

Title: Intro to Supersymmetry 8

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Abstract:

Perturbative SUSY in $d=3+1$

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Group symmetry \Rightarrow Poincaré group

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time translations $\Rightarrow x^m \rightarrow x^m + \epsilon^m$

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time translations $\Rightarrow x^\mu \rightarrow x^\mu + \epsilon^\mu$ + Lorentz rotations

\curvearrowright space rotations + boosts

$$x^\mu \rightarrow x'^\mu = \Lambda^\mu{}_\nu x^\nu \quad \Lambda^\mu{}_\nu \in SO(3,1)$$

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→ Group of symmetries noncompact



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unitary reprs are infinite dimensional.

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⇒ use Wigner trick

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(i) fix noncompact generators

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Perturbative SUSY in $d=3+1$

Group symmetry \Rightarrow Poincaré group

time translations $\Rightarrow x^{\mu} \rightarrow x^{\mu} + \epsilon^{\mu}$ + Lorentz rotations

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$$x^{\mu} \rightarrow x^{\mu'} = \Lambda^{\mu}_{\nu} x^{\nu} \quad \Lambda \in \underline{SO(3,1)}$$

2) fix boost generators.

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~~Massive~~ Massive particles.

⇒ to a rest frame of a massive particle.

$$p^m = (m, 0, 0, 0)$$

⇒ fix boost generators.

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$$P^{\mu} = (m, \underbrace{0, 0, 0}_{SO(3)})$$

A little group is $SO(3) \subset SO(3,1)$

⇒ $so(3)$ representations

→ $so(3)$ representations

$|j, j_z\rangle$
↑ ↑
spin z -component.

$$j \in \frac{1}{2}\mathbb{Z}$$

→ $so(3)$ representations

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spin z -component.

$$j \in \frac{1}{2}\mathbb{Z}$$

$$-j \leq j_z \leq j$$

$$\dim \{ |j, j_z\rangle \} = 2j + 1$$

⇒ massive particles

⇒ massive particles

• mass



⇒ massive particles

- mass
- spin
- internal quantum numbers.

⇒ massive particles

- mass

- spin

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Massless particles

⇒ massive particles

- mass

- spin

- internal quantum numbers.

Massless particles

$$P^\mu = (E, 0, 0, E)$$

$$P^\mu P_\mu = 0$$

⇒ massive particles

- mass
- spin
- internal quantum numbers.

Massless particles

$$P^\mu = (E, \underbrace{0, 0, 0}_{SO(2)}, E)$$

$$P^\mu P_\mu = 0$$

Little group is $SO(2) \sim U(1)$

$\Rightarrow \mathfrak{so}(3)$ representations

$|j, j_z\rangle$
 \uparrow spin \uparrow z-component.

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Little group is $SO(2) \sim U(1)$

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⇒ massive particles

- mass
- spin
- internal quantum numbers.

Massless particles

$$P^\mu = (E, \underbrace{0, 0}_{SO(2)}, E) \quad (fix the charge).$$

$$P^\mu P_\mu = 0$$

Little group is $SO(2) \sim U(1)$

⇒ massive particles

- mass
- spin
- internal quantum numbers.

Massless particles

$$P^\mu = (E, \underbrace{0, 0}_{SO(2)}, E) \quad (P^\mu P_\mu = 0) \quad (\text{fix the energy}).$$

Little group is $SO(2) \sim U(1)$

$|\lambda\rangle$ is a helicity.

Algebraically λ can be anything

Algebraically \mathbb{H} can be anything

$$SO(3,1) \sim SL(2, \mathbb{H})$$

Algebraically λ can be anything

$$SO(3,1) \sim SL(2, \mathbb{C})$$

$\lambda \gg \rightarrow$

Algebraically λ can be anything

$$SO(3,1) \sim SL(2, \mathbb{C})$$

$$|\lambda\rangle \longrightarrow$$

$$\psi$$

Algebraically λ can be anything

$$SO(3,1) \sim SL(2, \mathbb{C})$$

$$|\lambda\rangle \rightarrow \psi_\lambda$$

Do rotation around z-axis by θ .

$$\psi_\lambda \rightarrow e^{i\theta\lambda} \psi_\lambda$$

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Q: What is a generator of z -rotations in $SL(2, \mathbb{C})$
repr

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representation of $SO(3, 1)$

Q: What is a generator of \mathbb{R} -rotations in $SL(2, \mathbb{C})$
representation of $SO(3, 1)$

$$M_\theta = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

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Let's assume that M_θ is z -rotations

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Let's assume that M_θ is z -rotations

$$M_{2\pi} \neq \mathbf{I} \quad \text{but} \quad M_{4\pi} = \mathbf{I} \quad \text{in } SL(2, \mathbb{C})$$

Algebraically λ can be anything

$$SO(3,1) \sim SL(2, \mathbb{C})$$

$$|\lambda\rangle \rightarrow \psi_\lambda$$

Do rotation around z-axis by θ .

$$\begin{matrix} \psi_\lambda \\ \psi_{\lambda'} \\ \psi_{\lambda''} \end{matrix} \rightarrow e^{i\theta\lambda} \psi_\lambda$$

Algebraically λ can be anything

$$SO(3,1) \sim SL(2, \mathbb{C})$$

$$|\lambda\rangle \longrightarrow \psi_\lambda$$

Do rotation around z-axis by θ .

$$\begin{array}{ccc} \psi_\lambda & \xrightarrow{e^{i\theta\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i4\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e} & \psi_\lambda \end{array} \quad \lambda \in \frac{1}{2}$$

Algebraically λ can be anything

$$SO(3,1) \sim SL(2, \mathbb{C})$$

$$|\lambda\rangle \longrightarrow \psi_\lambda$$

Do rotation around z-axis by θ .

