Title: Bigravity and Lorentz-violating Massive Gravity

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Abstract:

Bigravity and Lorentz-violating Massive Gravity

(Do we really understand how to get rid of the vDVZ discontinuity ?)

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Astroparticules et Cosmologie

Frontiers of Modern Cosmology

PI- September 2007

PART 1: Some words about « massive gravity »

PART 2: Some words about Static Spherically Symmetric Solutions (S^4) of bigravity

Why being interested in « massive gravity »?

One way to modify gravity at « large distances »
… and get rid of dark matter and/or dark energy ?

$$H^2 = \frac{8\pi G}{3}\rho$$

Dark matter or dark energy ?

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Changing the dynamics of gravity ?

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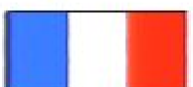

A French (appropriate in a place called Waterloo !) hero personifying the success/failure of both approaches is Le Verrier

- The discovery of Neptune
- The non discovery of Vulcan… but that of GR

1. What is massive gravity and some general things about S^4 ?

1.1. Pauli-Fierz theory and the vDVZ discontinuity

1.2. Non linear Pauli-Fierz and its Pathologies

1.3. DGP Theory

1.4. Bigravity

1.1. Quadratic massive gravity: the Pauli-Fierz theory and the vDVZ discontinuity

Pauli-Fierz action: second order action for a massive spin two $h_{\mu\nu}$

$$\underbrace{\int d^4x \sqrt{g} R_g}_{} + m^2 \int d^4x h_{\mu\nu} h_{\alpha\beta} \left(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

second order in $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$

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The propagators read

propagator for $m=0$ $$D_0^{\mu\nu\alpha\beta}(p) = \frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\alpha}\eta^{\nu\alpha}}{2p^2} - \frac{\eta^{\mu\nu}\eta^{\alpha\beta}}{2p^2} + \mathcal{O}(p)$$

propagator for $m \neq 0$ $$D_m^{\mu\nu\alpha\beta}(p) = \frac{\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\alpha}\eta^{\nu\alpha}}{2(p^2-m^2)} - \frac{\eta^{\mu\nu}\eta^{\alpha\beta}}{3(p^2-m^2)} + \mathcal{O}(p)$$

Coupling the graviton with a conserved energy-momentum tensor

$$S_{int} = \int d^4x \, \sqrt{g}\, h_{\mu\nu} T^{\mu\nu}$$

$$h^{\mu\nu} = \int D^{\mu\nu\alpha\beta}(x - x')T_{\alpha\beta}(x')d^4x'$$

The amplitude between two conserved sources T and S is given by

$$A = \int d^4x \, S^{\mu\nu}(x) h_{\mu\nu}(x)$$

For a massless graviton: $A_0 = \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T\right) S^{\mu\nu}$

In Fourier space

For a massive graviton: $A_m = \left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}T\right) S^{\mu\nu}$

e.g. amplitude between two non relativistic sources:

$$\left.\begin{array}{l} T^{\mu}_{\nu} \propto \mathrm{diag}(m_1, 0, 0, 0) \\ S^{\mu}_{\nu} \propto \mathrm{diag}(m_2, 0, 0, 0) \end{array}\right\} \quad \mathcal{A} \sim \tfrac{2}{3} m_1 m_2 \quad \text{Instead of} \quad \mathcal{A} \sim \tfrac{1}{2} m_1 m_2$$

Rescaling of Newton constant $G_{\mathrm{Newton}} = \frac{4}{3} G_{(4)}$

defined from Cavendish experiment

appearing in the action

but amplitude between an electromagnetic probe and a non-relativistic source is the same as in the massless case (the only difference between massive and massless case is in the trace part) ⇨ wrong light bending! (factor ¾)

NB, the PF mass term reads

$$M_P^2 m^2 \int d^4x \, (h_{ij}h_{ij} - 2h_{0i}h_{0i} - h_{ii}h_{jj} + 2h_{ii}h_{00})$$

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5 propagating d.o.f. in the quadratic PF

1.2. Non linear Pauli-Fierz theory and its pathologies

Can be defined by an action of the form

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Keep all order in $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$

Some background metric

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Look at Spherically Symmetric Solutions of this theory

$$ds^2 = -e^{\nu(\rho)}dt^2 + e^{\lambda(\rho)}d\rho^2 + e^{\mu(\rho)}\rho^2 d\Omega_2^2$$

$$\nu(r) = -\frac{r_S}{r}(1 \quad + \ldots$$

$$\lambda(r) = +\frac{1}{2} \quad \frac{r_S}{r}(1 \quad + \ldots$$

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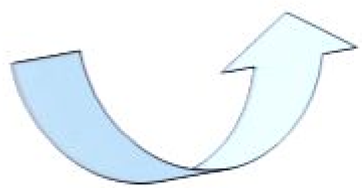
Wrong light bending!



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with $\epsilon = \frac{r_S}{m^4 r^5}$

Vainshtein '72

In some kind of non linear **PF**

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In some kind of non linear PF

Introduces a new length scale r_v in the problem below which the perturbation theory diverges!

For the sun: bigger than solar system! ⇐ with $r_v = (r_S m^{-4})^{1/5}$

So, what is going on at smaller distances?

Vainshtein's answer (1972):

There exists an other perturbative expansion at smaller distances, reading:

$$\nu(r) = -\frac{r_g}{r}\left\{1 + \mathcal{O}\left(r^{5/2}/r_v^{5/2}\right)\right\}$$

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No warranty that this solution can be matched with the other for large r!

Boulware, Deser '72

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This polarization can be described by the following action:

Arkani-Hamed, Georgi, and Schwartz

$$\frac{1}{2}(\nabla\phi)^2 - \frac{1}{M_P}\phi T + \frac{1}{\Lambda^5}\left\{(\nabla^2\phi)^3 + \underbrace{\ldots}\right\}$$

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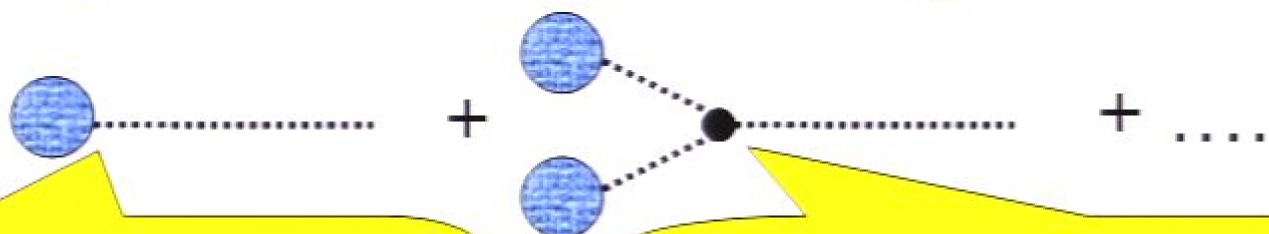
E.g. around a heavy source: of mass M



Interaction M/M_P of the external source with ϕ

The cubic interaction above generates O(1) coorrection at $r = r_v \equiv (r_S m^{-4})^{1/5}$

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$$M_P^2 \int d^4x \left\{ \left(\pi^{ij}\dot{g}_{ij} - NR^0 - N_i R^i\right) - m^2 \left(h_{ij}h_{ij} - 2N_iN_i - h_{ii}h_{jj} + 2h_{ii}\underbrace{\left(1 - N^2 + N_k g^{kl} N_l\right)}\right)\right\}$$

