Title: Quantum parameter estimation with atomic spins

Date: Oct 24, 2007 02:00 PM

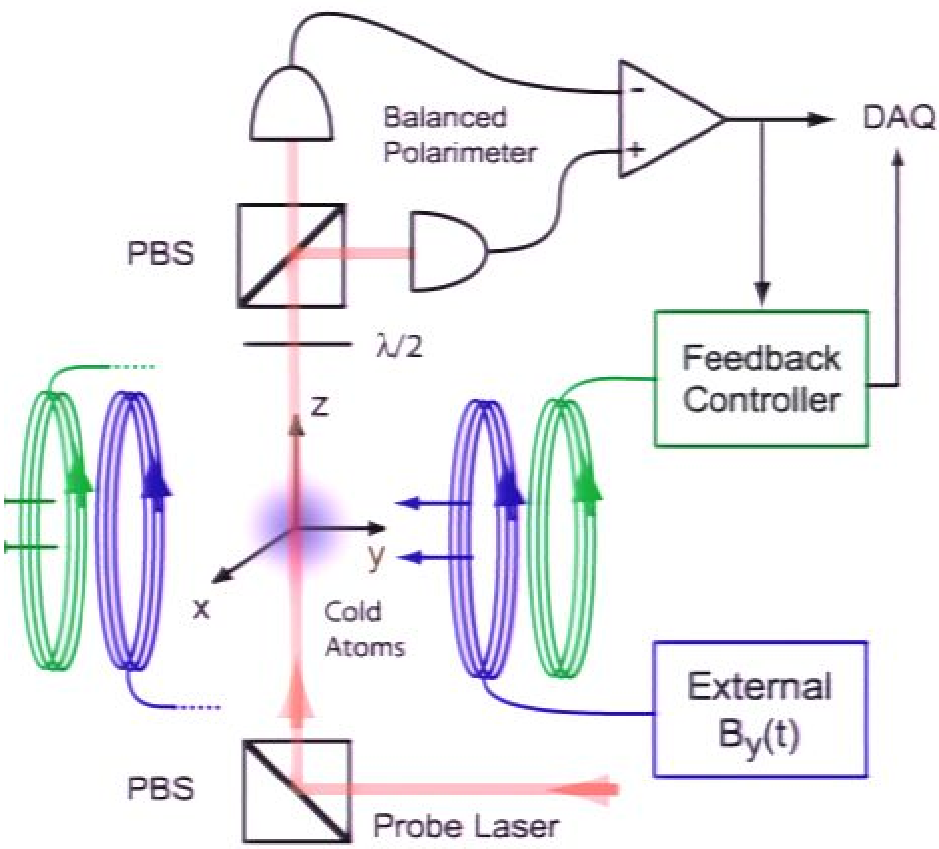
URL: http://pirsa.org/07100006

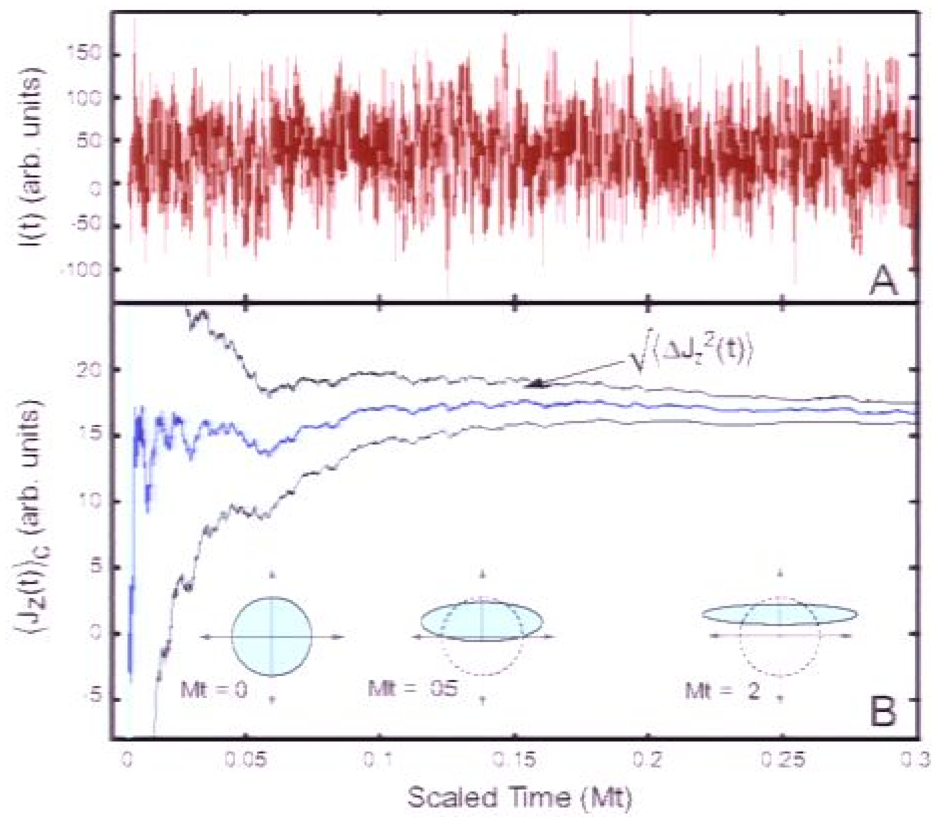
Abstract: Laser cooling and precision spectroscopy provide powerful tools for exploring quantum measurement and metrology using atoms as sensors. In this talk I will discuss our ongoing work to bring together abstract ideas of quantum parameter estimation and concrete physical details of atom-photon interactions in the specific context of magnetometry. I will also present some new ideas on how laser probing of cold atoms could provide a basis for developing entanglement-enhanced spin gyroscopes.

Quantum parameter estimation with atomic spins

Anthony Miller, John Stockton, Orion Crisafulli, Mike Armen, Magnus Hsu,
Gopal Sarma, Luc Bouten, Andrew Silberfarb and Hideo Mabuchi

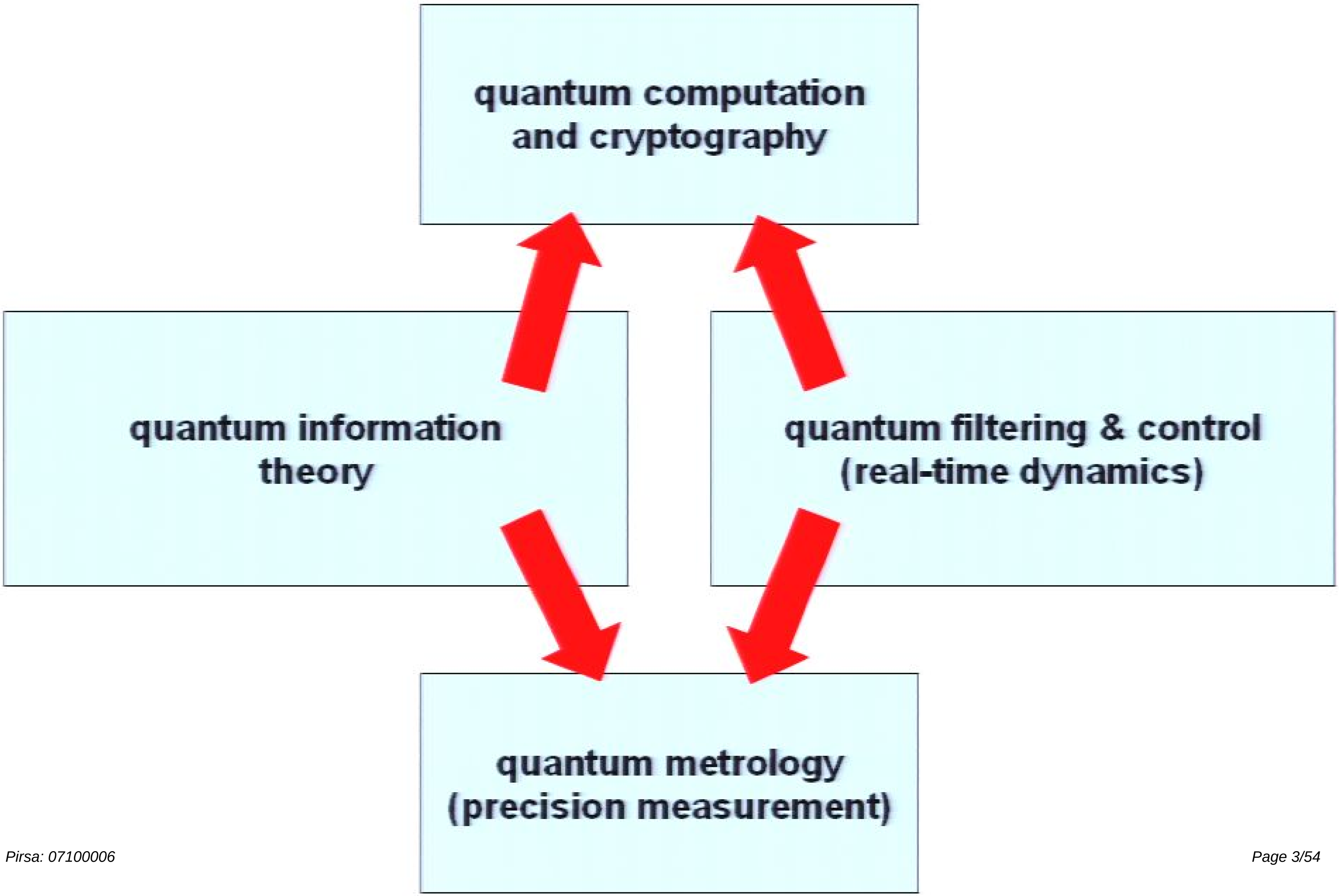
Physics and Control & Dynamical Systems, California Institute of Technology
Department of Applied Physics, Stanford University





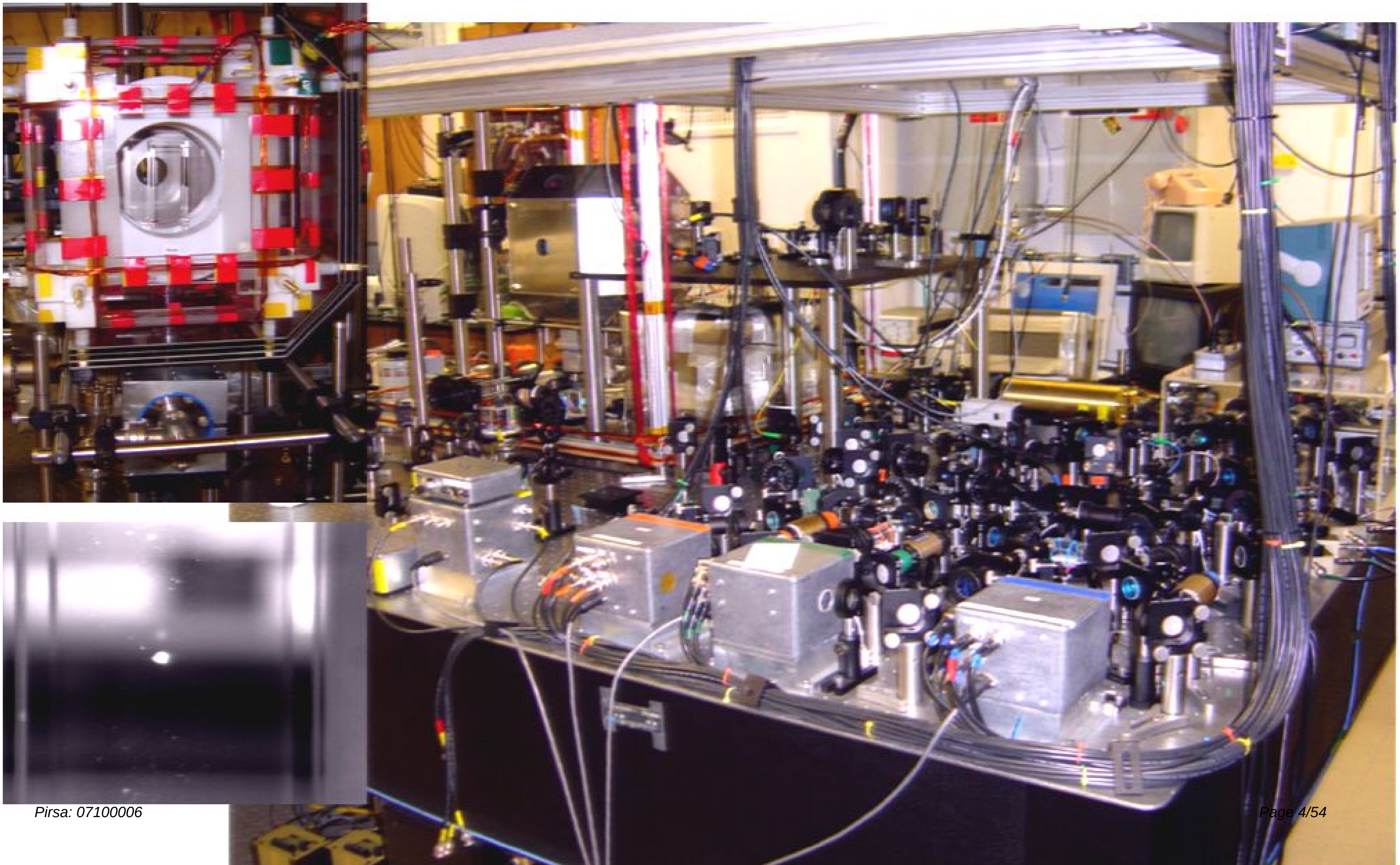
ONR, ARO, NSF

Quantum technologyology



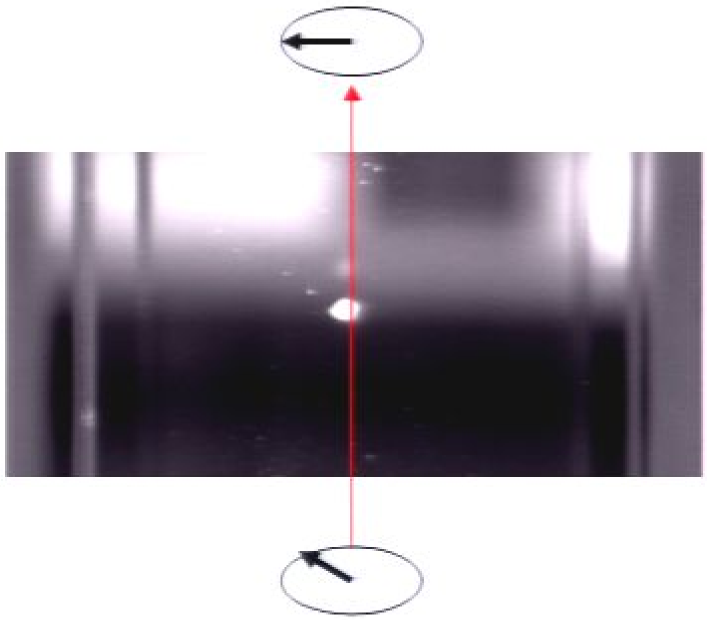
Quantum measurement/metrology with cold atoms

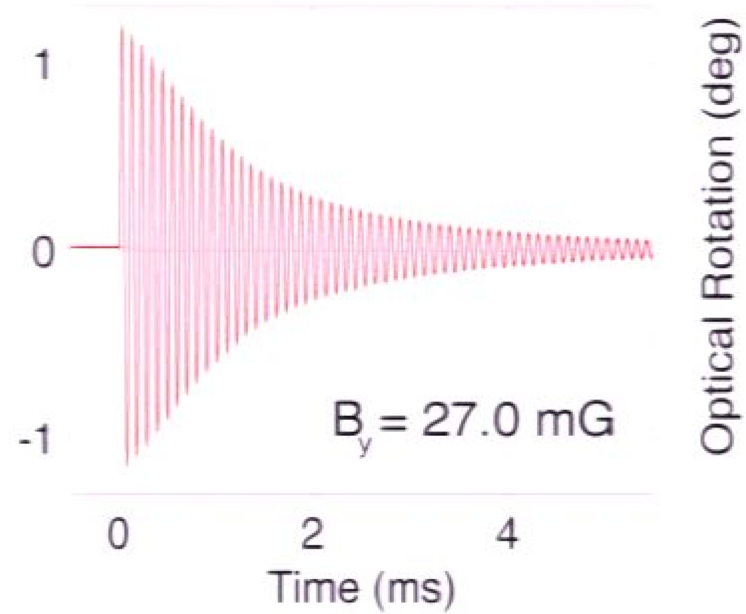
John K. Stockton, JM Geremia

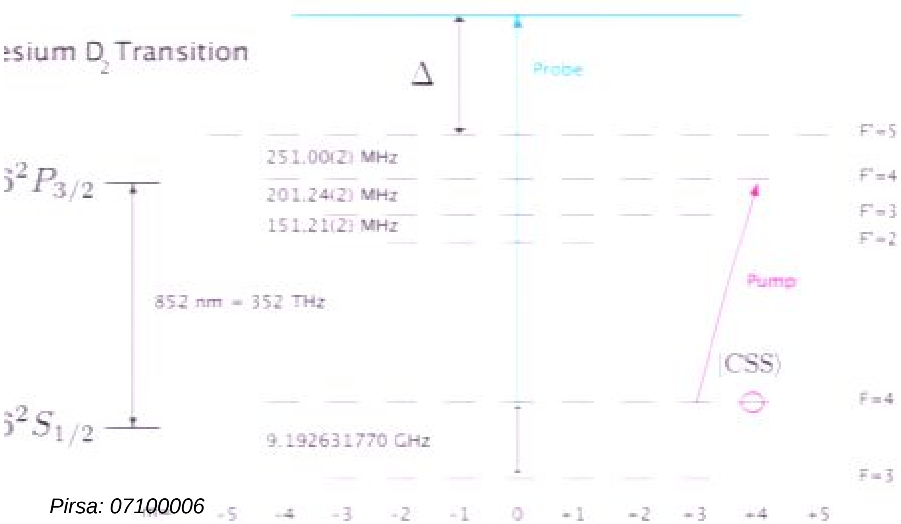


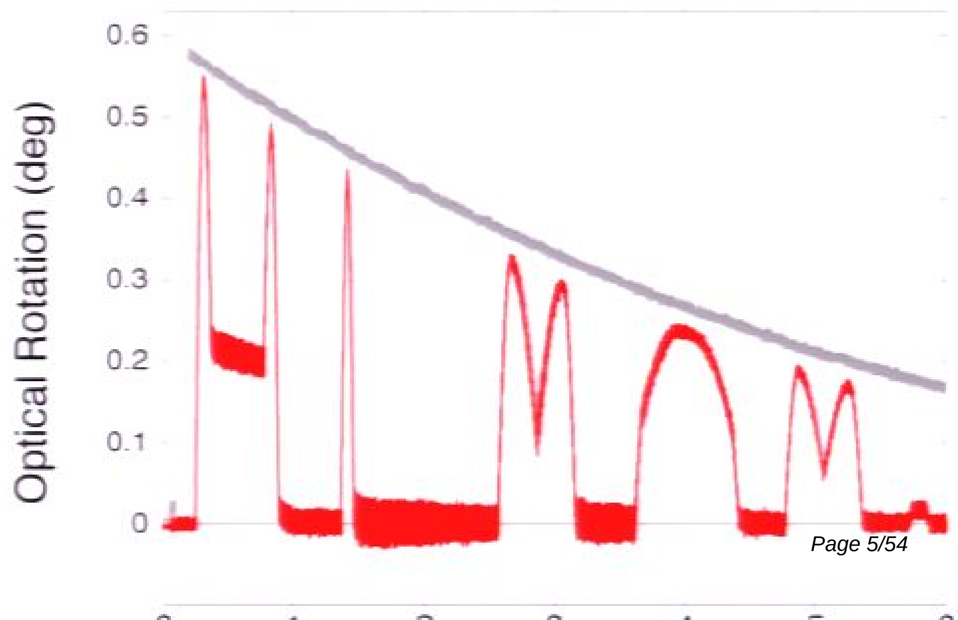
Faraday rotation: QND measurement of *collective* spin

Kuzmich/Bigelow/Mandel, Takahashi/Yabuzaki, Polzik*, Jessen*, …*



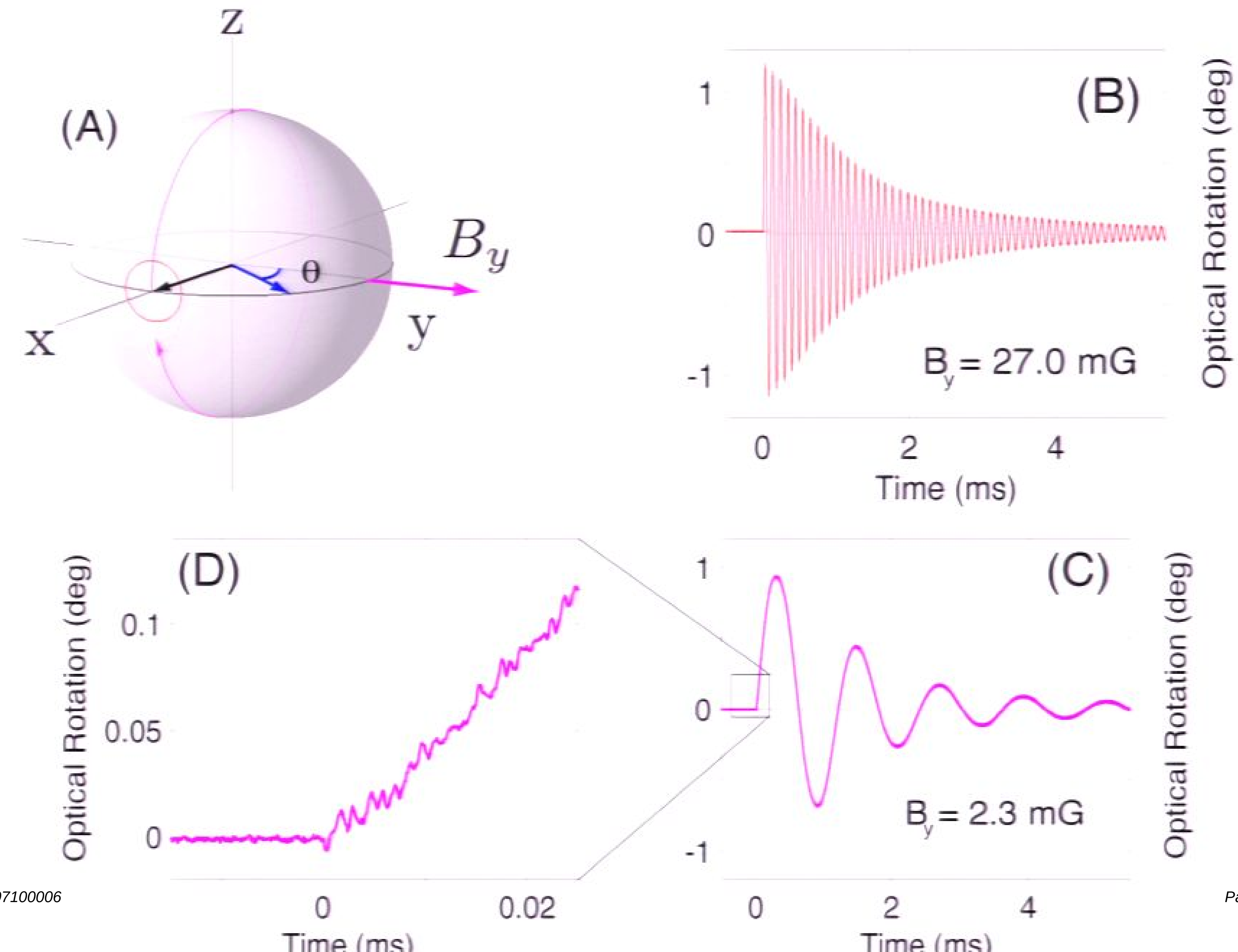






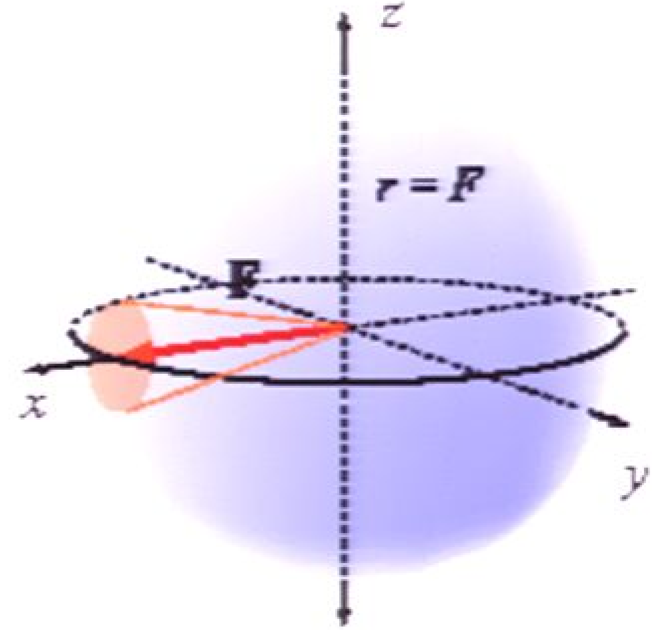
*tensor-coupling and propagation effects

