Title: Intro to Supersymmetry 8

Date: Oct 11, 2007 12:30 PM

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Abstract:

Perturbative SUSY in $d = 3+1$

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Group symmetry $\Rightarrow$ Poincaré group

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time translations $\Rightarrow$ $x^{\mu} \rightarrow x^{\mu} + \varepsilon^{\mu}$

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time translations $\Rightarrow$ $x^{\mu} \to x^{\mu} + \varepsilon^{\mu}$ + Lorentz rotations

$\to$ space rotations + boosts

$$x^{\mu} \to x^{\mu\prime} = \Lambda^{\mu}{}_{\nu} x^{\nu} \qquad \Lambda^{\mu}{}_{\nu} \in SO(3,1)$$

Perturbative SUSY in $d = 3+1$

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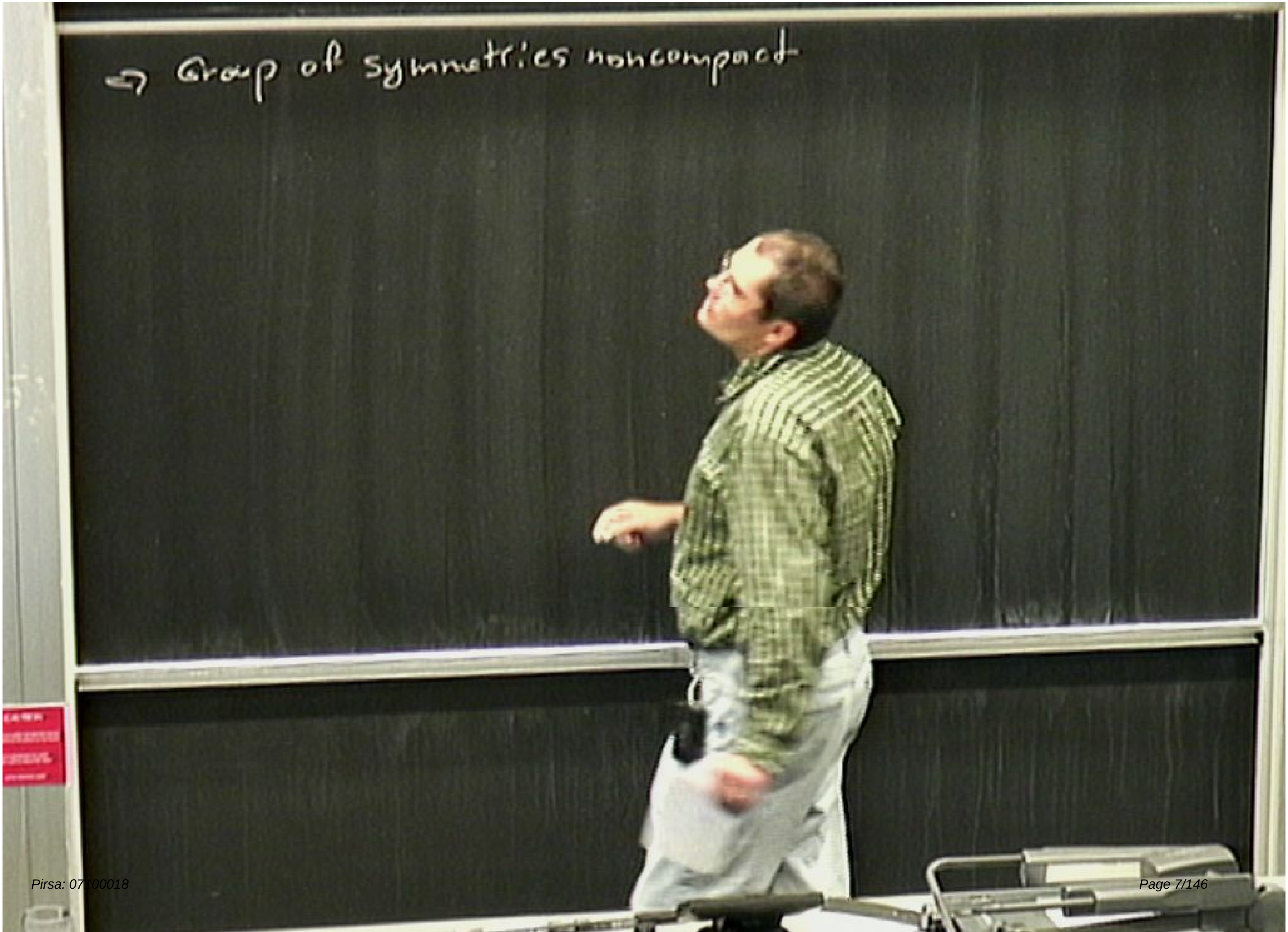

Group of symmetries noncompact

$\Rightarrow$ Group of symmetries noncompact

$\Downarrow$

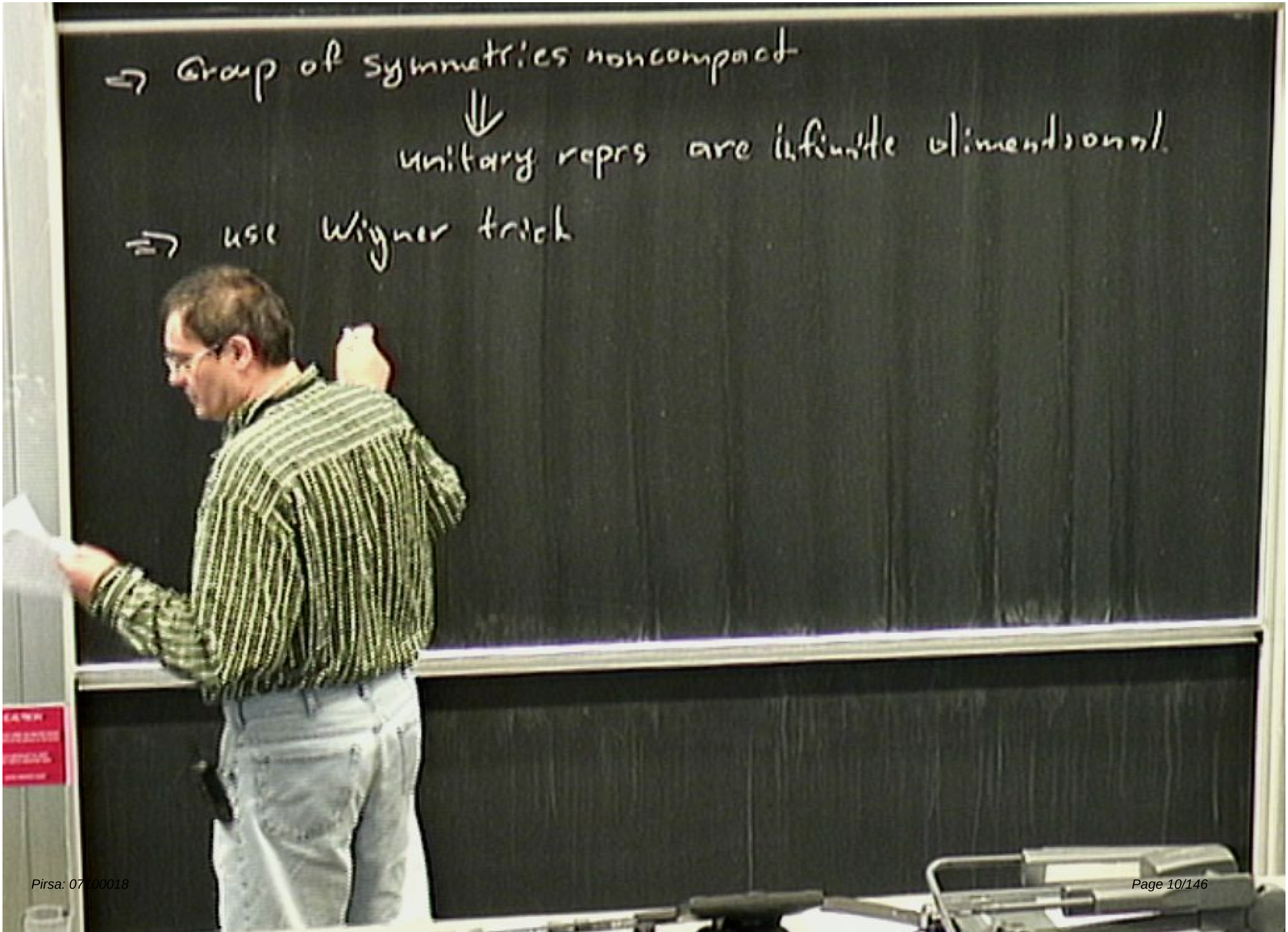
unitary reprs are infinite dimensional.

⇒ Group of symmetries noncompact

$\Downarrow$

unitary reprs are infinite dimensional

⇒ use Wigner trick


⇒ Group of symmetries noncompact
⇓
unitary reprs are infinite dimensional
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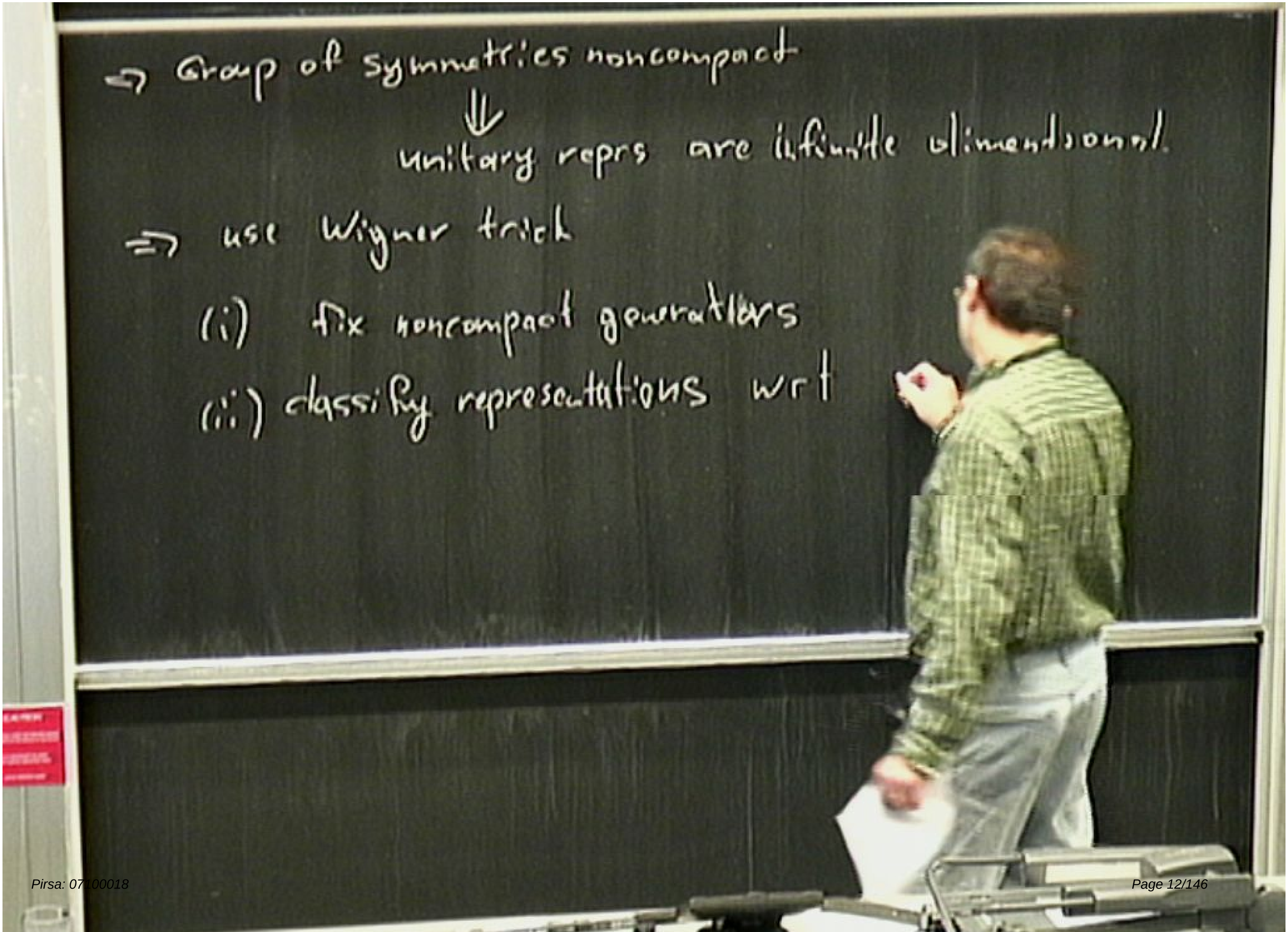
$\Downarrow$

unitary reprs are infinite dimensional

⇒ use Wigner trick

(i) fix noncompact generators


⇒ Group of symmetries noncompact
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(i) fix noncompact generators
(ii) classify representations wrt

⇒ Group of symmetries noncompact

$\Downarrow$

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(little group)

Perturbative SUSY in $d = 3+1$

Group symmetry $\Rightarrow$ Poincaré group

time translations $\Rightarrow$ $x^\mu \to x^\mu + \varepsilon^\mu$ + Lorentz rotations

$\hookrightarrow$ space rotations + boosts

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unitary reprs are infinite dimensional

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(i) fix noncompact generators

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(little group)

Perturbative SUSY in $d = 3+1$

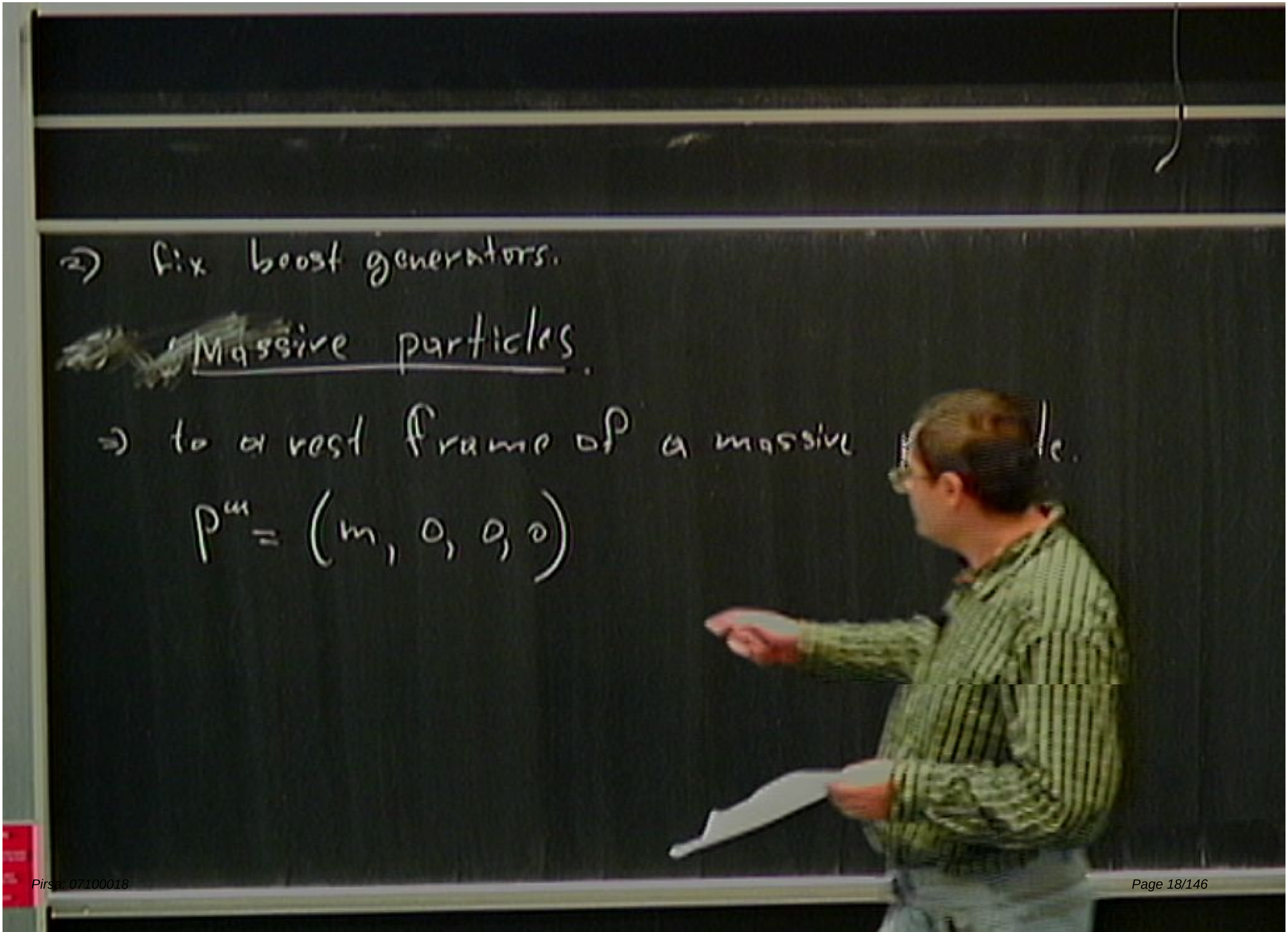
Group symmetry $\Rightarrow$ Poincaré group

time translations $\Rightarrow$ $x^\mu \to x^\mu + \varepsilon^\mu$ + Lorentz rotations

$\to$ space rotations ... sts

$x^\mu \to x^{\mu\prime} = \Lambda^\mu$ ... $\Lambda^\mu{}_\nu \in \underline{SO(3,1)}$

2) fix boost generators.


⇒ fix boost generators.
Massive particles
⇒ to a rest frame of a massive
$p^\mu = (m, 0, 0, 0)$

2) fix boost generators.

Massive particles.

⇒ to a rest frame of a massive particle.

$$p^{\mu} = (m, \underbrace{0, 0, 0}_{SO(3)})$$

A little group is $SO(3) \subset SO(3,1)$


⇒ SO(5) representations

$\Rightarrow$ SO(3) representations

$|j, j_z\rangle$ $\quad\quad$ $j \in \frac{1}{2}\mathbb{Z}$

spin $\quad$ z-component.