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Boulware, Deser '72

→ 6 propagating d.o.f., corresponding to the g_{ij}

Moreover, the reduced Lagrangian for those propagating d.o.f. read

Boulware, Deser '72

$$M_P^2 \quad \int d^4x \left\{ \pi^{ij}\dot{g}_{ij} - m^2\left(h_{ij}h_{ij} - h_{ii}h_{jj}\right) - \frac{1}{8m^2} R^l \left(\eta - h_{ii} g\right)^{-1}_{lm} R^m \right.$$
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Indeed the action for the scalar polarization

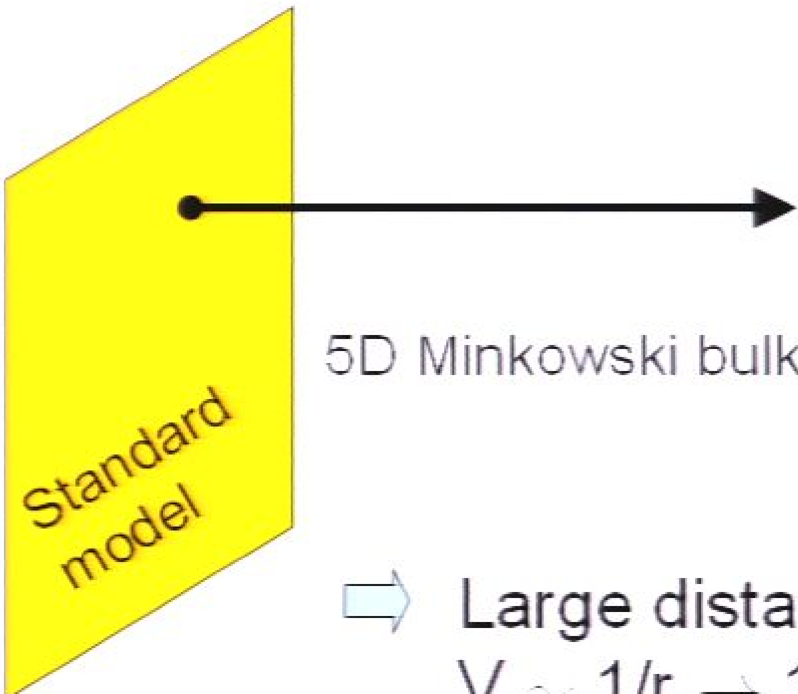
$$\frac{1}{2}(\nabla\phi)^2 - \frac{1}{M_P}\phi T + \frac{1}{\Lambda^5}\left\{ (\nabla^2\phi)^3 + \quad \ldots \ \right\}$$

Leads to order 4 E.O.M. $\Rightarrow$, it describes two scalar fields, one being ghost-like

1.3. The DGP model (or brane-induced gravity).

Dvali, Gabadadze, Porrati

$$S = M_{(5)}^3 \int d^5x \sqrt{g} \left(\tilde{R} + \cdots \right) + \int_{\rm brane} d^4x \sqrt{g} \mathcal{L}_{\rm matter} + \int_{\rm brane} d^4x \sqrt{g} \left(M_P^2 \, R + \cdots \right)$$



⇨ Large distance modification of gravity,
V ~ 1/r → 1/r²
with the crossover distance $r_c = \frac{M_{(4)}^2}{2M_{(5)}^3}$

⇨ But the tensorial structure of the graviton propagator is that of a massive graviton

Leads to the van Dam-Veltman-Zakharov discontinuity on Minkowski background!

Solutions of DGP gravity and the vDVZ discontinuity

- Exact cosmological solutions provide an explicit example of interpolation between theories with different tensor structure for the graviton propagator.

C.D.,Gabadadze, Dvali, Vainshtein (2002)

$$H^2 = \frac{\rho}{3M_P^2} \quad \xleftarrow[\text{small } r_c]{\text{large } r_c} \quad H^2 = \frac{\rho^2}{36 M_{(5)}^6}$$

Solution of 4D GR with cosmic fluid

$$S_0 = M_P^2 \int d^4x \sqrt{-g} R$$

Solution of 5D GR with a brane source

$$S_5 = M_{(5)}^3 \int d^5x \sqrt{-g} R$$

↳ Comes in support of a « Vainshtein mechanism » [non perturbative recovery of the « massless » solutions] at work in DGP…… Recently an other exact solution found by Kaloper for localized relativistic source showing the same recovery…..

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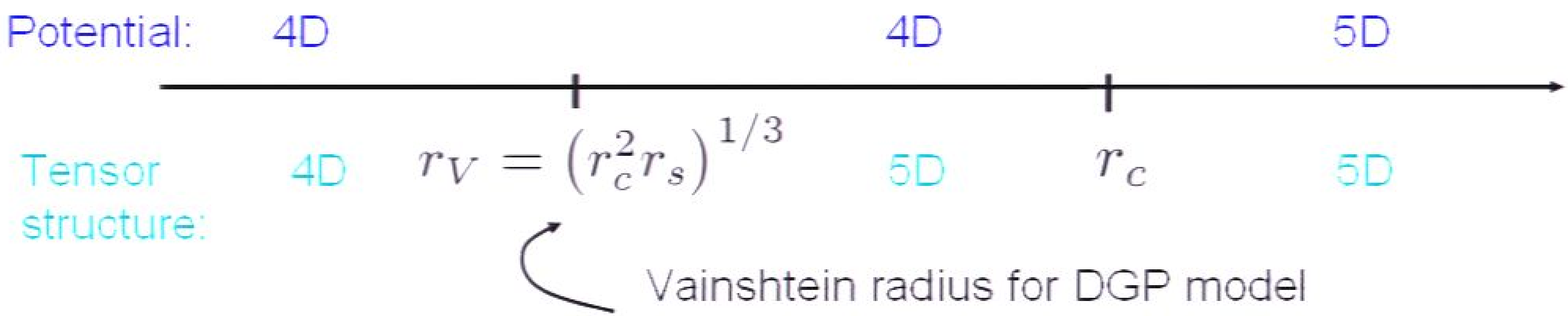
Gruzinov, Porrati, Lue, Lue & Starkman, Tanaka



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Potential:	4D		4D		5D
Tensor structure:	4D	$r_V = \left(r_c^2 r_s\right)^{1/3}$	5D	r_c	5D

Vainshtein radius for DGP model

Related to strong self interaction of the brane bending sector

C.D.,Gabadadze, Dvali, Vainshtein; Arkani-Hamed, Georgi Schwartz; Rubakov; Luty, Porrati, Rattazzi.

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Non linearities responsible for the « Vainsthein radius »

Tanaka; Nicolis Rattazzi;Damour

However

- Cosmological solutions play a tricky role for the vDVZ discontinuity: No vDVZ discontinuity on AdS (and dS) (Higuchi; Porrati; Kogan, Mouslopoulos, Papazoglou)

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Need for a better understanding of those issues, in particular:
• Validity of the linear pert. theory (linearization instability ? See Gababadze, Iglesias; C.D. ,Gabadadze, Iglesias)
• Theories where Vainshtein's mechanism does work vs those where it does not.

$\Rightarrow$ **Provides some motivation for a better understanding of S^4 in similar theories... e.g. bigravity**

However

• Cosmological solutions play a tricky role for the vDVZ discontinuity: No vDVZ discontinuity on AdS (and dS) (Higuchi; Porrati; Kogan, Mouslopoulos, Papazoglou)

• Known solutions of bigravity theory, with « cosmological asymptotics » which are arbitrarily close to « massless » Schwarzschild solutions (Salam, Strathdee '77; Isham Storey '78; Damour, Kogan, Papazoglou '03). See PART 2.

• As far as asymptotically flat S^4 are concerned, results obtained in some approximation scheme, surprises can arise trying to find the exact solution... similarly to what was recently studied by Damour, Kogan and Papazoglou in massive gravity or by Gabadadze and Iglesias for DGP!

