

Title: Intro to Supersymmetry 6

Date: Oct 04, 2007 12:30 PM

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Abstract:

3QM

$$H = \frac{1}{2} p^2 + \frac{1}{2} (\phi')^2 - \frac{1}{2} [\psi^*, \psi] f''$$

↓
D+1 dim QFT.

$$SQM \quad H = \frac{1}{2} \pi^2 + \frac{1}{2} (\dot{\varphi})^2 - \frac{1}{2} [\psi^*, \psi] f''$$

↑ q -generalized, coordinate.

↓ $D+1$ dim QFT.

$$S[\varphi(t)]$$

$$SQM \quad H = \frac{1}{2} \pi^2 + \frac{1}{2} (\varphi')^2 - \frac{1}{2} [\psi^* \psi] \varphi''$$

↖ φ - generalized coordinate.

↓ $D+1$ dim QFT.

$$S[\varphi(t), \psi(t), \psi^*(t)] = \int dt \left[\frac{1}{2} \dot{\varphi}^2 + i \psi^* \dot{\psi} - \frac{1}{2} (\varphi')^2 \right]$$

$$S_{QM} \quad H = \left(\frac{1}{2} \pi^2 + \frac{1}{2} (\dot{\varphi})^2 - \frac{1}{2} [\psi^\dagger, \psi] \dot{\varphi} \right)$$

\nwarrow φ - generalized coordinate

\downarrow $D+1$ dim QFT.

$$S[\varphi(t), \psi(t), \psi^\dagger(t)] = \int dt \left[\frac{1}{2} \dot{\varphi}^2 + i \psi^\dagger \dot{\psi} - \frac{1}{2} (\dot{\varphi})^2 + \frac{1}{2} \dot{\varphi} [\psi^\dagger, \psi] \right]$$

$\varphi(t)$ is a real scalar field

$\psi(t), \psi^\dagger(t)$ - fermionic anticommuting

Intro Grassmann calculus

θ_1, θ_2 - complex Grassmann variables.

$$\{\theta_1, \theta_2\} = 0 \Rightarrow \theta_1 \theta_2 = -\theta_2 \theta_1$$

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Taylor series expansion in θ .

$$f(\theta) = f_0 + f_1 \theta$$

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Differentiation & integration.

$\frac{d}{d\theta}$



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$$\frac{\partial}{\partial \theta_1} [\theta_1 \theta_2] = \frac{\partial \theta_1}{\partial \theta_1} \theta_2$$

Taylor series expansion in θ .

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$$(\theta^2 = 0)$$

$$\left\{ \frac{\partial}{\partial \theta_1} \theta \right\} = 1$$

Differentiation & integration.

$$\frac{\partial}{\partial \theta_1} [\theta_1, \theta_2] = \frac{\partial \theta_1}{\partial \theta_1} \theta_2 = \theta_2$$

$$\frac{\partial}{\partial \theta_2} [\theta_1, \theta_2] = -\theta_1 \frac{\partial \theta_2}{\partial \theta_2} = -\theta_1$$

$$H = \frac{1}{2} p^2 + \frac{1}{2} (q')^2 - \frac{1}{2} [\psi^*, \psi] f''$$

$$\left\{ \frac{\partial}{\partial \theta}, \theta \right\} f(\theta) = \frac{\partial}{\partial \theta} [\theta f] + \theta \frac{\partial}{\partial \theta} f$$

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$$= f - \theta \frac{\partial f}{\partial \theta} + \theta \frac{\partial f}{\partial \theta} = f$$

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$$= f + \theta \frac{\partial f}{\partial \theta} + \theta \frac{\partial f}{\partial \theta} = f + 2\theta \frac{\partial f}{\partial \theta}$$

Similarly;

$$\left[\frac{\partial}{\partial \theta^*}, \theta^* \right] = 1$$

$$\left[\theta, \theta^* \right] = 0$$

Integration.

$$\int d\theta \theta = \frac{d}{d\theta} \theta = 1$$

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$$\int d\theta \theta \stackrel{\text{def}}{=} \frac{\partial}{\partial \theta} \theta = 1$$

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$$(-)^A = \begin{cases} +1, & \text{if } A \text{ is bosonic} \\ -1, & \text{if } A \text{ is fermionic} \end{cases}$$

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$$\mathcal{L}(p, 4, 4^*) = \frac{1}{2} p^2 + i4 * 4$$

$$L(p, \psi, \psi^*) = \frac{1}{2} p \dot{\psi}^2 + i \psi^* \dot{\psi} - \frac{1}{2} (f')^2 + \frac{1}{2} f''[\psi^*, \psi]$$

$$\mathcal{L}(q, \dot{q}, \ddot{q}) = \frac{1}{2} m \dot{q}^2 + i \psi^* \dot{\psi} - \frac{1}{2} (F')^2 + \frac{1}{2} f''[\psi^*, \psi]$$

Remind:

$$\mathcal{L}(q_i, \dot{q}_i) \Rightarrow \pi_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$$

$$\mathcal{L}(q, \dot{q}, q^*) = \frac{1}{2} m \dot{q}^2 + i q^* \dot{q} - \frac{1}{2} (F')^2 + \frac{1}{2} f''[q^*, q]$$

Remind:

$$\mathcal{L}(q_i, \dot{q}_i) \Rightarrow \pi_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$$

$$H = \sum_i \dot{q}_i \pi_i - \mathcal{L}$$

$$L(q, \dot{q}, t) = \frac{1}{2} m \dot{q}^2 + V(q) - \frac{1}{2} (F')^2 + \frac{1}{2} F'' [q^*, q]$$

Remind:

$$L(q_i, \dot{q}_i) \Rightarrow \pi_i = \frac{\partial L}{\partial \dot{q}_i}$$

$$H = \sum_i \dot{q}_i \pi_i - L$$

$$\Pi_{\phi} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}$$

$$\Pi_{\psi} = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = 0$$

$$\Pi_{\psi^*} = 0$$

$$\Pi_{\phi} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}$$

$$\Pi_{\psi} = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \frac{d}{dt} (\psi^* \dot{\psi}) = -i \psi^*$$

$$\Pi_{\psi^*} = 0$$

$$\pi_{\dot{\phi}} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}$$

$$\pi_{\dot{\psi}} = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \frac{\partial}{\partial \dot{\psi}} \left(\dot{\psi} \psi^* \dot{\psi} \right) = \psi^*$$

$$\pi_{\dot{\psi}^*} = 0$$

$$H = \dot{\phi} \pi_{\dot{\phi}} + \dot{\psi} \pi_{\dot{\psi}} - \mathcal{L}$$

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$$H = \dot{\phi} \pi_{\dot{\phi}} + \dot{\psi} \pi_{\dot{\psi}} + \dot{\psi}^* \pi_{\dot{\psi}^*} - \mathcal{L}$$

$$\pi_p = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}$$

$$\pi_\psi = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \frac{\partial}{\partial \dot{\psi}} \left(\psi \psi^* \dot{\psi} \right) = -i \psi^*$$

