Title: Relative States and the Environment

Date: Sep 21, 2007 11:40 AM

URL: http://pirsa.org/07090070

Abstract: Everett explained Â“collapse of the wavepacketÂ” by noting that observer will perceive the state of the measured quantum system relative to the state of his own records. Two elements (missing in this simple and compelling explanation of effective collapse) are required to complete relative state interpretation: (i) A preferred basis for states of at least some systems in the wholly quantum Universe must be identified, so that apparatus pointers and other recording devices can persist over time. This implies breaking of the unitary symmetry in the original (more egalitarian) relative state interpretation, so that it can successfully account for classicality of macroscopic objects in accord with BohrÂ’s view of the role of measuring apparatus, and with our everyday experience. It is now widely accepted that decoherence (caused by the monitoring of systems by their environments) leads to einselection of pointer states, accounting for the emergence of preferred states. However, tools used by decoherence rely on the second missing link between quantum substrate and reality; (ii) A prescription that connects probabilities of outcomes with amplitudes of quantum states Â– such as BornÂ’s rule is still needed. BornÂ’s rule could be in principle postulated, but as Everett noted fifty years ago this should not be necessary. I show that both (i) einselection and (ii) BornÂ’s rule follow from symmetries of entangled quantum states. Entanglement represents information transfer between the to-be-classical quantum systems and their environments. Information transfer in course of decoherence produces multiple copies of the state of the system: its redundant imprints in the environment. This multiplicity of records can account for the objective existence of preferred pointer states: (iii) Quantum Darwinism singles out the Â“fittest observableÂ” of the system (the observable that produces the most information-theoretic Â“offspringÂ” of its state, i.e. the most copies in the environment). These fittest observables exist objectively: information about them can be found out indirectly, from the environment, without perturbing the underlying state of the system. The objective existence of pointer states is the foundation of the existential interpretation. The existential interpretation recognizes with Everett the relative nature of quantum states, but accounts for the effectively classical states (which unlike quantum states of isolated systems Â– can be found out without getting disrupted in the process) through quantum Darwinism

Relative States and the Environment

Wojciech Hubert Zurek
Theory Division, Los Alamos

Relative States and the Environment

Wojciech Hubert Zurek
Theory Division, Los Alamos

Relative States and the Environment

EVERETT '57

Wojciech Hubert Zurek
Theory Division, Los Alamos

Relative States and the Environment

EVERETT '57

Wojciech Hubert Zurek
Theory Division, Los Alamos

"BEYOND DECOHERENCE"

Relative States and the Environment

EVERETT '57

Wojciech Hubert Zurek
Theory Division, Los Alamos

"BEYOND DECOHERENCE"

WHZ, quant-ph arXiv:0707.2832 **"Relative states & the environment:…"**
Phys. Today **44**, 36-44 (1991) (quant-ph/0306072)
Rev. Mod. Phys. **75**, 715 (2003) (quant-ph/0105127)
M. Schlosshauer, "**Decoherence**" (Springer, 2007)

Textbook Quantum Theory

Textbook Quantum Theory

I. Quantum states of a system are represented by vectors in its Hilbert space. (“**Quantum Superposition Principle**”)

Textbook Quantum Theory

1. Quantum states of a system are represented by vectors in its Hilbert space. **("Quantum Superposition Principle")**
2. Evolutions are unitary (e.g. generated by Schroedinger equation). **("Unitarity")**

Textbook Quantum Theory

1. Quantum states of a system are represented by vectors in its Hilbert space. **("Quantum Superposition Principle")**
2. Evolutions are unitary (e.g. generated by Schroedinger equation). **("Unitarity")**
3. Immediate repetition of a measurement yields the same outcome. **("Predictability")**

Textbook Quantum Theory

1. Quantum states of a system are represented by vectors in its Hilbert space. (**"Quantum Superposition Principle"**)
2. Evolutions are unitary (e.g. generated by Schroedinger equation). **("Unitarity")**
3. Immediate repetition of a measurement yields the same outcome. **("Predictability")**
4. Outcomes restricted to orthonormal states $\{|s_k\rangle\}$ (eigenstates of the measured observable). Just one outcome is seen each time. **("Collapse Postulate")**

Textbook Quantum Theory

1. Quantum states of a system are represented by vectors in its Hilbert space. (**"Quantum Superposition Principle"**)
2. Evolutions are unitary (e.g. generated by Schroedinger equation). **("Unitarity")**
3. Immediate repetition of a measurement yields the same outcome. **("Predictability")**
4. Outcomes restricted to orthonormal states $\{|s_k>\}$ (eigenstates of the measured observable). Just one outcome is seen each time**. ("Collapse Postulate")**
5. Probability of finding an outcome $|s_k>$ given a state $|f>$ is $p_k=|<s_k|f>|^2$. **("Born's Rule")**

Textbook Quantum Theory

1. Quantum states of a system are represented by vectors in its Hilbert space. **("Quantum Superposition Principle")**
2. Evolutions are unitary (e.g. generated by Schroedinger equation). **("Unitarity")**
3. Immediate repetition of a measurement yields the same outcome. **("Predictability")**
4. Outcomes restricted to orthonormal states $\{|s_k>\}$ (eigenstates of the measured observable). Just one outcome is seen each time. **("Collapse Postulate")**
5. Probability of finding an outcome $|s_k>$ given a state $|f>$ is $p_k=|<s_k|f>|^2$. **("Born's Rule")**

Bohr, Dirac, "Copenhagen" -- 4&5 require "classical apparatus", etc.

Textbook Quantum Theory

1. Quantum states of a system are represented by vectors in its Hilbert space. **("Quantum Superposition Principle")**
2. Evolutions are unitary (e.g. generated by Schroedinger equation). **("Unitarity")**
3. Immediate repetition of a measurement yields the same outcome. **("Predictability")**
4. Outcomes restricted to orthonormal states $\{|s_k\rangle\}$ (eigenstates of the measured observable). Just one outcome is seen each time. **("Collapse Postulate")**
5. Probability of finding an outcome $|s_k\rangle$ given a state $|f\rangle$ is $p_k=|\langle s_k|f\rangle|^2$. **("Born's Rule")**

Bohr, Dirac, "Copenhagen" -- 4&5 require "classical apparatus", etc.

Everett: Relative states -- no need for explicit collapse!

Textbook Quantum Theory

1. Quantum states of a system are represented by vectors in its Hilbert space. **("Quantum Superposition Principle")**
2. Evolutions are unitary (e.g. generated by Schroedinger equation). **("Unitarity")**
3. Immediate repetition of a measurement yields the same outcome. **("Predictability")**
4. Outcomes restricted to orthonormal states $\{|s_k\rangle\}$ (eigenstates of the measured observable). Just one outcome is seen each time. **("Collapse Postulate")**
5. Probability of finding an outcome $|s_k\rangle$ given a state $|f\rangle$ is $p_k = |\langle s_k|f\rangle|^2$. **("Born's Rule")**

Bohr, Dirac, "Copenhagen" -- 4&5 require "classical apparatus", etc.

Everett: Relative states -- no need for explicit collapse!

$$|\Psi_{17}\rangle \cong |I_{17}\rangle\langle I_{17}|\Psi_{Univ...}$$

Textbook Quantum Theory

0. State of a composite system is a vector in the tensor product of the constituent Hilbert spaces. ("**Complexity**")
1. Quantum states of a system are represented by vectors in its Hilbert space. **("Quantum Superposition Principle")**
2. Evolutions are unitary (e.g. generated by Schroedinger equation). **("Unitarity")**
3. Immediate repetition of a measurement yields the same outcome. **("Predictability")**
4. Outcomes restricted to orthonormal states $\{|s_k\rangle\}$ (eigenstates of the measured observable). Just one outcome is seen each time. **("Collapse Postulate")**
5. Probability of finding an outcome $|s_k\rangle$ given a state $|f\rangle$ is $p_k=|\langle s_k|f\rangle|^2$. **("Born's Rule")**

Bohr, Dirac, "Copenhagen" -- 4&5 require "classical apparatus", etc.

Everett: Relative states -- no need for explicit collapse!

$|\Psi_{17}\rangle \cong |I_{17}\rangle\langle I_{17}|\Psi$

Textbook Quantum Theory

0. State of a composite system is a vector in the tensor product of the constituent Hilbert spaces. ("**Complexity**")
1. Quantum states of a system are represented by vectors in its Hilbert space. **("Quantum Superposition Principle")**
2. Evolutions are unitary (e.g. generated by Schroedinger equation). **("Unitarity")**
3. Immediate repetition of a measurement yields the same outcome. **("Predictability")**
4. Outcomes restricted to orthonormal states $\{|s_k>\}$ (eigenstates of the measured observable). Just one outcome is seen each time. **("Collapse Postulate")**
5. Probability of finding an outcome $|s_k>$ given a state $|f>$ is $p_k=|<s_k|f>|^2$. **("Born's Rule")**

Bohr, Dirac, "Copenhagen" -- 4&5 require "classical apparatus", etc.

Everett: Relative states -- no need for explicit collapse!

But need a preferred basis

$|\Psi_{17}\rangle \cong |I_{17}\rangle\langle I_{17}|\Psi_{Universe}\rangle$

Textbook Quantum Theory

0. State of a composite system is a vector in the tensor product of the constituent Hilbert spaces. ("**Complexity**")
1. Quantum states of a system are represented by vectors in its Hilbert space. **("Quantum Superposition Principle")**
2. Evolutions are unitary (e.g. generated by Schroedinger equation). **("Unitarity")**
3. Immediate repetition of a measurement yields the same outcome. **("Predictability")**
4. Outcomes restricted to orthonormal states $\{|s_k\rangle\}$ (eigenstates of the measured observable). Just one outcome is seen each time**. ("Collapse Postulate")**
5. Probability of finding an outcome $|s_k\rangle$ given a state $|f\rangle$ is $p_k=|\langle s_k|f\rangle|^2$. **("Born's Rule")**

Bohr, Dirac, "Copenhagen" -- 4&5 require "classical apparatus", etc.

Everett: Relative states -- no need for explicit collapse!

But need a preferred basis!!! $|\Psi_{17}\rangle \cong |I_{17}\rangle\langle I_{17}|\Psi_{Univ...}$

0. State of a composite system is a vector in the tensor product of the constituent Hilbert spaces. ("**Complexity**")

1. Quantum states of a system are represented by vectors in its Hilbert space. **("Quantum Superposition Principle")**
2. Evolutions are unitary (e.g. generated by Schroedinger equation). **("Unitarity")**
3. Immediate repetition of a measurement yields the same outcome. **("Predictability")**
4. Outcomes restricted to orthonormal states $\{|s_k\rangle\}$ (eigenstates of the measured observable). Just one outcome is seen each time. **("Collapse Postulate")**
5. Probability of finding an outcome $|s_k\rangle$ given a state $|f\rangle$ is $p_k=|\langle s_k|f\rangle|^2$. **("Born's Rule")**

Bohr, Dirac, "Copenhagen" -- 4&5 require "classical apparatus", etc.

Everett: Relative states -- no need for explicit collapse!

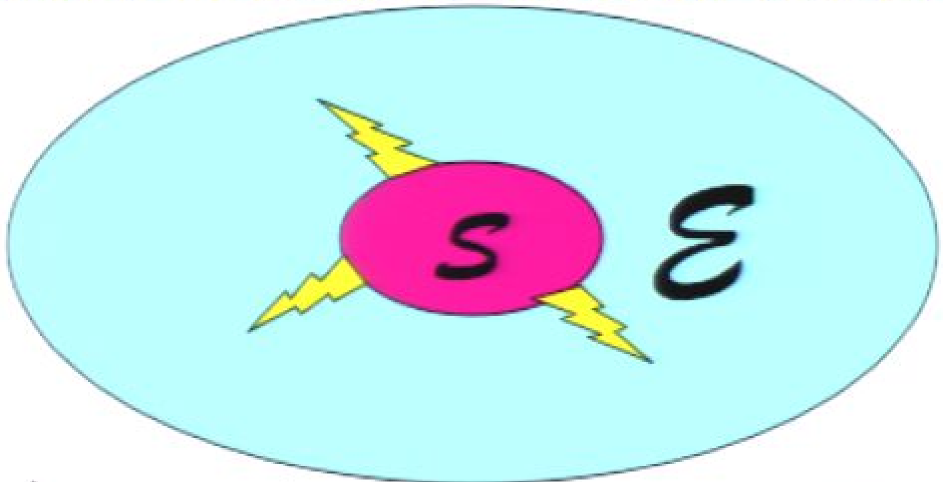
But need a preferred basis!!!

$|\Psi_{17}\rangle \cong |I_{17}\rangle\langle I_{17}|\Psi$

Plan of the Lecture

- Decoherence 101; the basic idea, and why it is not basic enough
- Preferred basis without decoherence and the origin of quantum jumps (orthogonality & collapse)
- Probability is objective in quantum theory -- "envariant" (entanglement - based) derivation of Born's rule $\left(p_k = |\psi_k|^2\right.$
- Role of decoherence & einselection: when pointer states define "events"
- Redundancy and quantum Darwinism ("environment as a witness" & origins of objective existence in a quantum world)
- Decoherence, environment - induced superselection, and predictability
- Existential Interpretation (Relative states + operational definition of "existence")

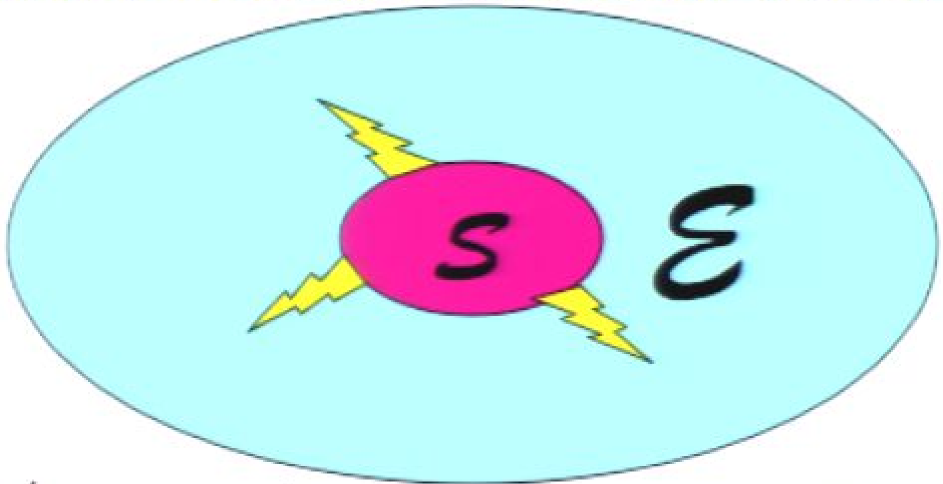
EINSELECTION*, POINTER BASIS, AND DECOHERENCE



$$|\Phi_{S\mathcal{E}}(0)\rangle = |\psi_S\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle \xrightarrow[\text{Entanglement}]{\text{Interaction}} \sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{S\mathcal{E}}(t$$

REDUCED DENSITY MATRIX $\rho_S(t) = Tr_{\mathcal{E}} |\Phi_{S\mathcal{E}}(t)\rangle\langle\Phi_{S\mathcal{E}}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle\sigma$

EINSELECTION*, POINTER BASIS, AND DECOHERENCE



$$\Phi_{SE}(0)\rangle = |\psi_S\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle \xrightarrow[\text{Entanglement}]{\text{Interaction}} \sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{SE}($$

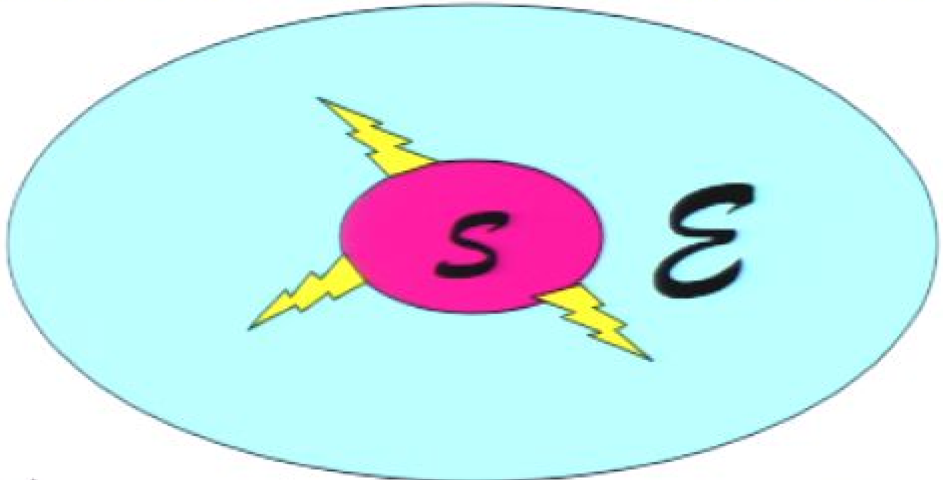
REDUCED DENSITY MATRIX $\rho_S(t) = Tr_{\mathcal{E}} |\Phi_{SE}(t)\rangle\langle\Phi_{SE}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle\sigma$

EINSELECTION* leads to POINTER STATES

(same states appear on the diagonal of $\rho_S(t)$ for times long compared to the decoherence time; **pointer states** are effectively classical!) Pointer states **left unperturbed** by the "environmental monitoring".

