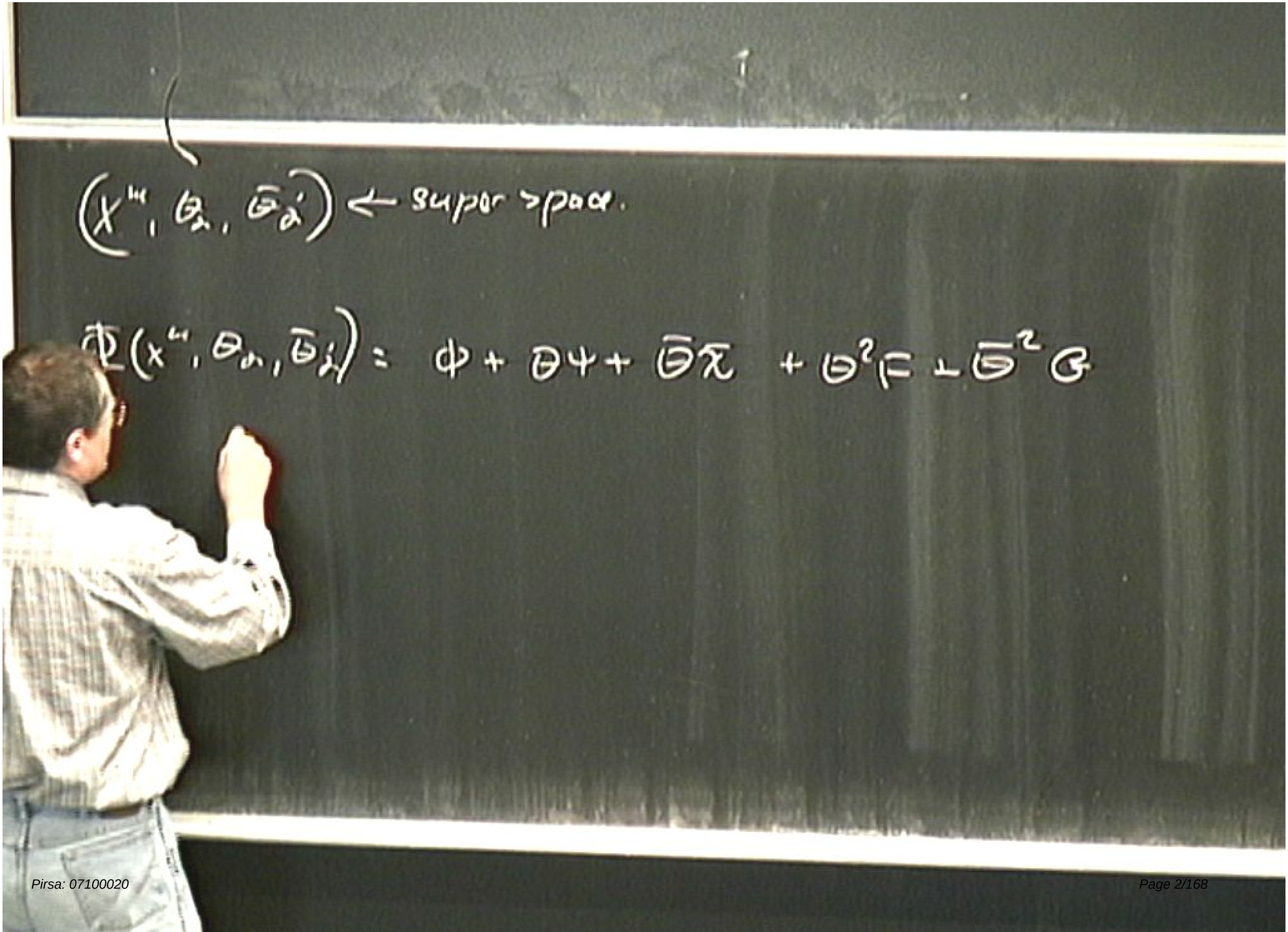
Title: Intro to Supersymmetry 10

Date: Oct 18, 2007 12:30 PM

URL: http://pirsa.org/07100020

Abstract:


$(x^\mu, \theta_\alpha, \bar\theta_{\dot\alpha}) \leftarrow$ super space.
$\Phi(x^\mu, \theta_\alpha, \bar\theta_{\dot\alpha}) = \phi + \theta\psi + \bar\theta\bar\chi + \theta^2 F + \bar\theta^2 G$

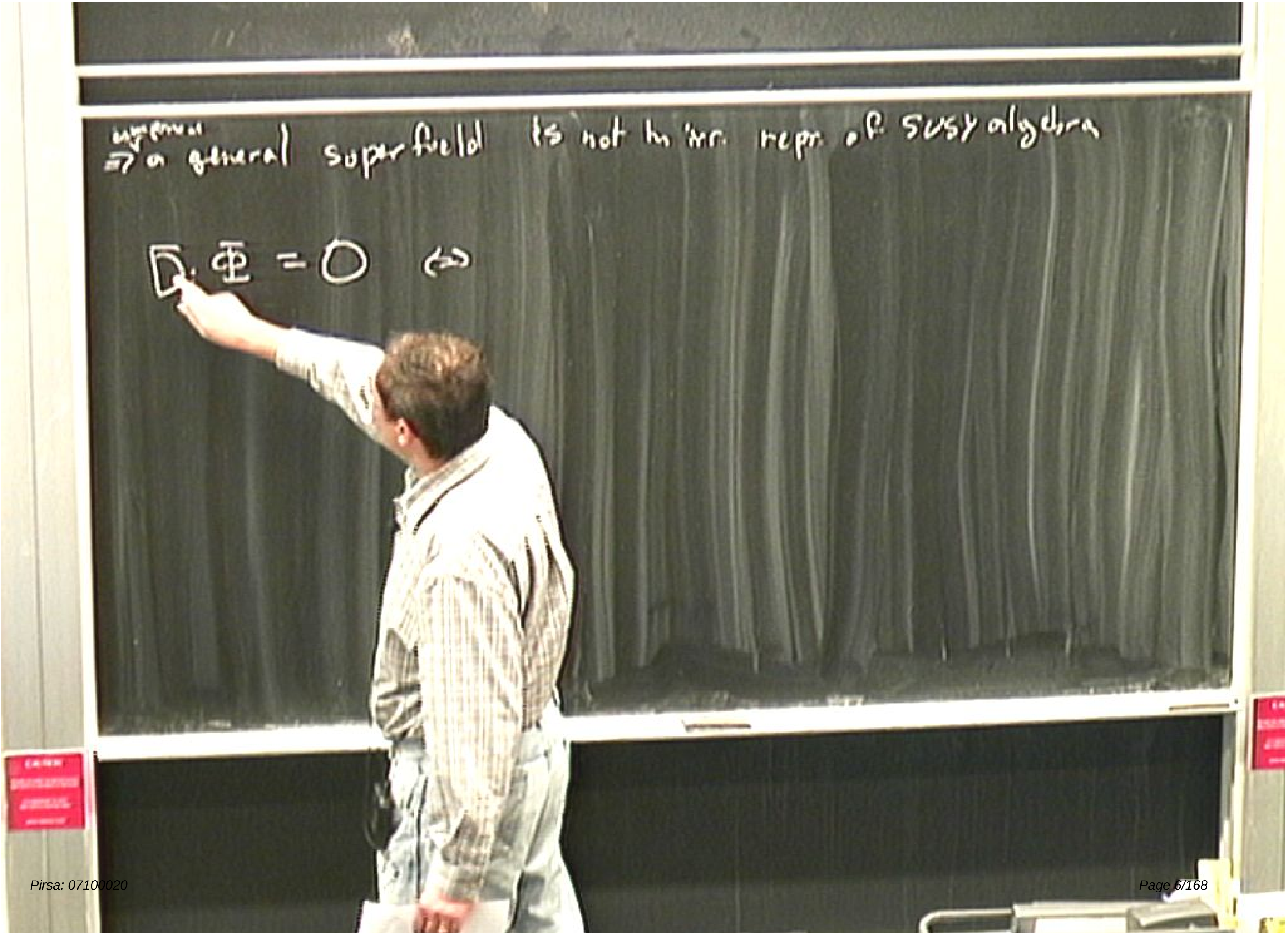
$(x^\mu, \theta_\alpha, \bar\theta_{\dot\alpha}) \leftarrow$ super space.

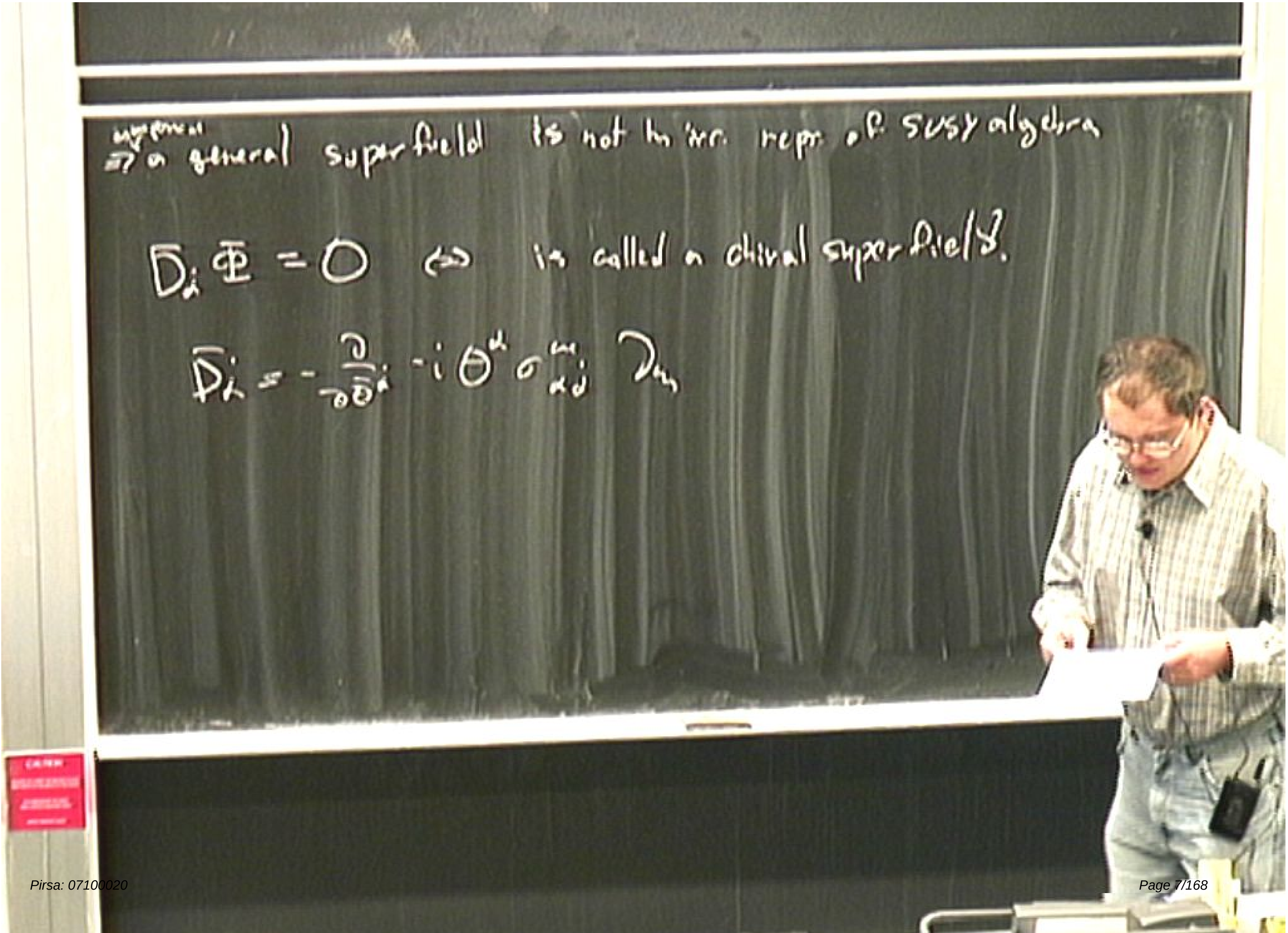
$$\Phi(x^\mu, \theta_\alpha, \bar\theta_{\dot\alpha}) = \phi + \theta\psi + \bar\theta\bar\chi + \theta^2 F + \bar\theta^2 G$$
$$+ \theta\sigma^\mu\bar\theta A_\mu + \theta^2\bar\theta\bar\lambda + \bar\theta^2\theta\rho + \theta^2\bar\theta^2 D$$

$\phi, \psi, \bar\chi, F, G, A_\mu, \lambda, \rho, D$ are fields

$\Rightarrow$ a general superfield is not an irr.

in general
$\Rightarrow$ a general superfield is not an irr. repr. of SUSY algebra


⇒ a general superfield is not an irr. repr. of susy algebra
D Φ = 0 ⟺


in general
⇒ a general superfield is not an irr. repr. of SUSY algebra
$\bar{D}_{\dot\alpha}\Phi = 0 \Leftrightarrow$ is called a chiral superfield.
$\bar{D}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot\alpha}\partial_{\mu}$

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$\bar{D}_{\dot\alpha} = -\frac{\partial}{\partial \bar\theta^{\dot\alpha}} - i \theta^{\alpha} \sigma^{m}_{\alpha\dot\alpha} \partial_m$

$\Rightarrow$ we look at "chir[illegible] [illegible]inates"

in general
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$\Rightarrow$ we looked at "chiral coordinates"

$\bar{D}_{\dot\alpha} \theta_\beta = 0 \qquad \bar{D}_{\dot\alpha} y^m = 0$

$\Rightarrow$ ~~a general~~ any general superfield is not an irr. repr. of SUSY algebra

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$\bar{D}_{\dot\alpha} \theta_\beta = 0 \qquad \bar{D}_{\dot\alpha} y^m = 0 \qquad y^m = x^m + i\theta\sigma^m\bar\theta$

in general
$\Rightarrow$ a general superfield is not an irr. repr. of SUSY algebra

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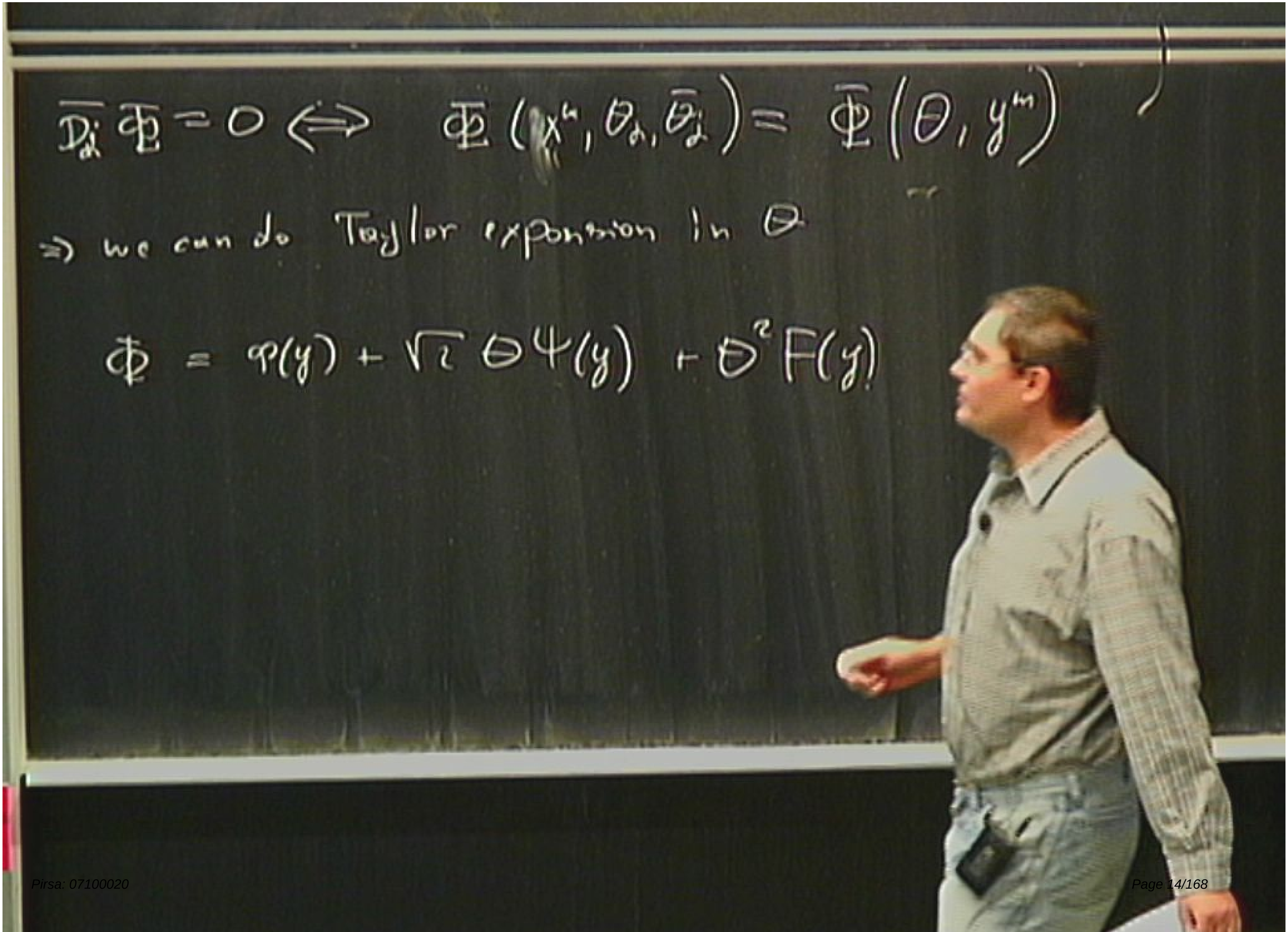
$\bar{D}_{\dot\alpha} \theta_\beta = 0 \qquad \bar{D}_{\dot\alpha} y^m = 0 \qquad y^m = x^m + i\theta\sigma^m\bar\theta$

$$\bar{D}_{\dot\alpha} \Phi = 0 \Longleftrightarrow \Phi(x^\mu, \theta_\alpha, \bar\theta_{\dot\alpha}) = \Phi(\theta, y^m)$$

$$\bar{D}_{\dot\alpha} \Phi = 0 \iff \Phi(x^\mu, \theta_\alpha, \bar\theta_{\dot\alpha}) = \Phi(\theta, y^m)$$

$\Rightarrow$ we can do Taylor expansion in $\theta$

$\Phi$


$\bar{D}_{\dot{\alpha}}\Phi = 0 \Longleftrightarrow \Phi(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}) = \Phi(\theta, y^m)$
⇒ we can do Taylor expansion in $\theta$
$\Phi = \varphi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$

$$\bar{D}_{\dot\alpha}\Phi = 0 \Longleftrightarrow \Phi(x^\mu, \theta_\alpha, \bar\theta_{\dot\alpha}) = \Phi(\theta, y^m)$$

$\Rightarrow$ we can do Taylor expansion in $\theta$

$$\Phi = \varphi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$$

a complex scalar

a Weyl fermion

$\bar{D}_{\dot{\alpha}} \Phi = 0 \iff \Phi(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}) = \Phi(\theta, y^m)$

$\Rightarrow$ we can do Taylor expansion in $\theta$

$$\Phi = \underbrace{\varphi(y)}_{\text{a complex scalar}} + \sqrt{2}\,\theta \underbrace{\psi(y)}_{\text{a Weyl fermion}} + \theta^2 \underbrace{F(y)}_{\text{a complex scalar (nondynamical)}}$$

$$\bar{D}_{\dot\alpha} \Phi = 0 \iff \Phi(x^\mu, \theta_\alpha, \bar\theta_{\dot\alpha}) = \Phi(\theta, y^m)$$

$\Rightarrow$ we can do Taylor expansion in $\theta$

$$\Phi = \varphi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$$

a complex scalar

a Weyl fermion

a complex scalar (nondynamical)

$\delta_{SUSY}$ ~~$\varphi$~~

$$\bar{D}_{\dot\alpha}\Phi = 0 \iff \Phi(x^\mu, \theta_\alpha, \bar\theta_{\dot\alpha}) = \Phi(\theta, y^\mu)$$

$\Rightarrow$ we can do Taylor expansion in $\theta$

$$\Phi = \varphi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$$

$\varphi$: a complex scalar

$\psi$: a Weyl fermion

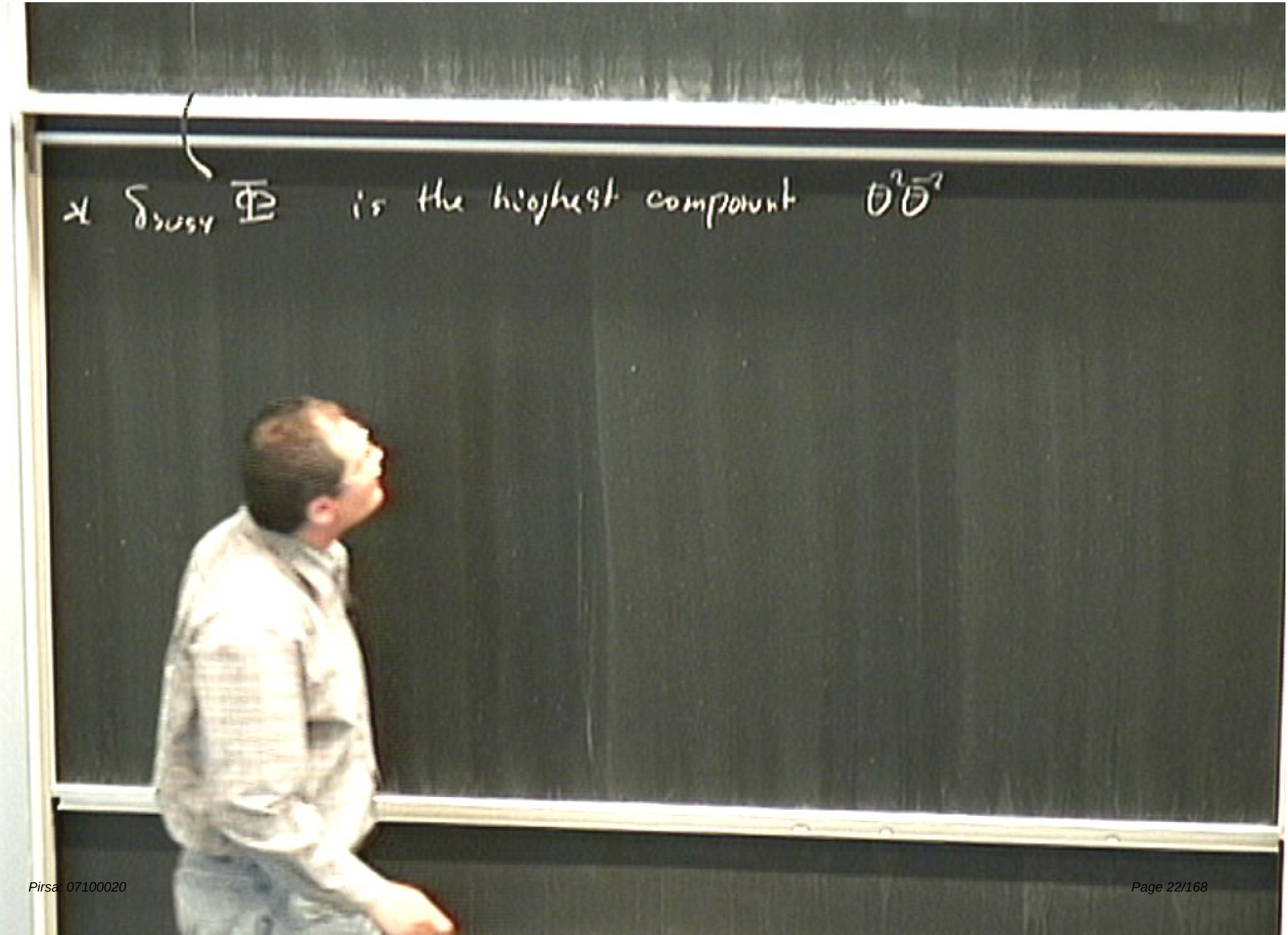
$F$: a complex scalar (nondynamical)

If $\bar{D}_{\dot\alpha}\Phi = 0 \Rightarrow \bar{D}_{\dot\alpha}\delta_{susy}\Phi = 0$

$\delta_{SUSY}\, \Phi$

$\delta_{susy}\ \bar{\Phi}$

$\delta_{susy}\, \Phi$ is the highest component


* $\delta_{susy}\bar{\Phi}$ is the highest component $\theta^2\bar{\theta}^2$

* $\delta_{susy}\, \Phi$ is the highest component $\theta^2\bar\theta^2$

$\Downarrow$

* the highest component of $\chi$sf is a total derivative

$\Downarrow$

* $\delta_{SUSY} \Phi$ is the highest component $\theta^2\bar{\theta}^2$

$\Downarrow$

* the highest component of xsf is a total derivative

$\Downarrow$

$$\mathcal{L}_{SUSY} = \int d^2\theta \, d^2\bar{\theta} \; \Phi$$

$\Downarrow$

* the highest component of xsf is a total derivative

$\Downarrow$

$$\mathcal{L}_{susy} = \int d^2\theta \, d^2\bar{\theta} \, \bar{\Phi}\Phi$$

$\delta_{susy}\mathcal{L}_{susy}$ = total derivative.