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$$H = \dot{\phi} \pi_p + \dot{\psi} \pi_\psi + \dot{\psi}^* \pi_{\psi^*} - \mathcal{L}$$

$$= \dot{\phi}^2 + i \psi^* \dot{\psi}$$

$$\Pi_{\dot{\phi}} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}$$

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$$= \dot{\phi}^2 + i \psi^* \dot{\psi} + 0$$

$$\pi_p = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}$$

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$$H = \dot{\phi} \pi_p + \dot{\psi} \pi_\psi + \dot{\psi}^* \pi_{\psi^*} - \mathcal{L}$$

$$= \dot{\phi}^2 + i \psi^* \dot{\psi} + 0 - \left[\frac{1}{2} \dot{\phi}^2 - i \psi^* \dot{\psi} + \frac{1}{2} (\psi)^2 \right] = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} (\psi)^2$$

||

$$\pi_p = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}$$

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$$\begin{aligned} H &= \dot{\phi} \pi_p + \dot{\psi} \pi_\psi + \dot{\psi}^* \pi_{\psi^*} - \mathcal{L} \\ &= \dot{\phi}^2 + \cancel{i\psi^* \dot{\psi}} + 0 - \left[\frac{1}{2} \dot{\phi}^2 - \cancel{i\psi^* \dot{\psi}} + \frac{1}{2} (\dot{\phi})^2 - \frac{1}{2} \rho'' [\psi^*, \psi] \right] \\ &= \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} (\rho')^2 - \frac{1}{2} \rho'' [\psi^*, \psi] \end{aligned}$$

$$\pi_p = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = \dot{\varphi}$$

$$\pi_\psi = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \frac{\partial}{\partial \dot{\psi}} (\varphi \psi^* \dot{\psi}) = -i \psi^*$$

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$$\begin{aligned} H &= \dot{\varphi} \pi_p + \dot{\psi} \pi_\psi + \dot{\psi}^* \pi_{\psi^*} - \mathcal{L} \\ &= \dot{\varphi}^2 + i \cancel{\psi^* \dot{\psi}} + 0 - \left(\frac{1}{2} \dot{\varphi}^2 - i \cancel{\psi^* \dot{\psi}} + \frac{1}{2} (\varphi')^2 - \frac{1}{2} \rho'' [\psi^*, \psi] \right) \\ &= \frac{1}{2} \dot{\varphi}^2 + \frac{1}{2} (\varphi')^2 - \frac{1}{2} \rho'' [\psi^*, \psi] \end{aligned}$$

$$\{f, g\}_{PB} \equiv \sum_i \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right)$$



$$\{f, g\}_{PB} = \sum_i \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right)$$

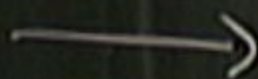
$$= \frac{1}{\hbar} [\hat{f}, \hat{g}]$$

q, p, q^*

$$\{f, g\}_{PB} \equiv \sum_i \frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i}$$

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$$q, p, p^* \longrightarrow q, p, p^*$$

$$\left. \begin{array}{l} \{p, q\} \\ \{p, p^*\} \end{array} \right\}$$

$$\{f, g\}_{PB} \equiv \sum_i \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right)$$

$$= \frac{1}{\hbar} [\hat{f}, \hat{g}]$$

$\varphi, \psi, \psi^* \quad \longrightarrow \quad \varphi, \psi, \psi^*$

$$\left. \begin{array}{l} \{f, g\} \\ \hbar \end{array} \right\} = \hat{1}$$

$$\{f, g\}_{PB} \equiv \sum_i \frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i}$$

$$= \frac{i}{\hbar} [\hat{f}, \hat{g}]$$

$$q, p, p^* \longrightarrow q, p, p^*$$

$$\left\{ \begin{matrix} p \\ = \\ p^* \end{matrix}, g \right\} = \mathcal{L} \Rightarrow [\hat{p}, \hat{g}] \frac{i}{\hbar} =$$

$$\{f, g\}_{PB} \equiv \sum_i \frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i}$$

$$= \frac{i}{\hbar} [\hat{f}, \hat{g}]$$

$$q, p, \psi \longrightarrow q, p, \psi^+$$

$$\left\{ \begin{array}{l} p \\ = \\ f \end{array} , g \right\} = \mathcal{L} \Rightarrow [\hat{p}, \hat{g}] \frac{i}{\hbar} = \mathcal{L}$$

canonical quantization

$$\left\{ \pi_{\varphi}, \varphi \right\}_{PB} = 1 \Rightarrow [\hat{\pi}_{\varphi}, \hat{\varphi}] = -i\hbar$$

$$\{ \pi_\psi, \psi \}_{PB} = 1 \Rightarrow [\hat{\pi}_\psi, \hat{\psi}] = -i\hbar$$

$$\{ \pi_\psi, \psi \}_{PB} = 1 \Rightarrow \{ -\psi^*, \psi \} = -i\hbar$$

$$\downarrow$$
$$-i\psi^*$$

$$\{ \pi_\psi, \psi \}_{PB} = 1 \Rightarrow [\hat{\pi}_\psi, \hat{\psi}] = -i\hbar$$

