

Title: Intro to Supersymmetry 7

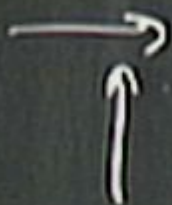
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Abstract:

$$(t, \theta, \theta^*)$$

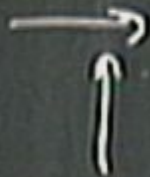
Superspace.



$$(t + i\varepsilon\theta^* + i\varepsilon^*\theta, \theta + \varepsilon, \theta^* + \varepsilon^*)$$

supersymmetry
transformation
as translations
on superspace.

(t, θ, θ^*)
Superspace



$(t + i\varepsilon^1\theta^* + i\varepsilon^2\theta, \theta + \varepsilon, \theta^* + \varepsilon^*)$

supersymmetry
transformation
as translations
on superspace

$$\Phi(t, \theta, \theta^*) = \varphi(t) + \theta\psi(t) - \theta^*\psi^*(t) + \theta\theta^*F(t)$$

↑ superfield

$t \rightarrow f + e \iff$

$\mathbb{H} \iff$

$$t \rightarrow t + \epsilon \quad \leftrightarrow \quad H \leftrightarrow i \partial_t$$

$$Q = \frac{\partial}{\partial \theta} + i \theta^* \frac{\partial}{\partial t}$$

$$Q^{\dagger} = \frac{\partial}{\partial \theta^*} + i \theta \frac{\partial}{\partial t}$$

$$t \rightarrow t + \epsilon \quad \longleftrightarrow$$

$$H \longleftrightarrow$$

$$1 \quad \partial_t$$

$$\delta \psi(t) = \epsilon \partial_t \psi(t)$$

$$Q = \frac{\partial}{\partial \theta} + i\theta^* \frac{\partial}{\partial t}$$

$$Q^+ = \frac{\partial}{\partial \theta^*} + i\theta \frac{\partial}{\partial t}$$



$t \rightarrow t + \epsilon \iff H \iff 1 \iff \psi$

$$Q = \frac{\partial}{\partial t} + i\theta^* \frac{\partial}{\partial \theta}$$

$$Q^+ = \frac{\partial}{\partial \theta^*} + i\theta \frac{\partial}{\partial t}$$

$$\delta \psi(t) = \epsilon i \gamma_4 \psi(t)$$

$$\delta \bar{\psi} = (\epsilon Q + \epsilon^* Q^+) \bar{\psi}$$

$$t \rightarrow t + \epsilon \iff H \iff i\partial_t$$

$$\delta \psi(t) = \epsilon i \partial_t \psi$$

$$\delta \bar{\psi} = (\epsilon Q + \epsilon^* Q^*) \bar{\psi}$$

$$Q = \frac{\partial}{\partial \theta} + i\theta^* \frac{\partial}{\partial t}$$

$$Q^+ = \frac{\partial}{\partial \theta^*} + i\theta \frac{\partial}{\partial t}$$

$$H = i\partial_t$$

$$\{Q, Q\} = \{Q^+, Q^+\} = 0 ; \{Q, Q^+\} = 2H$$

$$\delta \varphi(t) = \varepsilon \psi - \varepsilon^* \psi^*$$

$$\delta \psi(t) = \varepsilon^* (F - i \dot{\varphi})$$

$$\delta \psi^*(t) = \varepsilon (F + i \dot{\varphi})$$

$$\delta F$$



$$\delta \varphi(t) = \varepsilon \psi - \varepsilon^* \psi^*$$

$$\delta \psi(t) = \varepsilon^* (F - i\dot{\varphi})$$

$$\delta \psi^*(t) = \varepsilon (F + i\dot{\varphi})$$

$$\delta F(t) = -i (\varepsilon \dot{\psi} + \varepsilon^* \dot{\psi}^*) \quad \leftarrow \text{Important.}$$



$$\delta \mathcal{L}(t) = \epsilon \dot{\psi} - \epsilon^* \dot{\psi}^*$$

$$\delta \psi(t) = \epsilon^* (F - i\dot{\Phi})$$

$$\delta \psi^*(t) = \epsilon (F + i\dot{\Phi})$$

$$\delta F(t) = -i (\epsilon \dot{\psi} + \epsilon^* \dot{\psi}^*)$$

← Important.

$$\delta_{\text{SUSY}} \int \bar{\Phi}_1$$

$$\delta \mathcal{L}(t) = \epsilon \dot{\psi} - \epsilon^* \dot{\psi}^*$$

$$\delta \psi(t) = \epsilon^* (F - i \dot{\Phi})$$

$$\delta \psi^*(t) = \epsilon (F + i \dot{\Phi})$$

$$\delta F(t) = -i (\epsilon \dot{\psi} + \epsilon^* \dot{\psi}^*) \quad \leftarrow \text{Important.}$$

$$\delta_{\text{SUSY}} [\bar{\Phi}_1 \quad \bar{\Phi}_2] =$$

$$\delta \varphi(t) = \varepsilon \psi - \varepsilon^* \psi^*$$

$$\delta \psi(t) = \varepsilon^* (F - i\dot{\varphi})$$

$$\delta \psi^*(t) = \varepsilon (F + i\dot{\varphi})$$

$$\delta F(t) = -i (\varepsilon \dot{\psi} + \varepsilon^* \dot{\psi}^*) \quad \leftarrow \text{Important.}$$

$$\delta_{\text{sym}} \left[\bar{\Phi}_1 \bar{\Phi}_2 \right] = \delta_{\text{sym}} \bar{\Phi}_1 \bar{\Phi}_2 + \bar{\Phi}_1 \delta_{\text{sym}} \bar{\Phi}_2$$



$$t \rightarrow t + \epsilon \quad \Leftrightarrow$$

$$H \leftrightarrow i \partial_t$$

$$\delta \psi(t) = \epsilon i \partial_t \psi(t)$$

$$Q = \frac{\partial}{\partial \theta} + i \theta^* \frac{\partial}{\partial t}$$

$$Q^+ = \frac{\partial}{\partial \theta^*} + i \theta \frac{\partial}{\partial t}$$

$$H = i \partial_t$$

$$\{Q, Q\} = \{Q^+, Q^+\} = 0; \quad \{Q, Q^+\} = 2H$$

$$\overline{\delta \Phi} = (\epsilon Q + \epsilon^* Q^*) \overline{\Phi}$$

$$\delta \varphi(t) = \varepsilon \psi - \varepsilon^* \psi^*$$

$$\delta \psi(t) = \varepsilon^* (F - i\dot{\varphi})$$

$$\delta \psi^*(t) = \varepsilon (F + i\dot{\varphi})$$

$$\delta F(t) = -i (\varepsilon \dot{\psi} + \varepsilon^* \dot{\psi}^*) \quad \leftarrow \text{Important.}$$

$$\delta_{\text{sym}} [\bar{\Phi}_1 \Phi_2] = \delta_{\text{sym}} \bar{\Phi}_1 \Phi_2 + \bar{\Phi}_1 \delta_{\text{sym}} \Phi_2$$

$$\Rightarrow \text{IP } \bar{\Phi}_1, \Phi_2$$

$$\delta \varphi(t) = \varepsilon \psi - \varepsilon^* \psi^*$$

$$\delta \psi(t) = \varepsilon^* (F - i\dot{\bar{\Phi}})$$

$$\delta \psi^*(t) = \varepsilon (F + i\dot{\Phi})$$

$$\delta F(t) = -i (\varepsilon \dot{\psi} + \varepsilon^* \dot{\psi}^*) \quad \leftarrow \text{Important.}$$

$$\delta_{\text{susy}} [\bar{\Phi}_1 \Phi_2] = \delta_{\text{susy}} \bar{\Phi}_1 \Phi_2 + \bar{\Phi}_1 \delta_{\text{susy}} \Phi_2$$

\Rightarrow If $\bar{\Phi}_1, \Phi_2 \Rightarrow \bar{\Phi}_1 \Phi_2$ is also a superfield

\Rightarrow an arbitrary polynomial of \mathbb{C} is a superfield



\Rightarrow an arbitrary polynomial of Φ is a superfield



How to construct SUSY invariant actions.

$\mathcal{L}(\Phi) \rightarrow$ an arbitrary polynomial of Φ

\Rightarrow an arbitrary polynomial of Φ is a superfield



How to construct SUSY invariant actions.

$\mathcal{L}(\Phi) \rightarrow$ an arbitrary polynomial of Φ

\Rightarrow If we extract the highest component of $\mathcal{L}(\Phi)$



\Rightarrow an arbitrary polynomial of Φ is a superfield



How to construct SUSY invariant actions.

$\mathcal{L}(\Phi) \rightarrow$ an arbitrary polynomial of Φ

\rightarrow If we extract the highest component of $\mathcal{L}(\Phi)$

\mathbb{F}

\Rightarrow an arbitrary polynomial of Φ is a superfield



How to construct SUSY invariant actions.

