Title: Scale-dependent growth of structures in viable f(R) theories

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Abstract: f(R) theories are an alternative approach at the phenomenon of cosmic acceleration, in which the Einstein-Hilbert action for gravity is modified by adding a function of the Ricci scalar, f(R). While at the background level viable f(R) models must closely mimic LCDM, the difference in their prediction for the growth of large scale structures can be sufficiently large to leave detectable signatures in future surveys. In this talk, after reviewing the conditions for the background viability of f(R) theories, I will focus on scalar perturbations. I will present in some details the dynamics of linear perturbations, showing what are the characteristic imprints of f(R) models in the growth of structures and consider possible observational tests

Scale-dependent Growth of Structure in Viable f(R) Theories

Perimeter Institute
October, 2nd 2007

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in collaboration with: M.Trodden, L.Pogosian, R.Bean and D.Bernat

references: astro-ph/0709.0296
astro-ph/0611321, PRD'07

Syracuse University

Outline

- Cosmic Acceleration : Modified Gravity ?

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- Can we distinguish it from the Cosmological Constant and (conventional) Dark Energy models?

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we need to study Large Scale Structure!

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- Background viability

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$w_{eff} \approx -1$

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f(R) theories

- Background viability

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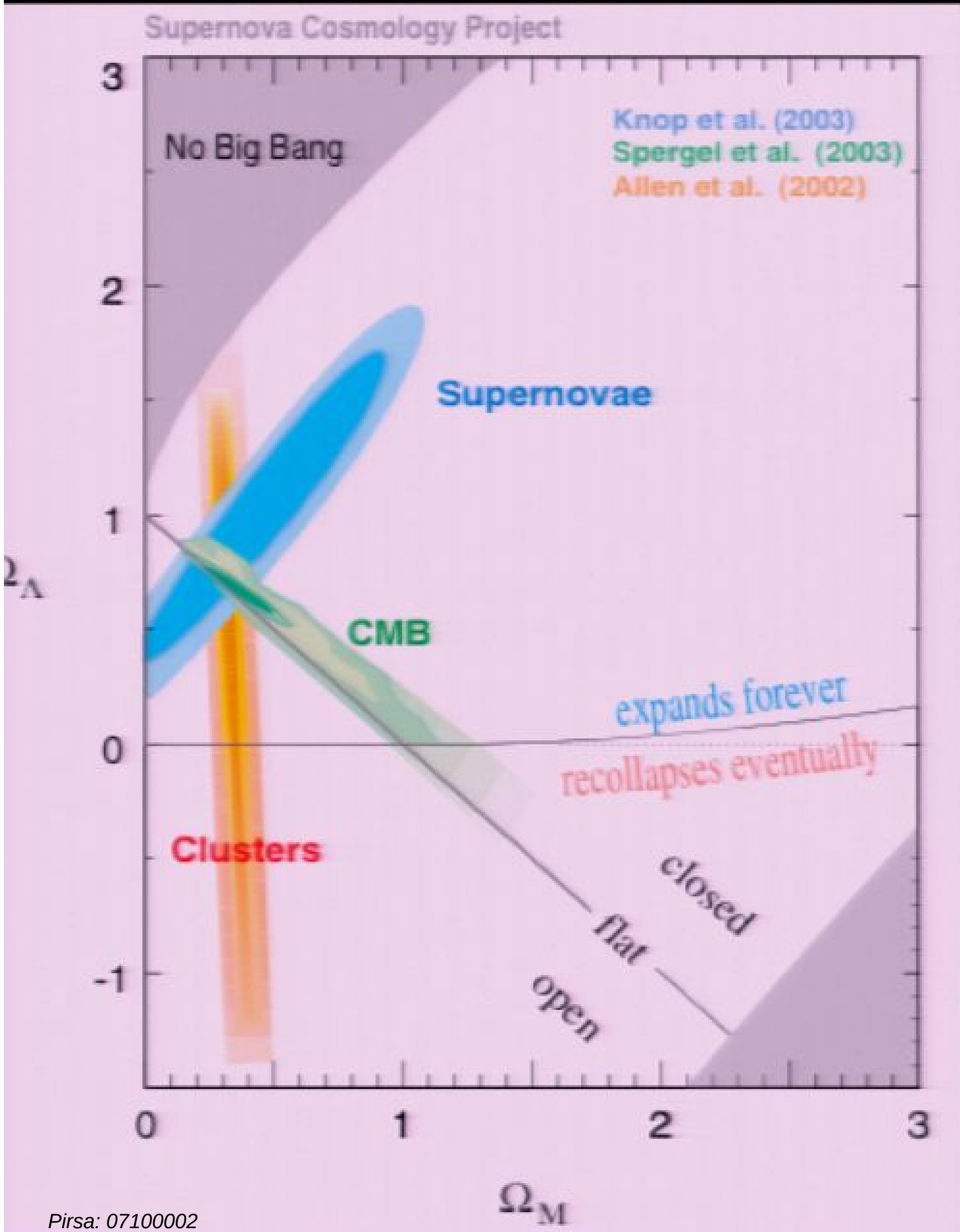
  $w_{eff} \approx -1$

- Growth of Structure

  ↓

  the dynamics is changed, leading to a characteristic scale-dependent pattern

Cosmic acceleration



SNeIa, CMB, LSS

+

standard GR applied to a homogeneous and isotropic Universe

↓

$$\Omega_m^0 \approx 0.3$$

$$\Omega_X^0 \approx 0.7$$

$$\left(\Omega^0 \equiv \frac{\rho^0}{3H_0^2 M_P^2} \right)$$

Cosmic acceleration

A very good fit to all these data is a Universe in which 70% of the energy budget is in the COSMOLOGICAL CONSTANT, LCDM

Anyhow it is important to explore the whole space of explanations that fit these data and could have testable features...

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Dark Energy

$$G_{\mu\nu} = \frac{1}{M_P^2} \tilde{T}_{\mu\nu}$$

X matter fields with dynamics such as to cause the late universe to accelerate (quintessence, k-essence, ...)

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Modified Gravity

$$\tilde{G}_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu}$$

modification of GR on large scales, admitting self-accelerating solutions

Modifying Gravity

- Modifications of the Friedmann equation, changing the dependence of the Hubble parameter on the matter density (Cardassian model,...)

  (K.Freese & M.Lewis, Phys.Lett.B540,(2002))

- Extra-dimensional models giving modified 4-dim equations (DGP model,...)

  (G.Dvali, M.Porrati & G.Gabadadze, Phys.Lett.B485(20

  ⟶ Degravitation

  (G.Dvali, S.Hofmann & J.Khoury, hep-th/0703027)

- Covariant modifications of the 4-dim Einstein-Hilbert action (f(R),f(R,P,Q),...)

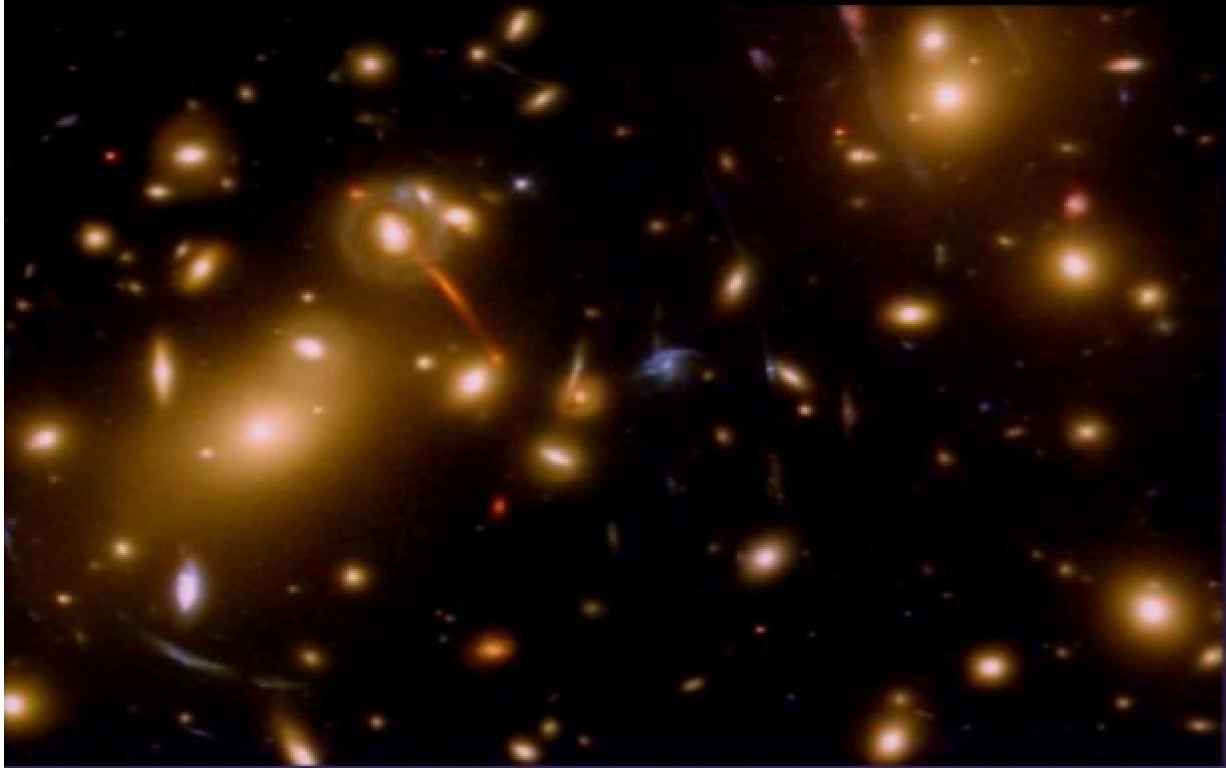
  (S.Capozziello, S.Carloni & A.Troisi, astro-ph/0303041
  S.Carroll, V.Duvvuri, M.Trodden & M.S.Turner, Phys.Rev.D70 043528 (2004))

- Cuscuton and Modified Source Gravity

  (N.Afshordi, D.J.H.Chung & G.Geshnizjani, Phys.Rev.D75 083513 (2007)
  S.Carroll, I.Sawicki, A.Silvestri & M.Trodden, New J Phys. 8 323 (2006))

Modifying Gravity

Typically, at the background level there is a degeneracy between Modified Gravity models and LCDM-Quintessence Dark Energy