$$\{ \pi_\psi, \psi \}_{PB} = 1 \Rightarrow \{ -i\psi^*, \psi \} = -i\hbar$$

$$\downarrow$$
$$-i\psi^*$$



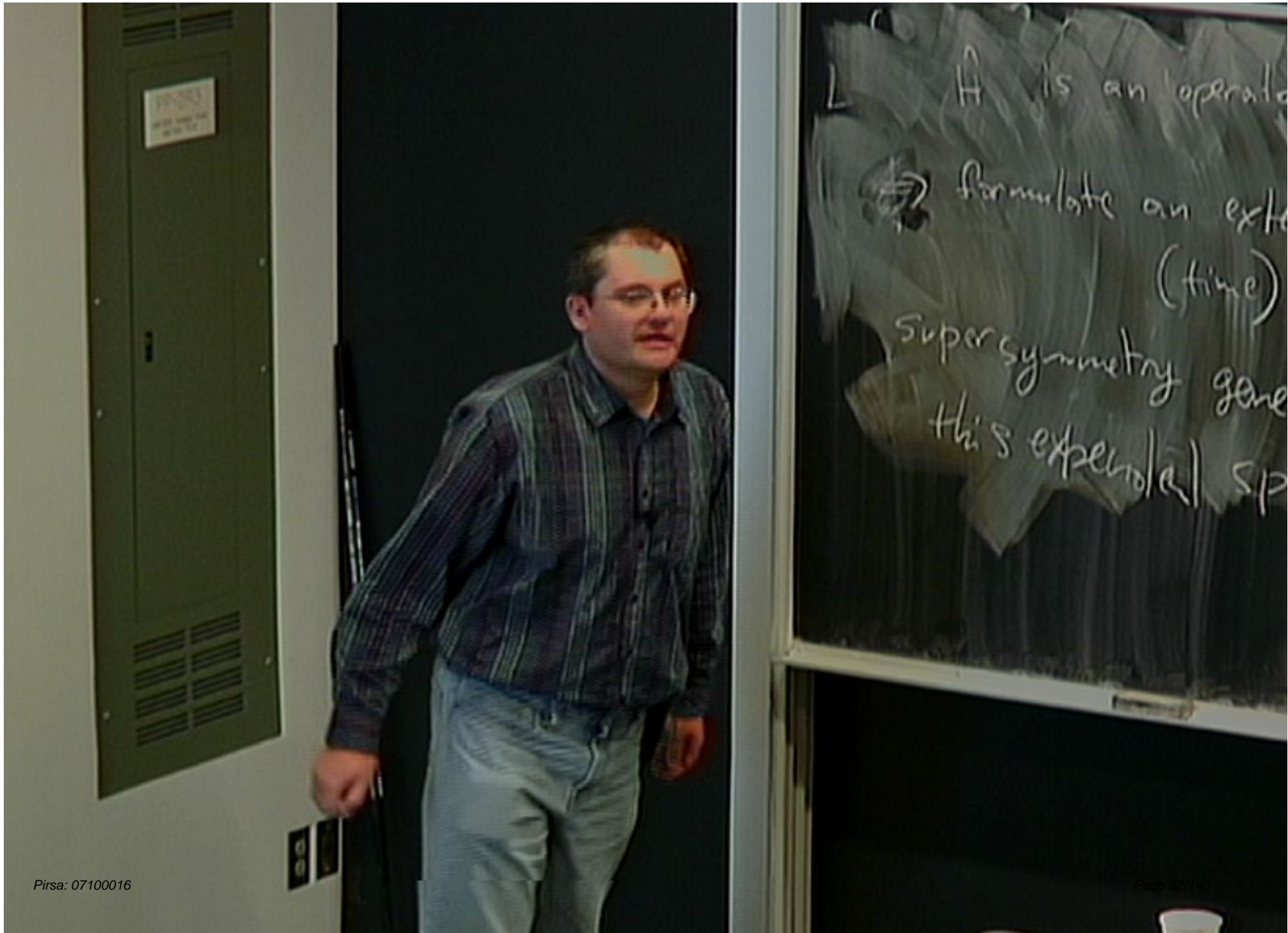
$$\{ \psi, \psi^* \} = \hbar$$

\hat{H} is an operator corresponding to time translation

\Rightarrow formulate an extension of space-time
(time) so that

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supersymmetry generators or translations on



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\Rightarrow formulate an extension of space-time
(time) so that we can realize

supersymmetry generators or translations on
this extended space-time \Leftrightarrow a superspace

Back to QM

\mathcal{O} - an operator in Schrödinger repr.

$\mathcal{O}(t)$

↑
Heisenberg
- representation

=

Back to QM

\mathcal{O} - an operator in Schrödinger repr.

$$\mathcal{O}(t) = e^{iP_0 t} \mathcal{O} e^{-iP_0 t}$$

↑
Heisenberg
representation

$$P_0 = -\hbar$$

Back to

\mathcal{O} - an operator in Schrödinger repr

$$\mathcal{O}(t) = e^{iP_0 t} \mathcal{O} e^{-iP_0 t} \quad P_0 = -H$$

↑
Heisenberg
- representation

\int

back to

\mathcal{O} - an operator in Schrödinger repr.

$$\mathcal{O}(t) = e^{iP_0 t} \mathcal{O} e^{-iP_0 t} \quad P_0 = -H$$

↑
Heisenberg
representation

$$\delta_t \mathcal{O} = \mathcal{O}(t+\epsilon) - \mathcal{O}(t)$$

back to

\mathcal{O} - an operator in Schrödinger repr.

$$\mathcal{O}(t) = e^{iP_0 t} \mathcal{O} e^{-iP_0 t} \quad P_0 = -H$$

↑
Heisenberg
representation

$$\delta_t \mathcal{O} = \mathcal{O}(t+\epsilon) - \mathcal{O}(t) = \epsilon [iP_0, \mathcal{O}(t)]$$

\mathcal{O} - an operator in Schrödinger repr.

$$\mathcal{O}(t) = e^{iP_0 t} \mathcal{O} e^{-iP_0 t} \quad P_0 = -H$$

↑
Heisenberg
representation

$$\delta_t \mathcal{O} = \mathcal{O}(t+\epsilon) - \mathcal{O}(t) = \epsilon [iP_0, \mathcal{O}(t)]$$

iP_0 is a generator of infinitesimal time translations

G_t

$$\psi(t) - \psi(t) = \epsilon [i P_0, \psi(t)]$$

generator of infinitesimal time translations.

$$P_0 = P_0^\dagger$$

\Rightarrow we want G_+ to be anti-hermitian.

$$S_t \theta = \varepsilon [G_+, \theta]$$

$$P_0 = P_0^\dagger$$

\Rightarrow we want G_\pm to be anti-hermitian

$$\delta_\pm \theta = \varepsilon [G_\pm, \theta]$$

$$\delta_\pm \theta^\dagger = \varepsilon [-G_\pm^\dagger, \theta^\dagger]$$

$$P_0 = P_0^\dagger$$

\Rightarrow we want G_+ to be anti-hermitian.


$$\delta_\pm \theta = \varepsilon [G_\pm, \theta]$$

$$\delta_\pm \theta^\dagger = \varepsilon [-G_\pm^\dagger, \theta^\dagger] \equiv \varepsilon [G_\pm, \theta^\dagger]$$

$$P_0 = P_0^+$$

\Rightarrow we want G_+ to be anti-hermitian.

$$\delta_+ \theta = \varepsilon [G_+, \theta]$$

$$\delta_+ \theta^+ = \varepsilon [-G_+^+, \theta^+] \equiv \varepsilon [G_+, \theta^+]$$


By analogy:

Q and Q^* → generators of supersymmetry transform.

$$\underbrace{\delta \mathcal{K}}_{\text{SUSY variation}} = \epsilon^*$$

$$P_0 = P_0^\dagger$$

\Rightarrow we want G_+ to be anti-hermitian

$$\delta_t \theta = \varepsilon [G_+, \theta]$$

$$\delta_t \theta^\dagger = \varepsilon [-G_+^\dagger, \theta^\dagger] \equiv \varepsilon [G_+, \theta^\dagger]$$

By analogy:

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$$\underbrace{\delta \chi}_{\text{SUSY variation}} = [\epsilon^* Q + \epsilon Q^*, \chi]$$

By analogy:

Q, Q^* → generators of supersymmetry transform.
generators of susy transformations

$$\underbrace{\delta \chi}_{\text{susy variation}} = \left[\overbrace{\epsilon^* Q + \epsilon Q^*}, \chi \right]$$
$$\underbrace{\delta \chi}_{\text{susy variation}} = \left[\epsilon^* Q + \epsilon Q^*, \chi \right]^{\dagger}$$

By analogy:

Q and Q^* → generators of supersymmetry transformations
generators of SUSY transformations

$$\underbrace{\delta \chi}_{\text{SUSY variation}} = \left[\overbrace{\varepsilon^* Q + \varepsilon Q^*}, \chi \right]$$
$$\left[\varepsilon^* Q + \varepsilon Q^* \right]^\dagger = Q^* \varepsilon$$

By analogy:

Q and Q^* → generators of supersymmetry transformations
generators of susy transformations

$$\underbrace{\delta \chi}_{\text{SUSY variation}} = \left[\overbrace{\epsilon^* Q + \epsilon Q^*}, \chi \right]$$

$$[\epsilon^* Q + \epsilon Q^*]^\dagger = Q^* \epsilon + Q \epsilon^*$$

By analogy:

Q and Q^* → generators of supersymmetry transformations
generators of susy transformations

$$\delta \chi = \left[\overbrace{\epsilon^* Q + \epsilon Q^*}, \chi \right]$$

susy variation

$$\begin{aligned} [\epsilon^* Q + \epsilon Q^*]^\dagger &= Q^* \epsilon + Q \epsilon^* \\ &= -(\epsilon Q^* + \epsilon^* Q) \end{aligned}$$

$$P_0 = P_0$$

\Rightarrow we want G_t to be anti-hermitian

$$\boxed{S_t \theta = \varepsilon [G_t, \theta]}$$

$$S_t \theta = O(t + \varepsilon) + O(t) = \varepsilon \partial_t \theta(t)$$

\Rightarrow we want G_t to be anti-hermitian

$$\boxed{S_t \theta = \varepsilon [G_t, \theta]}$$

$$S_t \theta = O(t+\varepsilon) + O(t) = \varepsilon \lambda_t \theta(t) + O(t)$$
$$\lambda_t \theta(t) \equiv [G_t, \theta]$$

\uparrow should identify

$$\hat{\theta} = \varepsilon [\hat{Q}_1, \hat{Q}_2] = \varepsilon \hat{Q}_0$$

$$\delta Q = \varepsilon [\hat{Q}, 0] = \varepsilon Q_{20} 0$$

$\chi \rightarrow$ is a symmetry variation.
 When:
 χ is time translation.

$$\delta Q = \epsilon \left[\hat{Q}_x, \hat{O} \right] = \epsilon \hat{Q}_x \hat{O}$$

$x \rightarrow$ is a symmetry variation.
 When:

x is time translation,

$$\hat{Q}_x = i P_0 \iff \hat{Q}_x = \frac{d}{dt}$$

$$i H$$

$$\delta Q = \epsilon \left[\dot{Q}_x, 0 \right] = \epsilon Q_0$$

$x \rightarrow$ is a symmetry variation.

When:

x is time translation,

$$\dot{Q}_x = i P_0 \iff Q_x = \frac{d}{dt} P_0$$

$$\delta Q = \epsilon \left[\hat{Q}_x, \hat{Q} \right] = \epsilon \hat{Q}_0$$

$x \rightarrow$ is a symmetry variation.
When:

x is time translation,

$$\hat{Q}_x = \begin{cases} P_0 \\ H \end{cases} \iff \hat{Q}_x = \frac{d}{dt} \begin{cases} Q \\ H \end{cases}$$

$$\delta \mathcal{L} = \epsilon \left[\hat{Q}_x, \theta \right] = \epsilon \hat{Q}_x \theta$$

$$\delta \psi = H \psi$$

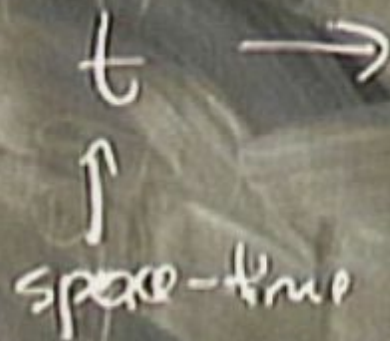
$x \rightarrow$ is a symmetry variation.

When

x is time translation,

$$\begin{array}{ccc} \hat{Q}_x = i\hat{P}_0 & \longleftrightarrow & \hat{Q}_x = \frac{\partial}{\partial t} \\ & \longleftrightarrow & \frac{\partial}{\partial t} \\ & \longleftrightarrow & i\hat{H} \end{array}$$

⇒ we now introduce or superspace



Here is a representation

$$\mathcal{D}_0 = \mathcal{O}(k, \ell) \oplus \mathcal{O}(k, \ell) \oplus \mathcal{O}(k, \ell) \oplus \mathcal{O}(k, \ell) \oplus \mathcal{O}(k, \ell)$$

\mathcal{D}_0 is a generator of \mathcal{G} (describing time)

⇒ we now introduce a superspace:

$$t \rightarrow \{t, P_0 t \theta, \theta^*\}$$

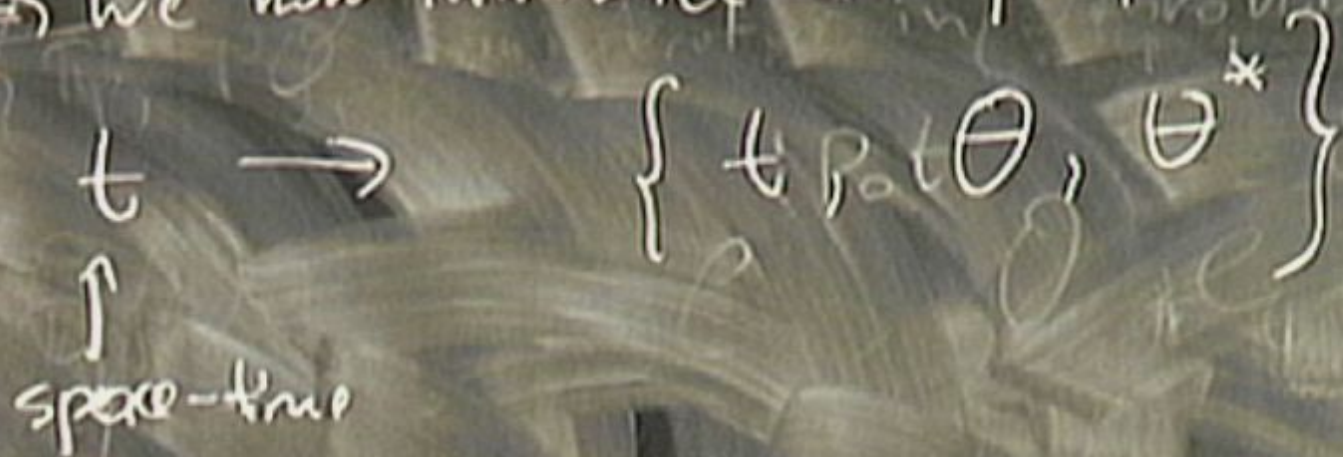
↑
space-time

Heisenberg representation

$$Q_t \theta = Q(\theta) \cdot (t, \theta)$$

(P_0) is a generator of (classical) time

⇒ we now introduce a superspace



$\varphi(t)$

P_0 is a generator of a desired time

⇒ we now introduce a superspace

t



$\{t, P_0, \theta, \theta^*\}$



space-time

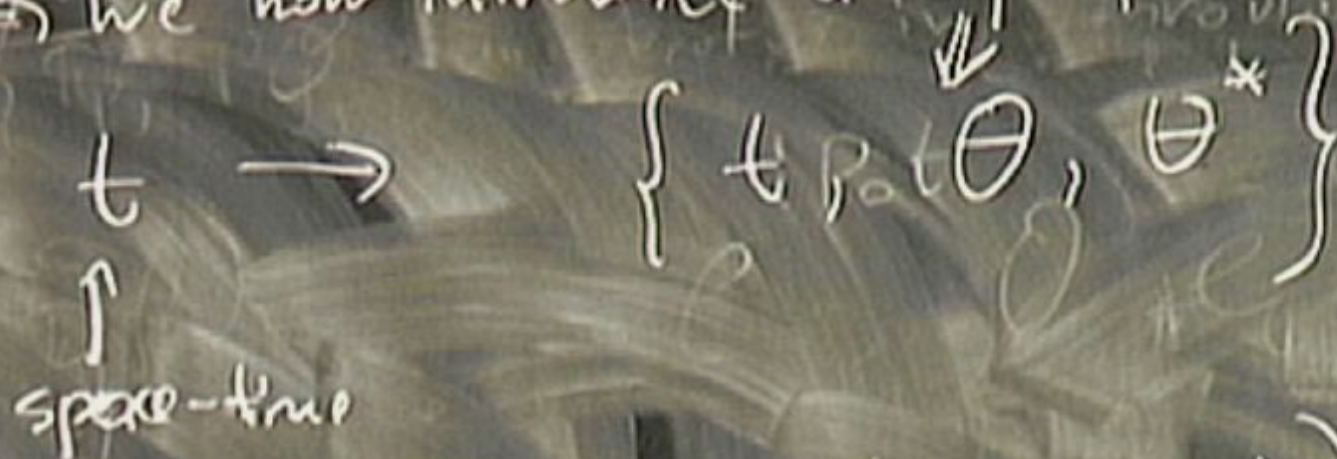
$\varphi(t)$
field



$\varphi(\theta)$

a superfield

⇒ we now introduce a superspace



$\varphi(t)$
field



$\varphi(t, \theta, \theta^*)$
a superfield

\Rightarrow a component expansion of a superfield

\Rightarrow do a Taylor series expansion.

