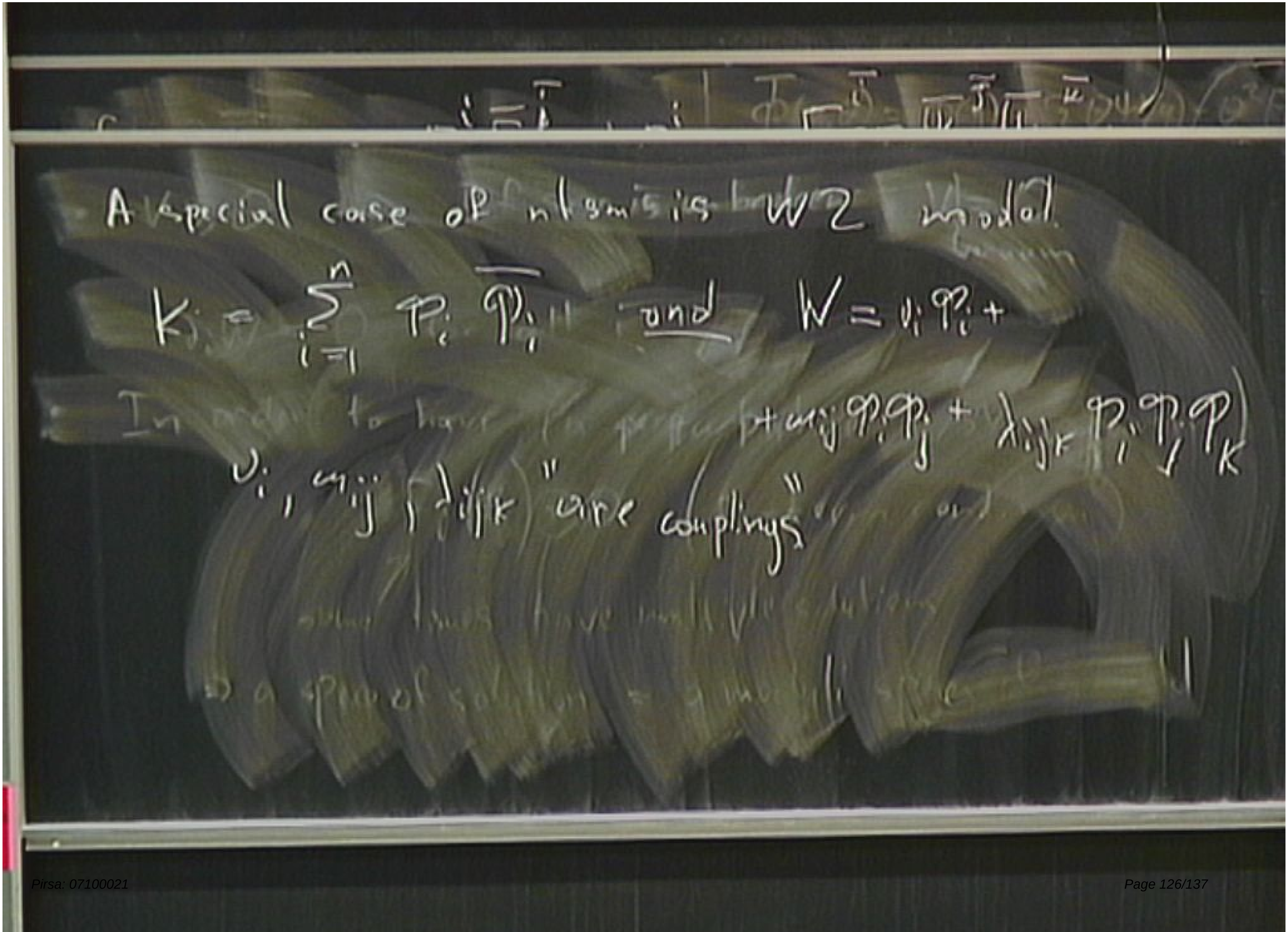
$F_i(\Phi) = 0$ (F-term cond)

these have only the solutions

⇒ a space of solutions = a moduli space of

A special case of nlσm is WZ model

$$K = \sum_{i=1}^{n} \varphi_i \bar{\varphi}_i \quad \underline{\text{and}} \quad W = l_i \varphi_i + m_{ij} \varphi_i \varphi_j + \lambda_{ijk} \varphi_i \varphi_j \varphi_k$$



A special case of nlsm's is WZ model.

$$K = \sum_{i=1}^{n} \varphi_i \bar{\varphi}_i \quad \underline{\text{and}} \quad W = v_i \varphi_i + m_{ij} \varphi_i \varphi_j + \lambda_{ijk} \varphi_i \varphi_j \varphi_k$$