$S = \int_{M_4} d^4x \, \mathcal{L}_{susy}$ is $\delta_{susy}$ invariant

* $\delta_{susy}\Phi$ is the highest component $\theta^2\bar\theta^2$

$\Downarrow$

* the highest component of xsf is a total derivative

$$\mathcal{L}_{susy} = \int d^2\theta\, d^2\bar\theta\, \Phi$$

$$\delta_{susy}\mathcal{L}_{susy} = \text{total derivative}$$

$$S = \int_{M_4} d^4x\, \mathcal{L}_{susy} \text{ is } \delta_{susy} \text{ invariant}$$

* $\delta_{susy}\,\Phi$ is the highest component $\theta^2\bar\theta^2$

$\Downarrow$

* the highest component of xsf is a total derivative

$\Downarrow$

$$\mathcal{L}_{susy} = \int d^2\theta\, d^2\bar\theta\, \Phi$$

$\Downarrow$

$$\delta_{susy}\mathcal{L}_{susy} = \text{total derivative}$$

$$S = \int_{M_4} d^4x\, \mathcal{L}_{susy} \text{ is } \delta_{susy} \text{ invariant}$$

* $\delta_{susy}\,\Phi \Rightarrow$ concentrate on highest component $\theta^2\bar\theta^2$

$\Downarrow$

* the highest component of vsf is a total derivative

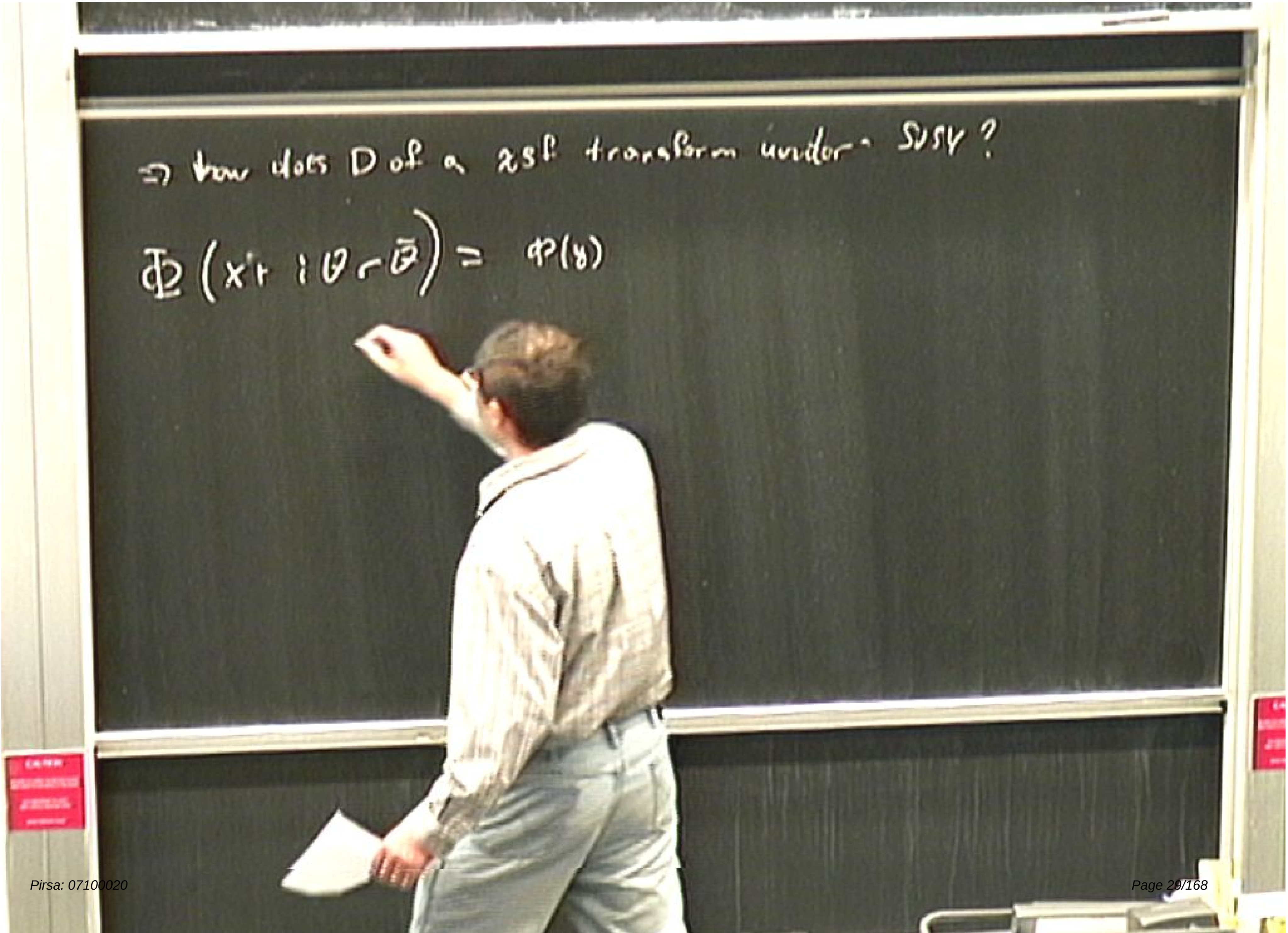
$\Downarrow$

$$\mathcal{L}_{susy} = \int d^2\theta\, d^2\bar\theta\, \Phi$$

$\Downarrow$

$$\delta_{susy}\mathcal{L}_{susy} = \text{total derivative}$$

$$S = \int_{M_4} d^4x\, \mathcal{L}_{susy} \text{ is } \delta_{susy} \text{ invariant}$$


⇒ how does D of a χSF transform under SUSY?
$\Phi(x^\mu + i\theta\sigma^\mu\bar{\theta}) = \Phi(y)$

⇒ how does D of a χSF transform under SUSY ?

$$\Phi(\underbrace{x + i\theta\sigma\bar{\theta}}_{y}) = \phi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$$

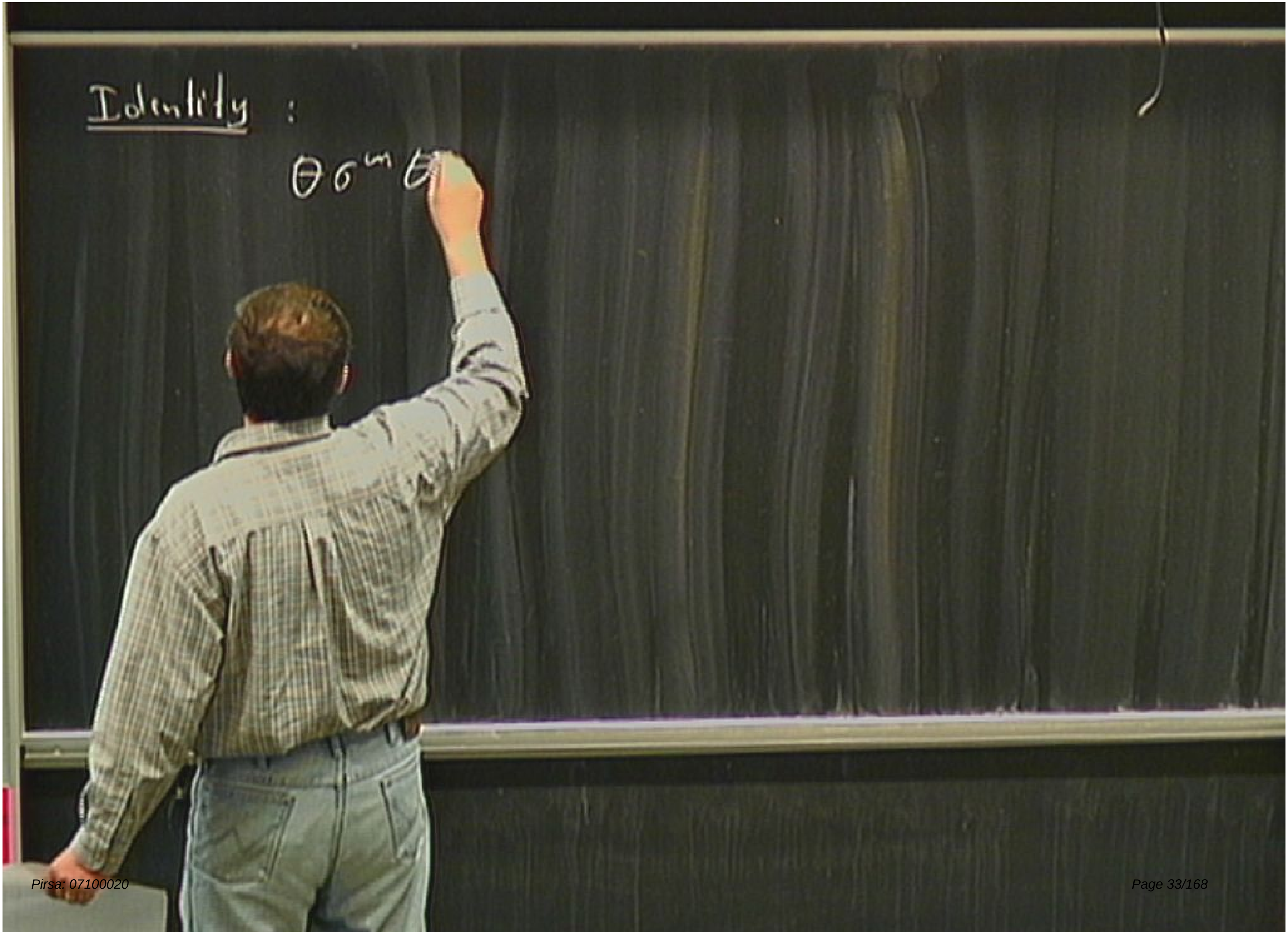
$$\Phi(y) = \phi(x) + i\theta\sigma^m\bar{\theta}\,\partial_m\phi$$

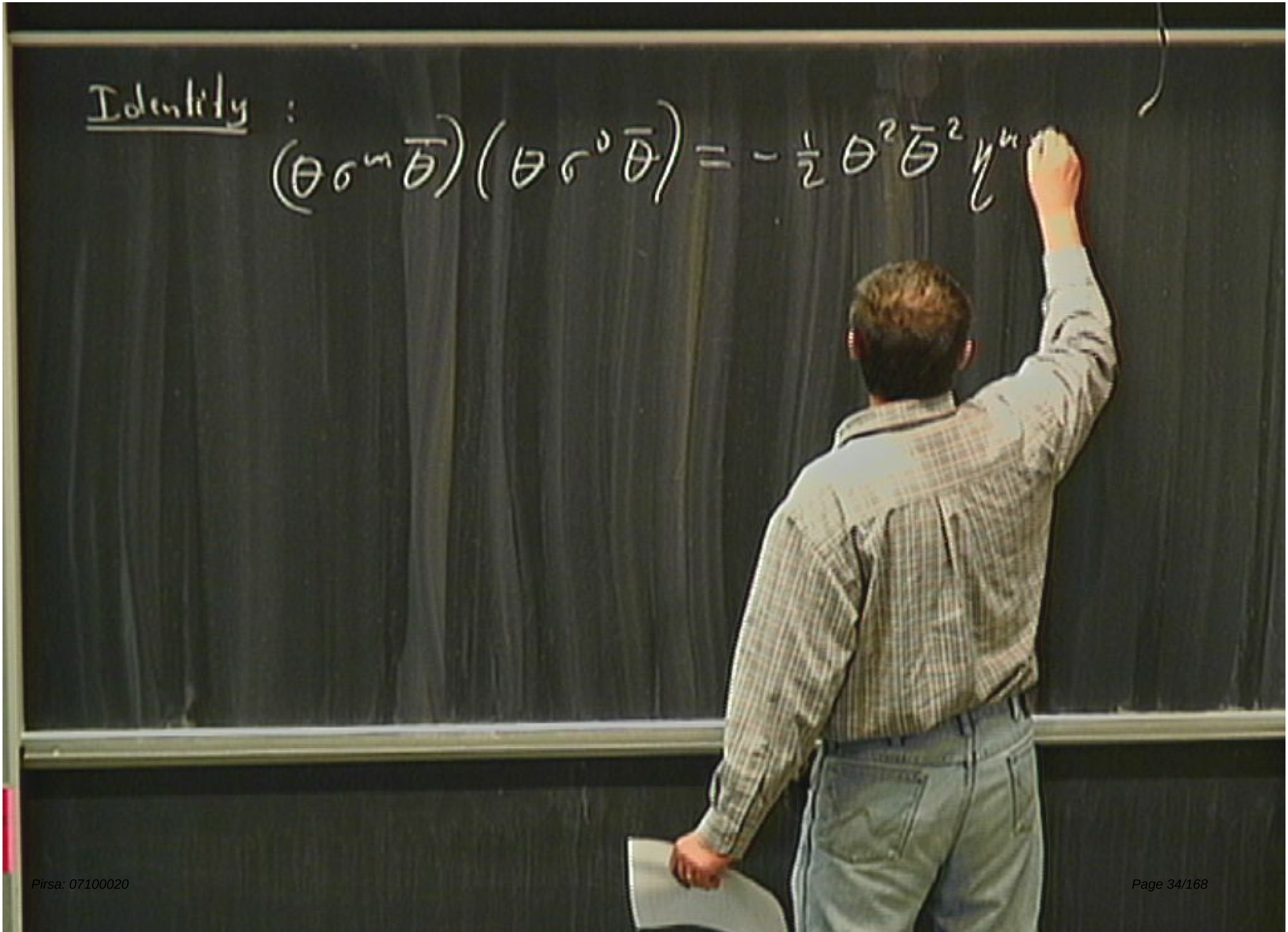
⇒ how does D of a χSF transform under SUSY?

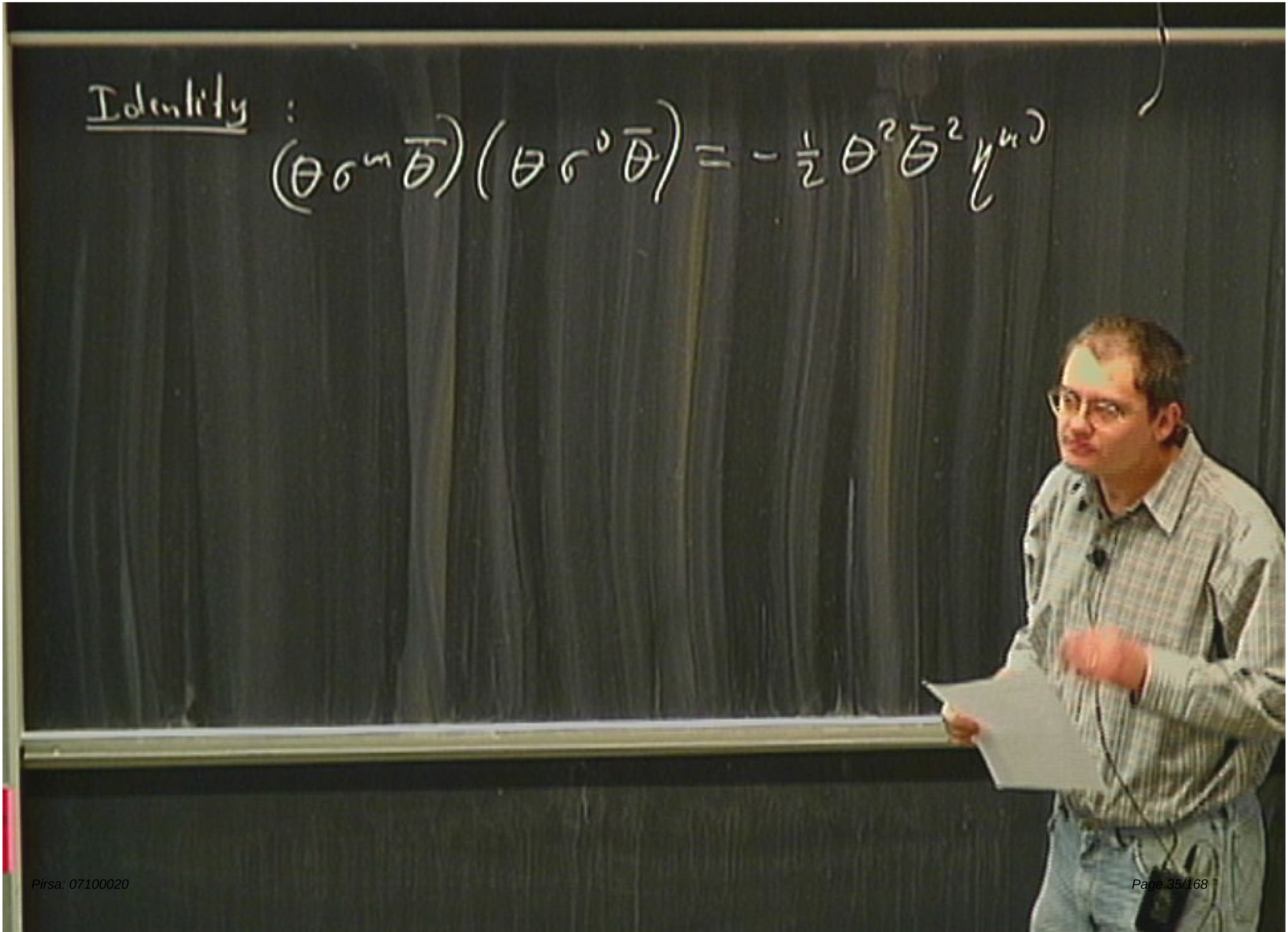
$$\Phi(\underbrace{x + i\theta\sigma\bar\theta}_{y}) = \phi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$$

$$\Phi(y) = \phi(x) + i\theta\sigma^\mu\bar\theta\,\partial_\mu\phi + \frac{1}{2!}(i\theta\sigma^\mu\bar\theta)(i\theta\sigma^\nu\bar\theta)\partial_\mu\partial_\nu\Phi$$

$x + i\theta\sigma\bar\theta$


Identity :
$\theta \sigma^m \theta$


Identity :
$(\theta\sigma^m\bar{\theta})(\theta\sigma^n\bar{\theta}) = -\frac{1}{2}\theta^2\bar{\theta}^2\eta^{mn}$


Identity :
$(\theta\sigma^{\mu}\bar{\theta})(\theta\sigma^{\nu}\bar{\theta}) = -\frac{1}{2}\theta^2\bar{\theta}^2\eta^{\mu\nu}$

$\Rightarrow$ how does D of a $\chi$SF transform under a SUSY?

$$\Phi(\underbrace{x + i\theta\sigma\bar{\theta}}_{y}, \theta) = \phi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y).$$

$$\Phi(y) = \phi(x) + i\theta\sigma^\mu\bar{\theta}\,\partial_\mu\phi + \underbrace{\frac{1}{2!}\left(i\theta\sigma^\mu\bar{\theta}\right)\left(i\theta\sigma^\nu\bar{\theta}\right)\partial_\mu\partial_\nu\Phi}_{-\frac{1}{2}\cdot\left(-\frac{1}{2}\right)\theta^2\bar{\theta}^2 g^{\mu\nu}\partial_\mu\partial_\nu\phi}$$

$\checkmark$ $x + i\theta\sigma\bar{\theta}$

$\Rightarrow$ how does D of a χSF transform under SUSY?

$$\Phi(\underbrace{x + i\theta\sigma\bar\theta}_{y}, \theta, \bar\theta) = \phi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$$

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$y = x + i\theta\sigma\bar\theta$

$$+\frac{1}{4}\theta^2\bar\theta^2\,\Box\Phi$$

$\Rightarrow$ how does D of a $\chi$SF transform under SUSY?