$$\varphi(t, \theta, \theta^*)$$

\Rightarrow a component expansion of a superfield

\Rightarrow do a Taylor series expansion.

$$\Phi(t, \theta, \theta^*) = \varphi(t) +$$

\Rightarrow a component expansion of a superfield

\Rightarrow do a Taylor series expansion.

$$\underbrace{\Phi(t, \theta, \theta^*)}_{\text{a bosonic}} = \underbrace{\varphi(t)}_{\text{a bosonic}} + \underbrace{\theta \psi(t) - \theta^* \psi^*(t)}_{\text{a fermionic}} + \theta \theta^* F(t)$$

$$\Phi \Rightarrow \left\{ \varphi(t), \psi(t), F(t) \right\}$$

\Rightarrow a component expansion of a superfield

\Rightarrow do a Taylor series expansion. + to be anti-hermitian

$$\underbrace{\Phi(t, \theta, \theta^*)}_{\text{a bosonic superfield}} = \underbrace{\varphi(t)}_{\text{a bosonic}} + \theta \psi(t) - \theta^* \psi^*(t) + \theta \theta^* F(t)$$

$$\Phi \Rightarrow \left\{ \varphi(t), \psi(t), F(t) \right\}$$

\Rightarrow a component expansion of a superfield

\Rightarrow do a Taylor series expansion

$$\underline{\Phi}(t, \theta, \theta^x) = \underbrace{\varphi(t)}_{\substack{\text{a bosonic} \\ \text{superfield}}} + \underbrace{\theta \psi(t) - \theta^x \psi^x(t)}_{\text{fermionic}} + \theta \theta^x F(t)$$

$$\underline{\Phi} \Rightarrow \left\{ \varphi(t), \psi(t), F(t) \right\}$$

\Rightarrow a component expansion of a superfield

\Rightarrow do a Taylor series expansion.

$$\underline{\Phi}(t, \theta, \theta^*) = \underbrace{\varphi(t)}_{\substack{\text{a bosonic} \\ \text{superfield}}} + \underbrace{\theta \psi(t)}_{\substack{\text{fermionic}}} - \underbrace{\theta^* \psi^*(t)}_{\substack{\text{fermionic}}} + \underbrace{\theta \theta^* F(t)}_{\substack{\text{a bosonic}}}$$

$$\underline{\Phi} \Rightarrow \left\{ \varphi(t), \psi(t), F(t) \right\}$$

⇒ a component expansion of a superfield

⇒ do a Taylor series expansion.

$$\underbrace{\Phi(t, \theta, \theta^*)}_{\substack{\text{a bosonic superfield} \\ \text{in } \mathbb{R}^4}} = \underbrace{\varphi(t)}_{\text{a bosonic}} + \underbrace{\theta \psi(t) - \theta^* \psi^*(t)}_{\text{fermionic}} + \theta \theta^* \underbrace{F(t)}_{\text{a bosonic}}$$

$$\Phi \Rightarrow \left\{ \varphi(t), \psi(t), F(t) \right\} \quad \psi^*(t) = \psi(t)$$

$$\delta_2 \Phi = [\epsilon^{\alpha\beta} \hat{Q} \quad 0 \quad 0]$$



$$\delta_2 \Phi = [\epsilon^* \hat{Q} + \epsilon Q^*, \mathbb{I}]$$

$$\delta_\varepsilon \Phi = [\varepsilon^* \hat{Q} + \varepsilon \hat{Q}^*, \mathbb{I}]$$

$$= (\varepsilon^* \hat{Q} + \varepsilon \hat{Q}^*) \Phi$$

$$\delta_\epsilon \Phi = [\epsilon^* \hat{Q} + \epsilon \hat{Q}^*, \Phi]$$

$$= (\epsilon^* Q + \epsilon Q^*) \Phi$$



differential operators on superspace

\Rightarrow a component expansion of a superfield

\Rightarrow do a Taylor series expansion.

$$\underline{\Phi}(t, \theta, \theta^*) = \underbrace{\varphi(t)}_{\substack{\text{a bosonic} \\ \text{superfield}}} + \underbrace{\theta \psi(t) - \theta^* \psi^*(t)}_{\text{fermionic}} + \theta \theta^* \underbrace{F(t)}_{\text{a bosonic}}$$

$$\underline{\Phi} \Rightarrow \left\{ \varphi(t), \psi(t), F(t) \right\} \quad \circ \quad F^*(t) = F(t)$$

$$\delta_\varepsilon \hat{\Phi} = \left[\underbrace{\varepsilon^* \hat{Q} + \varepsilon \hat{Q}^*}_{\text{bosonic}}, \hat{\Phi} \right]$$

$$= \left(\varepsilon^* Q + \varepsilon Q^* \right) \hat{\Phi}$$

differential operators

$$[\hat{p}, \hat{q}] = -i\hbar$$



$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$\hat{q} = x$$



$$[\hat{p}, \hat{q}] = -i\hbar$$



$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$\hat{q} = x$$

different

$$\left[-i\hbar \frac{\partial}{\partial x}, x \right] = -i\hbar$$

$$\delta_\epsilon \Phi = \left[\underbrace{\epsilon^* \hat{Q} + \epsilon \hat{Q}^*}_{\text{bosonic}}, \hat{Q} \right]$$

$$= \left(\epsilon^* \hat{Q} + \epsilon \hat{Q}^* \right) \Phi$$

$$\left[\hat{p}, \hat{q} \right] = -i\hbar$$

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$\hat{q} = x$$

differential operators on superspace

$$\left[-i\hbar \frac{\partial}{\partial x}, x \right] = -i\hbar$$

$$\delta_\varepsilon \mathbb{I} = \underbrace{[\varepsilon^* \hat{Q} + \varepsilon \hat{Q}^*, \mathbb{I}]}_{\text{bosonic}},$$

$$= (\varepsilon^* \hat{Q} + \varepsilon \hat{Q}^*) \mathbb{I}$$

$$\{\hat{Q}, \hat{Q}^*\} = 2\hbar$$

$$\{\hat{Q}, \hat{Q}\} = 0$$

$$\{\hat{Q}^*, \hat{Q}^*\} = 0$$

$$\{\hat{p}, \hat{q}\} = -i\hbar$$

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$\hat{q} = x$$

differential operators on superspace

$$\left[-i\hbar \frac{\partial}{\partial x}, x \right] = -i\hbar$$

$$\{\hat{Q}_1, \hat{Q}^x\} = 2\hbar$$

$$\{\hat{Q}, \hat{Q}^n\} = 0$$

$$\{\hat{Q}^n, \hat{Q}^x\} = 0$$

Expansion of a super field

$$H = \dots + \frac{\partial}{\partial \theta}$$

$$Q = \frac{\partial}{\partial \theta} + i \theta^* \frac{\partial}{\partial t}$$

