

Title: Intro to Supersymmetry 10

Date: Oct 18, 2007 12:30 PM

URL: <http://pirsa.org/07100020>

Abstract:

$(X^{\text{III}}, \theta_{\alpha}, \bar{\theta}_{\dot{\alpha}}) \leftarrow \text{super space.}$

$$\Phi(X^{\text{IV}}, \theta_{\alpha}, \bar{\theta}_{\dot{\alpha}}) = \phi + \theta\psi + \bar{\theta}\chi + \theta^2 f + \bar{\theta}^2 g$$

$(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}) \leftarrow \text{super space.}$

$$\Phi(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}) = \phi + \theta\psi + \bar{\theta}\bar{\chi} + \theta^2 F + \bar{\theta}^2 G \\ + \theta\sigma^\mu\bar{\theta} A_\mu + \theta^2\bar{\theta}\lambda + \bar{\theta}^2\theta\rho + \theta^2\bar{\theta}^2 D$$

$\phi, \psi, \bar{\chi}, F, G, A_\mu, \lambda, \rho, D$ are fields

\Rightarrow a general superfield is not in \mathfrak{m} .

\Rightarrow a general superfield is not in ir. repr. of SUSY algebra

\Rightarrow a general superfield is not in ir. repr. of SUSY algebra

$$\square \cdot \Phi = 0 \Leftrightarrow$$

^{important}
⇒ a general superfield is not in ir. repr. of SUSY algebra

$\bar{D}_i \Phi = 0 \Leftrightarrow$ is called a chiral superfield.

$$\bar{D}_i = -\frac{\partial}{\partial \theta^i} - i \theta^k \sigma_{ki}^m \partial_m$$



^{important}
 \Rightarrow a general superfield is not in the repr. of SUSY algebra

$\bar{D}_i \Phi = 0 \Leftrightarrow$ is called a chiral superfield.

$$\bar{D}_i = -\frac{\partial}{\partial \bar{\theta}^i} - i \theta^k \sigma_{ki}^m \partial_m$$

\Rightarrow we look at "chiral lines"

^{important}
 \Rightarrow a general superfield is not in the repr. of SUSY algebra

$\bar{D}_i \Phi = 0 \Leftrightarrow$ is called a chiral superfield.

$$\bar{D}_i = -\frac{\partial}{\partial \bar{\theta}^i} - i \theta^k \sigma_{ki}^m \partial_m$$

\Rightarrow we look at "chiral coordinates"

$$\bar{D}_i \theta_j = 0 \quad \bar{D}_i y^m = 0$$

\Rightarrow a general superfield is not in ir. repr. of SUSY algebra

$\bar{D}_i \Phi = 0 \Leftrightarrow$ is called a chiral superfield.

$$\bar{D}_i = -\frac{\partial}{\partial \bar{\theta}^i} - i \theta^m \sigma_{mi}^n \partial_n$$

\Rightarrow we look at "chiral coordinates"

$$\bar{D}_i \theta_j = 0 \quad \bar{D}_i y^n = 0 \quad y^n = x^n + i \theta \sigma^n \bar{\theta}$$

\Rightarrow a general superfield is not in the repr. of SUSY algebra

$\bar{D}_i \Phi = 0 \Leftrightarrow$ is called a chiral superfield.

$$\bar{D}_i = -\frac{\partial}{\partial \bar{\theta}^i} - i \theta^m \sigma_{mi}^n \partial_n$$

\Rightarrow we look at "chiral coordinates"

$$\bar{D}_i \theta_p = 0 \quad \bar{D}_i y^m = 0 \quad y^m = x^m + i \theta \sigma^m \bar{\theta}$$

$$\bar{D}_i \Phi = 0 \iff \Phi(x^n, \theta_n, \bar{\theta}_i) = \bar{\Phi}(\theta, y^n)$$

$$\bar{D}_i \Phi = 0 \Leftrightarrow \Phi(x^k, \theta_k, \bar{\theta}_i) = \bar{\Phi}(\theta, y^k)$$

\Rightarrow we can do Taylor expansion in θ

ϕ

$$\bar{D}_i \Phi = 0 \Leftrightarrow \Phi(x^k, \theta_k, \bar{\theta}_k) = \bar{\Phi}(\theta, y^k)$$

\Rightarrow we can do Taylor expansion in θ

$$\bar{\Phi} = \varphi(y) + \sqrt{\epsilon} \theta \psi(y) + \theta^2 F(y)$$

$$\bar{D}_i \Phi = 0 \Leftrightarrow \Phi(x^{\mu}, \theta_{\alpha}, \bar{\theta}_{\dot{\alpha}}) = \bar{\Phi}(\theta, y^m)$$

\Rightarrow we can do Taylor expansion in θ

$$\Phi = \underbrace{\varphi(y)}_{\substack{\uparrow \\ \text{a complex} \\ \text{scalar}}} + \sqrt{2} \theta \underbrace{\psi(y)}_{\substack{\uparrow \\ \text{a Weyl} \\ \text{fermion}}} + \theta^2 \underbrace{F(y)}_{\uparrow}$$

$$\bar{D}_i \Phi = 0 \Leftrightarrow \Phi(x^m, \theta_a, \bar{\theta}_i) = \bar{\Phi}(\theta, y^m)$$

\Rightarrow we can do Taylor expansion in θ

$$\Phi = \varphi(y) + \sqrt{2} \theta \psi(y) + \theta^2 F(y)$$

↑
a complex scalar

↑
a Weyl fermion

↑
a complex scalar (nondynamical)

$$\bar{D}_i \Phi = 0 \Leftrightarrow \Phi(x^a, \theta_a, \bar{\theta}_i) = \bar{\Phi}(\theta, y^m)$$

\Rightarrow we can do Taylor expansion in θ

$$\Phi = \varphi(y) + \sqrt{2} \theta \psi(y) + \theta^2 F(y)$$

↑
a complex scalar

↑
a Weyl fermion

↑
a complex scalar (nondynamics)

S_{SUSY}

$$\bar{D}_\alpha \bar{\Phi} = 0 \Leftrightarrow \Phi(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}) = \bar{\Phi}(\theta, y^m)$$

\Rightarrow we can do Taylor expansion in θ

$$\Phi = \varphi(y) + \sqrt{2} \theta \psi(y) + \theta^2 F(y)$$

\uparrow
a complex
scalar

\uparrow
a Weyl
fermion

\uparrow
a complex scalar (nondynamical)

IF $\bar{D}_\alpha \bar{\Phi} = 0 \Rightarrow \bar{D}_\alpha \delta_{\text{susy}} \bar{\Phi} = 0$

85084 ②

85094 Φ

$\mathbb{R} \times \mathbb{S}^1 \times \mathbb{S}^1$ is the highest component

χ $\delta_{\text{SU}(4)}$ \mathbb{E} is the highest component $\theta^2 \bar{\theta}^2$



x δ_{usy} \mathbb{E} is the highest component $\theta^2 \bar{\theta}^2$

\Downarrow
 x the highest component of $x\theta f$ is a total derivative

\Downarrow

χ $\delta_{\text{SUSY}} \Phi$ is the highest component $\theta^2 \bar{\theta}^2$

\Rightarrow
 χ the highest component of $\chi \psi$ is a total derivative

\Rightarrow

$$\mathcal{L}_{\text{SUSY}} = \int d^2\theta d^2\bar{\theta} \Phi$$

\Downarrow
* the highest component of χ_{sf} is a total derivative

\Downarrow

$$\mathcal{L}_{\text{susy}} = \int d^2\theta d^2\bar{\theta} \Phi$$

$\delta_{\chi_{sf}} \mathcal{L}_{\text{susy}} = \text{total derivative.}$

$S = \int_{M_4} d^4x \mathcal{L}_{\text{susy}}$ is δ_{susy} invariant

$\chi \delta_{\text{susy}} \Phi$ is the highest component $\theta^2 \bar{\theta}^2$

χ the highest component of $\chi \psi$ is a total derivative

$$\mathcal{L}_{\text{susy}} = \int d^2\theta d^2\bar{\theta} \Phi$$

$\delta_{\text{susy}} \mathcal{L}_{\text{susy}} = \text{total derivative}$

$S = \int_{M_4} \mathcal{L}_{\text{susy}}$ is δ_{susy} invariant

* $\delta_{\text{susy}} \Phi$ is the highest component $\theta^2 \bar{\theta}^2$

↓
* the highest component of $\chi \psi$ is a total derivative

⇓

$$\mathcal{L}_{\text{susy}} = \int d^2\theta d^2\bar{\theta} \Phi$$

$\delta_{\text{susy}} \mathcal{L}_{\text{susy}} = \text{total derivative}$

$S = \int_{\mathcal{M}_4} d^4x \mathcal{L}_{\text{susy}}$ is δ_{susy} invariant

* $\delta_{\text{susy}} \Phi \Rightarrow$ concentrate on highest component $\theta^2 \bar{\theta}^2$

* \Downarrow
* the highest component of $\chi \psi$ is a total derivative

\Downarrow

$$\mathcal{L}_{\text{susy}} = \int d^2\theta d^2\bar{\theta} \Phi$$

$\delta_{\text{susy}} \mathcal{L}_{\text{susy}} = \text{total derivative}$

$S = \int_{M_4} d^4x \mathcal{L}_{\text{susy}}$ is δ_{susy} invariant

\Rightarrow how does D of a 2SP transform under Sisy?

$$\Phi(x_t; \theta, \bar{\theta}) = \varphi(y)$$

\Rightarrow how does D of a $2SP$ transform under $SUSY$?

$$\Phi(\underbrace{x + i\theta\sigma^3\bar{\theta}}_y) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$$\bar{\Phi}(y) = \bar{\phi}(y) + i\theta\sigma^3\bar{\theta}\partial_y\bar{\phi}$$

\Rightarrow how does D of a 2SP transform under S/SY?

$$\Phi(\underbrace{x + i\theta c \bar{\theta}}_y) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$$\pi(y) = \phi(x) + i\theta c^n \bar{\theta} \partial_n \phi + \frac{1}{2!} (i\theta c^n \bar{\theta})(i\theta c^m \bar{\theta}) \partial_n \partial_m \phi$$

\swarrow
 $x + i\theta c \bar{\theta}$



$00^{\circ} \theta$

$$\text{If } D_x \psi = 0 \Rightarrow D_y \epsilon_{xy} \psi = 0$$

I think :

$00^{\circ} \theta) \psi$

Identity :