$$\begin{array}{ccc} \psi_\lambda & \xrightarrow{e^{i\theta\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i4\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i8\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i12\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i16\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i20\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i24\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i28\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i32\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i36\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i40\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i44\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i48\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i52\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i56\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i60\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i64\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i68\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i72\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i76\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i80\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i84\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i88\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i92\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i96\pi\lambda}} & \psi_\lambda \\ \psi_\lambda & \xrightarrow{e^{i100\pi\lambda}} & \psi_\lambda \end{array} \quad \lambda \in \frac{1}{2} \mathbb{Z}$$

$$M_0 = e^{i\frac{\theta}{2}\sigma_3}$$

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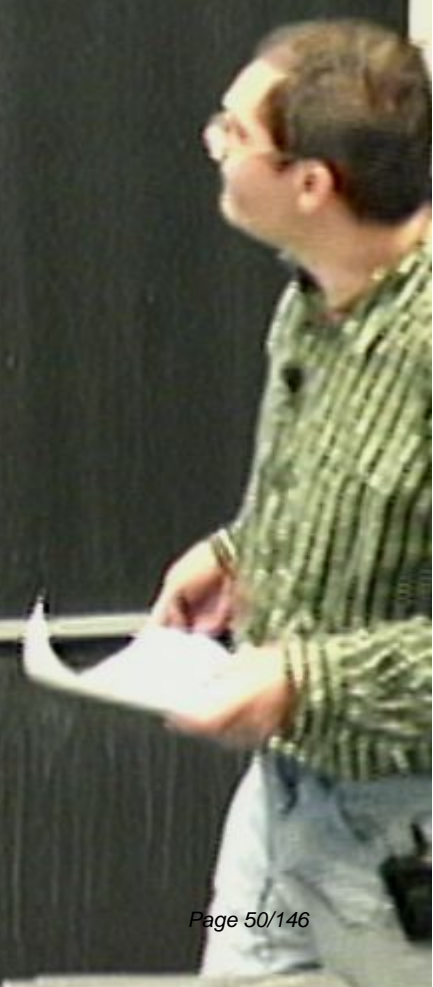
~~P~~ →

$$\cancel{P} = P_{\mu\nu} \sigma^{\mu\nu}$$

$$M_G = e^{i\frac{\theta}{2}\sigma_3}$$

$$P \rightarrow P' = MPM^\dagger$$

$$P = P_m \sigma^m$$



$$M_\theta = e^{\frac{i\theta}{2}\sigma_3} = \cos \frac{\theta}{2} + i\sigma_3 \sin \frac{\theta}{2}$$

$$P \rightarrow P' = MPM^\dagger$$

$$P = P_{\mu\nu} \sigma^{\mu\nu}$$

$$M_\theta = e^{i\frac{\theta}{2}\sigma_3} = \cos\frac{\theta}{2} + i\sigma_3 \sin\frac{\theta}{2}$$

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$$P = P_n \sigma^n$$

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$$P \rightarrow P' = MPM^\dagger = \left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\sigma_3 \right) \left(P_m \sigma^m \right)$$

$$P = P_m \sigma^m \quad \left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\sigma_3 \right) =$$

$$P = (P_0, P_1, P_2, P_3) \quad = P'_m \sigma^m$$

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$$(P_0, P_1, P_2, P_3) \rightarrow (P_0, P_1', P_2', P_3)$$

$$(P_0, P_1, P_2, P_3) \rightarrow (P_0, P_1', P_2', P_3)$$

$$P_1' = P_1 \cos \theta + \sin \theta P_2$$

$$P_2' = P_2 \cos \theta - \sin \theta P_1$$

$$(P_0, P_1, P_2, P_3) \rightarrow (P_0, P_1', P_2', P_3)$$

$$P_1' = P_1 \cos \theta + \sin \theta P_2$$

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} as a rotation
by " θ "
around z-axis

Q: What is a generator of z -rotations in $SL(2, \mathbb{C})$
representation of $SO(3, 1)$

$$M_\theta = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

Let's assume that M_θ is z -rotations

$$M_{2\pi} \neq \mathbf{I}$$

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$$e^{\frac{i\theta\sigma_3}{2}} = \sum_{h=0}^{\infty} \left(\frac{i\theta\sigma_3}{2} \right)^h \frac{1}{h!}$$
$$\sigma_3^{2n} = \mathbb{I}$$

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$$P \rightarrow (P' = MPM^\dagger) = \left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\sigma_3 \right) \left(P_m \sigma^m \right)$$

$$P = P_m \sigma^m \quad \det P = P_0 P^m$$

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$$\sigma^x = (\sigma^0, \sigma_x, \sigma_y, \sigma_z) \quad \sigma^0 = -I$$

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$$\psi_\lambda \xrightarrow{e^{i\theta\lambda}} \psi_\lambda \quad \lambda \in \frac{1}{2}\mathbb{Z}$$

$$\psi_\lambda \xrightarrow{e^{i4\pi\lambda}} \psi_\lambda = \psi_\lambda \quad e^{i4\pi\lambda} = 1$$