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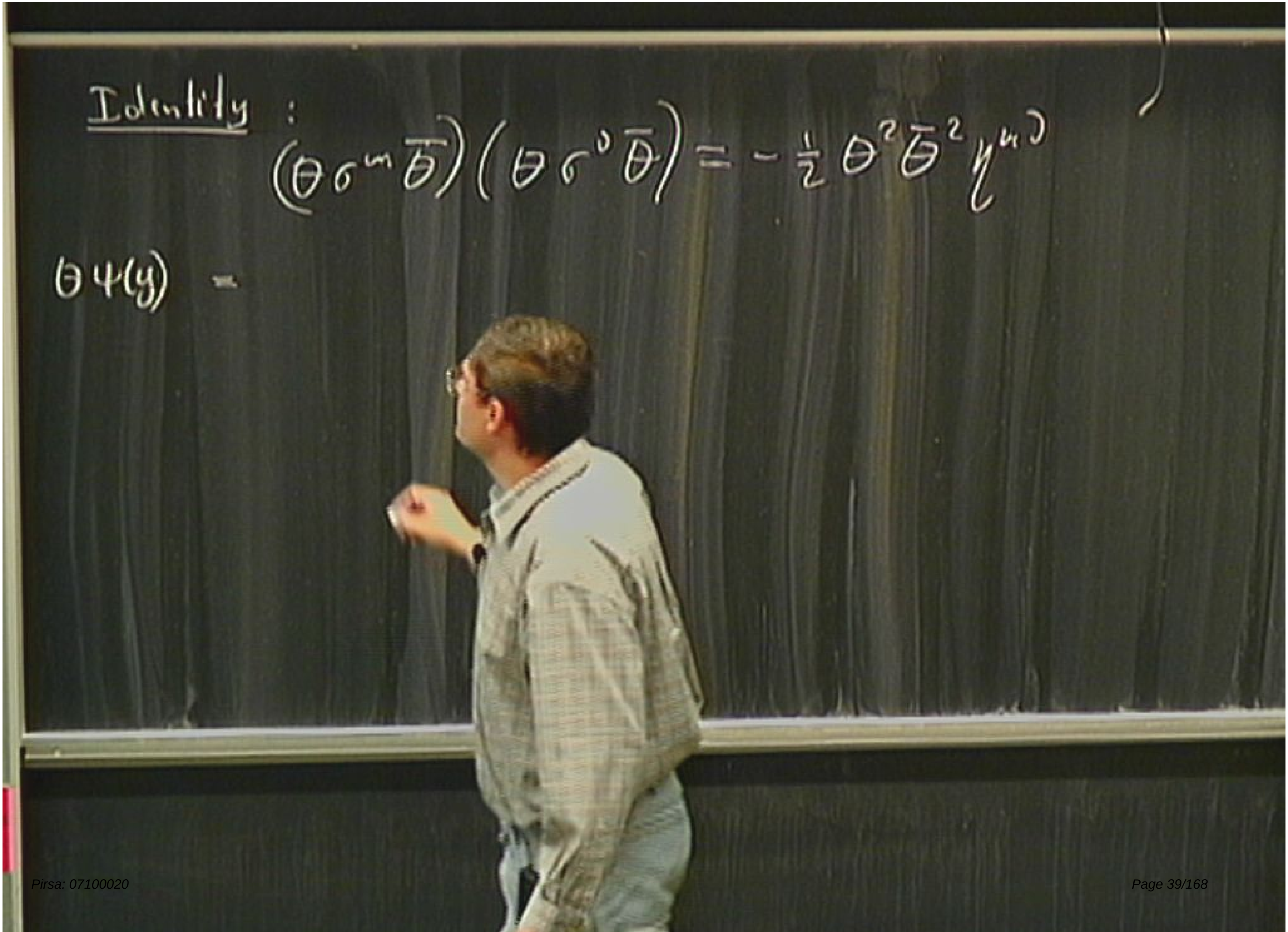
$$+\frac{1}{4}\theta^2\bar\theta^2\,\Box\Phi$$


Identity :
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$\theta\psi(y) =$

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$x + \underline{i\theta\sigma\bar\theta}$

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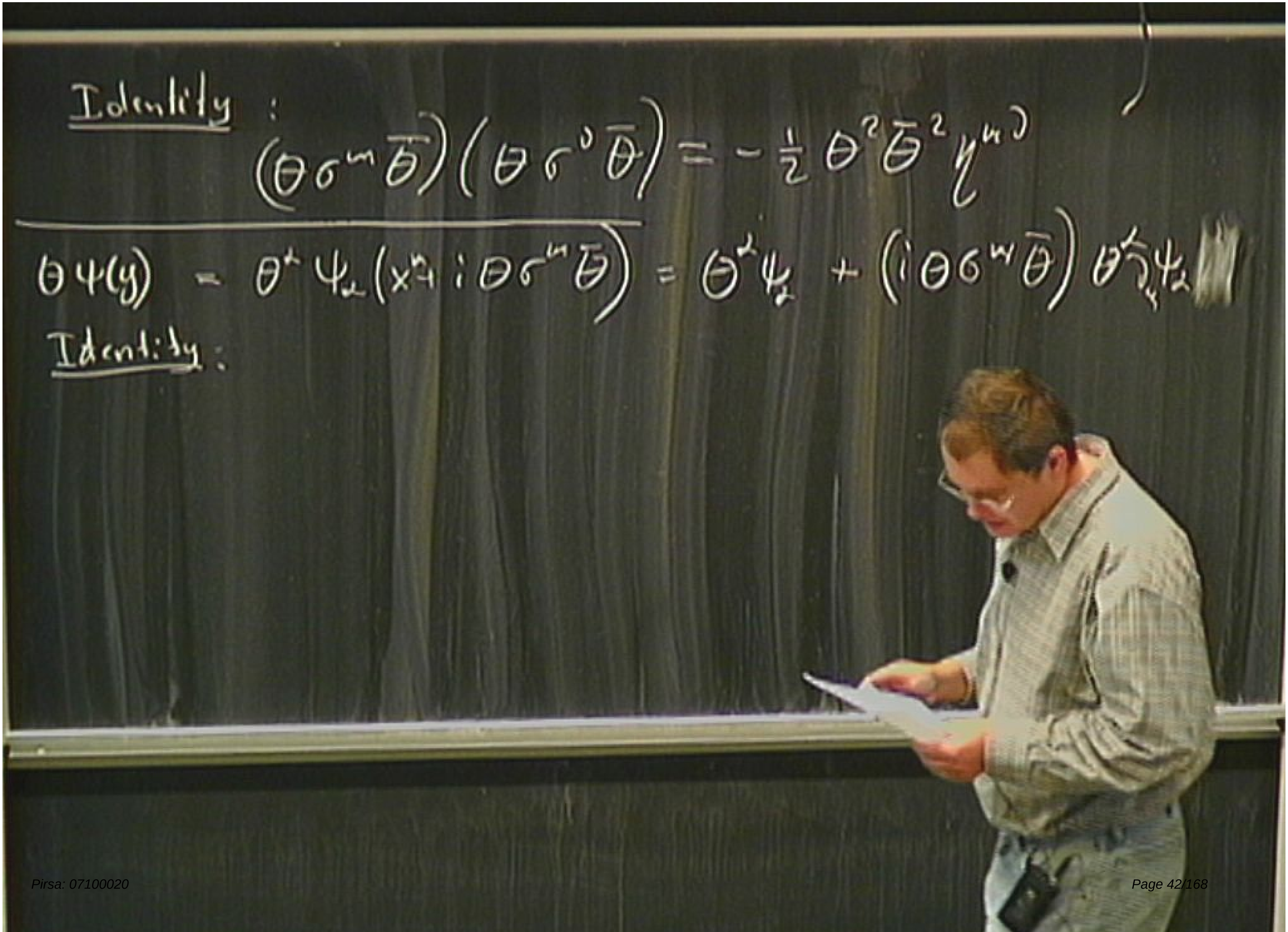
Identity :
$(\theta\sigma^{\mu}\bar{\theta})(\theta\sigma^{\nu}\bar{\theta}) = -\frac{1}{2}\theta^2\bar{\theta}^2\eta^{\mu\nu}$
$\theta\psi(y) = \theta^2\psi_2(x^{\mu} ; \theta\sigma^{\mu}\bar{\theta}) = \theta^2\psi_2 + (i\theta\sigma^{\mu}\bar{\theta})\,\theta^2\partial_{\mu}\psi_2$
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Identity:

$$(\theta\varphi)(\theta\psi) = \theta^\alpha\varphi_\alpha$$

Identity :

$$(\theta\sigma^{\mu}\bar{\theta})(\theta\sigma^{\nu}\bar{\theta}) = -\frac{1}{2}\theta^2\bar{\theta}^2\eta^{\mu\nu}$$

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Identity :

$$(\theta\varphi)(\theta\psi) = (\theta^\alpha\varphi_\alpha)(\theta^\beta\psi_\beta)$$

$$= -\frac{1}{2}(\varphi\psi)\theta^2 = -\frac{1}{2}(\varphi^\alpha\psi_\alpha)\cdot\theta^2$$

Identity :

$$(\theta\sigma^{\mu}\bar\theta)(\theta\sigma^{\nu}\bar\theta) = -\tfrac{1}{2}\theta^2\bar\theta^2\eta^{\mu\nu}$$

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Identity :

$$(\theta\varphi)(\theta\psi) = (\theta^{\alpha}\varphi_{\alpha})(\theta^{\beta}\psi_{\beta})$$

Homework

$$= -\tfrac{1}{2}(\varphi\psi)\,\theta^2 = -\tfrac{1}{2}(\varphi^{\alpha}\psi_{\alpha})\cdot\theta^2$$

Identity :

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Identity :

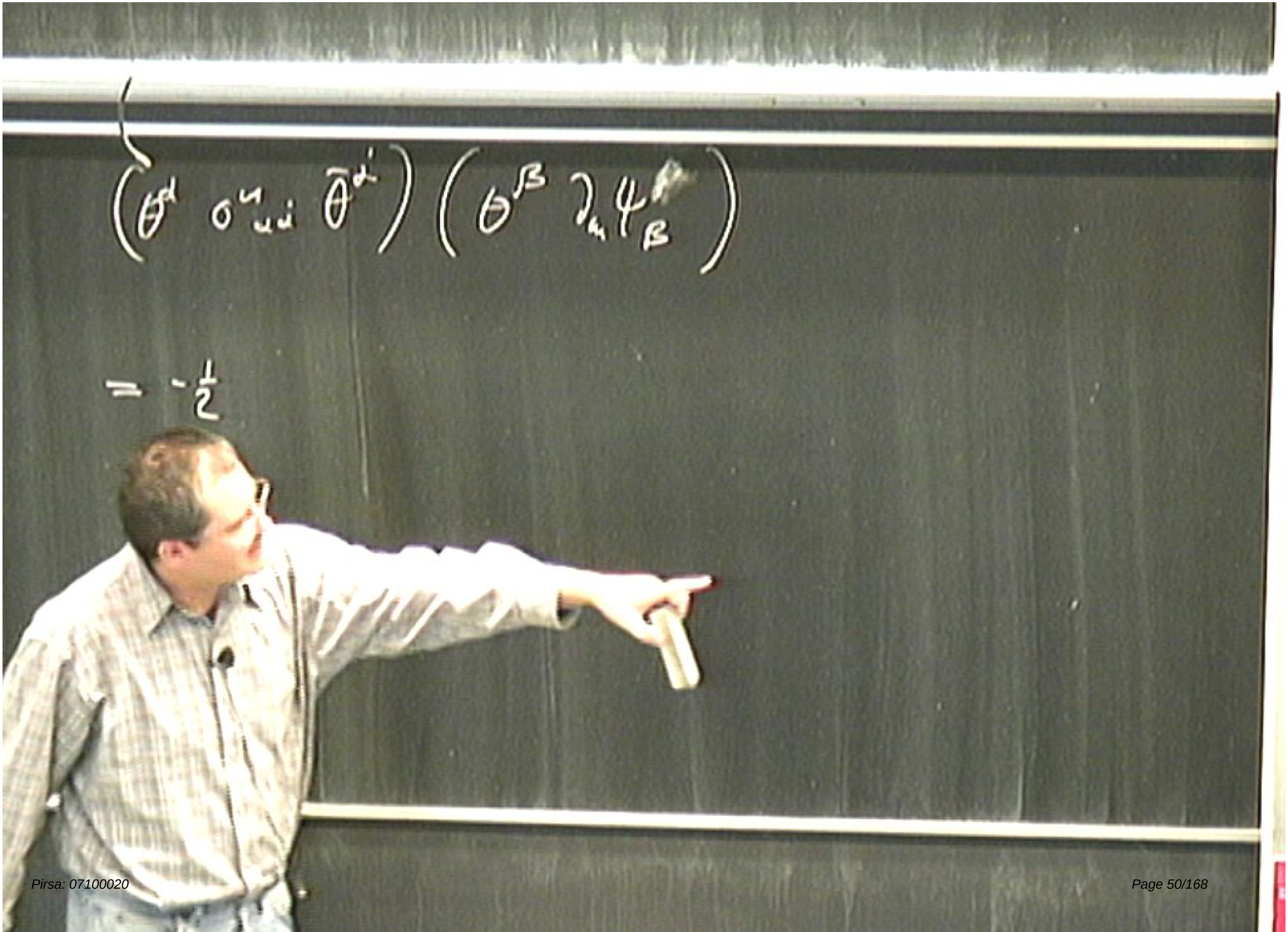
$$(\theta\varphi)(\theta\psi) = (\theta^\alpha\varphi_\alpha)(\theta^\beta\psi_\beta)$$

Homework

$$= -\frac{1}{2}(\varphi\psi)\,\theta^2 = -\frac{1}{2}(\varphi^\alpha\psi_\alpha)\,\theta^2$$


$\theta^{\alpha} \sigma^{\mu}_{\alpha\dot{\alpha}}$

$$\left(\theta^{\alpha}\ \sigma^{\mu}_{\alpha\dot{\alpha}}\ \bar{\theta}^{\dot{\alpha}}\right)\left(\theta^{\beta}\ \partial_{\mu}\psi_{\beta}\right.$$



$(\theta^{\alpha} \sigma^{\mu}_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}})(\theta^{\beta} \partial_{\mu} \psi_{\beta})$
$= -\frac{1}{2}$

$$\left(\theta^{\alpha}\underbrace{\sigma^{\mu}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}}\right)\left(\theta^{\beta}\underbrace{\partial_{\mu}\psi_{\beta}}\right)$$

$$= -\frac{1}{2}$$

$$\left(\theta^{\alpha}\,\underbrace{\sigma^{m}_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}}\right)\left(\theta^{\beta}\,\underbrace{\partial_m\psi_{\beta}}\right)$$

$$=\left(\theta\,\underbrace{\partial_m\psi}\right)\left(\theta\,\underbrace{\sigma^{m}\bar\theta}_{\psi}\right)=-\frac{1}{2}\theta^{2}$$

$m_\psi$

$$\left(\theta^\alpha \underbrace{\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}}\right)\left(\theta^\beta \underbrace{\partial_m\psi_\beta}\right)$$

$$= \left(\theta\, \underbrace{\partial_m\psi}_{\Phi}\right)\left(\theta \underbrace{\sigma^m\bar\theta}_{\psi}\right) = -\tfrac{1}{2}\theta^2\ \partial_m\psi^\alpha\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}$$

Identity:

$$(\theta\sigma^m\bar\theta)(\theta\sigma^n\bar\theta) = -\frac{1}{2}\theta^2\bar\theta^2\eta^{mn}$$

$$\theta\Psi(y) = \theta^\alpha\psi_\alpha(x^m + i\theta\sigma^m\bar\theta) = \theta^\alpha\psi_\alpha + \underbrace{(i\theta\sigma^m\bar\theta)\,\theta^\alpha\partial_m\psi_\alpha}$$

Identity:

$$(\theta\varphi)(\theta\psi) = (\theta^\alpha\varphi_\alpha)(\theta^\beta\psi_\beta)$$

Homework

$$= -\frac{1}{2}(\varphi\psi)\,\theta^2 = -\frac{1}{2}(\varphi \ldots)\ldots^2$$

Identity:

$$(\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) = -\frac{1}{2}\theta^2\bar\theta^2\eta^{\mu\nu}$$

$$\theta\psi(y) = \theta^\alpha\psi_\alpha(x^\mu + i\theta\sigma^\mu\bar\theta) = \theta^\alpha\psi_\alpha + (i\theta\sigma^\mu\bar\theta)\,\theta^\alpha\partial_\mu\psi_\alpha$$

Identity:

$$(\theta\varphi)(\theta\psi) = (\theta^\alpha\varphi_\alpha)(\theta^\beta\psi_\beta)$$

Homework

$$= -\frac{1}{2}(\varphi\psi)\,\theta^2 = -\frac{1}{2}(\varphi\ldots)\,\theta^2$$

Identity :

$$(\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) = -\tfrac{1}{2}\theta^2\bar\theta^2\eta^{\mu\nu}$$

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Identity :

$$(\theta\varphi)(\theta\psi) = (\theta^\alpha\varphi_\alpha)(\theta^\beta\psi_\beta)$$

Homework

$$= -\tfrac{1}{2}(\varphi\psi)\,\theta^2 = -\tfrac{1}{2}(\varphi^\alpha\psi_\alpha)\,\theta^2$$

$M_4$

$$\left(\theta^\alpha \underbrace{\sigma^m_{\alpha\dot\alpha}\, \bar\theta^{\dot\alpha}}\right)\left(\theta^\beta \underbrace{\partial_m \psi_\beta}\right)$$

$$= \left(\theta\, \underbrace{\partial_m \psi}_{\phi}\right)\left(\theta \underbrace{\sigma^m \bar\theta}_{\psi}\right) = -\tfrac{1}{2}\theta^2\ \partial_m \psi^\alpha\, \sigma^m_{\alpha\dot\alpha}\, \bar\theta^{\dot\alpha}$$

$M_4$

$$\left(\theta^\alpha \underbrace{\sigma^m_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}}\right)\left(\theta^\beta \underbrace{\partial_m \psi_\beta}\right)$$

$$= \Big(\theta\,\underbrace{\partial_m\psi}_{\phi}\Big)\Big(\theta\,\underbrace{\sigma^m\bar\theta}_{\psi}\Big) = -\tfrac{1}{2}\theta^2\,\partial_m\psi^\alpha\,\sigma^m_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}$$

$$= -\tfrac{1}{2}\left(\partial_m\psi\,\sigma^m\bar\theta\right)\theta^2$$

$$\theta\psi(y) = \theta^\alpha \psi_\alpha (x^m + i\theta\sigma^m\bar\theta) = \theta^\alpha\psi_\alpha + \underbrace{(i\theta\sigma^m\bar\theta)\,\theta^\alpha \partial_m \psi_\alpha}$$

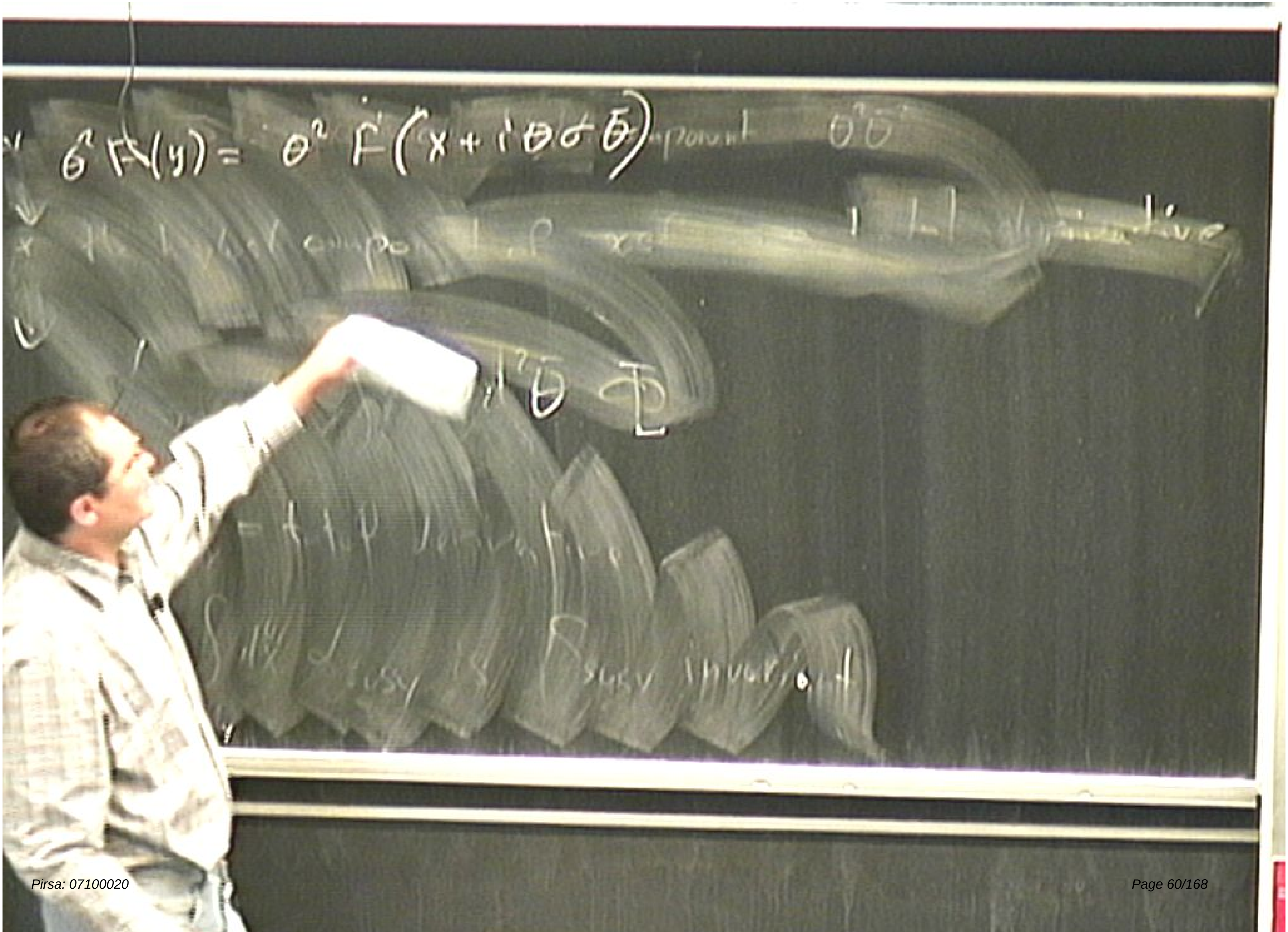
Identity:

$$(\theta\varphi)(\theta\psi) = (\underline{\theta^\alpha\varphi_\alpha})(\theta^\beta\psi_\beta)$$

Homework

$$= -\frac{1}{2}(\varphi\psi)\,\theta^2 \pm -\frac{1}{2}(\varphi^\alpha\psi_\alpha)\,\theta^2$$

$$= \boxed{\theta^\alpha\psi_\alpha - \frac{i}{2}(\partial_m\psi\,\sigma^m\bar\theta)\,\theta^2}$$


θ² F(y) = θ² F(x + iθσθ̄)
θ²θ̄²
derivative
SUSY invariant

$$\theta^2 F(y) = \theta^2 F(x + i\theta\sigma\bar\theta) = \theta^2 F(x)$$

$$\theta^2 F(y) = \theta^2 F(x + i\theta\sigma\bar\theta) = \theta^2 F(x)$$

$$\Phi(x^m, \theta_\alpha, \bar\theta_{\dot\alpha}) = \phi + i\theta\sigma^m\bar\theta\,\partial_m\phi + \frac{1}{4}\theta^2\bar\theta^2\,\Box\phi$$

depends on $x^m$

$$+\sqrt{2}\,\psi - \frac{i}{\sqrt{2}}\theta^2\,\partial_m\psi\,\sigma^m\bar\theta + \theta^2 F$$