*Environment INduced superSELECTION

EINSELECTION*, POINTER BASIS, AND DECOHERENCE



$$\Phi_{\mathcal{SE}}(0)\rangle = |\psi_{\mathcal{S}}\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle \xrightarrow[\text{Entanglement}]{\text{Interaction}} \sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{\mathcal{SE}}(t$$

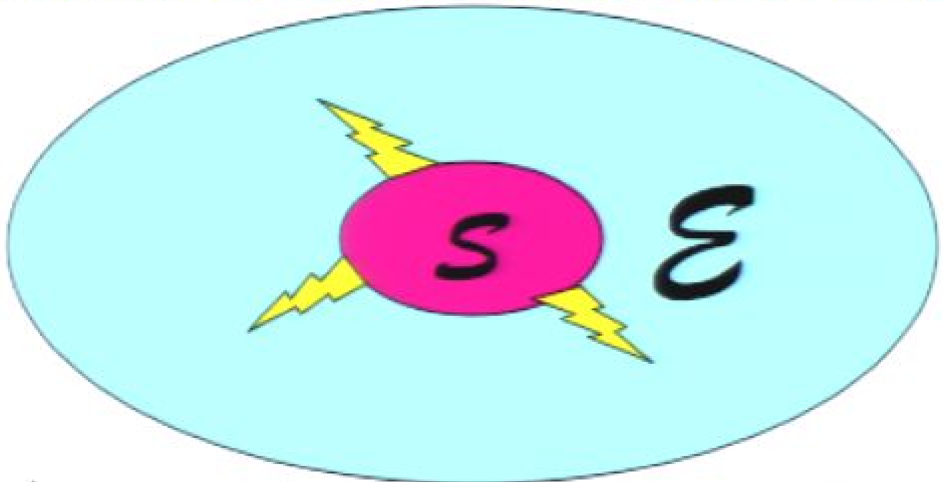
REDUCED DENSITY MATRIX $\rho_{\mathcal{S}}(t) = Tr_{\mathcal{E}} |\Phi_{\mathcal{SE}}(t)\rangle\langle\Phi_{\mathcal{SE}}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle\sigma$

EINSELECTION* leads to POINTER STATES

(same states appear on the diagonal of $\rho_{\mathcal{S}}(t)$ for times long compared to the decoherence time; **pointer states** are effectively classical!) Pointer states **left unperturbed** by the "environmental monitoring".

*Environment INduced superSELECTION

EINSELECTION*, POINTER BASIS, AND DECOHERENCE



$$|\Phi_{S\mathcal{E}}(0)\rangle = |\psi_S\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle \xRightarrow[\text{Entanglement}]{\text{Interaction}} \sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{S\mathcal{E}}(t$$

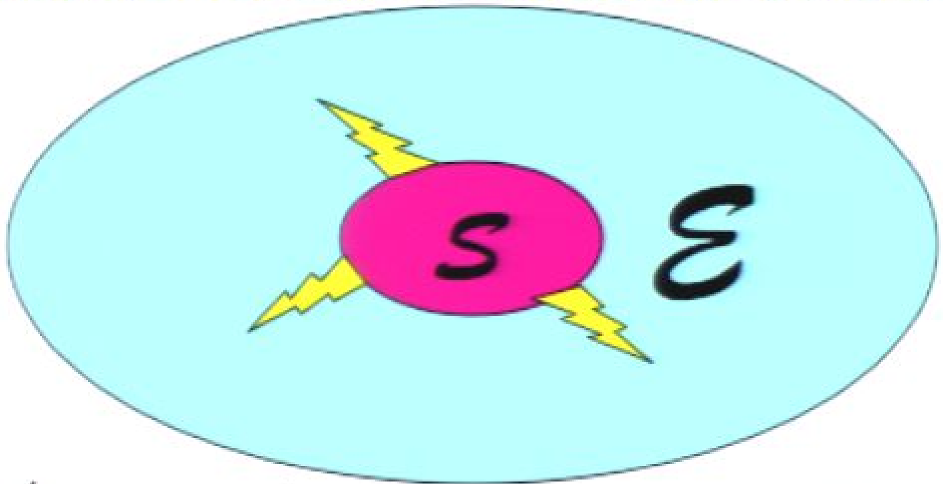
REDUCED DENSITY MATRIX $\rho_S(t) = Tr_{\mathcal{E}} |\Phi_{S\mathcal{E}}(t)\rangle\langle\Phi_{S\mathcal{E}}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle\sigma$

EINSELECTION* leads to POINTER STATES

(same states appear on the diagonal of $\rho_S(t)$ for times long compared to the decoherence time; **pointer states** are effectively classical!) Pointer states **left unperturbed** by the "environmental monitoring".

*Environment INduced superSELECTION

EINSELECTION*, POINTER BASIS, AND DECOHERENCE



$$|\Phi_{\mathcal{SE}}(0)\rangle = |\psi_{\mathcal{S}}\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle \xrightarrow[\text{Entanglement}]{\text{Interaction}} H_{SE} \; \sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{\mathcal{SE}}(t$$

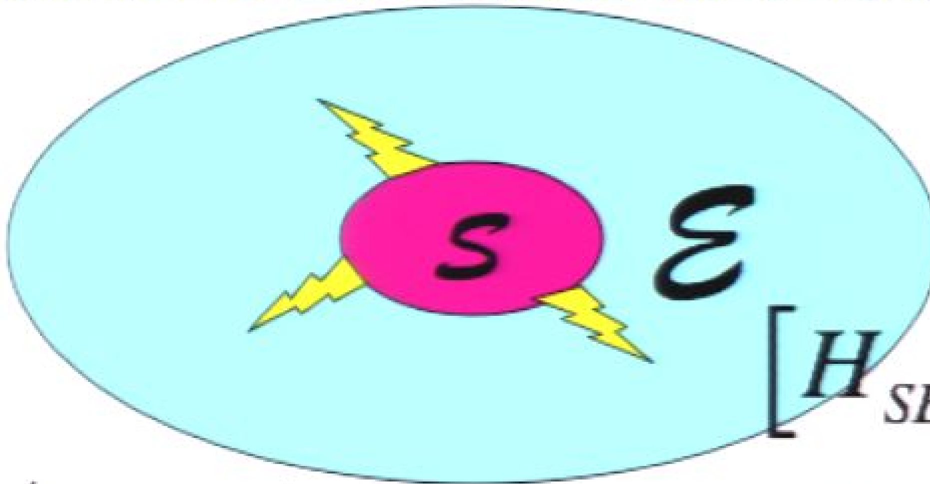
REDUCED DENSITY MATRIX $\rho_{\mathcal{S}}(t) = Tr_{\mathcal{E}} |\Phi_{\mathcal{SE}}(t)\rangle\langle\Phi_{\mathcal{SE}}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle \sigma$

EINSELECTION* leads to POINTER STATES

(same states appear on the diagonal of $\rho_{\mathcal{S}}(t)$ for times long compared to the decoherence time; **pointer states** are effectively classical!)
Pointer states **left unperturbed** by the "environmental monitoring".

*Environment INduced superSELECTION

EINSELECTION*, POINTER BASIS, AND DECOHERENCE



$$\left[H_{SE}, |\sigma_i\rangle\langle\sigma_i|\right] = $$

$$|\Phi_{S\mathcal{E}}(0)\rangle = |\psi_S\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle \xrightarrow[\text{Entanglement}]{\text{Interaction}} H_{SE} \quad \sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{S\mathcal{E}}(t$$

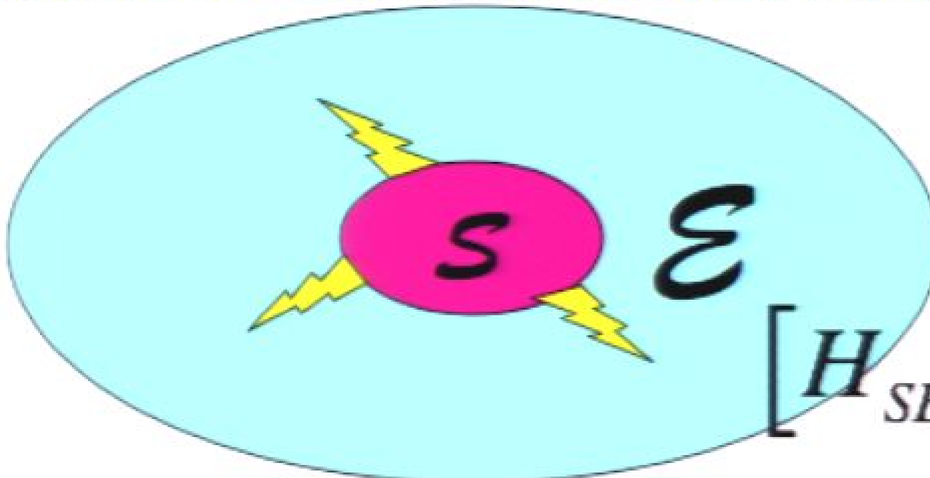
REDUCED DENSITY MATRIX $\rho_S(t) = Tr_{\mathcal{E}} |\Phi_{S\mathcal{E}}(t)\rangle\langle\Phi_{S\mathcal{E}}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle\sigma$

EINSELECTION* leads to POINTER STATES

(same states appear on the diagonal of $\rho_S(t)$ for times long compared to the decoherence time; **pointer states** are effectively classical!) Pointer states **left unperturbed** by the "environmental monitoring".

*Environment INduced superSELECTION

EINSELECTION*, POINTER BASIS, AND DECOHERENCE



$$\left[H_{SE}, |\sigma_i\rangle\langle\sigma_i|\right] = $$

$$\Phi_{S\mathcal{E}}(0)\rangle = |\psi_S\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle \xrightarrow[\text{Entanglement}]{\text{Interaction}\ H_{SE}} \sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{S\mathcal{E}}($$

EDUCED DENSITY MATRIX $\rho_S(t) = Tr_{\mathcal{E}} |\Phi_{S\mathcal{E}}(t)\rangle\langle\Phi_{S\mathcal{E}}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle$

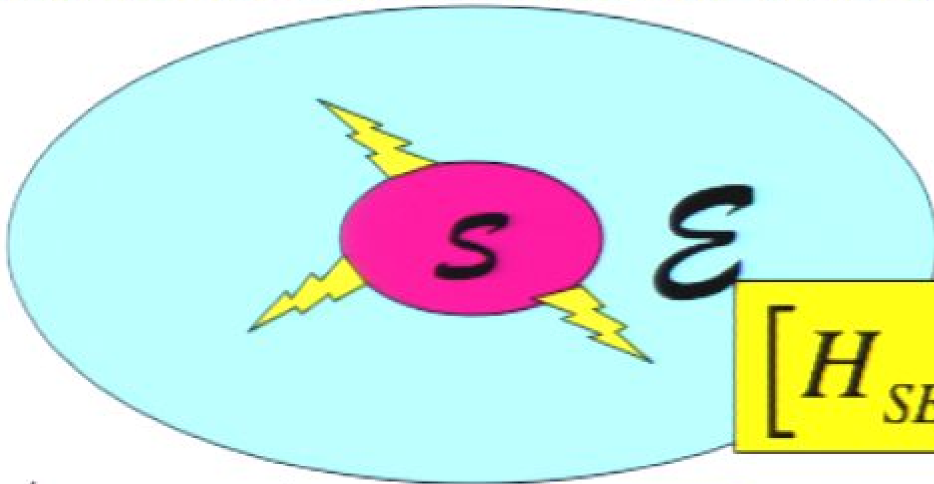
EINSELECTION* leads to POINTER STATES

(same states appear on the diagonal of $\rho_S(t)$ for times long compared to the decoherence time; **pointer states** are effectively classical!)

Pointer states **left unperturbed** by the "environmental monitoring".

*Environment INduced superSELECTION

EINSELECTION*, POINTER BASIS, AND DECOHERENCE



$$\left[H_{SE}, |\sigma_i\rangle\langle\sigma_i|\right] = ($$

$$|\Phi_{S\mathcal{E}}(0)\rangle = |\psi_S\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle \xrightarrow[\text{Entanglement}]{\text{Interaction}\ H_{SE}} \sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{S\mathcal{E}}(t$$

REDUCED DENSITY MATRIX $\rho_S(t) = Tr_{\mathcal{E}} |\Phi_{S\mathcal{E}}(t)\rangle\langle\Phi_{S\mathcal{E}}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle\sigma$

EINSELECTION* leads to POINTER STATES

(same states appear on the diagonal of $\rho_S(t)$ for times long compared to the decoherence time; **pointer states** are effectively classical!) Pointer states **left unperturbed** by the "environmental monitoring".

*Environment INduced superSELECTION

DECOHERENCE AND EINSELECTION

Thesis: Quantum theory can explain emergence of the classica

Principle of superposition loses its validity in "open" systems, that is, systems interacting with their environments.

Decoherence restricts stable states (states that can persist, and, therefore "exist") to the exceptional…

Pointer states that exist or evolve predictably in spite of the immersion of the system in the environment.

Predictability sieve can be used to 'sift' through the Hilbert space of any open quantum system in search of these pointer states.

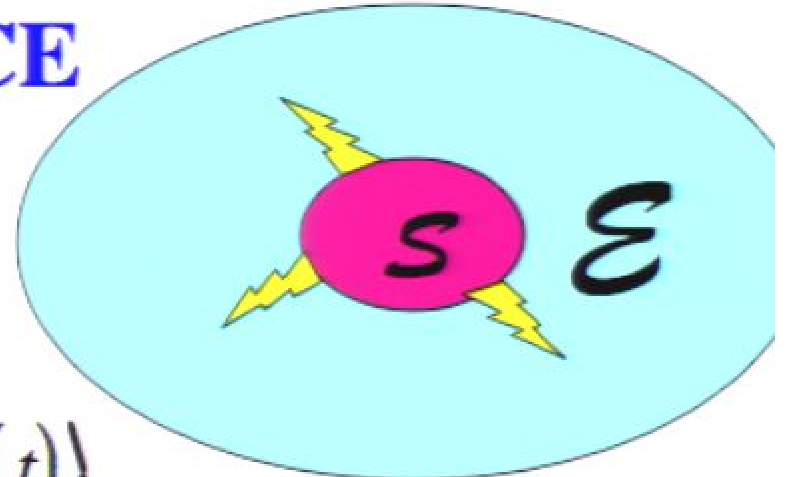
EINSELECTION (or Environment INduced superSELECTION) is the process of selection of these preferred pointer states.

For macroscopic systems, decoherence and einselection can be very effective, enforcing ban on Schroedinger cats.