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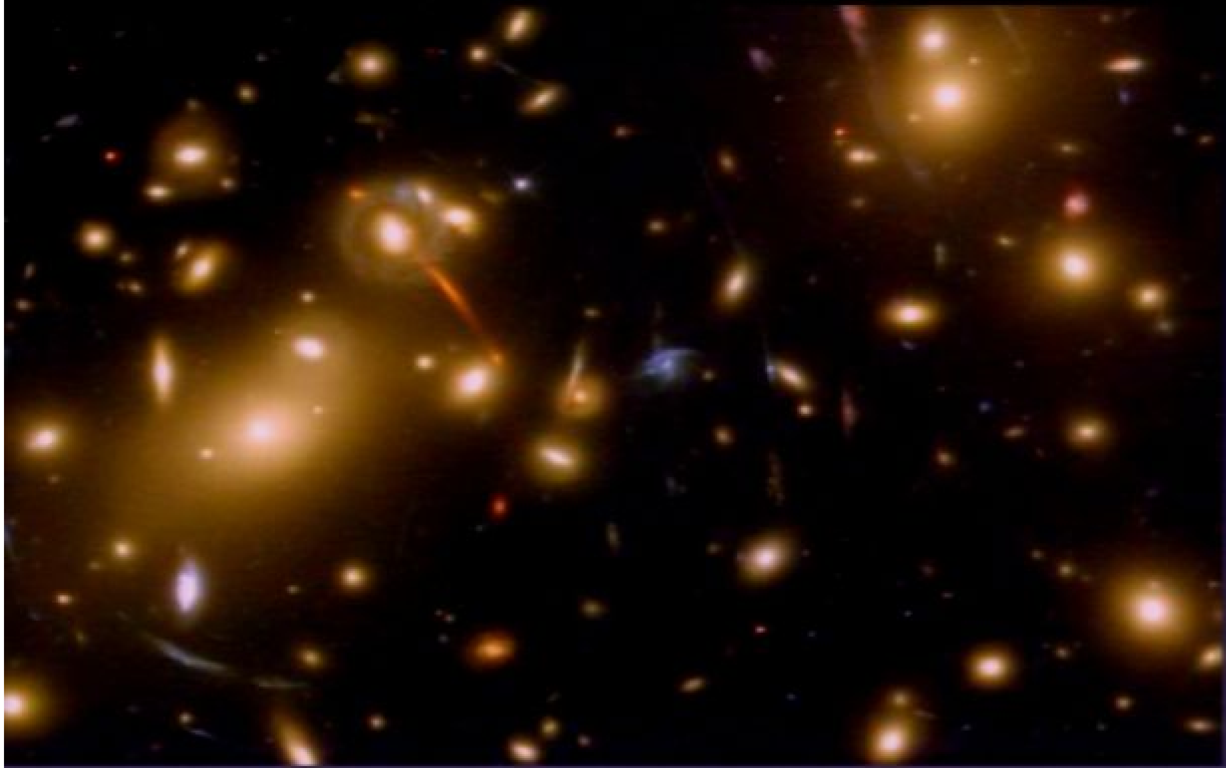

We need to move to the perturbations to break the degeneracy

In Modified Gravity the equation for the perturbations are changed, leading possibly to a reacher dynamics

f(R) Gravity

$$S = \frac{M_P^2}{2}\int dx^4\sqrt{-g}\,[R + f(R)] + \int d^4x\sqrt{-g}\mathcal{L}_m[\chi_i, g_{\mu\nu}]$$

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The Einstein equations are fourth order.

The trace-equation becomes:

$$(1 - f_R)R + 2f - 3\Box f_R = \frac{T}{M_P^2}$$

The scalaron

$$\Box f_R = \frac{1}{3}(R + 2f - Rf_R) - \frac{\kappa^2}{3}(\rho - 3P) \equiv \frac{\partial V_{eff}}{\partial f_R}$$

$$m^2_{f_R} \equiv \frac{\partial^2 V_{eff}}{\partial f_R^2} = \frac{1}{3}\left[\frac{1+f_R}{f_{RR}} - R\right]$$

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$|Rf_{RR}| \ll 1$

$f_R \to 0$

$$m_{f_R}^2 \approx \frac{1 + f_R}{3f_{RR}} \approx \frac{1}{3f_{RR}}$$

$$\lambda_C \equiv \frac{2\pi}{m_{f_R}} \approx 2\pi\sqrt{\frac{3f_{RR}}{1 + f_R}}$$

Designer f(R)

The fourth order nature of f(R) provides enough freedom to reproduce any cosmological background history by an appropriate choice of the f(R) function

We fix the expansion history

$$E \equiv \frac{H^2}{H_0^2} = \Omega_m a^{-3} \Omega_r a^{-4} + \frac{\rho_{eff}}{\rho_{cr}}$$

$$(w_{eff}(a))$$

and solve the Friedmann eq. as a second order differential equation for f(R)

$$y'' - \left(1 + \frac{E'}{2E} + \frac{R''}{R'}\right) y' + \frac{E'}{2E} y = \frac{E'}{2E} E_{eff}$$

$$y \equiv \frac{f(R)}{H_0^2}$$

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4. $|f_R^0| \leq 10^{-6}$ must be small at recent epochs to pass LGC

(Hu and Sawicki astro-ph/0705.1158)

$$w_{eff} \simeq -1$$

(Dolgov & Kawasaki, Phys.Lett.B 573 (2003), Navarro et al. gr-qc/0611127, Sawicki and Hu astro-ph/0702278
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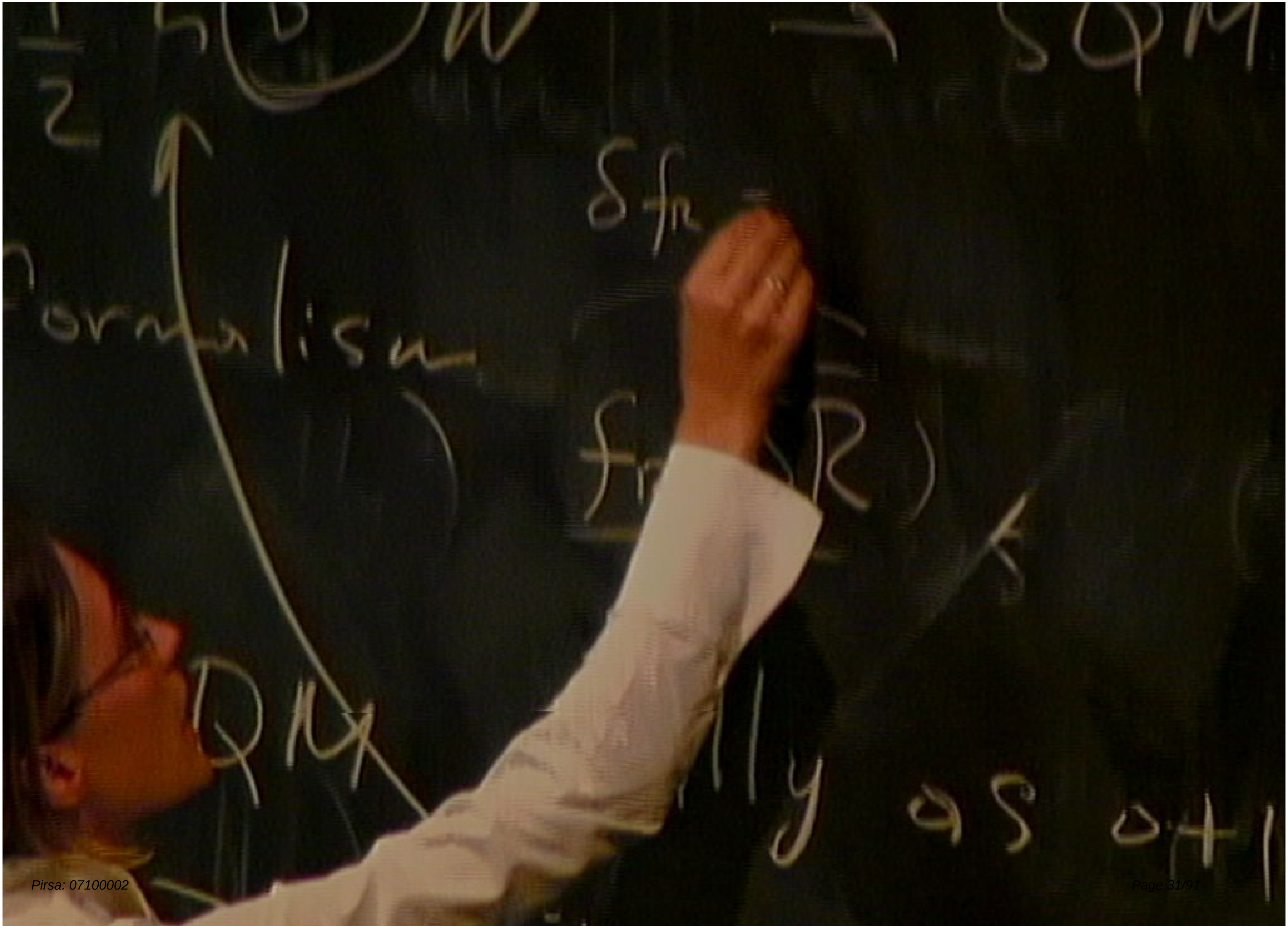
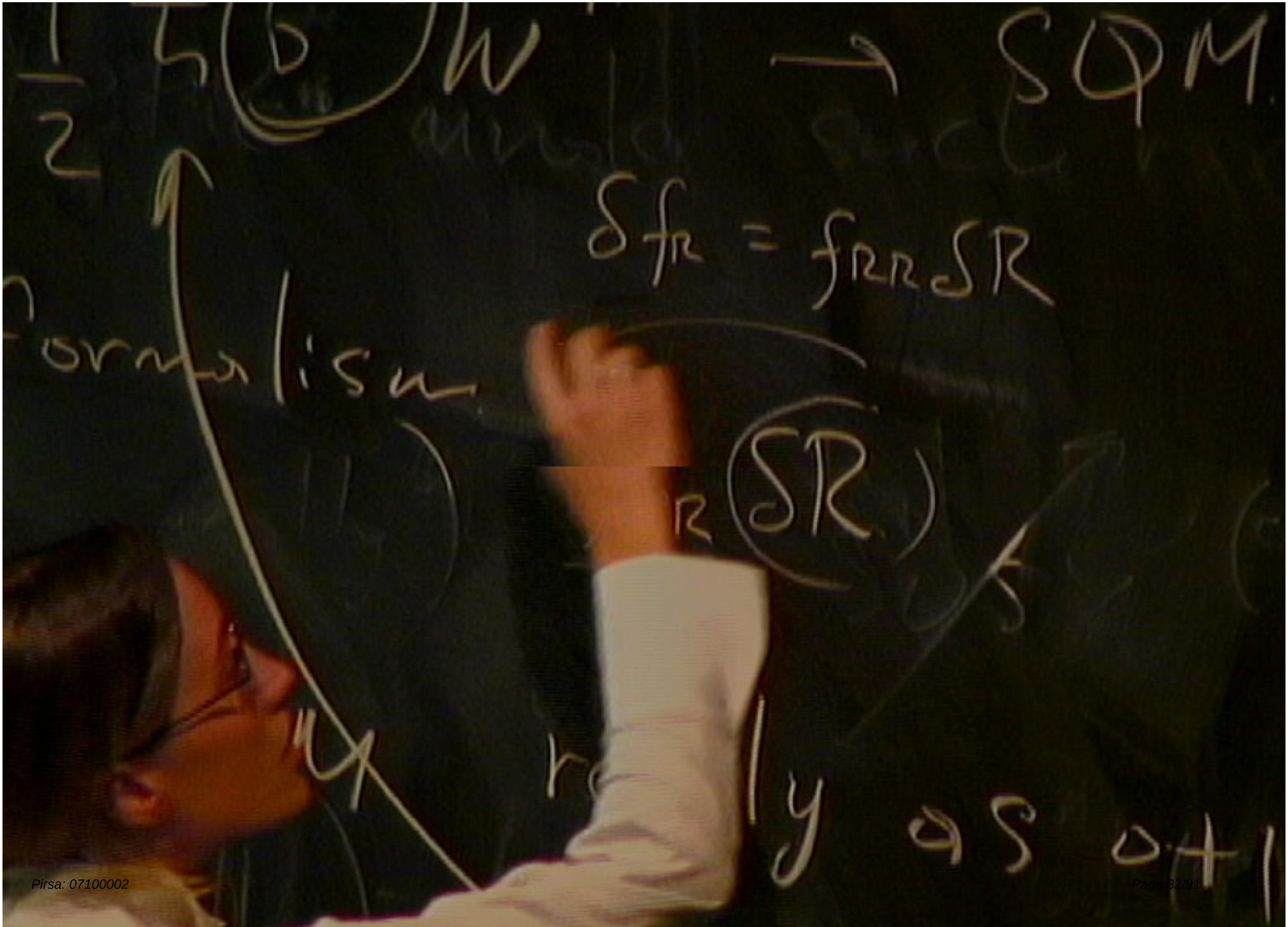

