Title: Solution(?) to the Everettian Evidential Problem

Date: Sep 23, 2007 04:30 PM

URL: http://pirsa.org/07090066

Abstract: This talk follows on from Wayne Myrvold\'s (and is based on joint work with Myrvold). I aim (and claim) to provide a unified account of theory confirmation that can deal with the (actual) situation in which we are uncertain whether the true theory is a probabilistic one or a branching-universe one, that does not presuppose the correctness of any particular physical theory, and that illuminates the connection between the decision-theoretic and the confirmation-theoretic roles of probabilities and their Everettian analogs. (The technique is to piggy-back on the existing body of physics-independent decision theory due to Savage, De Finetti and others, and to exploit the pervasive structural analogy between probabilistic theories and branching-universe theories in arguing for a particular application of that same mathematics to the branching case.) One corollary of this account is that ordinary empirical evidence (such as observed outcomes of relative-frequency trials) confirms Everettian QM in precisely the same way that it confirms a probabilistic QM; I claim that this result solves the Evidential Problem discussed by Myrvold. I will also briefly discuss the relationship between this approach and the Everettian \'derivation of the Born rule\' due to Deutsch and Wallace.

Solution(?) to the Everettian Evidential Problem

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Many Worlds at 50
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23 September 2007

"Everett and Evidence" (2007). Hilary Greaves and Wayne Myrvold.

I agree (with Myrvold) that…

- …The Evidential Problem is one that Everettians should take seriously.
- …What is needed is: either a way of making sense of probability in an Everettian multiverse, or a surrogate for probability-talk, on which observed relative frequencies that closely match Born-rule chances still count as evidence in favor of the theory, achieved in a way that does not presuppose the correctness of QM.

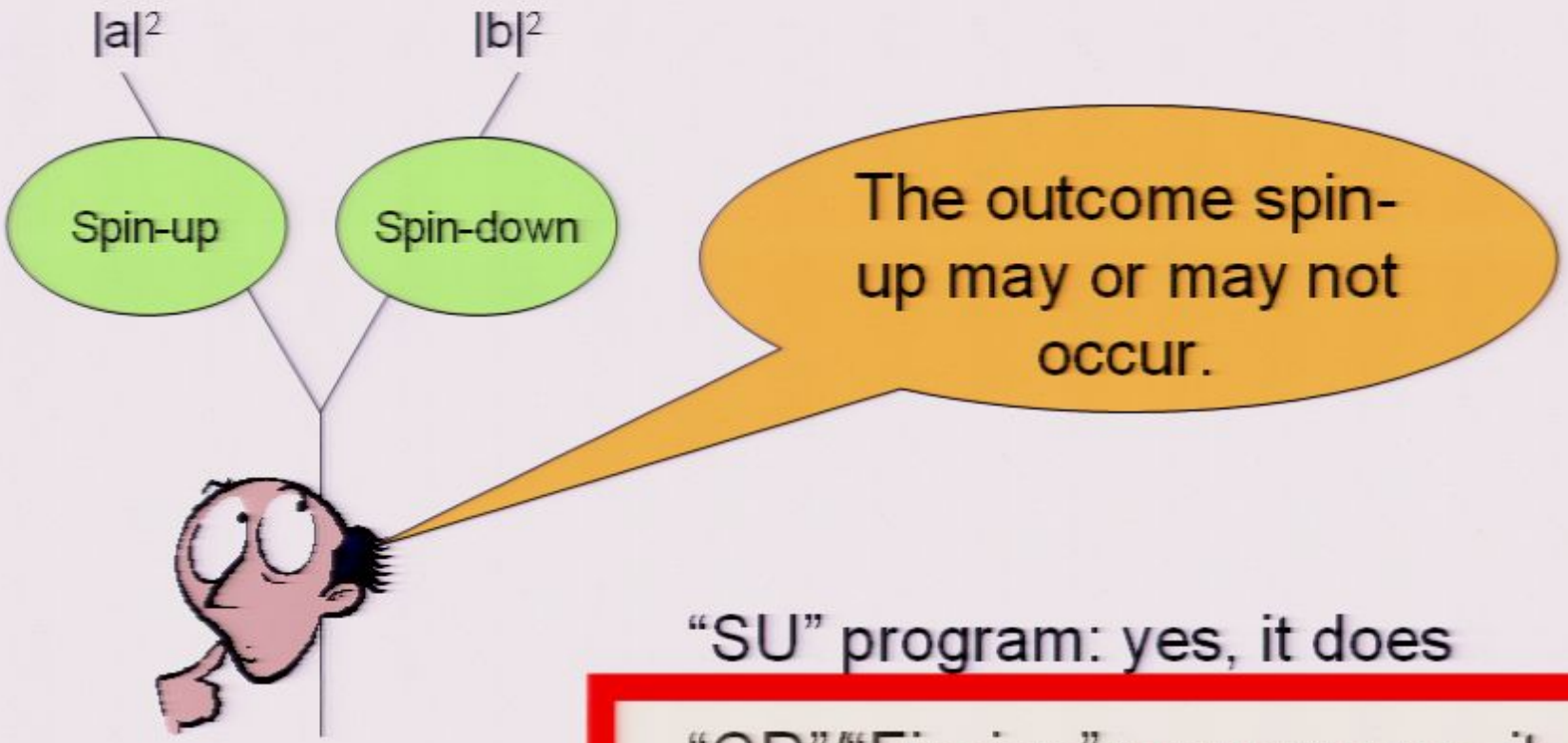
Preliminary remark 1: Incoherence and quantitative problems

- **Incoherence problem**: It doesn't seem to make sense to talk of Everettian probability at all, since all "possible" outcomes will be realized. So what can the amplitude-squares even mean?
- **Quantitative problem**: If it does make sense to talk about probability at all, how can/why must the probability-numbers be those of the Born rule?

Deutsch/ Wallace argument

Preliminary remark 2: “SU” and “OD”

- Q: Does it make sense to say:



“SU” program: yes, it does

“OD”/“Fission” program: no, it does not

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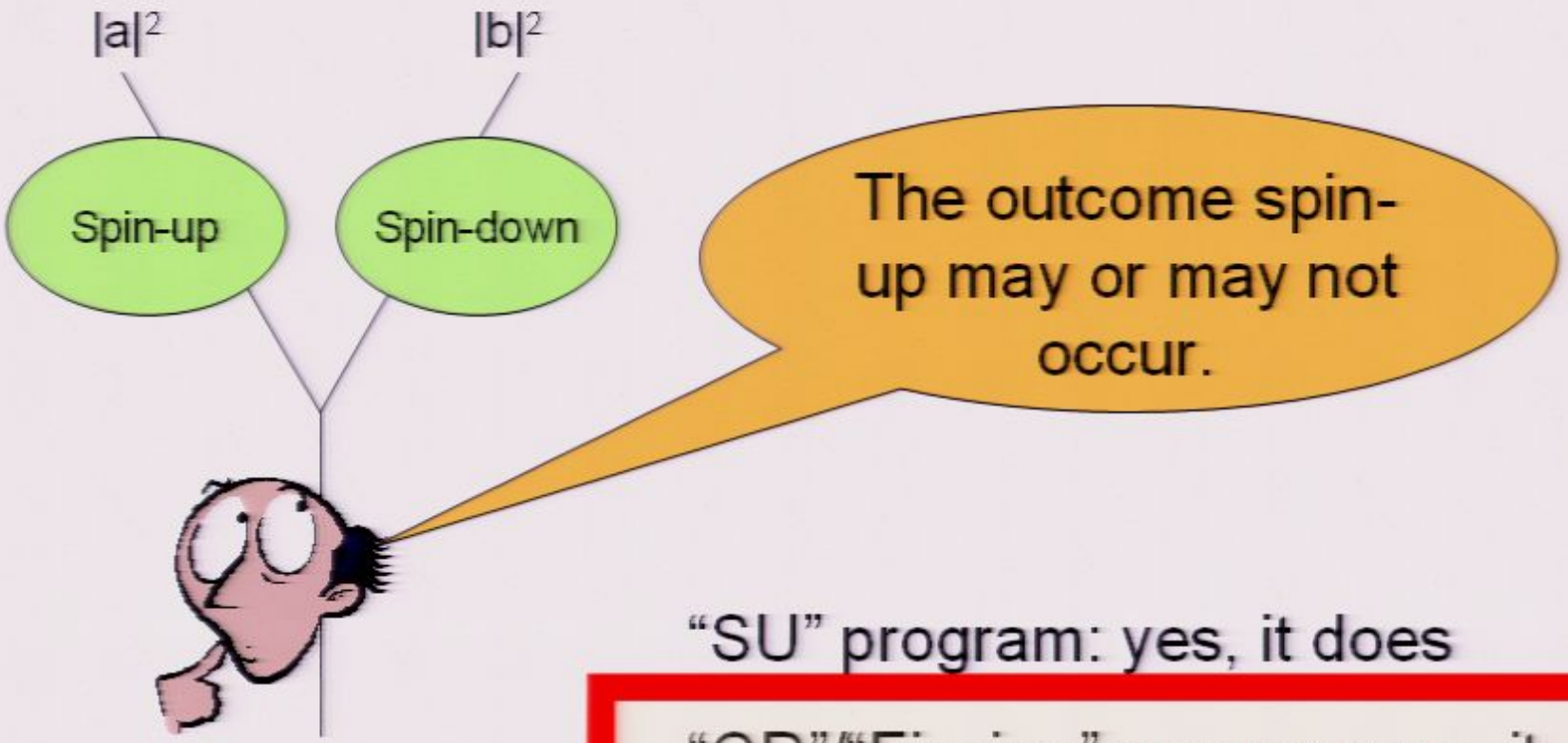