Einselection enforces an effective border that divides quantum from classical, making a point of view similar to Bohr's Copenhagen Interpretation possible, although starti from a rather different standpoint (i. e., no *ab initio* classical domain of the universe

Zeh, Joos, Paz, Caldeira, Leggett, Kiefer, Gell-Mann, Hartle, Omnes, Dalvit, Dziarmaga, Cucchietti …

EINSELECTION, POINTER BASIS, AND DECOHERENCE



$$\Phi_{S\mathcal{E}}(0)\rangle = |\psi_S\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i|\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle$$

$$\sum_i \alpha_i|\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{S\mathcal{E}}(t)\rangle$$

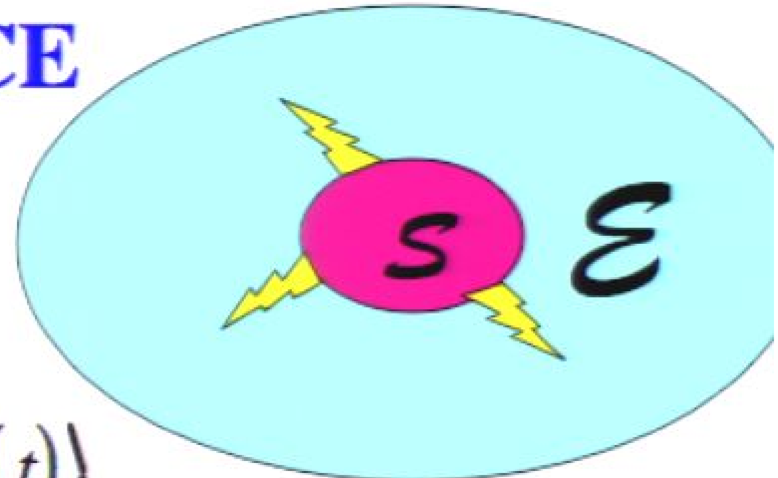
REDUCED DENSITY MATRIX

$$\rho_S(t) = Tr_{\mathcal{E}}|\Phi_{S\mathcal{E}}(t)\rangle\langle\Phi_{S\mathcal{E}}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle\sigma_i|$$

INSELECTION leads to POINTER STATES

same states appear on the diagonal of $\rho_S(t)$ for times long compared o the decoherence time)

EINSELECTION, POINTER BASIS, AND DECOHERENCE



$$\Phi_{S\mathcal{E}}(0)\rangle = |\psi_S\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle$$

Interaction / Entanglement $\Longrightarrow$ $$\sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{S\mathcal{E}}(t)\rangle$$

REDUCED DENSITY MATRIX

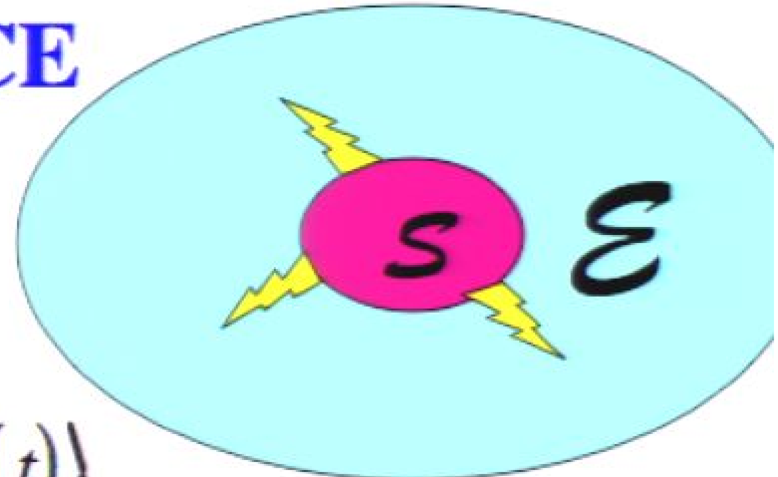
........Depends on Born's Rule!!!

$$\rho_S(t) = Tr_{\mathcal{E}} |\Phi_{S\mathcal{E}}(t)\rangle\langle\Phi_{S\mathcal{E}}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle\sigma_i|$$

INSELECTION leads to POINTER STATES

same states appear on the diagonal of $\rho_S(t)$ for times long compared o the decoherence time)

EINSELECTION, POINTER BASIS, AND DECOHERENCE



$$|\Phi_{S\mathcal{E}}(0)\rangle = |\psi_S\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle$$

$$\xrightarrow[\text{Entanglement}]{\text{Interaction}} \quad \sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{S\mathcal{E}}(t)\rangle$$

REDUCED DENSITY MATRIX

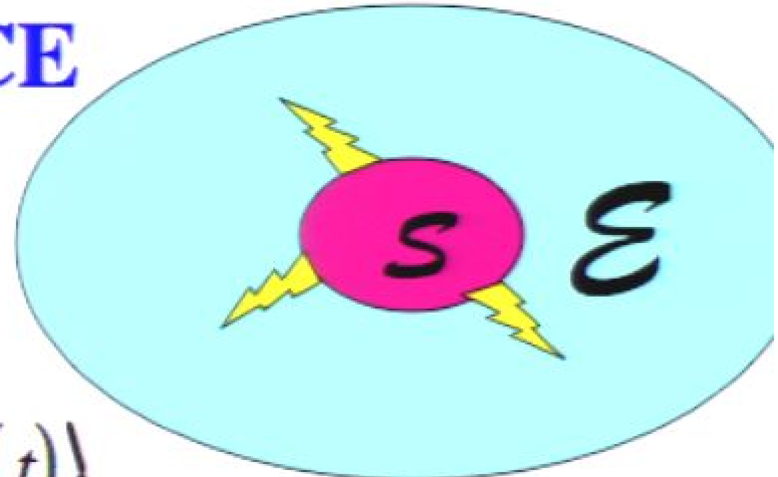
........Depends on Born's Rule!!!

$$\rho_S(t) = Tr_{\mathcal{E}} |\Phi_{S\mathcal{E}}(t)\rangle\langle\Phi_{S\mathcal{E}}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle\sigma_i|$$

EINSELECTION leads to POINTER STATES

same states appear on the diagonal of $\rho_S(t)$ for times long compared o the decoherence time)

EINSELECTION, POINTER BASIS, AND DECOHERENCE



$$|\Phi_{S\mathcal{E}}(0)\rangle = |\psi_S\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle$$

Interaction / Entanglement $\Rightarrow$ $\sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{S\mathcal{E}}(t)\rangle$

REDUCED DENSITY MATRIX

........Depends on Born's Rule!!!

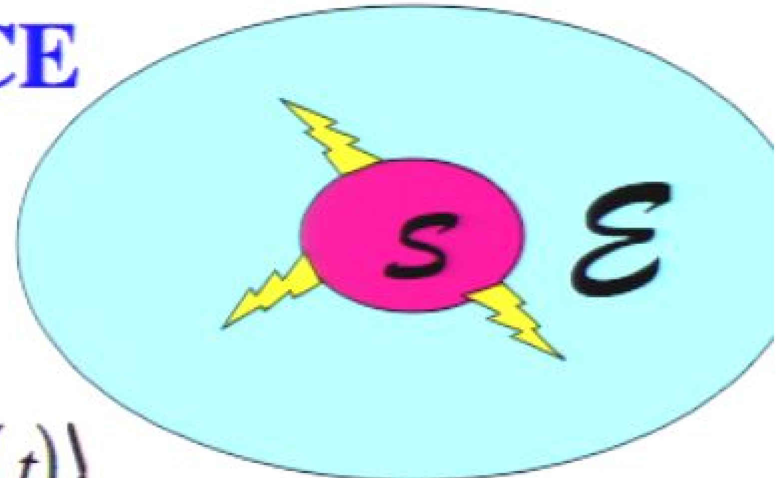
$$\rho_S(t) = Tr_{\mathcal{E}} |\Phi_{S\mathcal{E}}(t)\rangle\langle\Phi_{S\mathcal{E}}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle\sigma_i|$$

EINSELECTION leads to POINTER STATES

(same states appear on the diagonal of $\rho_S(t)$ for times long compared to the decoherence time)

$$[H_{SE}, |\sigma_i\rangle\langle\sigma_i|] = 0$$

EINSELECTION, POINTER BASIS, AND DECOHERENCE



$$|\Phi_{S\mathcal{E}}(0)\rangle = |\psi_S\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle$$

$\xrightarrow[\text{Entanglement}]{\text{Interaction}}$ $$\sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{S\mathcal{E}}(t)\rangle$$

REDUCED DENSITY MATRIX

........Depends on Born's Rule!!!

$$\rho_S(t) = Tr_{\mathcal{E}} |\Phi_{S\mathcal{E}}(t)\rangle\langle\Phi_{S\mathcal{E}}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle\sigma_i|$$

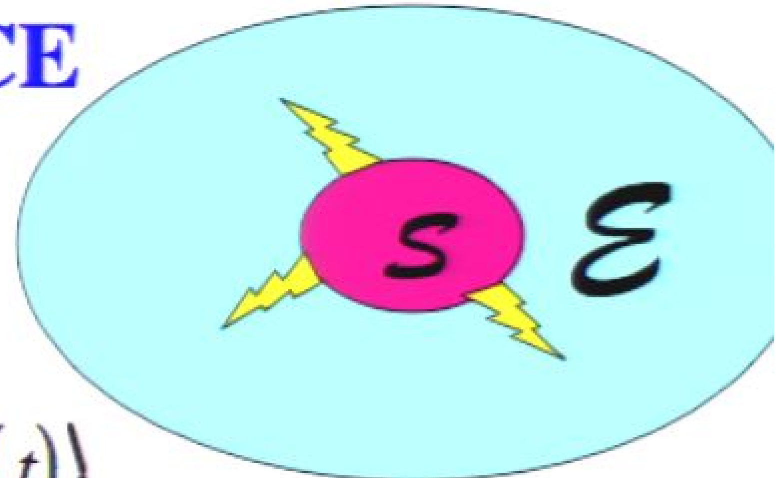
EINSELECTION leads to POINTER STATES

same states appear on the diagonal of $\rho_S(t)$ for times long compared to the decoherence time

????????

$$[H_{SE}, |\sigma_i\rangle\langle\sigma_i|] = 0$$

EINSELECTION, POINTER BASIS, AND DECOHERENCE



$$|\Phi_{SE}(0)\rangle = |\psi_S\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle$$

Interaction / Entanglement $\Longrightarrow$ $$\sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{SE}(t)\rangle$$

REDUCED DENSITY MATRIX

........Depends on Born's Rule!!!

$$\rho_S(t) = Tr_E |\Phi_{SE}(t)\rangle\langle\Phi_{SE}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle\sigma_i|$$

EINSELECTION leads to POINTER STATES

(same states appear on the diagonal of $\rho_S(t)$ for times long compared to the decoherence time)

$$[H_{SE}, |\sigma_i\rangle\langle\sigma_i|] = 0$$

??????????

Goal

Justify axioms 4&5 using the noncontroversial 0-3

PLAN:

Goal

Justify axioms 4&5 using the noncontroversial 0-3

PLAN:

- **Why are the measurement outcomes limited to an orthogonal subset of all the possible states in the Hilbert states? (as in "Collapse")**

Goal

Justify axioms 4&5 using the noncontroversial 0-3

PLAN:

- Why are the measurement outcomes limited to an orthogonal subset of all the possible states in the Hilbert states? (as in "Collapse")
- Why does "Born's rule" yield probabilities?

Goal

Justify axioms 4&5 using the noncontroversial 0-3

Understand emergence of "objective classical reality" -- how real states that can be found out by us arise from quantum substrate.

PLAN:

- Why are the measurement outcomes limited to an orthogonal subset of all the possible states in the Hilbert states? (as in "Collapse")
- Why does "Born's rule" yield probabilities?

Goal

Justify axioms 4&5 using the noncontroversial 0-3

Understand emergence of "objective classical reality" -- how real states that can be found out by us arise from quantum substrate.

PLAN:

- Why are the measurement outcomes limited to an orthogonal subset of all the possible states in the Hilbert states? (as in "Collapse")
- Why does "Born's rule" yield probabilities?
- How can "objective classical reality" -- states we can find out -- arise from the fragile quantum states that are perturbed by measurements? ("Quantum Darwinism")

Goal

Justify axioms 4&5 using the noncontroversial 0-3

Understand emergence of "objective classical reality" -- how real states that can be found out by us arise from quantum substrate.

PLAN:

- Why are the measurement outcomes limited to an orthogonal subset of all the possible states in the Hilbert states? (as in "Collapse")
- Why does "Born's rule" yield probabilities?
- How can "objective classical reality" -- states we can find out -- arise from the fragile quantum states that are perturbed by measurements? ("Quantum Darwinism")

States that can survive "being found out" intact must be orthogonal.

Consider two states that can be "found out":

$$|u\rangle|A_0\rangle \Rightarrow |u\rangle|A_u\rangle$$

$$|v\rangle|A_0\rangle \Rightarrow |v\rangle|A_v\rangle$$

Consider an initial **superposition** of these two states:

$$\left(\alpha|u\rangle + \beta|v\rangle\right)|A_0\rangle \Rightarrow \alpha|u\rangle|A_u\rangle + \beta|v\rangle|A_v\rangle$$

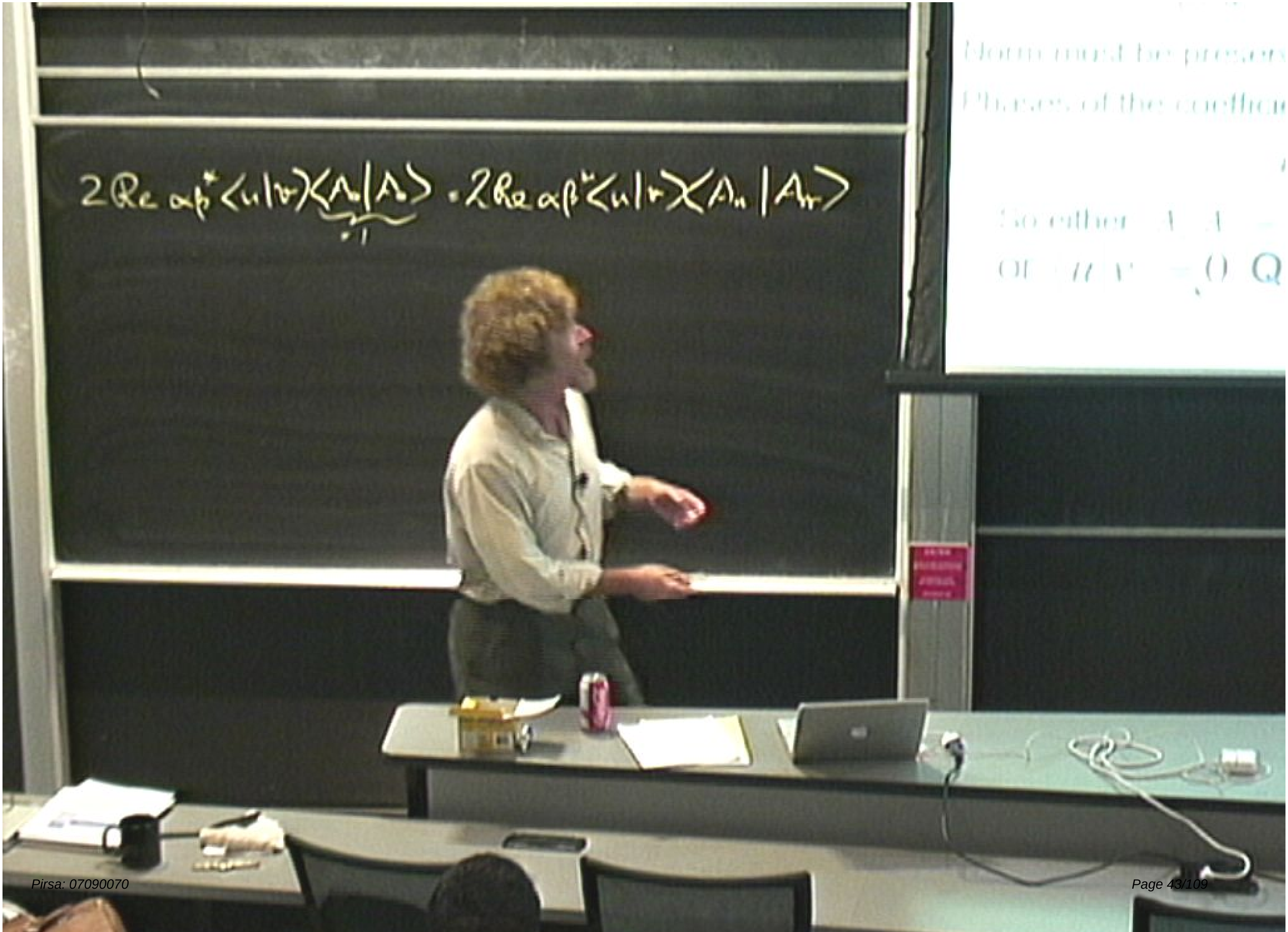
Norm must be preserved. Hence: $\mathrm{Re}(\alpha^*\beta\langle u|v\rangle) = \mathrm{Re}(\alpha^*\beta\langle u|v\rangle\langle A_u|A_v$