formalism
QM
as


→ SQM
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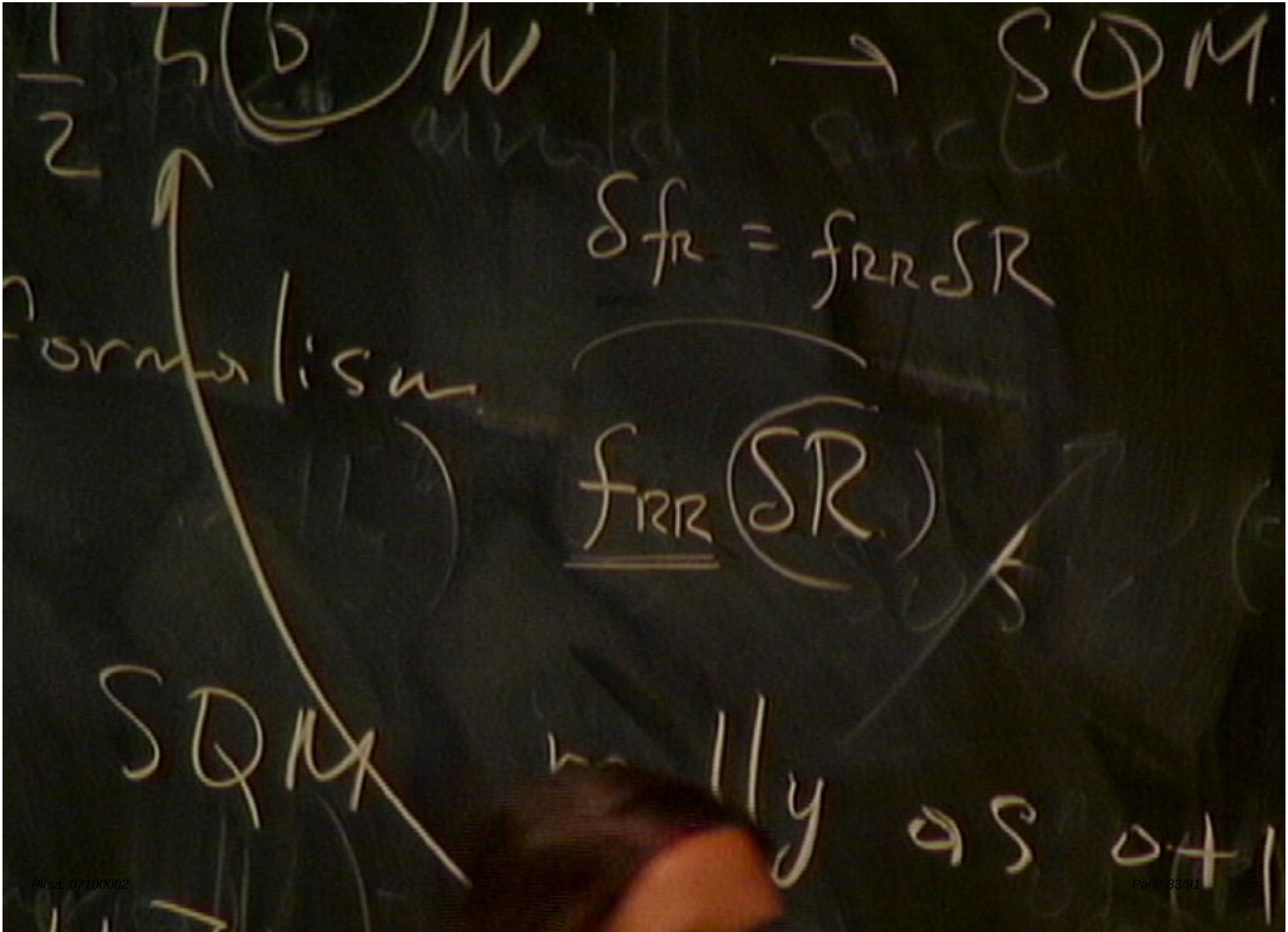
δfR = fRRδR
δR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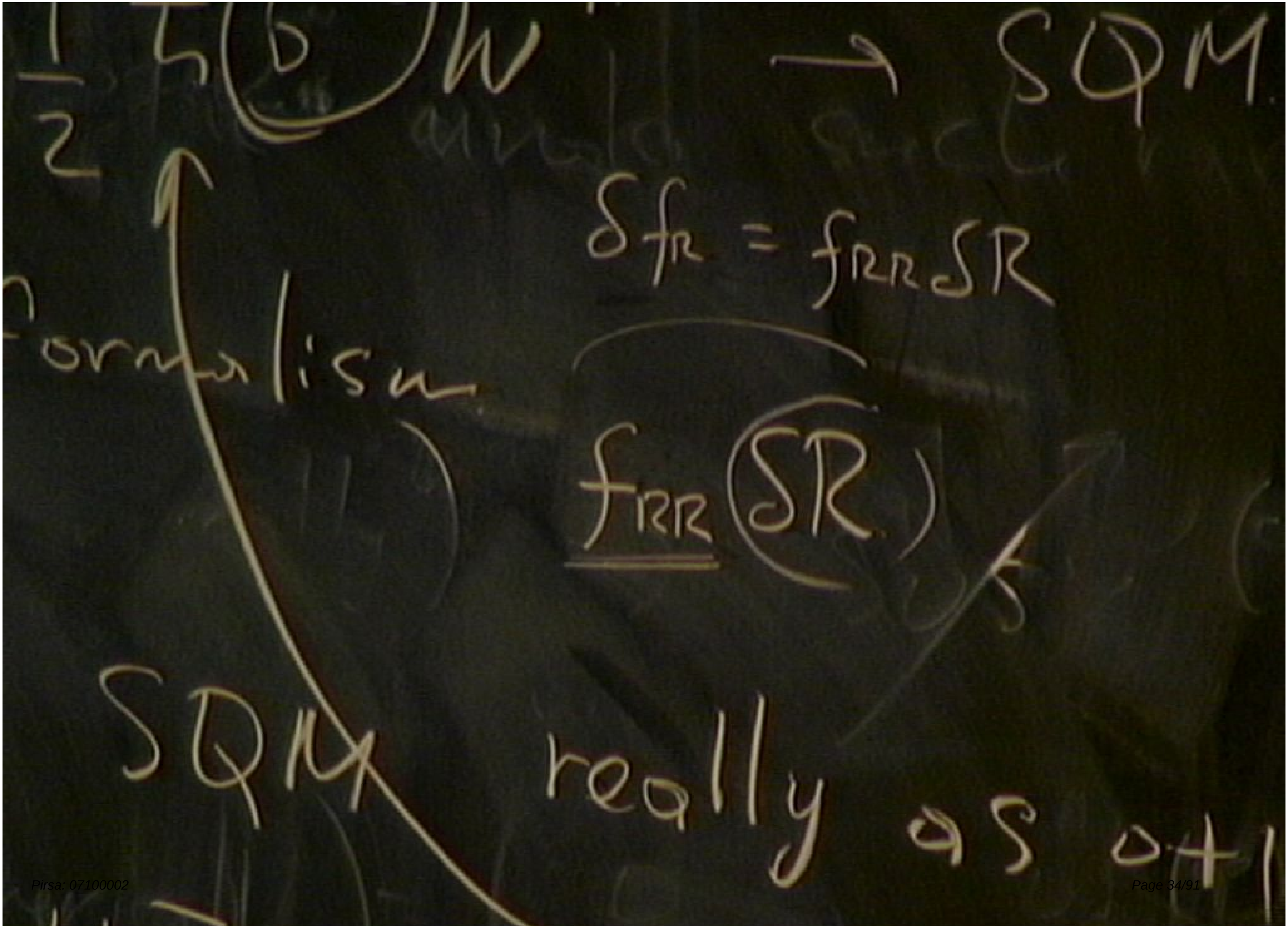
δf_R = f_RR δR
formalism
f_RR (δR)
SQM
lly as ot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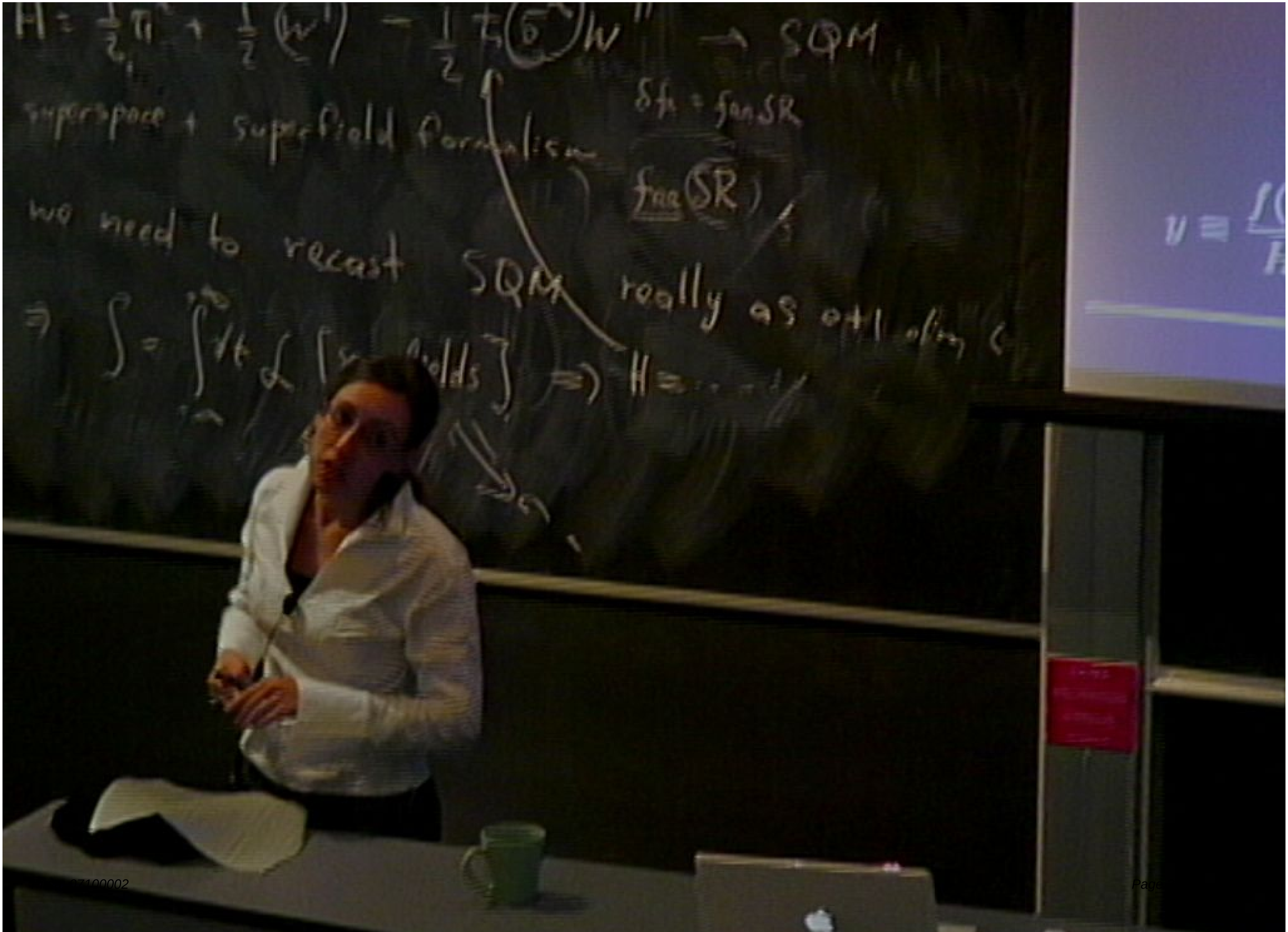
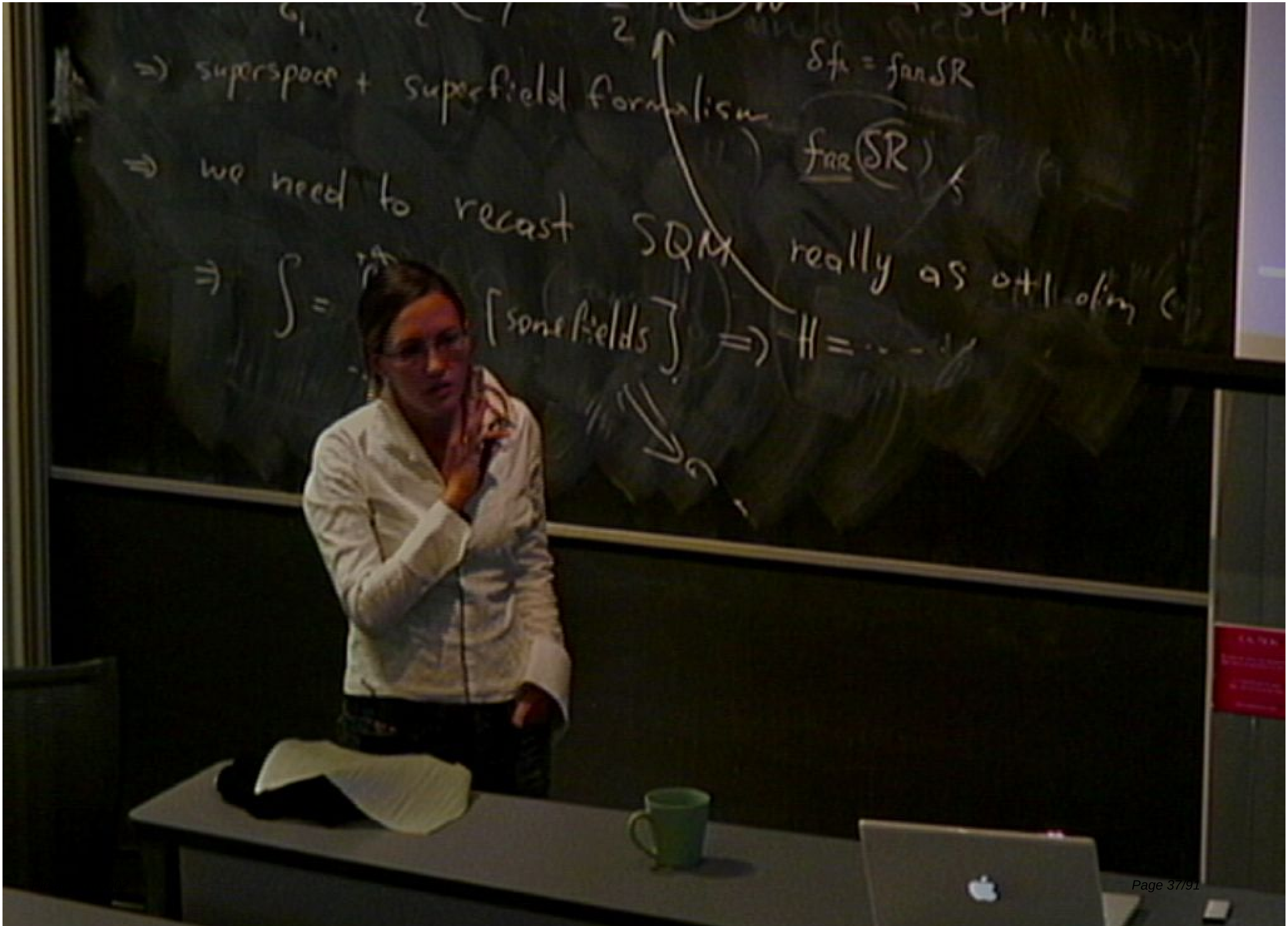
SQM really as


