Title: Intro to Supersymmetry 7

Date: Oct 09, 2007 12:30 PM

URL: http://pirsa.org/07100017

Abstract:

$$(t, \theta, \theta^*) \longrightarrow (t + i\varepsilon\theta^* + i\varepsilon^*\theta,\ \theta + \varepsilon,\ \theta^* + \varepsilon^*).$$

superspace

supersymmetry transformation as translations on superspace

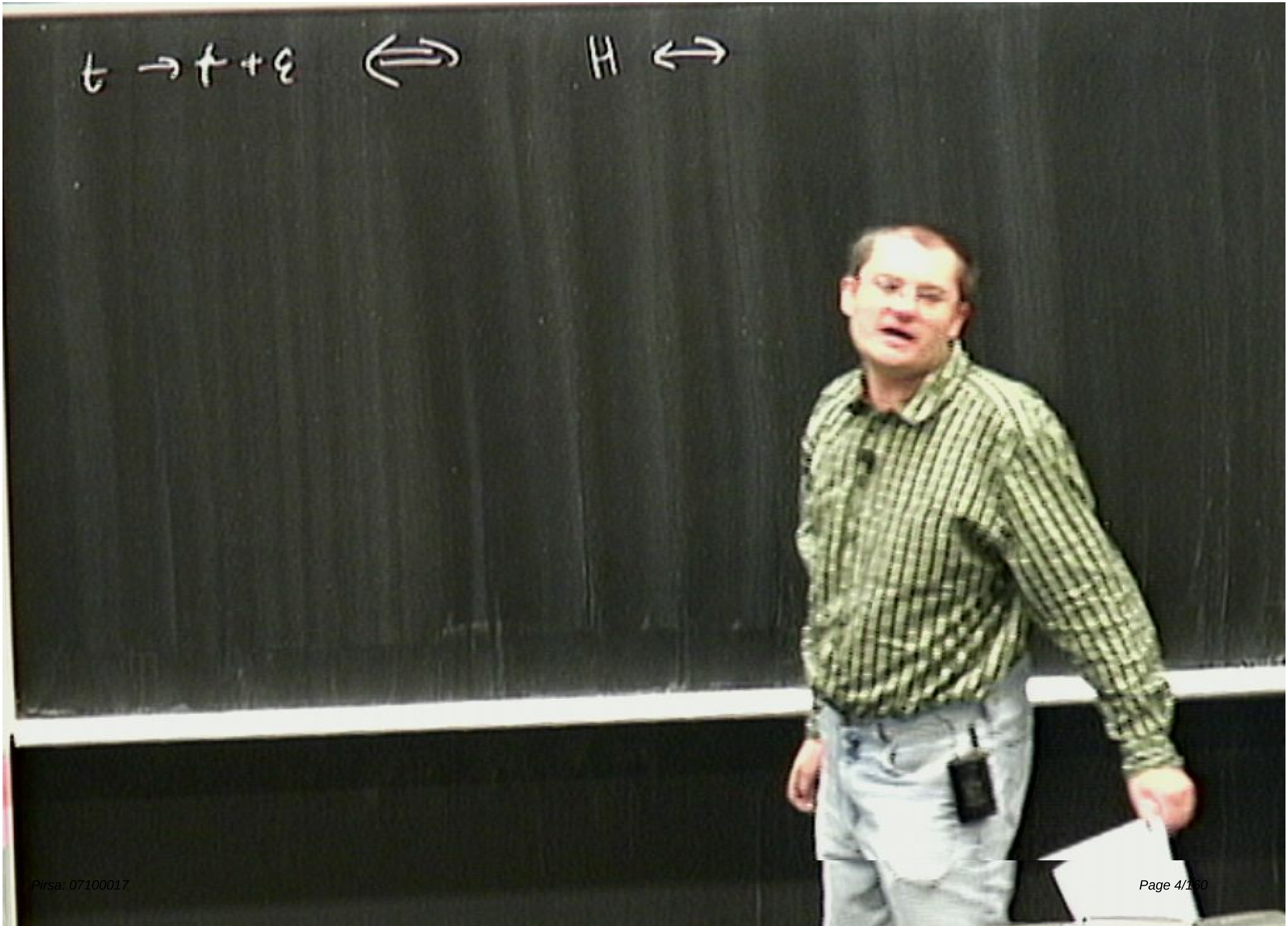
$$(t, \theta, \theta^*) \longrightarrow (t + i\varepsilon\theta^* + i\varepsilon^*\theta,\ \theta + \varepsilon,\ \theta^* + \varepsilon^*)$$

superspace

supersymmetry transformation as translations on superspace

$$\Phi(t, \theta, \theta^*) = \phi(t) + \theta\psi(t) - \theta^*\psi^*(t) + \theta\theta^* F(t)$$

superfield


t → t + ε ⟺ H ↔

$$t \to t + \varepsilon \iff H \leftrightarrow i\,\partial_t$$

$$Q = \frac{\partial}{\partial\theta} + i\theta^* \frac{\partial}{\partial t}$$

$$Q^\dagger = \frac{\partial}{\partial\theta^*} + i\theta \frac{\partial}{\partial t}$$

$$t \to t + \varepsilon \quad \Longleftrightarrow \quad H \Longleftrightarrow i\partial_t$$

$$\delta \psi(t) = \varepsilon\, i\, \partial_t \psi$$

$$Q = \frac{\partial}{\partial \theta} + i\theta^* \frac{\partial}{\partial t}$$

$$Q^\dagger = \frac{\partial}{\partial \theta^*} + i\theta \frac{\partial}{\partial t}$$

$$t \to t + \varepsilon \quad \Longleftrightarrow \quad H \longleftrightarrow i\partial_t$$

$$\delta \psi(t) = \varepsilon i \partial_t \psi(t)$$

$$\delta_{susy} \Phi = (\varepsilon Q + \varepsilon^* Q^*) \Phi$$

$$Q = \frac{\partial}{\partial \theta} + i\theta^* \frac{\partial}{\partial t}$$

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$$t \to t + \varepsilon \iff H \leftrightarrow i\partial_t$$

$$\delta \psi(t) = \varepsilon i \partial_t \psi(t)$$

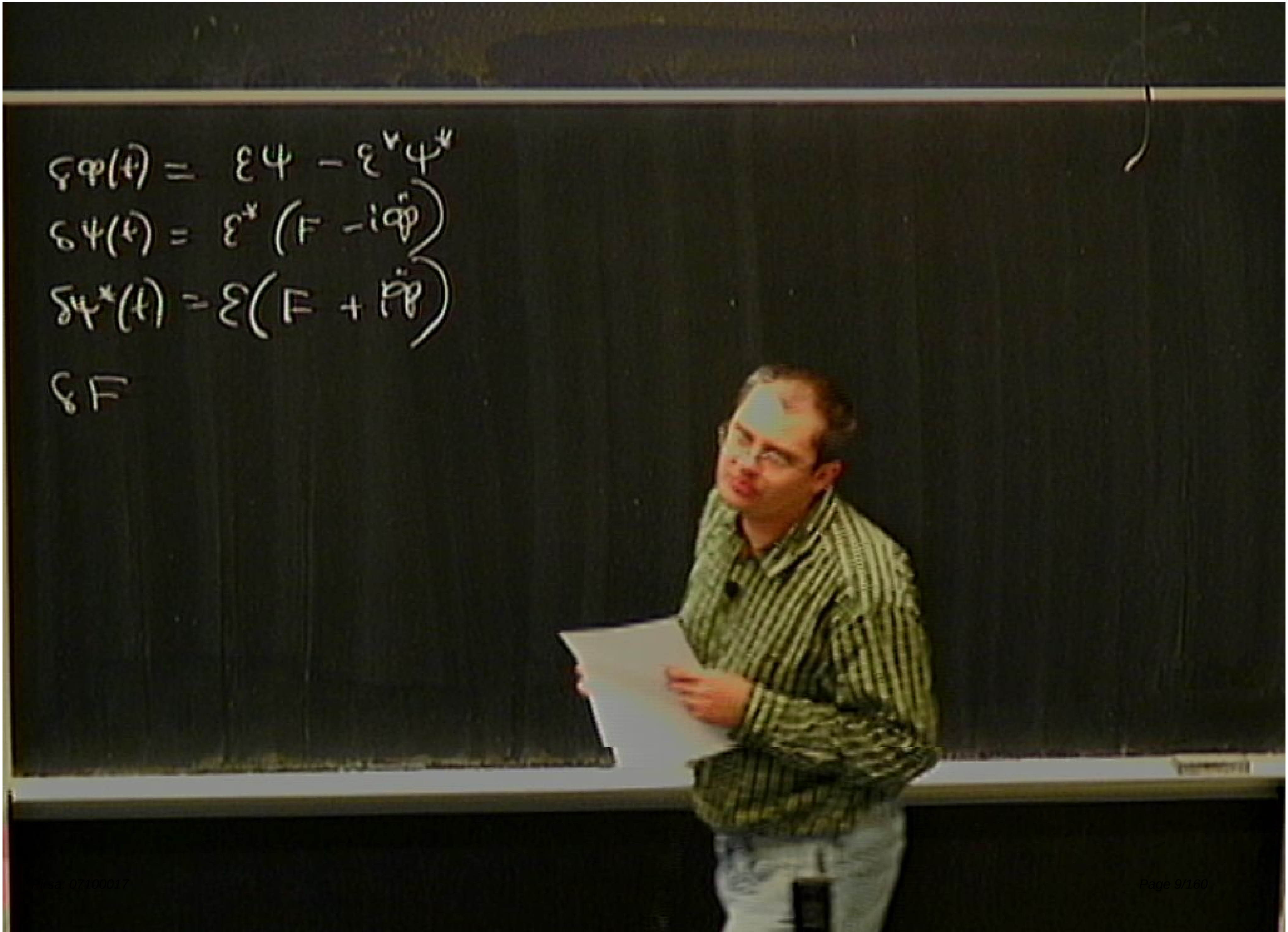
$$Q = \frac{\partial}{\partial \theta} + i\theta^* \frac{\partial}{\partial t}$$

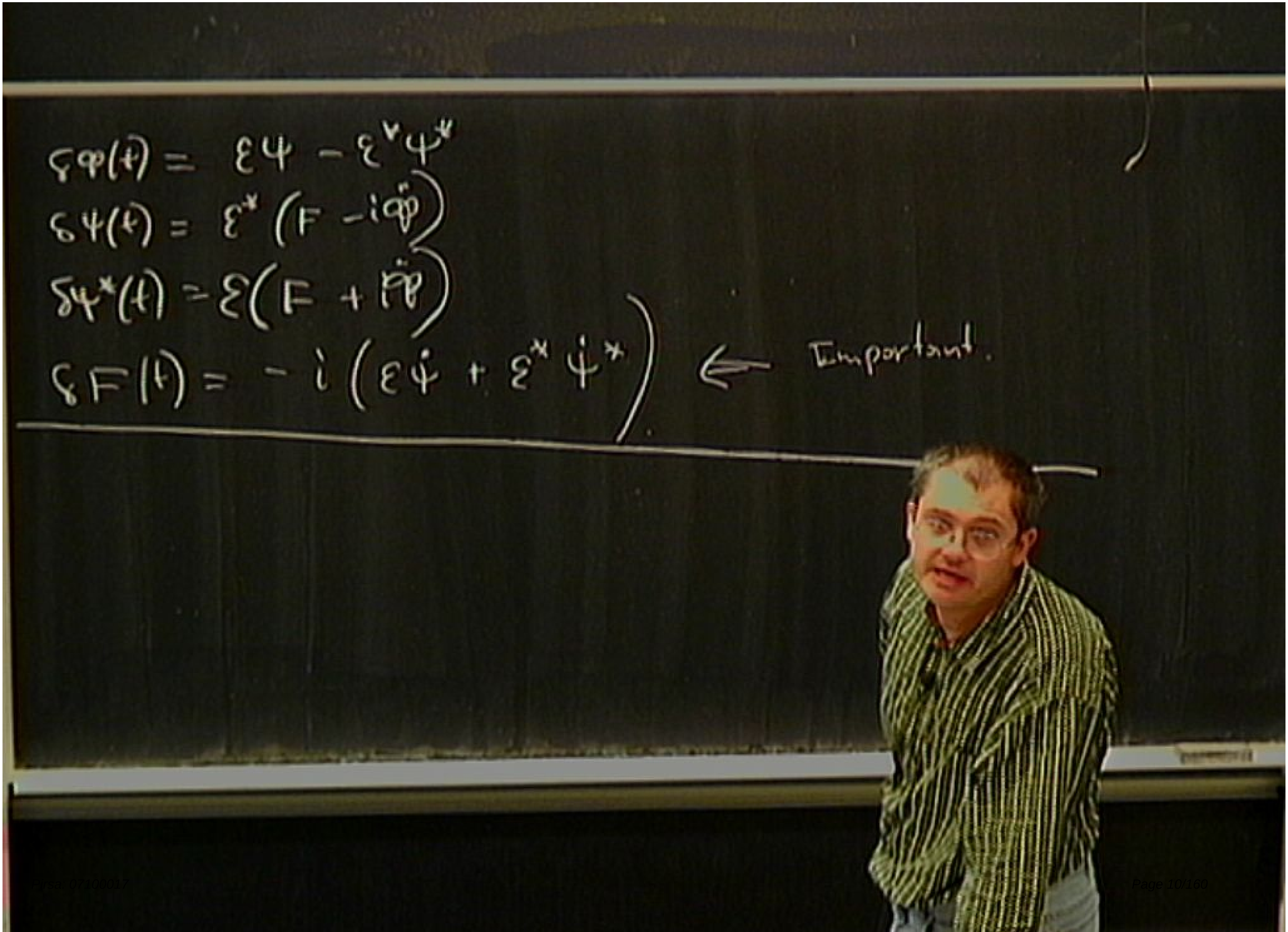
$$Q^+ = \frac{\partial}{\partial \theta^*} + i\theta \frac{\partial}{\partial t}$$

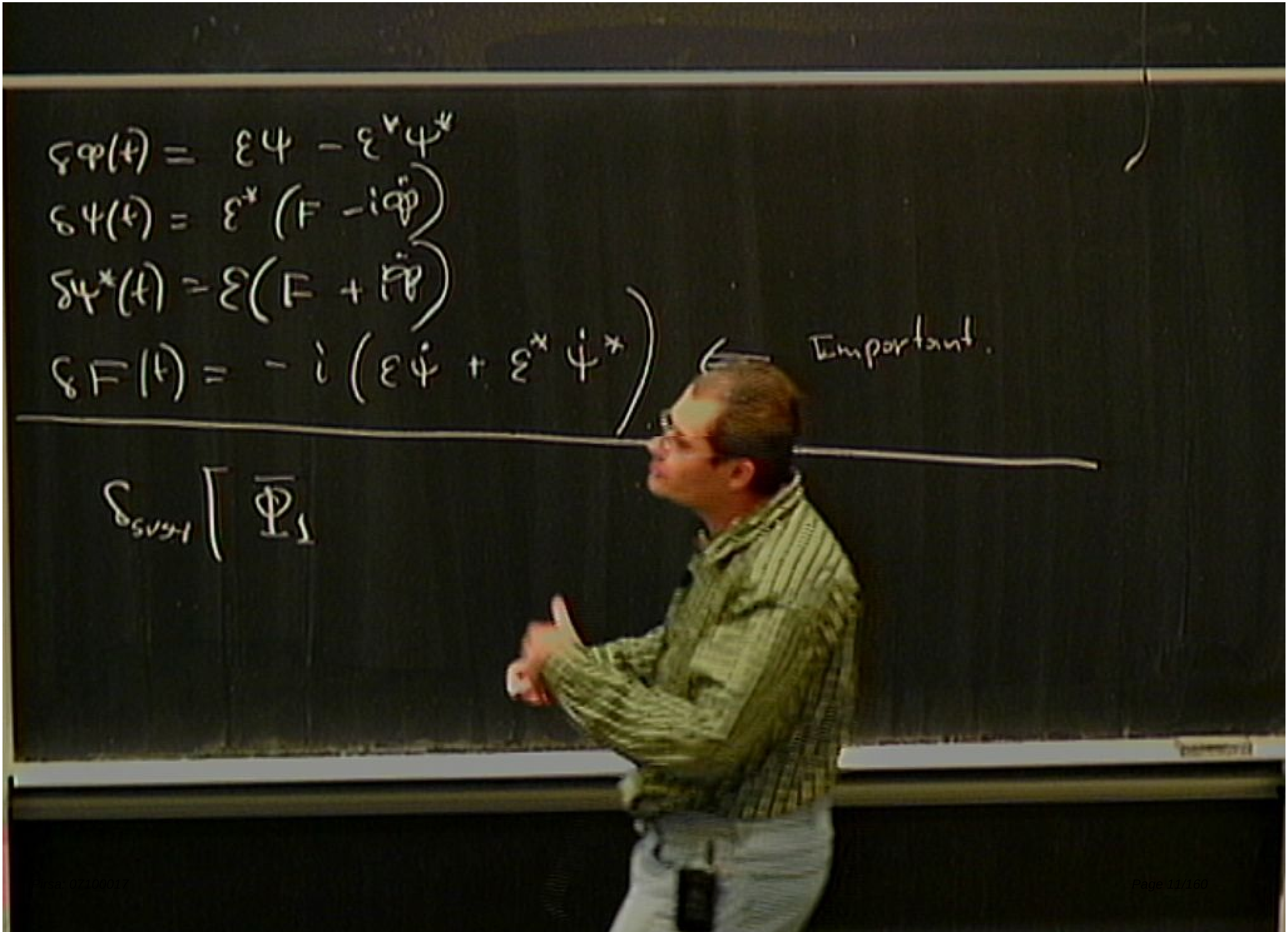
$$H = i\partial_t$$

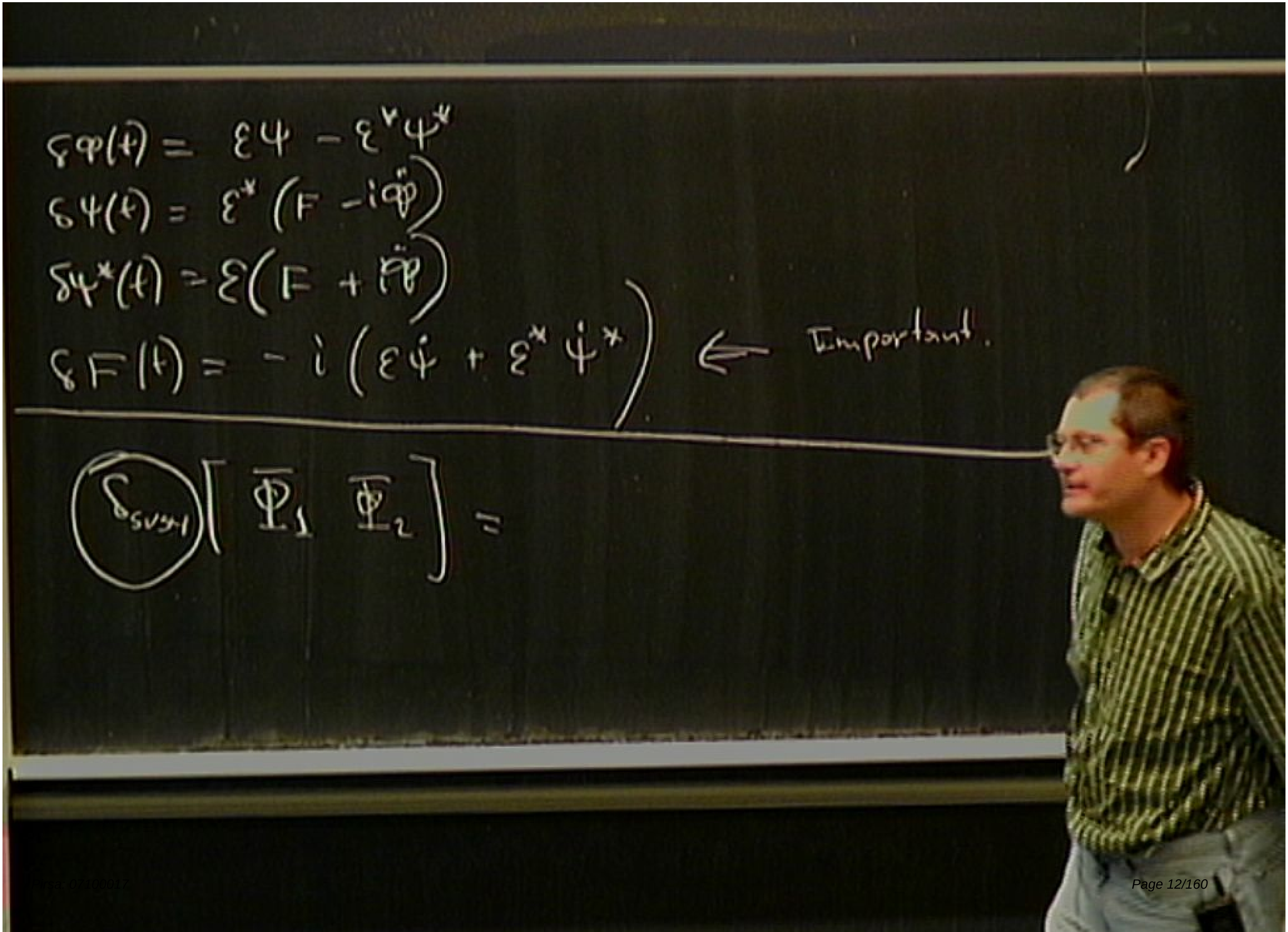
$$\{Q, Q\} = \{Q^+, Q^+\} = 0 \; ; \; \{Q, Q^+\} = 2H$$

$$\delta_{\theta,\varepsilon^*} \Phi = (\varepsilon Q + \varepsilon^* Q^*) \Phi$$


$\delta\varphi(t) = \varepsilon\psi - \varepsilon^*\psi^*$
$\delta\psi(t) = \varepsilon^*(F - i\dot{\varphi})$
$\delta\psi^*(t) = \varepsilon(F + i\dot{\varphi})$
δF


$\delta\varphi(t) = \varepsilon\psi - \varepsilon^*\psi^*$
$\delta\psi(t) = \varepsilon^*(F - i\dot{\varphi})$
$\delta\psi^*(t) = \varepsilon(F + i\dot{\varphi})$
$\delta F(t) = -i(\varepsilon\dot{\psi} + \varepsilon^*\dot{\psi}^*)$ ⟸ Important.


δφ(t) = εψ − ε*ψ*
δψ(t) = ε*(F − iφ̇)
δψ*(t) = ε(F + iφ̇)
δF(t) = − i (εψ̇ + ε*ψ̇*)
← Important.
S_susy [Φ_1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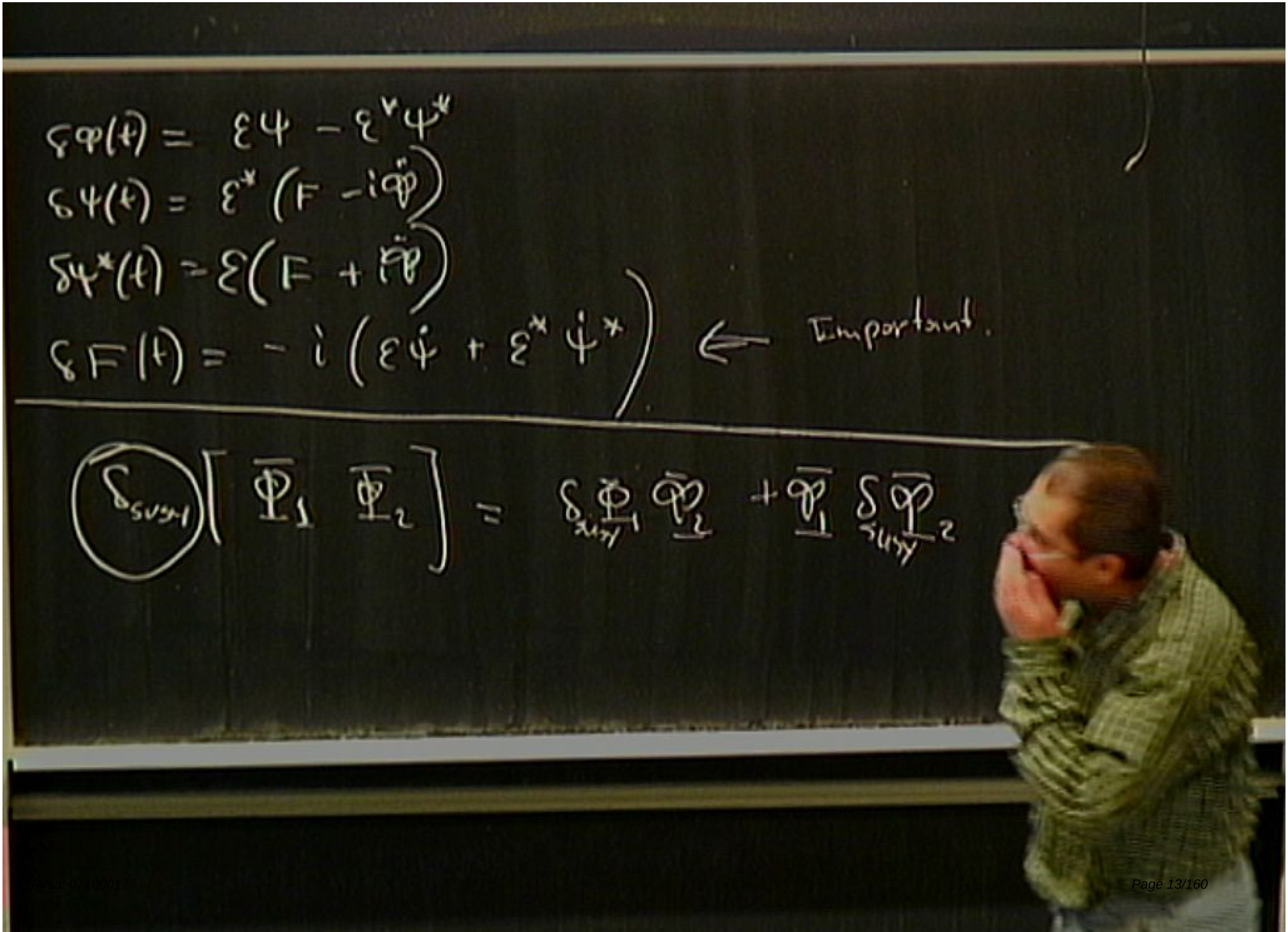
$\delta\psi^*(t) = \varepsilon(F + i\dot{\varphi})$
$\delta F(t) = -i(\varepsilon\dot{\psi} + \varepsilon^*\dot{\psi}^*)$ $\Leftarrow$ Important.
$\delta_{SUSY}[\Phi_1 \; \Phi_2] =$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$\delta\psi^*(t) = \varepsilon(F + i\dot{\varphi})$
$\delta F(t) = -i(\varepsilon\dot{\psi} + \varepsilon^*\dot{\psi}^*)$ ⟸ Important.
$\delta_{susy}[\Phi_1 \Phi_2] = \delta_{susy}\Phi_1 \Phi_2 + \Phi_1 \delta_{susy}\Phi_2$

$$t \to t + \varepsilon \iff H \leftrightarrow i\,\partial_t$$

$$\delta\,\psi(t) = \varepsilon\, i\,\partial_t\,\psi$$

$$Q = \frac{\partial}{\partial\theta} + i\,\theta^* \frac{\partial}{\partial t}$$

$$Q^\dagger = \frac{\partial}{\partial\theta^*} + i\,\theta\,\frac{\partial}{\partial t}$$

$$H = i\,\partial_t$$

$$\{Q,Q\} = \{Q^\dagger,Q^\dagger\} = 0\;;\quad \{Q,Q^\dagger\} = 2H$$

$$\delta_{susy}\Phi = (\varepsilon Q + \varepsilon^* Q^*)\,\Phi$$

$$\delta\varphi(t) = \varepsilon\psi - \varepsilon^*\psi^*$$

$$\delta\psi(t) = \varepsilon^*(F - i\dot{\varphi})$$

$$\delta\psi^*(t) = \varepsilon(F + i\dot{\varphi})$$

$$\delta F(t) = -i(\varepsilon\dot{\psi} + \varepsilon^*\dot{\psi}^*) \quad \Leftarrow \text{Important.}$$

$$\delta_{susy}[\bar{\Phi}_1 \bar{\Phi}_2] = \delta_{susy}\bar{\Phi}_1 \bar{\Phi}_2 + \bar{\Phi}_1 \delta_{susy}\bar{\Phi}_2$$