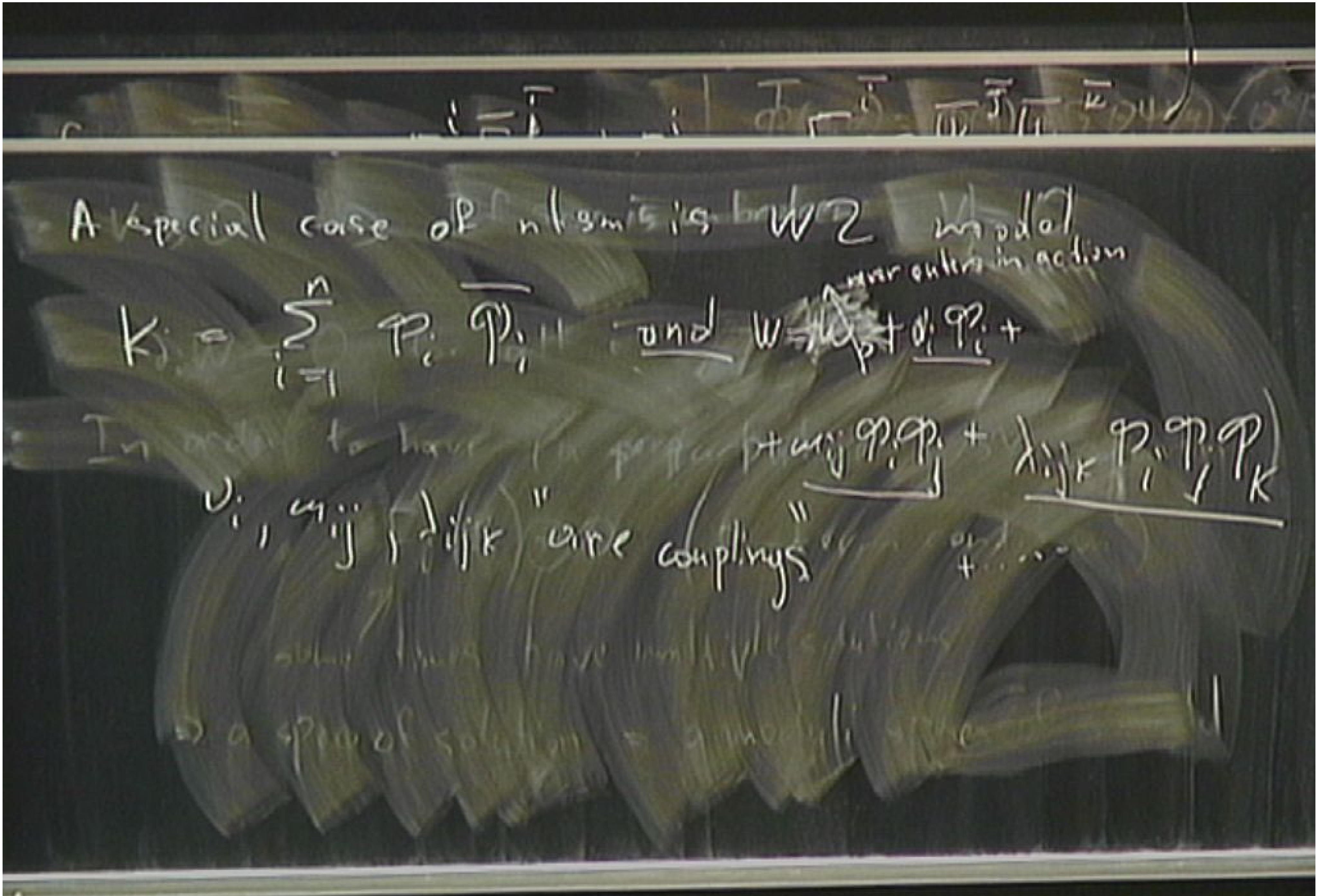
v_i, m_{ij}, λ_{ijk} "are couplings"

A special case of nlsm's is WZ model

renormalis in action

$$K = \sum_{i=1}^{n} \varphi_i \bar{\varphi}_i \quad \underline{\text{and}} \quad W = \sigma_i \varphi_i + m_{ij}\varphi_i\varphi_j + \lambda_{ijk}\varphi_i\varphi_j\varphi_k)$$

σ_i, m_{ij}, λ_{ijk} "are couplings"