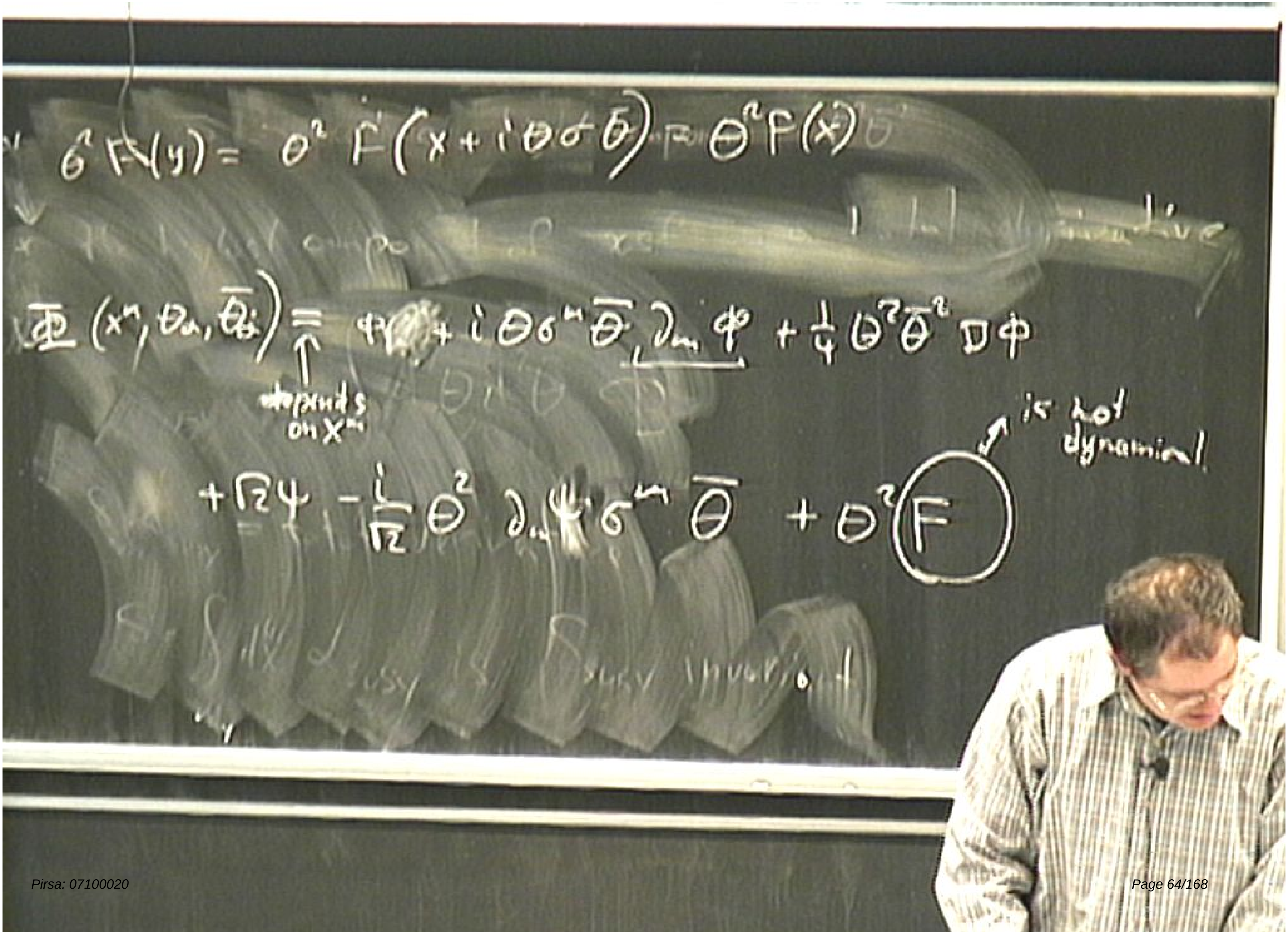
⇒ how does D of a χSF transform under SUSY?

$$\Phi(\underbrace{x + i\theta\sigma\bar\theta}_{y}) = \overset{\text{spin }0}{\phi(y)} + \sqrt{2}\,\theta\,\overset{\text{spin-}\frac{1}{2}}{\psi(y)} + \theta^2 F(y)$$

$$\phi(y) = \phi(x) + i\theta\sigma^\mu\bar\theta\,\partial_\mu\phi + \underbrace{\frac{1}{2!}(i\theta\sigma^\mu\bar\theta)(i\theta\sigma^\nu\bar\theta)\,\partial_\mu\partial_\nu\phi}_{-\frac{1}{2}\cdot\left(-\frac{1}{2}\right)\theta^2\bar\theta^2 g^{\mu\nu}\ldots}$$

$$y = x + i\theta\sigma\bar\theta$$

$$+\frac{1}{4}\theta^2\bar\theta^2\,\Box\phi$$


$\theta^2 F(y) = \theta^2 F(x + i\theta\sigma\bar{\theta}) = \theta^2 F(x)$
$\Phi(x^m, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}) = \phi(x) + i\theta\sigma^m\bar{\theta}\,\partial_m\phi + \frac{1}{4}\theta^2\bar{\theta}^2\Box\phi$
depends on $x^m$
$+\sqrt{2}\psi - \frac{i}{\sqrt{2}}\theta^2\,\partial_m\psi\sigma^m\bar{\theta} + \theta^2 F$
is not dynamical

$$\theta^2 F(y) = \theta^2 F(x + i\theta\sigma\bar\theta) = \theta^2 F(x)$$

$$\Phi(x^\mu, \theta_\alpha, \bar\theta_{\dot\alpha}) = \phi + i\theta\sigma^\mu\bar\theta\,\partial_\mu\phi + \frac{1}{4}\theta^2\bar\theta^2\Box\phi$$

depends on $x^\mu$

$$+\sqrt2\,\theta\psi - \frac{i}{\sqrt2}\theta^2\,\partial_\mu\psi\,\sigma^\mu\bar\theta + \theta^2 F$$

is not dynamical

$$\theta^2 F(y) = \theta^2 F(x + i\theta\sigma\bar\theta) = \theta^2 F(x)$$

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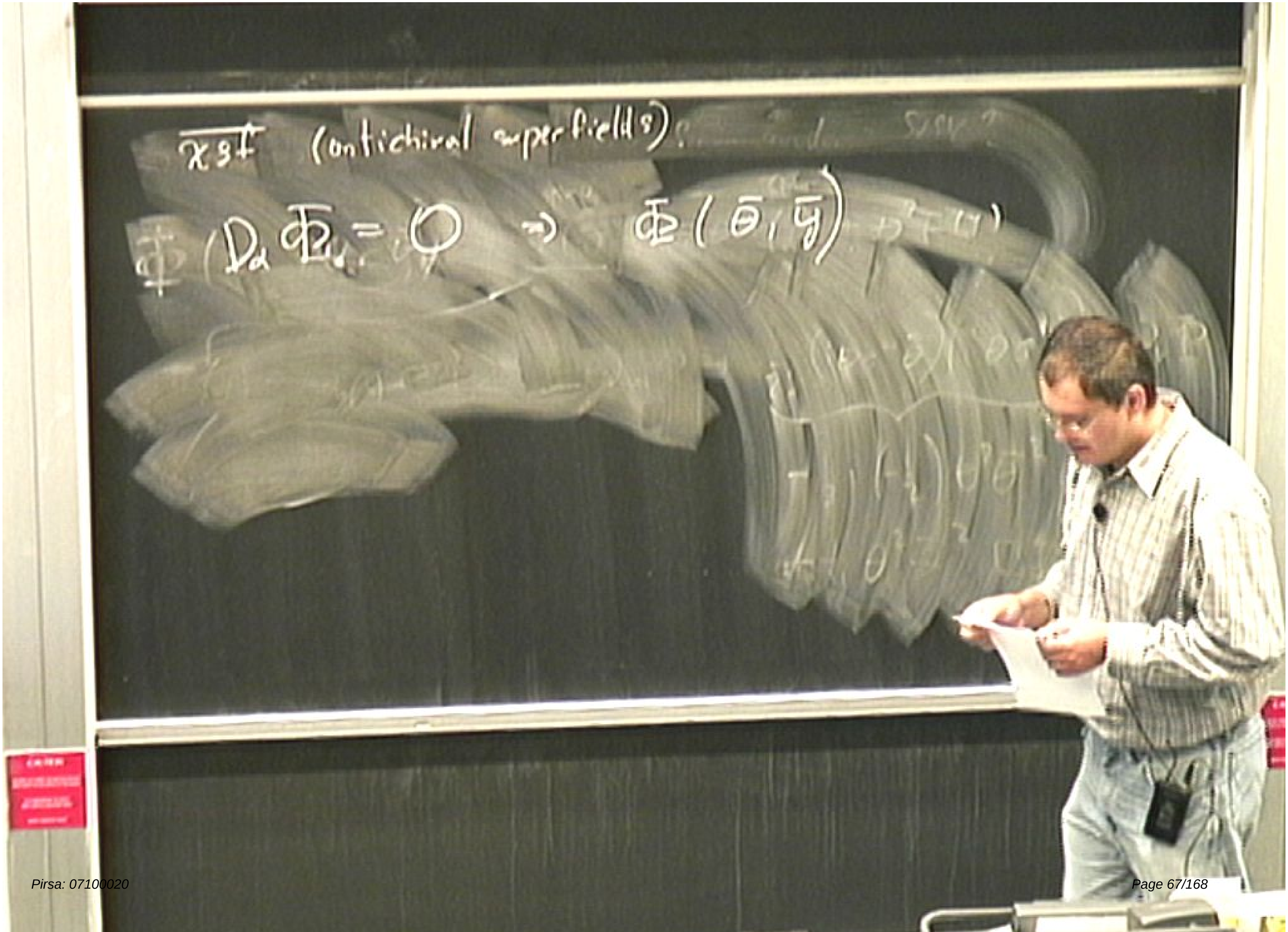
depends on $x^m$

$$+\sqrt{2}\,\psi - \frac{i}{\sqrt{2}}\theta^2\,\partial_m\psi\,\sigma^m\bar\theta + \theta^2 F$$

is not dynamical


(antichiral superfields)

$\overline{xs\,f}$ (antichiral superfields):

$D_\alpha \bar{\Phi} = 0 \Rightarrow \bar{\Phi}(\bar{\theta}, \bar{y})$

$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu$

$\overline{\chi s f}$ (antichiral superfield s):

$$D_\alpha \bar{\Phi} = 0 \Rightarrow \bar{\Phi}(\bar{\theta}, \bar{y})$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma^\mu_{\alpha \dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu$$

$\bar{y}^\mu$

$\overline{\chi SF}$ (antichiral superfields):

$$D_\alpha \bar{\Phi} = 0 \Rightarrow \bar{\Phi}(\bar{\theta}, \bar{y})$$

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$D_\alpha \bar{\theta}$

$$\bar{y}^\mu = x^\mu - i \theta \sigma^\mu \bar{\theta}$$

$\overline{x\, s\, t}$ (antichiral superfields):

$$D_\alpha \bar{\Phi} = 0 \Rightarrow \bar{\Phi}(\bar\theta, \bar y)$$

$$\bar y^m = x^m - i\theta\sigma^m\bar\theta$$

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m$$

$$D_\alpha\bar\theta = 0 \quad D_\alpha \bar y = 0$$

$\overline{\chi sf}$ (antichiral superfields):

$\bar{D}_\alpha \Phi = 0 \Rightarrow \Phi(\bar{\theta}, \bar{y})$

$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu$ $\qquad \bar{y}^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$

$D_\alpha \bar{\theta} = 0 \quad D_\alpha \bar{y} = 0$

If $\varphi_1$ & $\varphi_2$ are $\chi sf \Rightarrow \varphi_1\varphi_2$ is $\chi sf$

$\overline{\chi sf}$ (antichiral superfields)

$$D_\alpha \bar{\Phi} = 0 \Rightarrow \bar{\Phi}(\bar{\theta}, \bar{y})$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma^m_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_m \qquad \bar{y}^m = x^m - i\theta\sigma^m\bar{\theta}$$

$$D_\alpha \bar{\theta} = 0 \quad D_\alpha \bar{y} = 0$$

If $\varphi_1$ & $\varphi_2$ are $\chi sf \Rightarrow \varphi_1\varphi_2$ is $\chi sf$

$\bar{D}_{\dot{\alpha}}$

$\overline{\chi s f}$ (antichiral superfields):

$$\bar{D}_{\alpha} \bar{\Phi} = 0 \Rightarrow \bar{\Phi}(\bar{\theta}, \bar{y})$$

$$\bar{D}_{\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \sigma^{m}_{\alpha \dot{\alpha}} \theta^{\alpha} \partial_m \qquad \bar{y}^m = x^m - i \theta \sigma^m \bar{\theta}$$

$$D_{\alpha} \bar{\theta} = 0 \quad D_{\alpha} y = 0$$

If $\varphi_1$ & $\varphi_2$ are $\chi s f \Rightarrow \varphi_1 \varphi_2$ is $\chi s f$

$$\bar{D}_{\dot{\alpha}}(\varphi_1 \varphi_2) = \varphi_1 \bar{D}_{\dot{\alpha}} \varphi_2 + (\bar{D}_{\dot{\alpha}} \varphi_1) \varphi_2 =$$

$\overline{\chi sf}$ (antichiral superfields):

$$D_\alpha \bar{\Phi} = 0 \Rightarrow \bar{\Phi}(\bar{\theta}, \bar{y})$$

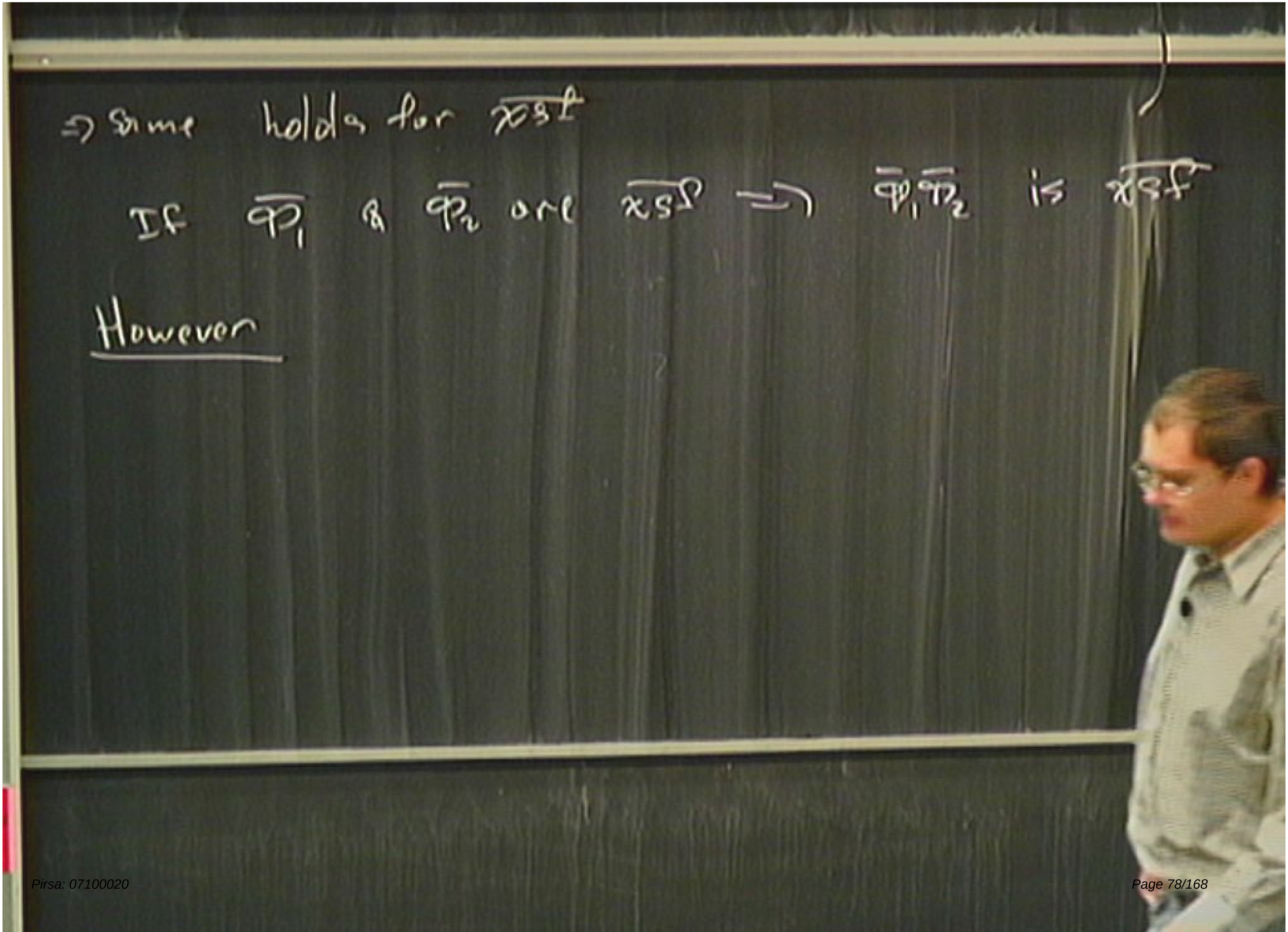
$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\sigma^m_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_m \qquad \bar{y}^m = x^m - i\theta\sigma^m\bar{\theta}$$

$$D_\alpha \bar{\theta} = 0 \quad D_\alpha \bar{y} = 0$$

If $\varphi_1$ & $\varphi_2$ are $\chi sf \Rightarrow \varphi_1\varphi_2$ is $\chi sf$

$$\bar{D}_{\dot{\alpha}}(\varphi_1\varphi_2) = \varphi_1 \bar{D}_{\dot{\alpha}}\varphi_2 + (\bar{D}_{\dot{\alpha}}\varphi_1)\varphi_2 = 0$$

$\Rightarrow$ same holds for $\overline{xst}$


⇒ same holds for $\overline{xsf}$
If $\overline{\varphi_1}$ & $\overline{\varphi_2}$ are $\overline{xsf}$ ⇒ $\overline{\varphi_1}\,\overline{\varphi_2}$ is $\overline{xsf}$
However

$\Rightarrow$ same holds for $\overline{xsf}$

If $\bar{\varphi}_1$ & $\bar{\varphi}_2$ are $\overline{xsf}$ $\Rightarrow$ $\bar{\varphi}_1 \bar{\varphi}_2$ is $\overline{xsf}$

<u>However</u> ... $\bar{\varphi}$ ( Assume $\bar{D}_{\dot{\alpha}} \varphi = 0$ xsf

$D_{\alpha} \bar{\varphi} = 0$ $\overline{xsf}$

$\Rightarrow$ same holds for $\overline{xsf}$

If $\bar\varphi_1$ & $\bar\varphi_2$ are $\overline{xsf}$ $\Rightarrow$ $\bar\varphi_1 \bar\varphi_2$ is $\overline{xsf}$

<u>However</u>

$\underbrace{\varphi\,\bar\varphi}_{\text{is a superfield}}$ (Assume $\bar D_{\dot\alpha}\varphi = 0$ xsf

$D_\alpha \bar\varphi = 0$ $\overline{xsf}$

$\Rightarrow$ same holds for $\overline{xsf}$

If $\bar{\varphi}_1$ & $\bar{\varphi}_2$ are $\overline{xsf}$ $\Rightarrow$ $\bar{\varphi}_1\bar{\varphi}_2$ is $\overline{xsf}$

<u>However</u>

$$\underbrace{\varphi\,\bar{\varphi}}_{\text{is a superfield}}$$

( Assume $\bar{D}_{\dot{\alpha}}\varphi = 0$ xsf

$D_\alpha \bar{\varphi} = 0$ $\overline{xsf}$ )

$\Rightarrow$ same holds for $\overline{xsf}$

If $\bar{\varphi}_1$ & $\bar{\varphi}_2$ are $\overline{xsf}$ $\Rightarrow$ $\bar{\varphi}_1\bar{\varphi}_2$ is $\overline{xsf}$

<u>However</u>

$\underbrace{\varphi\,\bar{\varphi}}$ is a superfield

neither xsf nor $\overline{xsf}$

(Assume $\bar{D}_{\dot{\alpha}}\varphi = 0$ xsf

$D_\alpha \bar{\varphi} = 0$ $\overline{xsf}$

$\bar{D}_{\dot{\alpha}}[\varphi\bar{\varphi}] \neq 0 \quad D_\alpha[\varphi\bar{\varphi}] \neq 0$

Lagrangians:

Recall:

Lagrangians.

Recall: $Q_r =$

Lagrangians:

Recall:

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i \sigma^m_{\alpha\dot\alpha} \bar\theta^{\dot\alpha} \partial_m$$

$$\bar Q_{\dot\alpha} = -\frac{\partial}{\partial \bar\theta^{\dot\alpha}} + i \sigma^m_{\alpha\dot\alpha} \theta^\alpha \partial_m$$

Lagrangians.

Recall.