$\mathcal{L}(\Phi) \rightarrow$ an arbitrary polynomial of Φ

\rightarrow If we extract the highest component of $\mathcal{L}(\Phi)$

$\int \mathcal{L} \rightarrow$ will transform as a total derivative.

$$S_{susy} = \int dt F_2$$

$$\delta_{susy} S_{susy} = \int dt \delta_{susy} F_2$$

$$K \psi = (i \not{\partial} - m) \psi$$

$$\bar{\psi} = (\psi^\dagger \gamma^0)$$

$$\begin{aligned}
 \int_{t_1}^{t_2} dt F_2 &= \int_{t_1}^{t_2} dt \left(\epsilon \dot{\psi} + \epsilon^* \dot{\psi}^* \right) \\
 \int_{t_1}^{t_2} dt \delta_{\text{usy}} F_2 &= \int_{t_1}^{t_2} dt \left(\delta_{\text{usy}} \left(\epsilon \dot{\psi} + \epsilon^* \dot{\psi}^* \right) \right)
 \end{aligned}$$

$$\int_{t_1}^{t_2} dt \left(\delta_{\text{usy}} \left(\epsilon \dot{\psi} + \epsilon^* \dot{\psi}^* \right) \right)$$

$$\begin{aligned}
 S_{\text{susy}} &= \int dt F_2 \\
 \delta_{\text{susy}} S_{\text{susy}} &= \int dt \delta_{\text{susy}} F_2 = \int dt \left[\frac{\delta}{\delta \psi} \left(\epsilon \psi + \epsilon^* \psi^* \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 \int_{-\infty}^{+\infty} dt F_2 &= \int_{-\infty}^{+\infty} dt \delta_{\text{usy}} F_2 \\
 &= \int_{-\infty}^{+\infty} dt \left[\frac{1}{2} (\epsilon \psi_2 + \epsilon^* \psi_2^*) \right] \\
 &= 0
 \end{aligned}$$

$$\theta\psi(t) - \theta^* \psi^*(t) + \theta\theta^* F(t)$$

\Rightarrow an arbitrary polynomial of \mathbb{QD} is a superfield



How to construct SUSY invariant actions.

$$S_{\text{susy}} = \int dt F_2$$

$$S_{\text{gusy}} S_{\text{usy}} = \int_{-\infty}^{+\infty} dt \delta_{\text{gusy}} F_2 = 0$$

$$F_2 = \alpha(\varphi) \Big|_{\theta\theta^*}$$

$$F_2 = \int d^2\theta \left(\epsilon \psi_2 + \epsilon^* \psi_2^* \right)$$

$$S_{\text{susy}} = \int dt F_2$$

$$S_{\text{gusy}} S_{\text{usy}} = \int_{-\infty}^{+\infty} dt \delta_{\text{gusy}} F_2 = \int_{-\infty}^{+\infty} dt \left[\frac{1}{2} (\epsilon \psi_2 + \epsilon^* \psi_2^*) \right]$$

$$= 0$$

$$F_2 = \alpha(\varphi) \Big|_{\theta\theta^*} = \int d\theta d\theta^* \mathcal{L}(\varphi)$$

$$S_{susy} = \int dt F_2$$

$$\delta_{susy} S_{susy} = \int dt \delta_{susy} F_2 = \int dt \left(\frac{\delta F_2}{\delta \psi} \delta \psi + \frac{\delta F_2}{\delta \psi^*} \delta \psi^* \right)$$

$$= 0$$

$$F_2 = \mathcal{L}(\varphi) \Big|_{\theta\theta^*} = \int d\theta d\theta^* \mathcal{L}(\varphi) = \frac{\partial \mathcal{L}}{\partial \theta} \frac{\partial \mathcal{L}}{\partial \theta^*} = -$$

$$\text{Sur}(\mathbb{P}_1, \mathbb{P}_1) = \mathbb{P}_1 \oplus \mathbb{P}_1 + \mathbb{P}_1 \oplus \mathbb{P}_1$$

\Rightarrow If $\mathbb{P}, \mathbb{Q}, \mathbb{P}_1 \Rightarrow \mathbb{P}, \mathbb{P}_1$ is also a superfield

$$\text{Sur} = \int \mathcal{L}(\mathbb{P})$$

$$\left[\int \mathcal{F}_i (\mathcal{E} \psi_2 + \mathcal{E}^c \psi_2^c) \right]$$

$$\begin{aligned} (\mathcal{F}) &= \\ \partial \cdot \mathcal{L}(\mathcal{F}) &= -F_d \end{aligned}$$



$$S_{\text{susy}} = \int \mathcal{L}(\bar{\Phi})$$



$$S_{\text{susy}} = \int \mathcal{L}(\Phi)$$

$$S_{\text{Susy}} = \int d^4x d^4\theta \mathcal{L}(\Phi)$$



$$S_{\text{usy}} = \int d^4x d^4\theta^* \mathcal{L}(\vec{\Phi})$$

$$S_{\text{Susy}} = \int d^4x d^4\theta^* \mathcal{L}(\Phi)$$

\Rightarrow enlarge "space" of superfields to
sup

$$S_{\text{susy}} = \int dt d\theta d\theta^* \mathcal{L}(\Phi)$$

\Rightarrow enlarge "space" of superfields to superfields that include derivatives.

$$\Phi(t, \theta, \theta^*) \xrightarrow{\text{naive}} \partial_t \Phi(t, \theta, \theta^*)$$

$$S_{\text{SUSY}} = \int d^4x d\theta d\theta^* \mathcal{L}(\bar{\Phi})$$

\Rightarrow enlarge "space" of superfields to superfields that include derivatives.

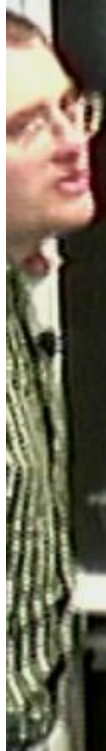
$$\bar{\Phi}(t, \theta, \theta^*) \xrightarrow{\text{naive}} \underbrace{\partial_t \bar{\Phi}(t, \theta, \theta^*)}_{\text{is NOT a superfield}}$$

$$S_{\text{susy}} = \int d^4x d^4\theta \mathcal{L}(\bar{\Phi})$$

\Rightarrow enlarge "space" of superfields to superfields that include derivatives.

$$\Phi(t, \theta, \theta^*) \xrightarrow{\text{naive}} \partial_t \Phi(t, \theta, \theta^*)$$

$$\mathcal{L}(\Phi) \rightarrow \mathcal{L}(\Phi, \partial_t \Phi) \quad \text{is NOT a superfield!}$$



$$(1, 0, 0) \left[\begin{matrix} \rightarrow \\ \rightarrow \\ \rightarrow \end{matrix} \right] \neq (1 + i\epsilon_0 + 0.3i + 0.3i + 0.1\epsilon, 0 + i\epsilon)$$

[The following text is heavily obscured by large, dark, curved chalk strokes.]

$$(1, 0, 0) \rightarrow (1 + i\epsilon_0, 0 + i\epsilon)$$

$$(L, \theta, \{ \delta_{\text{susy}}, \gamma_t \} \neq (0, i\epsilon^0 + i\epsilon^1, 0, i\epsilon, \theta^0 + \epsilon^1))$$

⇒ we need to introduce susy-covariant derivatives.

D,

$$\theta^0 + \epsilon^1 \rightarrow \theta^0 + \epsilon^1 \gamma_t \rightarrow \theta^0 + \epsilon^1 \gamma_t$$

$$(L, U, \{S_{\text{susy}}, \tau_t\} \neq 0 \quad (i\epsilon D + i\epsilon^* D^\dagger, 0 + \epsilon, 0^* + \epsilon^*))$$

\Rightarrow we need to introduce susy-covariant derivatives.

D, D^\dagger in such a way that

$$[D, S_{\text{susy}}] = 0; \quad [D^\dagger, S_{\text{susy}}] = 0$$



$t \Rightarrow t \quad D\Phi$

FTS on superfield

$$S_{SUSY}(D\Phi) = D[S_{SUSY}\Phi]$$

$\psi(\psi) = \dots$

$\Rightarrow D\Phi$ FTS on superfield $\psi(\theta) = \psi_0 + \theta\psi_1 + \theta^2\psi_2$

$$S_{SUSY}(D\Phi) = D[S_{SUSY}\Phi]$$

$$[D, \epsilon Q + \epsilon^* Q^*] = 0$$

$F_L = \begin{pmatrix} \theta \\ \theta^2 \end{pmatrix}$ θ θ^* Q

$t \rightarrow t$ $D\Phi$ Higgs on superfield? $\psi(\sigma) = \dots$

$$S_{\text{SUSY}}(D\Phi) = D [S_{\text{SUSY}}\Phi]$$

$$[D, \epsilon Q, + \epsilon^* Q^*] = 0 \Rightarrow \{D, Q\} = 0 \quad \{D, Q^*\}$$

$F_L = \dots$
 $\theta^0 \quad \theta^+$

$\Rightarrow D\Phi$ is on superfield Φ | $\psi(\theta) = \psi_0 + \psi_1\theta + \psi_2\theta^2$

$$S_{SUSY}(D\Phi) = D[S_{SUSY}\Phi] \quad (\psi_0, \psi_1, \psi_2)$$

$$[D, \epsilon Q + \epsilon^* Q^*] = 0 \Rightarrow \{D, Q\} = 0 \quad \{D, Q^*\}$$

