Title: The Atomic Picture of Extremal Black Holes

Date: Oct 23, 2007 11:00 AM

URL: http://pirsa.org/07100009

Abstract: Resent research seems to indicate that charged extremal black holes in D=4 supersymmetric theories should be most naturally described in terms of more primitive atomic constituents. I will briefly describe what I mean by these atomic constituents and how they appear to play a role in both BPS and non-BPS extremal black holes.

The Atomic theory of Extremal Black Holes

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w/ Finn Larsen, Joan Simon in progress

Motivation

+ Intro

Extremal Black Holes are interesting laboratory for quantum gravity because S comes from a degeneracy of "ground states" $T=0$

Recent work points to the importance of decomposing charge sector Γ into parts $\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3 \ldots$ which have almost no entropy, "atoms"

Want to extend results from BPS to new solutions which are extremal non-BPS (related to Reissner-Nordström)

First Step

Want to find mass formula and complete flow of scalars.

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- Define theory

  $N=8$, $N=2$ STU, BPS vs non-BPS

- Define notion of atoms, how they extend to other theories

- Review BPS black holes
  - Counting
  - Mass formula
  - multi-center solutions (briefly)

- non-BPS solutions (extremal)
  - new features
  - mass formula
  - Comparison to BPS bounds / Solutions.

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We are interested in IIA string theory compactified on $T^6 = T^2 \times T^2 \times T^2$

The low-energy theory is $\mathcal{N}=8$ SUGRA with symmetric scalar manifold $G/H = E_{7(7)}/SU(8)$ (dim = 70)

For the purposes of our paper/talk interested in truncation to $\mathcal{N}=2$ S-T-U theory with four vector fields and scalar-manifold $SL(2,R)^3/U(1)^3$

The charges of STU are D6 on T^6 D0
D4 on T^4 D2 on T^2

Generalizes well to

IIA on CY with symmetric cubic intersection form.

The $\mathcal{N}=8$ theory has a quartic invariant of the charges I_4 which helps classify the various single center solutions:

$I_4 > 0 \iff \frac{1}{8}$ BPS with $S = \pi\sqrt{I_4}$

$I_4 = 0$, $\partial_p I_4 \neq 0 \iff \frac{1}{8}$ BPS with $S_{\text{classical}} = 0$

$I_4 = 0$, $\partial_p I_4 = 0 \iff \frac{1}{4}$ BPS with $S_{\text{class.}} = 0$

$I_4 = 0$, $\partial_p I_4 = 0$, $\underbrace{\partial_p\partial_p}_{\text{adj}} I_4 = 0 \iff \frac{1}{2}$ BPS with $S_{\text{class}} = 0$

$I_4 < 0$ non-BPS single center only with extremal sol $S = \pi\sqrt{|I_4|}$

Possibility of multi-center BPS solutions in parts of moduli space.

$\mathcal{N}=2$ STU

The STU theory inherits a quartic invariant from $\mathcal{N}=8$ theory:

$$I_4 = -(p^0 Q_0 + p^i Q_i)^2 + 2p^1 Q_1 p^2 Q_2 + 2p^3 Q_3 p^2 Q_2 + 2p^1 Q_1 p^3 Q_3$$
$$+ \underset{D0}{Q_0}\, \underbrace{p^1 p^2 p^3}_{D4} - \underset{D6}{p^0}\, \underbrace{Q_1 Q_2 Q_3}_{D2}$$

Similar classification to $\mathcal{N}=8$ except that some of the $I_4>0$ solutions are no-longer BPS (They preserve the "wrong" super-charges)

Orbits:

$$I_4(\Gamma) > 0 \Leftrightarrow \Gamma \in \frac{SL(2,\mathbb{R})^3}{U(1)\times U(1)}$$ part of compact subgroup

$$I_4(\Gamma) < 0 \Leftrightarrow \Gamma \in \frac{SL(2,\mathbb{R})^3}{SO(1,1)\otimes SO(1,1)}$$ non-com

A general $\mathcal{N}=2$ extremal BH

have a metric of the form

$$ds^2 = -e^{2U(r)}(dt+\omega)^2 + e^{-2U(r)}(dr^2 + r^2 d\Omega_{S^2}^2)$$

Solutions for moduli z^i and warp factor e^{2U} come from solving effective Lagrangian

$$ds^2 = (\dot{U})^2 + G_{i\bar{j}}\dot{z}^i\dot{\bar{z}}^{\bar{j}} + e^{2U}V_{BH}(z,\bar{z})$$

function of Γ

For extremal BH, $\dot{z}^i = 0$ at horizon

Values for z^i can be found by extremizing V_{BH}

For BPS solutions z^i completely fixed (attractor)