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m$$

$$\bar Q_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\sigma^m_{\alpha\dot\alpha}\theta^\alpha\partial_m$$

$$\delta_{susy}\Phi \stackrel{def}{\equiv}$$

Lagrangians:

Recall: $Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m$

$\bar Q_{\dot\alpha} = -\frac{\partial}{\partial \bar\theta^{\dot\alpha}} + i\theta\sigma^m_{\alpha\dot\alpha}\partial_m$

$$\delta_{susy}\bar\Phi \stackrel{def}{\equiv} (\varepsilon Q + \bar\xi \bar Q)\bar\Phi \qquad \bar\xi_{\dot\alpha}\bar Q^{\dot\alpha}$$

$$= (\xi^\alpha Q_\alpha - \bar\xi^{\dot\alpha}\bar Q_{\dot\alpha})\bar\Phi$$


$\xi^\mu \Big( \frac{\partial}{\partial\theta^\mu}$

Lagrangians:

(Find how $\delta_{susy}$ $D = ?$

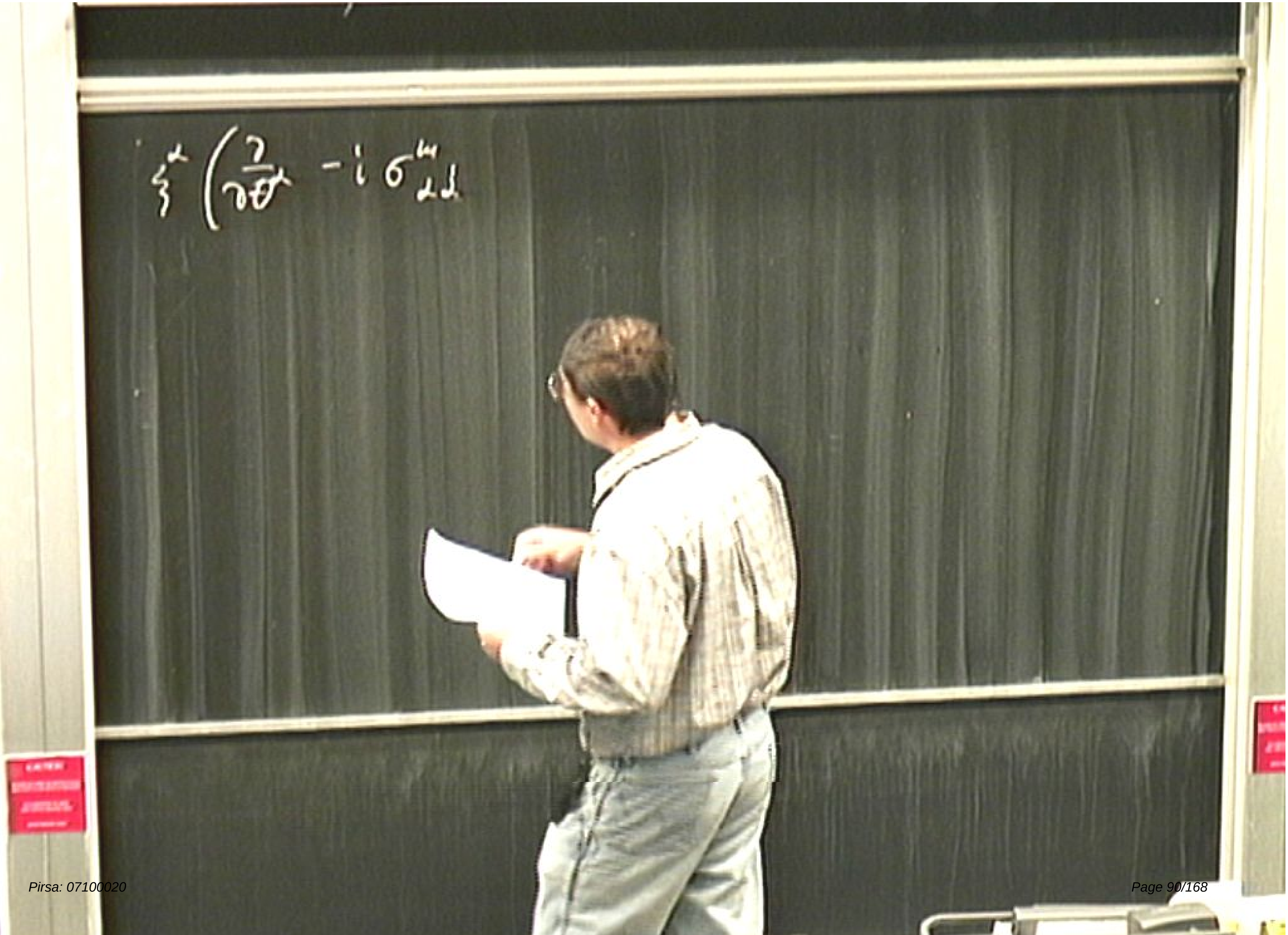
Recall:

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu$$

$$\bar Q_{\dot\alpha} = -\frac{\partial}{\partial \bar\theta^{\dot\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

$$\delta_{susy}\bar\Phi \overset{\text{def}}{\equiv} (\xi Q + \bar\xi\bar Q)\bar\Phi \qquad \bar\xi_{\dot\alpha}\bar Q^{\dot\alpha}$$

$$= (\xi^\alpha Q_\alpha - \bar\xi^{\dot\alpha}\bar Q_{\dot\alpha})\bar\Phi$$


$\xi^a \left( \frac{\partial}{\partial \theta^a} - i\, \sigma^{m}_{ab} \right.$

$$\xi^{\alpha}\left(\frac{\partial}{\partial\theta^{\alpha}} - i\,\sigma^{m}_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}\,\partial_m\right) - \bar\xi^{\dot\alpha}\left(-\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\,\theta^{\alpha}\sigma^{m}_{\alpha\dot\alpha}\,\partial_m\right)$$

$$\xi^{\alpha}\left(\frac{\partial}{\partial\theta^{\alpha}} - i\,\sigma^{m}_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}\,\partial_m\right) - \bar\xi^{\dot\alpha}\left(-\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\,\theta^{\alpha}\sigma^{m}_{\alpha\dot\alpha}\,\partial_m\right)$$

$$\Big[\varphi + \theta\psi + \bar\theta\bar\chi + \theta^2 F + \bar\theta^2 G + \theta\sigma^m\bar\theta\,A_m$$
$$+\,\theta^2\bar\theta\bar\lambda + \bar\theta^2\theta\rho + \theta^2\bar\theta^2 D\Big]$$

$$\delta_{\xi}\Phi = \xi^{\alpha}\left(\frac{\partial}{\partial\theta^{\alpha}} - i\,\sigma^{\mu}_{\alpha\dot{\alpha}}\,\bar{\theta}^{\dot{\alpha}}\,\partial_{\mu}\right) - \bar{\xi}^{\dot{\alpha}}\left(-\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\,\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\,\partial_{\mu}\right)$$

$$\Big[\varphi + \theta\psi + \bar{\theta}\bar{\chi} + \theta^{2}F + \bar{\theta}^{2}G + \theta\sigma^{\mu}\bar{\theta}\,A_{\mu}$$

$$+\,\theta^{2}\bar{\theta}\bar{\lambda} + \bar{\theta}^{2}\theta\rho + \theta^{2}\bar{\theta}^{2}D\Big]$$

$$\delta_{\xi\eta}\Phi = \xi^{\alpha}\left(\frac{\partial}{\partial\theta^{\alpha}} - i\sigma^{m}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_m\right) - \bar{\xi}^{\dot{\alpha}}\left(-\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^{\alpha}\sigma^{m}_{\alpha\dot{\alpha}}\partial_m\right)$$

$$\Big[\varphi + \theta\psi + \bar{\theta}\bar{\chi} + \theta^2 F + \bar{\theta}^2 G + \theta\sigma^m\bar{\theta}A_m$$

$$+ \theta^2\bar{\theta}\bar{\lambda} + \bar{\theta}^2\theta\rho + \theta^2\bar{\theta}^2 D\Big]$$

$$= \cdots\cdots + \theta^2\bar{\theta}^2\left[\ ?\ \right]$$

$$\xi Q \Phi = \xi^\alpha \left( \cancel{\frac{\partial}{\partial \theta^\alpha}} - i \sigma^m_{\alpha\dot\alpha} \bar\theta^{\dot\alpha} \partial_m \right) - \bar\xi^{\dot\alpha} \left( - \cancel{\frac{\partial}{\partial \bar\theta^{\dot\alpha}}} + i \theta^\alpha \sigma^m_{\alpha\dot\alpha} \partial_m \right)$$

$$\Big[ \varphi + \theta\psi + \bar\theta\bar\chi + \theta^2 F + \bar\theta^2 G + \theta\sigma^m\bar\theta A_m$$

$$+ \theta^2 \bar\theta \bar\lambda + \bar\theta^2 \theta \rho + \theta^2 \bar\theta^2 D \Big]$$

$$= \cdots\cdots + \theta^2 \bar\theta^2 [\,?\,]$$

$$\delta_{\xi}\Phi = \xi^{\alpha}\left(\cancel{\frac{\partial}{\partial\theta^{\alpha}}} - i\,\underbrace{\sigma^{\mu}_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}}\,\partial_{\mu}\right) - \bar\xi^{\dot\alpha}\left(-\cancel{\frac{\partial}{\partial\bar\theta^{\dot\alpha}}} + i\,\theta^{\alpha}\sigma^{\mu}_{\alpha\dot\alpha}\partial_{\mu}\right)$$

$$\Big[\varphi + \theta\psi + \bar\theta\bar\chi + \theta^{2}F + \bar\theta^{2}G + \theta\sigma^{\mu}\bar\theta A_{\mu}$$

$$+ \underbrace{\theta^{2}\bar\theta}\bar\lambda + \bar\theta^{2}\theta\rho + \theta^{2}\bar\theta^{2}D\Big]$$

$$= \cdots\cdots + \theta^{2}\bar\theta^{2}\,[\;?\;]$$

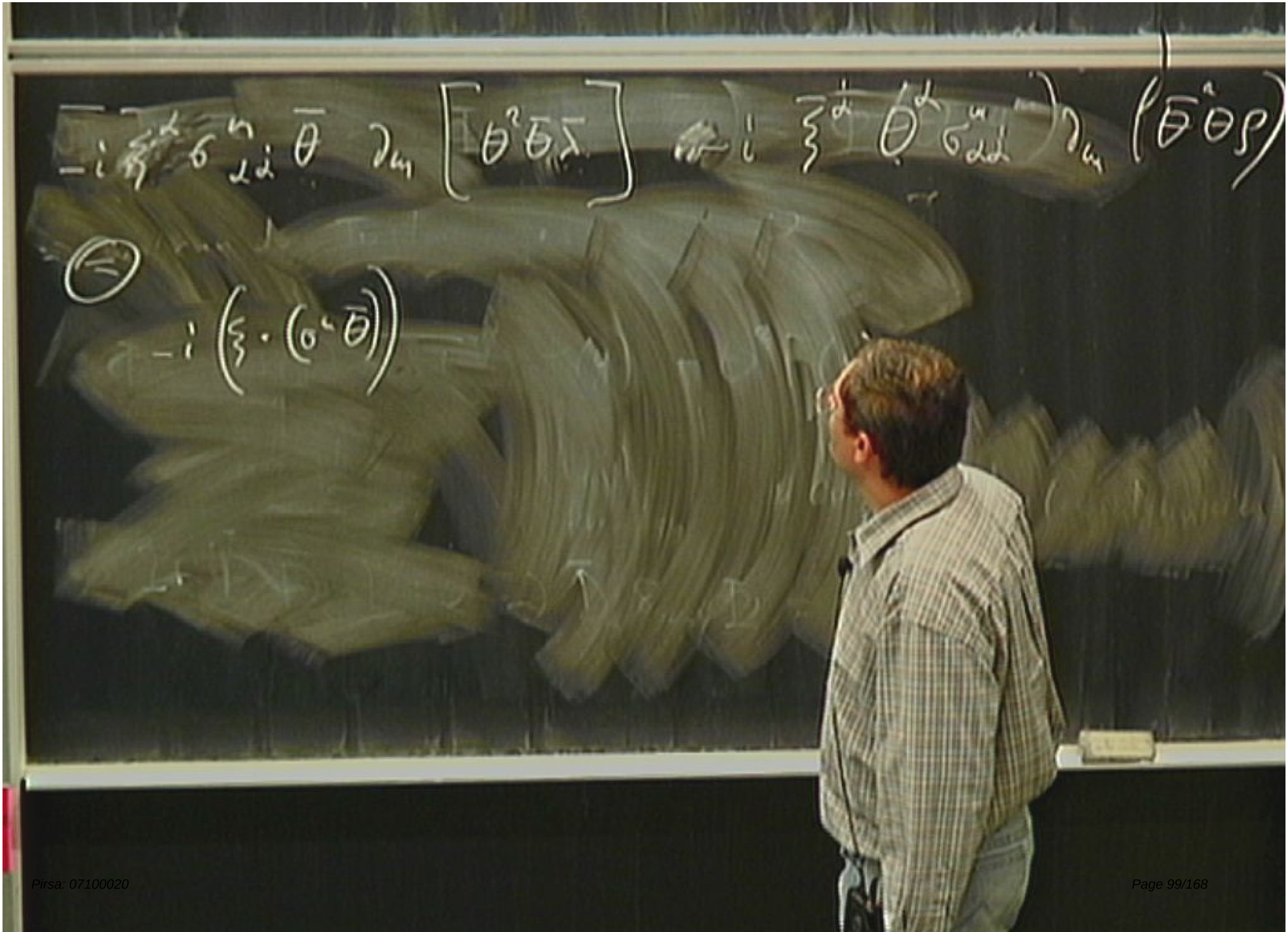
$$\delta_{\xi}\Phi = \xi^{\alpha}\left(\cancel{\frac{\partial}{\partial\theta^{\alpha}}} - i\,\sigma^{m}_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}\,\partial_m\right) - \bar\xi^{\dot\alpha}\left(-\cancel{\frac{\partial}{\partial\bar\theta^{\dot\alpha}}} + i\,\theta^{\alpha}\sigma^{m}_{\alpha\dot\alpha}\,\partial_m\right)$$

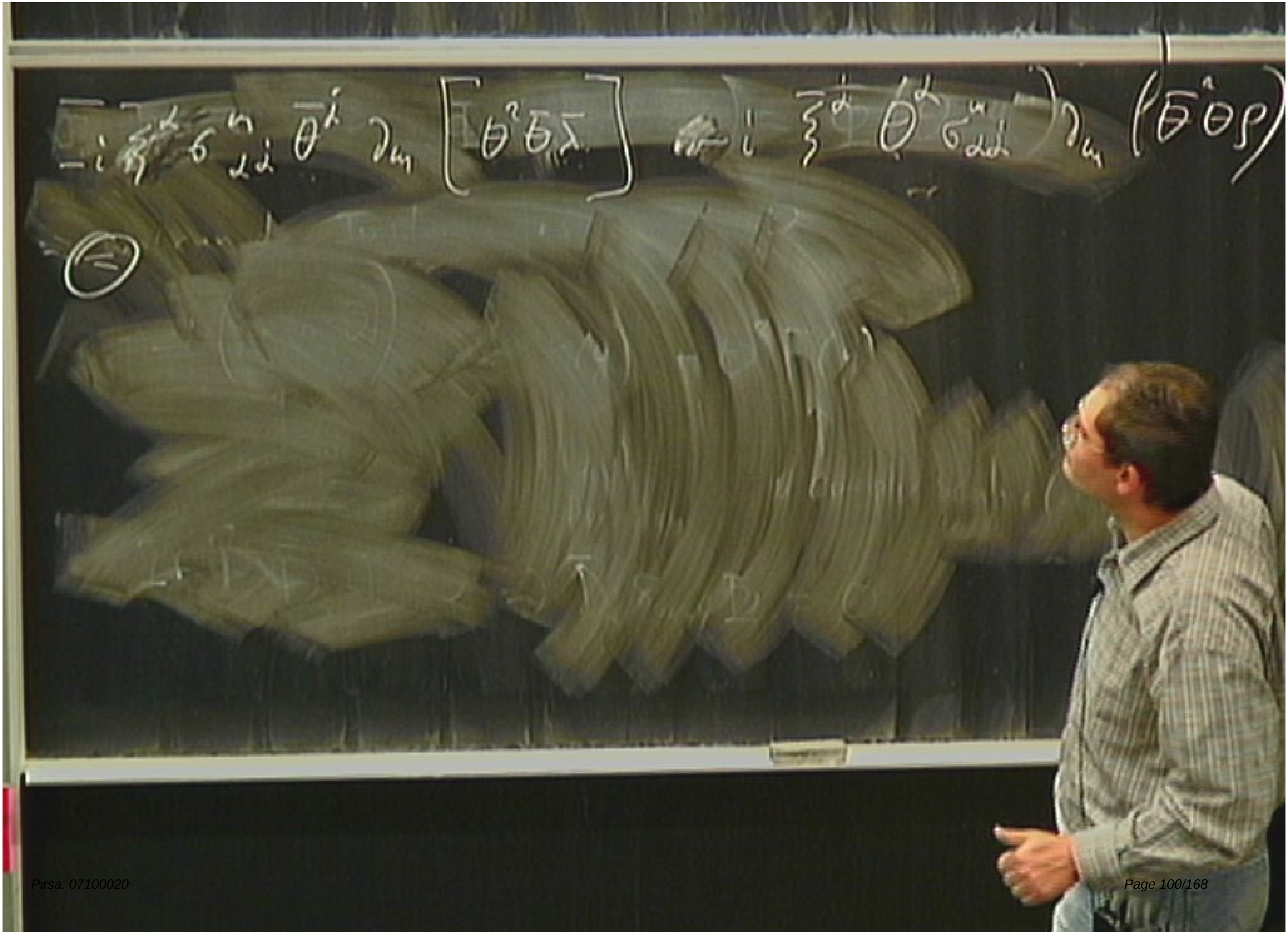
$$\Big[\varphi + \theta\psi + \bar\theta\bar\chi + \theta^2 F + \bar\theta^2 G + \theta\sigma^m\bar\theta\, A_m$$

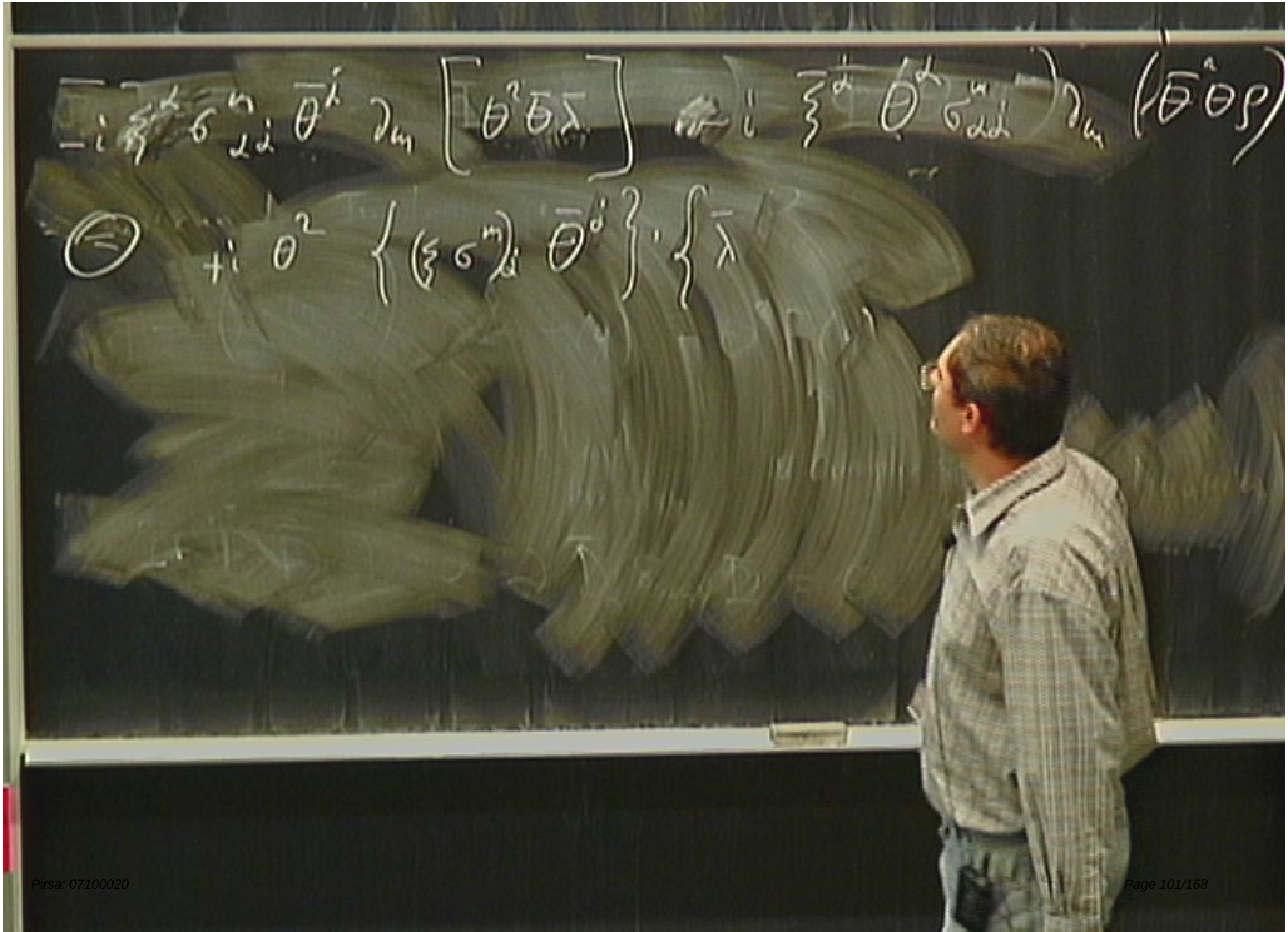
$$+ \theta^2\bar\theta\bar\lambda + \bar\theta^2\theta\rho + \theta^2\bar\theta^2 D\Big]$$

$$= \cdots\cdots + \theta^2\bar\theta^2\left[\ ?\ \right]$$

$$-i\,\xi^{\alpha}\sigma^{m}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\,\partial_m\left[\theta^2\bar\theta_{\dot\lambda}\right] - i\,\xi^{\dot\alpha}\,\theta^{\alpha}\sigma^{m}_{\alpha\dot\alpha}\,\partial_m\left(\bar\theta\,\theta\,\rho\right)$$






$-i\,\xi^{\alpha}\,\sigma^{m}_{\alpha\dot{\alpha}}\,\bar{\theta}^{\dot{\alpha}}\,\partial_m \left[\theta^2\bar{\theta}\bar{\lambda}\right] \quad i\,\xi^{\alpha}\,\theta^{\alpha}\,\sigma^{m}_{\alpha\dot{\alpha}}\,)\,\partial_m\,(\bar{\theta}\theta\rho)$
$\Theta \quad +i\,\theta^2\,\{(\xi\sigma^m)_{\dot{\alpha}}\,\bar{\theta}^{\dot{\alpha}}\}\cdot\{\bar{\lambda}$

$$-i\,\xi^{\alpha}\sigma^{m}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\,\partial_m\left[\theta^2\bar\theta\bar\lambda\right] - i\,\bar\xi^{\dot\alpha}\,\theta^{\alpha}\sigma^{m}_{\alpha\dot\alpha}\,\partial_m\left(\bar\theta^{\dot\alpha}\theta\rho\right)$$

$$\Theta \quad +i\,\theta^2\,\left\{(\xi\sigma^m)_{\dot\alpha}\,\bar\theta^{\dot\alpha}\right\}\cdot\left\{\bar\lambda_{\dot\beta}\,\bar\theta^{\dot\beta}\right\} + \cdots$$

$$-i\,\xi^{\alpha}\sigma^{m}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\,\partial_m\left[\theta^2\bar\theta\bar\lambda\right] \quad i\,\xi^{\alpha}\ \theta^{\alpha}\sigma^{m}_{\alpha\dot\alpha}\,)\,\partial_m\,(\bar\sigma\theta\rho)$$

$$\ominus \quad +i\,\theta^2\left\{(\xi\sigma^m)_{\dot\alpha}\bar\theta^{\dot\alpha}\right\}\cdot\left\{\bar\lambda_{\dot\beta}\bar\theta^{\dot\beta}\right\} + \cdots$$

$$-i\,\xi^{\alpha}\sigma^{m}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\,\partial_m\left[\theta^2\bar\theta\bar\lambda\right] + i\,\xi^{\alpha}\,\theta^{\alpha}\sigma^{m}_{\alpha\dot\alpha}\,)\,\partial_m\,(\bar\sigma^{\hat\ell}\theta\rho)$$

$$\ominus \quad +i\,\theta^2\left\{(\xi\sigma^m)_{\dot\alpha}\,\bar\theta^{\dot\alpha}\right\}\cdot\left\{\bar\lambda_{\dot\beta}\,\bar\theta^{\dot\beta}\right\} + \cdots$$

Identity (from homework)

$-i\,\xi^{\alpha}\sigma^{m}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\,\partial_m\left[\theta^2\bar{\theta}\bar{\lambda}\right]\;+\;i\,\bar{\xi}^{\dot{\alpha}}\left(\theta^{\alpha}\sigma^{m}_{\alpha\dot{\alpha}}\right)\partial_m\left(\bar{\theta}\dot{\theta}\rho\right)$

$\ominus\quad +i\,\theta^2\left\{(\xi\sigma^m)_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\right\}\cdot\left\{\bar{\lambda}_{\dot{\beta}}\bar{\theta}^{\dot{\beta}}\right\}\;+\;-\;-\;-$