$\Rightarrow$ If $\bar{\Phi}_1$ & $\bar{\Phi}_2$

$$\delta\varphi(t) = \varepsilon\psi - \varepsilon^*\psi^*$$

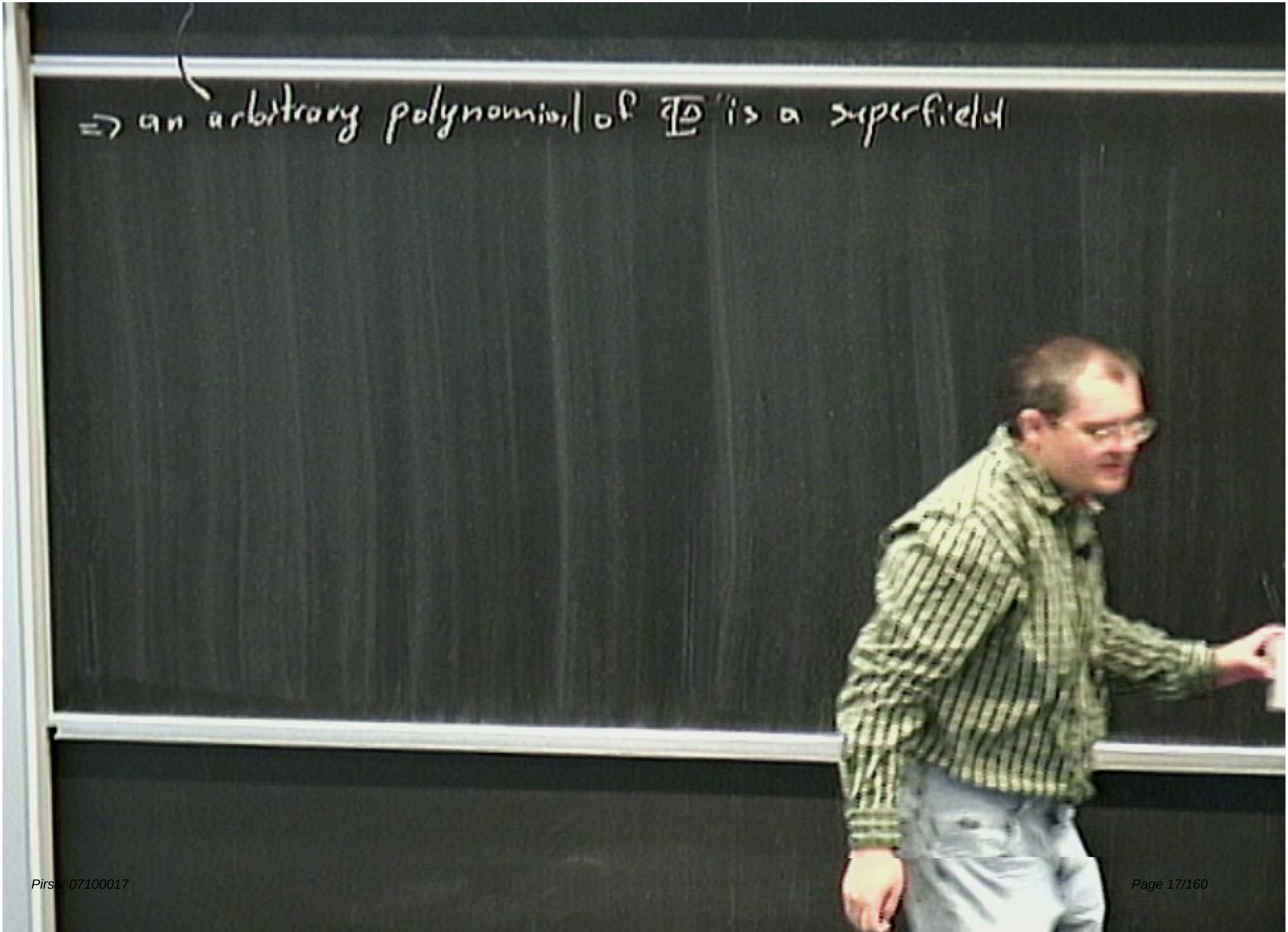
$$\delta\psi(t) = \varepsilon^*(F - i\dot{\varphi})$$

$$\delta\psi^*(t) = \varepsilon(F + i\dot{\varphi})$$

$$\delta F(t) = -i(\varepsilon\dot{\psi} + \varepsilon^*\dot{\psi}^*)$$ ⟸ Important.

$$\delta_{susy}[\Phi_1 \Phi_2] = \delta_{susy}\Phi_1 \Phi_2 + \Phi_1 \delta_{susy}\Phi_2$$

⇒ If Φ_1 & Φ_2 ⇒ $\Phi_1\Phi_2$ is also a superfield

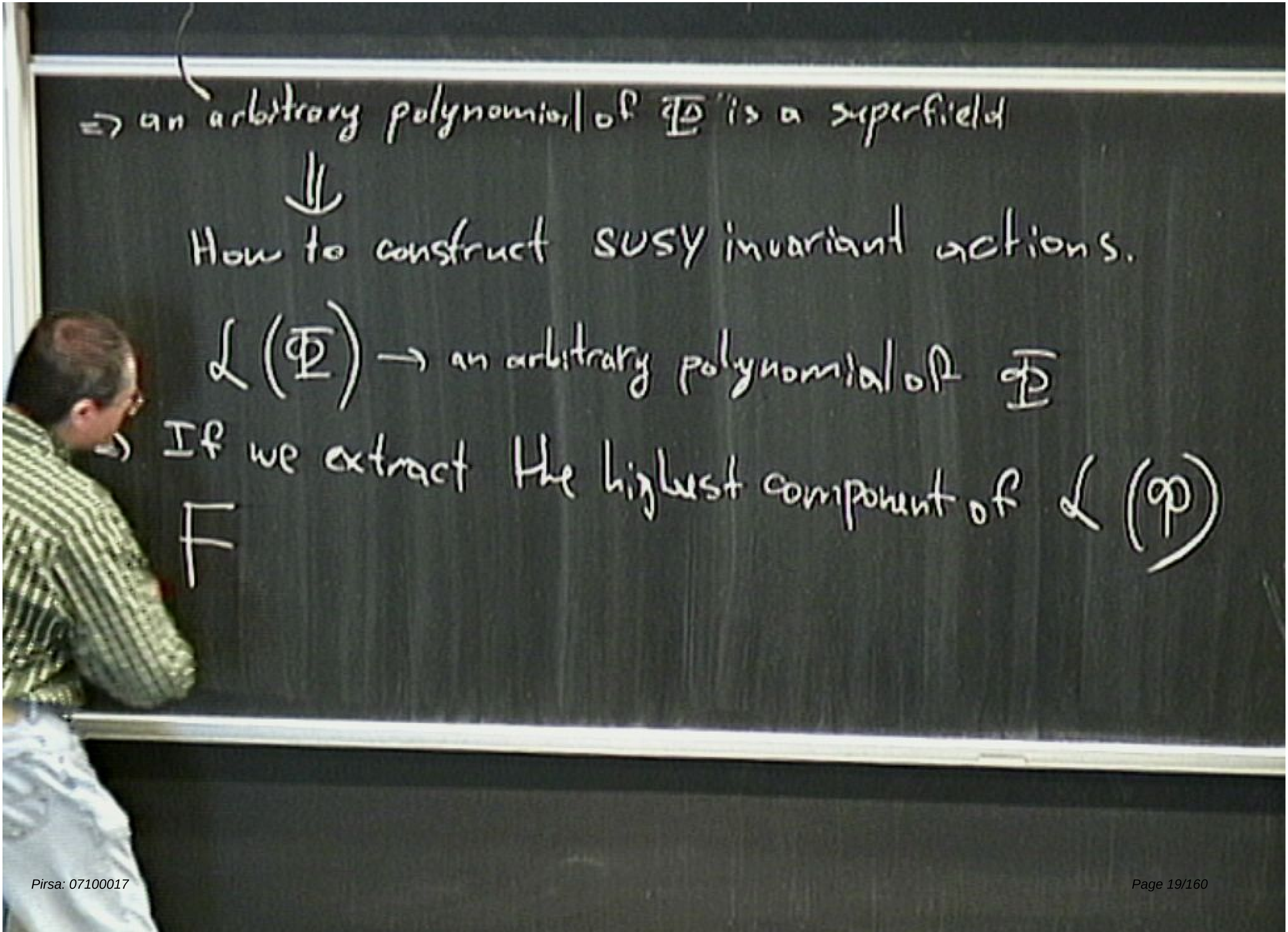

⇒ an arbitrary polynomial of Φ is a superfield

$\Rightarrow$ an arbitrary polynomial of Φ is a superfield

$$\Downarrow$$

How to construct SUSY invariant actions.

$\mathcal{L}(\Phi) \rightarrow$ an arbitrary polynomial of Φ



⇒ an arbitrary polynomial of Φ is a superfield
⇓
How to construct SUSY invariant actions.
L(Φ) → an arbitrary polynomial of Φ
⇒ If we extract the highest component of L(Φ)
F

$\Rightarrow$ an arbitrary polynomial of Φ is a superfield

$\Downarrow$

How to construct SUSY invariant actions.

$\mathcal{L}(\Phi) \rightarrow$ an arbitrary polynomial of Φ

$\Rightarrow$ If we extract the highest component of $\mathcal{L}(\Phi)$

F

$\Rightarrow$ an arbitrary polynomial of Φ is a superfield

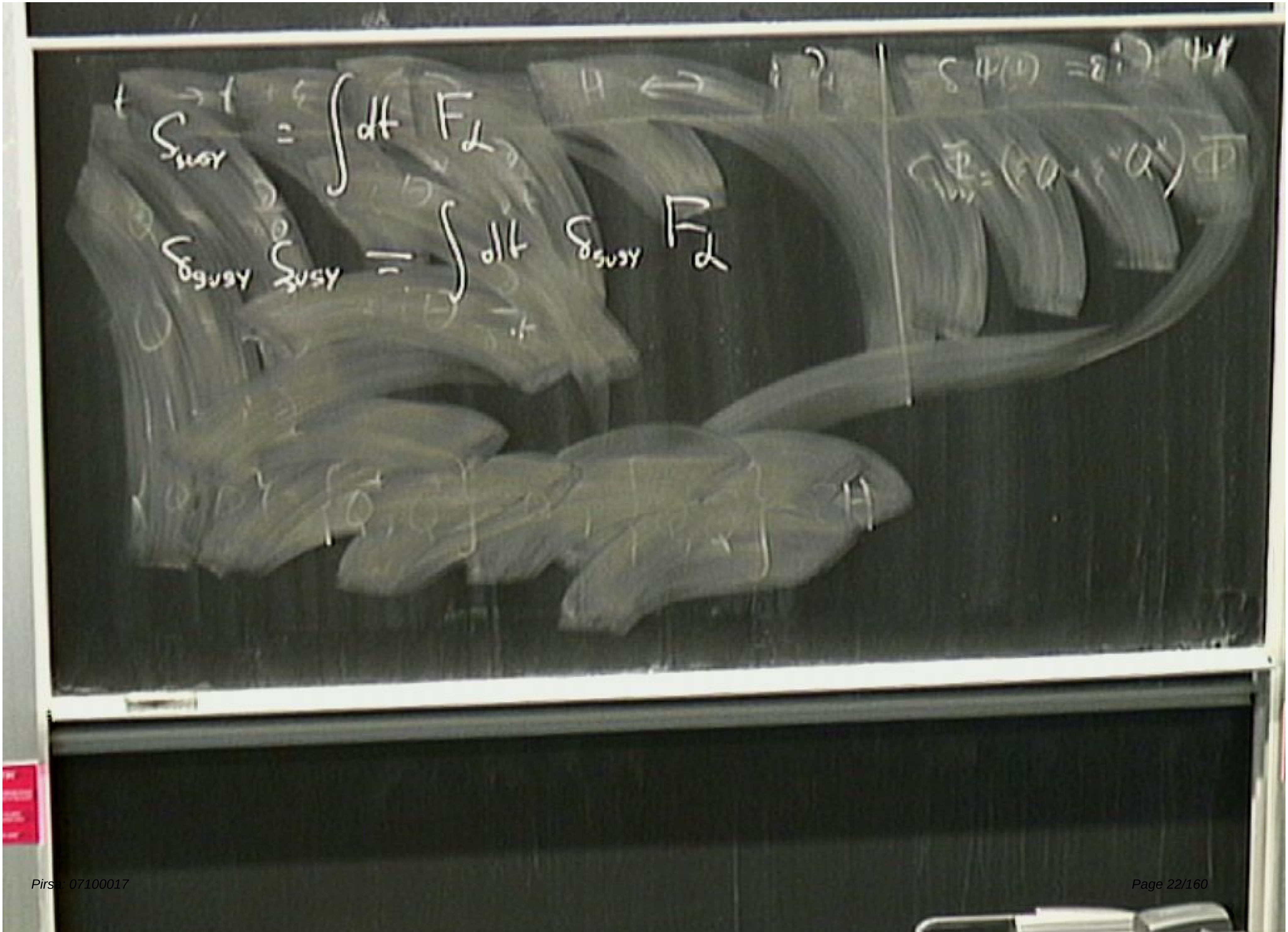
$$\Downarrow$$

How to construct SUSY invariant actions.

$\mathcal{L}(\Phi) \longrightarrow$ an arbitrary polynomial of Φ

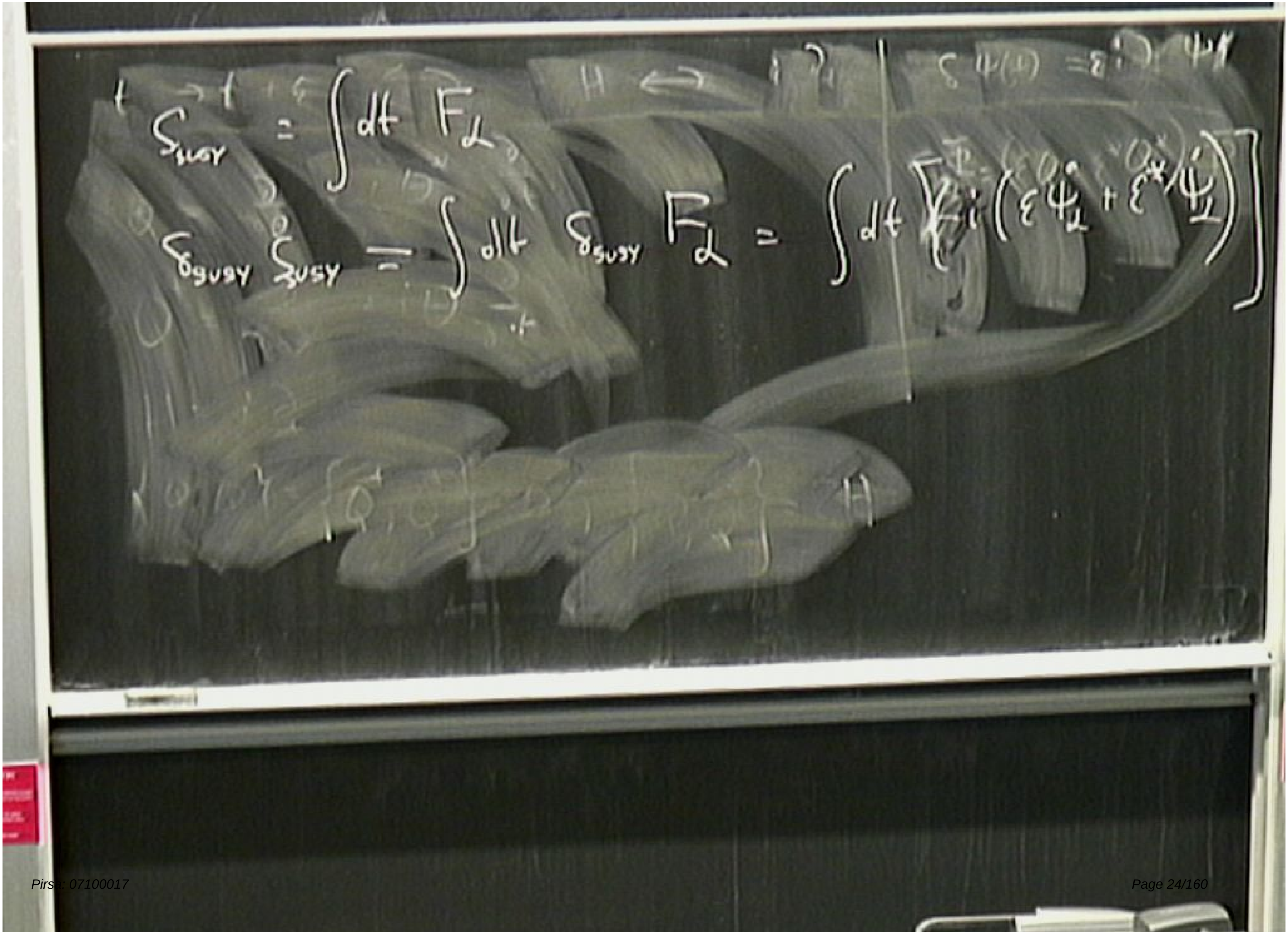
$\Rightarrow$ If we extract the highest component of $\mathcal{L}(\Phi)$

$F_{\mathcal{L}} \longrightarrow$ will transfor as a total derivative.

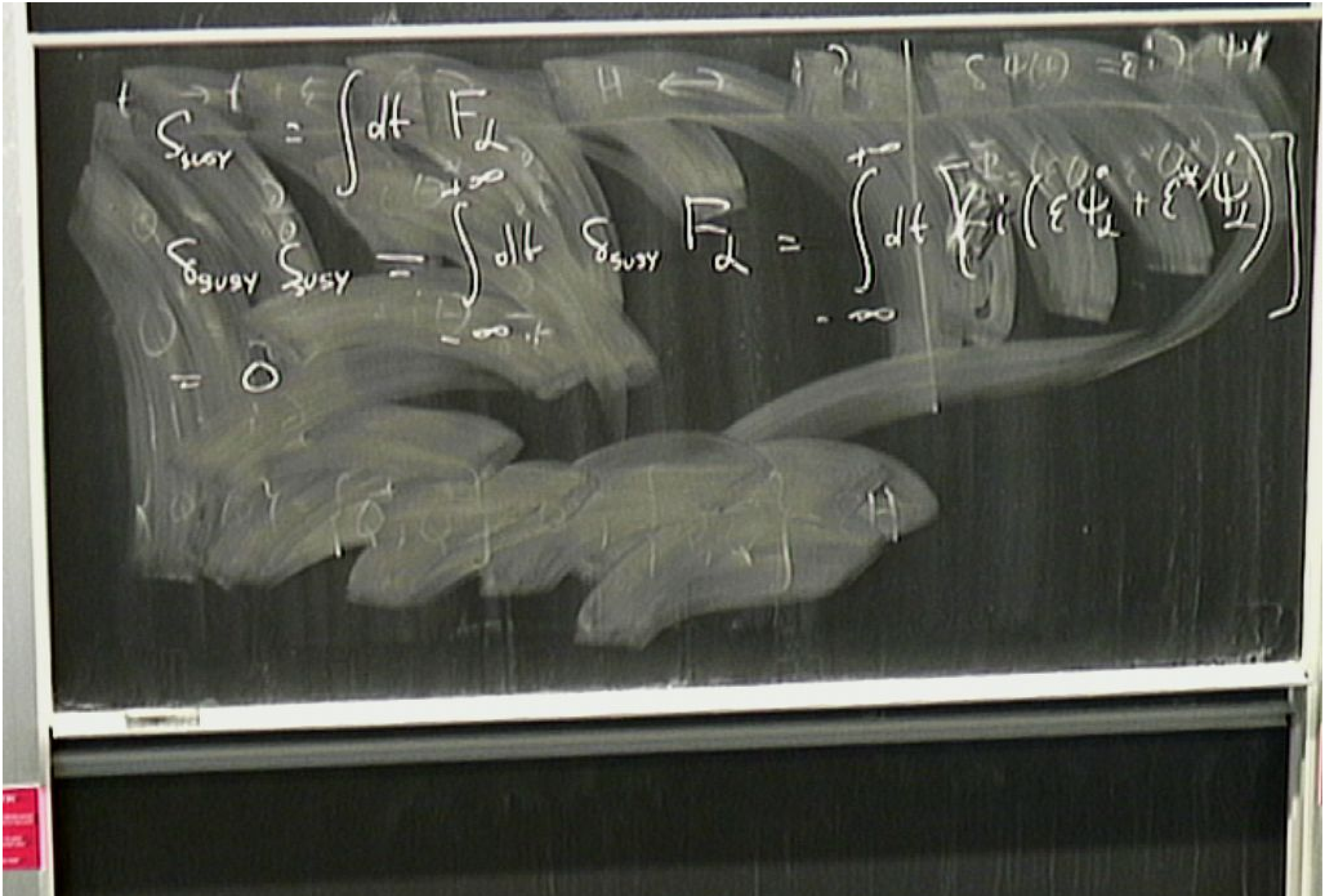

S_SUSY = ∫dt F_L
δ_SUSY S_SUSY = ∫dt δ_SUSY F_L

$$S_{SUSY} = \int dt \, F_L$$

$$\delta_{SUSY} S_{SUSY} = \int dt \, \delta_{SUSY} F_L = \int dt \left\{ i \left(\varepsilon \dot{\psi}_L + \varepsilon^* \dot{\psi}_L^* \right) \right.$$



S_SUSY = ∫ dt F_L
δ_SUSY S_SUSY = ∫ dt δ_SUSY F_L = ∫ dt [i(ε ψ̇_L + ε* ψ̇*_L)]


$S_{SUSY} = \int dt\, F_L$
$\delta_{SUSY} S_{SUSY} = \int dt\, \delta_{SUSY} F_L = \int_{-\infty}^{+\infty} dt \left[i\left(\varepsilon \dot{\psi}_L + \varepsilon^* \dot{\bar{\psi}}_L\right) \right]$
$= 0$

$$\ldots \; \theta\psi(t) - \theta^*\psi^*(t) + \theta\theta^* F(t)$$

$\Rightarrow$ an arbitrary polynomial of Φ is a superfield

$$\Downarrow$$

How to construct SUSY invariant actions.

$$S_{susy} = \int dt\, F_\alpha$$

$$\delta_{susy} S_{susy} = \int_{-\infty}^{+\infty} dt\, \delta_{susy} F_\alpha = \int_{-\infty}^{+\infty} dt\, i\left(\varepsilon \dot{\psi}_\alpha + \varepsilon^* \dot{\psi}^*_\alpha\right)$$

$$= 0$$

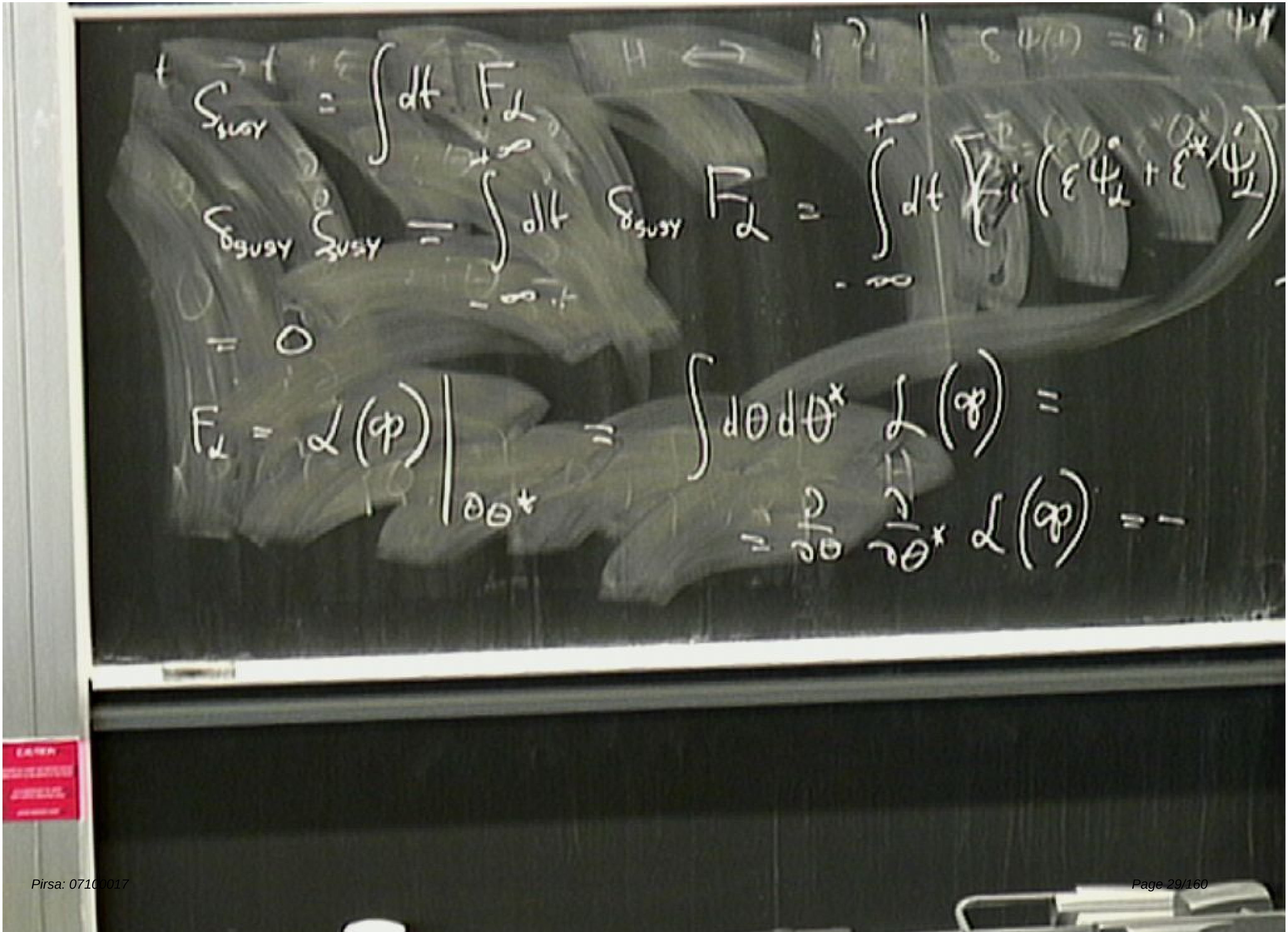
$$F_\alpha = \alpha(\varphi)\Big|_{\theta\theta^*}$$

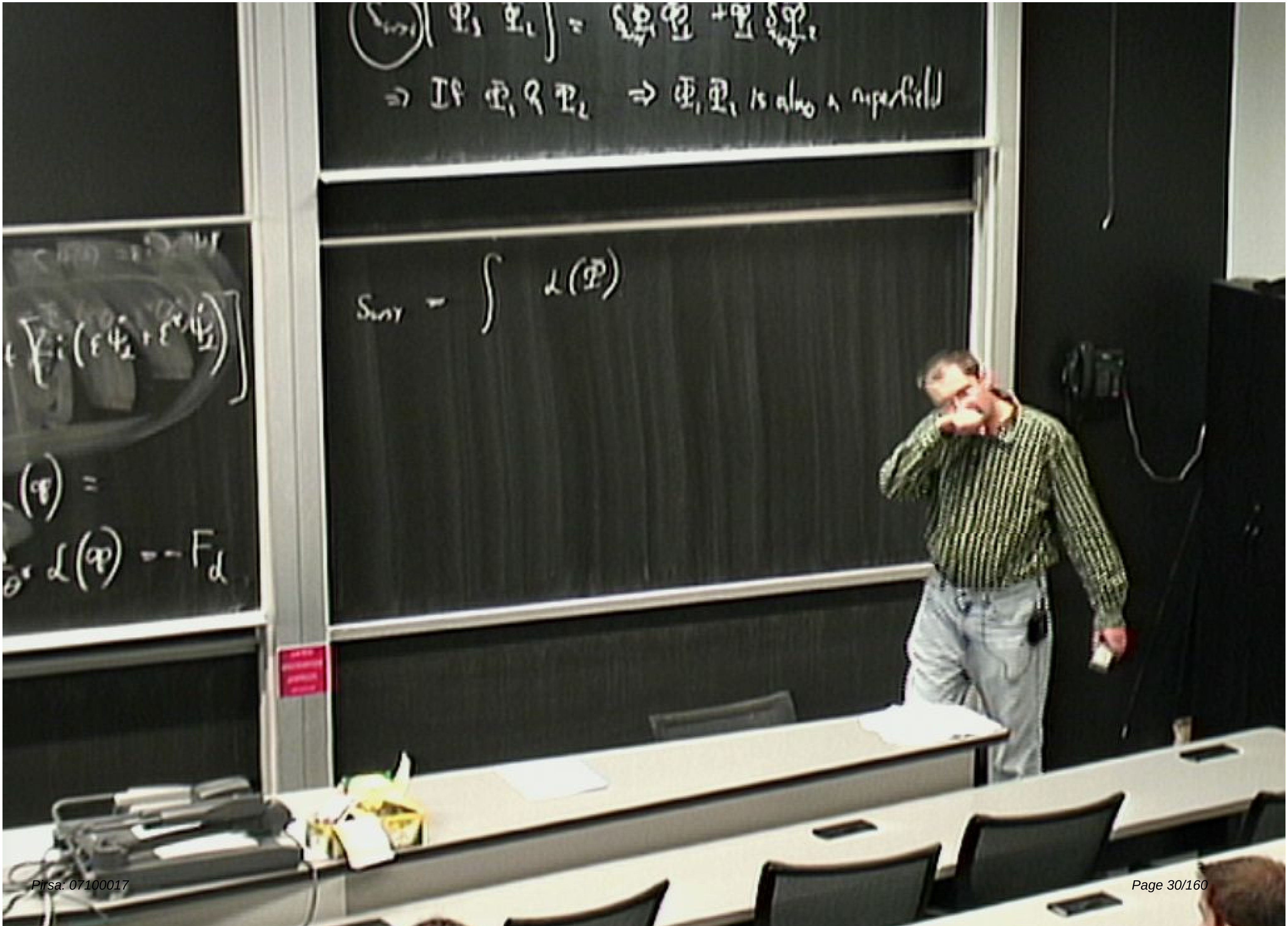
$$S_{SUSY} = \int dt \, F_{\mathcal{L}}$$