For non-BPS can have some flat directions (at least at quadratic order)

There is an easy way to spot Γ in STU which is $\frac{1}{2}$ BPS in $\mathcal{N}=8$:

$$e^{2U(r)} \to \frac{\sqrt{I_4(\Gamma)}}{r^2} \qquad \text{for general } I_4>0 \ \tfrac{1}{8}\text{ BPS}$$

$$\to \frac{\#}{r^{\frac{1}{2}}} \qquad \text{for } \tfrac{1}{2}\text{ BPS}$$

possibly generalizes? $\rightsquigarrow$

atom

Extension to other $\mathcal{N}=2$ theories

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$$e^{2U(r)} \to \frac{\sqrt{I_4(\Gamma)}}{r^2}$$ for general $I_4>0$ $\frac{1}{2}$ BPS

A prototype BPS solution for STU is

$Q_0, p^1, p^2, p^3 > 0 \quad ; \quad p^0, Q_i = 0$

This has

horizon area $\frac{A}{4\pi} = Q_0 p^1 p^2 p^3 = G_N \, \underbrace{q_0 p^1 p^2 p^3}_{\text{integers}}$

Fixed scalars

$z^1 = -i\sqrt{\frac{Q_0 p^1}{p^2 p^3}}, \quad z^2 = -i\sqrt{\frac{Q_0 p^2}{p^1 p^3}}, \quad z^3 = -i\sqrt{\frac{Q_0 p^3}{p^1 p^2}}$

The counting of states is simple

D0 - D4 - D4 - D4 $\leftrightarrow$ P - $\underbrace{\text{M5 - M5 - M5}}$

effective long string of length $p^1 p^2 p^3 \cdot l_{p_{5}}$

Put momentum $\overset{q_0}{\overset{\shortparallel}{P}}$ quantized in units of $\frac{1}{p^1 p^2 p^3}$ on right side of (4,0) CFT

$\Rightarrow \quad S = \pi \sqrt{P \cdot p^1 p^2 p^3} = \pi\sqrt{I_4}$

BPS Solutions

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$$z^1 = -i\sqrt{\frac{Q_0 p^1}{p^2 p^3}} \, , \quad z^2 = -i\sqrt{\frac{Q_0 p^2}{p^1 p^3}} \, , \quad z^3 = -i\sqrt{\frac{Q_0 p^3}{p^1 p^2}}$$

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$$D0\text{-}D4\text{-}D4\text{-}D4 \iff P - \underbrace{M5 - M5 - M5}_{\substack{\text{effective long string} \\ \text{of length } p^1 p^2 p^3 \cdot l_{p5}}}$$

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BPS Solutions

A prototype BPS solution for STU is

$q_0, p^1, p^2, p^3 > 0$; $p^0, q_i = 0$

This has

horizon area $\frac{A}{4\pi} = 4 q_0 p^1 p^2 p^3 = 4 G_N \underbrace{q_0 p^1 p^2 p^3}_{\text{integers}}$

Fixed scalars

$z^1 = -i\sqrt{\frac{q_0 p^1}{p^2 p^3}}$, $z^2 = -i\sqrt{\frac{q_0 p^2}{p^1 p^3}}$, $z^3 = -i\sqrt{\frac{q_0 p^3}{p^1 p^2}}$

The counting of states is simple

$D0 - D4 - D4 - D4 \leftrightarrow P - \underbrace{M5 - M5 - M5}$

effective long string

of length $p^1 p^2 p^3 \cdot l_{p3s}$

Put momentum $\overset{q_0}{\overset{=}{P}}$ quantized

in units of $\frac{1}{p^1 p^2 p^3}$ on

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$\Rightarrow \quad S = \pi \cdot \sqrt{P \cdot p^1 p^2 p^3} = \pi \sqrt{I_4}$

We can ask what the mass of our BPS black hole is as a function of the volumes V_i of the T^2's and the B-field densities $B_i = \frac{1}{V_i}\int_{T^2} B_{\text{[illegible]}}$

For zero - B-fields

$$M = Q_0 + P_1 + P_2 + P_3 \quad \leftarrow \text{marginal bound stat[e]}$$

But for non-zero B-fields

$$M = |Z| = \Big| Q_0 + P_1(1+iB_2)(1+iB_3) + P_2(1+iB_1)(1+iB_3) + P_3(1+iB_1)(1+iB_2) \Big|$$

no longer a marginal bound state.