Identity (from homework)

$$(\bar{\theta}\bar{\phi})(\bar{\theta}\bar{\psi}) = -\frac{1}{2}(\bar{\phi}\bar{\psi})\,\bar{\theta}^2$$

$$-i\,\xi^{\alpha}\,\sigma^{m}_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}\,\partial_m\left[\theta^2\bar\theta\bar\lambda\right] + i\,\xi^{\alpha}\,\theta^{\alpha}\,\sigma^{m}_{\alpha\dot\alpha}\,\partial_m\,(\bar\xi\bar\theta\rho)$$

$$\ominus\; i\,\theta^2\,\{(\xi\sigma^m)_{\dot\alpha}\,\bar\theta^{\dot\alpha}\}\cdot\{\bar\lambda_{\dot\beta}\,\bar\theta^{\dot\beta}\} + \cdots$$

Identity (from homework)

$$(\bar\theta\bar\phi)(\bar\theta\bar\psi) = -\frac{1}{2}(\bar\phi\bar\psi)\,\bar\theta^2$$

$$(\bar\phi\bar\theta)(\bar\psi\bar\theta) = -\frac{1}{2}(\bar\phi\bar\psi)\,\bar\theta^2$$

$-i\,\xi^{\alpha}\,\sigma^{m}_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}\,\partial_m\left[\theta^2\bar\theta\bar\lambda\right] + i\,\xi^{\alpha}\,\theta^{\alpha}\,\sigma^{m}_{\alpha\dot\alpha}\,\partial_m\,(\bar\sigma\theta\rho)$

$\ominus \quad i\,\theta^2\,\{(\xi\sigma^m)_{\dot\alpha}\,\bar\theta^{\dot\alpha}\}\cdot\{\bar\lambda_{\dot\beta}\,\bar\theta^{\dot\beta}\} + \cdots$

$-i\theta^2\,\xi^{\alpha}\sigma^{m}_{\alpha\dot\alpha}$

Identity (from homework)

$$(\bar\theta\bar\phi)(\bar\theta\bar\psi) = -\frac{1}{2}(\bar\phi\bar\psi)\,\bar\theta^2$$

$$(\bar\phi\bar\theta)(\bar\psi\bar\theta) = -\frac{1}{2}(\bar\phi\bar\psi)\,\bar\theta^2$$

$$= \boxed{\theta^\alpha \psi_L - \frac{i}{2}\left(\partial_m \psi \sigma^m \bar\theta\right)\theta^2}$$

$$-i\xi^\alpha \sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\,\partial_m\left[\theta^2\bar\theta\bar\lambda\right] - i\,\xi^\alpha\,\theta^\alpha\sigma^m_{\alpha\dot\alpha}\,\partial_m\left(\bar\xi^{\dot\alpha}\theta\rho\right)$$

$$\ominus\, i\,\theta^2\left\{(\xi\sigma^m)_{\dot\alpha}\bar\theta^{\dot\alpha}\right\}\cdot\left\{\bar\lambda_{\dot\beta}\bar\theta^{\dot\beta}\right\} + \cdots$$

$$-i\theta^2\,\xi^\alpha\sigma^m_{\alpha\dot\alpha}\,\partial_m\bar\lambda^{\dot\alpha}$$

Identity (from homework)

$$(\bar\theta\bar\phi)(\bar\theta\bar\psi) = -\frac{1}{2}(\bar\phi\bar\psi)\,\bar\theta^2$$

$$(\bar\phi\bar\theta)(\bar\psi\bar\theta) = -\frac{1}{2}(\bar\phi\bar\psi)\,\bar\theta^2$$

$$= \boxed{\theta^\alpha \psi_\alpha - \frac{i}{2}\left(\partial_m \psi \sigma^m \bar{\theta}\right)\theta^2}$$

$$-i\,\xi^\alpha \sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\,\partial_m\left[\theta^2\bar\theta\bar\lambda\right] \quad -i\,\xi^\alpha\,\theta^\alpha\sigma^m_{\alpha\dot\alpha}\,\partial_m\left(\bar\theta^{\dot\alpha}\bar\theta\rho\right)$$

$$\Theta \quad -i\,\theta^2\left\{(\xi\sigma^m)_{\dot\alpha}\bar\theta^{\dot\alpha}\right\}\cdot\left\{\bar\lambda_{\dot\beta}\bar\theta^{\dot\beta}\right\} + \cdots$$

$$-i\,\theta^2\,\xi^\alpha\sigma^m_{\alpha\dot\alpha}\,\partial_m\bar\lambda^{\dot\alpha}\left(-\frac{\bar\theta^2}{2}\right)$$

Identity (from homework)

$$(\bar\theta\bar\phi)(\bar\theta\bar\psi) = -\frac{1}{2}(\bar\phi\bar\psi)\,\bar\theta^2$$

$$(\bar\phi\bar\theta)(\bar\psi\bar\theta) = -\frac{1}{2}(\bar\phi\bar\psi)\,\bar\theta^2$$

$$= \frac{i}{2}\,\theta^2\bar\theta^2\;\xi\sigma^m\partial_m\bar\lambda$$

$\theta \quad \delta_{susy} D \doteq \partial_m \left[ \text{something} \right]$

$\delta_{susy} D \doteq \partial_m[\text{something}]$

$S = \int d^4x\, D$

$$\theta \quad \delta_{susy}\, D \doteq \partial_m\left[\text{something}\right]$$

$$\underset{\text{action}}{S} = \int d^4x\, D$$

$\theta \quad \delta_{susy} D \doteq \partial_m [\text{something}]$

$S = \int d^4x \, D$

$\uparrow$ action

$\delta_{susy} S = \int d^4x \; \delta_{susy} D = \int d^4x$

$$\delta_{susy} D \doteq \partial_m \left[ \text{something} \right]$$

$$S = \int d^4x \, D$$

$$\delta_{susy} S = \int_{M_4} d^4x \, \delta_{susy} D = \int_{M_4} d^4x \, \partial_m [\cdot\cdot]$$

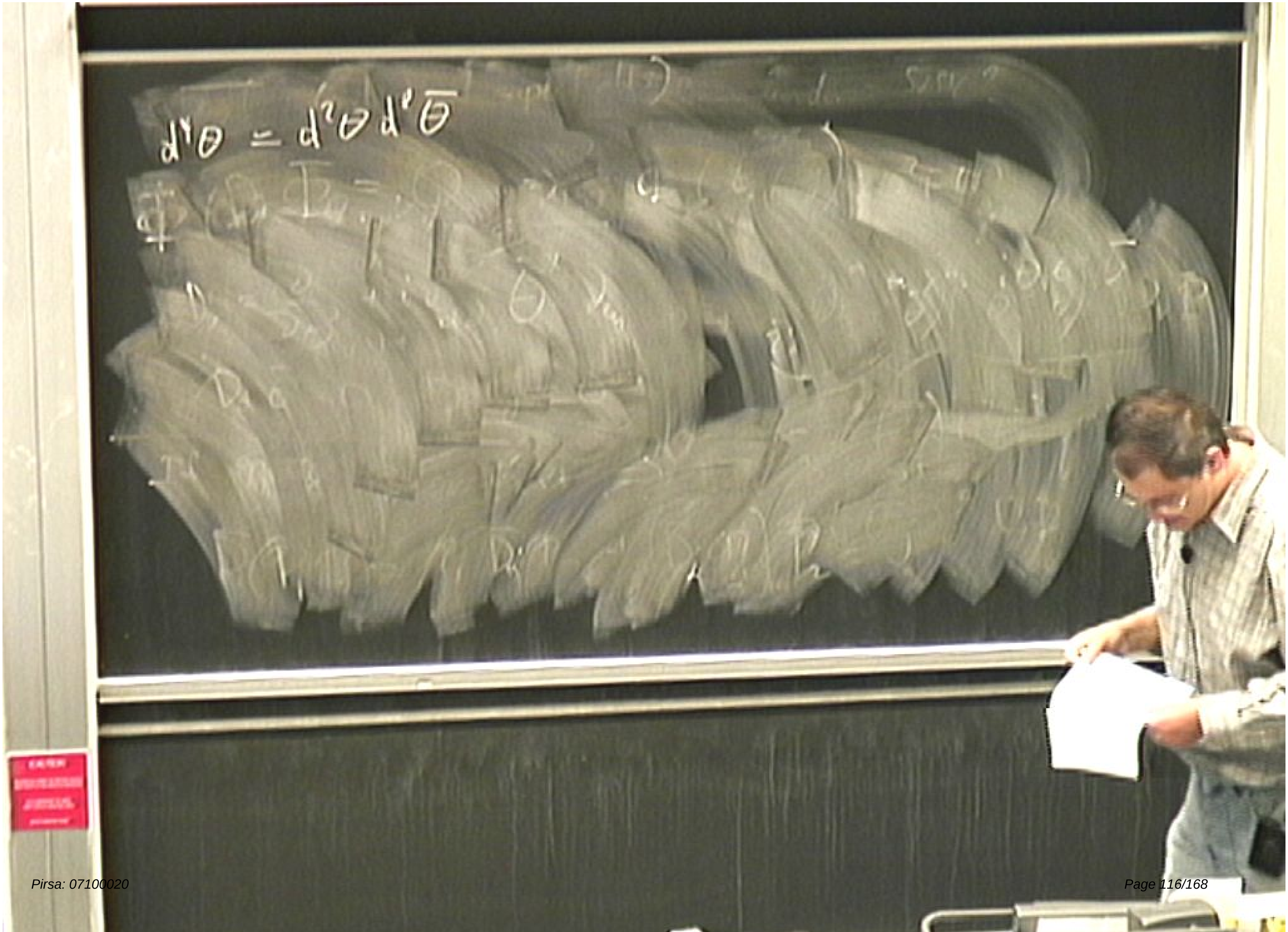
$$= \int_{\partial M_4} [\cdots]$$

$$\delta_{susy} D \stackrel{!}{=} \partial_m[\text{something}]$$

$$\underset{\text{action}}{S} = \int d^4x\, D$$

$$\delta_{susy} S = \int_{M_4} d^4x\, \delta_{susy} D = \int_{M_4} d^4x\, \partial_m[\cdots]$$

$$= \int_{\partial M_4} [\cdots] = 0$$


$d^4\theta = d^2\theta\, d^2\bar{\theta}$

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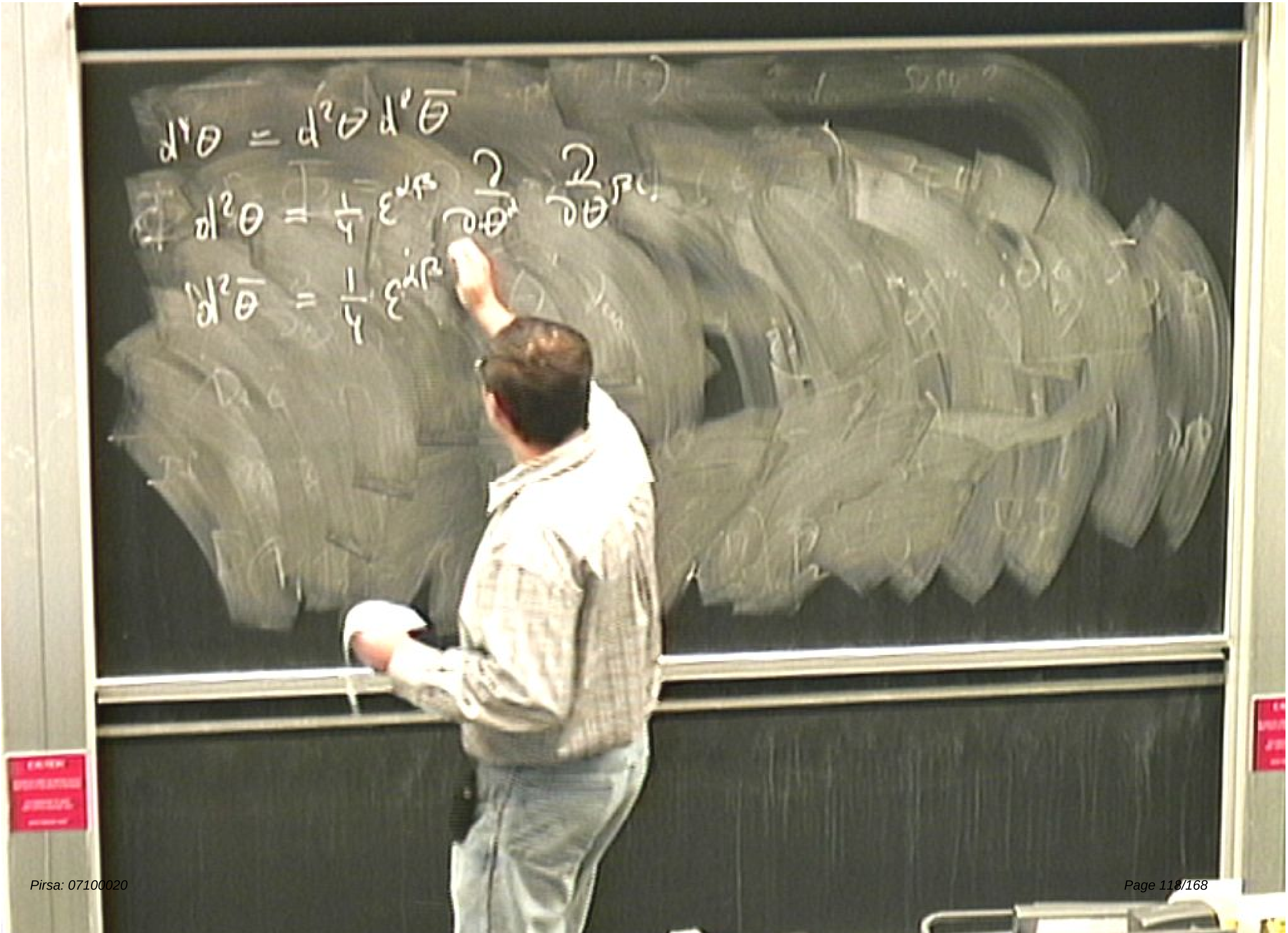
$$d^2\theta = \frac{1}{4}\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}$$


$d^4\theta = d^2\theta \, d^2\bar{\theta}$
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Le[t] is complete

$$4\int d^2\theta\,\theta^2$$

$$d^4\theta = d^2\theta \, d^2\bar{\theta}$$

$$d^2\theta = \frac{1}{4}\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}$$

$$d^2\bar{\theta} = \frac{1}{4}\varepsilon_{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar{\theta}_{\dot\alpha}}\frac{\partial}{\partial\bar{\theta}_{\dot\beta}}$$

Let is compute

$$4\int d^2\theta \, \theta^2 =$$

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$$d^2\bar{\theta} = \frac{1}{4}\varepsilon_{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar{\theta}_{\dot\alpha}}\frac{\partial}{\partial\bar{\theta}_{\dot\beta}}$$

Let us compute

$$4\int d^2\theta\,\theta^2 = \varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\;\theta$$

$d^4\theta = d^2\theta\, d^2\bar\theta$

$d^2\theta = \frac{1}{4}\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}$

$d^2\bar\theta = \frac{1}{4}\varepsilon_{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}$

Let's compute

$4\int d^2\theta\,\theta^2 = \varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\,\theta^{\alpha_1}\theta^{\alpha_2}\,\varepsilon_{\alpha_1\alpha_2}$

$$d^4\theta = d^2\theta\, d^2\bar\theta$$

$$d^2\theta = \frac{1}{4}\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}$$

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Let's compute

$$4\int d^2\theta\,\theta^2 = \varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\left[\theta^{\alpha_1}\theta^{\alpha_2}\right]\varepsilon_{\alpha_1\alpha_2}$$

$$= \varepsilon^{\alpha\beta}\varepsilon_{\alpha_1\alpha_2}\left[\delta^{\alpha_1}_{\beta}\delta^{\alpha_2}_{\alpha}\right.$$

$$d^4\theta = d^2\theta\, d^2\bar\theta$$

$$d^2\theta = \frac{1}{4}\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}$$

$$d^2\bar\theta = \frac{1}{4}\varepsilon_{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}$$

Let is complete

$$4\int d^2\theta\,\theta^2 = \varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\left[\theta^{\alpha_1}\theta^{\alpha_2}\right]\varepsilon_{\alpha_1\alpha_2}$$

$$= \varepsilon^{\alpha\beta}\varepsilon_{\alpha_1\alpha_2}\left[\delta^{\alpha_1}_{\beta}\delta^{\alpha_2}_{\alpha} - \delta^{\alpha_2}_{\beta}\delta\right.$$

$$d^4\theta = d^2\theta\, d^2\bar{\theta}$$

$$d^2\theta = \frac{1}{4}\,\varepsilon^{\alpha\beta}\,\frac{\partial}{\partial\theta^\alpha}\,\frac{\partial}{\partial\theta^\beta}$$

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Let's complete

$$4\int d^2\theta\,\theta^2 = \varepsilon^{\alpha\beta}\,\frac{\partial}{\partial\theta^\alpha}\,\frac{\partial}{\partial\theta^\beta}\left[\theta^{\alpha_1}\,\theta^{\alpha_2}\right]\varepsilon_{\alpha_1\alpha_2}$$

$$= \varepsilon^{\alpha\beta}\,\varepsilon_{\alpha_1\alpha_2}\left[\delta_\beta^{\alpha_1}\,\delta_\alpha^{\alpha_2} - \delta_\beta^{\alpha_2}\,\delta_\alpha^{\alpha_1}\right]$$

$$d^4\theta = d^2\theta\, d^2\bar\theta$$

$$d^2\theta = \frac{1}{4}\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}$$

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Let's compute

$$4\int d^2\theta\,\theta^2 = \varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\left[\theta^{\alpha_1}\theta^{\alpha_2}\right]\varepsilon_{\alpha_1\alpha_2}$$

$$= \varepsilon^{\alpha\beta}\varepsilon_{\alpha_1\alpha_2}\left[\delta^{\alpha_1}_{\beta}\delta^{\alpha_2}_{\alpha} - \delta^{\alpha_2}_{\beta}\delta^{\alpha_1}_{\alpha}\right] = 4$$

$$d^4\theta = d^2\theta\, d^2\bar\theta$$

$$\boxed{\int d^2\theta\, \theta^2 = 1}$$

$$d^2\theta = \frac{1}{4}\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}$$

$$d^2\bar\theta = \frac{1}{4}\varepsilon_{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}$$

Let is complete

$$4\int d^2\theta\, \theta^2 = \varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\left[\theta^{\alpha_1}\theta^{\alpha_2}\right]\varepsilon_{\alpha_1\alpha_2}$$

$$= \varepsilon^{\alpha\beta}\varepsilon_{\alpha_1\alpha_2}\left[\delta_\beta^{\alpha_1}\delta_\alpha^{\alpha_2} - \delta_\beta^{\alpha_2}\delta_\alpha^{\alpha_1}\right] = 4$$

$$d^4\theta = d^2\theta\, d^2\bar{\theta}$$

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$$\boxed{\int d^2\theta\,\theta^2 = 1}$$

Similarly:

$$\boxed{\int d^2\bar{\theta}\,\bar{\theta}^2 = 1}$$

Let's compute

$$4\int d^2\theta\,\theta^2 = \varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\left[\theta^{\alpha_1}\theta^{\alpha_2}\right]\varepsilon_{\alpha_1\alpha_2}$$

$$= \varepsilon^{\alpha\beta}\varepsilon_{\alpha_1\alpha_2}\left[\delta^{\alpha_1}_{\beta}\delta^{\alpha_2}_{\alpha} - \delta^{\alpha_2}_{\beta}\delta^{\alpha_1}_{\alpha}\right] = 4$$

$$d^4\theta = d^2\theta\, d^2\bar{\theta}$$

$$d^2\theta = \frac{1}{4}\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}$$

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Let is complete

$$4\int d^2\theta\, \theta^2 = \varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\left[\theta^{\alpha_1}\theta^{\alpha_2}\right]\varepsilon_{\alpha_1\alpha_2}$$

$$= \varepsilon^{\alpha\beta}\varepsilon_{\alpha_1\alpha_2}\left[\delta^{\alpha_1}_{\beta}\delta^{\alpha_2}_{\alpha} - \delta^{\alpha_2}_{\beta}\delta^{\alpha_1}_{\alpha}\right] = 4$$