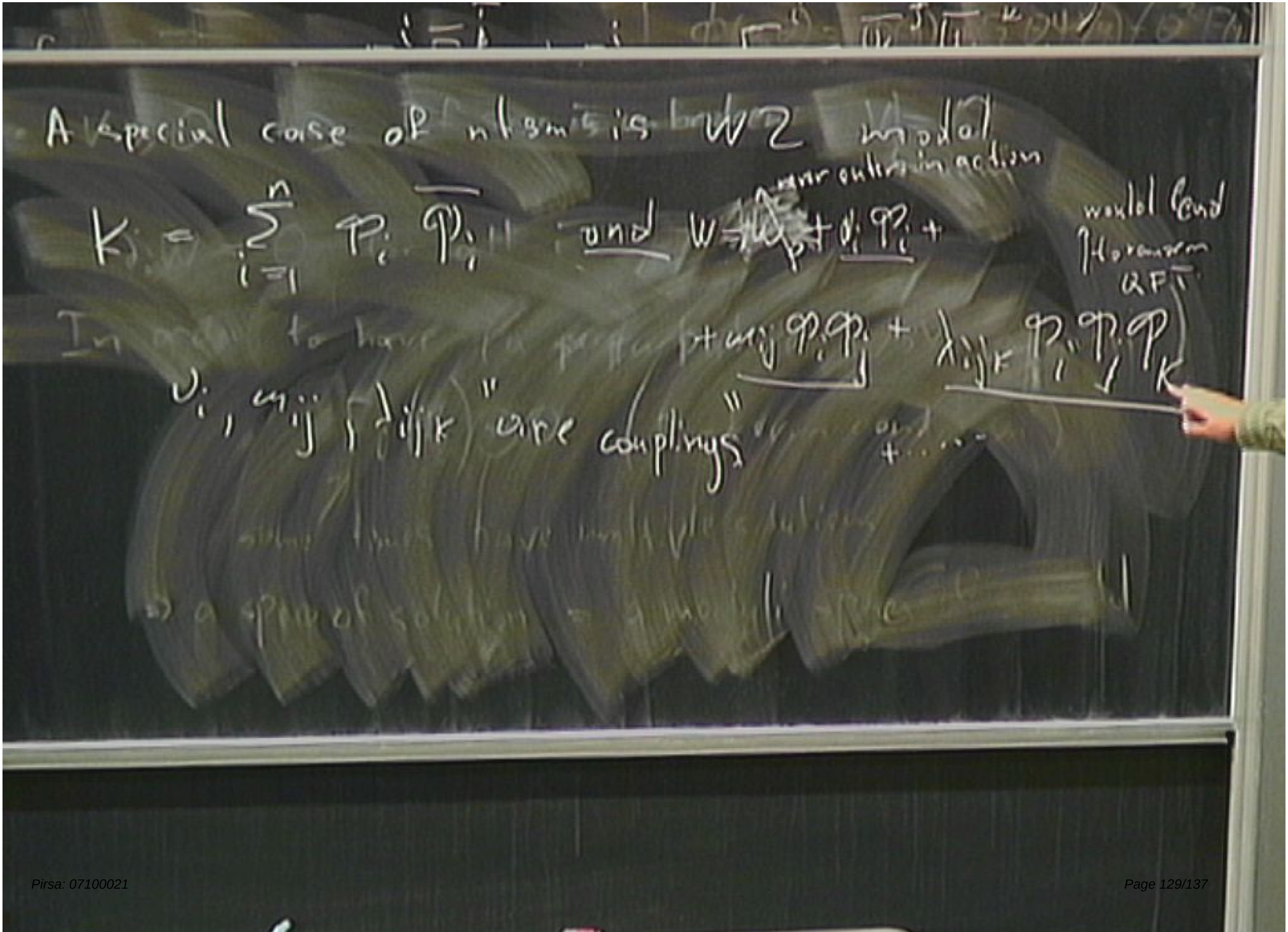


A special case of $nl\sigma m$ is WZ model

[illegible] in action

$$K = \sum_{i=1}^{n} \varphi_i \bar{\varphi}_i \quad \underline{\text{and}} \quad W = \underline{\sigma_i \varphi_i} + \underline{m_{ij} \varphi_i \varphi_j} + \underline{\lambda_{ijk} \varphi_i \varphi_j \varphi_k}$$

$\sigma_i, m_{ij}, \lambda_{ijk}$ "are couplings" + . . .



