Title: A Jigsaw Puzzle of the Cosmic Web

Date: Sep 12, 2007 10:30 AM

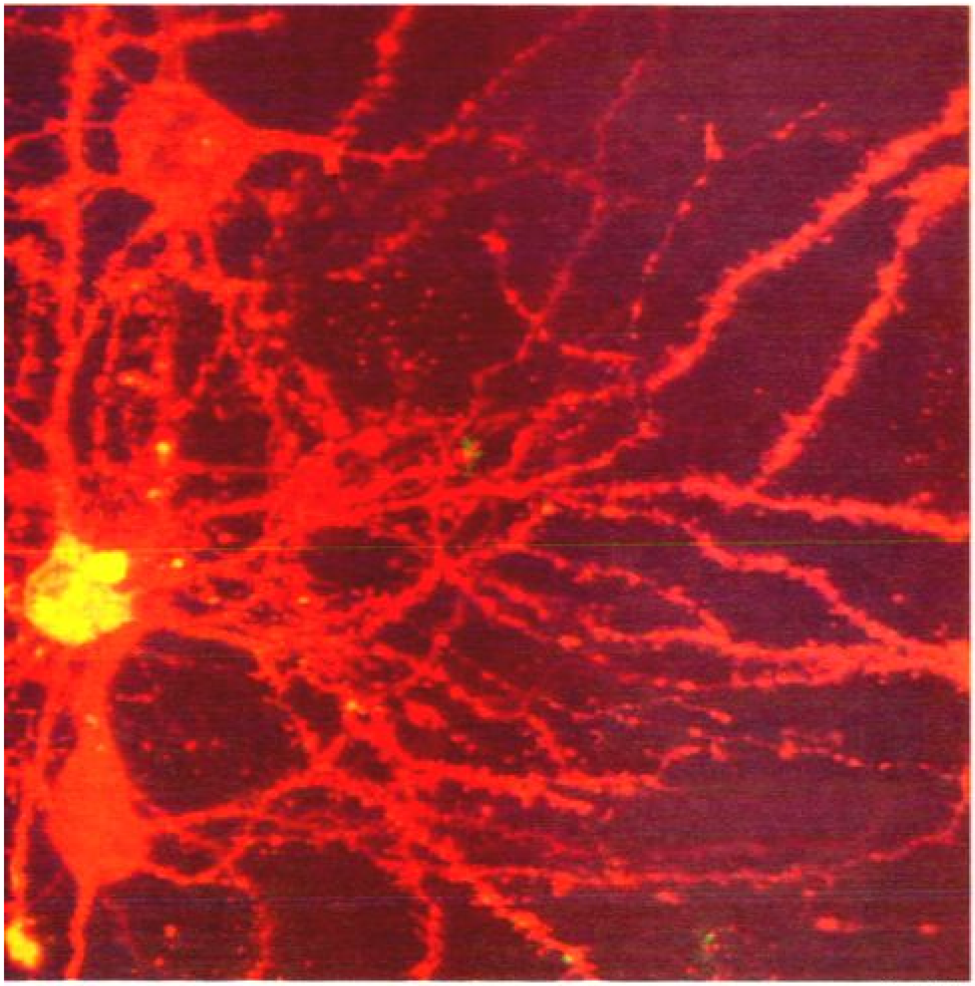
URL: http://pirsa.org/07090040

Abstract:

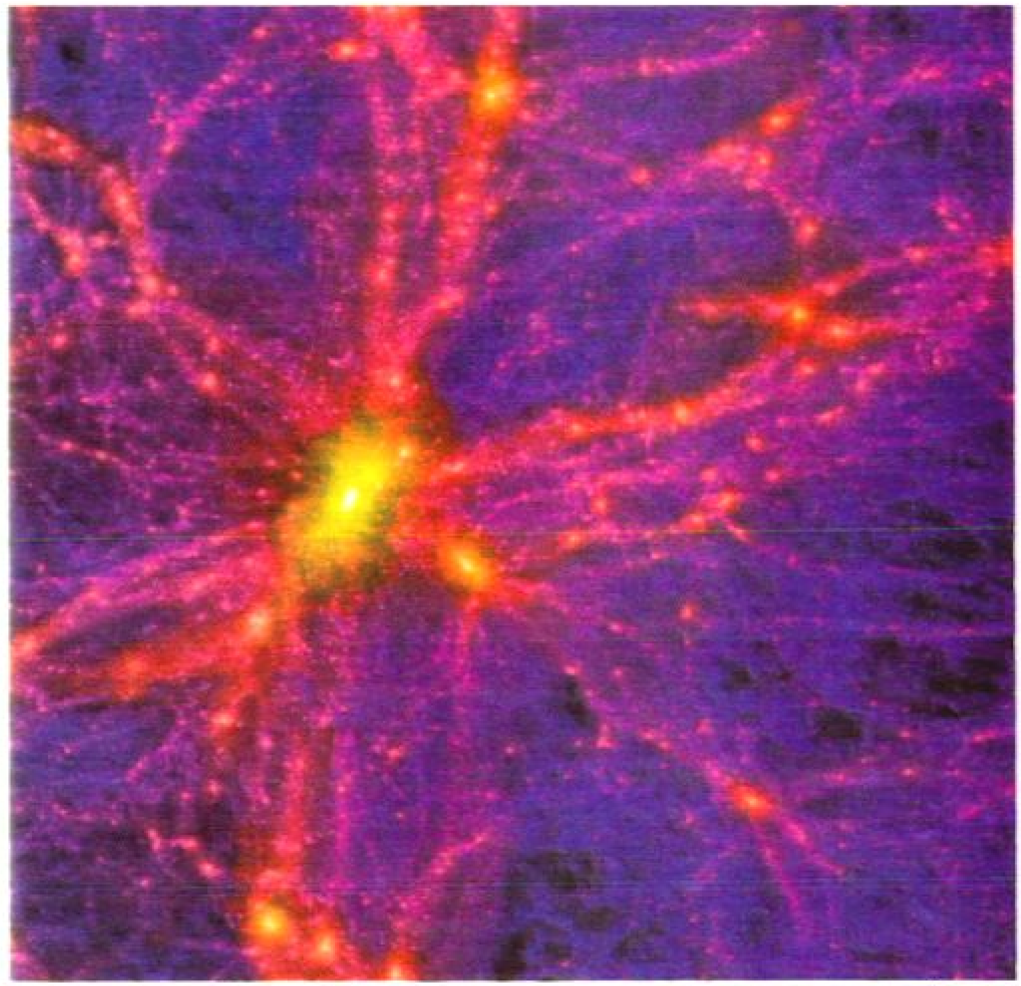
Outline

- Introduction
- Overview of dynamical models
- Analyzing the structure
- From point distributions to density field
- Clumps, filaments, walls
- Conclusion

Cosmic web and brain nerve cell

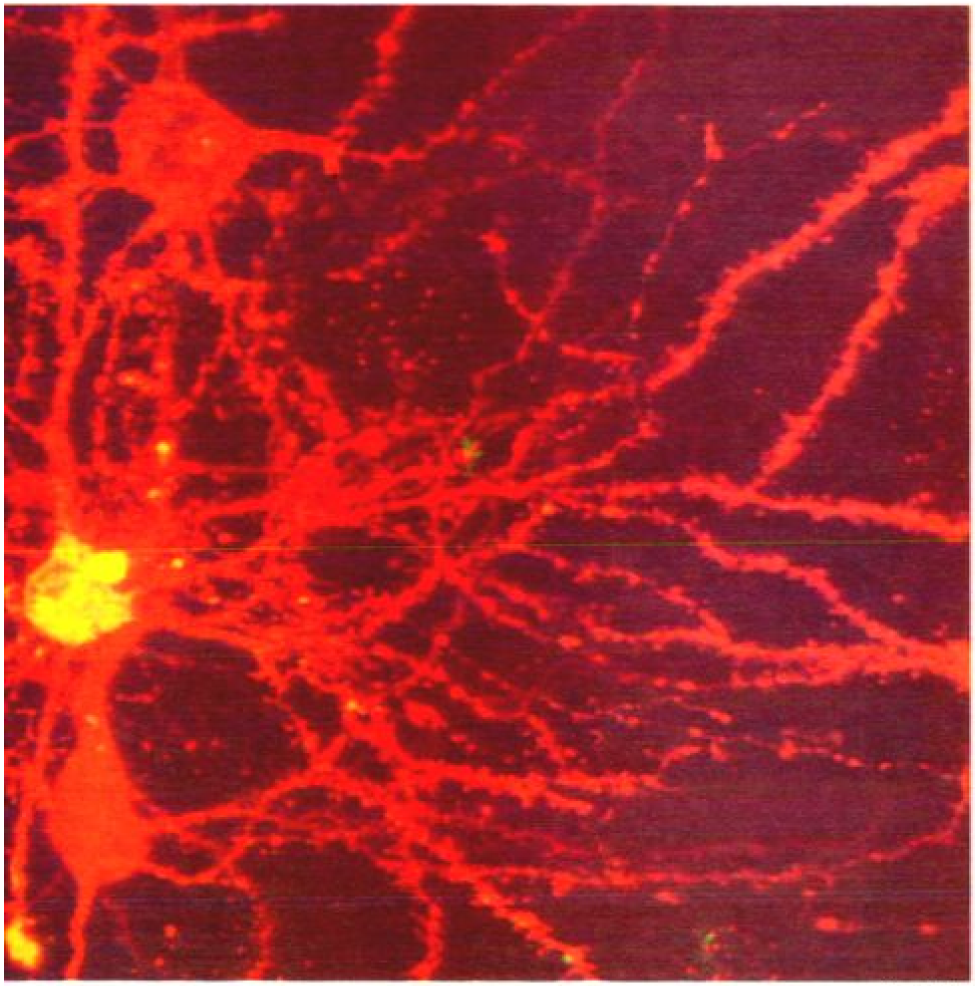


Mark Miller



Virgo Consortium

Cosmic web and brain nerve cell



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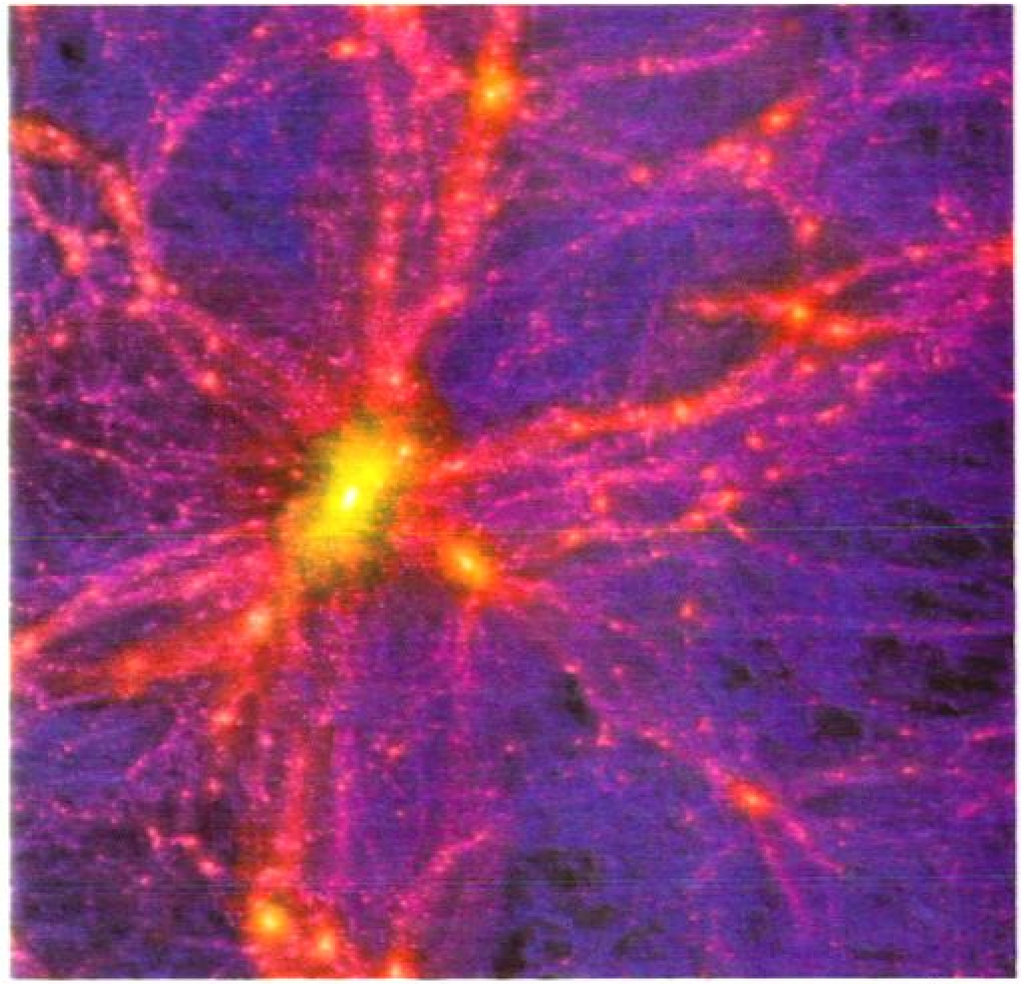
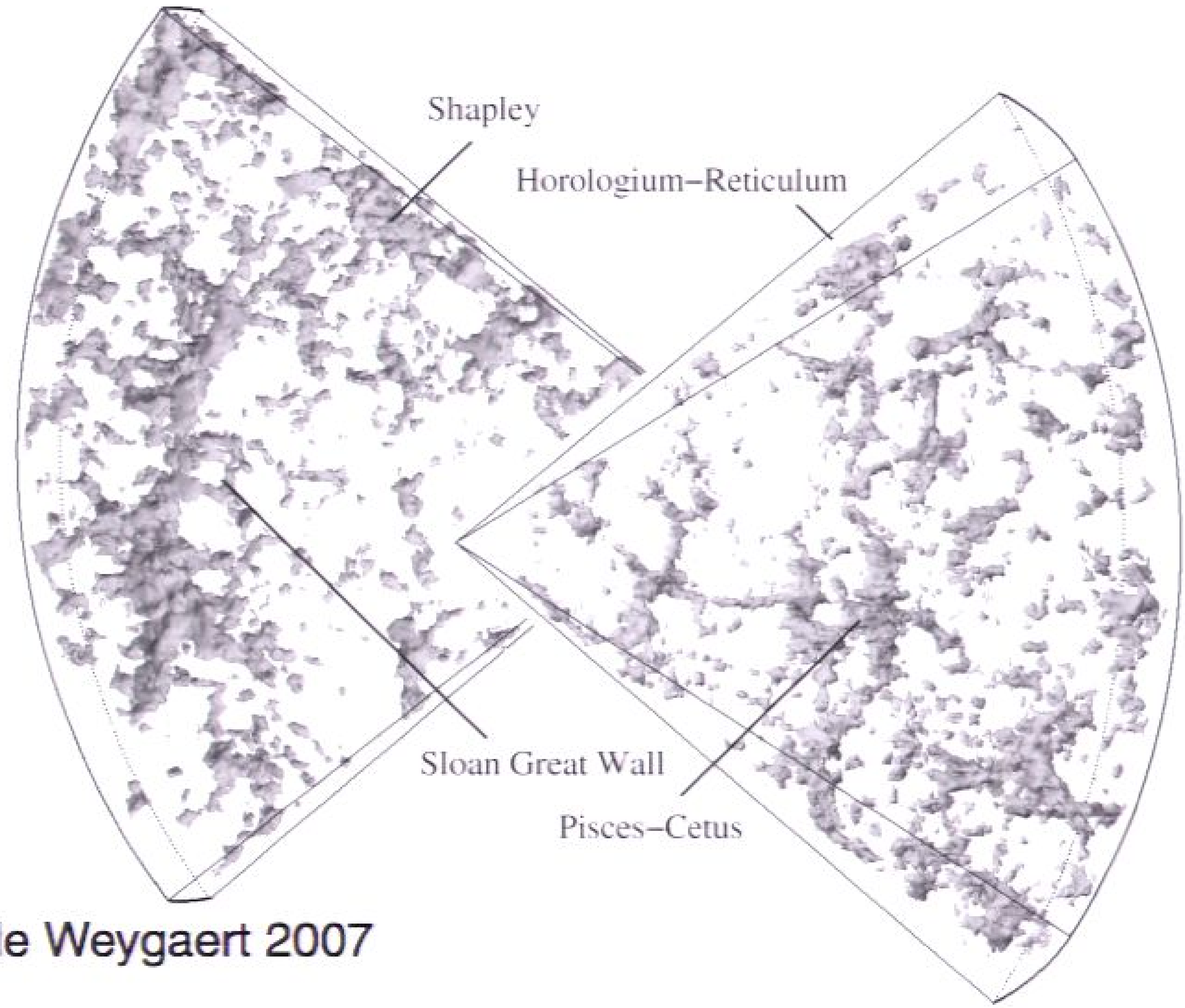


Virgo Consortium

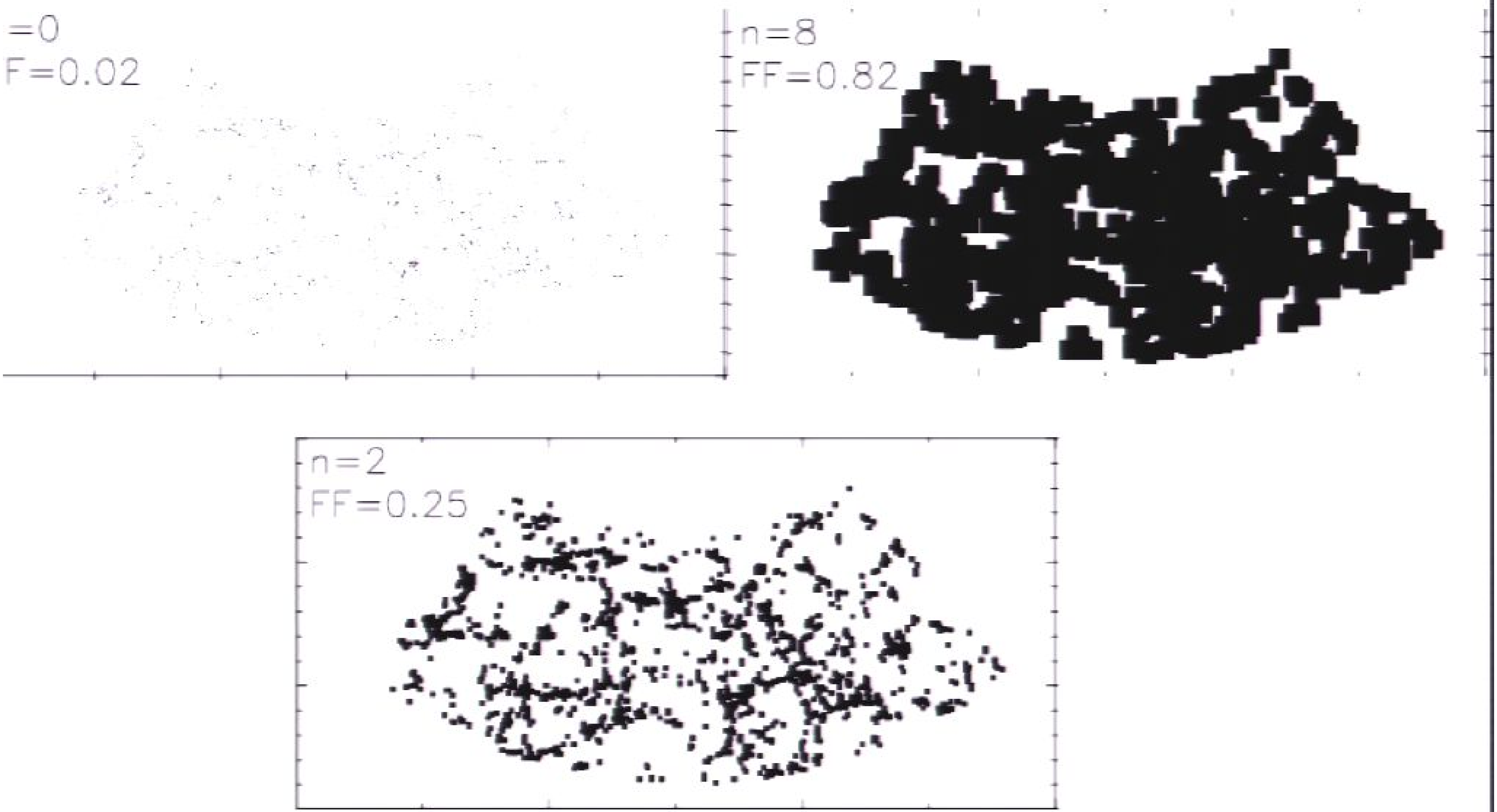
DTFE reconstruction of 2dFGRS: objective, much better than dot plots

Far better than dot plots!



Schaap, van de Weygaert 2007

What is your favorite size of dots?



LCRS =Las Campanas Redshift Survey

Ingredients needed for explaining cosmic web

- Background cosmology: H_0, Λ, Ω_{CDM}
- Initial perturbations: fluctuation mode (e.g. adiabatic), statistics (e.g. Gaussian), spectrum (e.g. $n \approx 1$), normalization (COBE or σ_8)
- Transfer function (Ω_{HDM}, Ω_b, Ω_r)
- Non-linear evolution: N-body simulations, **analytic or semianalytic models**
- Prescription for relating mass to light (galaxy formation)

Gravitational instability in expanding Universe

Comoving coordinates $\mathbf{x}$ and peculiar velocities $\mathbf{v}_p$

$$\mathbf{x} = \frac{1}{a(t)}, \quad \mathbf{v}_p = \mathbf{v} - H(t)\mathbf{r} = a(t)\frac{d\mathbf{x}}{dt}$$

$a(t) = (1+z)^{-1}$ is the scale factor; $H(t) = \dot{a}/a$

Evolution of DM is described by the Vlasov-Poisson model that usually is approximated by the Euler-Poisson model

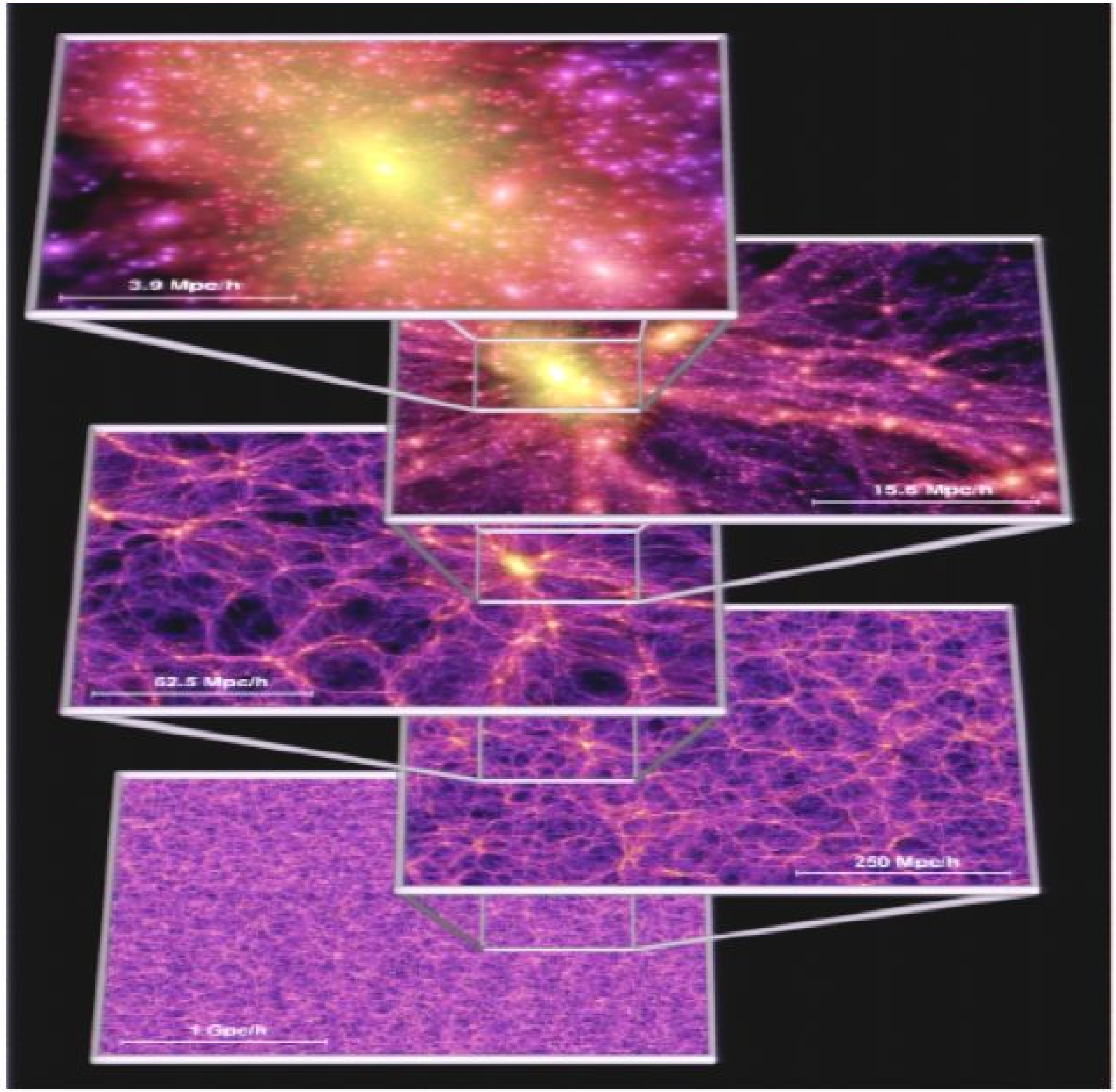
$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{1}{a}\nabla \cdot (\rho \mathbf{v}_p) &= -3H\rho \\ \frac{\partial \mathbf{v}_p}{\partial t} + \frac{1}{a}(\mathbf{v}_p \cdot \nabla)\mathbf{v}_p &= -\frac{1}{a}\nabla\varphi - H\mathbf{v}_p \\ \frac{1}{a}\nabla^2\varphi &= 4\pi G(\rho - \bar{\rho}) \end{aligned}$$

plus initial conditions

Millennium simulation (Springel et al 2005)