Broadband atomic magnetometry

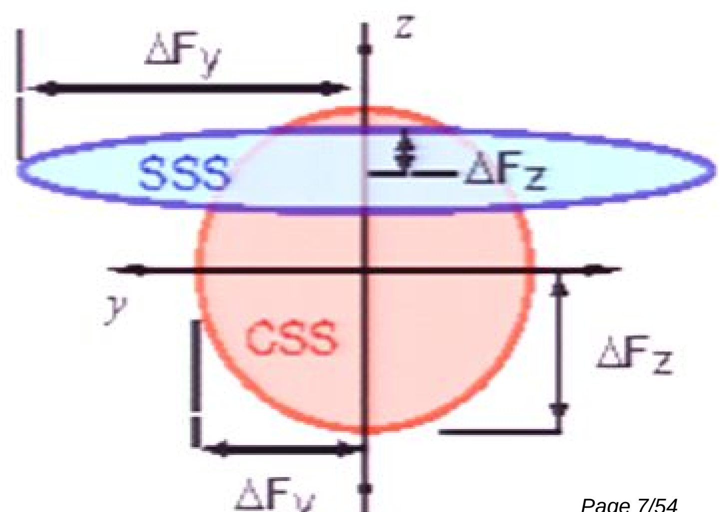


Quantum uncertainty of atomic magnetization

Cold cloud of Cs atoms
(gas phase, non-interacting)



Each atom carries a "spin" and associated magnetic moment



$$|\psi\rangle \in \mathcal{H}, \qquad \widehat{\mathbf{F}} = [\widehat{F}_x \; \widehat{F}_y \; \widehat{F}_z]$$

$$\Delta\widehat{F}_y \Delta\widehat{F}_z \geq \frac{1}{2}|\langle\widehat{F}_x\rangle|$$

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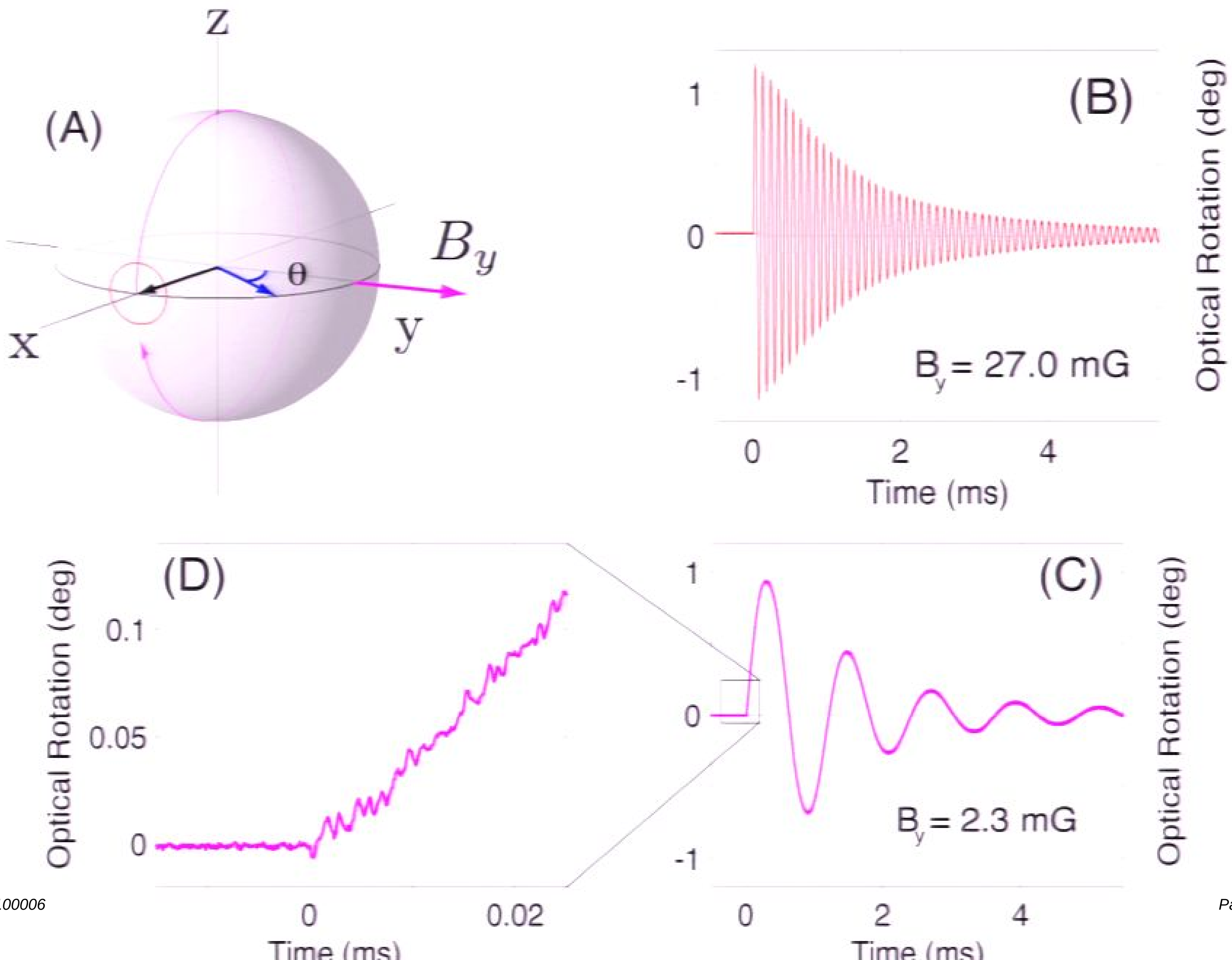

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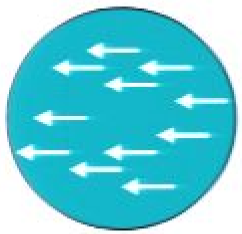

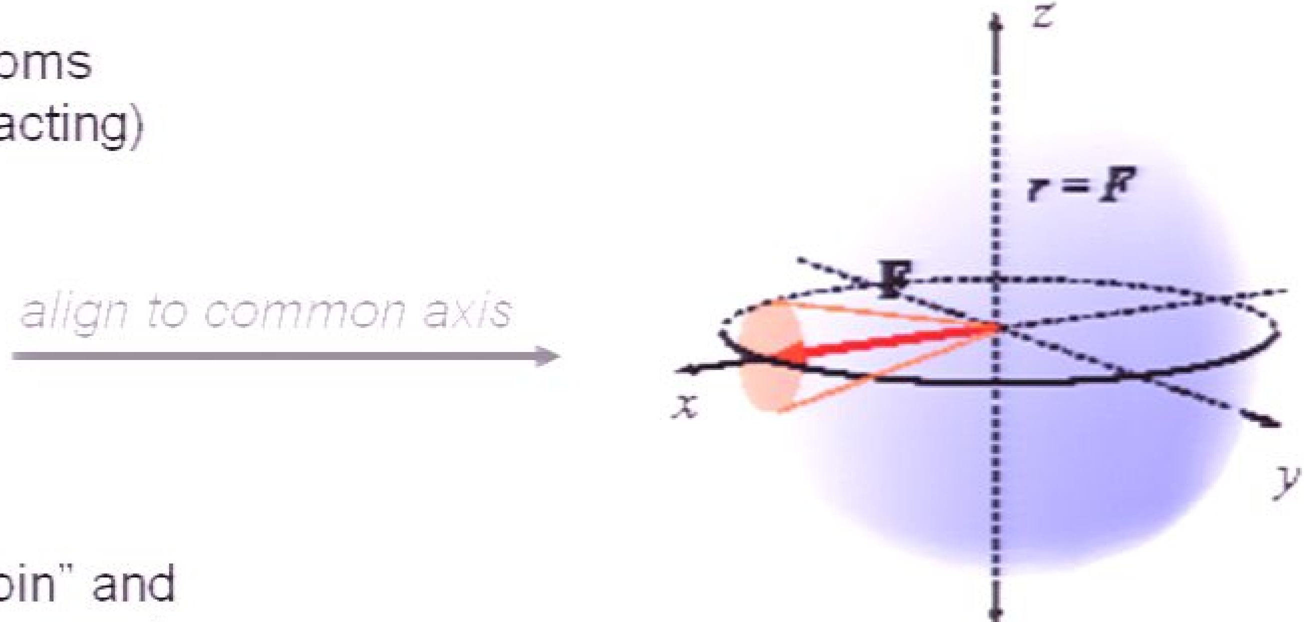

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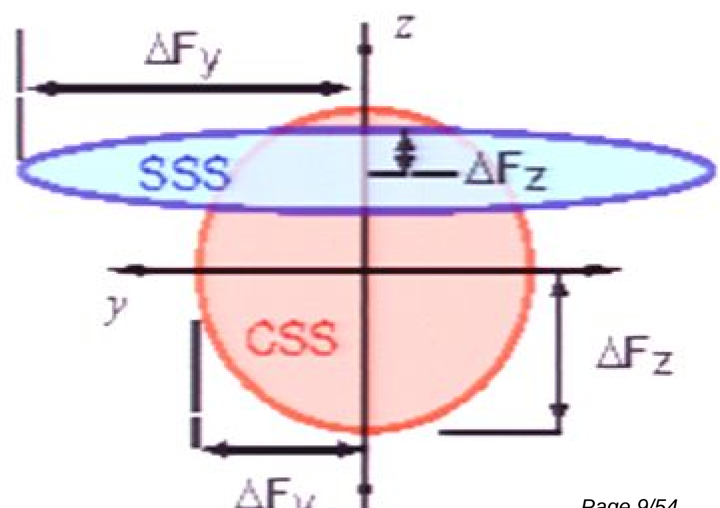
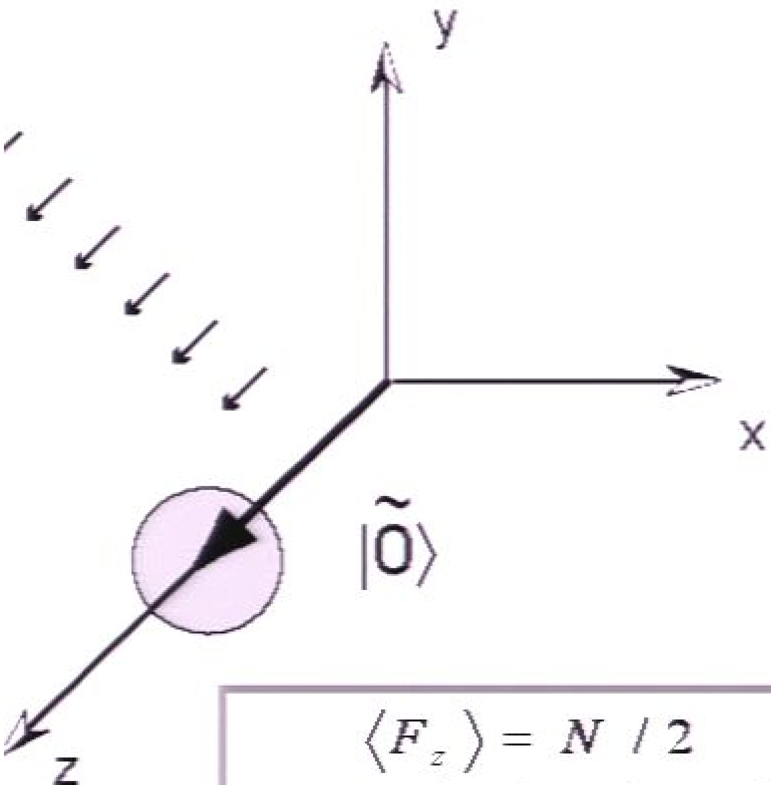
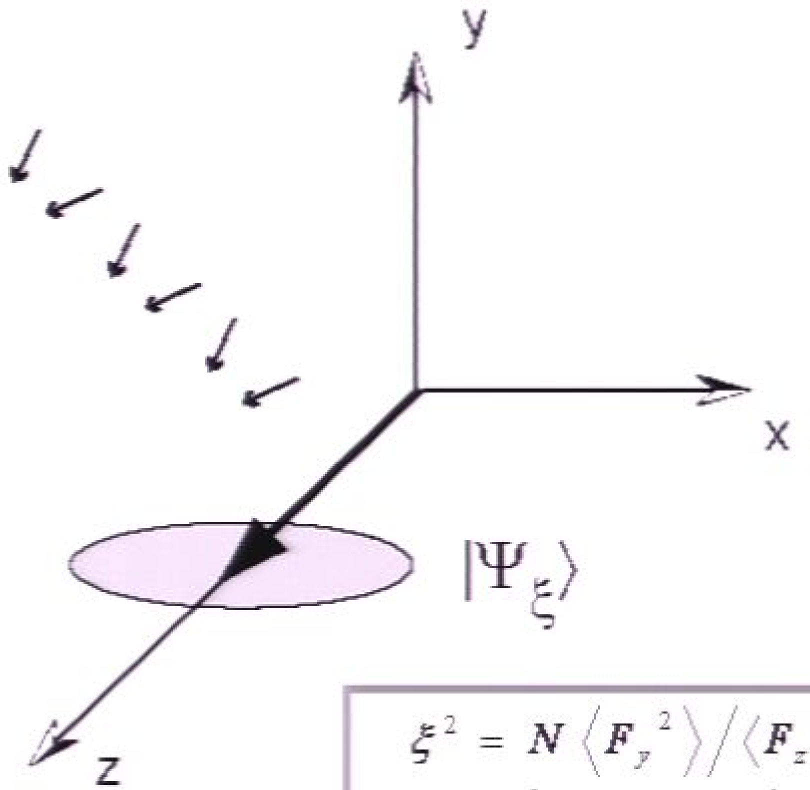


Spin-squeezing, symmetry and entanglement

John Stockton, JM Geremia, Andrew Doherty and HM, Phys. Rev. A **67**, 022112 (2003)



$$\langle F_z \rangle = N / 2$$
completely polarized



$$\xi^2 = N \langle F_y^{\,2} \rangle / \langle F_z \rangle^2 < 1$$
implies entanglement

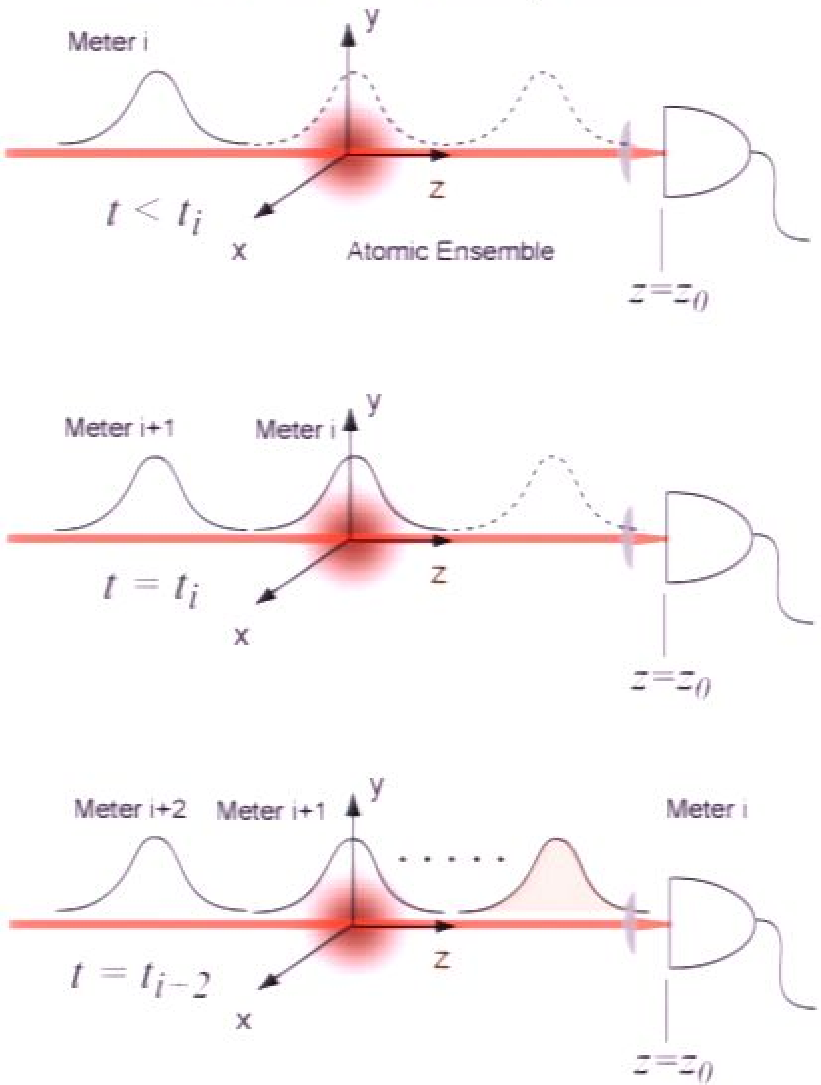
Measurement-induced quantum correlations

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle + |\uparrow\rangle)$$

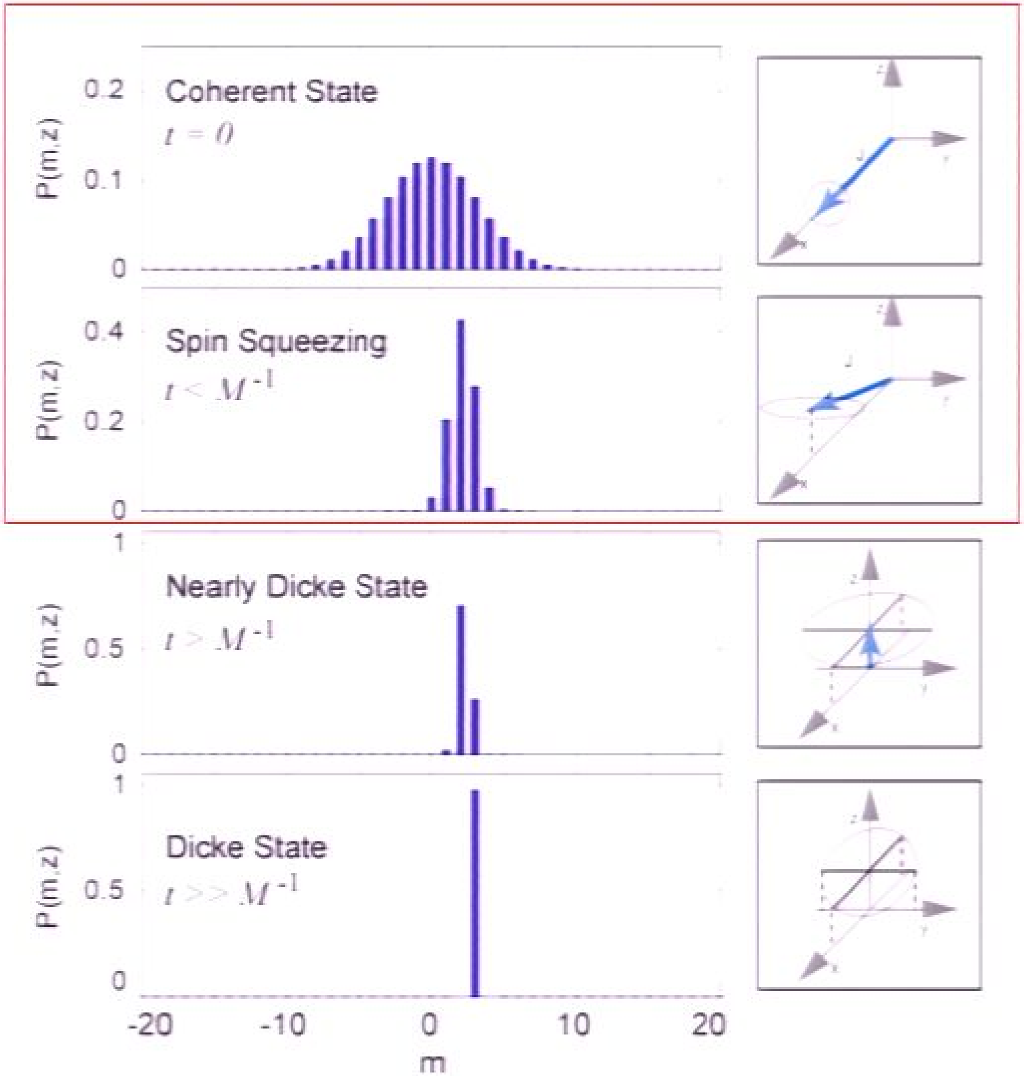
Continuous Faraday rotation - quantum filtering

JM Geremia, John Stockton and HM, Science **304**, 270 (2004)

Quantum Markov process



Quantum filtering; gradual "collapse"



Separation approach to quantum control *(Belavkin, ...)*

R. van Handel, J. K. Stockton and HM, quant-ph/0402136, IEEE-TAC

Stochastic Master Equation - conditional spin state

L.K. Thomsen, S. Mancini, and H. M. Wiseman, PRA **65** 061801(R) (2002)

$$\begin{aligned} d\rho_b(t) &= -i[\gamma F_y b, \rho_b(t)]dt + \mathcal{D}[\sqrt{M}F_z]\rho_b(t)dt \\ &\quad + \sqrt{\eta}\mathcal{H}[\sqrt{M}F_z]\left(2\sqrt{M\eta}[y(t)dt - \langle F_z\rangle_b dt]\right)\rho_b(t) \end{aligned}$$