Need for a better understanding of those issues, in particular:
• Validity of the linear pert. theory (linearization instability ? See Gababadze, Iglesias; C.D. ,Gabadadze, Iglesias)
• Theories where Vainshtein's mechanism does work vs those where it does not.

$\Rightarrow$ **Provides some motivation for a better understanding of S^4 in similar theories... e.g. bigravity**

1.4. Bigravity (f-g gravity)

Salam et al. '77
Isham, Storey '78

Let the background metric of non linear PF be dynamical, one is led to consider actions of the type

$$S = \int \mathrm{d}^4x\sqrt{-g}\ \left(\frac{-R_g}{2\kappa_g} + L_g\right) + \int \mathrm{d}^4x\sqrt{-f}\left(\frac{-R_f}{2\kappa_f} + L_f\right) + S_{int}[f,g]$$

Where $S_{int}[f,g]$ is some interaction term for the two metrics.

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and u+v = 1/2

This ensures that the theory is invariant under common diffeomorphisms

$$\left\{ \begin{array}{lcl} g_{\mu\nu} & \rightarrow & \partial_\mu x^\sigma \partial_\nu x^\rho g_{\rho\sigma} \\ f_{\mu\nu} & \rightarrow & \partial_\mu x^\sigma \partial_\nu x^\rho f_{\rho\sigma} \end{array} \right.$$

$$
\begin{aligned}
S &= \int \mathrm{d}^4x\sqrt{-g}\,\left(\frac{-R_g}{2\kappa_g}+L_g\right)+\int \mathrm{d}^4x\sqrt{-f}\left(\frac{-R_f}{2\kappa_f}+L_f\right)\\
&\quad -\frac{\zeta}{4}(-g)^u(-f)^{\frac{1}{2}-u}(f^{\mu\nu}-g^{\mu\nu})(f^{\sigma\tau}-g^{\sigma\tau})(g_{\mu\sigma}g_{\mu\tau}-g_{\mu\nu}g_{\sigma\tau})
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⇨ A way to investigate various properties of massive gravity

Other motivations

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Equations of motions read

$$\begin{cases} G^g_{\mu\nu} = T^g_{\mu\nu}[g,f] + T^g_{\mu\nu}[\text{matter}] \\ G^f_{\mu\nu} = T^f_{\mu\nu}[g,f] + T^f_{\mu\nu}[\text{matter}] \end{cases}$$

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Damour, Kogan, Papazoglou '03

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Damour, Kogan, Papazoglou '03

⇨ Some S^4 solutions breaks spontaneously Lorentz-invariance and give a Lorentz-violating mass term to the graviton…(similar to ghost condensate and Co.)

Blas, C.D., Garriga '07, Berezihani, Comelli, Nesti, Pilo '07

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Among which the vDVZ discontinuity and the Boulware Deser type of instability…

… and others to be discussed thereafter

2. Some aspects of Static Spherically Symmetric Solutions of bigravity

D. Blas, C.D., J.Garriga CQG 2006
D. Blas, C.D., J.Garriga, arXiv:0705.1982

Consider a bigravity theory

$$S = \int \mathrm{d}^4x\sqrt{-g}\,\left(\frac{-R_g}{2\kappa_g} + L_g\right) + \int \mathrm{d}^4x\sqrt{-f}\left(\frac{-R_f}{2\kappa_f} + L_f\right) + S_{int}[f,g]$$

E.g. in the PF « universality class »

$$S_{int} = -\frac{\zeta}{4}\int \mathrm{d}^4x(-g)^u(-f)^v(f^{\mu\nu} - g^{\mu\nu})(f^{\sigma\tau} - g^{\sigma\tau})(g_{\mu\sigma}g_{\mu\tau} - g_{\mu\nu}g_{\sigma\tau})$$

and u+v = 1/2

Then look at S^4 for the system (f,g)

Those solutions are of the form:

$$g_{\mu\nu}dx^\mu dx^\nu = Jdt^2 - Kdr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$f_{\mu\nu}dx^\mu dx^\nu = Cdt^2 - 2Ddtdr - Adr^2 - B\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

With *J*, *K*, *C*, *D*, *A*, *B* function of *r*

Salam, Strathdee '77

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Salam, Strathdee '77

Then one has either $B = \frac{2}{3}r^2$ ← Type I solutions

or $D = 0$ ← Type II solutions

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So Vainshtein mechanism seems not to work for non linear PF!

• Letting g be dynamical, exact (and boring) solutions can be found (both metric are proportional) and again no Yukawa asymptotics, Blas, C.D., Garriga

2.2. Type I solutions

Salam, Strathdee '77
Isham, Storey, '78
Blas, C.D., Garriga

Type I solutions are known analytically

$$g_{\mu\nu}dx^\mu dx^\nu = (1-q)dt^2 - (1-q)^{-1}dr^2 - r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)$$

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With
$$\begin{cases} A = \frac{2}{3\beta}(1-q)^{-2}\left(p+\beta-q-\beta q\right) \\ D^2 = \left(\frac{2}{3\beta}\right)^2(1-q)^{-2}(p-q)(p+\beta-1-\beta q) \end{cases}$$

and
$$\begin{cases} p = \frac{2M_f}{r} + \frac{2\Lambda_f}{9}r^2 \\ q = \frac{2M_g}{r} + \frac{\Lambda_g}{3}r^2 \end{cases}$$

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Namely, the change of variable $d\tilde{t} = \frac{1}{\sqrt{\beta}}\left\{dt \mp \frac{\sqrt{(p-q)(p+\beta-1-\beta q)}}{(1-q)(1-p)}dr\right\}$

Put the metric $f_{\mu\nu}$ in the usual static form of S(A)dS:

$$f_{\mu\nu}dx^\mu dx^\nu = \frac{2}{3}\left\{(1-p)d\tilde{t}^2 - (1-p)^{-1}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right\}$$

The (A)dS curvature radii of the solutions are determined by an integration constant β (and constrained if ρ_f and ρ_g both vanish)

$$\frac{\Lambda_f}{\kappa_f} = \frac{\zeta}{4}\left(\frac{3}{2}\right)^{4u} \beta^u \{3v + 9\beta(1-v)\} + \rho_f,$$
$$\frac{\Lambda_g}{\kappa_g} = \frac{\zeta}{4}\left(\frac{2}{3}\right)^{4v} \beta^{-v} \{3u - 9\beta(1+u)\} + \rho_g.$$

Vacuum energy density entering into the action

Similar to unimodular gravity

Some pathologies of those solutions:

- Causal structure

- Extension and global hyperbolicity ?

One virtue, and one interesting property:

- Spontaneous breaking of Lorentz invariance (and no vDVZ discontinuity), linearization instability ?