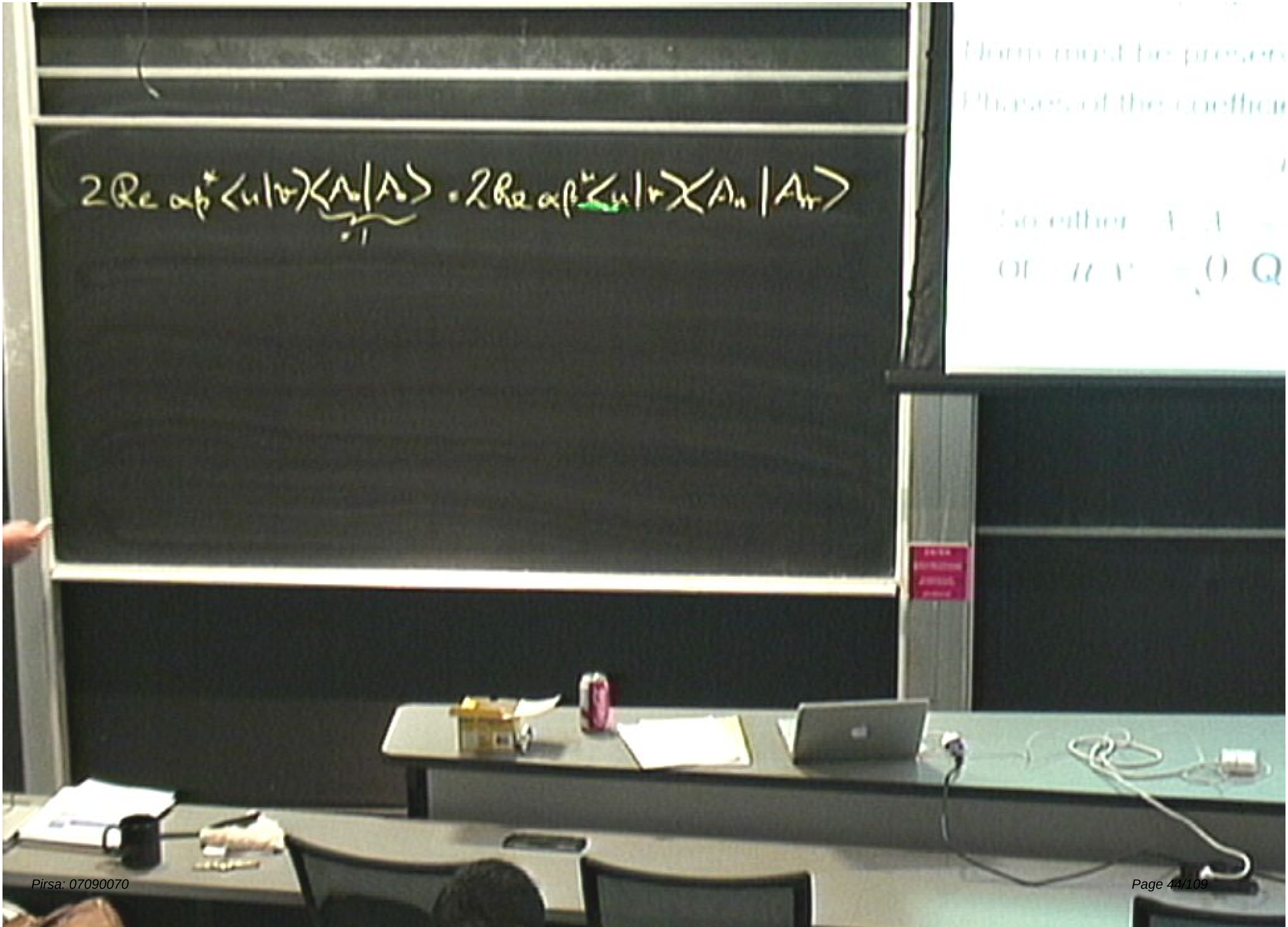
Phases of the coefficients can be adjusted at will. So:

$$\langle u|v\rangle = \langle u|v\rangle\langle A_u|A_v\rangle$$

So either $\langle A_u|A_v\rangle = 1$ (measurement was not successful)

or $\langle u|v\rangle = 0$ **QED!!!!**


2Re αβ* ⟨u|v⟩⟨A_u|A_v⟩ = 2Re αβ* ⟨u|v⟩⟨A_u|A_v⟩
=1


2Re αβ* ⟨u|v⟩⟨A_u|A_v⟩ = 2Re αβ* ⟨u|v⟩⟨A_u|A_v⟩

States that can survive "being found out" intact must be orthogonal.

Consider two states that can be "found out":

$$|u\rangle|A_0\rangle \Rightarrow |u\rangle|A_u\rangle$$

$$|v\rangle|A_0\rangle \Rightarrow |v\rangle|A_v\rangle$$

Consider an initial **superposition** of these two states:

$$\left(\alpha|u\rangle + \beta|v\rangle\right)|A_0\rangle \Rightarrow \alpha|u\rangle|A_u\rangle + \beta|v\rangle|A_v\rangle$$

Norm must be preserved. Hence: $\mathrm{Re}(\alpha^*\beta\langle u|v\rangle) = \mathrm{Re}(\alpha^*\beta\langle u|v\rangle\langle A_u|A_v$

Phases of the coefficients can be adjusted at will. So:

$$\langle u|v\rangle = \langle u|v\rangle\langle A_u|A_v\rangle$$

So either $\langle A_u|A_v\rangle = 1$ (measurement was not successful)

or $\langle u|v\rangle = 0$ **QED!!!!** "collapse"

PREDICTABILITY IMPLIES RESTRICTION TO ORTHOGONAL STATES!!!

Consequences and extensions

WHZ, arXiv:quant-ph/070316

Consequences and extensions

WHZ, arXiv:quant-ph/070316

Consequences and extensions

- Derivation of the key to **Collapse Postulate** from Axioms 1-3: explains why in general one cannot "find out" preexisting states

WHZ, arXiv:quant-ph/070316

Consequences and extensions

- Derivation of the key to **Collapse Postulate** from Axioms 1-3: explains why in general one cannot "find out" preexisting states
- Implies that **observables are Hermitean** (given an extra assumption that eigenvalues are real).

WHZ, arXiv:quant-ph/070316

Consequences and extensions

- Derivation of the key to **Collapse Postulate** from Axioms 1-3: explains why in general one cannot “find out” preexisting states
- Implies that **observables are Hermitean** (given an extra assumption that eigenvalues are real).
- Proof similar to “**no cloning theorem**” -- **information** about preexisting states cannot be found out -- passed on. (Cloning means making a “perfect copy”. Here the copy need not be perfect; “information - disturbance”)

WHZ, arXiv:quant-ph/070316

Consequences and extensions

- Derivation of the key to **Collapse Postulate** from Axioms 1-3: explains why in general one cannot "find out" preexisting states
- Implies that **observables are Hermitean** (given an extra assumption that eigenvalues are real).
- Proof similar to "**no cloning theorem**" -- **information** about preexisting states cannot be found out -- passed on. (Cloning means making a "perfect copy". Here the copy need not be perfect; "information - disturbance")
- Proof can be extended to the case when **apparatus (or environment) is initially in a mixed state**.

WHZ, arXiv:quant-ph/070316

Consequences and extensions

- Derivation of the key to **Collapse Postulate** from Axioms 1-3: explains why in general one cannot "find out" preexisting states
- Implies that **observables are Hermitean** (given an extra assumption that eigenvalues are real).
- Proof similar to "**no cloning theorem**" -- **information** about preexisting states cannot be found out -- passed on. (Cloning means making a "perfect copy". Here the copy need not be perfect; "information - disturbance")
- Proof can be extended to the case when **apparatus (or environment) is initially in a mixed state**.
- Axiom 3 -- **predictability** -- is the key to the proof!

WHZ, arXiv:quant-ph/070316

Consequences and extensions

- Derivation of the key to **Collapse Postulate** from Axioms 1-3: explains why in general one cannot "find out" preexisting states
- Implies that **observables are Hermitean** (given an extra assumption that eigenvalues are real).
- Proof similar to "**no cloning theorem**" -- **information** about preexisting states cannot be found out -- passed on. (Cloning means making a "perfect copy". Here the copy need not be perfect; "information - disturbance")
- Proof can be extended to the case when **apparatus (or environment) is initially in a mixed state**.
- Axiom 3 -- **predictability** -- is the key to the proof!
- **Information transfer need not be due to a deliberate measurement: any information transfer that does not perturb outcome states will have to abide by this rule: Pointer states, predictability sieve, and DECOHERENCE.**

WHZ arXiv:quant-ph/070316

Summary: Observables are Hermitean

Theorem: Outcomes of a measurement that satisfy postulates 1-3 must be orthogonal.

Proof (another version): measurement is an information transfer from a quantumsystem *S* to a **quantum** apparatus *A*. So, for any two possible **repeatable (predictable)** (Axiom 3) outcome states of the same measurement it must be true that:

$$|u\rangle|A_0\rangle \Rightarrow |u\rangle|A_u\rangle$$

$$|v\rangle|A_0\rangle \Rightarrow |v\rangle|A_v\rangle$$

By **unitarity** (Axiom 2) scalar product of the total (*S*+*A*) state before and after must be the same. So:

$$\langle u|v\rangle\langle A_0|A_0\rangle = \langle u|v\rangle\langle A_u|A_v\rangle$$

But $\langle A_0|A_0\rangle = 1$**. So either** $\langle A_u|A_v\rangle = 1$ **(measurement was not successful) or** $\langle u|v\rangle = 0$ **. QED!!!!**

Summary: Observables are Hermitean

Theorem: Outcomes of a measurement that satisfy postulates 1-3 must be orthogonal.

Proof (another version): measurement is an information transfer from a quantumsystem *S* to a **quantum** apparatus *A*. So, for any two possible **repeatable (predictable)** (Axiom 3) outcome states of the same measurement it must be true that:

$$|u\rangle|A_0\rangle \Rightarrow |u\rangle|A_u\rangle$$

$$|v\rangle|A_0\rangle \Rightarrow |v\rangle|A_v\rangle$$

By **unitarity** (Axiom 2) scalar product of the total (*S*+*A*) state before and after must be the same. So:

"information gain implies disturbance (Bennett, Brassard,& Mermi

$$\langle u|v\rangle\langle A_0|A_0\rangle = \langle u|v\rangle\langle A_u|A_v\rangle$$

But $\langle A_0|A_0\rangle = 1$**. So either** $\langle A_u|A_v\rangle = 1$ **(measurement was not successful) or** $\langle u|v\rangle = 0$ **. QED!!!!**

Summary: Observables are Hermitean

Theorem: Outcomes of a measurement that satisfy postulates 1-3 must be orthogonal.

Proof (another version): measurement is an information transfer from a quantumsystem *S* to a **quantum** apparatus *A*. So, for any two possible **repeatable (predictable)** (Axiom 3) outcome states of the same measurement it must be true that:

DISTINGUISHABLE "EVENTS"

$$|u\rangle|A_0\rangle \Rightarrow |u\rangle|A_u\rangle$$

$$|v\rangle|A_0\rangle \Rightarrow |v\rangle|A_v\rangle$$

By **unitarity** (Axiom 2) scalar product of the total (*S*+*A*) state before and after must be the same. So:

$$\langle u|v\rangle\langle A_0|A_0\rangle = \langle u|v\rangle\langle A_u|A_v\rangle$$

"information gain implies disturbance (Bennett, Brassard,& Mermi

But $\langle A_0|A_0\rangle = 1$**. So either** $\langle A_u|A_v\rangle = 1$ **(measurement was not successful) or** $\langle u|v\rangle = 0$ **. QED!!!!**

Summary: Observables are Hermitean

Theorem: Outcomes of a measurement that satisfy postulates 1-3 must be orthogonal.

Proof (another version): measurement is an information transfer from a quantumsystem S to a **quantum** apparatus A. So, for any two possible **repeatable (predictable)** (Axiom 3) outcome states of the same measurement it must be true that:

DISTINGUISHABLE "EVENTS"

$$|u\rangle|A_0\rangle \Rightarrow |u\rangle|A_u\rangle$$

$$|v\rangle|A_0\rangle \Rightarrow |v\rangle|A_v\rangle$$

NOTE: IN CONTRAST WITH DECOHERENCE, WE DO NOT INVOKE BORN'S RULE!!!

By **unitarity** (Axiom 2) scalar product of the total (S+A) state before and after must be the same. So:

"information gain implies disturbance (Bennett, Brassard,& Mermi

$$\langle u|v\rangle\langle A_0|A_0\rangle = \langle u|v\rangle\langle A_u|A_v\rangle$$

But $\langle A_0|A_0\rangle = 1$. **So either** $\langle A_u|A_v\rangle = 1$ **(measurement was not successful) or** $\langle u|v\rangle = 0$ **. QED!!!!**

Plan

Derive controversial axioms 4&5 from the noncontroversial 0-3.
Understand emergence of "objective classical reality" -- how rea
states that can be found out by us arise from quantum substrate

Plan

Derive controversial axioms 4&5 from the noncontroversial 0-3.
Understand emergence of "objective classical reality" -- how rea
states that can be found out by us arise from quantum substrate

- Why the measurement outcomes are limited to an **orthogonal subset** of all the possible states in the Hilbert states?

Plan

Derive controversial axioms 4&5 from the noncontroversial 0-3.
Understand emergence of "objective classical reality" -- how rea
states that can be found out by us arise from quantum substrate

- Why the measurement outcomes are limited to an **orthogonal subset** of all the possible states in the Hilbert states?
- Why does "**Born's rule**" yield probabilities?

Plan

Derive controversial axioms 4&5 from the noncontroversial 0-3.
Understand emergence of "objective classical reality" -- how rea
states that can be found out by us arise from quantum substrate

- Why the measurement outcomes are limited to an **orthogonal subset** of all the possible states in the Hilbert states?
- Why does "**Born's rule**" yield probabilities?
- How can "objective classical reality" -- states we can find out -- arise from the fragile quantum states that are perturbed by measurements? ("**Quantum Darwinism**")

Plan

Derive controversial axioms 4&5 from the noncontroversial 0-3.
Understand emergence of "objective classical reality" -- how rea
states that can be found out by us arise from quantum substrate

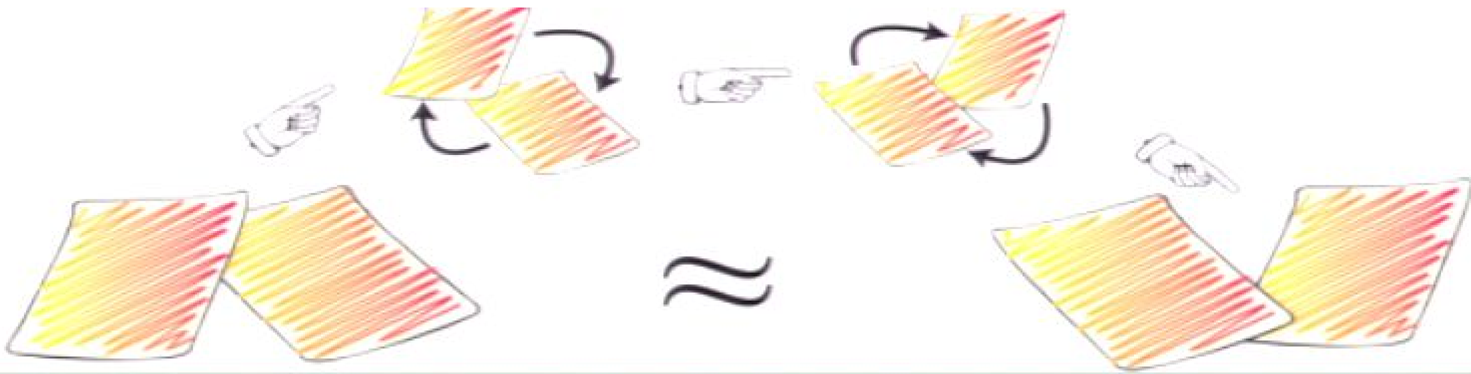
- Why the measurement outcomes are limited ✔ to an **orthogonal subset** of all the possible states in the Hilbert states? WE HAVE "EVENTS"!
- Why does "**Born's rule**" yield probabilities?
- How can "objective classical reality" -- states we can find out -- arise from the fragile quantum states that are perturbed by measurements? ("**Quantum Darwinism**")