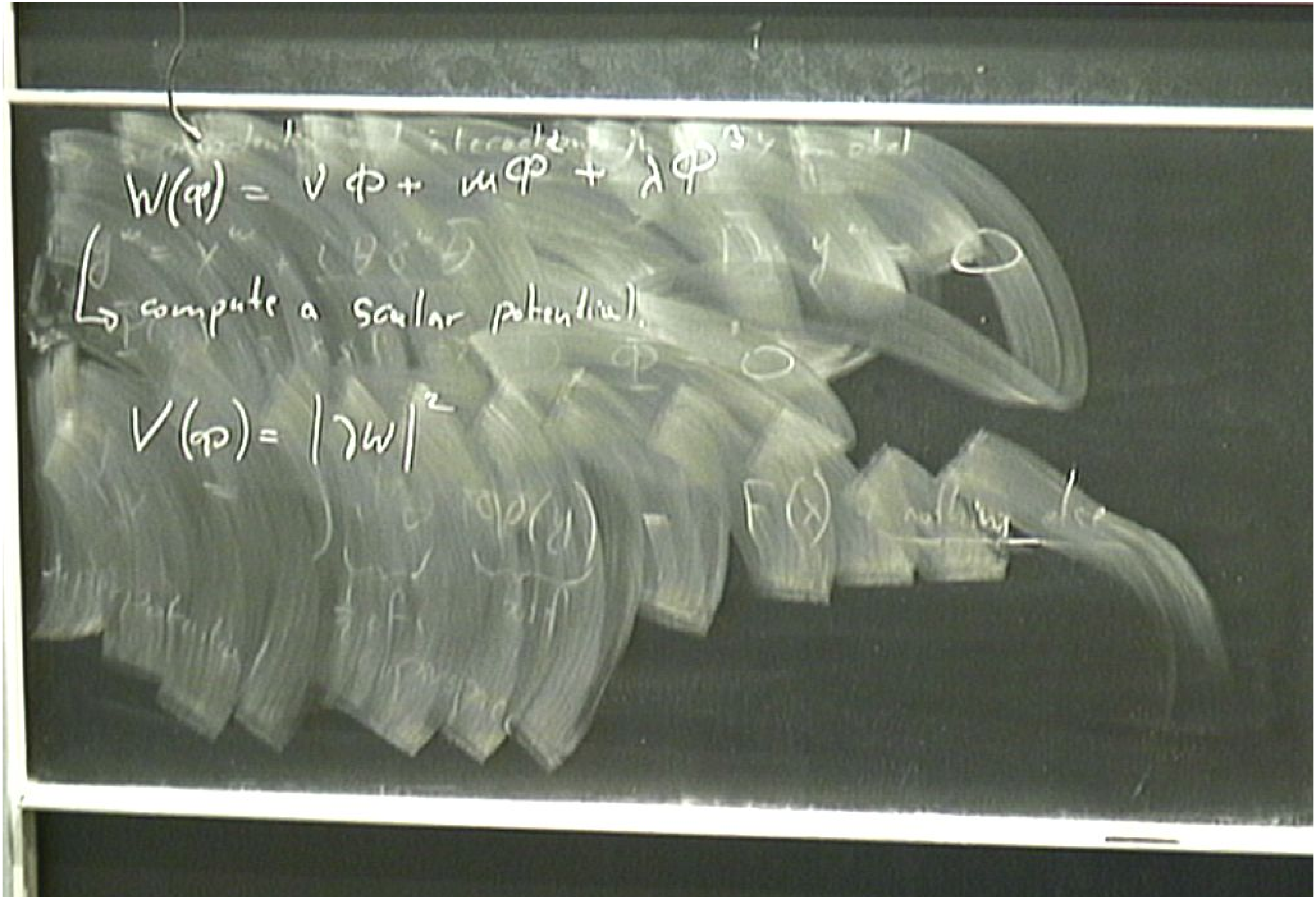
A special case of nlσm is WZ model

[illegible] in action

$$K = \sum_{i=1}^{n} \varphi_i \bar{\varphi}_i \quad \underline{\text{and}} \quad W = \underline{\nu_i \varphi_i} + \underline{m_{ij} \varphi_i \varphi_j} + \underline{\lambda_{ijk} \varphi_i \varphi_j \varphi_k} + \ldots$$

would find [illegible] QFT

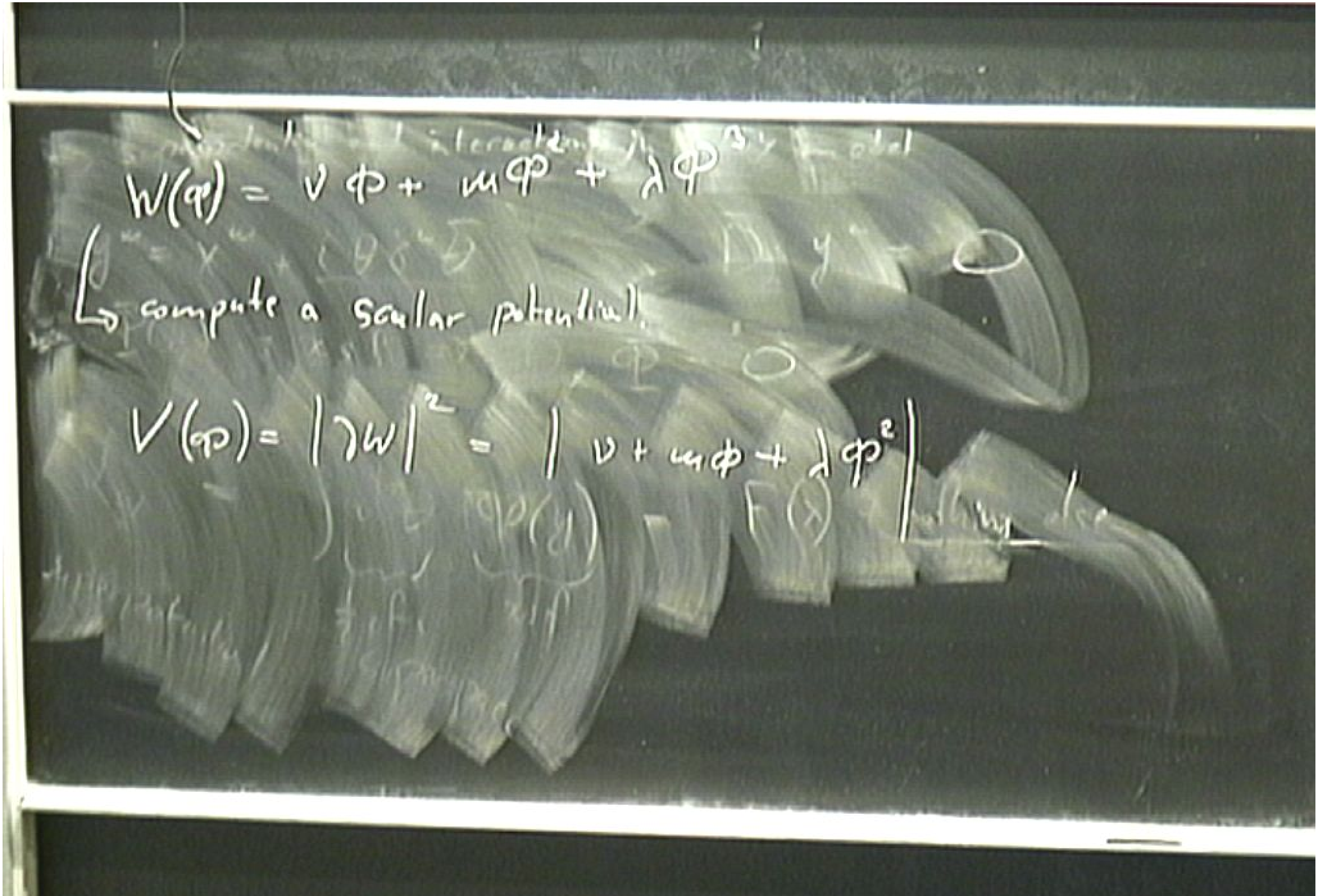
ν_i, m_{ij}, λ_{ijk} "are couplings"