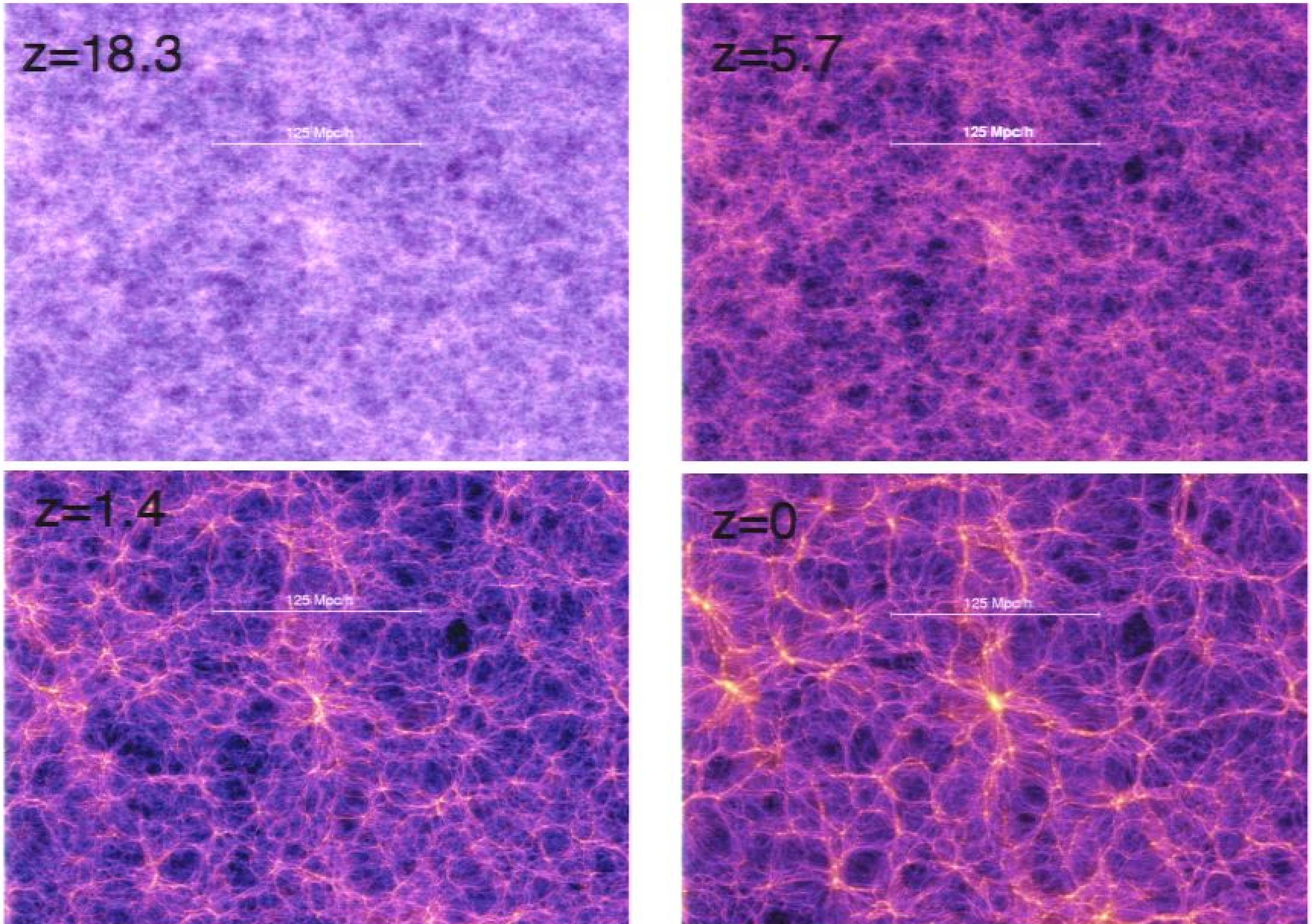
$$\Omega_m = 0.25,\ \Omega_b = 0.045$$
$$\Omega_\Lambda = 0.75,\ h = 0.73$$
$$n = 1,\ \sigma_8 = 0.9$$

$$N_p = 2160^3 \approx 10^{10}$$
$$m_p = 8.6 \times 10^8 h^{-1} M_\odot$$



Millennium simulation: evolution

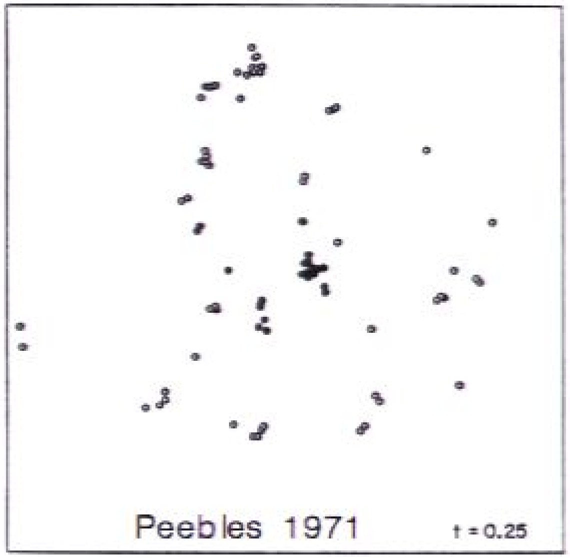
Springel et al 2005



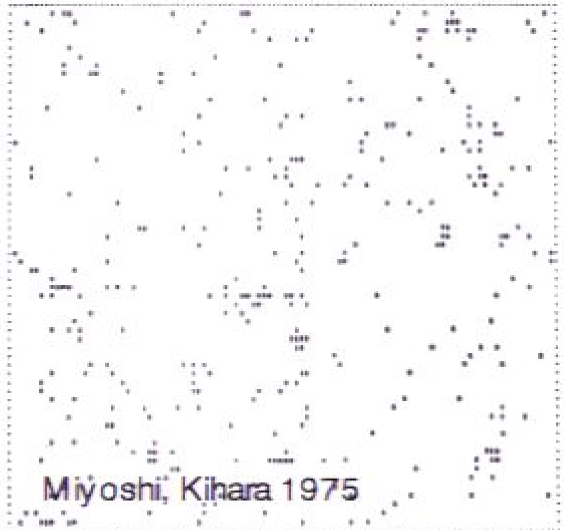
Theoretical models of cosmic web

- Hierarchical clustering
  - Spherical top-hat collapse (?)
  - Press-Schechter mass function (1974)
- Fragmentation
  - Zel'dovich approximation (1970)
- Adhesion approximation (1985)
  - Burgers' equation
- Truncated Zel'dovich approximation (1993)
  - Zel'dovich approximation
- "Cosmic web" (1996)
  - Zel'dovich approximation

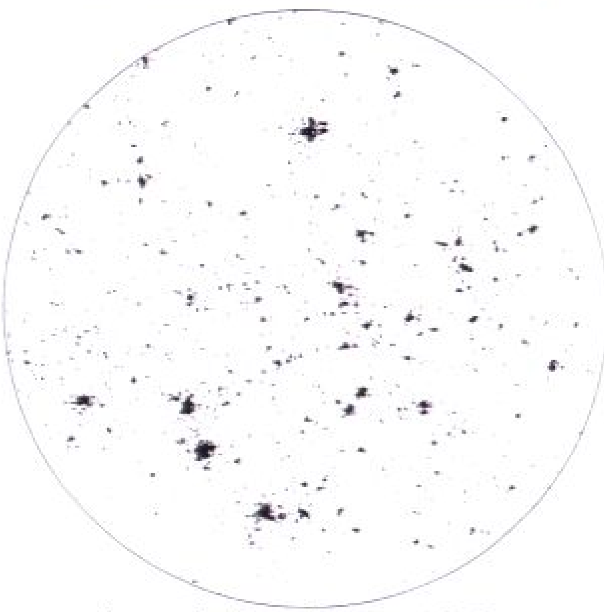
Western visions of structure in the Universe: Hierarchical clustering scenario



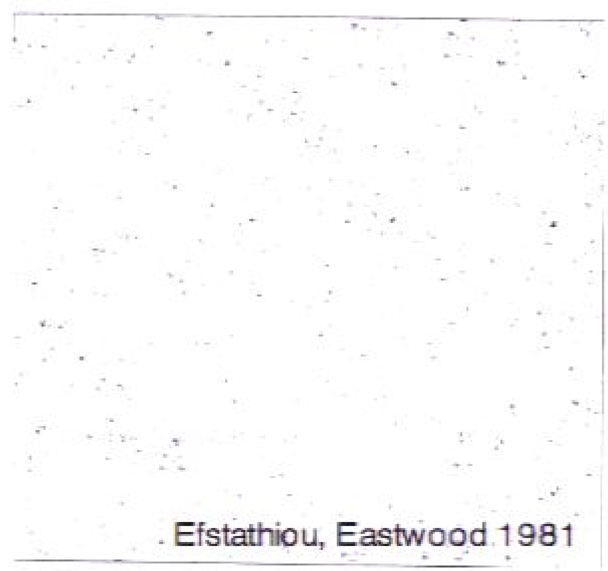
Peebles 1971


Miyoshi, Kihara 1975

White 1976

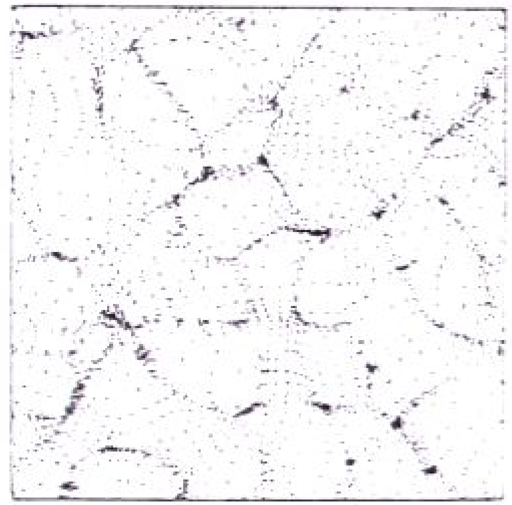

Aarseth, Gott, Turner 1979

Efstathiou, Jones 1979


Efstathiou, Eastwood 1981

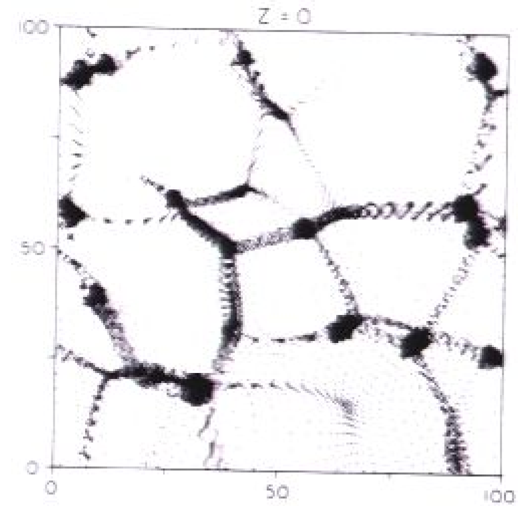
Eastern (Soviet) visions of structure in the Universe: Fragmentation scenario

Shandarin 1975 (ZA, 2D)

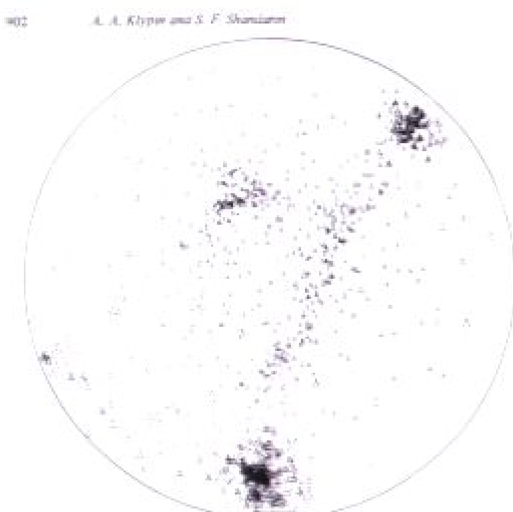


(b)

Doroshkevich et al 1978, 1980 (NB, 2D)



Melott 1983 (NB, 2D)



Klypin, Shandarin 1981, 1983 (NB, 3D)

Klypin, Shandarin 1984 (NB, 3D)

Hierarchical clustering vs Fragmentation

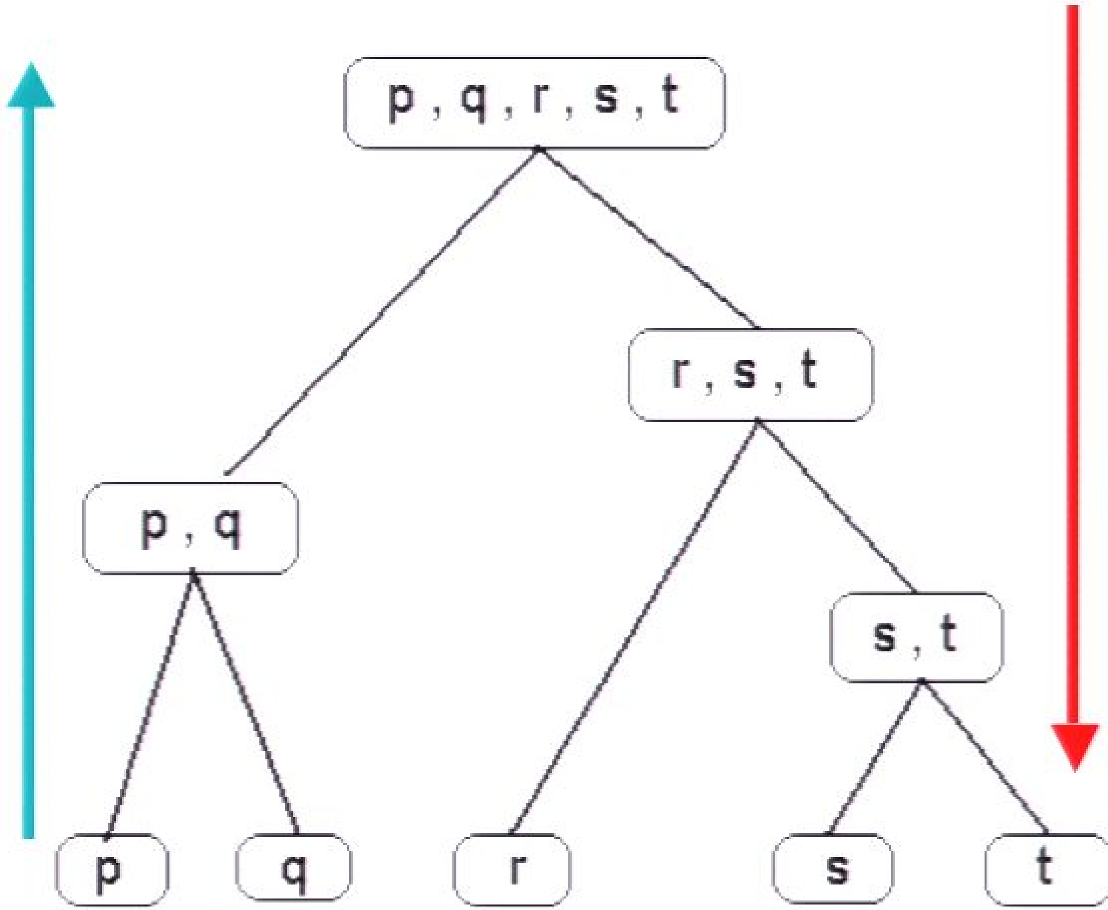
HC

Legacy:

* mass function,

* correlation hierarchy,

* merging history



F

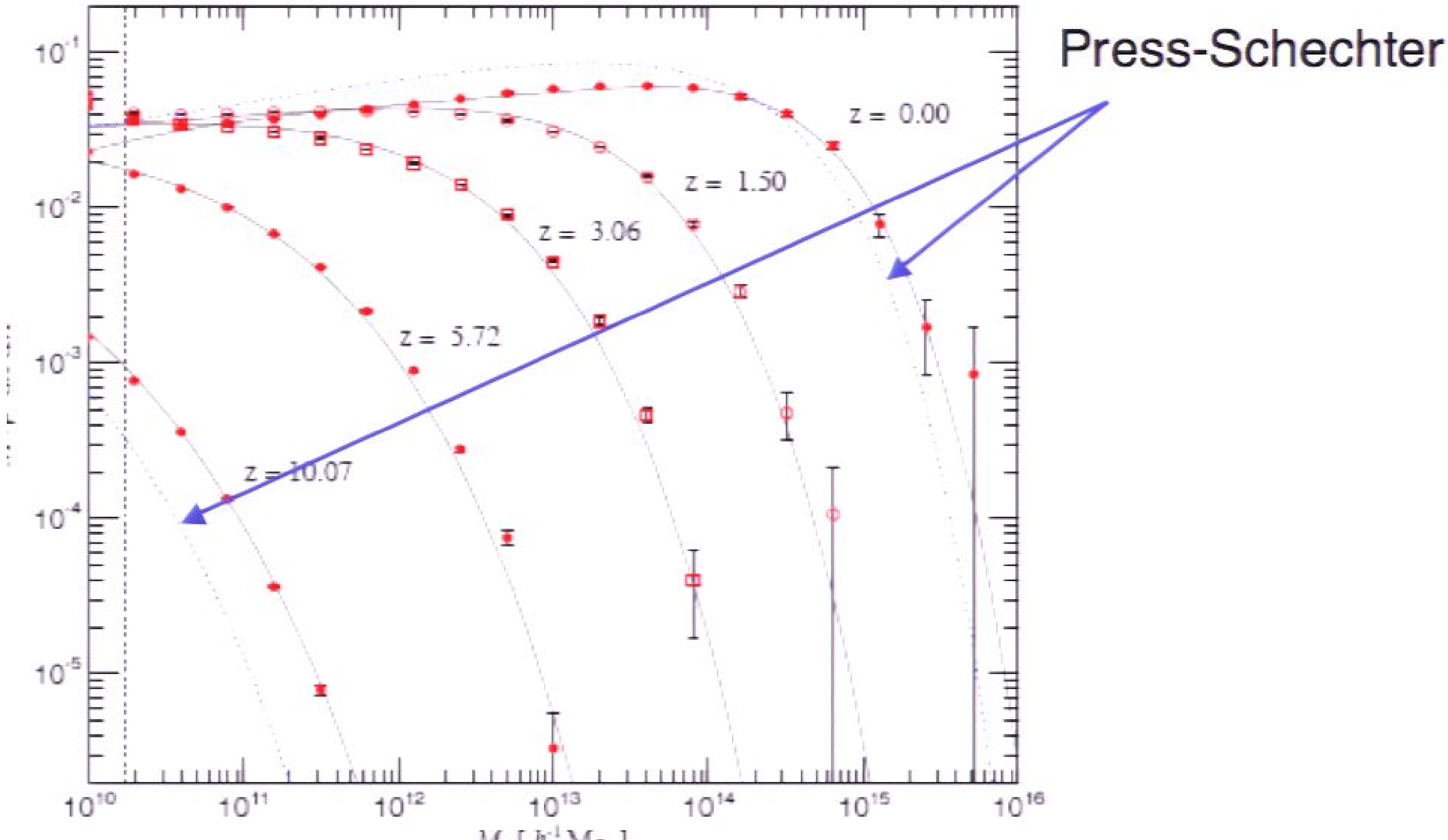
Legacy:

•pancakes,

* filaments

* connectedness

Millennium simulation: mass function



Mon. Not. R. Astron. Soc. **265**, 689–705 (1993)

Galaxy formation and the peaks formalism

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ABSTRACT

A large *N*-body simulation is used to compare the galaxies found by tagging peaks in the linear density field with the haloes that actually form. A variety of filters on the density field are tried in order to improve this correspondence, but none seems to do particularly well. The correlation function and velocity dispersion of the tagged peaks and the actual haloes also do not correspond very well. These comparisons bring into question the results of any study of galaxy formation that assumes that galaxies form at peaks in the initial density field, or simulations of large-scale structure that use the high-peak model to determine the galaxy distribution.

CONCLUSIONS

)ur main conclusion is that peaks in the linear density field re not good indicators of the sites of galaxy formation as etermined by the dissipationless collapse of haloes. It is

Zel'dovich approximation (1970)

Comoving coordinates: $x_i = r_i/a(t)$, where $a(t)$ is a scale factor.

Zel'dovich approximation is a map: $x(\mathbf{q},t) = q_i + D(t)s_i(\mathbf{q})$

If $\varphi(\mathbf{q})$ is the linear perturbation of grav. potential then $s_i(\mathbf{q}) = -\partial\varphi/\partial q_i$

Density can be found from the conservation of mass

$$\rho(\mathbf{q},t) = \bar{\rho}(t)\left|\frac{\partial x_i}{\partial q_k}\right| = \bar{\rho}\left[(1 - D(t)\alpha(\mathbf{q})\right]^{-1}\left[(1 - D(t)\beta(\mathbf{q})\right]^{-1}\left[(1 - D(t)\gamma(\mathbf{q})\right]^{-1}$$

$\alpha(\mathbf{q}) \geq \beta(\mathbf{q})$ and $\beta(\mathbf{q}) \geq \gamma(\mathbf{q})$ are the eigen values of the deformation tensor

$$d_{ik}(\mathbf{q}) = \frac{\partial s_i}{\partial q_k} = -\frac{\partial\varphi}{\partial q_i \partial q_k}$$

Linear density fluctuations: $\delta\rho/\rho = D(t)(\alpha + \beta + \gamma)$.

The Zel'dovich approximation describes anisotropic collapse and motion.
Major fields are initial velocity perturbation $v_\propto s_i$ and deformation tensor $d_{ik}(\mathbf{q})$.

Truncated Zel'dovich approximation

- The TZA (Truncated Zel'dovich Approximation) is a map

$$\mathbf{x}(\mathbf{q}, t) = q + D(t)\mathbf{s}_f(\mathbf{q}).$$

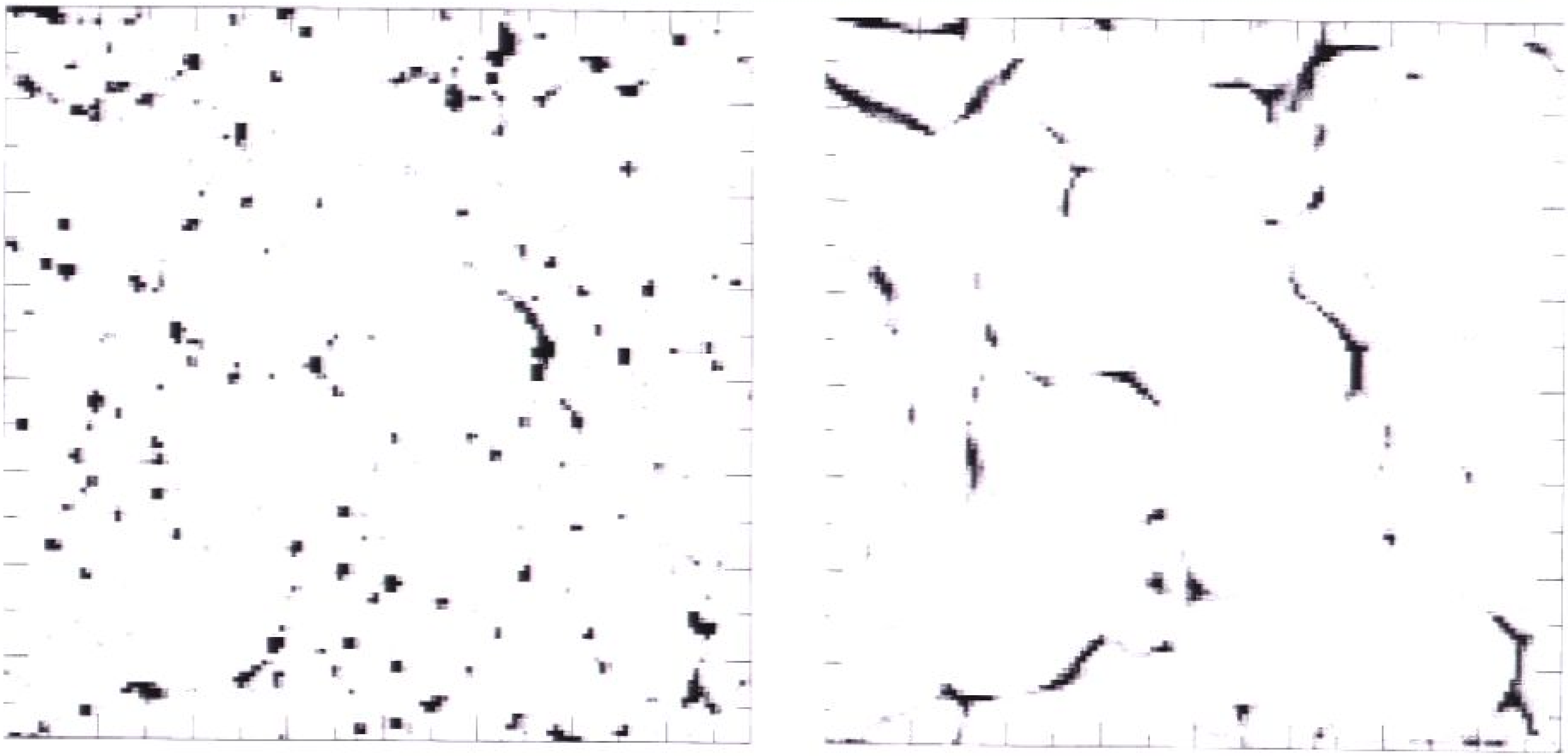
- The displacement field $\mathbf{s}_f(\mathbf{q})$ is a filtered version of the linear velocity perturbation field.
- Field $\mathbf{s}_f$ is smoothed over a scale R_f such that $\sigma(R_f, t) \approx 1$.
- Out of 6 tested models [1) lognormal, 2) truncated lognormal, 3) linear, 4) truncated linea, 5) ZA, 6) TZA] the TZA is absolutely the best.
- The largest large-scale structures are directly related to the generic singularities of the map as described by Arnol'd, Shandarin and Zel'dovich 1982

Coles, Melott, Shandarin 1993

Zel'dovich approximation in 2D

N-body v.s. TZA

(Truncated Zel'dovich Approximation = ZA for smoothed initial density conditions obtained by truncation of spectrum)



$$P(k) \propto k^{-1}$$

Melott, Shandarin, Weinberg 1994

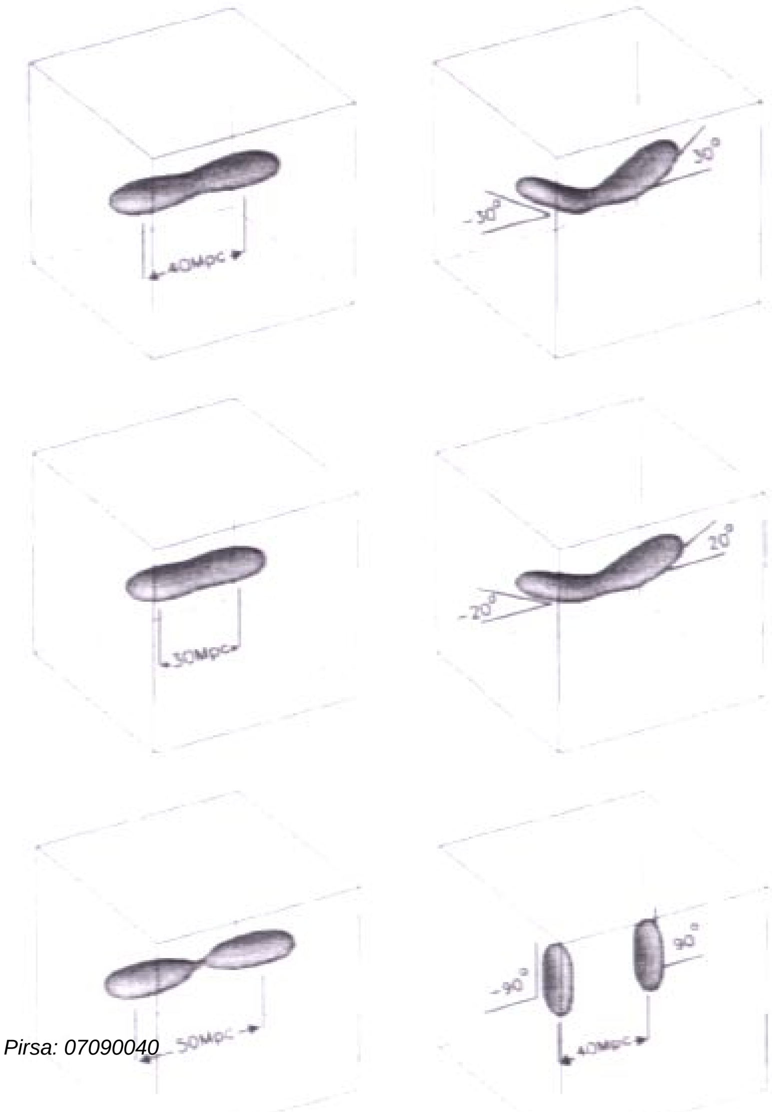
Cosmic web

- The model of the web is based on a map $\mathbf{x}(\mathbf{q}, t) = q + \mathbf{s}(\mathbf{q}, t)$.
- The displacement field $\mathbf{s}(\mathbf{q}, t)$ is split $\mathbf{s} = \mathbf{s}_b + \mathbf{s}_f$ between background and fluctuating fields.
- Field $\mathbf{s}_b$ is smoothed over a large scale R_b such that $\sigma(R_b, t) < 1$.
- The linear approximation (i.e. Zel'dovich approximation) $\mathbf{s}_b = D(t)\mathbf{s}_b(\mathbf{q})$ gives a reasonable description of the overall pattern on N-body simulations.