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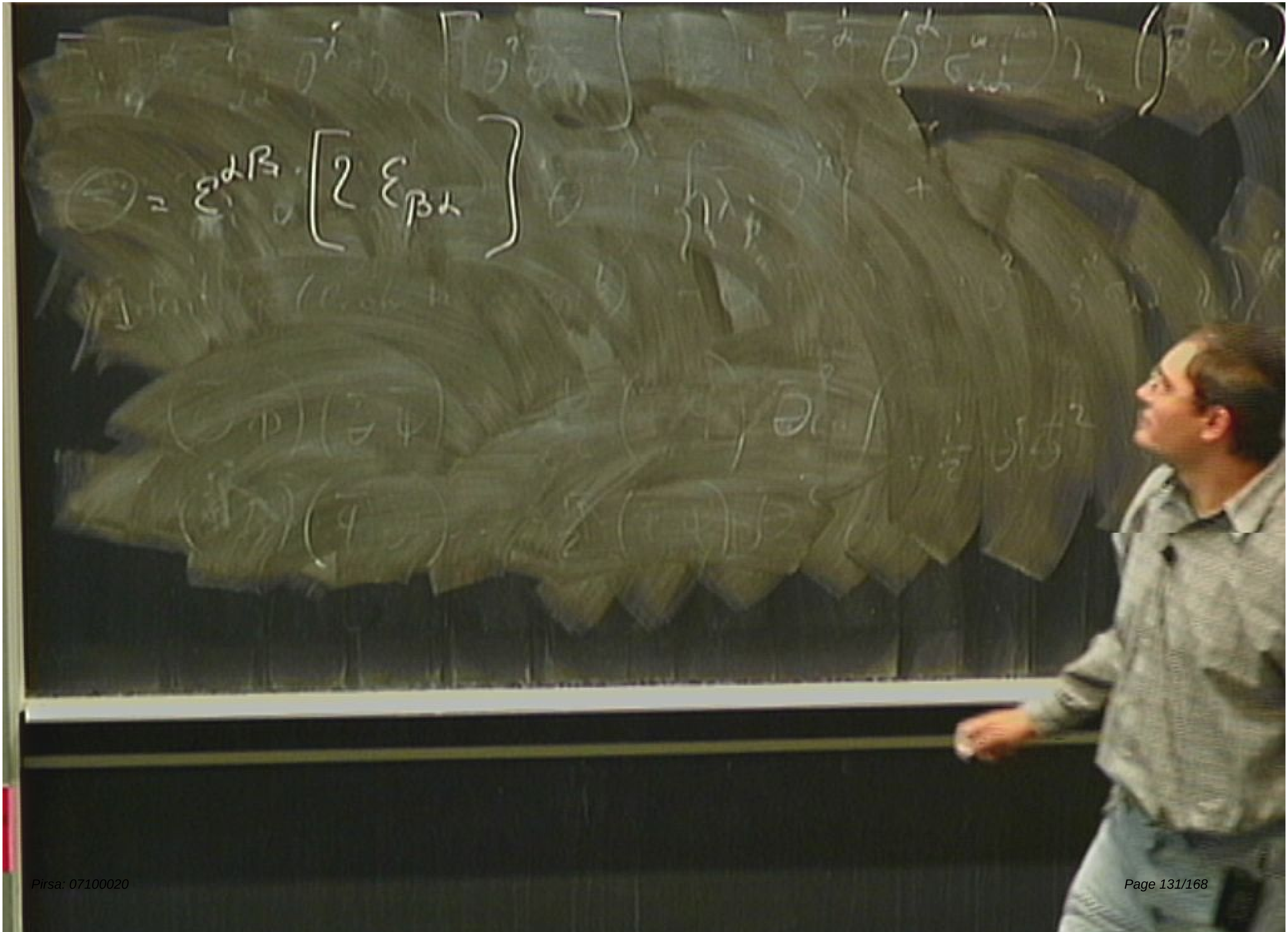
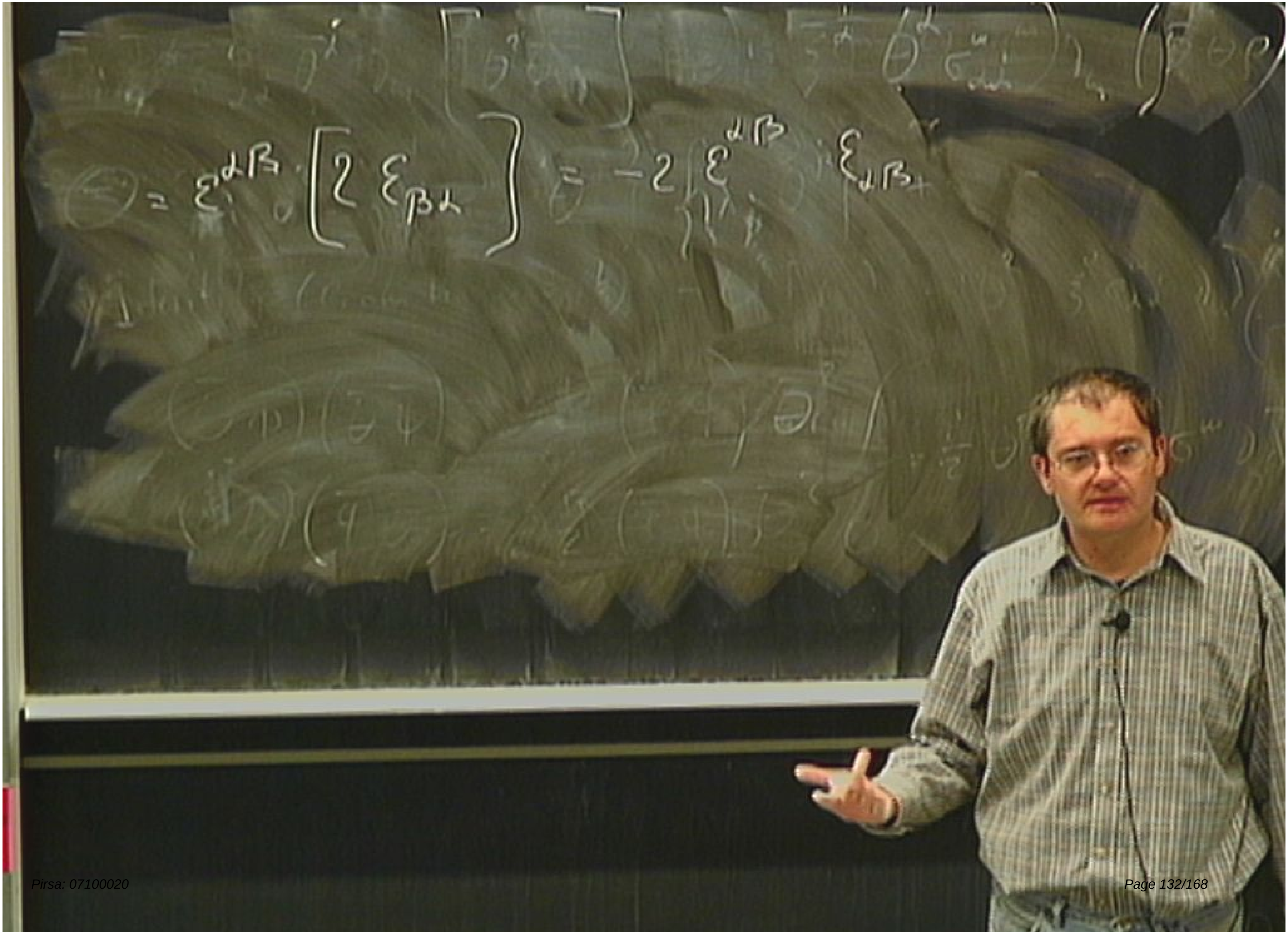
$$\boxed{\int d^2\bar\theta\, \bar\theta^2 = 1}$$


$\Theta = \varepsilon^{\alpha\beta} \cdot \left[ 2\, \varepsilon_{\beta\alpha} \right]$


$= \varepsilon^{\alpha\beta}\left[2\,\varepsilon_{\beta\alpha}\right] = -2\,\varepsilon^{\alpha\beta}\,\varepsilon_{\alpha\beta}$

$$\Theta = \varepsilon^{\alpha\beta} \cdot \left[ 2\, \varepsilon_{\beta\alpha} \right] = -2\, \varepsilon^{\alpha\beta}\, \varepsilon_{\alpha\beta} +$$

$$\Rightarrow \quad D = \int d^4\theta\ \Phi$$

$$= \varepsilon^{\alpha\beta}\left[2\,\varepsilon_{\beta\alpha}\right] = -2\,\varepsilon^{\alpha\beta}\,\varepsilon_{\alpha\beta}$$

$$\Rightarrow D = \int d^4\theta\, \Phi$$

$$= \varepsilon^{\alpha\beta}\left[2\,\varepsilon_{\beta\alpha}\right] = -2\,\varepsilon^{\alpha\beta}\,\varepsilon_{\alpha\beta}$$

$$\Rightarrow \quad D = \int d^4\theta\, \Phi$$

$\Rightarrow$ for a chiral superfield xsf: $\Phi(\theta, y) = \cdots + \frac{1}{4}\theta^2\bar{\theta}^2 D$

$$D = \frac{1}{4}\Box\phi \;\Rightarrow\; S = \int d^4x\, D_{xsf} = \frac{1}{4}\int d^4x\, \Box\phi = 0$$

$$\ominus = \varepsilon^{\alpha\beta} \left[ 2\, \varepsilon_{\beta\alpha} \right] = -2\, \varepsilon^{\alpha\beta}\, \varepsilon_{\alpha\beta}$$

$$\Rightarrow D = \int d^4\theta\, \Phi$$

$\Rightarrow$ for a chiral superfield $x s f$; $\Phi(\theta, y) = \cdots + \frac{1}{4}\bar{\theta}^2 \theta^2\, \Box\phi$

$$D = \frac{1}{4} \Box \phi \rightarrow S = \int d^4x\, D_{xsf} = \frac{1}{4}\int d^4x\, \Box\phi = 0$$

$\Rightarrow$ to get something we need both $x s \ell$ and $\overline{x s \ell}$

K

$\Rightarrow$ to get something we need both $x$ s.f. and $\overline{x \, s.f.}$

$$K = K(\Phi_i, \bar{\Phi}_j)$$

$\Rightarrow$ to get something we need both $\chi_{S}f$ and $\overline{\chi_{S}f}$

$$K \equiv K(\Phi_i, \overline{\Phi}_j)$$

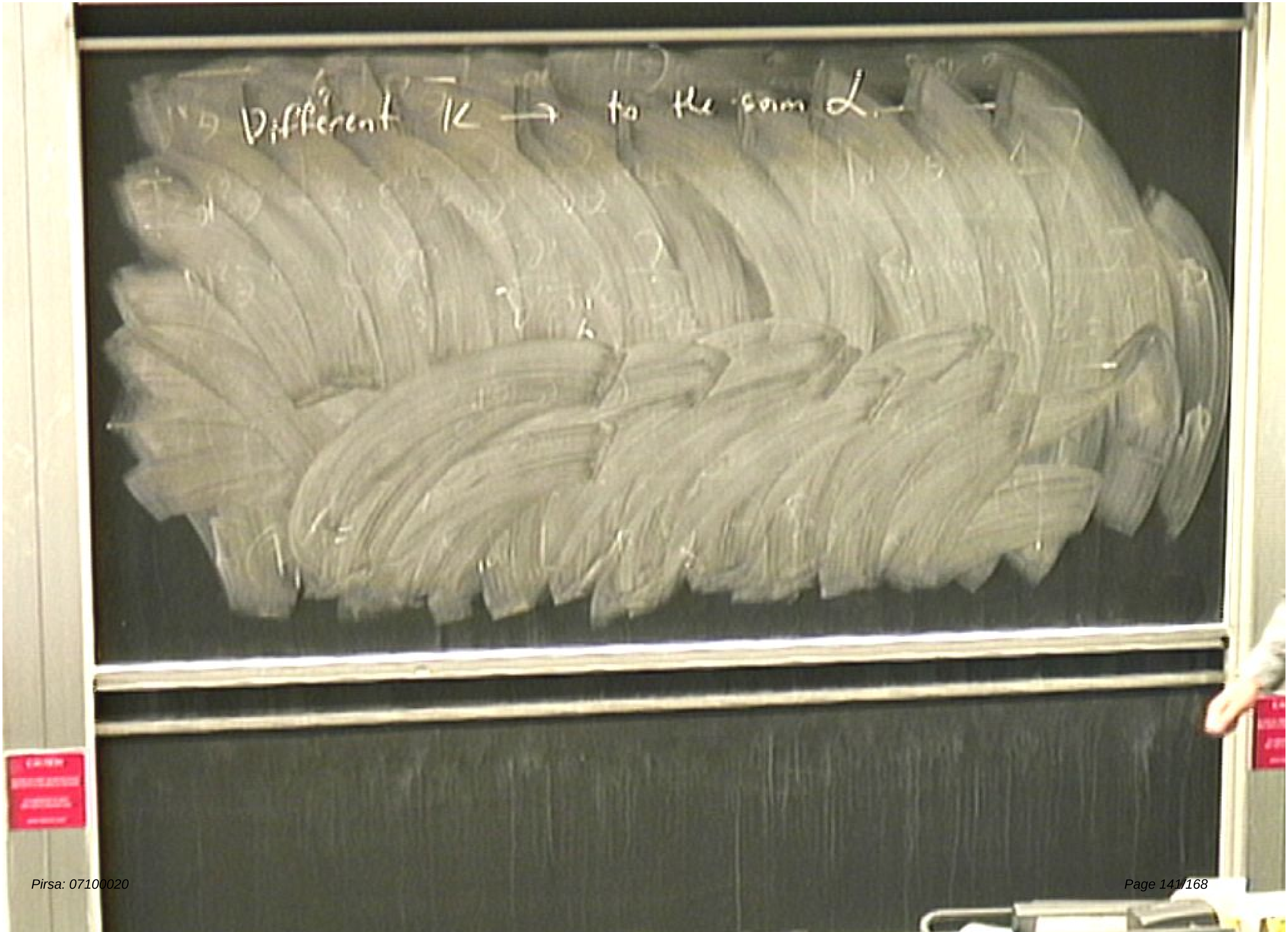
is a Kähler potential

$\Rightarrow$ to get something we need both $\chi$sf and $\overline{\chi\text{sf}}$

$$K \equiv K(\Phi_i, \overline{\Phi}_j)$$

is a Kähler potential

$$\mathcal{L} = \int d^4\theta\, K \rightarrow \text{automatically susy invariant.}$$


Different K → to the same L

Different $K \to$ to the same $\mathcal{L}$

$$K(\varphi_i, \bar{\varphi}_i) \Rightarrow K(\varphi_i, \bar{\varphi}_i) + f(\varphi_i)$$

Different $\mathbb{K} \to$ to the same $\mathcal{L}$

$$K(\varphi_i, \bar{\varphi}_i) \Rightarrow K(\varphi_i, \bar{\varphi}_i) + f(\varphi_i) + g(\bar{\varphi}_i)$$

$1^{st}$) Different $K \longrightarrow$ to the same $\mathcal{L}$ — x.s.f

$$K(\varphi_i, \bar{\varphi}_i) \Rightarrow K(\varphi_i, \bar{\varphi}_i) + \overbrace{f(\varphi_i)}^{\text{x.s.f}} + \underbrace{g(\bar{\varphi}_i)}_{\text{x.s.f}}$$

Different $K$ → to the same $\mathcal{L}$ — x.s.f

$$K(\Phi_i, \bar{\Phi}_i) \Rightarrow K(\Phi_i, \bar{\Phi}_i) + \overbrace{f(\Phi_i)}^{} + \underbrace{g(\bar{\Phi}_i)}_{x.s.f}$$

$$\mathcal{L} \longrightarrow \mathcal{L} + \int d^4\theta \, D$$

Different K → to the same $\mathcal{L}$ — xsf

$$K(\Phi_i, \bar{\Phi}_i) \Rightarrow K(\Phi_i, \bar{\Phi}_i) + f(\Phi_i)$$

$$+\, \underbrace{g(\bar{\Phi}_i)}_{xsf}$$

$$\mathcal{L} \to \mathcal{L} + \int d^4\theta\, D_{xsf}$$

Different K → to the same $\mathcal{L}$ — xsf

$$K(\Phi_i, \bar{\Phi}_i) \Rightarrow K(\Phi_i, \bar{\Phi}_i) + \overbrace{f(\Phi_i)}^{} + \underbrace{g(\bar{\Phi}_i)}_{\overline{xsf}}$$

$$\mathcal{L} \to \mathcal{L} + \underbrace{\int d^4\theta \, D_{xsf}} + \int d^4\theta \, D_{\overline{xsf}}$$

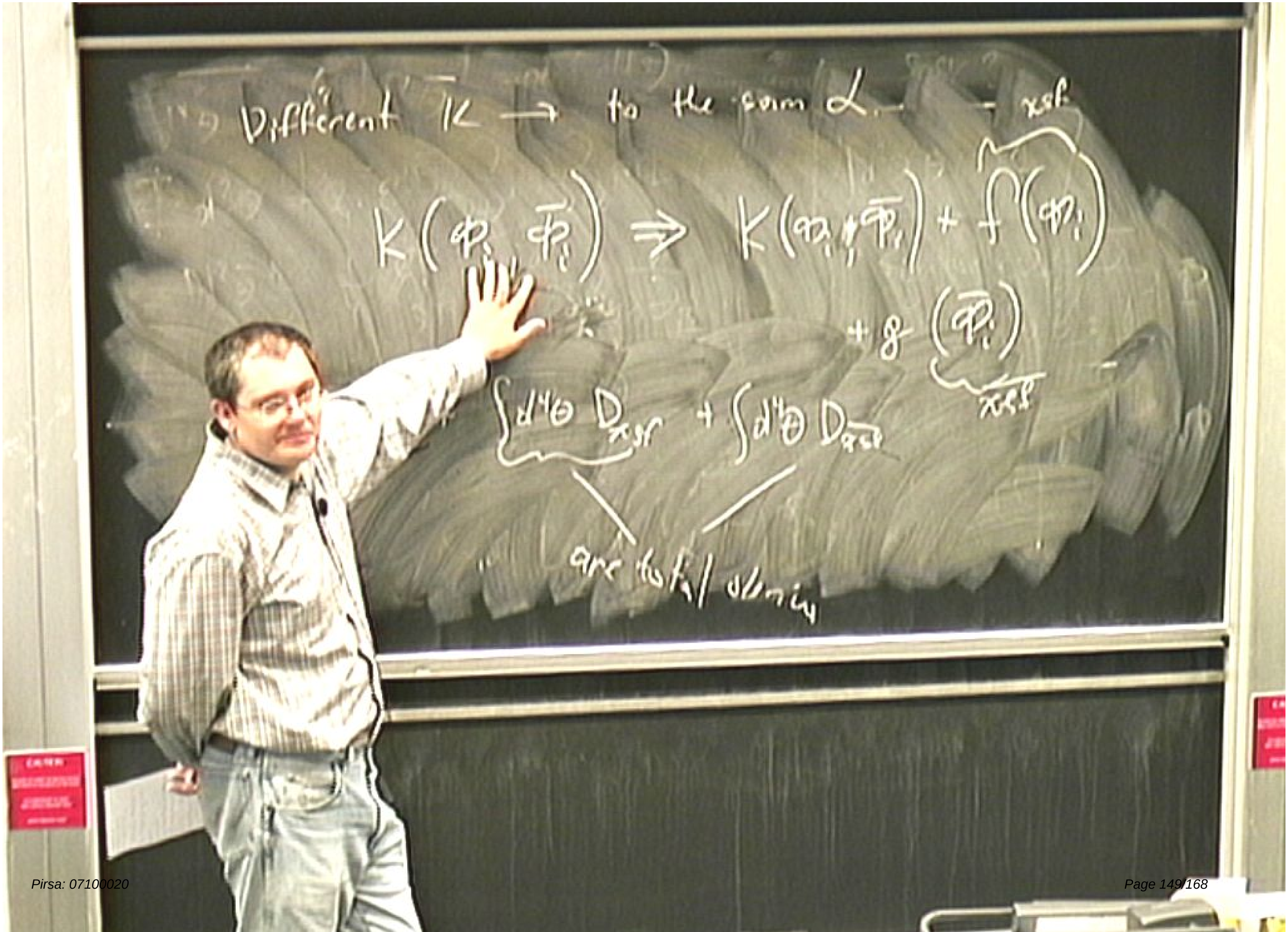
are total derivs

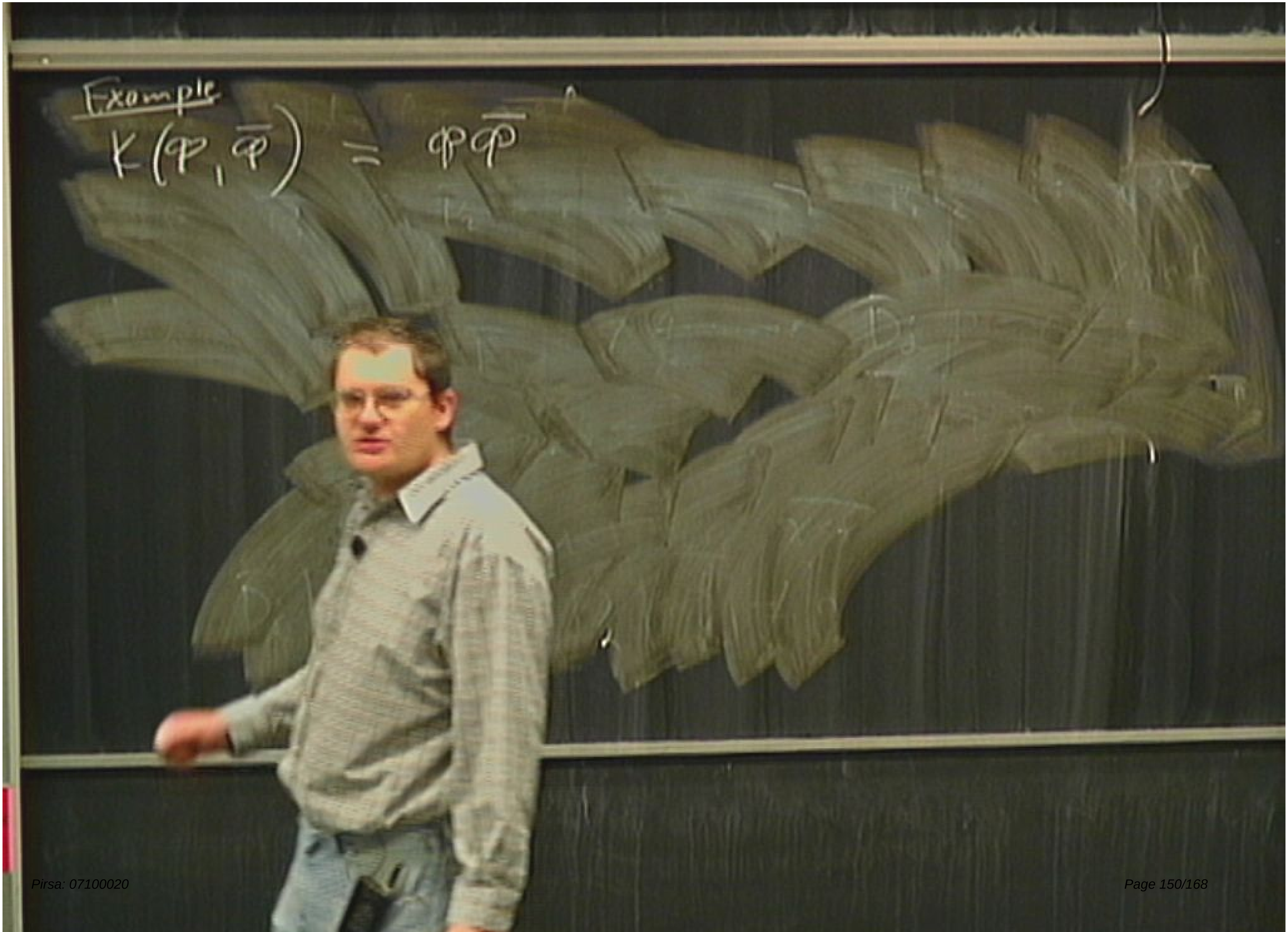
Different $K$ → to the same $\mathcal{L}$ — xsf

$$K(\Phi_i, \bar{\Phi}_i) \Rightarrow K(\Phi_i, \bar{\Phi}_i) + \overbrace{f(\Phi_i)}^{xsf} + \underbrace{g(\bar{\Phi}_i)}_{\overline{xsf}}$$

$$\mathcal{L} \to \mathcal{L} + \int d^4\theta \, D_{xsf} + \int d^4\theta \, D_{\overline{xsf}}$$

are total derivs


Different K → to the same L
K(Φi, Φ̄i) ⇒ K(Φi, Φ̄i) + f(Φi) + g(Φ̄i)
∫d⁴θ D + ∫d⁴θ D
are total


Example
$K(\varphi, \bar{\varphi}) = \varphi\bar{\varphi}$

Example

$$K(\varphi, \bar{\varphi}) = \varphi\bar{\varphi}$$

$\uparrow$ $\chi_{st}$ $\quad$ $\uparrow$ $\overline{\chi_{st}}$

Example

$$K(\varphi, \bar{\varphi}) = \varphi\bar{\varphi}$$

xst $\qquad$ $\overline{\text{xst}}$

$K = \varphi$

Example

$$K(\varphi, \bar{\varphi}) = \varphi\bar{\varphi}$$

$\uparrow$ $\chi$ s f $\quad$ $\uparrow$ $\overline{\chi \text{ s f}}$

$$K = \varphi + i\theta\sigma^{\mu}\bar{\theta}\,\partial_{\mu}\varphi + \frac{1}{4}\theta^{2}\bar{\theta}^{2}$$

Example

$$K(\varphi, \bar{\varphi}) = \varphi\bar{\varphi} \qquad K\Big|_{\theta^2\bar{\theta}^2} = ?$$

$\uparrow$ $\varphi$: $\chi$st, $\uparrow$ $\bar{\varphi}$: $\overline{\chi\text{st}}$

$$K = \varphi + i\theta\sigma^\mu\bar{\theta}\,\partial_\mu\varphi + \frac{1}{4}\theta^2\bar{\theta}^2$$