$D = \frac{\partial}{\partial\theta} + i\theta^* \frac{\partial}{\partial t}$		$Q = \frac{\partial}{\partial\theta} + i\theta^* \frac{\partial}{\partial t}$
$D^+ = \frac{\partial}{\partial\theta^*} - i\theta \frac{\partial}{\partial t}$		$Q^+ = \frac{\partial}{\partial\theta} + i\theta \frac{\partial}{\partial t}$

$$\{D, D\} = \{D^+, D^+\}$$

$$\{D, D\} = \{D^+, D^+\} = 0$$

$$\{D, D^+\} = -2H$$

$$\{D, D\} = \{D^\dagger, D^\dagger\} = 0$$

$$\{D, D^\dagger\} = -2H$$

$$\mathcal{L}(\Phi, D\Phi, D^\dagger\Phi^*)$$

$$\{D, D\} = \{D^\dagger, D^\dagger\} = 0$$

$$\{D, D^\dagger\} = -2H$$

$\mathcal{L}(\Phi, D\Phi, D^\dagger\Phi^*) \rightarrow$ an arbitrary polynomial.

$$\{D, D\} = \{D^+, D^+\} = 0$$

$$\{D, D^+\} = -2H$$

$\mathcal{L}(\Phi, D\Phi, D^+\Phi^*) \rightarrow$ an arbitrary polynomial.

$$S_{\text{susy}} = \int dt d\theta d\theta^* \mathcal{L}(\varphi, D\varphi, \overline{D\varphi})$$

Consider

$$\mathcal{L}(\phi, D\phi, D^+\bar{\phi}) =$$

Consider

$$\mathcal{L}(\phi, D\phi, D^+\bar{\phi}) = \left\{ -\frac{1}{2} D\phi D^+\bar{\phi} + f(\phi) \right\}$$

Consider

$$\mathcal{L}(\Phi, D\Phi, D^+\bar{\Phi}) = \left\{ -\frac{1}{2} D\Phi D^+\bar{\Phi} + f(\Phi) \right\}$$

$\bar{\Phi}$ is a real superfield

and $f(\Phi)$ is an arbitrary real function.

Consider

$$\mathcal{L}(\Phi, D\Phi, D^+\bar{\Phi}) = \left\{ -\frac{1}{2} D\Phi D^+\bar{\Phi} + f(\Phi) \right\}$$

$\bar{\Phi}$ is a real superfield

and $f(\Phi)$ is an arbitrary real function.

Consider

$$\mathcal{L}(\Phi, D\Phi, D^+\bar{\Phi}) = \left\{ -\frac{1}{2} D\Phi D^+\bar{\Phi} + f(\Phi) \right\}$$

$\bar{\Phi}$ is a real superfield

and $f(\Phi)$ is an arbitrary real ^{analytic} function.

Consider

$$\mathcal{L}(\Phi, D\Phi, D^+\bar{\Phi}) = \left\{ -\frac{1}{2} D\Phi D^+\bar{\Phi} + f(\Phi) \right\}$$

$\bar{\Phi}$ is a real superfield

and $f(\Phi)$ is an arbitrary real ^{analytic} function.

$$f(\Phi) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} \Phi^n$$

Consider

$$\mathcal{L}(\Phi, D\Phi, D^+\Phi) = \left\{ -\frac{1}{2} D\Phi D^+\Phi + f(\Phi) \right\}$$

Φ is a real superfield

and $f(\Phi)$ is an arbitrary real ^{analytic} function } $f(\Phi) \sim \frac{1}{\Phi}$

$$f(\Phi) = \sum_{h=0}^{\infty} \frac{g_h}{h!} \Phi^h$$

DGP =



$$D_{\theta} \Psi = \left(\frac{\partial}{\partial \theta} - i \theta^{\dagger} \frac{\partial}{\partial \psi} \right) \left[\Psi + \theta \psi - \theta^{\dagger} \psi^{\dagger} + \theta \theta^{\dagger} F \right]$$

$$D\psi = \left(\frac{\partial}{\partial \theta} - i\theta^* \frac{\partial}{\partial \theta} \right) \left[\psi + \theta \psi - \theta^* \psi^* + \theta \theta^* F \right]$$

$$= \psi(\theta)$$

$$D\varphi = \left(\frac{\partial}{\partial \theta} - i\theta^* \frac{\partial}{\partial \theta} \right) \left[\varphi + \theta \psi - \theta^* \psi^* + \theta \theta^* F \right]$$

$$= \psi(\theta) + \theta^* F$$

$$D\varphi = \left(\frac{\partial}{\partial \theta} - i\theta^\nu \frac{\partial}{\partial \psi} \right) \left[\varphi + \theta\psi - \theta^* \psi^\nu + \theta\theta^* F \right]$$

$$= \psi(\theta) + \theta^* F - i\theta^\nu \dot{\varphi}$$

$$D\varphi = \left(\frac{\partial}{\partial \theta} - i\theta^\nu \frac{\partial}{\partial x^\nu} \right) \left[\varphi + \theta\psi - \theta^* \psi^\dagger + \theta\theta^* F \right]$$

$$= \psi(\theta) + \theta^* F - i\theta^\nu \dot{\varphi} + i\theta\theta^* \dot{\psi}$$

$$D\varphi = \left(\frac{\partial}{\partial \theta} - i\theta^\nu \frac{\partial}{\partial x^\nu} \right) \left[\varphi + \theta\psi - \theta^* \psi^\dagger + \theta\theta^* F \right]$$

$$= \psi(\theta) + \theta^* F - i\theta^\nu \dot{\varphi} + i\theta\theta^* \dot{\psi}$$

$$D\varphi = \left(\frac{\partial}{\partial t} - i\theta^\nu \frac{\partial}{\partial x^\nu} \right) \left[\psi + \theta^\nu \psi_\nu - \theta^\nu \psi_\nu + \theta \theta^\nu F \right]$$

$$= \psi(t) + \theta^\nu F - i\theta^\nu \dot{\varphi} + i\theta \theta^\nu \dot{\psi}$$

$$= \psi(t) + \theta^\nu \left[F(t) - i\dot{\varphi} \right] + i\theta \theta^\nu \dot{\psi}$$

$$D\varphi = \left(\frac{\partial}{\partial t} - i\theta^\nu \frac{\partial}{\partial x^\nu} \right) \left[\varphi + \theta\psi - \theta^\nu \psi^\nu + \theta\theta^\nu F \right]$$

$$= \psi(t) + \theta^\nu F - i\theta^\nu \dot{\varphi} + i\theta\theta^\nu \dot{\psi}$$

$$= \psi(t) + \theta^\nu [F(t) - i\dot{\varphi}] + i\theta\theta^\nu \dot{\psi}$$

$$D^\dagger\varphi = -\psi^\nu(t) - \theta(F + i\dot{\varphi}) + i\theta\theta^\nu \dot{\psi}^*$$

Consider

$$\mathcal{L}(\Phi, D\Phi, D^+\bar{\Phi}) = \left\{ -\frac{1}{2} D\Phi D^+\bar{\Phi} + f(\Phi) \right\}$$

$\bar{\Phi}$ is a real superfield

and $f(\Phi)$ is an arbitrary real ^{analytic} function.

$$f(\Phi) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} \Phi^n$$

$$\begin{aligned}
D\varphi &= \left(\frac{\partial}{\partial t} - i\theta^\vee \frac{\partial}{\partial t} \right) \left[\varphi + \theta\psi - \theta^\vee \psi^\vee + \theta\theta^\vee F \right] \\
&= \psi(t) + \theta^\vee F - i\theta^\vee \dot{\varphi} + i\theta\theta^\vee \dot{\psi} \\
&= \psi(t) + \theta^\vee [F(t) - i\dot{\varphi}] + i\theta\theta^\vee \dot{\psi} \\
D^\dagger\varphi &= -\psi^\vee(t) - \theta(F + i\dot{\varphi}) + i\theta\theta^\vee \dot{\psi}
\end{aligned}$$



$$D\psi = \left(\frac{\partial}{\partial t} - i\theta^x \frac{\partial}{\partial x} \right) \left[\psi + \theta\dot{\psi} - \theta^x \psi^x + \theta\theta^x F \right]$$

$$= \psi(t) + \theta^x F - i\theta^x \dot{\psi} + i\theta\theta^x \dot{\psi}$$

$$= \psi(t) + \theta^x [F(t) - i\dot{\psi}] + i\theta\theta^x \dot{\psi}$$

$$D^{\dagger}\psi = -\psi^x(t) - \theta(F + i\dot{\psi}) + i\theta\theta^x \dot{\psi}^*$$

$$D\Phi \quad D^+\Phi \quad \Big|_{\theta\theta^+} \quad \downarrow \text{WIB} \quad \mathcal{L}(\Phi)$$

term

⇒ enlarge "space" of superfields to
superfields that include derivatives

$$\Phi(\theta, \theta^+) \xrightarrow{\text{add}} \Phi(t, \theta, \theta^+)$$

$$\mathcal{L}(\Phi) \rightarrow \mathcal{L}(\Phi, \partial\Phi)$$

is NOT a superfield

$$D\bar{\Phi} D^+\Phi \Big|_{\theta\theta^+} \left(\downarrow \text{term} = \Phi \right).$$