$$Q^+ = \frac{\partial}{\partial \theta^*} + i \theta \frac{\partial}{\partial t}$$

$$\dot{H} = i \frac{\partial}{\partial t}$$

$$Q = \frac{\partial}{\partial \theta} + i \theta^* \frac{\partial}{\partial t}$$

$$Q^\dagger = \frac{\partial}{\partial \theta^*} + i \theta \frac{\partial}{\partial t}$$

$$[Q]^\dagger = Q^\dagger$$

$$\dot{H} = i \frac{\partial}{\partial t}$$

superoperator

$$Q = \frac{\partial}{\partial \theta} + i \theta^* \frac{\partial}{\partial t}$$

$$Q^\dagger = \frac{\partial}{\partial \theta^*} + i \theta \frac{\partial}{\partial t}$$

$$[Q]^\dagger = Q^\dagger$$

$$\{Q, Q^\dagger\} = Q Q^\dagger + Q^\dagger Q =$$

$\dot{H} = i \frac{\partial}{\partial t}$ + expansion of a superfield

$$[Q]^\dagger = Q^\dagger$$

$$Q = \frac{\partial}{\partial \theta} + i \theta^* \frac{\partial}{\partial t}$$

$$Q^\dagger = \frac{\partial}{\partial \theta^*} + i \theta \frac{\partial}{\partial t}$$

$$\{Q, Q^\dagger\} = Q Q^\dagger + Q^\dagger Q =$$

$$\dot{H} = i \frac{\partial}{\partial t}$$

$$Q = \frac{\partial}{\partial \theta} + i \theta^* \frac{\partial}{\partial t}$$

$$Q^\dagger = \frac{\partial}{\partial \theta^*} + i \theta \frac{\partial}{\partial t}$$

$$[Q]^\dagger = Q^\dagger$$

$$\{Q, Q^\dagger\} = Q Q^\dagger + Q^\dagger Q = i \frac{\partial}{\partial t} + i \frac{\partial}{\partial t} = 2i \frac{\partial}{\partial t} = 2 \dot{H}$$

$$\dot{H} = i \frac{\partial}{\partial t}$$

$$Q = \frac{\partial}{\partial \theta} + i \theta^* \frac{\partial}{\partial t}$$

$$Q^\dagger = \frac{\partial}{\partial \theta^*} + i \theta \frac{\partial}{\partial t}$$

$$[Q]^\dagger = Q^\dagger$$

$$\{Q, Q^\dagger\} = Q Q^\dagger + Q^\dagger Q = i \frac{\partial}{\partial t} + i \frac{\partial}{\partial t} = 2i \frac{\partial}{\partial t} = 2\dot{H}$$

$$\{Q, Q\} = 0$$

because $\frac{\partial}{\partial Q} \Theta^x = 0$

CAUTION

Do not use the same symbol for different quantities.

It is essential to use the same symbol for the same quantity.

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$$(\varepsilon, \chi, \Phi(t, \theta, \theta^*))$$

$$P, \varphi, \varphi^*$$

$$\sqrt{P}$$

$$P$$

$$P$$

$$P$$

$$P$$

$$P$$

$$P$$

$$P$$

$$P$$

$$P$$

$$P$$

$$(\varepsilon Q + \varepsilon Q^+) \oplus (\varepsilon, \theta, \theta^*)$$

$$P, \varphi, \varphi^+$$

$$\Gamma P$$

$$\Gamma^+$$

$$\Gamma$$



$$L, \varphi, \varphi^+$$

$$\Gamma P$$

$$\Gamma^+$$

$$\Gamma$$

$$\Gamma P$$

$$\Gamma^+$$

$$\Gamma$$

$$\Gamma P$$

$$\Gamma^+$$

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$$\Gamma^+$$

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$$\Gamma P$$

$$\Gamma^+$$

$$\Gamma$$

$$\Gamma P$$

$$\Gamma^+$$

$$\Gamma$$

$$\left(\varepsilon Q + \varepsilon Q^+ \right) \Phi(t, \theta, \theta^*)$$

$$\varepsilon Q_t \varphi(t) = \varepsilon \frac{d}{dt} \varphi(t)$$

$$t \rightarrow t + \varepsilon$$

$$(\varepsilon Q + \varepsilon Q^+) \Phi(t, \theta, \theta^*)$$

$$\varepsilon Q_t \varphi(t) = \varepsilon \frac{\partial}{\partial t} \varphi(t)$$

$$t \rightarrow t + \varepsilon$$

$\varphi(t + \varepsilon)$

$$(\varepsilon Q + \varepsilon Q^*) \Phi(t, \theta, \theta^*) = \left[\varepsilon^* \left(\frac{\partial}{\partial t} + i\theta^* \frac{\partial}{\partial \theta} \right) + \left(\varepsilon \frac{\partial}{\partial t} + i\varepsilon\theta \frac{\partial}{\partial \theta} \right) \right] \Phi$$

$$\varepsilon Q_t \varphi(t) = \varepsilon \frac{\partial}{\partial t} \varphi(t)$$

$$t \rightarrow t + \varepsilon$$

$$(\varepsilon Q + \varepsilon Q^*) \Phi(t, \theta, \theta^*) = \left[\varepsilon \left(\frac{\partial}{\partial t} + i\theta^* \frac{\partial}{\partial \theta} \right) + \left(\varepsilon^* \frac{\partial}{\partial t} + i\theta \frac{\partial}{\partial \theta^*} \right) \right] \Phi(t, \theta, \theta^*)$$

$$\varepsilon Q_t \varphi(t) = \varepsilon \frac{\partial}{\partial t} \varphi(t)$$

$$t \rightarrow t + \varepsilon$$

$$\varphi(t + \varepsilon)$$

$$= i\varepsilon$$

$$(\varepsilon Q + \varepsilon Q^*) \Phi(t, \theta, \theta^*) = \left[\varepsilon \left(\frac{\partial}{\partial t} + i\theta^* \frac{\partial}{\partial \theta} \right) + \left(\varepsilon^+ \frac{\partial}{\partial \theta^*} + i\varepsilon \theta \frac{\partial}{\partial t} \right) \right] \Phi$$

$$\varepsilon Q_t \varphi(t) = \varepsilon \frac{\partial}{\partial t} \varphi(t) = (\varepsilon \theta^* + i\varepsilon \theta) \frac{\partial}{\partial t} \varphi$$

