Title: Subjective Probability and Many Worlds

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Abstract: Probability is often regarded as a problem for the many-worlds interpretation: if all branches of the splitting wavefunction are equally real, what sense does it make to say that the branches have different probabilities? In the decision-theoretic approach due to Deutsch and Wallace, probabilities acquire a meaning through the preferences of a rational agent. This talk reviews the decision-theoretic approach to probability in classical physics and quantum mechanics and shows that its application to the many-world interpretation creates a new difficulty for the latter.

Subjective probability and many worlds

Rüdiger Schack

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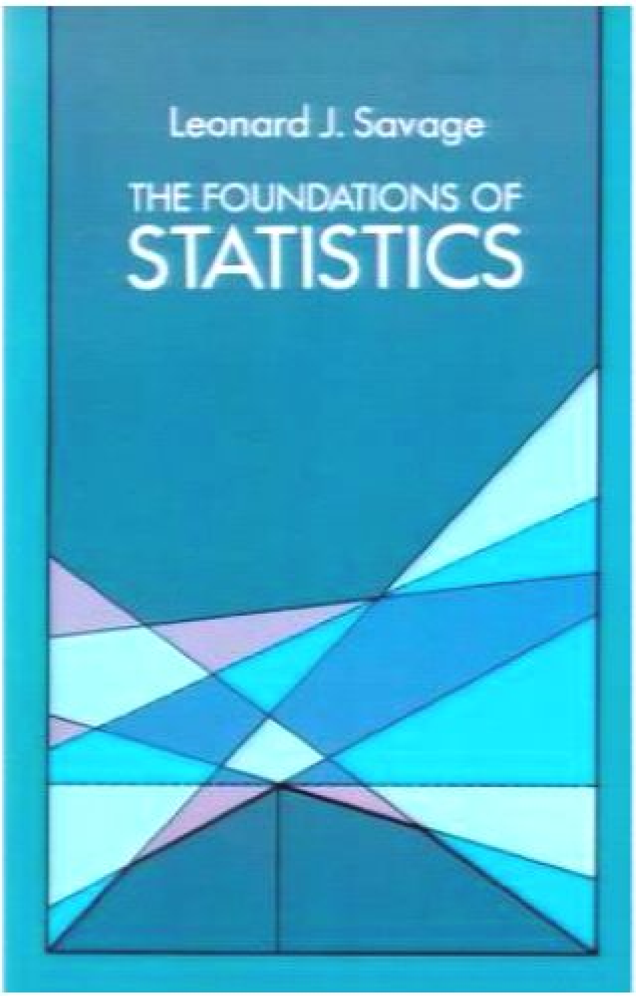
Decision theory and Everett

Deutsch (1999)

Barnum, Caves, Finkelstein, Fuchs, RS (1999)

Wallace (2002,2003,2004)

Decisions, acts, and consequences



Buy or Sell?

$\boxed{\text{Worth \$1 if } E \text{ is true}}$ ticket price $\$q$

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Operational definition of probability:

A assigns $\Pr(E) = q$ to the event E

$\Longleftrightarrow$

A regards $\$q$ as the fair price for the ticket.

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Dutch book coherence

A's probability assignments (i.e., ticket valuations) are incoherent if they can lead to a sure loss.

Coherence alone implies

(i) $\Pr \geq 0$

(ii) $\Pr(E) = 1$ if A believes that E is certain to occur.

(iii) $\Pr(E \vee F) = \Pr(E) + \Pr(F)$ if A believes that E and F are mutually exclusive.

(iv) $\Pr(E \wedge F) = \Pr(E|F)\,\Pr(F)$

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Single trial versus long run

The usual argument: If A does not obey the probability rules, she will lose in the long run.

The Dutch book argument: If A does not obey the probability rules, she will lose here and now.

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Rule (i)

(i) $P \geq 0$

$\boxed{\text{Worth \$1 if } E \text{ is true}}$ fair price $\$q < 0$

A is willing to sell the ticket for a negative amount of money. Sure loss!

Rule (ii)

(ii) $P(E) = 1$ if A believes that E is certain to occur.

$\boxed{\text{Worth \$1 if } E \text{ is true}}$ fair price $\$q < \$1$

A is willing to sell a ticket—which is definitely worth $\$1$ to her—for less than $\$1$. Sure loss!

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Rule (iii)

(iii) $P(E \vee F) = P(E) + P(F)$ if A believes that E and F are mutually exclusive.

Let $H = E \vee F$, $E \wedge F = \emptyset$.

Worth \$1 if H is true	fair price \$q
Worth \$1 if E is true	fair price \$r
Worth \$1 if F is true	fair price \$s

E.g., A would buy the blue ticket for $\$q$ and sell the red tickets for $\$r + \$s$. If $q > r + s$, sure loss!.

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Worth \$1 if $E \wedge F$ Worth $\$P(E|F)$ if $\neg F$	price $\$P(E|F)$
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Coherence implies
$\$P(E|F) = \$P(E \wedge F) + P(E|F)P(\neg F)$

Rule (iv) follows using $P(\neg F) = 1 - P(F)$.

