Title: A New Perspective on DGP Gravity

Date: Sep 11, 2007 05:00 PM

URL: http://pirsa.org/07090038

Abstract:

DGP Gravity: Dvali, Gabadadze & Porrati (hep-th/0005016)

- 4-D gravity on a brane in 5-D Minkowski space:

$$= 2M_5^3 \int_{bulk} d^5x\sqrt{-G}\,R(G) + \int_{brane} d^4x\,\sqrt{-g}\,\left(M_4^2\,\mathcal{R}(g) + \mathcal{L}_{\rm matter}\right)$$

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- may explain cosmic acceleration **(without Λ!)**

→ brane is self-accelerated

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  - DGP is stable/ghosts irrelevant (Koyama; Deffayet et al; Dvali `06)

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As it stands, DGP gravity is a **BAD** model !!

New perspective: take a 5-D point of view!

DGP model is 5-D Einstein gravity
coupled to certain class of matter/branes

Solutions of DGP Gravity:

Equations of motion:

Bulk: $$R_{ab} - \tfrac{1}{2} R\, G_{ab} = 0$$

Brane: $$2M_5^3\, K_{\mu\nu} = -M_4^2 \left(\mathcal{R}_{\mu\nu} - \tfrac{1}{6}\mathcal{R}\, g_{\mu\nu}\right)$$

Israel junction conditions

- brane stress tensor modified by geometric terms
- assumed Z_2 symmetry about the brane

$$\longrightarrow \Gamma_{\mu\nu} = K^{+}_{\mu\nu} + K^{-}_{\mu\nu} = 2K_{\mu\nu}$$

- assumed no additional matter on brane

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"cut-and-paste" to build symmetric solutions: (Deffayet '01)

- choose a known solution of 5-D Einstein equations, eg,

  Minkowski: $ds^2 = -dt^2 + dr^2 + r^2 d\Omega_3^2$ X 2

- choose a simple symmetric brane geometry, eg,

  FRW: $ds^2 = -d\tau^2 + a(\tau)^2\, d\Omega_3^2$

- choose simple embedding ansatz, eg, $r = a(\tau)$

  and cut-and-paste with Israel conditions solving trajectory

(Deffayet '01)

solution: $$a(\tau) = \frac{1}{H_0}\cosh(H_0\tau)$$

with $$H_0 = 2M_5^3/M_4^2$$

Brane geometry corresponds to 4-D deSitter space!

Self-Accelerating brane!

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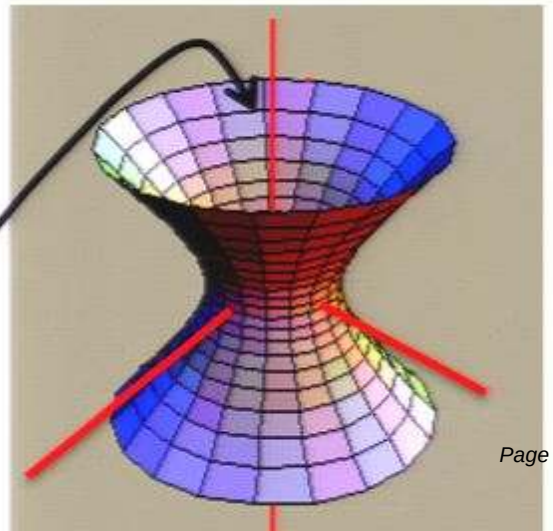
hyperboloid in 5-D Minkowski:

$$r^2 - t^2 = H_0^{-2}$$

remove center of space



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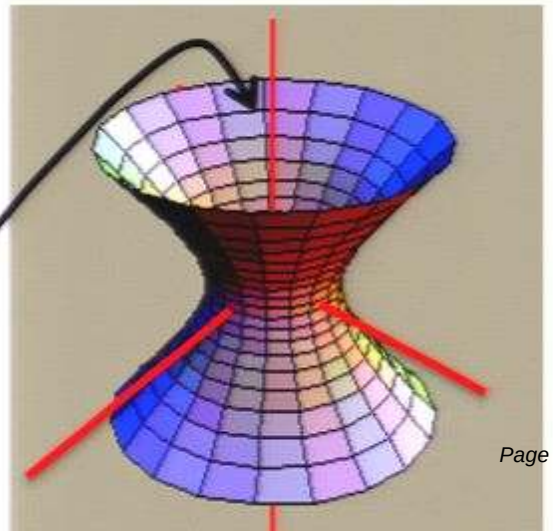
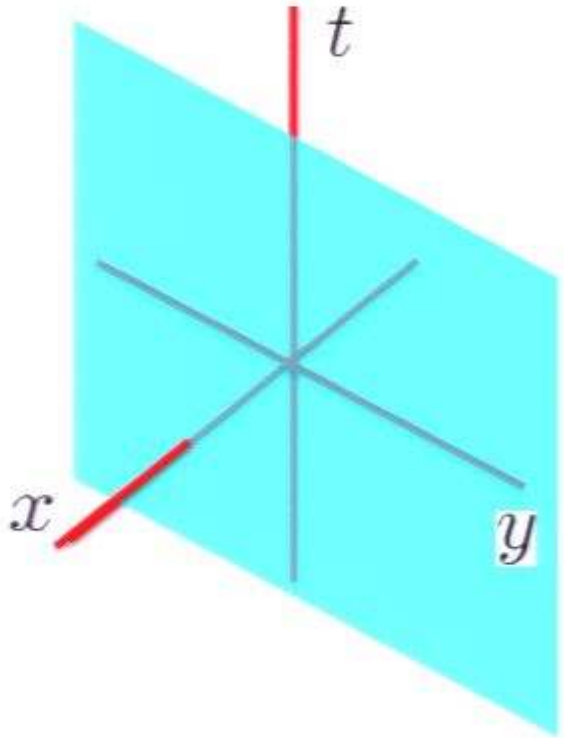


(also find: Normal branch)

Normal branch: $r \to \infty$ (that is $x = const.$)

flat 4-D brane



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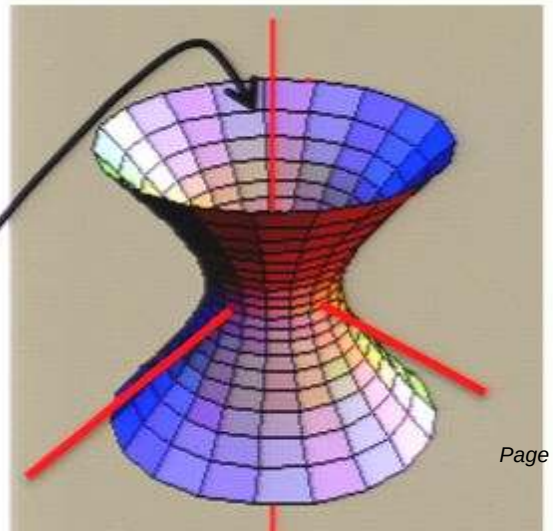

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DGP phenomenology versus fine-tuning:

$$H_0 = \frac{2M_5^3}{M_4^2} \sim \text{Hubble scale} \sim 1/(10^{26}\ metres)$$

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  effective stress-tensor for DGP brane:

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Sign !!

$$\sigma_{\rm eff}\equiv -6\,M_4^2H^2$$

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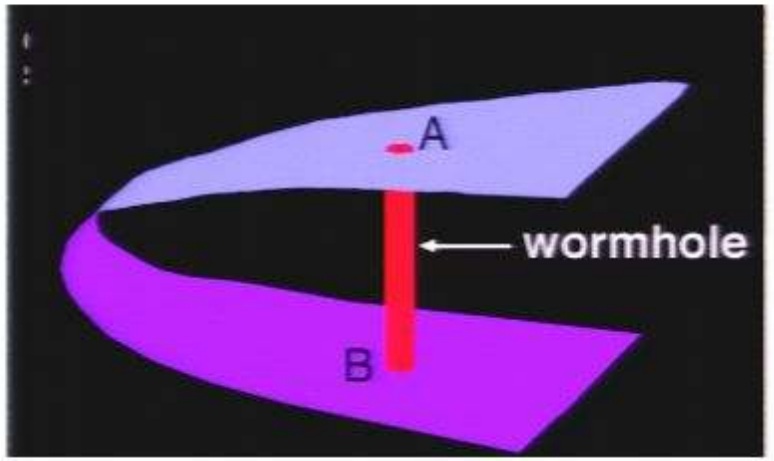
  yes, even though wormhole mouth rapidly expands here

  → colliding branes



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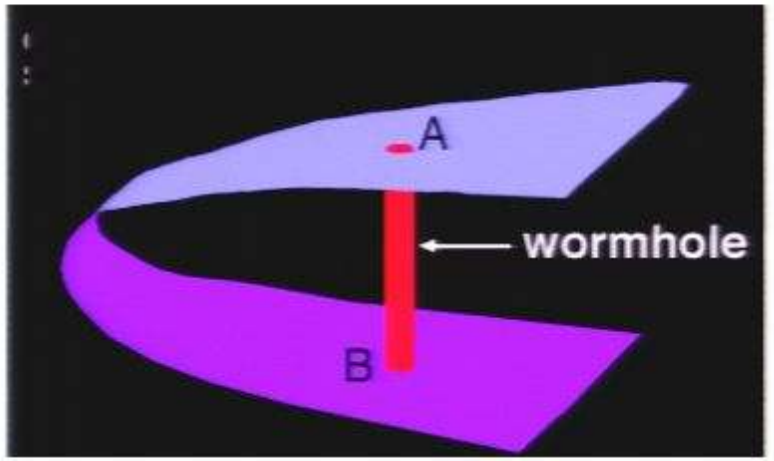

- wormhole issues ameliorated if brane is Z_2-orbifold

(negative energies? colliding branes? . . .)