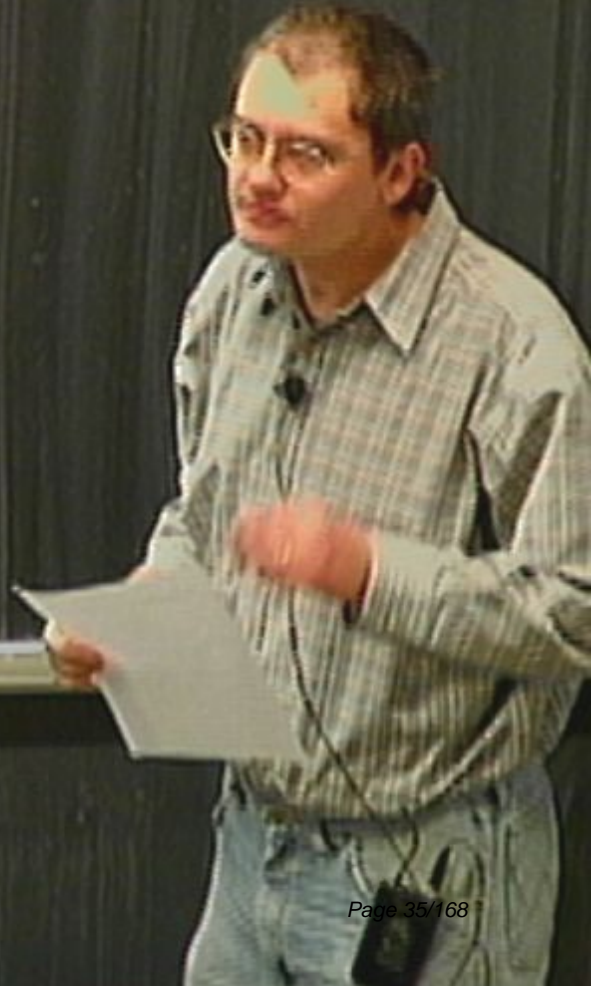
$$\theta \circ \theta = \theta$$

Identity :

$$(\theta \sigma^m \bar{\theta})(\theta \sigma^0 \bar{\theta}) = -\frac{1}{2} \theta^2 \bar{\theta}^2 \psi^m$$

Identity :

$$(\theta \sigma^m \bar{\theta})(\theta \sigma^0 \bar{\theta}) = -\frac{1}{2} \theta^2 \bar{\theta}^2 \gamma^{\mu\nu}$$



⇒ How does D of a 2SP transform under SUSY?

$$\Phi(\underbrace{x + i\theta\sigma^m\bar{\theta}}_y) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$$\begin{aligned} \Phi(y) = & \phi(x + i\theta\sigma^m\bar{\theta}) + i\theta\sigma^m\bar{\theta}\partial_m\phi + \underbrace{\frac{1}{2!}(i\theta\sigma^m\bar{\theta})(i\theta\sigma^n\bar{\theta})\partial_m\partial_n\phi}_{-\frac{1}{2}(-\frac{1}{2})\theta^2\bar{\theta}^2\eta^{mn}\partial_m\partial_n\phi} \end{aligned}$$

⇒ How does D of a 2SP transform under SUSY?

$$\Phi(\underbrace{x + i\theta\sigma^{\mu\nu}\bar{\theta}}_y) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$$\begin{aligned} \Phi(y) = \phi(x + i\theta\sigma^{\mu\nu}\bar{\theta}) &+ i\theta\sigma^{\mu\nu}\bar{\theta}\partial_{\mu\nu}\phi + \underbrace{\frac{1}{2!}(i\theta\sigma^{\mu\nu}\bar{\theta})(i\theta\sigma^{\rho\sigma}\bar{\theta})}_{-\frac{1}{2}(-\frac{1}{2})\theta^2\bar{\theta}^2\gamma^{\mu\nu\rho\sigma}}\partial_{\mu\nu}\partial_{\rho\sigma}\phi \\ &+ \frac{1}{4}\theta^2\bar{\theta}^2\Box\phi \end{aligned}$$

⇒ How does D of a 2SP transform under SUSY?

$$\Phi(\underbrace{x + i\theta\sigma^{\mu\nu}\bar{\theta}}_y) = \underline{\phi(y)} + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$$\begin{aligned} \Phi(y) = \phi(x + i\theta\sigma^{\mu\nu}\bar{\theta}) &+ i\theta\sigma^{\mu\nu}\bar{\theta}\partial_{\mu\nu}\phi + \underbrace{\frac{1}{2!}(i\theta\sigma^{\mu\nu}\bar{\theta})(i\theta\sigma^{\rho\sigma}\bar{\theta})}_{-\frac{1}{2}(-\frac{1}{2})\theta^2\bar{\theta}^2\gamma^{\mu\nu\rho\sigma}}\partial_{\mu\nu}\partial_{\rho\sigma}\phi \\ &+ \frac{1}{4}\theta^2\bar{\theta}^2\Box\phi \end{aligned}$$

Identity :

$$(\theta \sigma^m \bar{\theta})(\theta \sigma^0 \bar{\theta}) = -\frac{1}{2} \theta^2 \bar{\theta}^2 \eta^{\mu\nu}$$

$$\theta \psi(y) =$$



Identity :

$$(\theta \sigma^\mu \bar{\theta}) (\theta \sigma^\nu \bar{\theta}) = -\frac{1}{2} \theta^2 \bar{\theta}^2 \eta^{\mu\nu}$$

$$\theta \psi(y) = \theta^\alpha \psi_\alpha(x^{\hat{a}}; \theta \sigma^\mu \bar{\theta}) = \theta^\alpha \psi_\alpha + (i \theta \sigma^\mu \bar{\theta}) \theta^\alpha \dot{\psi}_\alpha$$

⇒ how does D of a 2SP transform under SUSY?

$$\Phi(\underbrace{x + i\theta\sigma^{\mu\nu}\bar{\theta}}_y) = \underbrace{\phi(y)} + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$$\begin{aligned} \Phi(y) &= \phi(x + i\theta\sigma^{\mu\nu}\bar{\theta}) + \frac{1}{2!} \underbrace{(i\theta\sigma^{\mu\nu}\bar{\theta})(i\theta\sigma^{\rho\sigma}\bar{\theta})}_{-\frac{1}{2}(-\frac{1}{2})\theta^2\bar{\theta}^2\gamma^{\mu\nu\rho\sigma}} \partial_{\mu}\partial_{\rho}\phi \\ &\quad + \frac{1}{4}\theta^2\bar{\theta}^2\Box\phi \end{aligned}$$

Identity :

$$(\theta \sigma^\mu \bar{\theta}) (\theta \sigma^\nu \bar{\theta}) = -\frac{1}{2} \theta^2 \bar{\theta}^2 \eta^{\mu\nu}$$

$$\theta \psi(y) = \theta^\alpha \psi_\alpha(x^{\mu\nu}; \theta \sigma^\mu \bar{\theta}) = \theta^\alpha \psi_\alpha + (i \theta \sigma^\mu \bar{\theta}) \theta^\alpha \dot{\psi}_\alpha$$

Identity :



Identity :

$$(\theta \sigma^{\mu} \bar{\theta}) (\theta \sigma^{\nu} \bar{\theta}) = -\frac{1}{2} \theta^2 \bar{\theta}^2 \eta^{\mu\nu}$$

$$\theta \psi(y) = \theta^{\alpha} \psi_{\alpha}(x^{\mu}; \theta \sigma^{\mu} \bar{\theta}) = \theta^{\alpha} \psi_{\alpha} + (i \theta \sigma^{\mu} \bar{\theta}) \theta^{\alpha} \dot{\psi}_{\alpha}$$

Identity :

$$(\theta \varphi) (\theta \psi) = \theta^{\alpha} \varphi_{\alpha}$$

Identity :

$$(\theta \sigma^\mu \bar{\theta})(\theta \sigma^\nu \bar{\theta}) = -\frac{1}{2} \theta^2 \bar{\theta}^2 \eta^{\mu\nu}$$

$$\theta \psi(y) = \theta^\alpha \psi_\alpha(x^{\mu\nu}; \theta \sigma^\mu \bar{\theta}) = \theta^\alpha \psi_\alpha + (i \theta \sigma^\mu \bar{\theta}) \theta^\alpha \dot{\psi}_\alpha$$

Identity :

$$(\theta \varphi)(\theta \psi) = \theta^\alpha \varphi_\alpha$$

Identity :

$$(\theta \sigma^\mu \bar{\theta}) (\theta \sigma^\nu \bar{\theta}) = -\frac{1}{2} \theta^2 \bar{\theta}^2 \eta^{\mu\nu}$$

$$\theta \psi(y) = \theta^\alpha \psi_\alpha(x^{\mu\nu}; \theta \sigma^\mu \bar{\theta}) = \theta^\alpha \psi_\alpha + (i \theta \sigma^\mu \bar{\theta}) \theta^\alpha \dot{\psi}_\alpha$$

Identity :

$$(\theta \varphi) (\theta \psi) = (\theta^\alpha \varphi_\alpha) (\theta^\beta \psi_\beta)$$

$$= -\frac{1}{2} (\varphi \psi) \theta^2 = -\frac{1}{2} (\varphi^\alpha \psi_\alpha) \theta^2$$

Identity :

$$(\theta \sigma^{\mu} \bar{\theta}) (\theta \sigma^{\nu} \bar{\theta}) = -\frac{1}{2} \theta^2 \bar{\theta}^2 \eta^{\mu\nu}$$

$$\theta \psi(y) = \theta^{\alpha} \psi_{\alpha}(x^{\mu}; \theta \sigma^{\mu} \bar{\theta}) = \theta^{\alpha} \psi_{\alpha} + \underbrace{(i \theta \sigma^{\mu} \bar{\theta}) \theta^{\alpha} \psi_{\alpha}}_{\text{}} \quad \text{|||}$$

Identity :

$$(\theta \varphi) (\theta \psi) = (\theta^{\alpha} \varphi_{\alpha}) (\theta^{\beta} \psi_{\beta})$$

Homework \nearrow

$$= -\frac{1}{2} (\varphi \psi) \theta^2 = -\frac{1}{2} (\varphi^{\alpha} \psi_{\alpha}) \theta^2$$

Identity :

$$(\theta \sigma^{\mu} \bar{\theta}) (\theta \sigma^{\nu} \bar{\theta}) = -\frac{1}{2} \theta^2 \bar{\theta}^2 \eta^{\mu\nu}$$

$$\theta \psi(y) = \theta^{\alpha} \psi_{\alpha}(x^{\mu}; \theta \sigma^{\mu} \bar{\theta}) = \theta^{\alpha} \psi_{\alpha} + \underbrace{(i \theta \sigma^{\mu} \bar{\theta}) \theta^{\alpha} \psi_{\alpha}}_{\text{}} \quad \text{|||}$$

Identity :

$$(\theta \varphi) (\bar{\theta} \psi) = (\theta^{\alpha} \varphi_{\alpha}) (\bar{\theta}^{\beta} \psi_{\beta})$$

Homework \nearrow

$$= -\frac{1}{2} (\varphi \psi) \theta^2 = -\frac{1}{2} (\varphi^{\alpha} \psi_{\alpha}) \theta^2$$