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Preliminary remark 3: Practical and epistemic/evidential problems

- **Practical problem:** How should I bet (act), insofar as I believe that Everettian quantum mechanics is true? ← Deutsch/ Wallace argument
- **Epistemic/Evidential problem:** Why should I believe Everettian quantum mechanics in the first place?

Review of the Evidential Problem

- We take ourselves to know how evidence bears on probabilistic (chance) theories
  - (CC: confirmation-theoretic role of chances): "If my theory assigned high chance to outcome X, and rival theories assigned low chance to X, then X confirms my theory relative to those rivals"
- QM is usually taken to be a chance theory, and as such has masses of evidence in its support
- But MWQM doesn't seem(?) to be a chance theory
- And it's *not clear* how that same evidence bears on it.
  - "Naïve conditionalization"... cannot be the correct general confirmation theory
  - But *what is*?

The Evidential Problem would be solved *if* we also had...

(CW: confirmation-theoretic role of branch weights.) "If my theory assigned high branch weight to outcome X, and rival theories assigned low branch weight to X, then X confirms my theory relative to those rivals."

- ...but *do* we/can we?

The Plan

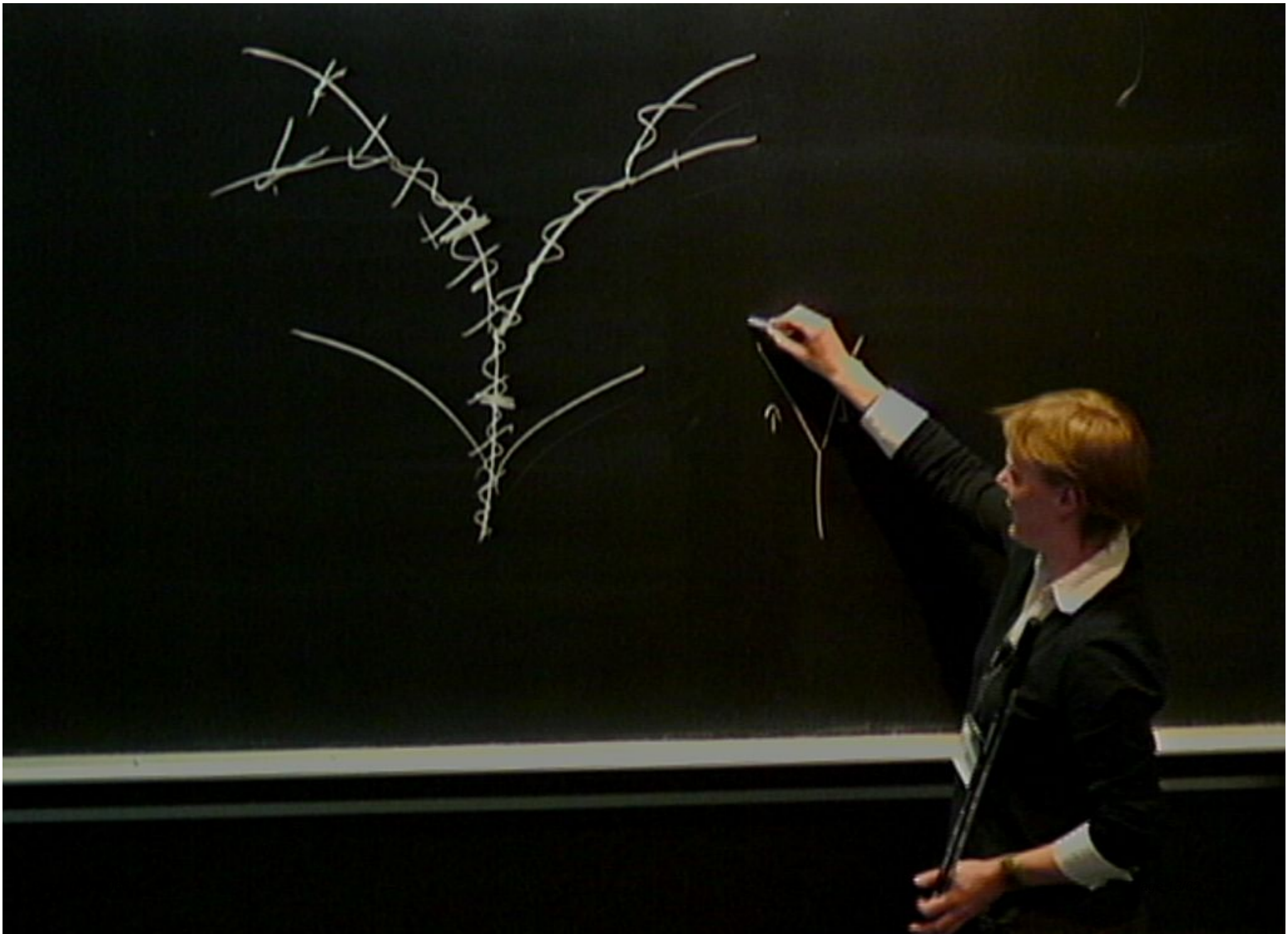
(1) Revisit the foundations of the confirmation-theoretic principle CC, for the case of chance theories only.

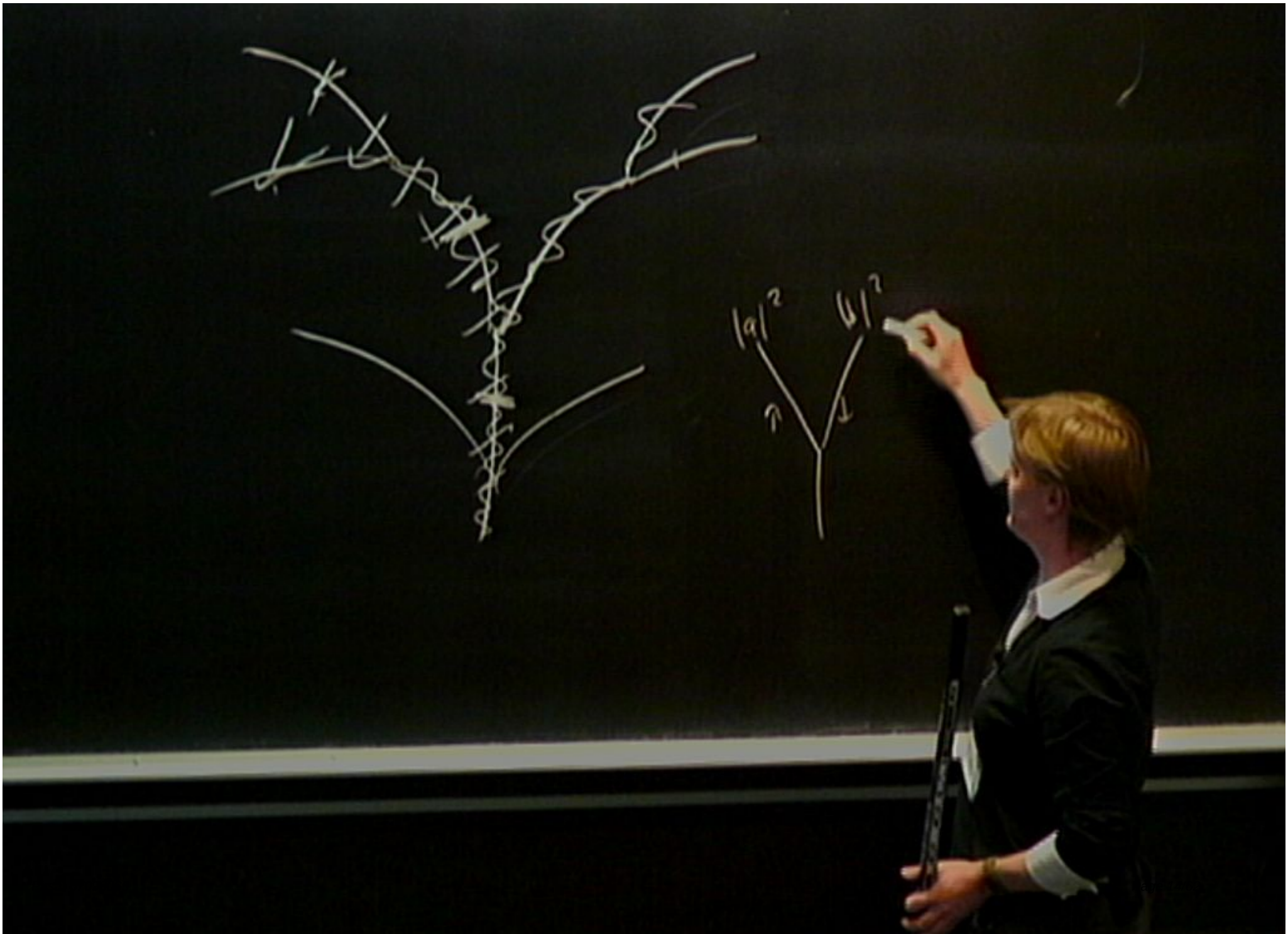
- This part of the discussion will be explicitly rationality-/decision-theoretic.
  - This *is* appropriate. (Cf:

    "We do not accept that the behaviour of a rational decision maker should play a role in modeling physical systems." (Richard Gill, 2005))

(2) Argue that exactly analogous foundations support CW in the branching case.

(3) Argue that this solves the Evidential Problem.

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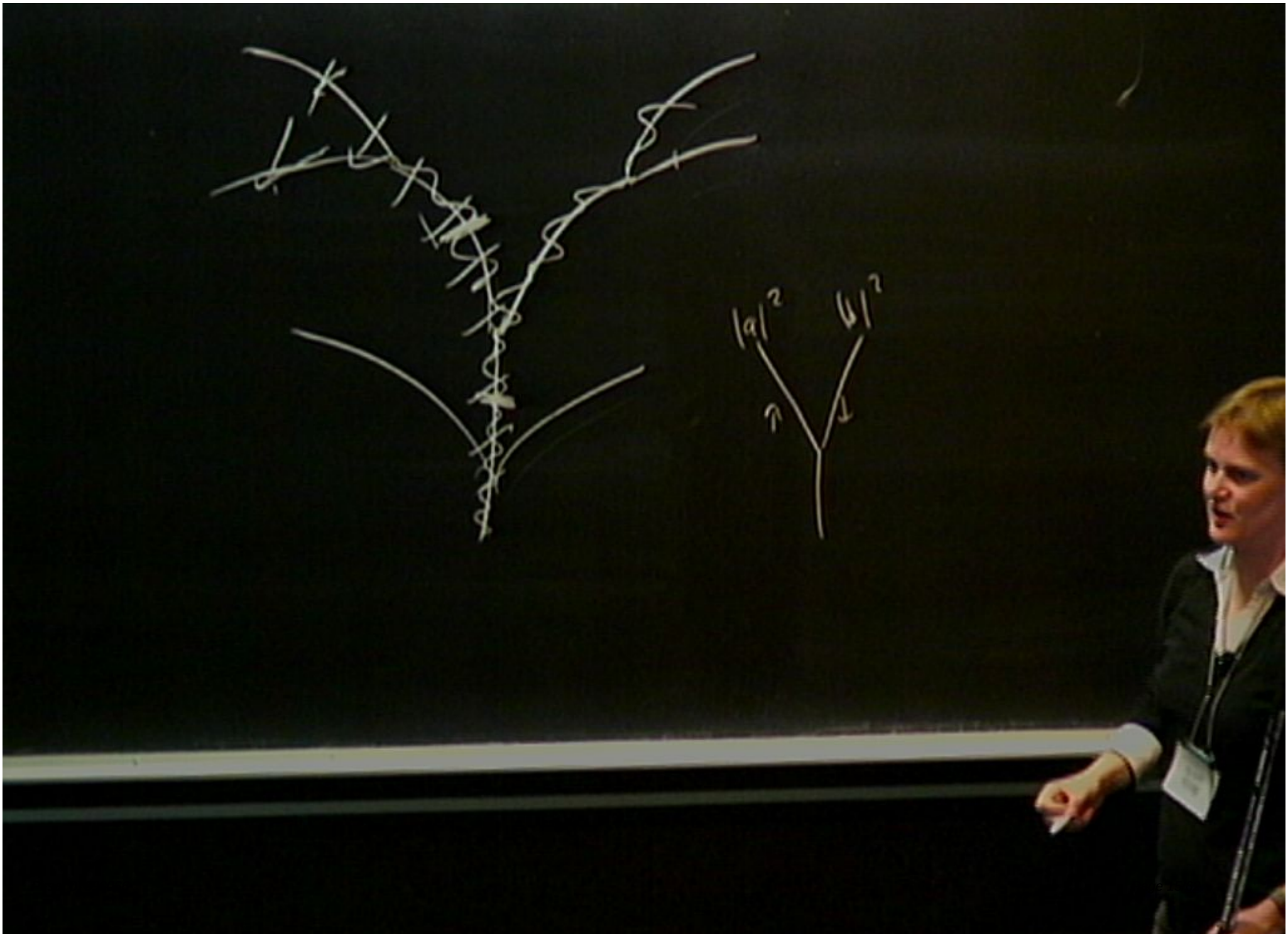
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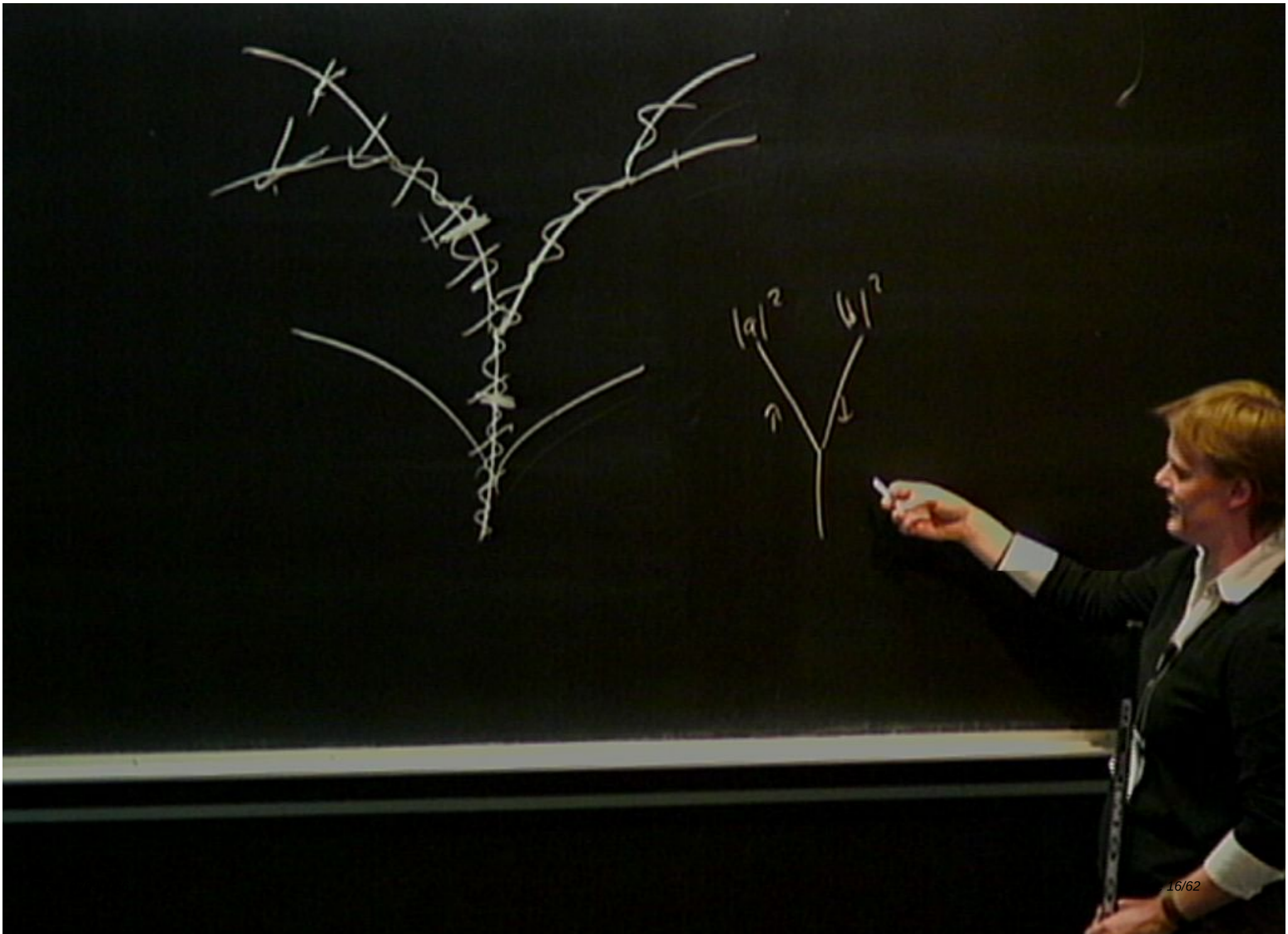