$$P_1' = P_1 \cos \theta + \sin \theta P_2$$

$$P_2' = P_2 \cos \theta - \sin \theta P_1$$

by " $-\theta$ "
around z-axis

do analytical continuation: $d=3,1$

$$SO(3,1) \rightarrow SO(4)$$

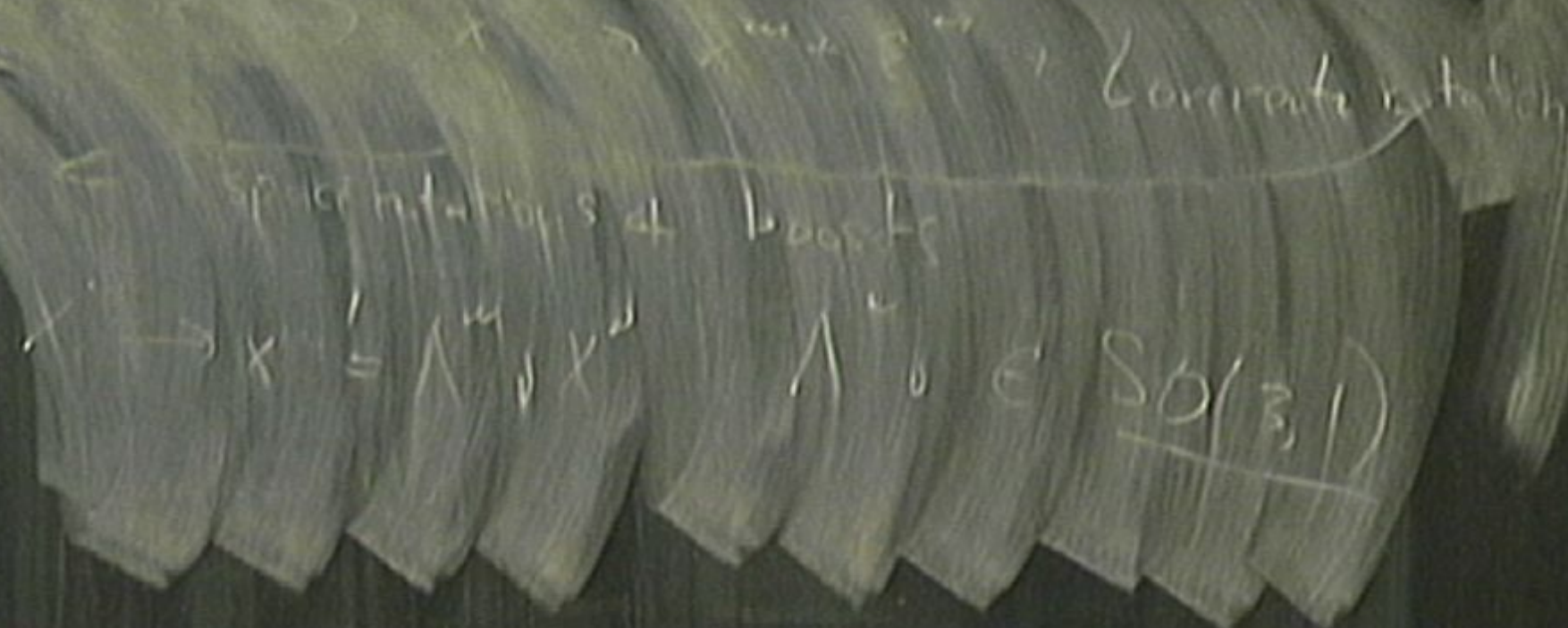
Lorentz rotation

spacetime rotations & boosts

$$x \rightarrow x' = \Lambda x \quad \Lambda \in SO(3,1)$$

do analytical continuation: $SO(3,1)$

$$SO(3,1) \rightarrow SO(4)$$



do analytical continuation: $d=3+1$

$$SO(3,1) \rightarrow SO(4) \sim SU(2)_L \times SU(2)_R$$

J_i : - generators of spatial rotations.

$$SO(3) \sim$$

$$\rightarrow x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\Lambda \in SO(3,1)$$

do analytical continuation: $d=3+1$

$$SO(3,1) \rightarrow SO(4) \sim SU(2)_L \times SU(2)_R$$



J_i - generators of spatial rotations.

$$SO(3) \sim SU(2)$$



$$x \rightarrow X = \Lambda \cdot x \cdot \Lambda^{-1} \quad \Lambda \in SO(3,1)$$

do analytical continuation: $d=3+1$

$$SO(3,1) \rightarrow SO(4) \sim SU(2)_L \times SU(2)_R$$

J_i - generators of spatial rotations. K_i - generators of boosts.

$$SO(3) \sim SU(2)$$

K_i - boost along spatial direction.

$$\Lambda \in SO(3,1)$$

do analytical continuation:

$$SO(3,1) \rightarrow SO(4) \sim SU(2)_L \times SU(2)_R$$



J_i - generators of spatial rotations.

$$e^{i\theta^i J_i}$$

$$SU(2)$$

- boost along spatial direction.

$$\in SO(3,1)$$



\downarrow J_i - generators of spatial rotations. $e^{i\theta^i J_i}$

$$SO(3) \sim SU(2)$$

K_i - boost along spatial direction. \rightarrow

$\underbrace{K_i}_{\text{another } SO(3)}$ $SO(3,1)$

$$\bar{J}_i \pm iK_i$$

J_i - generators of spatial rotations. $e^{i\theta^i J_i}$

$$SO(3) \sim SU(2)$$

K_i - boost along spatial direction. $\rightarrow \underbrace{iK_i}_{\text{another } SO(3)}$ $SO(3,1)$

$J_i \pm iK_i$ are generators of $SU(2)_L$ $SU(2)_R$

J_i - generators of spatial rotations. $e^{i\theta^i J_i}$

$$SO(3) \sim SU(2)$$

K_i - boost along spatial direction. $\rightarrow \underbrace{iK_i}_{\text{another } SO(3)}$ $SO(3,1)$

$J_i \pm iK_i$ are generators of $SU(2)_L$

$[SU(2)_L \times SU(2)_R]$ diagonal

J_i - generators of spatial rotations. $e^{i\theta^i J_i}$

$$SO(3) \sim SU(2)$$

K_i - boost along spatial direction. $\rightarrow \underbrace{K_i}_{\text{another } SO(3)}$ $SO(3,1)$

$J_i \pm iK_i$ are generators of $SU(2)_L$
 $[SU(2)_L \times SU(2)_R]_{\text{diagonal}} = \{J_i\} \sim SO(3)_R$

Any representation of $SU(2)_L \times SU(2)_R$

$$(j_L, j_R)$$

$SO(3)$ spin

scalar ϕ

\Rightarrow

(1)

(2)

Any representation of $SU(2)_L \times SU(2)_R$

$$(j_L, j_R)$$

$SO(3)$ spin

$$(0, 0)$$

0

scalar ϕ

$$(\frac{1}{2}, 0)$$

$\frac{1}{2}$

ψ_2

Any representation of $SU(2)_L \times SU(2)_R$

$$(j_L, j_R)$$

$SO(3)$ spin

$$(0, 0)$$

0

$$(\frac{1}{2}, 0)$$

$\frac{1}{2}$

$$(0, \frac{1}{2})$$

$\frac{1}{2}$

scalar ϕ

left-handed
spinor ψ_L

right-handed
spinor $\bar{\psi}_L$

Any representation of $SU(2)_L \times SU(2)_R$

$$(j_L, j_R)$$

$SO(3)$ spin

$$(0, 0)$$

0

scalar ϕ

left-handed
spinor ψ_L

$$(\frac{1}{2}, 0)$$

$\frac{1}{2}$

right handed
spinor $\bar{\psi}_R$

$$(0, \frac{1}{2})$$

$\frac{1}{2}$

a vector
(bispinor)