New solutions (and new problems):

5-D Schwarzschild

$$ds^2 = -\left(1 + \frac{\mu}{r^2}\right) dt^2 + \frac{dr^2}{1 + \frac{\mu}{r^2}} + r^2 d\Omega_3^2$$

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5-D Schwarzschild (with negative mass):

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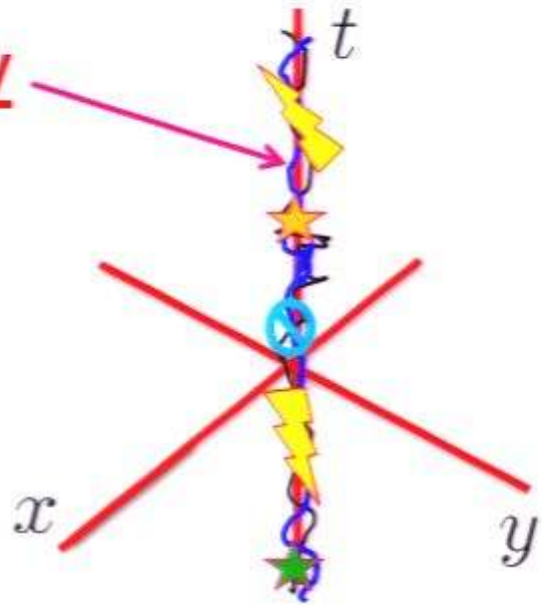
Mass: $M = -12\pi^2 M_5^3 \mu$



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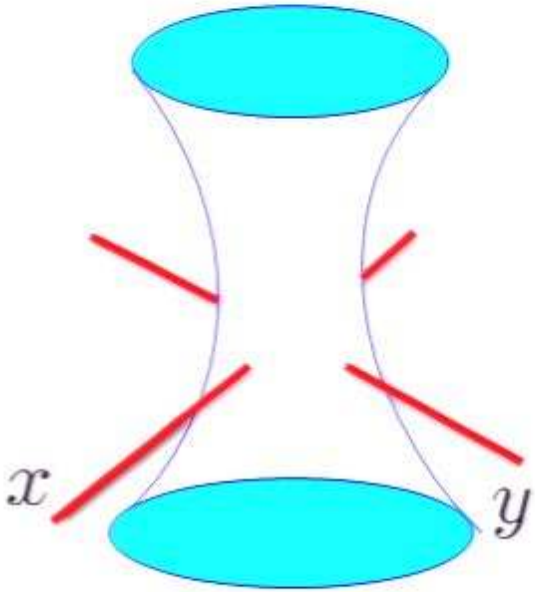
But cut-and-paste will remove center of space with SA brane!



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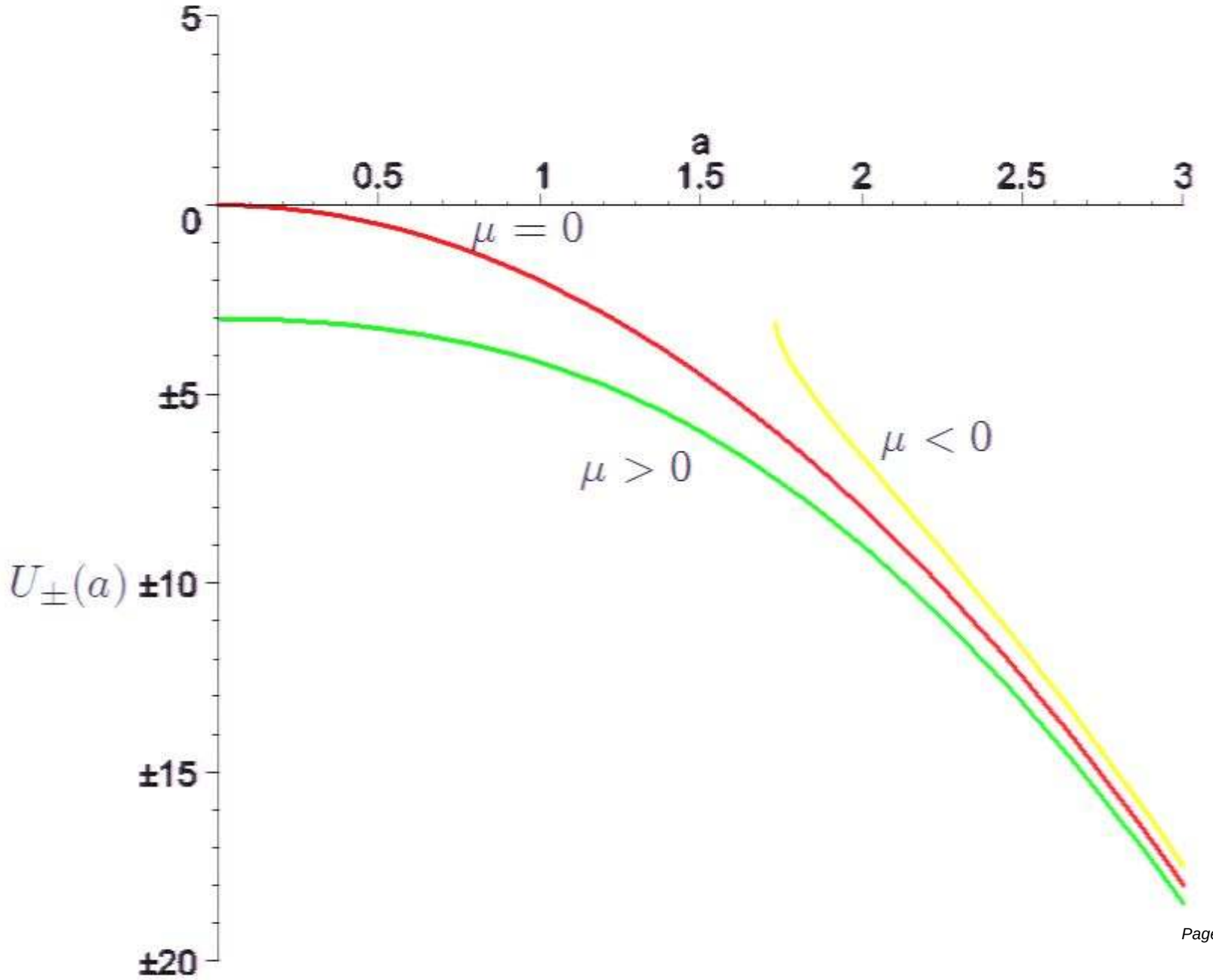

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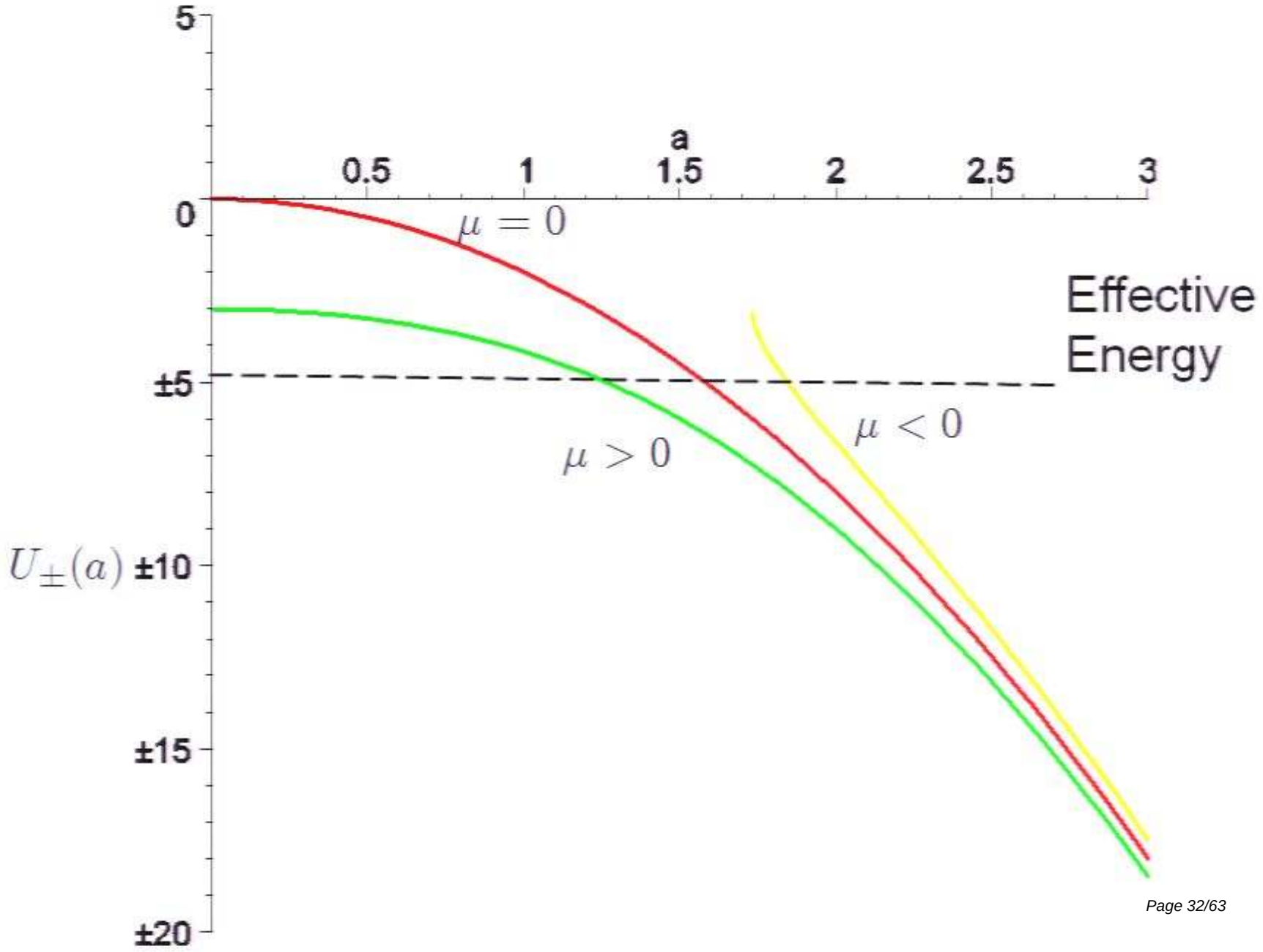