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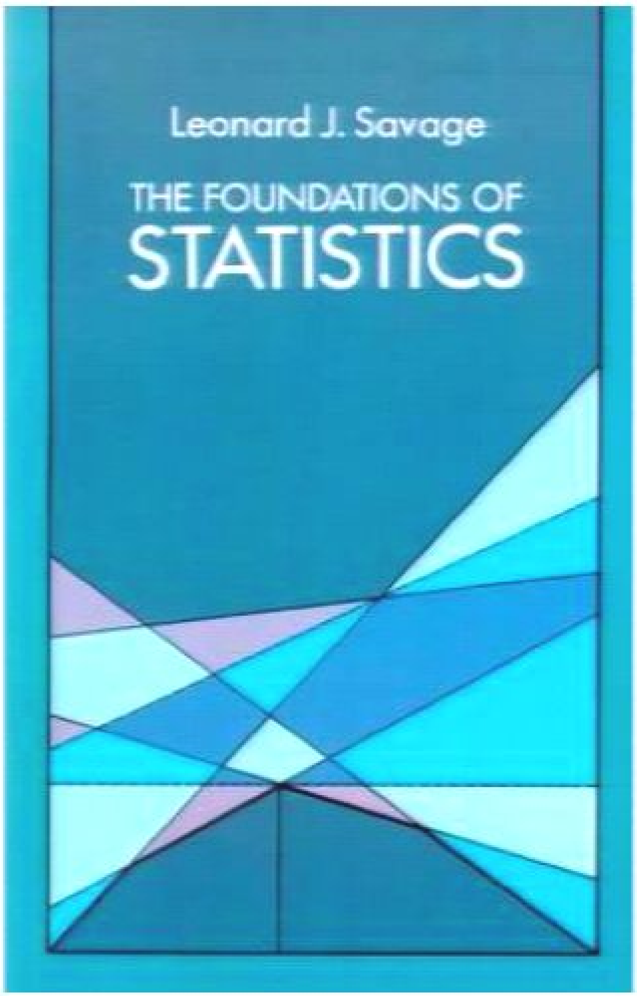
Dutch book in many worlds

Buy (or sell) now.

Branch 1: payoff_1
Branch 2: payoff_2
Branch 3: payoff_3

Coherence: at least one payoff is non-negative.

Probabilities as subjective degrees of belief



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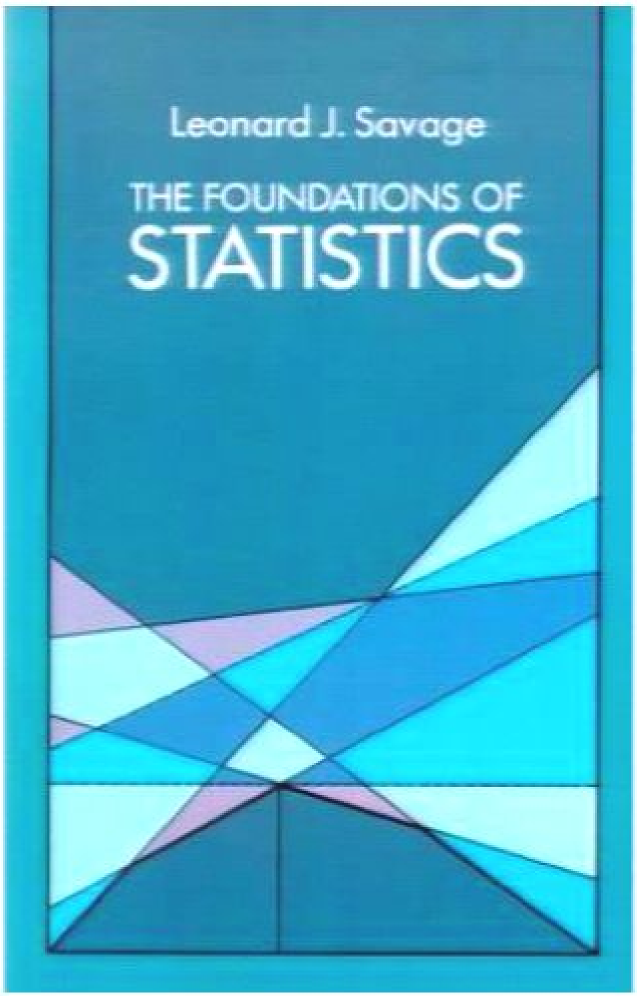

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Connection to frequency

Let $x_k \in \{0, 1\}$ be binary random variables.