A_μ

$$(\frac{1}{2}, \frac{1}{2})$$

Any representation of $SU(2)_L \times SU(2)_R$

$SO(3)$ spin
 ϕ

(j_L, j_R)
 $(0, 0)$

scalar ϕ

left-handed
 spinor ψ_L

$(\frac{1}{2}, 0)$

$\frac{1}{2}$

right handed
 spinor ψ_R

$(0, \frac{1}{2})$

$\frac{1}{2}$

a vector
 (bispinor) A_{ij}

$(\frac{1}{2}, \frac{1}{2})$

scalar ψ

left-handed
spinor ψ_L

right-handed
spinor ψ_R

a vector
(bispinor)

$A_{\alpha\beta}$
 $F_{\alpha\beta}^+$
symmetric

$$\left(\frac{1}{2}, 0\right)$$

$$\left(0, \frac{1}{2}\right)$$

$$\left(\frac{1}{2}, \frac{1}{2}\right)$$

$$(1, 0)$$

$$\frac{1}{2}$$

$$\frac{1}{2}$$

$$1$$

scalar φ

$$(0, 0)$$

left-handed spinor ψ_L

$$\left(\frac{1}{2}, 0\right)$$

$$\frac{1}{2}$$

right-handed spinor ψ_R

$$\left(0, \frac{1}{2}\right)$$

$$\frac{1}{2}$$

a vector (bispinor)

$$A_{\mu\nu}$$

$$\left(\frac{1}{2}, \frac{1}{2}\right)$$

$$1$$

$$F_{\alpha\beta}^+$$

symmetric

$$(1, 0)$$



scalar ϕ

$$(0, 0)$$

left-handed spinor ψ_L

$$(\frac{1}{2}, 0)$$

$$\frac{1}{2}$$

right handed spinor $\bar{\psi}_L$

$$(0, \frac{1}{2})$$

$$\frac{1}{2}$$

vector (bispinor)

$$A_{\mu\nu}$$

$$(\frac{1}{2}, \frac{1}{2})$$

$$1$$

self-dual 2-form

$$F^+_{(\alpha\beta)}$$

$$(1, 0)$$

$$1$$

anti-self dual 2-form

$$F^-_{(\alpha\beta)}$$

symmetric

$$(0, 1)$$

$$1$$

Recall:

$$\psi_\alpha \rightarrow \psi'_\alpha = M_\alpha^\beta \psi_\beta$$

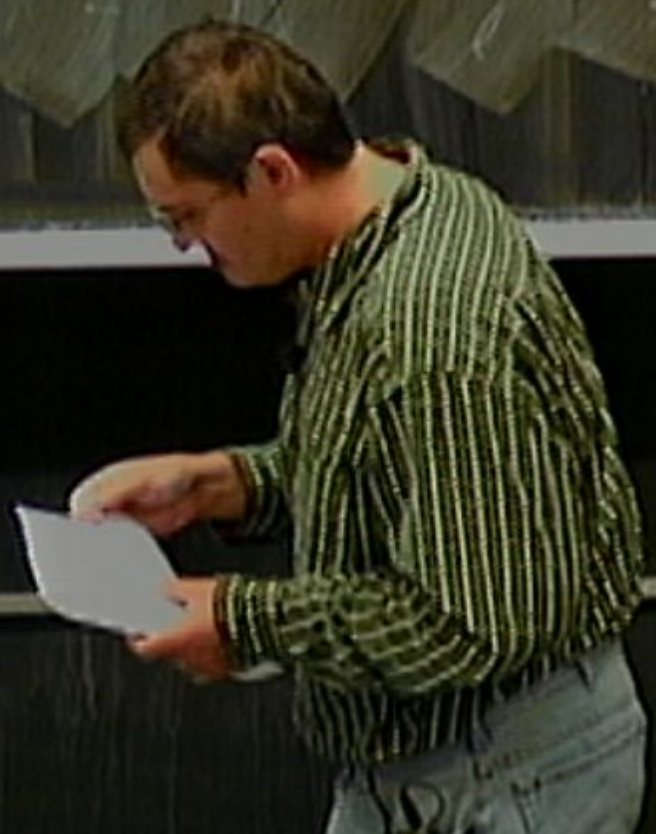
$$\bar{\psi}_\alpha \rightarrow \bar{\psi}'_\alpha = M_\alpha^\beta{}^* \bar{\psi}_\beta$$

$$A_{\alpha\beta} \rightarrow A'_{\alpha\beta} = M_\alpha^\gamma M_\gamma^\beta{}^* A_{\beta\dot{\beta}}$$

$$\bar{\Psi}_i \rightarrow \Psi_i' = M_{\alpha}^{* \beta} \bar{\Psi}_{\beta}$$

$$A_{\alpha i} \rightarrow A'_{\alpha i} = M_{\alpha}^{\beta} M_{\gamma}^{* \delta} A_{\beta \gamma}$$

A general $SL(2, \mathbb{C})$



$$\bar{\Psi}_i \rightarrow \Psi_i' = M_{\alpha}^{* \dot{\beta}} \bar{\Psi}_{\dot{\beta}}$$

$$A_{\alpha\dot{\alpha}} \rightarrow A'_{\alpha\dot{\alpha}} = M_{\alpha}^{\beta} M_{\dot{\alpha}}^{* \dot{\beta}} A_{\beta\dot{\beta}}$$

A general $SL(2, \mathbb{C})$ representation with Euclidean

Any representation of $SU(2)_L \times SU(2)_R$

$$(j_L, j_R)$$

$SO(3)$ spin ϕ

scalar ϕ

$$(0, 0)$$

left-handed spinor ψ_L

$$(\frac{1}{2}, 0)$$

$\frac{1}{2}$

right-handed spinor $\bar{\psi}_R$

$$(0, \frac{1}{2})$$

$\frac{1}{2}$

vector (bispinor) $A_{\mu\nu}$

$$(\frac{1}{2}, \frac{1}{2})$$

1

self-dual 2-form F^+

$$F^+(\alpha, \beta)$$

$$(1, 0)$$

1

anti-self dual 2-form F^-

$$F^-(\alpha, \beta)$$

symmetric

$$(0, 1)$$

1

$$\psi_\alpha \rightarrow \psi'_\alpha = M_\alpha^{\beta} \psi_\beta$$

$$\bar{\psi}_i \rightarrow \bar{\psi}'_i = M^{*j}_i \bar{\psi}_j$$

$$A_{\alpha i} \rightarrow A'_{\alpha i} = M_\alpha^{\beta} M^{*j}_i A_{\beta j}$$