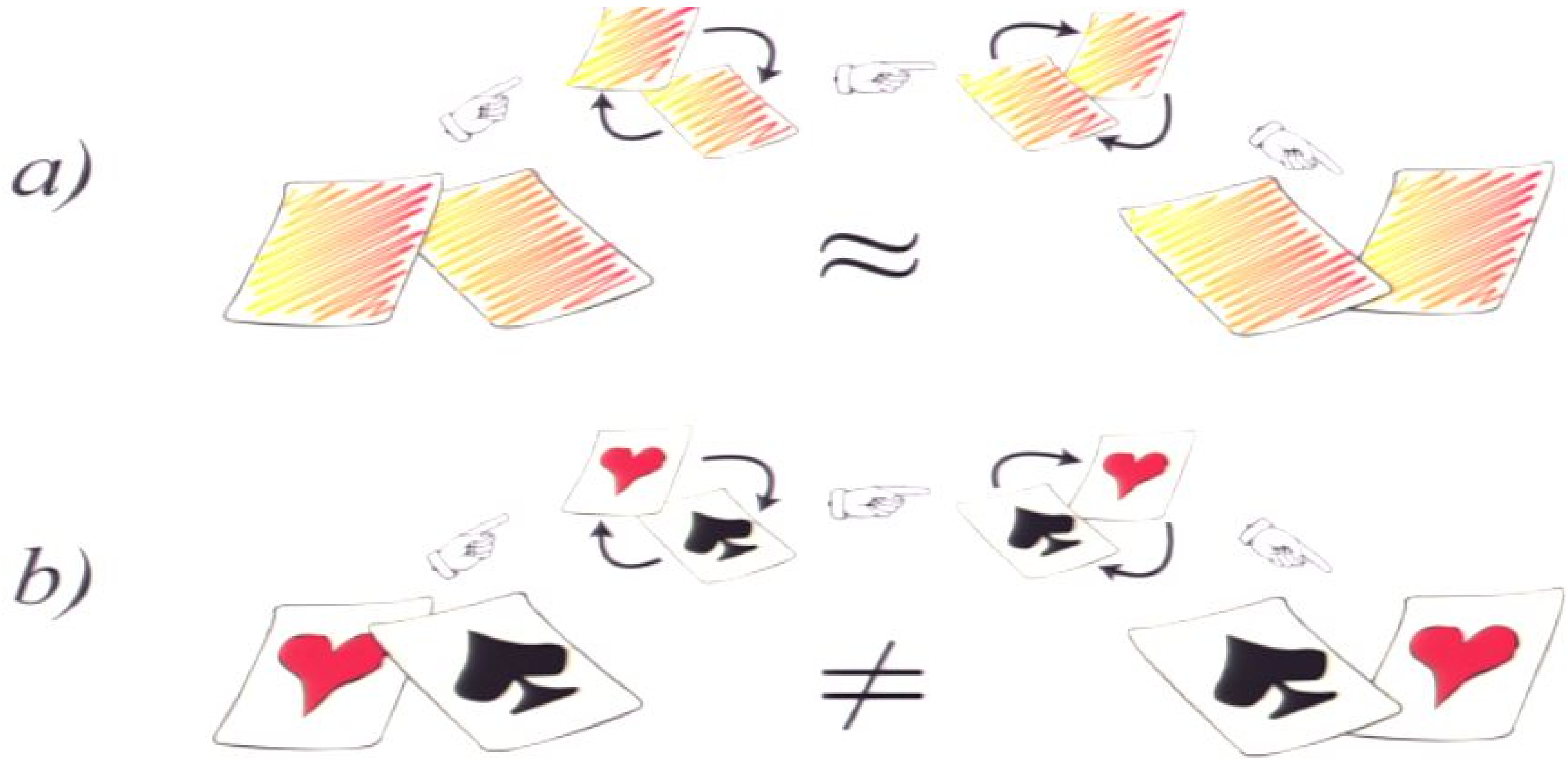
Plan

Derive controversial axioms 4&5 from the noncontroversial 0-3.
Understand emergence of "objective classical reality" -- how rea
states that can be found out by us arise from quantum substrate

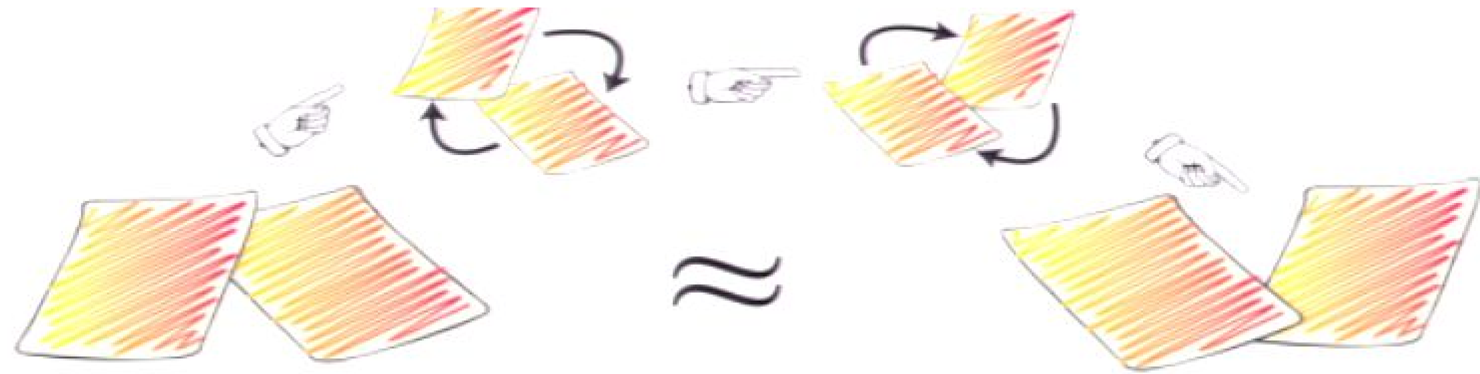
- ✓ Why the measurement outcomes are limited to an **orthogonal subset** of all the possible states in the Hilbert states? **WE HAVE "EVENTS"!**
- Why does "**Born's rule**" yield probabilities?
- How can "objective classical reality" -- states we can find out -- arise from the fragile quantum states that are perturbed by measurements? ("**Quantum Darwinism**")

a)

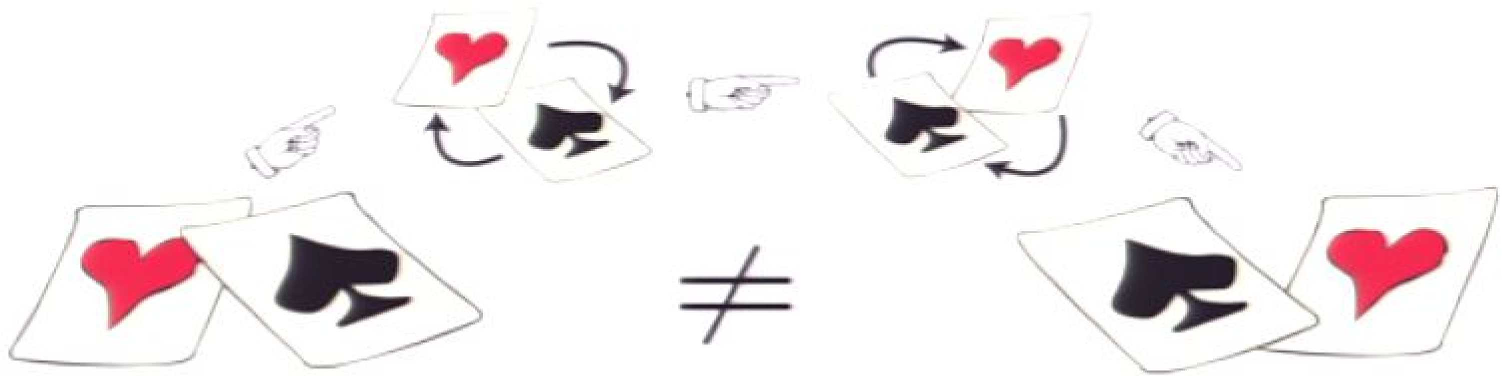



a)
b)

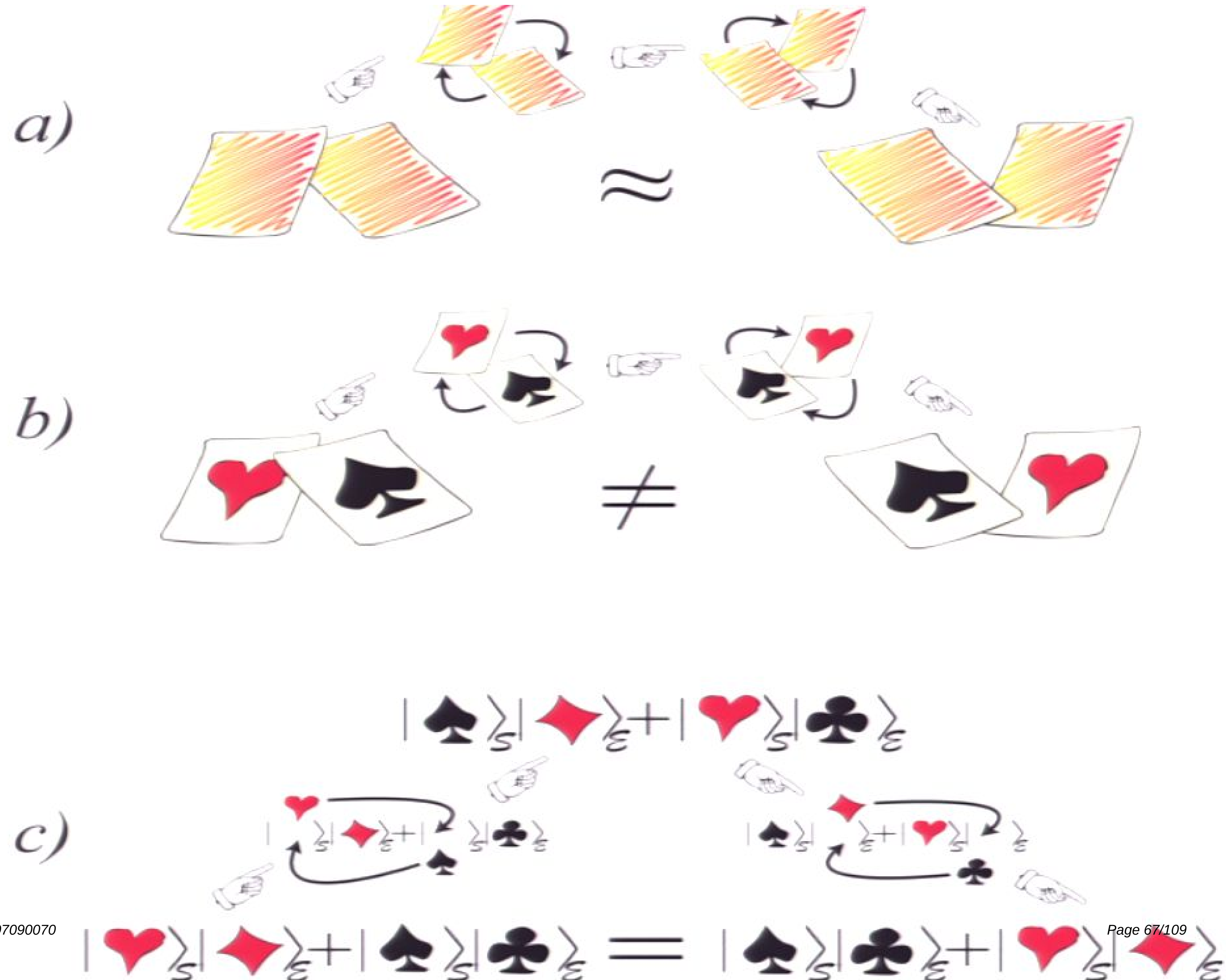
a)



b)



c)


a)
b)
c)

ENVARIANCE
(Entanglement-Assisted Invariance)

DEFINITION:

Consider a composite quantum object consisting of system $\mathcal{S}$ and environment $\mathcal{E}$. **When the combined state $\psi_{\mathcal{SE}}$ is transformed by:**

$$U_{\mathrm{S}} = u_{\mathrm{S}} \otimes \mathbf{1}_{\mathrm{E}}$$

but can be "untransformed" by acting solely on $\mathcal{E}$, that is, if there exists:

$$U_{\mathcal{E}} = \mathbf{1}_{\mathcal{S}} \otimes u_{\mathcal{E}}$$

then $\psi_{\mathcal{SE}}$ is ENVARIANT with respect to $u_{\mathcal{S}}$.

$$U_{\mathcal{E}}(U_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) = U_{\mathcal{E}}|\varphi_{\mathcal{SE}}\rangle = |\psi_{\mathcal{SE}}\rangle$$

Envariance is a property of U_{S} and the joint state $\psi_{\mathcal{SE}}$ of two systems, $\mathcal{S}$ & $\mathcal{E}$.

ENTANGLED STATE AS AN EXAMPLE OF ENVARIANCE:

Schmidt decomposition:

$$|\psi_{\mathcal{SE}}\rangle = \sum_{k=1}^{N} \alpha_k |s_k\rangle|\varepsilon_k\rangle$$

Above Schmidt states $|s_k\rangle$, $|\varepsilon_k\rangle$ are orthonormal and α_k complex.

Lemma 1: Unitary transformations with Schmidt eigenstates:

$$u_{\mathcal{S}}(s_k) = \sum_{k=1} \exp(i\phi_k)|s_k\rangle\langle s_k|$$

leave $\psi_{\mathcal{SE}}$ envariant.

Proof: $u_{\mathcal{S}}(s_k)|\psi_{\mathcal{SE}}\rangle = \sum_{k=1} \alpha_k \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle$, $u_{\mathcal{E}}(\varepsilon_k) = \sum_{k=1} \exp\{i(-\phi_k + 2\pi l_k)\}|\varepsilon_k\rangle\langle\varepsilon_k|$

$$u_{\mathrm{E}}(\varepsilon_k)\{u_{\mathrm{S}}(s_k)|\psi_{\mathrm{SE}}\rangle\} = \sum_{k=1} \alpha_k \exp\{i(\phi_k - \phi_k + 2\pi l_k)|s_k\rangle|\varepsilon_k\rangle = \sum_{k=1} \alpha_k \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle = |\psi_{\mathrm{SE}}$$

ENTANGLED STATE AS AN EXAMPLE OF ENVARIANCE:

Schmidt decomposition:

$$|\psi_{\mathcal{SE}}\rangle = \sum_{k=1}^{N} \alpha_k |s_k\rangle |\varepsilon_k\rangle$$

Above Schmidt states $|s_k\rangle$, $|\varepsilon_k\rangle$ are orthonormal and α_k complex.

Lemma 1: Unitary transformations with Schmidt eigenstates:

$$u_{\mathcal{S}}(s_k) = \sum_{k=1} \exp(i\phi_k)|s_k\rangle\langle s_k|$$

leave $\psi_{\mathcal{SE}}$ envariant.

Proof: $u_{\mathcal{S}}(s_k)|\psi_{\mathcal{SE}}\rangle = \sum_{k=1} \alpha_k \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle$, $u_{\mathcal{E}}(\varepsilon_k) = \sum_{k=1} \exp\{i(-\phi_k + 2\pi l_k)\}|\varepsilon_k\rangle\langle\varepsilon_k|$

$$u_{\mathrm{E}}(\varepsilon_k)\{u_{\mathrm{S}}(s_k)|\psi_{\mathrm{SE}}\rangle\} = \sum_{k=1} \alpha_k \exp\{i(\phi_k - \phi_k + 2\pi l_k)|s_k\rangle|\varepsilon_k\rangle = \sum_{k=1} \alpha_k \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle = |\psi_{\mathrm{SE}}$$

OCALLY

ENTANGLED STATE AS AN EXAMPLE OF ENVARIANCE:

Schmidt decomposition:

$$|\psi_{\mathcal{SE}}\rangle = \sum_{k=1}^{N} \alpha_k |s_k\rangle |\varepsilon_k\rangle$$

Above Schmidt states $|s_k\rangle$, $|\varepsilon_k\rangle$ are orthonormal and α_k complex.

Lemma 1: Unitary transformations with Schmidt eigenstates:

$$u_{\mathcal{S}}(s_k) = \sum_{k=1} \exp(i\phi_k) |s_k\rangle\langle s_k|$$

leave $\psi_{\mathcal{SE}}$ envariant.

Proof: $u_{\mathcal{S}}(s_k)|\psi_{\mathcal{SE}}\rangle = \sum_{k=1} \alpha_k \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle$, $u_{\mathcal{E}}(\varepsilon_k) = \sum_{k=1} \exp\{i(-\phi_k + 2\pi\, l_k)\}|\varepsilon_k\rangle\langle\varepsilon_k|$

$$u_{\mathrm{E}}(\varepsilon_k)\{u_{\mathrm{S}}(s_k)|\psi_{\mathrm{SE}}\rangle\} = \sum_{k=1} \alpha_k \exp\{i(\phi_k - \phi_k + 2\pi\, l_k)|s_k\rangle|\varepsilon_k\rangle = \sum_{k=1} \alpha_k \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle = |\psi_{\mathrm{SE}}$$

LOCALLY, SCHMIDT PHASES DO NOT MATTER

ENTANGLED STATE AS AN EXAMPLE OF ENVARIANCE:

Schmidt decomposition:

$$|\psi_{\mathcal{SE}}\rangle = \sum_{k=1}^{N} \alpha_k |s_k\rangle|\varepsilon_k\rangle$$

Above Schmidt states $|s_k\rangle$, $|\varepsilon_k\rangle$ are orthonormal and α_k complex.