Bond, Kofman, Pogosyan 1996

Cosmic web: properties/predictions of the model

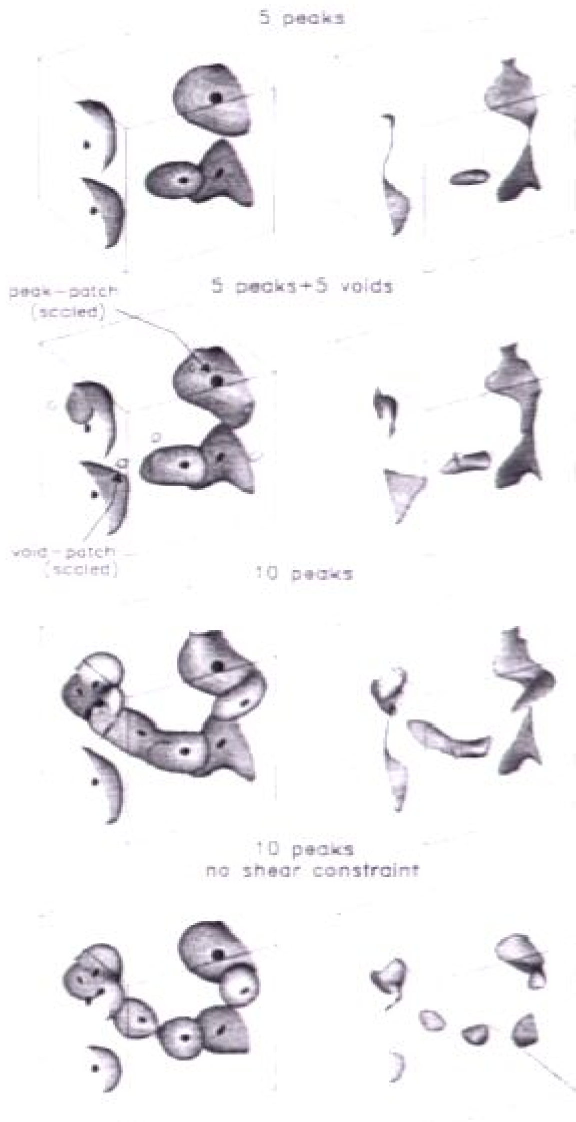
... the filamentary web is a consequence of the distribution and spatial coherence of the strain field in the medium.



Zel'dovich-mapped correlation functions constrained by shear at two peaks.

The bridge between two clusters gradually weakens
as the separation increases, breaking by 50/h Mpc (left column)
or
as the shear tensor orientations become misaligned (right column)

Bond, Kofman, Pogosyan 1996

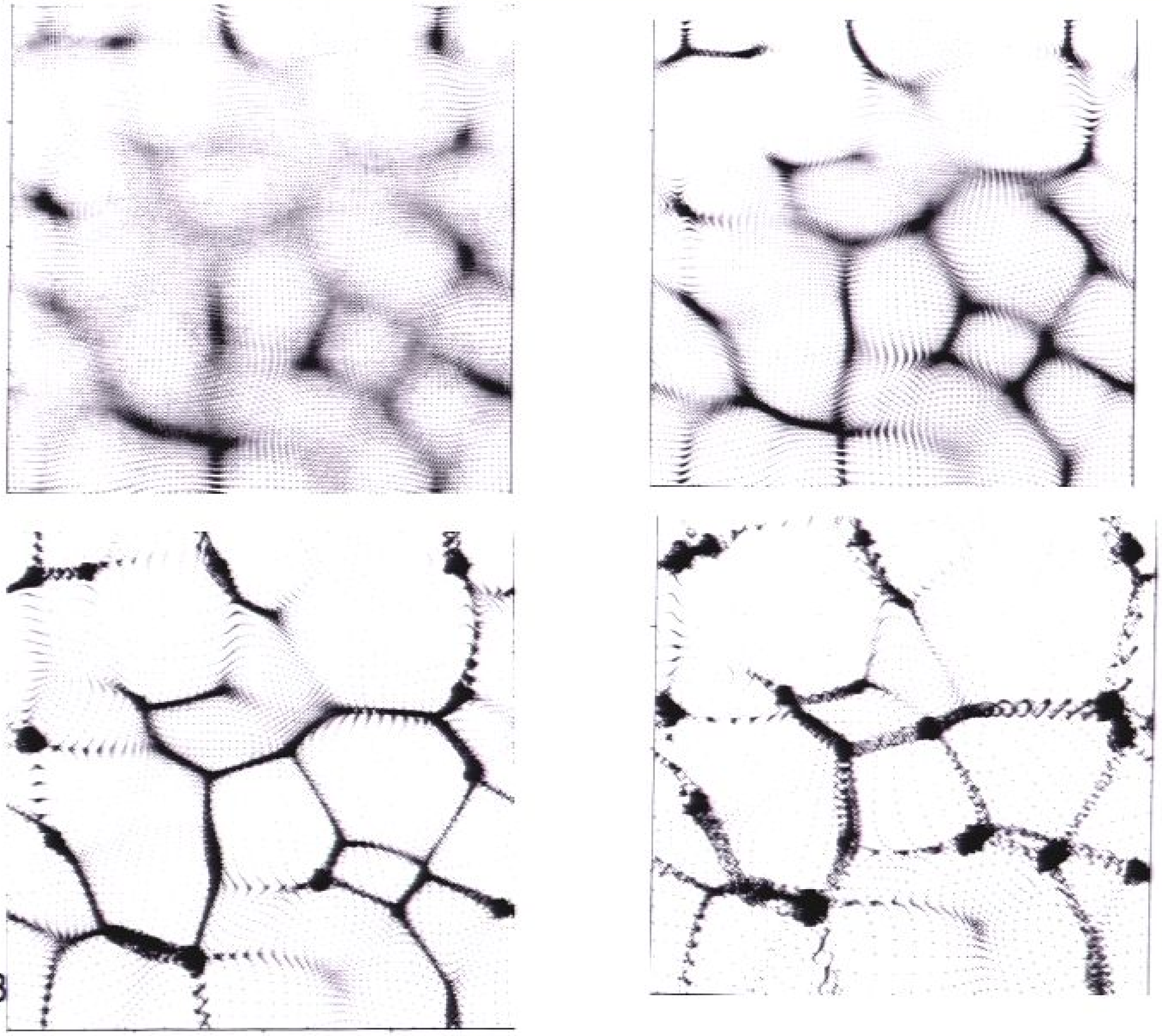


New language:
conditional multiple-point correlation functions in lagrangian space - that is, statistically averaged density and the displacement fields subject to various constraints on the the shear at multiple points

It seems to be too much of simplification: sequential structures only.

Higher order derivatives are also needed

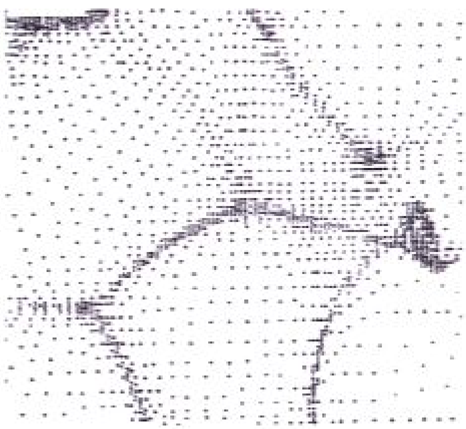
Evolution of 2D web



Coarse graining effect

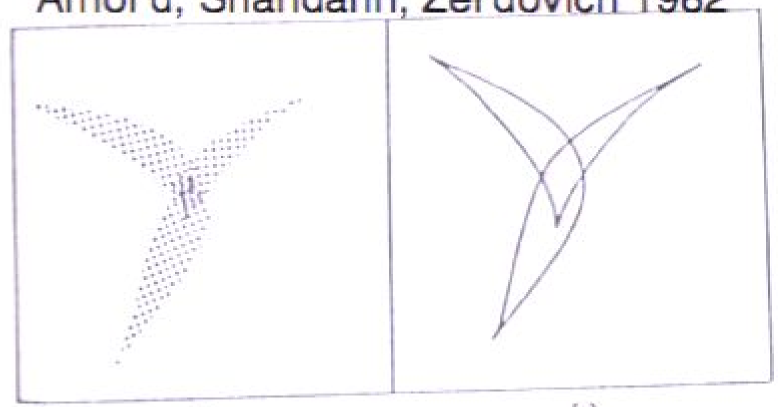
Melott 1983

For building cosmic web one needs bifurcation of filaments and walls

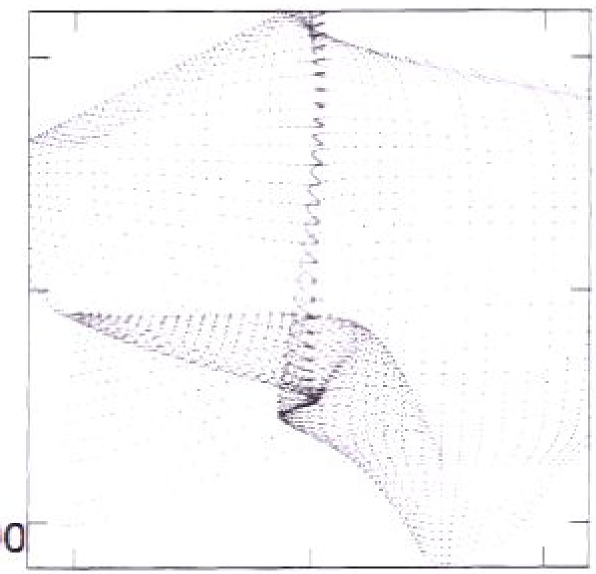


Shandarin 1975

Arnol'd, Shandarin, Zel'dovich 1982



A4 (swallow tail) singularity



Nusser, Dekel 1990

Generic singularities in 3D

$A_2 : \rho \propto \delta r^{-1/2}$ Surfaces (2D) at a generic instant of time

$A_3 : \rho \propto \delta r^{-2/3}$ Lines (1D) at a generic instant of time

$A_4 : \rho \propto \delta r^{-3/4}$ Points (0D) at a generic instant of time
Candidate to be a filament

$A_5 : \rho \propto \delta r^{-4/5}$ Points (0D) at particular instants of time

$D_4 : \rho \propto \delta r^{-1}$ Points (0D) at a generic instant of time
Candidate to be a filament

$D_5 : \rho \propto \delta r^{-1} \ln \delta r$ Points (0D) at particular instants of time

Arnol'd, Shandarin, Zel'dovich 1982

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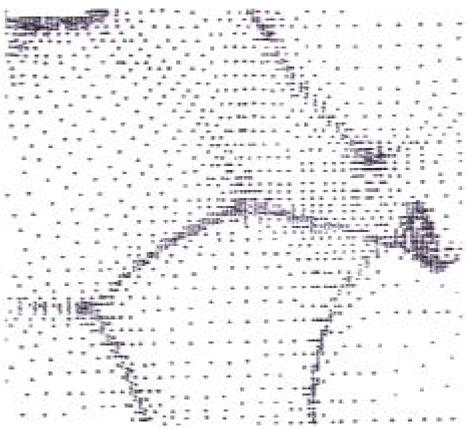

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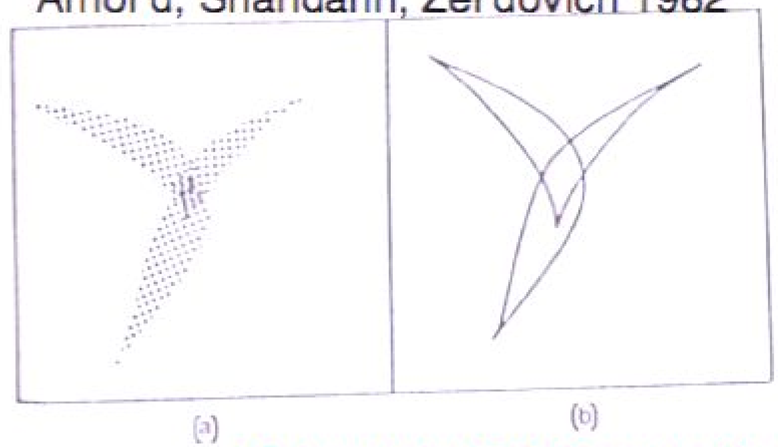

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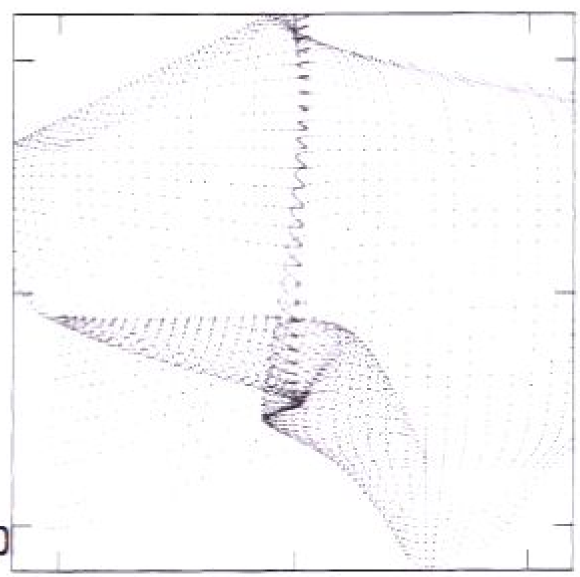
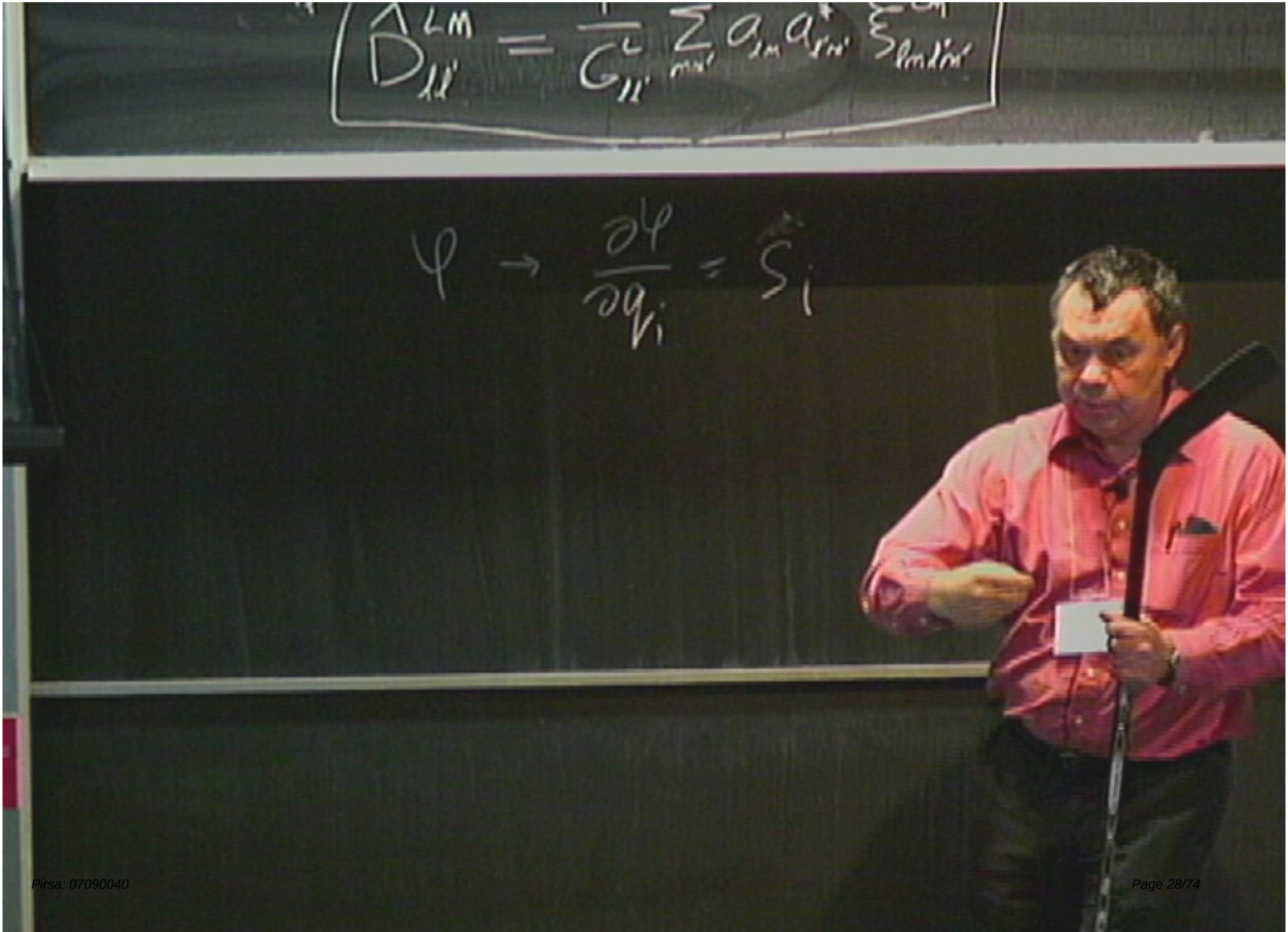
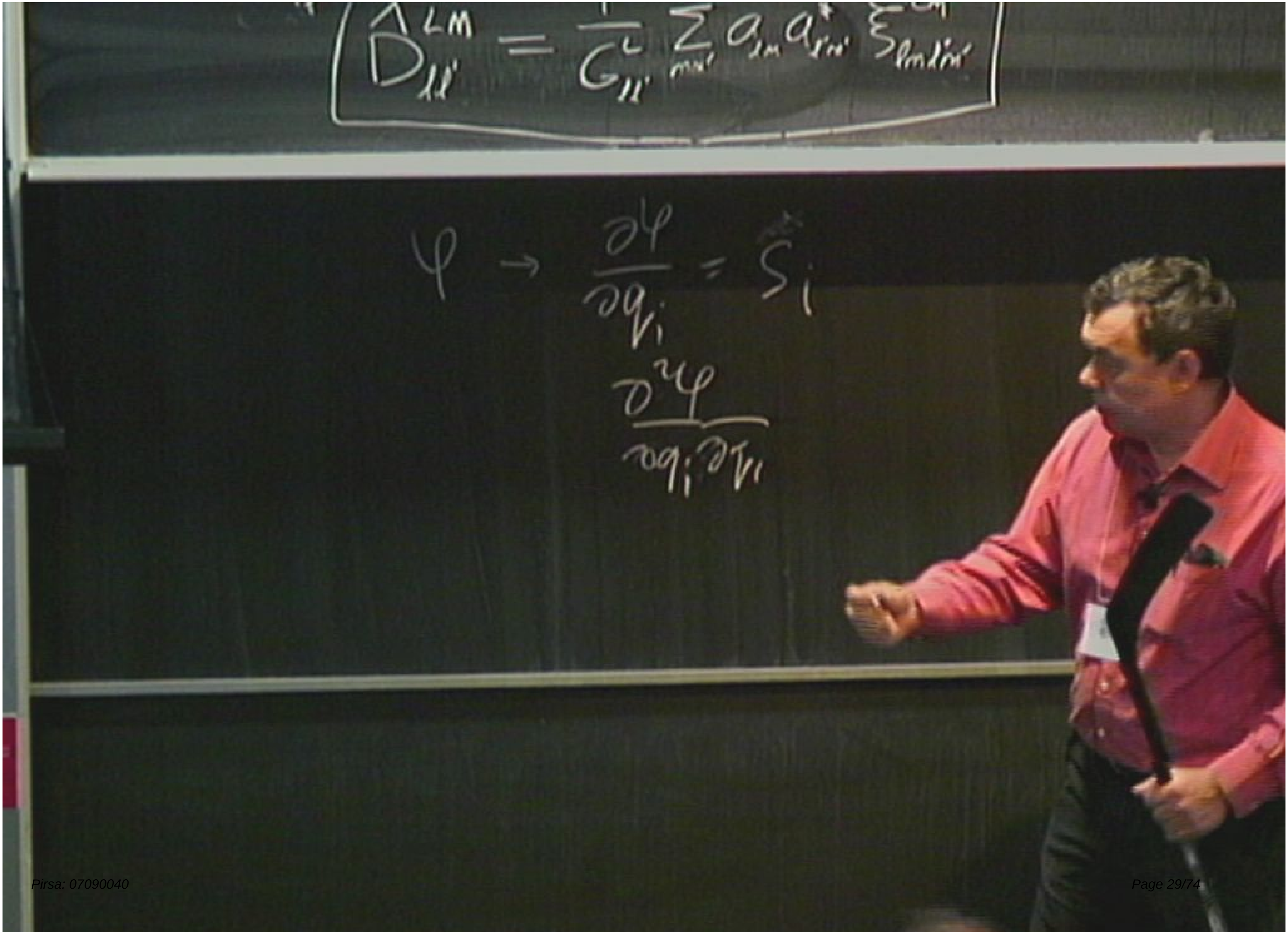


Nusser, Dekel 1990


$D^{LM}_{ll'} = \frac{1}{C^{L}_{ll'}} \sum_{mm'} a_{lm} a^{*}_{l'm'} \xi^{LM}_{lml'm'}$
$\varphi \rightarrow \frac{\partial \varphi}{\partial q_i} = \hat{S}_i$



$\hat{D}^{LM}_{ll'} = \frac{1}{G^{L}_{ll'}} \sum_{mm'} a_{lm} a^{*}_{l'm'} \xi_{lml'm'}$
$\varphi \to \frac{\partial \varphi}{\partial q_i} = \hat{S}_i$
$\frac{\partial^2 \varphi}{\partial q_i \partial q_{i'}}$

$$\hat{D}^{LM}_{\lambda\lambda'} = \frac{1}{G^{L}_{\lambda\lambda'}} \sum_{mm'} a_{\lambda m} a^{*}_{\lambda' m'} \xi^{LM}_{lm\lambda' m'}$$

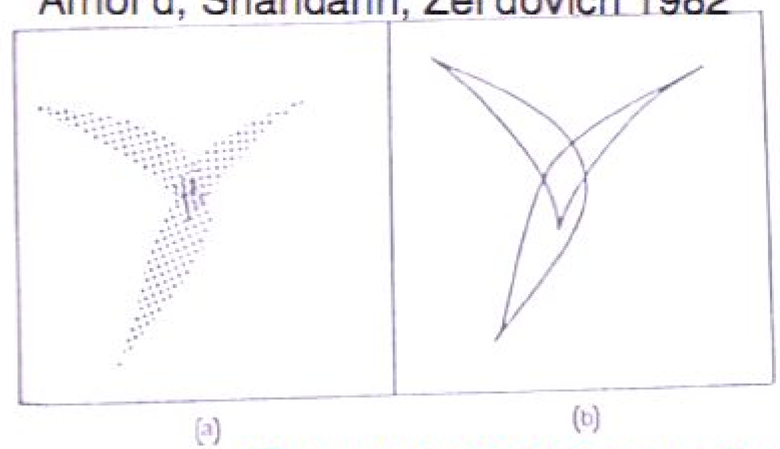
$$\varphi \rightarrow \frac{\partial \varphi}{\partial q_i} = \hat{S}_i$$

$$\frac{\partial^2 \varphi}{\partial q_i \partial q_l}, \quad \frac{\partial^3 \varphi}{\partial q_i \partial q_l \partial q_k}$$

For building cosmic web one needs bifurcation of filaments and walls

Shandarin 1975

Arnol'd, Shandarin, Zel'dovich 1982



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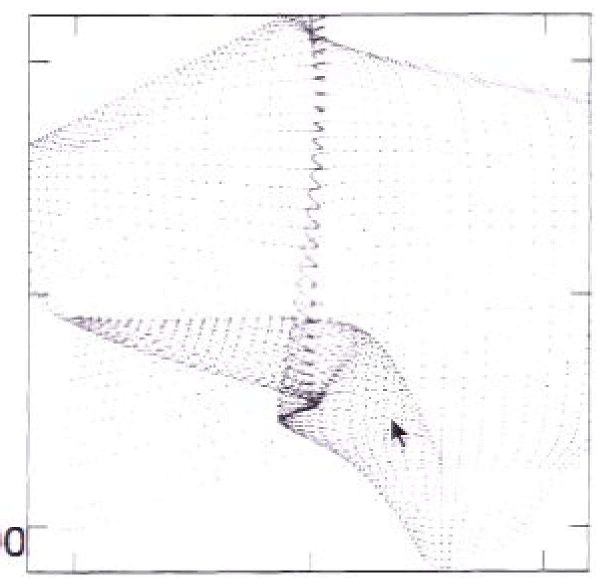