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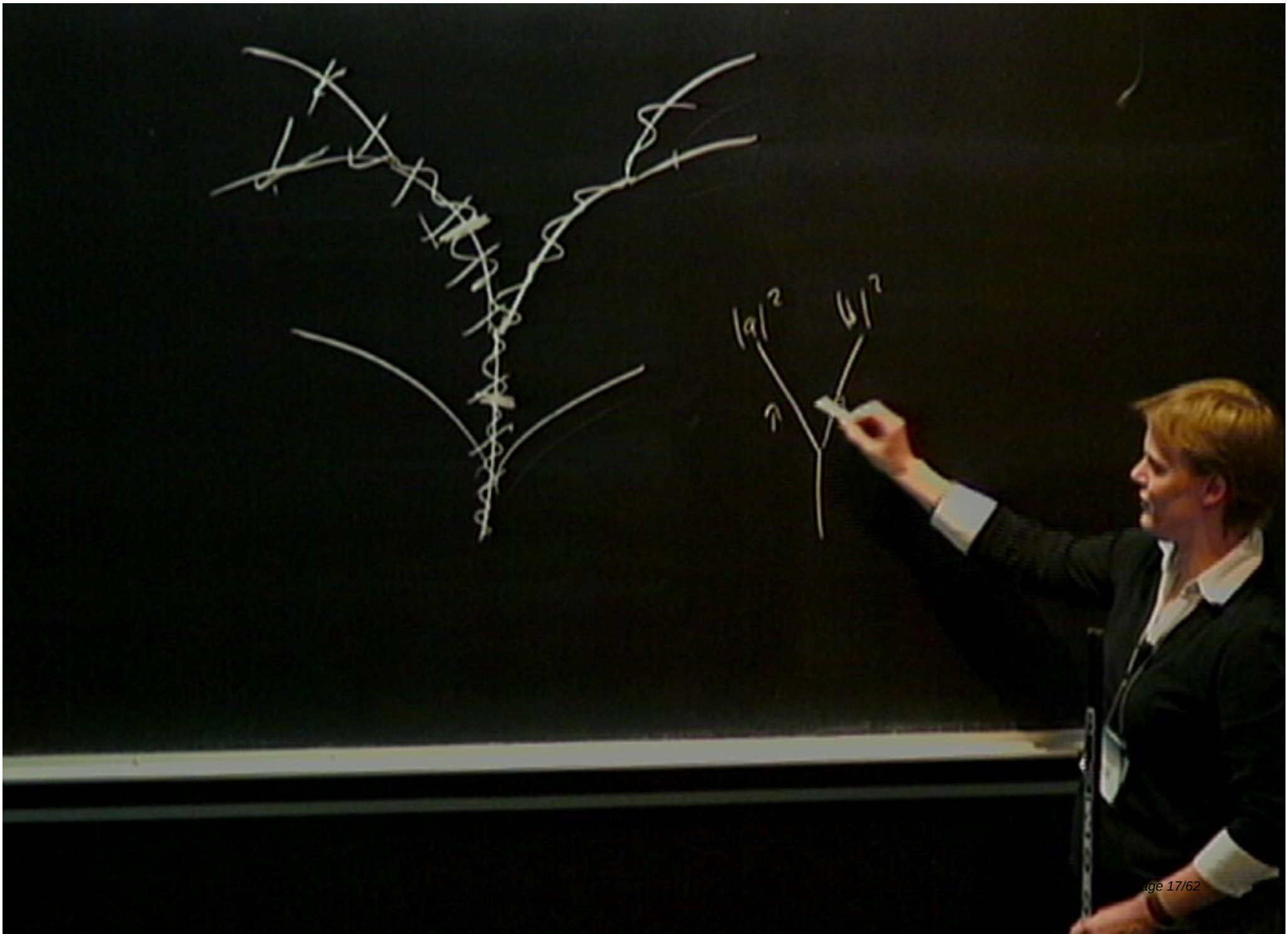
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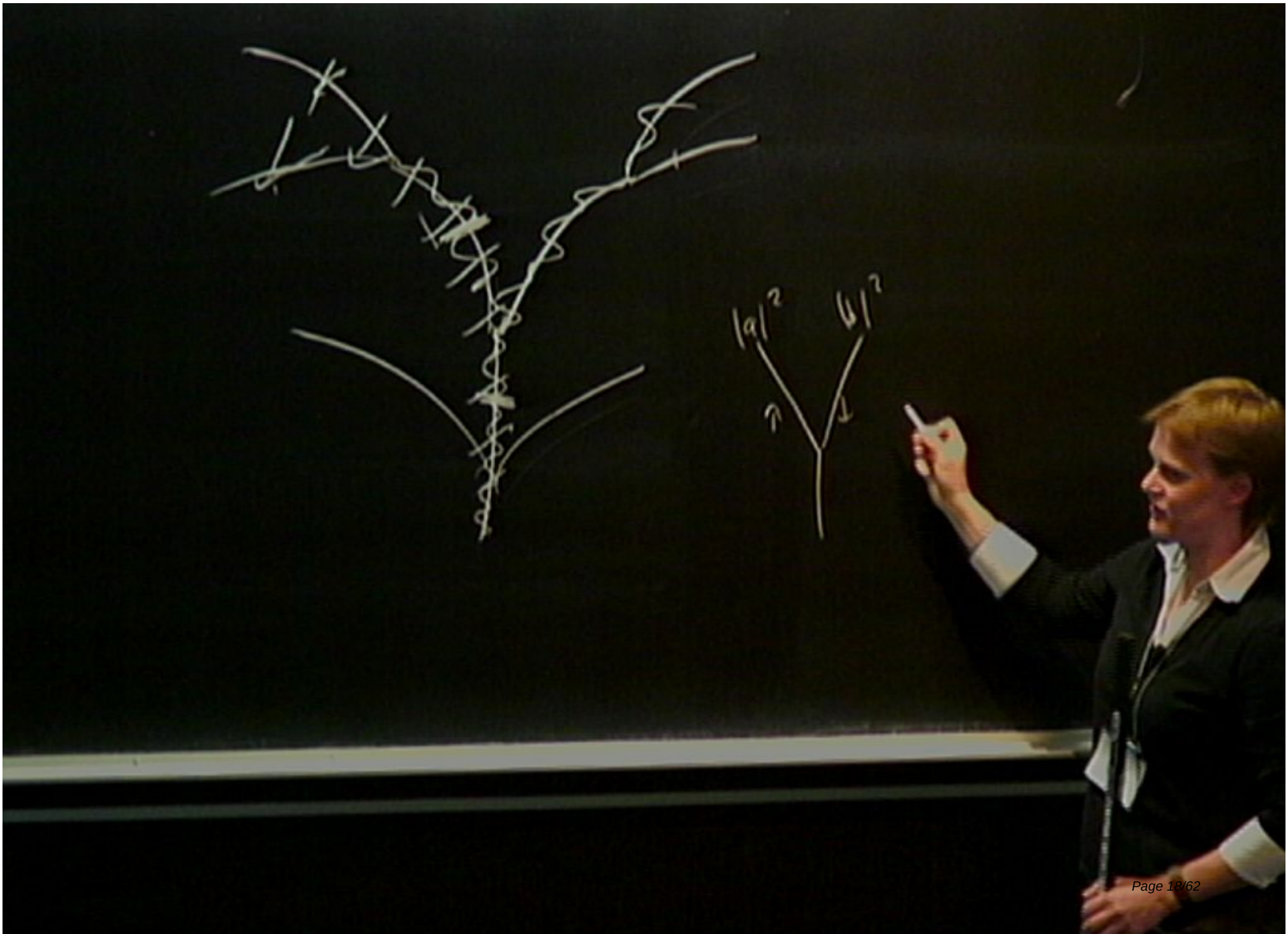
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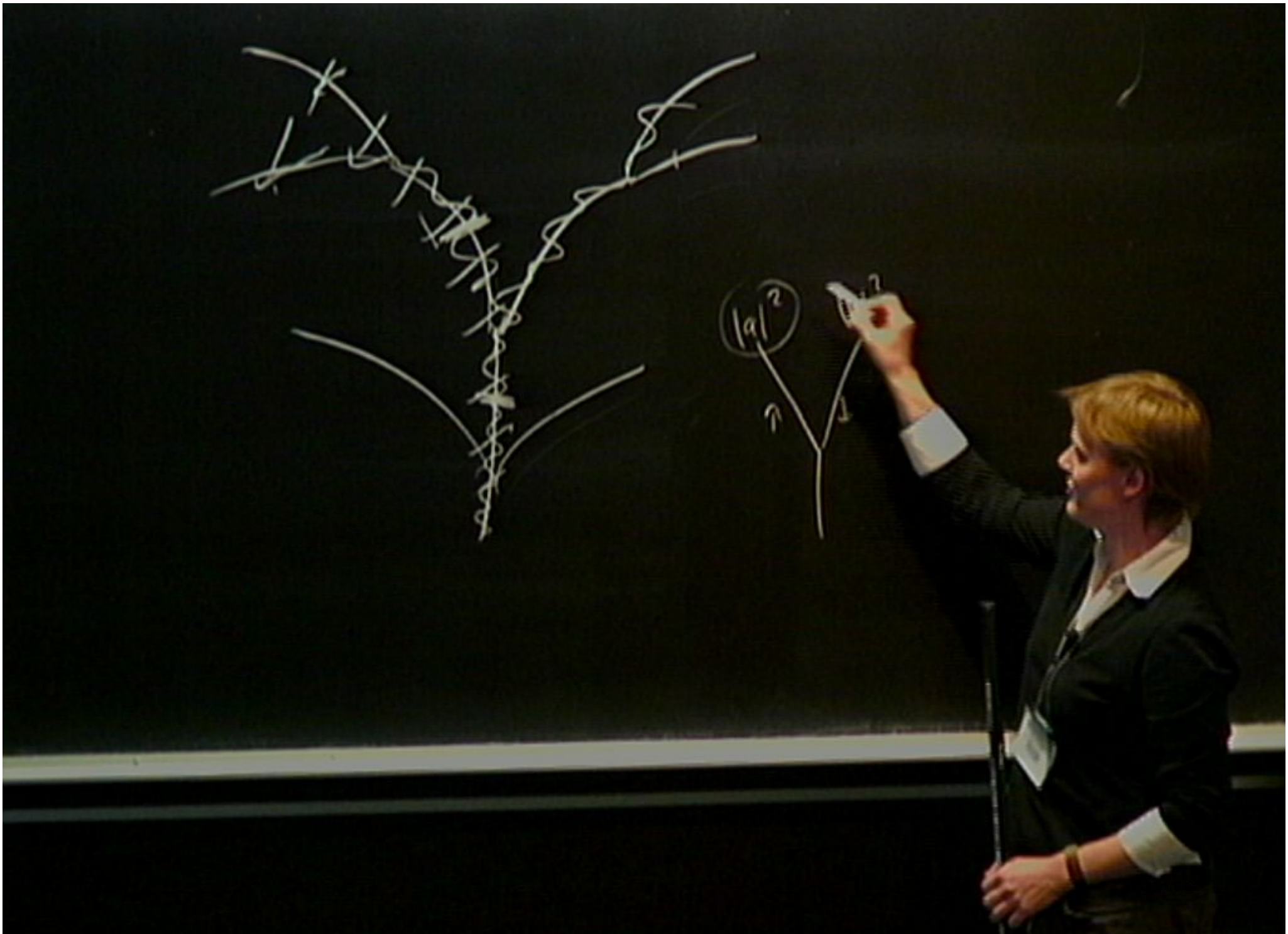
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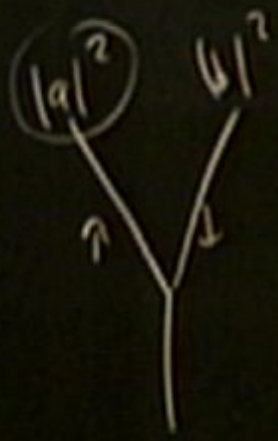
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(1) bayesian Foundations of CC