$\Rightarrow$ SO(3) representations

$|j, j_z\rangle$

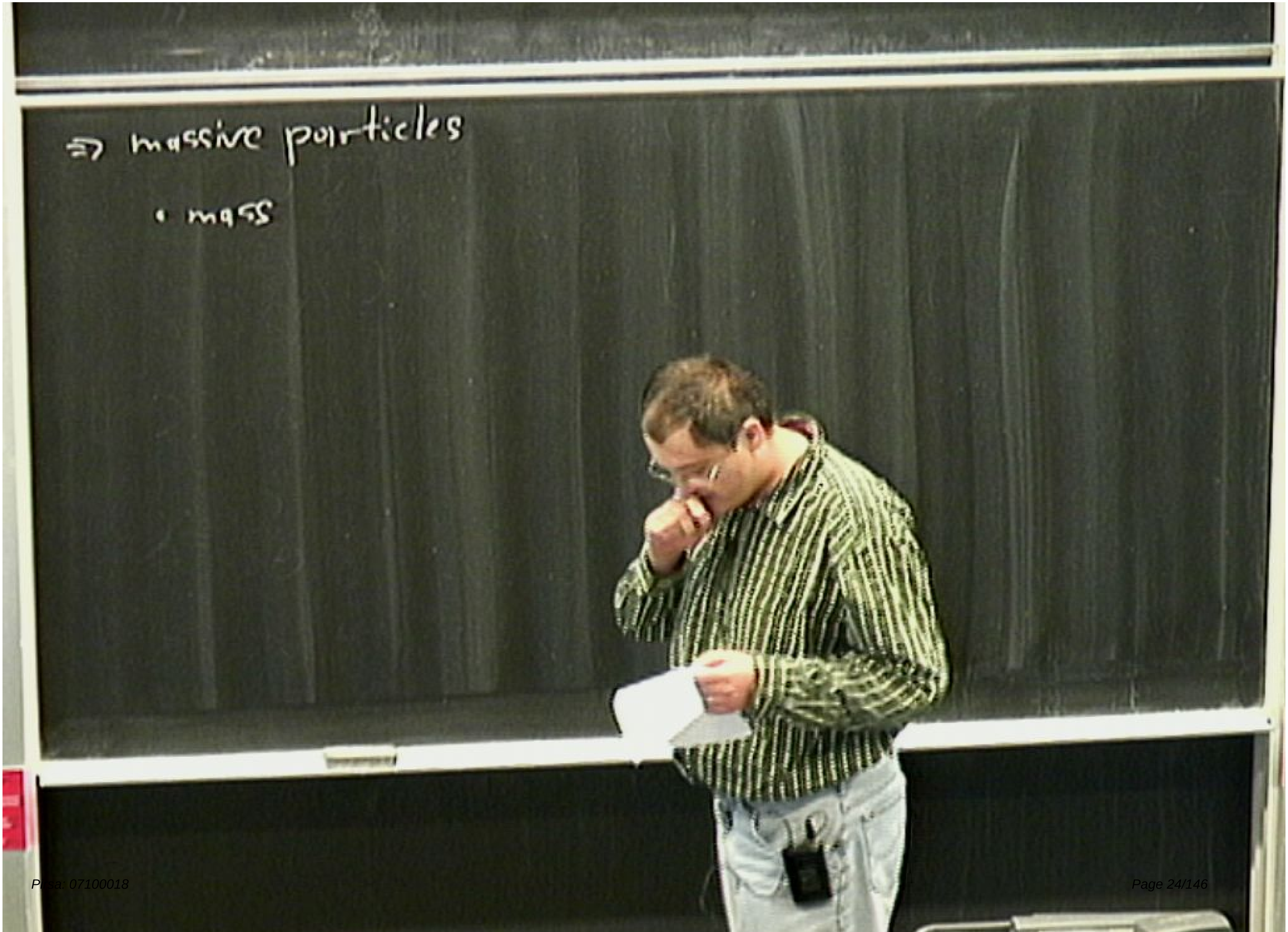
spin $\quad$ z-component.

$$j \in \tfrac{1}{2}\mathbb{Z}$$

$$-j \leq j_z \leq j$$

$$\dim\{\,|j, j_z\rangle\} = 2j+1$$

$\Rightarrow$ massive particles


⇒ massive particles
• mass

⇒ massive particles

- mass
- spin
- internal quantum numbers.

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Massless particles

⇒ massive particles

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Massless particles

$p^{\mu} = (E, 0, 0, E)$ $\qquad$ $p^{\mu} p_{\mu} = 0$

⇒ massive particles

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- spin
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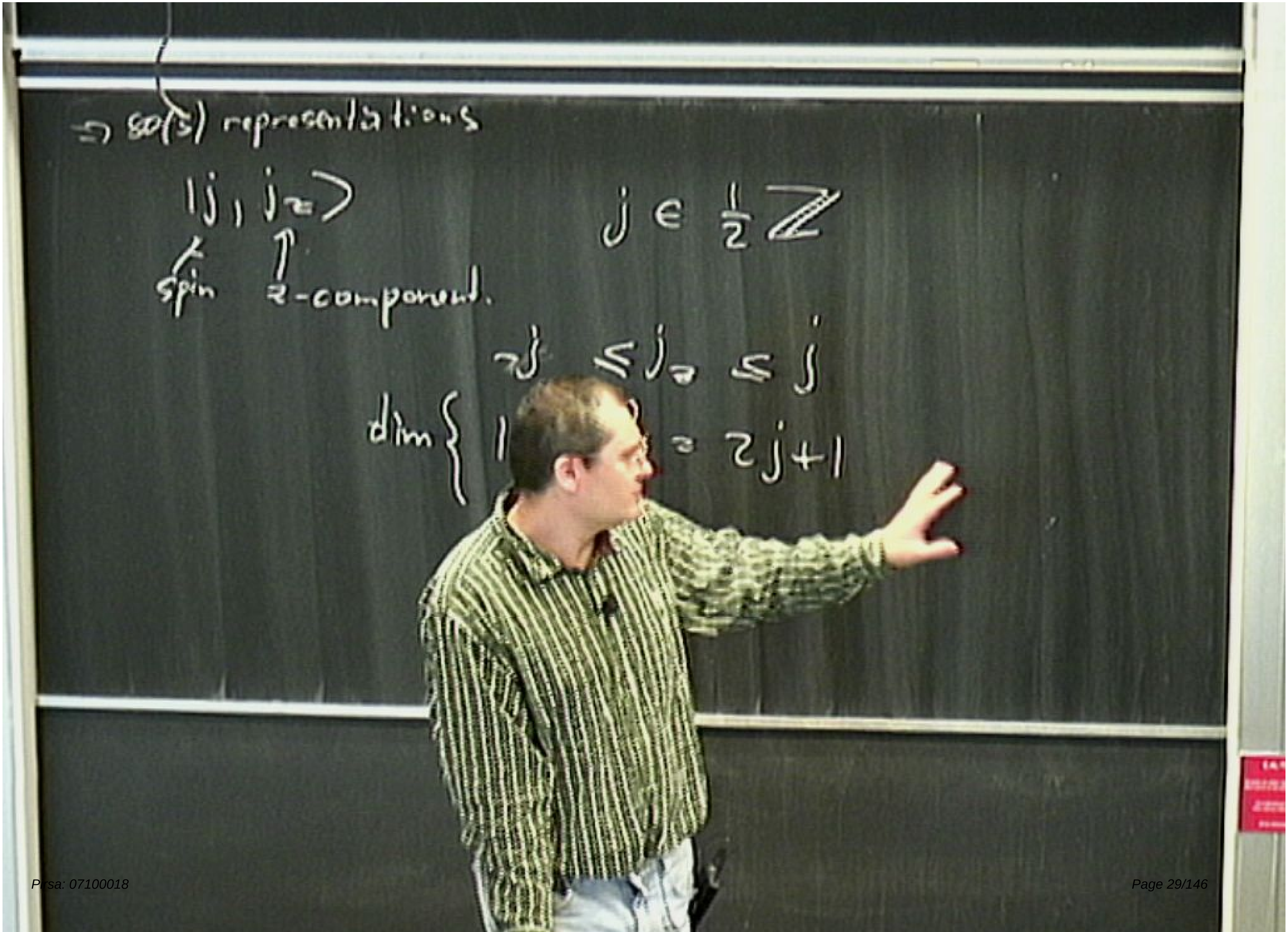
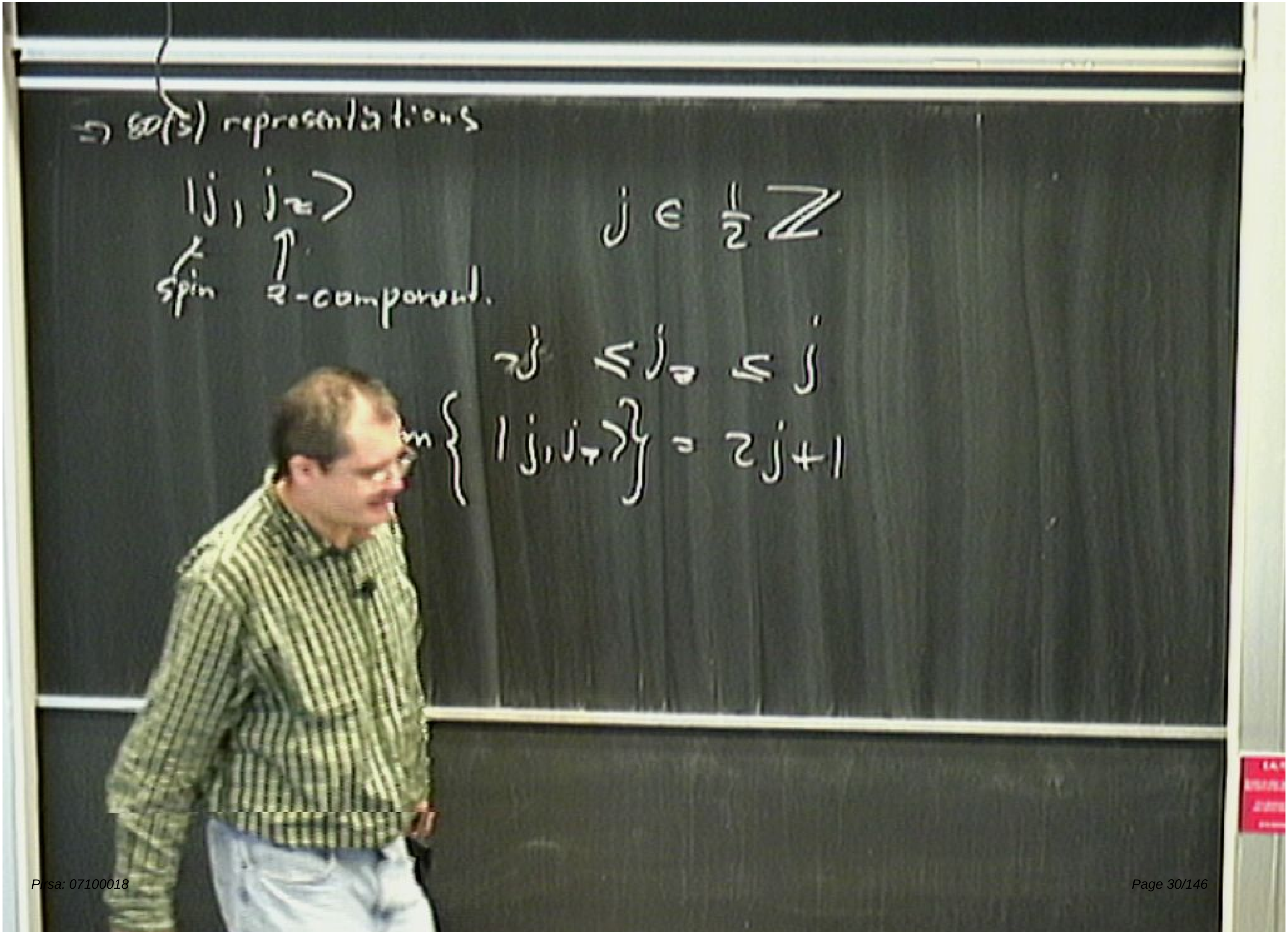
Massless particles

$$p^\mu = (E, \underbrace{0, 0,}_{SO(2)} E)$$

$$p^\mu p_\mu = 0$$

Little group is $SO(2) \sim U(1)$


⇒ SO(3) representations
|j, jz⟩
spin
z-component.
j ∈ ½ℤ
-j ≤ jz ≤ j
= 2j+1


⇒ SO(3) representations
$|j, j_z\rangle$
spin
z-component.
$j \in \frac{1}{2}\mathbb{Z}$
$-j \leq j_z \leq j$
m$\{ |j, j_z\rangle \} = 2j+1$

$\Rightarrow$ massive particles

- mass
- spin
- internal quantum numbers.

Massless particles

Little group is $SO(2) \sim U(1)$

$$p^{\mu} = (E, \underbrace{0, 0}_{SO(2)}, E)$$

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Little group is $SO(2) \sim U(1)$

$$p^{\mu} = (E, \underbrace{0, 0,}_{SO(2)} E) \qquad p^{\mu} p_{\mu} = 0$$

(fix the energy).

⇒ massive particles

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- internal quantum numbers.

<u>Massless particles</u>

$$p^{\mu} = (E, \underbrace{0, 0}_{SO(2)}, E)$$

$p^{\mu} p_{\mu} = 0$

(fix the energy).

Little group is $SO(2) \sim U(1)$

$|\lambda\rangle$ is a helicity.

Algebraically $\mathcal{A}$ can be anything

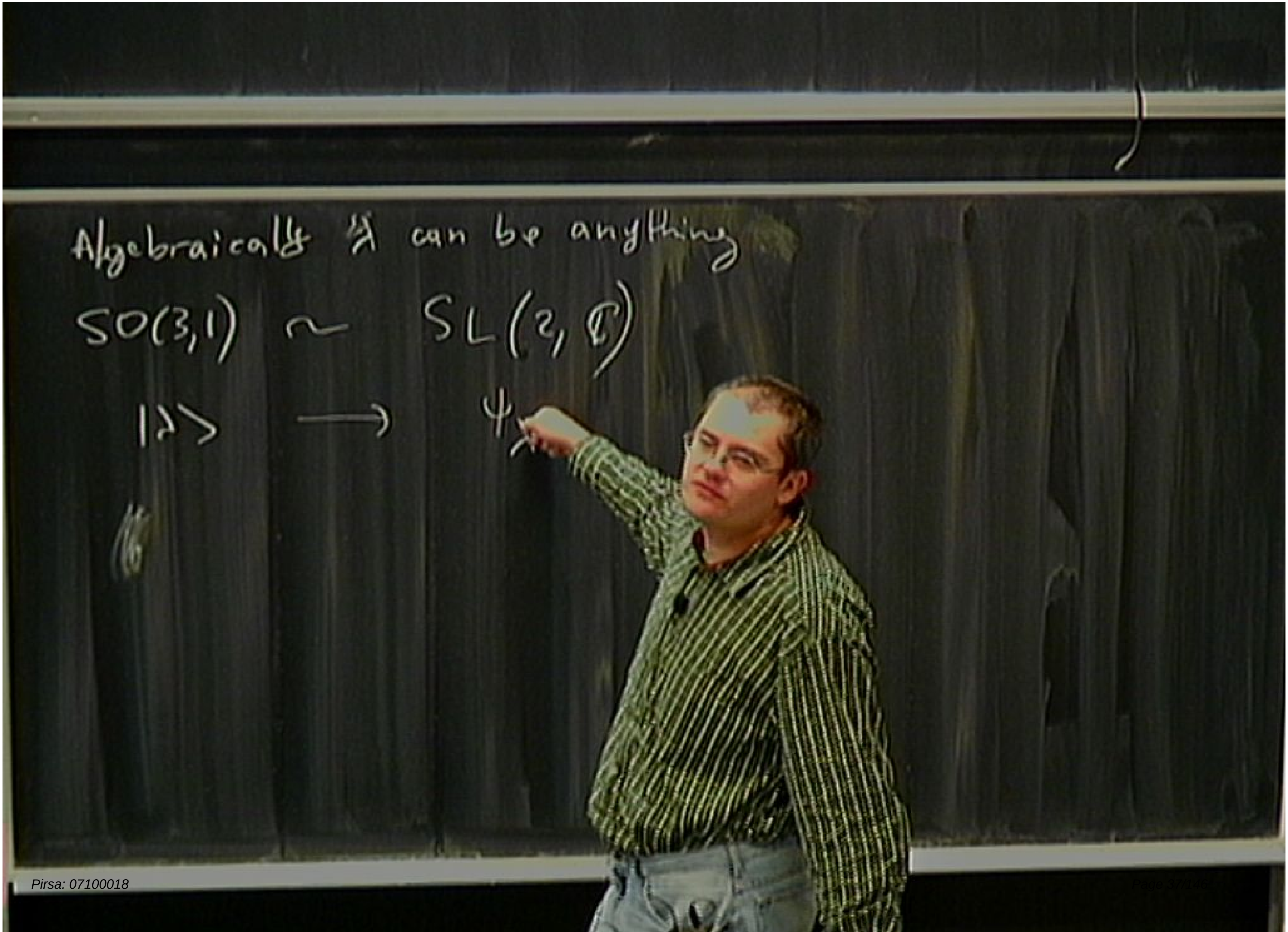
Algebraically λ can be anything

$$SO(3,1) \sim SL(2,\mathbb{C})$$

Algebraically λ can be anything

$SO(3,1) \sim SL(2, \mathbb{C})$

$|\lambda\rangle \longrightarrow$


Algebraically λ can be anything
SO(3,1) ~ SL(2, C)
|λ⟩ ⟶ ψ

Algebraically λ can be anything

$$SO(3,1) \sim SL(2,\mathbb{C})$$

$$|\lambda\rangle \longrightarrow \psi_\lambda$$

Do rotation around z-axis by θ.

$$\psi_\lambda \longrightarrow e^{i\theta\lambda}\psi_\lambda$$

Algebraically λ can be anything

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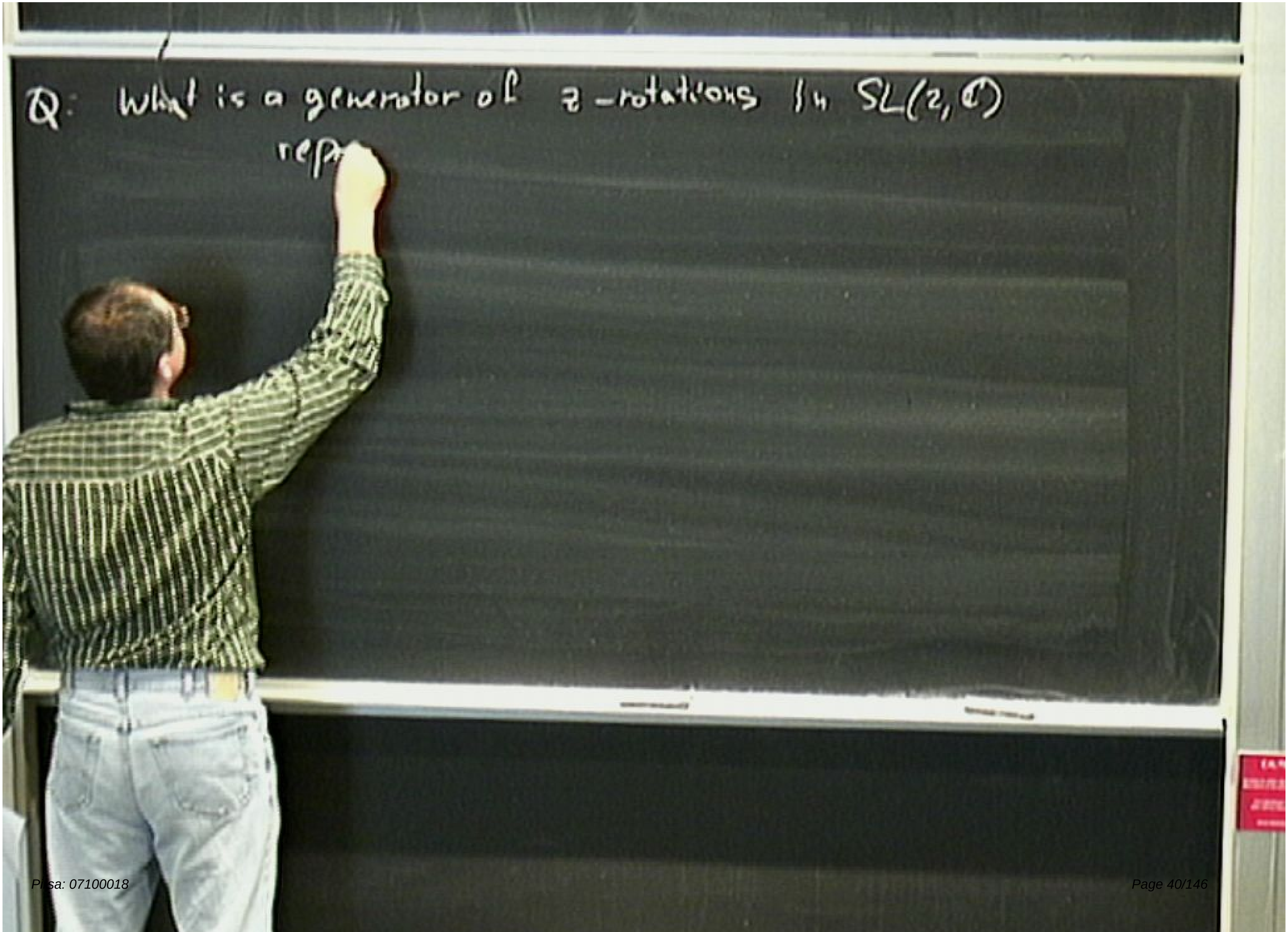
Q: What is a generator of z-rotations in SL(2, C)
rep