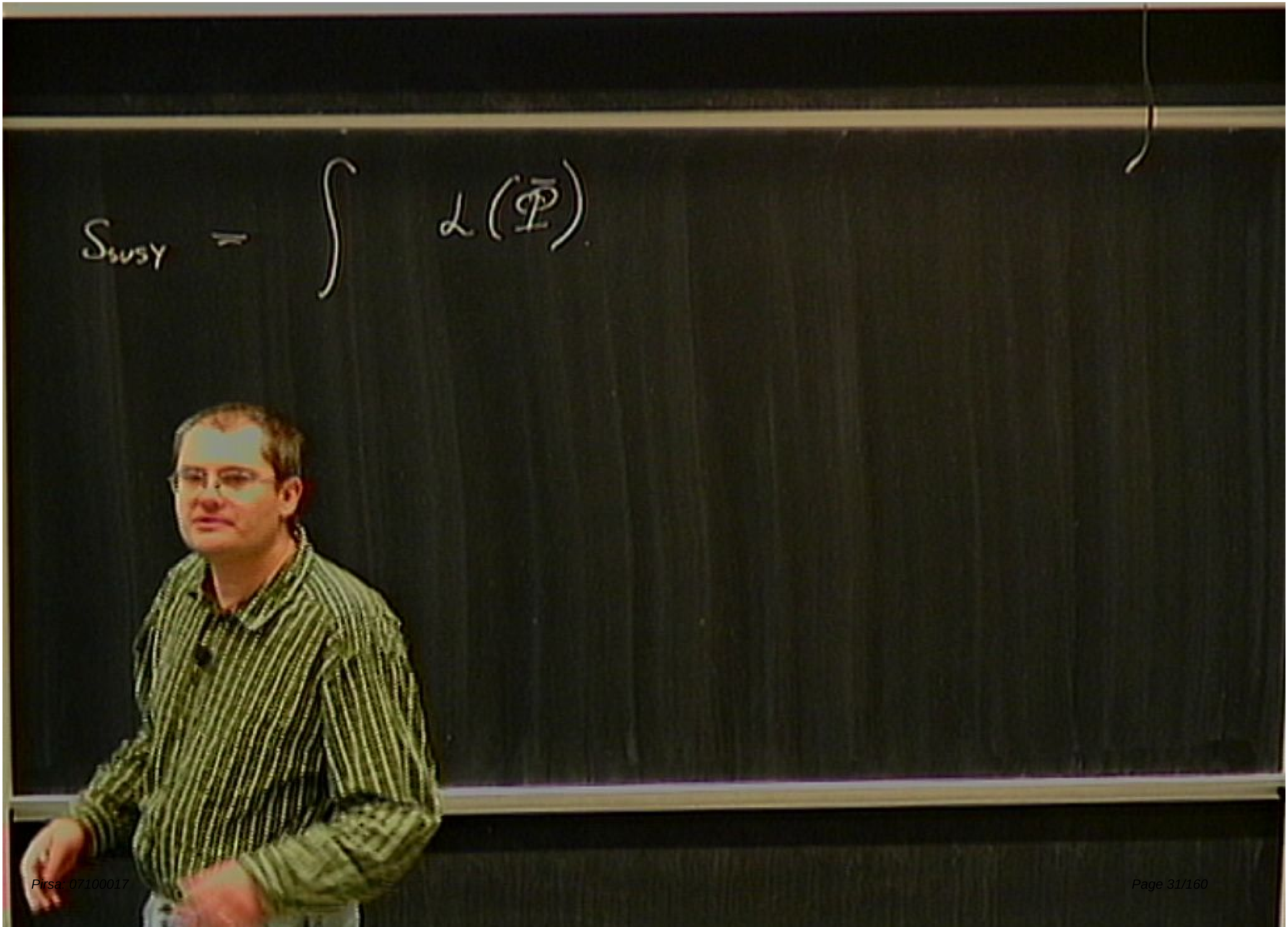
$$\delta_{SUSY} S_{SUSY} = \int_{-\infty}^{+\infty} dt \, \delta_{SUSY} F_{\mathcal{L}} = \int_{-\infty}^{+\infty} dt \, \frac{d}{dt} i\left(\varepsilon \psi_{\mathcal{L}} + \varepsilon^{*} \bar{\psi}_{\mathcal{L}}\right)$$

$$= 0$$

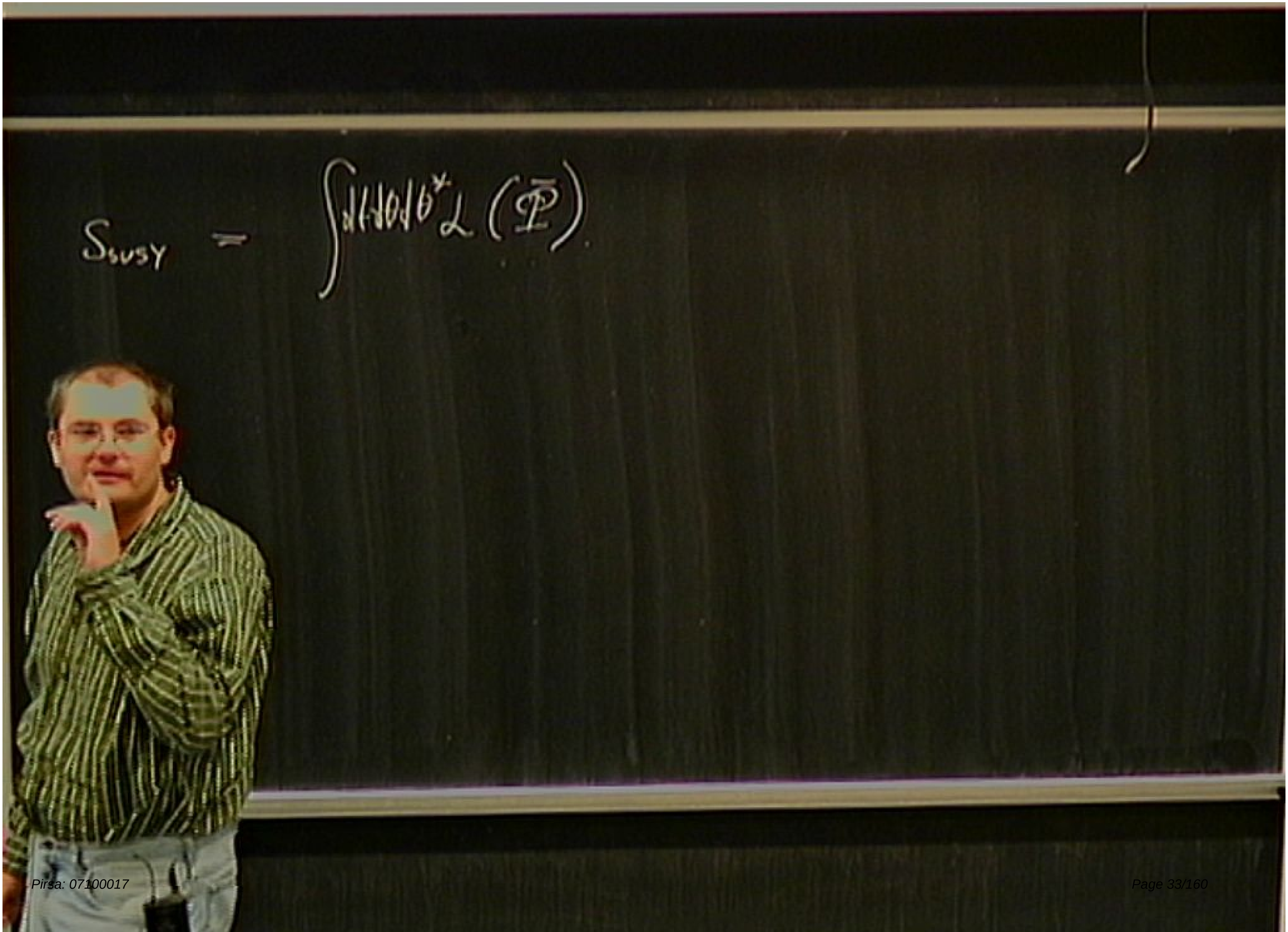
$$F_{\mathcal{L}} = \mathcal{L}(\varphi)\Big|_{\theta\theta^{*}} = \int d\theta \, d\theta^{*} \, \mathcal{L}(\varphi)$$


S_SUSY = ∫ dt F_L
δ_SUSY S_SUSY = ∫ dt δ_SUSY F_L = ∫ dt i(ε ψ_L + ε* ψ_L)
= 0
F_L = L(φ)|_θθ* = ∫ dθ dθ* L(φ) =
= ∂/∂θ ∂/∂θ* L(φ) = ...

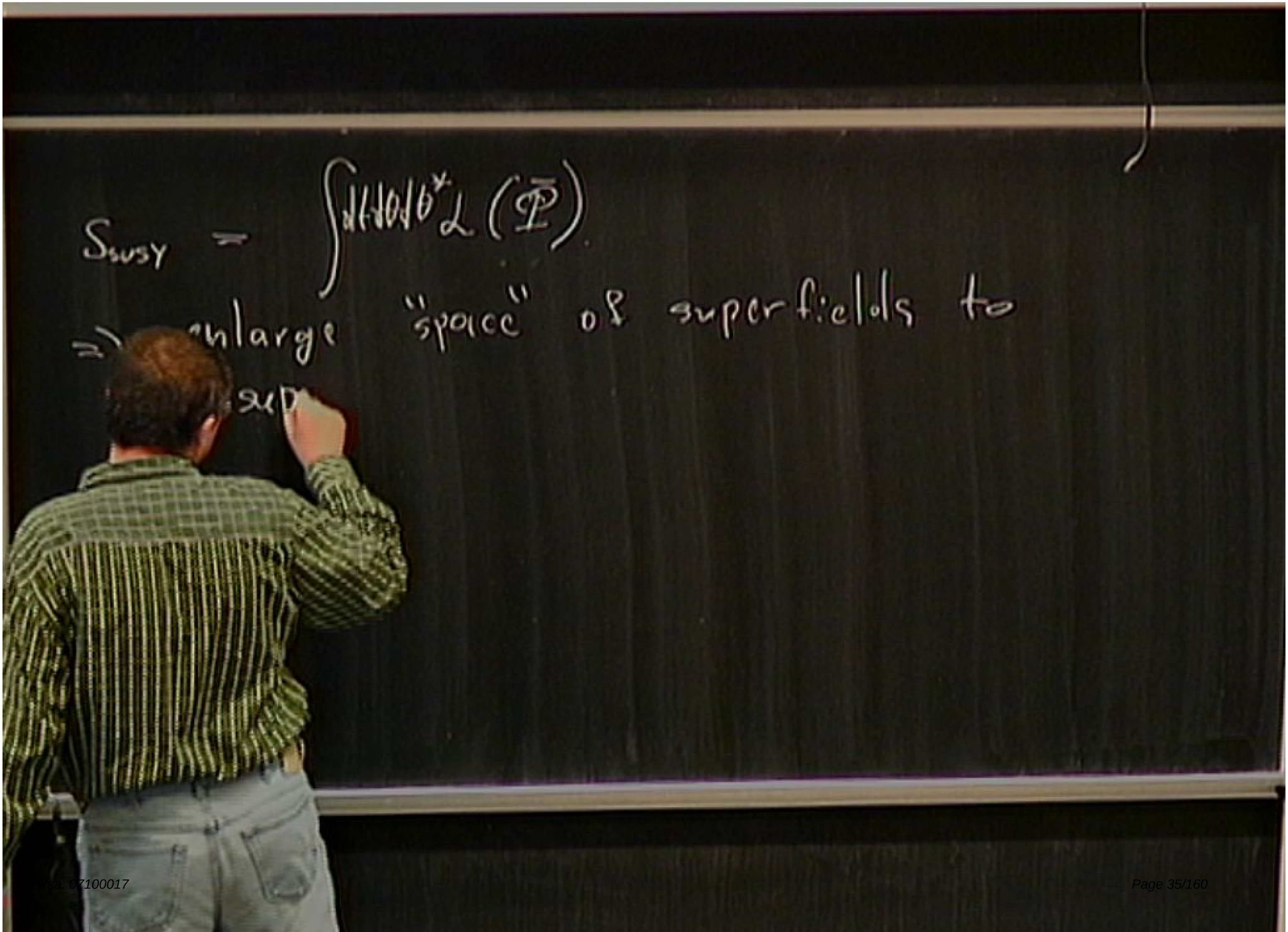

⇒ If Φ1 & Φ2 ⇒ Φ1Φ2 is also a superfield
Ssusy = ∫ L(Φ)


$S_{SUSY} = \int \mathcal{L}(\bar{\Phi})$

$$S_{SUSY} = \int dx\,dt\,d\theta\,d\theta^{*}\,\mathcal{L}(\bar{\Phi})$$


S_{SUSY} =

$$S_{SUSY} = \int dx\,dt\,d\theta\,d\theta^{*}\,\mathcal{L}(\bar{\Phi})$$



S_SUSY = ∫d⁴x dθ dθ* L(Φ̄)
⇒ enlarge "space" of superfields to

$$S_{SUSY} = \int dt\, d\theta\, d\theta^{*}\, \mathcal{L}(\Phi)$$

$\Rightarrow$ enlarge "space" of superfields to superfields that include derivatives.

$$\Phi(t, \theta, \theta^{*}) \xrightarrow{\text{naive}} \partial_t \Phi(t, \theta, \theta^{*})$$

$$S_{SUSY} = \int dt\, d\theta\, d\theta^* \, \mathcal{L}(\Phi)$$

$\Rightarrow$ enlarge "space" of superfields to superfields that include derivatives.

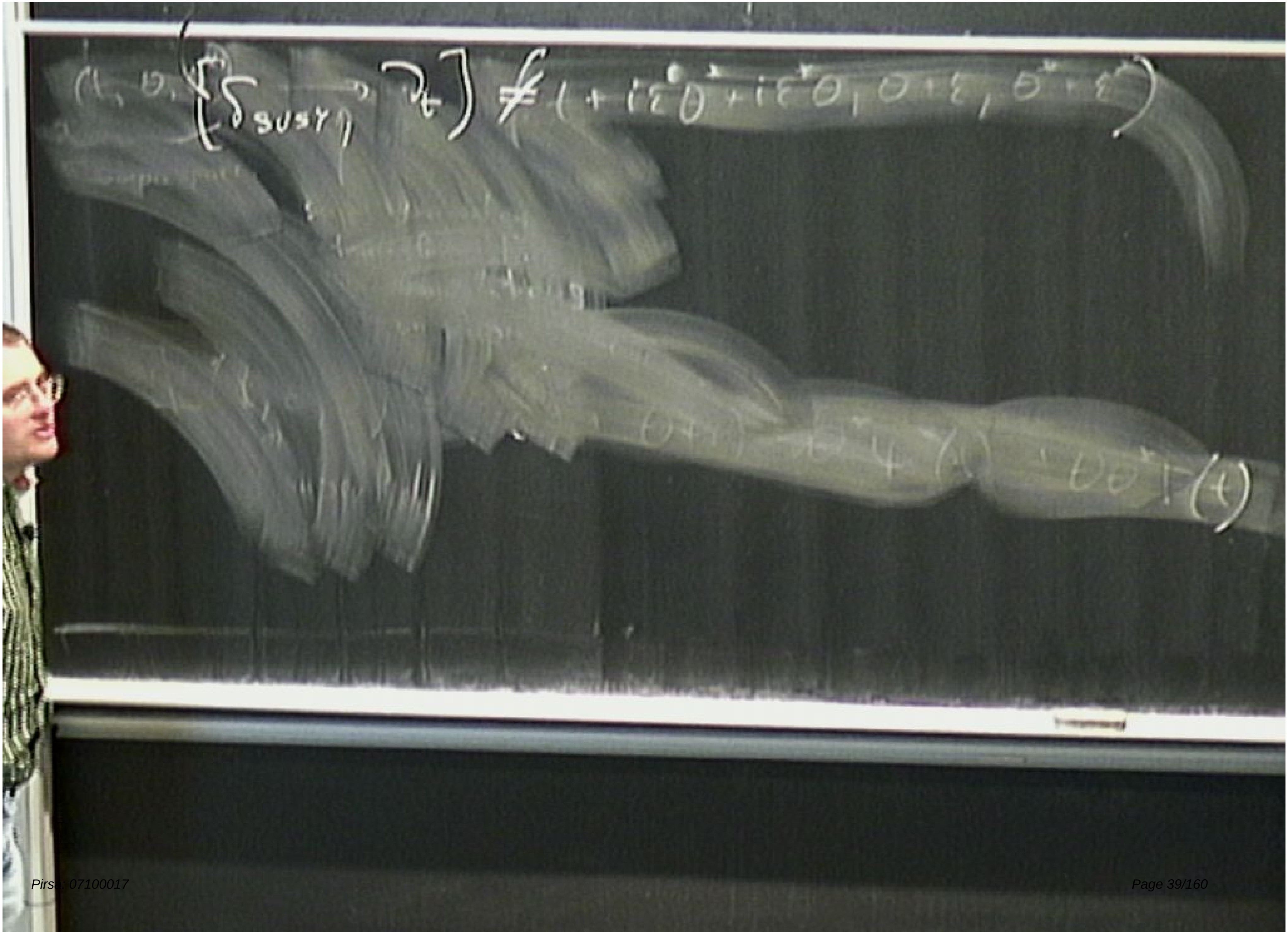
$$\Phi(t, \theta, \theta^*) \xrightarrow{\text{naive}} \underbrace{\partial_t \Phi(t, \theta, \theta^*)}_{\text{is NOT a superfield}}$$

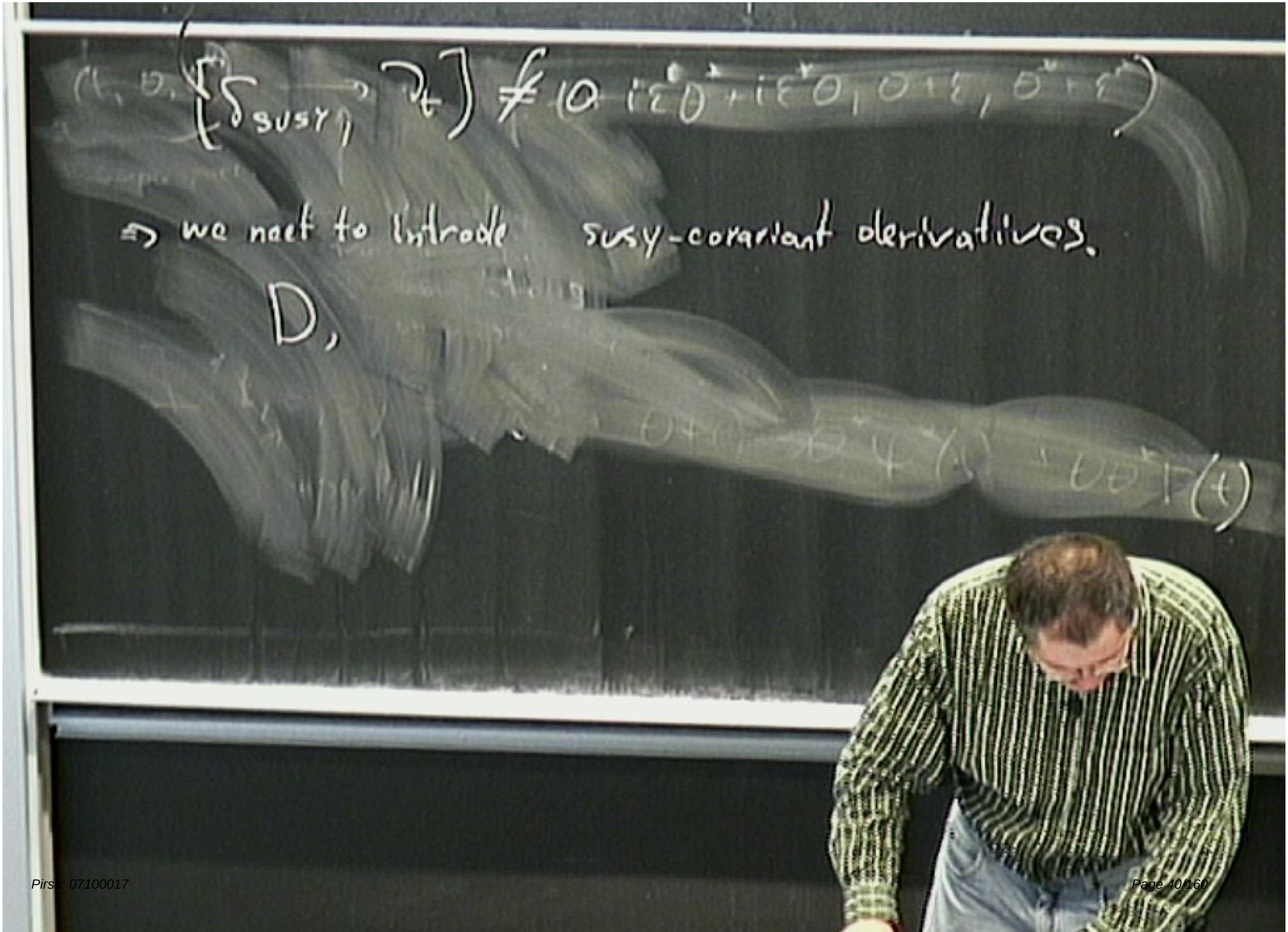
$$S_{SUSY} = \int dt\, d\theta\, d\theta^* \mathcal{L}(\Phi)$$

$\Rightarrow$ enlarge "space" of superfields to superfields that include derivatives.

$$\Phi(t, \theta, \theta^*) \xrightarrow{\text{naive}} \underbrace{\partial_t \Phi(t, \theta, \theta^*)}_{\text{is NOT a superfield}}$$

$$\mathcal{L}(\Phi) \longrightarrow \mathcal{L}(\Phi, \partial_t \Phi)$$


[δ_SUSY, ∂_t] ≠ (t + iε̄θ + iεθ̄, θ + ε, θ̄ + ε̄)

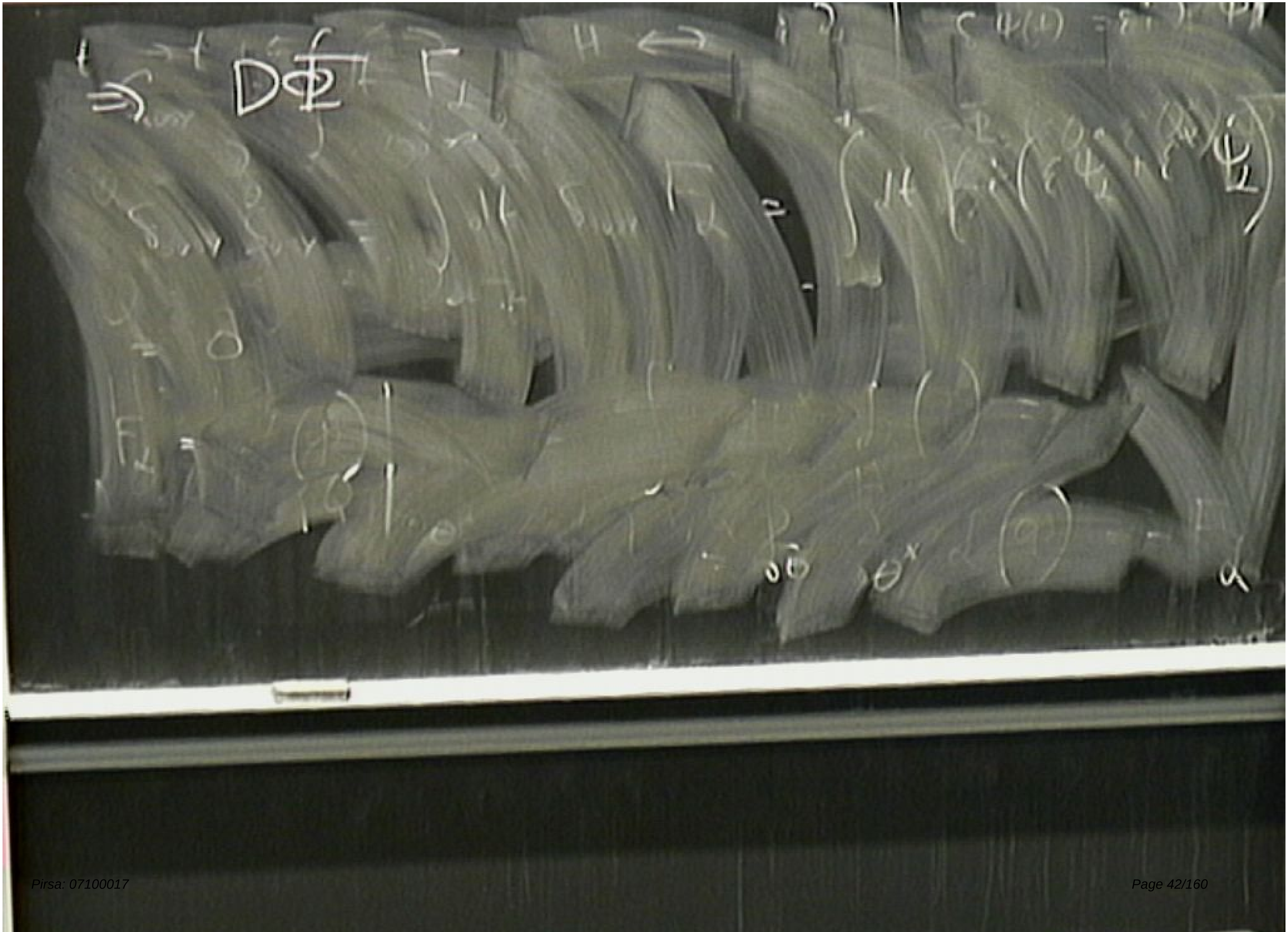

[δsusy, ∂t] ≠ 0
⇒ we need to introduce susy-covariant derivatives.
D,

$[\delta_{susy}, \partial_t] \neq 0$

$\Rightarrow$ we need to introduce susy-covariant derivatives.