θ^d 0^u u^i

$$\left(\theta^{\alpha} \quad 0^{\mu} \quad \bar{\theta}^{\dot{\alpha}} \right) \left(\theta^{\beta} \quad \eta_{\mu} \quad \psi^{\beta} \right)$$

$$\left(\theta^\alpha \quad 0 \quad \omega_{\alpha\beta} \quad \bar{\theta}^{\dot{\alpha}} \right) \left(\theta^\beta \quad \gamma_{\alpha\beta} \quad \psi^\alpha \right)$$

$$= -\frac{1}{2}$$

$$\left(\theta^{\alpha} \underbrace{\sigma^{\mu}_{\alpha\beta}}_{\gamma^{\mu}} \right) \left(\theta^{\beta} \underbrace{\gamma^{\mu}}_{\gamma^{\mu}} \right)$$

$$= -\frac{1}{2}$$

$$\left(\theta^\alpha \underbrace{\sigma^\mu_{\alpha\beta}}_{\psi} \bar{\theta}^{\beta'} \right) \left(\theta^\beta \underbrace{\gamma_\mu \psi^\beta}_{\psi} \right)$$

$$= \left(\theta \cdot \underbrace{\gamma_\mu \psi}_{\psi} \right) \left(\theta \underbrace{\sigma^\mu \bar{\theta}}_{\psi} \right) = -\frac{1}{2} \theta^2$$

u_4

$$\left(\theta^{\alpha} \underbrace{\sigma^{\mu}_{\alpha\beta}}_{\psi} \bar{\theta}^{\beta} \right) \left(\theta^{\beta} \underbrace{\gamma_{\mu} \psi^{\alpha}}_{\psi} \right)$$

$$= \left(\theta^{\alpha} \underbrace{\gamma_{\mu} \psi^{\alpha}}_{\psi} \right) \left(\theta^{\beta} \underbrace{\sigma^{\mu}_{\beta\alpha}}_{\psi} \right) = -\frac{1}{2} \theta^2 \gamma_{\mu} \psi^{\alpha} \sigma^{\mu}_{\alpha\beta} \bar{\theta}^{\beta}$$

Identity :

$$(\theta \sigma^m \bar{\theta}) (\theta \sigma^0 \bar{\theta}) = -\frac{1}{2} \theta^2 \bar{\theta}^2 \psi^{\psi}$$

$$\theta \psi(y) = \theta^{\alpha} \psi_{\alpha}(x^{\mu}; \theta \sigma^{\mu} \bar{\theta}) = \theta^{\alpha} \psi_{\alpha} + \underbrace{(\theta \sigma^{\mu} \bar{\theta}) \theta^{\beta} \psi_{\beta}}_{\text{?}}$$

Identity :

$$(\theta \varphi) (\theta \psi) = (\theta^{\alpha} \varphi_{\alpha}) (\theta^{\beta} \psi_{\beta})$$

Homework \rightarrow

$$= -\frac{1}{2} (\varphi \psi) \theta^2 = -\frac{1}{2} (\varphi \psi) \theta^2 \quad ?$$

Identity :

$$(\theta \sigma^m \bar{\theta}) (\theta \sigma^0 \bar{\theta}) = -\frac{1}{2} \theta^2 \bar{\theta}^2 \gamma^m$$

$$\theta \psi(y) = \theta^A \psi_A(x^m; \theta \sigma^m \bar{\theta}) = \theta^A \psi_A + \underbrace{(i \theta \sigma^m \bar{\theta}) \theta^A \psi_A}_{\text{cross terms}}$$

Identity :

$$(\theta \varphi) (\varphi \psi) = (\theta^A \varphi_A) (\varphi^B \psi_B)$$

Homework \nearrow

$$= -\frac{1}{2} (\varphi \psi) \theta^2 = -\frac{1}{2} (\varphi^A \psi_A) \theta^2$$

Identity :

$$(\theta \sigma^m \bar{\theta}) (\theta \sigma^0 \bar{\theta}) = -\frac{1}{2} \theta^2 \bar{\theta}^2 \gamma^m$$

$$\theta \psi(y) = \theta^{\alpha} \psi_{\alpha}(x^{\mu}; \theta \sigma^{\mu} \bar{\theta}) = \theta^{\alpha} \psi_{\alpha} + \underbrace{(i \theta \sigma^{\mu} \bar{\theta}) \theta^{\beta} \psi_{\beta}}_{\text{}} \quad \text{|||}$$

Identity :

$$(\theta \varphi) (\bar{\theta} \psi) = (\theta^{\alpha} \varphi_{\alpha}) (\bar{\theta}^{\beta} \psi_{\beta})$$

Homework \nearrow

$$= -\frac{1}{2} (\varphi \psi) \theta^2 = -\frac{1}{2} (\varphi^{\alpha} \psi_{\alpha}) \theta^2$$

114

$$\left(\theta^\alpha \underbrace{\sigma^m_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}}}_{\psi} \right) \left(\theta^\beta \underbrace{\gamma_m \psi^\beta}_{\psi} \right)$$

$$= \left(\theta^\alpha \underbrace{\gamma_m \psi^\alpha}_{\psi} \right) \left(\theta^\beta \underbrace{\sigma^m_{\beta\dot{\beta}} \bar{\theta}^{\dot{\beta}}}_{\psi} \right) = -\frac{1}{2} \theta^2 \gamma_m \psi^\alpha \sigma^m_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}}$$

u_4

$$\left(\theta^\alpha \underbrace{\sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}}}_{\psi} \right) \left(\theta^\beta \underbrace{\partial_\mu \psi^\mu}_\psi \right)$$

$$\begin{aligned} &= \left(\theta \cdot \underbrace{\partial_\mu \psi}_\psi \right) \left(\theta \underbrace{\sigma^\mu \bar{\theta}}_\psi \right) = -\frac{1}{2} \theta^2 \partial_\mu \psi^\mu \sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \\ &= -\frac{1}{2} \left(\partial_\mu \psi^\mu \sigma^\mu \bar{\theta} \right) \theta^2 \end{aligned}$$

$$\theta \psi(y) = \theta^\dagger \psi_L(x^\mu; \theta \sigma^\mu \bar{\theta}) = \theta^\dagger \psi_L + \underbrace{(i \theta \sigma^\mu \bar{\theta}) \theta^\dagger}_{\text{}} \psi_L$$

Identity:

$$(\theta \varphi) (\theta \psi) = (\theta^\dagger \varphi) (\theta^\dagger \psi)$$

Homework \nearrow

$$= -\frac{1}{2} (\varphi \psi) \theta^2 = -\frac{1}{2} (\varphi^\dagger \psi^\dagger) \theta^2$$

$$= \boxed{\theta^\dagger \psi_L - \frac{i}{2} (\varphi \psi \sigma^\mu \bar{\theta}) \theta^2}$$

$$\delta^2 F(x) = \theta^2 F''(x + i\theta\sigma\theta) = \theta^2 F''(x)$$

The Taylor expansion of x is $x = x_0 + i\theta\sigma\theta + \dots$

$\delta^2 F(x) = \theta^2 F''(x)$
 The Taylor expansion of x is $x = x_0 + i\theta\sigma\theta + \dots$
 The Taylor expansion of x is $x = x_0 + i\theta\sigma\theta + \dots$

$$\sigma^2 F(y) = \theta^2 F(x + i\theta\sigma\bar{\theta}) = \theta^2 F(x)$$

x is the only component of x that is invariant

$$\Phi(x^n, \theta, \bar{\theta}) = \Phi + i\theta\sigma^m\bar{\theta}\partial_m\Phi + \frac{1}{4}\theta^2\bar{\theta}^2\Box\Phi$$

depends on x^m

$$+ \sqrt{2}\psi - \frac{i}{\sqrt{2}}\theta^2\partial_m\psi\sigma^m\bar{\theta} + \theta^2 F$$

⇒ how does D of a 2st transform under SUSY?

$$\Phi(x + i\theta\sigma^3\bar{\theta}) = \underbrace{\phi(y)}_{\text{tree}} + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$$\phi(y) = \phi(x) + i\theta\sigma^3\bar{\theta}\partial_n\phi + \frac{1}{2!} \underbrace{(i\theta\sigma^3\bar{\theta})(i\theta\sigma^3\bar{\theta})}_{\text{tree}} \partial_n^2\phi$$

$$x + i\theta\sigma^3\bar{\theta}$$

$$-\frac{1}{2} \left(-\frac{1}{2}\right) \theta^2\bar{\theta}^2 \partial_n^2\phi$$

$$+\frac{1}{4}\theta^2\bar{\theta}^2 \square\phi$$

$$\sigma^2 F(y) = \theta^2 F(x + i\theta\sigma\bar{\theta}) \approx \theta^2 F(x)$$

$$\Phi(x^n, \theta_n, \bar{\theta}_n) = \Phi + i\theta\sigma^n \bar{\theta} \partial_n \Phi + \frac{1}{4} \theta^2 \bar{\theta}^2 \square \Phi$$

depends on x^n

is not dynamical

$$+ \sqrt{2} \psi - \frac{i}{\sqrt{2}} \theta^2 \partial_n \psi \sigma^n \bar{\theta} + \theta^2 \mathbb{F}$$

$$\sigma^2 F(y) = \theta^2 F(x + i\theta\sigma\bar{\theta}) = \theta^2 F(x)$$

$$\Phi(x^n, \theta, \bar{\theta}) = \Phi + i\theta\sigma^m \bar{\theta} \partial_m \Phi + \frac{1}{4} \theta^2 \bar{\theta}^2 \square \Phi$$

depends on x^n

is not dynamical

$$+ \sqrt{2} \psi - \frac{i}{\sqrt{2}} \theta^2 \partial_m \psi \sigma^m \bar{\theta} + \theta^2 \mathbb{F}$$

$$\sigma^2 \Gamma(y) = \theta^2 \Gamma(x + i\theta\sigma\bar{\theta}) = \theta^2 \Gamma(x)$$

$$\bar{\Phi}(x^n, \theta_n, \bar{\theta}_i) = \Phi + i\theta\sigma^n \bar{\theta}_i \partial_m \Phi + \frac{1}{4} \theta^2 \bar{\theta}^2 \square \Phi$$

depends on x^n

is not dynamical

$$+ \sqrt{2} \psi - \frac{i}{\sqrt{2}} \theta^2 \partial_m \psi \sigma^m \bar{\theta} + \theta^2 \mathbb{F}$$

$\overline{\chi_{94}}$ (antichiral superfields): $SUSY?$

$$\Phi(D_\alpha \overline{\Phi}_\alpha = 0) \rightarrow \Phi(\overline{\theta}, \overline{y})$$



$\overline{\chi_{94}}$ (antichiral superfields):

$$\Phi(D_\alpha \overline{\Phi}_\alpha = 0) \rightarrow \Phi(\overline{\theta}, \overline{y})$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma_{\alpha\dot{\alpha}}^{\mu} \overline{\theta}^{\dot{\alpha}} \partial_\mu$$

$\overline{\chi}_{94}$ (antichiral superfields):

$$\Phi(D_\alpha \overline{\Phi}_\alpha = 0) \rightarrow \Phi(\overline{\theta}, \overline{y})$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma_{\alpha i}^{\mu} \overline{\theta}^i \partial_\mu$$