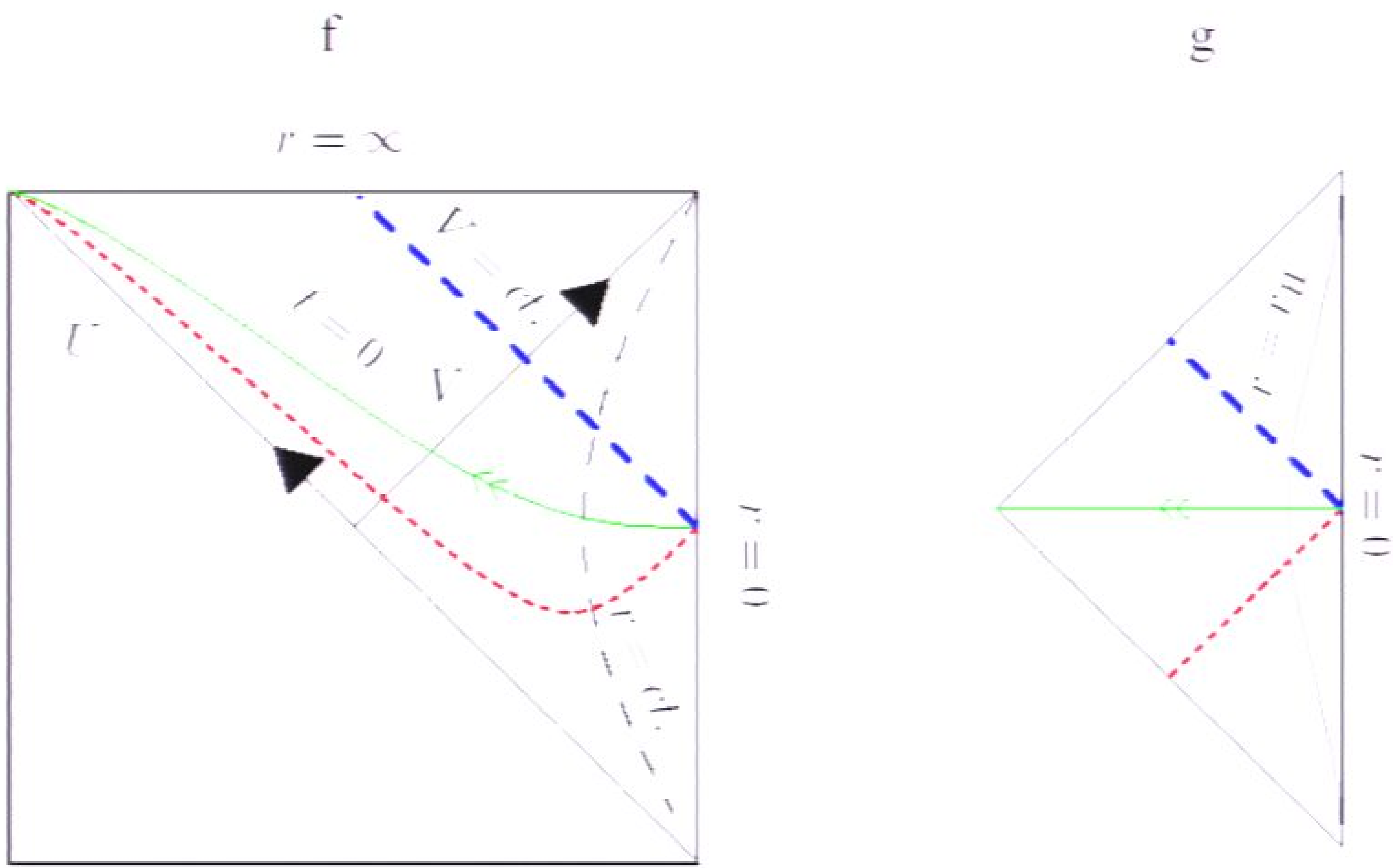
E.g. De Sitter with Minkowski (with β =1, $p = \frac{2\Lambda_f}{9} r^2$)

$$g_{\mu\nu} dx^\mu dx^\nu = dt^2 - dr^2 - r^2 d\Omega_2^2$$

$$f_{\mu\nu} dx^\mu dx^\nu = \underbrace{\frac{2}{3}(1-p)dt^2 - \frac{4}{3}p\ dtdr - \frac{2}{3}(1+p)dr^2}_{\text{The metric is not singular at p=1}} - \frac{2}{3}r^2 d\Omega_2^2$$

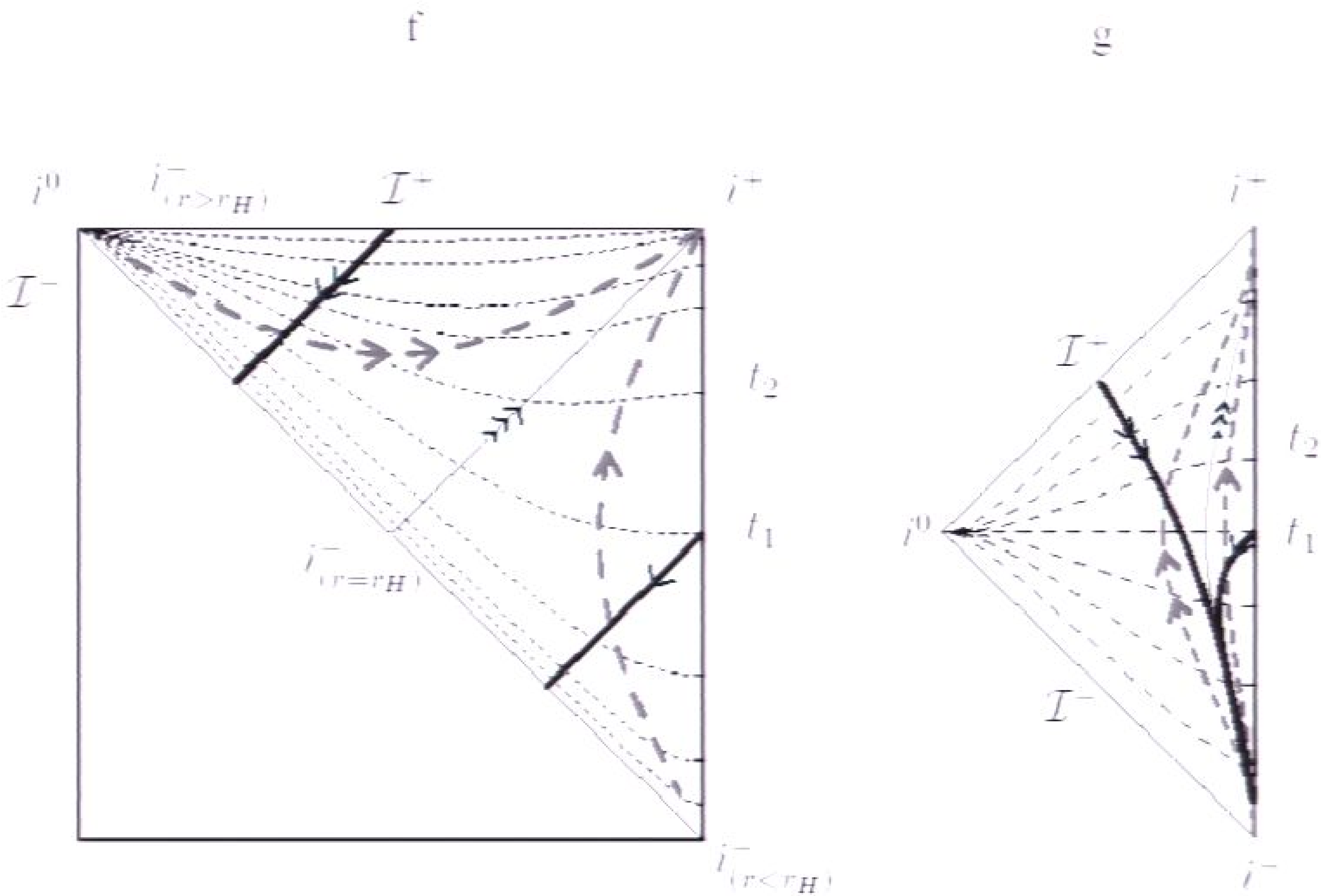
Minkowski coordinates are Eddington-Finkelstein type of coordinates for the dS metric…

Carter-Penrose diagrams of both metrics



Obviously the two metrics do no share the same light cone !