Also no idea how states counted before extend in gravity regime where $A \gg G_N$

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For zero-B fields

$$M = Q_0 + P_1 + P_2 + P_3$$ ← marginal bound state

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no longer a marginal bound state.

Also no idea how states counted before extend ~~in~~ gravity regime where $A \gg G_N$

With T. Levi and VJ Balasubramanian,
we proposed a different decomposition
into atoms.

$$\overset{\frac{1}{8}\text{BPS}}{\Gamma_{D0/D4}} = \overset{\frac{1}{2}\text{BPS}}{\Gamma^1_{D6+\text{flux}} + \Gamma^2_{\overline{D6}+\text{flux}} + \Gamma^3_{D0}}$$

For example $\Gamma_{D0/D4} = \Gamma^1_{D6+\text{flux}} + \Gamma^2_{\overline{D6}+\text{flux}} + \Gamma^3_{D0}$

with $\langle\Gamma^1,\Gamma^2\rangle, \langle\Gamma^1,\Gamma^3\rangle, \langle\Gamma^2,\Gamma^3\rangle \neq 0$!

BPS ground states come from
closed quiver quantum mechanics

$\Downarrow$ leads to capped scaling
geometries

The prototypical solution is

the D6-D0 extremal black hole

$$I_4 = -(p^0 q_0)^2 < 0$$

2 Interesting features to keep in mind

1) Can now carry angular momentum

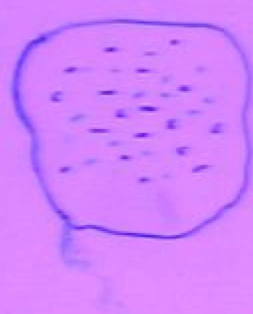
Two regimes:

1) $J \leq |q_0 p^0|$ $\quad S = \pi\sqrt{(p^0 q_0)^2 - J^2}$ all M equal

2) $J \geq |q_0 p^0|$ $\quad S = \pi\sqrt{J^2 - (p^0 q_0)^2}$ M changes

2) Exist 2-center solutions (BPS) made up of just 2 atomic types D6, D0 for $\mathrm{Re}[(1+iB_1)(1+iB_2)(1+iB_3)] < 0$

• D0
• D6



← Bound state with $|J| < |q_0 p^0|$

Action of $SO(1,1) \otimes SO(1,1)$

The attractor values for the D6-D0 BH are $B_1 = B_2 = B_3 = 0$ and $V_1 V_2 V_3 = \frac{q_0}{p^0} \times (2\pi l_s)^6$

This does not completely fix the V_i's

Two non-compact directions left $\rightsquigarrow SO(1,1)^2$

This reflects the fact that we only expect the mass formula to be a function of p^0, q_0, B_1, B_2, B_3.

For zero initial B-fields, the mass is just

$$M = \left[(p^0)^{\frac{2}{3}} + (q_0)^{\frac{2}{3}} \right]^{\frac{3}{2}}$$

We can rewrite the mass formula as the marginal sum of the masses for four D6-branes with flux $\Lambda = \left(\frac{q_0}{p^0}\right)^{\frac{1}{3}}$

$$
\begin{aligned}
M_{D6-D0} = \; & \frac{1}{4} p^0 \left(1+\Lambda^2\right)^{\frac{1}{2}} \left(1+\Lambda^2\right)^{\frac{1}{2}} \left(1+\Lambda^2\right)^{\frac{1}{2}} \\
& + \frac{1}{4} p^0 \left(1+\Lambda^2\right)^{\frac{1}{2}} \left(1+(-\Lambda)^2\right)^{\frac{1}{2}} \left(1+(-\Lambda)^2\right)^{\frac{1}{2}} \\
& + \frac{1}{4} p^0 \left(1+(-\Lambda)^2\right)^{\frac{1}{2}} \left(1+\Lambda^2\right)^{\frac{1}{2}} \left(1+(-\Lambda)^2\right)^{\frac{1}{2}} \\
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\end{aligned}
$$

This is related to the fact that we can dualize to a $\overline{D0}$ - D4 - D4 - D4 non-BPS BH $q_0 < 0$, $p^i > 0$ $\rightarrow$ $I_4 = 4 q_0 p^1 p^2 p^3 < 0$

and then

$$M = |q_0| + p^1 + p^2 + p^3 .$$

[note $M > M_{BPS} = \left| -|q_0| + p^1 + p^2 + p^3 \right|$]

Dualizing to $\overline{D0}-D4-D4-D4$ allow us to "guess" at a counting

$$D0-D4-D4-D4 \leftrightarrow P-M5-M5-M5 \quad (4,0) \uparrow \Sigma_P$$

$$\overline{D0}-D4-D4-D4 \leftrightarrow \overline{P}-M5-M5-M5 \quad (4,0) \uparrow \Sigma_{\overline{P}}$$

If you are in a regime where ground state spins don't mix then counting might explain why we get the correct entropy.