$\overline{\chi_{9f}}$ (antichiral superfields):

$$\Phi(D_\alpha \overline{\Phi}_\alpha = 0) \rightarrow \Phi(\overline{\theta}, \overline{y})$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma_{\alpha i}^{\mu} \overline{\theta}^i \partial_\mu$$

$$\overline{y}^{\mu} = x^{\mu} - i \theta \sigma^{\mu} \overline{\theta}$$

$\overline{\chi_{9f}}$ (antichiral superfields):

$$\Phi(D_\alpha \overline{\Phi}) = 0 \rightarrow \Phi(\overline{\theta}, \overline{y})$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma_{\alpha i}^{\mu} \overline{\theta}^i \partial_\mu$$

$$D_\alpha \overline{\theta}$$

$$\overline{y}^{\mu} = x^{\mu} - i \theta^i \sigma^{\mu} \overline{\theta}^i$$

$\overline{\chi_{94}}$ (antichiral superfields)

$$\Phi(D_\alpha \overline{\Phi}_\alpha = 0) \rightarrow \Phi(\overline{\theta}, \overline{y})$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma_{\alpha\dot{\alpha}}^{\mu} \overline{\theta}^{\dot{\alpha}} \partial_\mu$$

$$D_\alpha \overline{\theta} = 0 \quad D_\alpha \overline{y} = 0$$

$$\overline{y}^{\dot{\alpha}} = x^\mu - i \theta^\mu \sigma^\mu \overline{\theta}^{\dot{\alpha}}$$

$\overline{\chi sf}$ (antichiral superfields):

$$\Phi(D_\alpha \overline{\Phi}_\alpha = 0 \Rightarrow \Phi(\overline{\theta}, \overline{y})$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma_{\alpha i}^{\mu} \overline{\theta}^i \partial_\mu$$

$$D_\alpha \overline{\theta} = 0 \quad D_\alpha \overline{y} = 0$$

If Φ_1 & Φ_2 are $\chi sf \Rightarrow \Phi_1 \Phi_2$ is χsf

$\overline{\chi}$ sf (antichiral superfields):

$$\Phi(D_\alpha \overline{\Phi}_\alpha = 0 \Rightarrow \Phi(\overline{\theta}, \overline{y})$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma_{\alpha i}^{\mu} \overline{\theta}^i \partial_\mu$$

$$D_\alpha \overline{\theta} = 0 \quad D_\alpha \overline{y} = 0$$

If Φ_1 & Φ_2 are χ sf $\Rightarrow \Phi_1 \Phi_2$ is χ sf

\overline{D}_α

$\overline{\chi}$ sf (antichiral superfields):

$$\Phi(D_\alpha \overline{\Phi}) = 0 \rightarrow \Phi(\overline{\theta}, \overline{y})$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma_{\alpha\dot{\alpha}}^{\mu} \overline{\theta}^{\dot{\alpha}} \partial_\mu$$

$$\overline{y}^{\dot{\alpha}} = x^\mu - i \theta^\mu \overline{\theta}^{\dot{\alpha}}$$

$$D_\alpha \overline{\theta} = 0 \quad D_\alpha \overline{y} = 0$$

If φ_1 & φ_2 are χ sf $\Rightarrow \varphi_1 \varphi_2$ is χ sf

$$D_\alpha(\varphi_1 \varphi_2) = \varphi_1 D_\alpha \varphi_2 + (D_\alpha \varphi_1) \varphi_2 =$$

$\overline{\chi sf}$ (antichiral superfields):

$$\Phi(D_\alpha \overline{\Phi}_i = 0) \rightarrow \Phi(\overline{\theta}, \overline{y})$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma_{\alpha\dot{\alpha}}^{\mu} \overline{\theta}^{\dot{\alpha}} \partial_\mu$$

$$D_\alpha \overline{\theta} = 0 \quad D_\alpha \overline{y} = 0$$

$$\overline{y}^{\dot{\alpha}} = x^\mu - i \theta^\mu \sigma^\mu \overline{\theta}^{\dot{\alpha}}$$

If φ_1 & φ_2 are $\chi sf \Rightarrow \varphi_1 \varphi_2$ is χsf

$$D_\alpha(\varphi_1 \varphi_2) = \varphi_1 D_\alpha \varphi_2 + (D_\alpha \varphi_1) \varphi_2 = 0$$

\Rightarrow same holds for $\overline{\text{cost}}$

⇒ same holds for \overline{XSP}

If $\overline{\varphi_1}$ & $\overline{\varphi_2}$ are \overline{XSP} ⇒ $\overline{\varphi_1 \wedge \varphi_2}$ is \overline{XSP}

However

\Rightarrow same holds for \overline{xSP}

If $\overline{\varphi_1}$ & $\overline{\varphi_2}$ are $\overline{xSP} \Rightarrow \overline{\varphi_1 \varphi_2}$ is \overline{xSP}

However

$\overline{\varphi}$

(Assume

$\overline{D}_\alpha \varphi = 0 \swarrow \overline{xSP}$

$\overline{D}_\alpha \overline{\varphi} = 0 \swarrow \overline{xSP}$

⇒ same holds for $\overline{\text{NSP}}$

If $\overline{\varphi}_1$ & $\overline{\varphi}_2$ are $\overline{\text{NSP}} \Rightarrow \overline{\varphi}_1 \overline{\varphi}_2$ is $\overline{\text{NSP}}$

However

$\varphi \overline{\varphi}$ (Assume
is a superfield)

$\overline{D}_\alpha \varphi = 0 \swarrow \overline{\text{NSP}}$
 $\overline{D}_\alpha \overline{\varphi} = 0 \searrow \overline{\text{NSP}}$

\Rightarrow same holds for \overline{XSP}

If $\overline{\varphi}_1$ & $\overline{\varphi}_2$ are $\overline{XSP} \Rightarrow \overline{\varphi}_1 \overline{\varphi}_2$ is \overline{XSP}

However

$\varphi \overline{\varphi}$ (Assume
is a superfield)

$\overline{D}_\alpha \varphi = 0 \swarrow \overline{XSP}$
 $\overline{D}_\alpha \overline{\varphi} = 0 \searrow \overline{XSP}$

⇒ same holds for \overline{xst}

If $\overline{\varphi_1}$ & $\overline{\varphi_2}$ are \overline{xst} ⇒ $\overline{\varphi_1 \varphi_2}$ is \overline{xst}

However

$\varphi \overline{\varphi}$ (Assume $\overline{D}_\alpha \varphi = 0 \swarrow \overline{xst}$
is a superfield $\overline{D}_\alpha \overline{\varphi} = 0 \swarrow \overline{xst}$)

↑ neither \overline{xst} nor \overline{xst}

$$\overline{D}_\alpha [\varphi \overline{\varphi}] \neq 0 \quad \overline{D}_\alpha [\overline{\varphi \varphi}] \neq 0$$

Lagrangian

Recall

[Faded handwritten notes, possibly including mathematical symbols like ∂ , ∇ , and δ]

Lagrangian

Recall,

$$Q_1 =$$

$$\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} k x^2 + \frac{1}{2} k y^2 - m g y$$

Lagrangian

Recall,

$$Q_L = \frac{\partial}{\partial \dot{\theta}^i} - i \sigma_{\alpha\beta}^{\mu\nu} \bar{\theta}^{\alpha} \partial_{\mu} \psi_{\nu}$$

$$\bar{Q}_L = -\frac{\partial}{\partial \theta^i} + i \sigma_{\alpha\beta}^{\mu\nu} \bar{\theta}^{\alpha} \partial_{\mu} \psi_{\nu}$$

Lagrangian

Recall,

$$Q_+ = \frac{\partial}{\partial \theta^{\dot{a}}} - i \sigma_{\dot{a} a}^{\mu} \bar{\theta}^{\dot{a}} \partial_{\mu}$$

$$\bar{Q}_{\dot{a}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{a}}} + i \sigma_{\dot{a} a}^{\mu} \theta^{\dot{a}} \partial_{\mu}$$

$\mathcal{S}_{\text{susy}} \mathcal{P} \stackrel{\text{def}}{=} \equiv$

Lagrangian

Recall,

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i \sigma_{\alpha\dot{\alpha}}^m \bar{\theta}^{\dot{\alpha}} \partial_m$$

$$\bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \sigma_{\alpha\dot{\alpha}}^m \theta^\alpha \partial_m$$

$S_{\text{susy}} \Phi \stackrel{\text{def}}{=} (\epsilon Q + \bar{\epsilon} \bar{Q}) \Phi$

$$= (\epsilon^\alpha Q_\alpha - \bar{\epsilon}^{\dot{\alpha}} \bar{Q}_{\dot{\alpha}}) \Phi$$

$$\frac{1}{2} \left(\frac{2}{20} \right)$$

LA 99
UNIVERSITY
OF TEXAS
AT AUSTIN

Lagrangian

(Final Hamiltonian system $D = ?$)

Recall,

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i \sigma_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_{\dot{\alpha}}$$

$$\bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \sigma_{\alpha\dot{\alpha}} \theta^\alpha \partial_\alpha$$

$S_{\text{usy}} \bar{\Phi} \stackrel{\text{def}}{=} \int \bar{\Phi}$

$$= \int (\epsilon Q + \bar{\epsilon} \bar{Q}) \bar{\Phi}$$

$$= \int \bar{Q}_{\dot{\alpha}} Q^{\dot{\alpha}}$$

$$= \int (\epsilon^\alpha Q_\alpha - \bar{\epsilon}^{\dot{\alpha}} \bar{Q}_{\dot{\alpha}}) \bar{\Phi}$$

$$\xi^* \left(\frac{\partial}{\partial \theta^*} - i \sigma_{21}^* \right)$$



$$\xi^\mu \left(\frac{\partial}{\partial \theta^\mu} - i \sigma_{\mu\nu}^\alpha \theta^\nu \partial_\alpha \right) - \bar{\xi}^{\dot{\mu}} \left(-\frac{\partial}{\partial \bar{\theta}^{\dot{\mu}}} + i \theta^\alpha \sigma_{\alpha\dot{\mu}}^\mu \partial_\mu \right)$$

$$\psi^{\dagger} \left(\frac{\partial}{\partial \theta^{\alpha}} - i \sigma_{\alpha\beta}^{\mu} \bar{\theta}^{\beta} \partial_{\mu} \right) - \bar{\zeta}^{\dot{\alpha}} \left(-\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \theta^{\alpha} \sigma_{\alpha\dot{\alpha}}^{\mu} \partial_{\mu} \right)$$

$$\left[\varphi + \theta\psi + \bar{\theta}\bar{\chi} + \theta^2 F + \bar{\theta}^2 G + \theta\sigma^{\mu}\bar{\theta} \Lambda_{\mu} + \theta^{\alpha}\bar{\theta}^{\dot{\alpha}}\lambda + \bar{\theta}^{\dot{\alpha}}\theta^{\alpha}\rho + \theta^{\alpha}\bar{\theta}^{\dot{\alpha}}D \right]$$

$$\frac{1}{2} \left(\frac{\partial}{\partial \theta^{\dot{\alpha}}} - i \sigma_{\mu\nu}^{\dot{\alpha}\beta} \bar{\theta}^{\beta} \partial_{\mu} \right) - \frac{1}{2} \left(-\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \theta^{\alpha} \sigma_{\mu\nu}^{\dot{\alpha}\beta} \partial_{\mu} \right)$$