The past timelike infinity of Minkowski is mapped into part of the de Sitter horizon



More fun: de Sitter with Schwarzschild

$$g_{\mu\nu}dx^\mu dx^\nu = (1-q)dt^2 - (1-q)^{-1}dr^2 - r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)$$
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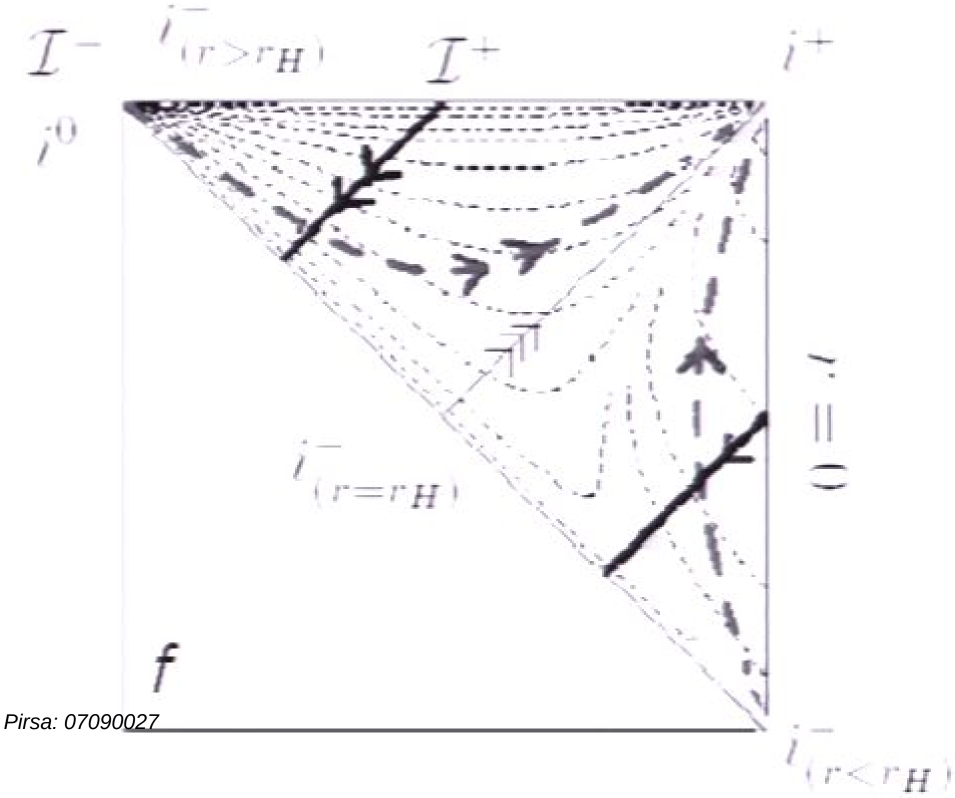
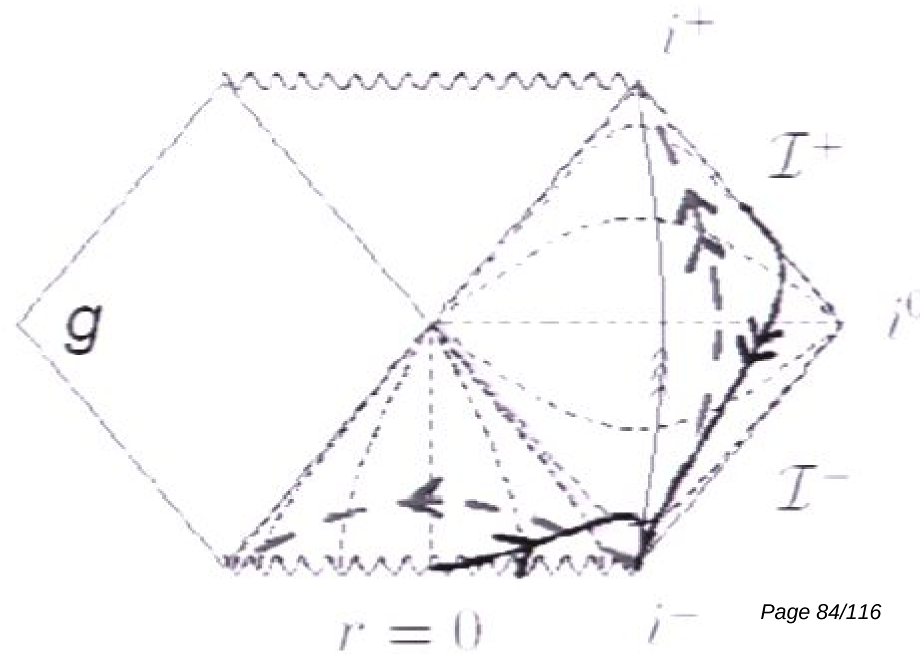
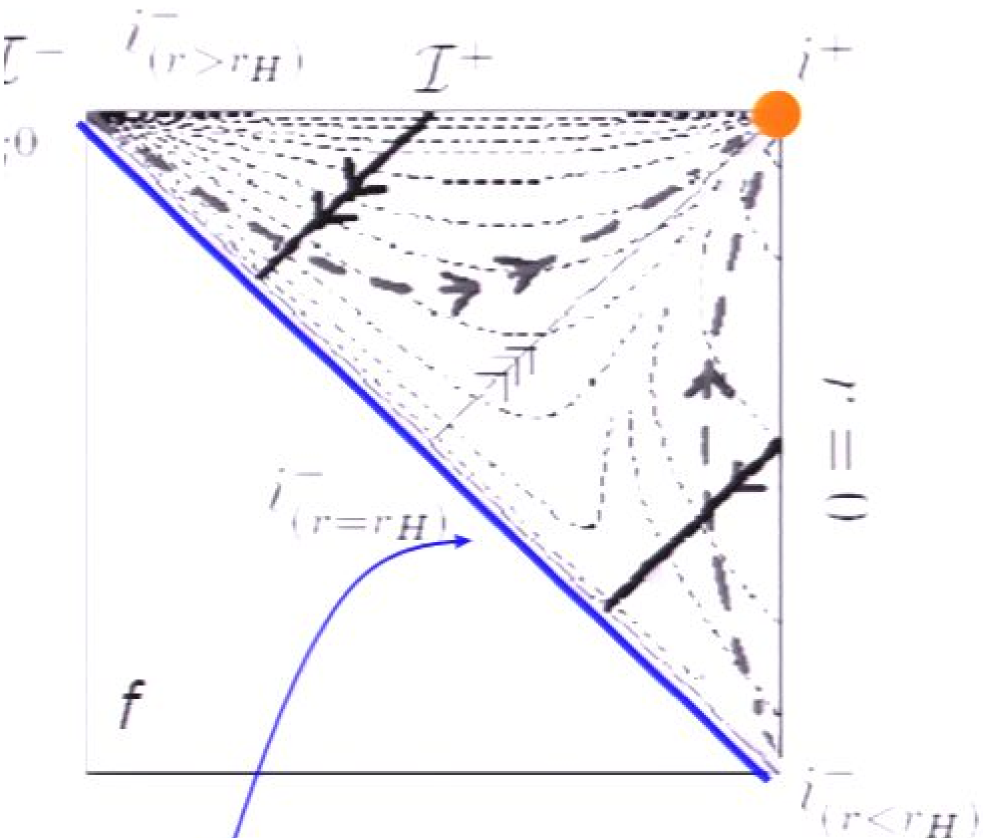
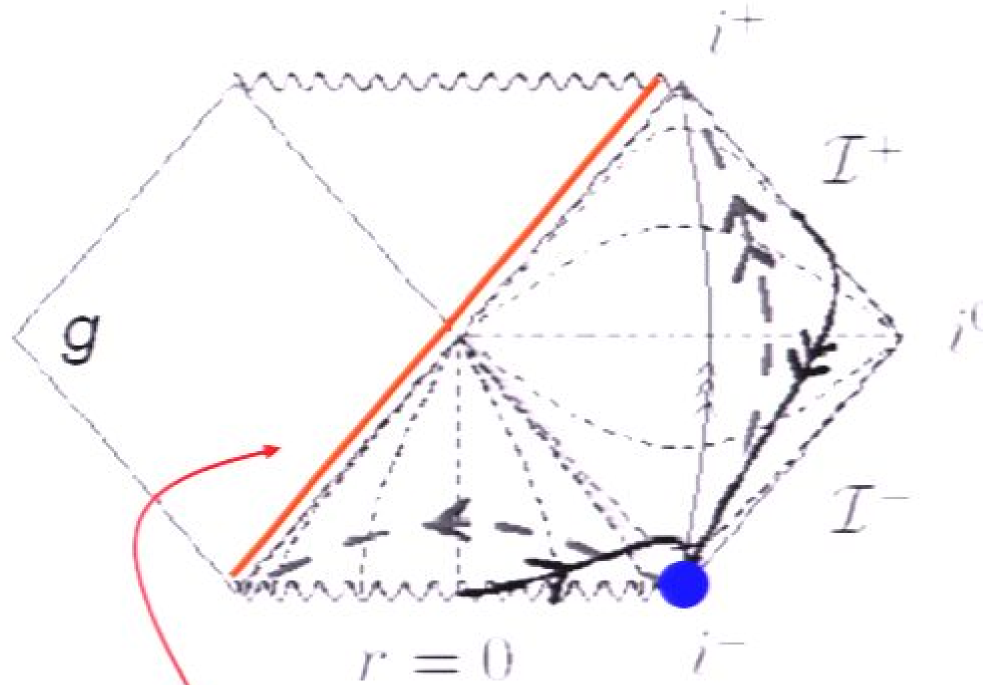
With $\begin{cases} A = \frac{2}{3}(1-q)^{-2}(p+1-2q) \\ D = \pm\frac{2}{3}(1-q)^{-1}\sqrt{(p-q)} \end{cases}$ and $\begin{cases} p = H^2r^2 \\ q = \frac{2M}{r} \end{cases}$