$t \rightarrow t + \varepsilon$
 $\varphi(t + \varepsilon)$

$$\mathbb{K} \left[\begin{pmatrix} \frac{e}{\theta} \\ \frac{e}{\theta} \end{pmatrix} + i \theta \begin{pmatrix} \frac{e}{\theta} \\ \frac{e}{\theta} \end{pmatrix} + \begin{pmatrix} \frac{e}{\theta} \\ \frac{e}{\theta} \end{pmatrix} + i \theta \begin{pmatrix} \frac{e}{\theta} \\ \frac{e}{\theta} \end{pmatrix} \right]$$

$$= \begin{pmatrix} \frac{e}{\theta} \\ \frac{e}{\theta} \end{pmatrix} + i \theta \begin{pmatrix} \frac{e}{\theta} \\ \frac{e}{\theta} \end{pmatrix} + \begin{pmatrix} \frac{e}{\theta} \\ \frac{e}{\theta} \end{pmatrix} + i \theta \begin{pmatrix} \frac{e}{\theta} \\ \frac{e}{\theta} \end{pmatrix}$$

$$\frac{\partial \mathbb{K}}{\partial \theta} + \frac{\partial \mathbb{K}}{\partial \theta} + \frac{\partial \mathbb{K}}{\partial \theta} + \frac{\partial \mathbb{K}}{\partial \theta}$$

$$\varphi(t) \begin{pmatrix} 3+t \\ 3+t \end{pmatrix}$$

$$(t, \theta, \theta^*)$$

susy
traslation.

$$\rightarrow (t + i\epsilon\theta^* + i\epsilon^*\theta,$$

$$(t, \theta, \theta^*)$$

→
susy
traslation.

$$\rightarrow (t + i\epsilon\theta^* + i\epsilon^*\theta, \theta + \epsilon, \theta^* + \epsilon^*)$$

$$\forall \epsilon > 0 \exists \delta > 0 \text{ such that } \forall t \in \mathbb{R} \text{ and } \forall \theta \in \mathbb{R} \text{ with } |\theta - \theta^*| < \delta \text{ we have } |\Phi(t, \theta, \theta^*) - \Phi(t, \theta^*, \theta^*)| < \epsilon$$

$$\Phi(t, \theta, \theta^*) = \left(\varepsilon^* Q + \varepsilon Q^* \right) \Phi$$

$$=$$

$$\left(\varepsilon^* Q + \varepsilon Q^* \right) \Phi$$

$$\begin{aligned}
 \text{So } \Phi(t, \theta, \theta^*) &= (\varepsilon^* Q + \varepsilon^* Q^+) \Phi \\
 &= [\varepsilon^* Q + \varepsilon^* Q^+, \Phi]
 \end{aligned}$$



$$\begin{aligned}
 \Phi(1, \hat{\theta}, \hat{\theta}^*) &= (\epsilon^* Q + \epsilon^* Q^*) \Phi \\
 &= [\epsilon^* \hat{Q} + \epsilon^* \hat{Q}^*, \Phi] \\
 [\hat{Q}, \Phi] &=
 \end{aligned}$$

$$\Phi(\hat{1}, \hat{0}, \hat{0}^*) = (\epsilon^* Q + \epsilon^* Q^*) \Phi$$

$$= [\epsilon^* \hat{Q} + \epsilon^* \hat{Q}^*, \Phi]$$

$$[Q, \Phi] = Q\Phi$$

$$\text{So } \Phi(\hat{1}, \hat{0}, \hat{0}^*) = (\epsilon^* Q + i^* Q^*) \Phi$$

$$= [\epsilon^* \hat{Q} + i^* \hat{Q}^*, \Phi]$$

$$[Q, \Phi] = Q\Phi \left(\frac{\partial}{\partial \theta} + i\theta \frac{\partial}{\partial \theta^*} \right)$$

$$\begin{aligned}
 \Omega \quad \Phi(t, \theta, \theta^*) &= (\varepsilon^* Q + \varepsilon^* Q^*) \Phi \\
 &= [\varepsilon^* \hat{Q} + \varepsilon^* \hat{Q}^*, \Phi] \\
 [\hat{Q}, \Phi] &= Q \Phi \left(\frac{\partial}{\partial \theta} + i \theta^* \frac{\partial}{\partial \theta^*} \right) (\psi + \theta \psi - \theta^* \psi^* + \theta \theta^*) \\
 &=
 \end{aligned}$$

$$\begin{aligned}
 \Omega \quad \Phi(t, \theta, \theta^*) &= (\varepsilon^* \hat{Q} + \varepsilon^* \hat{Q}^*) \Phi \\
 &= [\varepsilon^* \hat{Q} + \varepsilon^* \hat{Q}^*, \Phi] \\
 [\hat{Q}, \Phi] &= \hat{Q} \Phi \left(\frac{\partial}{\partial \theta} + i \theta^* \frac{\partial}{\partial \theta^*} \right) (\psi + \theta \psi - \theta^* \psi^* + \theta \theta^*) \\
 &=
 \end{aligned}$$

$$\begin{aligned}
 \Omega \quad \Phi(t, \theta, \theta^*) &= (\varepsilon^* Q + \varepsilon^* Q^*) \Phi \\
 &= [\varepsilon^* \hat{Q} + \varepsilon^* \hat{Q}^*, \Phi] \\
 [\hat{Q}, \Phi] &= Q \Phi \left(\frac{\partial}{\partial \theta} + i \theta^* \frac{\partial}{\partial \theta^*} \right) (\psi + \theta \psi - \theta^* \psi^* + \theta \theta^*) \\
 &= \psi + \theta^* F
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{Q}(\psi, \theta, \theta^*) &= (\varepsilon^* \hat{Q} + \varepsilon \hat{Q}^*) \psi \\
 &= [\varepsilon^* \hat{Q} + \varepsilon \hat{Q}^*, \psi] \\
 [\hat{Q}, \psi] &= \hat{Q} \psi + \left(\frac{\partial}{\partial \theta} + i\theta^* \frac{\partial}{\partial \theta^*} \right) (\psi + \theta \psi - \theta^* \psi^* + \theta \theta^*) \\
 &= \psi + \theta^* \psi + i\theta^* \dot{\psi}
 \end{aligned}$$

$$\begin{aligned}
\Omega \quad \Phi(t, \theta, \theta^*) &= (\varepsilon^* \hat{Q} + \varepsilon^* \hat{Q}^*) \Phi \\
&= [\varepsilon^* \hat{Q} + \varepsilon^* \hat{Q}^*, \Phi] \\
[\hat{Q}, \Phi] &= \hat{Q} \Phi \left(\frac{\partial}{\partial \theta} + i \theta^* \frac{\partial}{\partial \theta^*} \right) (\psi + \theta \psi - \theta^* \psi^* + \theta \theta^*) \\
&= \psi + \theta^* \psi - i \theta^* \dot{\psi} - i \theta \theta^* \dot{\psi}
\end{aligned}$$

$$[\hat{Q}, \Phi] = \Theta^+ \Phi = \left(\frac{\partial}{\partial t} + i\Theta \frac{\partial}{\partial t} \right) \left(\psi + \Theta \psi - \Theta^* \psi^* + \right. \\ \left. + G\Theta^* P \right)$$

$$[P_0, \Theta(t)]$$

describes time translations.

$$[\hat{Q}, \Phi] = \theta^+ \Phi = \left(\frac{\partial}{\partial t} + i\theta \frac{\partial}{\partial t} \right) \begin{pmatrix} \psi + \theta \psi - \theta^* \psi \\ + \theta \psi^* F \\ - \psi^* - \theta F + i\theta \psi - i\theta \theta^* F \end{pmatrix}$$