⇒ enlarge space of superfields to
superfields that include derivatives

$$\Phi(\theta, \theta^+) \xrightarrow{\text{shift}} \Phi(\theta + \epsilon, \theta^+)$$

$$\mathcal{L}(\Phi) \rightarrow \mathcal{L}(\Phi, \partial\Phi) \quad \text{is NOT invariant}$$

$$D\phi = \left(\frac{\partial}{\partial t} - i\theta^x \frac{\partial}{\partial x} \right) \left[\phi + \theta\psi - \theta^x \psi^x + \theta\theta^x F \right]$$

$$= \psi(t) + \theta^x F - i\theta^x \dot{\phi} + i\theta\theta^x \dot{\psi}$$

$$= \psi(t) + \theta^x [F(t) - i\dot{\phi}] + i\theta\theta^x \dot{\psi}$$

$$D^\dagger\phi = -\psi^x(t) - \theta(F + i\dot{\phi}) + i\theta\theta^x \dot{\psi}^*$$

$$D\bar{\Phi} D^+\Phi \Big|_{\theta\theta^*} \text{ term} = (\mathcal{L}). \quad \theta\theta^* [F^2 + \dot{\phi}^2]$$

⇒ enlarge "space" of superfields to
superfields that include derivatives

$$\mathcal{L}(\Phi, \theta, \theta^*) \xrightarrow{\text{rule}} \mathcal{L}(\Phi, \theta, \theta^*)$$

$$\mathcal{L}(\Phi) \rightarrow \mathcal{L}(\Phi, \partial\Phi) \quad \text{is NOT } \mathcal{L}(\Phi, \partial\Phi)$$

$$D\bar{\Phi} D^{\dagger}\Phi \Big|_{\theta\theta^* \text{ term}} = (\mathcal{L}) \cdot \theta\theta^* \left[F^2 + \dot{\phi}^2 + i\psi\dot{\psi}^* - i\dot{\psi}\psi^* \right]$$

⇒ enlarge "space" of superfields to
superfields that include derivatives

$$\Phi(x, \theta, \theta^*) \xrightarrow{\text{rule}} \int_t \Phi(x, \theta, \theta^*)$$

$$\mathcal{L}(\Phi) \rightarrow \mathcal{L}(\Phi, \partial\Phi) \quad \text{is NOT } \mathcal{L}(\Phi)$$

$$D\bar{\Phi} D^{\dagger}\Phi \Big|_{\theta\theta^{\dagger} \text{ term}} = (\mathcal{L}). \quad \theta\theta^{\dagger} \left[F^2 + \dot{\phi}^2 + i\psi\dot{\psi}^{\dagger} - i\dot{\psi}\psi^{\dagger} \right]$$

$\int d\theta d\theta^{\dagger} \theta\theta^{\dagger} \mathcal{L}$ is the action of superfields to include derivatives

$$\Phi(\theta, \theta^{\dagger})$$

$$\Phi(\theta, \theta^{\dagger})$$

IS NOT

$$\Phi(\theta)$$

$$D\bar{\Phi} D^{\dagger}\Phi \Big|_{\theta\theta^* \text{ term}} = (\mathcal{L}). \quad \theta\theta^* \left[F^2 + \dot{\phi}^2 + i\psi\dot{\psi}^* - i\dot{\psi}\psi^* \right]$$

$$= \int d\theta d\theta^* \theta\theta^* \left(\frac{\partial}{\partial\theta} \frac{\partial}{\partial\theta^*} \theta\theta^* \right) = \int d\theta d\theta^* \left(\frac{\partial}{\partial\theta} \frac{\partial}{\partial\theta^*} \right) = -1$$

super fields must include derivatives

$$\Phi(\theta, \theta^*) \xrightarrow{\text{shift}} \Phi(\theta + \epsilon, \theta^*)$$

$$\mathcal{L}(\Phi) \rightarrow \mathcal{L}(\Phi, \delta\Phi) \quad \text{is NOT invariant}$$

$$D\bar{\Phi} D^+\Phi \Big|_{\theta\theta^* \text{ term}} = (\mathcal{L}). \quad \theta\theta^* \left[F^2 + \dot{\phi}^2 + i\psi\dot{\psi}^* - i\dot{\psi}\psi^* \right]$$

$$= \int d\theta d\theta^* \theta\theta^* \stackrel{\text{include derivatives}}{=} \frac{\partial}{\partial\theta} \frac{\partial}{\partial\theta^*} \theta\theta^* = \left(\frac{\partial\theta}{\partial\theta} - \frac{\partial\theta^*}{\partial\theta^*} \right) = -1$$

$$\int d\theta d\theta^* \left[-\frac{1}{2} D\Phi D^+\Phi \right]$$

$$\mathcal{L}(\Phi, \theta, \theta^*)$$

$$\mathcal{L}(\Phi, \theta, \theta^*)$$

$$\mathcal{L}(\Phi)$$

$$\mathcal{L}(\Phi, \lambda\Phi)$$

is NOT

$$\bar{D}\Phi D^+\Phi \Big|_{\theta\theta^* \text{ term}} = (\mathcal{F}). \quad \theta\theta^* \left[F^2 + \dot{\phi}^2 + i\psi\dot{\psi}^* - i\dot{\psi}\psi^* \right]$$

$$= \int d\theta d\theta^* \theta\theta^* \stackrel{\text{poles}}{=} \frac{\partial}{\partial\theta} \frac{\partial}{\partial\theta^*} \theta\theta^* = \left[\frac{\partial}{\partial\theta} \frac{\partial}{\partial\theta^*} \right] = -1$$

$$\int d\theta d\theta^* \left[-\frac{1}{2} D\Phi D^+\Phi \right] = \frac{1}{2} (F^2 + \dot{\phi}^2 + i\psi\dot{\psi}^* - i\dot{\psi}\psi^*)$$

$\mathcal{L}(\Phi, \partial\Phi) \rightarrow \mathcal{L}(\Phi, \mathcal{F})$ is NOT

$$\overline{D}\Phi D^+\Phi \Big|_{\theta\theta^* \text{ term}} = \mathcal{L}(\Phi). \quad \theta\theta^* \left[F^2 + \dot{\phi}^2 + i\psi\dot{\psi}^* - i\dot{\psi}\psi^* \right]$$

$$= \int d\theta d\theta^* \theta\theta^* \left[\frac{\partial}{\partial\theta} \frac{\partial}{\partial\theta^*} \theta\theta^* = \frac{\partial}{\partial\theta} \frac{\partial}{\partial\theta} - \frac{\partial}{\partial\theta^*} \frac{\partial}{\partial\theta^*} \right]$$

$$\int d\theta d\theta^* \left[-\frac{1}{2} D\Phi D^+\Phi \right] = +\frac{1}{2} \left(F^2 + \dot{\phi}^2 + i\psi\dot{\psi}^* - i\dot{\psi}\psi^* \right)$$

$\mathcal{L}(\Phi, \theta, \theta^*) \rightarrow \mathcal{L}(\Phi, \theta, \theta^*)$ is NOT a total derivative

$f(x)$

$$f(\theta) |_{\theta\theta^*} = f[\]$$

$$f(\theta) |_{\theta^*} = f[\phi + \theta\psi - \theta^y\psi^y]$$

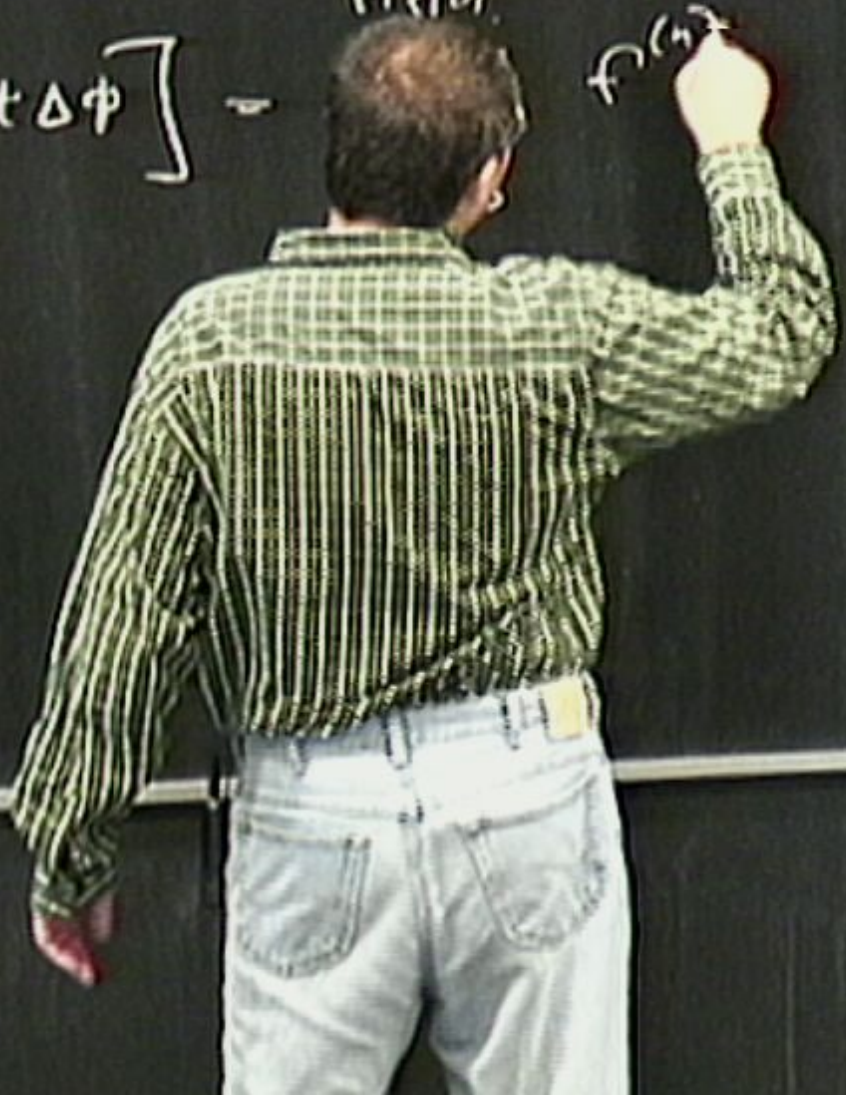


$$f(\theta) \Big|_{\theta^*} = f[\phi + \theta\psi - \theta^y\psi^y + \theta\theta^x F]$$