$D, D^\dagger$ in such a way that

$[D, \delta_{susy}] = 0 \; ; \; [D^\dagger, \delta_{susy}] = 0$



$\Rightarrow$ $D\Phi_L$ F's in superfield

$$\delta_{susy}(D\Phi) = D\left[\delta_{susy}\Phi_L\right]$$

$\Rightarrow$ $D\Phi$ is a superfield

$$\delta_{susy}(D\Phi) = D\left[\delta_{susy}\Phi\right]$$

$$\left[D, \epsilon Q + \epsilon^* Q^*\right] = 0$$

$\Rightarrow \quad D\Phi$ is a superfield

$$\delta_{susy}(D\Phi) = D\left[\delta_{susy}\Phi\right]$$

$$[D, \varepsilon Q + \varepsilon^* Q^*] = 0 \Rightarrow \{D, Q\} = 0, \quad \{D, Q^*\}$$

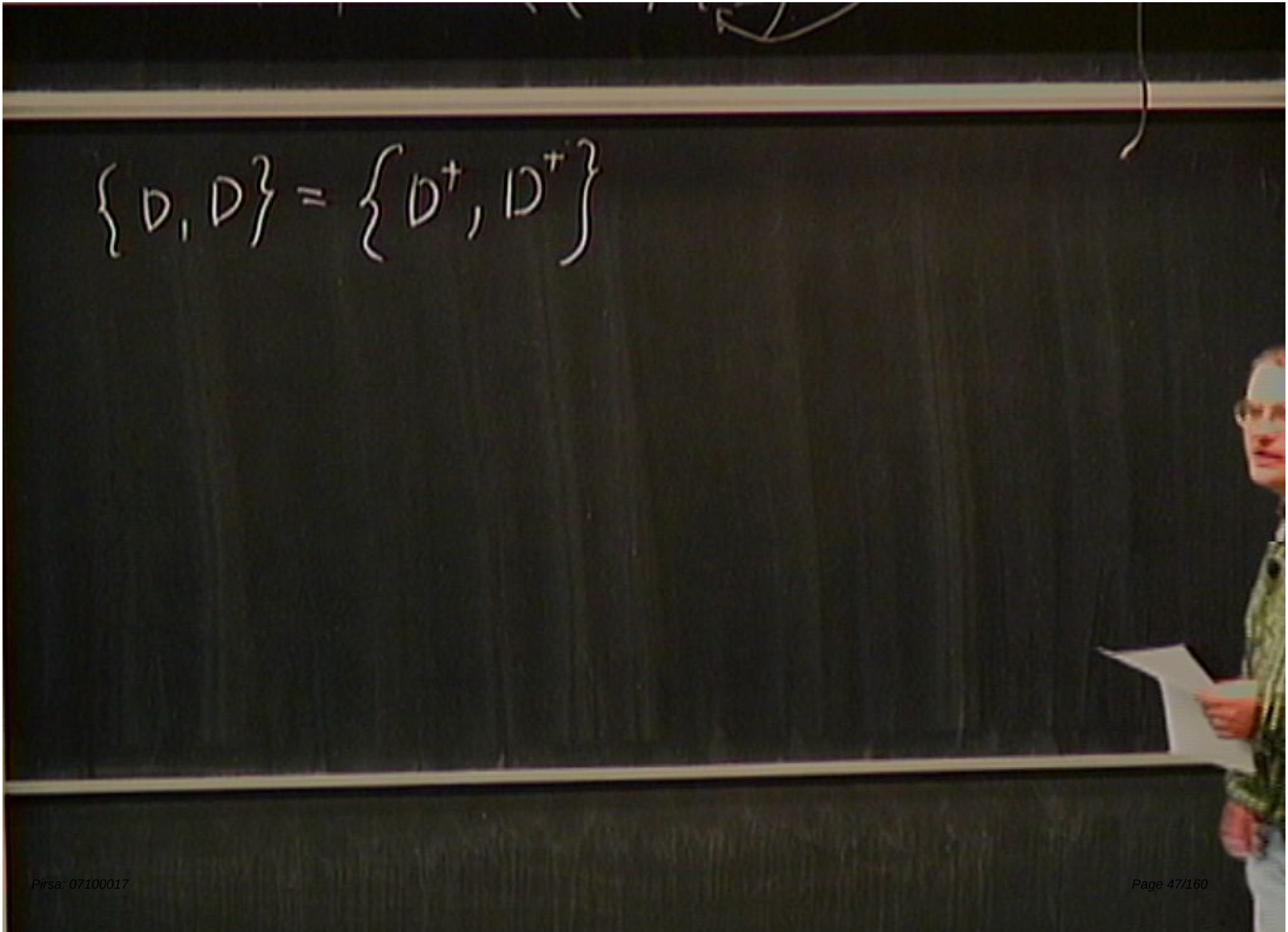
$\Rightarrow$ $D\Phi$ is a superfield

$$\delta_{susy}(D\Phi) = D\left[\delta_{susy}\Phi\right]$$

$$[D, \varepsilon Q + \varepsilon^* Q^*] = 0 \Rightarrow \{D, Q\} = 0 \quad \{D, Q^\dagger\}$$

$$D \equiv \frac{\partial}{\partial\theta} - i\theta^* \frac{\partial}{\partial t} \qquad Q = \frac{\partial}{\partial\theta} + i\theta^* \frac{\partial}{\partial t}$$

$$D^\dagger = \frac{\partial}{\partial\theta^*} - i\theta \frac{\partial}{\partial t} \qquad Q^\dagger = \frac{\partial}{\partial\theta} + i\theta \frac{\partial}{\partial t}$$



$$\{D, D\} = \{D^{+}, D^{+}\}$$

$$\{D, D\} = \{D^+, D^+\} = 0$$

$$\{D, D^+\} = -2H$$

$$\{D, D\} = \{D^\dagger, D^\dagger\} = 0$$

$$\{D, D^\dagger\} = -2H$$

$$\mathcal{L}\left(\Phi,\ D\Phi,\ D^\dagger\Phi^*\right.$$

$$\{D, D\} = \{D^{+}, D^{+}\} = 0$$

$$\{D, D^{+}\} = -2H$$

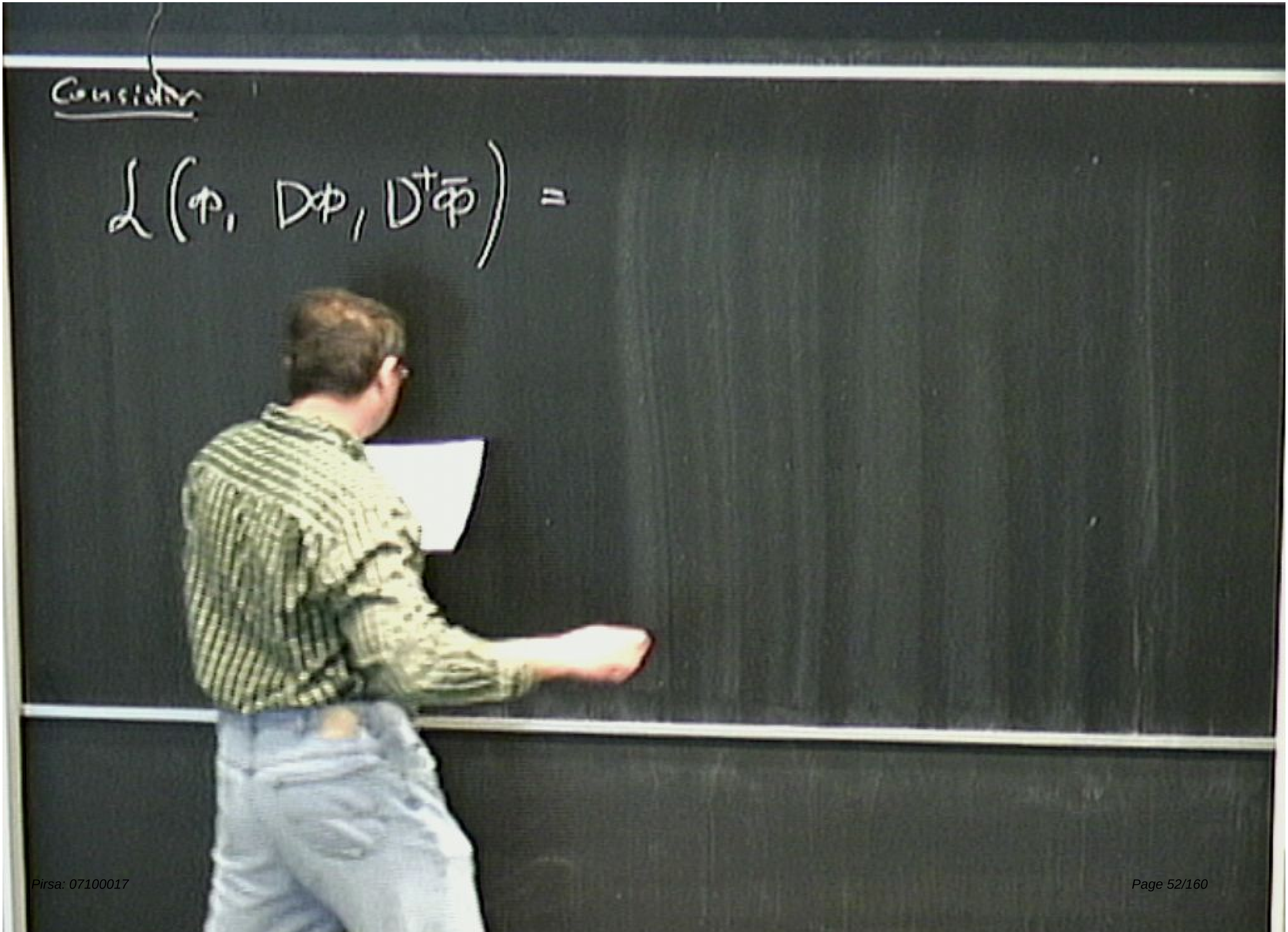
$$\mathcal{L}(\Phi, D\Phi, D^{+}\Phi^{*}) \rightarrow \text{an arbitrary polynomial}$$

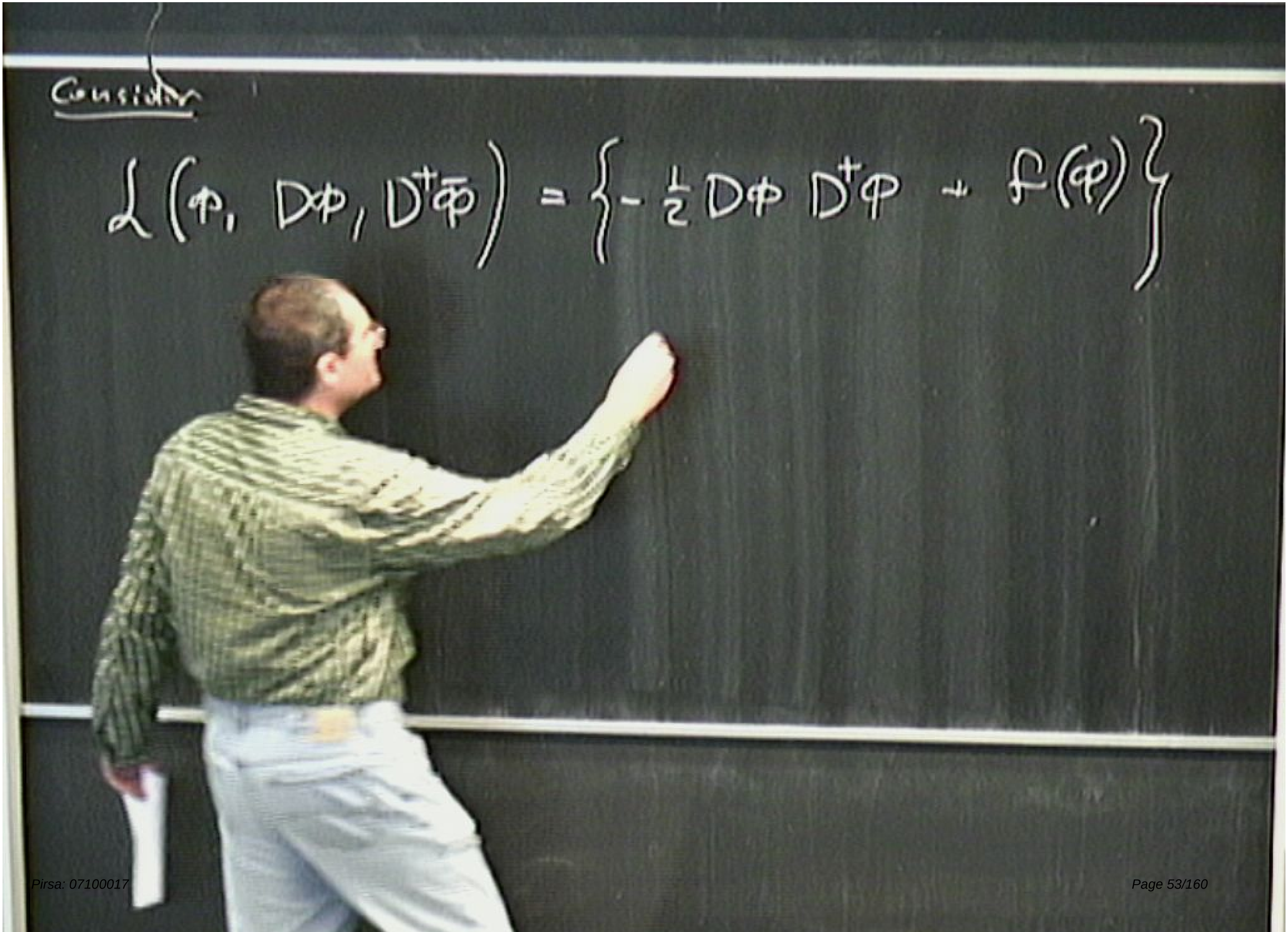
$$\{D, D\} = \{D^\dagger, D^\dagger\} = 0$$

$$\{D, D^\dagger\} = -2H$$

$$\mathcal{L}(\Phi, D\Phi, D^\dagger \Phi^*) \rightarrow \text{an arbitrary polynomial}$$

$$S_{susy} = \int dt\, d\theta\, d\theta^* \, \mathcal{L}(\varphi, D\varphi, \overline{D\varphi})$$


Consider
$\mathcal{L}(\phi, D\phi, D^{\dagger}\bar{\phi}) =$


Consider
$\mathcal{L}(\varphi, D\varphi, D^{+}\bar{\varphi}) = \{ -\frac{1}{2} D\varphi D^{+}\varphi + f(\varphi) \}$

Consider

$$\mathcal{L}(\Phi, D\Phi, D^{\dagger}\Phi) = \left\{ -\frac{1}{2} D\Phi\, D^{\dagger}\Phi + f(\Phi) \right\}$$

Φ is a real superfield

and $f(\Phi)$ is an arbitrary real function.

Consider

$$\mathcal{L}(\Phi, D\Phi, D^{+}\Phi) = \left\{ -\frac{1}{2} D\Phi \, D^{+}\Phi + f(\Phi) \right\}$$

Φ is a real superfield

and $f(\Phi)$ is an arbitrary real function.

Consider

$$\mathcal{L}\left(\phi,\ D\phi,\ D^{+}\phi\right) = \left\{ -\tfrac{1}{2} D\Phi\, D^{+}\Phi + f(\Phi) \right\}$$

Φ is a real superfield

and $f(\Phi)$ is an arbitrary real analytic function

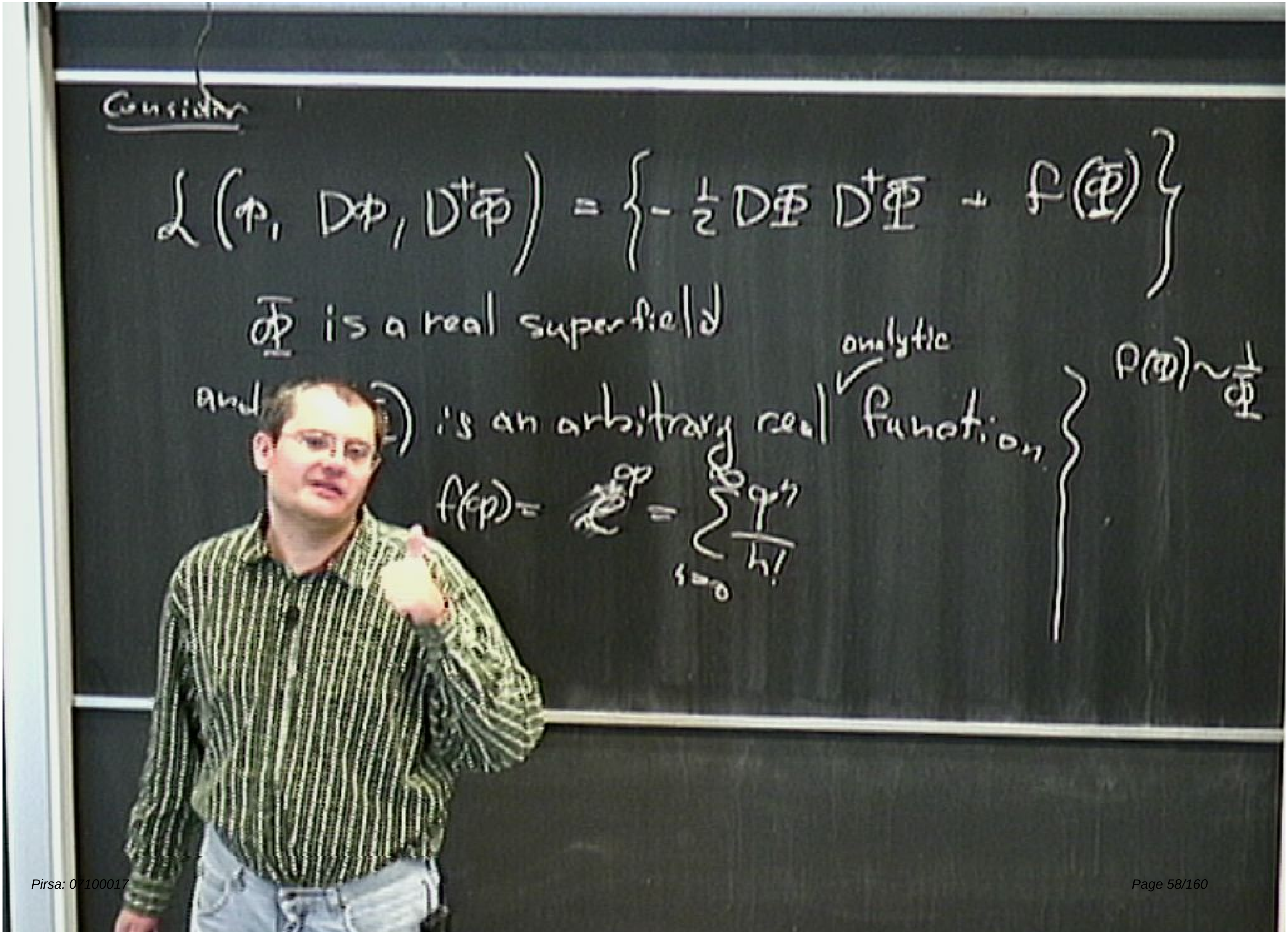
Consider

$$\mathcal{L}\left(\Phi, D\Phi, D^{\dagger}\Phi\right) = \left\{-\frac{1}{2} D\Phi\, D^{\dagger}\Phi + f(\Phi)\right\}$$

Φ is a real superfield

and $f(\Phi)$ is an arbitrary real analytic function.

$$f(\Phi) = e^{\Phi} = \sum_{n=0}^{\infty} \frac{\Phi^n}{n!}$$


Consider
$\mathcal{L}(\Phi, D\Phi, D^{+}\Phi) = \{-\frac{1}{2} D\Phi D^{+}\Phi + f(\Phi)\}$
Φ is a real superfield
and
is an arbitrary real function
analytic
$f(\Phi) \sim \frac{1}{\Phi}$
$f(\Phi) = e^{\Phi} = \sum_{n=0}^{\infty} \frac{\Phi^n}{n!}$


Dφ =

$$D\Phi = \left(\frac{\partial}{\partial\theta} - i\theta^{*}\frac{\partial}{\partial t}\right)\left[\phi + \theta\psi - \theta^{*}\psi^{*} + \theta\theta^{*}F\right)$$

$$D\Phi = \left(\frac{\partial}{\partial\theta} - i\theta^* \frac{\partial}{\partial t}\right)\left[\phi + \theta\psi - \theta^*\psi^* + \theta\theta^* F\right)$$

$$= \psi(\theta)$$

$$D\Phi = \left(\frac{\partial}{\partial\theta} - i\theta^* \frac{\partial}{\partial t}\right)\left[\phi + \theta\psi - \theta^*\psi^* + \theta\theta^* F\right)$$

$$= \psi(t) + \theta^* F$$

$$D\Phi = \left(\frac{\partial}{\partial\theta} - i\theta^* \frac{\partial}{\partial t}\right)\left[\phi + \theta\psi - \theta^*\psi^* + \theta\theta^* F\right)$$

$$= \psi(t) + \theta^* F - i\theta^* \dot{\phi}$$

$$D\Phi = \left(\frac{\partial}{\partial\theta} - i\theta^{*}\frac{\partial}{\partial t}\right)\left[\varphi + \theta\psi - \theta^{*}\psi^{*} + \theta\theta^{*}F\right)$$

$$= \psi(t) + \theta^{*}F - i\theta^{*}\dot{\varphi} + i\theta\theta^{*}\dot{\psi}$$

$$D\Phi = \left(\frac{\partial}{\partial\theta} - i\theta^* \frac{\partial}{\partial t}\right)\left[\phi + \theta\psi - \theta^*\psi^* + \theta\theta^* F\right)$$

$$= \psi(t) + \theta^* F - i\theta^* \dot{\phi} + i\theta\theta^* \dot{\psi}$$

$$D\Phi = \left(\frac{\partial}{\partial\theta} - i\theta^* \frac{\partial}{\partial t}\right)\left[\varphi + \theta\psi - \theta^*\psi^* + \theta\theta^* F\right)$$

$$= \psi(t) + \theta^* F - i\theta^* \dot{\varphi} + i\theta\theta^* \dot{\psi}$$

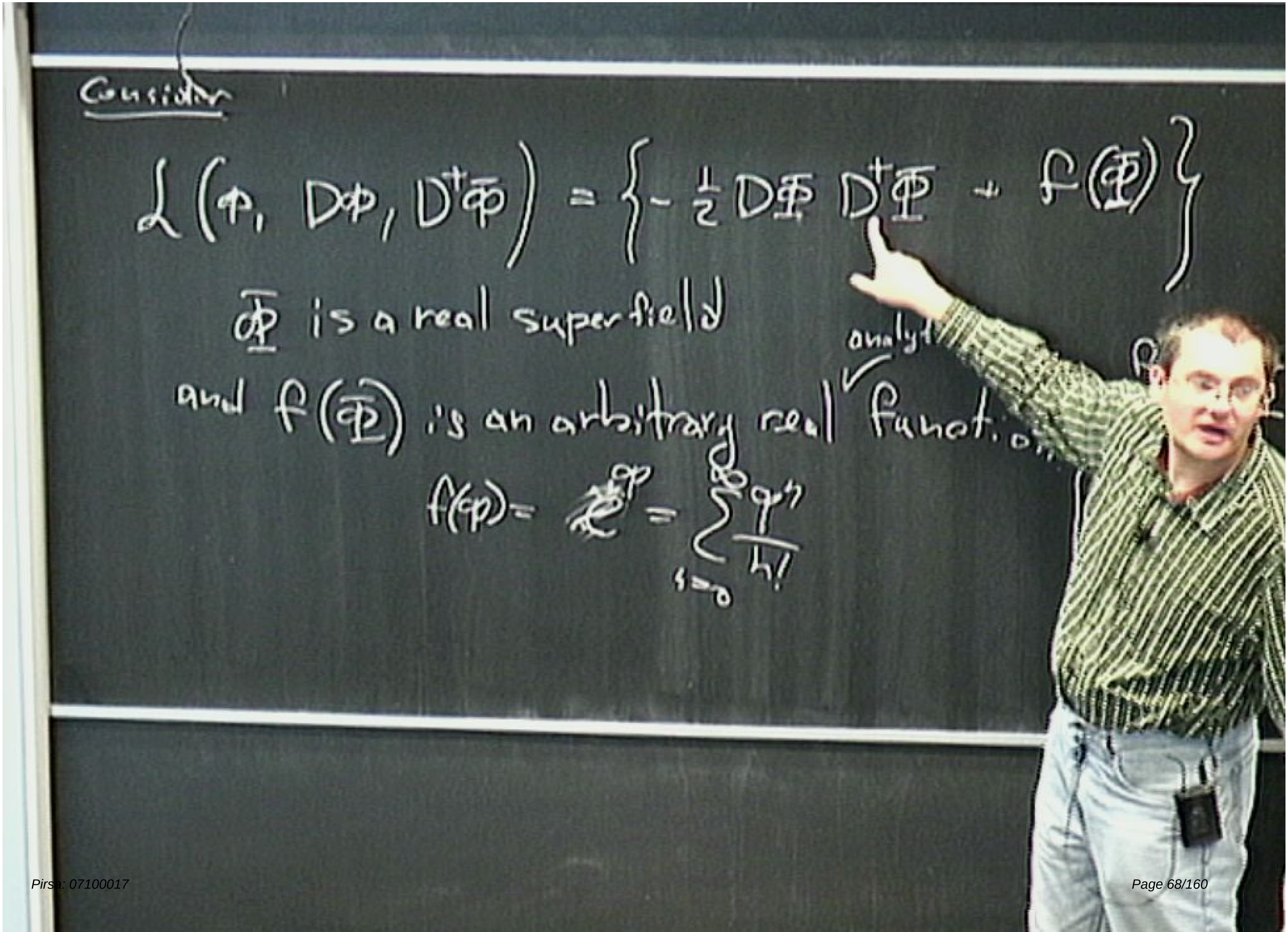
$$= \psi(t) + \theta^*\left[F(t) - i\dot{\varphi}(t)\right] + i\theta\theta^* \dot{\psi}$$

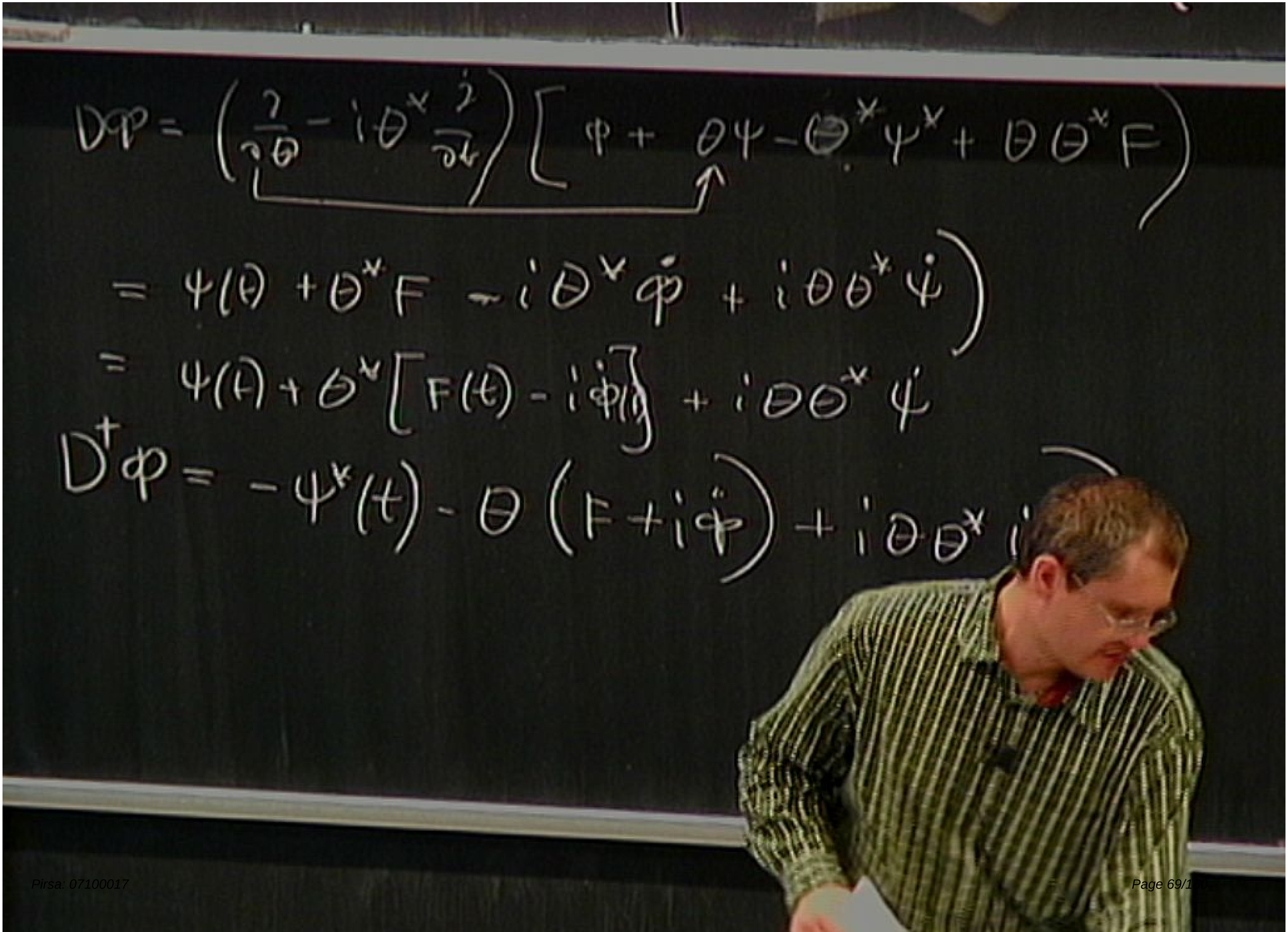
$$D\varphi = \left(\frac{\partial}{\partial\theta} - i\theta^* \frac{\partial}{\partial t}\right)\left[\varphi + \theta\psi - \theta^*\psi^* + \theta\theta^* F\right)$$

$$= \psi(t) + \theta^* F - i\theta^*\dot{\varphi} + i\theta\theta^*\dot{\psi}$$

$$= \psi(t) + \theta^*\left[F(t) - i\dot{\varphi}(t)\right] + i\theta\theta^*\dot{\psi}$$

$$D^\dagger\varphi = -\psi^*(t) - \theta\left(F + i\dot{\varphi}\right) + i\theta\theta^*\dot{\psi}^*$$


Consider
$\mathcal{L}(\varphi, D\varphi, D^{\dagger}\bar{\varphi}) = \left\{ -\frac{1}{2} D\Phi\, D^{\dagger}\Phi + f(\Phi) \right\}$
Φ is a real superfield
and $f(\Phi)$ is an arbitrary real analytic function
$f(\varphi) = e^{\varphi} = \sum_{n=0}^{\infty} \frac{\varphi^n}{n!}$


DΦ = (∂/∂θ − iθ* ∂/∂t)[φ + θψ − θ*ψ* + θθ*F)
= ψ(t) + θ*F − iθ*φ̇ + iθθ*ψ̇)
= ψ(t) + θ*[F(t) − iφ̇(t)] + iθθ*ψ̇
D†φ = −ψ*(t) − θ(F + iφ̇) + iθθ*

$$D\Phi = \left(\frac{\partial}{\partial\theta} - i\theta^* \frac{\partial}{\partial t}\right)\left[\varphi + \theta\psi - \theta^*\psi^* + \theta\theta^* F\right)$$

$$= \psi(t) + \theta^* F - i\theta^*\dot{\varphi} + i\theta\theta^*\dot{\psi}$$

$$= \psi(t) + \theta^*\left[F(t) - i\dot{\varphi}(t)\right] + i\theta\theta^*\dot{\psi}$$

$$D^\dagger\Phi = -\psi^*(t) - \theta\left(F + i\dot{\varphi}\right) + i\theta\theta^*\dot{\psi}^*$$

$D\bar{\Phi}\, D^\dagger \bar{\Phi} \Big|_{\theta\theta^*}$ term

$\Rightarrow$ enlarge "space of superfields" to superfields that include derivatives.

$\Phi(t, \theta, \theta^*) \to \partial_t \Phi(t, \theta, \theta^*)$ is NOT superfield

$L(\Phi) \to L(\Phi, \partial_t \Phi)$

$D\bar{\Phi}\, D^{\dagger}\Phi \Big|_{\theta\theta^{*}\ \text{term}} \neq (\dot{\varphi})$

SUSY

$\Rightarrow$ enlarge space of superfields to superfields that include derivatives.