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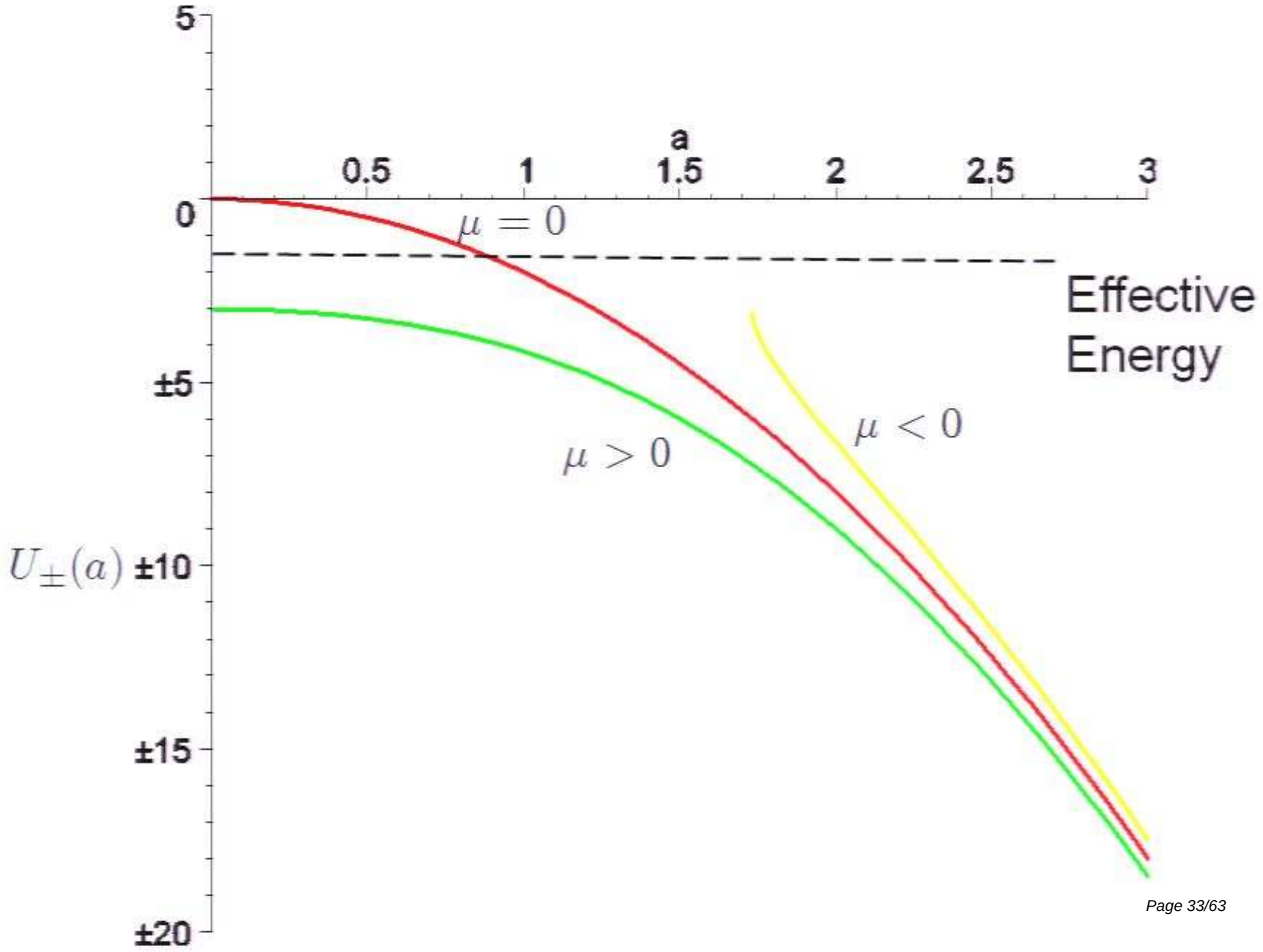

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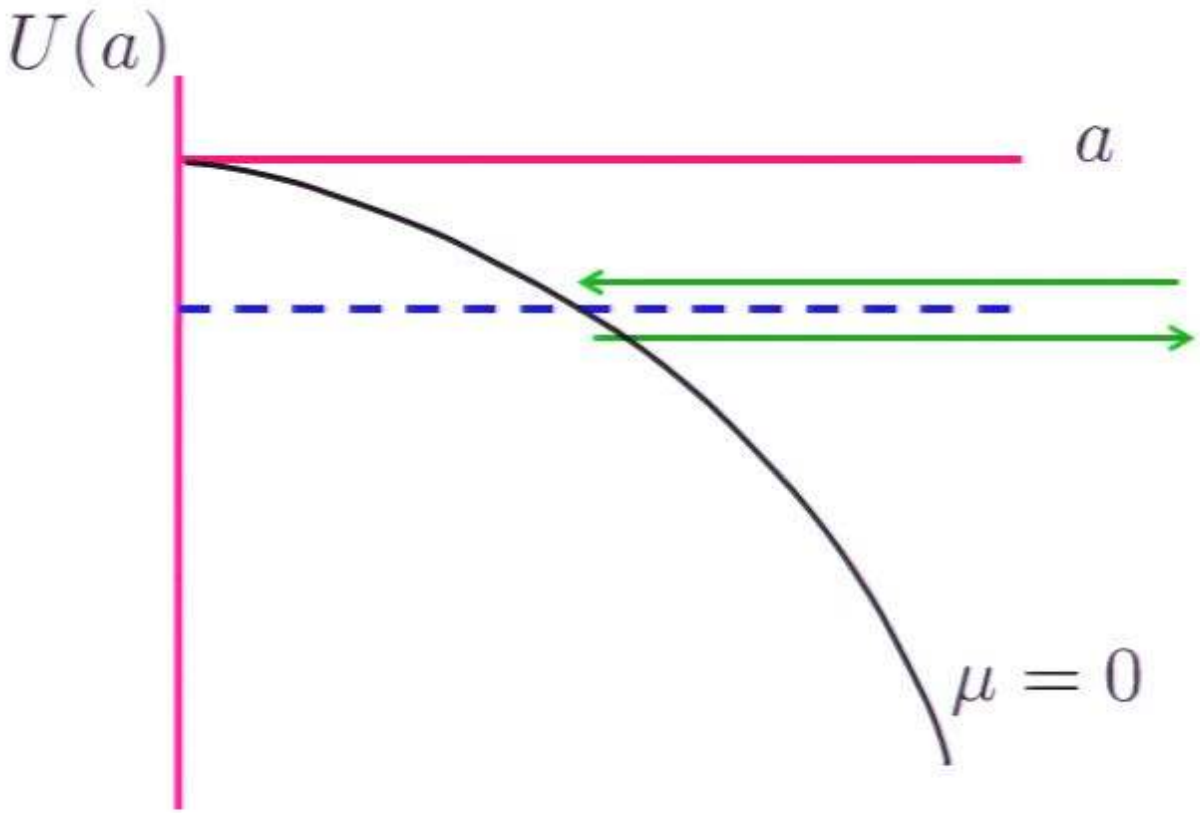

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Bouncing Cosmology

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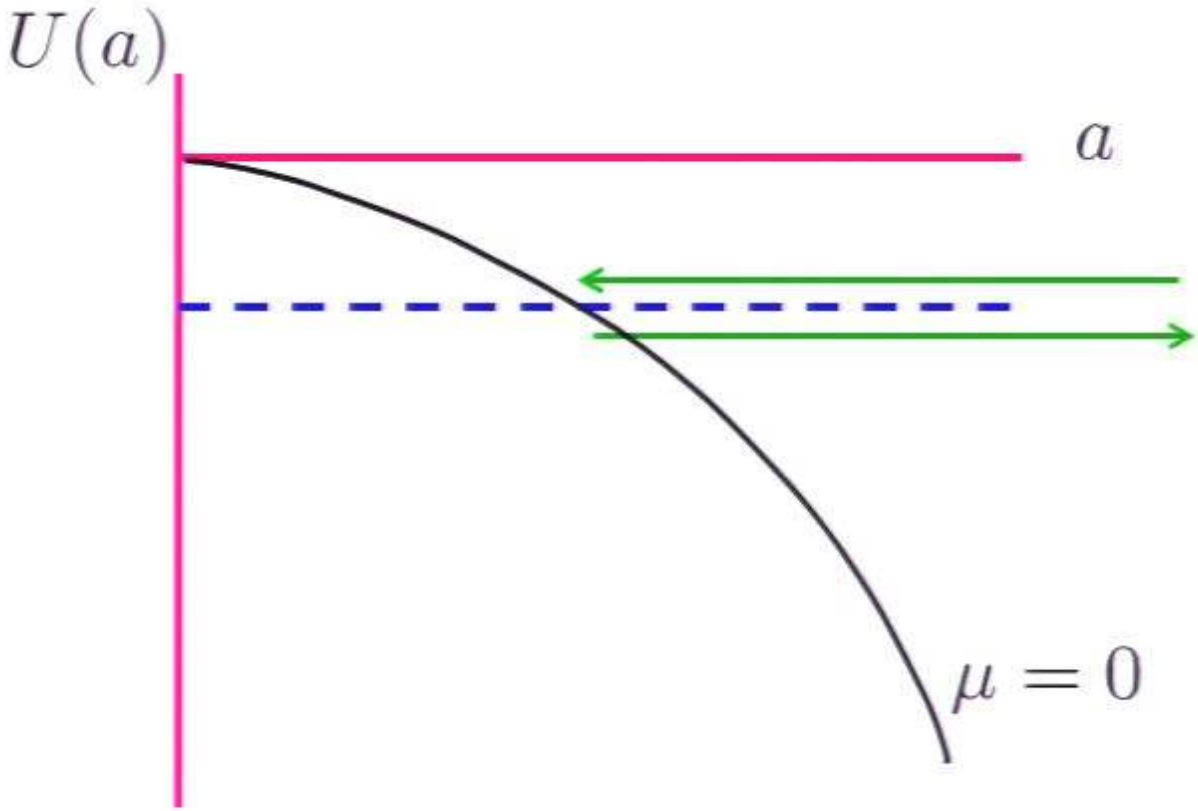