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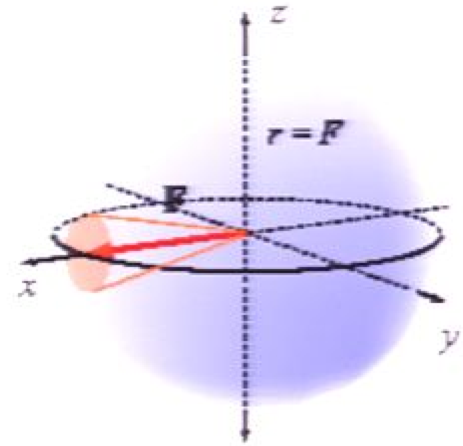
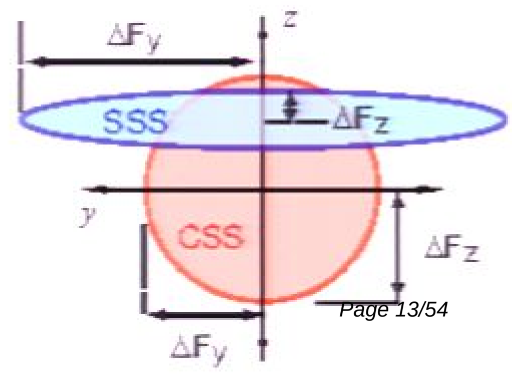
Hilbert space dimension very large $\sim (2f+1)^N$

Reduction by symmetry to $\sim 2f \times N$





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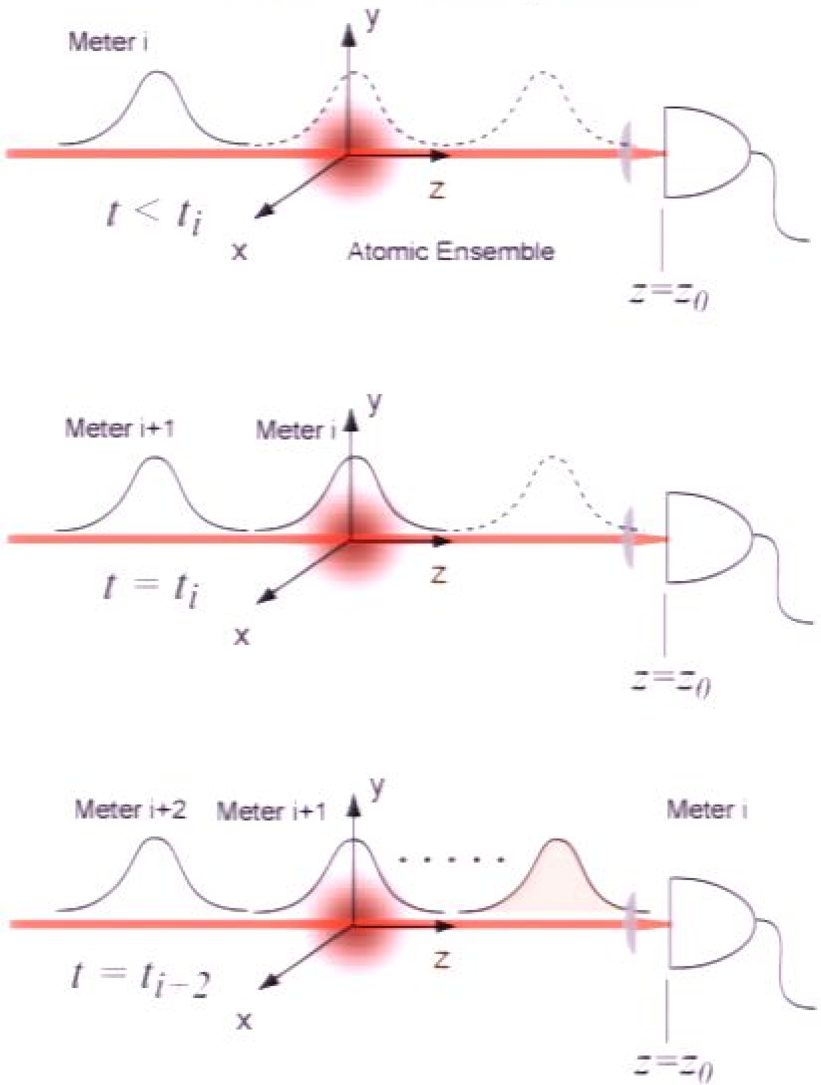

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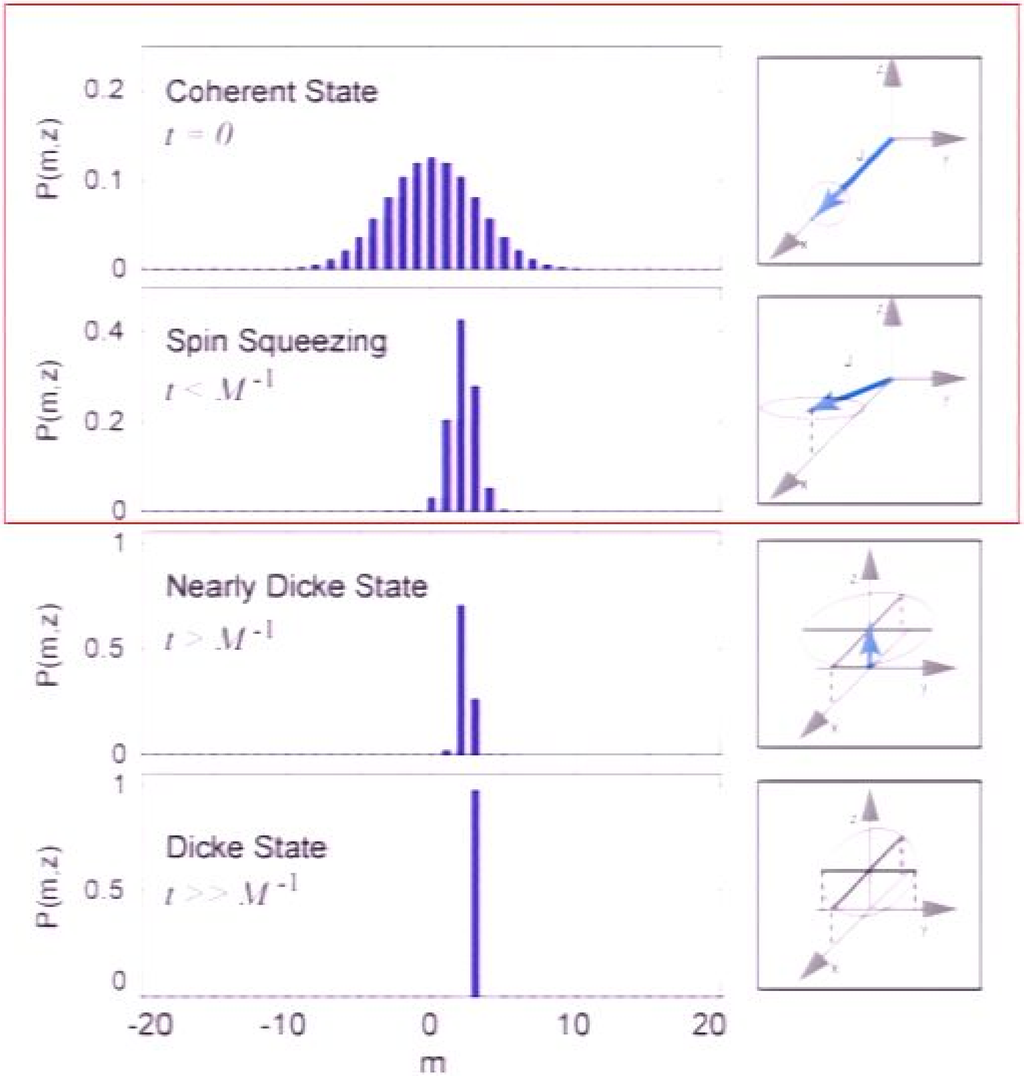

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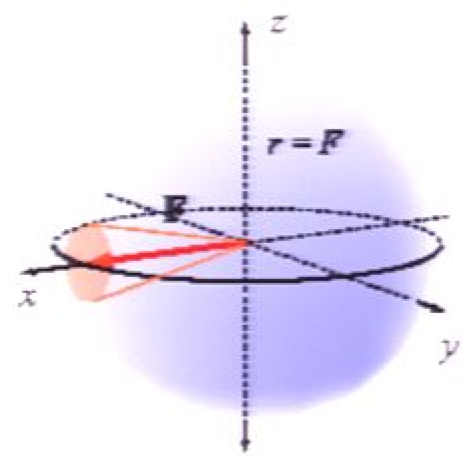

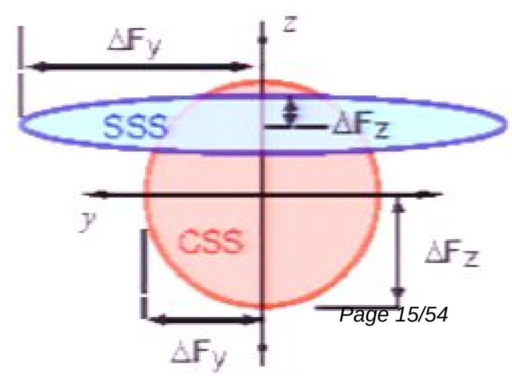

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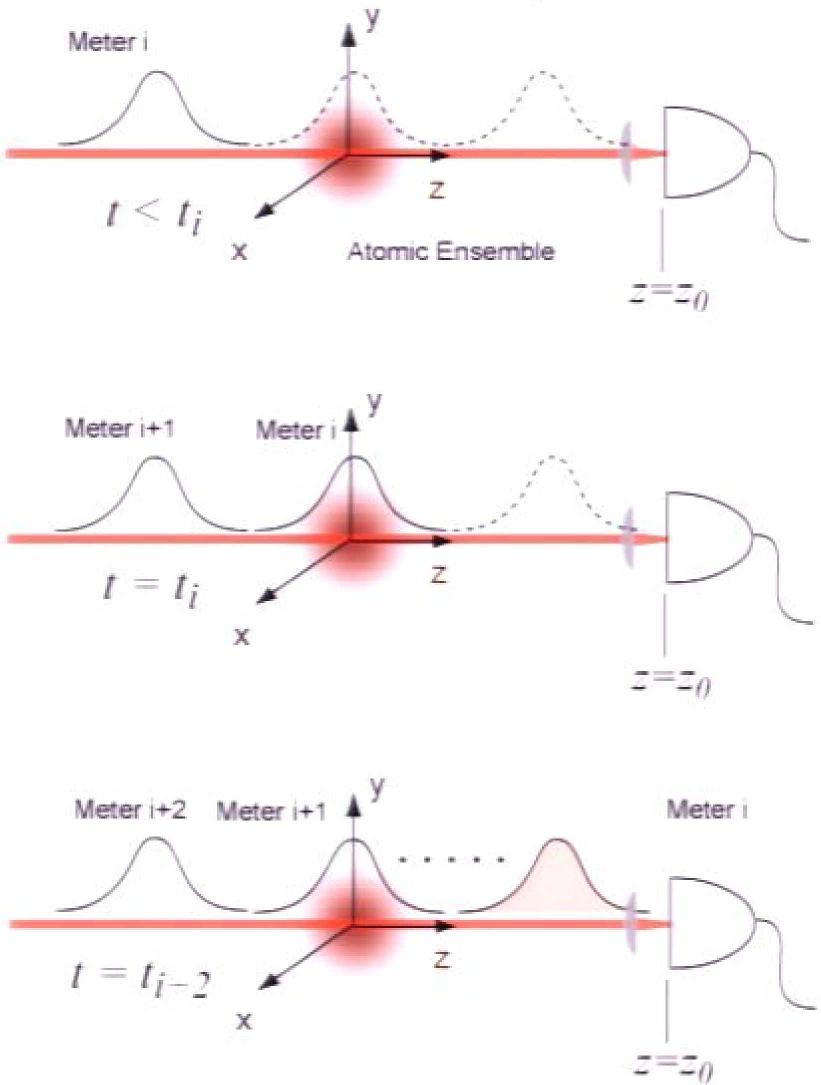

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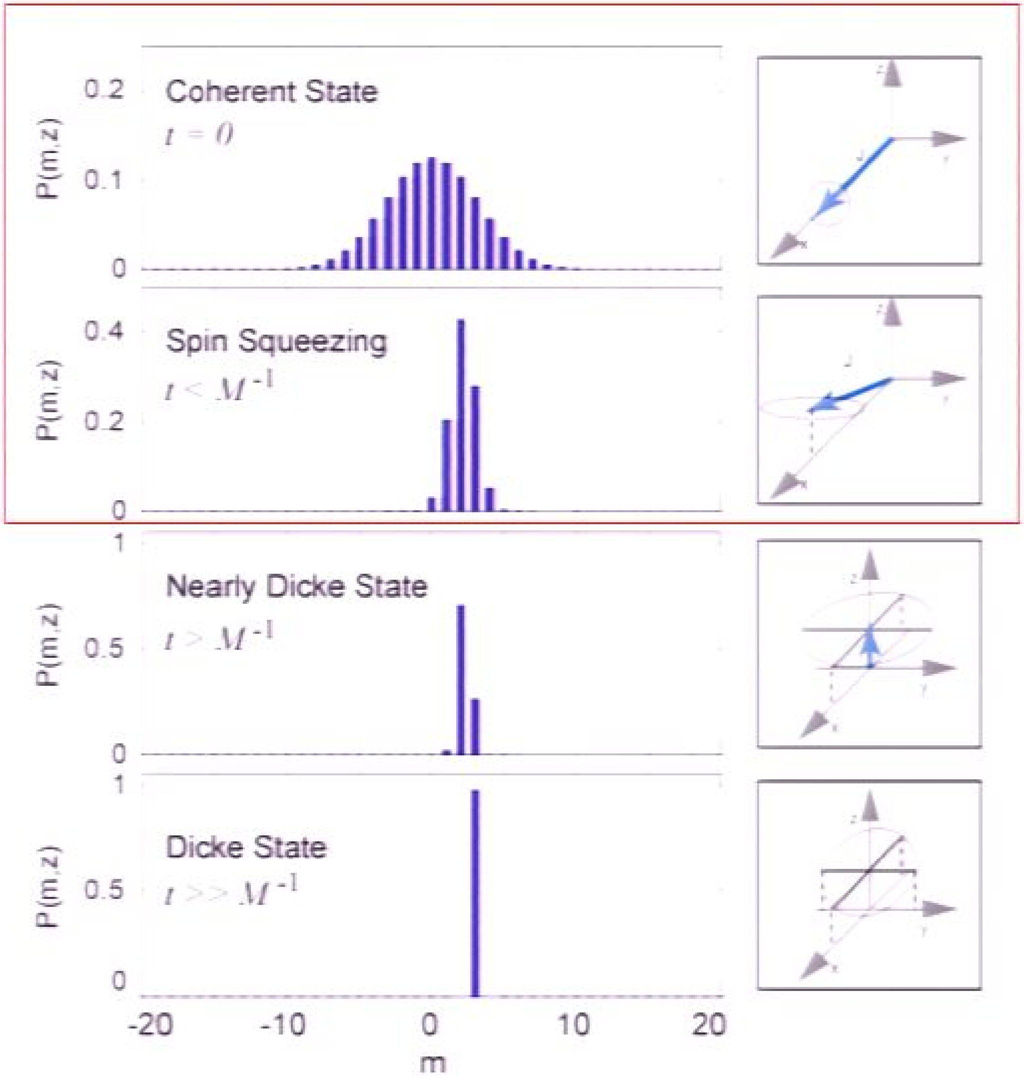

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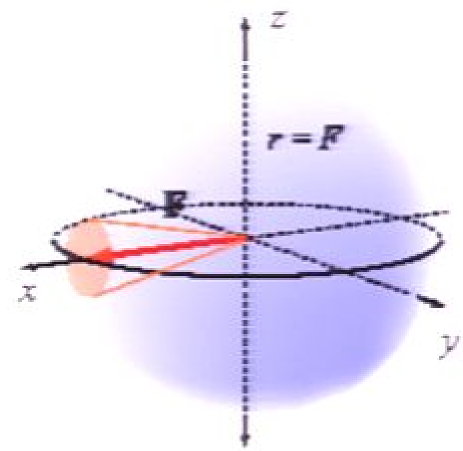

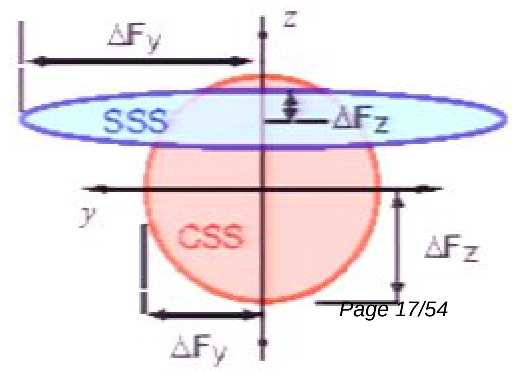

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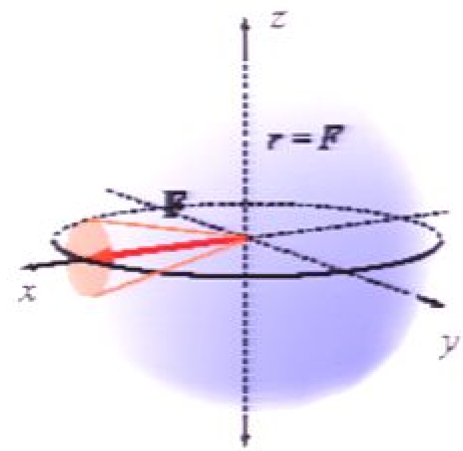
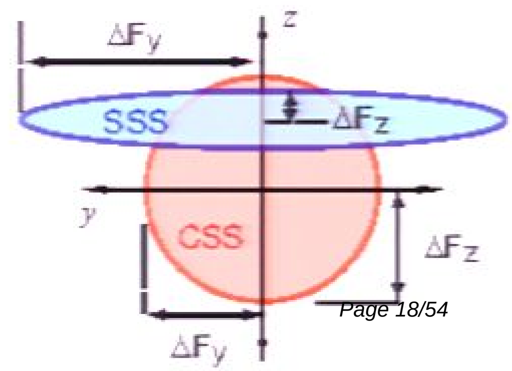


Hilbert space dimension very large $\sim (2f+1)^N$

Reduction by symmetry to $\sim 2f \times N$

- restriction to "back-action evading" operators
- flat approximation to local phase space
- moment expansion in one operator of interest
- Gaussian truncation...



"Quantum Kalman filter" for atomic spins

JM Geremia, John Stockton, Andrew Doherty and HM, Phys. Rev. Lett. **91**, 250801 (2003)

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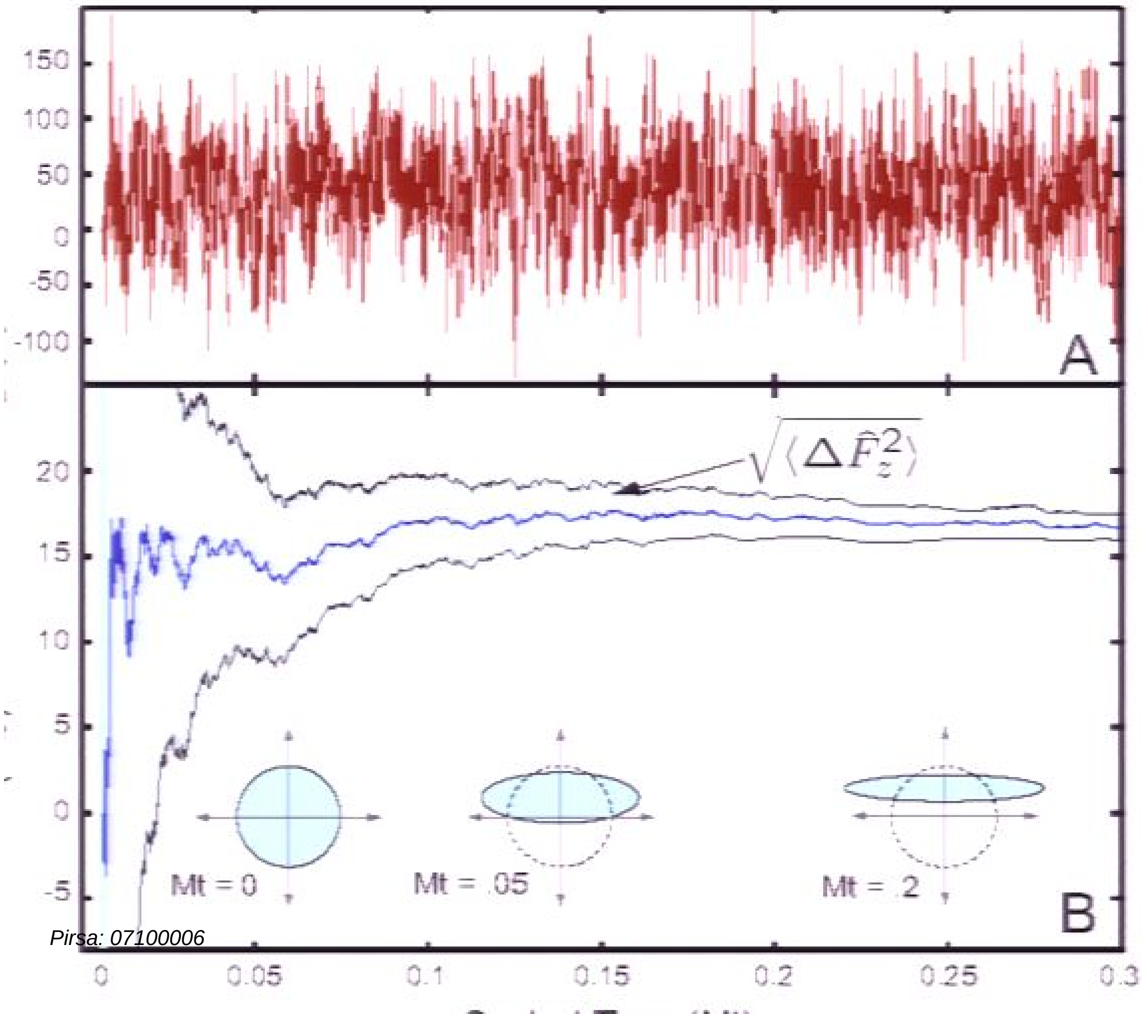
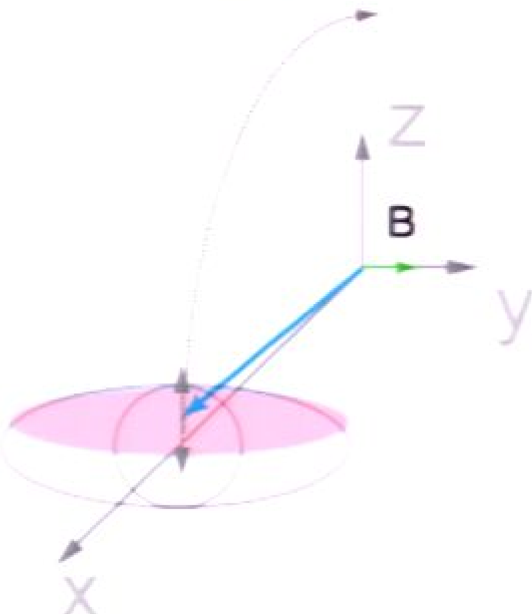
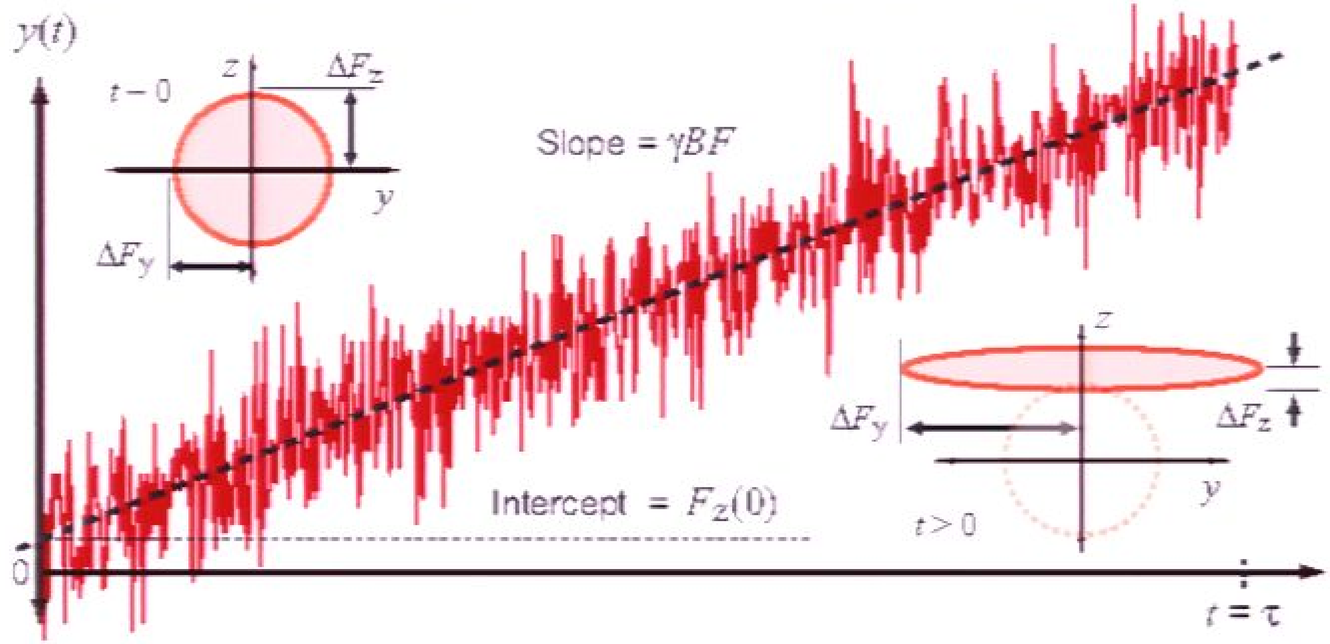