a) The framework
b) Savage axioms and representation theorem (MEU)
c) Combined experiments and learning (conditionalization)
d) Exchangeability and De Finetti representation theorem (beliefs about chances)
e) Recovering CC

(a) The framework

- **A**, set of *experiments* (e.g. this coin-flip)
- Set S^A of *outcomes* of each $A \in \mathbf{A}$
- Partitions Π^A of outcome-spaces S^A (e.g. heads, tails)
- Set **P** of *payoffs* (e.g. receive \$10)
- Set **W** of *wagers* (e.g. [H→\$100, T→-\$50])
  - Any assignment of payoffs to finite partitions of outcome-spaces is a wager in **W**
- Preference ordering ≤ on set **W** of wagers

(b) Savage axioms and representation theorem

- Impose 'rationality constraints' (P1-P6) on the agent's preference ordering ≤.
- Prove that any preference ordering obeying P1-P6 has an MEU representation. That is:
  - There exists a *u* on payoffs, and a function α on subsets of experimental outcome-spaces, such that for any two wagers f,g, f≤g iff V(f)≤V(g), where, for a wager $f:[D_i \rightarrow a_i]$ ($D_i \subseteq S^A, a_i \in \mathbf{P}$), V(f) is defined by

    $$V(f) := \Sigma_i\, \alpha(F_i)\, u(a_i).$$
- So: The agent "acts as if" α is her credence function, u is her utility function, and she is maximizing expected utility.

(c) Combined experiments and learning

- Q: “OK, but what does this have to do with epistemology?”
- A:
  - A preference ordering on wagers induces a preference ordering on belief-updating strategies.
  - Given P1-P6, the preferred strategy is always updating by conditionalization.

Combined experiments and learning (cont'd)

- Example: Betting on a new die
  - Prior experimentation is advantageous
- Combined experiments
  - For any two compatible experiments A, B, there is a third experiment C=A+B, with outcome-space $S^A \times S^B$
  - Require **A** (and hence **W**) to be closed under combination of compatible experiments

Suppose we have:

- Suppose we have:
  - Two compatible experiments A, B
  - An n-element partition $\{D^A_i\}$ of S^A
  - Wagers f, g on B
  - A choice to make: f or g?
  - The possibility of making this choice after learning the outcome of A, and hence
  - The choice of an updating strategy
- The choice of updating strategy is a choice of wager on the combined experiment A+B (2^n possible strategies)
- *So,* ***the agent's existing preference ordering on wagers induces a preference ordering on updating strategies.***

Conditionalization

- Strategy for updating on the outcome of A: an assignment of posteriors α_i to elements $D_i \in \Pi^A$, such that, after observing D_i, she will prefer f to g iff the EU of f *with respect to* α_i is greater than that of g.
- Conditionalization: $\alpha_i(E^B_j) = \alpha(D^A_i \,\&\, E^B_j)/\alpha(D^A_i)$.
- Theorem: If ≤ obeys P1-P6 and α is the credence function representing ≤, then ≤ strictly prefers conditionalization of α to any other updating strategy.

Do what you wanted to do!

(P7: Intertemporal consistency.) For times $t_1<t_2$, if at t_1 the agent preferred that at t_2 she update according to updating strategy $\alpha=\{\alpha_i\}$, then at t_2 she does update according to this strategy.

