Title: K-Essence, superluminal propagation, causality and emergent geometry

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Abstract: The k-essence theories admit the superluminal propagation of the perturbations on classical nontrivial backgrounds. In this talk I will review our arguments from arXiv:0708.0561v1 and show that in spite of the superluminal propagation the causal paradoxes do not arise in these theories and in this respect they are not less safe than General Relativity.

k-essence, SUPERLUMINAL PROPAGATION, CAUSALITY and emergent geometry

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based on arXiv:0708.0561

in collaboration with : V. Mukhanov, E. Babichev

General formalism

Sc

Action: $S_\phi = \int d^4x \sqrt{-g}\, P(\phi, X)$

where $X = \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi$ sign $(+,---)$

EMT

$$T_{\mu\nu} = P_{,X} \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} P$$

NEC: $T_{\mu\nu} n^\mu n^\nu \geq 0 \leftrightarrow \boxed{P_{,X} \geq 0}$ null

Hydrodynamics: (if $X > 0$)

$$\underset{\text{pressure}}{P = P(\phi, X)}, \quad \underset{\text{energy dens.}}{\varepsilon = 2X P_{,X} - P}$$

$$\underset{\text{4 velocity}}{U_\mu = \frac{\nabla_\mu \phi}{\sqrt{2X}}}, \quad \underset{\text{sound speed}}{\boxed{C_s^2 = \frac{P_{,X}}{\varepsilon_{,X}}}}$$ [Mukhanov Garriga 99]

• General formalism

Sca.

Action: $S_\phi = \int d^4x \sqrt{-g}\, \overset{\mathcal{L}}{P}(\phi, X)$

where $X = \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi$ $(+, \overset{\text{sign}}{-}--)$

EMT
$$T_{\mu\nu} = P_{,X} \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} P$$

NEC: $T_{\mu\nu} \underset{\text{null}}{n^\mu} n^\nu \geqslant 0 \leftrightarrow \boxed{P_{,X} \geqslant 0}$

Hydrodynamics: (if $X > 0$)

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EoM:

$$\mathbb{G} \quad \nabla_\mu \nabla_\nu \quad \cdots \quad \frac{\quad}{P_{,X}} = 0$$

where

$$\mathbb{G}^{\mu\nu} = g^{\mu\nu} + \frac{P_{,XX}}{P_{,X}} \nabla^\mu \phi \nabla^\nu \phi$$

EoM is hyperbolic if

$$\boxed{1 + \frac{P_{,XX}}{P_{,X}} 2X > 0}$$

$$\Updownarrow$$

$$C_s^2 > 0$$

Mukhanov, Garriga 99

$$\left[C_s^2 = \left[1 + \frac{P_{,XX}}{P_{,X}} 2X \right]^{-1} \right]$$

Aharonov, Komar, Susskind, 69

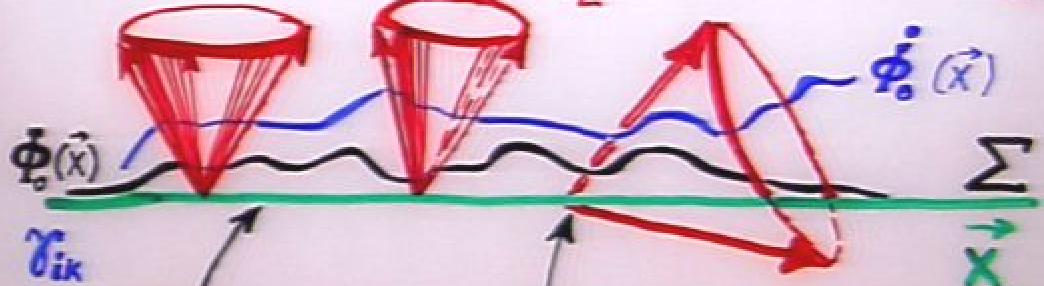
Armendaric-Picon, Lim 05

Rendall 05

NOT all initial data are allowed for well posed **Cauchy** problem.