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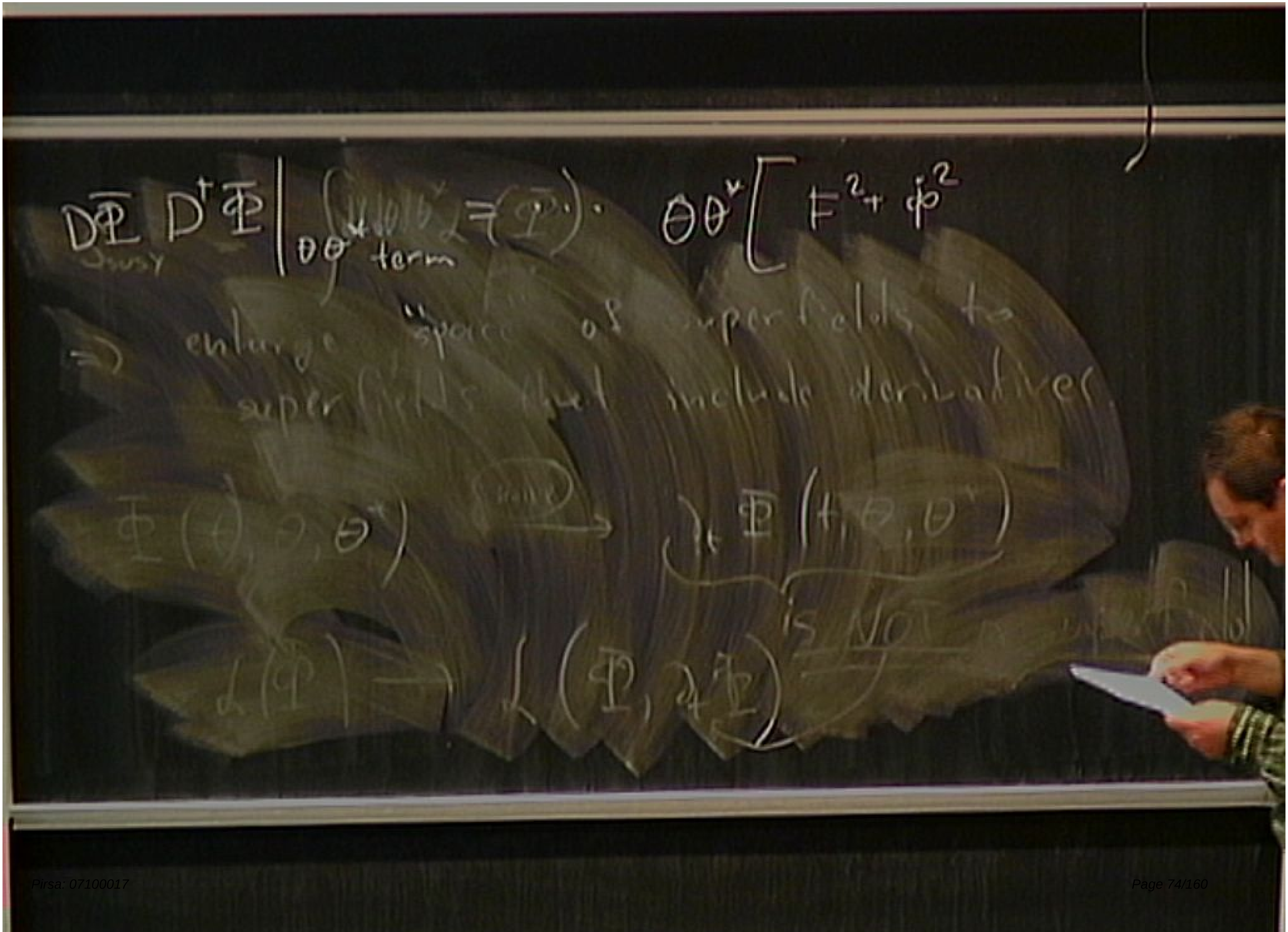
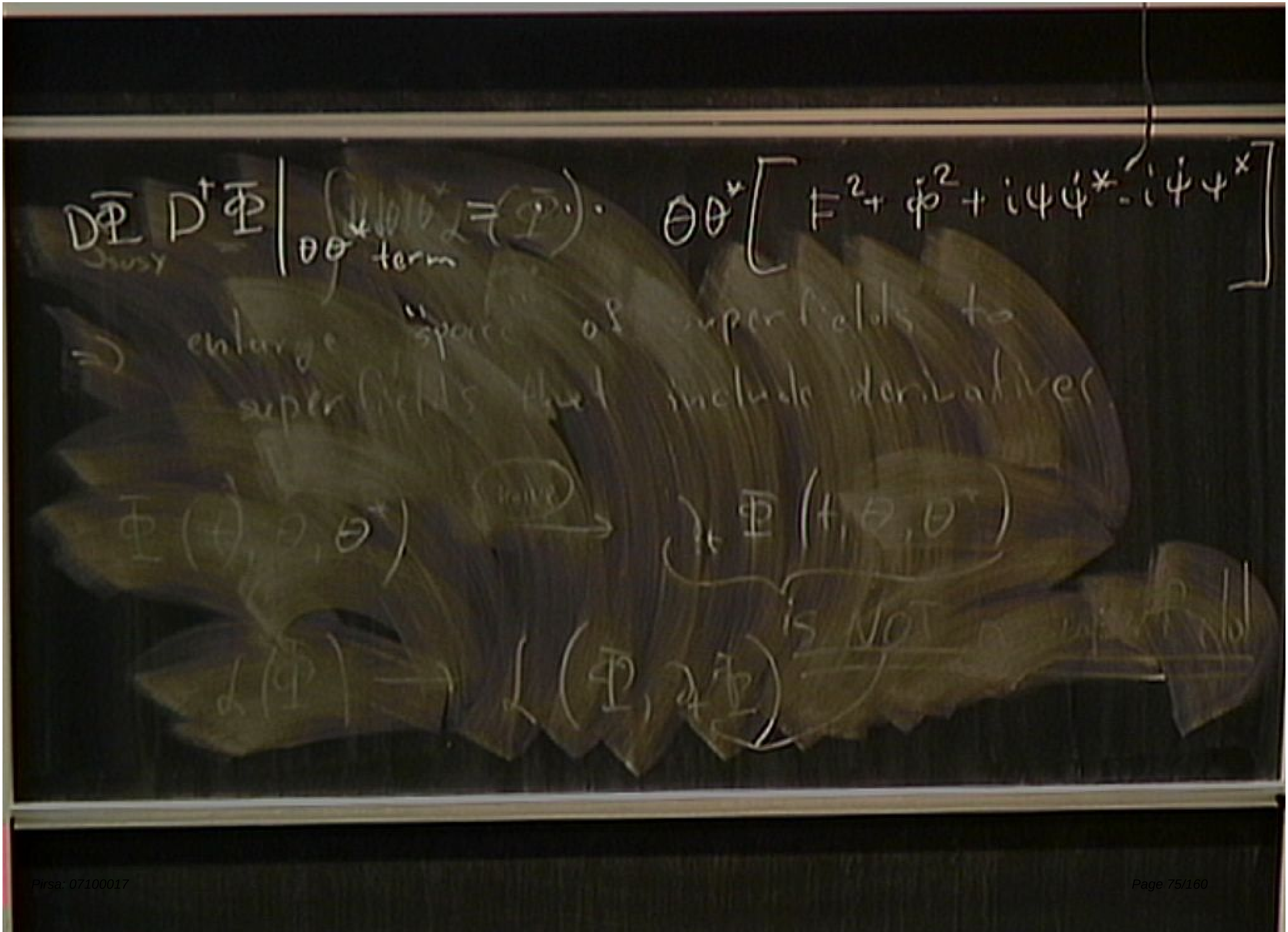
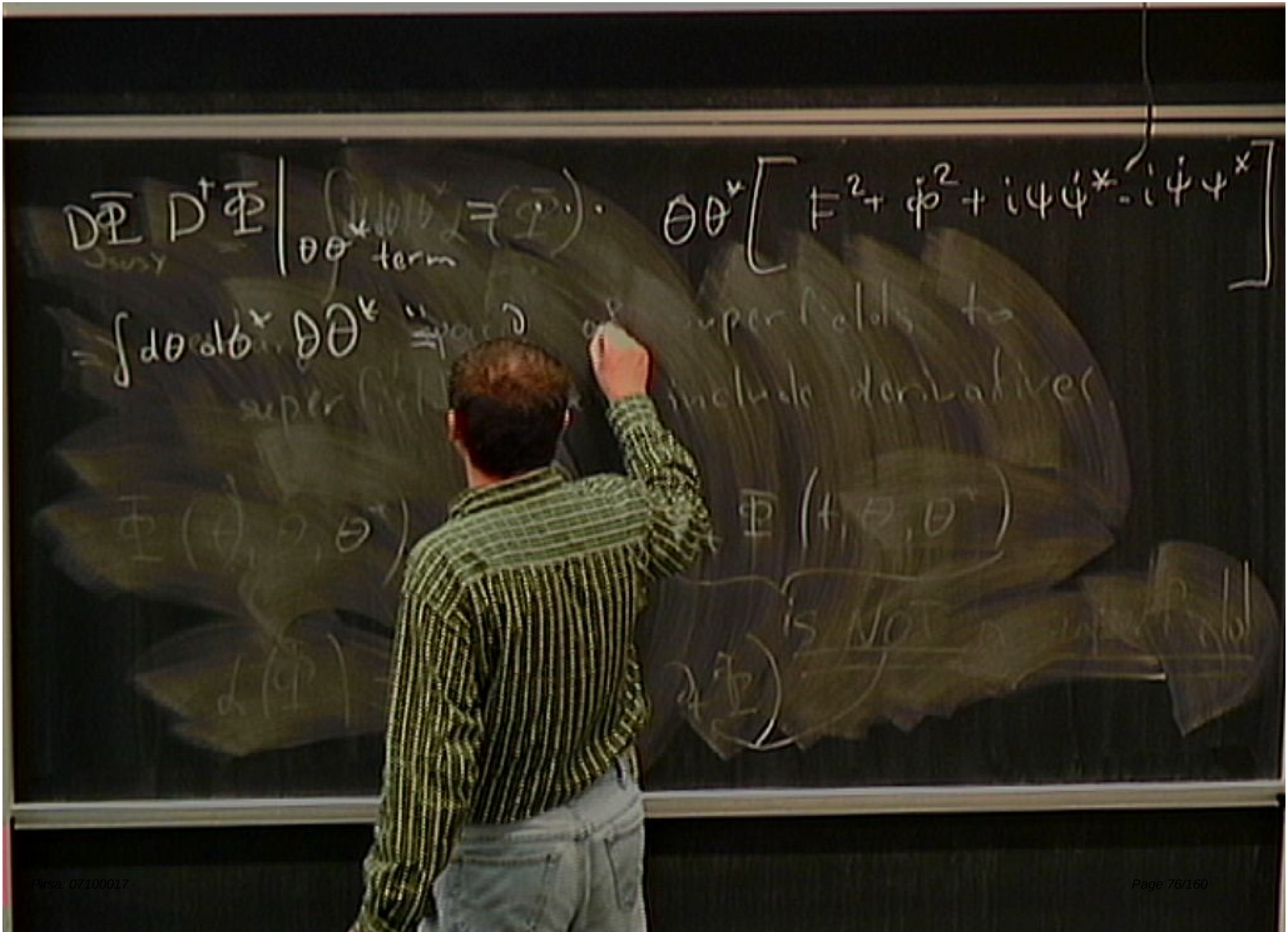
$$D^\dagger\varphi = -\psi^*(t) - \theta\left(F + i\dot{\varphi}\right) + i\theta\theta^*\dot{\psi}^*$$


θθ* term
enlarge "space" of superfields to
superfields that include derivatives
is NOT



$D\bar{\Phi}\, D^\dagger \bar{\Phi} \Big|_{\theta\theta^* \text{ term}}$
$\theta\theta^* \left[F^2 + \dot{\phi}^2 + i\psi\dot{\psi}^* - i\dot{\psi}\psi^* \right]$


θθ* term
include derivatives
is NOT

$$D\bar{\Phi}\, D^\dagger \Phi \Big|_{\theta\theta^* \text{ term}} \qquad \theta\theta^*\left[F^2 + \dot{\phi}^2 + i\psi\dot{\psi}^* - i\dot{\psi}\psi^* \right]$$

$$\int d\theta\, d\theta^*\, \theta\theta^* = \frac{\partial}{\partial\theta}\frac{\partial}{\partial\theta^*}\theta\theta^* = \frac{\partial\theta}{\partial\theta}\,\frac{\partial\theta^*}{\partial\theta^*} = -1$$

$$D\Phi\, D^\dagger\Phi \Big|_{\theta\theta^* \text{ term}} \qquad \theta\theta^*\left[F^2 + \dot\phi^2 + i\psi\dot\psi^* - i\dot\psi\psi^*\right]$$

$$\int d\theta\, d\theta^*\, \theta\theta^* = \frac{\partial}{\partial\theta}\frac{\partial}{\partial\theta^*}\,\theta\theta^* = -1 \qquad \frac{\partial\theta}{\partial\theta} = \frac{\partial\theta^*}{\partial\theta^*} = 1$$

$$\int d\theta\, d\theta^* \left[-\tfrac{1}{2} D\Phi\, D^\dagger\Phi\right]$$

$$D\Phi\, D^\dagger\Phi \Big|_{\theta\theta^* \text{ term}} = \theta\theta^*\left[F^2 + \dot\varphi^2 + i\psi\dot\psi^* - i\dot\psi\psi^*\right]$$

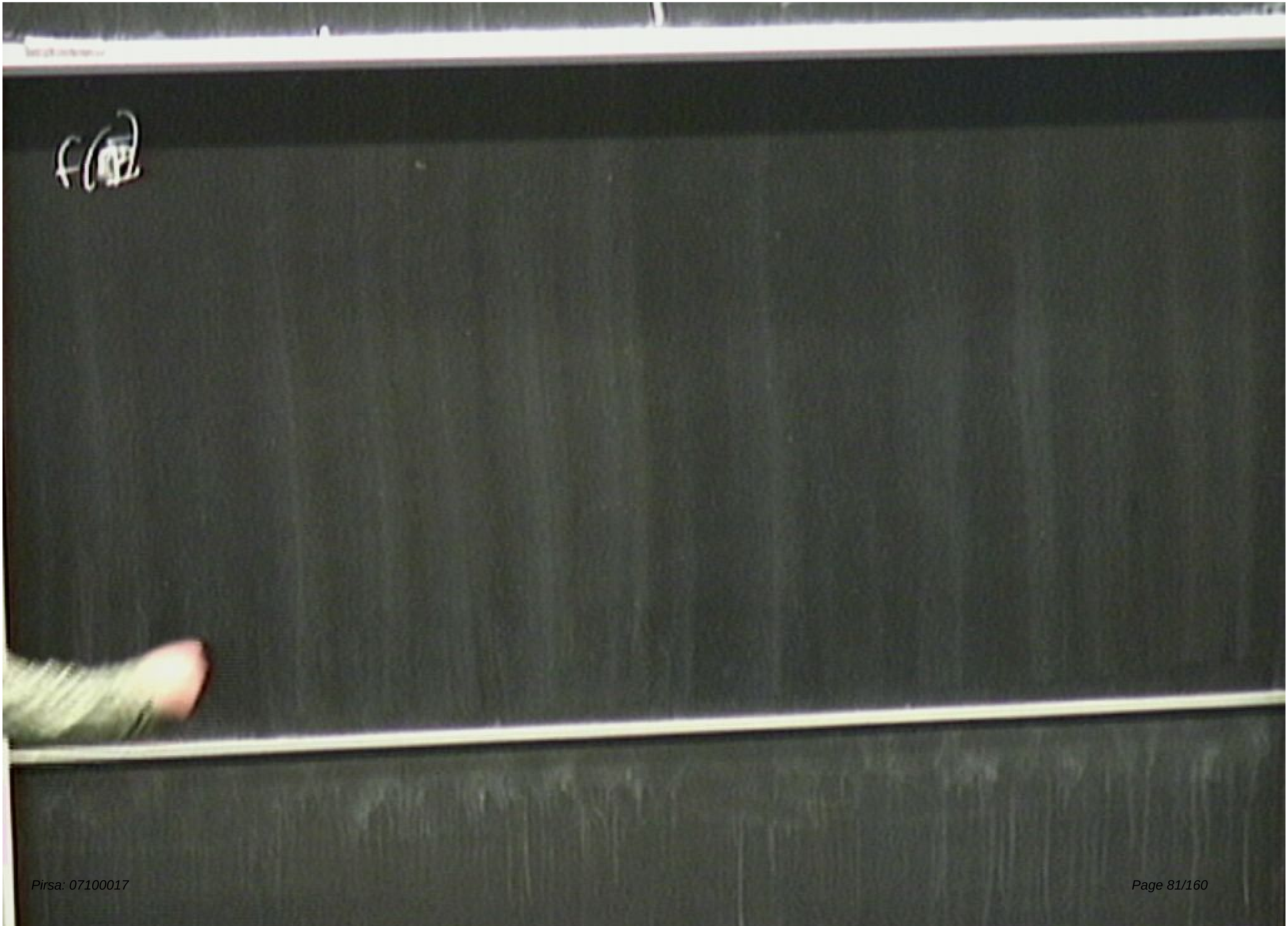
$$\int d\theta\, d\theta^*\, \theta\theta^* = \frac{\partial}{\partial\theta}\frac{\partial}{\partial\theta^*}\,\theta\theta^* = -1 \qquad \frac{\partial\theta}{\partial\theta}\ \ \frac{\partial\theta^*}{\partial\theta^*} = -1$$

$$\int d\theta\, d\theta^*\left[-\frac{1}{2} D\Phi\, D^\dagger\Phi\right] = \frac{1}{2}\left(F^2 + \dot\varphi^2 + i\psi\dot\psi^* - i\dot\psi\psi^*\right)$$

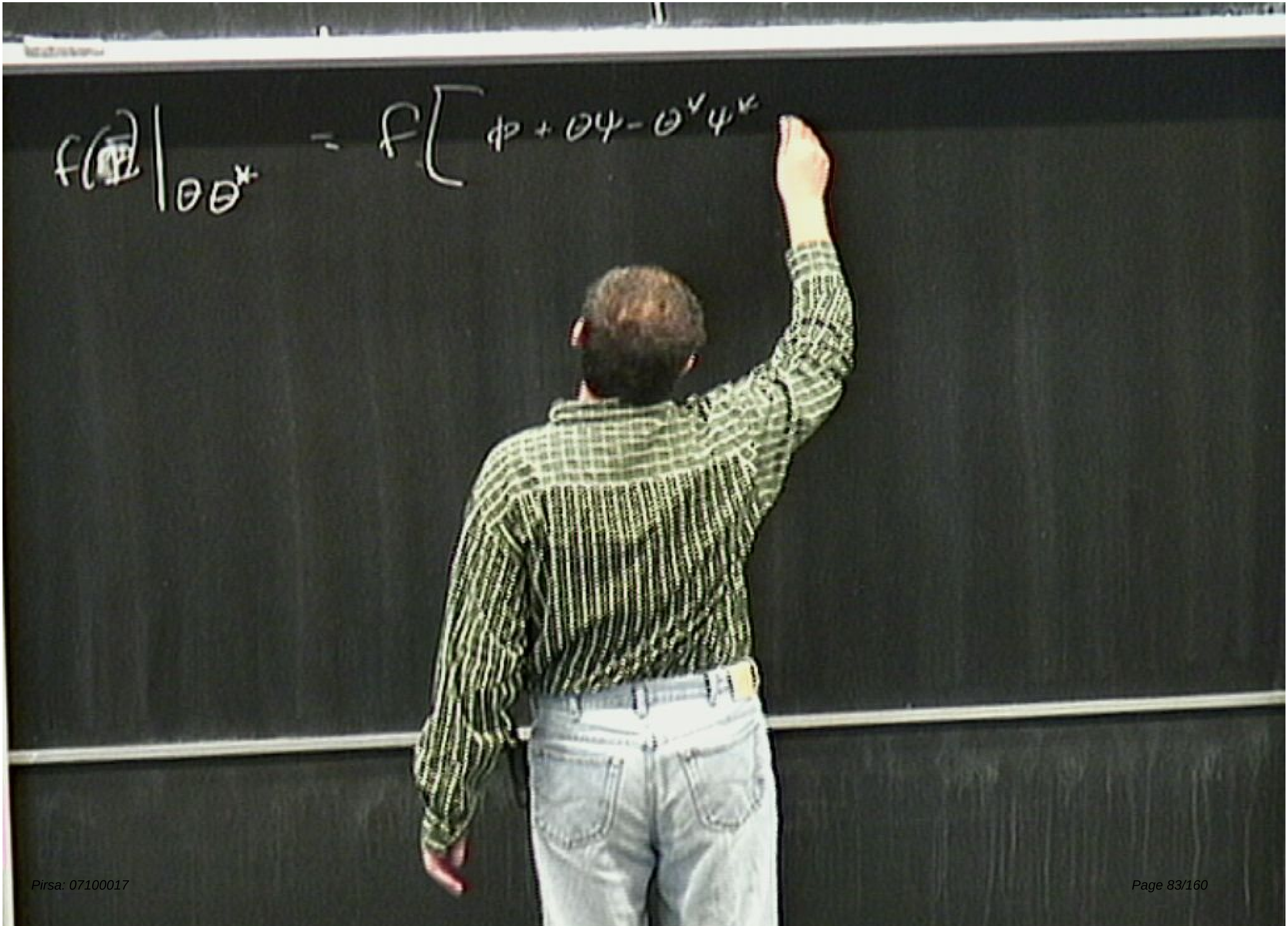
$$D\bar{\Phi}\, D^\dagger \Phi \Big|_{\theta\theta^* \text{ term}} \qquad \theta\theta^* \left[F^2 + \dot\phi^2 + i\psi\dot\psi^* - i\dot\psi\psi^* \right]$$

$$\int d\theta\, d\theta^*\, \theta\theta^* = \frac{\partial}{\partial\theta}\frac{\partial}{\partial\theta^*}\,\theta\theta^* = -\frac{\partial\theta}{\partial\theta}\,\frac{\partial\theta^*}{\partial\theta^*} = -1$$

$$\int d\theta\, d\theta^* \left[-\frac{1}{2} D\Phi\, D^\dagger \Phi \right] = +\frac{1}{2}\left(F^2 + \dot\phi^2 - i\psi\dot\psi^* - i\dot\psi\psi^* \right)$$



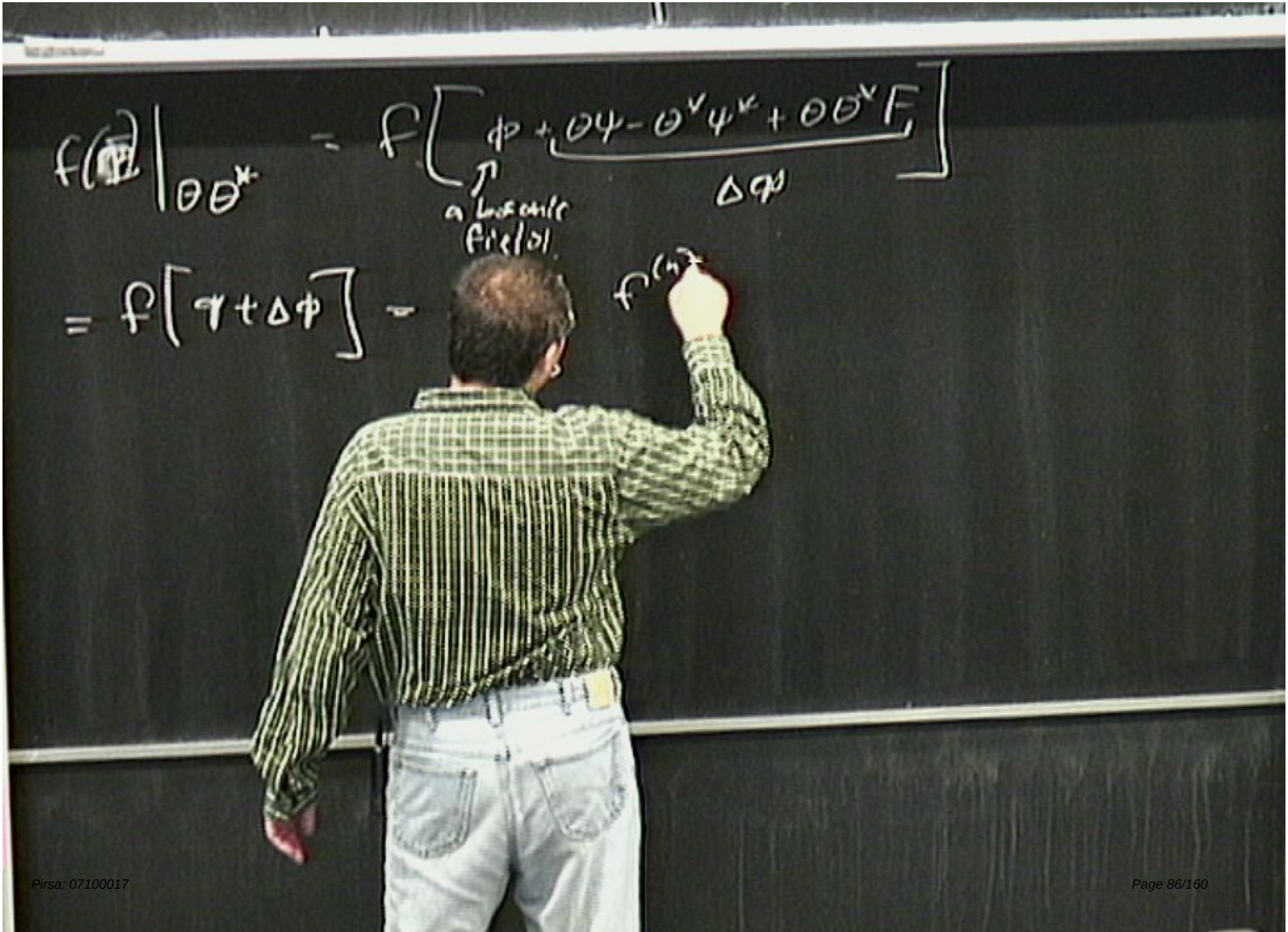
$$f(\partial)\Big|_{\theta\theta^*} = f\Big[$$


= f[φ + θψ − θ*ψ*

$$f(\Phi)\Big|_{\theta\theta^*} = f\left[\phi + \theta\psi - \theta^*\psi^* + \theta\theta^* F\right]$$

$$f(\Phi)\Big|_{\theta\theta^*} = f\Big[\underset{\substack{\uparrow \\ \text{a bosonic} \\ \text{field}}}{\phi} + \underbrace{\theta\psi - \theta^*\psi^* + \theta\theta^* F}_{\Delta\phi}\Big]$$

$$= f\left[\phi + \Delta\phi\right]$$


a bosonic field
Δφ

$$f(\Phi)\Big|_{\theta\theta^*} = f\Big[\phi + \underbrace{\theta\psi - \theta^*\psi^* + \theta\theta^* F}_{\Delta\phi}\Big]$$