$p^{(n)}(x_1, \ldots, x_n)$, $n = 1, 2, \ldots$ form an exchangeable sequence if

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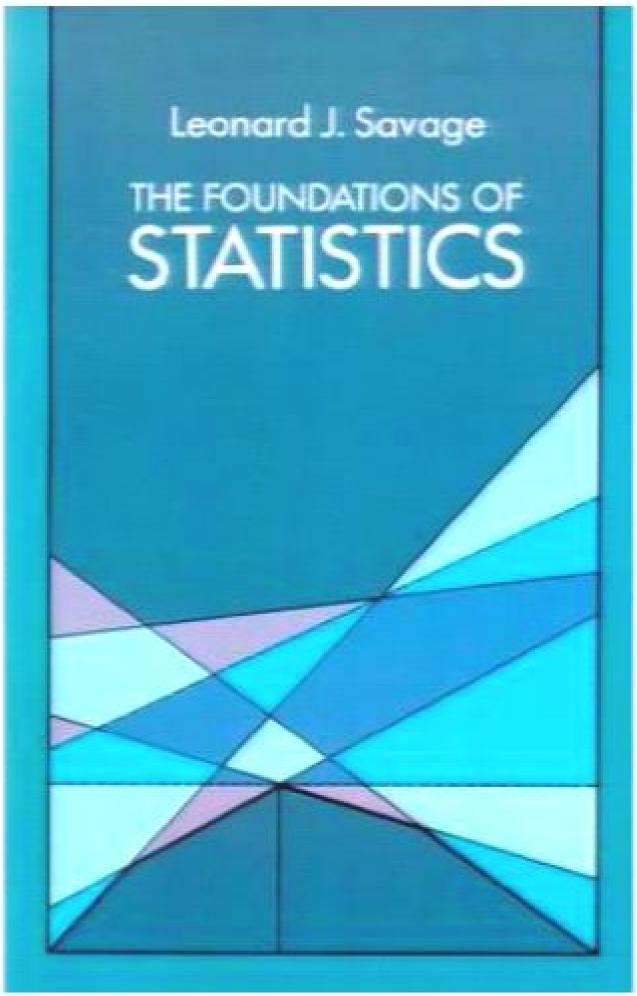

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For given N, we say that $p^{(N)}(x_1, \ldots, x_N)$ is exchangeable if it is part of an exchangeable sequence.

De Finetti's representation theorem (binary case)

$p^{(N)}(x_1, \ldots, x_N)$ is exchangeable

if and only if

$$p^{(N)}(x_1, \ldots, x_N) = \int_0^1 P(q)dq\, q^k (1-q)^{N-k}$$

where k is the number of zeroes in $(x_1, \ldots, x_N)$.

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Teaching elementary probability

Question: Find the probability that, in a family with two children, exactly one child is a boy.

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Answer: 1/3.

Proof: There are three possibilities:

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Hence $\Pr(1) = 1/3$.

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Teaching elementary probability

The conventional reply: There really are four possibilities:

(0) no boys: GG
(1) one boy: GB or BG
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They are exhaustive:
$\Pr(GG) + \Pr(GB) + \Pr(BG) + \Pr(BB) = 1$

Hence
$\Pr(\text{1 boy}) = \Pr(GB) + \Pr(BG) = 1/4 + 1/4 = 1/2$

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Teaching elementary probability

Both are correct!

They just start from different prior probabilities

(1) $\Pr(\text{0 boys}) = \Pr(\text{1 boys}) = \Pr(\text{2 boys}) = 1/3$

(2) $\Pr(GG) = \Pr(GB) = \Pr(BG) = \Pr(BB) = 1/4$

Empiricist approach

There are 2,588,192 two-children families in the UK. Of these, 1,270,110 have exactly one boy.

Thus $\mathrm{Pr}(\text{one boy}) = \dfrac{1,270,110}{2,588,192} = 49.07\%$.

Empiricist approach

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Thus $\Pr(\text{one boy}) = \dfrac{1,270,110}{2,588,192} = 49.07\%$.

Correct only when we select a family at random from all two-children families.