Lemma 1: Unitary transformations with Schmidt eigenstates:

$$u_{\mathcal{S}}(s_k) = \sum_{k=1} \exp(i\phi_k)|s_k\rangle\langle s_k|$$

leave $\psi_{\mathcal{SE}}$ envariant.

Proof: $u_{\mathcal{S}}(s_k)|\psi_{\mathcal{SE}}\rangle = \sum_{k=1}\alpha_k \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle$, $u_{\mathcal{E}}(\varepsilon_k) = \sum_{k=1}\exp\{i(-\phi_k + 2\pi\, l_k)\}|\varepsilon_k\rangle\langle\varepsilon_k|$

$$u_{\mathrm{E}}(\varepsilon_k)\{u_{\mathrm{S}}(s_k)|\psi_{\mathrm{SE}}\rangle\} = \sum_{k=1}\alpha_k \exp\{i(\phi_k - \phi_k + 2\pi\, l_k)|s_k\rangle|\varepsilon_k\rangle = \sum_{k=1}\alpha_k \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle = |\psi_{\mathrm{SE}}$$

LOCALLY, SCHMIDT PHASES DO NOT MATTER: DECOHERENCE!!!

ENVARIANCE -- SOME PROPERTIES

$$U_{\mathcal{E}}(U_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) = U_{\mathcal{E}}|\varphi_{\mathcal{SE}}\rangle = \exp(i\phi)|\psi_{\mathcal{SE}}\rangle$$

- Envariant $\psi_{\mathcal{SE}}$ is an eigenstate of two unitary transformations with a unit (or unimodular) eigenvalue.
- Envariance can be defined for density matrices of $\mathcal{SE}$, but this will not be necessary, as one can instead purify the state of $\mathcal{SE}$ in the usual way, by introducing $\mathcal{E}'$, so the density matrix of $\mathcal{SE}$ is given by: $\rho_{\mathcal{SE}} = Tr_{\mathcal{E}'}|\Psi_{\mathcal{SEE}'}\rangle\langle\Psi_{\mathcal{SEE}'}|$
- A product of envariant transformations of $\psi_{\mathcal{SE}}$ is an envariant transformation of $\psi_{\mathcal{SE}}$.
- All envariant transformations have Schmidt eigenstates.
- There may be many environments that undo an effect of the same unitary transformation on the system

For additional discussion, see WHZ, quant-ph/0211037, PRL, 90, 120404 (2003) also *Decoherence, einselection, and the quantum origin of the classical* RMP, 75, 715 (2003); and especially *Probabilities from entanglement, ...* quant-ph/040516

PHASE ENVARIANCE THEOREM

Fact 1: Unitary transformations must act on the system to alter its state (if they act only somewhere else, system is not effected).

Fact 2: The state of the system is all that is necessary/available to predict measurement outcomes (including their probabilities).

Fact 3: A state of the composite system is all that is needed/available to determine the state of the system.

Moreover, "entanglement happens":

$$|\psi_{\mathcal{SE}}\rangle \propto \sum_{k=1}^{N} \alpha_k |s_k\rangle|\varepsilon_k\rangle$$

THEOREM 1: State (and probabilities) of $\mathcal{S}$ alone can depend only on the absolute values of Schmidt coefficients $|\alpha_k|$, and **not on their phases**

Proof: Phases of α_k can be changed by acting on $\mathcal{S}$ alone. But the state of the whole can be restored by acting only on $\mathcal{E}$. So change of phases of Schmidt coefficients could not have affected $\mathcal{S}$! QED.

$\therefore$ By phase envariance, $\{|\alpha_k|, |s_k\rangle\}$ must provide a complete local description of the system alone.

Same info as reduced density matrix!!!

Envariance of entangled states: the case of **equal coefficients**

$$|\psi_{\mathcal{SE}}\rangle \propto \sum_{k=1}^{N} \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle$$

In this case ANY orthonormal basis is Schmidt. In particular, in the Hilbert subspace spanned by any two $\{|s_k\rangle, |s_l\rangle\}$ one can define a Hadamard basis;

$$|\pm\rangle = (|s_k\rangle \pm |s_l\rangle)/\sqrt{2}$$

This can be used to generate 'new kind' of envariant transformations:

A SWAP: $u_{\mathcal{S}}(k \leftrightarrow l) = \exp(i\varphi_{kl})|s_k\rangle\langle s_l| + h.c.$

Can be 'undone' by the COUNTERSWAP:

$$u_{\mathcal{E}}(k \leftrightarrow l) = \exp\{i(-\varphi_{kl} - \varphi_k + \varphi_l)\}|\varepsilon_l\rangle\langle\varepsilon_k| + h.c.$$

LEMMA 3: Swaps of states are envariant when Schmidt coefficients have same absolute value.

PHASE ENVARIANCE THEOREM

Fact 1: Unitary transformations must act on the system to alter its state (if they act only somewhere else, system is not effected).

Fact 2: The state of the system is all that is necessary/available to predict measurement outcomes (including their probabilities).

Fact 3: A state of the composite system is all that is needed/available to determine the state of the system.

Moreover, "entanglement happens":

$$|\psi_{\mathcal{SE}}\rangle \propto \sum_{k=1}^{N} \alpha_k |s_k\rangle|\varepsilon_k\rangle$$

THEOREM 1: State (and probabilities) of $\mathcal{S}$ alone can depend only on the absolute values of Schmidt coefficients $|\alpha_k|$, and **not on their phases**

Proof: Phases of α_k can be changed by acting on $\mathcal{S}$ alone. But the state of the whole can be restored by acting only on $\mathcal{E}$. So change of phases of Schmidt coefficients could not have affected $\mathcal{S}$! QED.

$\therefore$ By phase envariance, $\{|\alpha_k|, |s_k\rangle\}$ must provide a complete local description of the system alone.

Same info as reduced density matrix!!!

Envariance of entangled states: the case of **equal coefficients**

$$|\psi_{\mathcal{SE}}\rangle \propto \sum_{k=1}^{N} \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle$$

In this case ANY orthonormal basis is Schmidt. In particular, in the Hilbert subspace spanned by any two $\{|s_k\rangle, |s_l\rangle\}$ one can define a Hadamard basis;

$$|\pm\rangle = (|s_k\rangle \pm |s_l\rangle)/\sqrt{2}$$

This can be used to generate 'new kind' of envariant transformations:

A SWAP: $u_{\mathcal{S}}(k \leftrightarrow l) = \exp(i\varphi_{kl})|s_k\rangle\langle s_l| + h.c.$

Can be 'undone' by the COUNTERSWAP:

$$u_{\mathcal{E}}(k \leftrightarrow l) = \exp\{i(-\varphi_{kl} - \varphi_k + \varphi_l)\}|\varepsilon_l\rangle\langle\varepsilon_k| + h.c.$$

LEMMA 3: Swaps of states are envariant when Schmidt coefficients have same absolute value.

“Probability from certainty”

Probabilities of Schmidt partners are the same (detecting 0 in S implies 0 in E, etc.).

|0>|0> + |1>|1> (initial state -- equal abs. values of coeff's)

SWAP on S

|1>|0> + |0>|1> (prob's in S **must have swapped**, so that after swap they are equal to the prob's of state in E that were not affected)

COUNTERSWAP on E

|1>|1> + |0>|0> (p's in S **must be the same** as they were to begin with -- global state is back to the "original")

Probabilities can "stay the same" and also "get exchanged" only when they are equal!!! (p(0)=p(1)) (Schlosshauer & Fine, Barnum, WHZ)

Probability of envariantly swappable states

$$|\psi_{\mathcal{SE}}\rangle \propto \sum_{k=1}^{N} \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle$$

By the Phase Envariance Theorem the set of pairs $|\alpha_k|, |s_k\rangle$ provides a complete description of $\mathcal{S}$. But all $|\alpha_k|$ are equal.

With additional assumption about probabilities, can prove

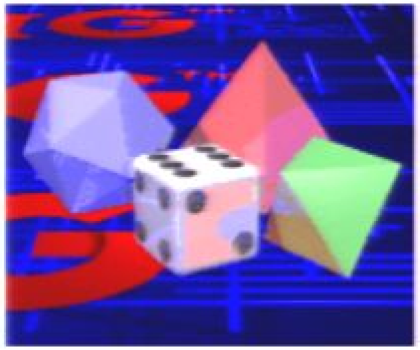
THEOREM 2: Probabilities of envariantly swappable states are equa

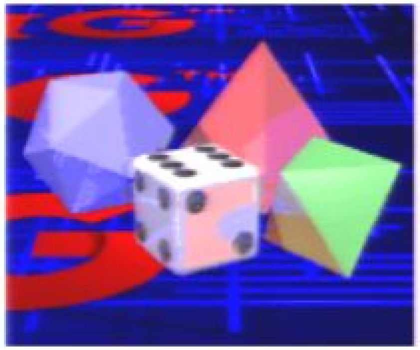
a) "Pedantic assumption"; when states get swapped, so do probabilitite

b) When the state of the system does not change under any unitary in a part of its Hilbert space, probabilities of any set of basis states are equa

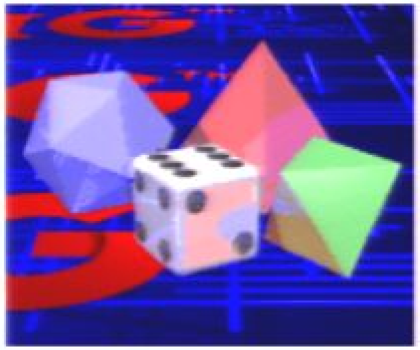
c) Because there is one-to-one correlation between $|s_k\rangle, |\varepsilon_k\rangle$

Therefore, by normalization: $$p_k = \frac{1}{N} \quad \forall_k$$

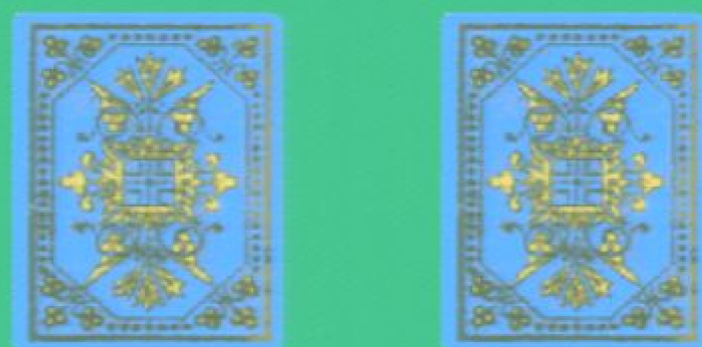


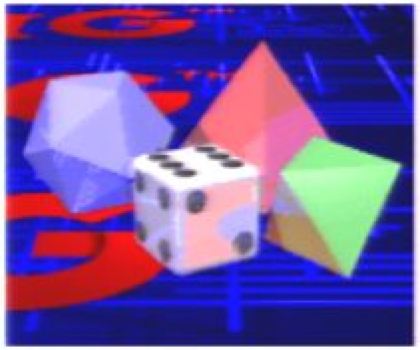


Symmetries can reflect ignorance

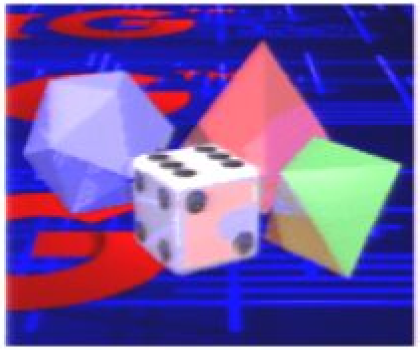


Symmetries can reflect ignorance

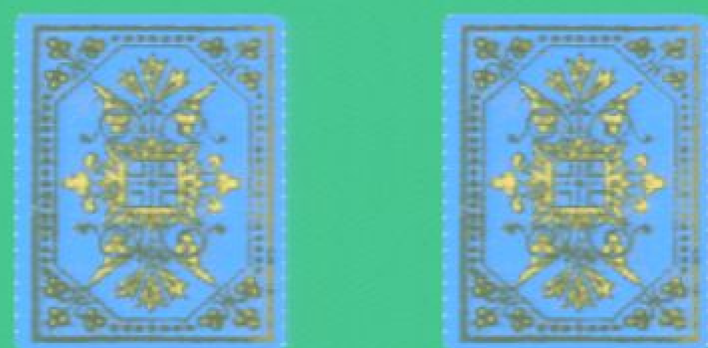


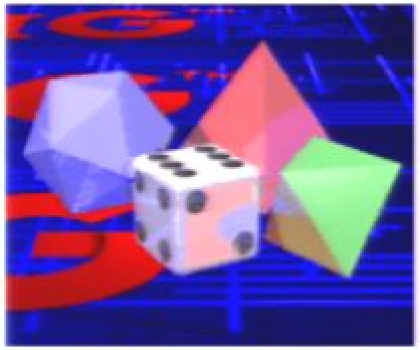


Symmetries can reflect ignorance

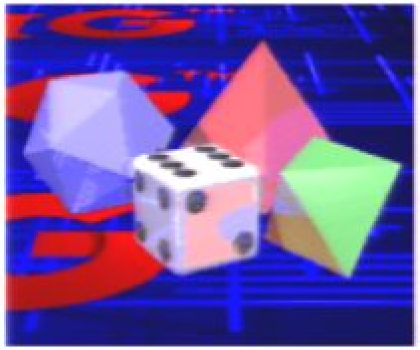


Symmetries can reflect ignorance

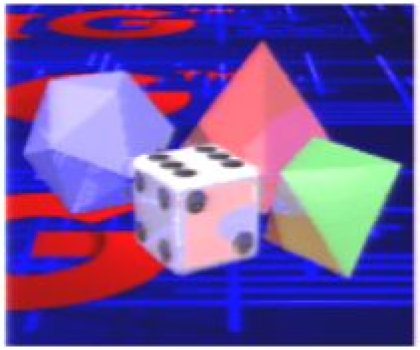




Symmetries can reflect ignorance



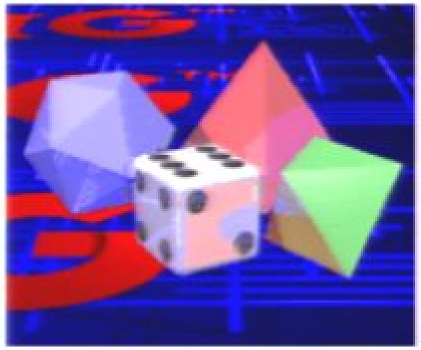
Symmetries can reflect ignorance



Symmetries can reflect ignorance

Probabilities from envariance

(Environment-assisted iNVARIANCE

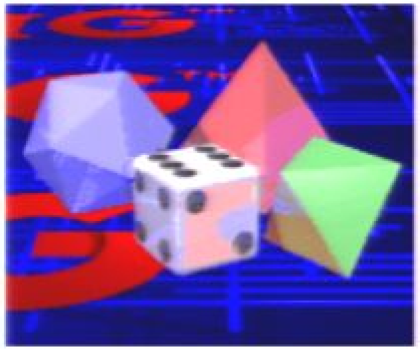


Symmetries can reflect ignorance

Probabilities from envariance

(Environment-assisted iNVARIANC

$$|0_S\rangle|0_E\rangle + |1_S\rangle|1_E\rangle \xrightarrow{\text{swap in } S} |1_S\rangle|0_E\rangle + |0_S\rangle|1_E\rangle \xrightarrow{\text{swap in } E} |1_S\rangle|1_E\rangle + |0_S\rangle|$$

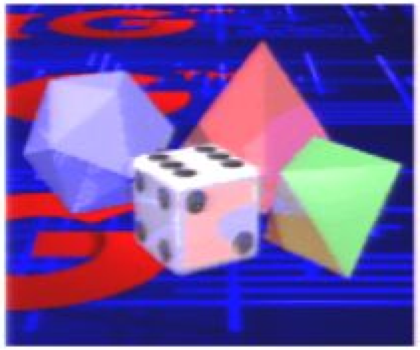


Symmetries can reflect ignorance

Probabilities from envarianc

(Environment-assisted iNVARIANC

$$|0_S\rangle|0_E\rangle + |1_S\rangle|1_E\rangle \xrightarrow{\text{swap in } S} |1_S\rangle|0_E\rangle + |0_S\rangle|1_E\rangle \xrightarrow{\text{swap in } E} |1_S\rangle|1_E\rangle + |0_S\rangle|$$