Assume β=1, $r_H > r_s$



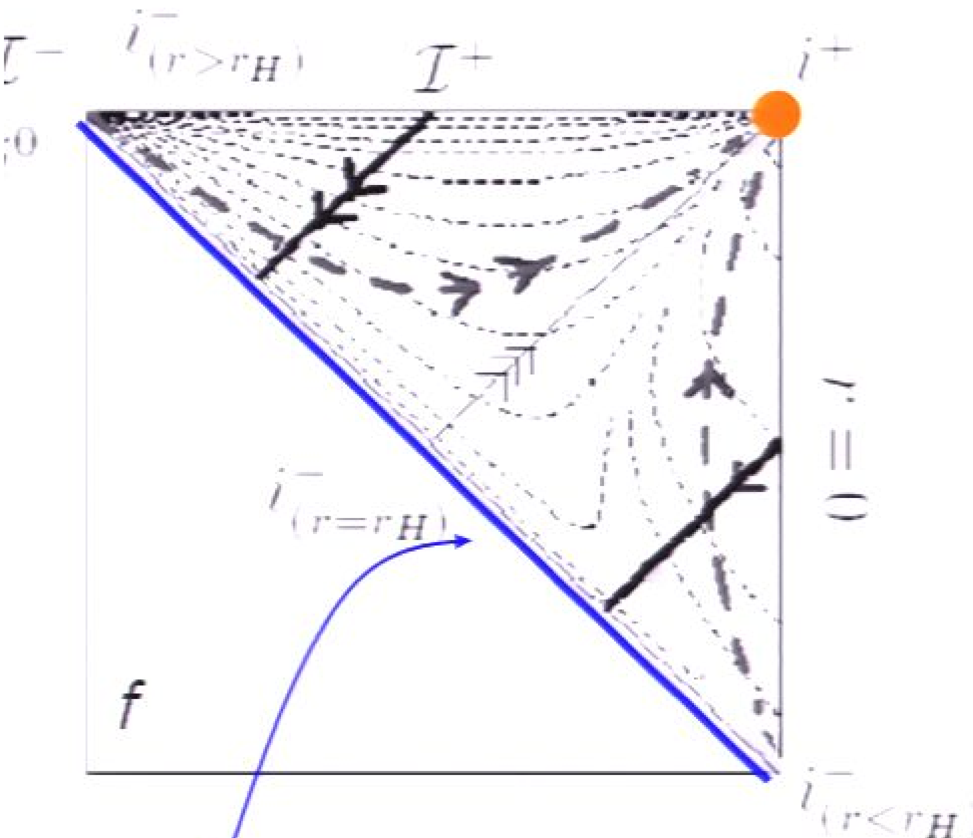


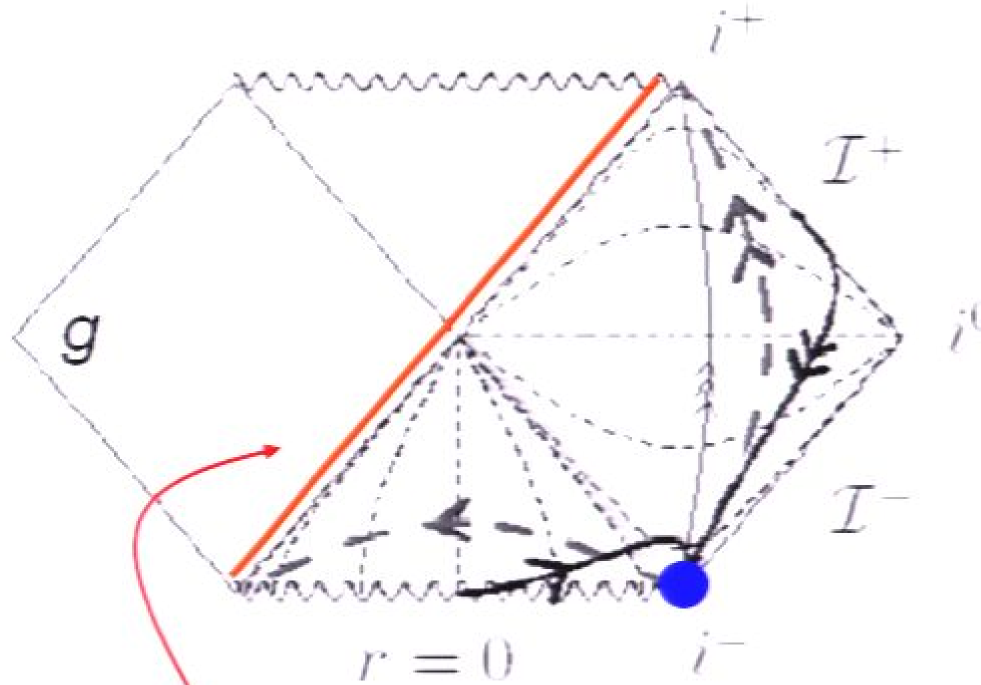




This part of the dS horizon is mapped into the past timelike infinity of $r=r_H$ 2-sphere of Schwarzschild

This part of the Schwarzshild horizon is mapped into the future timelike infinity of $r=r_s$ 2-sphere of de Sitter