QND, spin-squeezing and sub-shotnoise metrology

Kitagawa & Ueda, Wineland *et al.*; JM Geremia *et al.* PRL **91**, 250801 (2003)





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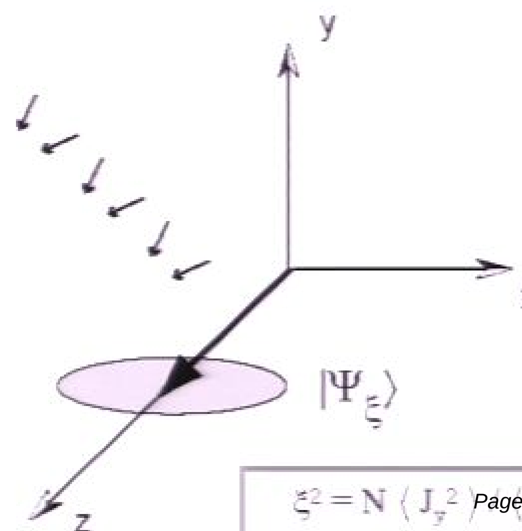
statistical regression uncertainty ***only***

$$\Delta\tilde{B}(\tau;N)_{\text{average}} = \frac{2}{\gamma F\tau}\sqrt{\Delta F_z^2(0) + \frac{\Delta\zeta_\tau^2}{M}}$$

"optical" **signal-to-noise** ratio

"shotnoise limit" $\sim \Delta F_z(0)/F\tau \propto F^{-1/2}$: **SNR** $\rightarrow \infty$



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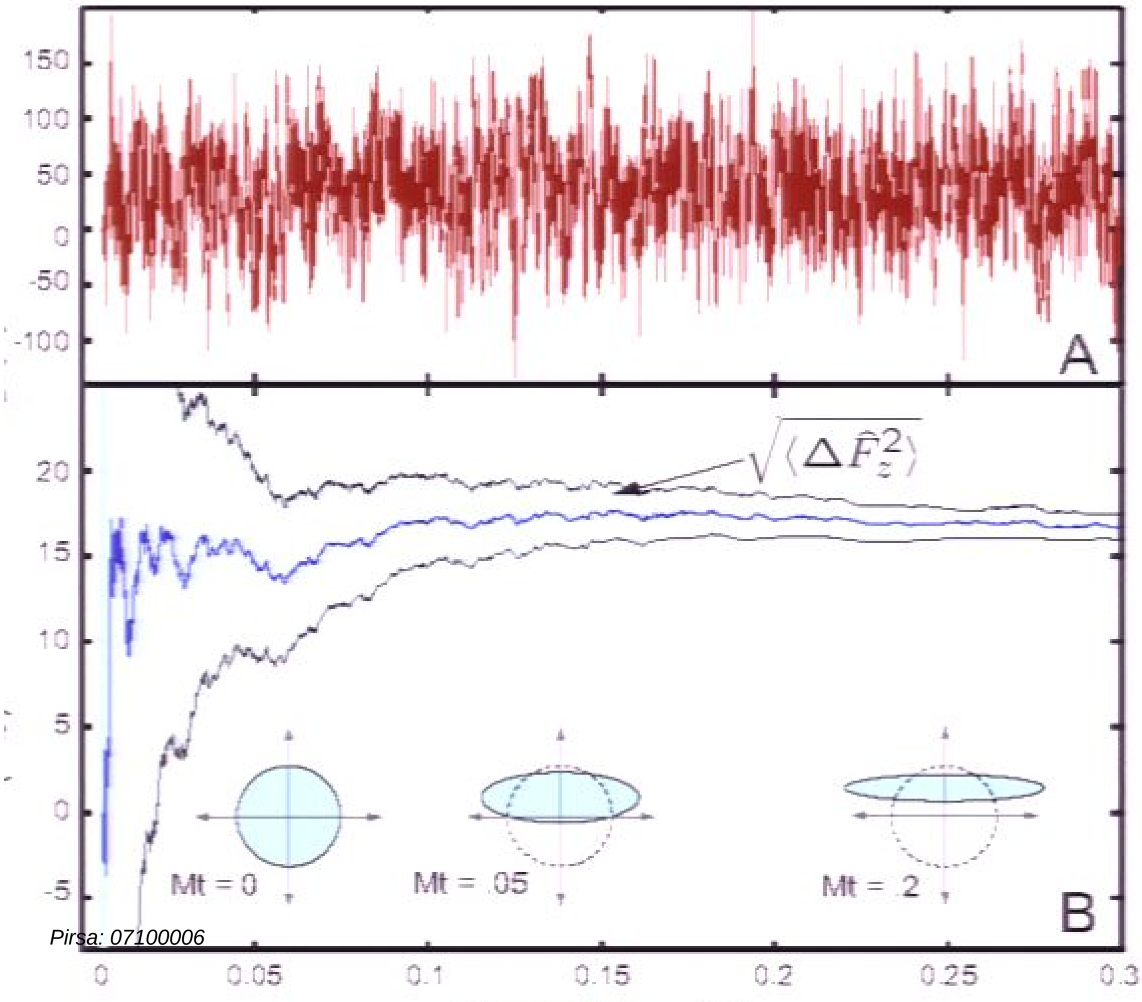


“classical equivalent” signal model

$$y(t)dt \propto f_z dt + \frac{1}{2\sqrt{M\eta}} dV_t$$

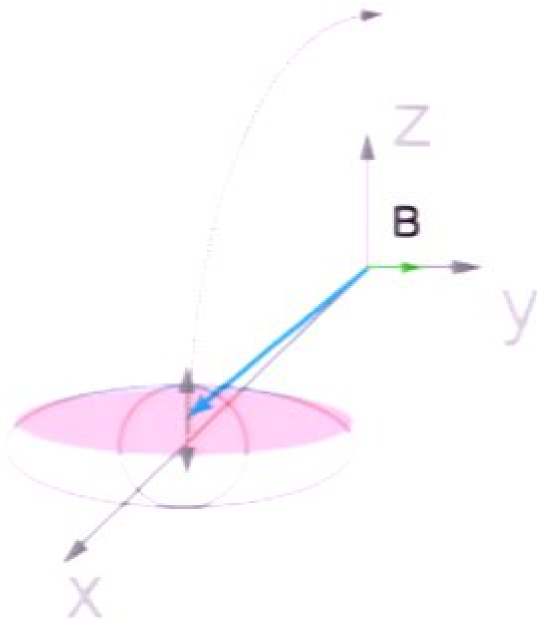
f_z a *classical* random variable

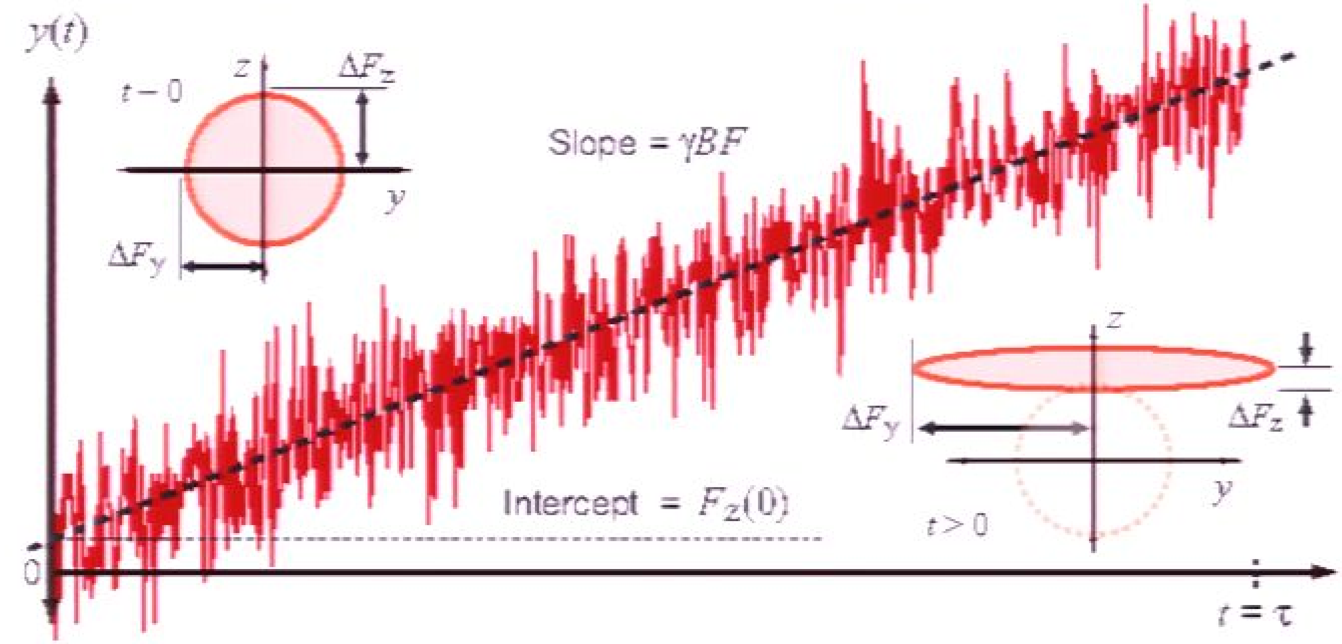
f_z “represents” $\hat{F}_z$ eigenvalue

$dV_t \sim$ gaussian white noise

QND, spin-squeezing and sub-shotnoise metrology

Kitagawa & Ueda, Wineland *et al.*; JM Geremia *et al.* PRL **91**, 250801 (2003)





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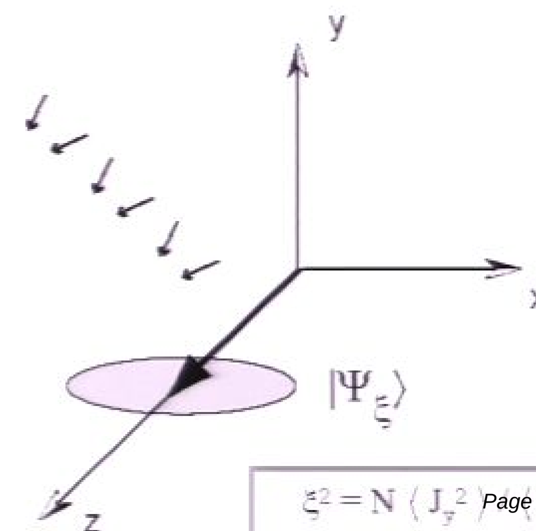

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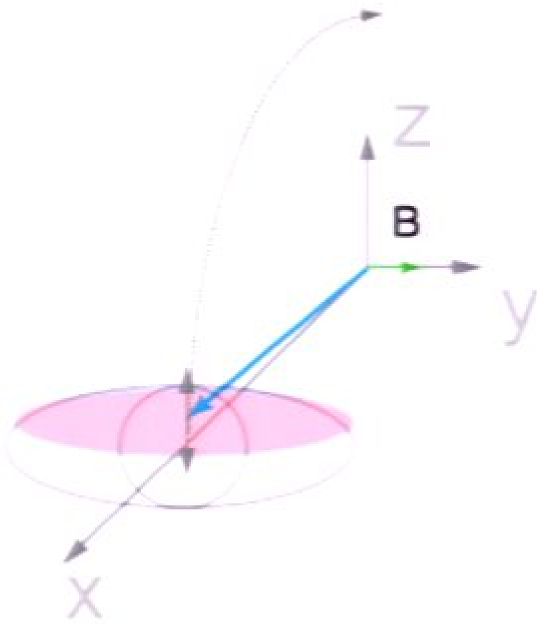

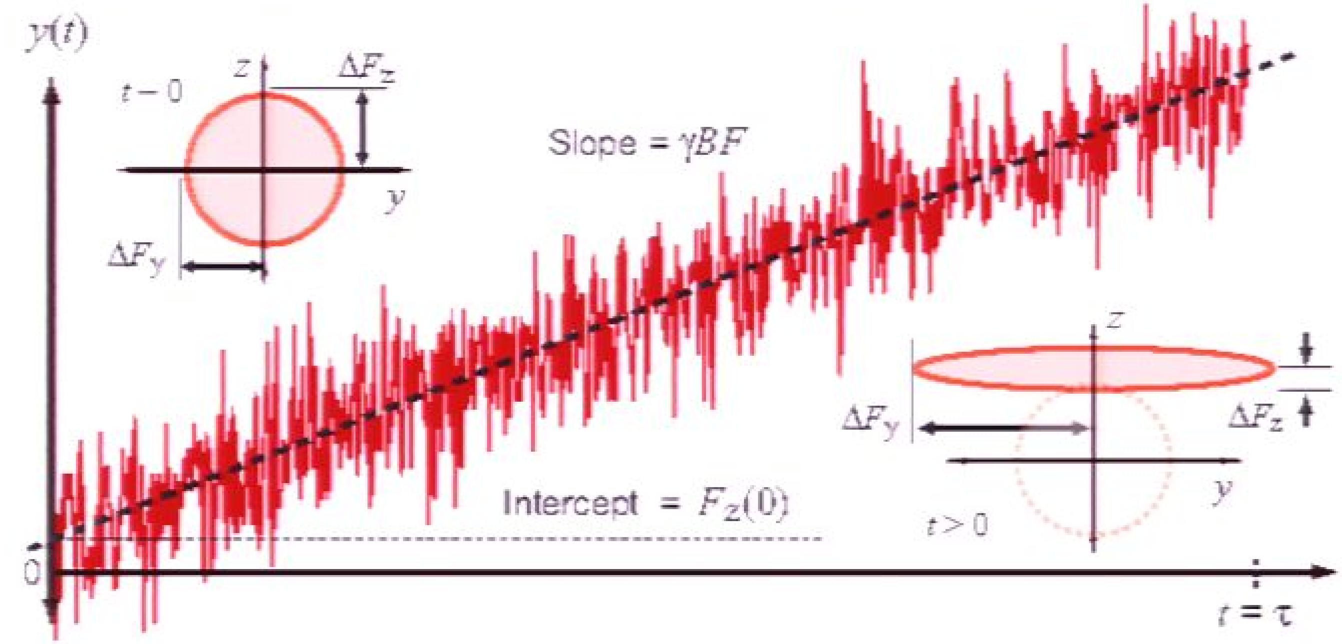

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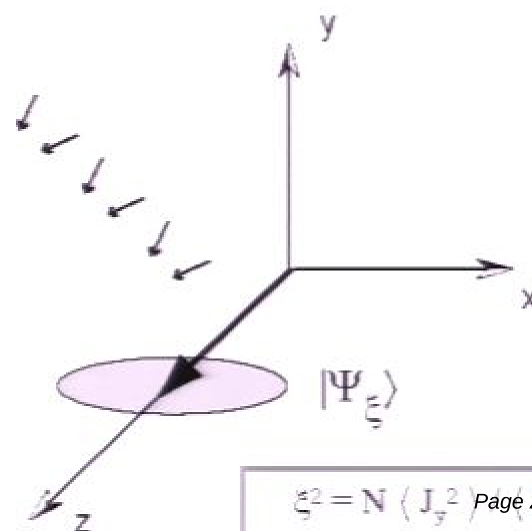

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Open-loop sensitivity to parameter variation

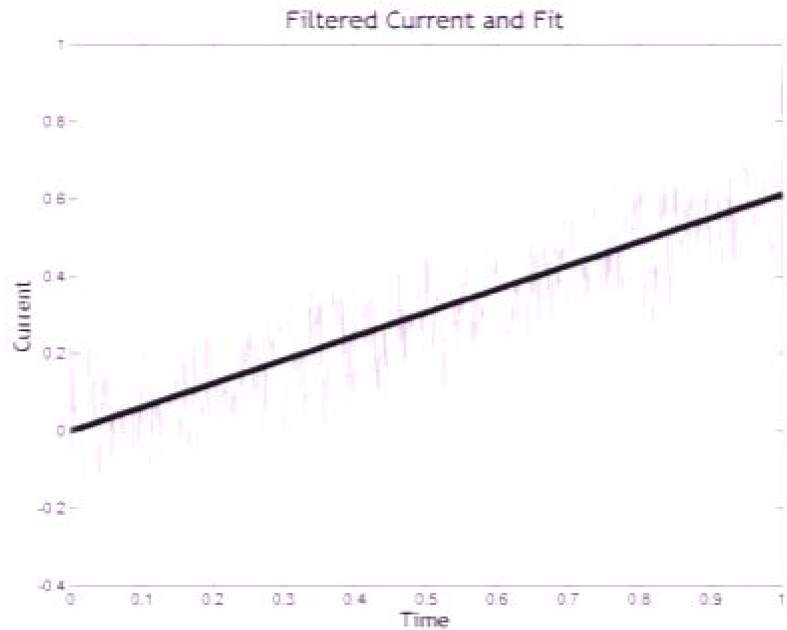
Canonical magnetometry is sensitive to atom number fluctuations

Photocurrent: $I = \gamma F b t + dW/dt$

Estimated Slope: $\hat{m}$

Correct Estimate: $\hat{b}_0 = \hat{m}/\gamma F$

Minimal Error: $E[(\hat{b}_0 - b)^2] \propto 1/F^2$



Incorrect assumed number: $\tilde{F} \neq F$

Incorrect estimate: $\hat{b} = \hat{m}/\gamma\tilde{F} = \hat{b}_0 F/\tilde{F}$

Resulting error: $E[(\hat{b} - b)^2] = E[(\hat{b}_0 - b)^2] + \frac{E[\Delta F^2]}{\tilde{F}^2}(b^2 + E[(\hat{b}_0 - b)^2])$

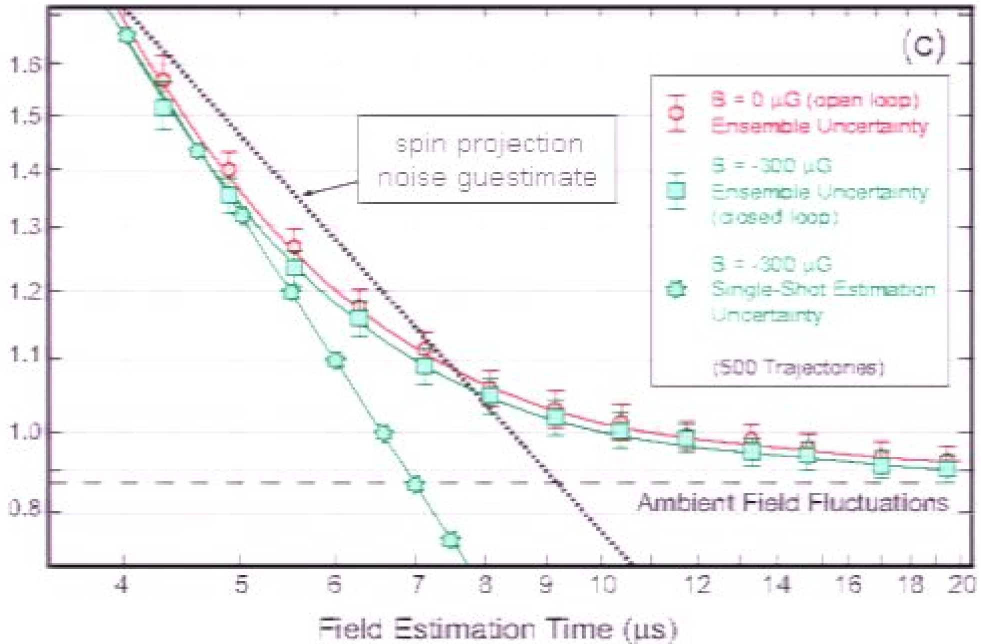
$E[\Delta F^2]$ represents shot-to-shot fluctuations of actual number

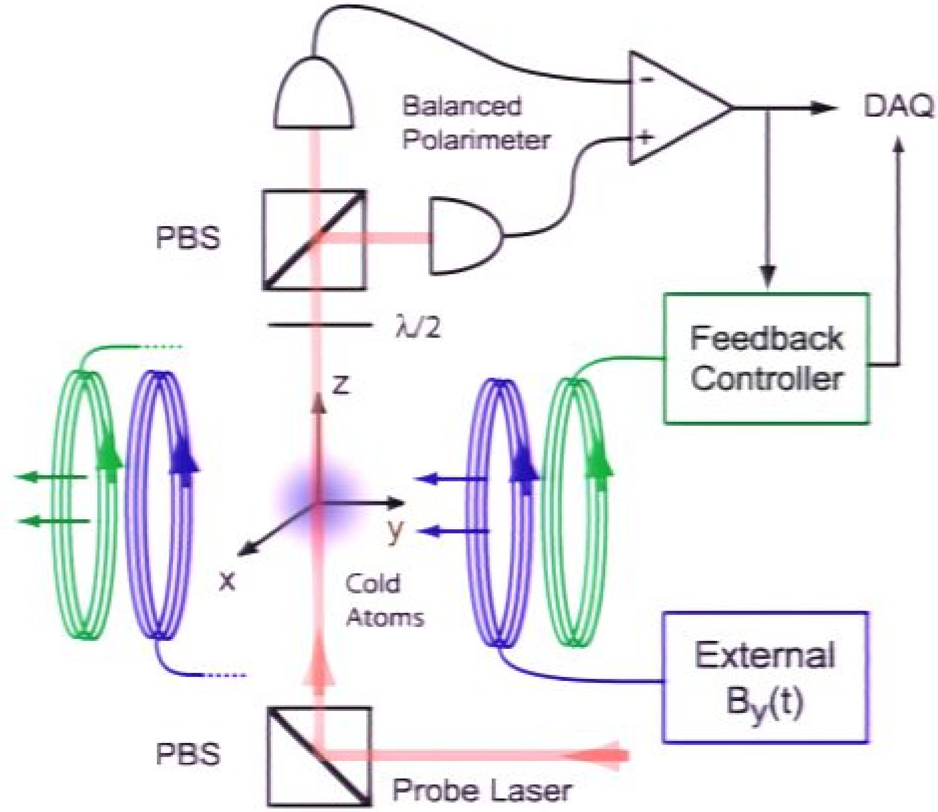
Feedback provides robustness to multiplicative uncertainty (~ gain margin)