① $1 + 2X \dfrac{P_{,XX}}{P_{,X}} > 0$ Hyperbolicity

② $G^{-1}_{\mu\nu}\left[\phi\big|_{\Sigma}, \nabla_\mu \phi\big|_{\Sigma}\right]$, Σ space like in $g_{\mu\nu}$



Σ space-like in $G^{-1}_{\mu\nu}$

Σ time-like in $G^{-1}_{\mu\nu}$

ill posed Cauchy problem!

⬇

allowed $(\Phi_0(\vec{x}), \dot{\Phi}_0(\vec{x}))$: Σ space-like in $G^{-1}_{\mu\nu}$ and $g_{\mu\nu}$.

„Good initial data"

„ = "

Well posed Cauchy problem

„ = " !

„Small" derivatives (+ Hyperbol.)

$$1 + c_s^2 (\vec{\nabla}\phi_0)^2 \frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}} > 0$$

$(\vec{\nabla}\phi_0)^2 = \gamma^{ik}\partial_i\phi_0\partial_k\phi_0$

$$\parallel$$

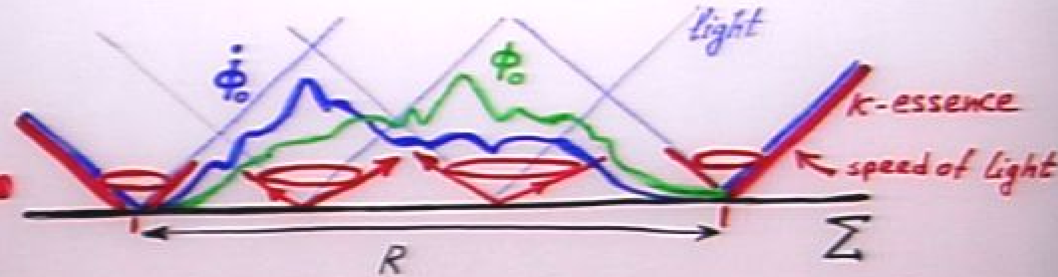
$$c_s^2 \left(1 + \dot{\phi}_0^2 \frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}\right) > 0$$

$$c_s^2 = \left(1 + 2X \frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}\right)^{-1} > 0$$

Hyperb.

Is $g^{\mu\nu}$

- $\mathcal{L}(X,\phi) \simeq V(\phi) + K_1(\phi) X + K_2(\phi) X^2 + \dots$
  for $X \approx 0$

- 

- if $K_1 = 0 \rightarrow C_s^2 = \dfrac{1}{1+2(n-1)} < 1$

- For $\mathcal{L}(X,\cancel{\phi}) = \mathcal{L}(X)$

$\exists \; \underline{\underline{\phi = f(x \pm t)}}$

$\Downarrow$

Causal Limit for localized configurations is given by $g_{\mu\nu}$!

Is $g_{\mu\nu}$

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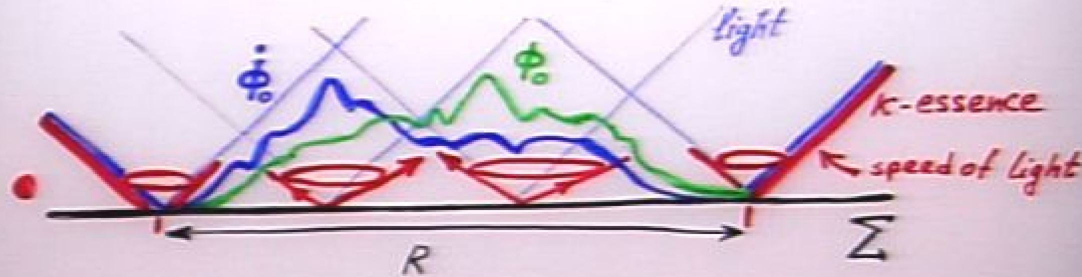
  for $X \simeq 0$

- 

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Causal Limit for localized configurations is given by $g_{\mu\nu}$!

and emergent geometry.

- In which metric do π's around $\phi(x^\mu)$ „live"?

(test field appr.)