SQM
superspace + superfield formalism
we need to recast SQM really as


⇒ superspace + superfield formalism
⇒ we need to recast SQM really as 0+1 dim
[some fields]
SR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Yes! While at the background level viable f(R) must closely mimic LCDM, the difference in their prediction for the growth of large scale structure can be significant

The scalaron will set a transition scale, inducing a characteristic scale-dependent pattern of growth

On scales below the Compton wavelength of the scalaron, the modifications contribute a slip between the Newtonian potentials and the growth is enhanced by the fifth-force.

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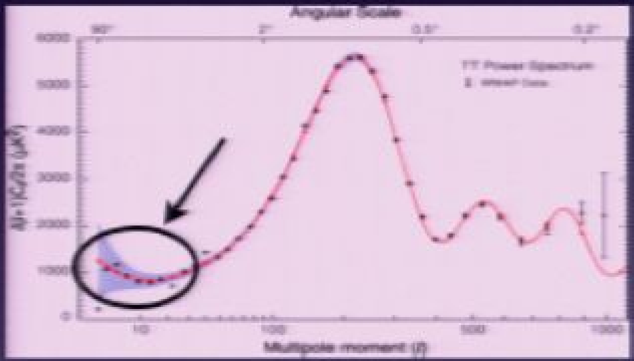

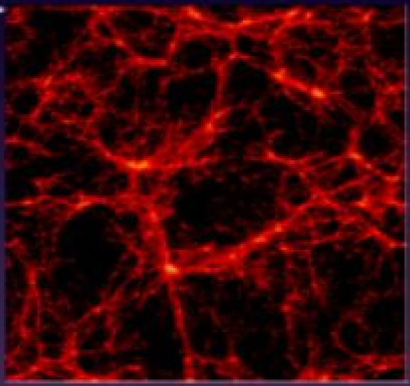

ISW, P(k), ISW-galaxy & WL

Dynamics of Linear Perturbations

Scalar perturbations in Newtonian gauge

$$ds^2 = -a^2(\tau)\,(1+2\Psi)\,d\tau^2 + a^2(\tau)\,(1-2\Phi)\,d\vec{x}^2$$

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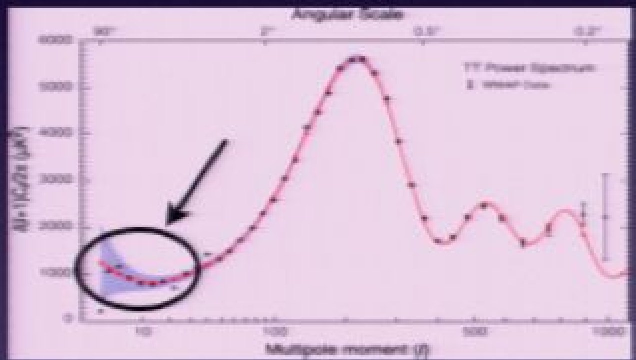

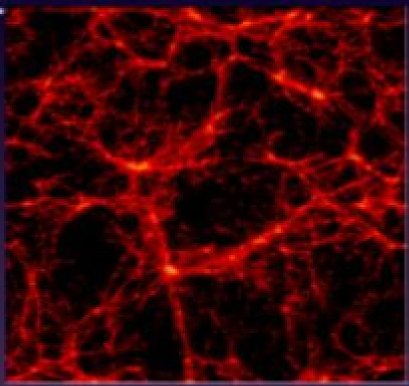

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anisotropy:

$$\Phi - \Psi = \frac{\delta(f_R)}{1+f_R} = \frac{f_{RR}}{1+f_R}\delta R$$

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$$\Phi - \Psi = \frac{\delta(f_R)}{1+f_R} = \frac{f_{RR}}{1+f_R}\delta R$$

$$\Phi - \Psi = 0$$

Poisson:

$$\frac{k^2}{a^2}\Phi = -\frac{3}{2}E_i\Delta_i - \Big[f_R\frac{k^2}{a^2}\Phi - \frac{1}{2}\frac{k^2}{a^2}f_{RR}\delta R + \frac{3}{2}H^2 f'_R\,(\Psi+\Phi') + \frac{3}{2}HH' f_{RR}\delta$$

Dynamics of Linear Perturbations

lew variables:

$$\begin{cases} \Phi_+ \equiv \dfrac{\Phi + \Psi}{2} \\ \chi \equiv f_{RR}\delta R = F(\Phi - \Psi) \end{cases}$$

Φ_+ ⟶ ISW, Weak Lensing

χ ⟶ effective shear

Dynamics of Linear Perturbations

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$F \equiv (1 + f_R)$

$$\Phi'_+ = \frac{3}{2}\frac{aE_mV_m}{HkF} - \left(1 + \frac{1}{2}\frac{F'}{F}\right)\Phi_+ + \frac{3}{4}\frac{F'}{F^2}\chi$$

$$\chi' = -\frac{2E_m\Delta_m}{H^2}\frac{F}{F'} + \left(1 + \frac{F'}{F} - 2\frac{H'}{H}\frac{F}{F'}\right)\chi - 2F\Phi'_+ - 2F\left(1 + \frac{2}{3}\frac{k^2}{a^2H^2}\frac{F}{F'}\right)\Phi_+$$