(d) Exchangeability and the De Finetti representation

- An agent might judge experiments in a sequence to be alike in all relevant respects
- Exchangeability (definition):
  - Let $A=\{A_1,A_2,\ldots\}$ be a sequence of mutually compatible experiments with isomorphic outcome-spaces. Suppose that for any wager on the outcomes of a finite subset of A, the value of the wager is unchanged if the payoffs attached to any two elements of the sequence are switched. Then, we say that the sequence is *exchangeable*. [Agent-relative.]
- Scope for learning is consistent with exchangeability

De Finetti representation

- $\{A_1, A_2, \ldots\}$: an sequence of experiments with common outcome-space S^A
- Consider an m-element subset of $\{A_1, A_2, \ldots\}$
- $F=\{F_i: i=1,\ldots,n\}$: partition of S^A (e.g. H,T)
- G: induced partition of combined outcome-space, $\{F_i\}x\{F_i\}x\ldots x\{F_i\}$ (m times)
- For each $s \in G$, let $k_i(s)$ denote the number of times F_i occurs in s
- Λ_n: (n-1)-dimensional simplex $\{\lambda=(\lambda_1,\ldots, \lambda_n): \Sigma_i \lambda_i=1\}$.
- α_λ: the probability function on G given by

  $$\alpha_\lambda(s) = \lambda_1^{k_1(s)} \ldots \lambda_n^{k_n(s)}.$$
- V_λ: the candidate value functions given by

  $$V_\lambda(f) = \Sigma_{s\in G}\, \alpha_\lambda(s)\, u(f(s)).$$
- De Finetti: If $A=\{A_1, A_2, \ldots\}$ is exchangeable, then there is a measure μ on Λ_n such that, for any wager f on the outcomes of a set of m experiments in A,

  $$V(f) = \int_{\Lambda n} d\mu(\lambda)\, V_\lambda(f).$$
- The agent's credences are "as if": she believes that there are objective chances satisfying the PP, and she has credences about the values of these chances, and she's maximizing expected utility with respect to the credence-average of the possible chance distributions.
  - (P8: Non-dogmatism)

(e) Recovering the relevance of RF data to beliefs about chances

- Consider a long run of coin-tosses
- You have a prior credence function about the outcomes of this long run (P1-P6)
- You start out uncertain what the chance of heads on a single toss is (i.e. μ is relatively flat) (P8)
- You observe a long run of outcomes, in which heads comes up approximately 2/3 of the time
- You conditionalize on this string of outcomes (P7)
- Result: μ becomes sharply peaked close to λ=2/3

(e cont'd) Recovering CC

- If my theory said that the chance of heads was 2/3 and yours said it was 1/3, then my theory assigned higher chance to the actual outcome than did yours.
- If our preferences obeyed P1-P6 and P8, and we obeyed P7, then we will have become more confident that my theory is true and that yours is false.

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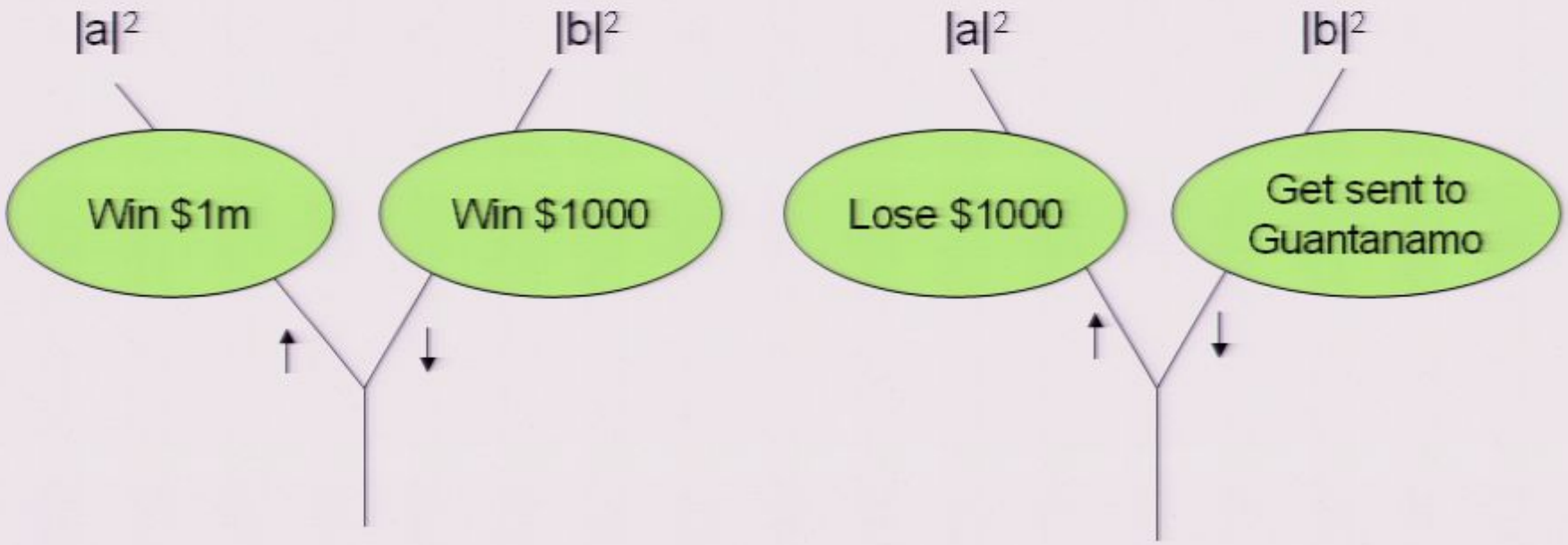
(2) Bayesian foundations for CW??

This confirmation theory seems fine…

…AND THEN A BRANCHING THEORY COMES ALONG.

Decisions in a branching universe…

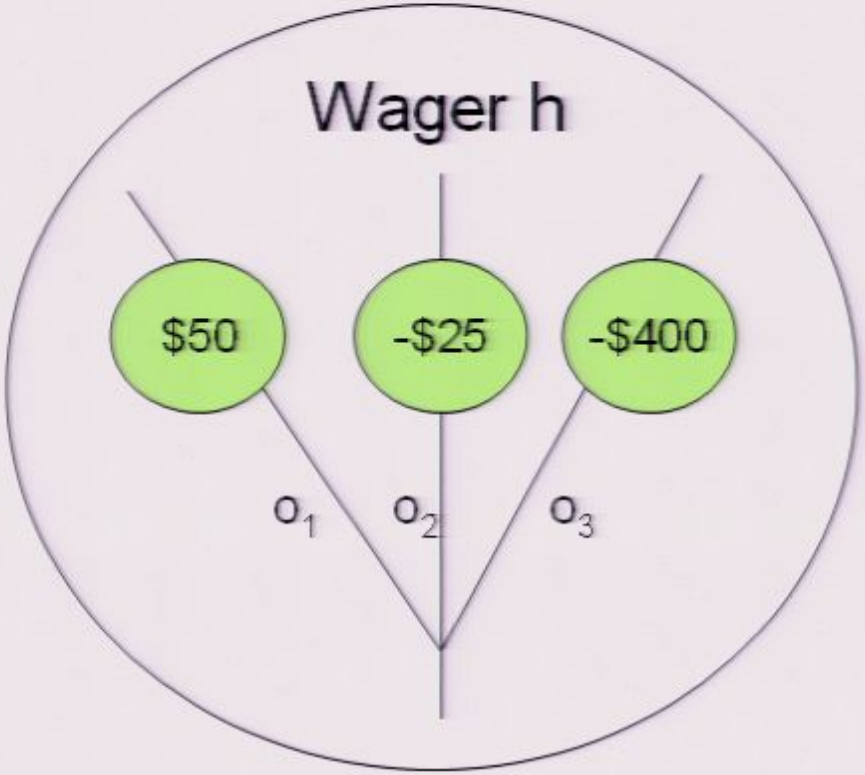
- … *still make sense.*
- Which tree would you rather realize?



Two ways to apply the framework to branching theories

- Application #1 (→ weaker decision theory):
  - An experiment has only one 'outcome'
  - A 'payoff' is a whole *assignment* of monetary rewards (etc) to branches within a tree
  - The only applicable rationality constraint on preferences between such assignments is the trivial P1 (transitivity and completeness)
- Application #2 (→ stronger decision theory):
  - An 'outcome' is an event-on-a-branch (e.g. spin-up)
  - A 'payoff' is a reward-on-a-branch
  - The non-trivial rationality constraints come into play
- Claim: The axioms P1-P8 are as plausible under Application #2 to the branching case as they are under the usual application to the non-branching case.

Exploring Application #2: Transitivity (P1a)



Quiz: If g≤f and h≤g then…?

Exploring Application #2: Dominance (P3)

- Suppose that I($1000)>I(-$5).

- Quiz: j>k, or k>j (or j≈k)?

“Justifying” Dominance

- Usual justification (for chance case):
  - “The two wagers give me the same reward if spin-up occurs, and k gives me a better reward if spin-down occurs. So, in choosing k over j, I can’t lose, and I might gain.”
- Justification under application #2 to the general case:
  - “If the world is branching then: The two wagers give me the same reward on the spin-up branch, and k gives me a better reward on the spin-down branch. So, in choosing k over j, I won’t lose on any branch, and I will gain on some branch.”