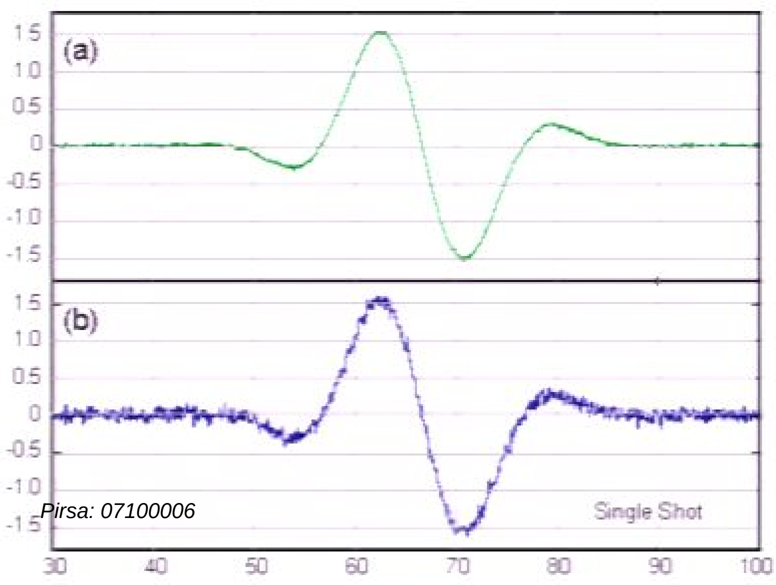
J. K. Stockton, JM Geremia, A. C. Doherty and HM, Phys. Rev. A **69**, 032109 (2004)

Robust broadband atomic magnetometry

JM Geremia, John Stockton and HM, PRL **94**, 203002 (2005)

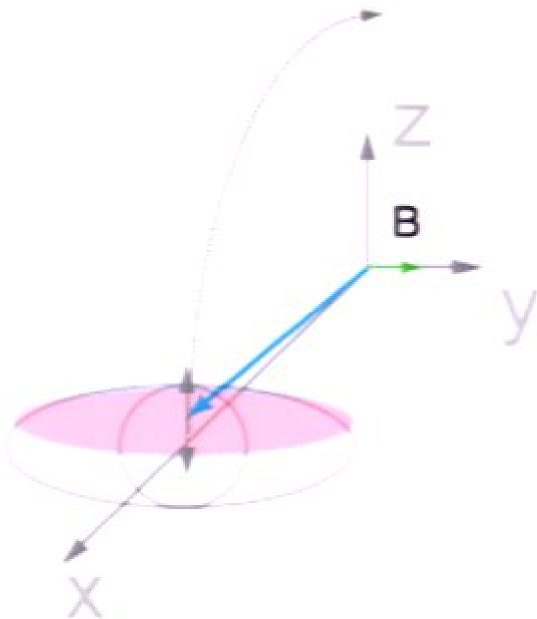


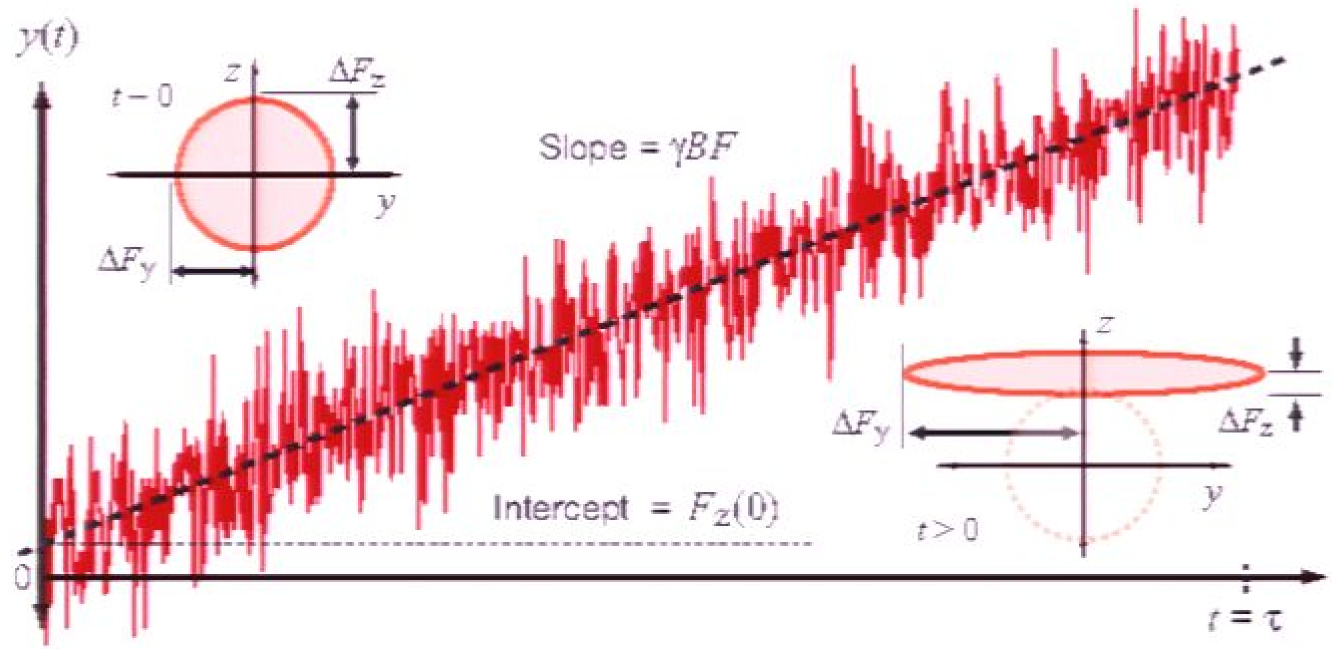




- Demonstrates use of *real-time feedback* to achieve robustness in a backaction-evading measurement scheme
- Method naturally accommodates broadband, non-stationary signals

The shotnoise limit in atomic magnetometry





need to show definitively:

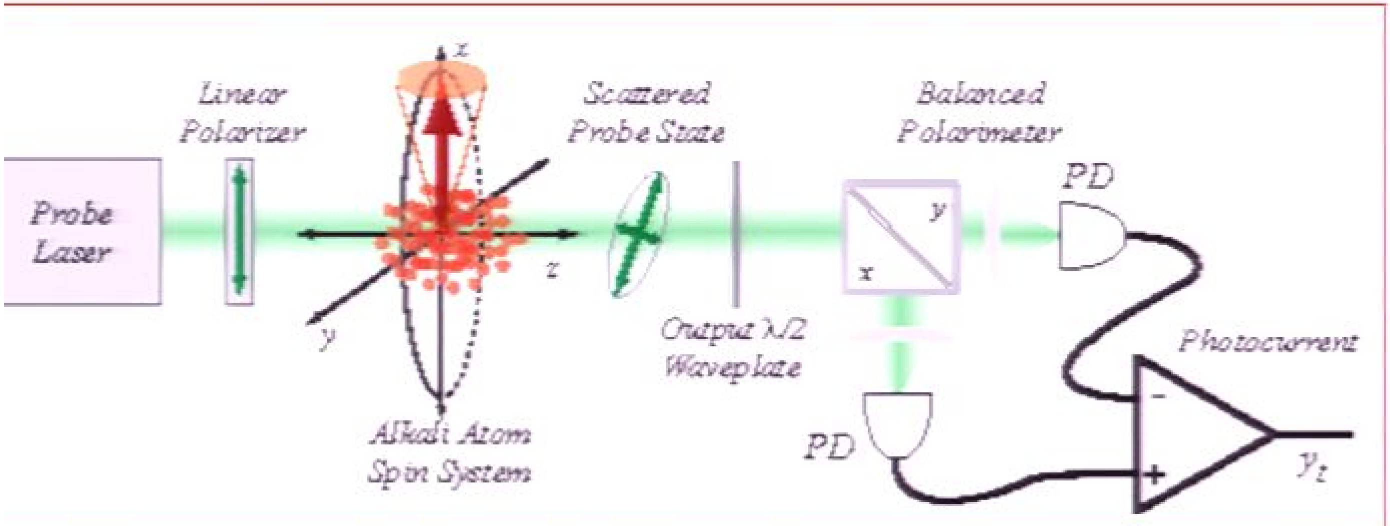
$$\underset{\text{experimental}}{\Delta\tilde{B}(\tau)} < \underset{\text{shotnoise-limited}}{\Delta\tilde{B}(\tau)} = \frac{2}{\gamma F \tau}\sqrt{\Delta F_z^2(0) + \frac{\Delta\zeta_\tau^2}{M}} = \frac{2}{\gamma\tau\sqrt{N}}$$

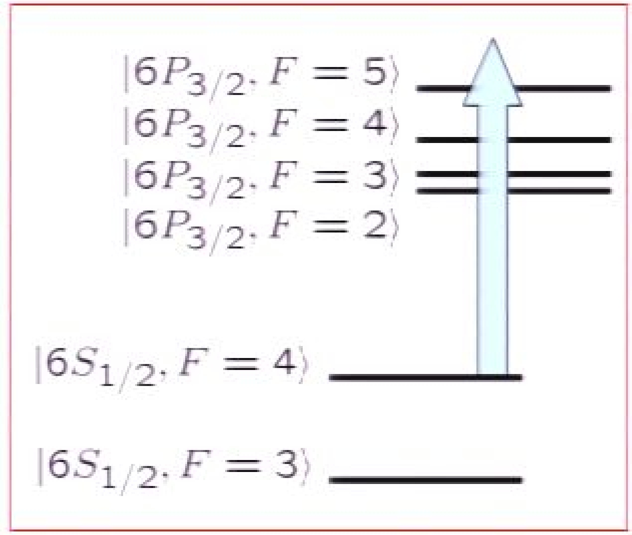
... where $\Delta F_z^2(0)$ is the projection noise of an ideal coherent spin state

$\Rightarrow \Delta\tilde{B}_{SNL}(\tau; N)$ does not depend on atomic physics, geometric factors

$$\Delta\tilde{B}_{SNL}(1\text{ms}; 10^9) \approx 30\text{nG}$$

Primary obstacles to sub-shotnoise magnetometry





ystematic

- Need to know effective atom number N to fix $\Delta\tilde{B}_{SNL}(\tau;N)$
- Need to suppress effects of tensor interaction

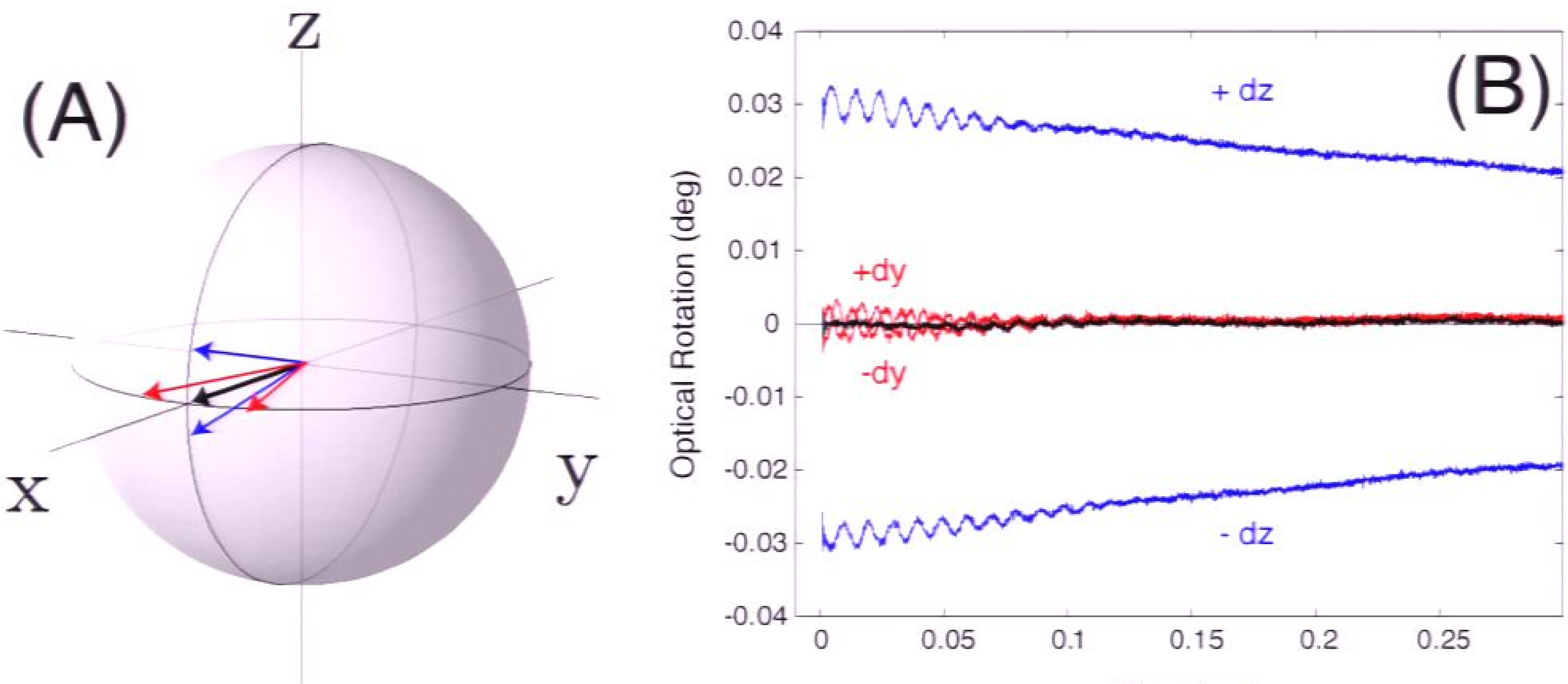
$$_{f,f'} = \hat{P}_f\,\hat{\mathbf{d}}\,\hat{P}_{f'}\,\hat{\mathbf{d}}^\dagger\,\hat{P}_f \rightarrow \hat{\alpha}^{(0)}_{f,f'} + \hat{\alpha}^{(1)}_{f,f'} + \hat{\alpha}^{(2)}_{f,f'} \qquad \hat{H}^{(j)} = \sum_{f,f'} \hat{\mathbf{E}}^{(-)} \cdot \frac{\hat{\alpha}^{(j)}_{f,f'}}{\hbar\Delta_{f,f'}} \cdot \hat{\mathbf{E}}^{(+)}$$

echnical

- Background field fluctuations comparable to $\Delta\tilde{B}_{SNL}(\tau;N)$
- Difficulties in achieving high polarization at high OD

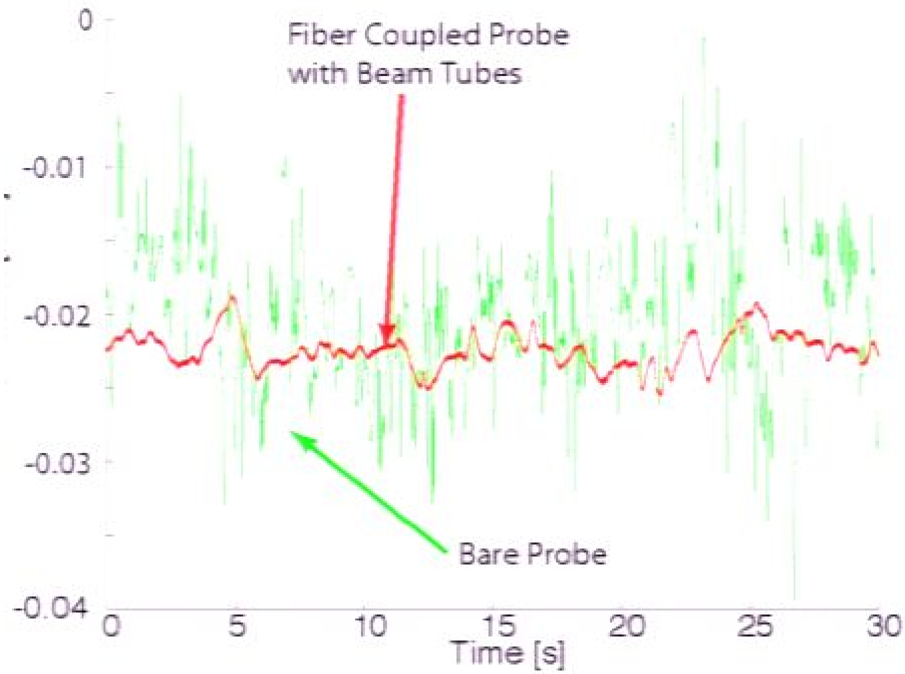
Tensor coupling-induced classical spin noise

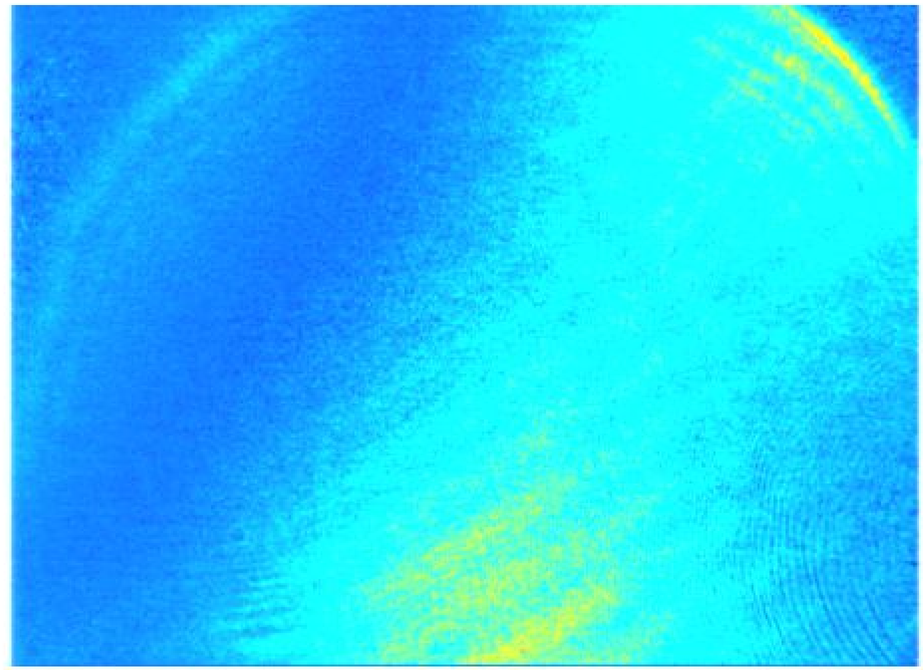
John Stockton, Caltech Ph.D. Thesis (2007)