Nusser, Dekel 1990

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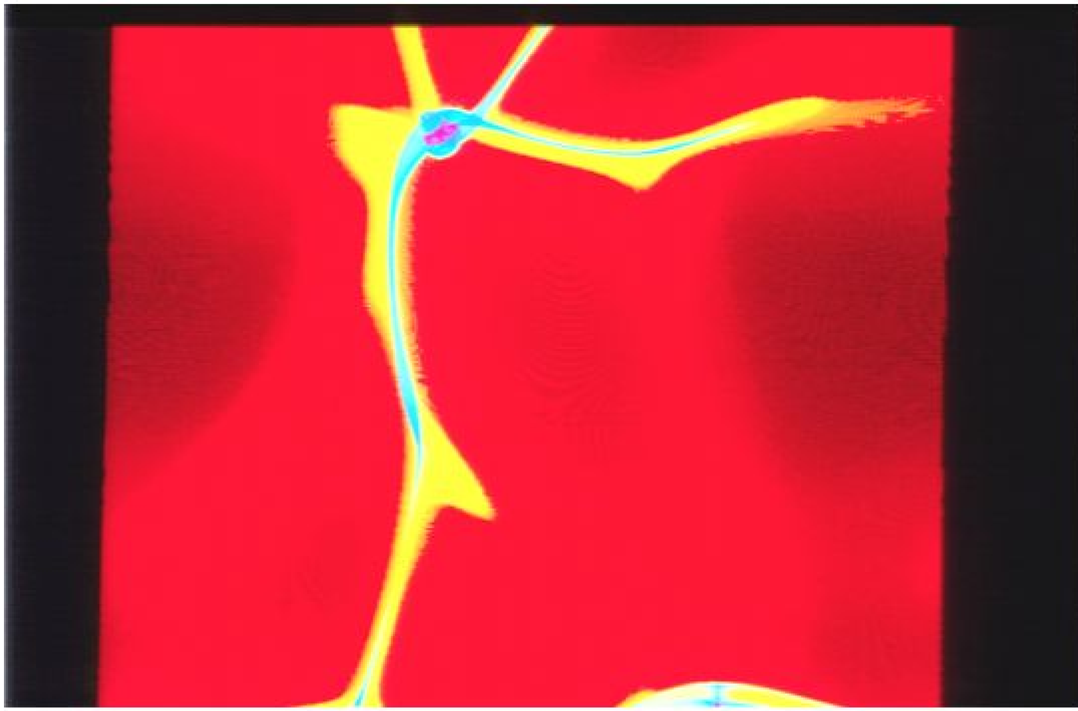
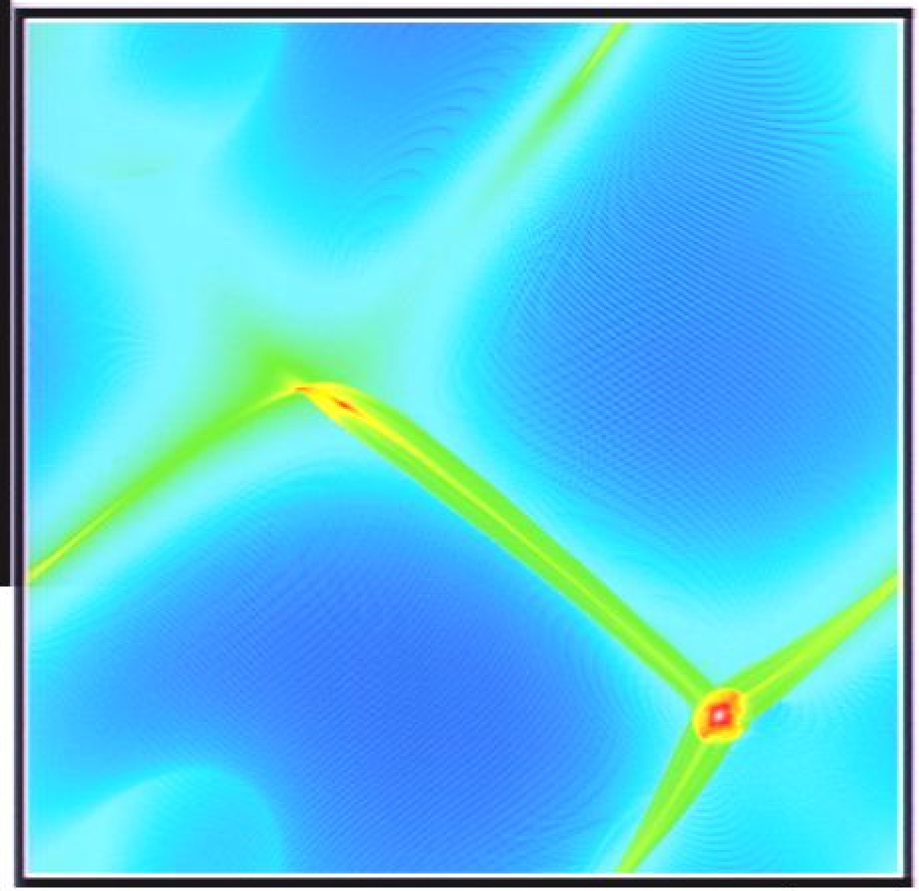
Arnol'd, Shandarin, Zel'dovich 1982

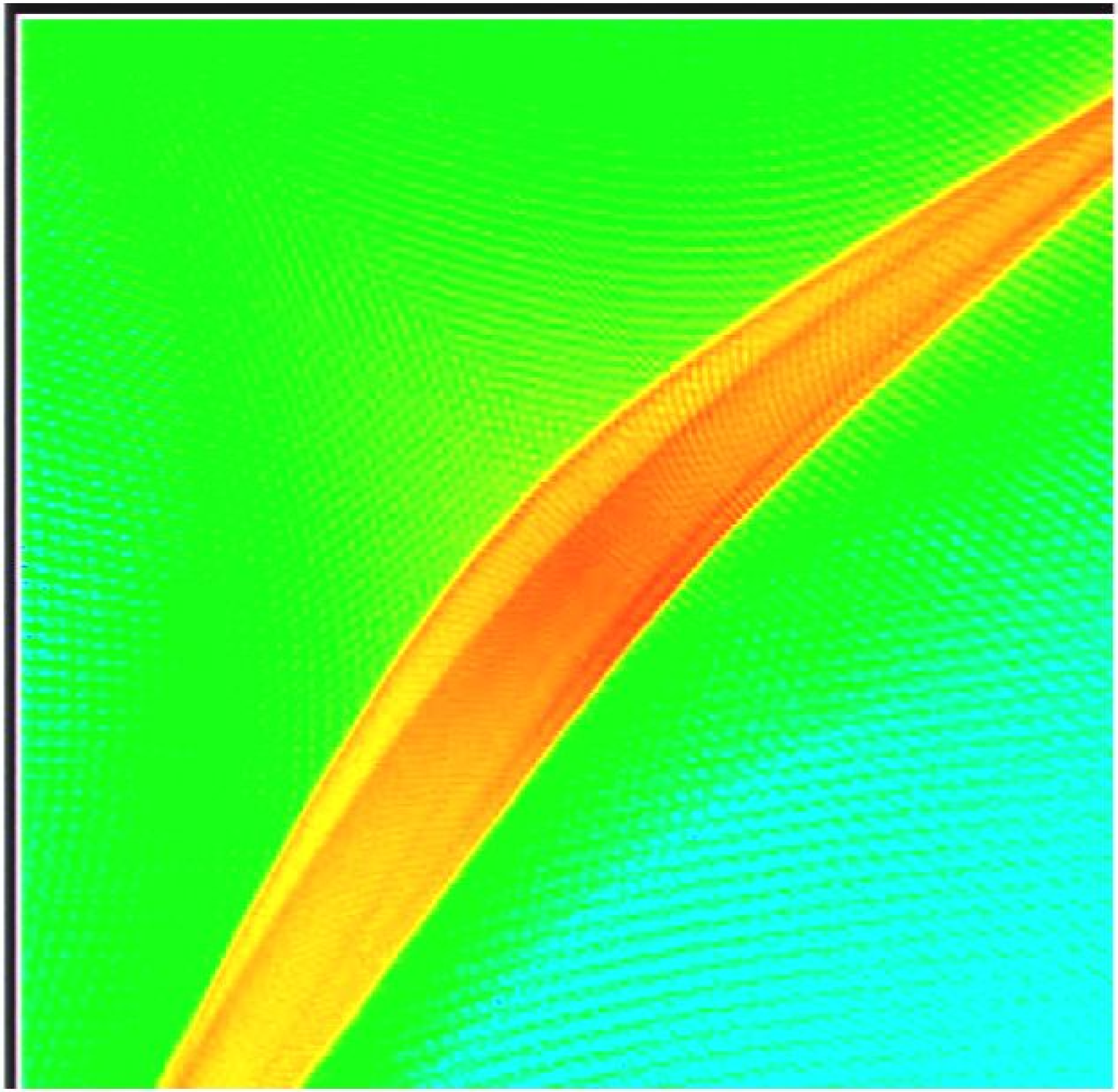
Melott, Shandarin 1990 (2D)



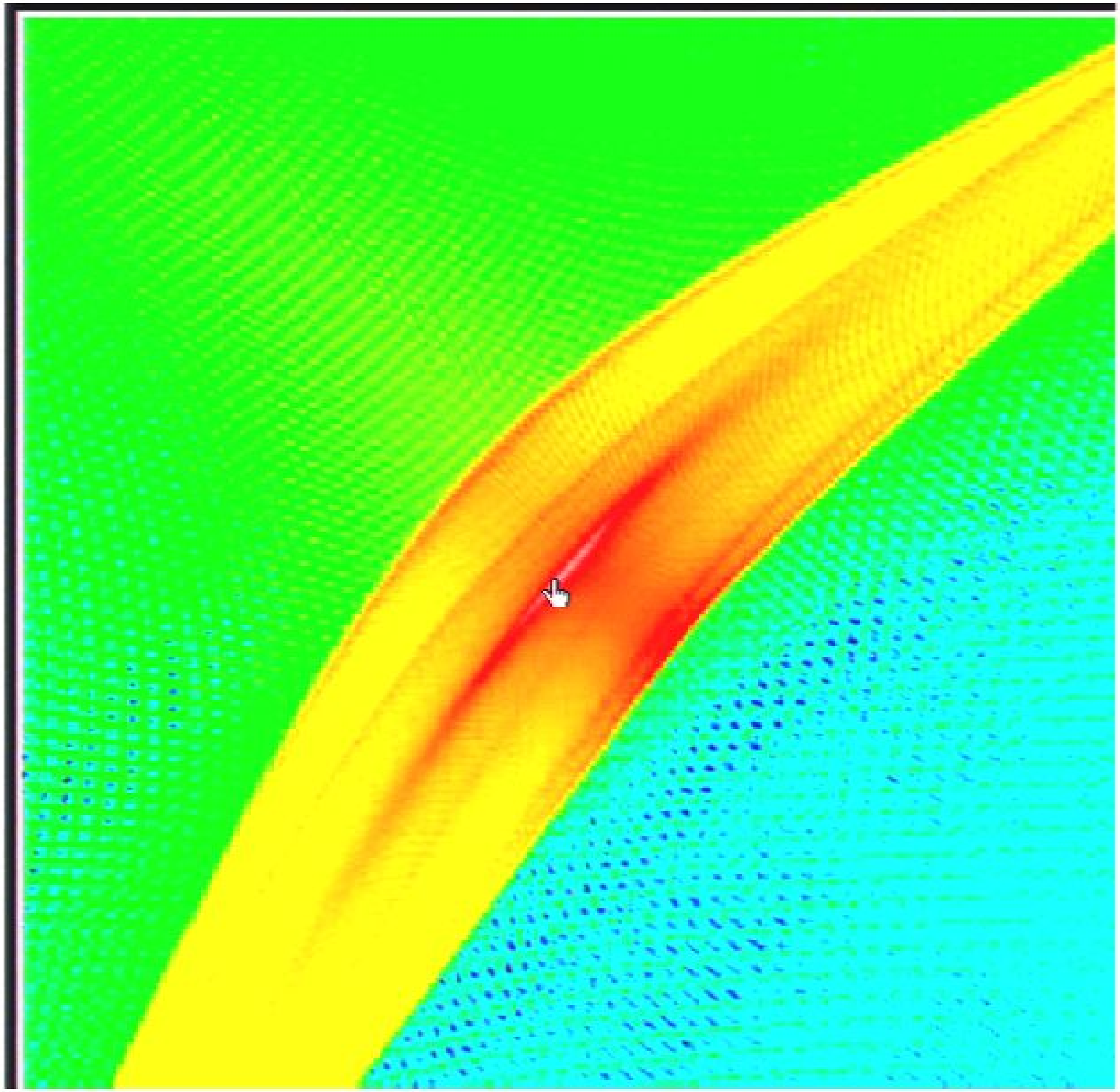


Shirokov, Bertschinger 2005 (3D)

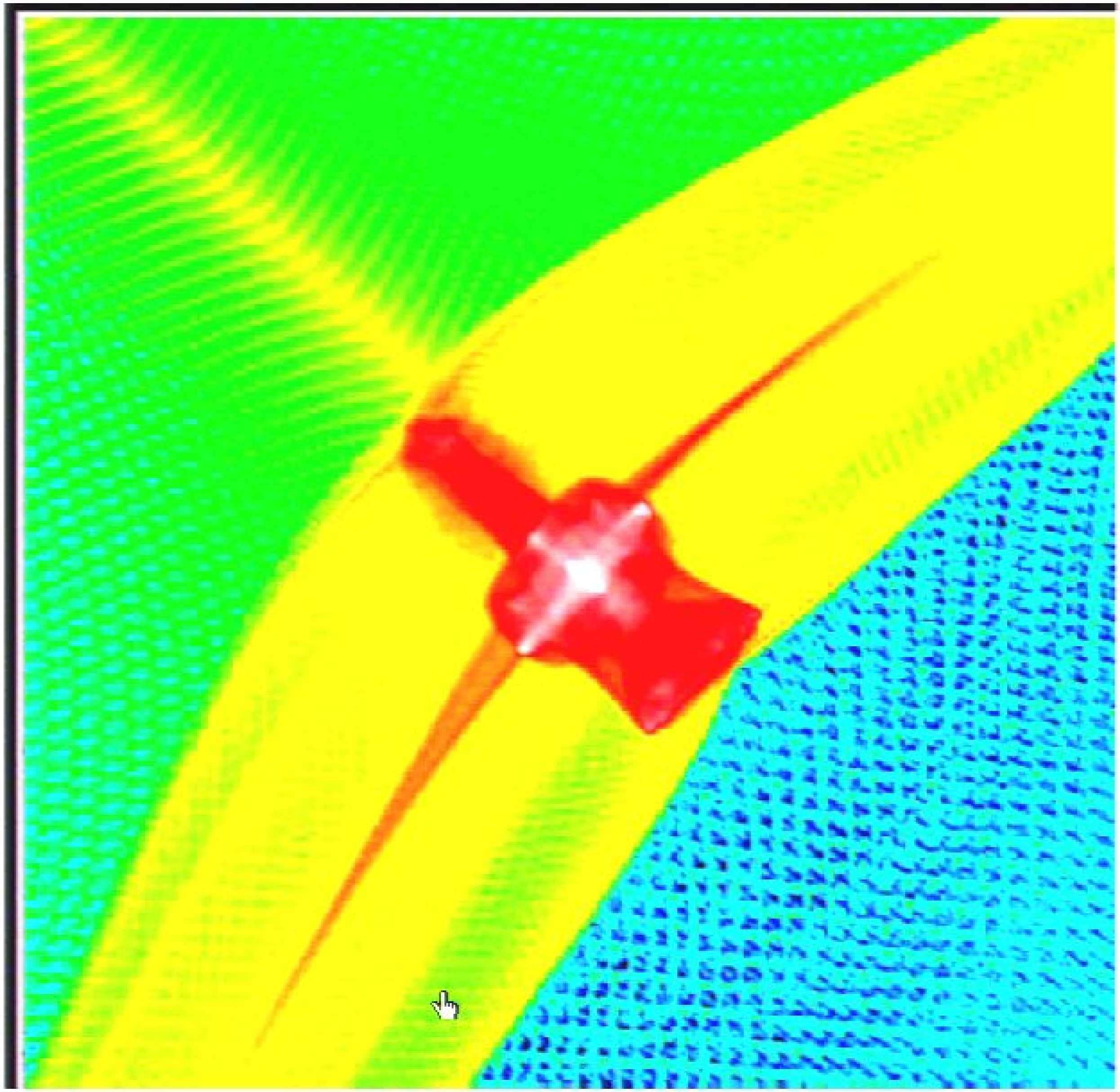
Shirokov, Bertschinger 2005: 3D N-B simulation



Shirokov, Bertschinger 2005: 3D N-B simulation



Shirokov, Bertschinger 2005: 3D N-B simulation



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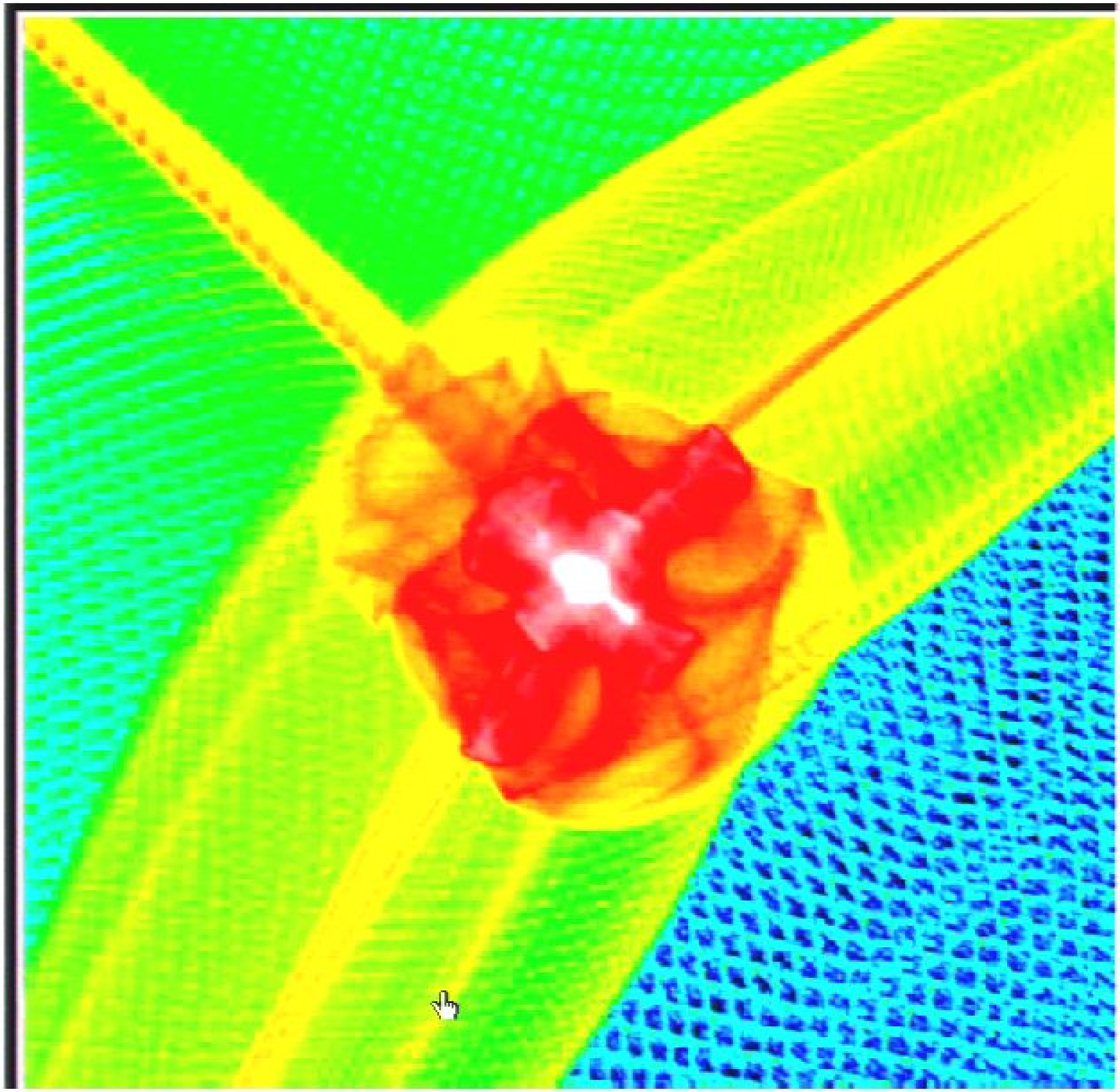
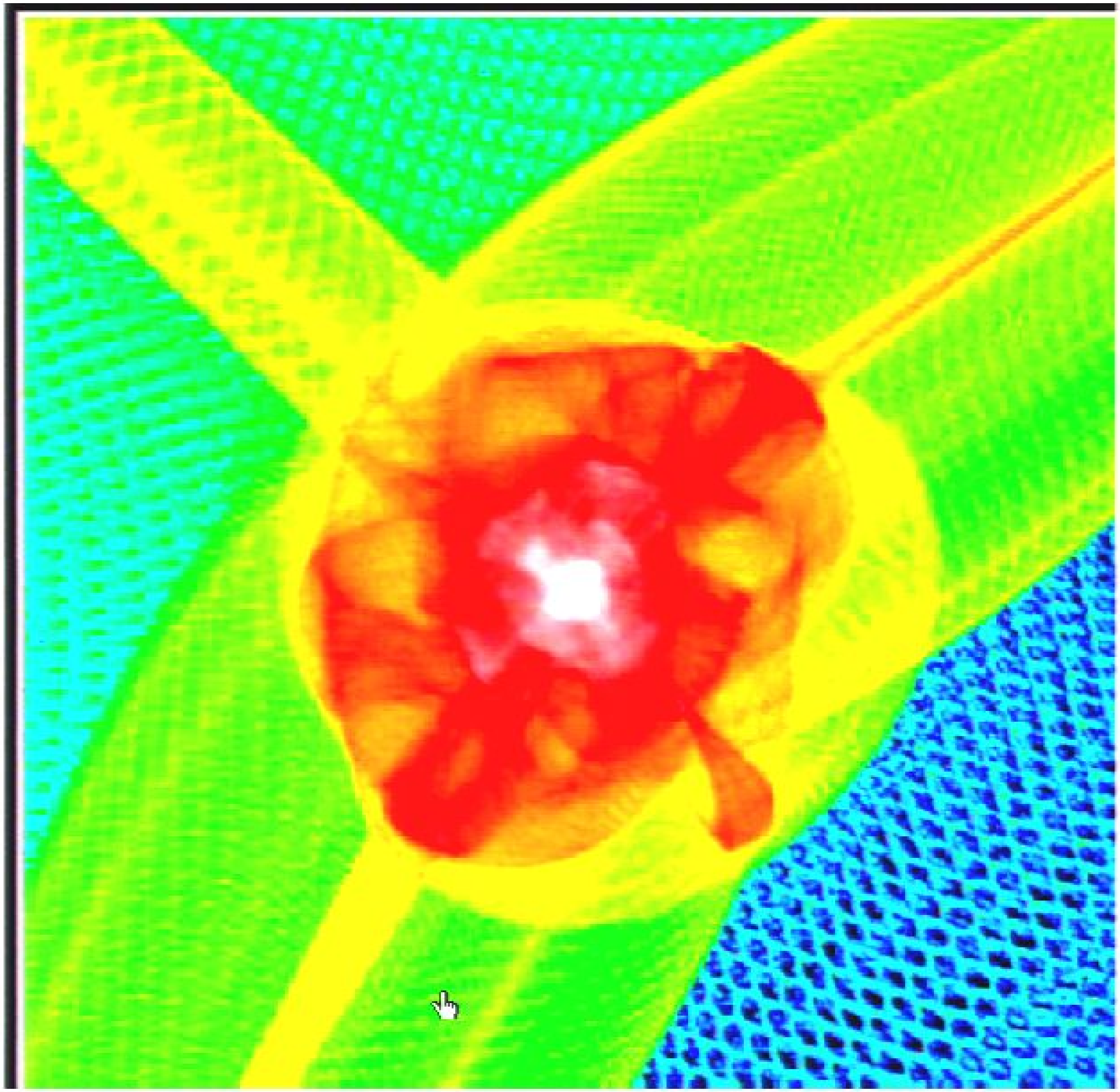