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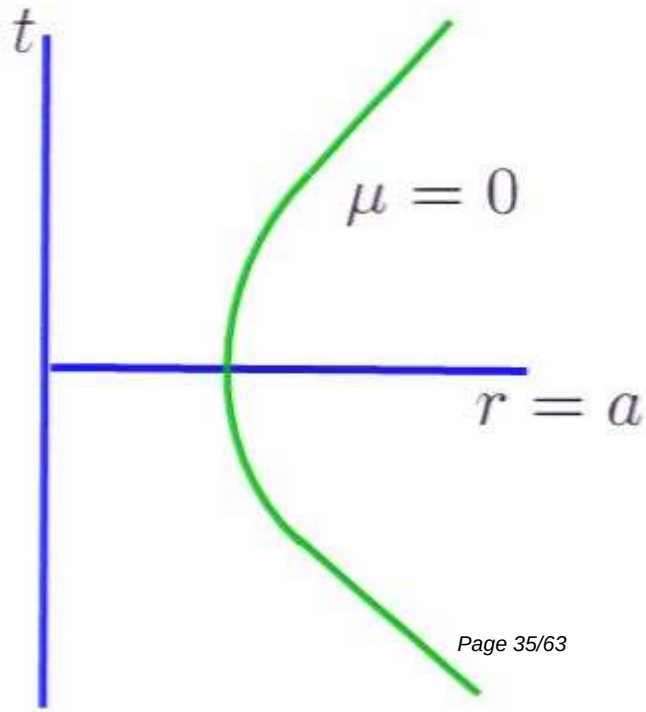

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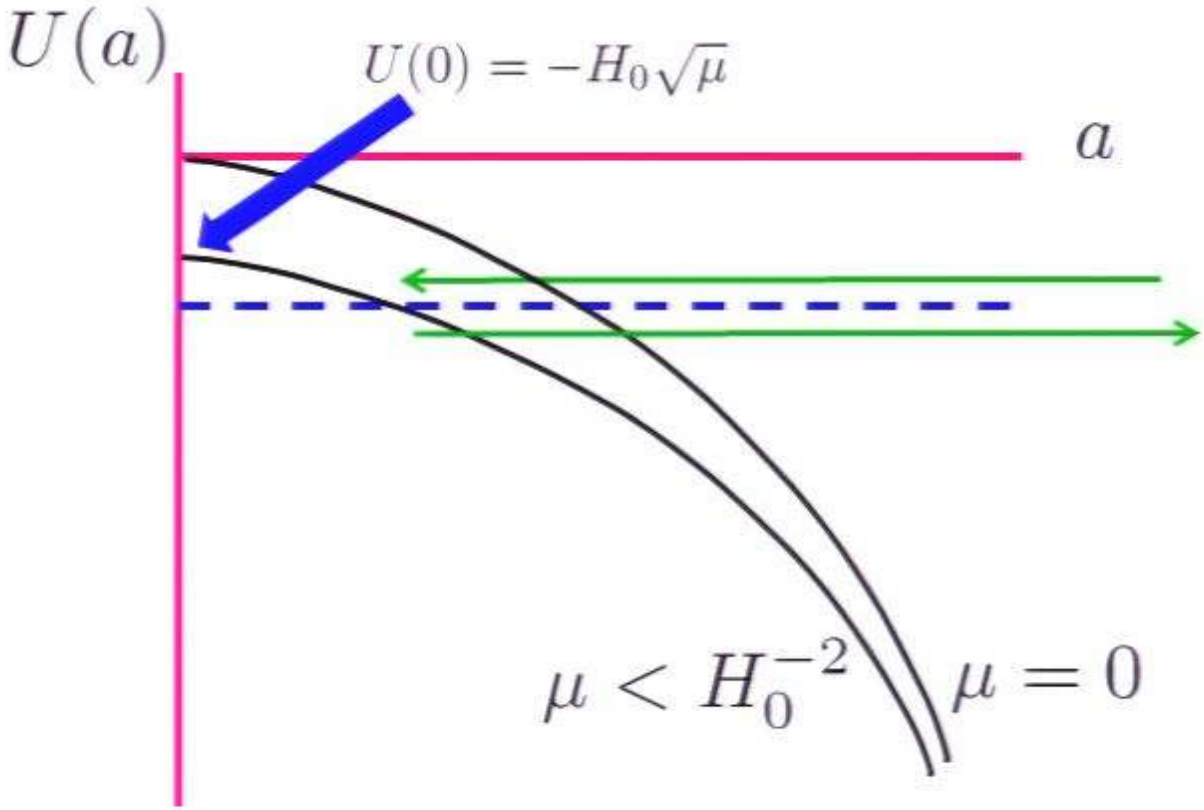

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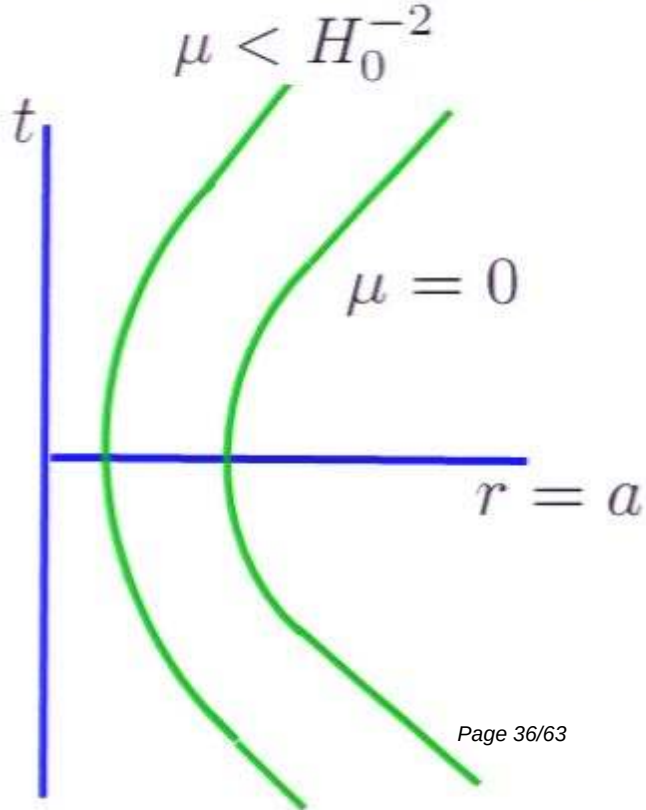

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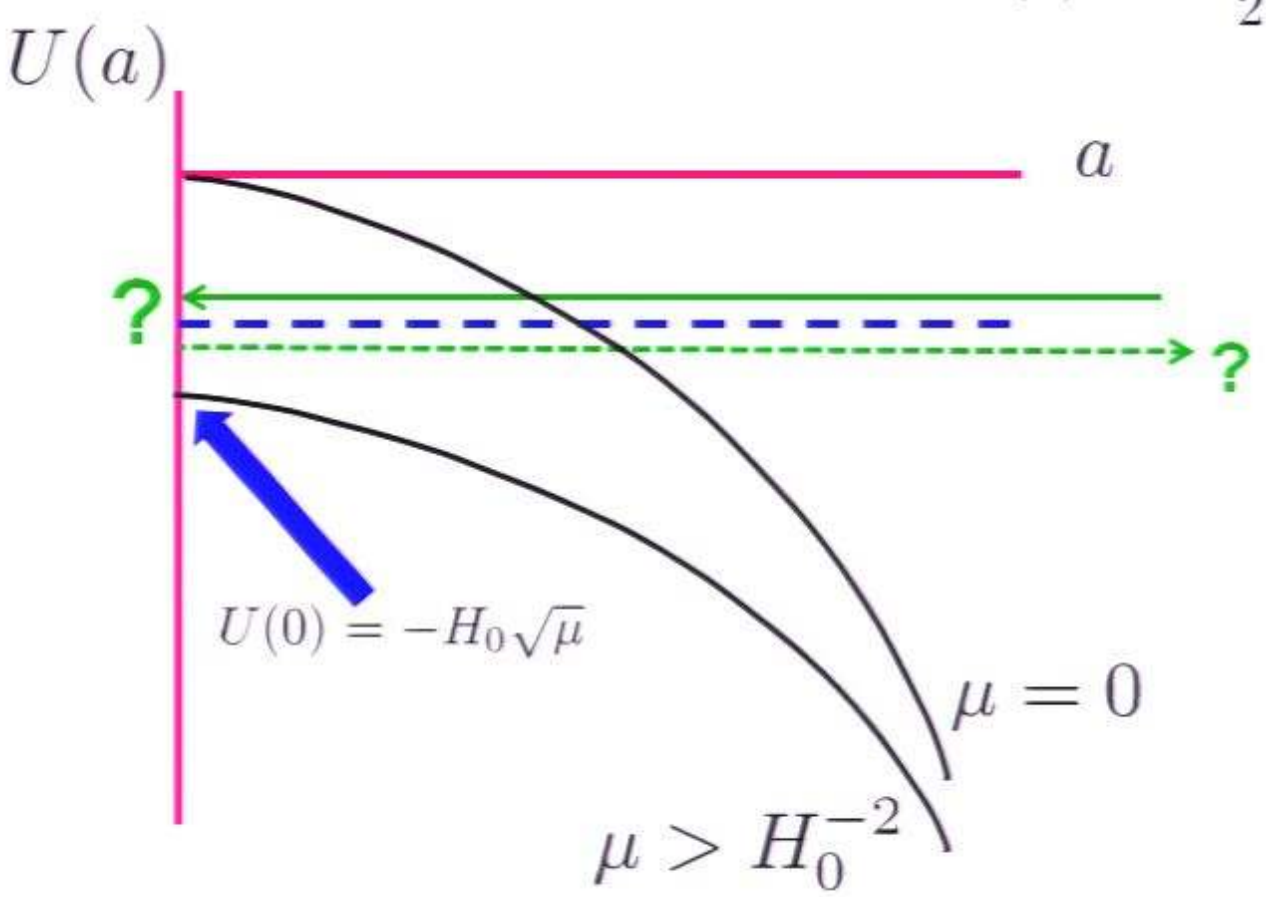