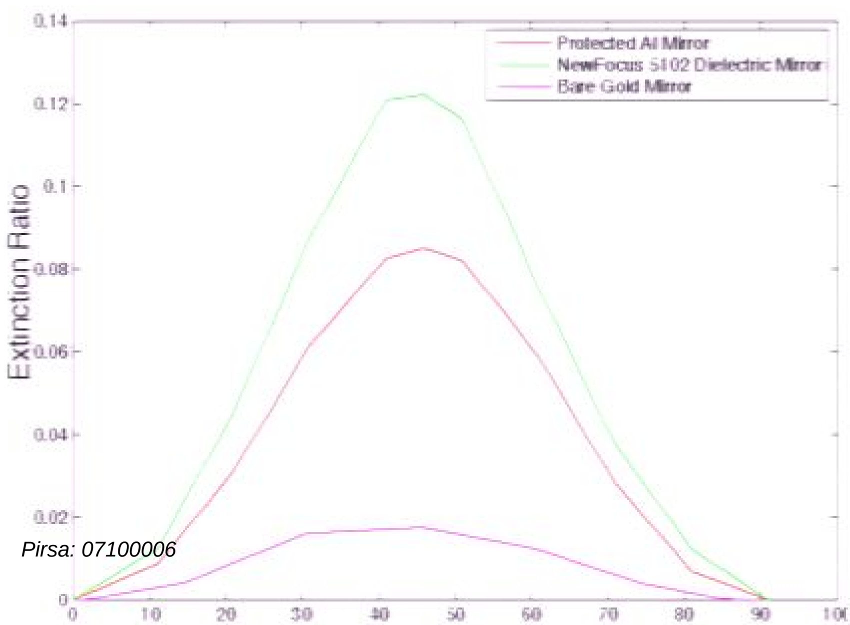


Polarization/pointing fluctuations couple tensor dynamics to Faraday variance

Polarization and pointing stability 2007



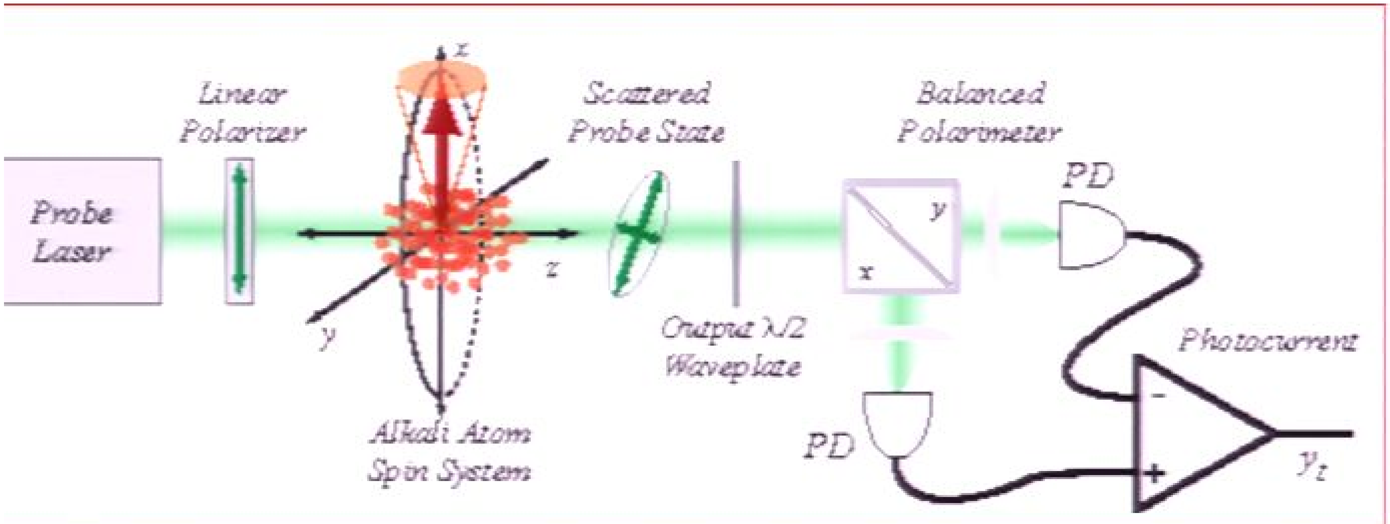


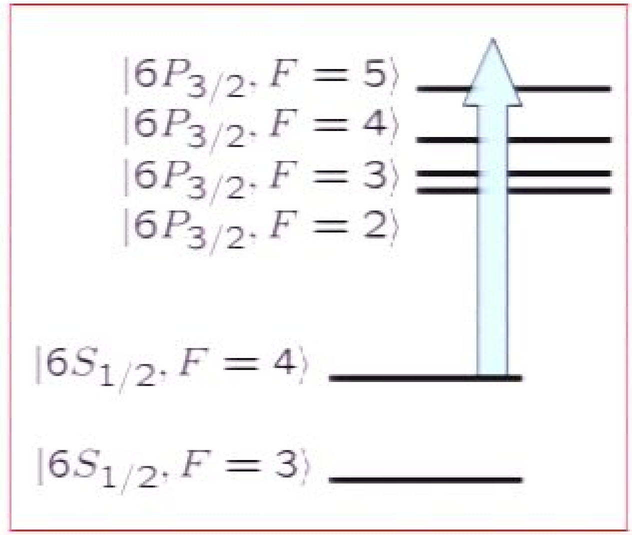


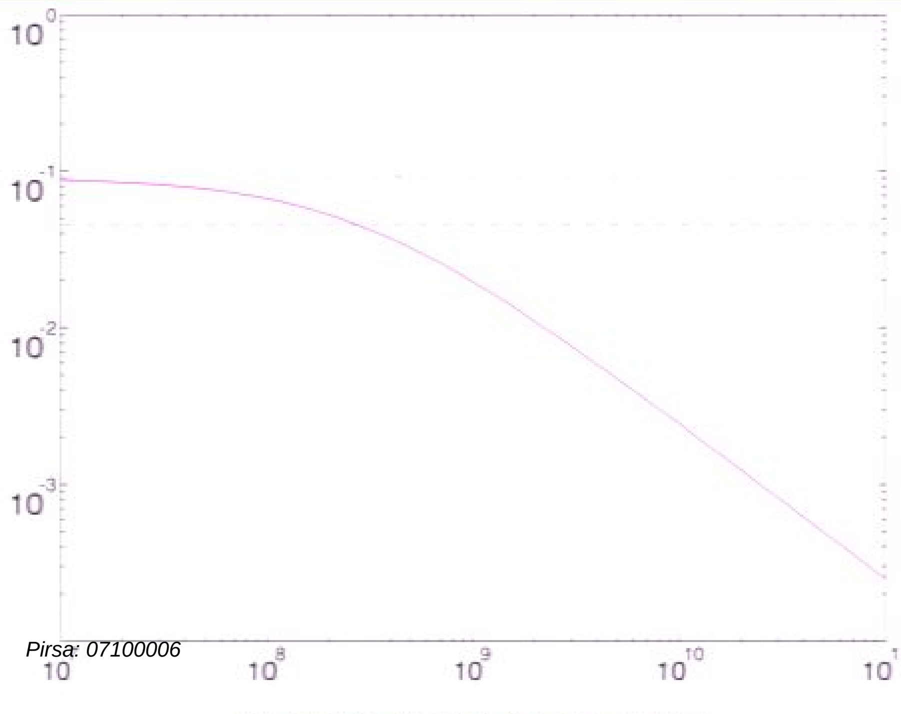
- slow fluctuations now 1.89×10^{-9} deg^2 (13 mW, 1 kHz bandwidth, 200s)
- compares with $\sim 10^{-4}$ deg^2 previously
- this improvement enables fundamental change in tensor suppression strategy

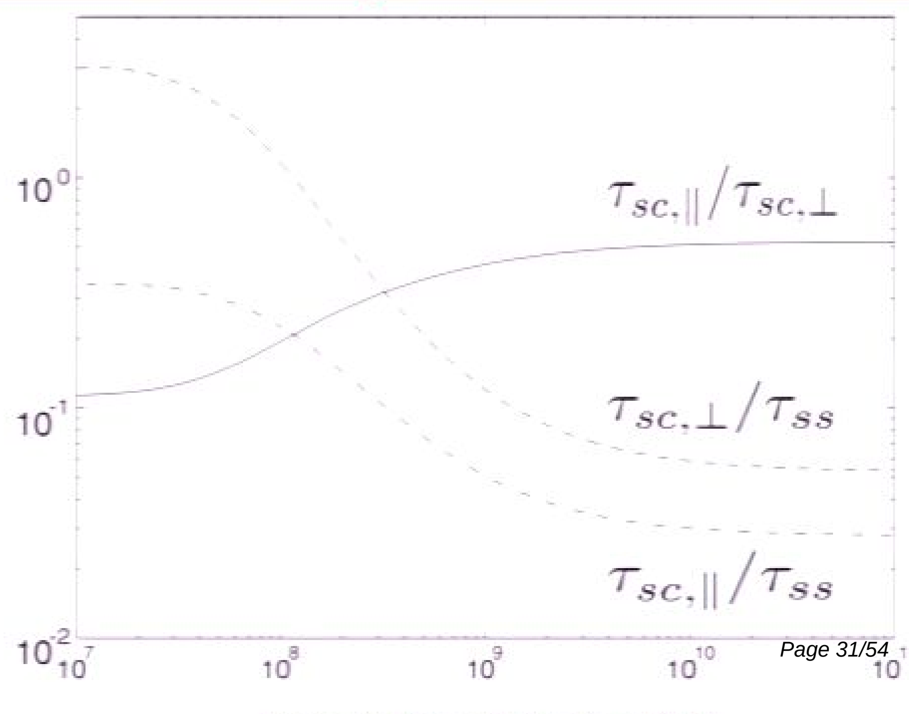
Vector and tensor coupling strengths vs. detuning

JM Geremia, John Stockton and HM, Phys. Rev. A **73**, 042112 (2006)

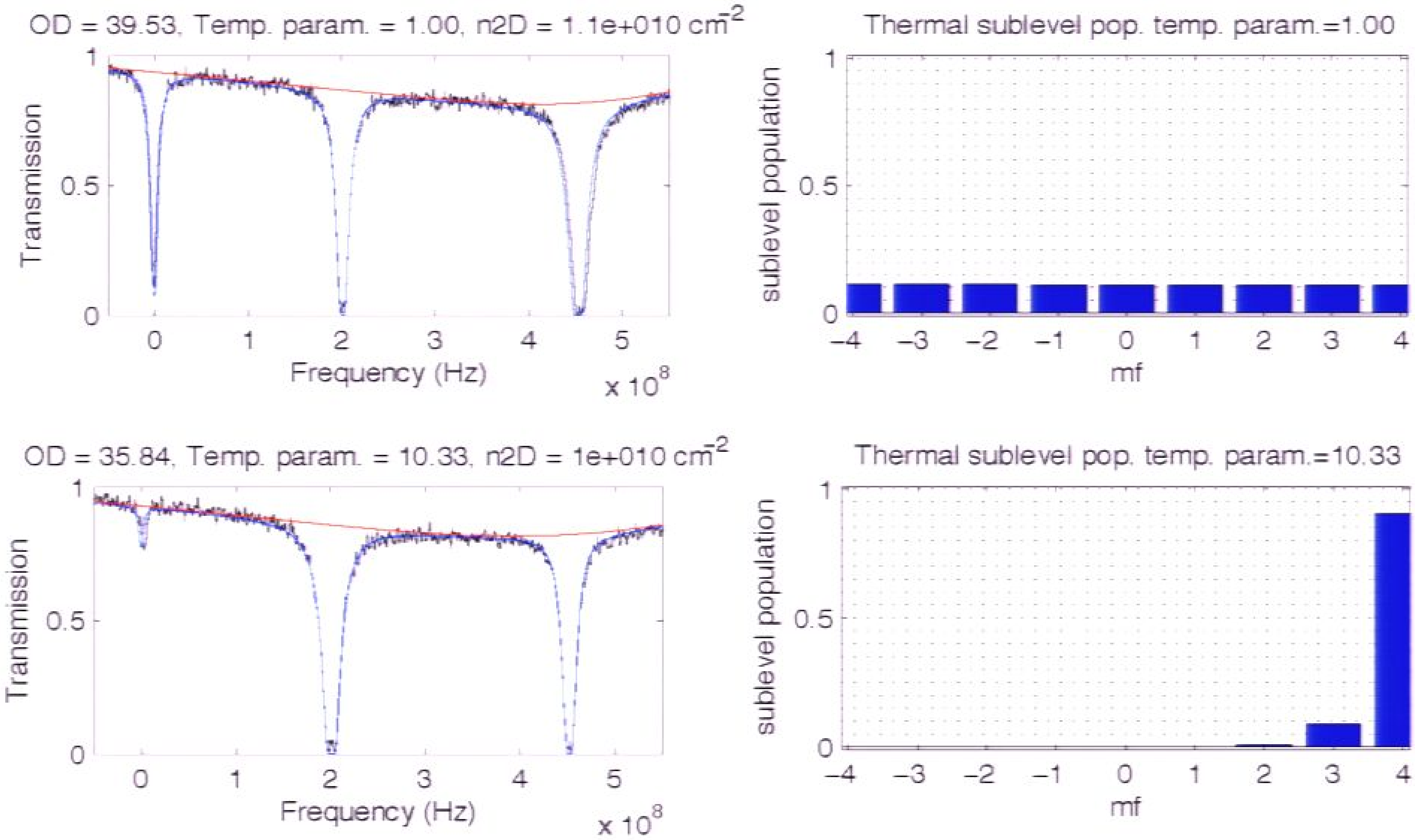






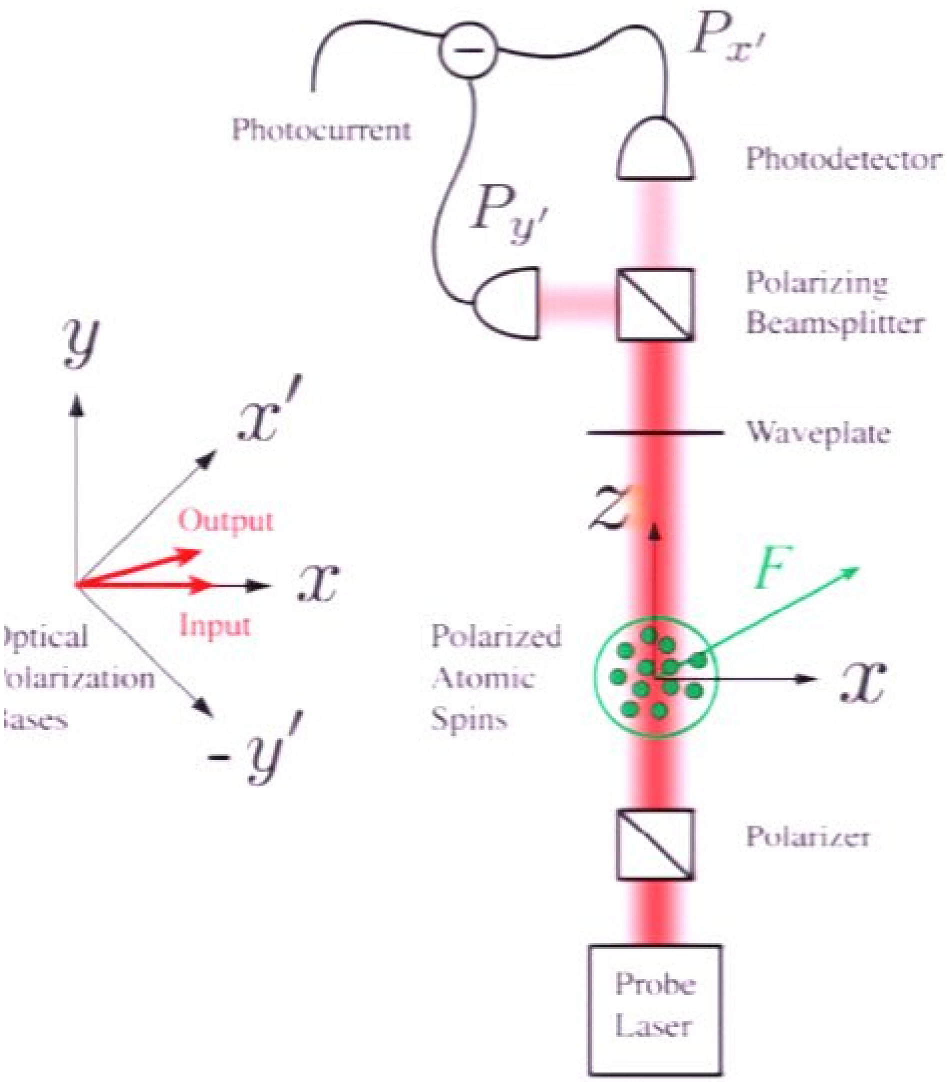


Knowing N: OD, polarization and beam spatial profile

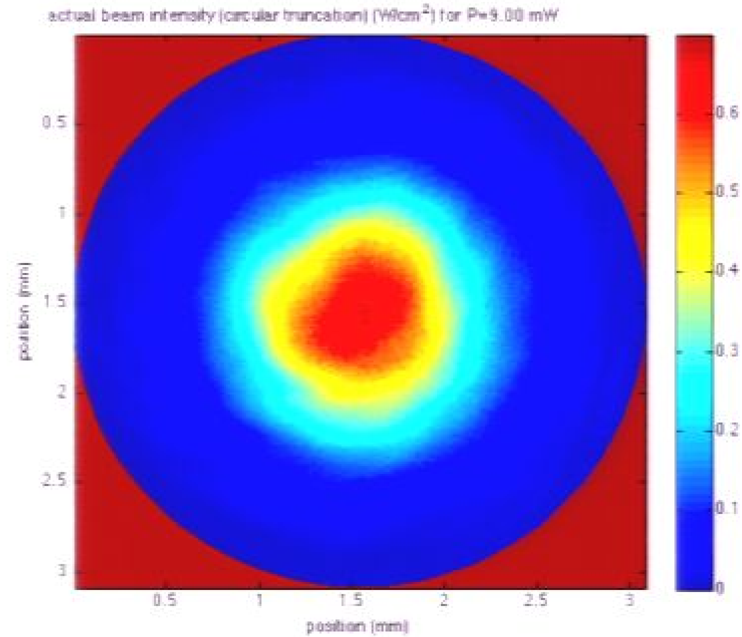


Effective OD given polarization ≈ 34; OD inferred from peak Faraday rotation is 33.8

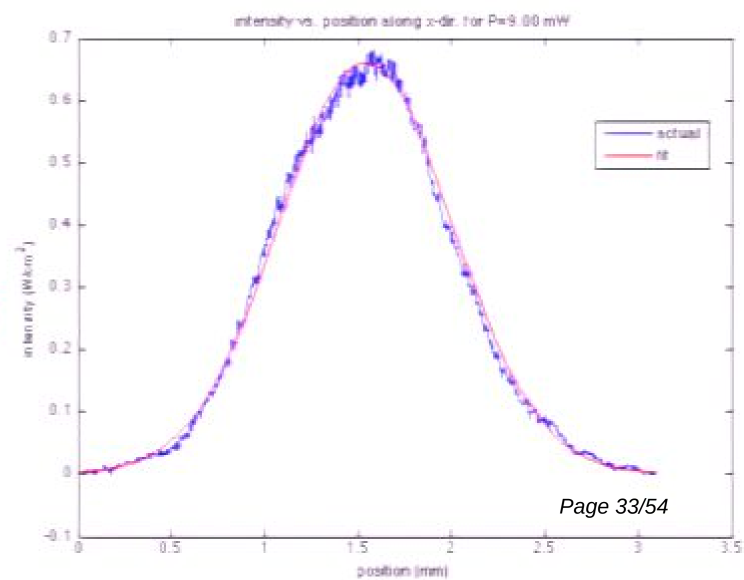
Measured transverse profile of probe beam



probe image at cloud:



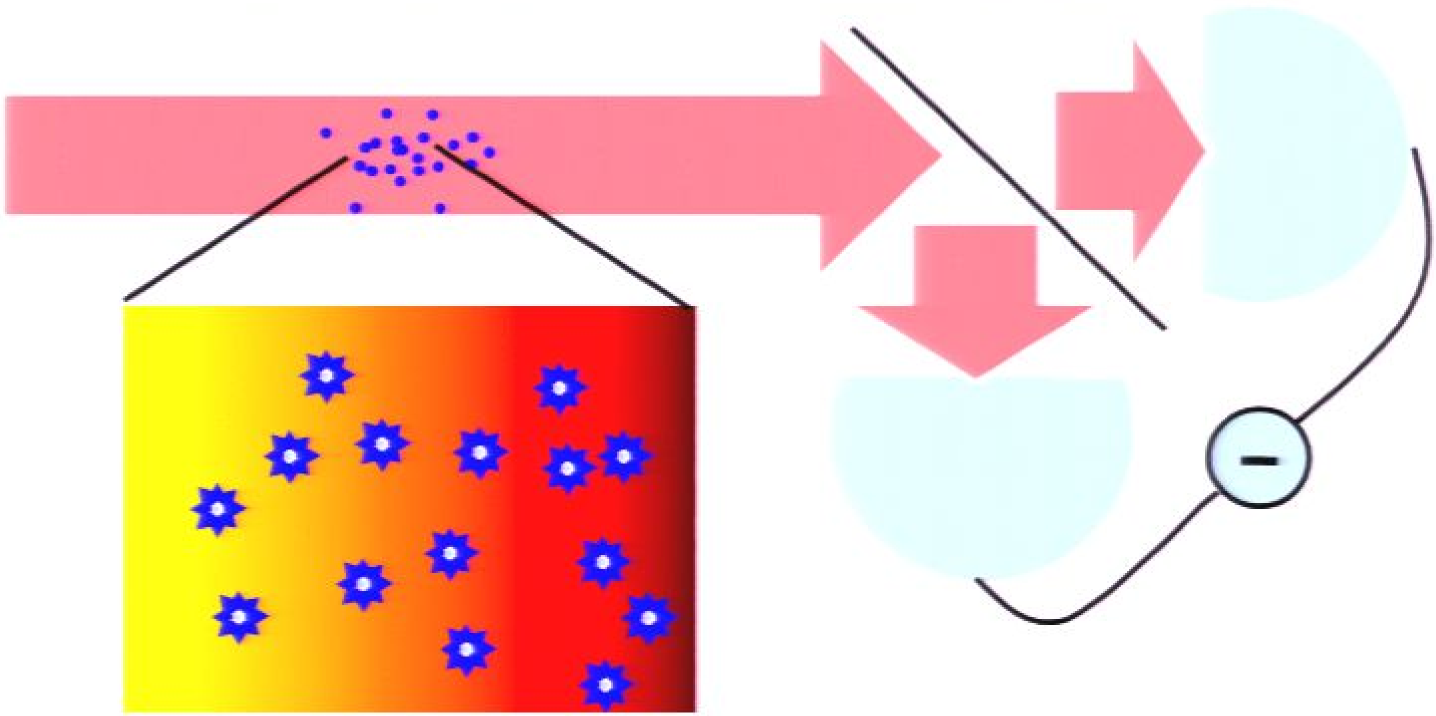
probe cross-section:



conclusion: probe profile Gaussian at the cloud

Full model: 3D optical modes, distributed atoms

Andrew Silberfarb and Luc Bouten, in preparation

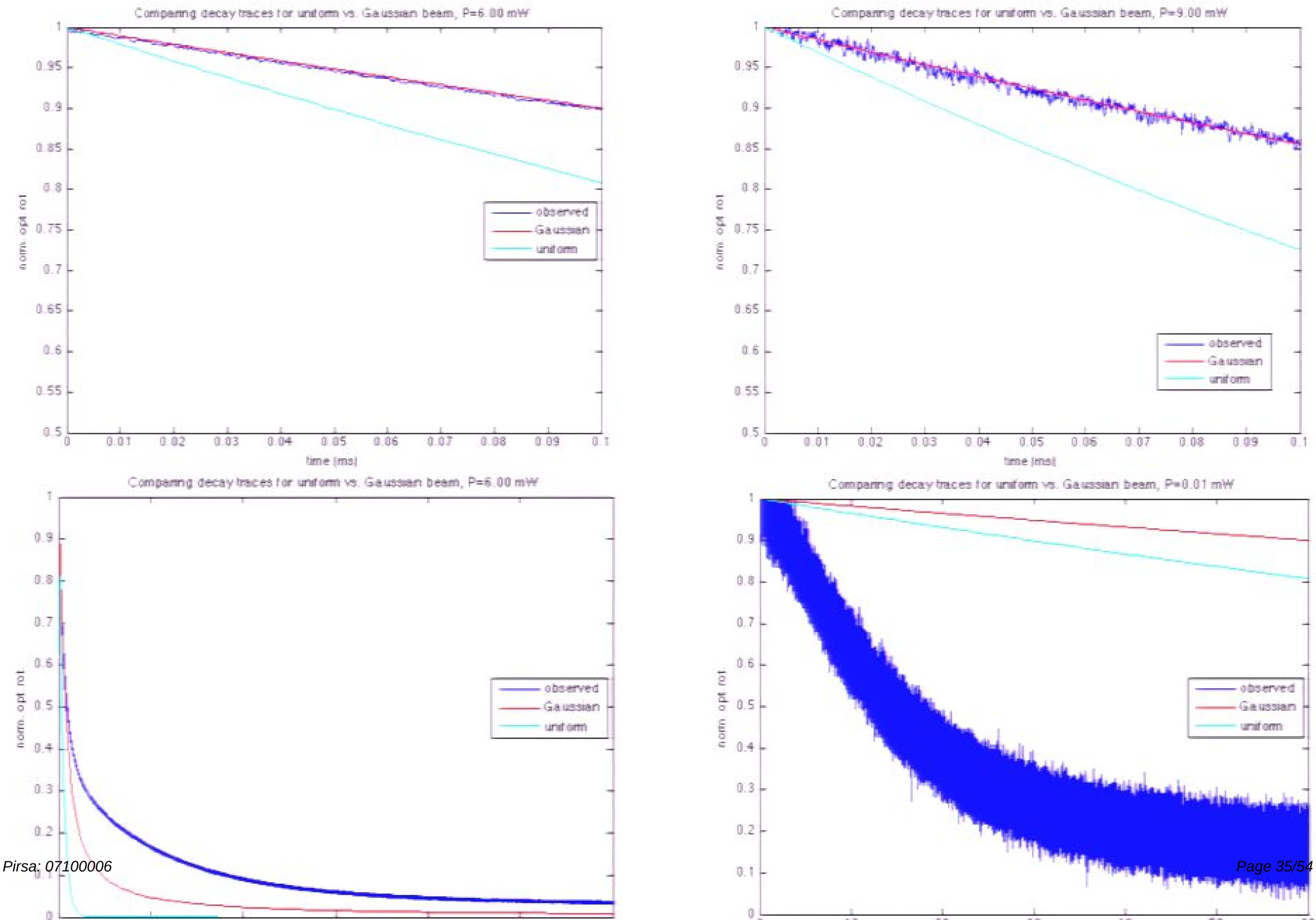


$$dU_t^\dagger B_t^{(\mathbf{n},a)} U_t = dB_t^{(\mathbf{n},a)} + \sum_l \sqrt{\gamma_l}\,\sigma_z^l(t)\, dB_t^{(\mathbf{n},a)} dB_t^{(l,y)\dagger}$$

$$S(t) \propto \sqrt{\pi}\frac{\sigma}{k} dW_t^L + \sum_l \sqrt{\gamma_l}\,\sigma_z^l(t)\, e^{ik(z-z_l)} \int_D dx\,dy \int d\mathbf{n}\, e^{-(x^2+y^2)/2\sigma^2} e^{-ik\mathbf{n}\cdot(\mathbf{x}-\mathbf{x}_l)} dt$$

$$S(t) \propto dW_t^L + \sqrt{\frac{1}{2}\gamma \mathrm{OD}}\, x\, dt \qquad x(t) = \frac{\sum_l e^{-(x_l^2+y_l^2)/\sigma^2}\sigma_z^l(t)}{\sqrt{\sum_l e^{-2(x_l^2+y_l^2)/\sigma^2}}}$$