Q: What is a generator of z-rotations in $SL(2,\mathbb{C})$ representation of $SO(3,1)$

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$$M_\theta = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

Q: What is a generator of z-rotations in $SL(2,\mathbb{C})$ representation of $SO(3,1)$?

$$M_\theta = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

Let's assume that M_θ is z-rotations

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$M_{2\pi} \neq \mathbb{I}$ <u>but</u> $M_{4\pi} = \mathbb{I}$ in $SL(2,\mathbb{C})$

Algebraically λ can be anything

$$SO(3,1) \sim SL(2,\mathbb{C})$$

$$|\lambda\rangle \longrightarrow \psi_\lambda$$

Do rotation around z-axis by θ.

$$\psi_\lambda \longrightarrow e^{i\theta\lambda}\,\psi_\lambda$$

$$\psi_\lambda \xrightarrow{4\pi}$$

Algebraically λ can be anything

$$SO(3,1) \sim SL(2,\mathbb{C})$$

$$|\lambda\rangle \longrightarrow \psi_\lambda$$

Do rotation around z-axis by θ

$$\psi_\lambda \longrightarrow e^{i\theta\lambda}\,\psi_\lambda \qquad \lambda \in \tfrac{1}{2}$$

$$\psi_\lambda \xrightarrow{4\pi} e^{i4\pi\lambda}\,\psi_\lambda = \psi_\lambda$$

Algebraically λ can be anything

$SO(3,1) \sim SL(2,\mathbb{C})$

$|\lambda\rangle \longrightarrow \psi_\lambda$

Do rotation around z-axis by θ.

$$\psi_\lambda \longrightarrow e^{i\theta\lambda}\psi_\lambda \qquad \lambda \in \tfrac{1}{2}\mathbb{Z}$$

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$$M_\sigma = e^{\frac{i\theta}{2}\sigma_3}$$

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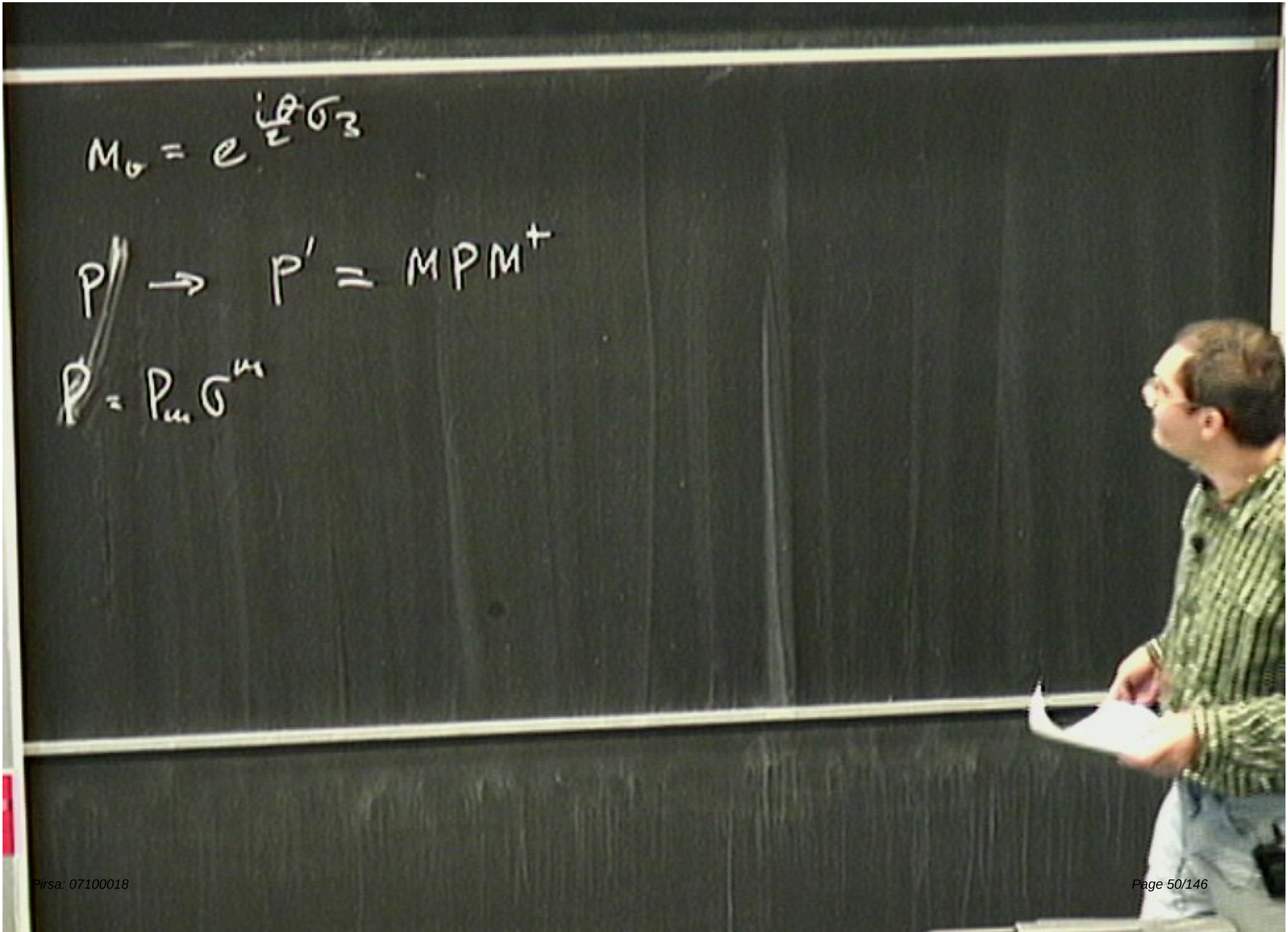
$$\not{P} \longrightarrow$$

$$\not{P} = P_\mu \sigma^\mu$$


$M_\theta = e^{\frac{i\theta}{2}\sigma_3}$
$\not{P} \to P' = MPM^\dagger$
$\not{P} = P_\mu \sigma^\mu$

$$M_\sigma = e^{\frac{i\theta}{2}\sigma_3} = \cos\frac{\theta}{2} + i\sigma_3 \sin\frac{\theta}{2}$$

$$P \to P' = MPM^+$$

$$P = P_\mu \sigma^\mu$$

$$M_\theta = e^{i\frac{\theta}{2}\sigma_3} = \cos\frac{\theta}{2} + i\sigma_3 \sin\frac{\theta}{2}$$

$$P \to P' = MPM^{+} = \left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\sigma_3\right)\left(P_\mu \sigma^\mu\right)\left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\sigma_3\right)$$

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$$M_\theta = e^{\frac{i\theta}{2}\sigma_3} = \cos\frac{\theta}{2} + i\sigma_3 \sin\frac{\theta}{2}$$

$$\cancel{P} \rightarrow P' = MPM^{+} = \left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\sigma_3\right)\left(P_\mu \sigma^\mu\right)$$

$$\left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\sigma_3\right) =$$

$$\cancel{P} = P_\mu \sigma^\mu$$

$$P = (P_0, P_1, P_2, P_3)$$

$$= P_\mu' \sigma^\mu$$

$$M_\theta = e^{i\frac{\theta}{2}\sigma_3} = \cos\frac{\theta}{2} + i\sigma_3 \sin\frac{\theta}{2}$$

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$$\cancel{P} = P_\mu \sigma^\mu$$

$$P = (P_0, P_1, P_2, P_3)$$

$$= P'_m \sigma^m$$

$$(P_0, P_1, P_2, P_3) \longrightarrow (P_0, P_1', P_2', P_3)$$

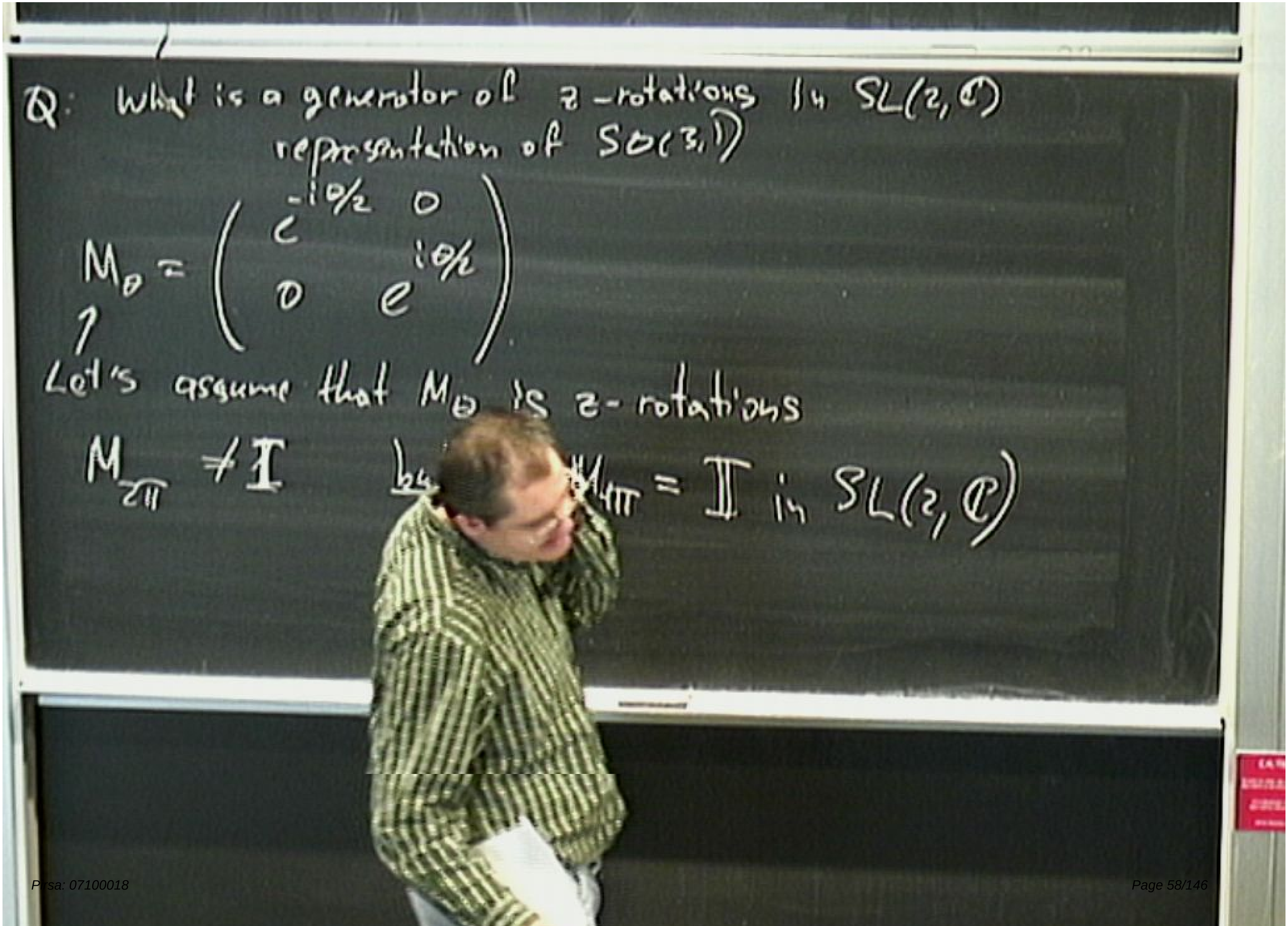
$$(P_0, P_1, P_2, P_3) \longrightarrow (P_0, P_1', P_2', P_3)$$

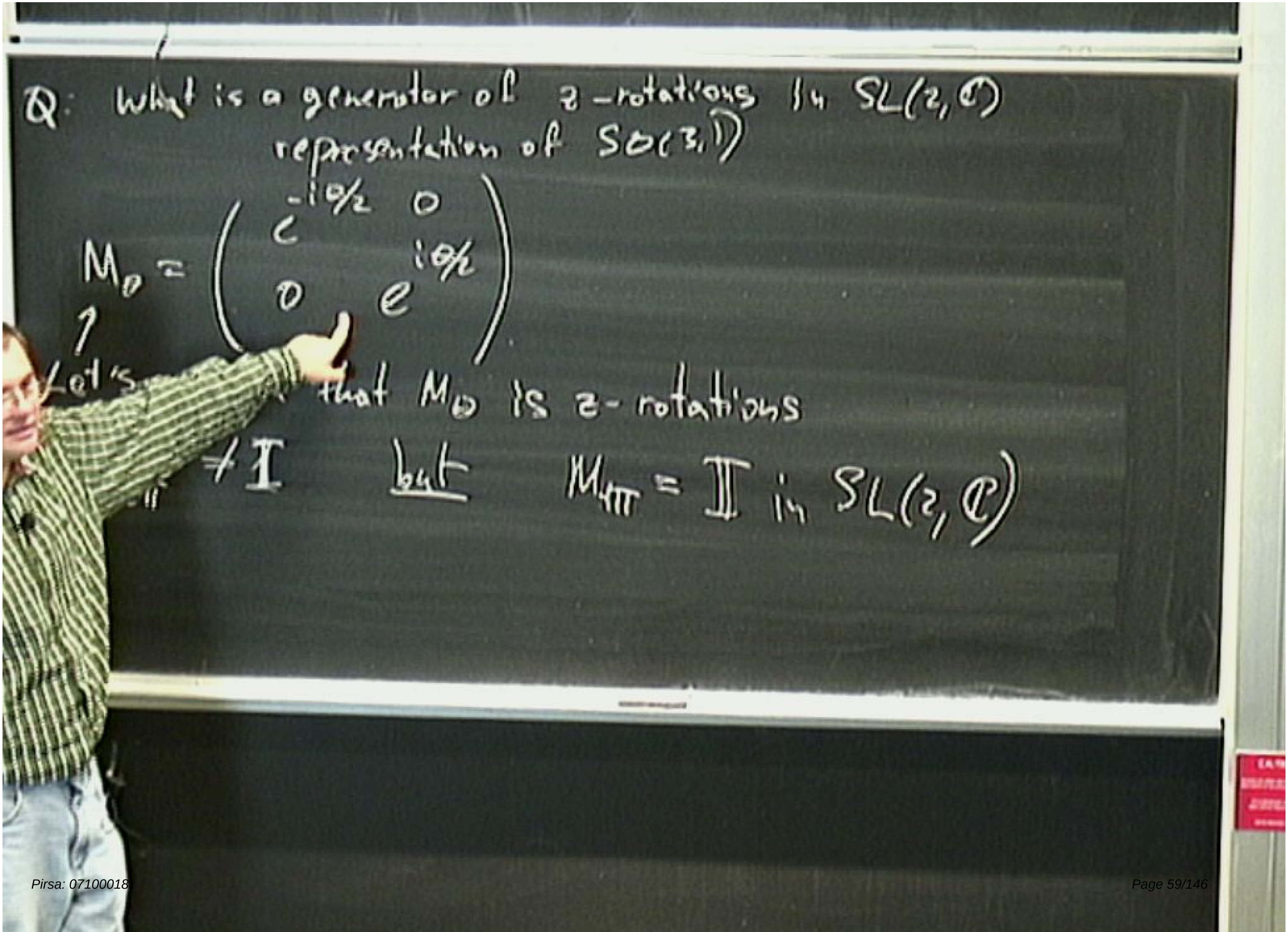
$$P_1' = P_1 \cos\theta + \sin\theta \, P_2$$

$$P_2' = P_2 \cos\theta - \sin\theta \, P_1$$

$$(P_0, P_1, P_2, P_3) \longrightarrow (P_0, P_1', P_2', P_3)$$

$$\left.\begin{aligned} P_1' &= P_1 \cos\theta + \sin\theta \, P_2 \\ P_2' &= P_2 \cos\theta - \sin\theta \, P_1 \end{aligned}\right\} \text{ as a rotation by } \text{“}-\theta\text{” around } z\text{-axis}$$


Q: What is a generator of z-rotations in SL(2,C) representation of SO(3,1)
M_θ = (e^{-iθ/2} 0 ; 0 e^{iθ/2})
Let's assume that M_θ is z-rotations
M_{2π} ≠ I but M_{4π} = I in SL(2,C)


Q: What is a generator of z-rotations in SL(2,C)
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$M_\theta = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$
that M_θ is z-rotations
$\neq \mathbb{I}$ but $M_{4\pi} = \mathbb{I}$ in SL(2,C)

Q: What is a generator of z-rotations in $SL(2,\mathbb{C})$ representation of $SO(3,1)$

$$M_\theta = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$$e^{\frac{i\theta\sigma_3}{2}} = \sum_{n=0}^{\infty}\left(\frac{i\theta\sigma_3}{2}\right)^n \frac{1}{n!}$$

$$\sigma_3^{2n} = \mathbb{I}$$

Let's assume that M_θ is z-rotations

$M_{2\pi} \neq \mathbb{I}$ <u>but</u> $M_{4\pi} = \mathbb{I}$ in $SL(2,\mathbb{C})$

$$M_\theta = e^{\frac{i\theta}{2}\sigma_3} = \underline{\underline{\cos\frac{\theta}{2}}} + i\sigma_3 \sin\frac{\theta}{2}$$

$$P \to P' = MPM^+ = \left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\sigma_3\right)\left(P_\mu \sigma^\mu\right)$$

$$P = P_\mu \sigma^\mu \quad \det P = P_\mu P^\mu \qquad \left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\sigma_3\right) =$$

$$P = (P_0, P_1, P_2, P_3) \qquad = P_m' \sigma^m$$

$$M_\theta = e^{\frac{i\theta}{2}\sigma_3} = \underline{\underline{\cos\frac{\theta}{2}}} + i\sigma_3 \sin\frac{\theta}{2}$$

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$$P = (P_0, P_1, P_2, P_3) \qquad = P'_\mu \sigma^\mu$$

$$M_\sigma = e^{i\frac{\theta}{2}\sigma_3} = \underline{\underline{\cos\frac{\theta}{2}}} + i\,\sigma_3 \sin\frac{\theta}{2} \qquad \sigma^0 = -I$$

$$\sigma^\mu = (\sigma^0, \sigma_x, \sigma_y, \sigma_z)$$

$$\cancel{P} \to P' = MPM^+ = \left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\sigma_3\right)\left(P_\mu\sigma^\mu\right)$$