$$W(\varphi) = v\,\Phi + m\Phi^2 + \lambda\Phi^3$$

↳ compute a scalar potential.

$$V(\varphi) = |\partial W|^2$$



$$W(\varphi) = v\,\Phi + m\Phi + \lambda\Phi$$

↳ compute a scalar potential!

$$V(\varphi) = |\partial W|^2 = |\, v + m\phi + \lambda\phi^2 \,|$$

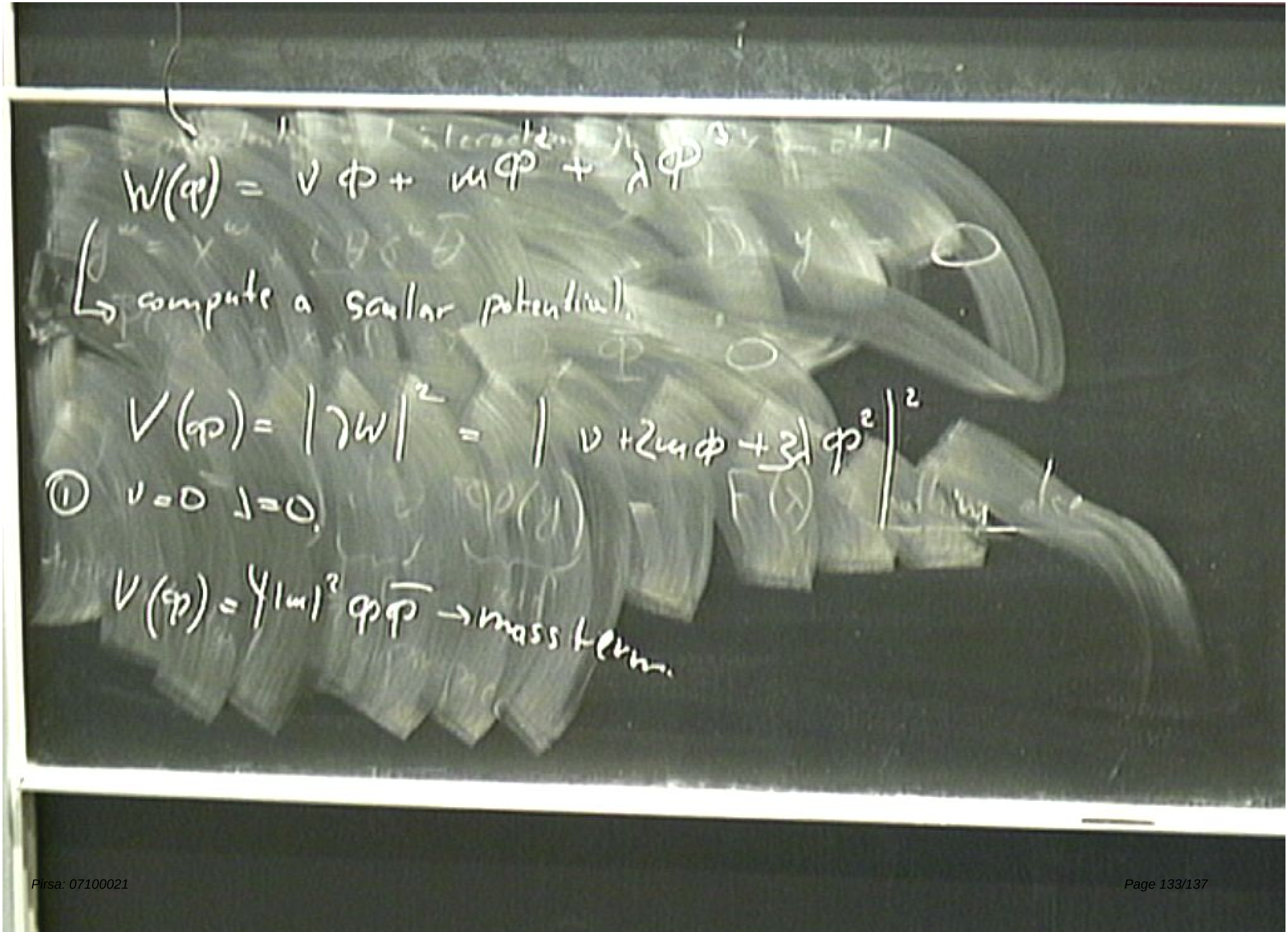
$$W(\varphi) = v\varphi + m\varphi^2 + \lambda\varphi^3$$