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I.C.s:

at z=1000

$$\Phi_+ = -1 \qquad \Delta_m = -\frac{2k^2}{3a^2H^2}\Phi_+$$

$$\chi = 0 \qquad v_m = \frac{2k}{3aH}\Phi_+$$

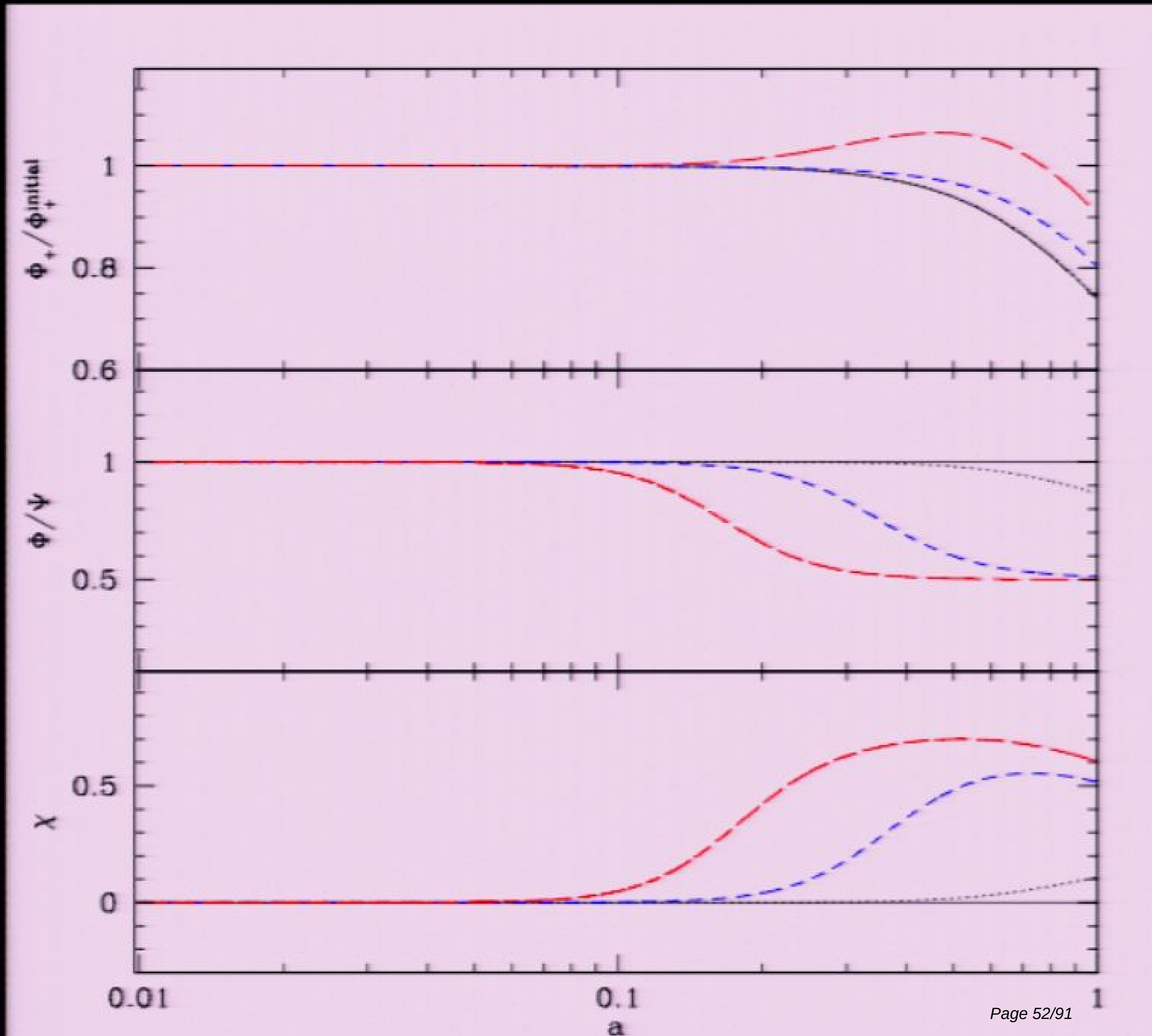
$w_{eff} = -1$

$f_R^0 = -10^{-4}$

$k = 0.01h/Mpc$ (dotted, black)

$k = 0.1h/Mpc$ (short dashed, blue)

$k = 0.5h/Mpc$ (long dashed, red)



Sub-Horizon

CDM equation:

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time and scale dependent rescaling of Newton constant $\equiv G_{eff}$

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$$\frac{k^2}{a^2}\frac{f_{RR}}{F}$$

There is a scale associated with the extra d.o.f. :

$$\lambda_C \equiv \frac{1}{m_{f_R}} \approx \sqrt{\frac{f_R R}{F}}$$

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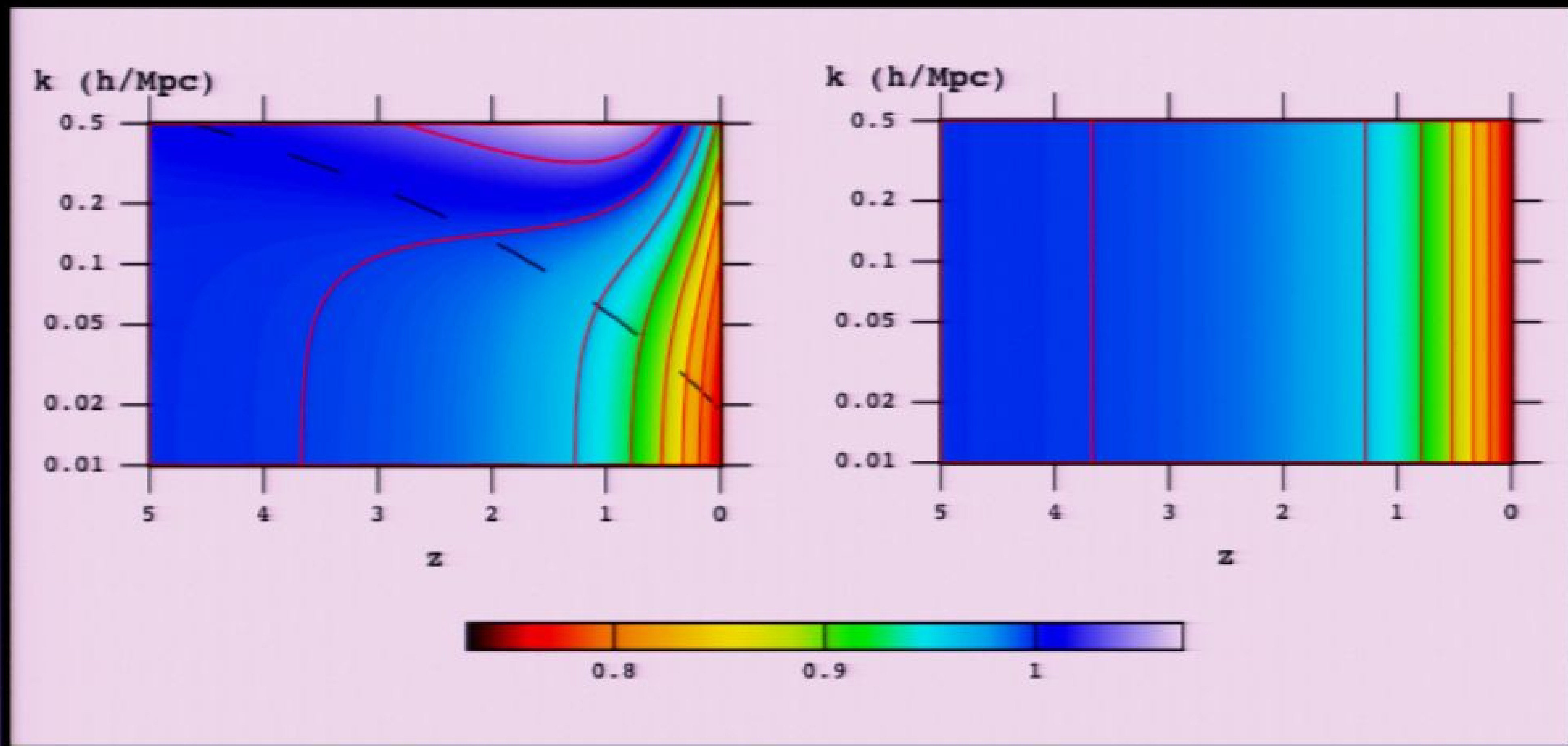

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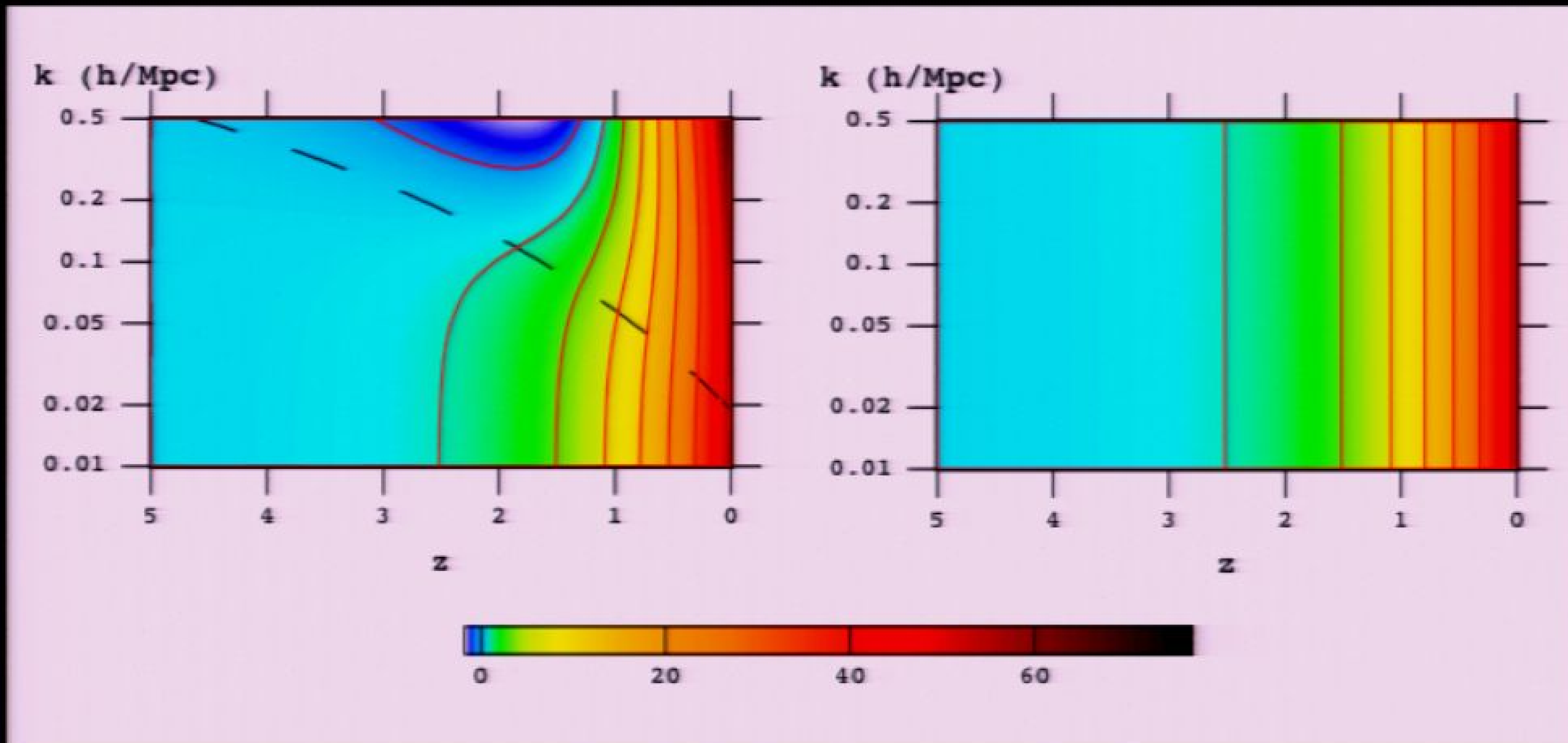
$$-\frac{d\Phi_+}{dz} \cdot \Delta_m$$



Effective shear and ratio between the potentials

$$\eta(k,z) \equiv \frac{\Phi}{\Psi}$$

(Zhang et al. astro-ph/0704.1932)

$$\bar{\omega}(k,z) \equiv \frac{\Psi - \Phi}{\Phi} = -\frac{\chi}{F\Phi}$$

(Caldwell et al. PRD76 023507(2007))