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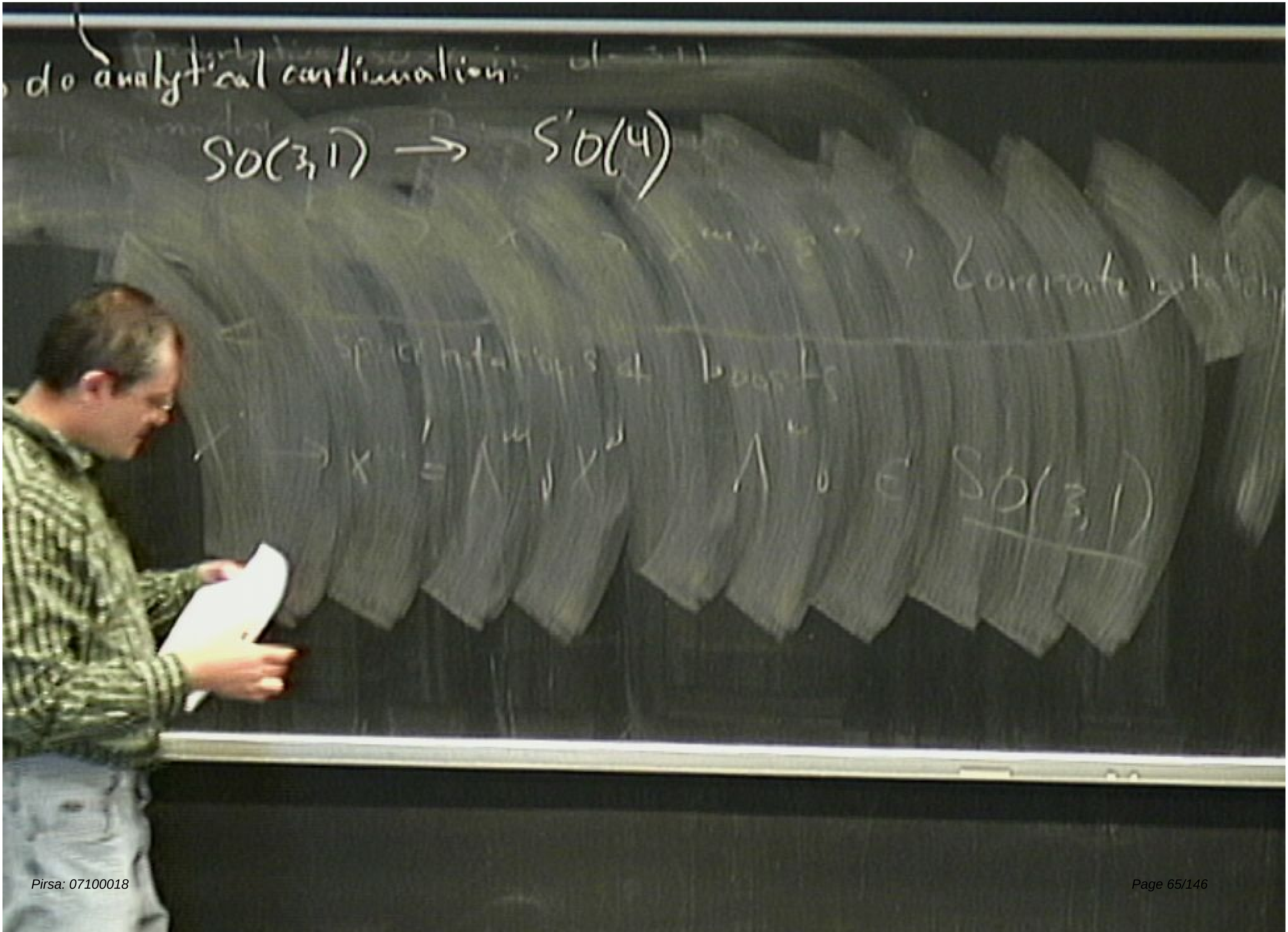
$$P = (P_0, P_1, P_2, P_3) \qquad = P_m'\sigma^\mu$$

$$\psi_\lambda \to e^{i\theta\lambda}\psi_\lambda \qquad \lambda \in \tfrac{1}{2}\mathbb{Z}$$

$$\psi_\lambda \xrightarrow{4\pi} e^{i4\pi\lambda}\psi_\lambda = \psi_\lambda \qquad e^{i4\pi\lambda} = 1$$

$$\left.\begin{aligned} P_1' &= P_1\cos\theta + \sin\theta\, P_2 \\ P_2' &= P_2\cos\theta - \sin\theta\, P_1 \end{aligned}\right\}$$

by "$-\theta$"
around z-axis


do analytical continuation:
SO(3,1) → SO(4)
Lorentz rotation
rotations & boosts
x → x' = Λ x
Λ ∈ SO(3,1)

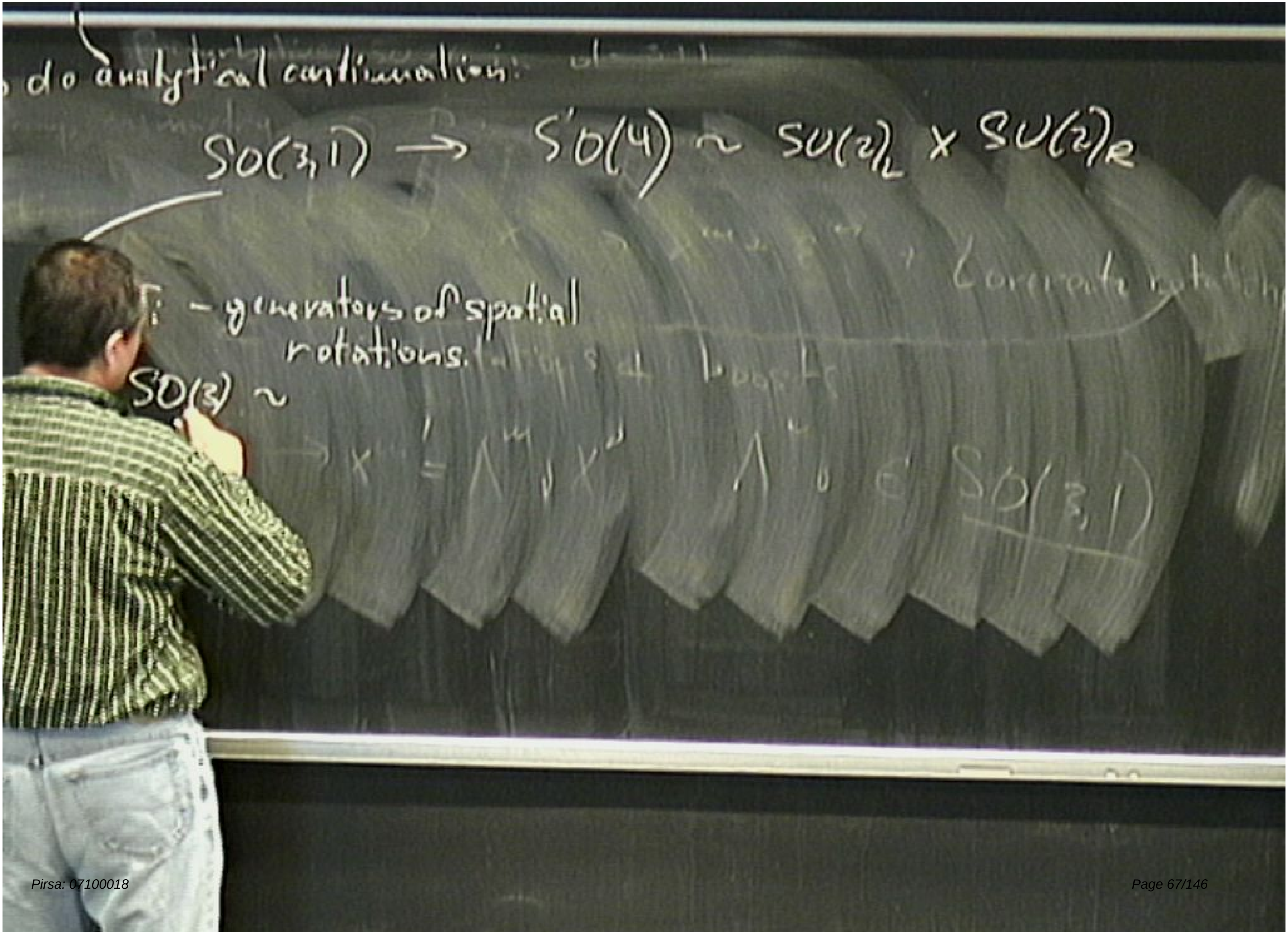
do analytical continuation:

$$SO(3,1) \to SO(4)$$

Lorentz rotations

spacetime rotations & boosts

$$x \to x' = \Lambda^{\mu}{}_{\nu} x^{\nu} \qquad \Lambda \in \underline{SO(3,1)}$$



do analytical continuation:
SO(3,1) → SO(4) ~ SU(2)_L × SU(2)_R
- generators of spatial rotations.
SO(3) ~

do analytical continuation:

$$SO(3,1) \longrightarrow SO(4) \sim SU(2)_L \times SU(2)_R$$

J_i – generators of spatial rotations.

$$SO(3) \sim SU(2)$$

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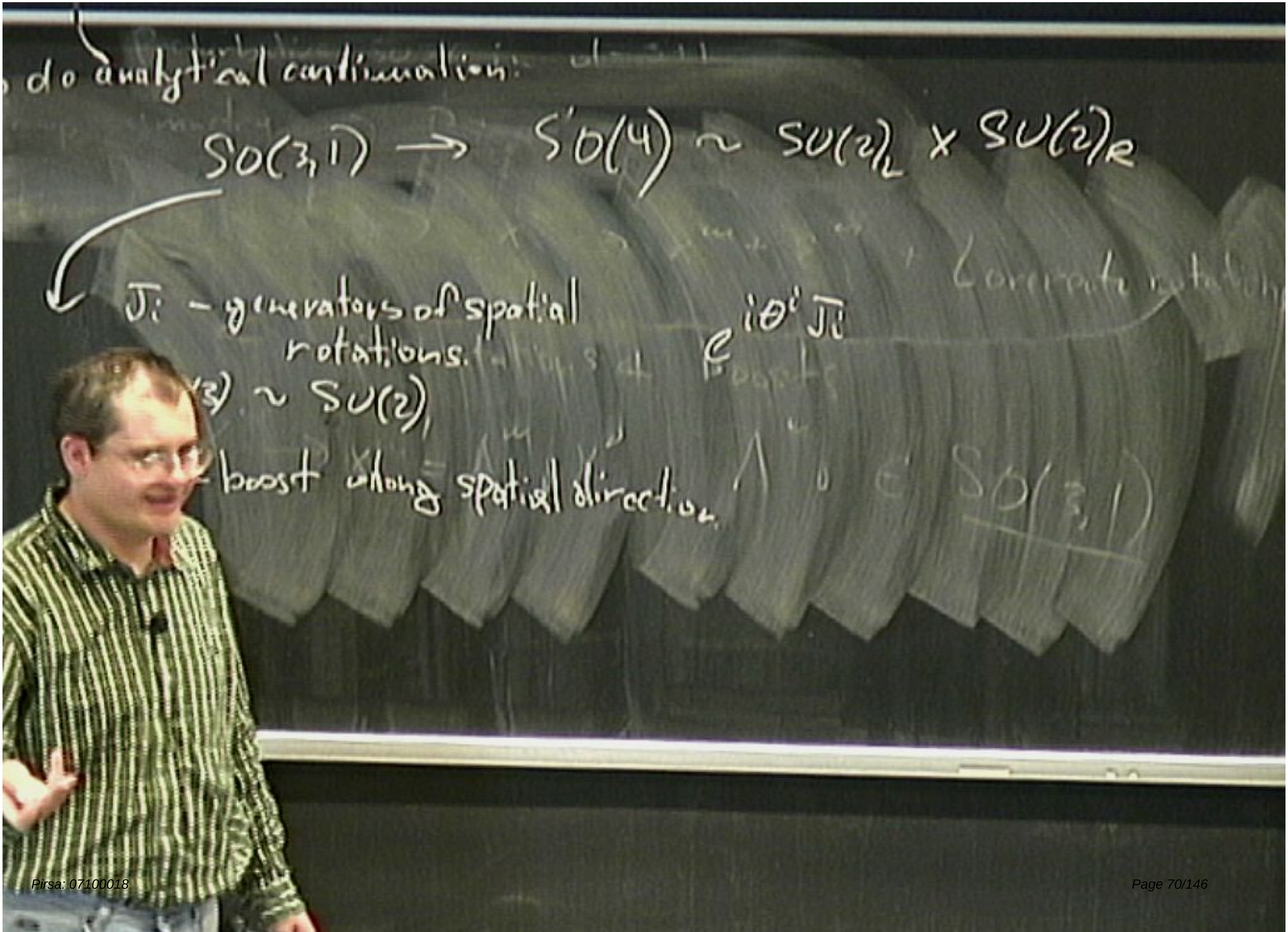
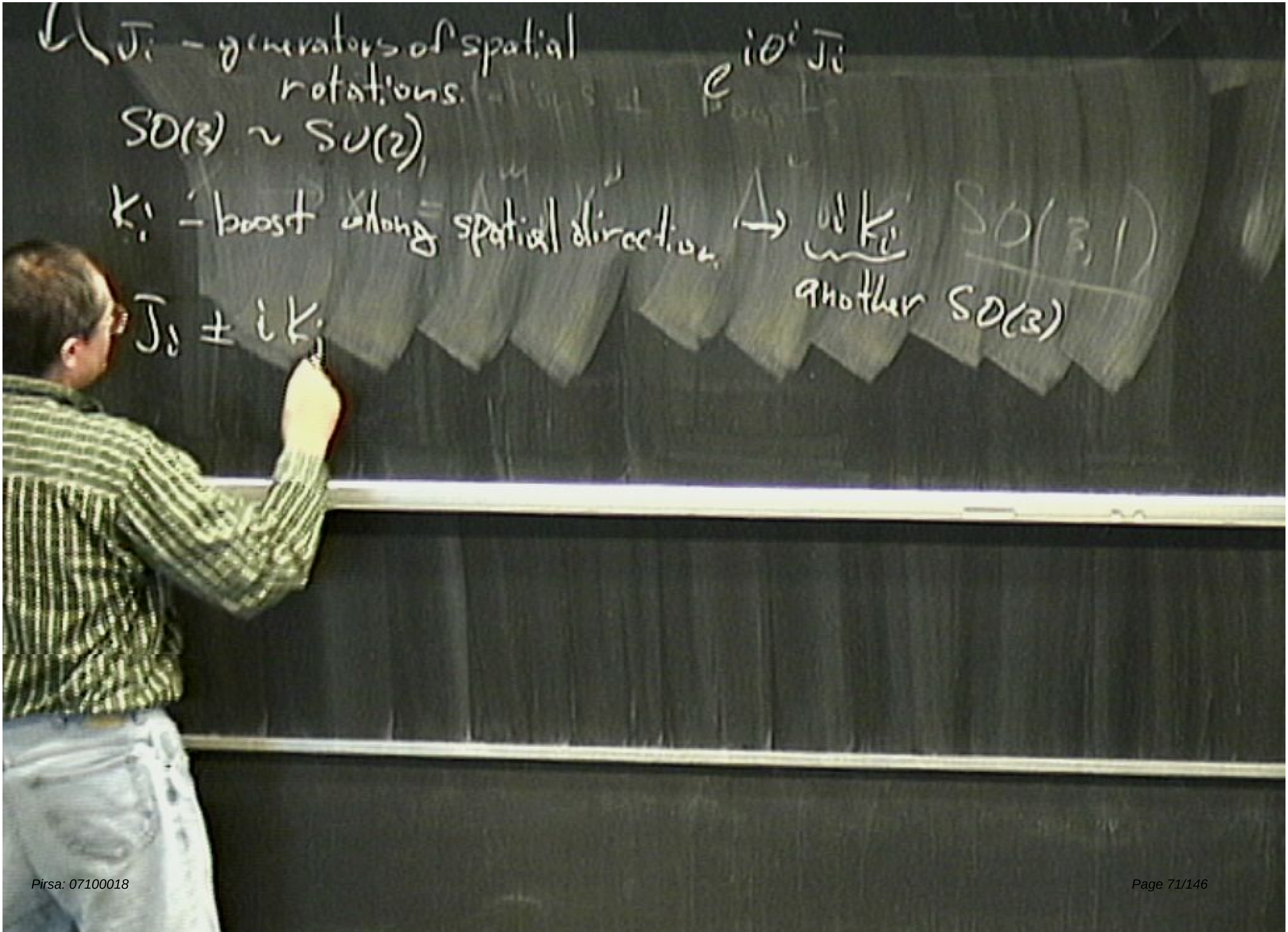
$$SO(3,1) \longrightarrow SO(4) \sim SU(2)_L \times SU(2)_R$$

J_i - generators of spatial rotations.

$SO(3) \sim SU(2)$

K_i - boost along spatial direction.


do analytical continuation:
SO(3,1) → SO(4) ~ SU(2)L × SU(2)R
Ji – generators of spatial rotations.
e iθi Ji
(3) ~ SU(2)
– boost along spatial direction.