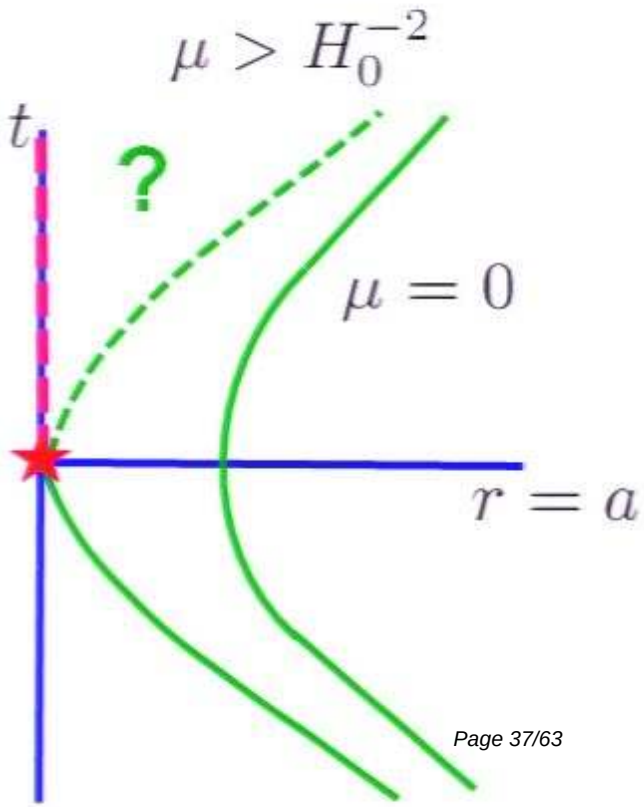

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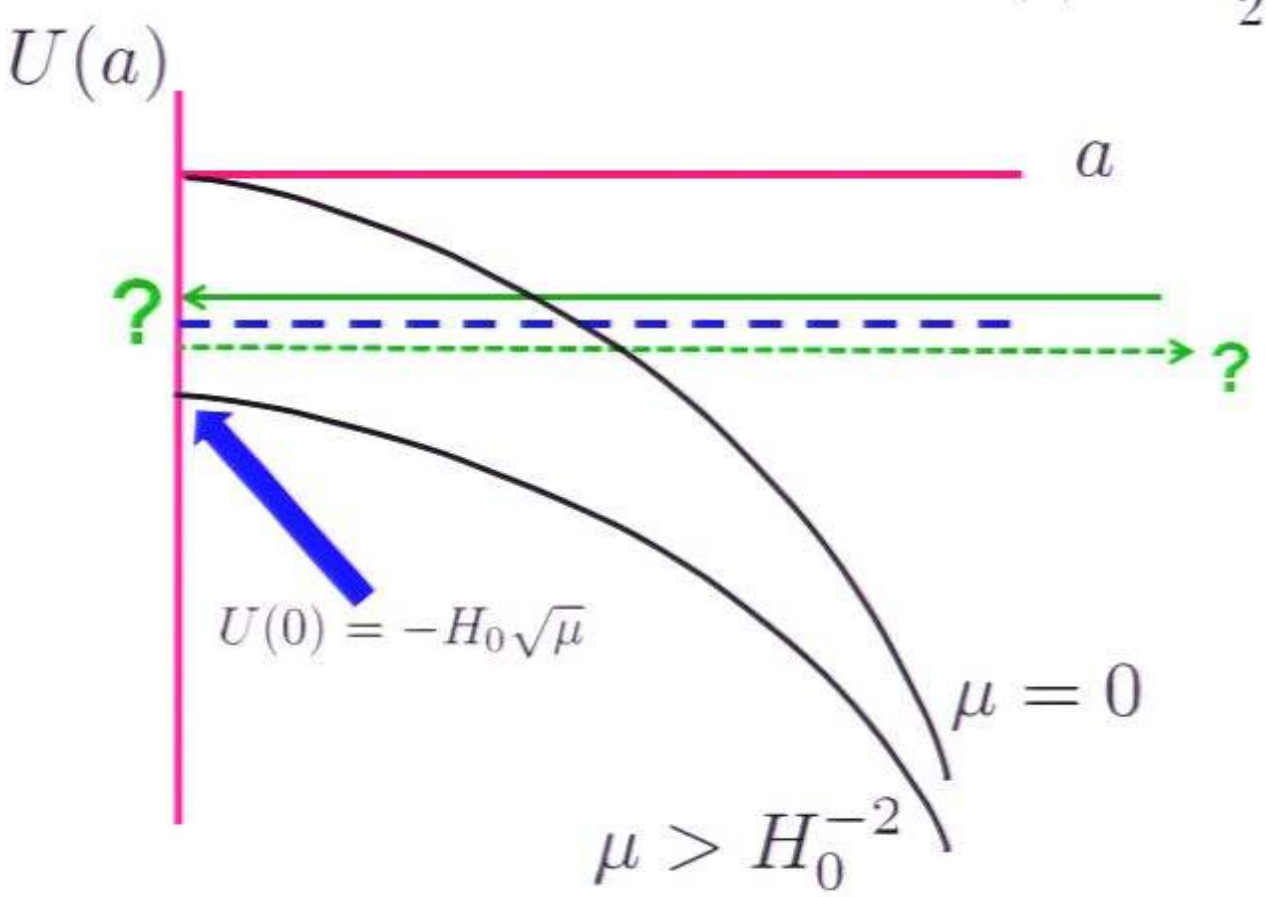
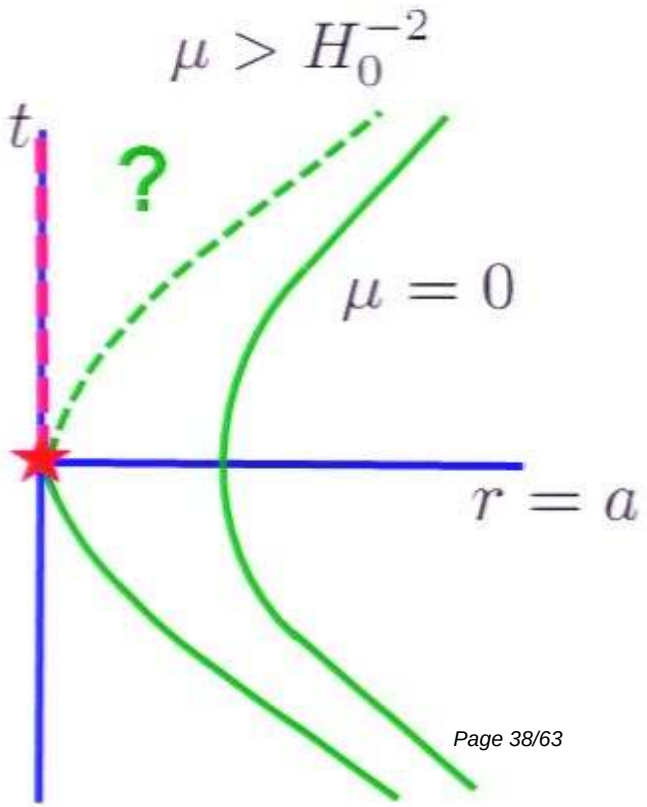
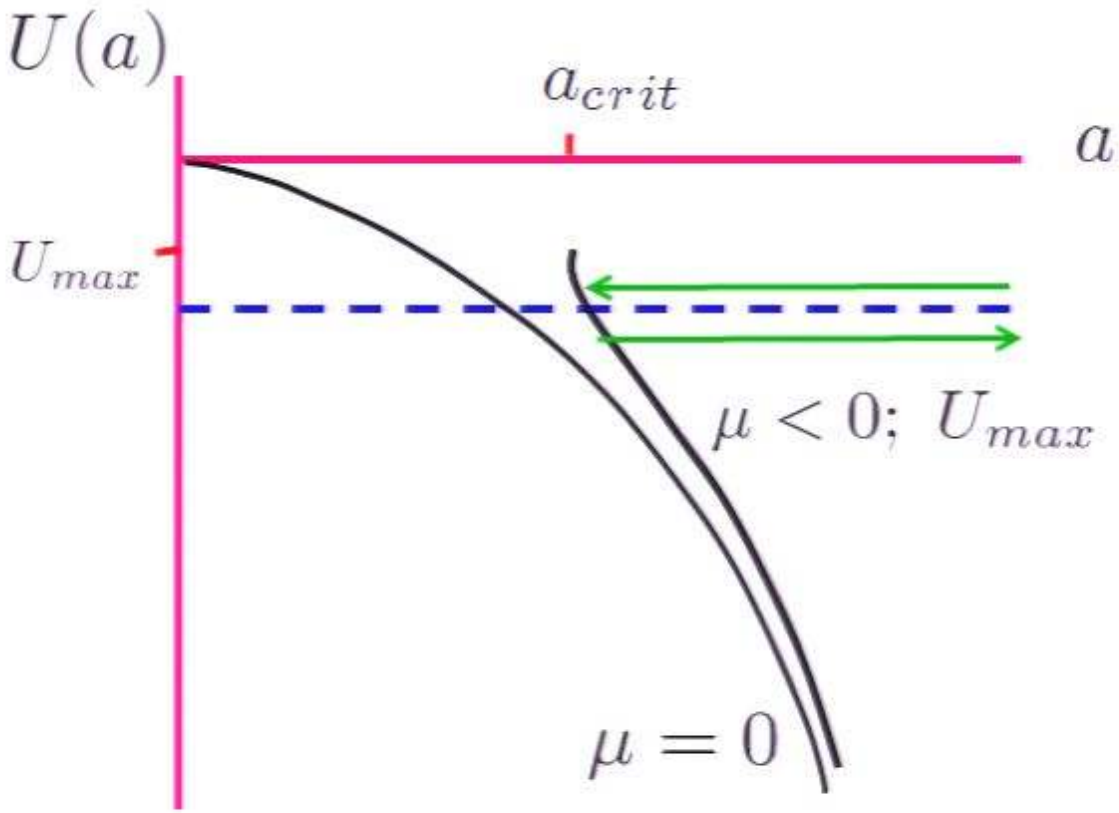