- EoM for π's generally covariant in $\tilde{G}^{-1}_{\mu\nu}$
- $\tilde{G}^{-1}_{\mu\nu}$ can be only conf. transf of $G^{-1}_{\mu\nu}$

$\Downarrow$

$$\tilde{G}^{-1}_{\mu\nu} = \frac{\mathcal{L}_{,X}}{c_s}\left[g^{\mu\nu} - c_s^2 \frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}} \nabla_\mu\phi_0 \nabla_\nu\phi_0\right]$$

$$\tilde{G}^{\mu\nu} D_\mu D_\nu \pi + M^2_{eff}\, \pi = 0$$

$$D_\mu \tilde{G}^{-1}_{\alpha\beta} = 0 \qquad \pi \text{ causal in } \tilde{G}^{\mu\nu}$$

$$M^2_{eff} = \frac{c_s}{\mathcal{L}_{,X}}\left(\mathcal{E}_{,\phi\phi} + \left(\mathcal{L}_{,X} G^{\mu\nu}\right)_{,\phi} \nabla_\mu \nabla_\nu \phi_0\right)$$

and emergent geometry.

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$$\Downarrow$$

$$\tilde{G}^{-1}_{\mu\nu} = \frac{\mathcal{L}_{,X}}{c_s}\left[g^{\mu\nu} - c_s^2\,\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}\nabla_\mu\phi_0\nabla_\nu\phi_0\right]$$

$$\tilde{G}^{\mu\nu} D_\mu D_\nu \pi + M^2_{eff}\,\pi = 0$$

π causal in $\tilde{G}^{\mu\nu}$

$$D_\mu \tilde{G}^{-1}_{\alpha\beta} = 0$$

$$M^2_{eff} = \frac{c_s}{\mathcal{L}_{,X}}\left(\mathcal{E}_{,\phi\phi} + \left(\mathcal{L}_{,X}\, G^{\mu\nu}\right)_{,\phi}\nabla_\mu\nabla_\nu\phi_0\right)$$

$$S^2 = \frac{1}{2}\int d^4x\sqrt{-\bar{G}}\left[\tilde{G}^{\mu\nu}\partial_\mu\pi\,\partial_\nu\pi - M_{eff}^2\,\pi^2\right]$$

also for general cosmological perturbations

different M_{eff}^2 but the same $\tilde{G}^{\mu\nu}$

for $\quad \pi \equiv \delta\phi + \frac{\dot{\phi}_0}{H}\psi$

in cosmology (flat Friedmann Universe)

$$\nu \equiv \sqrt{\varepsilon_{,X}}\,a\,\pi$$

$$z = \frac{\dot{\phi}}{H}\sqrt{\varepsilon_{,X}}\,a$$

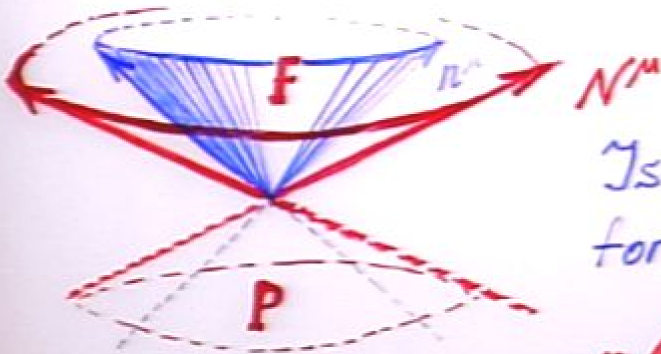
$$S^2 = \frac{1}{2}\int d\eta\, d^3x\left(\nu'^2 - c_s^2(\vec{\nabla}\nu)^2 + \frac{z''}{z}\right)$$

(Garriga, Mukhanov 1999)

$S^2 \rightarrow$ Quantization

Chronology Protection

Hawking 19



Is this a problem for causality?