J_i – generators of spatial rotations.
$e^{i\theta^i J_i}$
$SO(3) \sim SU(2)$
K_i – boost along spatial direction.
$\rightarrow \underbrace{iK_i}$
another $SO(3)$
$J_i \pm iK_i$

J_i - generators of spatial rotations. $e^{i\theta^i J_i}$

$SO(3) \sim SU(2)$

K_i - boost along spatial direction $\to \underbrace{i K_i}_{\text{another } SO(3)}$

$J_i \pm i K_i$ are generators of $SU(2)_R^L$

J_i - generators of spatial rotations. $\quad e^{i\theta^i J_i}$

$SO(3) \sim SU(2)$

K_i - boost along spatial direction. $\rightarrow \underbrace{iK_i}_{\text{another } SO(3)}$ $\quad \frac{SO(3,1)}{}$

$J_i \pm iK_i$ are generators of $SU(2)_L$, $SU(2)_R$

$[SU(2)_L \times SU(2)_R]_{\text{diagonal}}$

J_i – generators of spatial rotations. $e^{i\theta^i J_i}$

$SO(3) \sim SU(2)$

K_i – boost along spatial direction. $\rightarrow$ $\underbrace{i K_i}$ another $SO(3)$

$SO(3,1)$

$J_i \pm i K_i$ are generators of $SU(2)^L$, $SU(2)^R$

$\left[SU(2)_L \times SU(2)_R\right]_{\text{diagonal}} = \{J_i\} \sim SO(3)$

$\Rightarrow$ Any representation of $SU(2)_L \times SU(2)_R$

(j_L, j_R) SO(3) spin

scalar φ $(0, 0)$

$\Rightarrow$ Any representation of $SU(2)_L \times SU(2)_R$

(j_L, j_R) — $SO(3)$ spin

scalar φ — $(0, 0)$ — 0

$\Rightarrow$ ψ_L — $(\frac{1}{2}, 0)$ — $\frac{1}{2}$

Any representation of $SU(2)_L \times SU(2)_R$

(j_L, j_R) — $SO(3)$ spin

	(j_L, j_R)	0
scalar φ	$(0, 0)$	
left-handed spinor ψ_L	$(\frac{1}{2}, 0)$	$\frac{1}{2}$
right handed spinor $\bar{\psi}_{\dot{\alpha}}$	$(0, \frac{1}{2})$	$\frac{1}{2}$

Any representation of $SU(2)_L \times SU(2)_R$ — $SO(3)$ spin

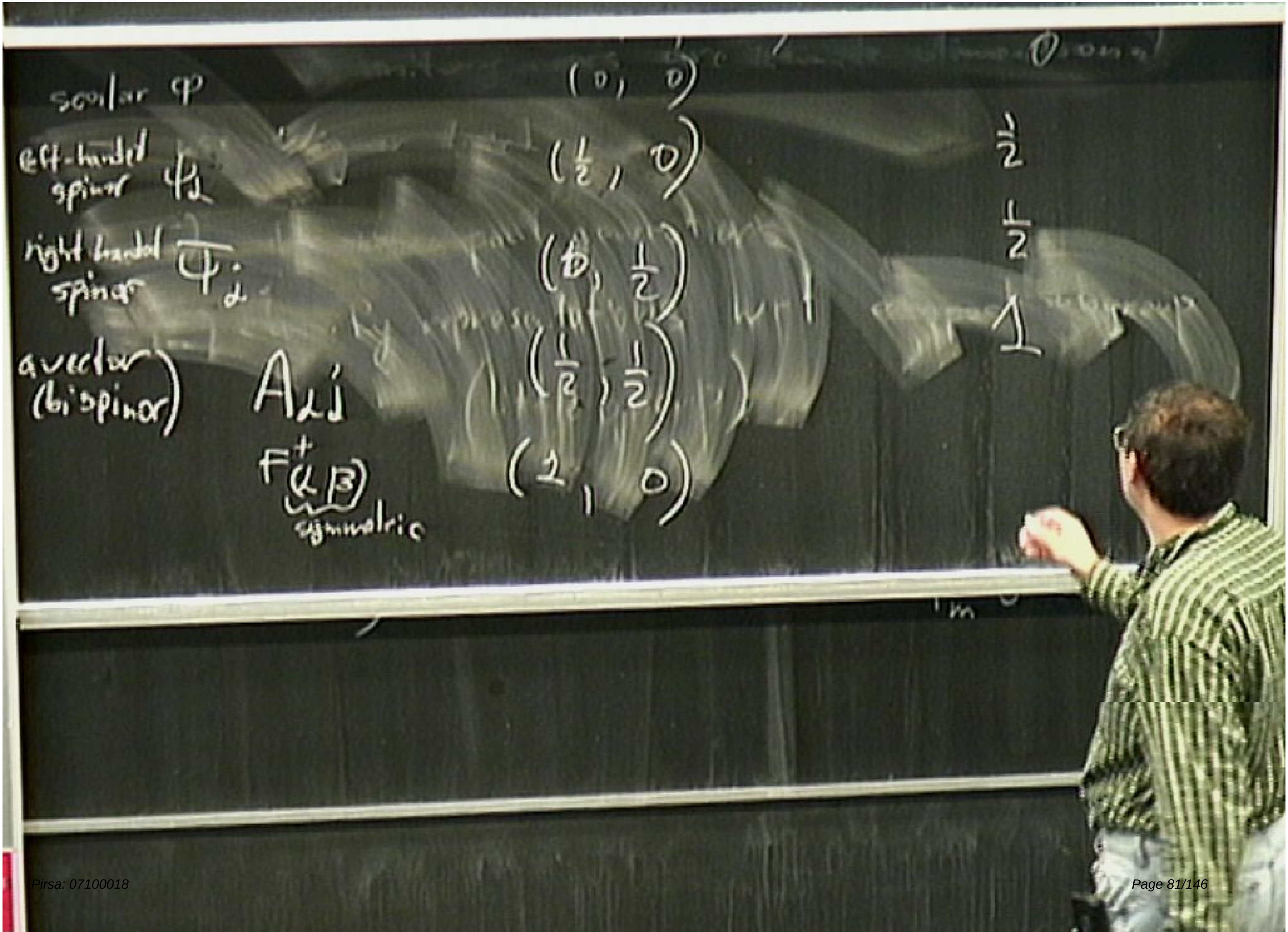
(j_L, j_R)

scalar φ	$(0, 0)$	0
left-handed spinor ψ_α	$(\frac{1}{2}, 0)$	$\frac{1}{2}$
right handed spinor $\bar{\psi}_{\dot\alpha}$	$(0, \frac{1}{2})$	$\frac{1}{2}$
a vector (bispinor) $A_{\alpha\dot\alpha}$	$(\frac{1}{2}, \frac{1}{2})$	

Any representation of $SU(2)_L \times SU(2)_R$

		(j_L, j_R)	$SO(3)$ spin
scalar	φ	$(0, 0)$	0
left-handed spinor	ψ_α	$(\frac{1}{2}, 0)$	$\frac{1}{2}$
right-handed spinor	$\bar{\psi}_{\dot\alpha}$	$(0, \frac{1}{2})$	$\frac{1}{2}$
a vector (bispinor)	$A_{\alpha\dot\alpha}$	$(\frac{1}{2}, \frac{1}{2})$	

scalar	φ	$(0, 0)$	
left-handed spinor	ψ_α	$(\frac{1}{2}, 0)$	$\frac{1}{2}$
right handed spinor	$\bar{\psi}_{\dot\alpha}$	$(0, \frac{1}{2})$	$\frac{1}{2}$
a vector (bispinor)	$A_{\alpha\dot\alpha}$	$(\frac{1}{2}, \frac{1}{2})$	1
	$F^{+}_{\underbrace{(\alpha\beta)}_{\text{symmetric}}}$	$(1, 0)$	


scalar φ $(0, 0)$ 0
left-handed spinor ψ_α $(\frac{1}{2}, 0)$ $\frac{1}{2}$
right handed spinor $\bar{\psi}_{\dot{\alpha}}$ $(0, \frac{1}{2})$ $\frac{1}{2}$
a vector (bispinor) $A_{\alpha\dot{\alpha}}$ $(\frac{1}{2}, \frac{1}{2})$ 1
$F^{+}_{(\alpha\beta)}$ symmetric $(1, 0)$

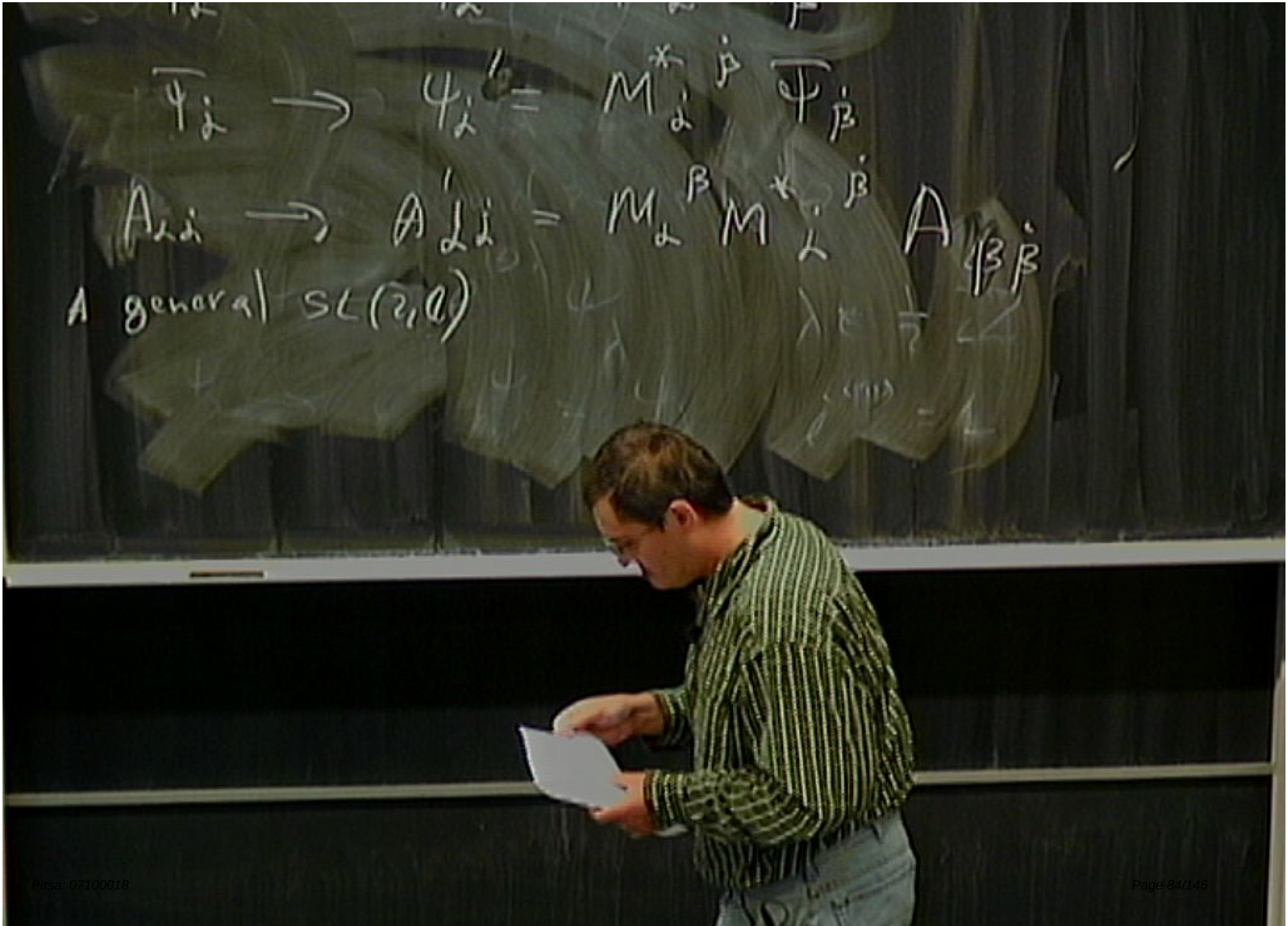
scalar	φ	$(0, 0)$	0
left-handed spinor	ψ_{α}	$(\frac{1}{2}, 0)$	$\frac{1}{2}$
right handed spinor	$\bar{\psi}_{\dot{\alpha}}$	$(0, \frac{1}{2})$	$\frac{1}{2}$
a vector (bi-spinor)	$A_{\alpha\dot{\alpha}}$	$(\frac{1}{2}, \frac{1}{2})$	1
self-dual 2-form	$F^{+}_{\underbrace{(\alpha\beta)}_{\text{symmetric}}}$	$(1, 0)$	1
anti-self dual 2-form	$F_{(\dot{\alpha}\dot{\beta})}$	$(0, 1)$	1

Recall:

$$\psi_\alpha \longrightarrow \psi'_\alpha = M_\alpha{}^\beta \psi_\beta$$

$$\bar{\psi}_{\dot\alpha} \longrightarrow \bar{\psi}'_{\dot\alpha} = M^*{}_{\dot\alpha}{}^{\dot\beta} \bar{\psi}_{\dot\beta}$$

$$A_{\alpha\dot\alpha} \longrightarrow A'_{\alpha\dot\alpha} = M_\alpha{}^\beta M^*{}_{\dot\alpha}{}^{\dot\beta} A_{\beta\dot\beta}$$


$\bar{\psi}_{\dot{\alpha}} \longrightarrow \psi'_{\dot{\alpha}} = M^{*\dot{\beta}}_{\dot{\alpha}} \bar{\psi}_{\dot{\beta}}$
$A_{\alpha\dot{\alpha}} \longrightarrow A'_{\alpha\dot{\alpha}} = M_{\alpha}^{\ \beta} M^{*\ \dot{\beta}}_{\dot{\alpha}} A_{\beta\dot{\beta}}$
A general SL(2,C)

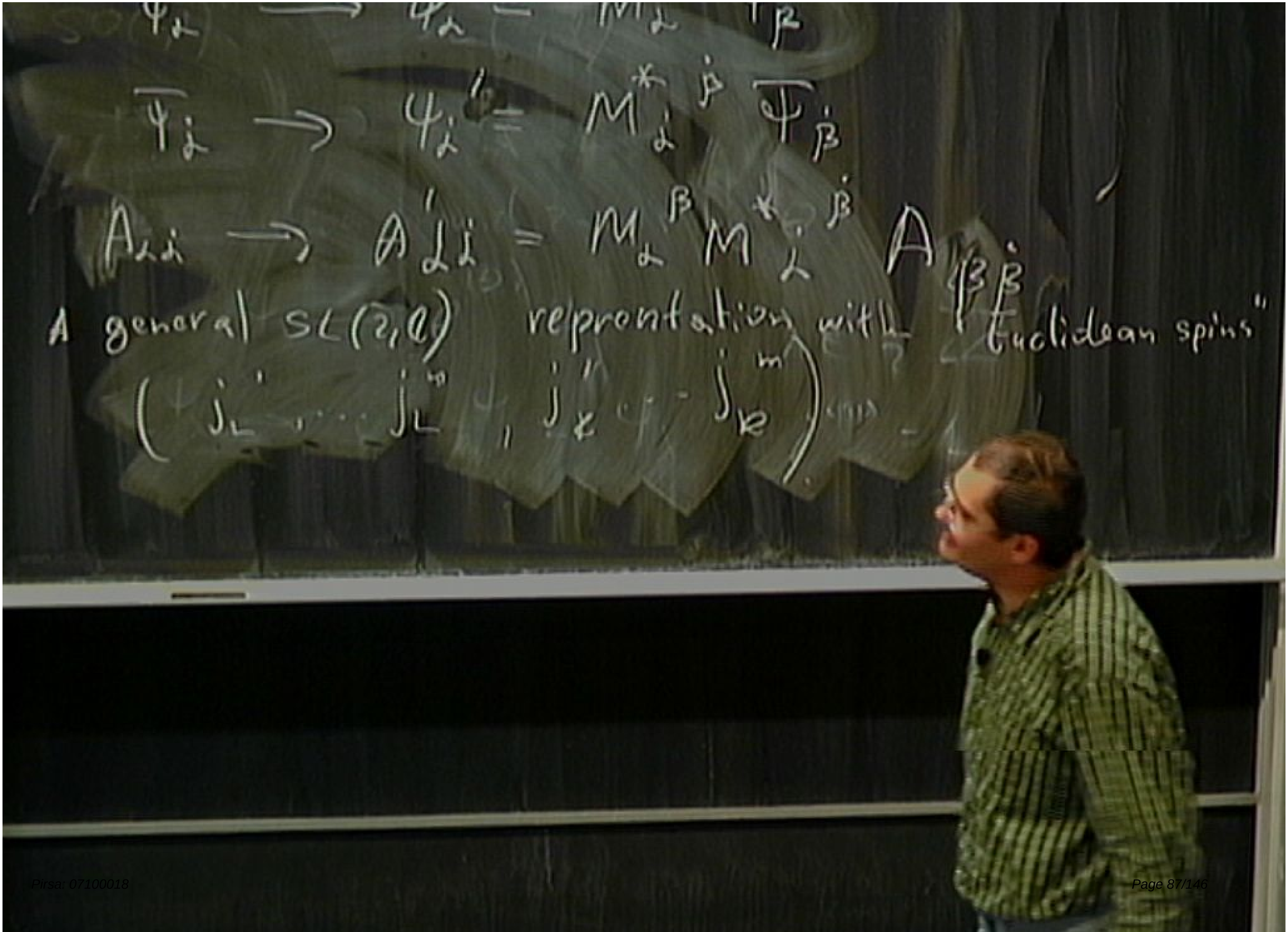
$$\bar{\psi}_{\dot\alpha} \longrightarrow \psi'_{\dot\alpha} = M^{*}{}_{\dot\alpha}{}^{\dot\beta}\, \bar{\psi}_{\dot\beta}$$

$$A_{\alpha\dot\alpha} \longrightarrow A'_{\alpha\dot\alpha} = M_\alpha{}^\beta M^{*}{}_{\dot\alpha}{}^{\dot\beta} A_{\beta\dot\beta}$$

A general $SL(2,\mathbb{C})$ repr[e]sentation with Euclidean

⇒ Any representation of $SU(2)_L \times SU(2)_R$

		(j_L, j_R)	SO(3) spi
scalar	φ	$(0, 0)$	0
ft-handed spinor	ψ_α	$(\frac{1}{2}, 0)$	$\frac{1}{2}$
ight handed spinor	$\bar{\psi}_{\dot\alpha}$	$(0, \frac{1}{2})$	$\frac{1}{2}$
vector (bispinor)	$A_{\alpha\dot\alpha}$	$(\frac{1}{2}, \frac{1}{2})$	1
elf-dual 2-form	$F^+_{(\alpha\beta)}$ symmetric	$(1, 0)$	1
nti-self dual 2-form	$F_{(\dot\alpha\dot\beta)}$	$(0, 1)$	1


Ψ̄α̇ → Ψ'α̇ = M*α̇β̇ Ψ̄β̇
Aαα̇ → A'αα̇ = Mαβ M*α̇β̇ Aββ̇
A general SL(2,C) repr[e]ntation with "Euclidean spins"
(jL', ..., jL'', jR', ..., jR''')

$$\bar{\psi}_{\dot\alpha} \longrightarrow \psi'_{\dot\alpha} = M^{*\,\dot\beta}_{\dot\alpha}\, \bar{\psi}_{\dot\beta}$$

$$A_{\alpha\dot\alpha} \longrightarrow A'_{\alpha\dot\alpha} = M_\alpha{}^{\beta} M^{*\,\dot\beta}_{\dot\alpha}\, A_{\beta\dot\beta}$$

A general $SL(2,\mathbb{C})$ reprentation with "Euclidean spins"

$$\left(j_L', \ldots j_L'' \;,\; j_R' \ldots j_R''' \right)$$

$$X_{\alpha_1 \ldots \alpha_m ; \dot\alpha_1 \ldots \dot\alpha_n}$$

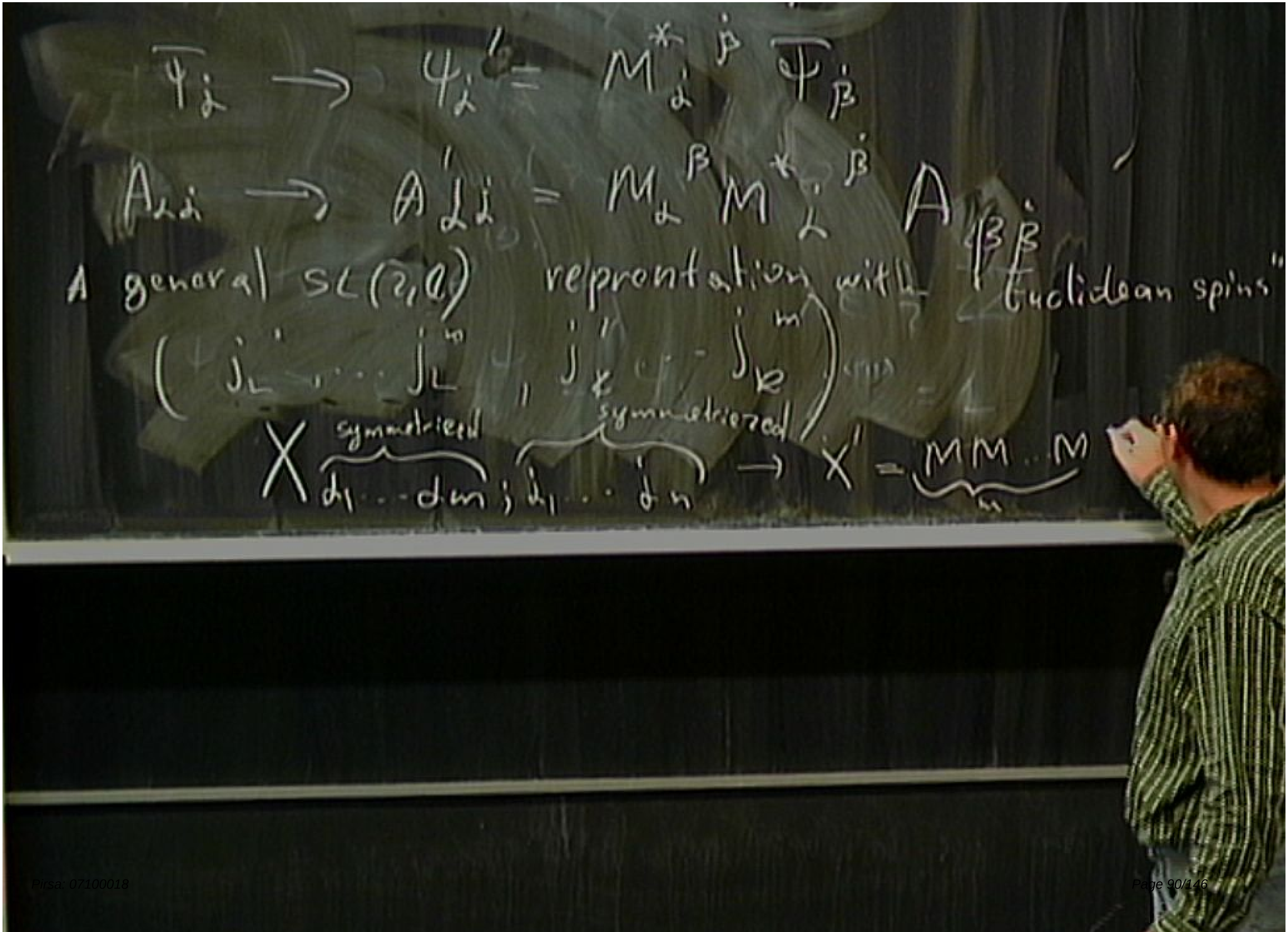
$$\bar{\psi}_{\dot{\alpha}} \longrightarrow \psi'_{\dot{\alpha}} = M^{*\ \dot{\beta}}_{\dot{\alpha}}\, \bar{\psi}_{\dot{\beta}}$$

$$A_{\alpha\dot{\alpha}} \longrightarrow A'_{\alpha\dot{\alpha}} = M_\alpha{}^\beta\, M^{*\ \dot{\beta}}_{\dot{\alpha}}\, A_{\beta\dot{\beta}}$$