Interestingly, marginal mass formula persists for all equal B-fields

$$M_{\overline{D0}-D4-D4-D4} = |Q_0| + \sum_i P^i (1+B^2) = \cancel{\ }$$

$$= M_{\overline{D0}} + \sum_i M_{D4} \quad !$$

All non-BPS configurations can be dualized to $\overline{D0}-D4-D4-D4$ + equal B-fields B

For example, the general D0-D6 mass is

$\rho^0, q_6 > 0$

$$M_{D6\text{-}D0} = \frac{1}{4}\rho^0 \left[1+(\Lambda_1+B_1)^2\right]^{\frac{1}{2}}\left[1+(\Lambda_2+B_2)^2\right]^{\frac{1}{2}}\left[1+(\Lambda_3+B_3)^2\right]^{\frac{1}{2}}$$
$$+\frac{1}{4}\rho^0 \left[1+(\Lambda_1+B_1)^2\right]^{\frac{1}{2}}\left[1+(\Lambda_2-B_2)^2\right]^{\frac{1}{2}}\left[1+(\Lambda_3-B_3)^2\right]^{\frac{1}{2}}$$
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Where we define the fluxes implicitly from the B-fields via:

- $\Lambda_1 + \Lambda_1^{-1}(1+B_1^2) = \Lambda_2 + \Lambda_2^{-1}(1+B_2^2) = \Lambda_3 + \Lambda_3^{-1}(1+B_3^2)$
- $\Lambda_1\Lambda_2\Lambda_3 = \frac{q_0}{\rho^0}$

Not obvious for B_i not equal!

But $M_{D6\text{-}D0} > M_{BPS} \Rightarrow$ always a gap

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- $\Lambda_1 \Lambda_2 \Lambda_3 = \dfrac{q_0}{p^0}$