$$f(\phi) \Big|_{\theta\theta^*} = f \left[\underbrace{\phi + \theta\psi - \theta^*\psi^* + \theta\theta^*F}_{\substack{\uparrow \\ \text{a local} \\ \text{field}}} \right] \\ = f[\tau + \Delta\phi]$$

$$f(\mathbf{r})|_{\theta\theta^*} = f\left[\underbrace{\mathbf{r} + \theta\psi - \theta^* \psi^*}_{\substack{\uparrow \\ \text{a local} \\ \text{field}}}, \underbrace{\theta\theta^*}_{\Delta\phi}\right]$$

$$= f[\mathbf{r} + \Delta\phi] - f^{(1)}$$



$$f(\varphi) \Big|_{\theta\theta^*} = f \left[\underbrace{\varphi + \theta\psi - \theta^*\psi^* + \theta\theta^*F}_{\Delta\varphi} \right]$$

\uparrow
 a local
 field.

$$= f[\varphi + \Delta\varphi] = \sum_{n=0}^{\infty} \frac{f^{(n)}(\varphi) (\Delta\varphi)^n}{n!}$$

$$f(\varphi) \Big|_{\theta\theta^*} = f \left[\underbrace{\varphi + \theta\psi - \theta^*\psi^* + \theta\theta^*F}_{\Delta\varphi} \right]$$

\uparrow
 a bosonic field

$$= f[\varphi + \Delta\varphi] = \sum_{n=0}^{\infty} \frac{f^{(n)}(\varphi) (\Delta\varphi)^n}{n!}$$

$$(\Delta\varphi)^3 \equiv 0$$

$$f(\varphi) \Big|_{\theta\theta^*} = f \left[\underbrace{\varphi + \theta\psi - \theta^*\psi^* + \theta\theta^*F}_{\Delta\varphi} \right]$$

\uparrow
 a local
 field

$$= f[\varphi + \Delta\varphi] = \sum_{n=0}^{\infty} \frac{f^{(n)}(\varphi) (\Delta\varphi)^n}{n!}$$

$$(\Delta\varphi)^3 \equiv 0$$

$$= f(\varphi) +$$

$$f(\varphi) \Big|_{\theta\theta^*} = f \left[\underbrace{\varphi + \theta\psi - \theta^* \psi^* + \theta\theta^* F}_{\Delta\varphi} \right]$$

\uparrow
 a local
 field

$$= f[\varphi + \Delta\varphi] = \sum_{n=0}^{\infty} \frac{f^{(n)}(\varphi) (\Delta\varphi)^n}{n!}$$

$$(\Delta\varphi)^3 \equiv 0$$

$$= f(\varphi) + f'(\varphi) \cdot \Delta\varphi + \frac{f''(\varphi)}{2} \Delta\varphi^2$$

$$f(\phi) \Big|_{\theta\theta^*} = f \left[\underbrace{\phi + \theta\psi - \theta^*\psi^* + \theta\theta^*F}_{\Delta\phi} \right]$$

\uparrow
 a local
 field

$$= f[\varphi + \Delta\phi] = \sum_{n=0}^{\infty} \frac{f^{(n)}(\varphi) (\Delta\phi)^n}{n!}$$

$$(\Delta\phi)^3 \equiv 0$$

$$= f(\varphi) + f'(\varphi) \cdot \Delta\phi + \frac{f''(\varphi)}{2} \Delta\phi^2 \Big|_{\theta\theta^* \text{ term}}$$

\Rightarrow an arbitrary polynomial of \mathbb{P}^1 is a superfield

$$\textcircled{=} f'(\varphi) \frac{\partial \varphi}{\partial \theta^x} + \frac{f''(\varphi)}{2} (\partial \varphi)^2$$

How to construct $SU(2)$ invariant actions

$L(\mathbb{P}^1)$ is a doublet polynomial of \mathbb{P}^1

If we contract the highest component of $L(\mathbb{P}^1)$
 $\mathbb{P}^1 \rightarrow$ will transform as a total derivative

⇒ an arbitrary polynomial of \mathbb{P}^1 is a superfield

$$\textcircled{=} f'(\varphi) \frac{\delta \varphi}{\delta \theta^x} + \frac{f''(\varphi)}{2} (\delta \varphi)^2$$

How to construct SUSY invariant actions

$$= f''$$

⇒ a differential polynomial of \mathbb{P}^1

If we extract the highest component of $\int (\varphi)$
→ will transform as a total derivative

\Rightarrow an arbitrary polynomial of \mathbb{R}^D is a superfield

$$\textcircled{=} f'(\varphi) \frac{\delta \varphi}{\delta \theta^x} + \frac{f''(\varphi)}{2} (\delta \varphi)^2$$

How to construct SUSY invariant actions

$$= f'(F) + \frac{f''(\varphi)}{2} \text{ arbitrary polynomial of } \varphi$$

If we extract the highest component of $\int (\varphi)$
 $\int \varphi \rightarrow$ will transform as a total derivative

⇒ an arbitrary polynomial of Φ is a superfield

$$\textcircled{=} f'(\Phi) \frac{\delta \Phi}{\delta \theta^{\dot{\alpha}}} + \frac{f''(\Phi)}{2} (\delta \Phi)^2$$

How to construct SUSY invariant actions

$$= f'(F) + \frac{f''(\Phi)}{2} \left(-\theta \psi \theta^{\dot{\alpha}} \psi^{\dot{\alpha}} - \theta^{\dot{\alpha}} \psi^{\dot{\alpha}} \theta \psi \right)$$

If we extract the highest component of $\int (\Phi)$
 $\int \int \rightarrow$ will transform as a total derivative

⇒ an arbitrary polynomial of Φ is a superfield

$$\textcircled{=} f'(\Phi) \frac{\delta \mathcal{L}}{\delta \theta \theta^*} + \frac{f''(\Phi)}{2} (\delta \Phi)^2$$

How to construct SUSY invariant actions

$$= \theta \theta^* f'(\Phi) + \frac{f''(\Phi)}{2} \left(-\theta \psi \theta^* \psi^* - \theta^* \psi^* \theta \psi \right)$$

$$= \theta \theta^* \left[f'(\Phi) + \frac{f''(\Phi)}{2} [\psi, \psi^*] \right] \text{ component of } \mathcal{L}(\Phi)$$

will transform as a total derivative

$$S_{\text{SUSY}} = \int d^4x \left[\frac{1}{2} F^2 + \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} i \psi \dot{\psi}^* - i \psi \psi^* \partial_0 F \right]$$

$$\psi \dot{\phi} + i \partial_0 \psi$$

$$\psi \dot{\phi} + i \partial_0 \psi$$

$$D_P = \psi \dot{\phi} + i \partial_0 \psi$$

$$S_{\text{SUSY}} = \int_{\text{vol}} \left[\frac{1}{2} F^2 + \frac{1}{2} \dot{\phi}^2 + \frac{i}{2} \psi \dot{\psi}^* - \frac{i}{2} \dot{\psi} \psi^* - F' \phi - \frac{F''}{2} [\psi, \psi^*] \right]$$

$$\psi(t) = \psi + i\theta \theta^* \psi$$

$$\dot{\psi}(t) = \dot{\psi} + i\theta \theta^* \dot{\psi}$$

$$D_t \psi = \dot{\psi}(t) - \theta (F + i\dot{\phi}) + i\theta \theta^* \dot{\psi}^*$$

⇒ an arbitrary polynomial of \mathbb{P}^1 is a superfield

$$\ominus f'(\varphi) \frac{\delta \varphi}{\delta \theta^x} + \frac{f''(\varphi)}{2} (\delta \varphi)^2$$

How to construct SUSY invariant actions

$$= \theta^x \partial'_x F + \frac{f''(\varphi)}{2} \left(-\theta \psi \theta^* \psi^* - \theta^* \psi^* \theta \psi \right)$$

$\frac{1}{2} [\psi, \psi^*]$ component of $\chi(\varphi)$
 will transform as a total derivative

$$S_{\text{susy}} = \int dt d^3x \left[\frac{1}{2} F^2 + \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{i}{2} \dot{\psi} \psi^* - F' \phi - \frac{F''}{2} [\psi, \psi^*] \right]$$

Before:

$$L = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} F^2 + i \psi \dot{\psi}^* - \frac{i}{2} \dot{\psi} \psi^* - F' \phi - \frac{F''}{2} [\psi, \psi^*]$$

$$S_{\text{SUSY}} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2} \dot{\phi}^2 + \frac{i}{2} \psi \dot{\psi}^* - \frac{i}{2} \dot{\psi} \psi^* - f' F - \frac{f''}{2} [\psi, \psi^*] \right]$$