A general $SL(2,\mathbb{C})$ reprentation with "Euclidean spins"

$$\left(j_L^{\,1} \ldots j_L^{\,m},\; j_R^{\,1} \ldots j_R^{\,m} \right)$$

$$X_{\underbrace{\alpha_1 \ldots \alpha_m}_{\text{symmetrized}};\,\underbrace{\dot{\alpha}_1 \ldots \dot{\alpha}_n}_{\text{symmetrized}}}$$


A general SL(2,C) representation with "Euclidean spins"
symmetrized
symmetrized

$$\bar{\psi}_{\dot\alpha} \longrightarrow \bar{\psi}'_{\dot\alpha} = M^{*\ \dot\beta}_{\dot\alpha}\, \bar{\psi}_{\dot\beta}$$

$$A_{\alpha\dot\alpha} \longrightarrow A'_{\alpha\dot\alpha} = M_\alpha^{\ \beta} M^{*\ \dot\beta}_{\dot\alpha} A_{\beta\dot\beta}$$

A general $SL(2,\mathbb{C})$ repr[es]ent[at]ion with "Euclidean spins"

$$\left(j_L^{\,'} \ldots j_L^{\,m},\ j_R^{\,'} \ldots j_R^{\,m} \right)$$

$$X_{\underbrace{\alpha_1 \ldots \alpha_m}_{\text{symmetrized}};\ \underbrace{\dot\alpha_1 \ldots \dot\alpha_n}_{\text{symmetrized}}} \longrightarrow X' = \underbrace{MM \ldots M}_{m}\ \underbrace{M^* \ldots M^*}_{n} X$$

K_i – boost along spatial direction $\rightarrow$ $\underbrace{i K_i}$ another $SO(3)$

R^{d} ... is a generator of 2 rotations in $SL(2,\mathbb{C})$

representation of $SO(3,1)$

A_i: $\rightarrow$ vector.

π_{ij} =

... that M_D is 2-rotations

... π = ... in $SL(2,\mathbb{C})$

K_i: - boost along spatial direction $\rightarrow$ $\underbrace{i K_i}$

another $SO(3)$

R^d $SO(d)$ $SL(2,C)$

A_i: $\rightarrow$ vector

T_{ij}

K_i - boost along spatial direction $\rightarrow$ $\underbrace{K_i}$

another $SO(3)$

R^d ... $SO(d)$ ...

$A_i \rightarrow$ vector

$$T_{ij} = T_{(ij)}^{\text{traceless}} + T_{[ij]} + T_k^k g_{ij}$$

K_i - boost along spatial direction $\rightarrow$ $\underbrace{iK_i}$

another $SO(3)$

R^d ... a $SO(d)$

$A_i \rightarrow$ vector

$$T_{ij} = T_{(ij)} + T_{[ij]} + T^k{}_k\, g_{ij}$$

traceless

K_i : - boost along spatial direction $\rightarrow$ $\underbrace{i K_i}$

another $SO(3)$

R^d $SO(d)$

A_i $\rightarrow$ vector

$$T_{ij} = T^{\text{traceless}}_{(ij)} + T_{[ij]} + T_k^{\ k} g_{ij}$$

$\rightarrow$ reason by it is symmetric traceless

K_i - boost along spatial direction $\to$ $\underbrace{iK_i}$

another $SO(3)$

$\mathbb{R}^d$ $SO(d)$

$A_i \to$ vector

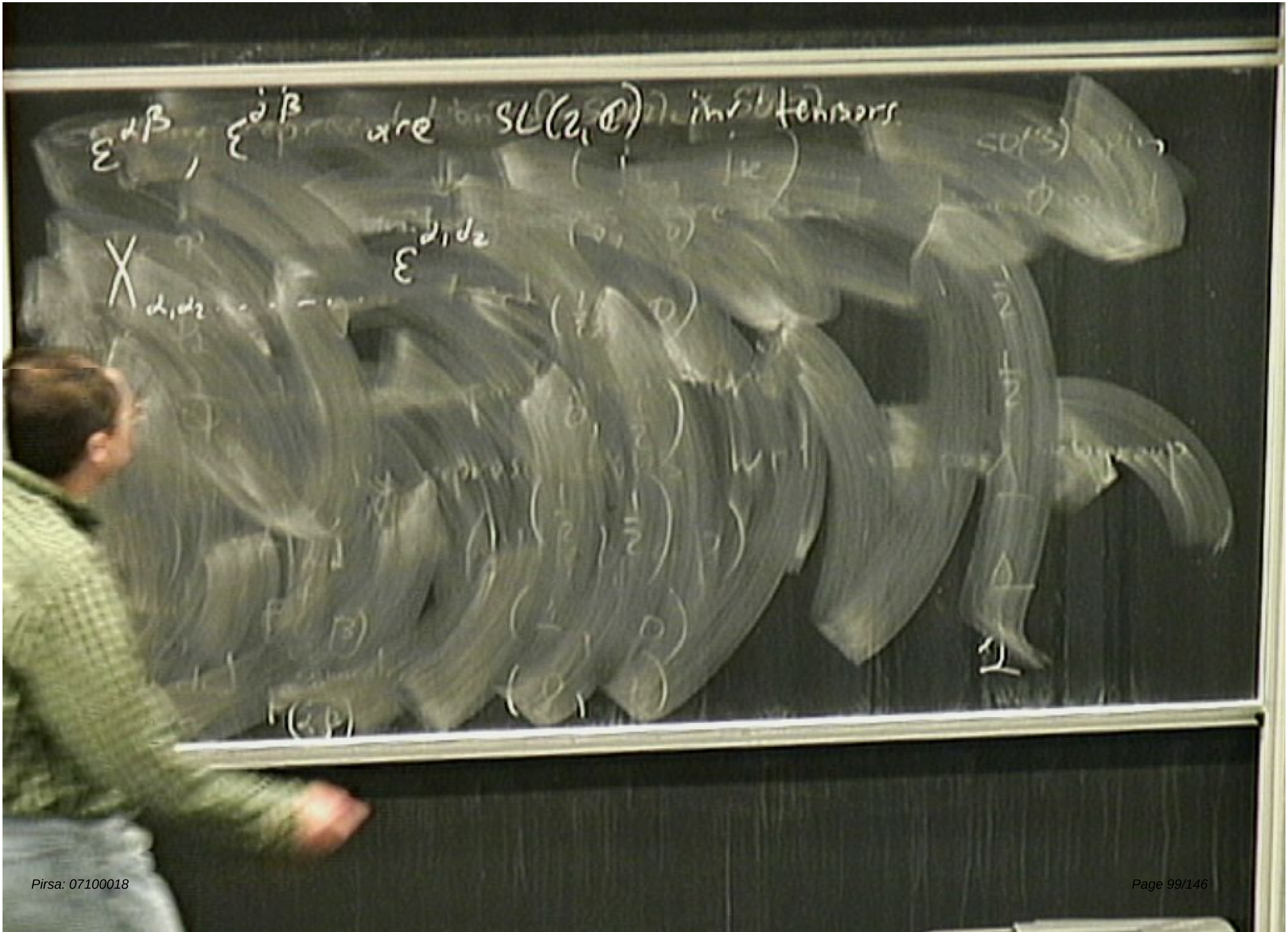
$$T_{ij} = T^{traceless}_{(ij)} + T_{[ij]} + T_k{}^k g_{ij}$$

$\to$ reason by it's symmetric traceless

$\delta_i^j \to$ an invariant tensor

$T_{ij} g^{ij}$

$\varepsilon^{\alpha\beta}$, $\varepsilon^{\dot\alpha\dot\beta}$ are $SL(2,\mathbb{C})$ inv. tensors


$\varepsilon^{\alpha\beta}$, $\varepsilon^{\dot\alpha\dot\beta}$ are $SL(2,\mathbb{C})$ inv tensors
$\varepsilon^{\alpha_1\alpha_2}$
$X_{\alpha_1\alpha_2}$

$\varepsilon^{\alpha\beta}$, $\varepsilon^{\dot\alpha\dot\beta}$ are $SL(2,\mathbb{C})$ inv. tensors

$X_{\alpha_1\alpha_2 \dots}$ $\varepsilon^{\alpha_1\alpha_2} = 0$ otherwise are

initial representation is reducible

$\varepsilon^{\alpha\beta}$, $\varepsilon^{\dot\alpha\dot\beta}$ are $SL(2,\mathbb{C})$ inv. tensors

$X_{\alpha_1\alpha_2\ldots}$ $\varepsilon^{\alpha_1\alpha_2}$ $= 0$ otherwise are

initial representation is reducible

$\varepsilon^{\alpha\beta}$, $\varepsilon^{\dot\alpha\dot\beta}$ are $SL(2,\mathbb{C})$ inv. tensors

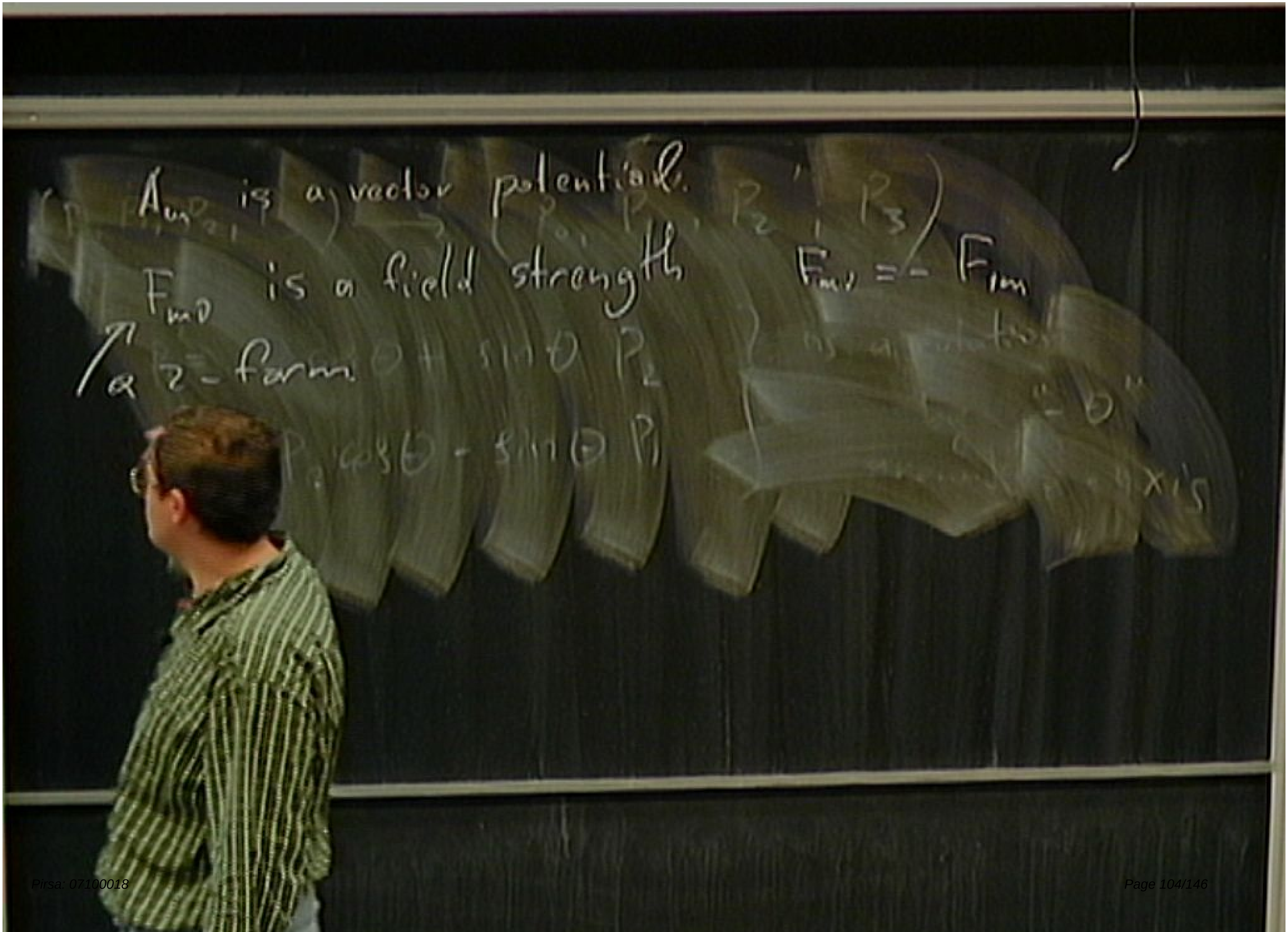
$X_{\underline{\alpha_1\alpha_2}\cdots}$ $\varepsilon^{\alpha_1\alpha_2} \neq 0$ otherwise are

$\Downarrow$ initial representation is reducible

$\Downarrow$ must be symmetric in $SL(2,\mathbb{C})$ spinor indecies

A_μ is a vector potential.

$F_{\mu\nu}$ is a field strength $\quad F_{\mu\nu} = - F_{\nu\mu}$



A_m is a vector potential.
F_mν is a field strength
F_mν = - F_νm
a 2-form

A_μ is a vector potential.

$F_{\mu\nu}$ is a field strength $\quad F_{\mu\nu} = -F_{\nu\mu}$

a 2-form

$\mu \longrightarrow$

$$\underbrace{\left(\tfrac{1}{2},\tfrac{1}{2}\right)}_{\mu} \otimes \underbrace{\left(\tfrac{1}{2},\tfrac{1}{2}\right)}_{\nu}$$

A_μ is a vector potential.

$F_{\mu\nu}$ is a field strength $\quad F_{\mu\nu} = -F_{\nu\mu}$

a 2-form

$\mu \longrightarrow$

$$\left[\underbrace{\left(\tfrac{1}{2},\tfrac{1}{2}\right)}_{\mu} \otimes \underbrace{\left(\tfrac{1}{2},\tfrac{1}{2}\right)}_{\nu}\right]_A$$

A_μ is a vector potential.

$F_{\mu\nu}$ is a field strength $\quad F_{\mu\nu} = - F_{\nu\mu}$

a 2-form

$\mu \longrightarrow$

$$\Big[\underbrace{\left(\tfrac{1}{2}, \tfrac{1}{2}\right)}_{\mu} \otimes \underbrace{\left(\tfrac{1}{2}, \tfrac{1}{2}\right)}_{\nu} \Big]_A =$$

A_μ is a vector potential.

$F_{\mu\nu}$ is a field strength $\quad F_{\mu\nu} = - F_{\nu\mu}$

$\nearrow$ a 2-form

$\mu \longrightarrow$

$$\left[\underbrace{\left(\tfrac{1}{2}, \tfrac{1}{2}\right)}_{\mu} \otimes \underbrace{\left(\tfrac{1}{2}, \tfrac{1}{2}\right)}_{\nu} \right]_A = \left(\left(\tfrac{1}{2} \otimes \tfrac{1}{2}\right)_{A}, \left(\tfrac{1}{2} \otimes \tfrac{1}{2}\right)_S \right)$$

A_μ is a vector potential.

$F_{\mu\nu}$ is a field strength $\quad F_{\mu\nu} = -F_{\nu\mu}$

a 2-form

$\mu \longrightarrow$

$$\Big[\underbrace{\left(\tfrac{1}{2},\tfrac{1}{2}\right)}_{\mu} \otimes \underbrace{\left(\tfrac{1}{2},\tfrac{1}{2}\right)}_{\nu}\Big]_A = \left(\left(\tfrac{1}{2}\otimes\tfrac{1}{2}\right)_{A,1}\left(\tfrac{1}{2}\otimes\tfrac{1}{2}\right)_S\right)$$

$A_{\mu\nu}$ is a vector potential.