A general $SL(2, \mathbb{C})$ representation with "Euclidean spins"

$$\left(\begin{matrix} j_L & \dots & j_L \\ j_R & \dots & j_R \end{matrix} \right)_{(m)}$$



$$\bar{\Psi}_i \rightarrow \Psi_i = M_{\alpha}^{*j} \bar{\Psi}_{\beta}$$

$$A_{\alpha i} \rightarrow A'_{\alpha i} = M_{\alpha}^{\beta} M_{\gamma}^{*j} A_{\beta j}$$

A general $SL(2,0)$ representation with "Euclidean spins"

$$\left(\underbrace{j_1, \dots, j_m}_{\text{spin } m}, \underbrace{j_1, \dots, j_n}_{\text{spin } n} \right)$$

$$X_{d_1 \dots d_m; d_1 \dots d_n}$$

$$\bar{\Psi}_i \rightarrow \Psi_i' = M_{\alpha}^{*j} \bar{\Psi}_{\dot{\beta}}$$

$$A_{\alpha i} \rightarrow A'_{\alpha i} = M_{\alpha}^{\beta} M_{\gamma}^{*j} A_{\beta \dot{\gamma}}$$

A general $SL(2, \mathbb{C})$ representation with "Euclidean spins"

$$\left(\begin{array}{c} j_1 \dots j_m \\ j_1 \dots j_n \end{array} \right)$$

\times $\underbrace{\quad}_{\text{symmetrized}} \underbrace{\quad}_{\text{symmetrized}}$
 $d_1 \dots d_m ; d_1 \dots d_n$

$$\bar{\Psi}_i \rightarrow \Psi_i' = M_{\alpha}^{* \dot{\beta}} \bar{\Psi}_{\dot{\beta}}$$

$$A_{\alpha i} \rightarrow A_{\dot{\alpha} i}' = M_{\alpha}^{\beta} M_{\dot{\alpha}}^{* \dot{\beta}} A_{\beta \dot{\beta}}$$

A general $SL(2,0)$ representation with "Euclidean spins"

$$\left(\begin{array}{c} j_1' \dots j_m' \\ j_1'' \dots j_n'' \end{array} \right)_{\text{symmetrized}}$$

$$X_{d_1 \dots d_m; i_1 \dots i_n} \rightarrow X' = \underbrace{M M \dots M}_m$$



$$\bar{\Psi}_i \rightarrow \Psi_i' = M_{\alpha}^{*j} \bar{\Psi}_{\beta}$$

$$A_{\alpha i} \rightarrow A_{\alpha i}' = M_{\alpha}^{\beta} M_{\gamma}^{*j} A_{\beta j}$$

A general $SL(2, \mathbb{C})$ representation with "Euclidean spins"

$$\left(\begin{array}{c} j_1 \dots j_m \\ \text{symmetrized} \end{array} ; \begin{array}{c} j_1' \dots j_n' \\ \text{symmetrized} \end{array} \right)$$

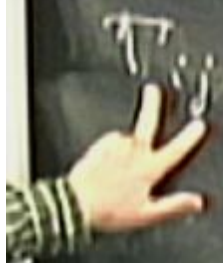
$$X_{d_1 \dots d_m ; d_1' \dots d_n'} \rightarrow X' = \underbrace{M M \dots M}_m \underbrace{M M^* \dots M^*}_n X$$

K_i - boost along spatial direction \rightarrow $\underbrace{K_i}_{\text{another } SO(2)}$ $SO(3,1)$

R^d is a generator of 2 rotations in $SL(2, \mathbb{C})$
 representation of $SO(3,1)$
 $A_i \rightarrow$ vector
 Π_{ij}
 Let's show that M_{ij} is 2-rotations
 $\Pi_{ij} = \Pi$ in $SL(2, \mathbb{C})$

K_i - boost along spatial direction \rightarrow $\underbrace{K_i}_{\text{another } SO(2)}$

\mathbb{R}^d is a $SO(d)$ representation of $SO(d)$
 $A_i \rightarrow$ vector



M_ω is 2-rotations
 $M_\pi = I$ in $SL(2, \mathbb{C})$

K_i - boost along spatial direction \rightarrow $\underbrace{K_i}_{\text{another } SO(2)}$

R^d is a $SO(d)$ of rotations in $SL(2, \mathbb{C})$
 representation of $SO(3,1)$
 $A_i \rightarrow$ vector

$$\mathbb{T}_{ij} = \mathbb{T}_{(ij)}^{\text{traceless}} + \mathbb{T}_{[ij]} + \mathbb{T}_{\#}^k g_{ij}$$

that M_{ij} is a rotation
 $\mathbb{T} \in SL(2, \mathbb{C})$

K_i - boost along spatial direction \rightarrow $\underbrace{K_i}_{\text{another } SO(2)}$

\mathbb{R}^d is a $SO(d)$ of rotations in $SL(2, \mathbb{C})$
 representation of $SO(3,1)$
 $A_i \rightarrow$ vector
 $\pi_{ij} = \pi_{(ij)} + \pi_{[ij]} + \pi_{ij}^k g_{ij}$

that M_0 is 2-rotations
 that $M_{\pi} = \pi$ in $SL(2, \mathbb{C})$

K_i - boost along spatial direction \rightarrow $\underbrace{K_i}_{\text{another } SO(2)}$

R^d is a $SO(d)$ representation of $SO(d)$
 $A_i \rightarrow$ vector

$$\pi_{ij} = \underbrace{\pi_{(ij)}}_{\text{traceless}} + \pi_{[ij]} + \pi_{ij}^k g_{ij}$$

reason by it's symmetric, traceless

$\pi \in SL(2, \mathbb{C})$

K_i - boost along spatial direction \rightarrow $\underbrace{K_i}_{\text{another } SO(2)}$

R^d \rightarrow $SO(d)$ \rightarrow rotations in $SL(2, \mathbb{C})$

A_i \rightarrow vector

$$\Pi_{ij} = \underbrace{\Pi_{(ij)}}_{\text{traceless}} + \Pi_{[ij]} + \Pi_{ij}^k g_{ij}$$

reason by it's symmetric, traceless

$S_i^i \rightarrow$ an invariant tensor

$$\Pi_{ij} g_{ij}$$

$\Pi \in SL(2, \mathbb{C})$

$\epsilon^{\alpha\beta}, \epsilon^{\dot{\alpha}\dot{\beta}}$ are $SL(2, \mathbb{C})$ inverters.