Magnetization decay: short/long timescales



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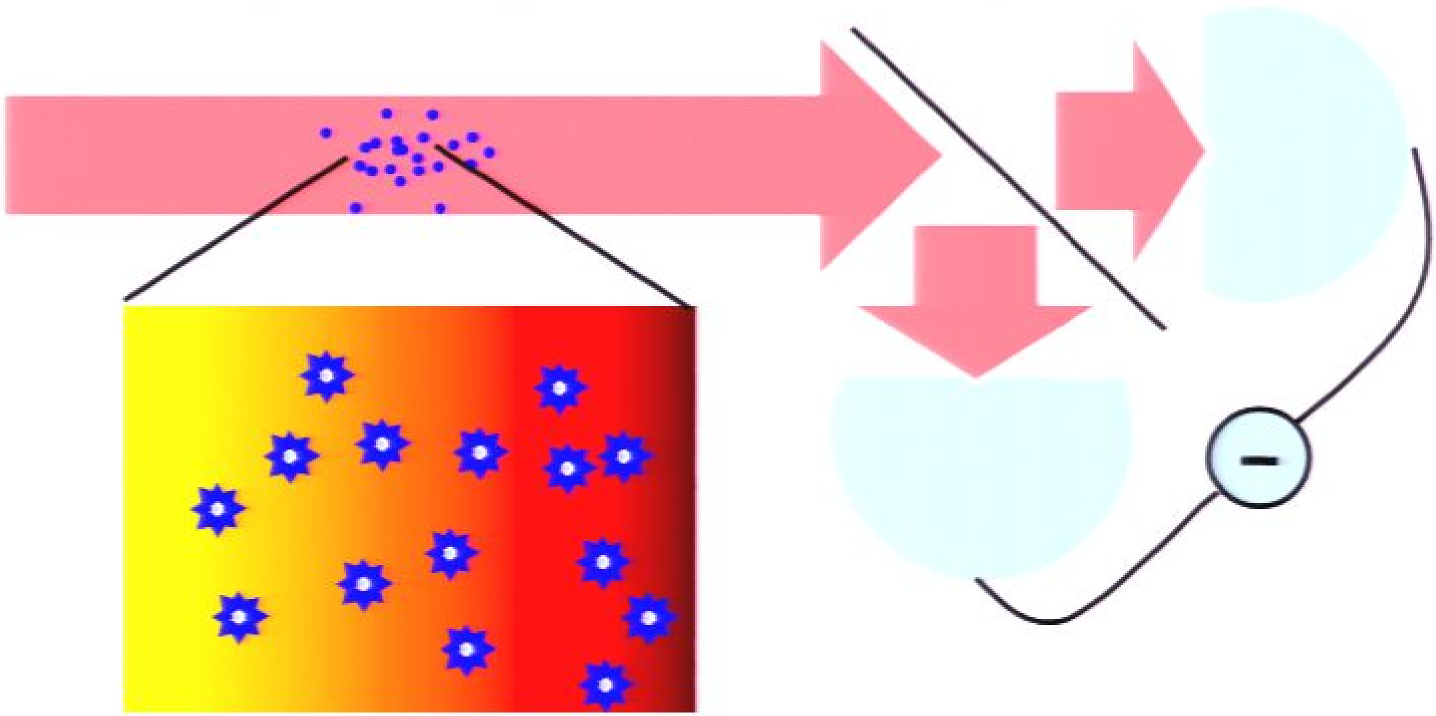

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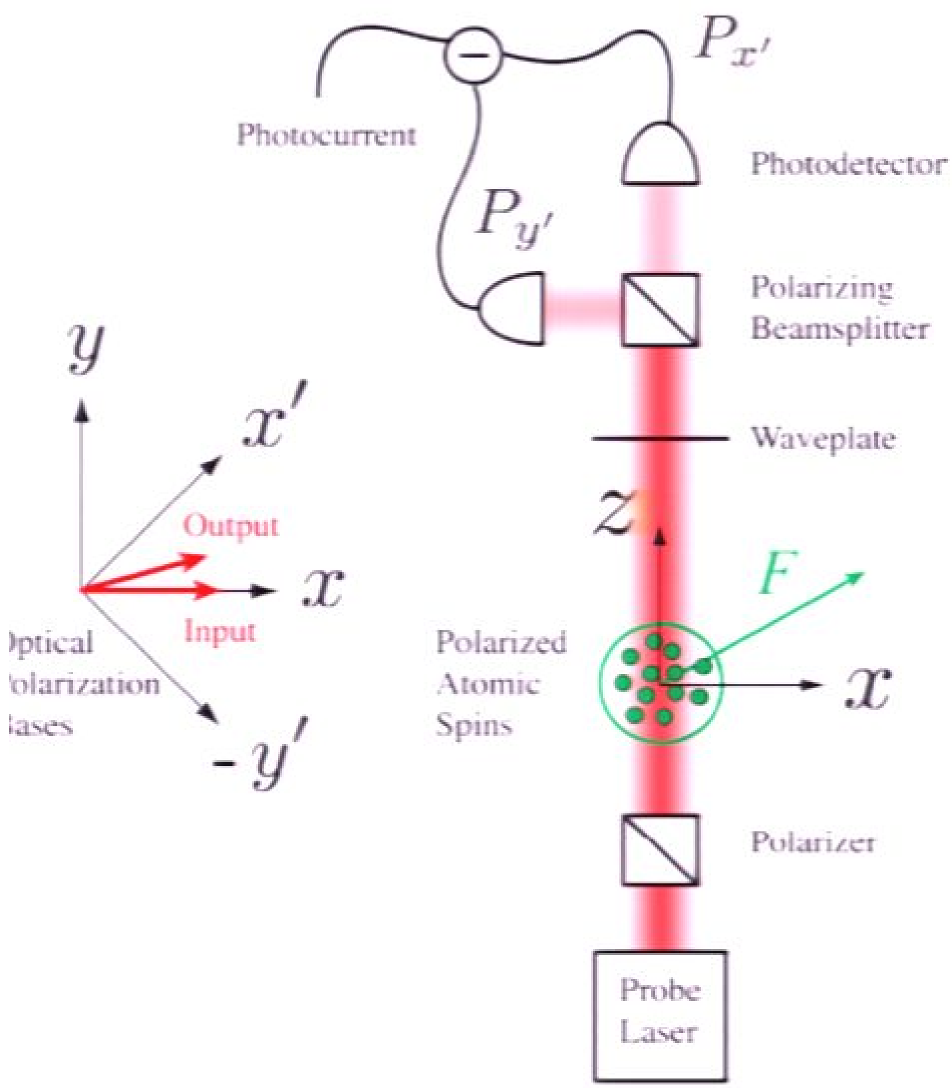

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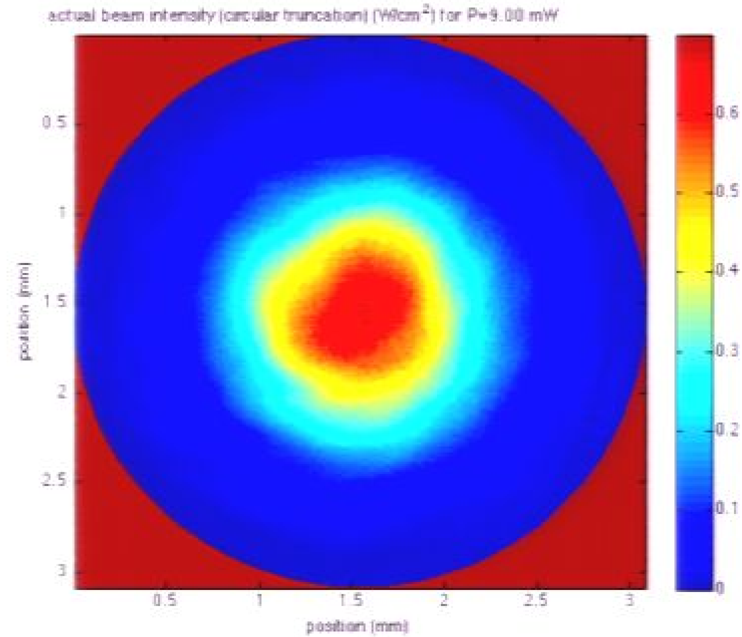

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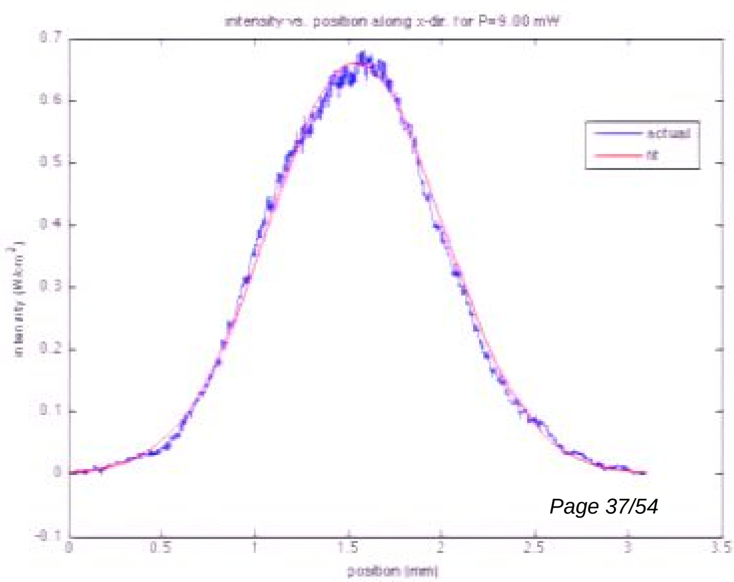

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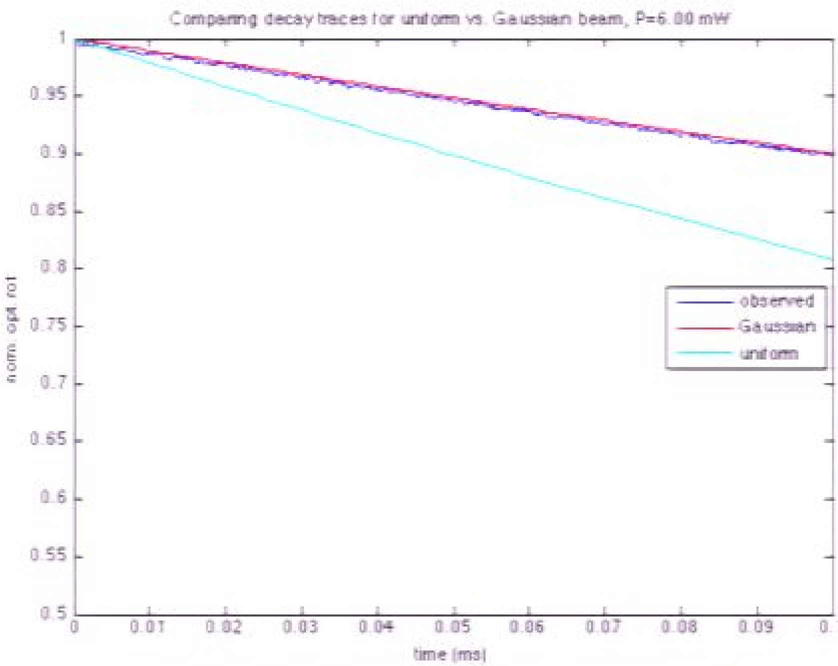
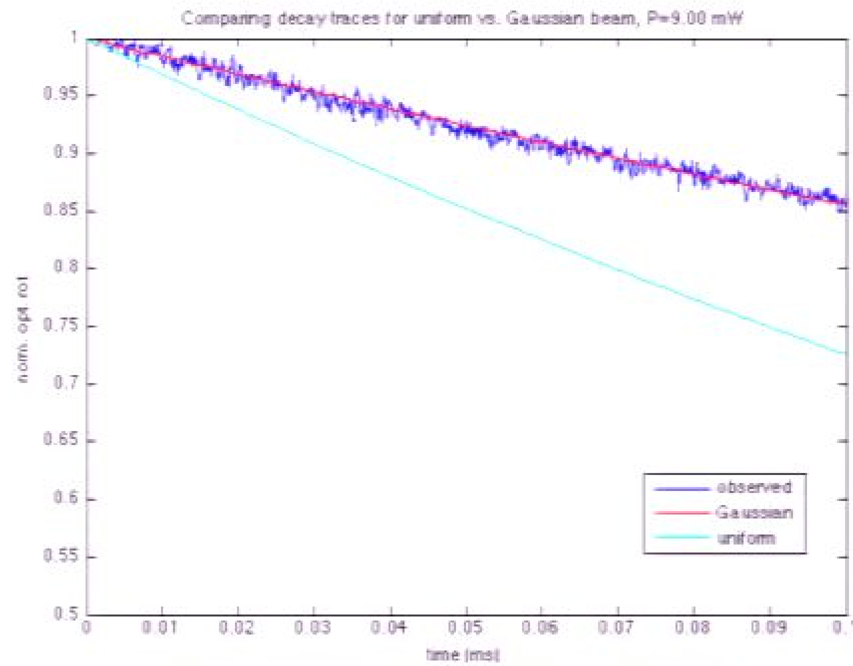
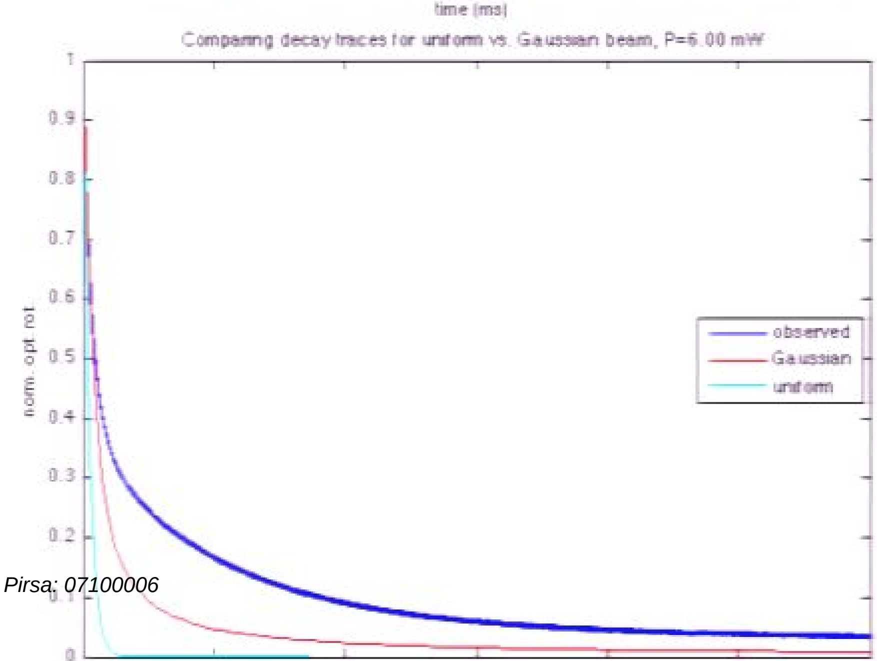
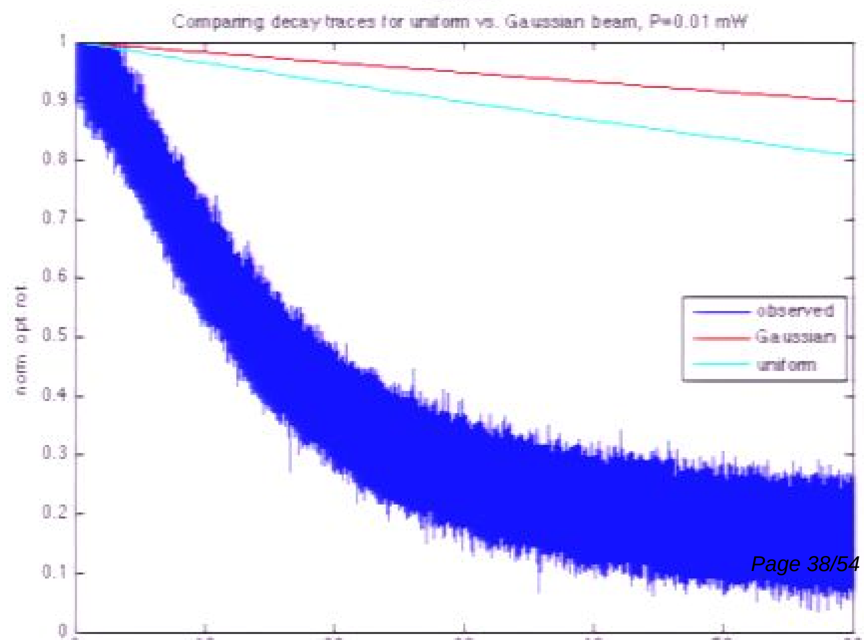
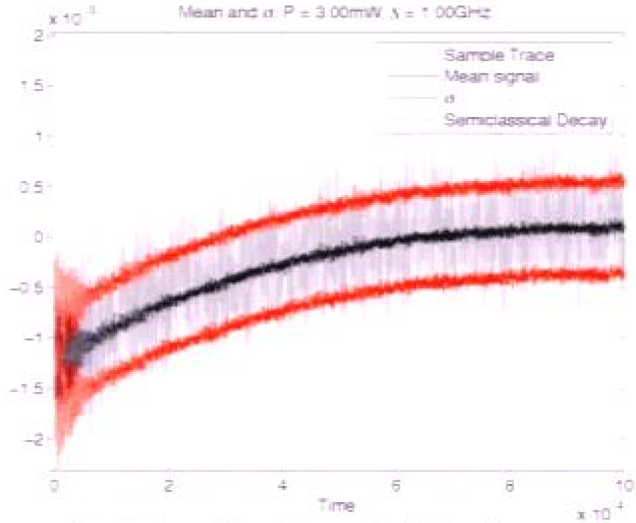


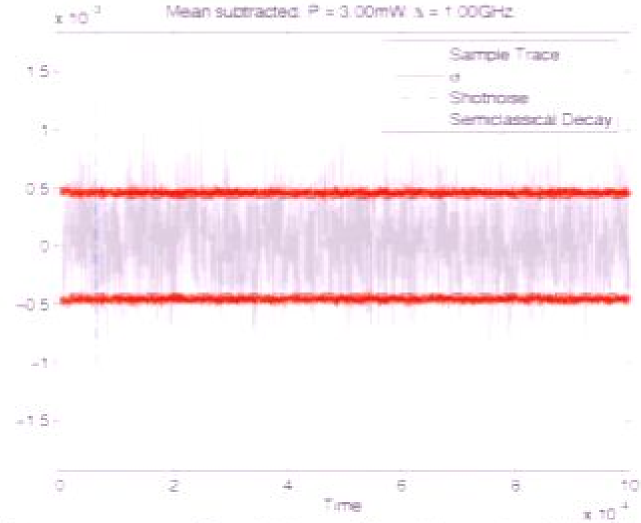


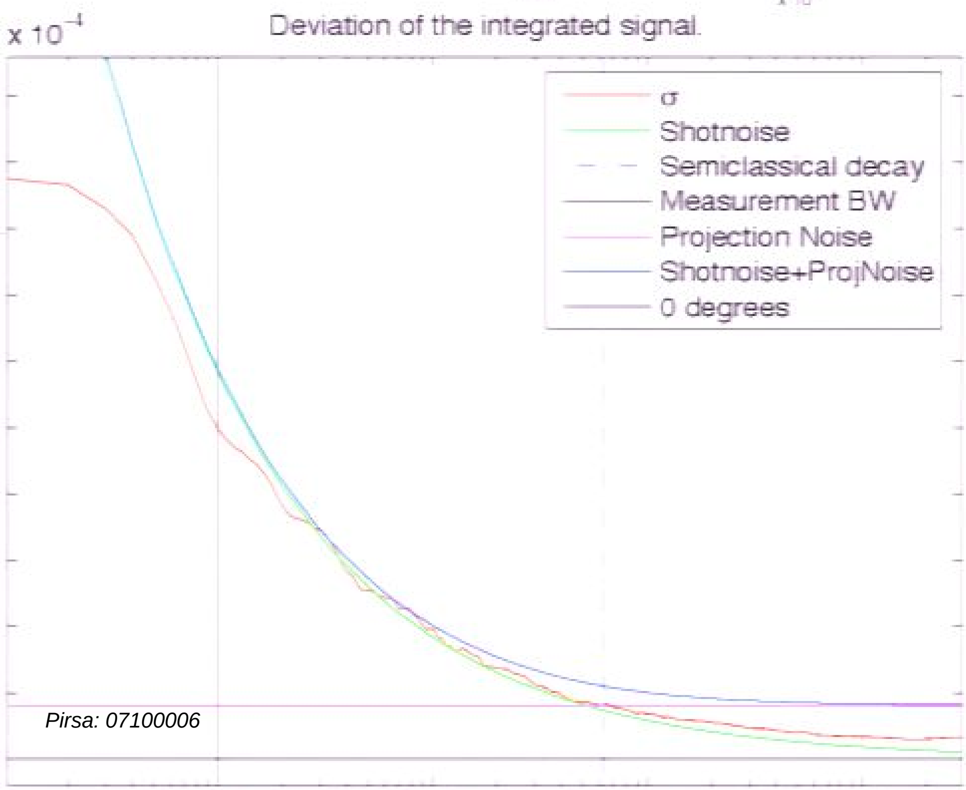


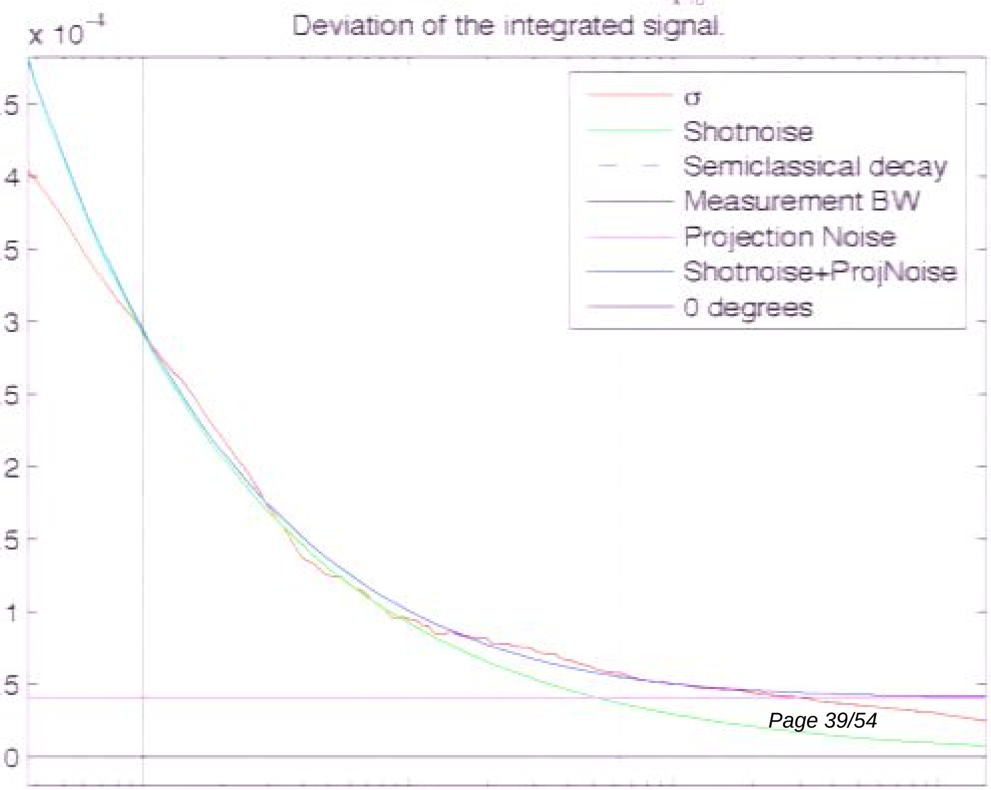


Recent dc Faraday variance data









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S. D. Bartlett, T. Rudolph and R. W. Spekkens, quant-ph/0610030 (2006)
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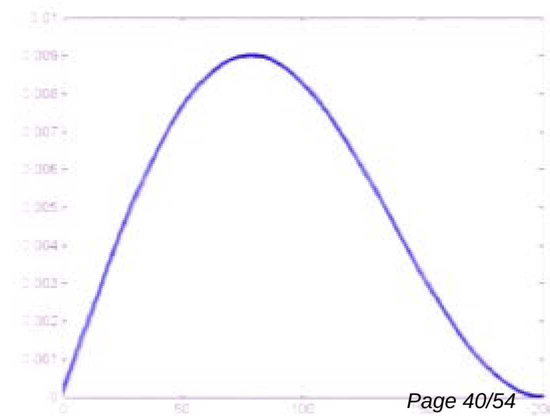