$F_{\mu\nu}$ is a field strength $\quad F_{\mu\nu} = - F_{\nu\mu}$

a 2-form

$M \longrightarrow \alpha\dot{\alpha}$

$$\Big[\underbrace{\left(\tfrac{1}{2},\tfrac{1}{2}\right)}_{\mu} \otimes \underbrace{\left(\tfrac{1}{2},\tfrac{1}{2}\right)}_{\nu}\Big]_A = \left(\left(\tfrac{1}{2}\otimes\tfrac{1}{2}\right)_A, \left(\tfrac{1}{2}\otimes\tfrac{1}{2}\right)_S\right) + \left(\left(\tfrac{1}{2}\otimes\tfrac{1}{2}\right)_S, \left(\tfrac{1}{2}\otimes\tfrac{1}{2}\right)_A\right)$$

g spatial direction.

another $SO(3)$

rotations in $SL(2,\mathbb{C})$

$SO(3,1)$

$= (0,1) \oplus$

less

$= I$ in $SL(2,\mathbb{C})$

another $SO(3)$

$$= \underline{(0,1)} \oplus \underline{(1,0)}$$

less

in $SL(2,\mathbb{C})$

another $SO(3)$

$$= \underline{(2,1)} \oplus \underline{(1,0)}$$

less

in $SL(2,\mathbb{C})$

another $SO(3)$

$$= \underbrace{(0, 1)}_{F^{-}_{\dot\alpha\dot\beta}} \oplus \underbrace{(1, 0)}_{F^{+}_{\alpha\beta}}$$

less

$SL(2, \mathbb{C})$

another $SO(3)$

$$= \underline{(0,1)} \oplus \underline{(1,0)}$$

$SO(3,1)$ $\quad F_{\dot\alpha\dot\beta}$ $\quad F^{+}_{\alpha\beta}$

$\varepsilon_{\mu\nu\lambda\rho} =$

less

another $SO(3)$

$$= \underline{(0,1)} \oplus \underline{(1,0)}$$

$SO(3,1)$ $\quad F_{\dot\alpha\dot\beta} \qquad F_{\alpha\beta}$

$\varepsilon_{\mu\nu\lambda\rho} = \pm 1$

$$\varepsilon_{0123} = -1$$

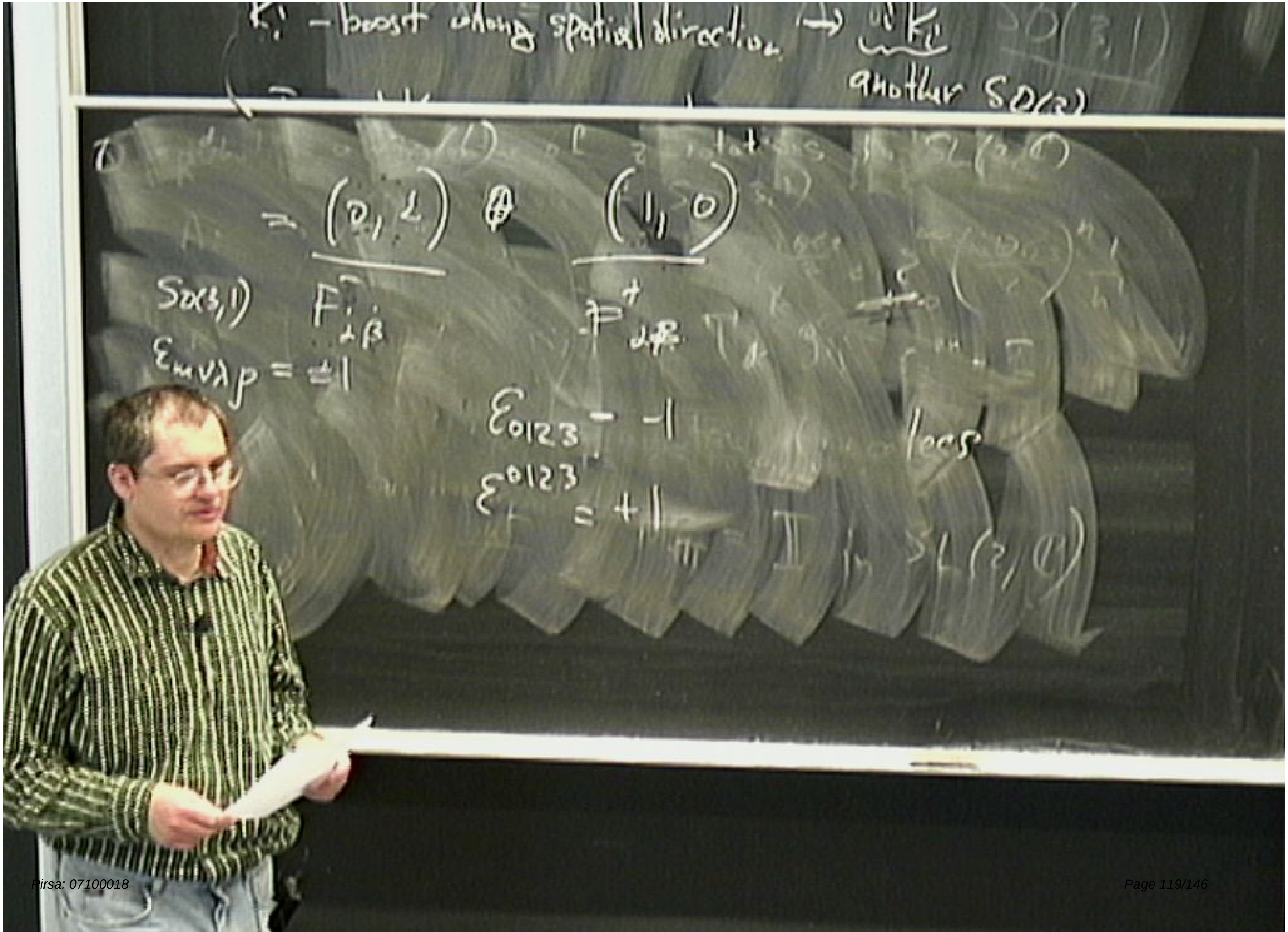
$$\varepsilon^{0123} = +1$$

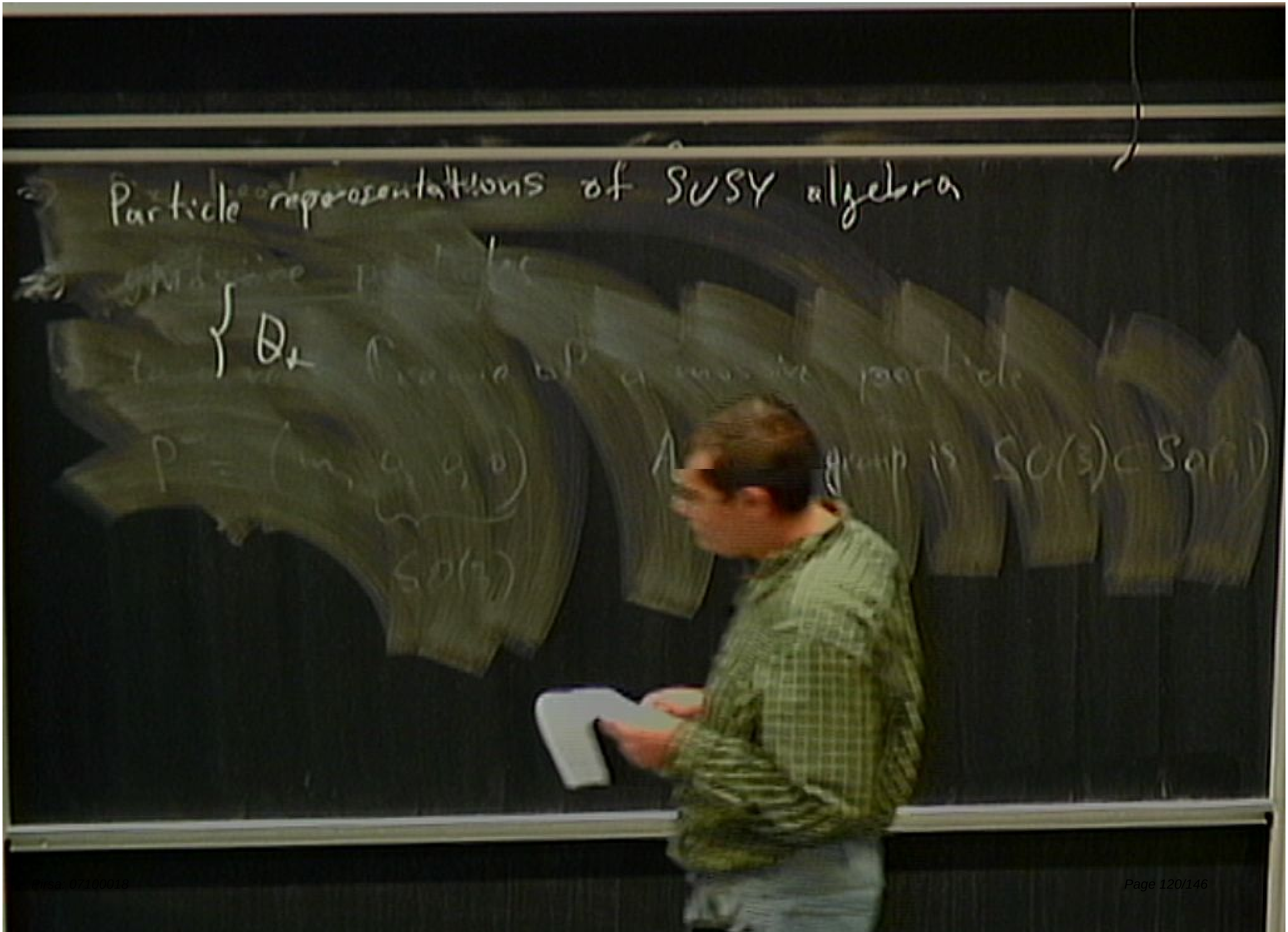
Given $F_{\mu\nu} \Longrightarrow \overset{*}{F}_{\mu\nu} = \frac{i}{\sqrt{2}} \varepsilon_{\mu\nu\lambda\rho} F^{\lambda\rho}$

Given $F_{\mu\nu} \longrightarrow \underset{\underset{\text{Hodge dual}}{\uparrow}}{F^{*}_{\mu\nu}} = \frac{i}{\sqrt{2}} \varepsilon_{\mu\nu\lambda\rho} F^{\lambda\rho}$

$F^{+} = (F + F^{*})\frac{1}{\sqrt{2}}$ — self dual

$F^{-} = (F - F^{*})\frac{1}{\sqrt{2}}$ — anti self dual


Ki - boost along spatial direction → iKi
another SO(3)
= (2, 1) ⊕ (1, 0)
SO(3,1)
εμνλρ = ±1
ε0123 = -1
ε0123 = +1


Particle representations of SUSY algebra

Particle representations of SUSY algebra

massive particles

$$\{Q_\alpha , \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu$$

$p^\mu = (m, 0, 0, 0)$

$\underbrace{0, 0, 0}_{SO(3)}$

A little group is $SO(3) \subset SO(3,1)$

Particle representations of SUSY algebra

$$\{ Q_\alpha , \bar{Q}_{\dot\alpha} \} = 2 \sigma^\mu_{\alpha\dot\alpha} P_\mu = 2 P_{\alpha\dot\alpha}$$

$$\{ Q_\alpha , Q_\beta \} = \{ \bar{Q}_{\dot\alpha} , \bar{Q}_{\dot\beta} \} = 0$$

$$[P_\mu , Q_\alpha] = [P_\mu , \bar{Q}_{\dot\alpha}] = 0$$

K_i - boost along spatial direction $\rightarrow$ $\underbrace{iK_i}$

another $SO(3)$

7) Boost massive

K_i - boost along spatial direction $\rightarrow$ $\underbrace{iK_i}$

another $SO(3)$

(E) Boost massive particle to a rest frame

$p^\mu = (m, 0, 0, 0)$ $\quad p_\mu = (-m, 0, 0, 0)$

K_i – boost along spatial direction $\rightarrow iK_i$ another $SO(3)$

$\Rightarrow$ Boost massive particle to a rest frame

$$p^{\mu} = (m, 0, 0, 0) \qquad P_{\mu} = (-m, 0, 0, 0)$$

$$\Rightarrow \quad \{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2m\,\delta_{\alpha\dot\alpha}$$

K_i - boost along spatial direction $\Rightarrow$ $\underbrace{iK_i}$

another $SO(3)$

$\Rightarrow$ Boost massive particle to a rest frame

$$p^{\mu} = (m, 0, 0, 0) \qquad P_{\mu} = (-m, 0, 0, 0)$$

$$\Rightarrow \quad \{Q_{\alpha}, \bar{Q}_{\dot{\alpha}}\} = 2m\,\delta_{\alpha\dot{\alpha}} \qquad \{Q_{\alpha}, Q_{\beta}\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

$$\Rightarrow \quad a_{\alpha} = \frac{1}{\sqrt{2m}} Q_{\alpha} \qquad \bar{a}_{\dot{\alpha}} = \frac{1}{\sqrt{2m}} \bar{Q}_{\dot{\alpha}}$$

$$\{\bar{a}_\alpha, a_\beta\} = \delta_{\alpha\beta},$$

$$\{a_\alpha, a_\beta\} = \{\bar{a}_\alpha, \bar{a}_\beta\} = 0$$

$$\left.\begin{array}{l}\{\bar{a}_\alpha, a_\beta\} = \delta_{\alpha\beta}, \\ \{a_\alpha, a_\beta\} = \{\bar{a}_\alpha, \bar{a}_\beta\} = 0\end{array}\right\},$$

4-dim

$$\{\bar{a}_\alpha, a_\beta\} = \delta_{\alpha\beta},$$

$$\{a_\alpha, a_\beta\} = \{\bar{a}_\alpha, \bar{a}_\beta\} = 0$$

4-dim Clifford algebra

$a_\alpha^2 = 0 \Rightarrow$ we can always find $|\Omega\rangle$

$$\{\bar{a}_\alpha, a_\beta\} = \delta_{\alpha\beta},$$

$$\{a_\alpha, a_\beta\} = \{\bar{a}_\alpha, \bar{a}_\beta\} = 0$$

4-dim Clifford algebra

$a_\alpha^2 = 0 \Rightarrow$ we can always find $|\Omega\rangle$

$$a_\alpha|\Omega\rangle = 0$$

$$\{\bar{a}_\alpha, a_\beta\} = \delta_{\alpha\beta},$$

$$\{a_\alpha, a_\beta\} = \{\bar{a}_\alpha, \bar{a}_\beta\} = 0$$

4-dim Clifford algebra

$a_\alpha^2 = 0 \Rightarrow$ we can always find $|\Omega\rangle$

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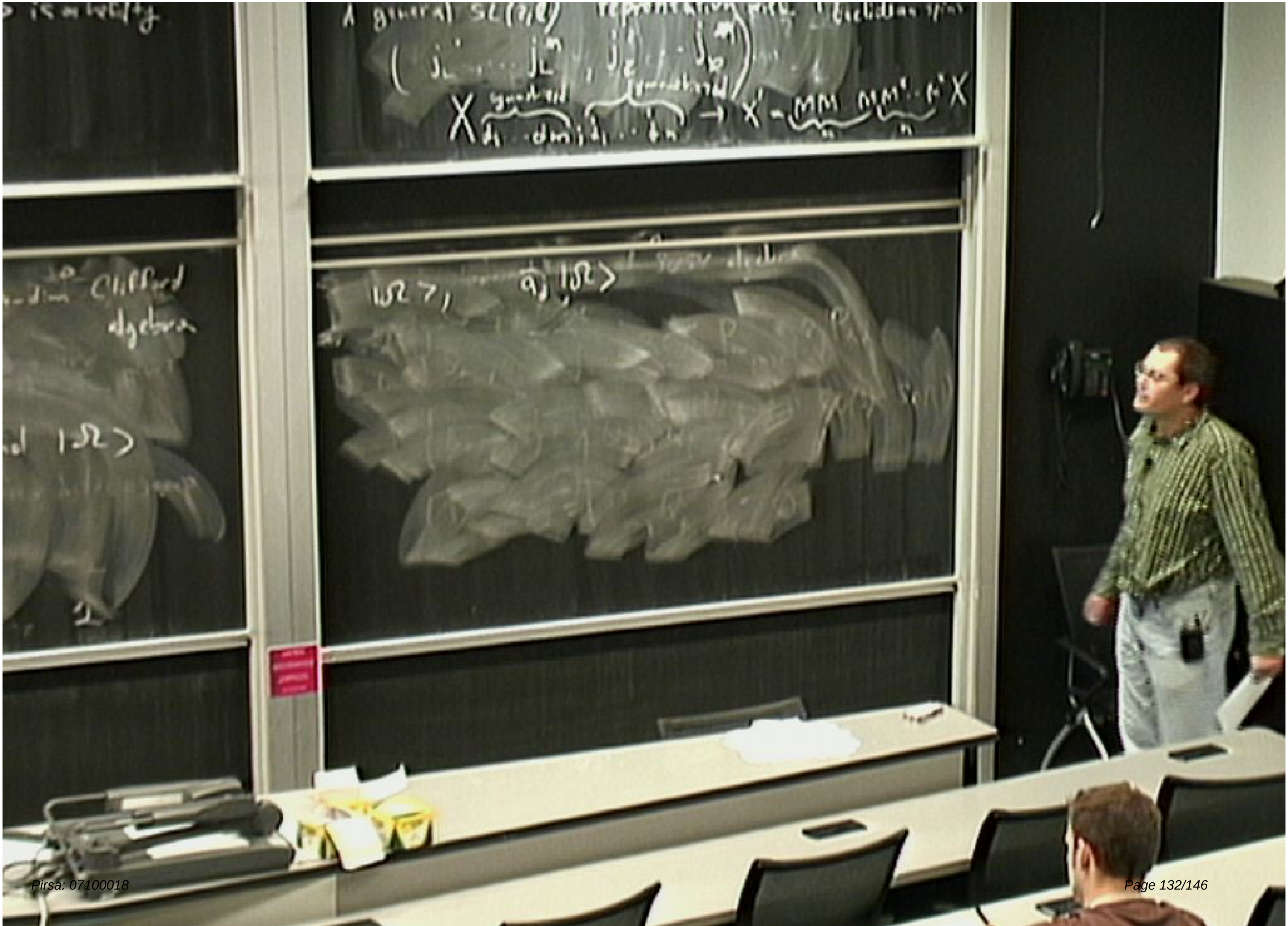
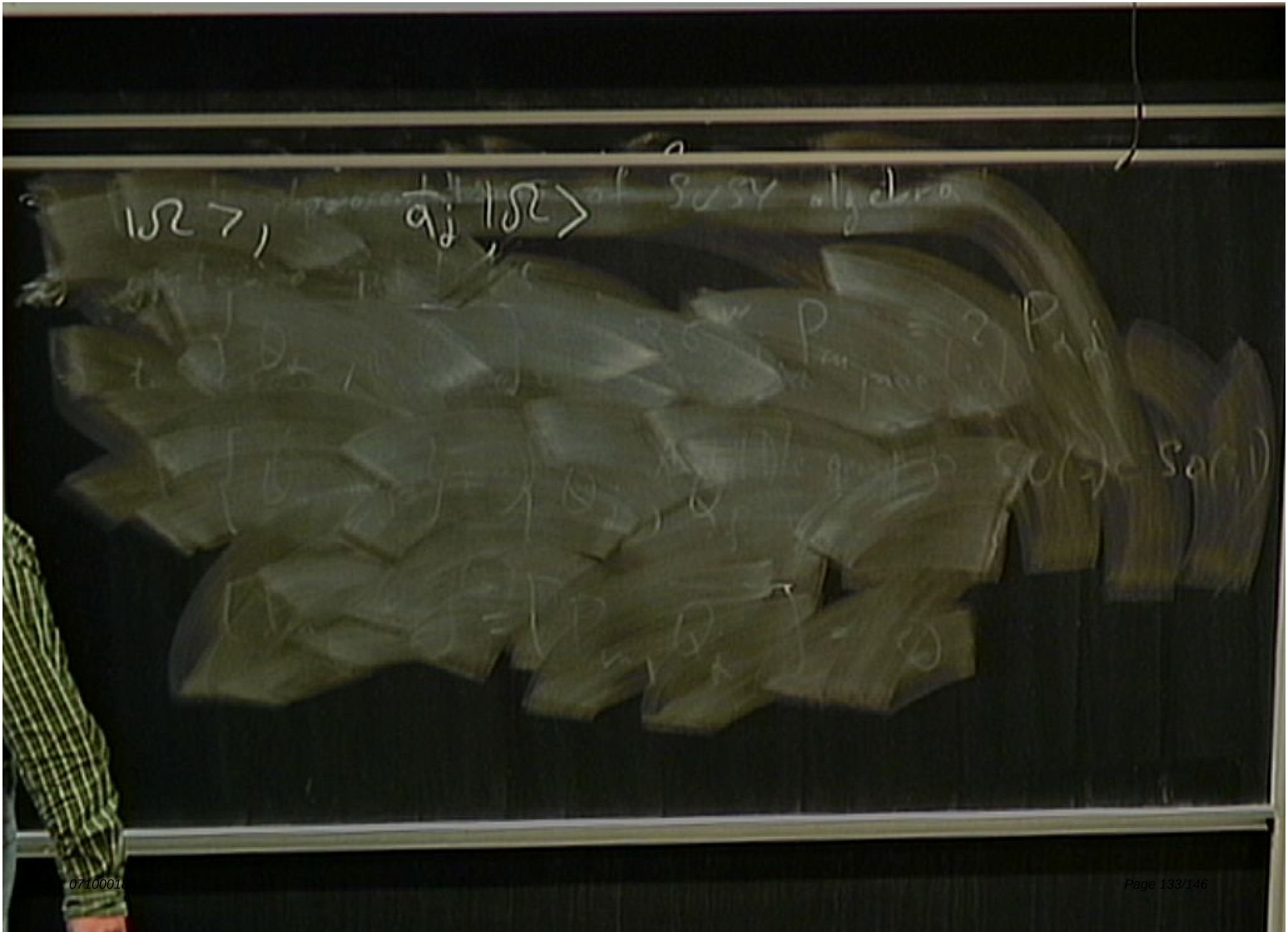