↳ compute a scalar potential.

$$V(\varphi) = |\partial W|^2 = |v + m\varphi + \lambda\varphi^2|^2$$

① $v=0$ $\lambda=0$.

$$V(\varphi) = |m|^2\varphi\bar{\varphi}$$



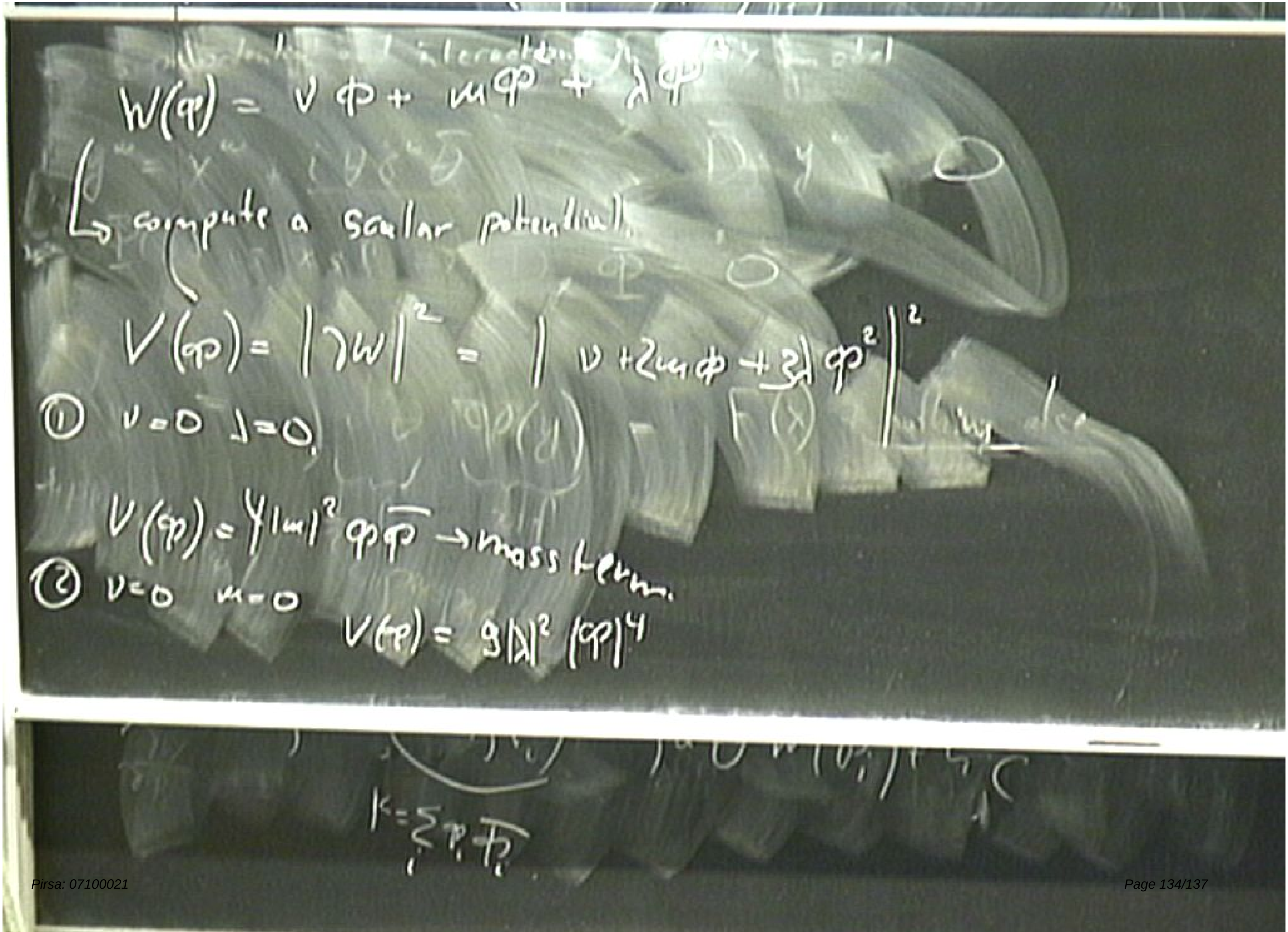
$$W(\varphi) = v\varphi + m\varphi + \lambda\varphi$$

↳ compute a scalar potential.

$$V(\varphi) = |\partial W|^2 = |v + 2m\varphi + 3\lambda\varphi^2|^2$$

① $v=0$ $\lambda=0$.