For large μ, solution singular but still have smooth initial data

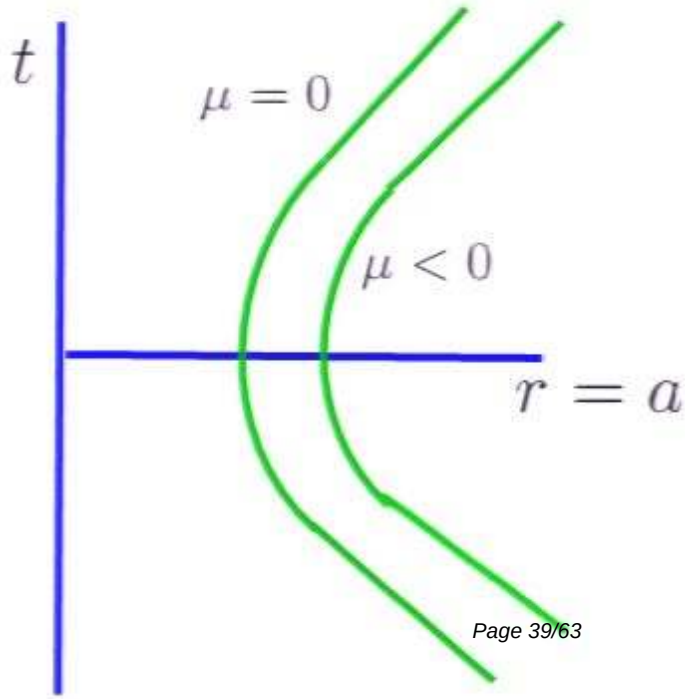
Aside: SA branch with μ<0 $(M > 0)$

$$\dot{a}^2 + U(a) = -1$$



$$a_{crit}^4 = \frac{4\mu}{H_0^2}$$

$$U_{max} = -H_0\,\mu^{1/2}$$



effect of μ<0 is repulsive

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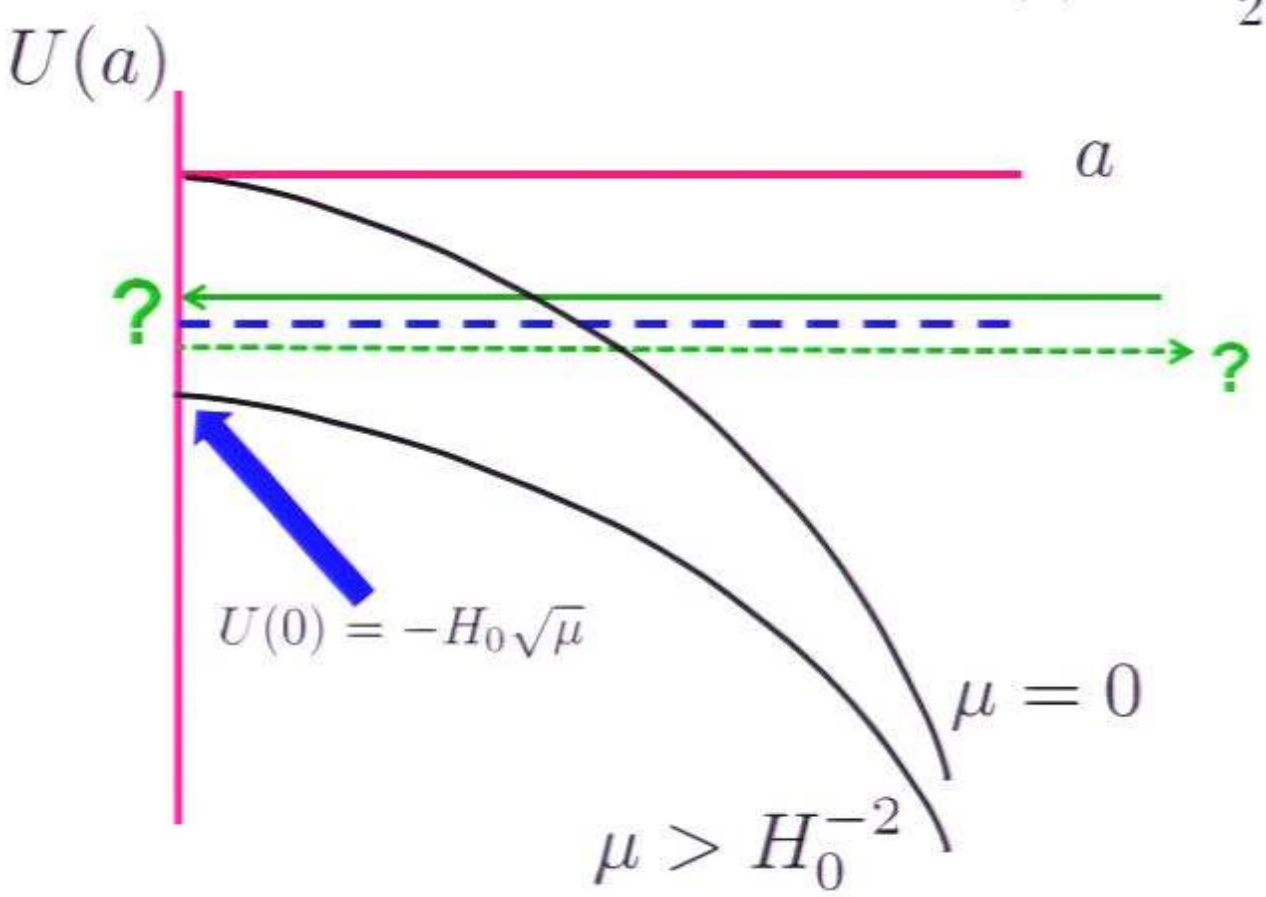
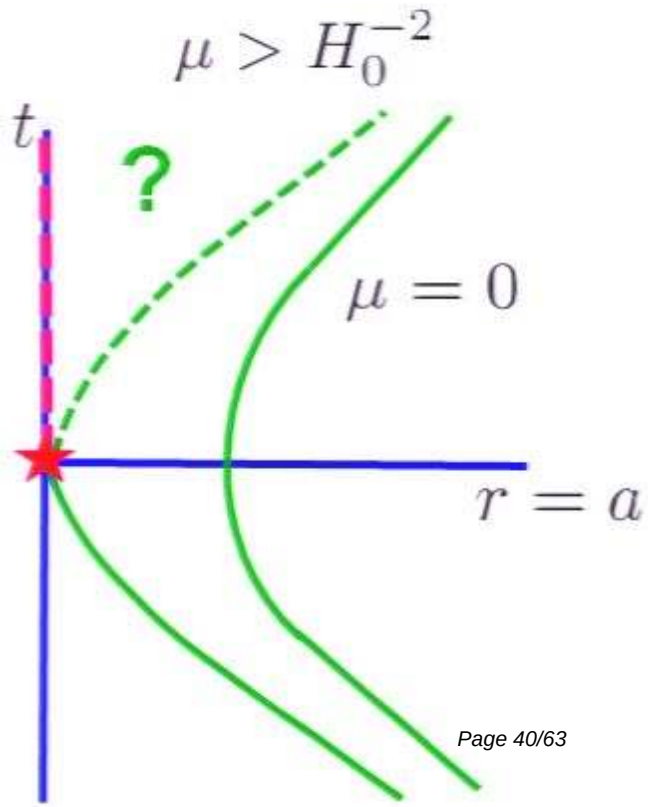
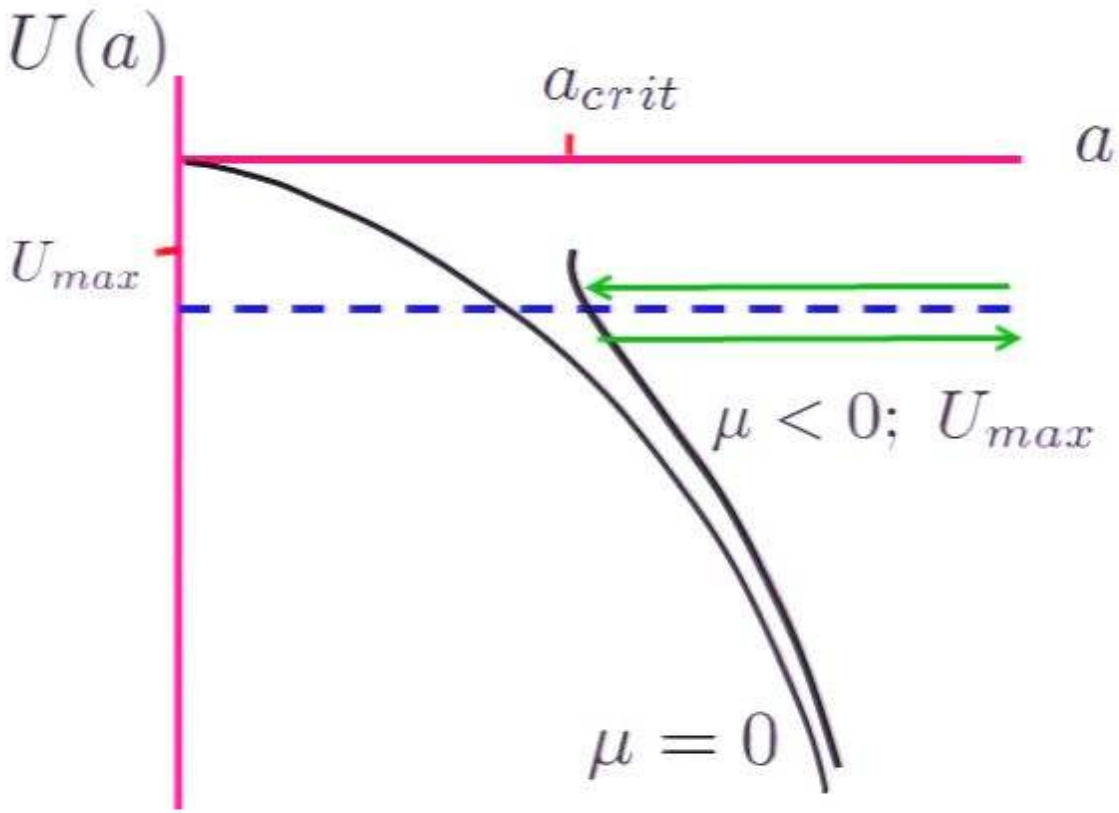