Shirokov, Bertschinger 2005: 3D N-B simulation



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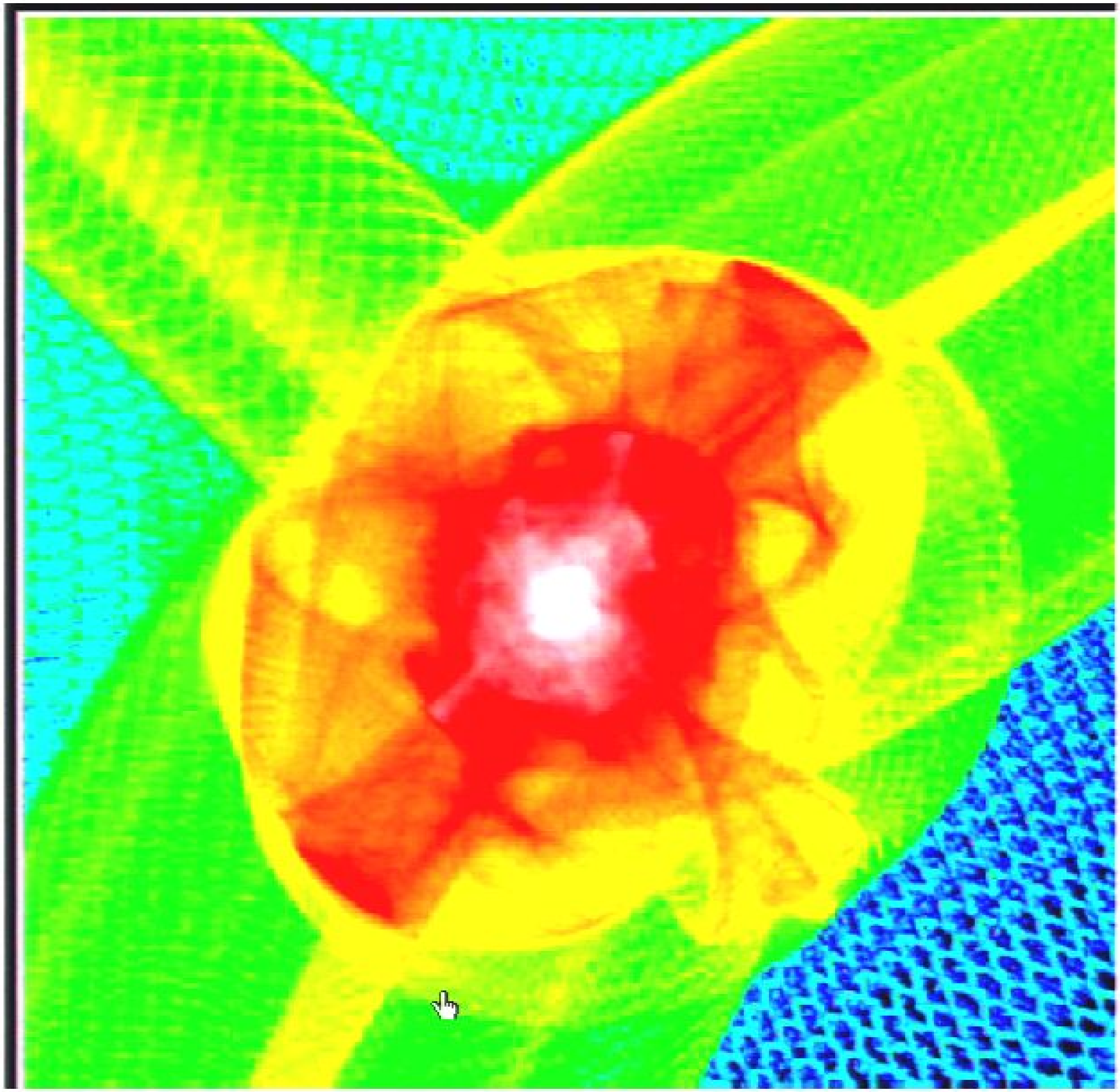
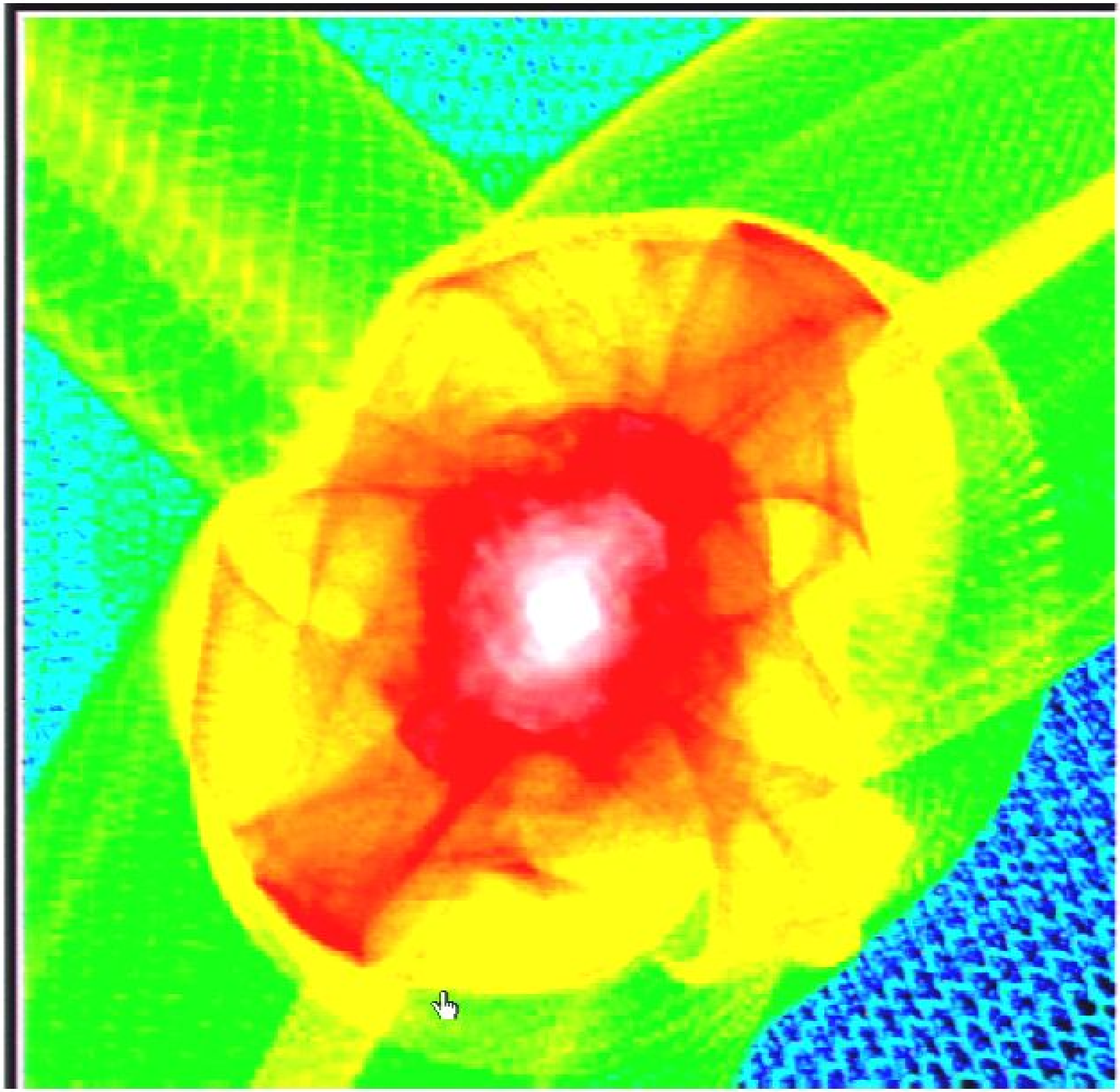
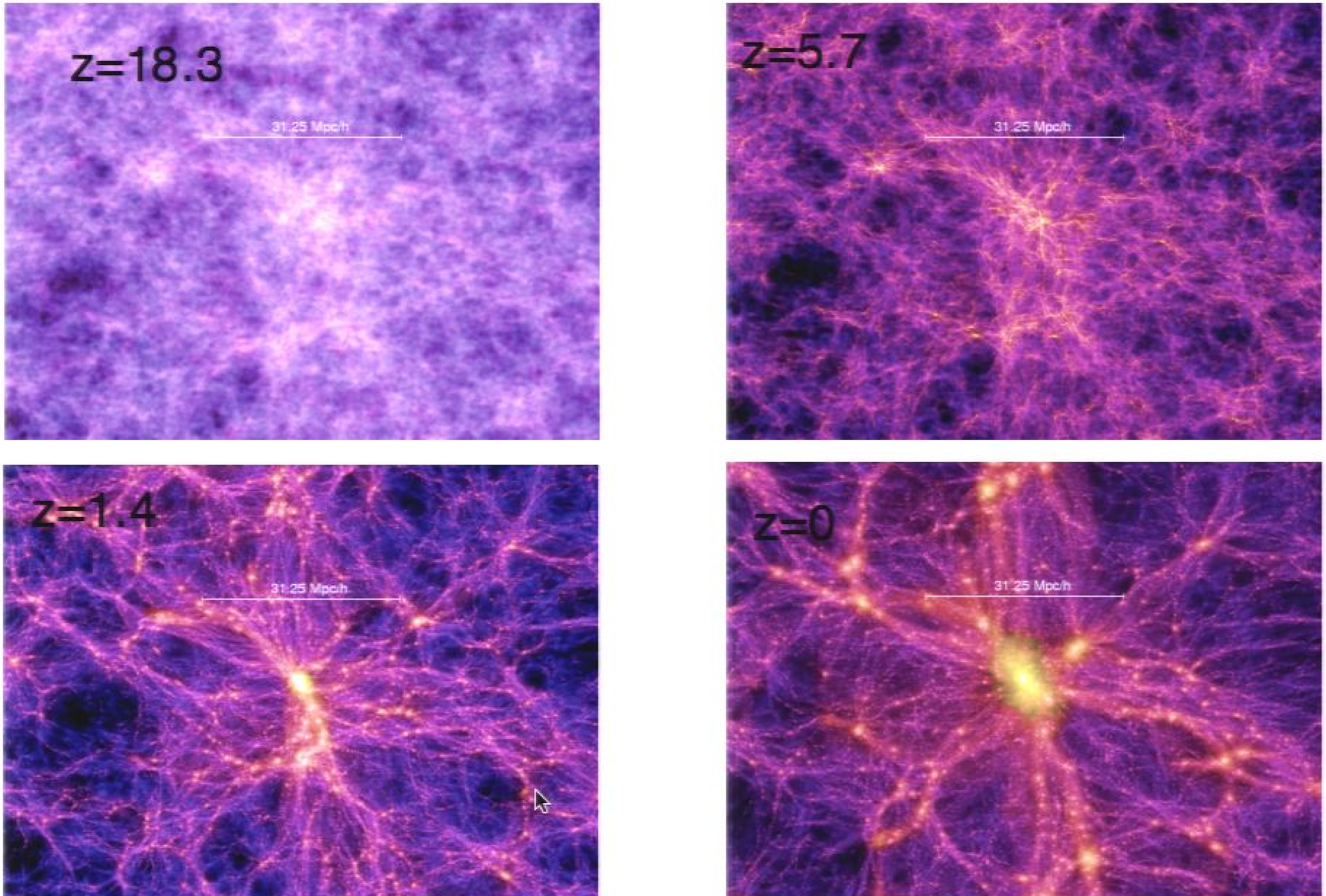


Shirokov, Bertschinger 2005: 3D N-B simulation



Millennium simulation: evolution

V.Springel et al 2005

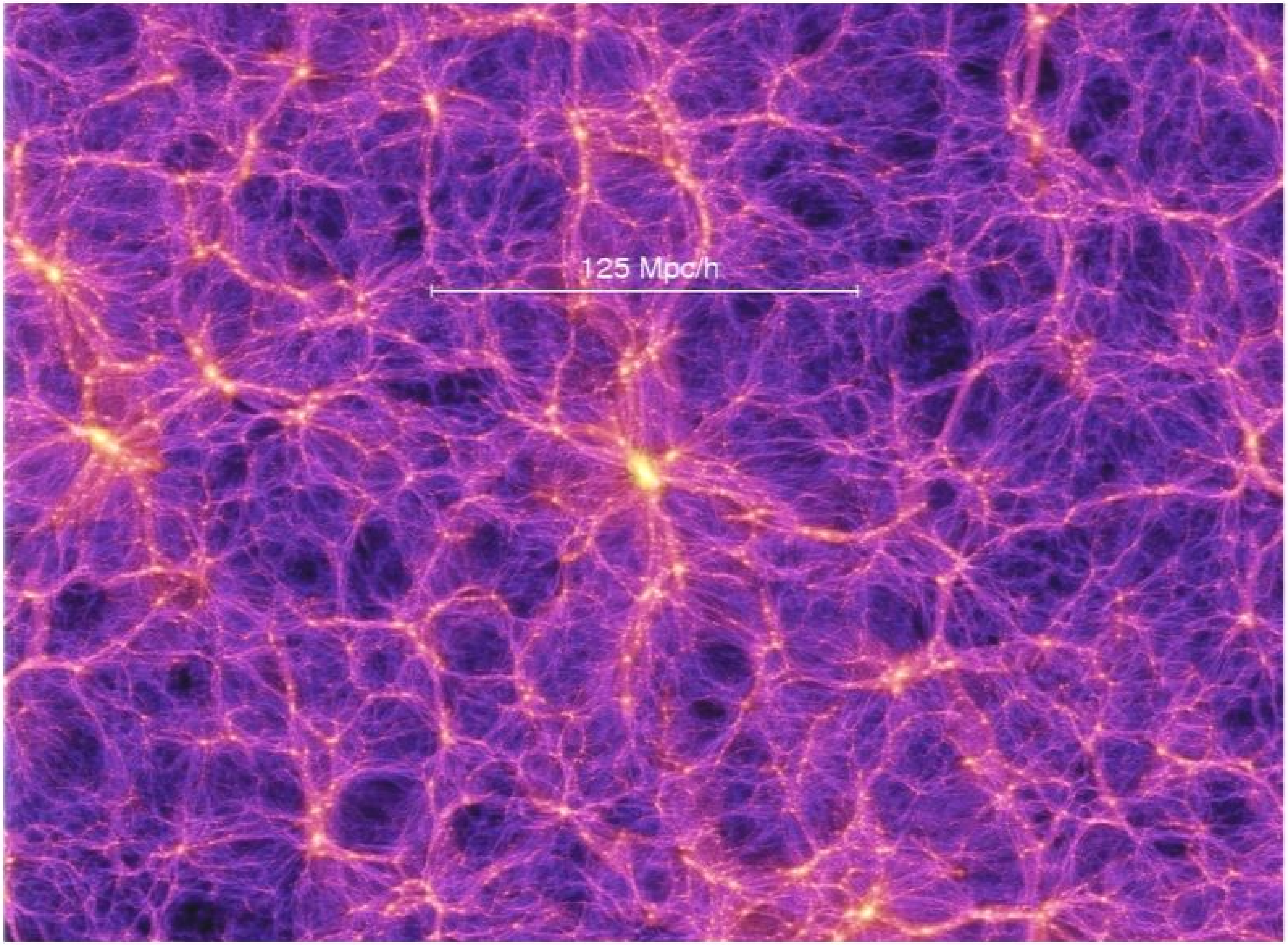


Cosmic web: properties/predictions of the model (4)

Superclusters are filamentary cluster-cluster bridges.
We predict that the most pronounced filaments will be found between clusters of galaxies that are aligned with each other and close together (BKP 96)

It seems to be not that obvious for DM in high-resolution simulations.

Some problems with probabilistic interpretation



Adhesion approximation

Particles follow the Zel'dovich approximation

$$x_i(\mathbf{q}, t) = q_i + D(t) s_i(\mathbf{q})$$

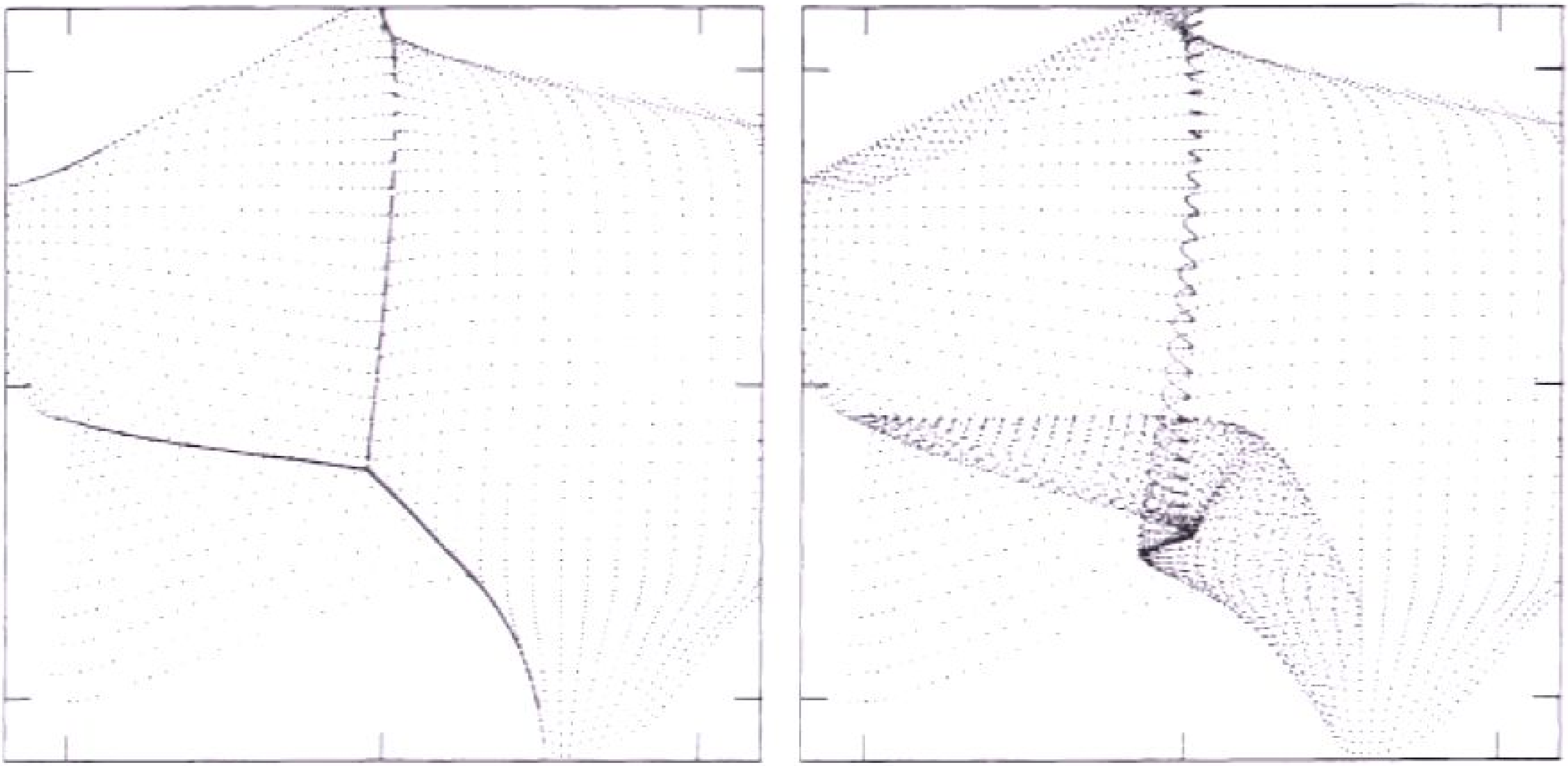
until they run into caustic regions, where viscosity causes them to stick together. Burgers' equation describes this stage

$$\frac{\partial \mathbf{v}}{\partial D} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \nu \nabla^2 \mathbf{v}$$

$$\text{where } \mathbf{v} = \frac{d\mathbf{x}}{dD} \quad \text{and} \quad \nu \to 0$$

Gurbatov, Saichev, Shandarin 1985,1989

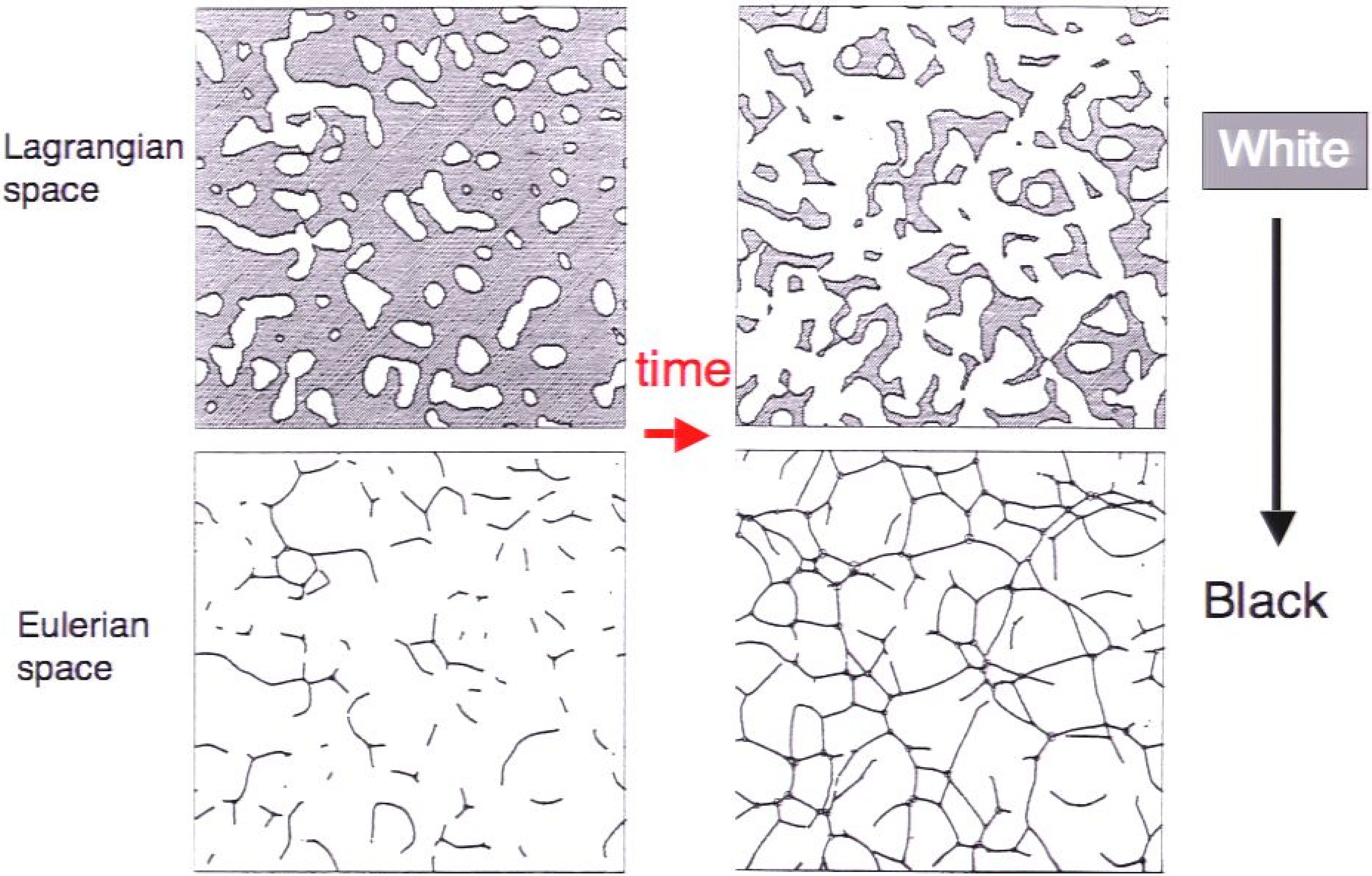
Adhesion and Zel'dovich approximations (2D)