„No!" / Global properties

Example: Friedmann universe

ϕ - cosmological scalar : $\overset{\circ}{\phi}(t)$

$$ds^2 = dt^2 - a^2(t)\, d\vec{x}^2 = g_{\mu\nu} dx^\mu dx^\nu$$

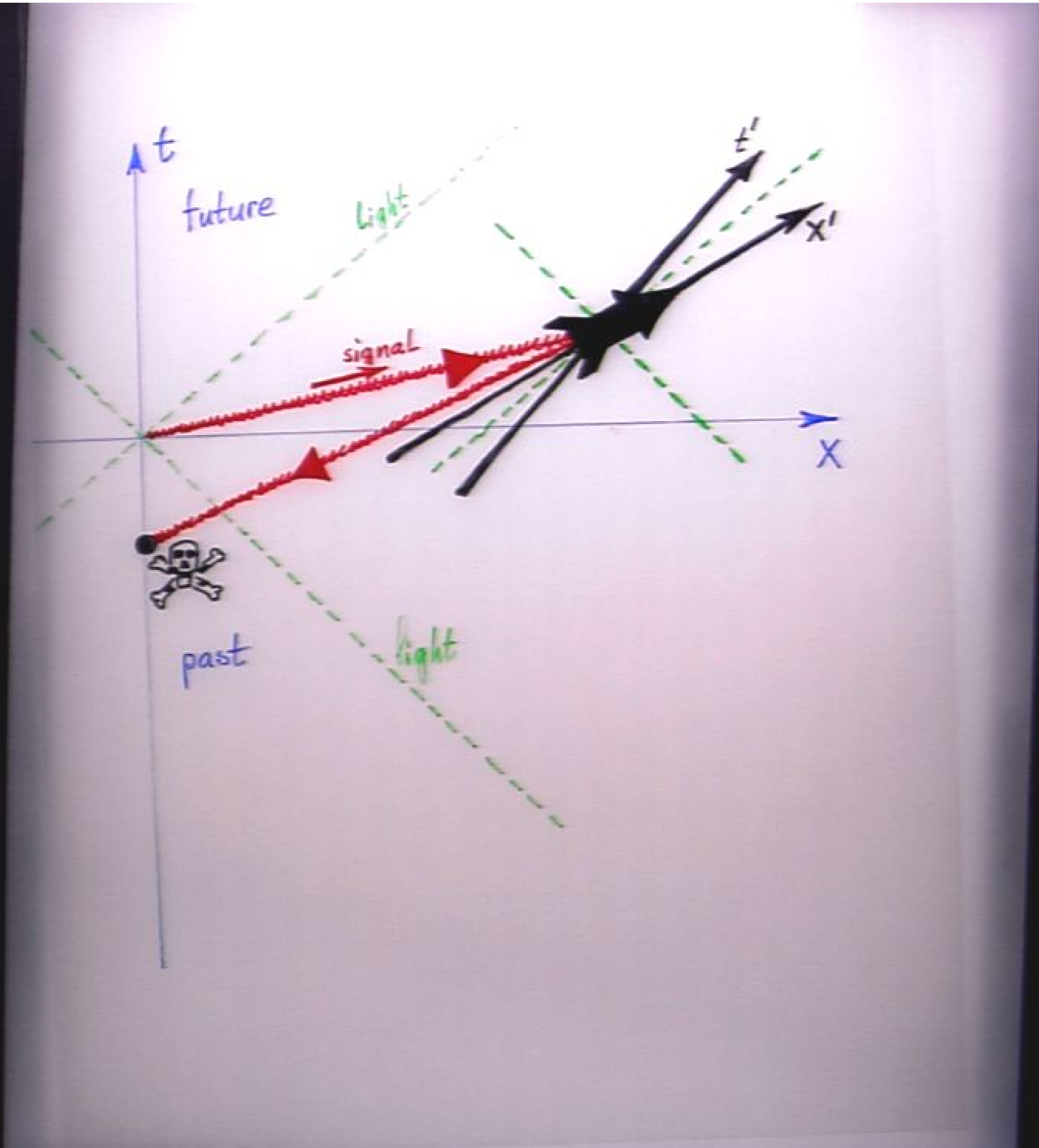
$$dl^2 = G^{-1}_{\mu\nu}\, dx^\mu dx^\nu = ds^2 -$$