For large μ, solution singular but still have smooth initial data

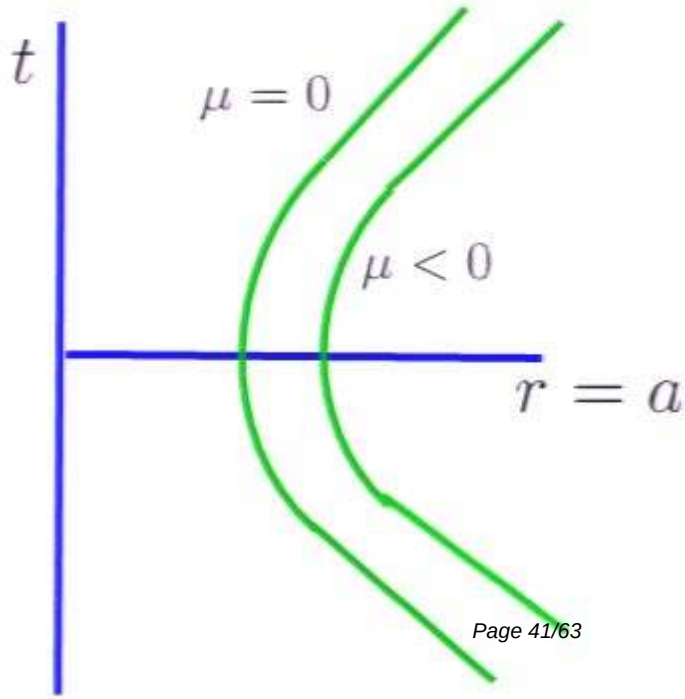
Aside: SA branch with μ<0 $(M > 0)$

$$\dot{a}^2 + U(a) = -1$$



$$a_{crit}^4 = \frac{4\mu}{H_0^2}$$

$$U_{max} = -H_0\,\mu^{1/2}$$



effect of μ<0 is repulsive

- spectrum of DGP is unbounded below!

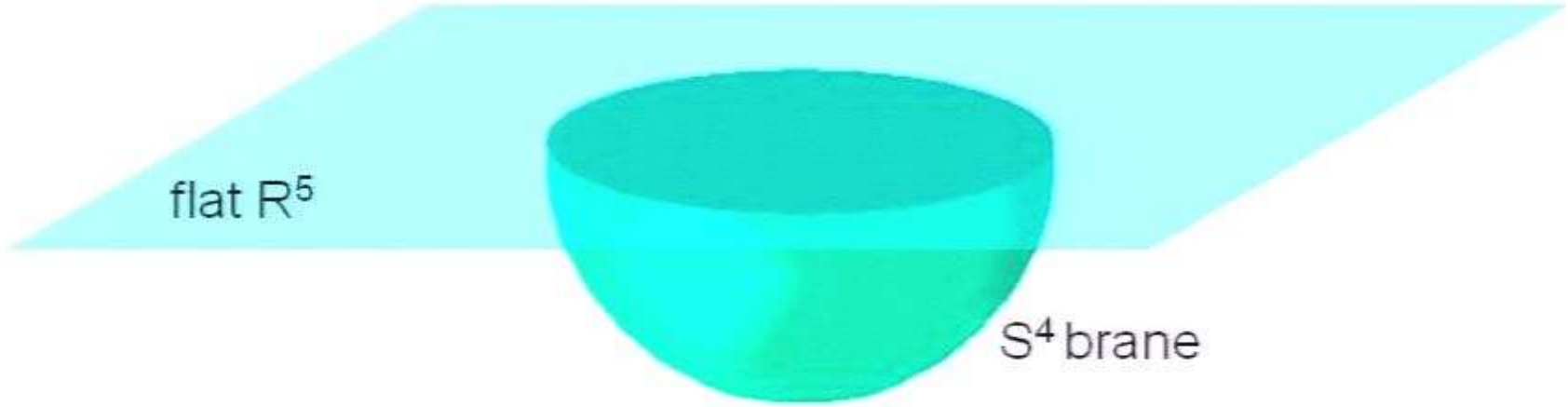
- many more general solutions with M<0

  → SA brane shields solution from bulk singularity

- expect theory unstable; no ground state

Tunneling (with M=0; so complementary to previous)

- Euclidean instanton describing appearance of SA brane



- tunneling probability $\propto \quad \exp\left(-\Delta S_E\right)$

$$\Delta S_E = S_{instanton} - S_{background}$$

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SA branes are source of problem.
So can we get rid of them?

DGP Gravity v2.2:

consider effects of UV completion – higher dimension terms

as toy model, add: $\alpha / M_4^{2n-4} \int d^4x \, \mathcal{R}^n$

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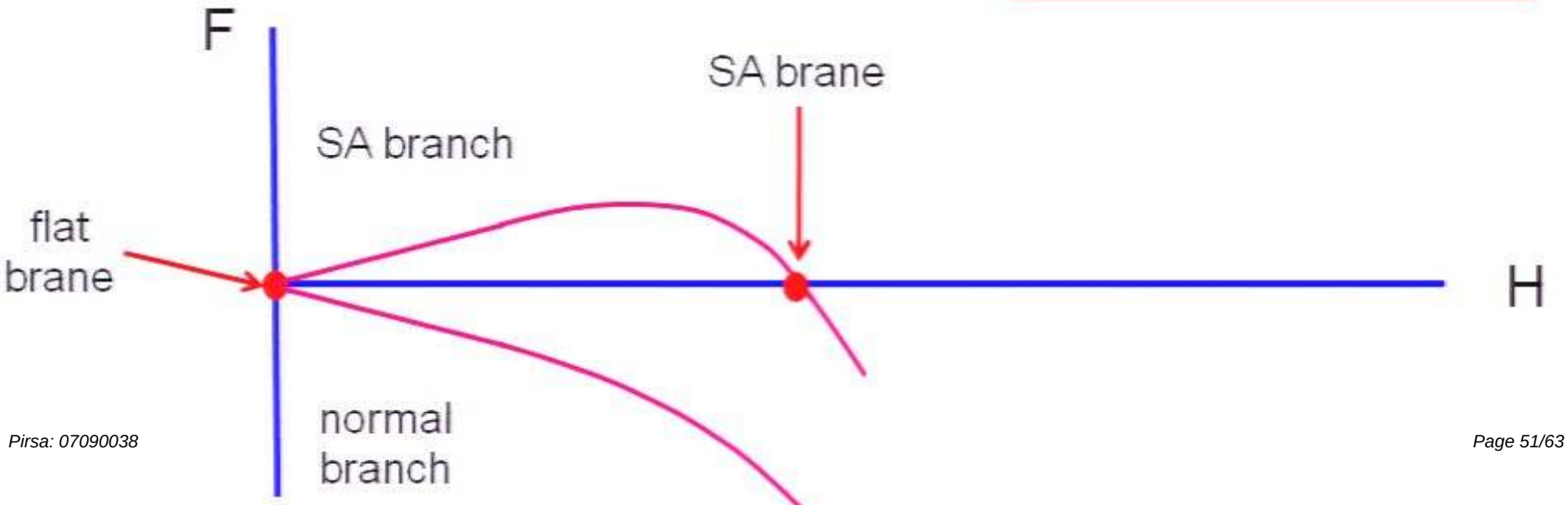
$$0 = F_{\pm}(H) \simeq \pm M_5^3 H - M_4^2 H^2 - \alpha M_4^4 \left(\frac{H}{M_4} \right)^{2n}$$



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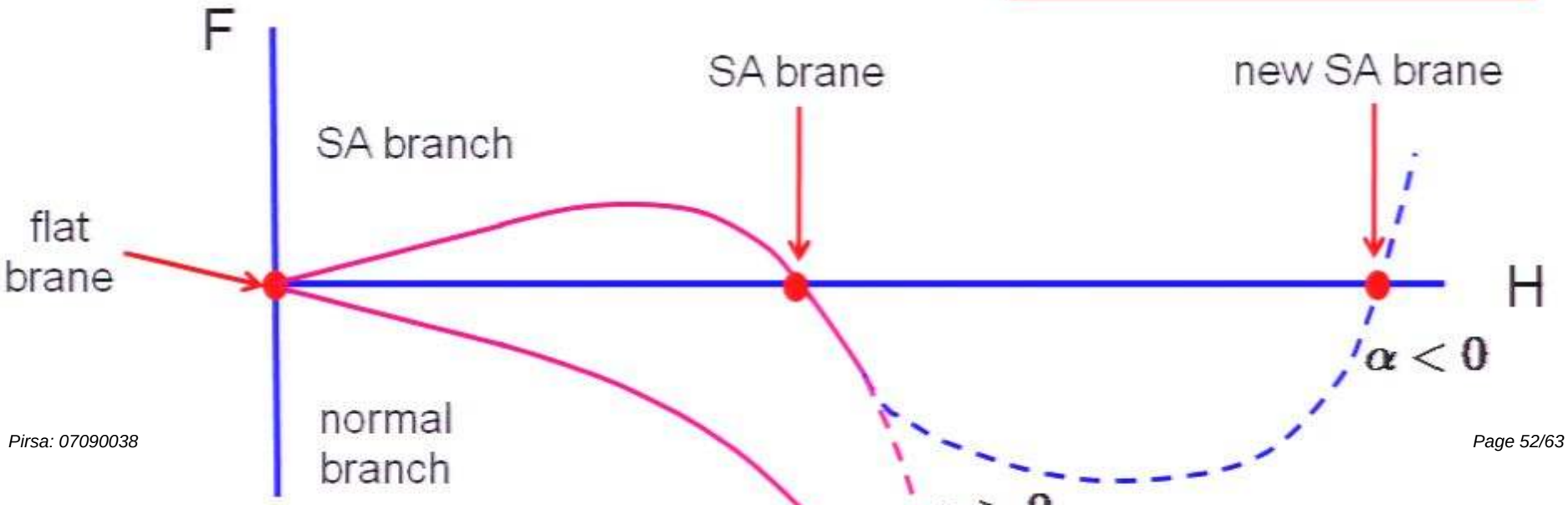