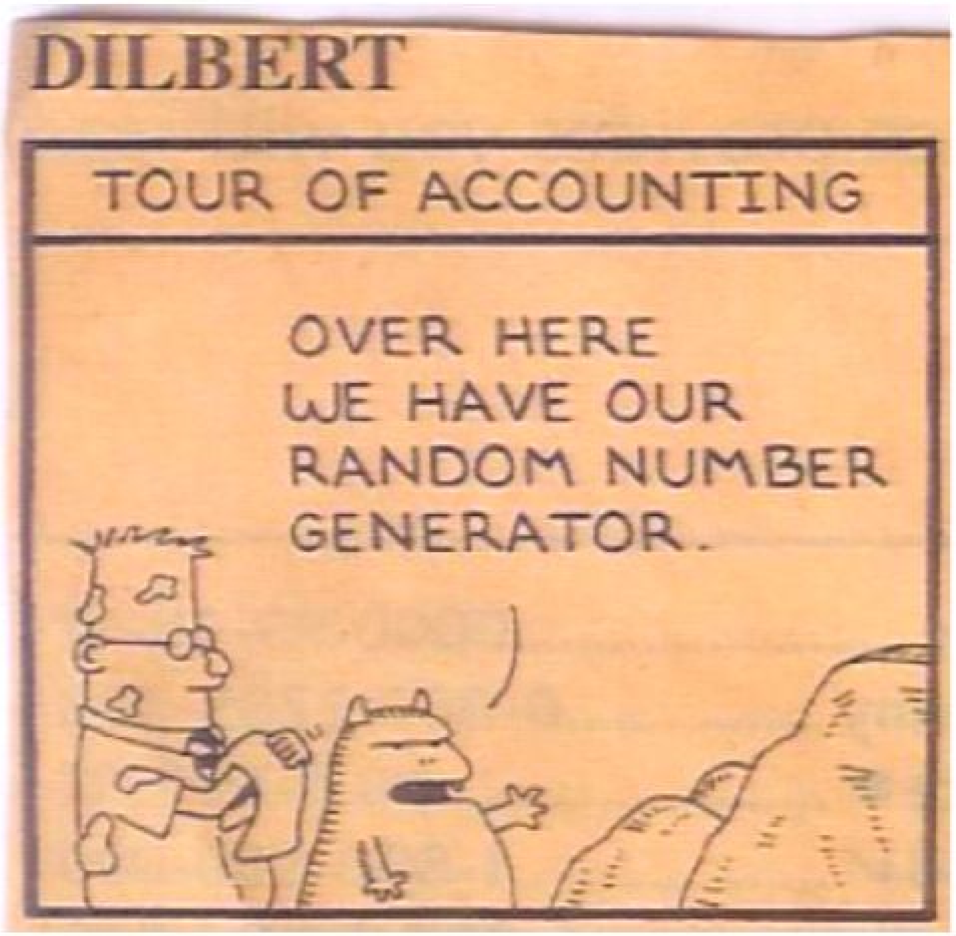
Empiricist approach

Real situation: the proportion $\dfrac{1,270,110}{2,588,192} = 49.07\%$

Prior probability: all families are equally likely to be selected

$\Pr($one boy$)$ DEPENDS ON BOTH.

www.dilbert.com



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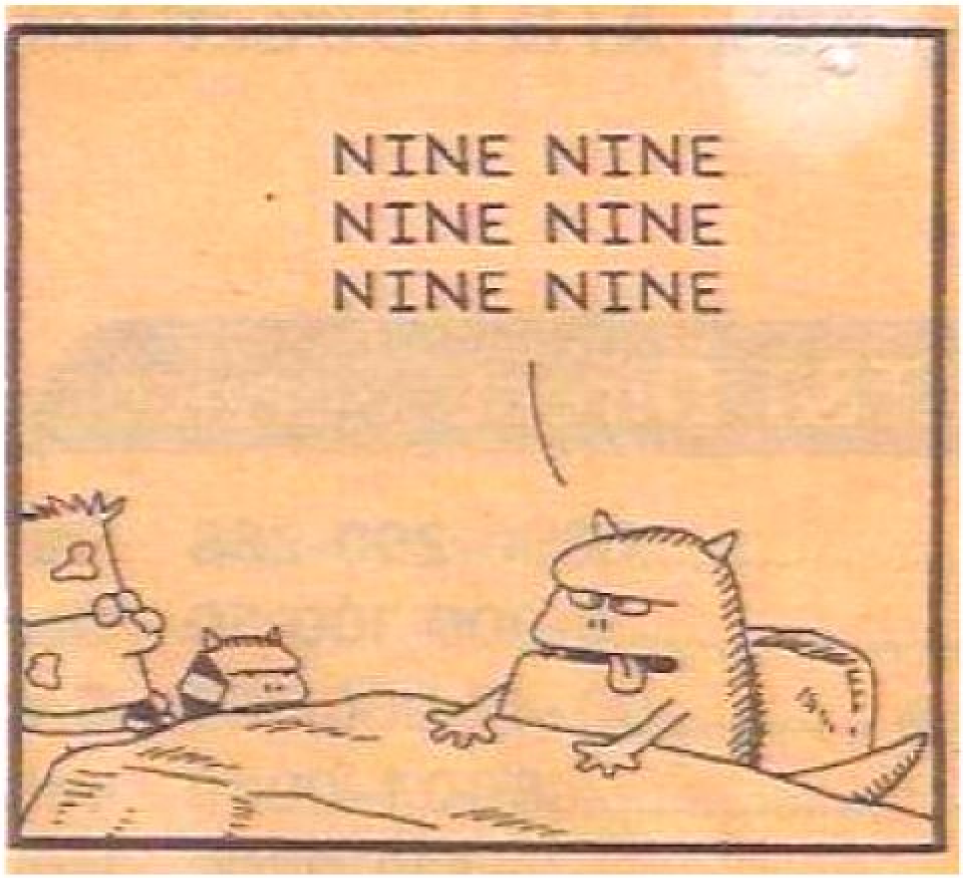