Example

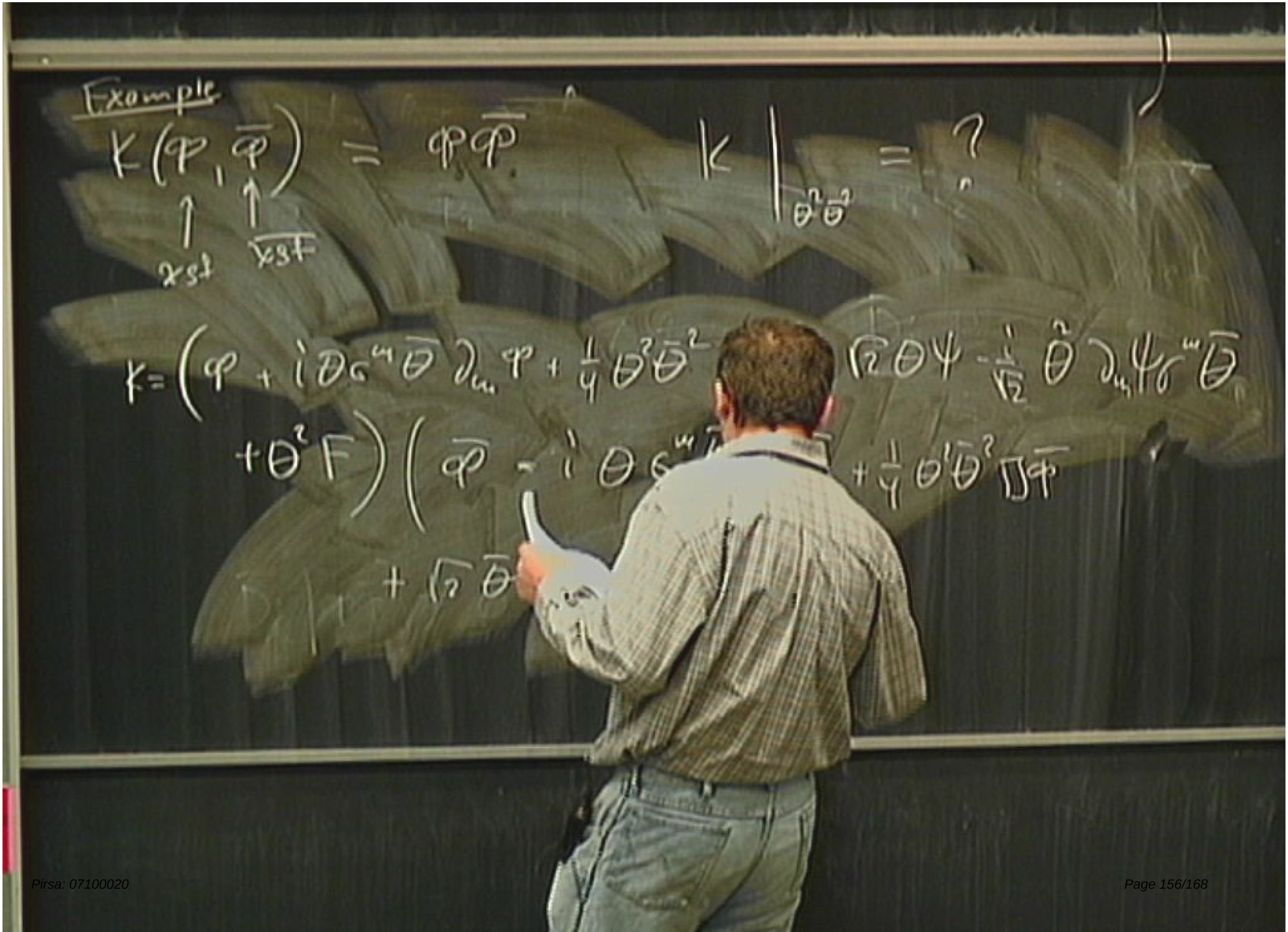
$$K(\varphi, \bar{\varphi}) = \varphi\bar{\varphi} \qquad K\Big|_{\theta^2\bar{\theta}^2} = ?$$

$\uparrow$ $\uparrow$

$x_{st}$ $\overline{x_{st}}$

$$K = \left(\varphi + i\theta\sigma^\mu\bar{\theta}\,\partial_\mu\varphi + \frac{1}{4}\theta^2\bar{\theta}^2\,\Box\varphi + \sqrt{2}\,\theta\psi - \frac{i}{\sqrt{2}}\,\theta^2\,\partial_\mu\psi\sigma^\mu\bar{\theta}\right.$$

$$\left. + \theta^2 F\right)\left(\bar{\varphi} - i\theta\right.$$


Example
K(φ, φ̄) = φφ̄
xsf
K|θ²θ̄² = ?

Example

$$K(\varphi, \bar{\varphi}) = \varphi\bar{\varphi} \qquad K\Big|_{\theta^2\bar{\theta}^2} = ?$$

$\uparrow$ $x s t$ $\quad$ $\uparrow$ $\overline{x s t}$

$$K = \Big(\varphi + i\theta\sigma^m\bar{\theta}\,\partial_m\varphi + \tfrac{1}{4}\theta^2\bar{\theta}^2\,\Box\varphi + \sqrt{2}\,\theta\psi - \tfrac{i}{\sqrt{2}}\,\theta^2\,\partial_m\psi\sigma^m\bar{\theta} + \theta^2 F\Big)\Big(\bar{\varphi} - i\,\theta\sigma^m\bar{\theta}\,\partial_m\bar{\varphi} + \tfrac{1}{4}\theta^2\bar{\theta}^2\,\Box\bar{\varphi} + \sqrt{2}\,\bar{\theta}\bar{\psi} + \tfrac{i}{\sqrt{2}}\,\bar{\theta}^2\,\theta\sigma^m\partial_m\bar{\psi} + \bar{\theta}^2\bar{F}\Big)$$

Example

$$K(\varphi, \bar{\varphi}) = \varphi\bar{\varphi} \qquad K\Big|_{\theta^2\bar{\theta}^2} = ?$$

$\uparrow$ xst $\quad \uparrow \overline{\text{xst}}$

$$K = \Big(\varphi + \underline{i\theta\sigma^\mu\bar{\theta}\,\partial_\mu\varphi} + \tfrac{1}{4}\theta^2\bar{\theta}^2\,\Box\varphi + \sqrt{2}\,\theta\psi - \tfrac{i}{\sqrt{2}}\theta^2\,\partial_\mu\psi\sigma^\mu\bar{\theta}$$

$$+ \underline{\theta^2 F}\Big)\Big(\bar{\varphi} - \underline{i\,\theta\sigma^\mu\bar{\theta}\,\partial_\mu\bar{\varphi}} + \tfrac{1}{4}\theta^2\bar{\theta}^2\,\Box\bar{\varphi}$$

$$+ \sqrt{2}\,\bar{\theta}\bar{\psi} + \tfrac{i}{\sqrt{2}}\bar{\theta}^2\,\theta\sigma^\mu\partial_\mu\bar{\psi} + \underline{\bar{\theta}^2\bar{F}}\Big)$$

Example

$$K(\varphi, \bar\varphi) = \varphi\bar\varphi \qquad K\Big|_{\theta^2\bar\theta^2} = ?$$

$\varphi$: $x s t$, $\bar\varphi$: $\overline{x s t}$

$$K = \Big(\varphi + i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi + \tfrac{1}{4}\theta^2\bar\theta^2\,\Box\varphi + \sqrt{2}\,\theta\psi - \tfrac{i}{\sqrt{2}}\,\theta^2\,\partial_\mu\psi\sigma^\mu\bar\theta$$
$$+\,\theta^2 F\Big)\Big(\bar\varphi - i\,\theta\sigma^\mu\bar\theta\,\partial_\mu\bar\varphi + \tfrac{1}{4}\theta^2\bar\theta^2\,\Box\bar\varphi$$
$$+\,\sqrt{2}\,\bar\theta\bar\psi + \tfrac{i}{\sqrt{2}}\,\bar\theta^2\,\theta\sigma^\mu\partial_\mu\psi + \bar\theta^2\bar F\Big)$$

$$\phi\bar{\phi}\Big|_{\theta^2\bar{\theta}^2} = \qquad \partial_\mu\phi\,\partial\bar{\phi} = \partial_\mu\phi\,\partial^\mu\bar{\phi}$$

$$= -\partial\phi\,\partial\bar{\phi} + i\,\partial_\mu\bar{\psi}\,\bar{\sigma}^\mu\psi + F\bar{F}$$

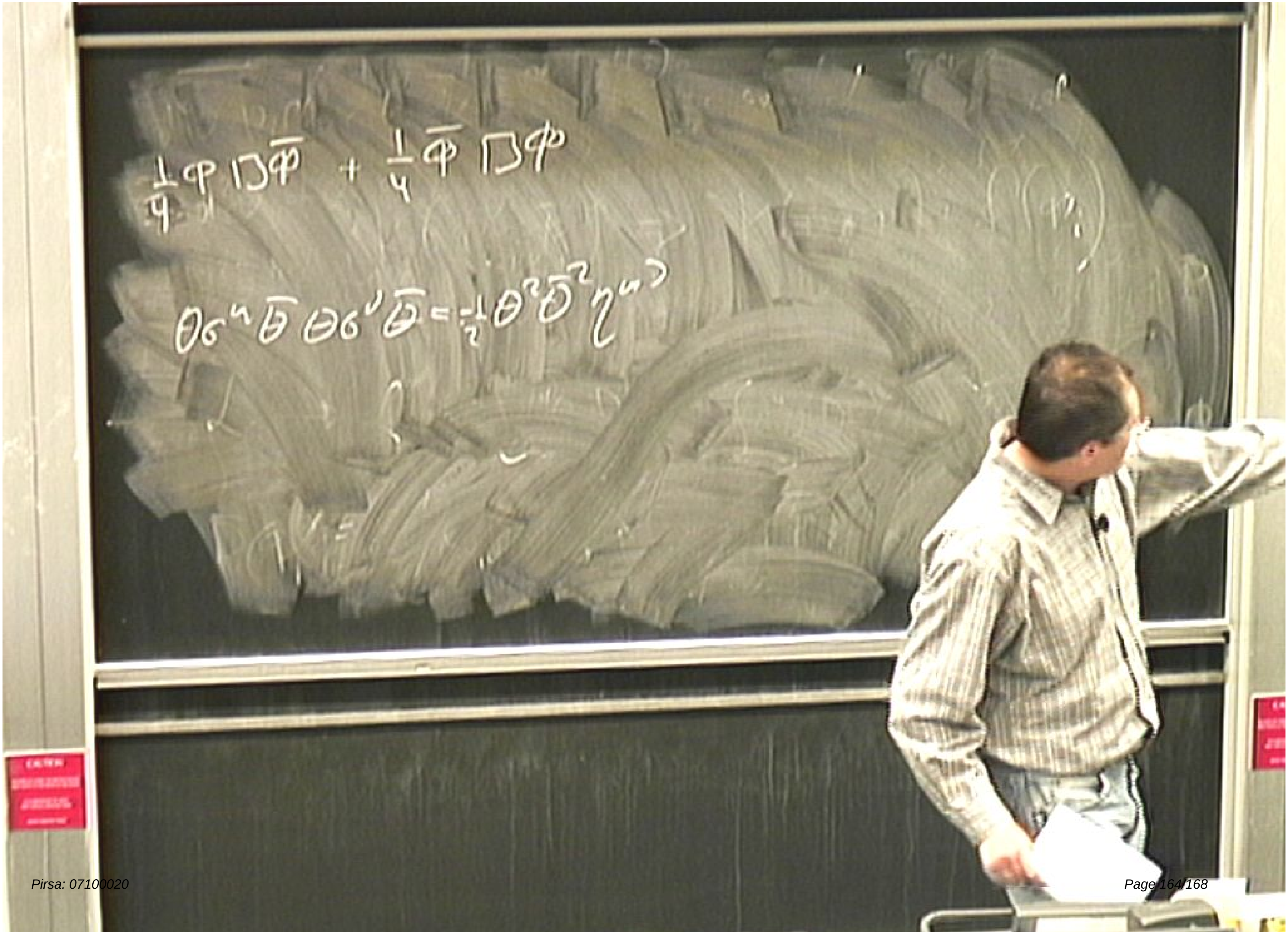
$$\Phi^\dagger \bar{\Phi} \Big|_{\theta^2\bar{\theta}^2} = \qquad \boxed{\partial_\mu \varphi\, \partial \bar{\varphi} = \partial_\mu \varphi\, \partial^\mu \bar{\varphi}}$$

$$= -\partial \varphi\, \partial \bar{\varphi} + i\, \partial_\mu \bar{\psi}\, \bar{\sigma}^\mu \psi + F\bar{F}$$

$$\phi\bar{\phi}\Big|_{\theta^2\bar{\theta}^2} = \boxed{\begin{array}{l}\partial_\mu\phi\,\partial^\mu\bar{\phi} = \partial_\mu\phi\,\partial^\mu\bar{\phi} \\ \partial_\mu\bar{\psi}\,\bar{\sigma}^\mu\psi = \partial_\mu\bar{\psi}_{\dot{\alpha}}\,\bar{\sigma}^{\mu\dot{\alpha}\alpha}\,\psi_\alpha\end{array}}$$

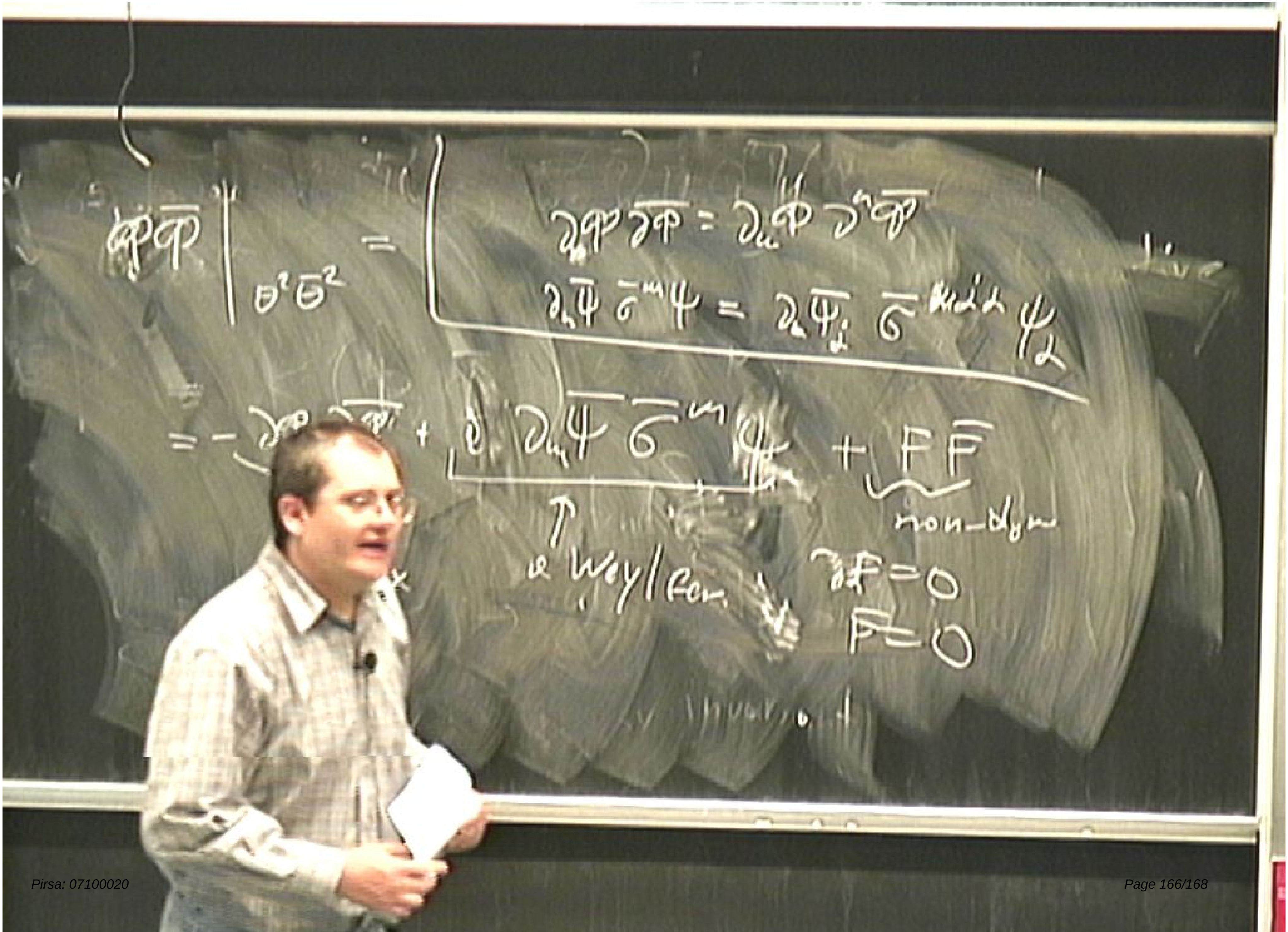
$$= -\partial\phi\,\partial\bar{\phi} + i\,\partial_\mu\bar{\psi}\,\bar{\sigma}^\mu\psi + F\bar{F}$$

$\frac{1}{2}\varphi \, \Box \, \bar{\varphi}$


$\frac{1}{4}\varphi\,\Box\bar{\varphi} + \frac{1}{4}\bar{\varphi}\,\Box\varphi$
$\theta\sigma^\mu\bar{\theta}\,\theta\sigma^\nu\bar{\theta} = -\frac{1}{2}\theta^2\bar{\theta}^2\eta^{\mu\nu}$

$$\frac{1}{4}\Phi\Box\bar\Phi + \frac{1}{4}\bar\Phi\Box\Phi - \frac{1}{2}\partial\Phi\partial\bar\Phi$$

$$\theta\sigma^\mu\bar\theta\,\theta\sigma^\nu\bar\theta = -\frac{1}{2}\theta^2\bar\theta^2\eta^{\mu\nu}$$

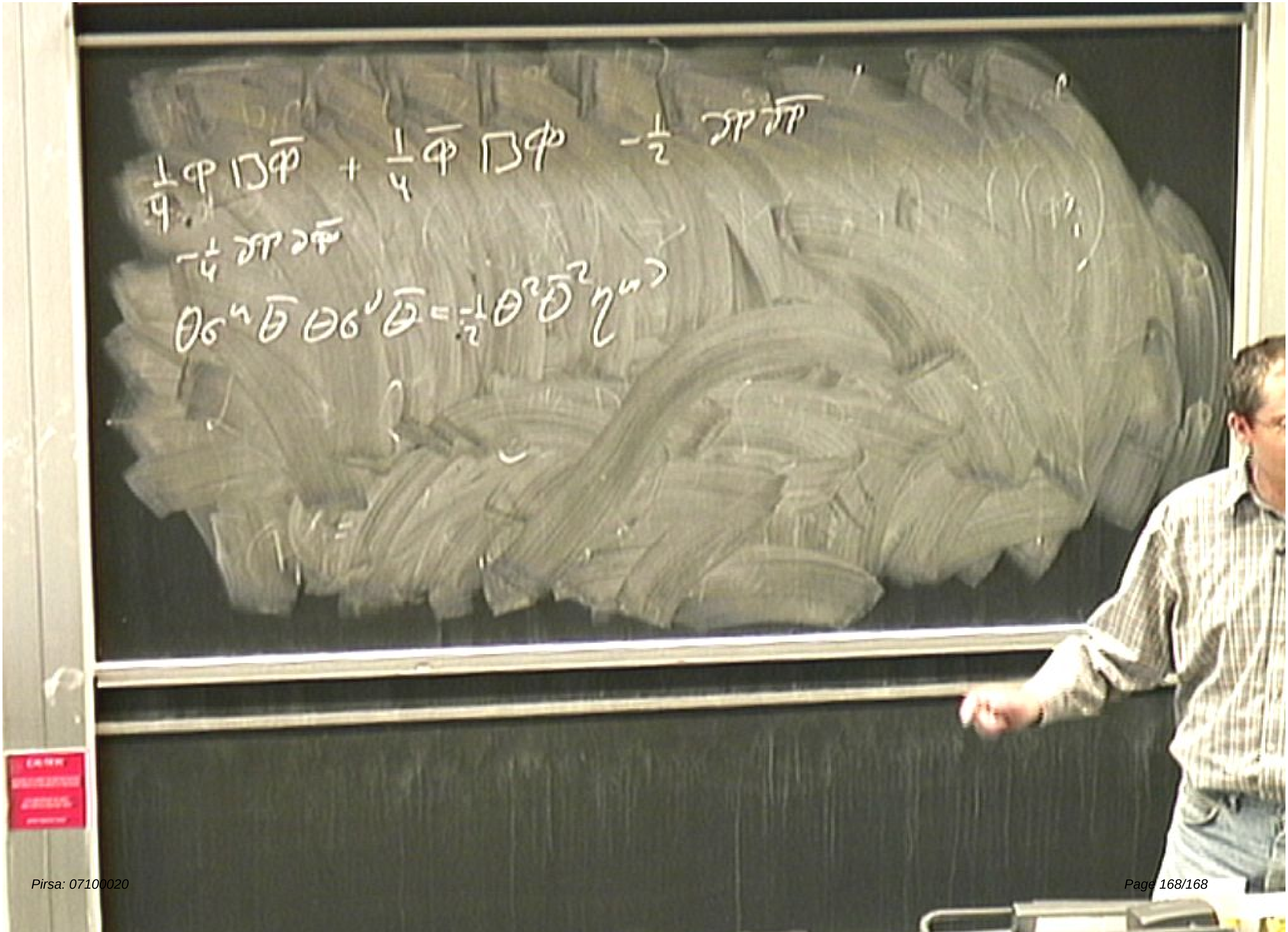

non-dyn
Weyl fer.
F = 0

$$\Phi\bar{\Phi}\Big|_{\theta^2\bar{\theta}^2} = \boxed{\begin{array}{l} \partial_m\Phi\,\bar{\partial}\bar{\Phi} = \partial_m\Phi\,\partial^m\bar{\Phi} \\ \partial_m\bar{\Psi}\,\bar{\sigma}^m\Psi = \partial_m\bar{\Psi}_{\dot\alpha}\,\bar{\sigma}^{m\dot\alpha\alpha}\,\psi_\alpha \end{array}}$$

$$= -\underbrace{\partial\varphi\,\partial\bar{\varphi}}_{\text{a complex}} + \underbrace{i\,\partial_m\bar{\Psi}\,\bar{\sigma}^m\,\psi}_{\text{a Weyl fer.}} + \cancel{\underbrace{F\bar{F}}_{\text{non-dyn}}}$$

$\partial F = 0$

$F = 0$


$\frac{1}{4}\Phi\,\Box\bar{\Phi} + \frac{1}{4}\bar{\Phi}\,\Box\Phi \; -\frac{1}{2}\,\partial\Phi\,\partial\bar{\Phi}$
$-\frac{1}{4}\,\partial\Phi\,\partial\bar{\Phi}$
$\theta\sigma^{\mu}\bar{\theta}\,\theta\sigma^{\nu}\bar{\theta} = -\frac{1}{2}\theta^{2}\bar{\theta}^{2}\eta^{\mu\nu}$