(ϕ ← a bosonic field)

$$= f[\phi + \Delta\phi] = \sum_{n=0}^{\infty} \frac{f^{(n)}(\phi)\,(\Delta\phi)^n}{n!}$$

$$f(\Phi)\Big|_{\theta\theta^*} = f\Big[\underset{\substack{\uparrow \\ \text{a bosonic} \\ \text{field}}}{\phi} + \underbrace{\theta\psi - \theta^*\psi^* + \theta\theta^* F}_{\Delta\phi}\Big]$$

$$= f\left[\phi + \Delta\phi\right] = \sum_{n=0}^{\infty} \frac{f^{(n)}(\phi)\,(\Delta\phi)^n}{n!}$$

$$(\Delta\phi)^3 \equiv 0$$

$$f(\Phi)\Big|_{\theta\theta^*} = f\Big[\underset{\substack{\uparrow \\ \text{a bosonic} \\ \text{field}}}{\phi} + \underbrace{\theta\psi - \theta^*\psi^* + \theta\theta^* F}_{\Delta\phi}\Big]$$

$$= f[\phi + \Delta\phi] = \sum_{n=0}^{\infty} \frac{f^{(n)}(\phi)\,(\Delta\phi)^n}{n!}$$

$$(\Delta\phi)^3 \equiv 0$$

$$= f(\phi) +$$

$$f(\Phi)\Big|_{\theta\theta^*} = f\Big[\underset{\uparrow}{\phi} + \underbrace{\theta\psi - \theta^*\psi^* + \theta\theta^* F}_{\Delta\phi}\Big]$$

a bosonic field

$$= f[\phi + \Delta\phi] = \sum_{n=0}^{\infty} \frac{f^{(n)}(\phi)\,(\Delta\phi)^n}{n!}$$

$$(\Delta\phi)^3 \equiv 0$$

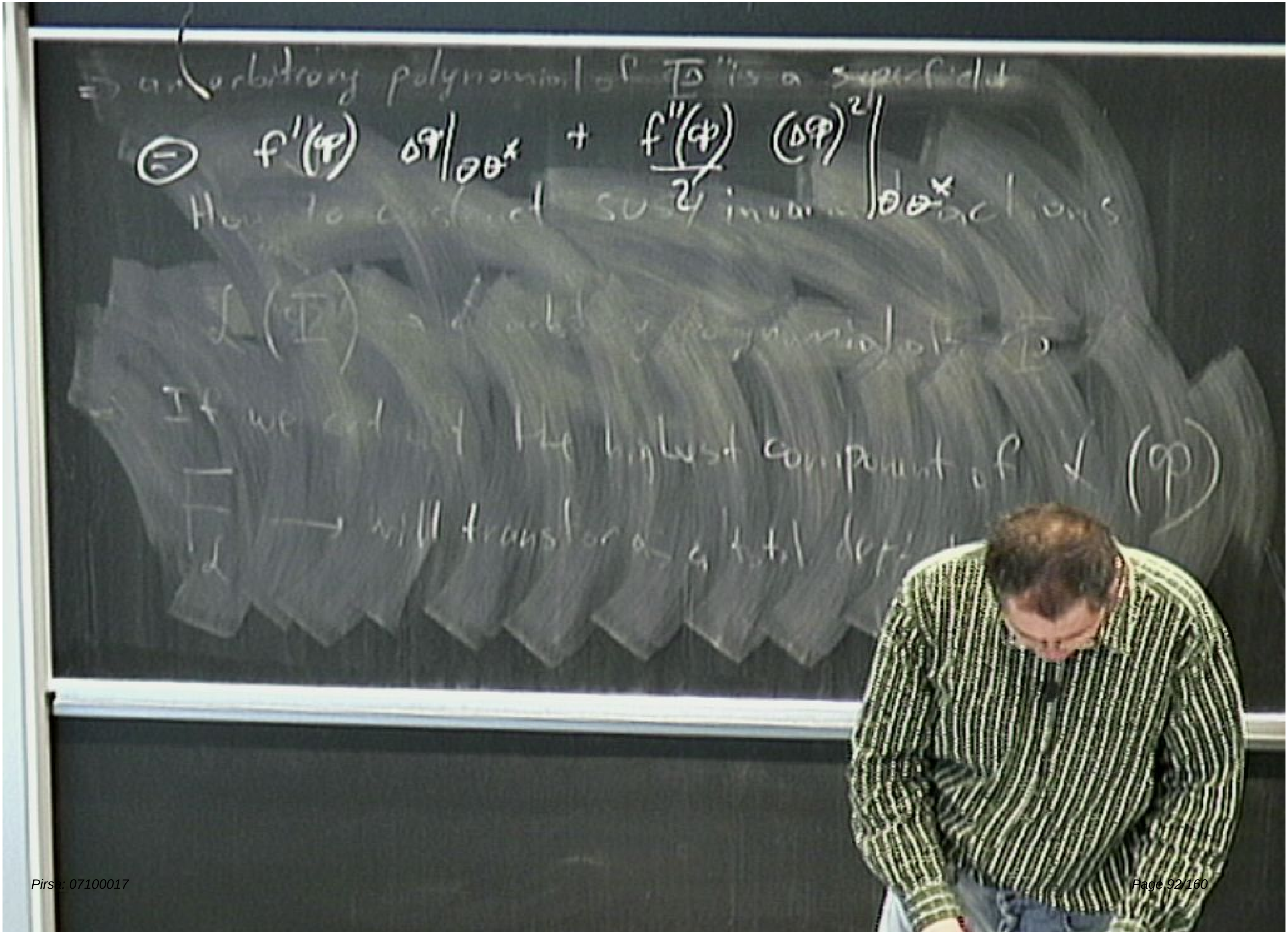
$$= f(\phi) + f'(\phi)\cdot\Delta\phi + \frac{f''(\phi)}{2}\cdot\Delta\phi^2$$

$$f(\Phi)\Big|_{\theta\theta^*} = f\Big[\underset{\substack{\uparrow \\ \text{a bosonic} \\ \text{field}}}{\phi} + \underbrace{\theta\psi - \theta^*\psi^* + \theta\theta^* F}_{\Delta\phi}\Big]$$

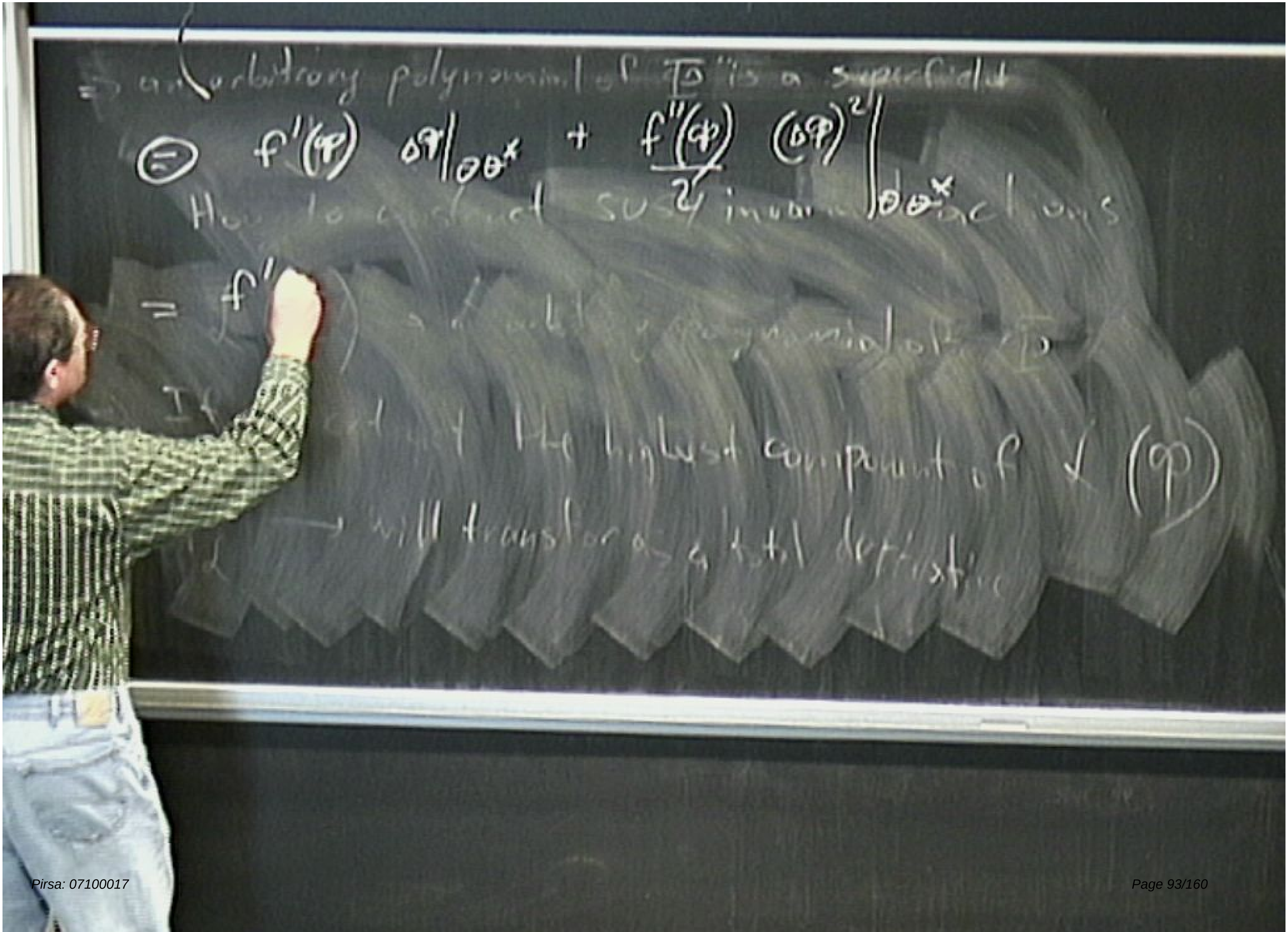
$$= f\left[\phi + \Delta\phi\right] = \sum_{n=0}^{\infty} \frac{f^{(n)}(\phi)\,(\Delta\phi)^n}{n!}$$

$$(\Delta\phi)^3 \equiv 0$$

$$= f(\phi) + f'(\phi)\cdot\Delta\phi + \frac{f''(\phi)}{2}\cdot\Delta\phi^2 \Bigg|_{\theta\theta^* \text{ term}}$$



an arbitrary polynomial of Φ is a superfield
f'(φ) ∂φ/∂θ^α + f''(φ)/2 (Dφ)^2 | ∂θ^α
How to construct SUSY invariant actions
If we ... the highest component of L(φ)
F_L → will transform as a total derivative


⇒ an arbitrary polynomial of Φ is a superfield
f'(φ) Dφ|θθ* + f''(φ)/2 (Dφ)² |θθ*
How to construct SUSY invariant actions
the highest component of ℒ(φ)
will transform as a total derivative

⇒ an arbitrary polynomial of "Φ" is a superfield

$$\ominus \quad f'(\varphi)\, D\varphi\big|_{\partial\theta^\alpha} + \frac{f''(\varphi)}{2}\,(D\varphi)^2\Big|_{\partial\theta^\alpha}$$

How to construct SUSY invariant actions

$$= f' F + \frac{f''(\varphi)}{2} \ldots$$

$\mathcal{L}(\Phi)$ will be polynomial of Φ

If we add only the highest component of $V(\varphi)$

$F_\alpha \longrightarrow$ will transform as a total derivative

⇒ an arbitrary polynomial of Φ is a superfield

$$\ominus \; f'(\varphi)\, \partial\varphi\big|_{\theta\theta^*} + \frac{f''(\varphi)}{2} (\partial\varphi)^2\Big|_{\theta\theta^*}$$

How to construct SUSY invariant actions

$$= f' F + \frac{f''(\varphi)}{2} \cdot \left(-\theta\psi\theta^*\psi^* - \theta^*\psi^*\theta\psi \right)$$

If we add ... the highest component of $V(\varphi)$

$F \longrightarrow$ will transform as a total derivative

$\mathcal{L}$

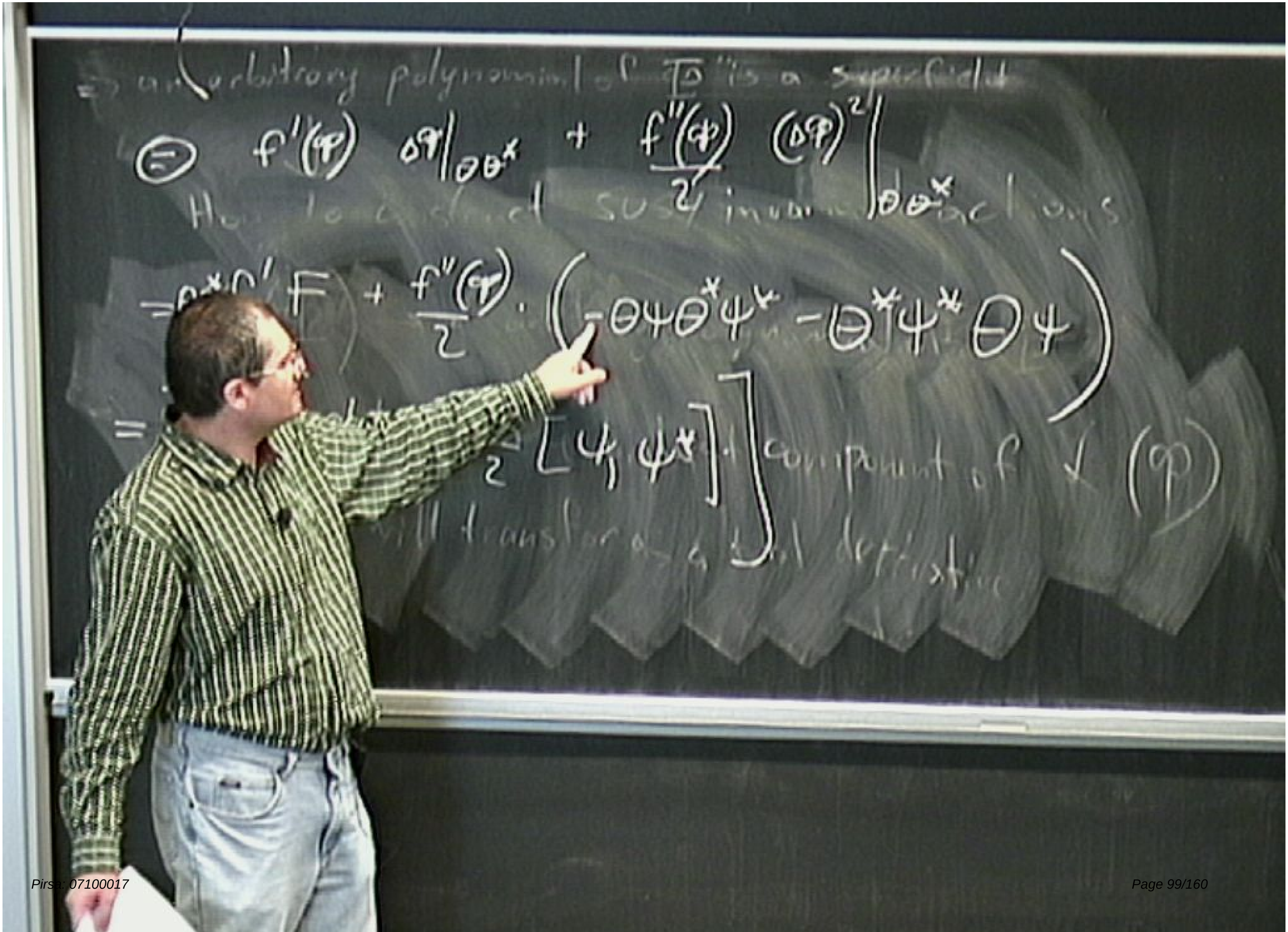
$$= f'(\varphi)\,\delta\varphi\big|_{\theta\theta^*} + \frac{f''(\varphi)}{2}(\delta\varphi)^2\Big|_{\theta\theta^*}$$

$$= \theta\theta^* f' F + \frac{f''(\varphi)}{2}\cdot\left(-\theta\psi\theta^*\psi^* - \theta^*\psi^*\theta\psi\right)$$

$$= \theta\theta^*\left[f' F + \frac{f''}{2}[\psi, \psi^*]\right]$$

$$S_{SUSY} = \int dt \Big[\tfrac{1}{2} F^2 + \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2}\, i \psi \dot{\psi}^* - i \dot{\psi} \psi^*$$

$$S_{SUSY} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{i}{2} \dot{\psi} \psi^* - f' F - \frac{f''}{2} [\psi, \psi^*] \right]$$


an arbitrary polynomial of Φ is a superfield
f'(φ) Dφ|θθ* + f''(φ)/2 (Dφ)²|θθ*
How to construct SUSY invariant actions
f''(φ)/2 · (-θψθ*ψ* - θ*ψ*θψ)
[ψ, ψ*]
component of
will transform

$$S_{SUSY} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{i}{2} \dot{\psi} \psi^* - f' F - \frac{f''}{2} [\psi, \psi^*] \right]$$

Before:

$$S_{SUSY} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{i}{2} \dot{\psi} \psi^* - f' F - \frac{f''}{2} [\psi, \psi^*] \right]$$

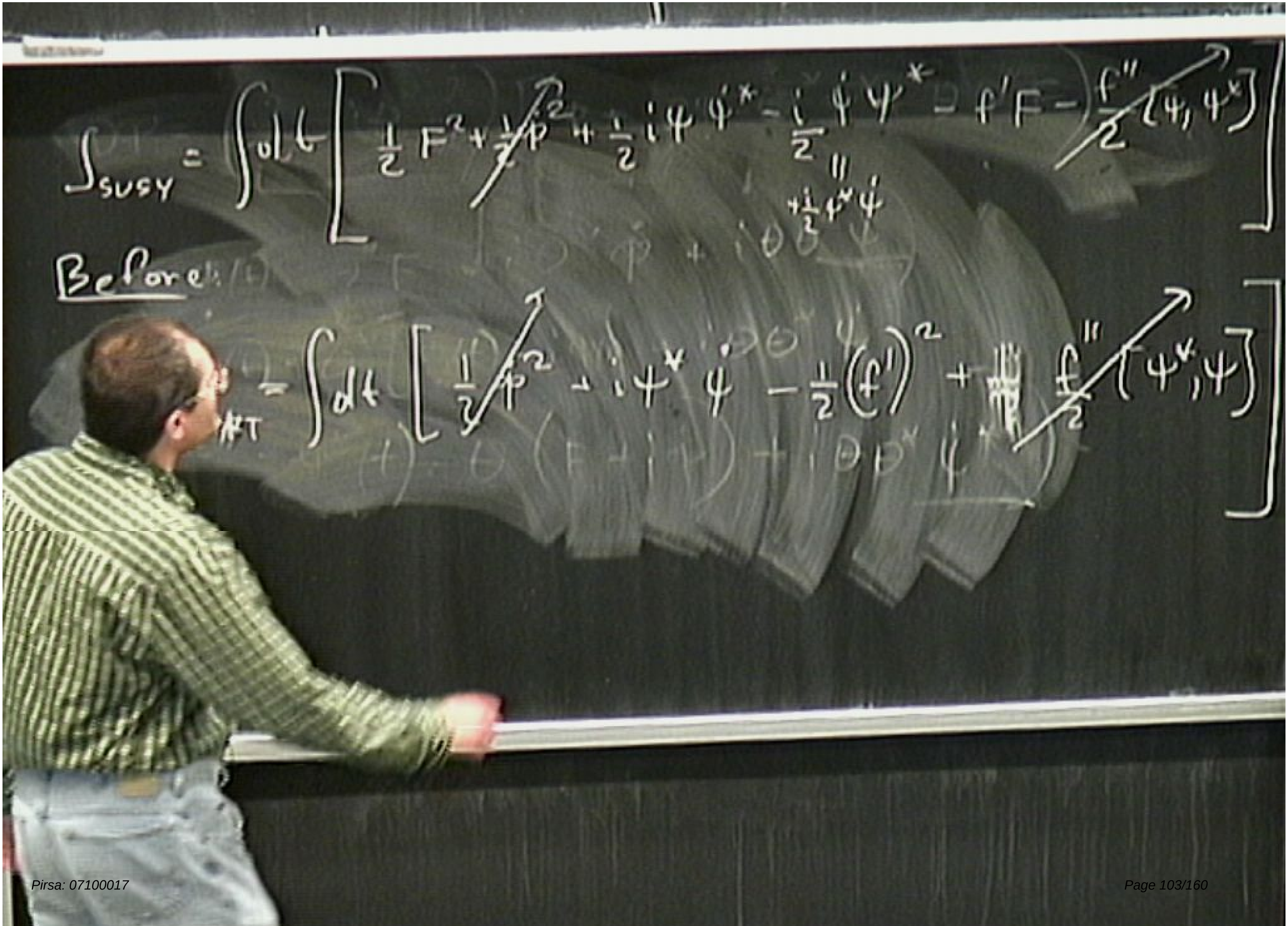
Before:

$$S_{0+1\ SQFT} = \int dt \left[\frac{1}{2} \dot{\phi}^2 + i \psi^* \dot{\psi} - \frac{1}{2} (f')^2 + \frac{1}{2} \frac{f''}{2} [\psi^*, \psi] \right]$$

$$S_{SUSY} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2}\dot\phi^2 + \frac{1}{2} i \psi \dot\psi^* - \frac{i}{2} \dot\psi \psi^* - f' F - \frac{f''}{2} [\psi, \psi^*] \right]$$

Before:

$$S_{0+1\ SQM} = \int dt \left[\frac{1}{2}\dot\phi^2 + i\psi^* \dot\psi - \frac{1}{2}(f')^2 + \frac{f''}{2} [\psi^*, \psi] \right]$$



$S_{SUSY} = \int dt \left[\frac{1}{2}F^2 + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{i}{2} \dot{\psi} \psi^* - f' F - \frac{f''}{2} [\psi, \psi^*] \right]$
$= +\frac{i}{2}\psi^* \dot{\psi}$
Before:
$S = \int dt \left[\frac{1}{2}\dot{\phi}^2 - i \psi^* \dot{\psi} - \frac{1}{2}(f')^2 + \frac{f''}{2} [\psi^*, \psi] \right]$

$$S_{SUSY} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{i}{2} \dot{\psi} \psi^* - f' F - \frac{f''}{2} [\psi, \psi^*] \right]$$

$$-\frac{i}{2}\dot{\psi}\psi^* \qquad = +\frac{i}{2}\psi^*\dot{\psi}$$

Before:

$$S_{old\ SQ/FT} = \int dt \left[\frac{1}{2}\dot{\phi}^2 + i \psi^* \dot{\psi} - \frac{1}{2}(f')^2 + \frac{f''}{2} [\psi^*, \psi] \right]$$

$$S_{SUSY} = \int dt \left[\frac{1}{2}F^2 + \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{i}{2} \dot{\psi} \psi^* - f' F - \frac{f''}{2}[\psi, \psi^*] \right]$$

$$\frac{1}{2} i \psi \dot{\psi}^* \to -\frac{i}{2}\dot{\psi}\psi^* \Downarrow +\frac{i}{2}\psi^*\dot{\psi} \qquad -\frac{i}{2}\dot{\psi}\psi^* = +\frac{i}{2}\psi^*\dot{\psi}$$

<u>Before:</u>

$$S_{0+1\ SQM} = \int dt \left[\frac{1}{2}\dot{\varphi}^2 + i\psi^* \dot{\psi} - \frac{1}{2}(f')^2 + \frac{f''}{2}[\psi^*, \psi] \right]$$

$$S_{SUSY} = \int dt \left[\frac{1}{2}F^2 + \frac{1}{2}\dot{\phi}^2 + \frac{i}{2}\psi\dot{\psi}^* - \frac{i}{2}\dot{\psi}\psi^* - f'F - \frac{f''}{2}[\psi,\psi^*] \right]$$

$$-\frac{i}{2}\dot{\psi}\psi^* \Updownarrow +\frac{i}{2}\psi^*\dot{\psi}$$

$$+\frac{i}{2}\psi^*\dot{\psi}$$

Before:

$$S_{old\ SQ\#T} = \int dt \left[\frac{1}{2}\dot{\phi}^2 - i\psi^*\dot{\psi} - \frac{1}{2}(f')^2 + \frac{f''}{2}[\psi^*,\psi] \right]$$

$$S_{SUSY} = \int dt \left[\frac{1}{2}F^2 + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2} i\psi\dot{\psi}^* - \frac{i}{2}\dot{\psi}\psi^* - f'F - \frac{f''}{2}[\psi,\psi^*] \right]$$

$$-\frac{i}{2}\dot{\psi}\psi^* \Downarrow +\frac{i}{2}\psi^*\dot{\psi}$$

Before:

$$S_{0+1\ SQM} = \int dt \left[\frac{1}{2}\dot{\phi}^2 - i\psi^*\dot{\psi} - \frac{1}{2}(f')^2 + \frac{f''}{2}[\psi^*,\psi] \right]$$

$$\Delta S_{SUSY} = \int dt \left[\frac{1}{2}F^2 - f'F \right]$$

$$S_{SUSY} = \int dt \left[\frac{1}{2}F^2 + \frac{1}{2}\dot{\phi}^2 + \frac{i}{2}\psi\dot{\psi}^* - \frac{i}{2}\dot{\psi}\psi^* - f'F - \frac{f''}{2}[\psi, \psi^*] \right]$$

$$-\frac{i}{2}\dot{\psi}\psi^* \quad +\frac{i}{2}\psi^*\dot{\psi}$$

$$+\frac{i}{2}\psi^*\dot{\psi}$$

Before:

$$S_{0+1\ SQM} = \int dt \left[\frac{1}{2}\dot{\phi}^2 + i\psi^*\dot{\psi} - \frac{1}{2}(f')^2 + \frac{f''}{2}[\psi^*, \psi] \right]$$

$$S_{susy} = \int dt \left[\frac{1}{2}F^2 - f'F \right]$$

$F(t)$

$$S_{SUSY} = \int dt \left[\frac{1}{2}F^2 + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{i}{2}\dot{\psi}\psi^* - f'F - \frac{f''}{2}[\psi, \psi^*] \right]$$

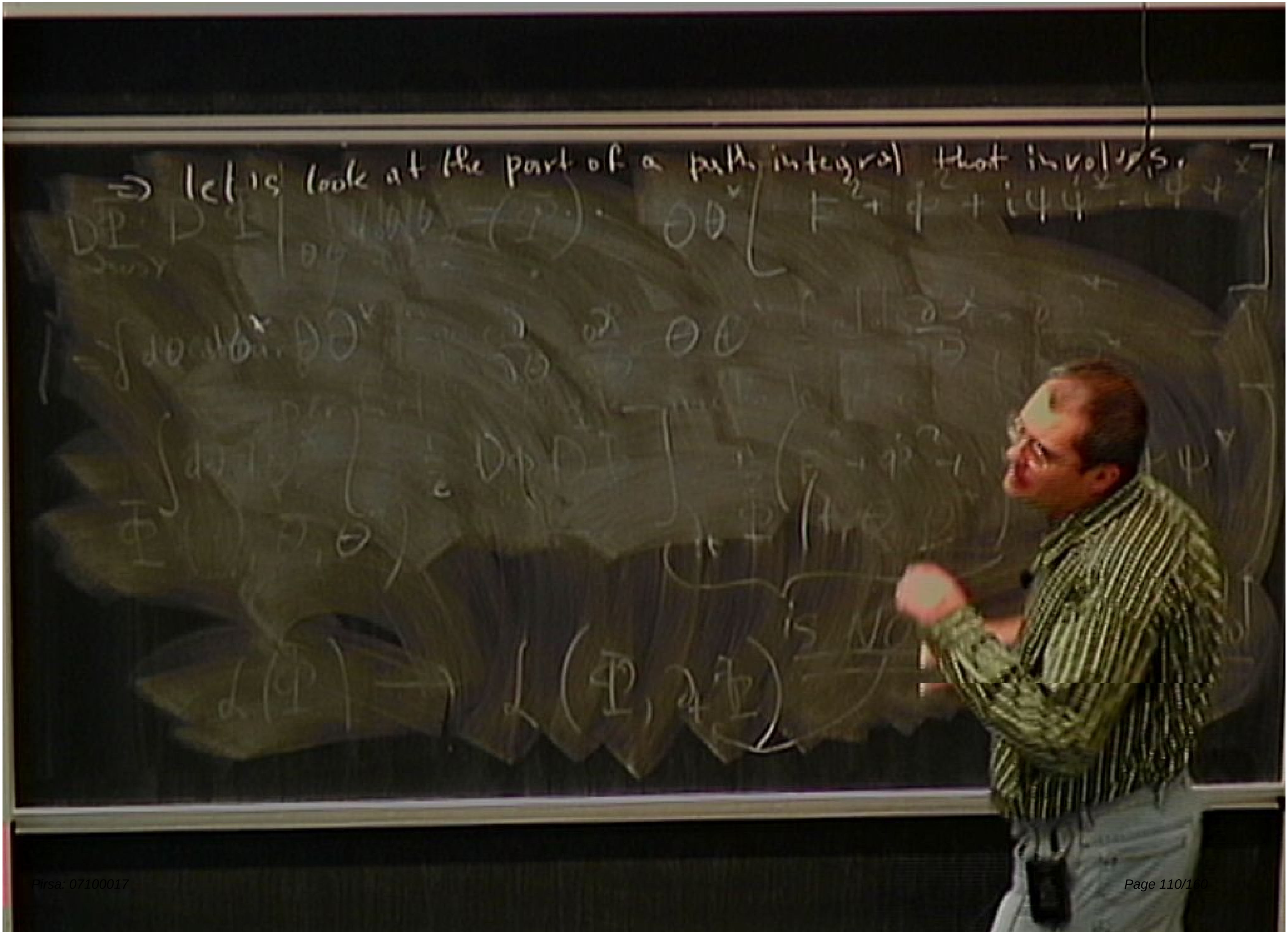
$$-\frac{i}{2}\dot{\psi}\psi^* \Downarrow +\frac{i}{2}\psi^*\dot{\psi} \qquad +\frac{i}{2}\psi^*\dot{\psi}$$

Before:

$$S_{\text{osc sq pot}} = \int dt \left[\frac{1}{2}\dot{\phi}^2 + i\psi^*\dot{\psi} - \frac{1}{2}(f')^2 + \frac{f''}{2}[\psi^*, \psi] \right]$$

$$\Delta S_{SUSY} = \int dt \left[\frac{1}{2}F^2 - f'F \right] \qquad F(t)$$

$\Rightarrow$ we can integrate out F **exactly**


let us look at the part of a path integral that involves

⇒ let's look at the part of a path integral that involves F (F enters quadratically into Lagrangian)

$$\int [dF]$$

⇒ let's look at the part of a path integral that involves F (F enters quadratically into Lagrangian)

$$\int [dF]\, e^{iS} = \int [dF]\, e^{i\int dt\,\left(\frac{1}{2}F^2 - F'f'\right)}$$

⇒ let's look at the part of a path integral that involves F (F enters quadratically into Lagrangian)

$$\int [dF]\, e^{iS} = \int [dF]\, e^{i\int dt\,\left(\frac{1}{2}F(t)^2 - F(t)f(t)\right)}$$

$$= \int [dF]\, e^{i\int dt\,\left[\frac{1}{2}(F-f)^2\right]}$$

⇒ let us look at the part of a path integral that involves F (F enters quadratically into Lagrangian)

$$\int [dF]\, e^{iS} = \int [dF]\, e^{i\int dt \left(\frac{1}{2}F(t)^2 - F(t) f(t)\right)}$$

$$= \int [dF]\, e^{i\int dt \left[\frac{1}{2}(F - f)^2\right] + i\int dt \ldots}$$

⇒ let us look at the part of a path integral that involves F (F enters quadratically into Lagrangian)

$$\int [dF]\, e^{iS} = \int [dF]\, e^{i\int dt\, \left(\frac{1}{2}F(t)^2 - F(t) f(t)\right)}$$

$$= \int [dF]\, e^{i\int dt \left[\frac{1}{2}(F - f)^2\right] + i\int dt \left(-\frac{f^2}{2}\right)}$$