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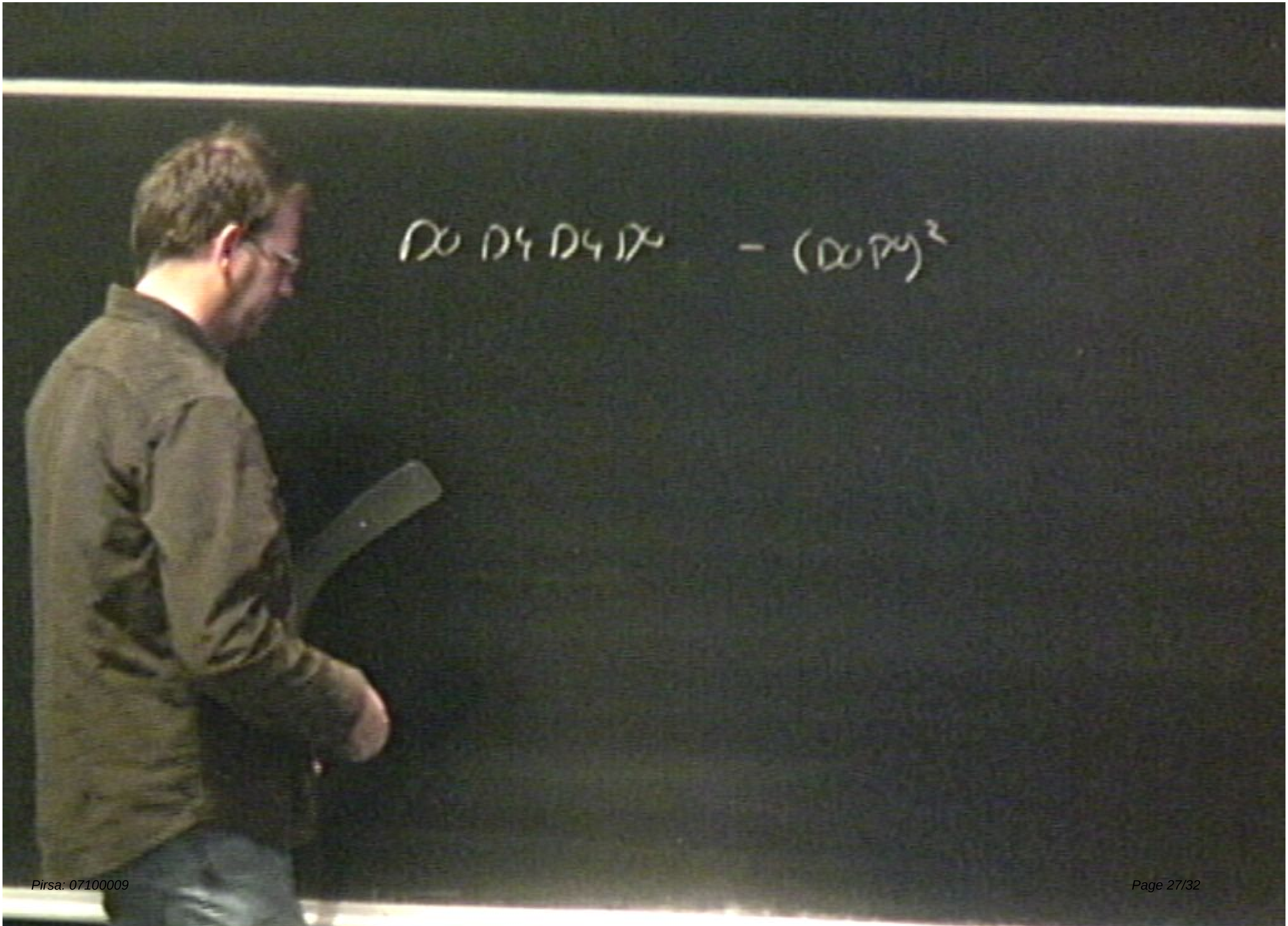
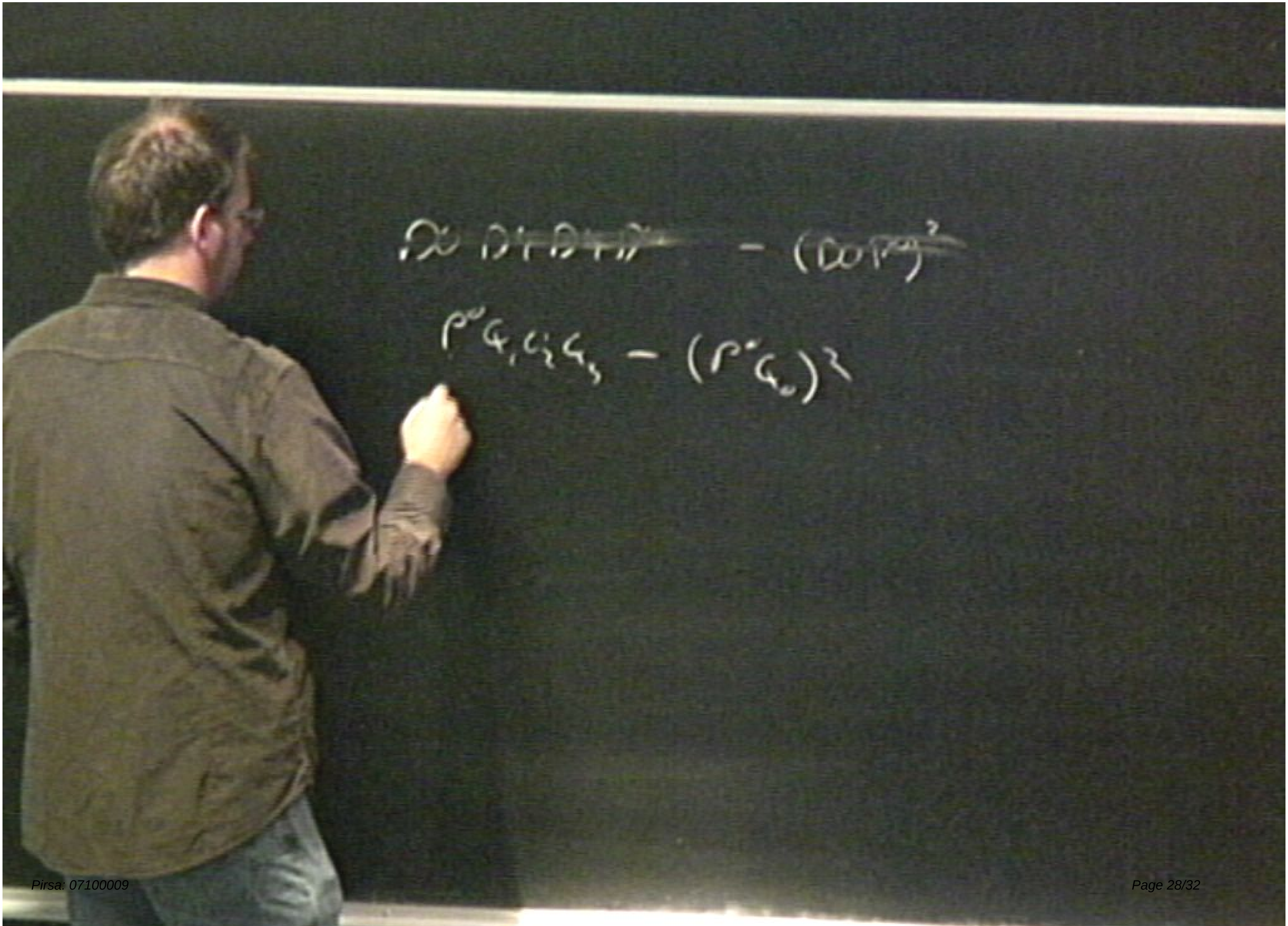
But $M_{D6\text{-}D0} > M_{BPS} \Rightarrow$ always a gap