$$\left[\varphi + \theta \psi + \bar{\theta} \bar{\chi} + \theta^2 F + \bar{\theta}^2 G + \theta \sigma^{\mu} \bar{\theta} \Delta_{\mu} + \theta^{\alpha} \bar{\theta}^{\dot{\alpha}} \lambda_{\dot{\alpha}} + \bar{\theta}^{\dot{\alpha}} \theta^{\alpha} \rho_{\alpha} + \theta^{\alpha} \bar{\theta}^{\dot{\alpha}} D_{\dot{\alpha}} \right]$$

$$\bar{\psi} = \bar{\chi} \left(\frac{\partial}{\partial \theta} - i \sigma_{\mu\nu}^{\alpha} \bar{\theta}^{\nu} \partial_{\mu} \right) - \bar{\chi}^{\dagger} \left(-\frac{\partial}{\partial \bar{\theta}} + i \theta^{\alpha} \sigma_{\mu\nu}^{\alpha} \partial_{\mu} \right)$$

$$\begin{aligned} & [\varphi + \theta \psi + \bar{\theta} \bar{\chi} + \theta^2 F + \bar{\theta}^2 G + \theta \sigma^{\mu} \bar{\theta} A_{\mu} \\ & \quad + \theta^{\nu} \bar{\theta}^{\lambda} + \bar{\theta}^2 \theta^{\rho} + \theta^2 \bar{\theta}^2 D] \\ & = \dots + \theta^2 \bar{\theta}^2 [?] \end{aligned}$$

$$\bar{\psi} = \frac{1}{\sqrt{2}} \left(\frac{\psi}{\sqrt{2}} - i \sigma_{23} \theta^i \partial_i \right) - \frac{1}{\sqrt{2}} \left(-\frac{\psi}{\sqrt{2}} + i \theta^i \sigma_{23}^i \partial_i \right)$$

$$\begin{aligned} & \left[\varphi + \theta \psi + \bar{\theta} \bar{\chi} + \theta^2 F + \bar{\theta}^2 G + \theta \sigma^{\mu\nu} \bar{\theta} \Lambda_{\mu\nu} \right. \\ & \quad \left. + \theta^i \bar{\theta}^j \lambda_{ij} + \bar{\theta}^i \theta^j \rho_{ij} + \theta^2 \bar{\theta}^2 D \right] \\ & = \dots + \theta^2 \bar{\theta}^2 [?] \end{aligned}$$

$$\bar{\psi} \left(\cancel{\not{\partial}} - i \underbrace{\sigma_{\mu\nu} \theta^\nu \partial_\mu} \right) - \bar{\chi} \left(-\cancel{\not{\partial}} + i \theta^\mu \sigma_{\mu\nu} \partial_\nu \right)$$

$$\begin{aligned} & \left[\varphi + \theta\psi + \bar{\theta}\bar{\chi} + \theta^2 F + \bar{\theta}^2 G + \theta\sigma^\mu\bar{\theta} A_\mu \right. \\ & \quad \left. + \underbrace{\theta^2\bar{\theta}\bar{\lambda}} + \bar{\theta}^2\theta\rho + \theta^2\bar{\theta}^2 D \right] \\ & = \dots + \theta^2\bar{\theta}^2 [?] \end{aligned}$$

$$\bar{\psi} \not{\partial} \psi - i \bar{\psi} \sigma_{\mu\nu} \theta^{\mu} \partial^{\nu} \psi - \bar{\psi} \left(-\not{\partial} + i \theta^{\alpha} \sigma_{\alpha\beta} \partial^{\beta} \right) \psi$$

$$\begin{aligned} & \left[\varphi + \theta\psi + \bar{\theta}\bar{\chi} + \theta^2 F + \bar{\theta}^2 G + \theta\sigma^{\mu}\bar{\theta} \Delta_{\mu} \right. \\ & \quad \left. + \theta^{\alpha} \bar{\theta}^{\beta} \lambda_{\alpha\beta} + \underline{\bar{\theta}^2 \theta^{\mu} P_{\mu}} + \theta^2 \bar{\theta}^2 D \right] \\ & = \dots + \theta^2 \bar{\theta}^2 [?] \end{aligned}$$

$$-i \int d^4x \bar{\psi} \gamma^\mu \theta \partial_\mu [\theta^2 \bar{\psi}] \rightarrow i \int d^4x \theta^2 \partial_\mu \bar{\psi} \gamma^\mu \psi \quad (\bar{\psi} \theta \psi)$$

$$-i \int \bar{\psi} \gamma^{\mu} \bar{\theta} \partial_{\mu} [\theta^{\nu} \bar{\psi}] \approx -i \int \bar{\psi} \gamma^{\mu} \bar{\theta} \partial_{\mu} \psi \approx (\bar{\theta} \theta) \int \bar{\psi} \gamma^{\mu} \partial_{\mu} \psi$$

①

$$-i (\bar{\xi} \cdot (\theta \cdot \bar{\theta}))$$



$$-i \sum_{\vec{k}} \sum_{\vec{l}} \bar{\theta}^i \partial_{\mu} \left[\theta^j \bar{\theta}^k \right] \rightarrow i \sum_{\vec{k}} \bar{\theta}^i \partial_{\mu} \left(\theta^j \bar{\theta}^k \right) \partial_{\mu} (\bar{\theta}^l \theta^m)$$

①

[The rest of the chalkboard is heavily scribbled out with white chalk.]



$$-i \xi^{\mu} \sigma_{\mu\nu} \bar{\theta}^{\nu} \partial_{\mu} [\theta^{\rho} \bar{\theta}^{\sigma} \lambda] \quad \text{or} \quad i \xi^{\mu} \theta^{\nu} \sigma_{\mu\nu} \partial_{\mu} (\bar{\theta}^{\rho} \theta^{\sigma} \rho)$$

$$\ominus +i \theta^{\rho} \{ (\xi \sigma^{\mu})_{\rho} \bar{\theta}^{\sigma} \} \cdot \{ \bar{\lambda} \}$$

$$-i \xi^{\mu} \sigma_{\mu\nu} \bar{\theta}^{\nu} \partial_{\mu} [\theta^{\rho} \bar{\theta}^{\lambda}] \rightarrow i \xi^{\mu} \theta^{\nu} \sigma_{\mu\nu} \partial_{\mu} (\bar{\theta}^{\rho} \theta^{\sigma})$$

$$\textcircled{=} +i \theta^2 \{ (\xi \sigma^{\mu})_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \} \cdot \{ \bar{\lambda}_{\dot{\beta}} \theta^{\dot{\beta}} \} + \dots$$

$$-i \xi^k \sigma_{\mu\nu}^m \bar{\theta}^i \partial_\mu \left[\theta^2 \bar{\theta} \lambda \right] \rightarrow i \xi^k \left(\theta^2 \sigma_{\mu\nu}^m \right) \partial_\mu (\bar{\theta} \theta \rho) \quad (5.09)$$

$$\ominus + i \theta^2 \left\{ (\xi \sigma^m)_i \bar{\theta}^i \right\} \cdot \left\{ \bar{\lambda}_j \theta^{j\beta} \right\} + \dots$$

$$-i \xi^{\lambda} \sigma^{\mu}{}_{\lambda} \bar{\theta}^i \partial_{\mu} [\theta^{\alpha} \bar{\theta}^{\lambda}] = i \xi^{\lambda} \theta^{\alpha} \sigma^{\mu}{}_{\lambda} \partial_{\mu} (\xi^{\rho} \theta^{\rho}) \quad (5'09)$$

$$\ominus + i \theta^2 \left\{ (\xi \sigma^{\mu})_{\lambda} \bar{\theta}^{\lambda} \right\} \cdot \left\{ \bar{\eta}_{\lambda} \bar{\theta}^{\lambda} \right\} + \dots$$

Identity (from homework)

$$-i \xi^{\mu} \sigma_{\mu i}^{\dot{a}} \bar{\theta}^{\dot{a}} \partial_{\mu} [\theta^{\alpha} \bar{\theta}_{\dot{\alpha}}] = i \xi^{\mu} \sigma_{\mu}^{\dot{a} b} \theta^{\dot{a}} \partial_{\mu} (\bar{\theta}^{\dot{b}}) \quad (5 \theta^{\alpha})$$

$$\Rightarrow +i \theta^2 \left\{ (\xi \sigma^{\mu})_{\dot{a}} \bar{\theta}^{\dot{a}} \right\} \cdot \left\{ \eta_{\lambda \dot{\beta}} \bar{\theta}^{\dot{\beta}} \right\} + \dots$$

Identity (from homework)

$$(\bar{\theta} \bar{\Phi})(\bar{\theta} \bar{\Psi}) = -\frac{1}{2} (\bar{\Phi} \bar{\Psi}) \bar{\theta}^2$$

$$-i \xi^m \sigma_{\mu\nu} \bar{\theta}^i \gamma_\mu [\theta^2 \bar{\theta} \lambda] \rightarrow i \xi^m \theta^{\alpha} \sigma_{\alpha\beta} \gamma_\mu (\bar{\theta}^i \theta^j)$$

$$\ominus \quad +i \theta^2 \left\{ (\xi \sigma^m)_i \bar{\theta}^i \right\} \cdot \left\{ \lambda_j \bar{\theta}^j \right\} + \dots$$

Identity (from homework)

$$(\bar{\theta} \Phi) (\bar{\theta} \Psi) = -\frac{1}{2} (\Phi \Psi) \bar{\theta}^2$$

$$(\bar{\Psi} \bar{\theta}) (\bar{\Psi} \bar{\theta}) = -\frac{1}{2} (\bar{\Psi} \bar{\Psi}) \bar{\theta}^2$$

$$-i \xi^{\alpha} \sigma_{\alpha\dot{\alpha}}^{\mu} \bar{\theta}^{\dot{\alpha}} \gamma_{\mu} \left[\theta^{\beta} \bar{\theta}_{\beta} \right] = -i \xi^{\alpha} \theta^{\beta} \sigma_{\alpha\dot{\alpha}}^{\mu} \gamma_{\mu} (\bar{\theta}_{\dot{\alpha}}) \quad (509)$$

$$\ominus \quad +i \theta^2 \left\{ (\xi \sigma^{\mu})_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \right\} \cdot \left\{ \bar{\eta}_{\dot{\beta}} \theta^{\beta} \right\} + \dots$$

Identity (from homework)

$$(\bar{\theta} \Phi) (\bar{\theta} \Psi) = -\frac{1}{2} (\Phi \Psi) \bar{\theta}^2$$

$$(\bar{\Psi} \bar{\theta}) (\bar{\Psi} \bar{\theta}) = -\frac{1}{2} (\bar{\Psi} \bar{\Psi}) \bar{\theta}^2$$

$$= \left| \theta^\nu \psi_\nu - \frac{i}{2} (\gamma_\mu \psi \sigma^\mu \bar{\theta}) \theta^2 \right|$$

$$-i \xi^\mu \sigma_{\mu i} \bar{\theta}^i \gamma_\mu \left[\theta^2 \bar{\psi} \right] \rightarrow -i \xi^\mu \theta^\nu \sigma_{\mu\nu} \gamma_\mu (\bar{\psi} \theta)$$

$$\Rightarrow +i \theta^2 \left\{ (\xi \sigma^\mu)_i \bar{\theta}^i \right\} \left\{ \bar{\psi} \gamma_\mu \theta \right\} + \dots$$