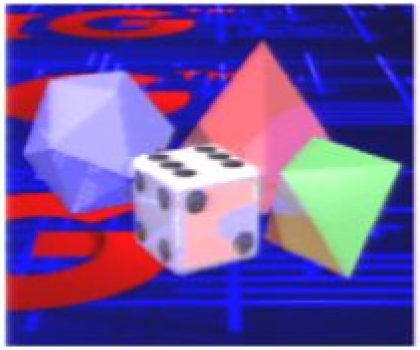


Symmetries can reflect ignorance

Probabilities from envarianc

(Environment-assisted iNVARIANC

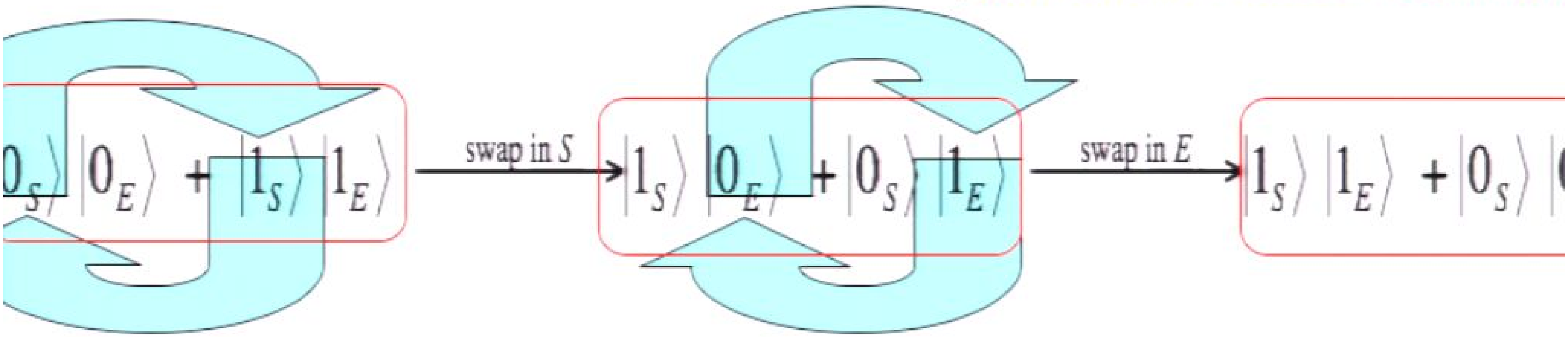
$$0_S\rangle |0_E\rangle + |1_S\rangle |1_E\rangle \xrightarrow{\text{swap in } S} |1_S\rangle |0_E\rangle + |0_S\rangle |1_E\rangle \xrightarrow{\text{swap in } E} |1_S\rangle |1_E\rangle + |0_S\rangle |$$



Symmetries can reflect ignorance

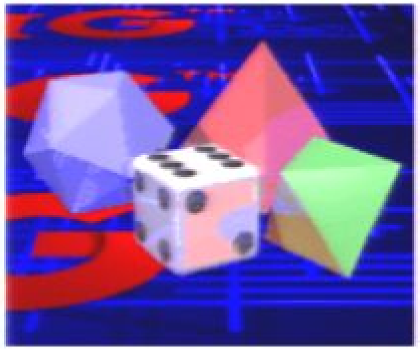
Probabilities from envariance

(Environment-assisted iNVARIANCE



$$p = |\psi|^2$$ follows!

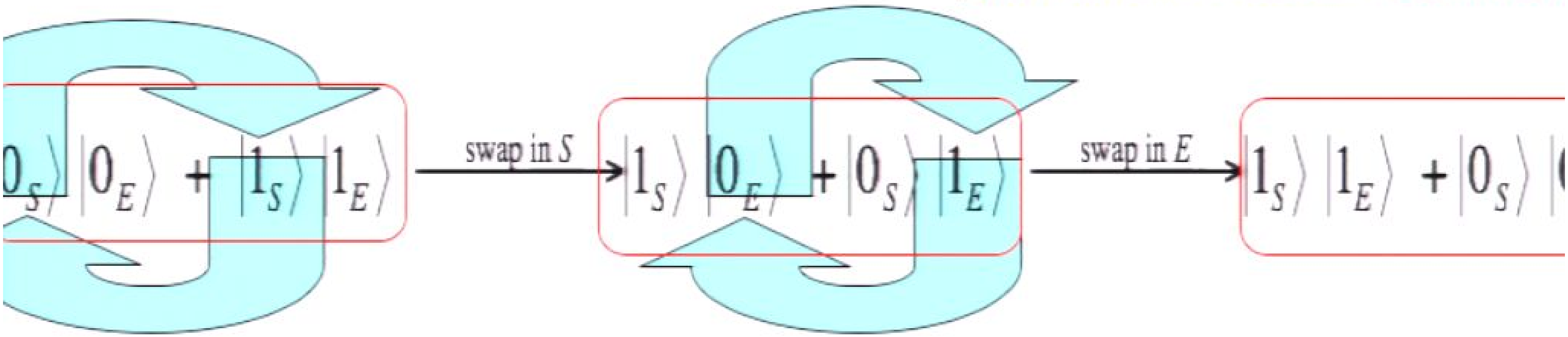
Note: Swaps do change unentangled states ! Phases matter!



Symmetries can reflect ignorance

Probabilities from envariance

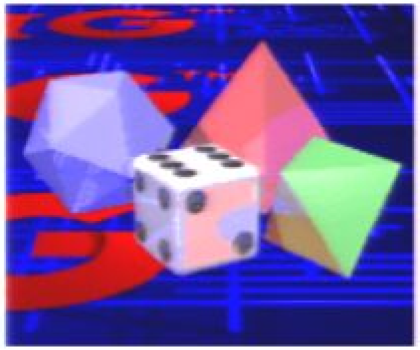
(Environment-assisted iNVARIANCE



$$p = |\psi|^2$$ follows!

Note: Swaps do change unentangled states ! Phases matter!

$|0\rangle + i|1\rangle$ IS ORTHOGONAL TO $|1\rangle + i|0\rangle$



Symmetries can reflect ignorance

Probabilities from envarianc

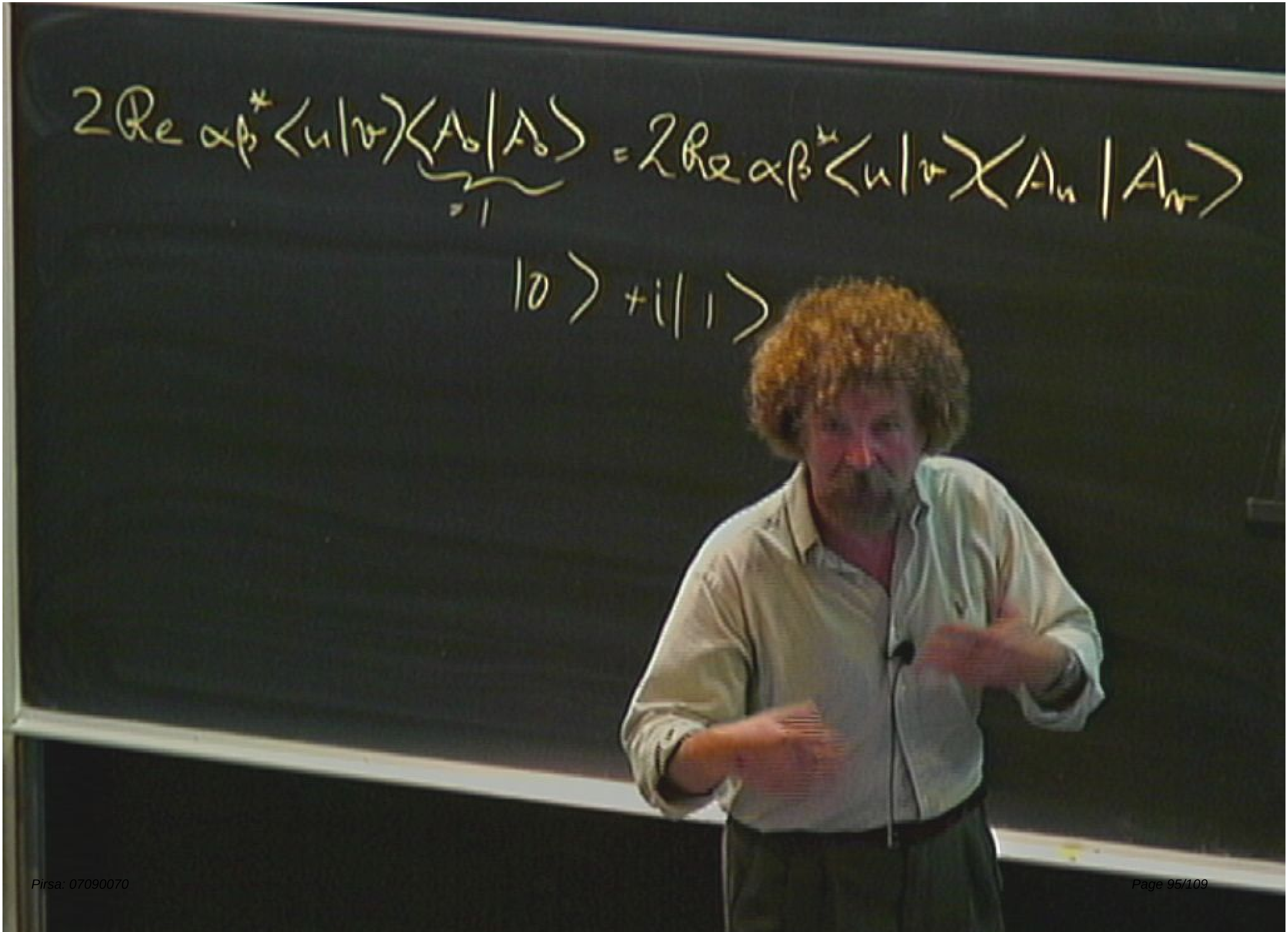
(Environment-assisted iNVARIANC

$$|0_S\rangle|0_E\rangle + |1_S\rangle|1_E\rangle \xrightarrow{\text{swap in } S} |1_S\rangle|0_E\rangle + |0_S\rangle|1_E\rangle \xrightarrow{\text{swap in } E} |1_S\rangle|1_E\rangle + |0_S\rangle|$$

$$p = |\psi|^2$$ follows!

ote: Swaps do change unentangled states ! Phases matter!

$|0\rangle + i|1\rangle$ **IS ORTHOGONAL TO** $|1\rangle + i|0\rangle$


$2\mathrm{Re}\,\alpha\beta^*\langle u|v\rangle\langle A_0|A_0\rangle = 2\mathrm{Re}\,\alpha\beta^*\langle u|v\rangle\langle A_u|A_v\rangle$
$=1$
$|0\rangle + i|1\rangle$

Special case with **unequal** coefficients

Consider system $\mathcal{S}$ with two states $\{|0\rangle, |2\rangle\}$

The environment $\mathcal{E}$ has three states $\{|0\rangle, |1\rangle, |2\rangle\}$ and $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$

$$\left|\psi_{\mathcal{SE}}\right\rangle = \sqrt{\frac{2}{3}}|0\rangle|+\rangle + \sqrt{\frac{1}{3}}|2\rangle|2\rangle$$

An auxilliary environment $\mathcal{E}'$ interacts with $\mathcal{E}$ so that:

$$\left|\psi_{\mathcal{SE}}\right\rangle\left|\varepsilon_0'\right\rangle = \left(\sqrt{\frac{2}{3}}|0\rangle|+\rangle + \sqrt{\frac{1}{3}}|2\rangle|2\rangle\right)|0\rangle \Rightarrow \sqrt{\frac{2}{3}}|0\rangle(|0\rangle|0\rangle + |1\rangle|1\rangle)/\sqrt{2} + \sqrt{\frac{1}{3}}|2\rangle|2\rangle|2\rangle$$

$$= \left(|0\rangle|0\rangle|0\rangle + |0\rangle|1\rangle|1\rangle + |2\rangle|2\rangle|2\rangle\right)/\sqrt{3}$$

States $|0\rangle|0\rangle$, $|0\rangle|1\rangle$, $|2\rangle|2\rangle$ have equal coefficients. Therefore, Each of them has probability of 1/3. Consequently:

$$p(0) = p(0,0)+p(0,1) = 2/3, \quad \text{and} \quad p(2) = 1/3.$$

..... BORN's RULE!!!

Probabilities from Envariance

The case of commensurate probabilities: $|\psi_{\mathcal{SE}}\rangle = \sum_{k=1}^{N} \underbrace{\sqrt{m_k/M}}_{\alpha_k} |s_k\rangle |\varepsilon_k\rangle$

Attach the auxiliary "counter" environment $\mathcal{C}$:

$$|\psi_{\text{SE}}\rangle|e'_0\rangle = \left(\sum_{k=1}^{N} \underbrace{\sqrt{m_k/M}}_{\alpha_k} |s_k\rangle \underbrace{\left(\sum_{j_k=1}^{m_k} \frac{1}{\sqrt{m_k}} |e_{j_k}\rangle \right)}_{|\varepsilon_k\rangle} \right) |c_0\rangle \Rightarrow$$

$$\Rightarrow \frac{1}{\sqrt{M}} \sum_{j=1}^{M} |s_{k(j)}\rangle |e_j\rangle |c_j\rangle$$

ŤHEOREM 3: The case with commensurate probabilities can be ŗeduced to the case with equal probabilities. **BORN's RULE follows**

$$p_j = \frac{1}{M}, \qquad p_k = \sum_{j_k=1}^{m_k} p_{j_k} = \frac{m_k}{M} = |\alpha_k|^2$$

Ğeneral case -- by continuity. QED

Probabilities from Envariance

The case of commensurate probabilities: $|\psi_{\mathcal{SE}}\rangle = \sum_{k=1}^{N} \underbrace{\sqrt{m_k/M}}_{\alpha_k} |s_k\rangle|\varepsilon_k\rangle$

Attach the auxiliary "counter" environment $\mathcal{C}$:

$$|\psi_{\mathrm{SE}}\rangle|e'_0\rangle = \left(\sum_{k=1}^{N} \underbrace{\sqrt{m_k/M}}_{\alpha_k} |s_k\rangle \left(\underbrace{\sum_{j_k=1}^{m_k} \frac{1}{\sqrt{m_k}} |e_{j_k}\rangle}_{|\varepsilon_k\rangle} \right) \right) |c_0\rangle \Rightarrow$$

$$\Rightarrow \frac{1}{\sqrt{M}} \sum_{j=1}^{M} |s_{k(j)}\rangle|e_j\rangle|c_j\rangle$$

THEOREM 3: The case with commensurate probabilities can be reduced to the case with equal probabilities. **BORN's RULE follows**

$$p_j = \frac{1}{M}, \qquad p_k = \sum_{j_k=1}^{m_k} p_{j_k} = \frac{m_k}{M} = |\alpha_k|^2$$

General case -- by continuity. QED

Why the proof works

- Need to know how to relate quantum states and "events". ("Symmetry breaking" induced by information transfer.)
- Need to prove that phases of the coefficients do not matter (otherwise swapping alters state even when absolute values of coeff's are equal). ("Decoherence without decoherence")
- How it fits within the usual decoherence framework: Schmidt states end up coinciding with pointer states (selected for their resilience)
- Cannot really start by assuming decoherence -- decoherence tools presume Born's rule

ENVARIANCE* -- SUMMARY

1. New symmetry - ENVARIANCE - of joint states of quantum systems. It is related to causality.

2. In quantum physics perfect knowledge of the whole may imply complete ignorance of a part.

3. **BORN's RULE** follows as a consequence of envariance.

4. Relative frequency interpretation of probabilities naturally follows.

5. Envariance supplies a new foundation for environment - induced superselection, decoherence, quantum statistical physics, etc., by justifying the form and interpretation of reduced density matrices.

*WHZ, PRL **90**, 120404; RMP **75**, 715 (2003); PRA **71**, 052105 (2005

Plan

Derive controversial axioms 4&5 from the noncontroversial 0-3.
Understand emergence of “objective classical reality” -- how rea
states that can be found out by us arise from quantum substrate

Plan

Derive controversial axioms 4&5 from the noncontroversial 0-3.
Understand emergence of "objective classical reality" -- how rea
states that can be found out by us arise from quantum substrate

- Why the measurement outcomes are limited to an orthogonal subset of all the possible states in the Hilbert states?