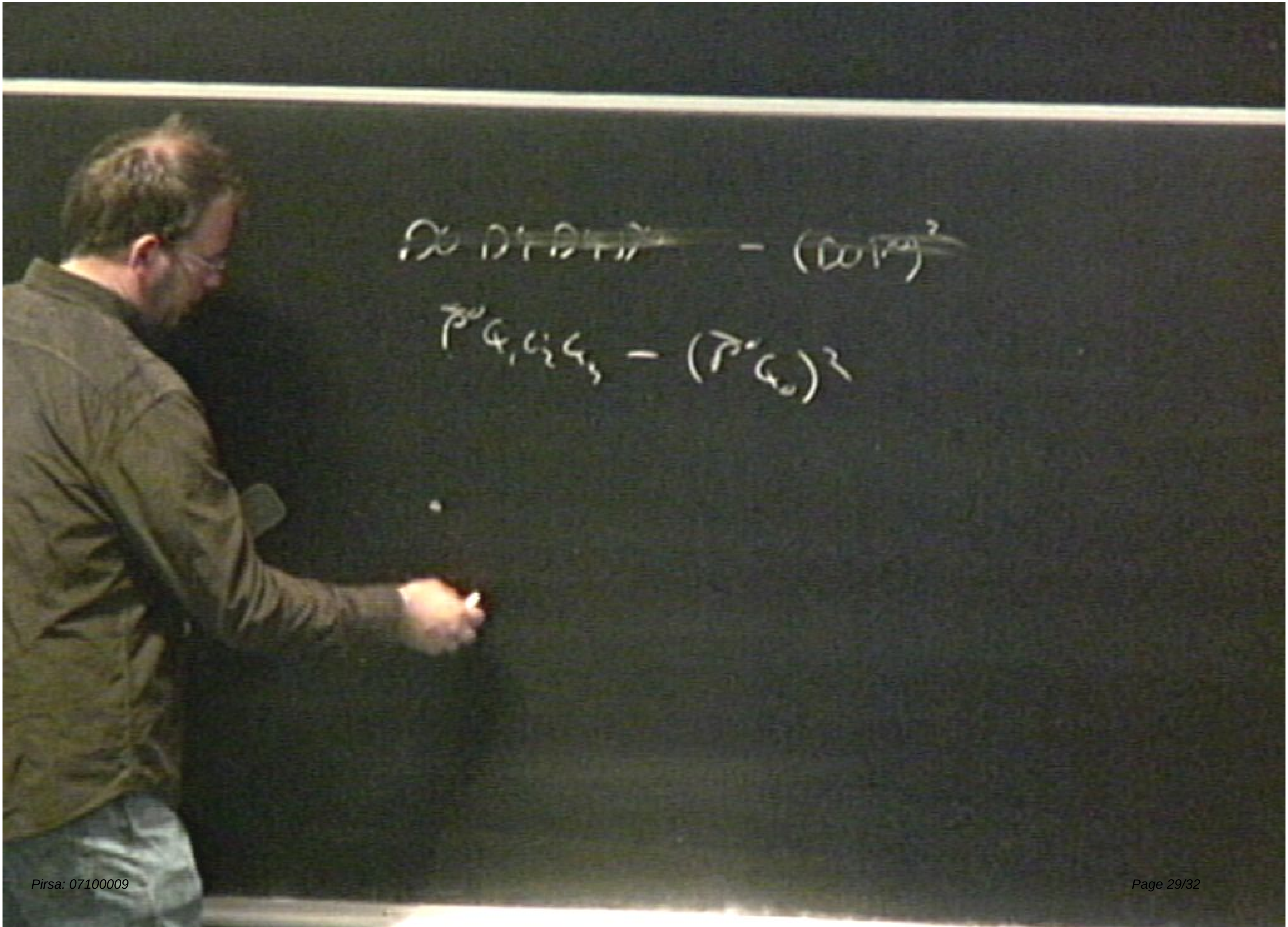
Future directions:

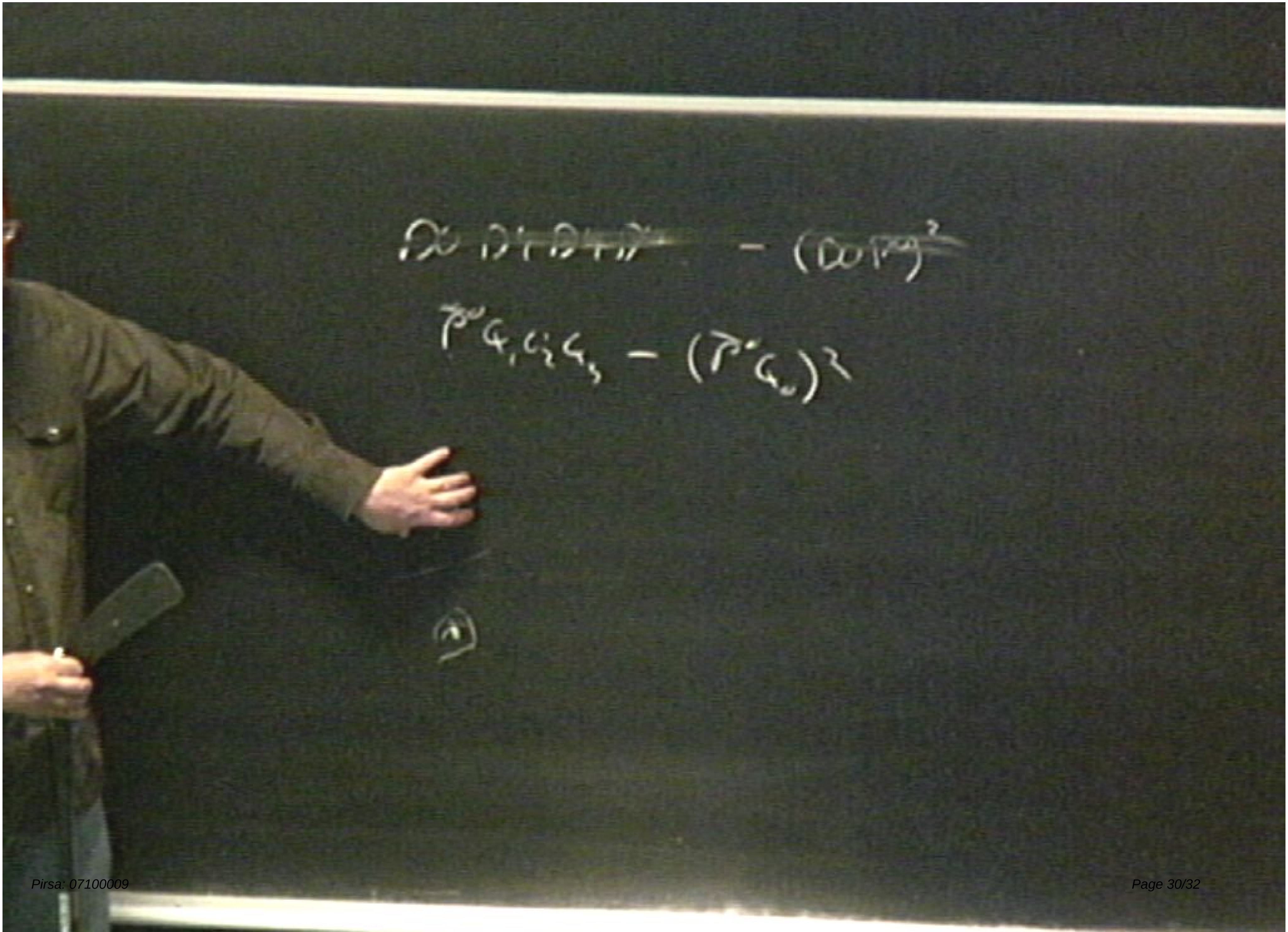
- understand marginality property
- non-BPS multi-center solutions
- relationship to multi-center BPS phase (compare entropies?)