$$- c_s^2 \frac{p_{,XX}}{p_{,X}} 2X\, dt^2 \Rightarrow$$

$$dl^2 = c_s^2(t)\, dt^2 - a^2(t)\, d\vec{x}^2$$

$$d\tau = c_s dt \longrightarrow dl^2 - \text{„Friedmann"}$$

$$dl^2 = d\tau^2 - a^2(\tau)\, d\vec{x}^2$$


t
future
light
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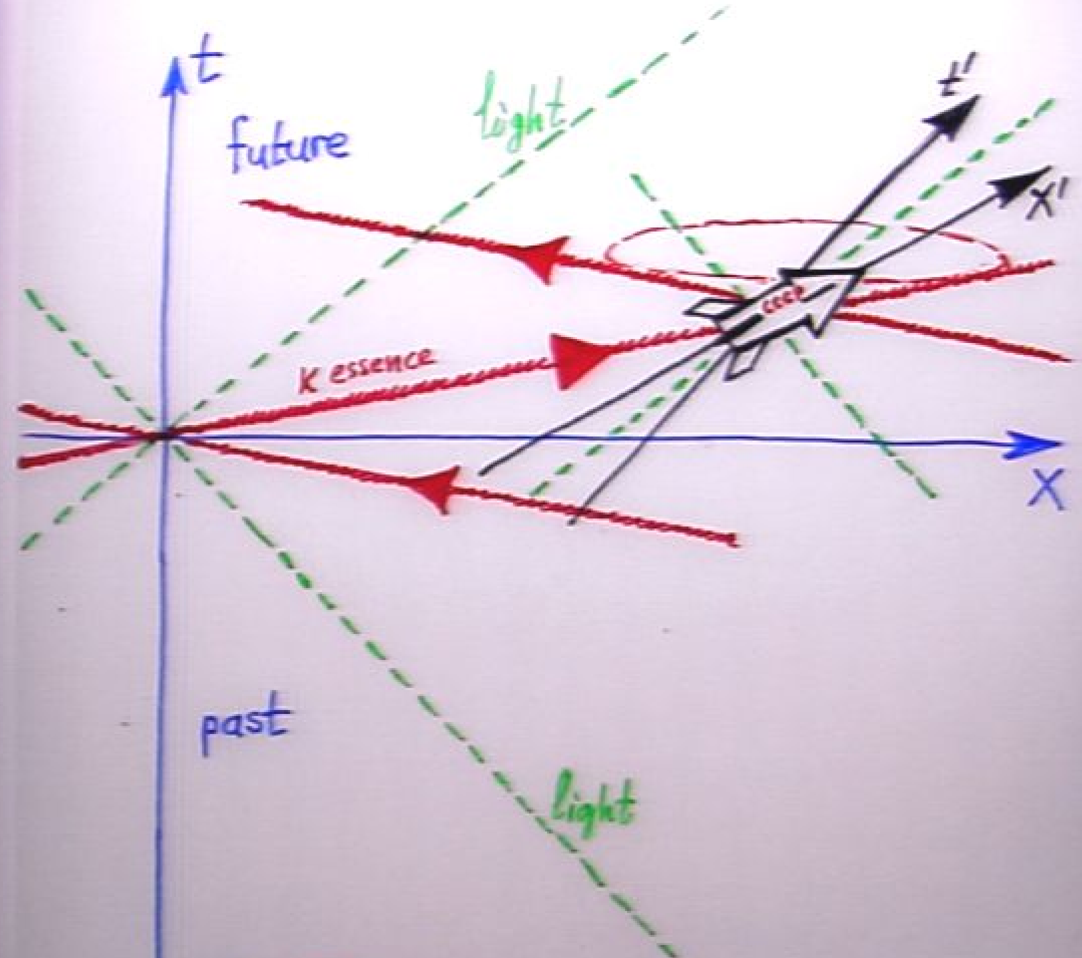
signal
X
past
light



$\exists$ **Global time** f $\rightarrow$
$\rightarrow$ **stable causality**

$\exists$ diff. f : $\nabla^\mu f$ is future directed timelike vector field. $\rightarrow$ No CCC in „the most interesting" backgrounds.

• Conclusion

Superluminal propagation does not lead to causal paradoxes in κ-essence.

Thanks for attention!

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