Exploring Application #2: 'Likelihood' (P4)

- The axiom:
  - Let A, B be experiments.
  - Let $F \subseteq S^A$, $G \subseteq S^B$.
  - Let $b<a$, $b'<a'$ (payoffs).
  - Suppose $[F \rightarrow a, \neg F \rightarrow b] \leq [G \rightarrow a, \neg G \rightarrow b]$.
  - Then, $[F \rightarrow a', \neg F \rightarrow b'] \leq [G \rightarrow a', \neg G \rightarrow b']$.
- What's going on:
  - "The agent regards G (on B) as more likely than F (on A)"
  - "The agent regards the G branch (on B) as more decision-theoretically important than the F branch (on A)"

…and so on.

- Right??

What follows from Application #2?

- Savage:
  - The agent's preference ordering over **W** has an MEU representation, *where u assigns numbers to payoffs-on-possible-branches and α assigns numbers to possible branches*
- Learning:
  - The agent's preferences for wagers on combined experiments are as if she prefers to update her α-function by conditionalization (defined formally as before, viz. $\alpha_i(E^B_j) = \alpha(D^A_i \,\&\, E^B_j)/\alpha(D^A_i)$.
- De Finetti:
  - For sequences of experiments that are exchangeable w.r.t. the agent's preferences, the agent's α-function has a representation as a mixture of α_λ's.

Reinterpretation

- The α-function cannot any longer be a *credence* function…
  - …but it is something that behaves like one, insofar as decision-making is concerned.
- The α_λ-functions cannot any longer be *chance* ascriptions…
  - …but they are things that play the same role in decision-making.
- Suggested terminology:
  - α: *"caring measure", or "quasicredence"*
  - α_λ: *chances or branch weights*
- In the branching case: The agent's preferences are "as if" she is maximizing expected utility according to a measure that quantifies how much she cares about what happens on each branch. This measure in turn is determined by her degrees of belief about what branch weights are, for each possible experiment. She treats the branch weights as giving the objective relative importances of the branches. Updating the α-function by the preferred strategy has the consequences that RF data boosts credence that the actual branch weights approximately match the observed relative frequencies, and that CW is true.

Example: Repeated coin-flips

- This coin is about to be flipped 100 times.
- Your preferences for wagers on this (combined) experiment treat the sequence of flips as exchangeable.
- Your alpha-function (let's say) assigns coefficient significantly above zero only to $\alpha_{1/3}$ and $\alpha_{2/3}$.
- You observe the first 50 flips. 15 come up heads.
- You update on this, by conditionalization.
- Result: You become more confident that the branch weight for heads is 1/3.
- Why this makes good sense:
  - In updating in this way, you knew that you would come to the wrong conclusion on *some* branches (e.g.: if the actual branch weights are {1/3,2/3} for H,T resp., you'll come to the wrong conclusion on branches with relative frequency close to {2/3,1/3}. But you also thought it likely that the branches on which you'd come to the wrong conclusion had low total weight.
    - No other available updating policy does better.

What does this have to do with Everettian QM?

- We can learn, empirically, that the chances-or-branch-weights are Born-rule ones.
- This will confirm Everettian QM *provided that* Everettian QM is a theory according to which the branch weights (in our sense) are Born-rule ones.
- Two options:
  - *Prove*, using independent characterizations of (i) branch weight and (ii) amplitude-squared measure, that the two numerically coincide. (Deutsch-Wallace argument)
  - Have Everettian QM *postulate* that the branch weights are given by the amplitude-squares.

The picture that emerges

- We have a unified account that...
  - ... illuminates the connection between the practical and evidential roles of probabilities
  - ... applies equally well to indeterministic theories with chances and to branching-universe theories with branch weights
  - ...and to situations (like the actual one) in which we don't know whether the world is indeterministic or branching
  - ... entails CW in the same way that it entails CC.
- Everettian QM can be understood as a theory according to which the branch weights are given by the Born rule.
- This has the consequence that our existing empirical evidence confirms Everettian QM just like any other version of QM.

On “explanation”

- “It is… frequencies, and not our betting behaviours, which make up the raw data of our experience. And it is those frequencies…, and not our betting behaviours, which primarily and in the first instance stand in need of a scientific explanation. And it is precisely here that the fission hypothesis famously and perenially and by its own admission falls short. … And the thought that one might be able to get away without explaining those frequencies… -- which is the central thought of the decision-theoretic strategy – is (when you think about it) mad.”

David Albert (this conference)

On “explanation”

- *If* RFs confirm Everettian QM, then:
  - It cannot be that Everettian QM doesn’t explain those RFs.
  - It must be (rather) that: insofar as ascriptions of relatively high probability count as ‘explanatory’, ascriptions of relatively high branch weight count as ‘explanatory’ too.
    - Also: who cares?
- *If* Everettian QM doesn’t explain RFs, then:
  - It must be that RFs don’t confirm Everettian QM.
  - So it must be that the confirmation theory I have offered is *false* (*not* that it’s irrelevant).
  - But then, we must ask:
    - Which axiom is false? And
    - What’s the true confirmation theory?