Identity (from homework)

$$(\bar{\theta} \not{\xi}) (\bar{\theta} \not{\psi}) = -\frac{1}{2} (\not{\xi} \not{\psi}) \bar{\theta}^2$$

$$(\not{\xi} \bar{\theta}) (\not{\psi} \bar{\theta}) = -\frac{1}{2} (\not{\xi} \not{\psi}) \bar{\theta}^2$$

$$-i \theta^2 \xi^\mu \sigma_{\mu i} \gamma_\mu \bar{\psi}^i$$

$$= \left[\bar{\theta} \psi - \frac{i}{2} (\gamma_\mu \psi \sigma^\mu \bar{\theta}) \theta^2 \right]$$

$$= i \xi^\mu \sigma_{\mu i} \bar{\theta}^i \gamma_\mu \left[\theta^2 \bar{\theta} \lambda \right] + i \xi^\mu \left(\theta^\mu \sigma_{\mu i} \bar{\theta}^i \right) \gamma_\mu (\bar{\psi} \theta \rho)$$

$$\ominus + i \theta^2 \left\{ (\xi \sigma^\mu)_i \bar{\theta}^i \right\} \cdot \left\{ \bar{\psi} \lambda \right\} \bar{\theta}^i + \dots$$

Identity (from homework)

$$\begin{aligned} (\bar{\theta} \bar{\theta}) (\bar{\theta} \bar{\psi}) &= -\frac{1}{2} (\bar{\psi} \bar{\psi}) \bar{\theta}^2 \\ (\bar{\psi} \bar{\theta}) (\bar{\psi} \bar{\theta}) &= -\frac{1}{2} (\bar{\psi} \bar{\psi}) \bar{\theta}^2 \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \begin{array}{l} -i \theta^2 \xi^\mu \sigma_{\mu i} \gamma_\mu \bar{\theta}^i \\ = \frac{i}{2} \theta^2 \bar{\theta}^2 \xi \sigma^\mu \gamma_\mu \bar{\lambda} \end{array}$$

$$\partial_{\mu} S^{\mu\nu} = \partial_{\mu} [\text{something}]$$

$$S = \int d^4x D$$

$$\theta \delta_{\text{susy}} D^2 \hat{\psi} = \partial_m [\text{something}]$$

$$S = \int d^4x \mathcal{L}$$

↑
action

susy invariant

$$\delta_{\text{SUSY}} D = \partial_m [\text{something}]$$

$$S = \int d^4x D$$

↑
action

$$\delta_{\text{SUSY}} S = \int d^4x \delta_{\text{SUSY}} D = \int d^4x$$

δ_{SUSY} invariant

$$\delta_{SUSY} D = \partial_m [\text{something}]$$

$$S = \int d^4x D$$

$$\delta_{SUSY} S = \int d^4x \delta_{SUSY} D = \int d^4x \partial_m [\dots]$$

$$= \int_{\partial M_4} [\dots]$$

δ_{SUSY} invariant

$$\delta_{SUSY} D = \partial_m [\text{something}]$$

$$S = \int d^4x D$$

↑
action

$$\delta_{SUSY} S = \int d^4x \delta_{SUSY} D = \int d^4x \partial_m [\dots]$$

$$= \int \delta_{SUSY} [\dots] = 0$$

δ_{SUSY}

SUSY invariant

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

[The rest of the chalkboard is heavily scribbled over with chalk, obscuring any text or equations that were previously written.]



EX-101
EX-101
EX-101

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

$$d^2\theta = \frac{1}{2} \epsilon^{\mu\nu} \frac{\partial^2}{\partial\theta^\mu \partial\theta^\nu}$$

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

$$d^2\theta = \frac{1}{4} \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu} \theta^\rho$$

$$d^2\bar{\theta} = \frac{1}{4} \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\bar{\theta}^\mu} \frac{\partial}{\partial\bar{\theta}^\nu} \bar{\theta}^\rho$$

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

$$d^2\theta = \frac{1}{4} \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu}$$

$$d^2\bar{\theta} = \frac{1}{4} \epsilon_{\mu\nu\rho} \frac{\partial}{\partial\bar{\theta}_\mu} \frac{\partial}{\partial\bar{\theta}_\nu}$$

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

$$d^2\theta = \frac{1}{4} \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu} \theta^\rho$$

$$d^2\bar{\theta} = \frac{1}{4} \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\bar{\theta}^\mu} \frac{\partial}{\partial\bar{\theta}^\nu} \bar{\theta}^\rho$$

Let us complete

$$4 \int d^2\theta \theta^2$$

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

$$d^2\theta = \frac{1}{4} \epsilon^{\mu\nu} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu}$$

$$d^2\bar{\theta} = \frac{1}{4} \epsilon^{\mu\nu} \frac{\partial}{\partial\bar{\theta}^\mu} \frac{\partial}{\partial\bar{\theta}^\nu}$$

Let us complete

$$4 \int d^2\theta \theta^2 =$$

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

$$d^2\theta = \frac{1}{4} \epsilon^{\mu\nu} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu}$$

$$d^2\bar{\theta} = \frac{1}{4} \epsilon^{\dot{\mu}\dot{\nu}} \frac{\partial}{\partial\bar{\theta}^{\dot{\mu}}} \frac{\partial}{\partial\bar{\theta}^{\dot{\nu}}}$$

Let us complete

$$4 \int d^2\theta \theta^2 = \epsilon^{\mu\nu} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu} \theta$$

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

$$d^2\theta = \frac{1}{4} \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu} \theta^\rho$$

$$d^2\bar{\theta} = \frac{1}{4} \epsilon_{\mu\nu\rho} \frac{\partial}{\partial\bar{\theta}^\mu} \frac{\partial}{\partial\bar{\theta}^\nu} \bar{\theta}^\rho$$

Let us complete

$$4 \int d^2\theta \theta^2 = \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\bar{\theta}^\mu} \frac{\partial}{\partial\bar{\theta}^\nu} \theta^{\mu_1} \theta^{\mu_2} \epsilon_{\mu_1\mu_2\rho}$$

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

$$d^2\theta = \frac{1}{4} \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu}$$

$$d^2\bar{\theta} = \frac{1}{4} \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\bar{\theta}^\mu} \frac{\partial}{\partial\bar{\theta}^\nu}$$

Let us complete

$$4 \int d^2\theta \theta^2 = \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu} [\theta^{\mu_1} \theta^{\mu_2}] \epsilon_{\mu_1\mu_2}$$

$$= \epsilon^{\alpha\beta} \epsilon_{\alpha_1\alpha_2} \left[\delta_{\beta}^{\alpha_1} \delta_{\alpha}^{\alpha_2} \right]$$

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

$$d^2\theta = \frac{1}{4} \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu} \theta^\rho$$

$$d^2\bar{\theta} = \frac{1}{4} \epsilon_{\mu\nu\rho} \frac{\partial}{\partial\bar{\theta}^\mu} \frac{\partial}{\partial\bar{\theta}^\nu} \bar{\theta}^\rho$$

Let θ is complete

$$4 \int d^2\theta \theta^2 = \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu} \left[\theta^{\mu_1} \theta^{\mu_2} \right] \epsilon_{\mu_1\mu_2\rho}$$

$$= \epsilon^{\mu\nu\rho} \left[\delta_{\mu_1\nu} \int \delta_{\mu_2\rho} - \delta_{\mu_2\nu} \int \delta_{\mu_1\rho} \right]$$

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

$$d^2\theta = \frac{1}{4} \epsilon^{\alpha\beta} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta}$$

$$d^2\bar{\theta} = \frac{1}{4} \epsilon_{\alpha\beta} \frac{\partial}{\partial\bar{\theta}^\alpha} \frac{\partial}{\partial\bar{\theta}^\beta}$$

Let t is complete

$$4 \int d^2\theta \theta^2 = \epsilon^{\alpha\beta} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta} \left[\begin{matrix} \theta^{\alpha_1} \\ \theta^{\alpha_2} \end{matrix} \right] \epsilon_{\alpha_1\alpha_2}$$

$$= \epsilon^{\alpha\beta} \epsilon_{\alpha_1\alpha_2} \left[\begin{matrix} \delta_{\beta}^{\alpha_1} \delta_{\alpha}^{\alpha_2} - \delta_{\beta}^{\alpha_2} \delta_{\alpha}^{\alpha_1} \end{matrix} \right]$$

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

$$d^2\theta = \frac{1}{4} \epsilon^{\alpha\beta} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta}$$

$$d^2\bar{\theta} = \frac{1}{4} \epsilon_{\alpha\beta} \frac{\partial}{\partial\bar{\theta}^\alpha} \frac{\partial}{\partial\bar{\theta}^\beta}$$

Let θ is complete

$$4 \int d^2\theta \theta^2 = \epsilon^{\alpha\beta} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta} \left[\theta^{\alpha_1} \theta^{\alpha_2} \right] \epsilon_{\alpha_1\alpha_2}$$

$$= \epsilon^{\alpha\beta} \epsilon_{\alpha_1\alpha_2} \left[\delta_{\beta}^{\alpha_1} \delta_{\alpha}^{\alpha_2} - \delta_{\beta}^{\alpha_2} \delta_{\alpha}^{\alpha_1} \right] = 4$$

$$d^4\theta = d^2\theta d^2\bar{\theta}$$

$$d^2\theta = \frac{1}{4} \epsilon^{\alpha\beta} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta}$$

$$d^2\bar{\theta} = \frac{1}{4} \epsilon_{\dot{\alpha}\dot{\beta}} \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} \frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}}$$

Let t is complete

$$4 \int d^2\theta d^2\bar{\theta} \theta^{\alpha_1} \theta^{\alpha_2} \bar{\theta}^{\dot{\alpha}_1} \bar{\theta}^{\dot{\alpha}_2} \epsilon_{\alpha_1\alpha_2} \epsilon_{\dot{\alpha}_1\dot{\alpha}_2} = 4$$

$$= \epsilon^{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} \left[\delta_{\beta}^{\alpha_1} \delta_{\alpha_2}^{\dot{\alpha}_1} - \delta_{\beta}^{\alpha_2} \delta_{\alpha_1}^{\dot{\alpha}_1} \right] = 4$$

$$\int d^4\theta \theta^2 = 1$$

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

$$d^2\theta = \frac{1}{4} \epsilon^{\mu\nu} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu}$$

$$d^2\bar{\theta} = \frac{1}{4} \epsilon_{\alpha\beta} \frac{\partial}{\partial\bar{\theta}^\alpha} \frac{\partial}{\partial\bar{\theta}^\beta}$$