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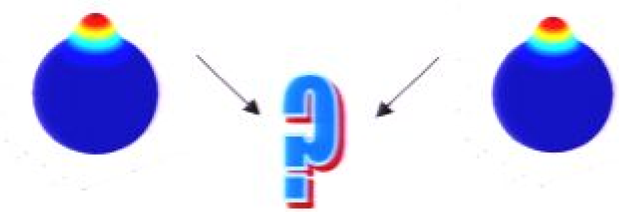

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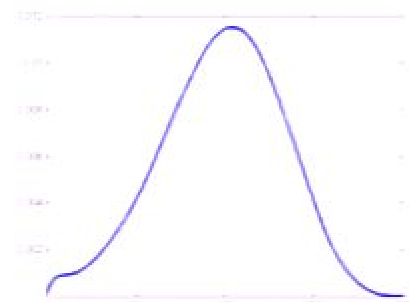

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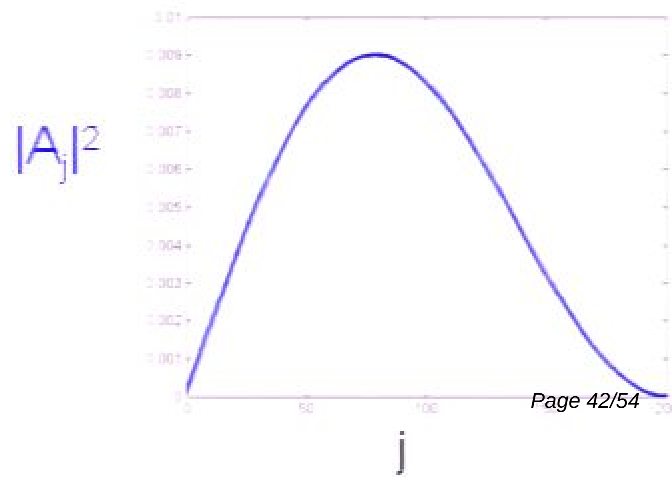

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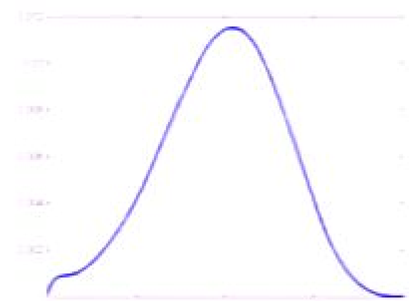

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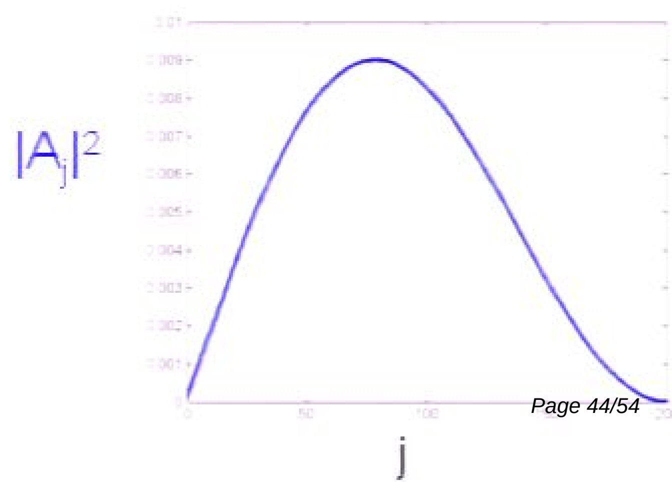

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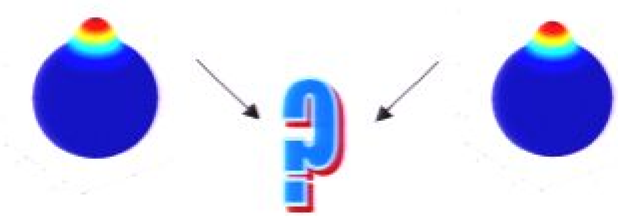

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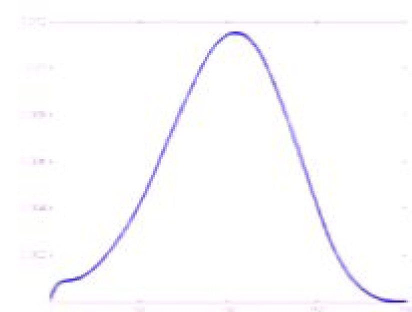

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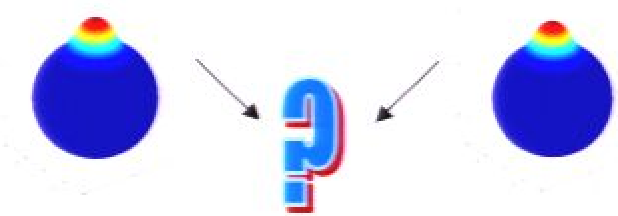

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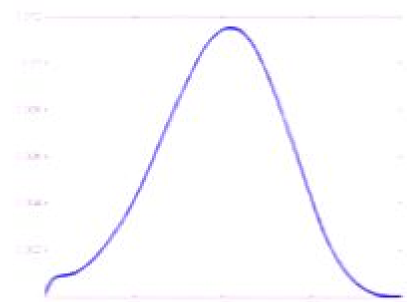
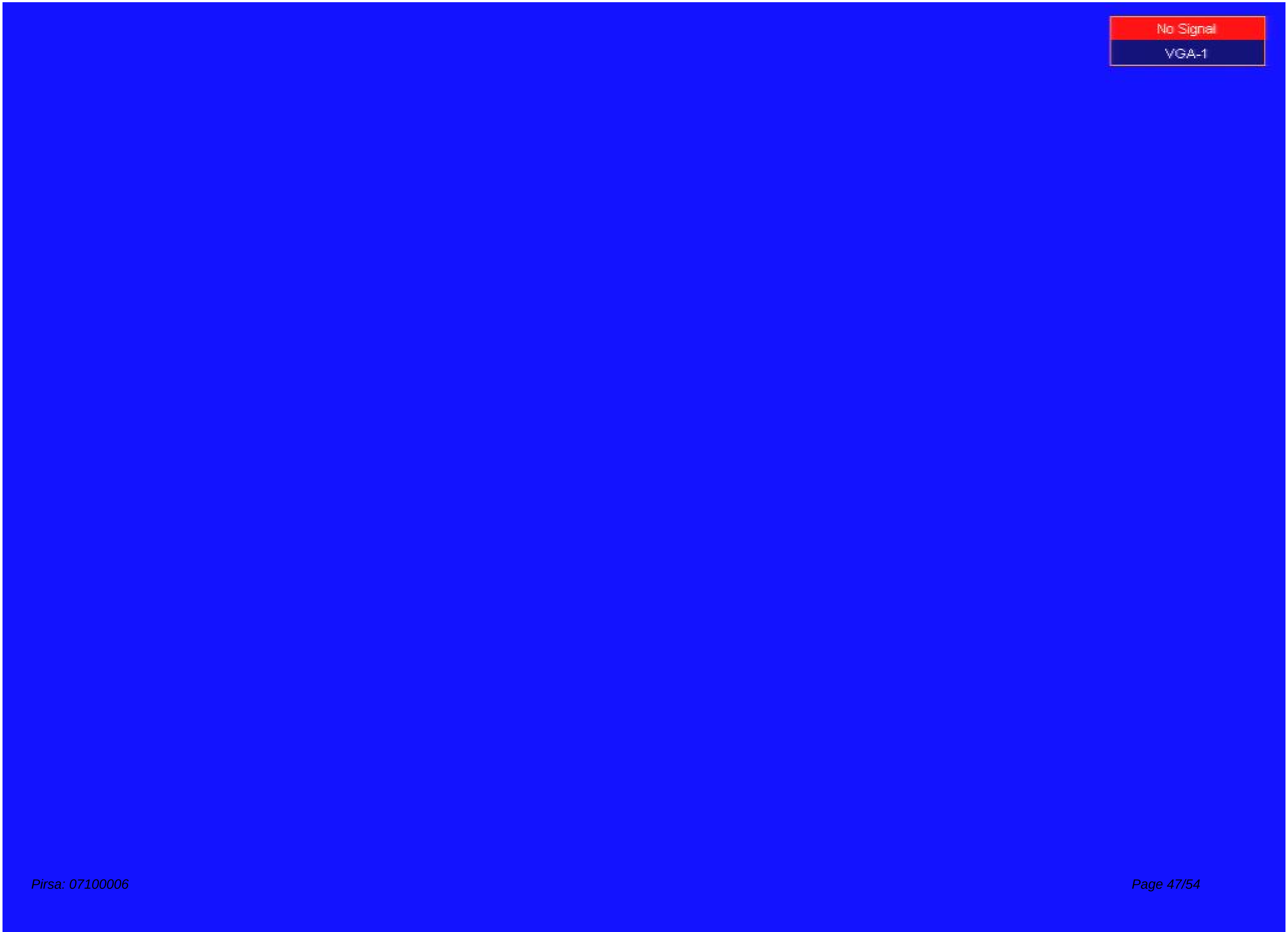
Rotate: $R = e^{-iJ_y\pi/3}$ $R = e^{-iJ_y 2\pi/3}$

"Squeeze": $(J_{z1} + J_{z2})|\psi\rangle = 0\,|\psi\rangle$



$|A_j|^2$ distribution

- Achieves N^{-2} scaling (numerically); deterministic squeezing via feedback control?
- Even more tractable Gaussian version achieves $N^{-7/5}$ (again *c.f.* Mølmer *et al.*)

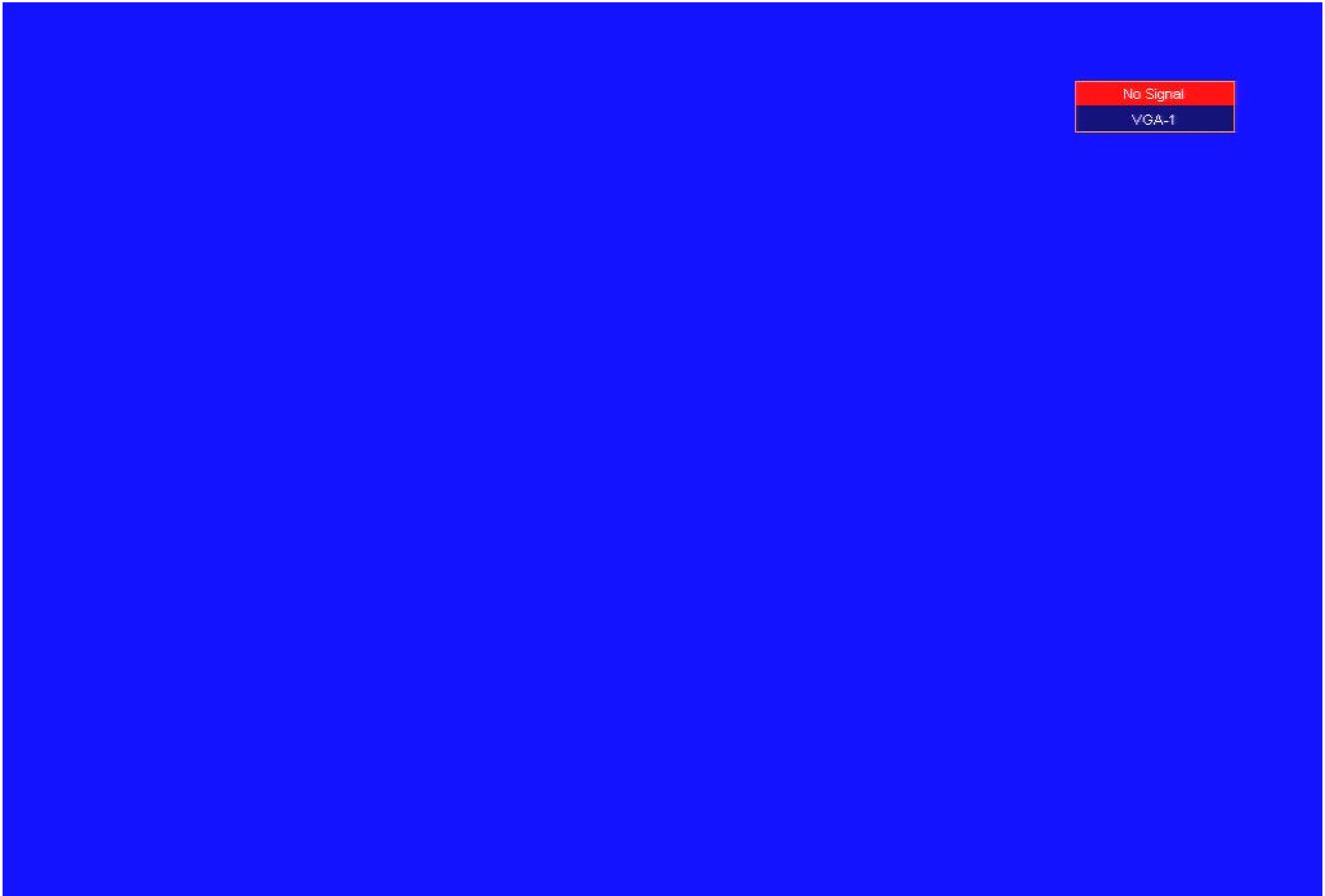

No Signal
VGA-1

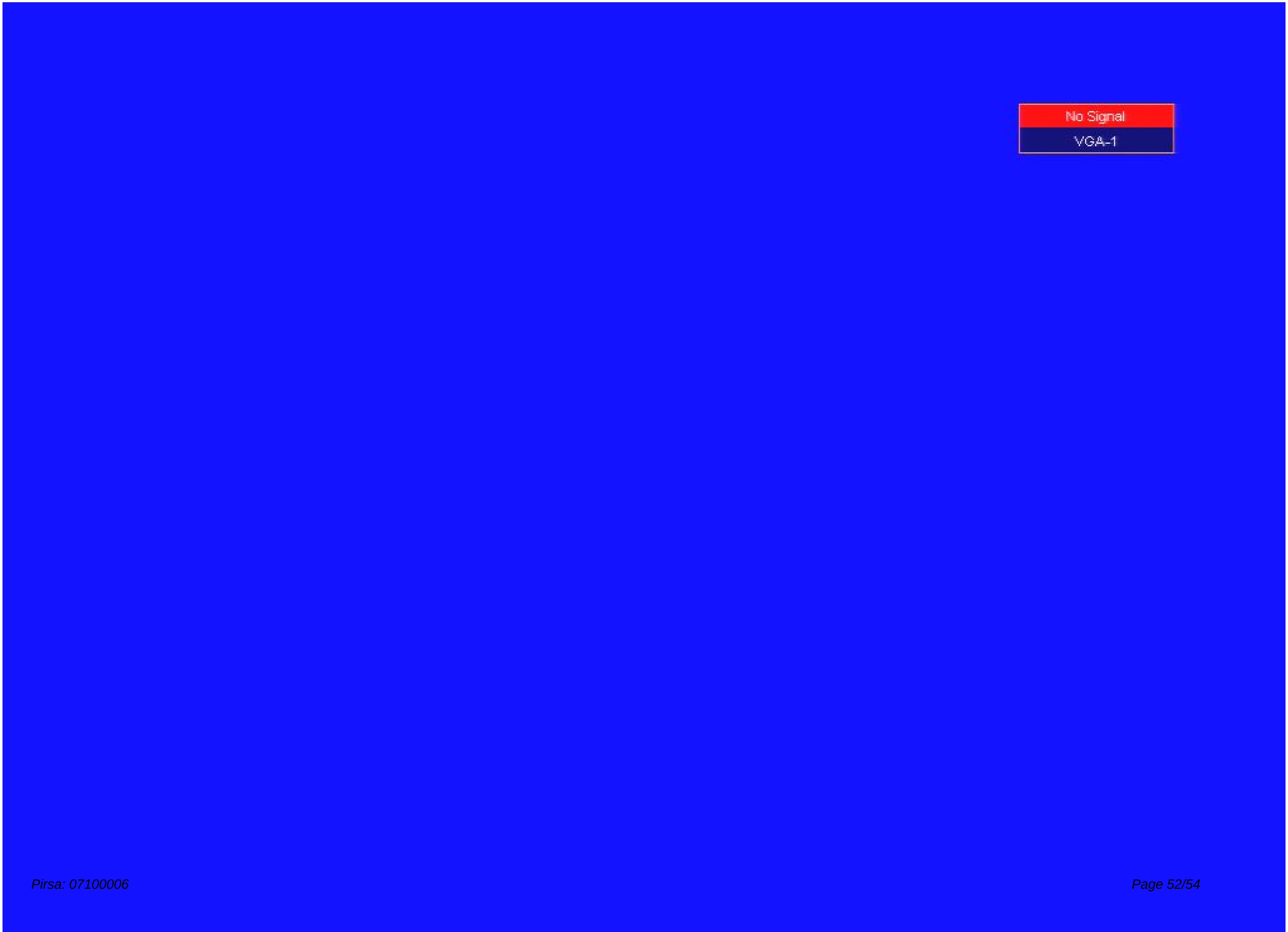
No Signal
VGA-1

No Signal
VGA-1

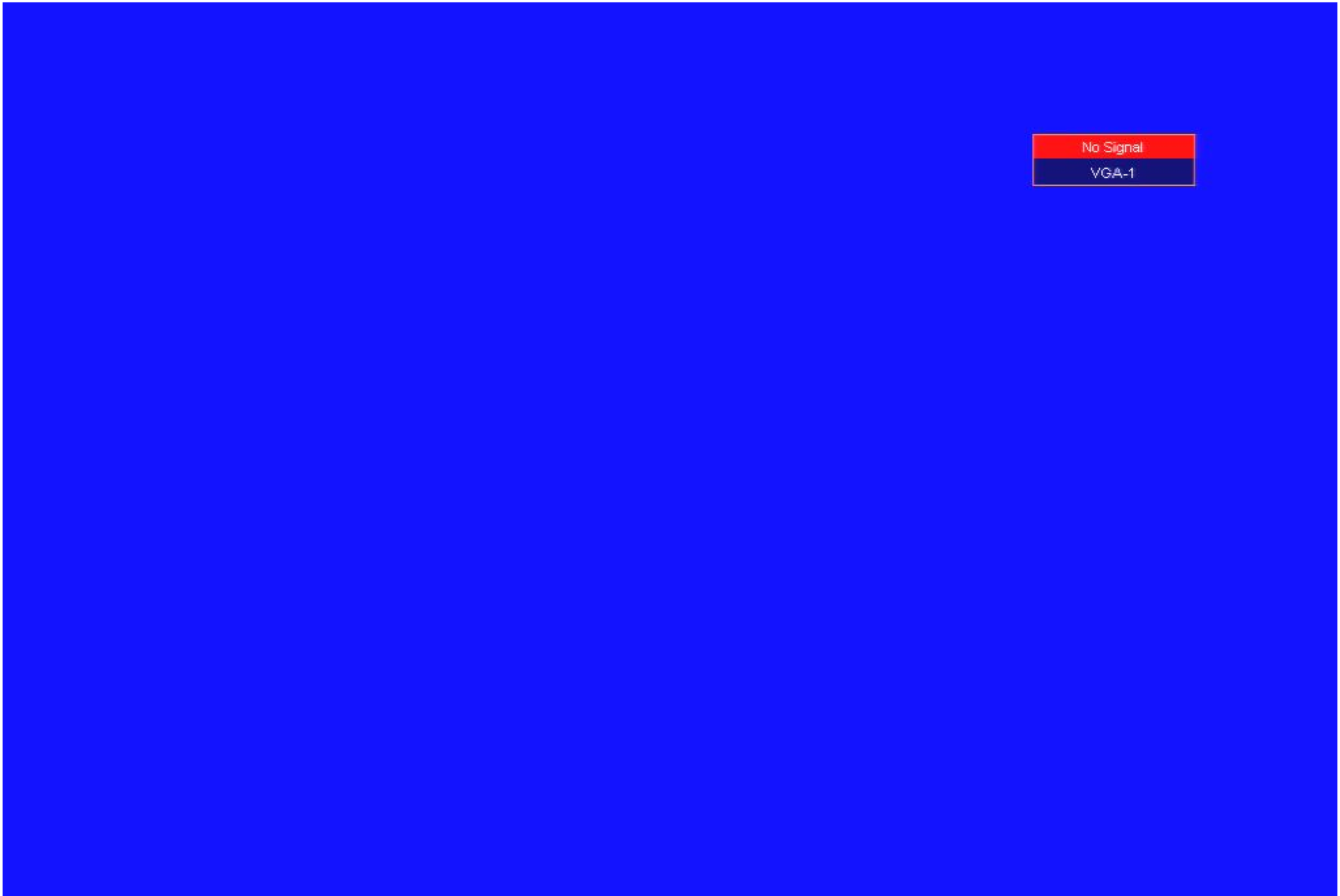
No Signal

VGA-1


No Signal
VGA-1


No Signal
VGA-1

No Signal
VGA-1


No Signal
VGA-1