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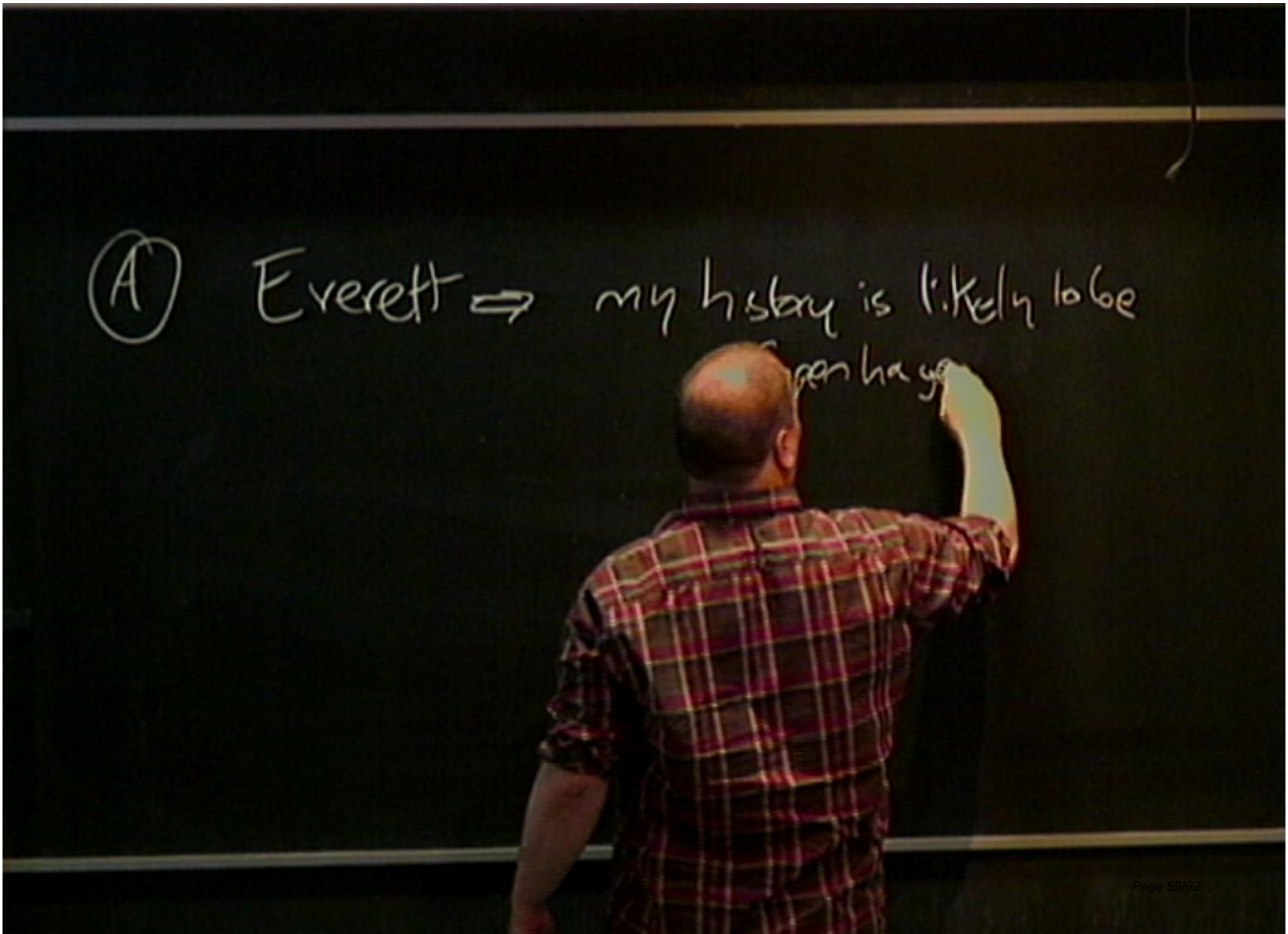
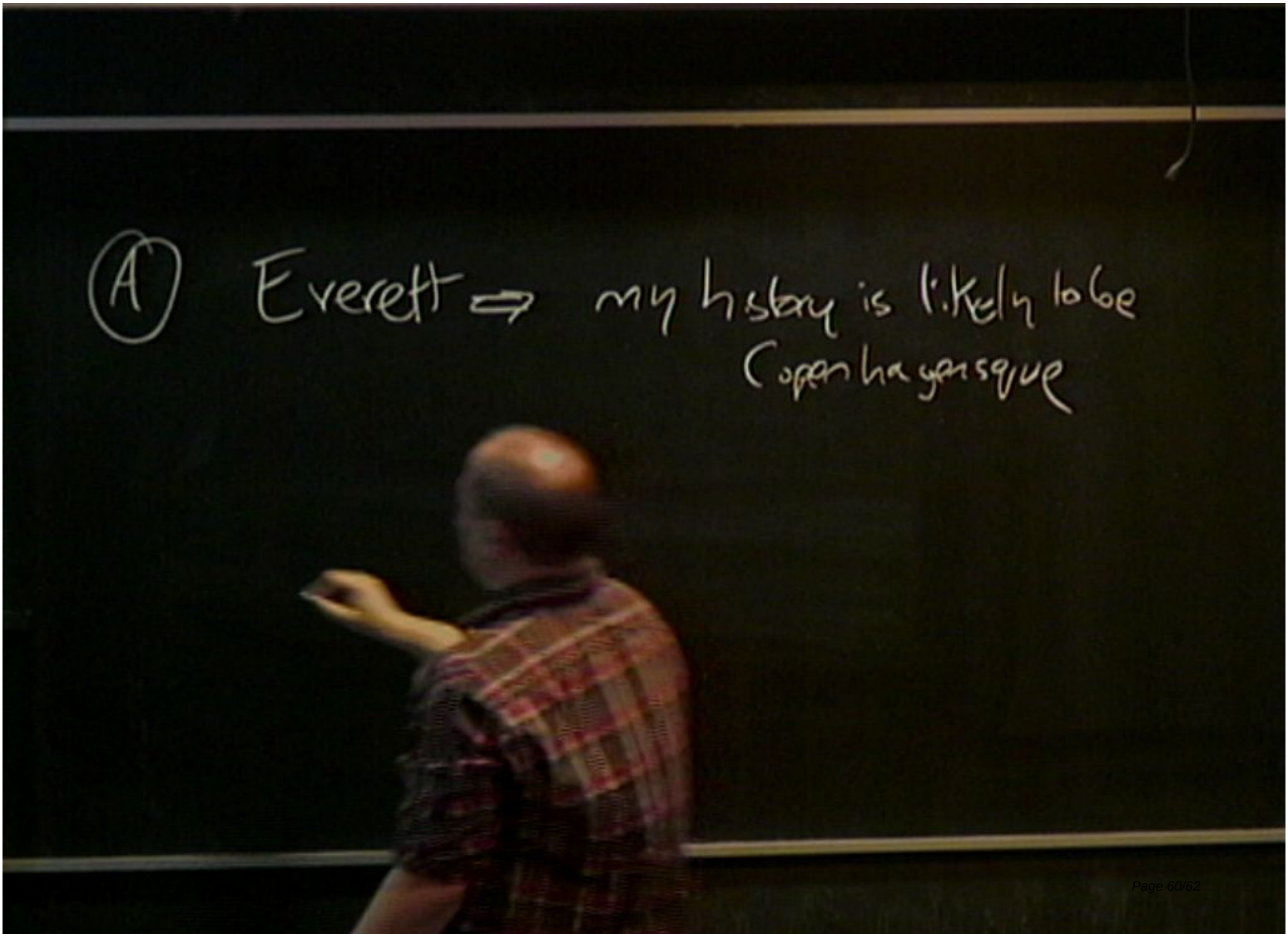
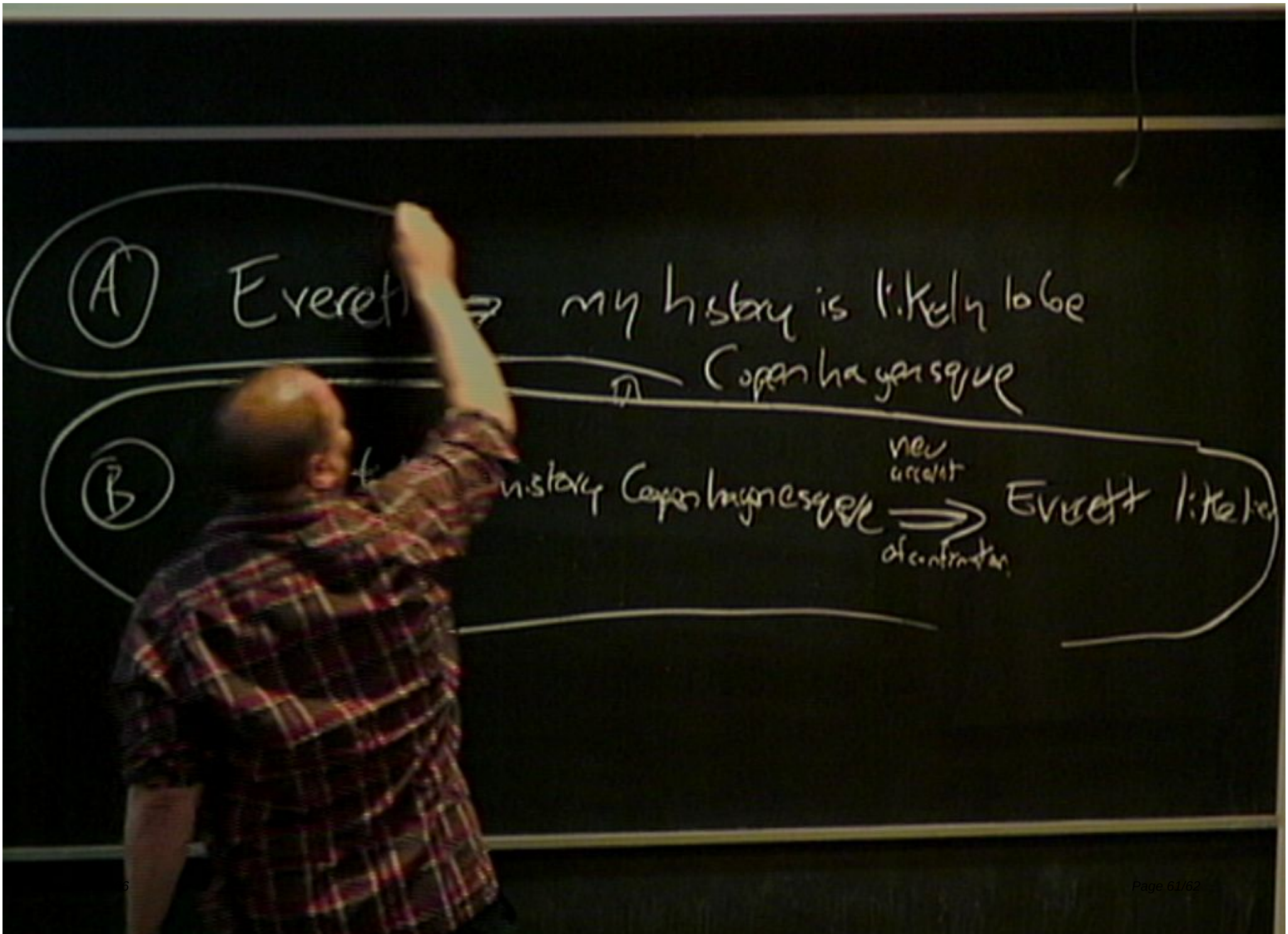

(A) Everett ⇒ my history is likely to be


(A) Everett ⇒ my history is likely to be
Copenhagensque


A
Everett
my history is likely to be
Copenhagensque
B
history Copenhagenesque
new
urgent
Everett likely
of confirmation

(A) Everett $\Rightarrow$ my history is likely to be Copenhagenesque

(B) If I find my history Copenhagenesque $\xrightarrow[\text{of confirmation}]{\text{new evidence}}$ Everett likely