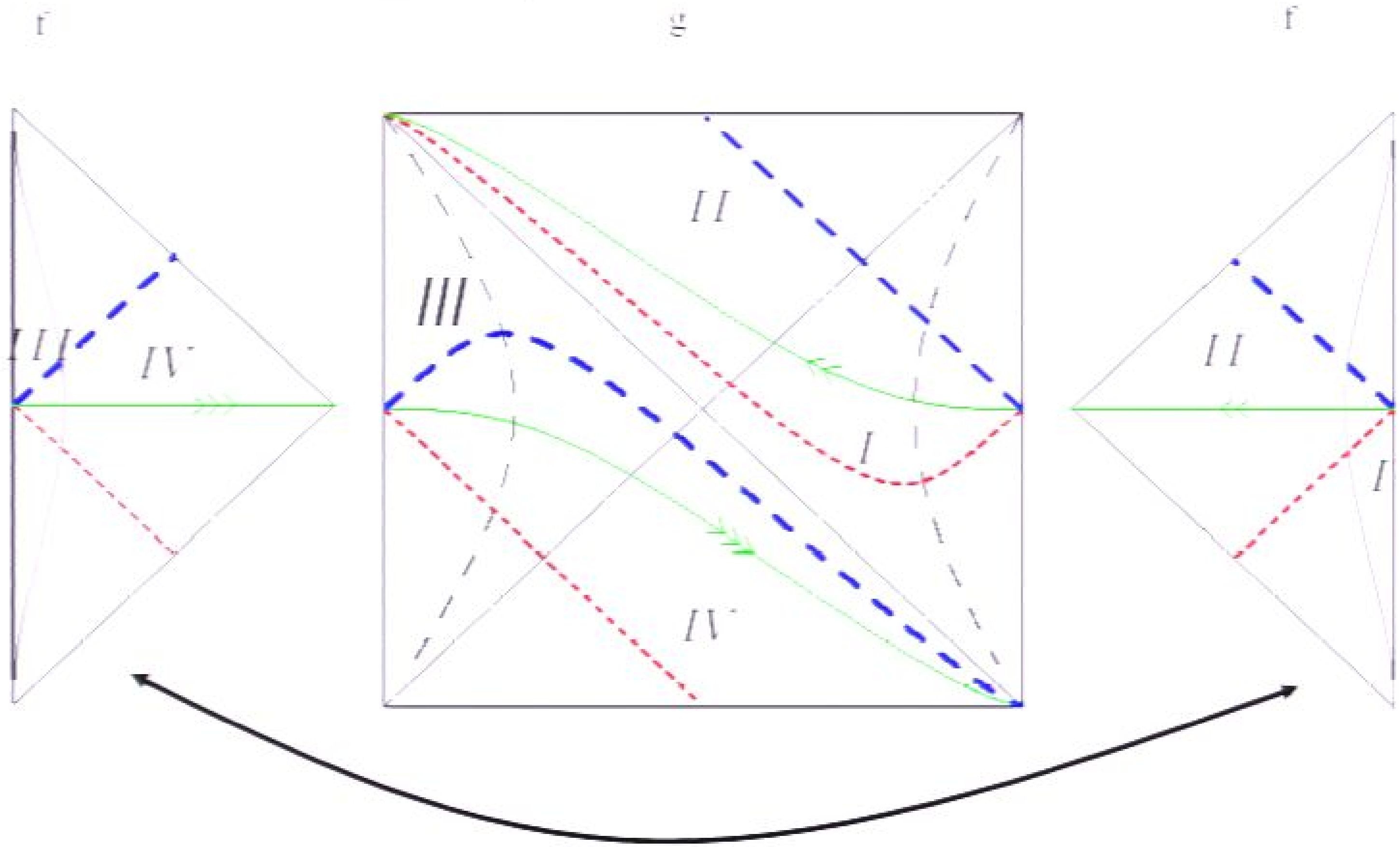




Can one cross the horizon ?

Can one cross the horizon ?

A geometric extension is possible



The Two Minkowski metrics are glued together along the time like infinities of $r=r_H$ spheres

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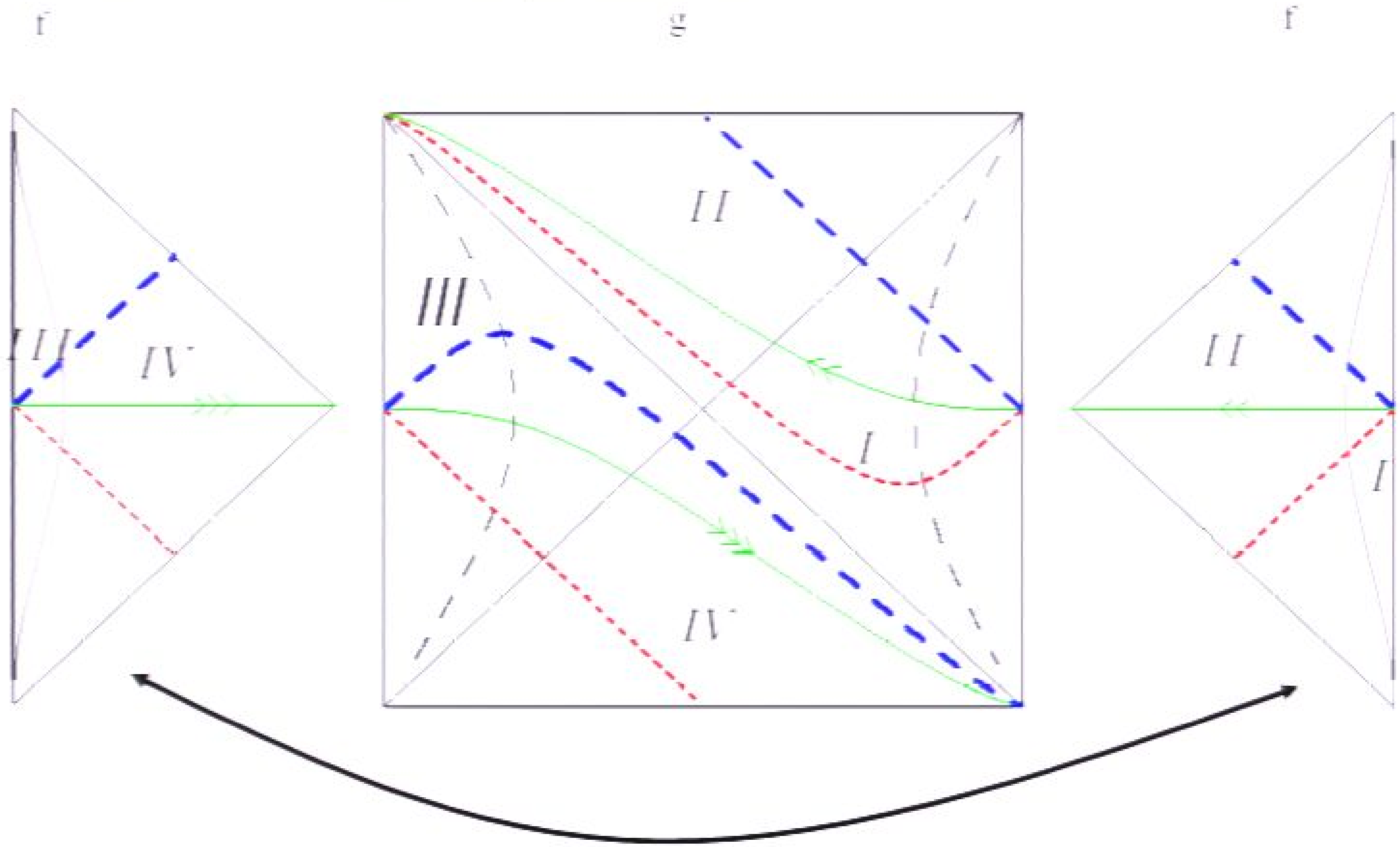