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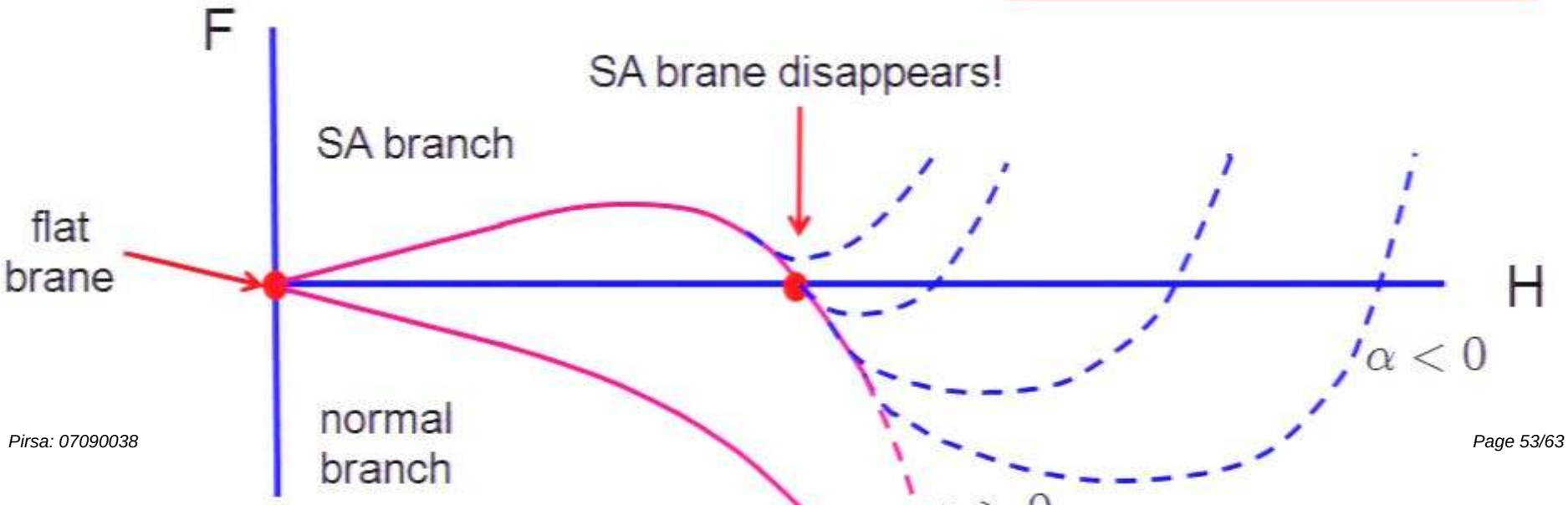

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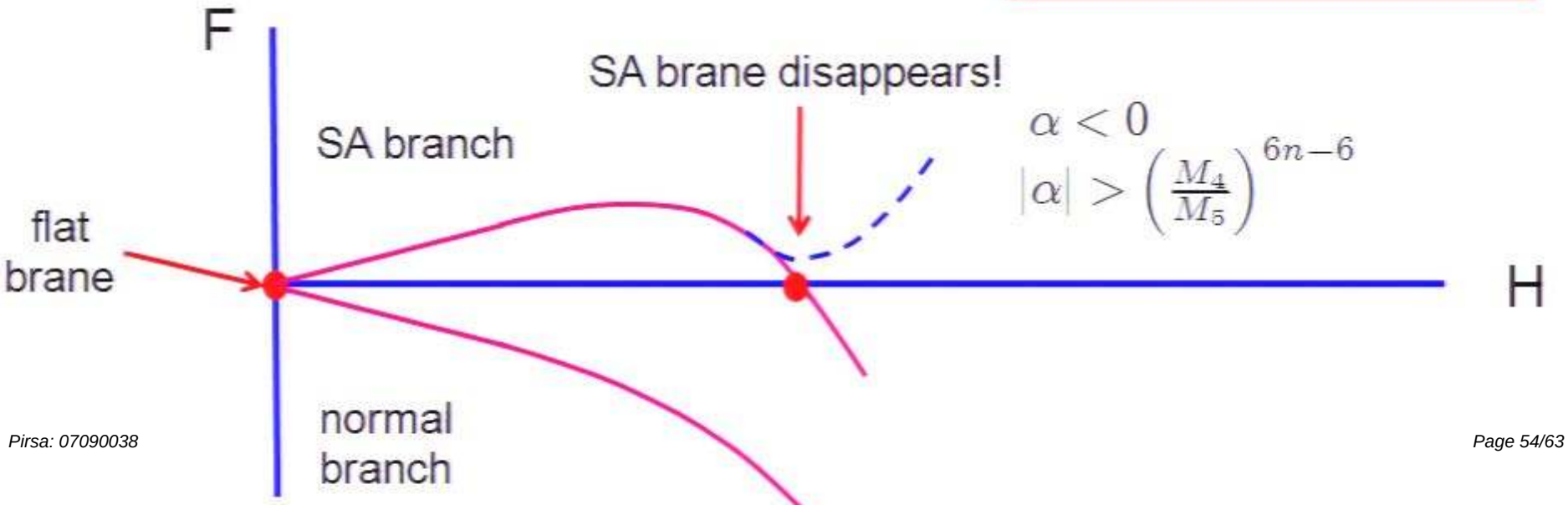

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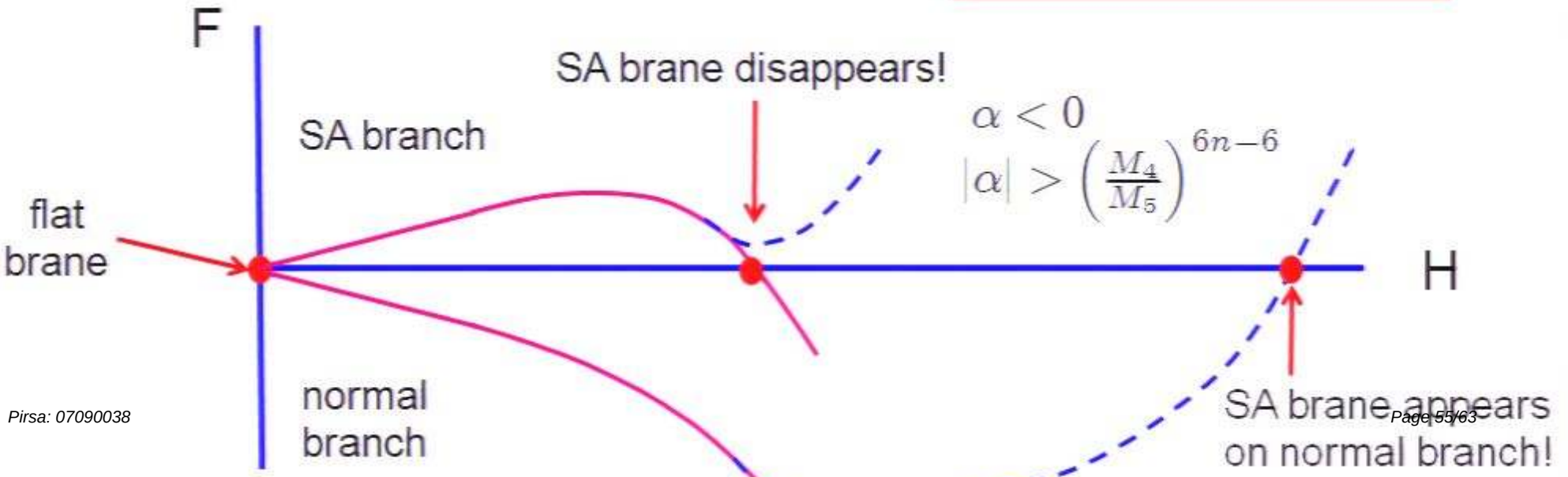

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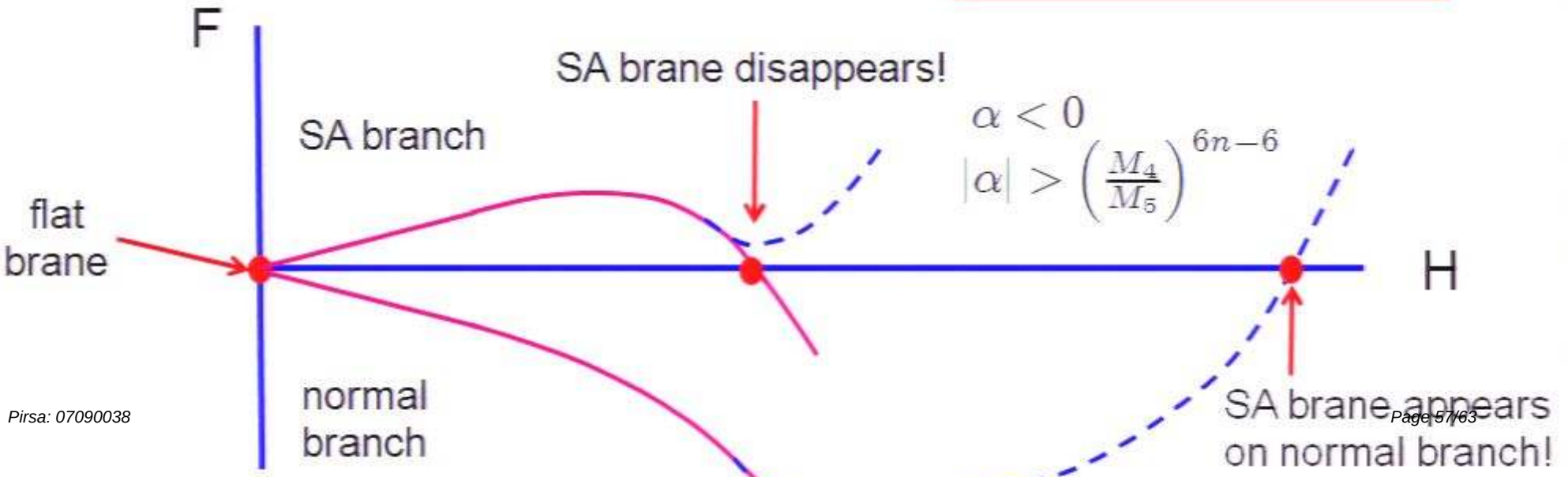

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Techniques used here provide simple diagnostic for other models