$SO(3)$

[The rest of the page is heavily scribbled out with chalk, obscuring most of the original text. Some faint traces of mathematical symbols and words like "unit", "anti", and "2-4" are visible.]

$\epsilon^{\alpha\beta}, \epsilon^{\dot{\alpha}\dot{\beta}}$ are $SL(2, \mathbb{C})$ invariants

$SO(3)$

X

ϵ



$\epsilon^{i\beta}, \epsilon^{j\beta}$ are $SL(2, \mathbb{C})$ invariants

$SO(3)$

$\chi_{d_1 d_2} \dots \epsilon^{d_1 d_2} \equiv 0$ otherwise are

initial representation is reducible.

[This section contains very faint and mostly illegible handwritten notes and diagrams, including some mathematical symbols and matrix structures.]

$\epsilon^{\alpha\beta}, \epsilon^{\dot{\alpha}\dot{\beta}}$ are $SL(2, \mathbb{C})$ infinitesimals

$SO(3)$

$\chi_{d_1 d_2} \dots \epsilon^{d_1 d_2} \neq 0$ otherwise are

initial representation is reducible.

[This section contains heavily scribbled-out text and faint mathematical symbols, including what appears to be a matrix structure with elements like $\frac{1}{2}$ and 0 .]

$\epsilon^{\alpha\beta}, \epsilon^{\dot{\alpha}\dot{\beta}}$ are $SL(2, \mathbb{C})$ invariants

$SO(3)$

$\chi_{d_1 d_2} \dots \epsilon^{d_1 d_2} \neq 0$ otherwise are

\Downarrow initial representation is reducible.

must be symmetric in $(\frac{1}{2}, \frac{1}{2})$ spinor indices.

anti-
2-
2-
anti-
2-
2-

$(\frac{1}{2}, \frac{1}{2})$

$(\frac{1}{2}, \frac{1}{2})$

$(\frac{1}{2}, \frac{1}{2})$

$A_{\mu\nu}$ is a vector potential.

$F_{\mu\nu}$ is a field strength

$$P_1 = \cos\theta + \sin\theta P_2$$

$$P_1' = P_2 \cos\theta - \sin\theta P_1$$

(P_2, P_3)

$$F_{\mu\nu} = -F_{\nu\mu}$$

is a matrix

"B"

axis

$A_{\mu\nu}$ is a vector potential.

$F_{\mu\nu}$ is a field strength

$$F_{\mu\nu} = -F_{\nu\mu}$$

\rightarrow a 2-form $\theta + \sin\theta P_2$

$$P_2 \cos\theta - \sin\theta P_1$$

as a rotation
"B"
axis

$A_{\mu\nu}$ is a vector potential.

$F_{\mu\nu}$ is a field strength

$$F_{\mu\nu} = -F_{\nu\mu}$$

α 2-form $\cos\theta P_2 + \sin\theta P_1$

$$M \rightarrow P_2 \cos\theta - P_1 \sin\theta$$

$$\underbrace{\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}}_M \otimes \underbrace{\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}}_M$$

$A_{\mu\nu}$ is a vector potential.

$F_{\mu\nu}$ is a field strength

$$F_{\mu\nu} = -F_{\nu\mu}$$

\rightarrow a 2-form $\theta + \sin\theta P_2$

$$M \rightarrow \theta + \sin\theta P_2$$

$$\left[\underbrace{\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}}_M \otimes \underbrace{\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}}_N \right]_A$$

$A_{\mu\nu}$ is a vector potential.

$F_{\mu\nu}$ is a field strength

$$F_{\mu\nu} = -F_{\nu\mu}$$

α 2-form $\cos\theta P_2$

$M \rightarrow \cos\theta P_2 - \sin\theta P_1$

$$\left[\underbrace{\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 2 & 2 \end{pmatrix}}_M \otimes \underbrace{\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 2 & 2 \end{pmatrix}}_N \right]_A =$$

$A_{\mu\nu}$ is a vector potential.

$F_{\mu\nu}$ is a field strength

$$F_{\mu\nu} = -F_{\nu\mu}$$

\rightarrow a 2-form $\theta + \sin\theta P_2$

$M \rightarrow P_2 \oplus P_1$

$$\left[\underbrace{\begin{pmatrix} \frac{1}{2} & 1 \\ 2 & \frac{1}{2} \end{pmatrix}}_M \otimes \underbrace{\begin{pmatrix} \frac{1}{2} & 1 \\ 2 & \frac{1}{2} \end{pmatrix}}_N \right]_A = \left(\begin{pmatrix} \frac{1}{2} & 1 \\ 2 & \frac{1}{2} \end{pmatrix}_{A_1} \oplus \begin{pmatrix} \frac{1}{2} & 1 \\ 2 & \frac{1}{2} \end{pmatrix}_S \right)^{X \cdot 5}$$

$A_{\mu\nu}$ is a vector potential.

$F_{\mu\nu}$ is a field strength

$$F_{\mu\nu} = -F_{\nu\mu}$$

α 2-form $\theta + \sin\theta P_2$

$$M \rightarrow \mathbb{R} \oplus \mathbb{R} - \sin\theta P_1$$

$$\left[\underbrace{\left(\frac{1}{2}, \frac{1}{2} \right)}_M \otimes \underbrace{\left(\frac{1}{2}, \frac{1}{2} \right)}_S \right]_A = \left(\left(\frac{1}{2} \otimes \frac{1}{2} \right)_{A_1}, \left(\frac{1}{2} \otimes \frac{1}{2} \right)_S \right)_{X^1 S}$$

$A_{\mu\nu}$ is a vector potential.