Let us complete

$$4 \int d^2\theta \theta^2 = \epsilon^{\mu\nu} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu} \left[\theta^{\alpha_1} \theta^{\alpha_2} \right] \epsilon_{\alpha_1\alpha_2}$$

$$= \epsilon^{\mu\nu} \epsilon_{\alpha_1\alpha_2} \left[\delta_{\beta}^{\alpha_1} \delta_{\mu}^{\alpha_2} - \delta_{\beta}^{\alpha_2} \delta_{\mu}^{\alpha_1} \right] = 4$$

$$\int d^1\theta \theta^2 = 1$$

Similarly:

$$\int d^1\bar{\theta} \bar{\theta}^2 = 1$$

$$d^1\theta = d^2\theta d^1\bar{\theta}$$

$$d^2\theta = \frac{1}{4} \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu} \theta^\rho$$

$$d^2\bar{\theta} = \frac{1}{4} \epsilon_{\mu\nu\rho} \frac{\partial}{\partial\bar{\theta}^\mu} \frac{\partial}{\partial\bar{\theta}^\nu} \bar{\theta}^\rho$$

Let θ is complete

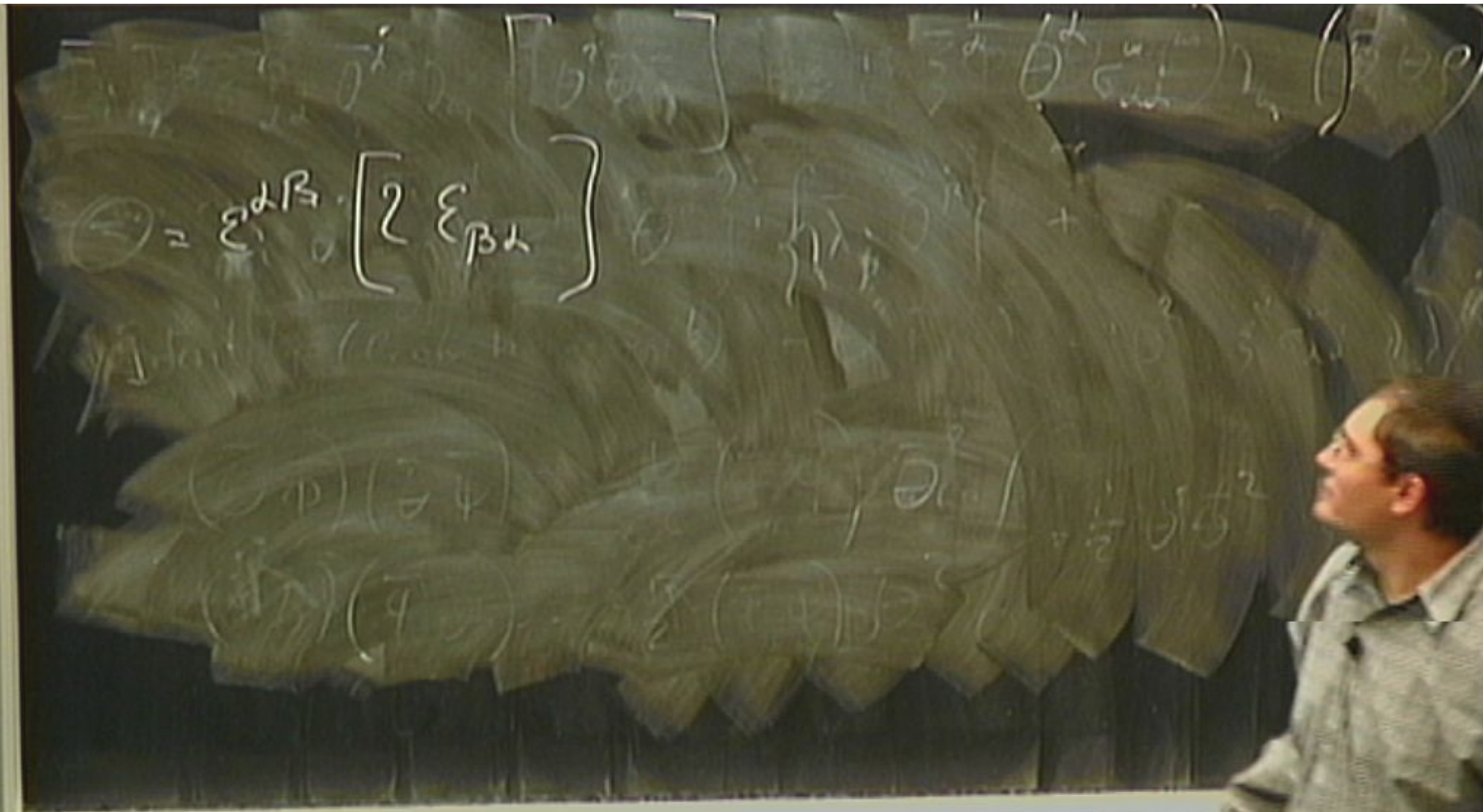
$$4 \int d^2\theta \theta^2 = \epsilon^{\mu\nu\rho} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\theta^\nu} \left[\theta^{\alpha_1} \theta^{\alpha_2} \right] \epsilon_{\alpha_1\alpha_2\rho}$$

$$= \epsilon^{\mu\nu\rho} \epsilon_{\alpha_1\alpha_2\rho} \left[\delta_{\beta}^{\alpha_1} \delta_{\mu}^{\alpha_2} - \delta_{\beta}^{\alpha_2} \delta_{\mu}^{\alpha_1} \right] = 4$$

$$\int d^1\theta \theta^2 = 1$$

Similarly:

$$\int d^1\bar{\theta} \bar{\theta}^2 = 1$$



$$\Theta = \epsilon^{\alpha\beta} \left[2 \epsilon_{\beta\alpha} \right] = -2 \left(\begin{array}{c} \epsilon^{\alpha\beta} \\ \epsilon_{\alpha\beta} \end{array} \right)$$



$$\ominus = \varepsilon^{\alpha\beta} \left[2 \varepsilon_{\beta\lambda} \right] = -2 \varepsilon^{\alpha\beta} \varepsilon_{\beta\lambda}$$

$$\Rightarrow \int D = \int d^4\theta \Phi$$

$$\mathcal{L} = \epsilon^{\alpha\beta} \left[2 \epsilon_{\beta\lambda} \right] = -2 \left\{ \begin{matrix} \alpha\beta \\ \lambda\mu \end{matrix} \right\} \epsilon_{\lambda\mu}$$

$$\Rightarrow \int D = \int d^4\theta \mathcal{L}$$

$$\ominus = \varepsilon^{\alpha\beta} \left[2 \varepsilon_{\beta\alpha} \right] = -2 \varepsilon^{\alpha\beta} \varepsilon_{\alpha\beta}$$

$$\Rightarrow \int D = \int d^4\theta \Phi$$

\Rightarrow for a chiral superfield χ_{st} : $\int D \Phi(\theta, y) = \dots + \frac{1}{4} \bar{\theta}^2 \theta^2 D$

$$D = \frac{1}{4} \bar{\theta}^2 \theta^2 \rightarrow S = \int d^4x D_{\chi_{st}} = \frac{1}{4} \int d^4x D \varphi = 0$$

$$\Rightarrow \epsilon^{\alpha\beta} \left[2 \epsilon_{\beta\alpha} \right] = -2 \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$\Rightarrow D = \int d^4\theta \Phi$$

\Rightarrow for a chiral superfield $\chi(x, \theta)$, $\Phi(x, \theta) = \dots + \frac{1}{4} \bar{\theta}^2 \theta^2 D\chi$

$$D = \frac{1}{4} \int d^4\theta \Phi \Rightarrow S = \int d^4x D_{\chi\chi} = \frac{1}{4} \int d^4x D\chi = 0$$

to get something we need both π and σ

K

$$S = \int dx \left(\frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \phi''^2 - V(\phi) \right)$$

invariant

⇒ to get something we need both π and σ

$$K = K(\Phi_1, \Phi_2)$$

\Rightarrow to get something we need both ω and $\bar{\omega}$

$$K = K(\Phi, \bar{\Phi})$$

is a Kähler potential

⇒ to get something we need both χ and $\bar{\chi}$

$$K = K(\Phi_i, \bar{\Phi}_j)$$

is a Kähler potential

$$\mathcal{L} = \int d^4\theta K \rightarrow \text{automatically susy invariant.}$$

11.9 Different $\mathbb{K} \rightarrow$ to the same \mathcal{L} .

[The rest of the chalkboard is heavily scribbled over with grey chalk, obscuring any text that might have been written below the first line.]

CAUTION
PULL TO OPEN

Different $K \rightarrow$ to the same α .

$$K(\alpha_i, \bar{\alpha}_i) \Rightarrow K(\alpha_i, \bar{\alpha}_i) + f(\alpha_i)$$

"Different $\mathbb{K} \rightarrow$ to the same \mathcal{L} "

$$\mathbb{K}(\varphi_i, \bar{\varphi}_i) \Rightarrow \mathbb{K}(\varphi_i, \bar{\varphi}_i) + f(\varphi_i) + g(\bar{\varphi}_i)$$

Different $\bar{K} \rightarrow$ to the same α xsf

$$K(\varphi_i, \bar{\varphi}_i) \Rightarrow K(\varphi_i, \bar{\varphi}_i) + \underbrace{f(\varphi_i)}_{\text{xsf}} + \underbrace{g(\bar{\varphi}_i)}_{\text{xsf}}$$

Different $K \rightarrow$ to the same \mathcal{L} - xsp

$$K(\varphi_i, \bar{\varphi}_i) \Rightarrow K(\varphi_i, \bar{\varphi}_i) + \underbrace{f(\varphi_i)}_{\text{xsp}}$$

$$+ \underbrace{g(\bar{\varphi}_i)}_{\text{xsp}}$$

$$\mathcal{L} \rightarrow \mathcal{L} + \int d^4x D$$

Different $K \rightarrow$ to the same L xsf

$$K(\varphi_i, \bar{\varphi}_i) \Rightarrow K(\varphi_i, \bar{\varphi}_i) + f(\varphi_i)$$

$$+ g(\bar{\varphi}_i)$$

$$L \rightarrow L + \int d^4x D_{xsf}$$

Different $\bar{K} \rightarrow$ to the same \mathcal{L} xsf

$$K(\varphi_i, \bar{\varphi}_i) \Rightarrow K(\varphi_i, \bar{\varphi}_i) + \underbrace{f(\varphi_i)}_{\text{xsf}} + g(\bar{\varphi}_i)$$

$$\mathcal{L} \rightarrow \mathcal{L} + \underbrace{\int d^4\theta D_{\text{xsf}}}_{\text{xsf}} + \underbrace{\int d^4\theta D_{\text{g}}}_{\text{xsf}}$$

are total deriv

Different $\bar{K} \rightarrow$ to the same \mathcal{L} xsf

$$K(\varphi_i, \bar{\varphi}_i) \Rightarrow K(\varphi_i, \bar{\varphi}_i) + \underbrace{f(\varphi_i)}_{\text{xsf}} + g(\bar{\varphi}_i)$$

$$\mathcal{L} \rightarrow \mathcal{L} + \underbrace{\int d^4x D_{\text{xsf}}}_{\text{are total deriv}} + \underbrace{\int d^4x D_{\text{gsg}}}_{\text{are total deriv}}$$

are total deriv

"Different $K \rightarrow$ to the same \mathcal{L} — \mathcal{L} is

$$K(\varphi_i, \bar{\varphi}_i) \Rightarrow K(\varphi_i, \bar{\varphi}_i) + \underbrace{f(\varphi_i)}_{\mathcal{L}}$$

$$+ \underbrace{g(\bar{\varphi}_i)}_{\mathcal{L}}$$

$$\underbrace{\int d^4\theta D_{\mathcal{L}}}_{\mathcal{L}} + \underbrace{\int d^4\theta D_{\mathcal{L}}}_{\mathcal{L}}$$

are total derivatives

Example

$$K(\varphi, \bar{\varphi}) = \varphi \bar{\varphi}$$

Example

$$K(\varphi, \overline{\varphi}) = \varphi \overline{\varphi}$$

$$\begin{array}{c} \uparrow \quad \uparrow \\ xst \quad \overline{xst} \end{array}$$

[The rest of the chalkboard contains faint, mostly illegible handwritten notes and diagrams.]