Nusser, Dekel 1990

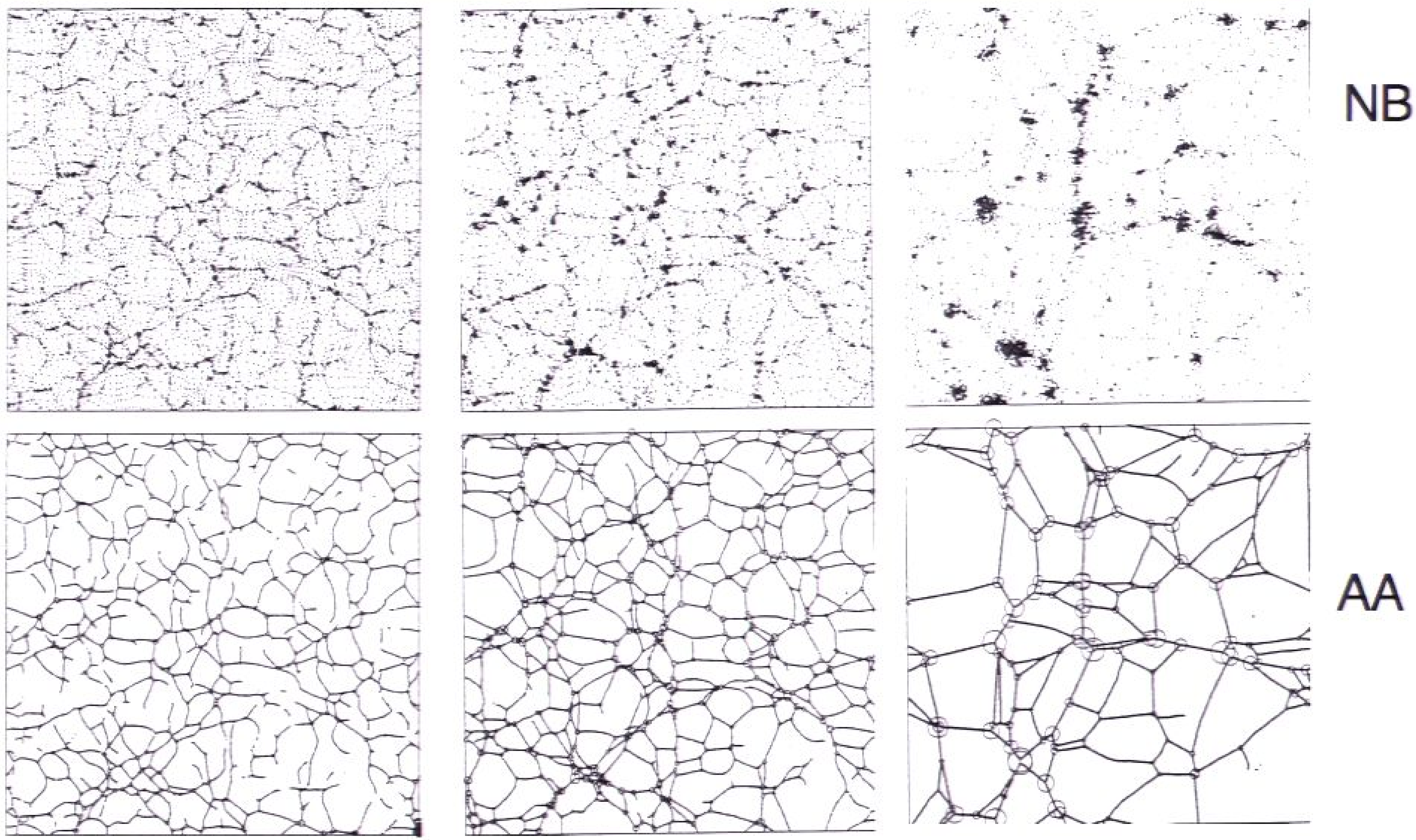
Adhesion model

Pogosyan PhD 1990

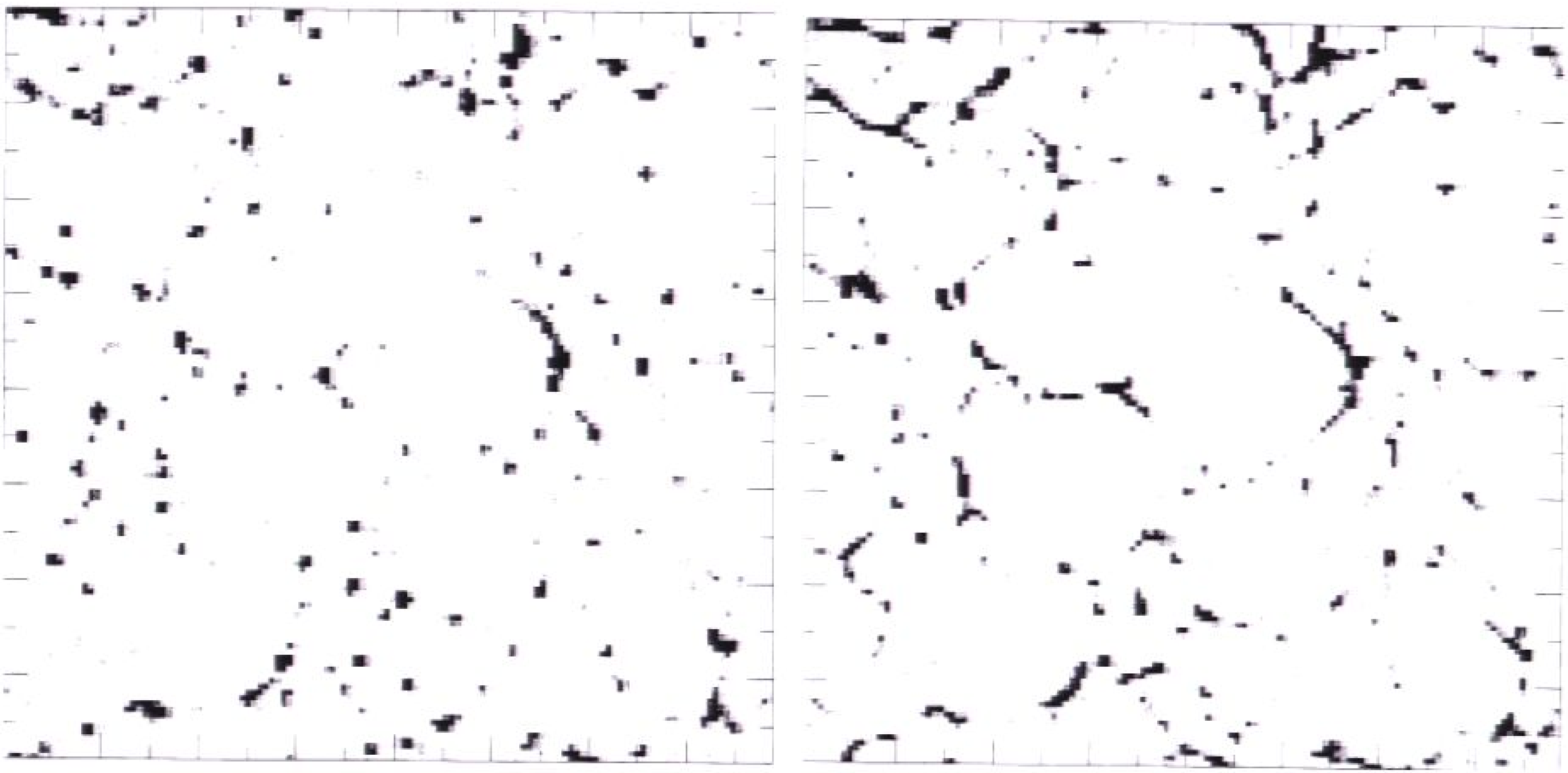


Evolution in adhesion model

Pogosyan PhD

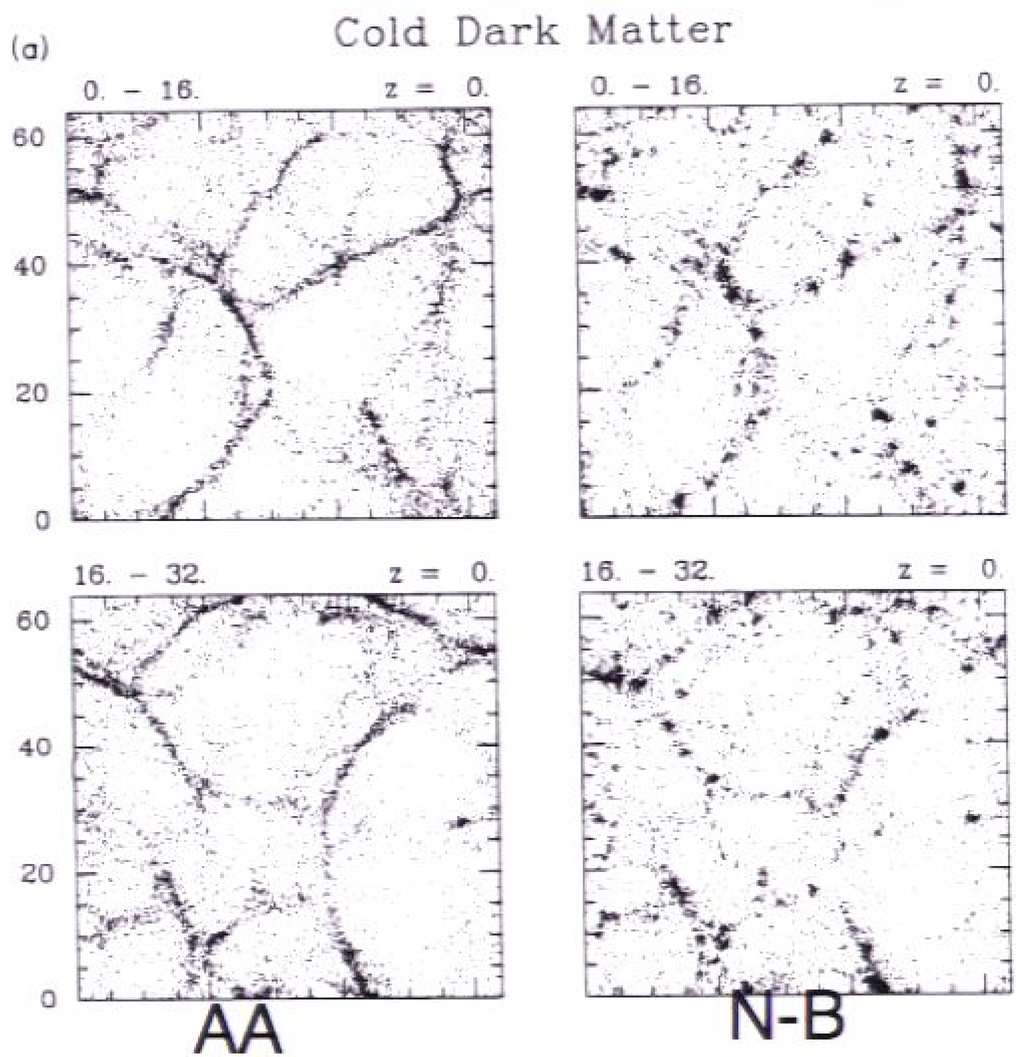


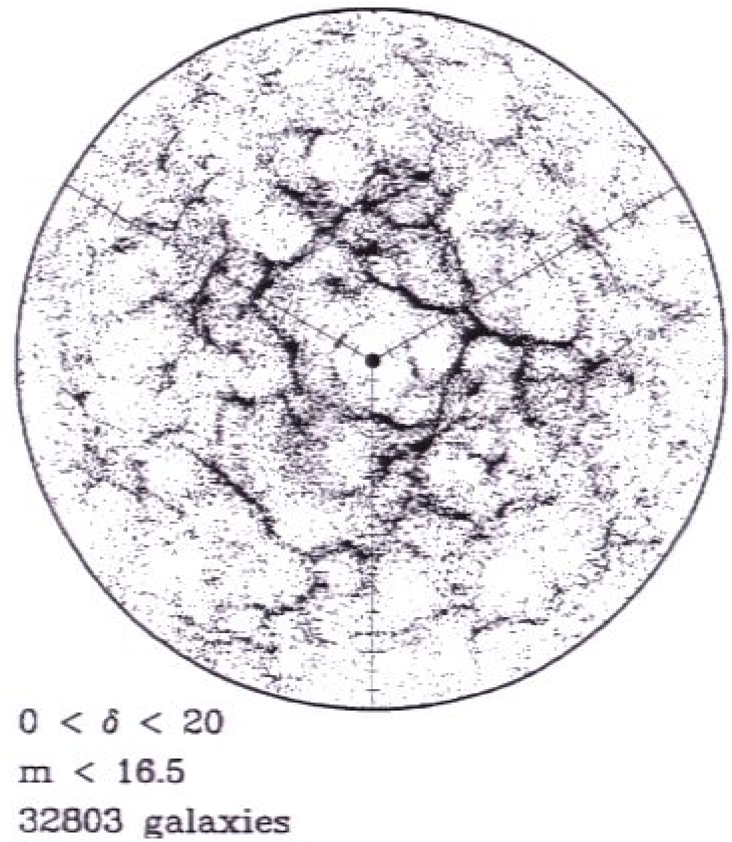
N-body v.s. Adhesion Apprximation (3D)



$P(k) \propto k^{-1}$

Adhesion approximation in 3D





Weinberg, Gunn 1990

Natural outcomes of Adhesion model

The structure is made of four types of structural elements: uncollapsed matter (3D), walls (~2D), filaments (~1D), clumps

Fraction of mass in these elements evolves with time, in particular walls loose mass very fast and are difficult to detect.

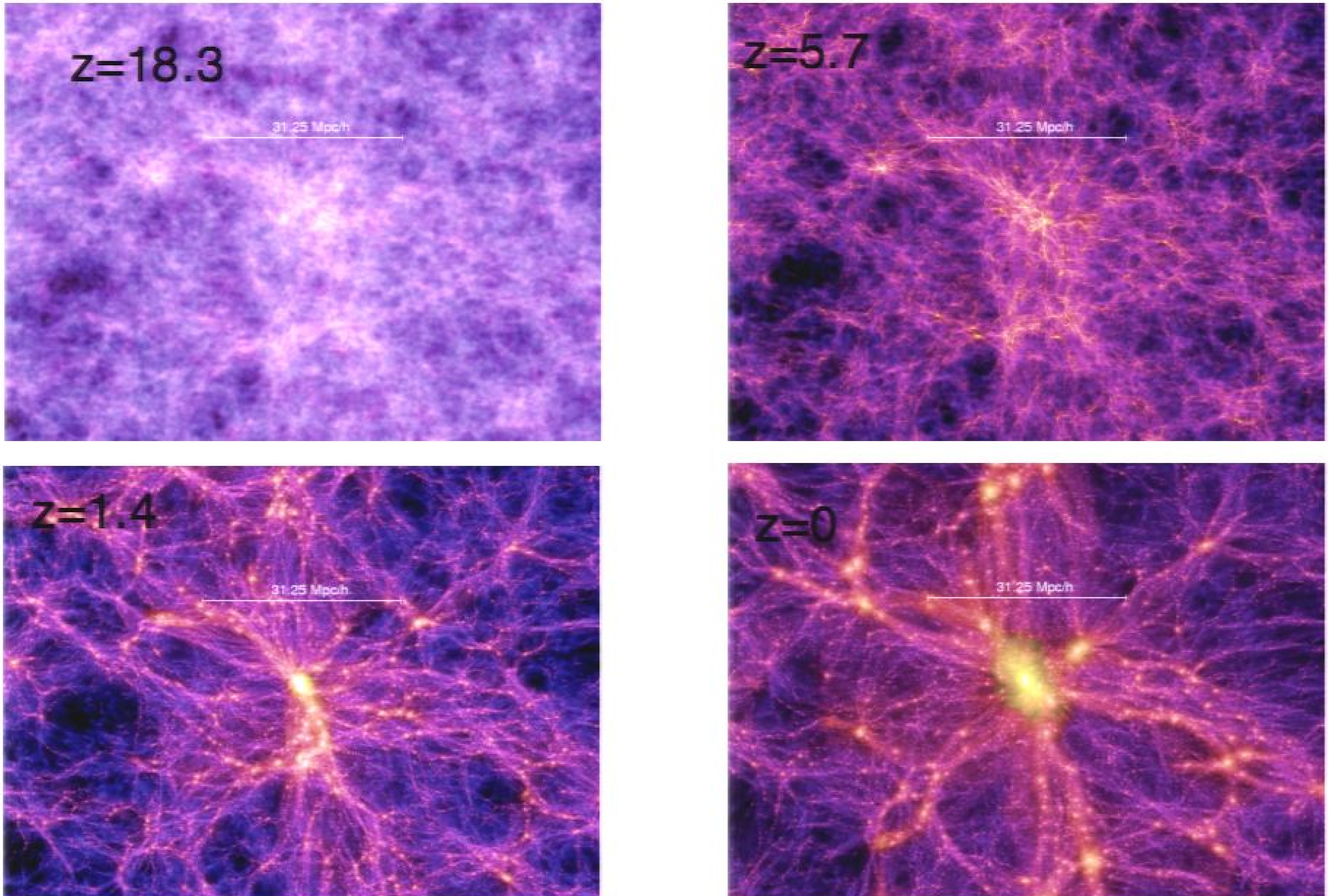
At the formation stage most of mass flow towards the walls; in walls towards the filaments, and in filaments towards the clumps.

All elements(walls, filaments and clumps) move and merge in the course of evolution. Some cells i.e. 3D "polyhedra" made of walls and 2D "polygons" made of filaments collapse resulting in: 1) larger cells, 2) multiple merging of clumps and 3) "multiarm" clumps.

Small cells in large voids evolve much slower and last much longer resulting in multiscale distribution of filaments

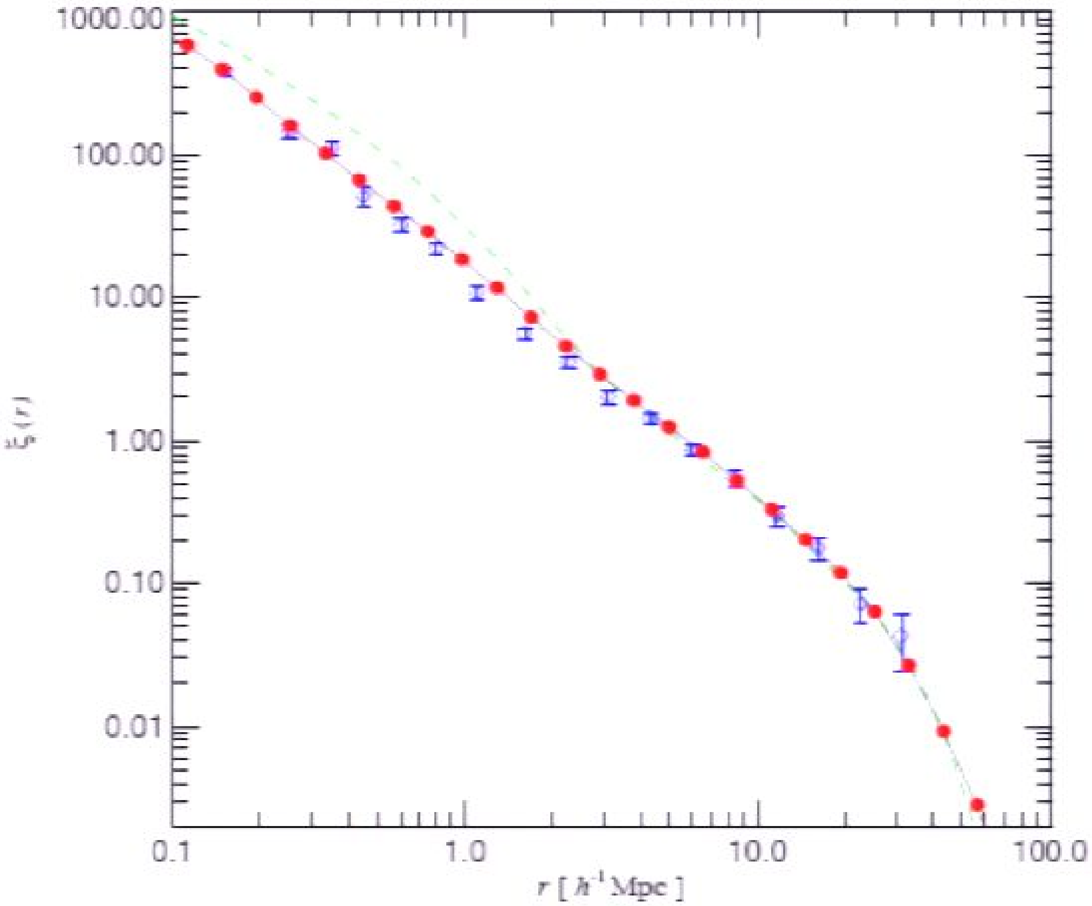
Millennium simulation: evolution

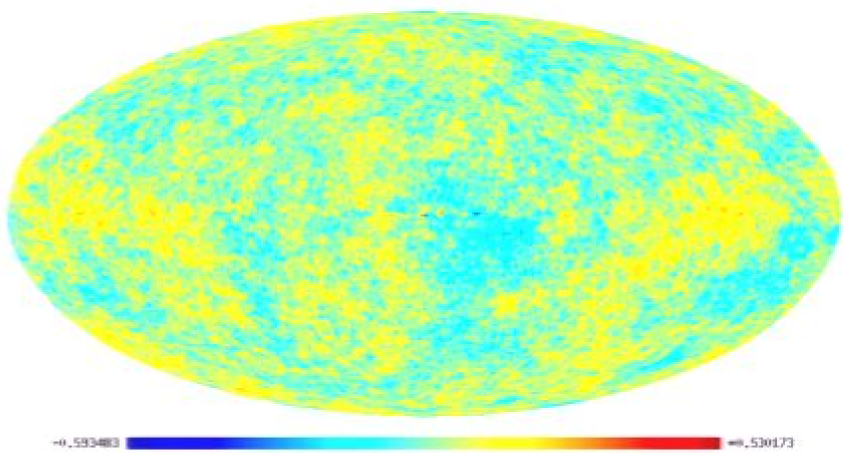
V.Springel et al 2005

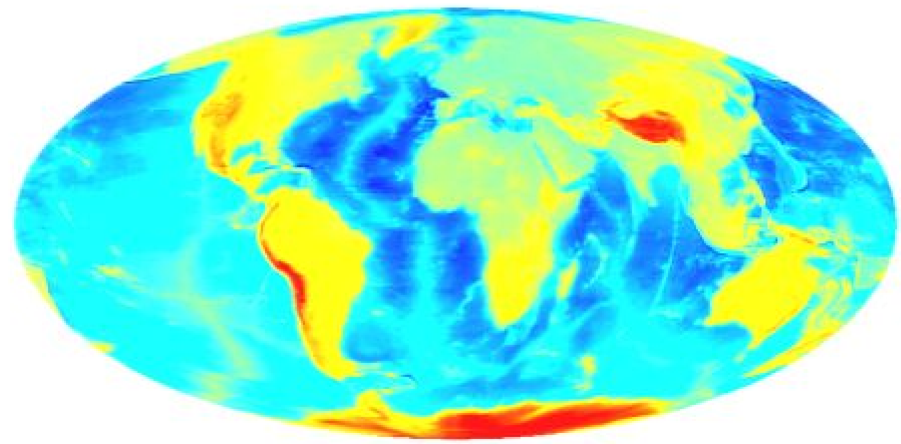


Analyzing the structure

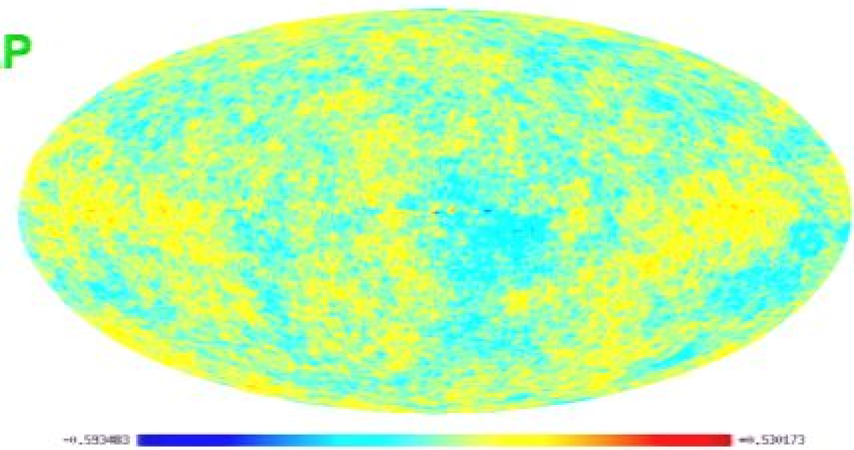
Millennium simulation: 2-point function



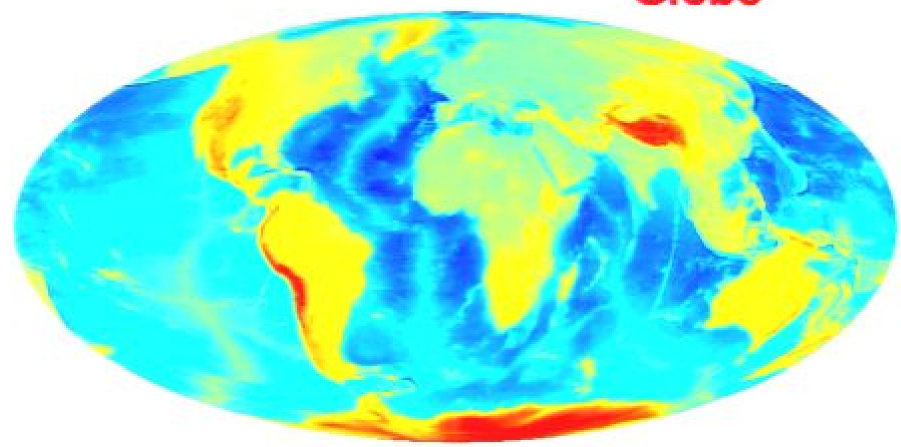

-0.593483
+0.530173

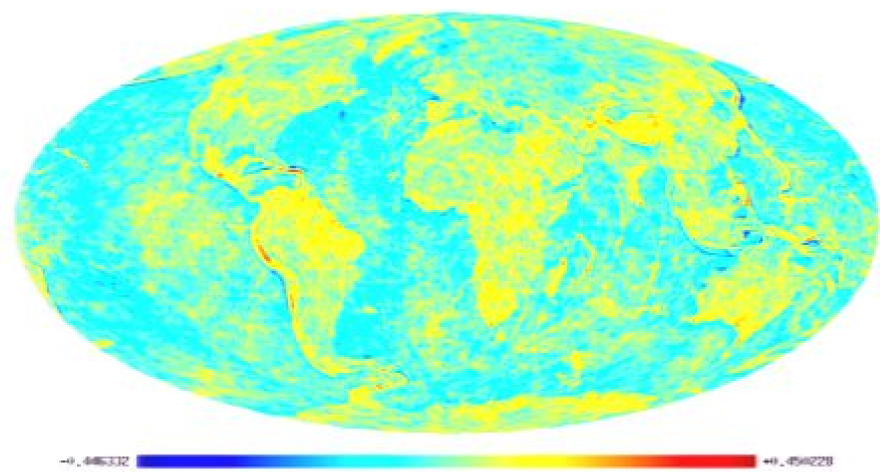


WMAP

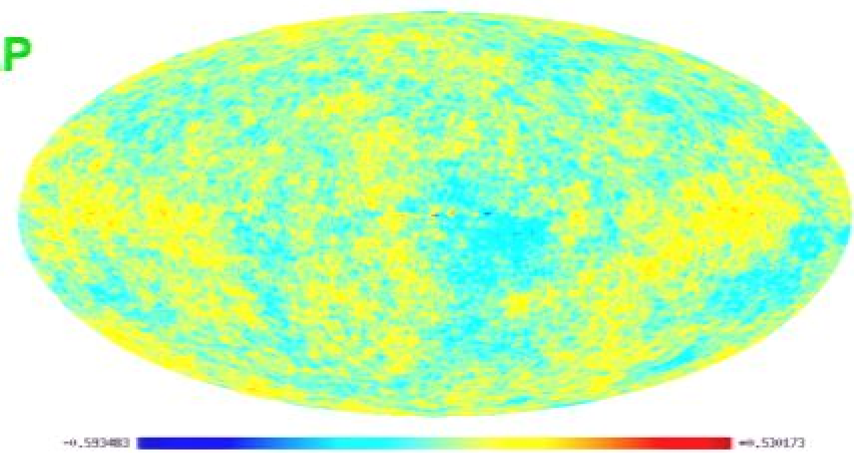


Globe

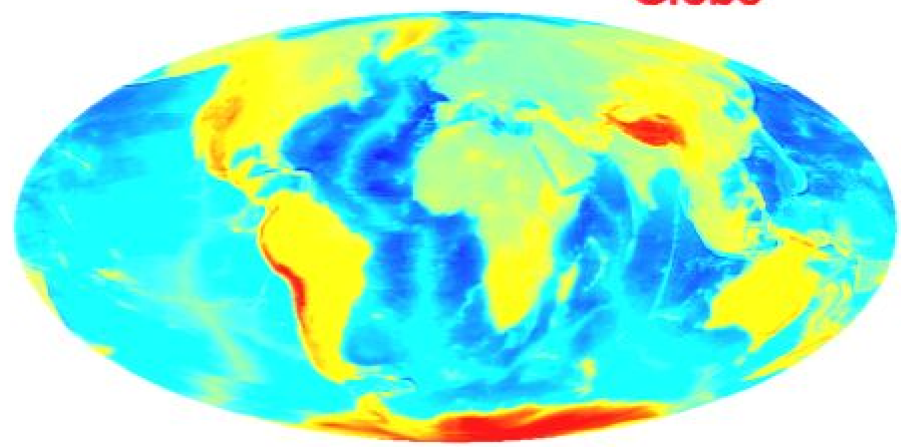


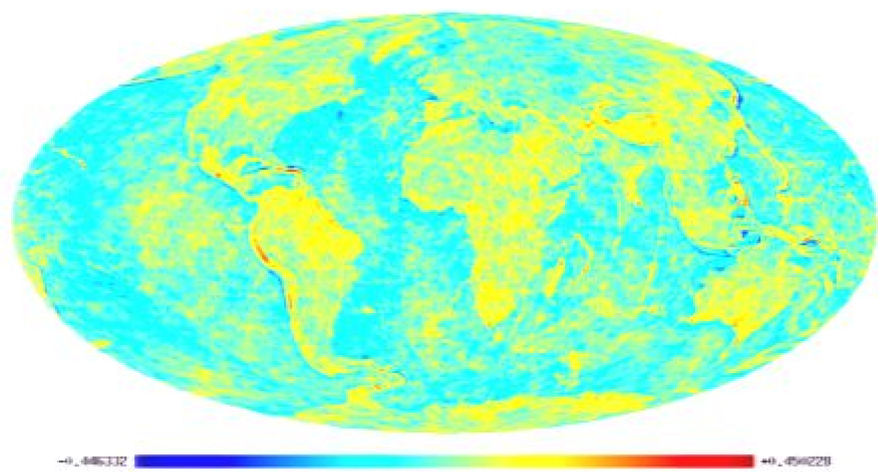


WMAP



Globe

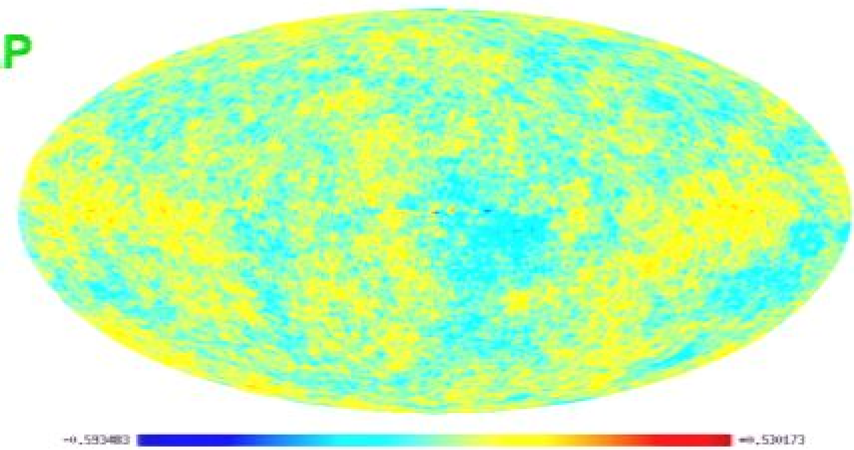




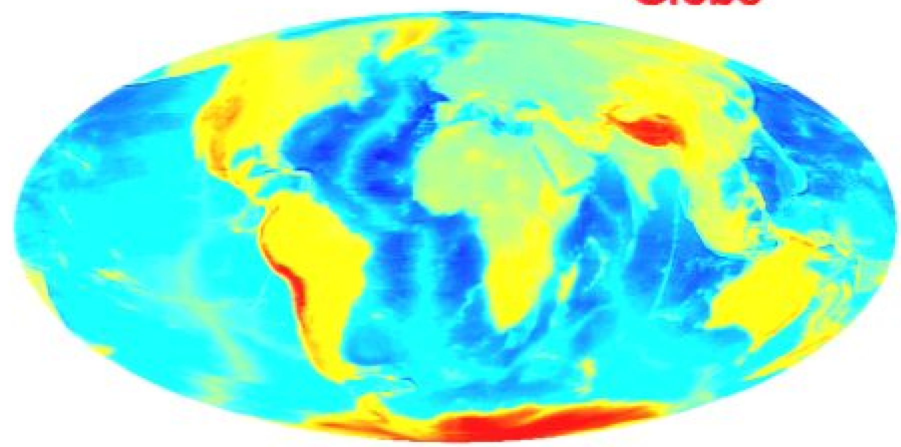
Globe phases
WMAP amplitudes

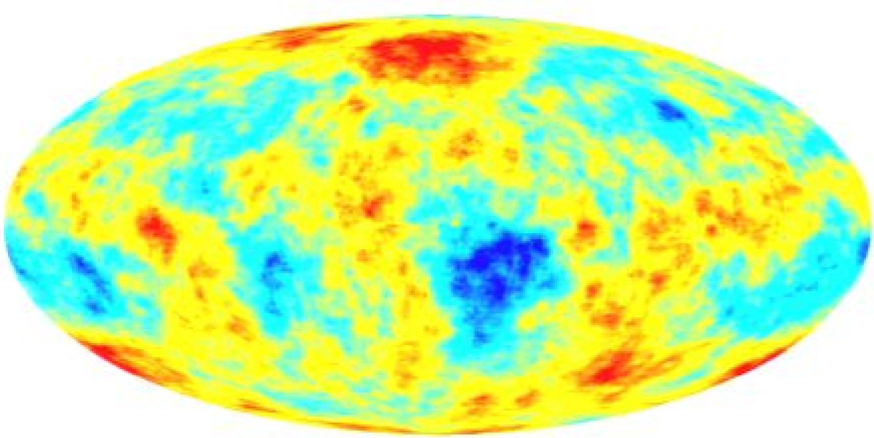
Courtesy of P. Coles

WMAP

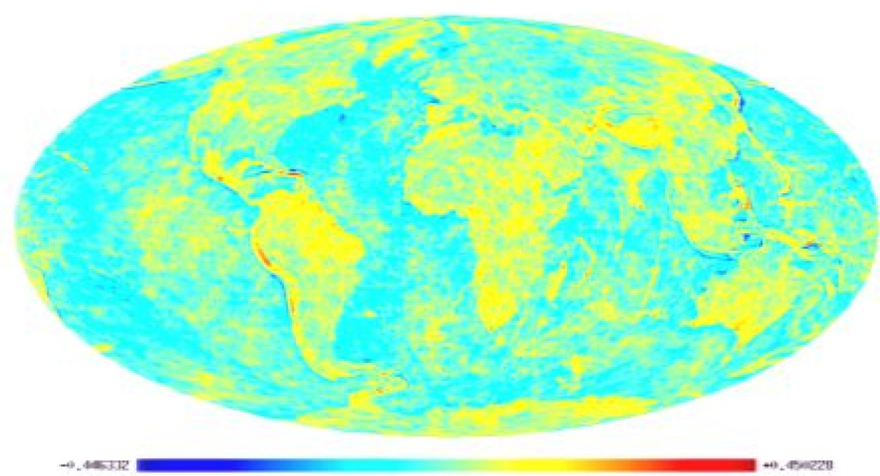


Globe





WMAP phases
Globe amplitudes



Globe phases
WMAP amplitudes

Three point function

Two and three point correlation functions are defined in cosmology as

$$\begin{aligned}\xi(r_{12}) &= <\delta(r_1)\delta(r_2)> \\ \zeta(r_{12}, r_{23}, r_{31}) &= <\delta(r_1)\delta(r_2)\delta(r_3)>\end{aligned}$$