Before:

$$S_{\text{SUSY}} = \int dt \left[\frac{1}{2} \dot{\phi}^2 + i \psi^* \dot{\psi} - \frac{1}{2} (f')^2 + \frac{1}{2} f'' [\psi^*, \psi] \right]$$

$$S_{\text{SUSY}} = \int dt \left[\frac{1}{2} \cancel{F^2} + \frac{1}{2} \cancel{p^2} + \frac{1}{2} i \psi \dot{\psi}^* - \frac{1}{2} \dot{\psi} \psi^* - f' F - \frac{f''}{2} (\psi, \psi^*) \right]$$

Before:

$$S_{\text{old SUSY}} = \int dt \left[\frac{1}{2} \cancel{p^2} + i \psi^* \dot{\psi} - \frac{1}{2} (f')^2 + \frac{f''}{2} (\psi^*, \psi) \right]$$

$$\int_{\text{SUSY}} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2} \dot{p}^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{1}{2} \dot{\psi} \psi^* - p' F - \frac{f''}{2} (\psi, \psi^*) \right]$$

Before:

$$\int_{\text{NT}} = \int dt \left[\frac{1}{2} \dot{p}^2 - i \psi^* \dot{\psi} - \frac{1}{2} (f')^2 + \frac{f''}{2} (\psi^*, \psi) \right]$$

$$S_{\text{susy}} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{1}{2} i \dot{\psi} \psi^* - f' F - \frac{f''}{2} (\psi, \psi^*) \right]$$

Before:

$$S_{\text{old susy}} = \int dt \left[\frac{1}{2} \dot{\phi}^2 - i \psi^* \dot{\psi} - \frac{1}{2} (f')^2 + \frac{f''}{2} (\psi^*, \psi) \right]$$

$$S_{\text{SUSY}} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2} \dot{p}^2 + \frac{1}{2} i \dot{\psi} \psi^* - \frac{1}{2} \dot{\psi} \psi^* - f' F - \frac{f''}{2} (\psi, \psi^*) \right]$$

\swarrow $\frac{1}{2} \dot{\psi} \psi^*$ \searrow $\frac{1}{2} \dot{\psi} \psi^*$
 \swarrow $\frac{1}{2} \dot{\psi} \psi^*$ \searrow $\frac{1}{2} \dot{\psi} \psi^*$

Before:

$$S_{\text{SUSY}} = \int dt \left[\frac{1}{2} \dot{p}^2 + i \dot{\psi}^* \psi - \frac{1}{2} (f')^2 + \frac{f''}{2} (\psi^*, \psi) \right]$$

$$S_{\text{susy}} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} i \psi \dot{\psi}^* - i \dot{\psi} \psi^* \right] = \int dt \left[\frac{1}{2} F^2 - \frac{1}{2} (\dot{\phi})^2 + \frac{1}{2} \psi \dot{\psi}^* - \frac{1}{2} \dot{\psi} \psi^* \right]$$

$\frac{1}{2} \psi \dot{\psi}^* - \frac{1}{2} \dot{\psi} \psi^* = \frac{1}{2} \psi \dot{\psi}^* + \frac{1}{2} \dot{\psi}^* \psi - \frac{1}{2} \dot{\psi} \psi^* - \frac{1}{2} \psi^* \dot{\psi}$

Before:

$$S_{\text{all}} = \int dt \left[\frac{1}{2} \dot{\phi}^2 - i \psi^* \dot{\psi} - \frac{1}{2} (F')^2 + \frac{1}{2} \psi \dot{\psi}^* - \frac{1}{2} \dot{\psi} \psi^* \right]$$



$$S_{\text{susy}} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2} p^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{1}{2} \dot{\psi} \psi^* - p' F - \frac{1}{2} (\psi, \psi^*) \right]$$

$\begin{array}{c} \text{integrate} \\ \frac{1}{2} \psi \dot{\psi}^* \\ \downarrow \\ \frac{1}{2} \dot{\psi} \psi^* \end{array}$

Before:

$$S_{\text{old}} = \int dt \left[\frac{1}{2} p^2 - i \psi^* \dot{\psi} - \frac{1}{2} (\dot{f})^2 + \frac{1}{2} f'' (\psi^*, \psi) \right]$$

$$\Delta S_{\text{susy}} = \int dt \left[\frac{1}{2} F^2 - p' F \right]$$

$$S_{\text{susy}} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2} p^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{1}{2} i \dot{\psi} \psi^* - p' F - \frac{f''}{2} (\psi, \psi^*) \right]$$

$\begin{matrix} \text{integrate} \\ \downarrow \\ -\frac{1}{2} \psi \dot{\psi}^* \\ \downarrow \\ +\frac{1}{2} \dot{\psi} \psi^* \end{matrix}$

Before:

$$S_{\text{old}} = \int dt \left[\frac{1}{2} p^2 - i \psi^* \dot{\psi} - \frac{1}{2} (f')^2 + \frac{f''}{2} (\psi^*, \psi) \right]$$

$$S_{\text{susy}} = \int dt \left[\frac{1}{2} F^2 - p' F \right]$$

$$S_{\text{susy}} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2} \dot{p}^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{i}{2} \dot{\psi} \psi^* \right] = \int dt \left[\frac{1}{2} F^2 - \frac{1}{2} \dot{F}^2 \right] \left[\psi, \psi^* \right]$$

$\begin{matrix} \text{integrate} \\ \downarrow \\ -\frac{i}{2} \psi \dot{\psi}^* \\ \downarrow \\ +\frac{i}{2} \dot{\psi} \psi^* \end{matrix}$

Before:

$$S_{\text{old}} = \int dt \left[\frac{1}{2} \dot{p}^2 - i \psi^* \dot{\psi} - \frac{1}{2} (F')^2 + \frac{1}{2} F'' \right] \left[\psi^*, \psi \right]$$

$\begin{matrix} \text{circle} \\ \downarrow \\ -\frac{1}{2} (F')^2 \\ \downarrow \\ F(F) \end{matrix}$

$$\Delta S_{\text{susy}} = \int dt \left[\frac{1}{2} F^2 - F' F \right]$$

\Rightarrow we can integrate out F exactly

\Rightarrow let us look at the part of a path integral that involves F (F enters quadratically into Lagrangian)

$$\int [dF]$$

$$L(\Phi) \rightarrow L(\Phi, \partial\Phi)$$

\Rightarrow let us look at the part of a path integral that involves F .
 (F enters quadratically into Lagrangian)

$$\int [dF] e^{iS} = \int [dF] e^{i \int dt \left(\frac{1}{2} \dot{F}^2 - F'F' \right)}$$

$$L(\Phi) \rightarrow L(\Phi, 2\Phi) \text{ is No.}$$

⇒ let us look at the part of a path integral that involves S .

$\int \mathcal{D}F$ (F enters quadratically into Lagrangian)

$$\int \mathcal{D}F e^{iS} = \int \mathcal{D}F e^{i \int dt \left(\frac{1}{2} \dot{F}^2 - F \dot{f} \right)}$$

$$\int \mathcal{D}F e^{i \int dt \left[\frac{1}{2} (F - f)^2 \right]}$$

$$L(\Phi) \rightarrow L(\Phi, \partial\Phi)$$

⇒ let us look at the part of a path integral that involves S .

$\int \mathcal{D}F$ (F enters quadratically into Lagrangian)

$$\int \mathcal{D}F e^{iS} = \int \mathcal{D}F e^{i \int dt \left(\frac{1}{2} \dot{F}^2 - F \ddot{F} \right)}$$

$$\int \mathcal{D}F e^{i \int dt \left[\frac{1}{2} (\dot{F} - \ddot{F})^2 \right] + i \int dt \dots}$$

$$\mathcal{L}(\Phi) \rightarrow \mathcal{L}(\Phi, \dot{\Phi}) \quad \text{is No}$$

⇒ let us look at the part of a path integral that involves

F (F enters quadratically into Lagrangian)

$$\int [dF] e^{iS} = \int [dF] e^{i \int dt \left(\frac{1}{2} \dot{F}^2 - F \dot{F} \right)}$$

$$\int [dF] e^{i \int dt \left[\frac{1}{2} (F - F')^2 \right] + i \int dt \left(- \frac{(F')^2}{2} \right)}$$

$L(\Phi) \rightarrow L(\Phi, \dot{\Phi})$ is No

\Rightarrow let us look at the part of a path integral that involves F .
 (F enters quadratically into Lagrangian)

$$\int [dF] e^{i \int dt \left(\frac{1}{2} \dot{F}^2 - F \dot{f}' \right)} = \int [dF] e^{i \int dt \left[\frac{1}{2} (F - f')^2 \right] + i \int dt \left(- \frac{(f')^2}{2} \right)}$$

$$\int [dF] e^{i \int dt \left(- \frac{(f')^2}{2} \right)} = \int [dF] e^{i \int dt \left[\frac{1}{2} (F - f')^2 \right] + i \int dt \left(- \frac{(f')^2}{2} \right)}$$

$$\int [d\hat{F}] \quad (\mathbb{R}, \mathbb{Z}) \quad \text{is No}$$

⇒ let us look at the part of a path integral that involves

F (F enters quadratically into Lagrangian)

$$\int [dF] e^{i \int dt \left(\frac{1}{2} \dot{F}^2 - F \dot{F} \right)} = \int [dF] e^{i \int dt \left(\frac{1}{2} (\dot{F} - F')^2 \right) + i \int dt \left(- \frac{(\dot{F})^2}{2} \right)}$$