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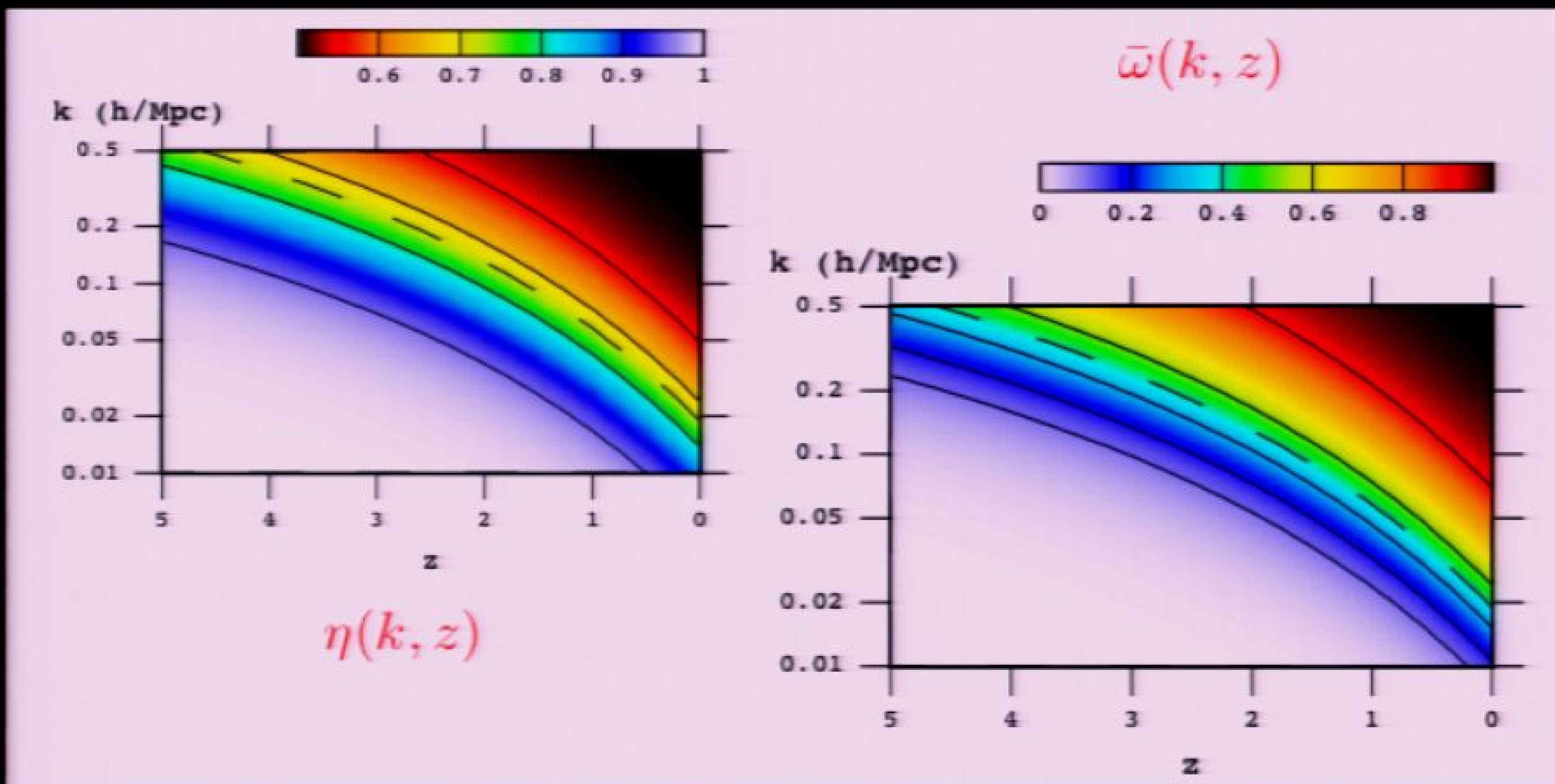

Effective Dark fluid

$$\tilde{G}_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu} \longrightarrow G_{\mu\nu} = \frac{1}{M_P^2}\left(T_{\mu\nu} + T_{\mu\nu}^{eff}\right)$$

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$\Pi_{eff} \propto \chi$

on sub-horizon scales

$$Q \equiv \frac{\lambda_C}{\lambda}$$

$$c_{eff}^2 \simeq \frac{2}{3}\frac{Q}{(1-F)(1+2Q)+QF} \xrightarrow{Q \ll 1} \simeq 0^+$$

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$$c_s^2 - c_{an}^2 \geq 0$$

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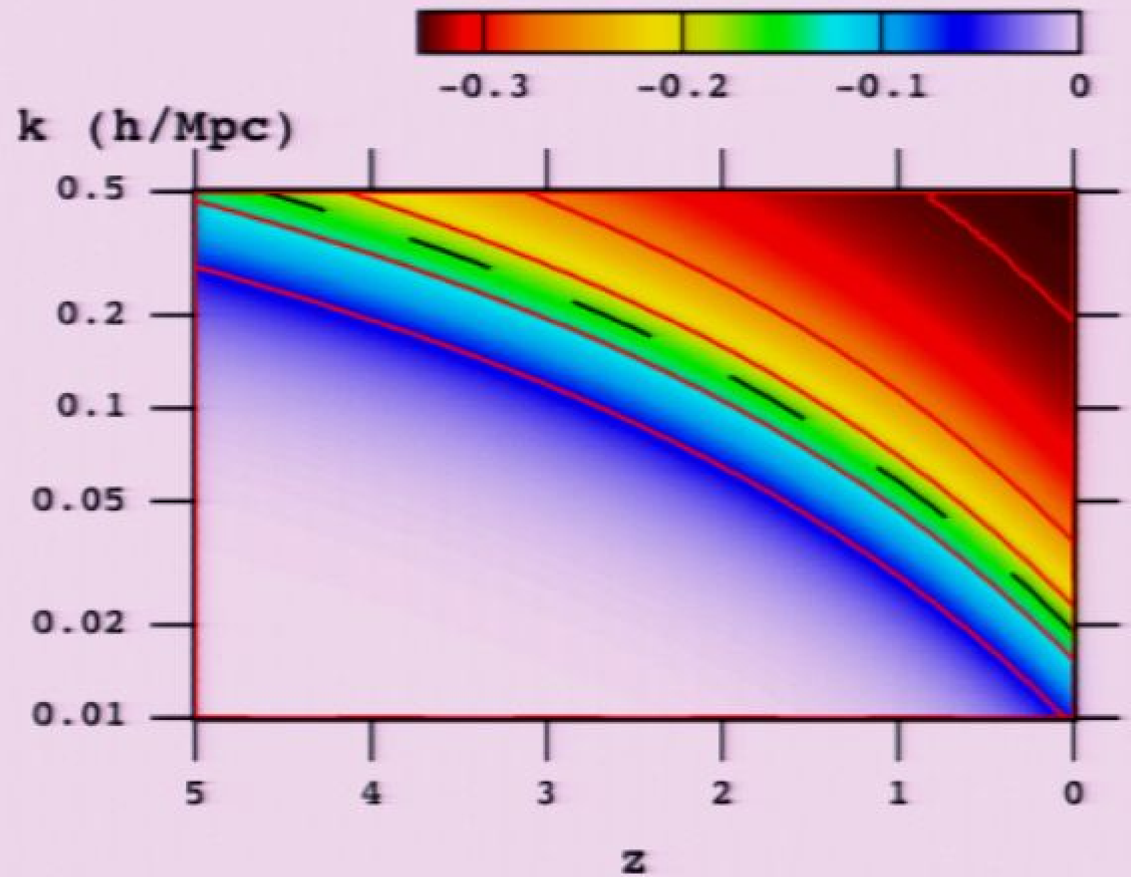

Summary I

- viable expansion history + Local Constraints of Gravity

↓

viable f(R) closely mimic LCDM with $w_{eff} \approx -1$

- the degeneracy is broken at the level of perturbations

- we observe a characteristic scale-dependent pattern, with the scalaron Compton wavelength separating two regimes

Summary II

- on scales $\lambda \gg \lambda_C$ the fifth force is exponentially suppressed and things behave similarly to standard GR

- on scales $\lambda \ll \lambda_C$ there is an enhancement of the growth due to the fifth force

$$\Downarrow$$

  - slip between Newtonian potentials $\Psi \to 2\Phi$
  - $G_{eff} \simeq \frac{4}{3}\frac{G}{F}$
  - the rate of growth depends on the balance between the fifth force and the acceleration of the background

Summary III

- the effect of modifications is maximized on modes below Compton waevelength during matter era

Φ_+ grows →

- ISW effect during matter era
- negatve ISW-galaxy

- Highly accurate 3D maps of Φ_+ from Weak Lensing Surveys

THANK YOU !

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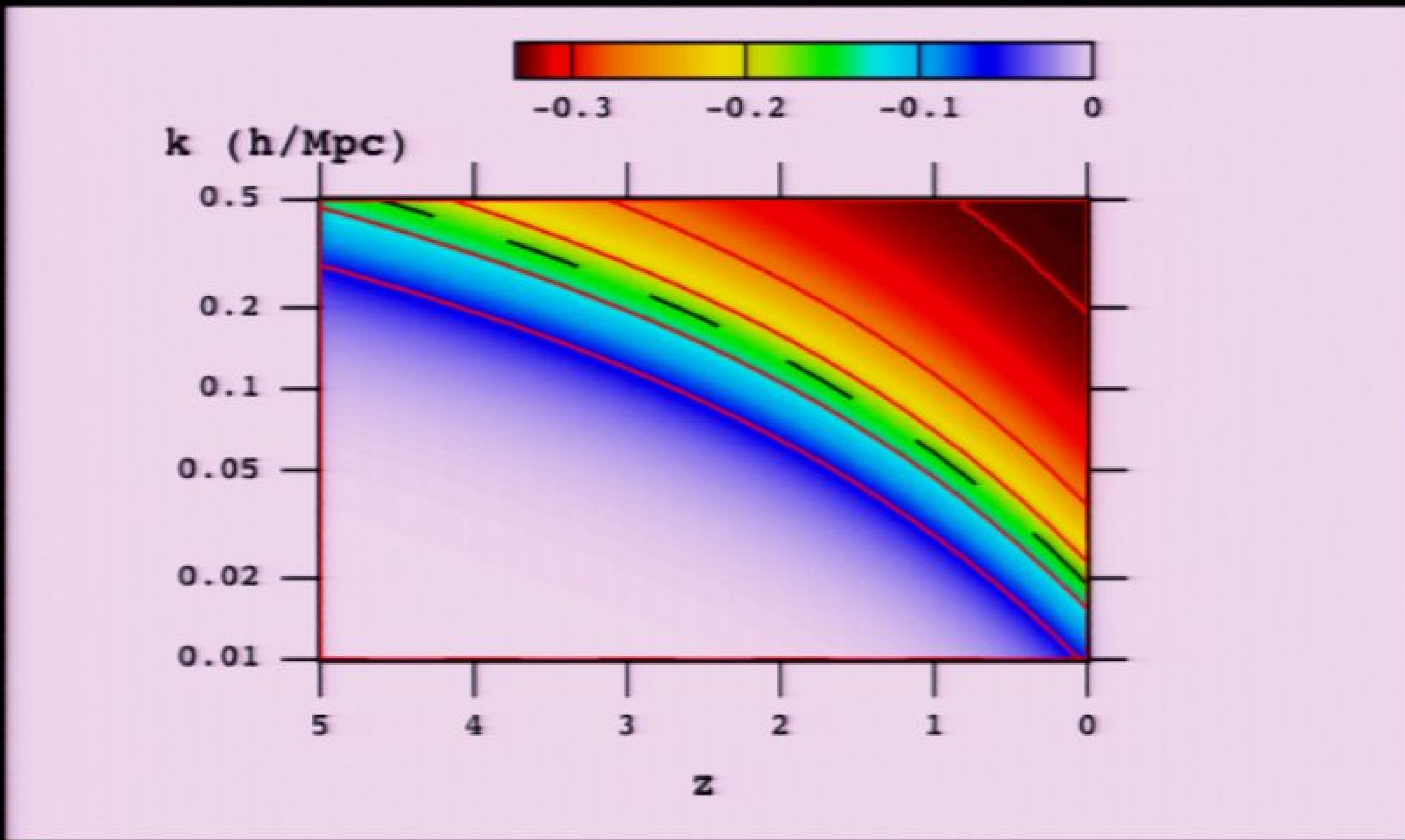