⇒ let us look at the part of a path integral that involves F (F enters quadratically into Lagrangian)

$$\int [dF]\, e^{iS} = \int [dF]\, e^{i\int dt \left(\frac{1}{2}F(t)^2 - F(t) f(t)\right)}$$

$$= \int [dF]\, e^{i\int dt \left[\frac{1}{2}(F - f)^2\right] + i\int dt \left(-\frac{(f)^2}{2}\right)}$$

$$= e^{i\int dt\left(-\frac{(f)^2}{2}\right)} \int [d\hat{F}]$$

⇒ let us look at the part of a path integral that involves F (F enters quadratically into Lagrangian)

$$\int [dF]\, e^{iS} = \int [dF]\, e^{i\int dt\,\left(\frac{1}{2}F(t)^2 - F(t) f'(t)\right)}$$

$$= \int [dF]\, e^{i\int dt\,\left[\frac{1}{2}(F - f')^2\right] + i\int dt\,\left(-\frac{(f')^2}{2}\right)}$$

$$\hat{F} \overset{\text{def}}{\equiv} F - f'$$

$$e^{i\int dt\left(-\frac{(f')^2}{2}\right)} \int [d\hat{F}]$$

⇒ let us look at the part of a path integral that involves

F (F enters quadratically into Lagrangian)

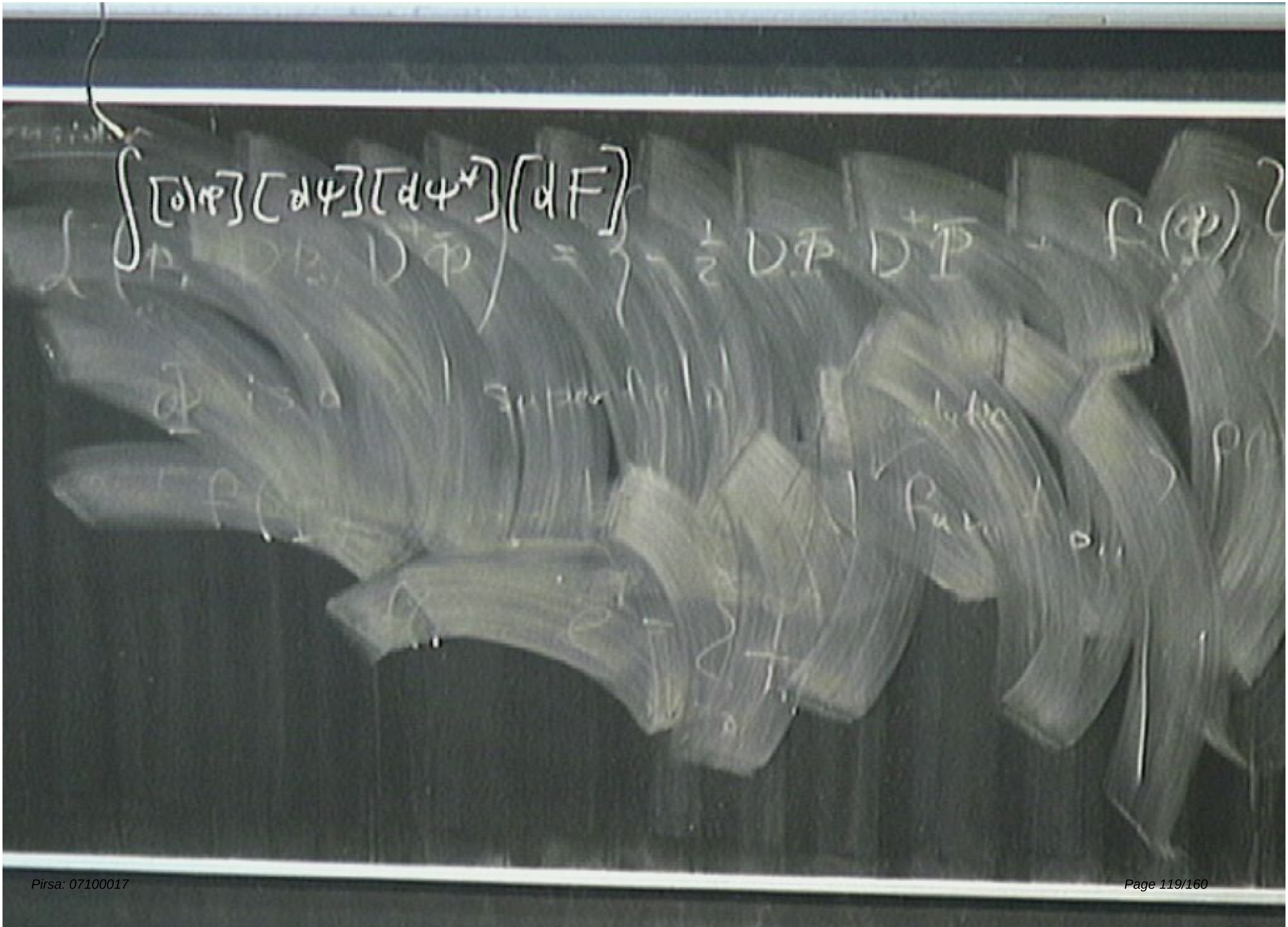
is

$$\int [dF]\, e^{\ldots} = \int [dF]\, e^{i\int dt \left(\frac{1}{2}F(t)^2 - F(t) f'(t)\right)}$$

$$= \int [dF]\, e^{i\int dt \left[\frac{1}{2}(F - f')^2\right] + i\int dt \left(-\frac{(f')^2}{2}\right)}$$

$$e^{i\int dt \left(-\frac{(f')^2}{2}\right)} \int [d\hat{F}]\, e^{i\int dt \frac{1}{2}\hat{F}^2} \Rightarrow \text{const} \cdot e^{i\int dt \left(-\frac{(f')^2}{2}\right)}$$

$$\hat{F} \overset{\text{def}}{\equiv} F - f'$$


∫[dφ][dψ][dψ*][dF]

$$\int [d\varphi][d\psi][d\psi^*][dF]\, e^{i S_{susy}}$$

$$\int [d\varphi][d\psi][d\psi^*][dF]\; e^{iS_{susy}}$$

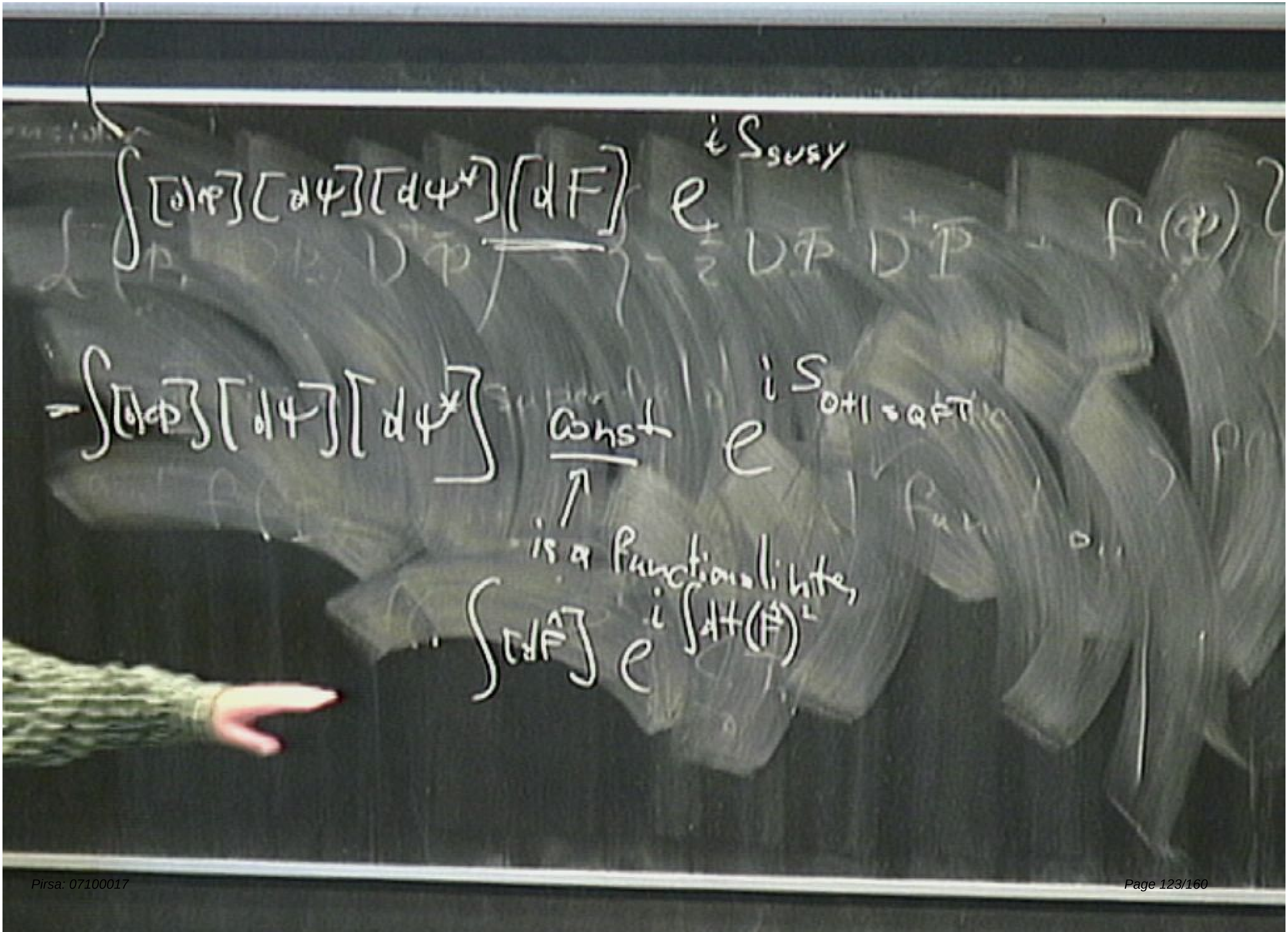
$$= \int [d\varphi][d\psi][d\psi^*]\ \underline{\text{const}}$$

$$\int [d\varphi][d\psi][d\psi^*][dF]\, e^{i S_{susy}}$$

$$= \int [d\varphi][d\psi][d\psi^*] \; \underline{\text{const}}$$

↑ is a functional integral

$$\int [d\hat{F}]\, e^{i\int dt\, (\hat{F})^2}$$


$\int [d\varphi][d\psi][d\psi^*][dF]\, e^{i S_{susy}}$
$= \int [d\varphi][d\psi][d\psi^*]\ \text{const}\ e^{i S_{0+1\ SQFT}}$
const is a functional integral,
$\int [d\hat{F}]\, e^{i \int dt (\hat{F})^2}$

Integrating out F, $D^{\dagger}$ (only exact if F is nondynamical and enters into $\mathcal{L}$ quadratically).

$$S_{new} = \int dt \, d\theta \, d\theta^{*} \, \mathcal{L}\left(\varphi, D\varphi, \overline{D\varphi}\right)$$

Integrating out F, D^* (only exact if F is nondynamical and enters into $\mathcal{L}$ quadratically).

* exact if F is infinitely massive.

Integrating out F. * (only exact if F is nondynamical and enters into $\mathcal{L}$ quadratically).

* exact if F is infinitely massive.

⇒ Solve EOM for F:

$F = f' = \phi$

$S = \int dt\, d\theta\, d\theta^* \, \mathcal{L}(\phi, D\phi, \overline{D\phi})$

Integrating out F

* (only exact if F is nondynamical and enters into $\mathcal{L}$ quadratically).

* exact if F is infinitely massive.

→ Solve EOM for F: (doing saddle point approximation in auxiliary path integral).

$F - f' = 0$

$F = f'$

Integrating out F

* (only exact if F is nondynamical and enters into $\mathcal{L}$ quadratically).
* exact if F is infinitely massive.

⇒ Solve EOM for F: (doing saddle point approximation in path integral).

$$F - f' = 0$$

$$F = f'$$

⇒ substitute back: $\int dt\, d\theta\, d\theta^* \, \mathcal{L}(\varphi, D\varphi, \bar{D}\varphi)$

Integrating out F:

* (only exact if F is nondynamical and enters into $\mathcal{L}$ quadratically).
* exact if F is infinitely massive.

⇒ Solve EOM for F: (doing saddle point approximation in evaluating path integral).

$$F - f' = 0$$

$$F = f'$$

⇒ substitute back: $\int \frac{1}{2}F^2 + f'' F = \left(+ \frac{1}{2}(f')^2 \right)$

$$S_{SUSY} = \int dt \left[\frac{1}{2} F^2 + \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} i \psi \dot{\psi}^* - \frac{i}{2} \dot{\psi} \psi^* - f' F - \frac{f''}{2} [\psi, \psi^*] \right]$$

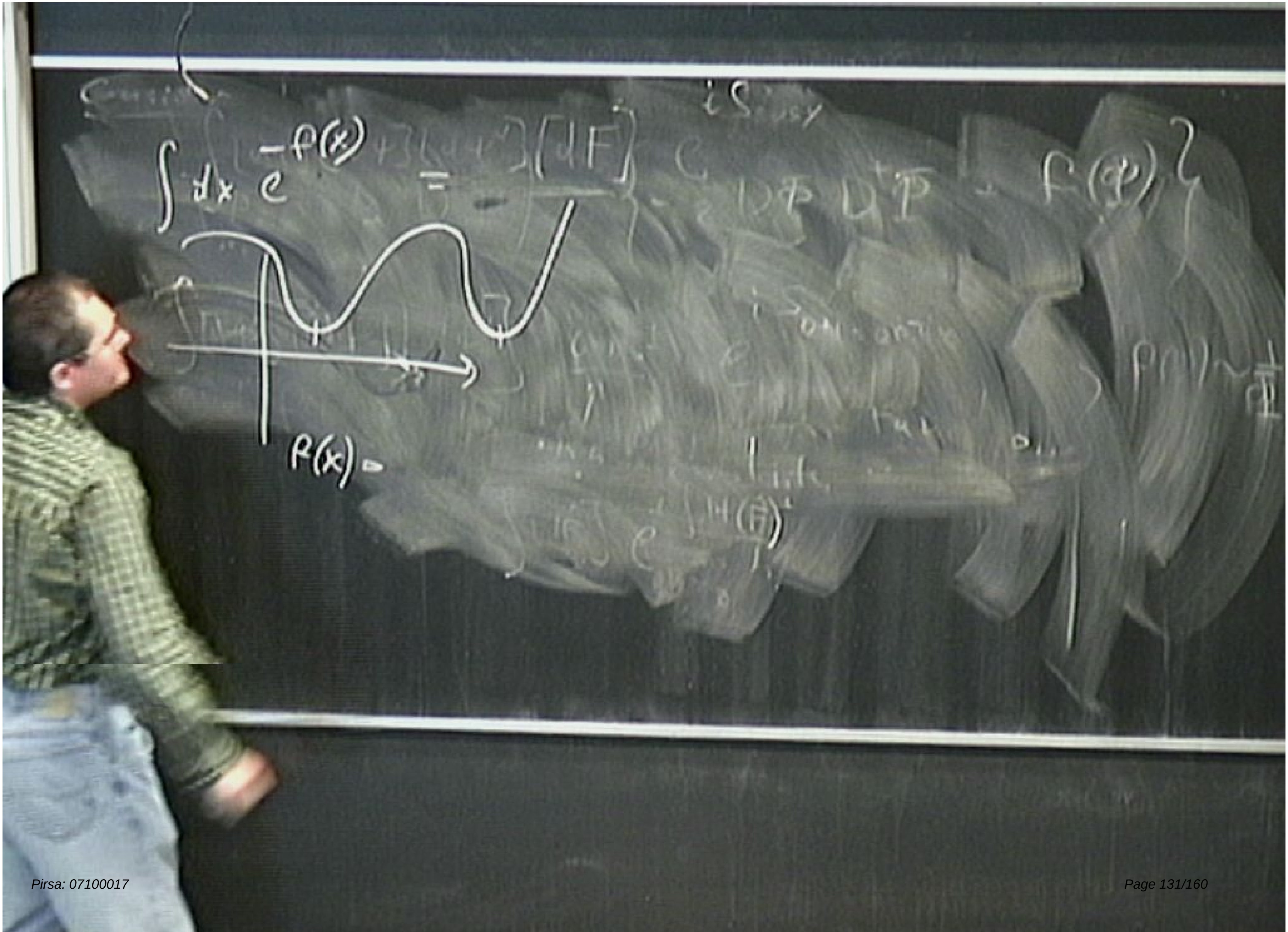
$$-\frac{i}{2}\dot{\psi}\psi^* = +\frac{i}{2}\psi^*\dot{\psi}$$

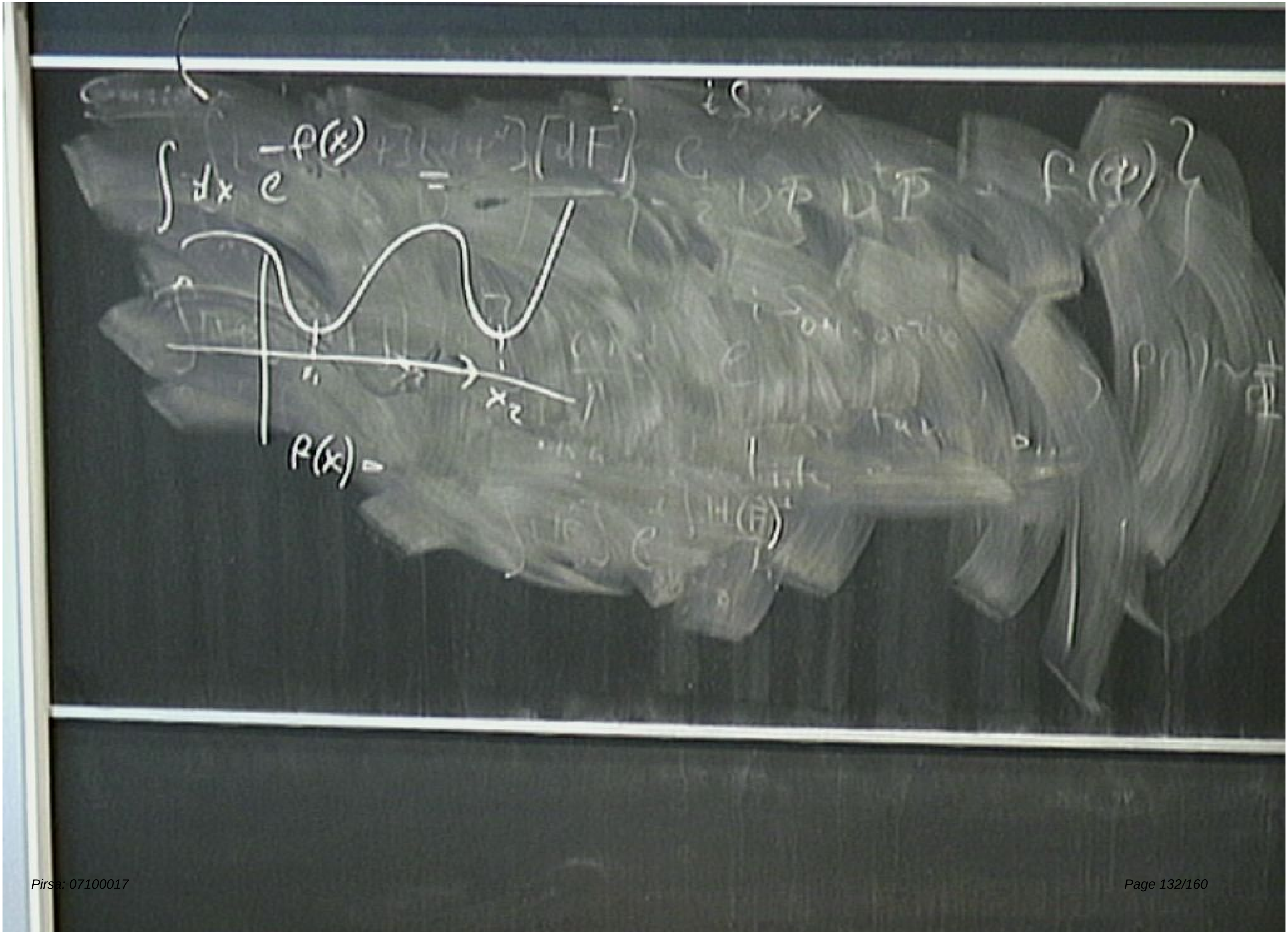
Before:

$$S_{0+1\, SQM} = \int dt \left[\frac{1}{2} \dot{\phi}^2 + i \psi^* \dot{\psi} - \frac{1}{2} (f')^2 + \frac{f''}{2} [\psi^*, \psi] \right]$$

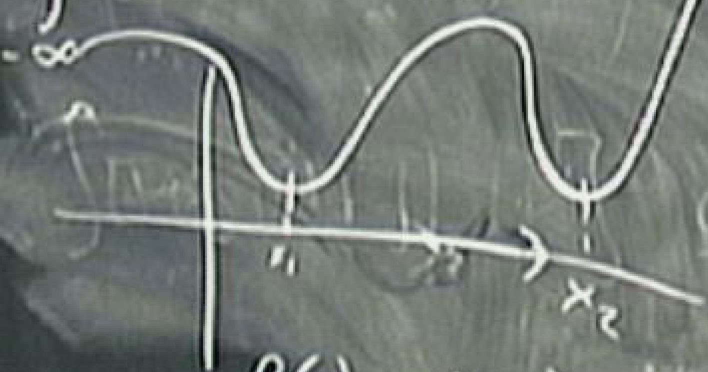
$$\Delta S_{susy} = \int dt \left[\frac{1}{2} F^2 - f' F \right] \qquad F(t)$$

$\Rightarrow$ we can integrate out F **exactly**


f(x)

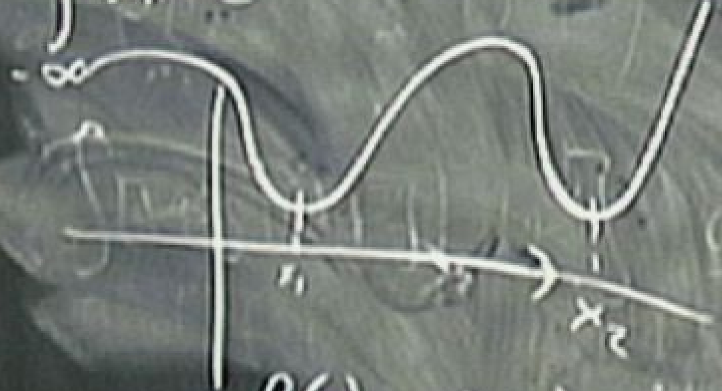

∫dx e^{-F(x)}
f(φ)
x_2
F(x) =

$$\int_{-\infty}^{+\infty} dx\, e^{-f(x)}$$

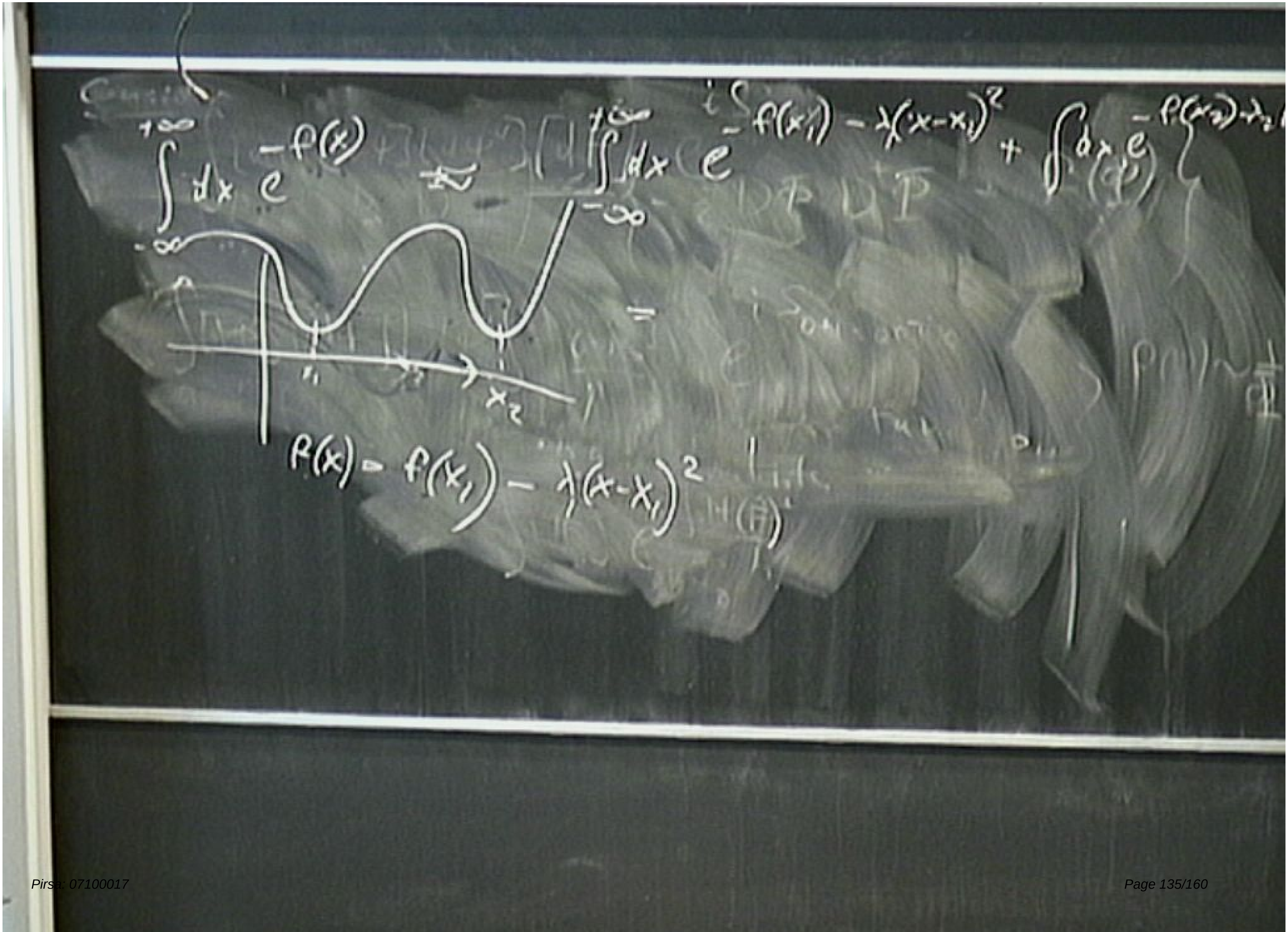


$$f(x) = f(x_1) - \lambda (x - x_1)^2$$

$$\int_{-\infty}^{+\infty} dx\, e^{-f(x)} \approx \int_{-\infty}^{+\infty} dx\, e^{-f(x_1) - \lambda_1 (x-x_1)^2} + \int dx\, e^{-f(x_2) - \lambda_2 \ldots}$$

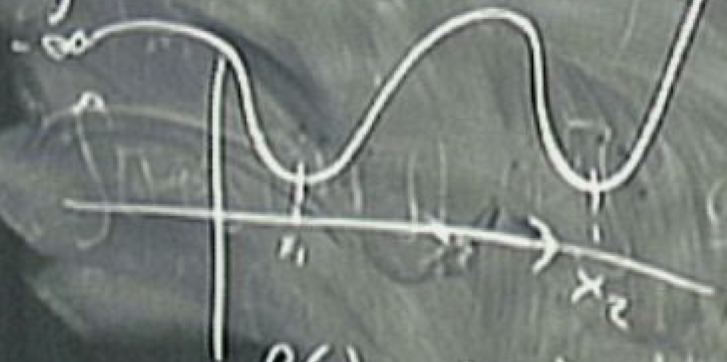


$$f(x) = f(x_1) - \lambda_1 (x - x_1)^2$$



x₁
x₂
f(x) = f(x₁) − λ(x−x₁)²

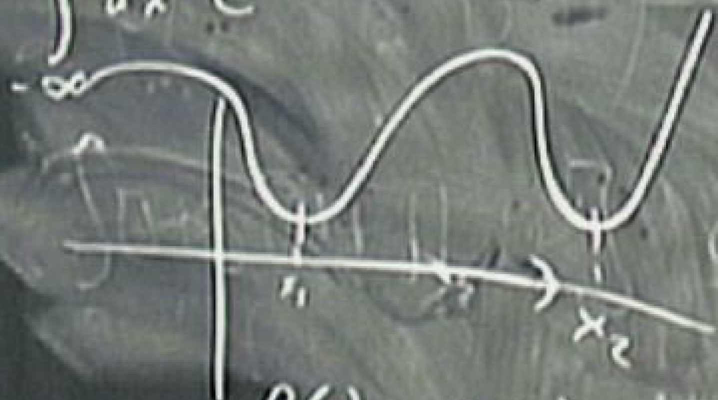
$$\int_{-\infty}^{+\infty} dx\, e^{-f(x)} \qquad \int_{-\infty}^{+\infty} dx\, e^{-f(x_1) - \lambda_1 (x-x_1)^2} + \int dx\, e^{-f(x_2) - \lambda_2 \ldots}$$



$$= \frac{e^{-f(x_1)}}{\sqrt{\lambda_1}} \int dy\, e^{-y^2}$$

$$f(x) = f(x_1) - \lambda_1 (x-x_1)^2$$

$$\int_{-\infty}^{+\infty} dx\, e^{-f(x)} \approx \int_{-\infty}^{+\infty} dx\, e^{-f(x_1) - \lambda_1(x-x_1)^2} + \int dx\, e^{-f(x_2) - \lambda_2(x-x_2)^2}$$



$$= \frac{e^{-f(x_1)}}{\sqrt{\lambda_1}} \int dy\, e^{-y^2} + \frac{e^{-f(x_2)}}{\sqrt{\lambda_2}} \int dy\, e^{-y^2}$$

$$f(x) = f(x_1) - \lambda_1(x-x_1)^2$$

Integrating out F: * (only exact if F is nondynamical and enters into $\mathcal{L}$ quadratically).