$V(\varphi) = 4|m|^2 \varphi\bar{\varphi}$ → mass term.



$$W(\varphi) = v\,\varphi + m\varphi^2 + \lambda\varphi^3$$

↳ compute a scalar potential.

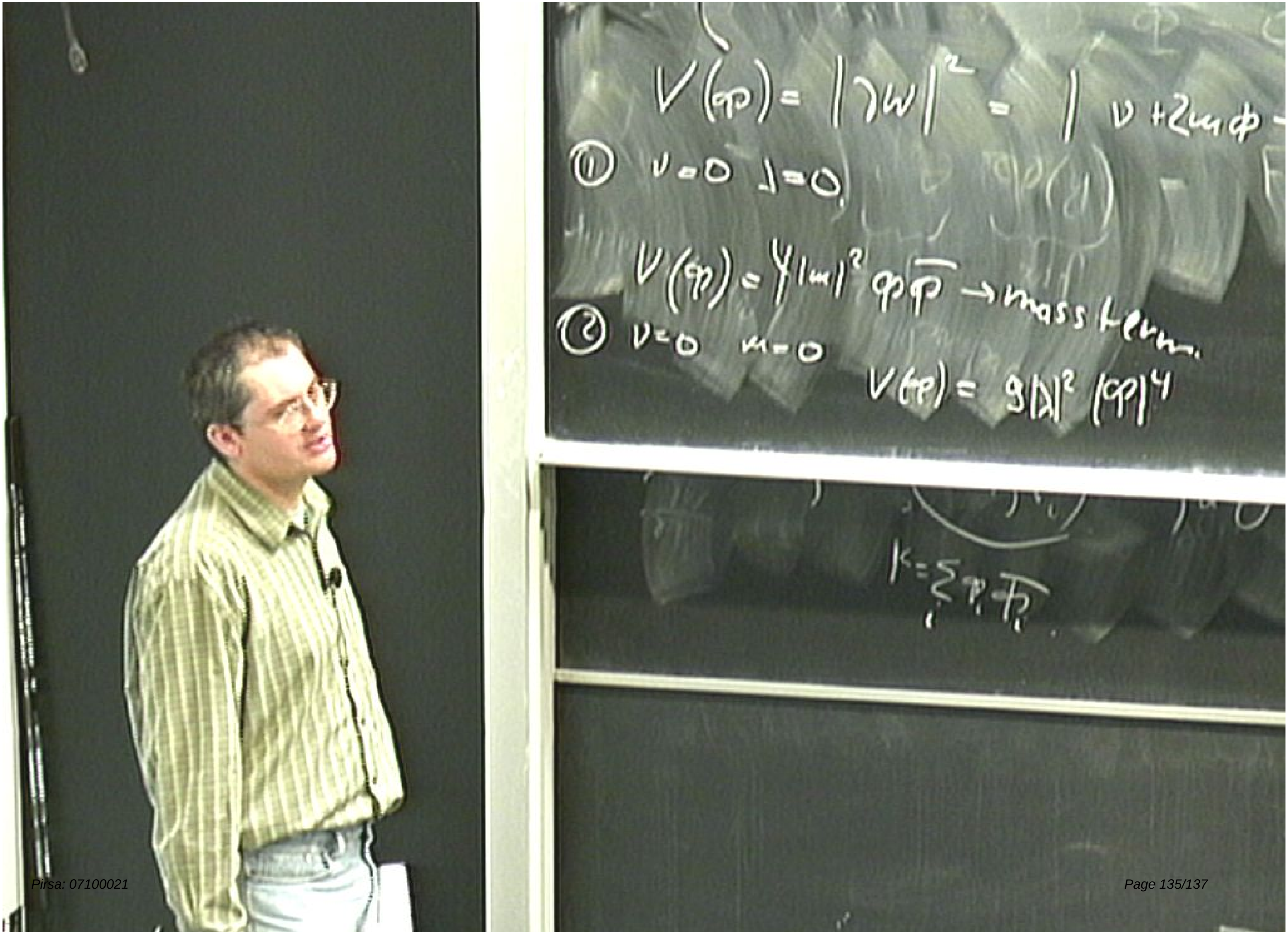
$$V(\varphi) = |\partial W|^2 = |\, v + 2m\varphi + 3\lambda\varphi^2 |^2$$