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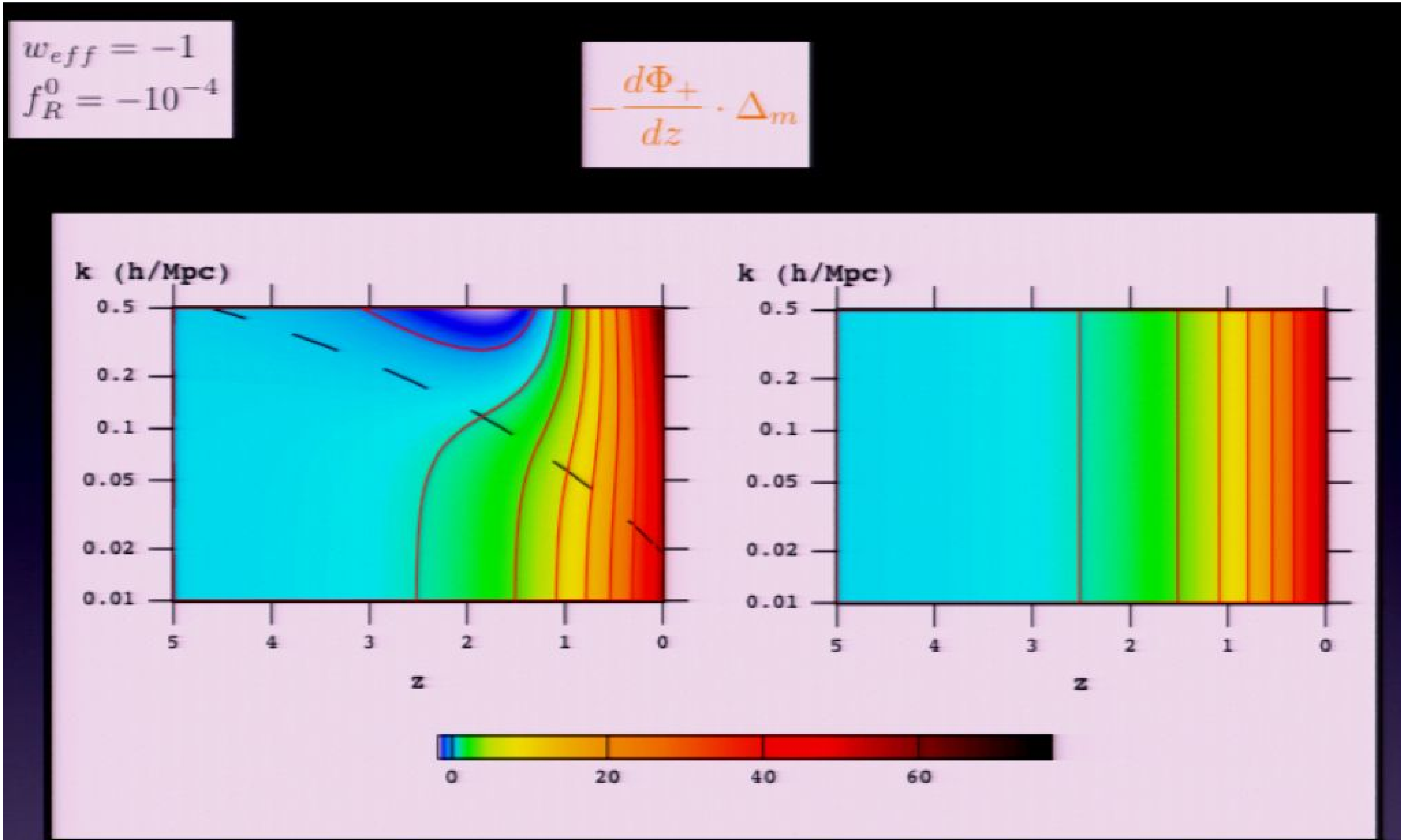

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k (h/Mpc)
0.5
0.2
0.1
0.05
0.02
0.01
5 4 3 2 1 0
z
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0.5
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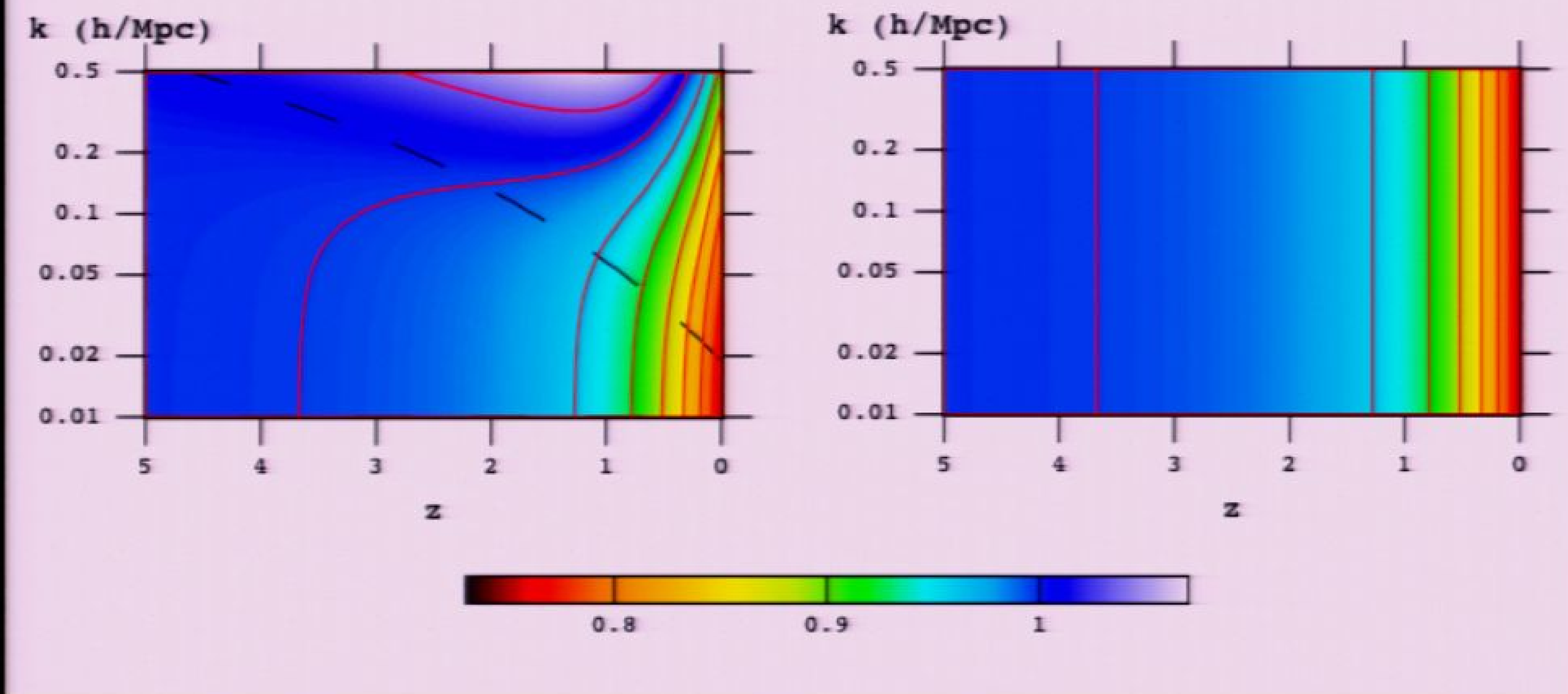

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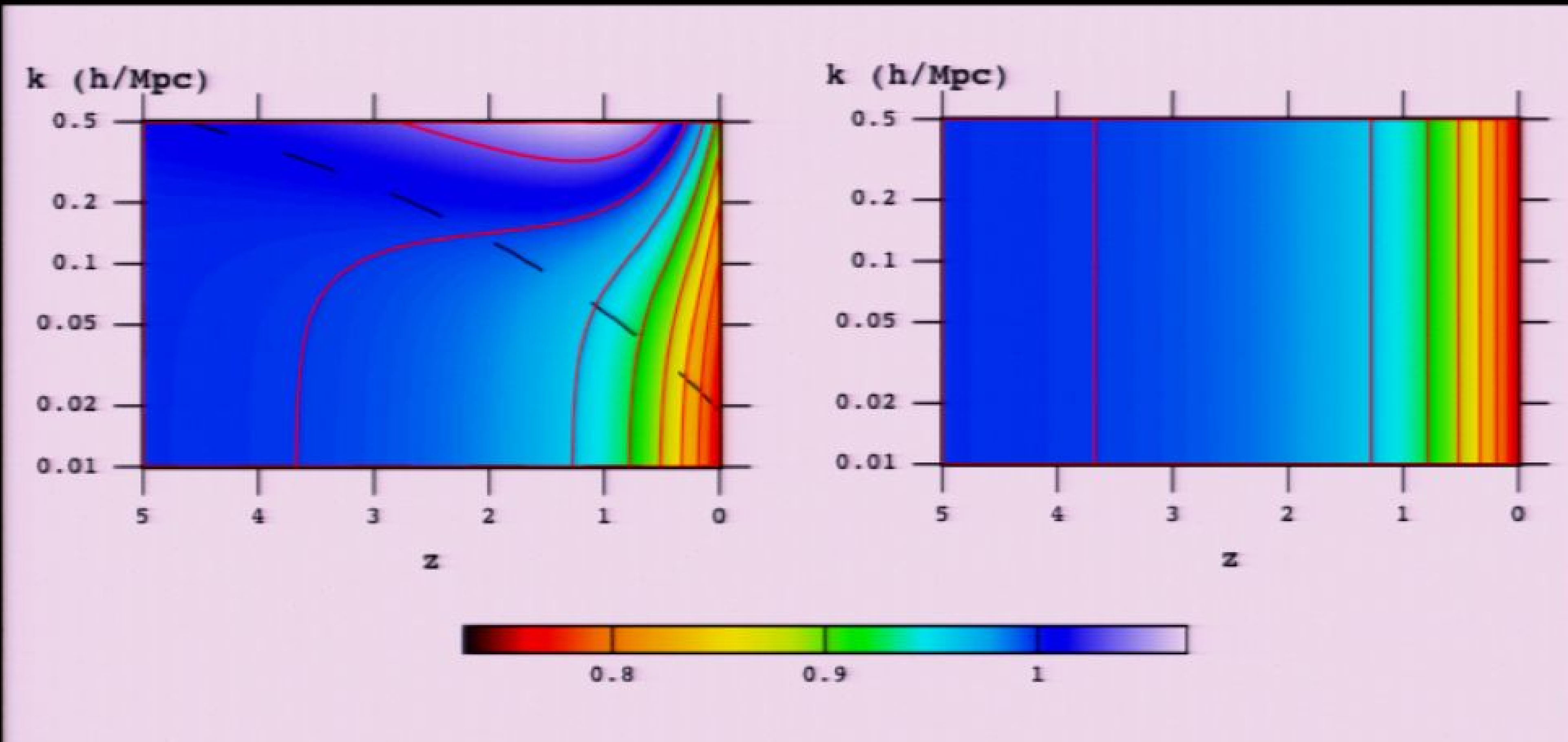