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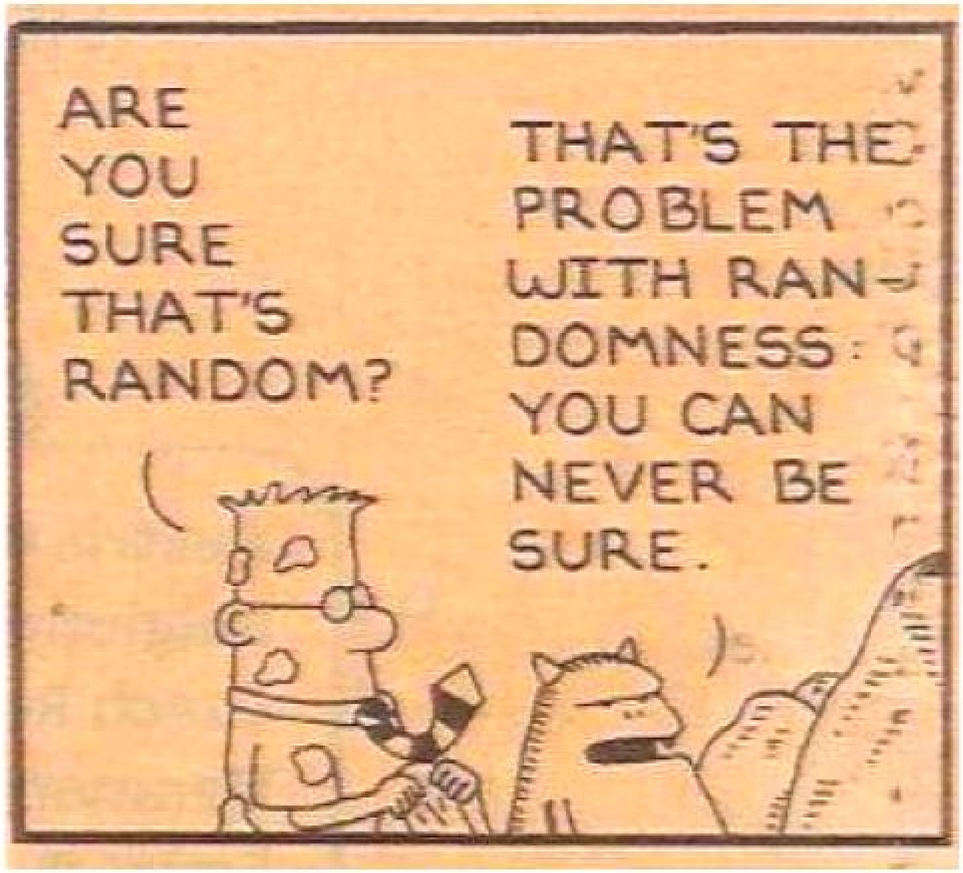

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A category distinction

Real situations, “facts”

versus

Probabilities = an agent’s degrees of belief

Surely this probability is objective

$$\Pr(15) = 1/37$$

BBC NEWS UK EDITION

Last Updated: Sunday, 5 December, 2004, 05:40 GMT

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'Laser scam' gamblers to keep £1m

A group of gamblers who won more than £1m at the Ritz Casino by using laser technology have been told by police they can keep their winnings.

A laser scanner was allegedly used by the gamblers

The trio - a Hungarian woman and two Serbian men - were arrested in March but police have apparently decided that they did not break the law.

A laser scanner linked to a computer was allegedly used to gauge numbers likely to come up on the roulette wheel.

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Chance AND Probability: the Principal Principle (PP)

$$\Pr\left(E \middle| \text{chance}(E) = q\right) = q \; .$$

where

E is an event,

$q \in [0, 1]$, and

$\Pr$ is an agent's degree of belief.

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PP attempts to give empirical content to chance

Flip a coin N times, let q be the chance of Heads, and let $Pr(q)$ be an agent's (subjective) prior for one flip.

Prior for N flips:

$$p^{(N)}(x_1, \ldots, x_N) = \int_0^1 Pr(q)dq\, q^k(1-q)^{N-k}$$

Now use frequency data to update $Pr(q)$.

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Chance of exactly what?

In a classical, deterministic theory, only situations corresponding to chance = 0 or 1 can be unambiguously defined.

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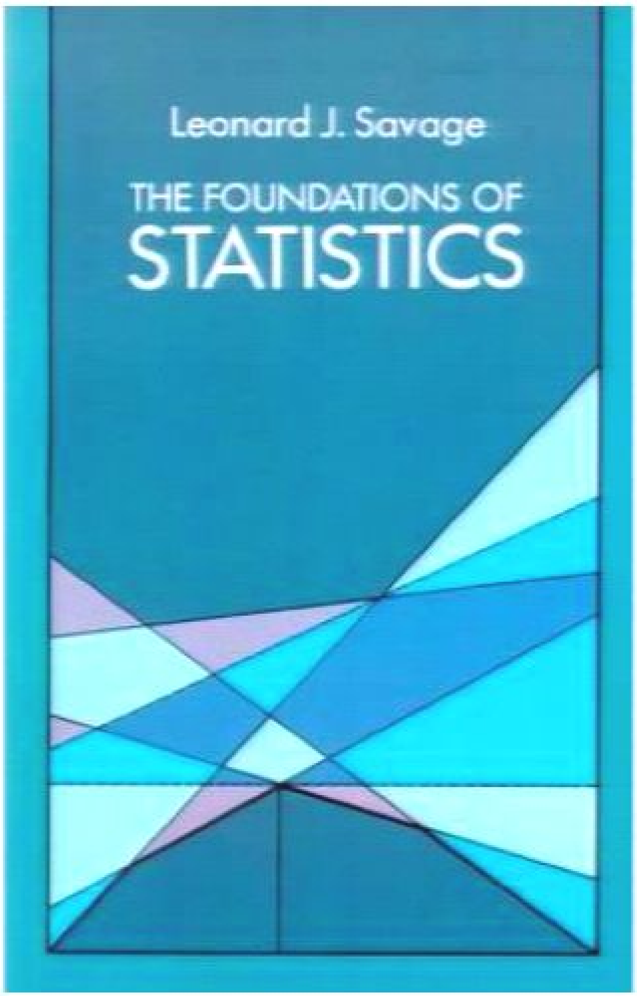
Savage's program in quantum mechanics?