$(\psi_0, \theta(t))$
 infinitesimal time translation

$$[\hat{Q}, \Phi] = \theta^+ \Phi = \left(\frac{\partial}{\partial \psi^*} + i\theta \frac{\partial}{\partial t} \right) \begin{pmatrix} \psi + \theta \psi - \theta^* \psi^* \\ + \theta \psi^* \end{pmatrix}$$

$$= -\psi^* - \theta F + i\theta (\psi - i\theta \theta^* \psi^*)$$

$(P_0, \theta(t))$
 infinitesimal time translation

$$[\hat{Q}, \Phi] = \theta^+ \Phi = \left(\frac{\partial}{\partial t} + i\theta \frac{\partial}{\partial t} \right) \left(\psi + \theta \psi - \theta^* \psi^* + \dots + \theta F \right)$$

$$= -\psi^* - \theta F + i\theta (\dot{\psi} - i\theta \theta^* \dot{\psi}^*)$$

$$\delta \Phi = (\epsilon Q + \epsilon^* \theta^+) \Phi =$$

$(P_0, \theta(\theta))$

generator time translations.

$$[\hat{Q}, \Phi] = O^+ \Phi = \left(\frac{\partial}{\partial x} + i\theta \frac{\partial}{\partial t} \right) \begin{pmatrix} \psi + \theta \psi - \theta^* \psi^* + \\ + \theta^* F \end{pmatrix}$$

$$= -\psi^* - \theta F + i\theta (\dot{\psi} - i\theta \theta^* \dot{\psi}^*)$$

$$\delta \Phi = (\varepsilon Q + \varepsilon^* O^+) \Phi =$$

$$= \begin{pmatrix} \varepsilon \psi - \varepsilon^* \psi^* \\ \varepsilon \theta^* (F + i\dot{\psi}) + \varepsilon \theta (i\dot{\psi} - F) \end{pmatrix} - i\theta \theta^* \begin{pmatrix} \varepsilon \dot{\psi} + \varepsilon^* \dot{\psi}^* \end{pmatrix}$$

$$[\hat{Q}, \Phi] = \hat{Q}^+ \Phi = \left(\frac{\partial}{\partial x} + i\theta \frac{\partial}{\partial t} \right) \begin{pmatrix} \psi + \theta \psi - \theta^* \psi^* + \\ + \theta^* F \end{pmatrix} \\ = \frac{-\psi^* - \theta F + i\theta(\dot{\psi} - i\theta^* \dot{\psi}^*)}{\dots}$$

$$\delta \Phi = (\varepsilon Q + \varepsilon^* \hat{Q}^+) \Phi = \\ = (\varepsilon \psi - \varepsilon^* \psi^*) + \varepsilon \theta^* (F + i\dot{\psi}) + \varepsilon \theta (i\dot{\psi} - F) \\ - i\theta \theta^* (\varepsilon \dot{\psi} + \varepsilon^* \dot{\psi}^*)$$

$$\bar{\Phi} = \varphi(t) + \theta \psi(t) - \theta^* \psi^*(t) + \theta \theta^* F(t)$$

$$\overline{\Phi} = \Phi(t) + \theta \psi(t) - \theta^* \psi^*(t) + \theta \theta^* F(t)$$

$$\delta \overline{\Phi} = \delta \Phi + \theta \delta \psi - \theta^* \delta \psi^* + \theta \delta \theta^* F$$

$$\delta \Phi = \varepsilon \psi - \varepsilon^* \psi^*$$

$$\delta \psi = -\varepsilon$$

$$\bar{\Phi} = \varphi(t) + \theta \psi(t) - \theta^* \psi^*(t) + \theta \theta^* F(t)$$

$$\delta \bar{\Phi} = \delta \varphi + \theta \delta \psi - \theta^* \delta \psi^* + \theta \delta \theta^* F$$

$$\delta \varphi = \varepsilon \psi - \varepsilon^* \psi^*$$

$$\delta \psi = -\varepsilon (i\dot{\varphi} - F)$$

$$\delta \psi^* = \varepsilon$$

$$\bar{\Phi} = \varphi(t) + \theta \psi(t) - \theta^* \psi^*(t) + \theta \theta^* F(t)$$

$$\delta \bar{\Phi} = \delta \varphi + \theta \delta \psi - \theta^* \delta \psi^* + \theta \theta^* \delta F$$

$$\delta \varphi = \varepsilon \psi - \varepsilon^* \psi^*$$

$$\delta \psi = -\varepsilon^* (i\dot{\varphi} - F)$$

$$\delta \psi^* = \varepsilon (F + i\dot{\varphi})$$

$$\delta F =$$

$$\bar{\Phi} = \varphi(t) + \theta \psi(t) - \theta^* \psi^*(t) + \theta \theta^* F(t)$$

$$\delta \bar{\Phi} = \delta \varphi + \theta \delta \psi - \theta^* \delta \psi^* + \theta \theta^* \delta F$$

$$\delta \varphi = \varepsilon \psi - \varepsilon^* \psi^*$$

$$\delta \psi = -\varepsilon^* (i\dot{\varphi} - F)$$

$$\delta \psi^* = \varepsilon (F + i\dot{\varphi})$$

$$\delta F = -i(\varepsilon^* \dot{\psi} + \varepsilon \dot{\psi}^*)$$

$$\bar{\Phi} = \varphi(t) + \theta \psi(t) - \theta^* \psi^*(t) + \theta \theta^* F(t)$$

$$\delta \bar{\Phi} = \delta \varphi + \theta \delta \psi - \theta^* \delta \psi^* + \theta \theta^* \delta F$$

$$\delta \varphi = \varepsilon \psi - \varepsilon^* \psi^*$$

δF

$$\delta \psi = -\varepsilon^* (i\dot{\varphi} - F)$$

$$\delta \psi^* = \varepsilon (F + i\dot{\varphi})$$

$$\delta F = -i(\varepsilon^* \dot{\psi} + \varepsilon \dot{\psi}^*)$$

$$\bar{\Phi} = \varphi(t) + \theta \psi(t) - \theta^\dagger \psi^*(t) + \theta \theta^\dagger F(t)$$

$$\delta \bar{\Phi} = \delta \varphi + \theta \delta \psi - \theta^\dagger \delta \psi^* + \theta \theta^\dagger \delta F$$

$$\delta \varphi = \epsilon \psi - \epsilon^\dagger \psi^*$$

$$\delta \psi = -\epsilon^\dagger (i\dot{\varphi} - F)$$

$$\delta \psi^* = \epsilon (F + i\dot{\varphi})$$

$$\delta F = -i(\epsilon^\dagger \dot{\psi} + \epsilon \dot{\psi}^*)$$

δF
 \uparrow
 highest component
 of a superfield

$$\delta \psi^* + \theta \theta^2 \delta F$$

$\delta F =$ total derivative.

highest component
of a superfield.

$$+ \zeta^j \psi^j$$

$$\overline{\Phi} = \varphi(t) + \theta \psi(t) - \theta^* \psi^*(t) + \theta \theta^* F(t)$$

$$\delta \overline{\Phi} = \delta \varphi + \theta \delta \psi - \theta^* \delta \psi^* + \theta \theta^* \delta F$$

$$\delta \varphi = \varepsilon \psi - \varepsilon^* \psi^*$$

$$\delta \psi = -\varepsilon^* (i\dot{\varphi} - F)$$

$$\delta \psi^* = \varepsilon (F + i\dot{\varphi})$$

$$\delta F = -i(\varepsilon^* \dot{\psi} + \varepsilon \dot{\psi}^*) = -\partial_t [i\varepsilon^* \psi + i\varepsilon \psi^*]$$

δF = total derivative.

highest component of a superfield.