* exact if F is infinitely massive.

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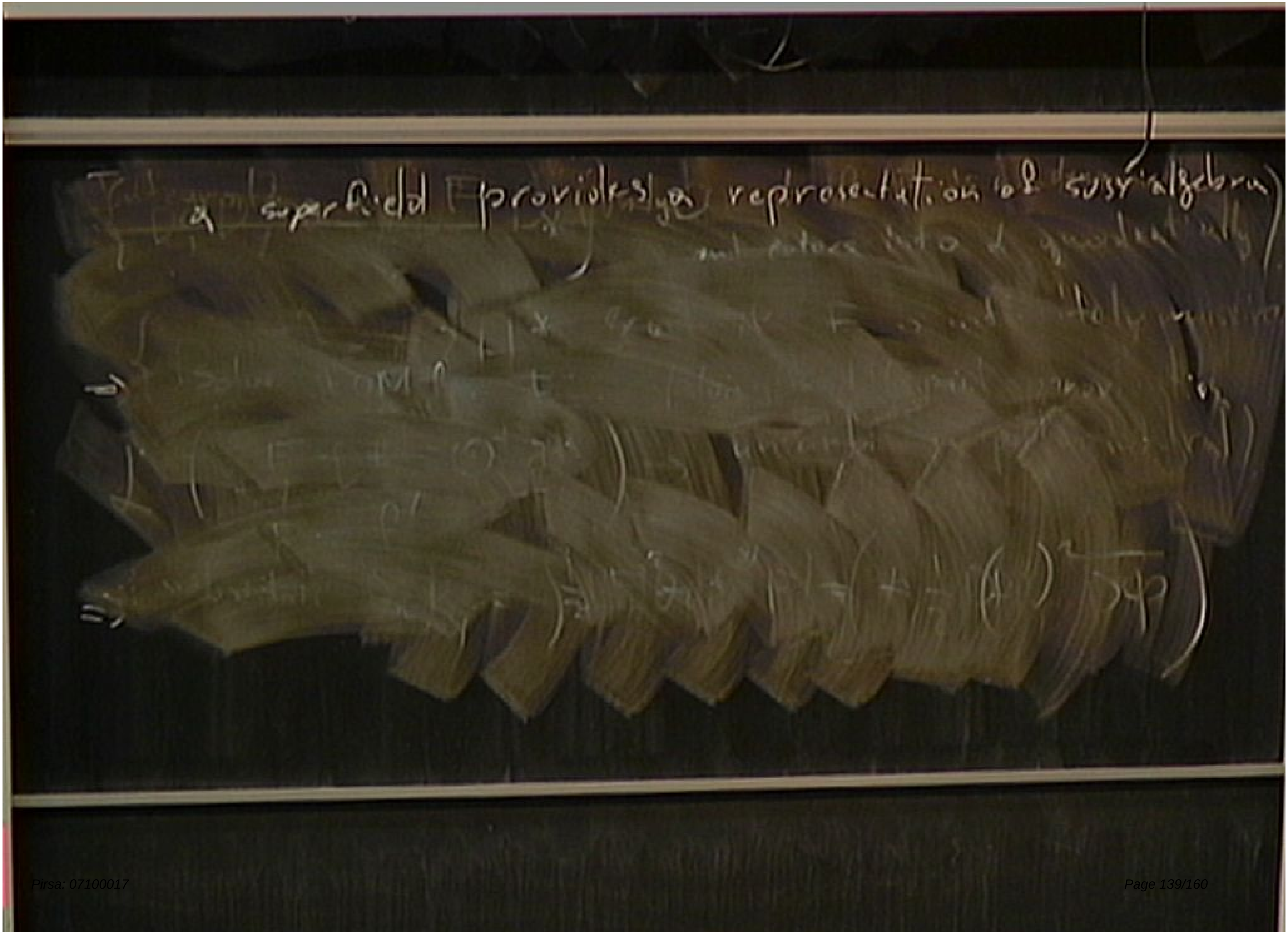
$\Rightarrow$ substitute back: $\frac{1}{2}F^2 + f'' F \to + \frac{1}{2}(f')^2$


a superfield provides a representation of susy algebra

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↑ in general → is reducible

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→ chiral superfields

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⇒ chiral superfields

def: X is χsf if $\bar{D}^{+} X = 0$

a **superfield** provides a representation of susy algebra

↑ in general ... is reducible

⇒ **chiral superfields**

def: X is χsf if $D^+ X = 0$

$D^+ = \frac{\partial}{\partial \theta^+} - i\bar\theta \frac{\partial}{\partial t}$ "chiral coordinates": $\{\theta, \tau = t - i\theta\bar\theta\}$

⇒ $D^+\theta = 0$, $D^+\tau = 0$

a vsf is an arbitrary function of chiral coordinates

$$X = X(\theta, \tau)$$

a vsf is an arbitrary function of chiral coordinates

$$X = X(\theta, \tau)$$

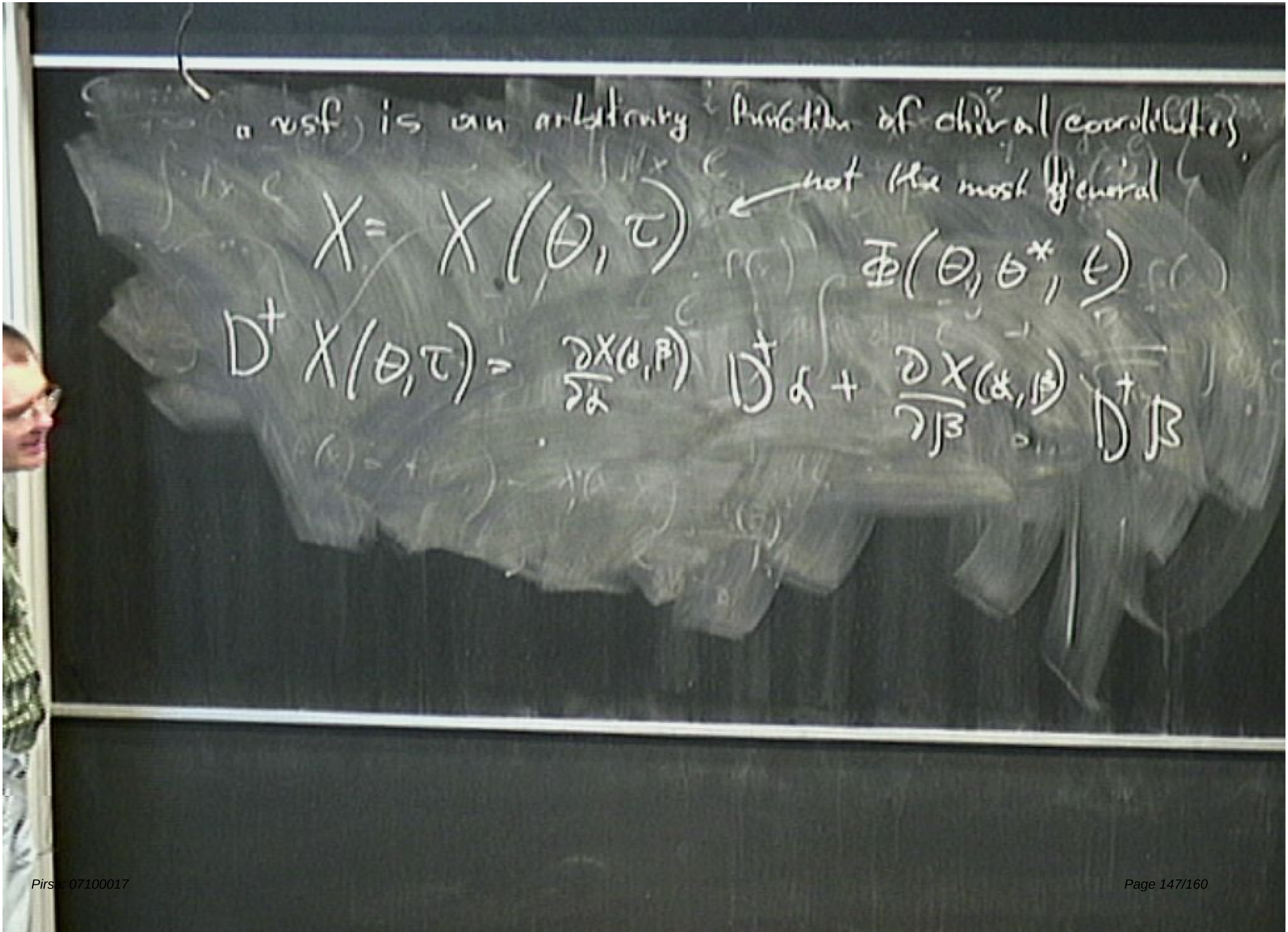
$$D^+ X(\theta,\tau) = \frac{\partial X(\alpha,\beta)}{\partial \alpha} D^+ \alpha + \frac{\partial X(\alpha,\beta)}{\partial \beta} D^+ \beta$$

a vsf is an arbitrary function of chiral coordinates

$$X = X(\theta, \tau) \quad \leftarrow \text{not the most general}$$

$$\Phi(\theta, \theta^*, t)$$

$$D^+ X(\theta,\tau) = \frac{\partial X(\alpha,\beta)}{\partial \alpha} D^+ \alpha + \frac{\partial X(\alpha,\beta)}{\partial \beta} D^+ \beta$$


a vsf is an arbitrary function of chiral coordinates
$X = X(\theta, \tau)$ ← not the most general
$\Phi(\theta, \theta^*, t)$
$D^+ X(\theta,\tau) = \frac{\partial X(\alpha,\beta)}{\partial\alpha} D^+\alpha + \frac{\partial X(\alpha,\beta)}{\partial\beta} D^+\beta$

a vsf is an arbitrary function of chiral coordinates

$$X = X(\theta, \tau)$$ ← not the most general

$$\Phi(\theta, \theta^*, t)$$

$$D^+ X(\theta,\tau) = \frac{\partial X(\alpha,\beta)}{\partial \alpha} D^+\alpha + \frac{\partial X(\alpha,\beta)}{\partial \beta} D^+\beta$$

$$X(\theta,\tau) = \varphi(\tau) + \theta\psi(\tau)$$

a vsf is an arbitrary function of chiral coordinates

$X = X(\theta, \tau)$ ← not the most general

$\Phi(\theta, \theta^*, t)$

$$D^+ X(\theta,\tau) = \frac{\partial X(\alpha,\beta)}{\partial \alpha} D^+ \alpha + \frac{\partial X(\alpha,\beta)}{\partial \beta} D^+ \beta$$

$$X(\theta,\tau) = \varphi(\tau) + \theta \psi(\tau) = \varphi(t) + \theta\psi(t) - i\theta\theta^* \dot{\varphi}(t)$$

$\tau = t - i\theta\theta^*$

$\int \bar{\Psi}\Phi$ get rid of

$\underbrace{D^{+}\Phi}_{\text{automatically chiral}}$

$$D^{+}\left[D^{+}\Phi\right] = (D^{+})^{2}\Phi = 0 \qquad \Phi'' \to 0$$

$\int \bar{\Phi} =$ get x s.f

$\underbrace{D^+ \bar{\Phi}}$ automatically chiral!

$D^+ \left[D^+ \bar{\Phi} \right] = (D^+)^2 \bar{\Phi} = 0 \quad \bar{\Phi} \to 0$

$\int_x = \int dt \, \underbrace{d\theta}$

½ of all Grassmann

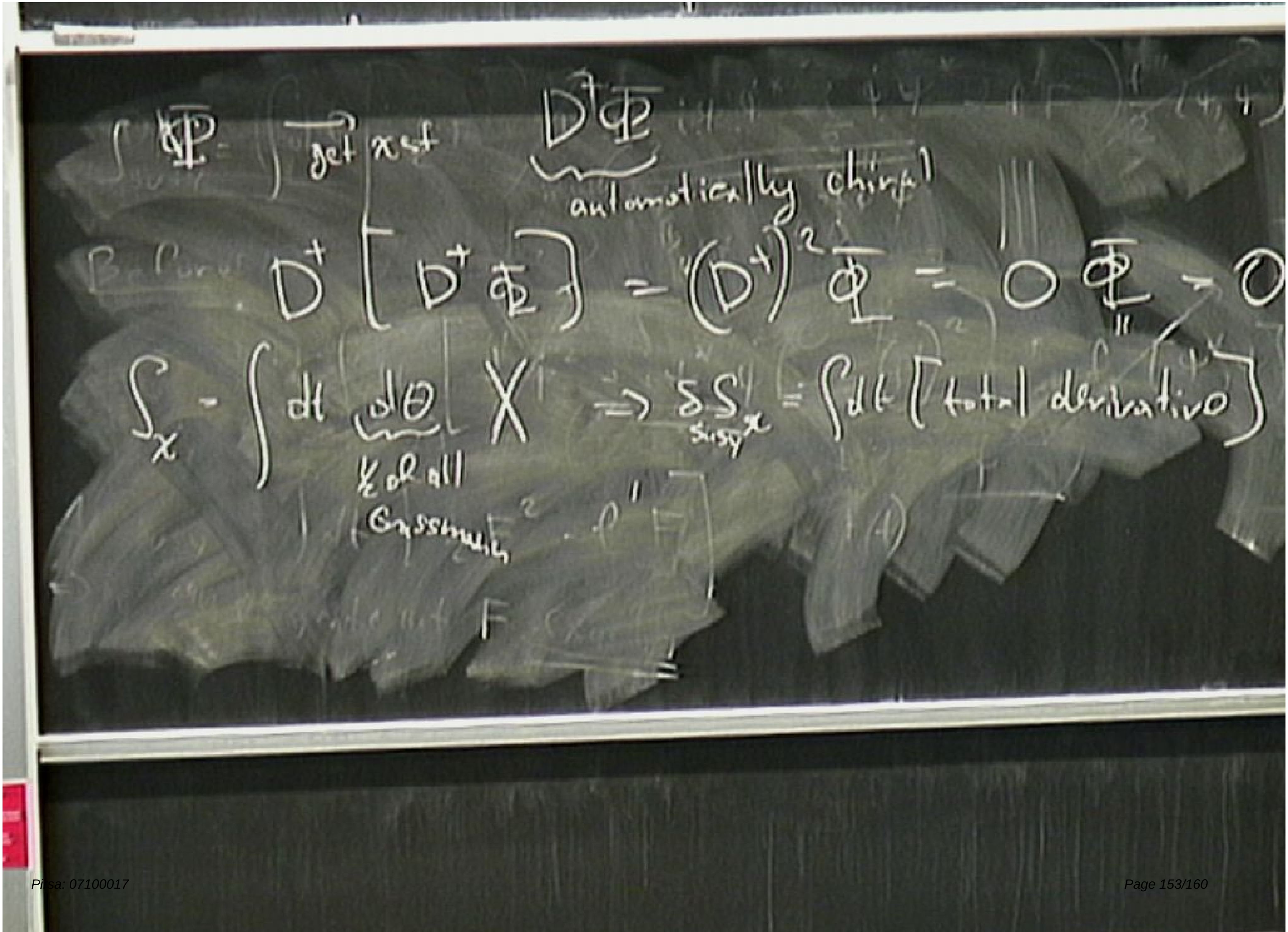
$\int \bar{\Psi}$ ... get $x \not\!f$

$D^{+}\bar{\Phi}$ automatically chiral

$D^{+}\left[D^{+}\bar{\Phi}\right] = (D^{+})^{2}\bar{\Phi} = 0$ $\bar{\Phi} \to 0$

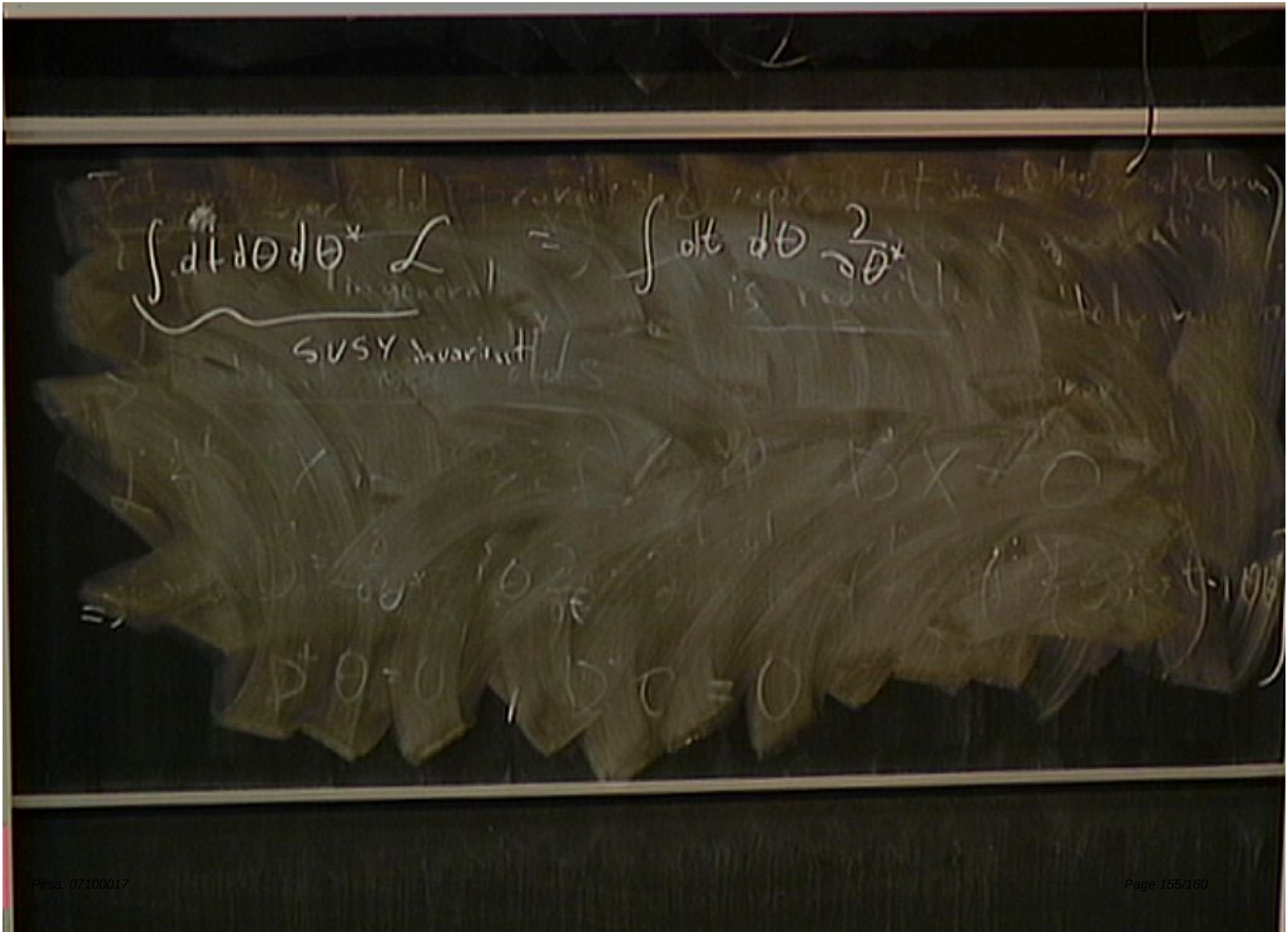
$\int_{x} = \int dt \, d\theta \left[X \right.$

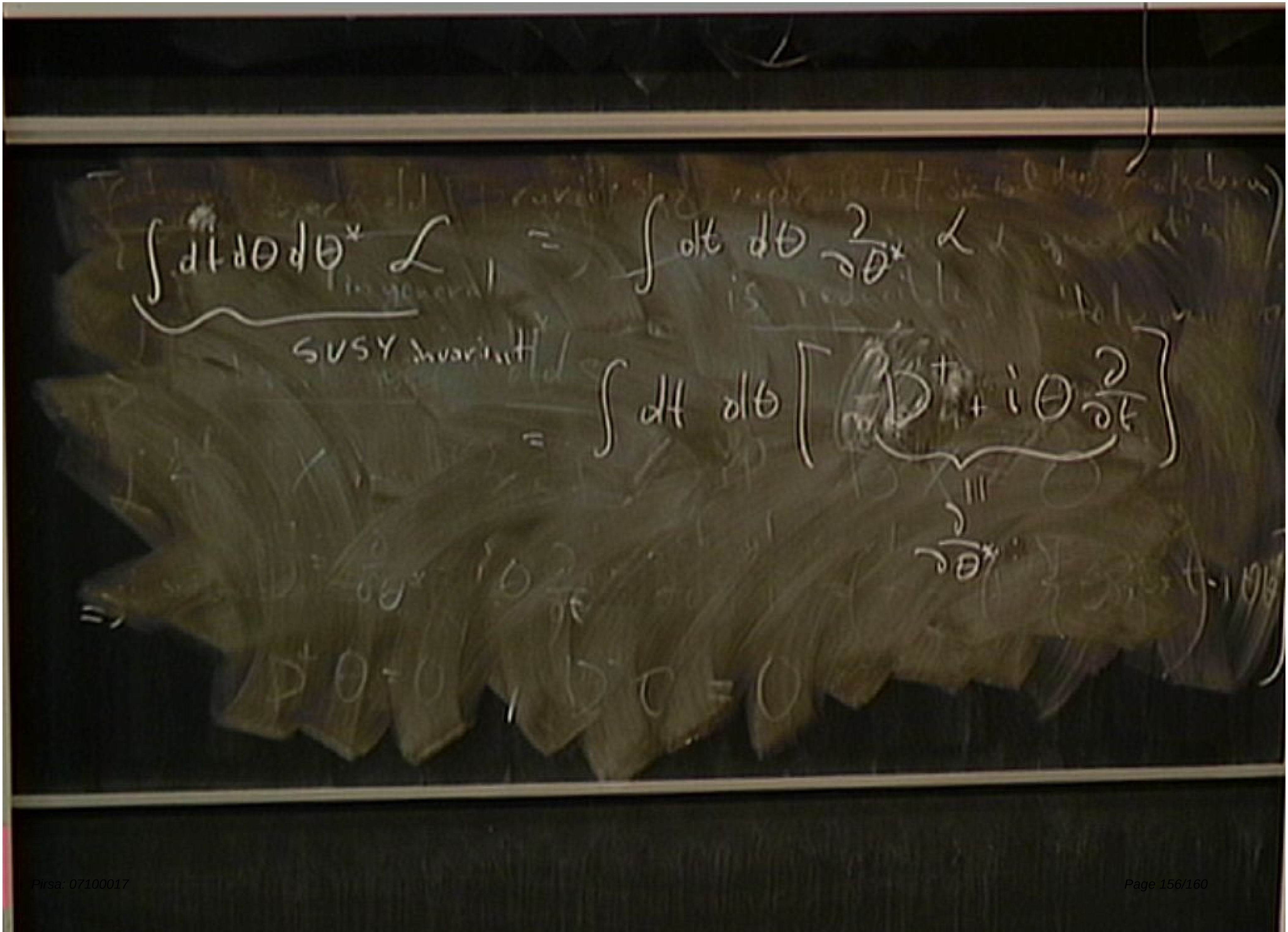
$d\theta$: for all Grassmann


automatically chiral
$D^+ [D^+ \bar{\Phi}] = (D^+)^2 \bar{\Phi} = 0$
$\int dt \, d\theta \, X \Rightarrow \delta S = \int dt$ [total derivative]
of all
Grassmann

$$\int dt\, d\theta\, d\theta^* \mathcal{L}$$

SUSY invariant


$\int dt\, d\theta\, d\theta^* \mathcal{L} = \int dt\, d\theta \frac{\partial}{\partial\theta^*}$
SUSY invariant


∫dt dθ dθ* L = ∫dt dθ ∂/∂θ* L
in general
is
SUSY invariant
= ∫dt dθ [D† + iθ ∂/∂t]
∂/∂θ*

$$\underbrace{\int dt\, d\theta\, d\theta^* \, \mathcal{L}}_{\text{SUSY invariant}} = \int dt\, d\theta \, \frac{\partial}{\partial \theta^*} \mathcal{L}$$

$$= \int dt\, d\theta \left[\underbrace{D^\dagger + i\theta \frac{\partial}{\partial t}}_{= \frac{\partial}{\partial \theta^*}} \right] \mathcal{L}$$

$$\underbrace{\int dt\, d\theta\, d\theta^*\, \mathcal{L}}_{\text{SUSY invariant}} = \int dt\, d\theta\, \frac{\partial}{\partial \theta^*}\, \mathcal{L}$$

in general

$$= \int dt\, d\theta \Big[\underbrace{D^\dagger + i\theta \frac{\partial}{\partial t}}_{\equiv\, \frac{\partial}{\partial \theta^*}} \Big] \mathcal{L}$$

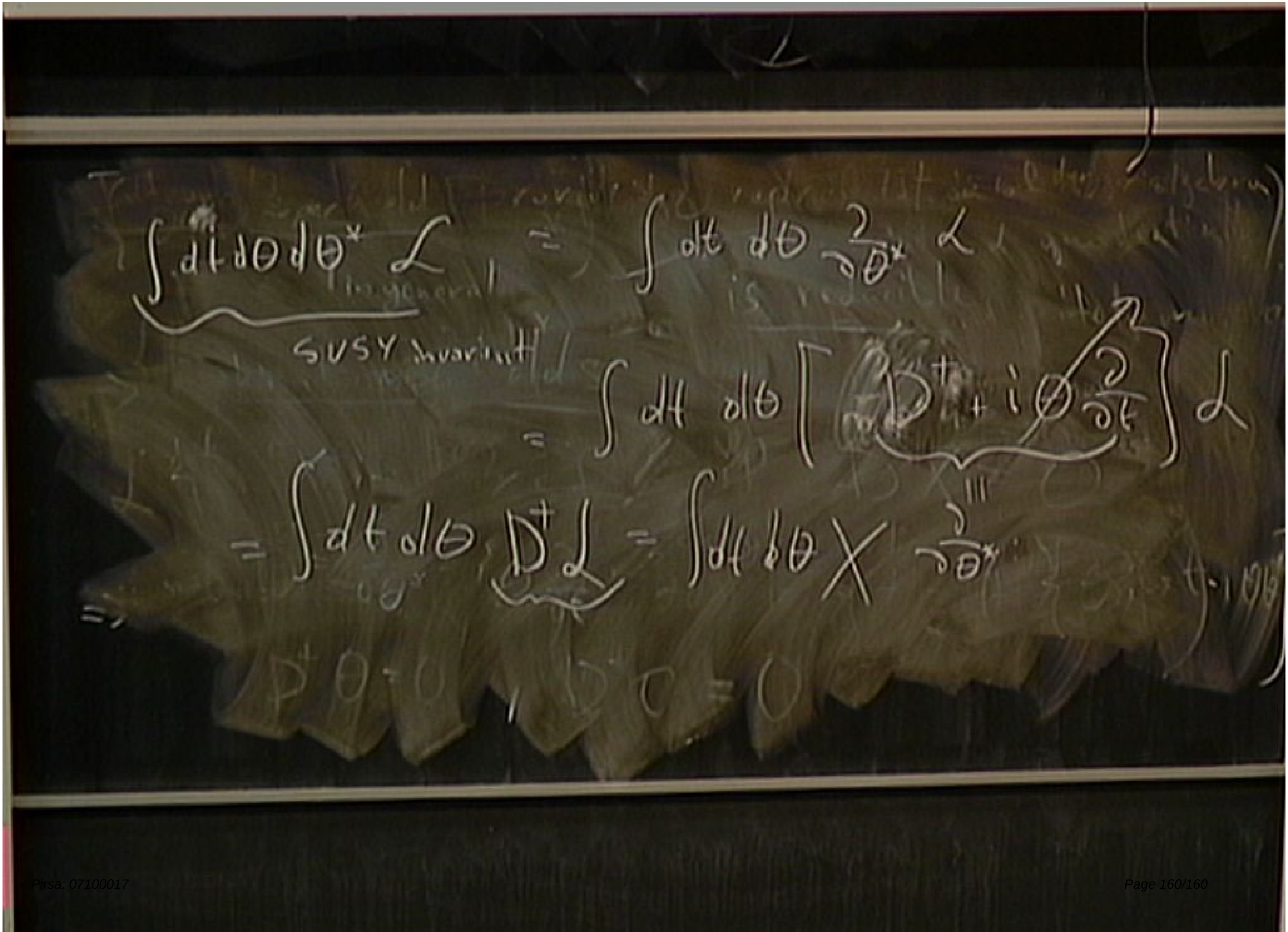
$$= \int dt\, d\theta\, D^\dagger \mathcal{L}$$

$$\underbrace{\int dt\, d\theta\, d\theta^* \, \mathcal{L}}_{\text{SUSY invariant}} = \int dt\, d\theta \, \frac{\partial}{\partial \theta^*} \mathcal{L}$$

(in general)

$$= \int dt\, d\theta \left[\underbrace{D^\dagger + i\theta \frac{\partial}{\partial t}}_{\equiv\, \frac{\partial}{\partial \theta^*}} \right] \mathcal{L}$$

$$= \int dt\, d\theta \, D^\dagger \mathcal{L}$$



in general
SUSY invariant