$F_{\mu\nu}$ is a field strength

$$F_{\mu\nu} = -F_{\nu\mu}$$

α 2-form $\Theta + \sin\theta P_2$

$$M \rightarrow P_2 \oplus d\Omega - \sin\theta P_1$$

$$\left[\underbrace{\left(\frac{1}{2}, \frac{1}{2}\right)}_M \otimes \underbrace{\left(\frac{1}{2}, \frac{1}{2}\right)}_S \right]_A = \left(\left(\frac{1}{2} \otimes \frac{1}{2}\right)_{A_1}, \left(\frac{1}{2} \otimes \frac{1}{2}\right)_S \right) + \left(\left(\frac{1}{2} \otimes \frac{1}{2}\right)_S, \left(\frac{1}{2} \otimes \frac{1}{2}\right)_A \right)$$

of partition direction.

another $SO(2)$

\mathbb{R}^2 rotations $SO(2)$
 $A = (0, 2) \oplus$

Factorless

$\pi = \mathbb{I} \in SO(2)$

of parallel direction.
 another $SO(2)$

$$\Rightarrow \underline{(0, 2)} \oplus \underline{(1, 0)}$$

leaves

another $S(2)$

$$= \frac{(0, 2)}{\oplus} \frac{(1, 0)}$$

less

another $SO(2)$

$$= \frac{(0, 2)}{F_{\alpha\beta}^-} \oplus \frac{(1, 0)}{F_{\alpha\beta}^+}$$

less

another SO(2)

$$A = \frac{(0, 1)}{F_{\alpha\beta}} \oplus \frac{(1, 0)}{F_{\alpha\beta}^+}$$

$$S_{\alpha\beta, 11} \\ \epsilon_{\mu\nu\lambda\rho} =$$

$$F_{\alpha\beta}^+$$

leaves

$$\pi = \mathbb{I} \text{ in } SO(2, 1)$$

another SO(2)

$$A = \frac{(0, 1)}{(1, 0)} \oplus$$

$$SO(3,1) \quad F_{\alpha\beta} \\ \epsilon_{\mu\nu\lambda\rho} = \pm 1$$

$$F_{\alpha\beta}^+$$

$$\epsilon_{0123} = -1 \\ \epsilon^{0123} = +1$$

less

SL(2, C)

Given $F_{\mu\nu} \rightarrow F_{\mu\nu} = \frac{i}{\sqrt{2}} \epsilon_{\mu\nu\lambda\rho} F^{\lambda\rho}$

[The rest of the board is heavily scribbled out with chalk, making the text illegible.]

Given $F_{\mu\nu} \rightarrow F^{\lambda\rho} = \frac{i}{\sqrt{2}} \epsilon^{\mu\nu\lambda\rho} F^{\lambda\rho}$

Hodge dual

$F^+ = \frac{1}{\sqrt{2}} (F + F^*)$ — self-dual

$F^- = \frac{1}{\sqrt{2}} (F - F^*)$ — anti-self-dual

self
2-
anti
2-to

(1, 2)
(2, 1)
(2, 1)
(1, 2)

K_i - boost along spatial direction \rightarrow $\underbrace{K_i}_{\text{another } SO(2)}$

\mathbb{R}^4 rotations $SO(3,1)$

$$= \frac{(0, 1)}{(1, 0)}$$

$SO(3,1)$

$$F_{\alpha\beta}$$

$$F^{\alpha\beta}$$

$$\epsilon_{\mu\nu\lambda\rho} = \pm 1$$

$$\epsilon_{0123} = -1$$

$$\epsilon^{0123} = +1$$

less



Particle representations of SUSY algebra

Majorana fermions
} \mathcal{D}_4

$$P = (m, \underbrace{0, 0, 0}_{SO(3)})$$

group is $SO(3) \subset SO(4)$



Particle representations of SUSY algebra

$$\{Q_i, \bar{Q}_i\} = 2\sigma_{ij} P_{ij}$$

$$P_{ij} = (m, \mathbf{a}, \mathbf{a}, 0)$$

$\underbrace{\hspace{10em}}_{SO(3)}$

A little group is $SO(3) \subset SO(4)$

Particle representations of SUSY algebra

$$\{Q_\alpha, \bar{Q}_i\} = 2\sigma_{\alpha i}^\mu P_\mu = 2P_{\alpha i}$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_i, \bar{Q}_j\} = 0 \quad SO(3) \subset SO(1,3)$$

$$[P_\mu, Q_\alpha] = [P_\mu, \bar{Q}_i] = 0$$

K_i - boost along spatial direction \rightarrow $\underbrace{u_i K_i}$ another $SO(2)$

b) Boost massive

$$(0, 1)$$

$$(1, 0)$$

$$SL(2, \mathbb{C})$$

$$\begin{pmatrix} \cosh \frac{\eta}{2} & \sinh \frac{\eta}{2} \\ \sinh \frac{\eta}{2} & \cosh \frac{\eta}{2} \end{pmatrix}$$

$$SL(2, \mathbb{R})$$

K_i - boost along spatial direction \rightarrow $\underbrace{SO(3,1)}$
another $SO(2)$

B) Boost massive particle to a rest frame $SL(2, \mathbb{C})$

$$p^\mu = (m, 0, 0, 0) \quad p_\mu = (-m, 0, 0, 0)$$

K_i - boost along spatial direction \rightarrow $\underbrace{SO(3,1)}$
another $SO(2)$

Boost massive particle to a rest frame $L(2,0)$

$$p^\mu = (m, 0, 0, 0) \quad , \quad p_\mu = (-m, 0, 0, 0)$$

$$\Rightarrow \{Q_a, \bar{Q}_a\} = 2m \delta_{ab}$$

K_i - boost along spatial direction \rightarrow $\underbrace{K_i}_{\text{another } SO(2)}$