Savage's program in quantum mechanics!

- Quantum states represent an agent's (decision-theoretic) degrees of belief (Caves, Fuchs, RS)
- Gleason's theorem
- The quantum de Finetti theorem
- Wallace's quantum version of Savage's axioms
- Etc.

Gleason's theorem

Assume there is a function h from the one-dimensional projectors acting on a Hilbert space of dimension greater than 2 to the unit interval, with the property that for each orthonormal basis $\{|\psi_k\rangle\}$,

$$\sum_k h\big(|\psi_k\rangle\langle\psi_k|\big) = 1 \; .$$

Then there exists a density operator ρ such that

$$h\big(|\psi\rangle\langle\psi|\big) = \langle\psi|\rho|\psi\rangle \; .$$

Exchangeability for quantum systems

A state $\rho^{(N)}$ of N systems is exchangeable if it is a member of an exchangeable sequence $\rho^{(n)}$, i.e.,

(i) (symmetry) each $\rho^{(n)}$ is invariant under permutations of the n systems on which it is defined; and

(ii) (extendibility) $\rho^{(n)} = \mathrm{tr}_{n+1}\rho^{(n+1)}$ for all n, where tr_{n+1} denotes the partial trace over the $(n+1)$th system.

Quantum de Finetti Theorem

$\rho^{(N)}$ is exchangeable

if and only if

$$\rho^{(N)} = \int d\rho \, p(\rho) \, \rho^{\otimes N} = \int d\rho \, p(\rho) \, \rho \otimes \cdots \otimes \rho \, .$$

(Hudson, Moody 1976; Caves, Fuchs, RS 2002)

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Bayesian quantum tomography

$$\boxed{\rho^{(N+M)} = \int d\rho\, p(\rho)\, \rho^{\otimes(N+M)}}$$

measure N subsystems

get outcome $\vec{\alpha}$

$$\boxed{\rho^{(M)} = \int d\rho\, p(\rho|\vec{\alpha})\, \rho^{\otimes M}}$$

$p(\rho|\vec{\alpha})$ given by a quantum Bayes rule.

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Two kinds of quantum states

"Belief states" are used for decision making.

"Real states" are real.

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A quantum Principal Principle

The "belief state" of a system, given that the "real state" is $|q\rangle$, is $|q\rangle$.

(This is the Deutsch-Wallace rationality constraint.)

Real state of exactly what?

In quantum theory, the situation giving rise to a putative real state $|q\rangle$ cannot be unambiguously defined.

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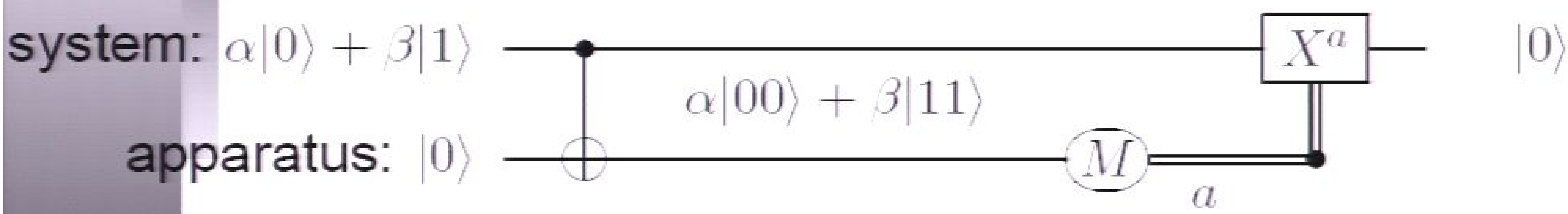
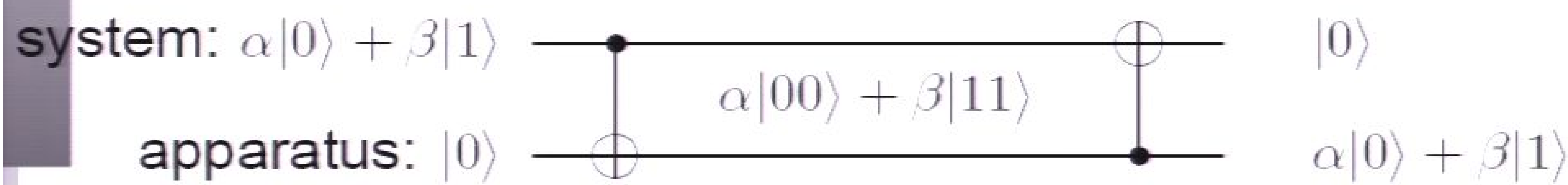
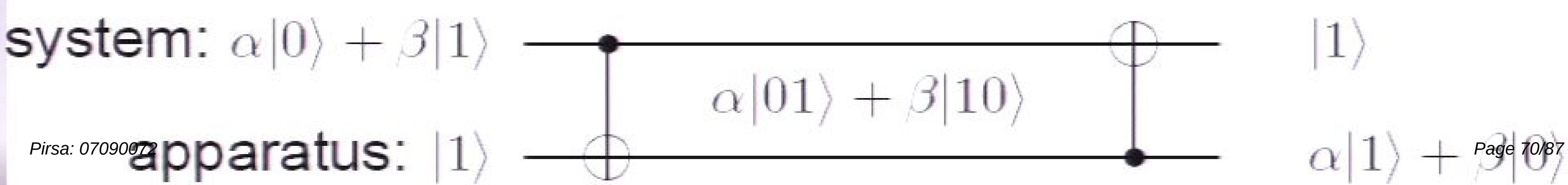
Quantum state preparation