$\hat{F} \equiv \dot{F} - F'$

$$\int [d\hat{F}] \int [dF] e^{i \int dt \left(\frac{1}{2} \hat{F}^2 \right)} e^{i \int dt \left(- \frac{(\dot{F})^2}{2} \right)}$$

(\mathbb{R}, \mathbb{Z}) is No

⇒ let us look at the part of a path integral that involves

F (F enters quadratically into Lagrangian)

$$\int [dF] e^{i \int dt \left(\frac{1}{2} \dot{F}^2 - F f'(t) \right)} = \int [dF] e^{i \int dt \left[\frac{1}{2} (\dot{F} - f'(t))^2 \right] + i \int dt \left(- \frac{(\dot{F})^2}{2} \right)}$$

$$\Rightarrow \int [d\hat{F}] e^{i \int dt \frac{1}{2} \hat{F}^2} \Rightarrow \text{const} \cdot e^{i \int dt \left(- \frac{(\dot{F})^2}{2} \right)}$$

$$\int [d\theta] [d\psi] [d\psi^\dagger] [dF] \left\{ \frac{1}{2} D\Phi D^\dagger\Phi - F(\Phi) \right\}$$

Φ is a superfield
 $F(\Phi)$ is a function of Φ

$$\int [d\theta] [d\psi] [d\psi^\dagger] [dF] e^{i S_{\text{susy}}} \left(\frac{1}{2} D\Phi D^\dagger\Phi - F(\Phi) \right)$$

Φ is a superfield
 $F(\Phi)$ is a function of Φ

$$\int [d\psi] [d\psi^*] [dF] e^{i S_{susy}}$$

$$- \int [d\psi] [d\psi^*] \omega_{inst}$$

$$\int [d\psi] [d\psi^*] [dF] e^{i S_{susy}} \left(\frac{1}{2} D\psi D\psi^* - F(\psi) \right)$$

$$- \int [d\psi] [d\psi^*] \omega_{\text{inst}}$$

\uparrow
 is a functional integral
 $\int [dF] e^{i \int dt (F)^2}$

$$\int [d\psi] [d\psi^*] [dF] e^{i S_{\text{Susy}}} \frac{1}{2} D\psi D\psi^* - F(\psi)$$

$$- \int [d\psi] [d\psi^*] \omega_{\text{hst}} e^{i S_{\text{Higgs}}}$$

is a functional hite,

$$\int [dF] e^{i \int d^4x (F)^2}$$

Integrating out F . D^* (only exact if F is nondynamical
and enters into L quadratically)

* exact if F is infinitely massive.

$L(t, \mathbb{D}^* \Phi)$ → an arbitrary polynomial

$$S_{\text{eff}} = \int dt d^3x^* L(t, \mathbb{D}^* \Phi)$$

Integrating out F \star (only exact if F is nondynamical and enters into \mathcal{L} quadratically)

\rightarrow \star exact if F is infinitely massive. Solve EOM for F :

$$F - F' = \Phi \quad \mathbb{D}$$

(arbitrary polynomial)

$$S_{\text{eff}} = \int dt \int d^3x \mathcal{L}(x, D\phi, \overline{D}\psi)$$

Integrating out F D_x^+ (only exact if F is nondynamical and enters into L quadratically)

\rightarrow solve EOM for F : * exact if F is infinitely massive (doing saddle point approximation)

$$F - F' = \Phi$$

$$F = \Phi'$$

$$S_{eff} = \int dt d\theta dt^* L(\tau, D\tau, D\psi)$$

Integrating out F \star (only exact if F is nondynamical and enters into \mathcal{L} quadratically)

\rightarrow solve EOM for F : \star exact if F is infinitely massive (doing saddle point approximation, unambiguously path integral)

$$F - F' = \Phi$$

$$F = F'$$

\Rightarrow substitute back, $= \int dt d\theta dt^* \mathcal{L}(t, D\theta, D\psi)$

Integrating out F \times (only exact if F is nondynamical and enters into \mathcal{L} quadratically)

\rightarrow solve EOM for F : (doing saddle point approximation, unphysical path integral)

$$F - F' = \Phi$$

$$F = f'$$

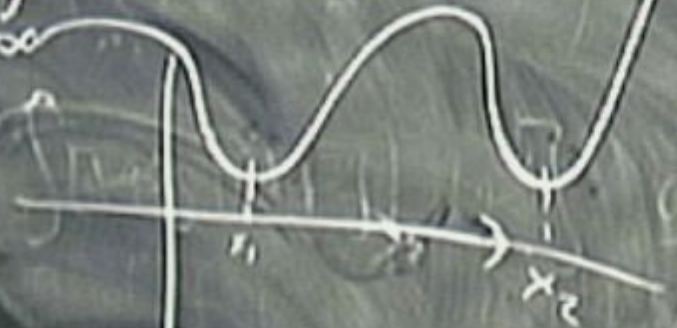
\Rightarrow substitute back: $\left(\frac{i}{2} F^2 + i f' F \right) = \left(\frac{i}{2} (f')^2 + \dots \right)$

$$\int_{-\infty}^{+\infty} dx e^{-\beta F(x)} = \int_{-\infty}^{+\infty} dx e^{-\beta [U(x) - TS(x)]}$$



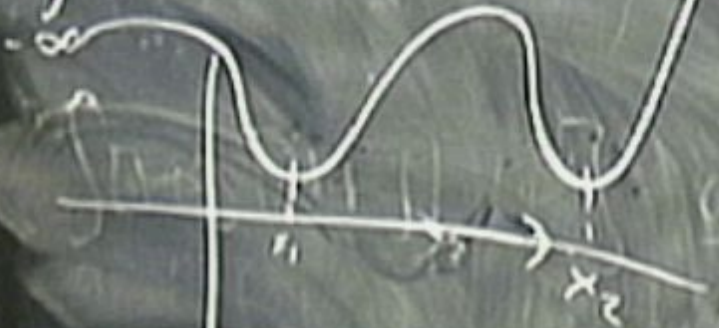
$$R(x) = R(x_1) - \lambda(x-x_1)^2$$

$$\int_{-\infty}^{+\infty} dx e^{-f(x)} \quad \int_{-\infty}^{+\infty} dx e^{-f(x_1) - \lambda(x-x_1)^2} + \int_{-\infty}^{+\infty} dx e^{-f(x_2) - \lambda(x-x_2)^2}$$



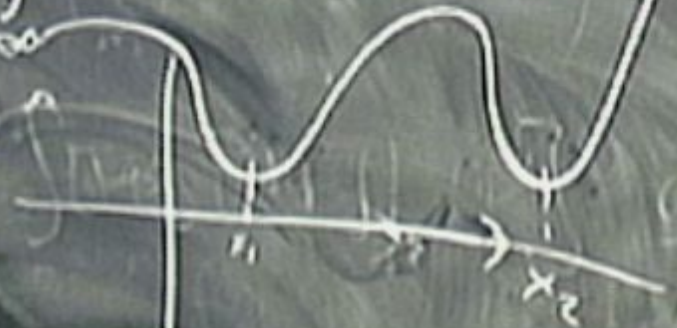
$$R(x) = f(x_1) - \lambda(x-x_1)^2$$

$$\int_{-\infty}^{+\infty} dx e^{-\lambda(x-x_1)^2} + \int_{-\infty}^{+\infty} dx e^{-\lambda(x-x_2)^2}$$



$$f(x) = f(x_1) - \lambda(x-x_1)^2$$

$$\int_{-\infty}^{+\infty} dx e^{-f(x)} = \int_{-\infty}^{x_1} dx e^{-f(x)} + \int_{x_1}^{x_2} dx e^{-f(x)} + \int_{x_2}^{+\infty} dx e^{-f(x)}$$



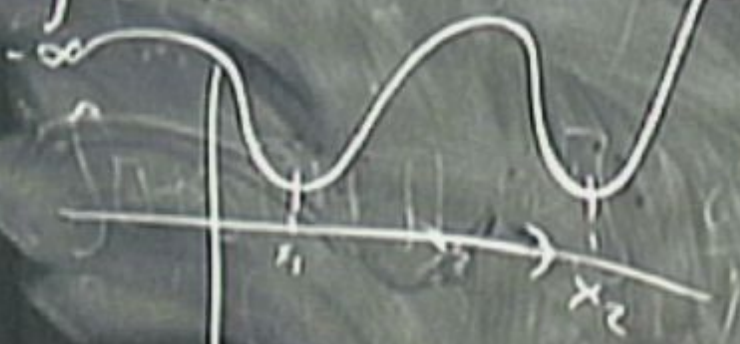
$$= \int_{-\infty}^{x_1} dx e^{-f(x_1) - \lambda(x-x_1)^2} + \int_{x_1}^{x_2} dx e^{-f(x)} + \int_{x_2}^{+\infty} dx e^{-f(x_2) - \lambda(x-x_2)^2}$$

$$= \int_{-\infty}^{x_1} dx e^{-f(x_1)} e^{-\lambda(x-x_1)^2} + \int_{x_1}^{x_2} dx e^{-f(x)} + \int_{x_2}^{+\infty} dx e^{-f(x_2)} e^{-\lambda(x-x_2)^2}$$

$$= e^{-f(x_1)} \int_{-\infty}^{x_1} dx e^{-\lambda(x-x_1)^2} + \int_{x_1}^{x_2} dx e^{-f(x)} + e^{-f(x_2)} \int_{x_2}^{+\infty} dx e^{-\lambda(x-x_2)^2}$$

$$R(x) = f(x_1) - \lambda(x-x_1)^2$$

$$\int_{-\infty}^{\infty} dx e^{-f(x)} = \int_{-\infty}^{x_1} dx e^{-f(x)} + \int_{x_1}^{x_2} dx e^{-f(x)} + \int_{x_2}^{\infty} dx e^{-f(x)}$$



$$\frac{1}{\sqrt{\lambda_1}} e^{-\frac{f(x_1)}{\lambda_1} - \lambda_1(x-x_1)^2}$$

$$\int dy e^{-y^2}$$

$$\frac{1}{\sqrt{\lambda_2}} e^{-\frac{f(x_2)}{\lambda_2} - \lambda_2(x-x_2)^2}$$

$$R(x) = f(x_1) - \lambda_1(x-x_1)^2$$

Integrating out F \times (only exact if F is nondynamical and enters into \mathcal{L} quadratically)