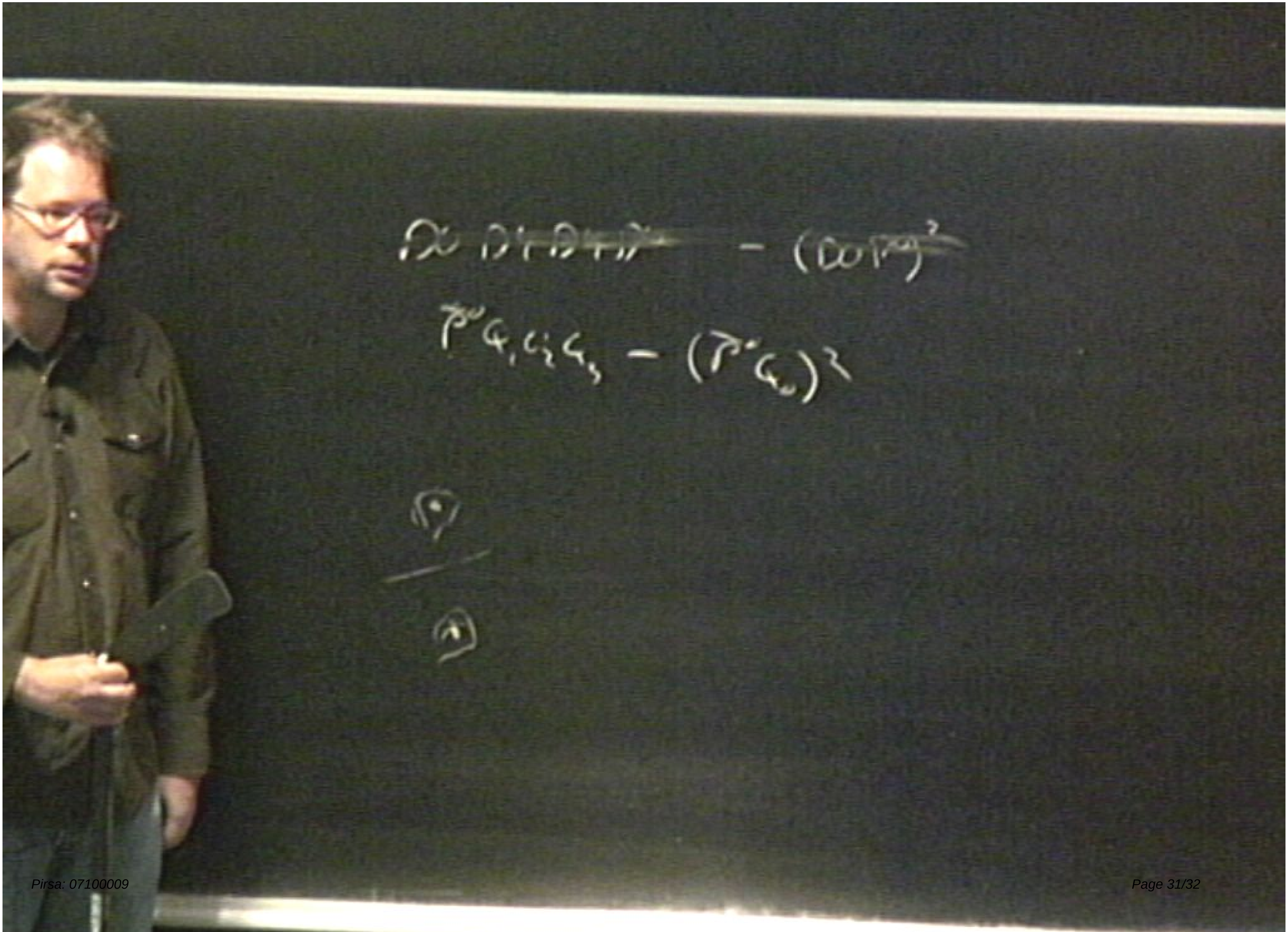
Thank You!


$D_x D_y D_y D_x - (D_x D_y)^2$


ρ⁰ q₁ q₂ q₃ − (ρ⁰ q₀)²







$$\psi_i + \sum_j \frac{\langle \Gamma_i \Gamma_j \rangle}{r_{ij}} = c$$