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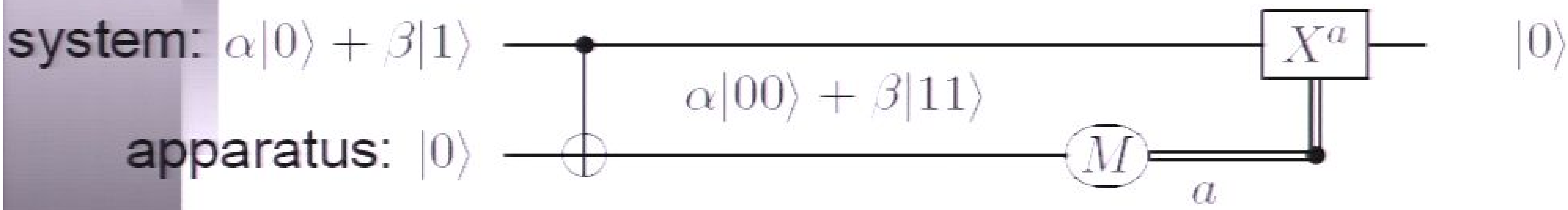

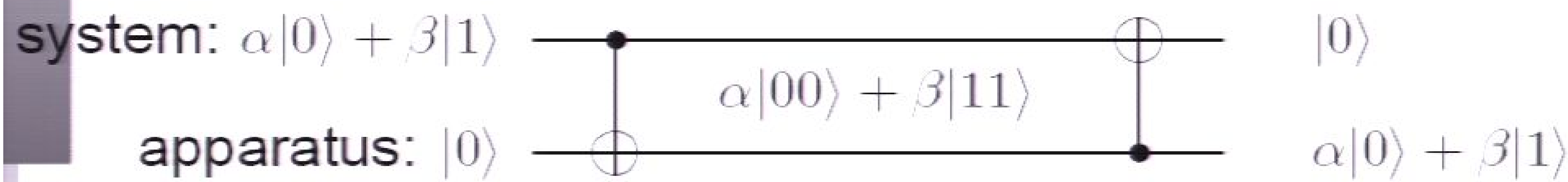

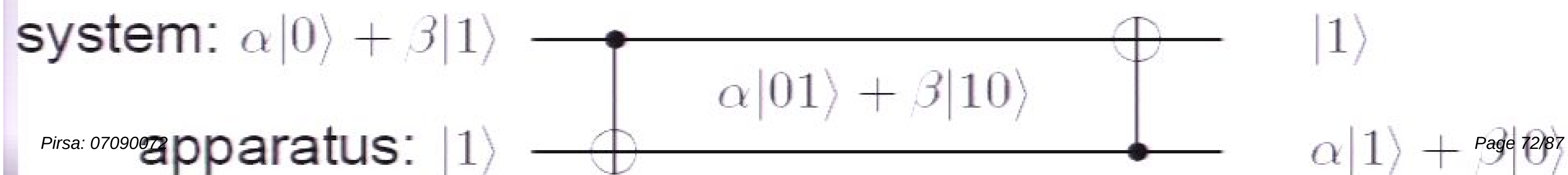

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From Probabilism to Quantum Bayesianism

Any probabilistic argument starts from a judgment in the form of a prior probability assignment.

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Conclusions

- Decision-theoretic approach to q.m.: YES
- Decision-theoretic approach to many worlds: NO

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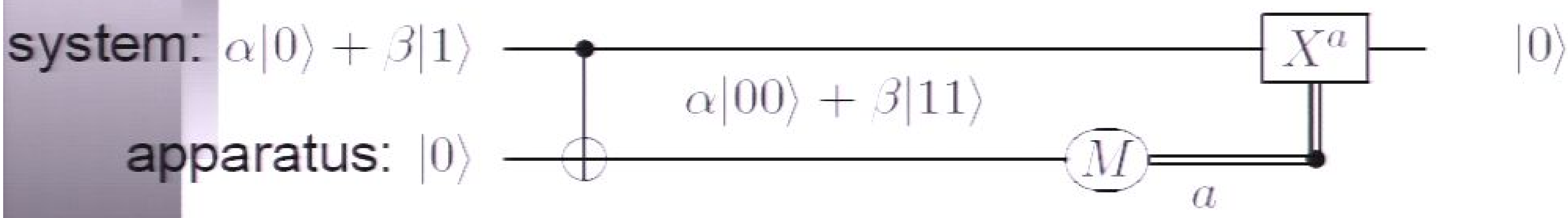