Clifford
algebra

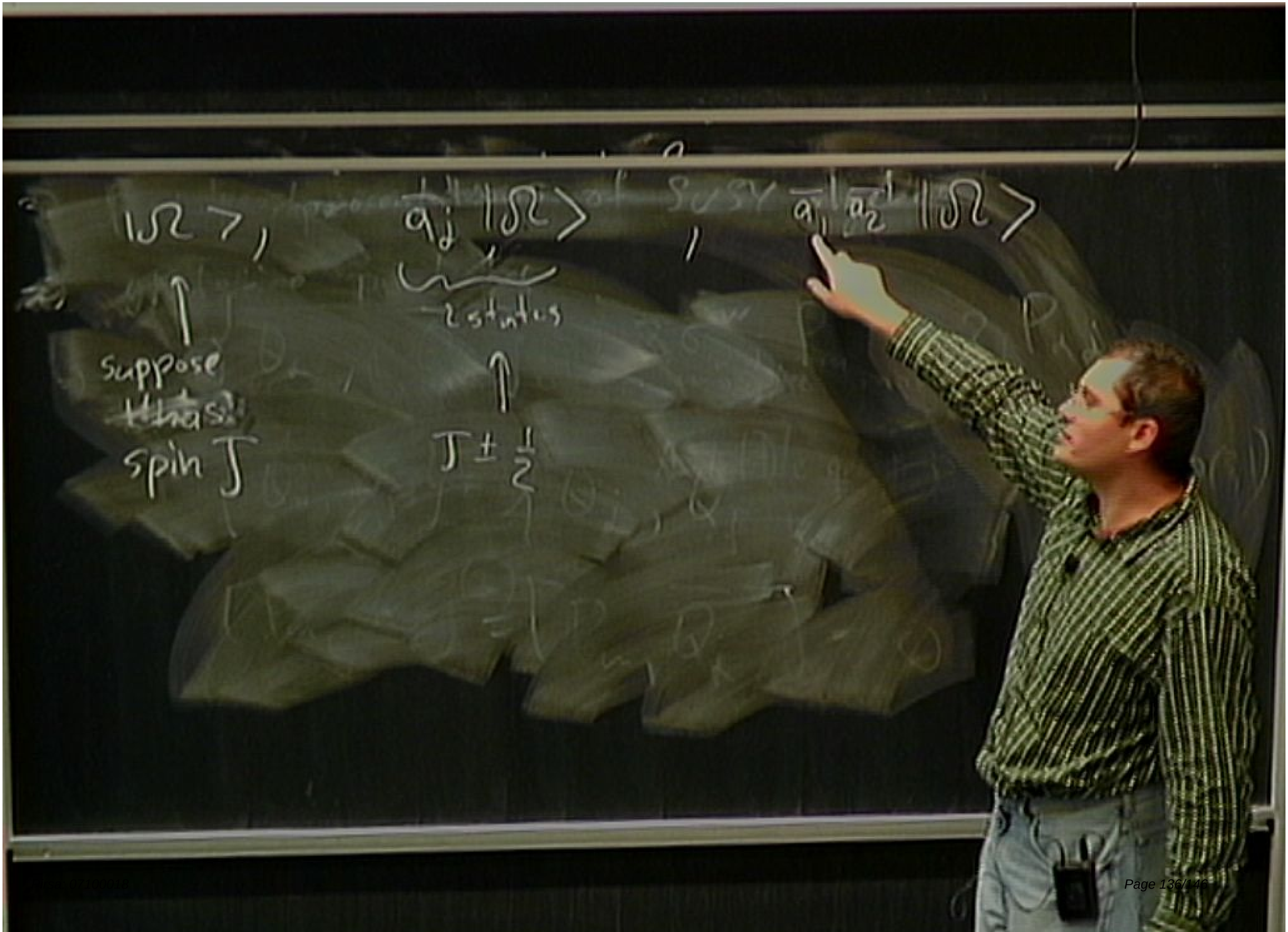


|Ω>, q_j |Ω> of SUSY algebra

$|\Omega\rangle, \quad \underbrace{a_j^\dagger |\Omega\rangle}_{2\ \text{states}}, \quad a_1^\dagger a_2^\dagger |\Omega\rangle$

$|\Omega\rangle$, $\underbrace{a_j^\dagger |\Omega\rangle}_{2\text{ states}}$, SUSY $a_1^\dagger a_2^\dagger |\Omega\rangle$

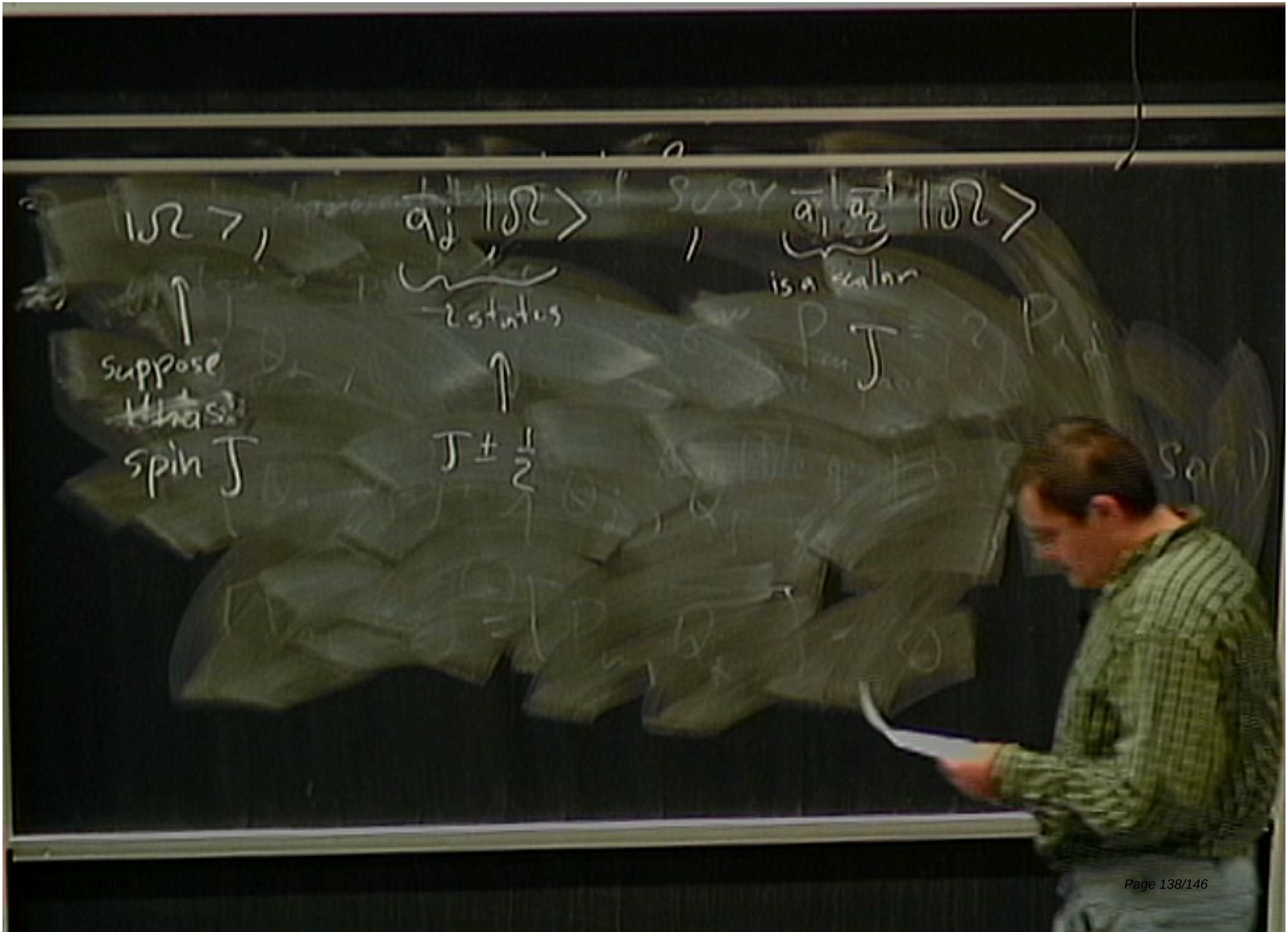
suppose ~~this~~ has: spin J


|Ω >,
a_j† |Ω >
2 states
suppose
has
spin J
J ± 1/2
a_1† a_2† |Ω >

$|\Omega\rangle$, $\underbrace{\bar{q}_j |\Omega\rangle}_{\text{2 states}}$, $\underbrace{\bar{q}_1 \bar{q}_2}_{\text{is a scalar}} |\Omega\rangle$

suppose ~~this~~ has spin J

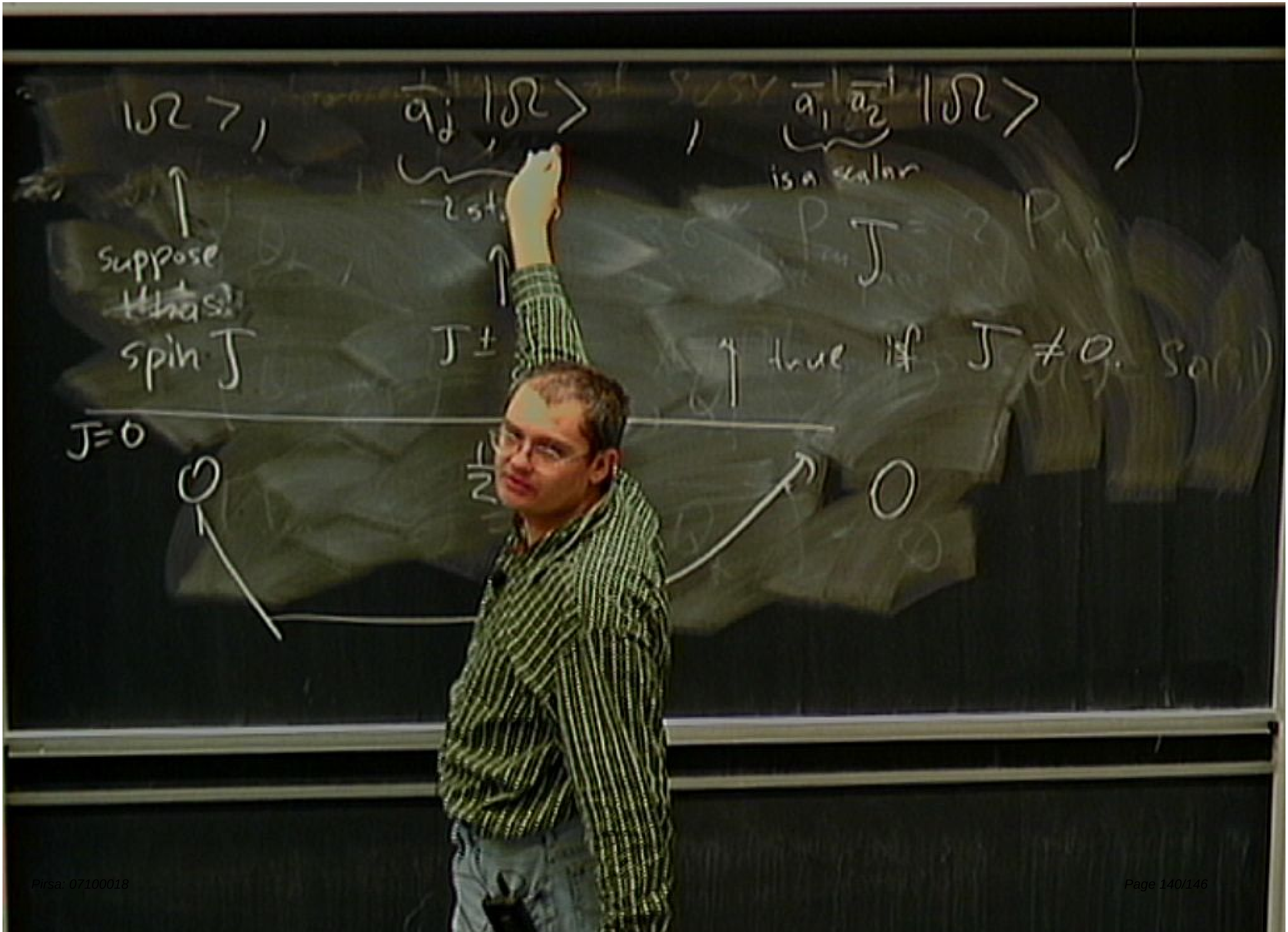
$J \pm \frac{1}{2}$


|Ω >,
a†_j |Ω >
2 states
a†_1 a†_2 |Ω >
is a scalar
suppose ~~it~~ has spin J
J ± 1/2
J

$|\Omega\rangle$, $\quad \underbrace{\bar{a}_j\,|\Omega\rangle}_{\text{2 states}}$, $\quad \underbrace{\bar{a}_1\bar{a}_2}_{\text{is a scalar}}|\Omega\rangle$

$\uparrow$ suppose ~~this~~ is: spin J $\qquad \uparrow$ $J \pm \frac{1}{2}$ $\qquad \uparrow$ true if $J \neq 0$.

$J=0$: $\quad 0 \qquad \frac{1}{2} \qquad 0$



|Ω >,
a_j |Ω >
1st
a_1 a_2 |Ω >
is a scalar
suppose that's spin J
J ±
true if J ≠ 0.
J=0
0
1/2
0

$$|\Omega\rangle, \quad \underbrace{\bar{a}_j |\Omega\rangle}_{2\ \text{states}}, \quad \underbrace{\bar{a}_1 \bar{a}_2}_{\text{is a scalar}} |\Omega\rangle$$

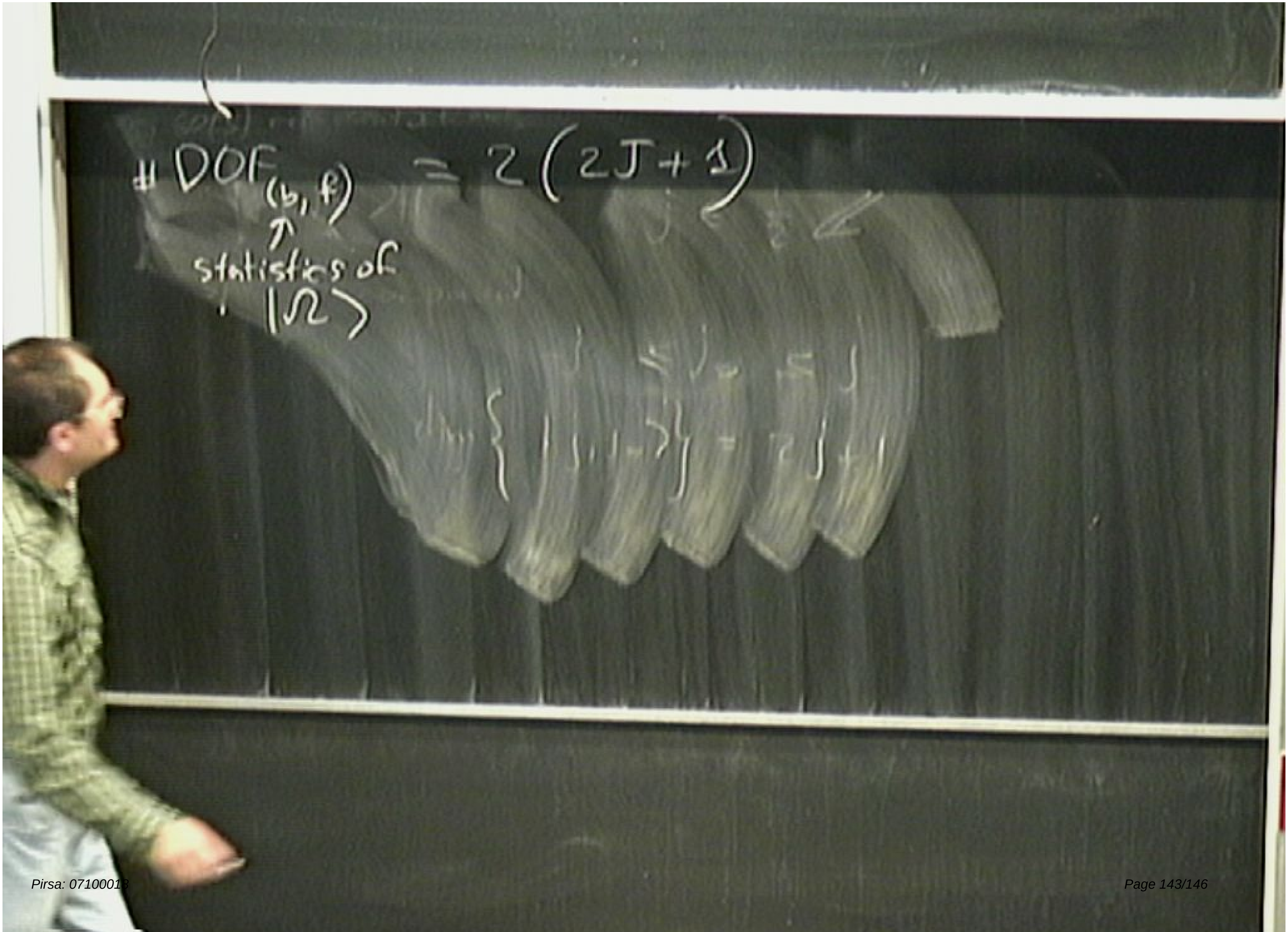
suppose ~~this~~ this:

spin J $\qquad J \pm \frac{1}{2}$ $\qquad$ true if $J \neq 0$. So

$J=0$

$0 \qquad \frac{1}{2} \qquad 0$

DOF$_{(b,f)}$



DOF(b, f) = 2(2J + 1)
statistics of
|Ω⟩

$$\# DOF_{(b,f)} = 2(2J+1) = 4J+2$$

$\uparrow$ statistics of $|\Omega\rangle$

$$\# DOF_{(f,b)}$$

$J \neq 0$

$$\# \, DOF_{(b,f)} = 2(2J+1) = 4J+2$$

$\uparrow$ statistics of $|\Omega\rangle$

$$\# \, DOF_{(f,b)} = \left(2\left(J-\tfrac{1}{2}\right)+1\right) + 2\left(J+\tfrac{1}{2}\right)+1 = 4J+2$$

$J \neq 0$

$$\# \, DOF_{(b,f)} = 2(2J+1) = 4J+2$$

statistics of $|\Omega\rangle$

$$\# \, DOF_{(f,b)} = 2\left(J-\tfrac{1}{2}\right)+1 + 2\left(J+\tfrac{1}{2}\right)+1 = 4J+2$$

Same for $J=0$