Example

$$K(\varphi, \overline{\varphi}) = \varphi \overline{\varphi}$$

$$\begin{array}{c} \uparrow \quad \uparrow \\ xst \quad \overline{xst} \end{array}$$

$$K = \varphi$$

Example

$$K(\varphi, \bar{\varphi}) = \varphi \bar{\varphi}$$

$$\begin{array}{c} \uparrow \quad \uparrow \\ x_{st} \quad \bar{x}_{st} \end{array}$$

$$K = \varphi + i \partial_{\alpha} \bar{\theta} \partial_{\alpha} \varphi + \frac{1}{4} \theta^2 \bar{\theta}^2$$

Example

$$K(\varphi, \bar{\varphi}) = \varphi \bar{\varphi}$$

\uparrow \uparrow
 x_{st} \bar{x}_{st}

$$K |_{\theta^2 \bar{\theta}^2} = ?$$

$$K = \varphi + i \partial_{\alpha} \bar{\theta} \partial_{\alpha} \varphi + \frac{1}{4} \theta^2 \bar{\theta}^2$$

Example

$$K(\varphi, \bar{\varphi}) = \varphi \bar{\varphi}$$

\uparrow \uparrow
 xst \bar{xst}

$$K \Big|_{\theta^2 \bar{\theta}^2} = ?$$

$$K = \left(\varphi + i\theta\sigma^{\mu\nu}\bar{\theta}\partial_{\mu\nu}\varphi + \frac{1}{4}\theta^2\bar{\theta}^2\Box\varphi + \sqrt{2}\theta\psi - \frac{i}{\sqrt{2}}\bar{\theta}^2\partial_{\mu\nu}\psi\sigma^{\mu\nu}\bar{\theta} + \theta^2 F \right) \left(\bar{\varphi} - i\theta$$

Example

$$K(\varphi, \bar{\varphi}) = \varphi \bar{\varphi}$$

\uparrow \uparrow
 xst \overline{xst}

$$K|_{\theta^2 \bar{\theta}^2} = ?$$

$$K = \left(\varphi + i \theta \sigma^{\mu\nu} \bar{\theta} \partial_{\mu\nu} \varphi + \frac{1}{4} \theta^2 \bar{\theta}^2 \right) \left(\bar{\varphi} - i \theta \sigma^{\mu\nu} \bar{\theta} \partial_{\mu\nu} \bar{\varphi} + \frac{1}{4} \theta^2 \bar{\theta}^2 \right) + \sqrt{2} \theta \bar{\theta}$$

Example

$$K(\varphi, \bar{\varphi}) = \varphi \bar{\varphi}$$

↑ xst ↑ xst

$$K|_{\theta^2 \bar{\theta}^2} = ?$$

$$K = \left(\varphi + i \theta \sigma^{\mu\nu} \bar{\theta} \partial_{\mu\nu} \varphi + \frac{1}{4} \theta^2 \bar{\theta}^2 \square \varphi + \sqrt{2} \theta \psi - \frac{i}{\sqrt{2}} \bar{\theta}^2 \partial_{\mu\nu} \psi \sigma^{\mu\nu} \bar{\theta} + \theta^2 F \right) \left(\bar{\varphi} - i \theta \sigma^{\mu\nu} \bar{\theta} \partial_{\mu\nu} \bar{\varphi} + \frac{1}{4} \theta^2 \bar{\theta}^2 \square \bar{\varphi} + \sqrt{2} \bar{\theta} \bar{\psi} + \frac{i}{\sqrt{2}} \bar{\theta}^2 \theta \sigma^{\mu\nu} \partial_{\mu\nu} \bar{\psi} + \bar{\theta}^2 \bar{F} \right)$$

Example

$$K(\varphi, \bar{\varphi}) = \varphi \bar{\varphi} \quad K \Big|_{\theta^2 \bar{\theta}^2} = ?$$

\uparrow \uparrow
 xst \bar{xst}

$$K = \left(\varphi + \frac{i \theta \sigma^{\mu\nu} \bar{\theta}}{4} \partial_{\mu\nu} \varphi + \frac{1}{4} \theta^2 \bar{\theta}^2 \square \varphi + \sqrt{2} \theta \psi - \frac{i}{\sqrt{2}} \bar{\theta}^2 \partial_{\mu\nu} \psi \sigma^{\mu\nu} \bar{\theta} + \theta^2 F \right) \left(\bar{\varphi} - \frac{i \theta \sigma^{\mu\nu} \bar{\theta}}{4} \partial_{\mu\nu} \bar{\varphi} + \frac{1}{4} \theta^2 \bar{\theta}^2 \square \bar{\varphi} + \sqrt{2} \bar{\theta} \bar{\psi} + \frac{i}{\sqrt{2}} \bar{\theta}^2 \theta \sigma^{\mu\nu} \partial_{\mu\nu} \bar{\psi} + \bar{\theta}^2 \bar{F} \right)$$

Example

$$K(\varphi, \bar{\varphi}) = \varphi \bar{\varphi}$$

↑
xst xst

$$K \Big|_{\theta^2 \bar{\theta}^2} = ?$$

$$K = \left(\varphi + \frac{i \theta \sigma^{\mu\nu} \bar{\theta}}{4} \partial_{\mu\nu} \varphi + \frac{1}{4} \theta^2 \bar{\theta}^2 \square \varphi + \sqrt{2} \theta \psi - \frac{i}{\sqrt{2}} \bar{\theta}^2 \partial_{\mu\nu} \psi \sigma^{\mu\nu} \bar{\theta} + \theta^2 F \right) \left(\bar{\varphi} - \frac{i \theta \sigma^{\mu\nu} \bar{\theta}}{4} \partial_{\mu\nu} \bar{\varphi} + \frac{1}{4} \theta^2 \bar{\theta}^2 \square \bar{\varphi} + \sqrt{2} \bar{\theta} \bar{\psi} + \frac{i}{\sqrt{2}} \bar{\theta}^2 \theta \sigma^{\mu\nu} \partial_{\mu\nu} \bar{\psi} + \bar{\theta}^2 \bar{F} \right)$$

$$\frac{\partial \bar{\phi}}{\partial t^2} = \frac{\partial \bar{\phi}}{\partial t} = \frac{\partial \bar{\phi}}{\partial t}$$

$$= -\frac{\partial \bar{\phi}}{\partial t} + \frac{\partial \bar{\phi}}{\partial t} + \frac{\partial \bar{\phi}}{\partial t}$$

$$\frac{\partial \bar{\psi} \psi}{\partial t} = \left[\frac{\partial \bar{\psi} \psi}{\partial t} = \frac{\partial \bar{\psi}}{\partial t} \psi + \bar{\psi} \frac{\partial \psi}{\partial t} \right]$$

$$= -\frac{\partial \bar{\psi}}{\partial t} \psi + \bar{\psi} \frac{\partial \psi}{\partial t} + F \bar{F}$$

$$\begin{aligned}
 \not{\partial} \not{\partial} &= \theta^2 \bar{\theta}^2 = \left[\begin{aligned} \not{\partial} \not{\partial} &= \not{\partial}_\mu \not{\partial}^\mu \\ \not{\partial} \not{\partial} \sigma^m &= \not{\partial}_\mu \not{\partial}^\mu \sigma^m \psi \end{aligned} \right] \\
 &= -\not{\partial} \not{\partial} + \not{\partial}_\mu \not{\partial}^\mu \sigma^m \psi + FF
 \end{aligned}$$

$\frac{1}{2} \varphi$ 13φ

CAUTION
DO NOT TOUCH
THIS SURFACE
UNLESS
AUTHORIZED

CAUTION
DO NOT TOUCH
THIS SURFACE
UNLESS
AUTHORIZED

$$\frac{1}{4} \phi \square \bar{\phi} + \frac{1}{4} \bar{\phi} \square \phi$$

$$\theta \epsilon^{\mu\nu} \bar{\theta} \theta \epsilon^{\rho\sigma} \bar{\theta} = -\frac{1}{2} \theta^2 \bar{\theta}^2 \eta^{\mu\nu\rho\sigma}$$

$$\frac{1}{4} \phi \square \bar{\phi} + \frac{1}{4} \bar{\phi} \square \phi - \frac{1}{2} \psi \bar{\psi}$$

$$\theta \epsilon^\mu \bar{\theta} \theta \epsilon^\nu \bar{\theta} = -\frac{1}{2} \theta^\mu \bar{\theta}^\nu \gamma_{\mu\nu}$$

$\psi\bar{\psi}$

$\theta^2\bar{\theta}^2$

$=$

$$\partial_\mu\psi\partial^\mu\bar{\psi} = \partial_\mu\psi\partial^\mu\bar{\psi}$$

$$\partial_\mu\bar{\psi}\partial^\mu\psi = \partial_\mu\bar{\psi}\partial^\mu\psi$$

$$= -\partial_\mu\psi\partial^\mu\bar{\psi} + \partial_\mu\bar{\psi}\partial^\mu\psi + \underbrace{FF}_{\text{non-dim}}$$

↑
a Weyl ferm.

$$\partial F = 0$$

$$F = 0$$

$$\psi \bar{\psi} \Big|_{\theta^2 \bar{\theta}^2} = \left[\begin{array}{l} \partial_{\mu} \psi \bar{\partial}^{\mu} \psi \\ \partial_{\mu} \bar{\psi} \bar{\partial}^{\mu} \psi \end{array} \right]$$

$$= - \underbrace{\partial_{\mu} \psi \bar{\partial}^{\mu} \psi}_{\text{complex}} + \underbrace{\partial_{\mu} \bar{\psi} \bar{\partial}^{\mu} \psi}_{\text{Weyl/Fer.}} + \cancel{\text{FF}}_{\text{non-0}}$$

↑
complex

↑
Weyl/Fer.

FF = 0
FF = 0

$$\frac{1}{4} \phi \square \bar{\phi} + \frac{1}{4} \bar{\phi} \square \phi - \frac{1}{2} \psi \bar{\psi}$$

$$-\frac{1}{4} \psi \psi$$

$$\theta \epsilon^{\mu\nu} \bar{\theta} \epsilon^{\rho\sigma} = -\frac{1}{2} \theta^{\mu} \bar{\theta}^{\nu} \eta^{\mu\nu}$$