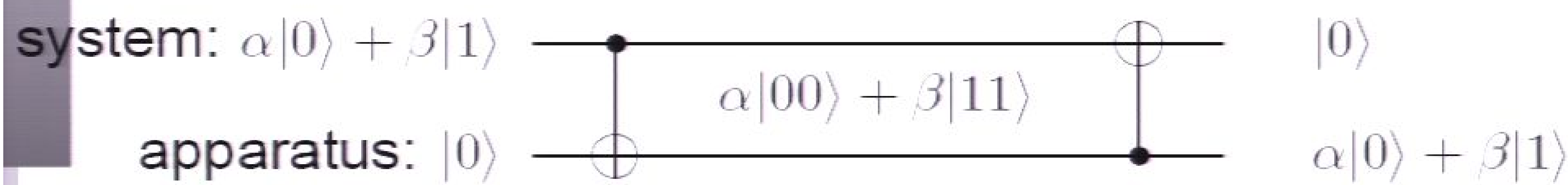

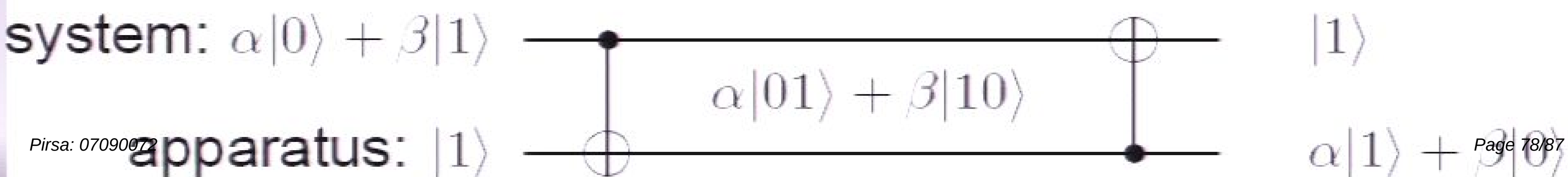

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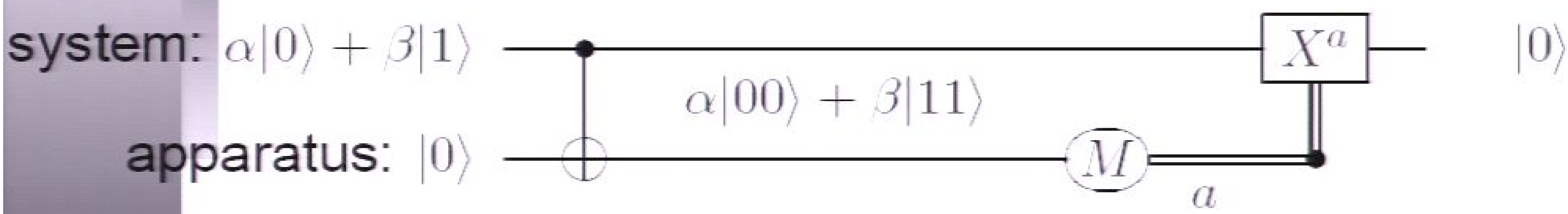
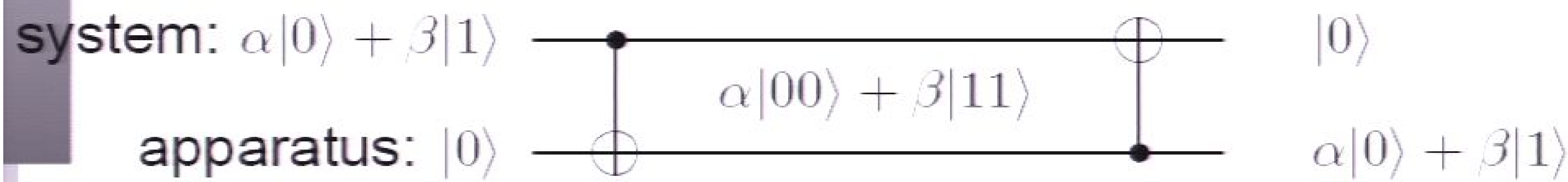
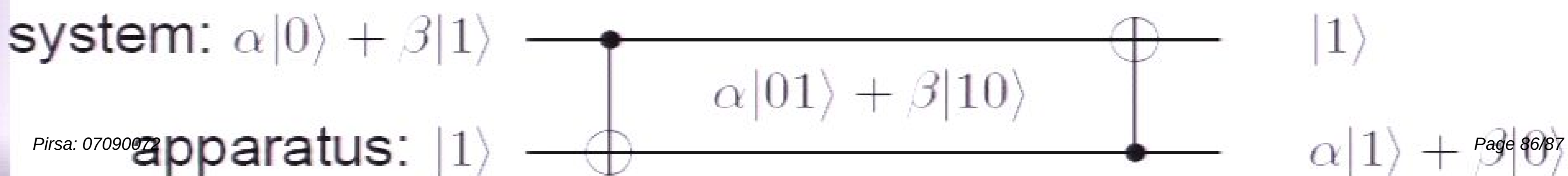
Any probabilistic argument starts from a judgment in the form of a prior probability assignment.

Quantum state preparation







Real state of exactly what?

In quantum theory, the situation giving rise to a putative real state $|q\rangle$ cannot be unambiguously defined.

(Case 1: there are no quantum microstates)

Case 2: the agent's initial "belief state" is given: $|q\rangle$ is subjective

Case 3: the initial "real state" is given: infinite regress