\times exact if F is infinitely massive

\Rightarrow solve EOM for F : (doing saddle point approximation, unphysical path integral)

$$F - f' = 0$$

$$F = f'$$

\Rightarrow substitute back $= \left(\frac{i}{2} (F^2 + f'' F) = \left(+ \frac{1}{2} (f')^2 \right) \right)$

A superfield \mathbb{F} provides a representation of susy algebra

[The following text is heavily obscured by dark, repetitive scribbles and is largely illegible.]

A superfield \mathbb{F} provides a representation of SUSY algebra
 (in general) \rightarrow is reducible

[The following text is heavily obscured by white scribbles and is largely illegible. Some faint words like "is reducible" and "SUSY algebra" are visible through the noise.]

a superfield \mathbb{F} provides a representation of susy algebra
↑ in general
is reducible

→ chiral superfields

A superfield \mathbb{F} provides a representation of susy algebra
↑ in general
is reducible

→ chiral superfields

def X is chiral if $D^+ X = 0$

\mathbb{R} superfield provides a representation of \mathbb{Z}_2 algebra
 (in general) \rightarrow is reducible

\rightarrow chiral superfields

def X is \mathbb{Z}_2 if $D^+ X = 0$

$$D^+ = \frac{\partial}{\partial \theta^+} - i \theta^+ \frac{\partial}{\partial t}$$

"chiral coordinates"

$$\left\{ \theta, \bar{\theta} = t - i\theta\bar{\theta} \right\}$$

$$D^+ \theta = 0, \quad D^+ \bar{\theta} = 0$$

a vst is an arbitrary function of chiral coordinates

$$X = X(\theta, \tau)$$

a vst is an arbitrary function of chiral coordinates

$$X = X(\theta, \tau)$$

$$D^+ X(\theta, \tau) = \frac{\partial X(\alpha, \beta)}{\partial \alpha} D^+ \alpha + \frac{\partial X(\alpha, \beta)}{\partial \beta} D^+ \beta$$

a vst is an arbitrary function of chiral coordinates

$$X = X(\theta, \tau)$$

← not the most general

$$\Phi(\theta, \theta^*, \tau)$$

$$D^+ X(\theta, \tau) = \frac{\partial X(\alpha, \beta)}{\partial \alpha} D^+ \alpha + \frac{\partial X(\alpha, \beta)}{\partial \beta} D^+ \beta$$

a vst is an arbitrary function of chiral coordinates

$$X = X(\theta, \tau)$$

← not the most general

$$\Phi(\theta, \theta^*, t)$$

$$D^+ X(\theta, \tau) = \frac{\partial X(\alpha, \beta)}{\partial \alpha} D^+ \alpha + \frac{\partial X(\alpha, \beta)}{\partial \beta} D^+ \beta$$

α vsf is an arbitrary function of chiral coordinates

$$X = X(\theta, \tau)$$

← not the most general

$$\Phi(\theta, \theta^*, t)$$

$$D^+ X(\theta, \tau) = \frac{\partial X(\alpha, \beta)}{\partial \alpha} D^+ \alpha + \frac{\partial X(\alpha, \beta)}{\partial \beta} D^+ \beta$$

$$X(\theta, \tau) = \varphi(\tau) + \theta \psi(\tau)$$

a vst is an arbitrary function of chiral coordinates

$$X = X(\theta, \tau) \quad \leftarrow \text{not the most general}$$

$$\Phi(\theta, \theta^*, t)$$

$$D^+ X(\theta, \tau) = \frac{\partial X(\alpha, \beta)}{\partial \alpha} D^+ \alpha + \frac{\partial X(\alpha, \beta)}{\partial \beta} D^+ \beta$$

$$X(\theta, \tau) = \varphi(\tau) + \theta \psi(\tau) = \varphi(t) + \theta \psi(t) - i\theta\theta^* \dot{\varphi}(t)$$

\uparrow
 $\tau = t - i\theta\theta^*$

$\int \mathbb{P}^1 \rightarrow \text{get rest}$

$D^+ \Phi$
 automatically chiral

Bo-Pur: $D^+ [D^+ \Phi] = (D^+)^2 \Phi = 0 \Phi \rightarrow 0$

$\int \mathcal{L} \rightarrow \text{get rest}$

$D^+ \Phi$
 automatically chiral

Be Careful: $D^+ [D^+ \Phi] = (D^+)^2 \Phi = 0 \Phi = 0$

$\int_x = \int dt \int d\theta$
 1/2 of all
 Grassmann

$\int \mathbb{P} \rightarrow \text{get rest}$

$D^+ \Phi$
 automatically chiral

Be Careful: $D^+ [D^+ \Phi] = (D^+)^2 \Phi = 0 \Phi = 0$

$\int_X = \int dt \int d\theta$

1/2 of all
 Grassmann

$X^2 = \rho^2$

$\int \mathcal{L}(\Phi) \rightarrow \text{get } \chi \text{ of } \underbrace{D^+ \Phi}_{\text{automatically chiral}}$

Euler-Lagrange: $D^+ [D^+ \Phi] = (D^+)^2 \Phi = 0 \Phi = 0$

$\int_{\mathcal{X}} - \int dt \underbrace{\frac{d\theta}{dt}}_{\substack{\text{1 of all} \\ \text{Casimir}}} X \Rightarrow \delta S_{\text{susy } \mathcal{X}} = \int dt [\text{total derivative}]$

F^2
 F

$$\int d^4x \mathcal{L}^*$$

SUSY invariant

is reducible

[Faint handwritten notes and diagrams, including various mathematical symbols and arrows, are visible in the background.]

$$\int d^4x d^4\theta d^4\theta^* \mathcal{L} = \int dt d^3x \frac{2}{\theta^*} \mathcal{L}$$

in general
is reducible

SUSY invariant

$$\underbrace{\int dt d\theta d\theta^* \mathcal{L}}_{\text{SUSY invariant}} = \int dt d\theta \frac{\partial}{\partial \theta^*} \mathcal{L} = \int dt d\theta \left[\underbrace{\mathcal{D}^+ + i\theta \frac{\partial}{\partial t}}_{\equiv \frac{\partial}{\partial \theta^*}} \right]$$

$$\underbrace{\int dt d\theta d\theta^* \mathcal{L}}_{\text{SUSY invariant}} = \int dt d\theta \frac{\partial}{\partial \theta^*} \mathcal{L} = \int dt d\theta \left[\underbrace{D^{\dagger} + i\theta \frac{\partial}{\partial t}}_{\frac{\partial}{\partial \theta^*}} \right] \mathcal{L}$$

$$D^{\dagger} \theta = 0, \quad \theta D = 0$$

$$\int dt d\theta d\theta^* \mathcal{L} \stackrel{\text{in general}}{=} \int dt d\theta \frac{\partial}{\partial \theta^*} \mathcal{L}$$

SUSY invariant

$$= \int dt d\theta \left[\underbrace{D^+ + i\theta \frac{\partial}{\partial t}}_{\equiv \frac{\partial}{\partial \theta^*}} \right] \mathcal{L}$$

$$= \int dt d\theta D^+ \mathcal{L}$$

$$D^+ \theta = 0, \quad \theta \cdot D = 0$$

$$\int dt d\theta d\theta^* \mathcal{L} \stackrel{\text{in general}}{=} \int dt d\theta \frac{\partial}{\partial \theta^*} \mathcal{L}$$

SUSY invariant

$$= \int dt d\theta \left[\underbrace{D^+ + i\theta \frac{\partial}{\partial t}}_{\equiv \frac{\partial}{\partial \theta^*}} \right] \mathcal{L}$$

$$= \int dt d\theta \underbrace{D^+}_{\equiv \frac{\partial}{\partial \theta^*}} \mathcal{L}$$

$$D^+ \theta = 0, \quad \theta \theta = 0$$

$$\int d^4x d\theta d\theta^* \mathcal{L} \stackrel{\text{in general}}{=} \int dt d\theta \frac{\partial}{\partial \theta^*} \mathcal{L}$$

SUSY invariant

$$= \int dt d\theta \left[\cancel{D^+} + i\theta \frac{\partial}{\partial t} \right] \mathcal{L}$$

$$= \int dt d\theta \underbrace{D^+}_{\text{SUSY}} \mathcal{L} = \int dt d\theta X \frac{\partial}{\partial \theta^*}$$

$$D^+ \theta = 0, \quad D^+ \theta^* = 0$$