Groth and Peebles (1977) suggested to use the reduced three point function

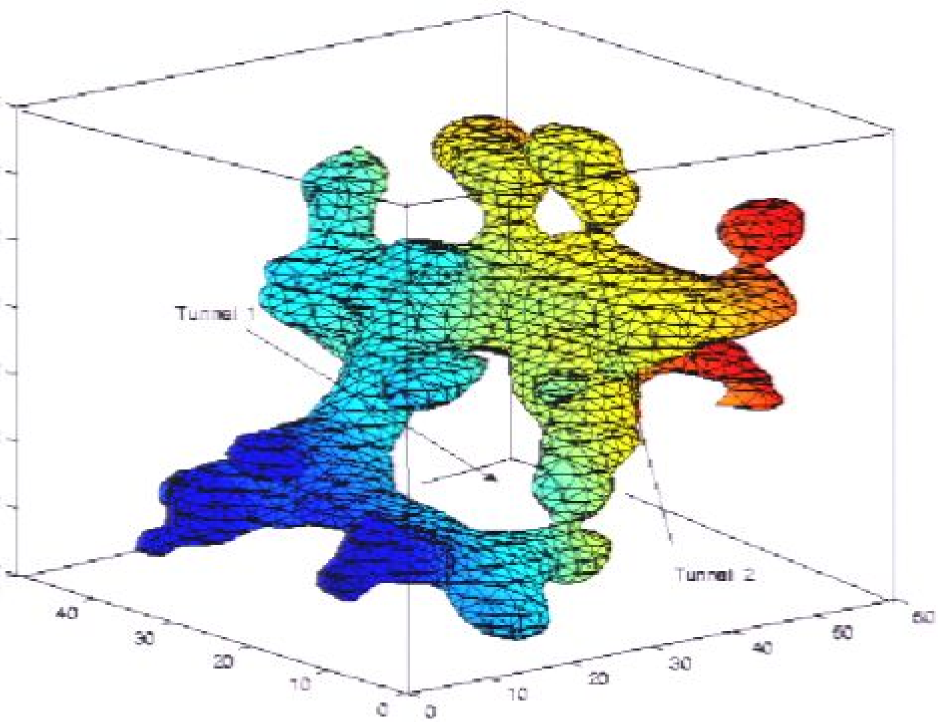
$$Q_3(r_{12}, r_{23}, r_{31}) = \frac{\zeta(r_{12}, r_{23}, r_{31})}{\xi(r_{12})\xi(r_{23}) + \xi(r_{23})\xi(r_{31}) + \xi(r_{31})\xi(r_{12})}$$

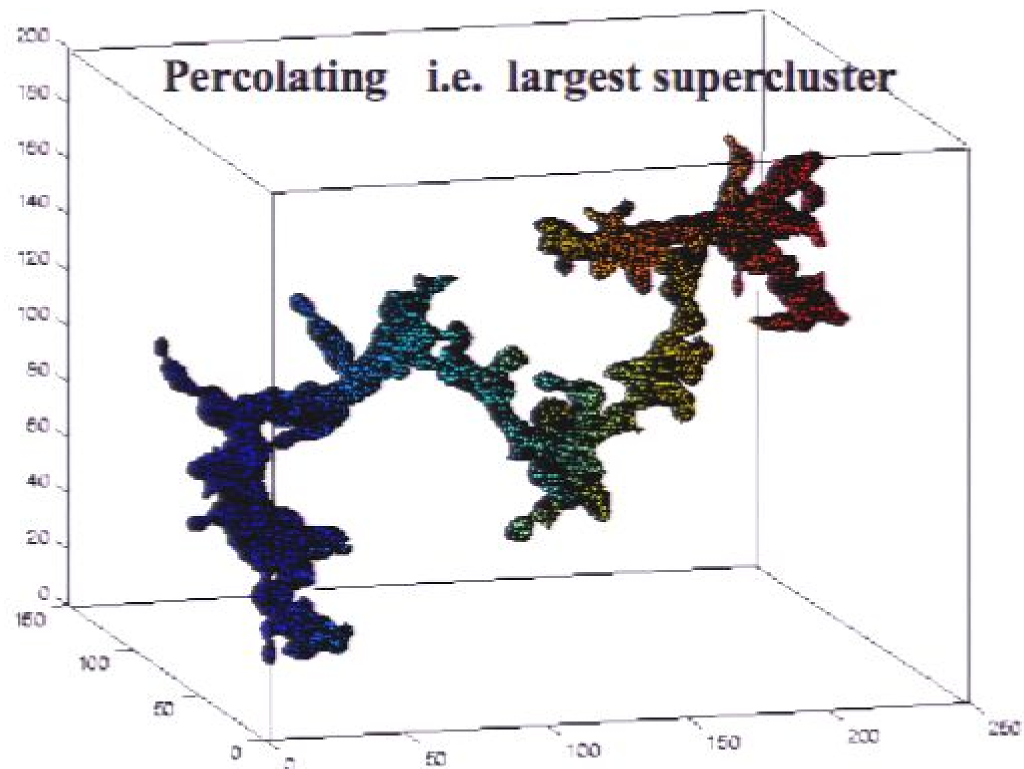
Q_3 is a slowly varying function: $Q_3 \sim 1$ (Nichol etal 2006, Gaztanaga etal 2005). Thus, if $\xi(r) \propto r^{-1.8}$ then

$$\zeta \propto r^{-3.6}$$

and therefore decays very fast with distance.

Examples of superclusters in LCDM simulation (VIRGO consortium) by SURFGEN

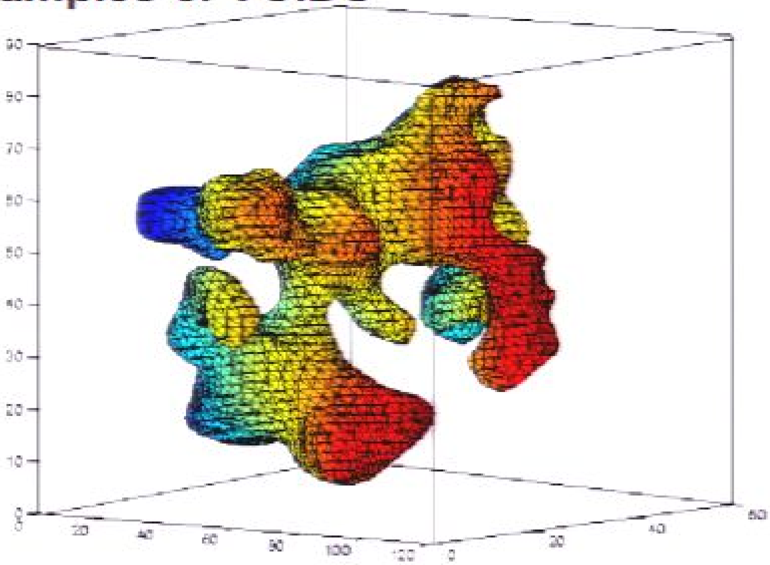


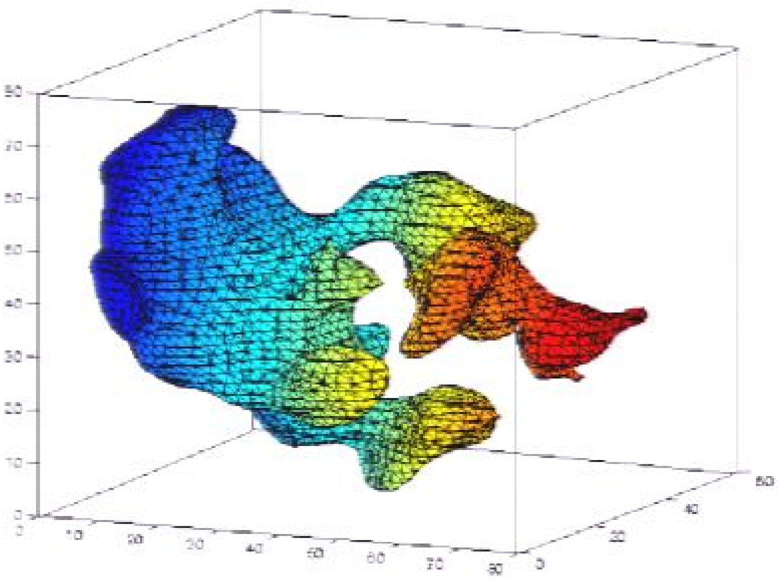


Sheth, Sahni, Sh, Sathyaprakash
2003, MN 343, 22

Examples of VOIDS

Shandarin et al. 2004, MNRAS,





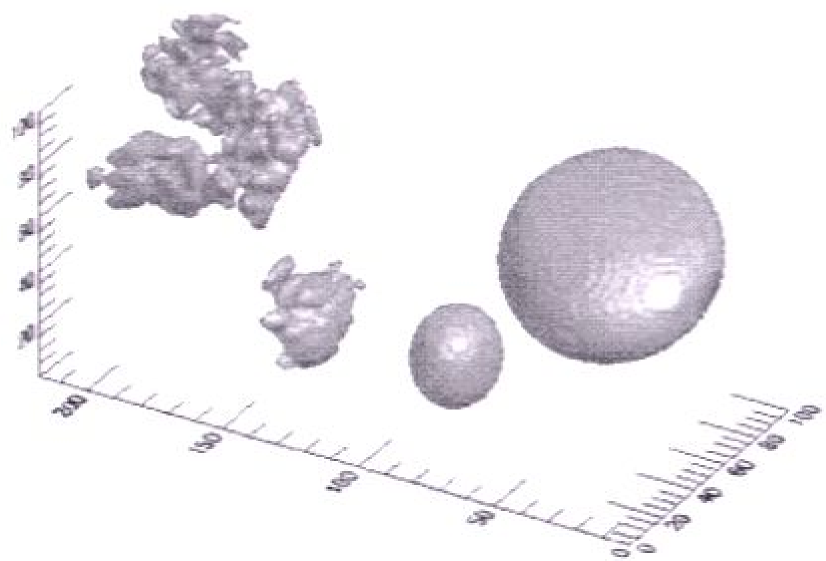
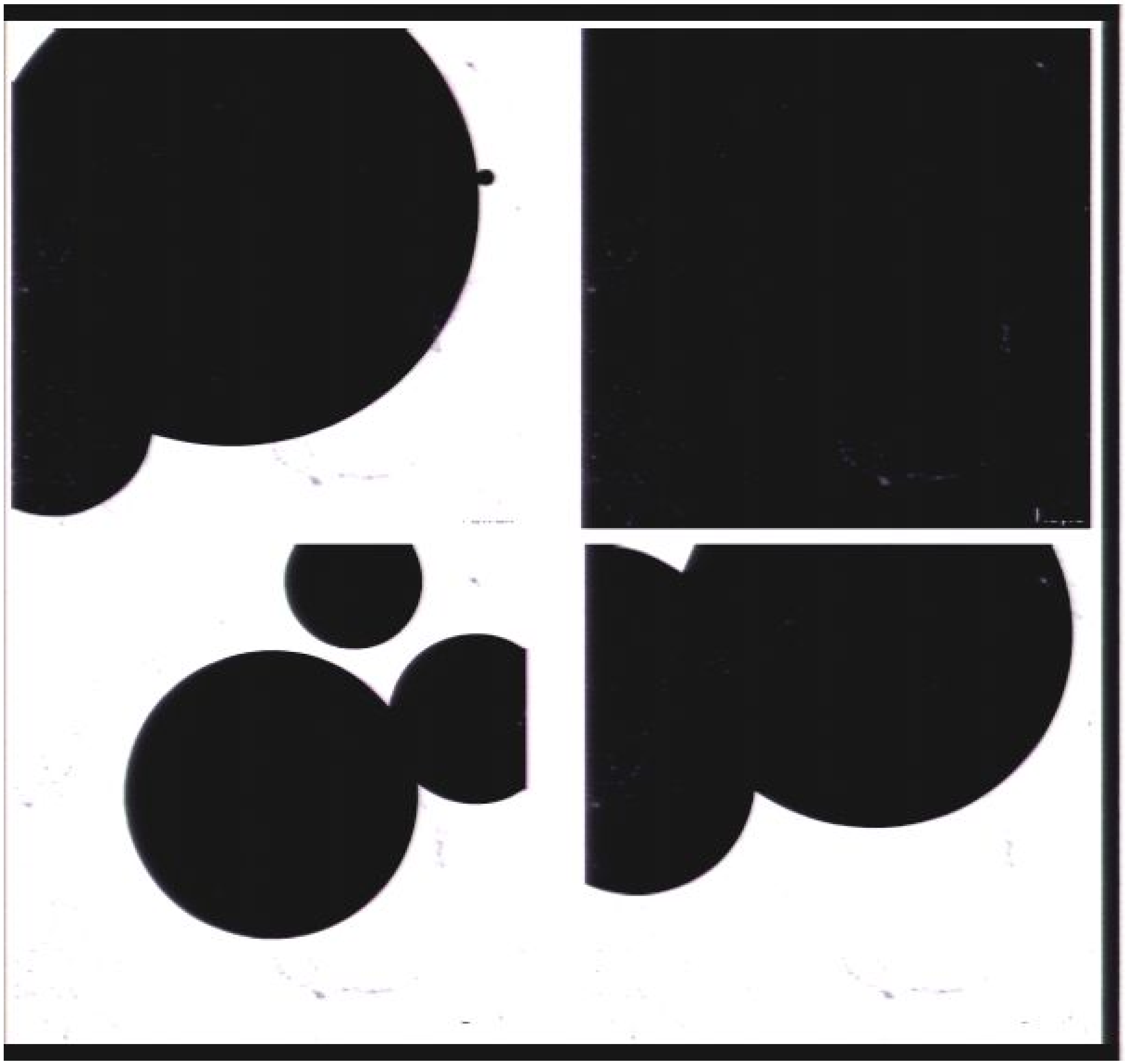


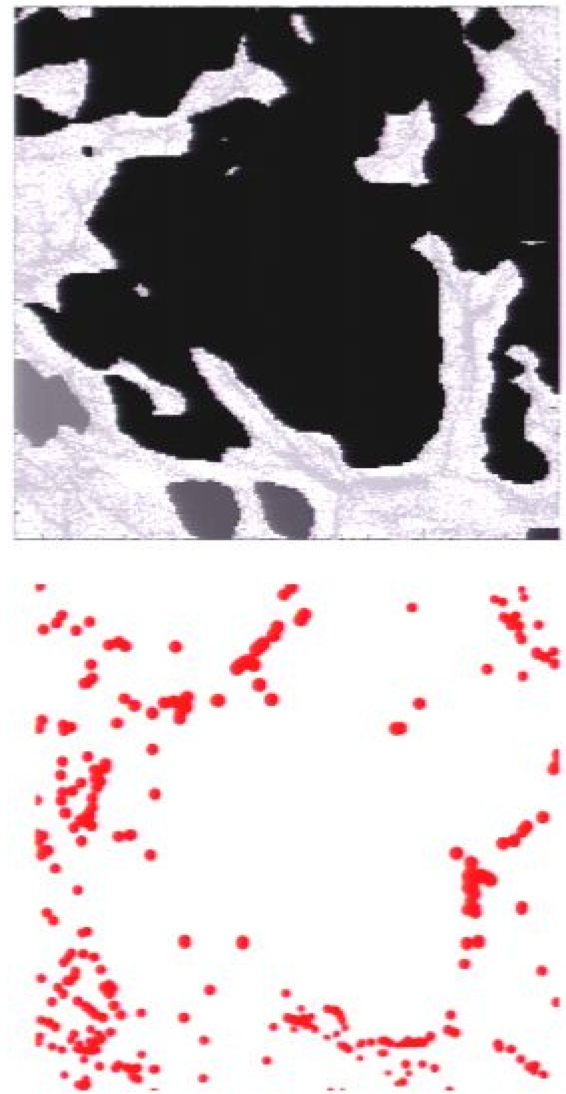
Figure 9. Comparison of an ellipsoid fit to high and low porosity voids. The low porosity small void (bottom left) has volume of $V = 7500$ cells, Porosity $P = V_E/V_V = 1.92$, and Inverse Porosity $IP = V_V/V_E = 0.52$. the fitting ellipsoid is at the bottom right. The high porosity large void (top left) has volume of $V = 24000$ cells, $P = 4.72$, and $IP = 0.21$.

Fitting voids by ellipsoids

Shandarin,Feldman,Heitmann,Habib 2006, MNRAS

Shapes of voids





Minkowski Functionals

Volume : V

Surface Area: $A = \iint_S da$

Integrated Mean Curvature : $C = \frac{1}{2}\iint_S \left(\frac{1}{R_1} + \frac{1}{R_2}\right) da$

Integrated Gaussian Curvature (EC): $\chi = \frac{1}{2\pi}\iint_S \frac{1}{R_1 R_2} da$

Genus: $G = 1 - \chi / 2$

where R_1 and R_2 are the principal curvature radii

Mecke, Buchert & Wagner 1994

Sizes and Shapes

Thickness: $T = \frac{3V}{A}$

Breadth: $B = \frac{A}{C}$

Length: $L = \frac{C}{4\pi}$

Sphere: T=B=L=R

Sahni, Sathyaprakash & Shandarin 1998

For each supercluster or void

SHAPEFINDERS

Planarity: $P = \frac{B - T}{B + T}$

Filamentarity: $F = \frac{L - B}{L + B}$

Sphere: P=F=0

LCDM Superclusters (red) vs Voids (blue)

R_f=5/h Mpc

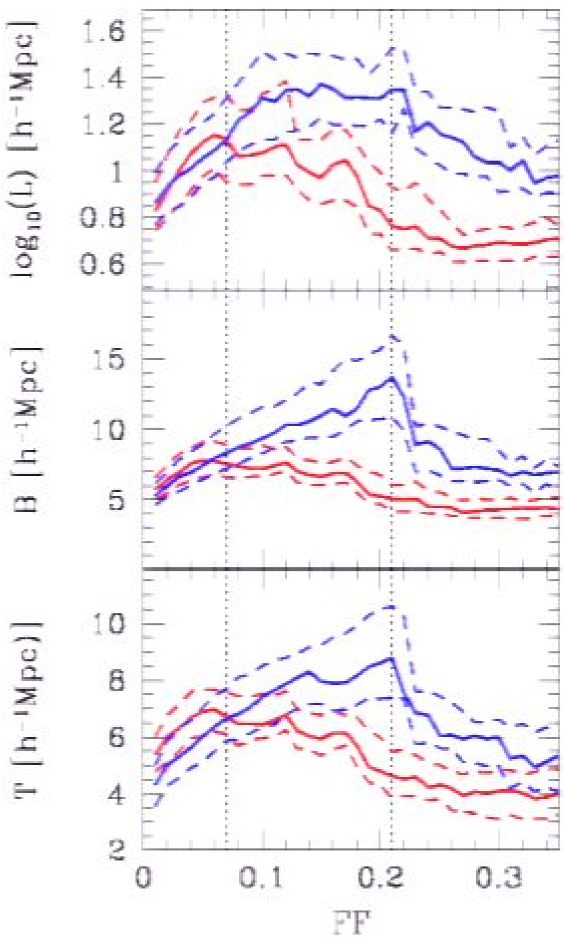
Median (+/-) 25%

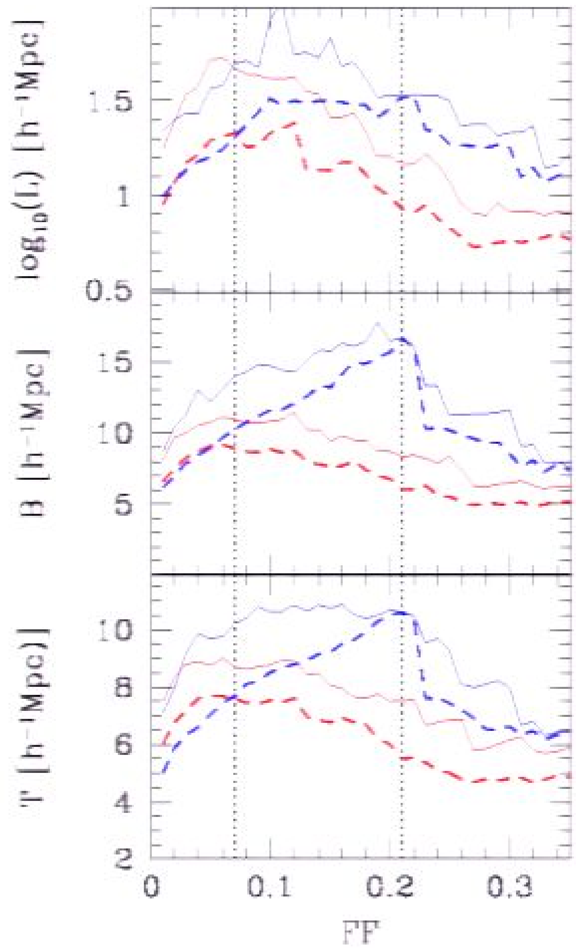
Top 25%

log(Length)
(radius)

Breadth

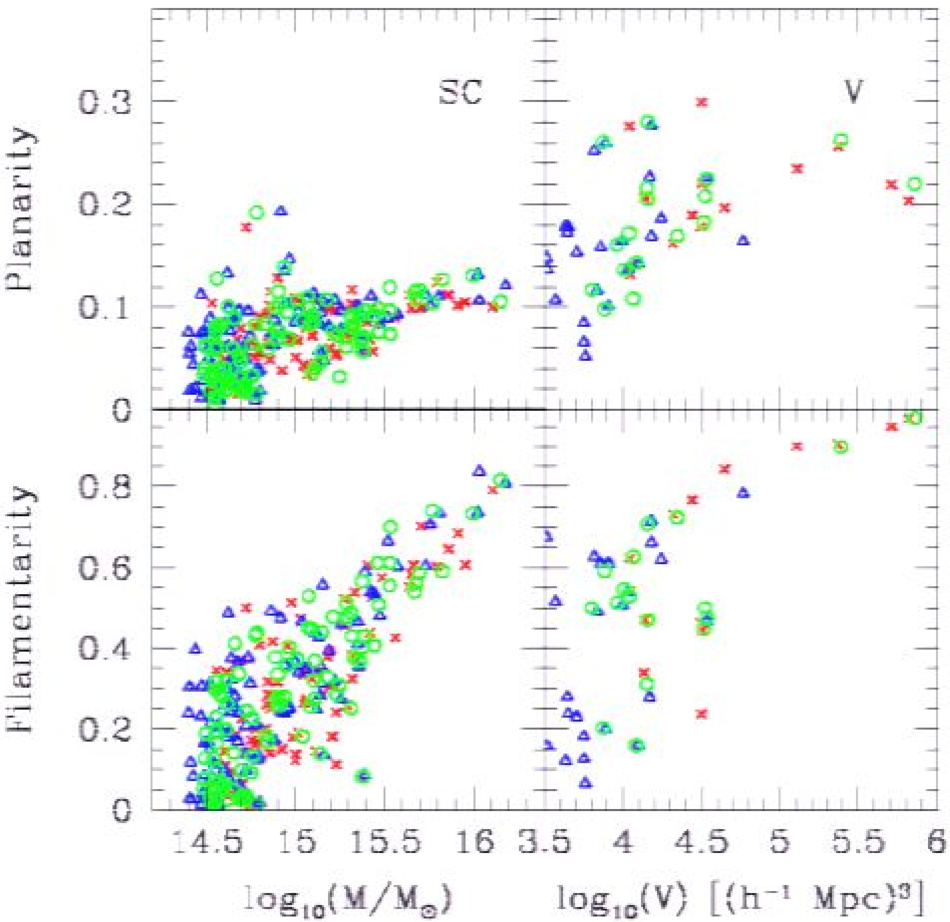
Thickness





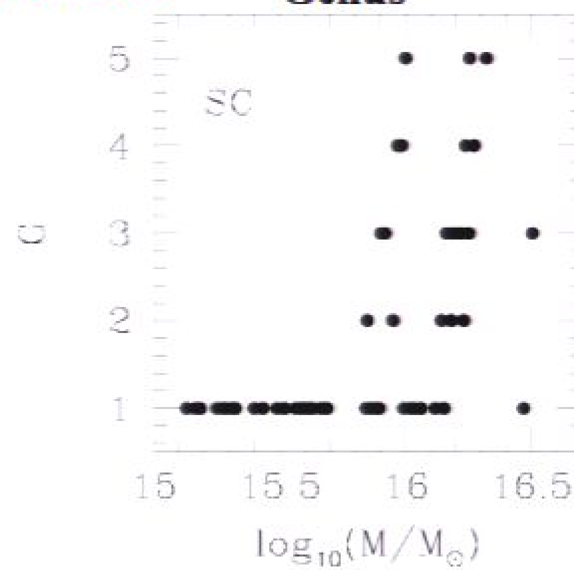
Shandarin, Sheth, Sahni 2004

Shapes of SC and V



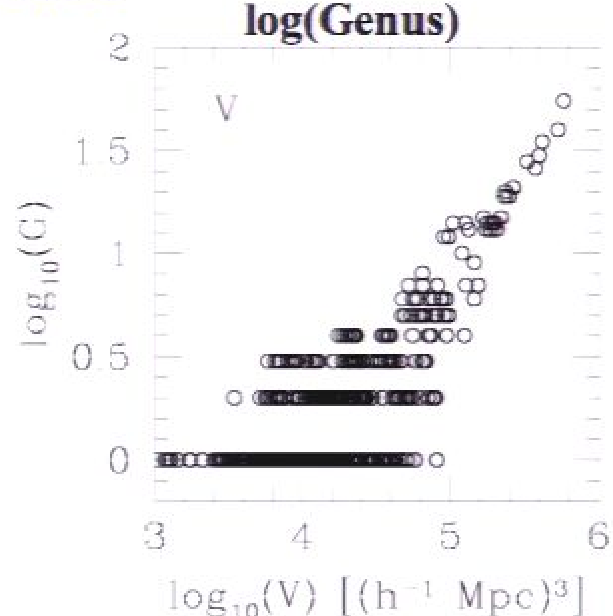
Superclusters

Genus



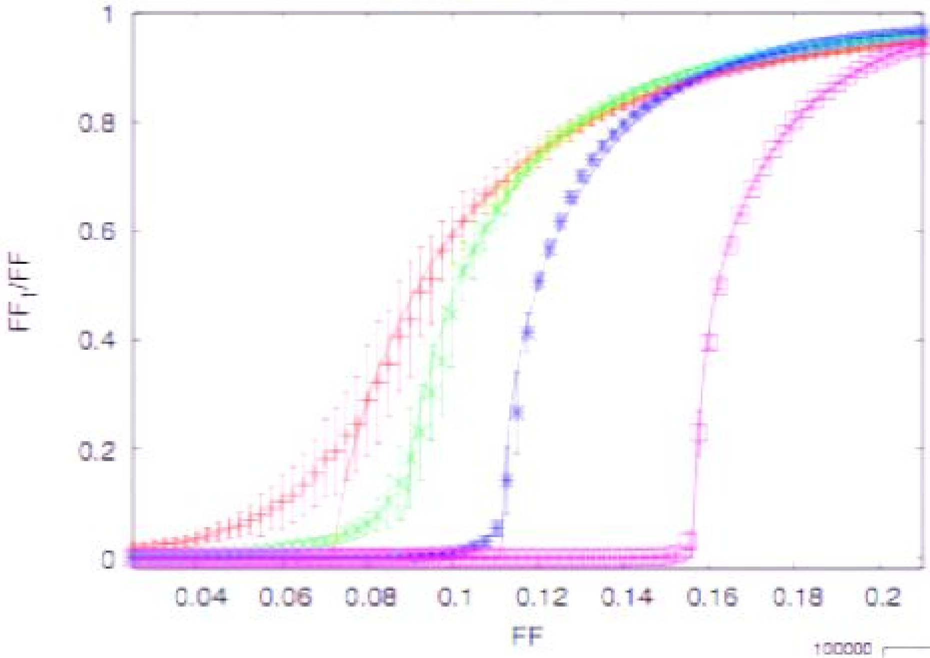
Voids

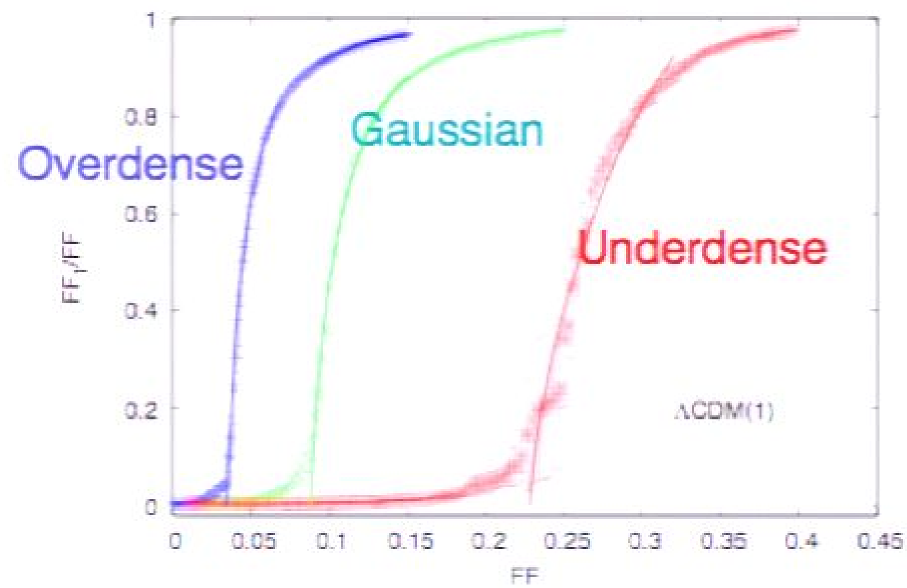
log(Genus)