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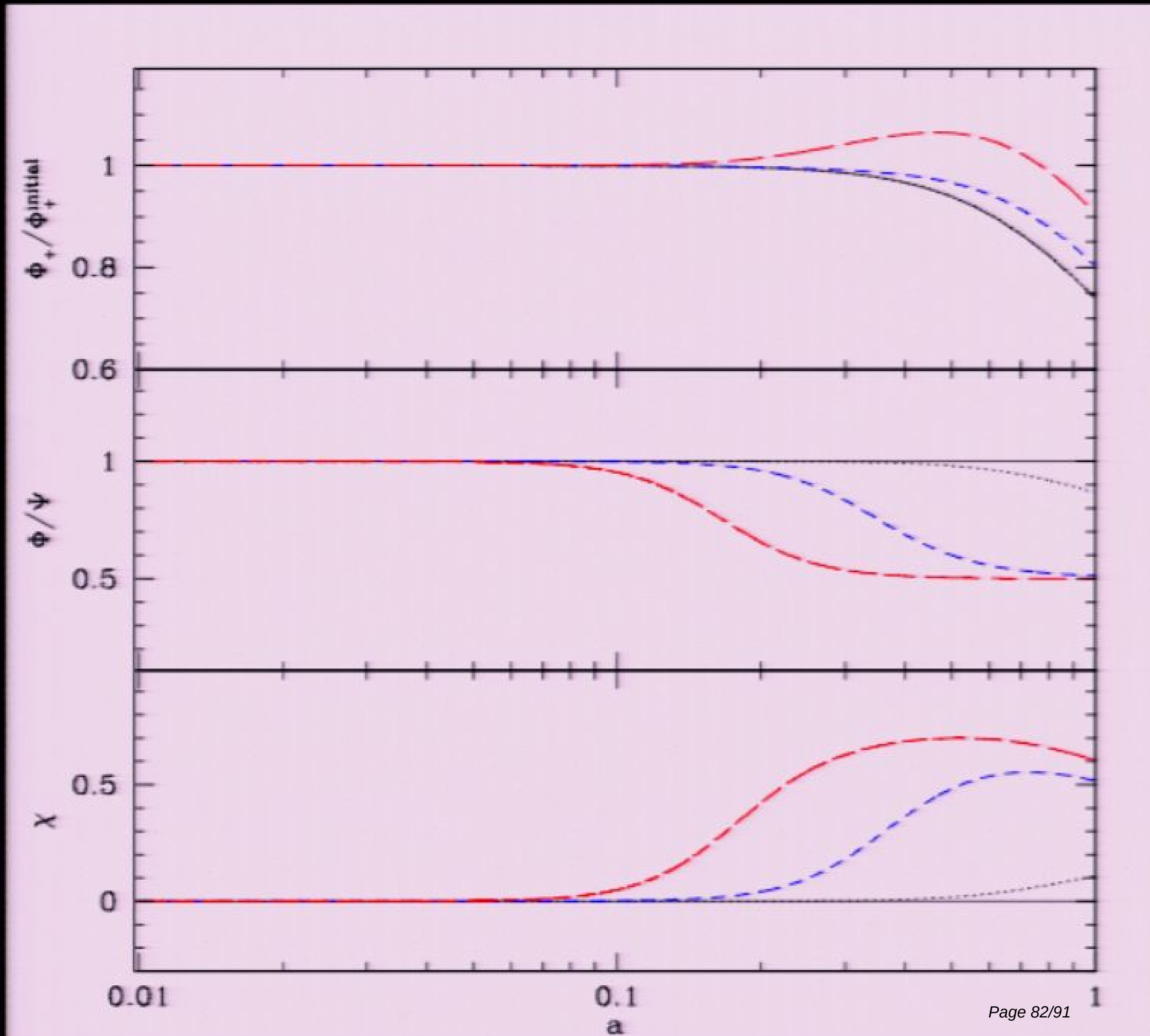

Dynamics of Linear Perturbations

lew variables:

$$\begin{cases} \Phi_+ \equiv \dfrac{\Phi + \Psi}{2} \\ \chi \equiv f_{RR}\delta R = F(\Phi - \Psi) \end{cases}$$

→ ISW, Weak Lensing

→ effective shear

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$$F \equiv (1 + f_R)$$

$$\Phi'_+ = \frac{3}{2}\frac{aE_mV_m}{HkF} - \left(1 + \frac{1}{2}\frac{F'}{F}\right)\Phi_+ + \frac{3}{4}\frac{F'}{F^2}\chi$$

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I.C.s:

at z=1000

$$\Phi_+ = -1 \qquad \Delta_m = -\frac{2k^2}{3a^2H^2}\Phi_+$$

$$\chi = 0 \qquad v_m = \frac{2k}{3aH}\Phi_+$$

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Φ_+ → ISW, Weak Lensing

χ → effective shear

The scalaron

$$\Box f_R = \frac{1}{3}\left(R + 2f - Rf_R\right) - \frac{\kappa^2}{3}(\rho - 3P) \equiv \frac{\partial V_{eff}}{\partial f_R}$$

$$m_{f_R}^2 \equiv \frac{\partial^2 V_{eff}}{\partial f_R^2} = \frac{1}{3}\left[\frac{1+f_R}{f_{RR}} - R\right]$$

Designer f(R)

The fourth order nature of f(R) provides enough freedom to reproduce any cosmological background history by an appropriate choice of the f(R) function

We fix the expansion history

$$E \equiv \frac{H^2}{H_0^2} = \Omega_m a^{-3} \Omega_r a^{-4} + \frac{\rho_{eff}}{\rho_{cr}}$$

$$(w_{eff}(a))$$

and solve the Friedmann eq. as a second order differential equation for f(R)

$$y'' - \left(1 + \frac{E'}{2E} + \frac{R''}{R'}\right) y' + \frac{E'}{2E} y = \frac{E'}{2E} E_{eff}$$

$$y \equiv \frac{f(R)}{H_0^2}$$

Background Viability

1. $f_{RR} > 0$ to have a stable high-curvature regime, to have a non-tachyonic scalar field

2. $1 + f_R > 0$ to have a positive effective Newton constant

3. $f_R < 0$ negative, monotonically increasing function of R that asymptotes to zero from below

4. $|f_R^0| \leq 10^{-6}$ must be small at recent epochs to pass LGC

(Hu and Sawicki astro-ph/0705.1158)

$$w_{eff} \simeq -1$$

(Dolgov & Kawasaki, Phys.Lett.B 573 (2003), Navarro et al. gr-qc/0611127, Sawicki and Hu astro-ph/0702278
Starobinsky astro-ph/0706.2041, Chiba, Smith, Erickcek astro-ph/0611867
Amendola et al. astro-ph/0603703-0612180, Amendola & Tsujikawa astro-ph/0705.0396)

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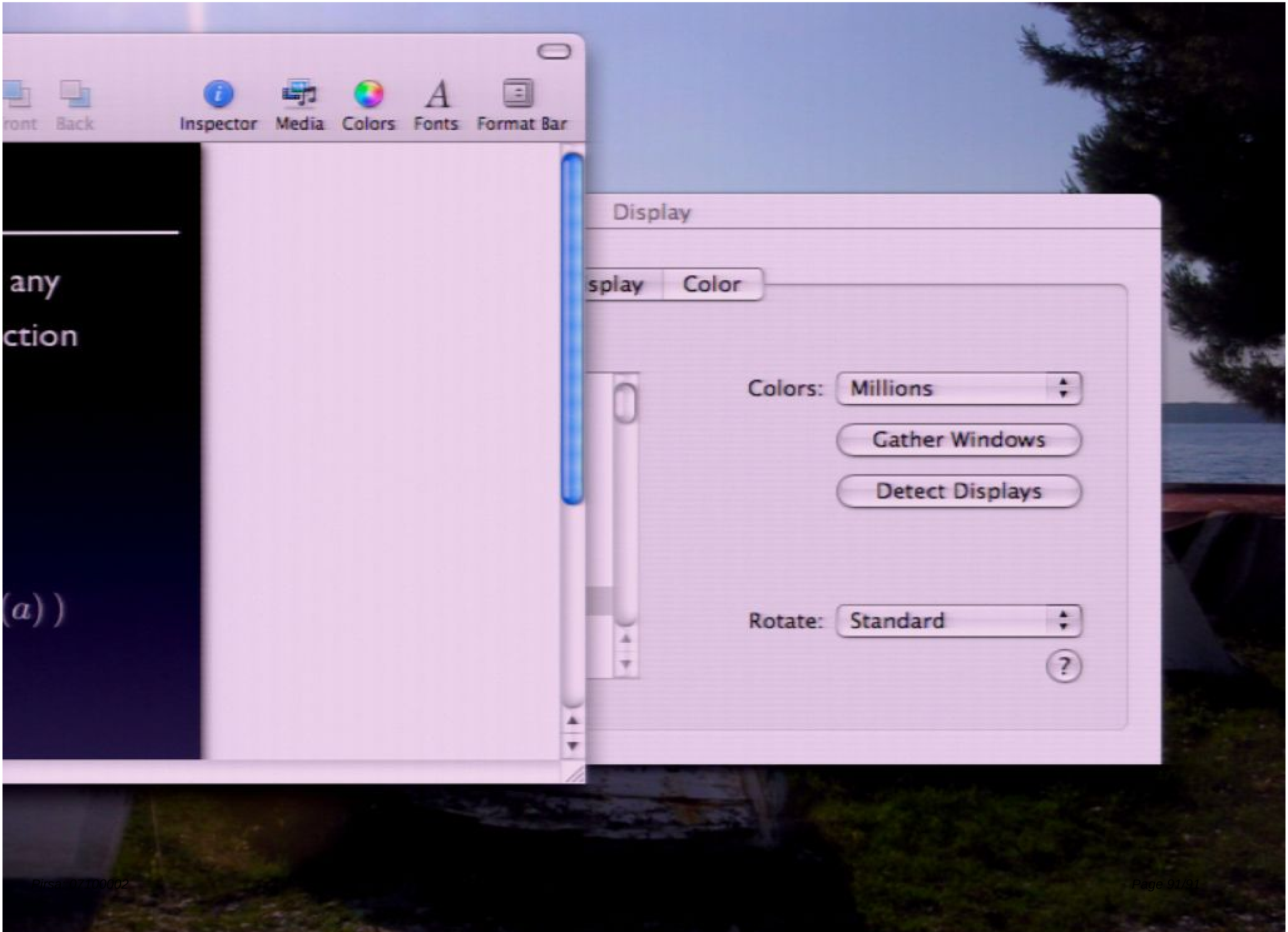
Back
Inspector
Media
Colors
Fonts
Format Bar
any
ction
Display
Color
Colors:
Millions
Gather Windows
Detect Displays
Rotate:
Standard