Plan

Derive controversial axioms 4&5 from the noncontroversial 0-3.
Understand emergence of "objective classical reality" -- how rea
states that can be found out by us arise from quantum substrate

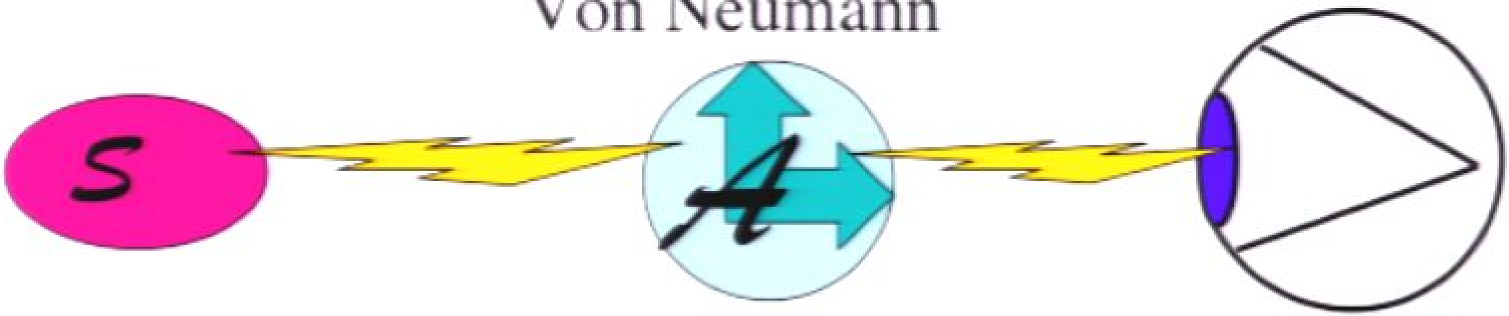
- Why the measurement outcomes are limited to an orthogonal subset of all the possible states in the Hilbert states?
- Why does "**Born's rule**" yield probabilities?

Plan

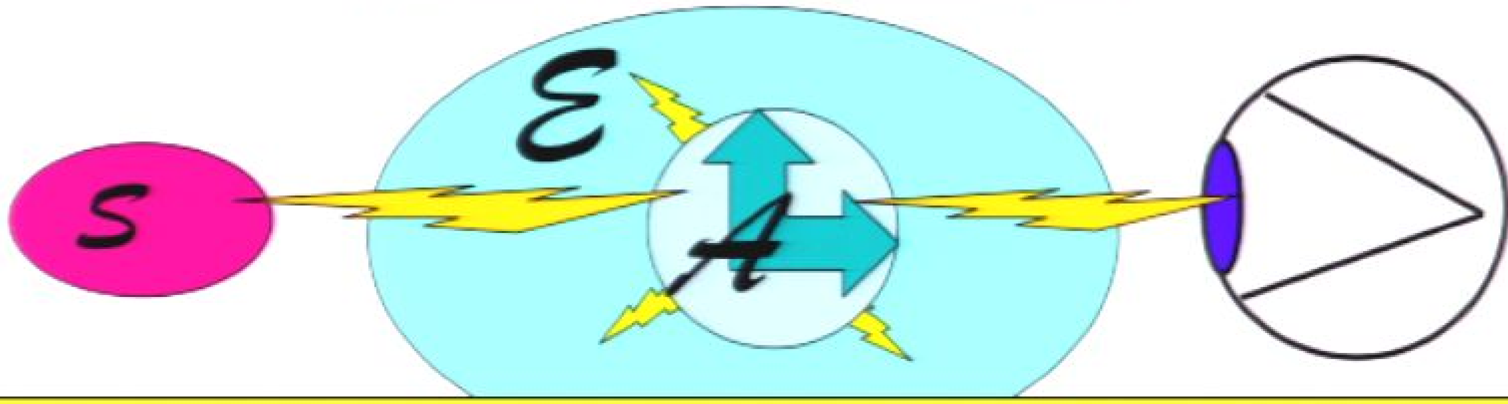
Derive controversial axioms 4&5 from the noncontroversial 0-3.
Understand emergence of "objective classical reality" -- how rea
states that can be found out by us arise from quantum substrate

- Why the measurement outcomes are limited to an orthogonal subset of all the possible states in the Hilbert states?
- Why does "**Born's rule**" yield probabilities?
- How can "objective classical reality" -- states we can find out -- arise from the fragile quantum states that are perturbed by measurements? ("**Quantum Darwinism**")

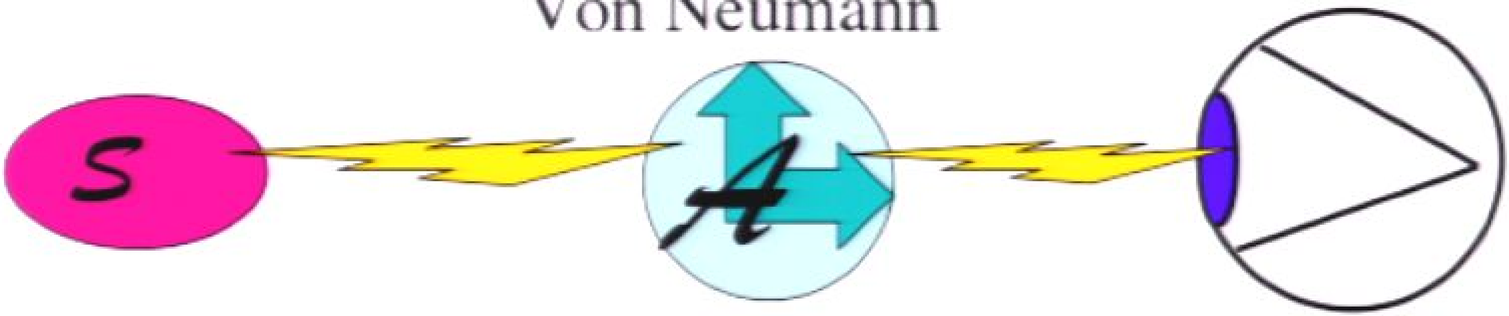
Von Neumann



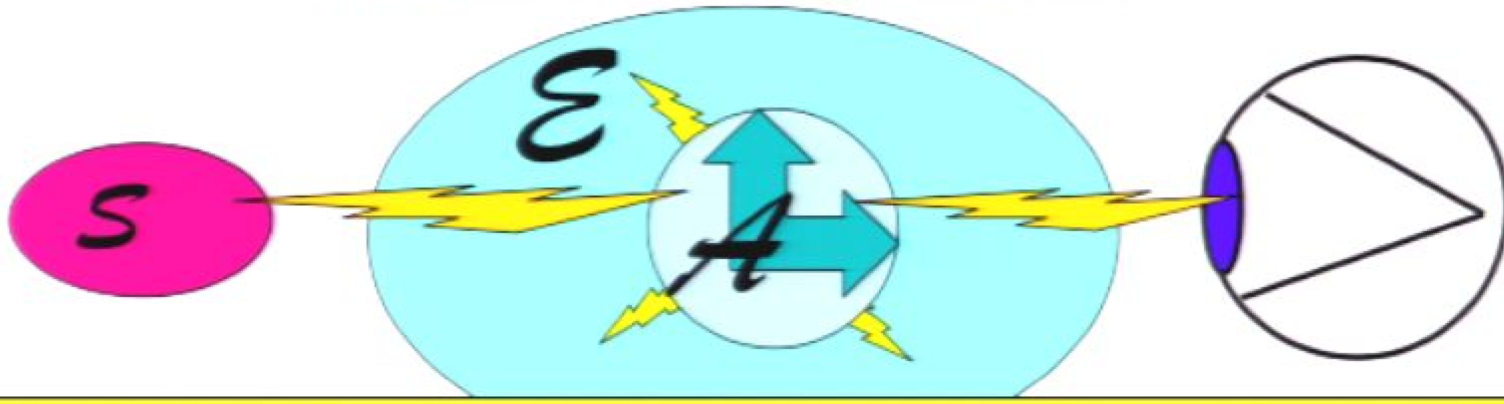
Decoherence & Einselection



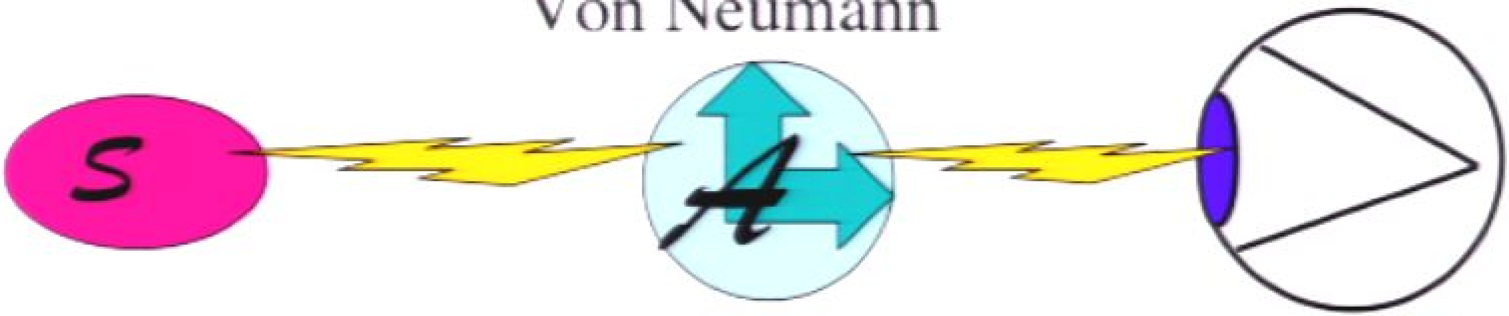
Von Neumann



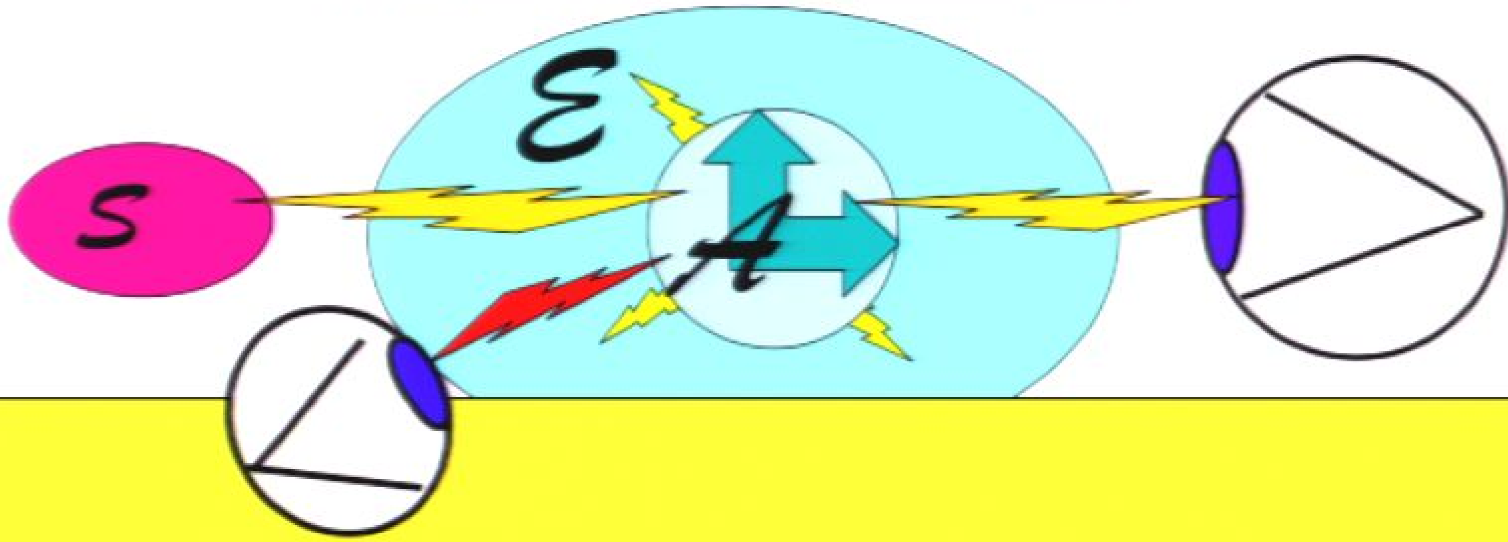
Decoherence & Einselection

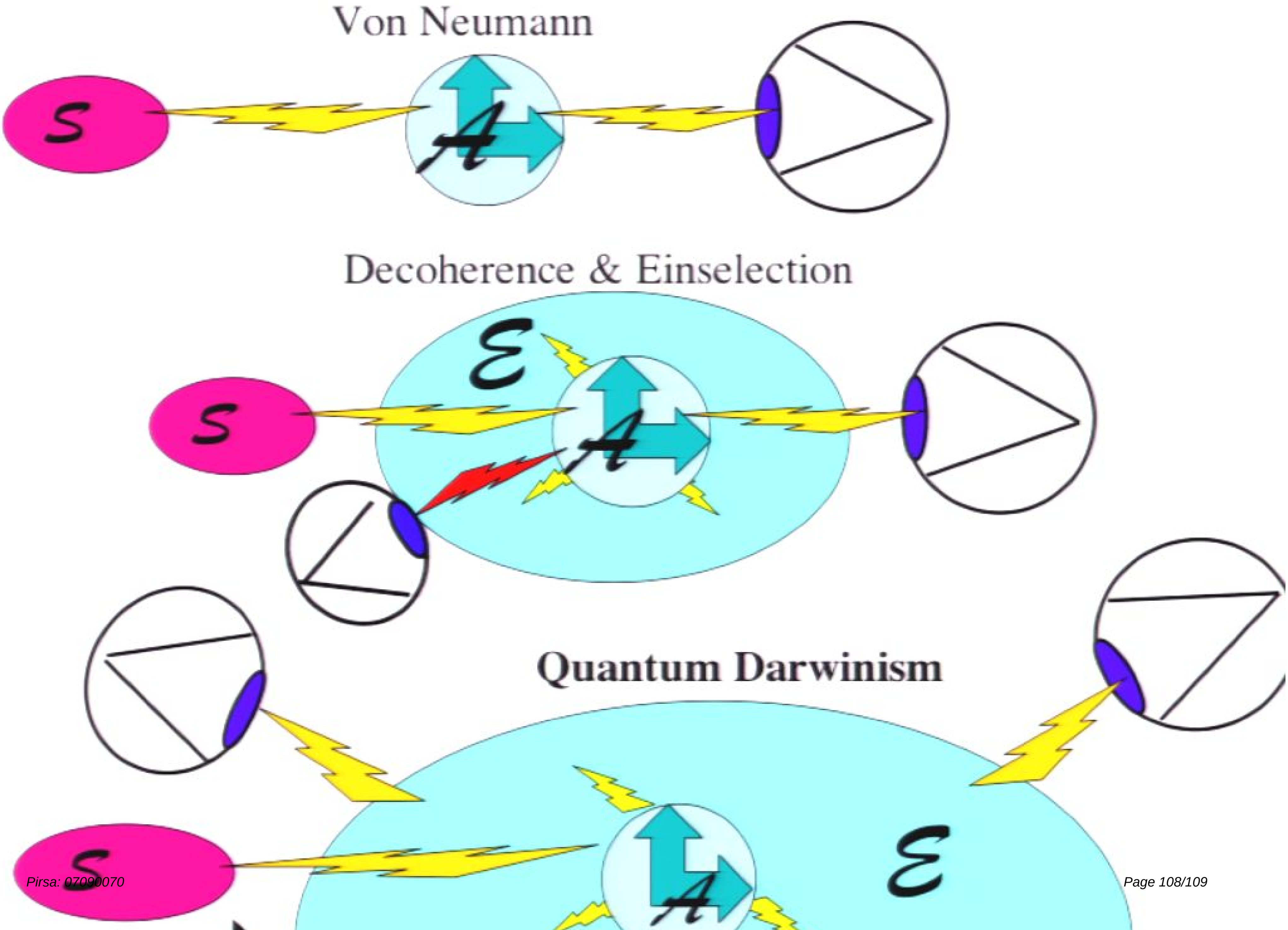


Von Neumann



Decoherence & Einselection




Von Neumann
S
A
Decoherence & Einselection
ℰ
S
A
Quantum Darwinism
S
A
ℰ

Envariance of entangled states: the case of **equal coefficients**

$$|\psi_{SE}\rangle \propto \sum_{k=1}^{N} \exp(i\phi_k)|s_k\rangle|\varepsilon_k\rangle$$

In this case ANY orthonormal basis is Schmidt. In particular, in the Hilbert subspace spanned by any two $\{|s_k\rangle, |s_l\rangle\}$ one can define a Hadamard basis;

$$|\pm\rangle = (|s_k\rangle \pm |s_l\rangle)/\sqrt{2}$$

This can be used to generate 'new kind' of envariant transformations:

A SWAP: $u_S(k \leftrightarrow l) = \exp(i\varphi_{kl})|s_k\rangle\langle s_l| + h.c.$

Can be 'undone' by the COUNTERSWAP:

$$u_{\mathcal{E}}(k \leftrightarrow l) = \exp\{i(-\varphi_{kl} - \varphi_k + \varphi_l)\}|\varepsilon_l\rangle\langle\varepsilon_k| + h.c.$$

LEMMA 3: Swaps of states are envariant when Schmidt coefficients have same absolute value.