Percolation in Gaussian and nonlinear fields

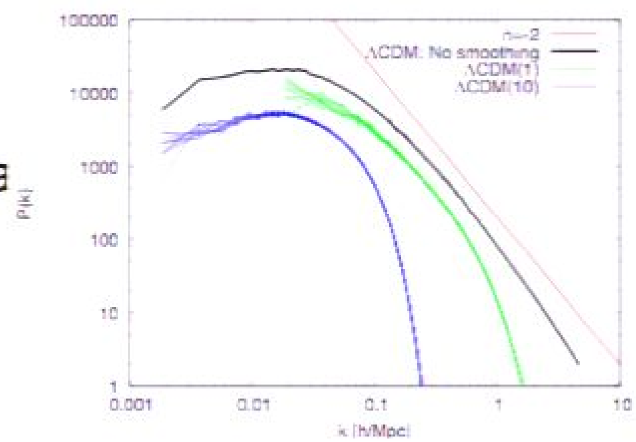
Gaussian fields





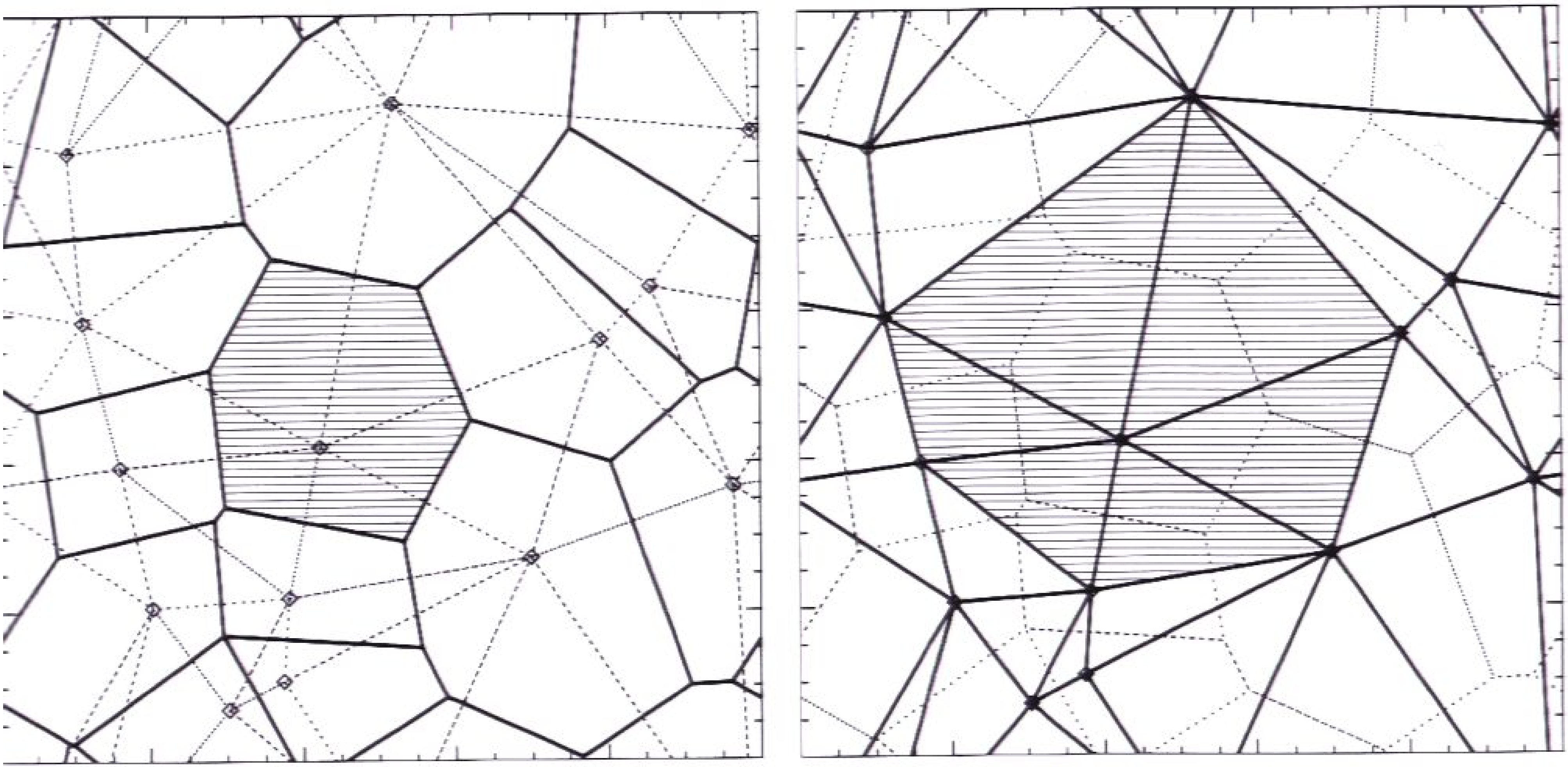
Nonlinear evolution

Power spectra



Density field from points

Voronoi and Delaunay tessellations (2D)

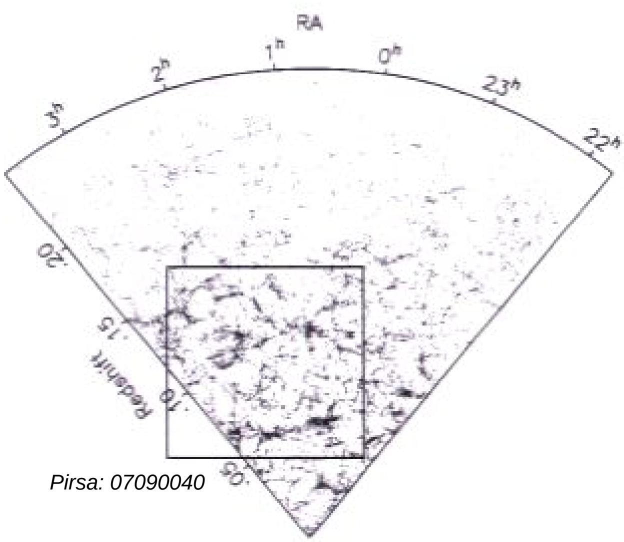


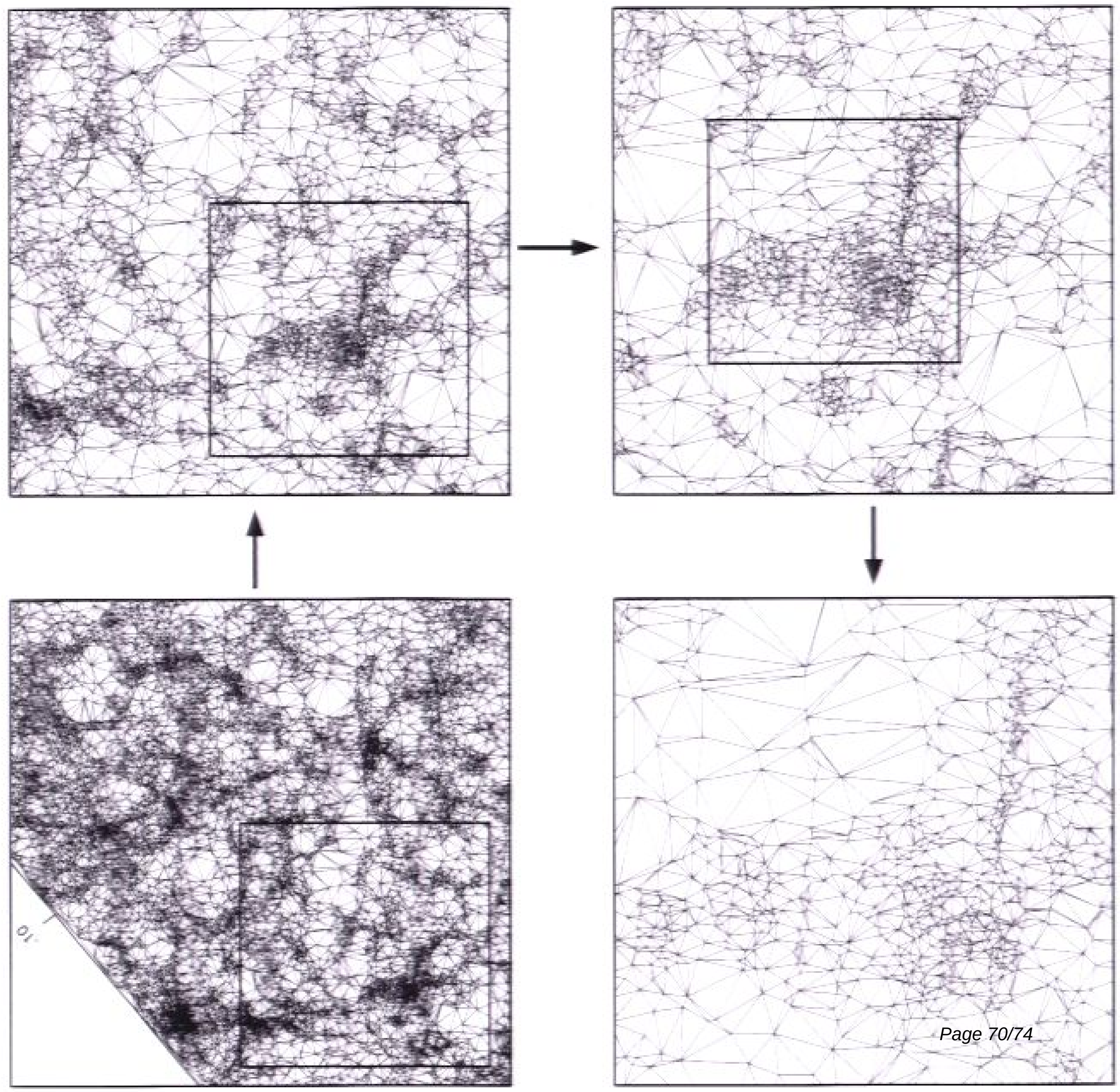
Voronoi tessellation Delaunay tessellation

Schaap, van de Weygaert 2007

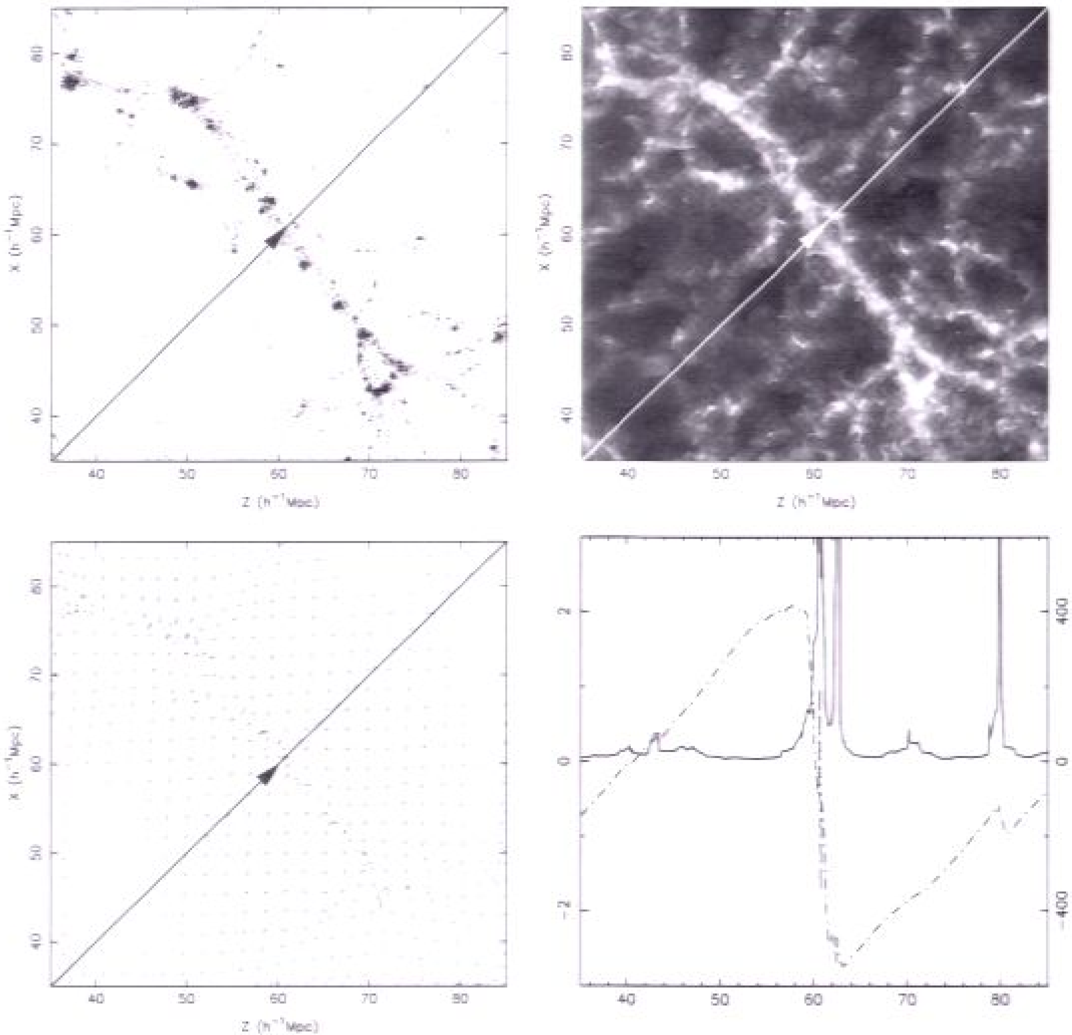
Spatial resolution of DTFE

Schaap, van de Weygaert 2007



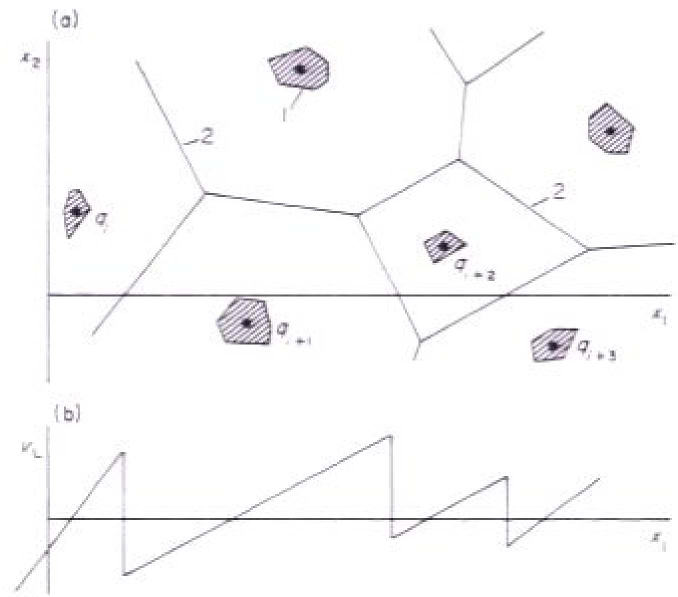


Structure of a filament



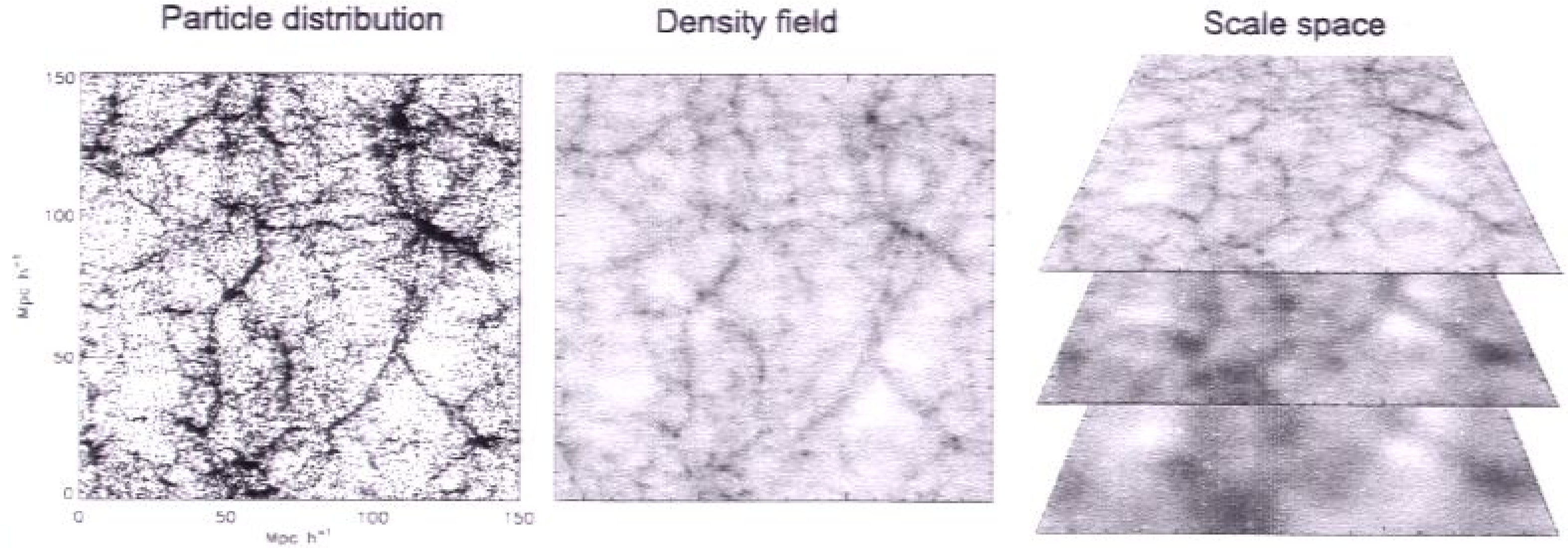
Schaap, van de Weygaert 2007

Adhesion appr.



Gurbatov, Saichev, Shandarin 1985

Multiscale Morphology Filter (MMF)



Aragon-Calvo, van de Weygaert 2007

At every scale

$$\rho(\mathbf{r}) = \rho(\mathbf{r}_0) + \frac{\partial \rho}{\partial r_i}(r_i - r_{0i}) + \frac{\partial^2 \rho}{\partial r_i \partial r_k}(r_i - r_{0i})(r_k - r_{0k}) + \ldots$$

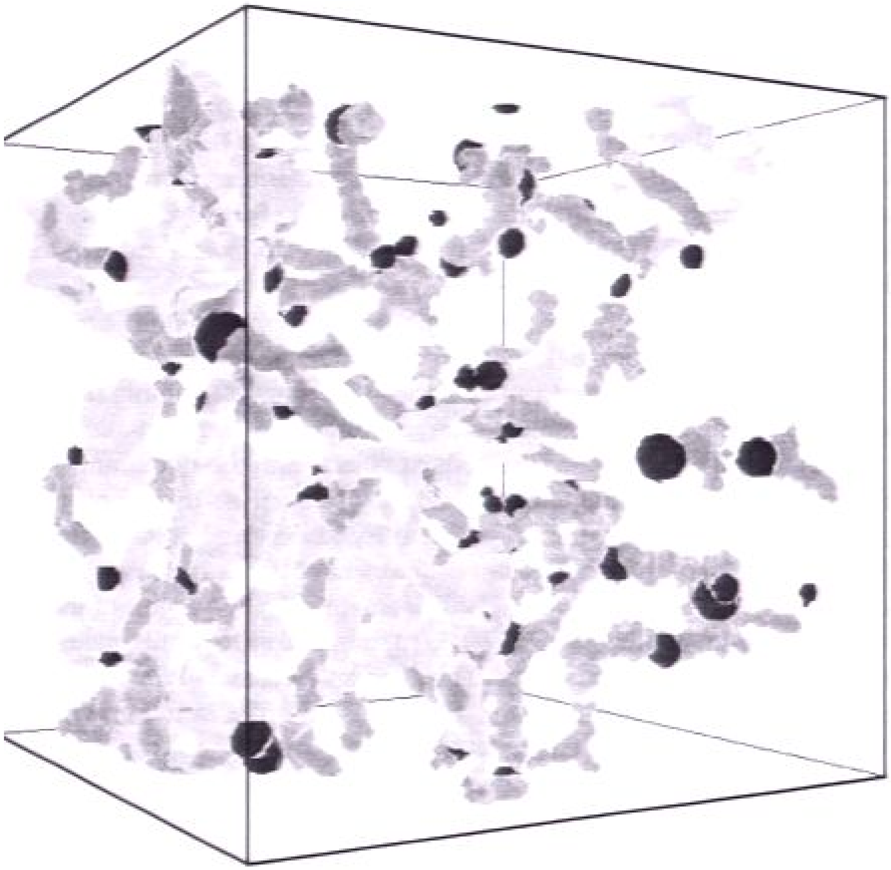
Eigen values $\lambda_1 > \lambda_2 > \lambda_3$ of the Hessian $\frac{\partial^2 \rho}{\partial r_i \partial r_k}$ determines the type of the mass element:

clump/blob: $\lambda_1 \approx \lambda_2 \approx \lambda_3$ and $\lambda_3 < 0$ $\lambda_2 < 0$, $\lambda_1 < 0$

filament/line: $\lambda_1 \approx \lambda_2 \gg \lambda_3$ and $\lambda_3 < 0$ $\lambda_2 < 0$,

wall/sheet: $\lambda_1 \gg \lambda_2 \approx \lambda_3$ and $\lambda_3 < 0$

Mass, volume and density of cosmic web



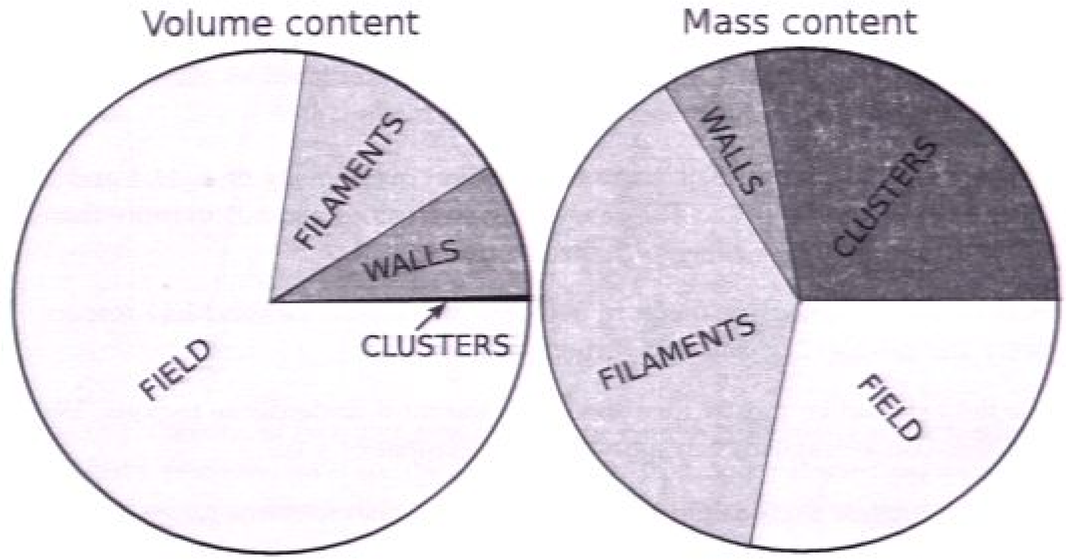


Figure 3.9: *Pie diagram showing an inventory of the Cosmic Web in terms of volume (left) and mass (right).*

	Clusters	filaments	walls	field
Volume filling (%)	0.38	8.79	4.89	85.94
Mass content (%)	28.1	39.2	5.45	27.25
Mean overdensity	73	4.45	1.11	0.31
Median overdensity	11.5	1.65	0.88	0.30

Aragon-Calvo, van de Weygaert 2007

Conclusions

Hierarchical clustering is the model of the evolution and merging of clumps;
* it is able to compute the mass functions of the clumps,
* it says that field on scales greater then R (sigma~1) is approximately linear (old)
* it says nothing about the filaments and walls (new)

TZA and 'Cosmic web' model by BKP 96 are similar addressing only the largest structure at a given epoch. BKP is of probabilistic while TZA of deterministic nature.

TZA supplemented by Arnol'd's theory of singularities provides a more detailed description of the largest structures. One can compute correlation functions from density/particle field, but cannot generate density field from correlation functions, It can only very crudely approximate it.

Both are unable to deal with multiscale nature of the structures at a given epoch.

Adhesion Approximation describes the evolution as merging of cells: clumps and filaments merge when a corresponding cell collapses.

AA predicts the existing of small cells inside large voids down to smallest scale present in the initial power spectrum as well as filaments and walls in large voids.