Boost massive particle to a rest frame $L(2,0)$

$$p^\mu = (m, 0, 0, 0) \quad , \quad p_\mu = (-m, 0, 0, 0)$$

$$\Rightarrow \{Q_\alpha, \bar{Q}_\alpha\} = 2m \delta_{\alpha\beta} \quad \{Q_\alpha, \bar{Q}_\beta\} = \{\bar{Q}_\alpha, \bar{Q}_\beta\} = 0$$

$$\Rightarrow Q_\alpha = \frac{1}{\sqrt{2m}} Q_{\alpha'} \quad \bar{Q}_\alpha = \frac{1}{\sqrt{2m}} \bar{Q}_{\alpha'}$$

$$\{\bar{a}_\alpha, a_\beta\} = \delta_{\alpha\beta}$$

$$\{a_\alpha, a_\beta\} = \{\bar{a}_\alpha, \bar{a}_\beta\} = 0$$

$$\{\bar{a}_\alpha, a_\beta\} = \delta_{\alpha\beta}$$

$$\{a_\alpha, a_\beta\} = \{\bar{a}_\alpha, \bar{a}_\beta\} = 0$$

4-dim

F

spinors

$$\{\bar{a}_\alpha, a_\beta\} = \delta_{\alpha\beta}$$

$$\{a_\alpha, a_\beta\} = \{\bar{a}_\alpha, \bar{a}_\beta\} = 0$$

} 4-dim Clifford algebra

$a_\alpha^2 = 0 \Rightarrow$ we can always find $|\Omega\rangle$

$$\{\bar{a}_\alpha, a_\beta\} = \delta_{\alpha\beta}$$

$$\{a_\alpha, a_\beta\} = \{\bar{a}_\alpha, \bar{a}_\beta\} = 0$$

4-dim Clifford algebra

$a_\alpha^2 = 0 \Rightarrow$ we can always find $|\Omega\rangle$

$$a_\alpha |\Omega\rangle = 0$$

$$\{\bar{a}_\alpha, a_\beta\} = \delta_{\alpha\beta}$$

$$\{a_\alpha, a_\beta\} = \{\bar{a}_\alpha, \bar{a}_\beta\} = 0$$

} 4-dim Clifford algebra

$a_\alpha^2 = 0 \Rightarrow$ we can always find $|\Omega\rangle$

$$a_\alpha |\Omega\rangle = 0$$

A general $SO(n)$ representation with Clifford spinors

$$\left(\begin{array}{c} j_1 \dots j_m \\ \vdots \\ j_1 \dots j_m \end{array} \right)$$

$X_{d_1 \dots d_m} \rightarrow X = \underbrace{M_{d_1} \dots M_{d_m}}_n \cdot X$

Clifford algebra

$\psi \rangle$

$\psi \rangle, \bar{\psi} \rangle$

Clifford algebra



$|\Omega\rangle,$

$a_j |\Omega\rangle$

of SUSY algebra

$P_{\text{min}} = 2 P_{\text{max}}$

$SO(2) \subset SO(3)$

$|\Omega\rangle,$

$a_j |\Omega\rangle$

$\underbrace{\hspace{2cm}}$
- 2 states

$a_1 a_2 |\Omega\rangle$

$P_{\text{par}} = ?$

$SO(2) \subset SO(3)$

$|\Omega\rangle,$

$a_j |\Omega\rangle$

$\underbrace{\hspace{2cm}}$
- 2 states

of SUSY $\bar{a}_1 \bar{a}_2 |\Omega\rangle$

↑
suppose
this is
spin J

$|\Omega\rangle,$

↑
suppose
this
spin J

$a_j |\Omega\rangle$

↖ ↗
2 states

↑
 $J \pm \frac{1}{2}$

$a_1 a_2 |\Omega\rangle$



$|\Omega\rangle,$

↑
suppose
this is
spin J

$a_j |\Omega\rangle$
↑
2 states

↑
 $J \pm \frac{1}{2}$

$\overline{a_1 a_2} |\Omega\rangle$
is a scalar

$|\Omega\rangle,$

↑
suppose
this is
spin J

$a_j |\Omega\rangle$
↑
2 states

↑
 $J \pm \frac{1}{2}$

$\underbrace{a_1 a_2}_{\text{is a scalar}} |\Omega\rangle$

P, J, P

So

$$|\Omega\rangle,$$

$$a_j |\Omega\rangle$$

$$\underbrace{a_1 a_2}_{\text{is a scalar}} |\Omega\rangle$$

↑
suppose
this is
spin J

↑
is a scalar

$$J \pm \frac{1}{2}$$

↑ true if $J \neq 0$. So

$$J=0$$

$$0$$

$$\frac{1}{2}$$

$$0$$

$|\Omega\rangle,$

$\underbrace{a_j}_{-2\text{st}} |\Omega\rangle$

$\underbrace{a_1 a_2}_{\text{is a scalar}} |\Omega\rangle$

↑
suppose
this
spin J

J_{\pm}

↑ true if $J \neq 0$. So

$J=0$

0

$\frac{1}{2}$

0

$|\Omega\rangle,$

$a_i |\Omega\rangle$

$\underbrace{a_1 a_2}_{\text{is a scalar}} |\Omega\rangle$

↑
suppose
this is
spin J

↑
2 status

↑
 $J \pm \frac{1}{2}$

↑ true if $J \neq 0$. So

$J=0$

0

$\frac{1}{2}$

0

DOF (b, f)

$\dim \{ \dots \} = 2j + 1$

$$\# \text{DOF}_{(b, f)} = 2(2J + 1)$$

\uparrow
 statistics of
 $|\Omega\rangle$



[The following text is heavily scribbled out with white chalk, making it illegible.]

$$\# \text{DOF}_{(b, f)} = 2(2J+1) = 4J+2$$

\uparrow
 statistics of
 $|\Omega\rangle$

$$\# \text{DOF}_{(r, b)} = \dim \{ |s, l, m\rangle \} = 2J+1$$

$$\frac{J+0}{\# \text{ DOF}}_{(b, f)} = 2(2J+1) = 4J+2$$

\uparrow
 statistics of
 $|\Omega\rangle$

$$\# \text{ DOF}_{(a, b)} = 1 \left(2 \left(J + \frac{1}{2} \right) + 1 \right) + 2 \left(J + \frac{1}{2} \right) + 1 = 4J+2$$

$$\frac{J+0}{\# \text{ DOF}} = 2(2J+1) = 4J+2$$

\uparrow
 statistics of
 $|\Omega\rangle$

$$\# \text{ DOF} = 1 \left(2\left(J + \frac{1}{2}\right) + 1 \right) + 2 \left(J + \frac{1}{2} \right) + 1 = 4J+2$$

Same for $J=0$