① $v = 0 \quad \lambda = 0$

$$V(\varphi) = 4|m|^2 \varphi\bar{\varphi} \rightarrow \text{mass term.}$$

② $v = 0 \quad m = 0 \qquad V(\varphi) = 9|\lambda|^2 |\varphi|^4$

$$K = \sum_i \varphi_i \bar{\varphi}_i$$



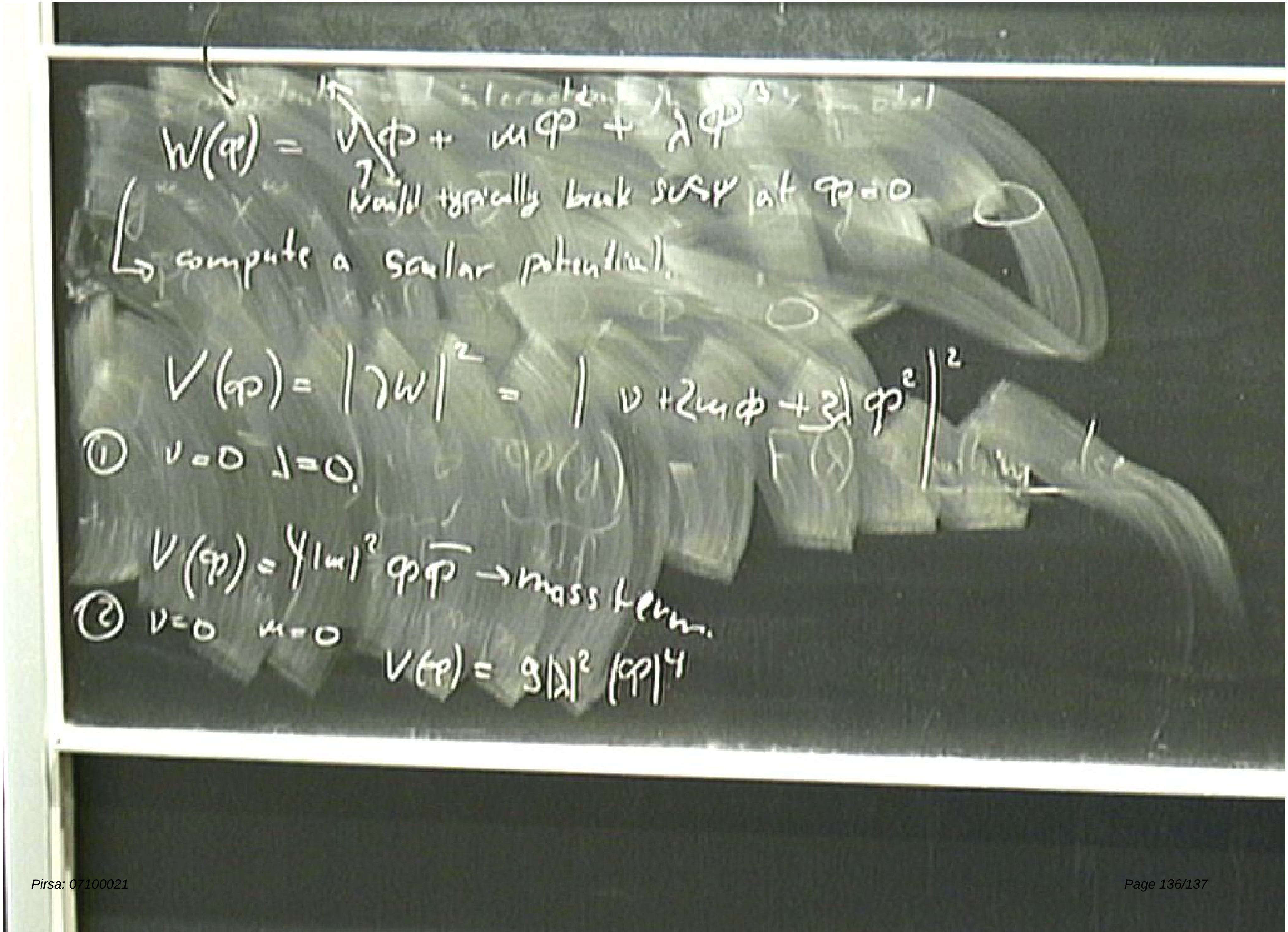
$$V(\varphi) = |\partial W|^2 = | \; v + 2m\phi$$

① $v=0 \quad \lambda=0$

$$V(\varphi) = 4|m|^2 \varphi\bar{\varphi} \rightarrow \text{mass term.}$$

② $v=0 \quad m=0 \qquad V(\varphi) = 9|\lambda|^2 |\varphi|^4$

$$K = \sum_i \varphi_i \bar{\varphi}_i \, .$$



$W(\varphi) = v\varphi + m\varphi^2 + \lambda\varphi^3$

would typically break SUSY at $\varphi = 0$

↳ compute a scalar potential.

$V(\varphi) = |\partial W|^2 = | v + 2m\varphi + 3\lambda\varphi^2 |^2$

① $v = 0$ $\lambda = 0$.

$V(\varphi) = 4|m|^2 \varphi\bar{\varphi}$ → mass term.

② $v = 0$ $m = 0$ $V(\varphi) = 9|\lambda|^2 |\varphi|^4$

$\Rightarrow$ v can be absorbed by a field redefinition $(\lambda = 0)$.

$$W = v\phi + m^{\cancel{2}}\phi^2 = \underbrace{m\left(\phi + \frac{v}{2m}\right)^2}_{\tilde{\phi}} - \frac{v^2}{4m}$$