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… but …

- Not unique

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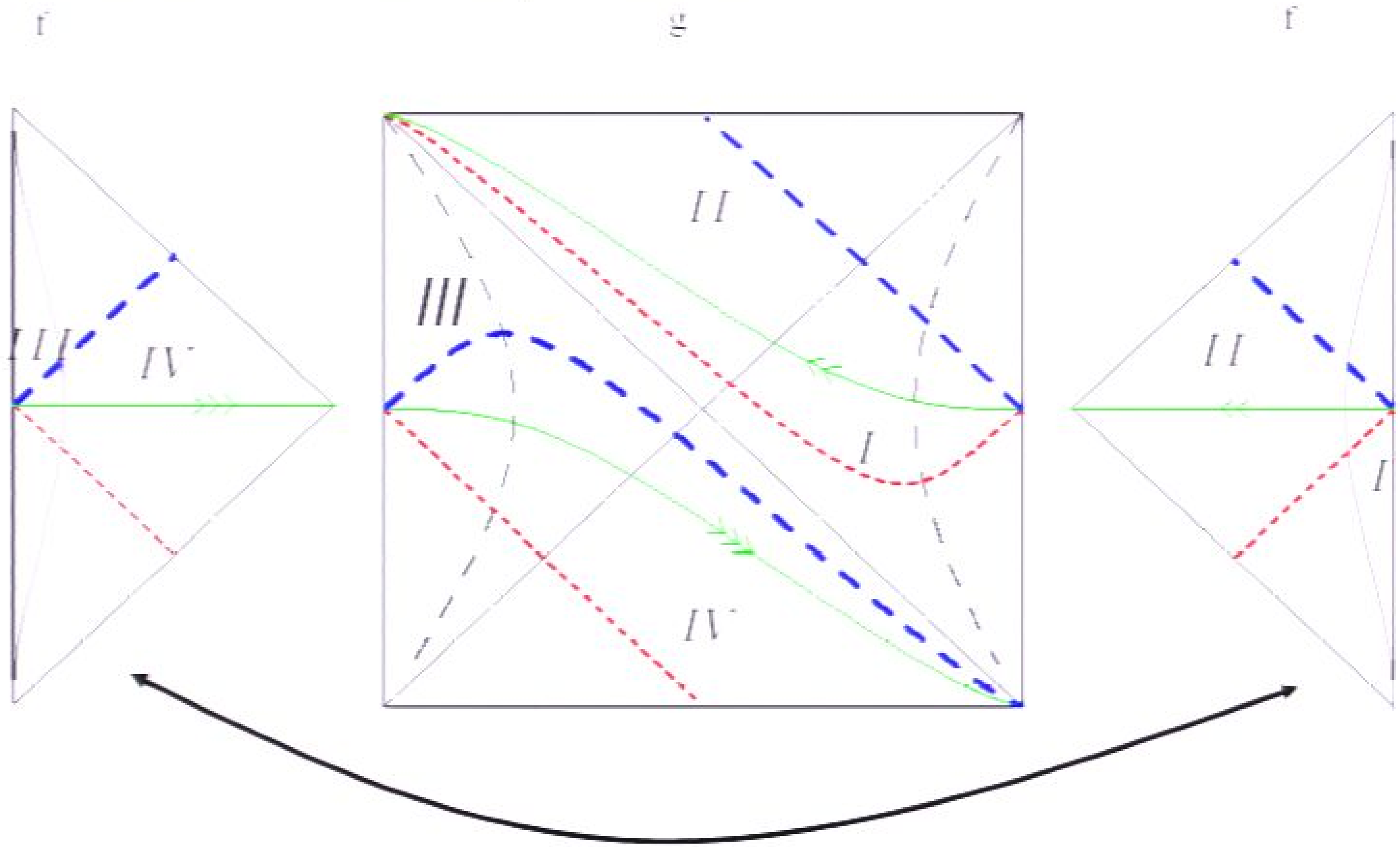

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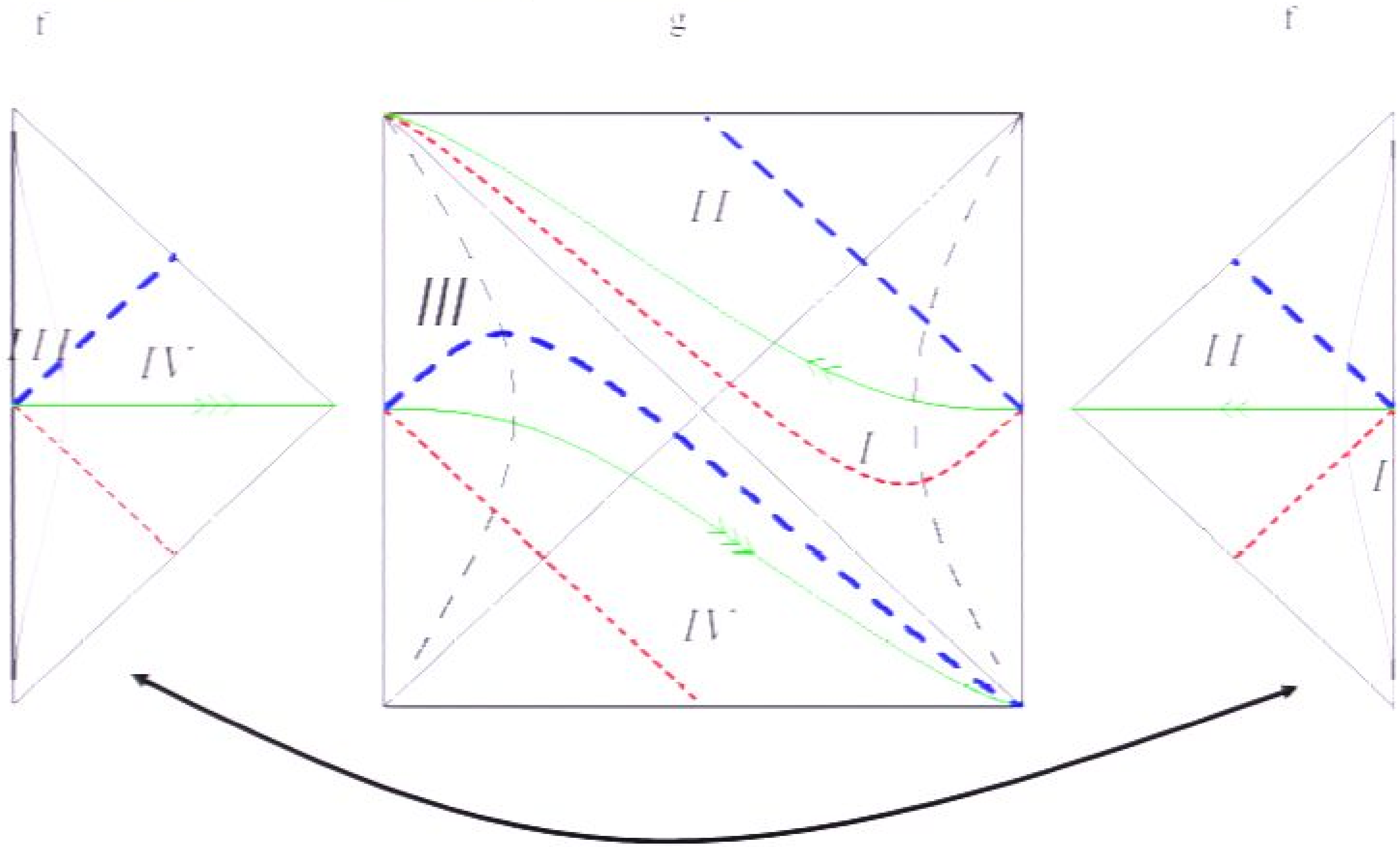
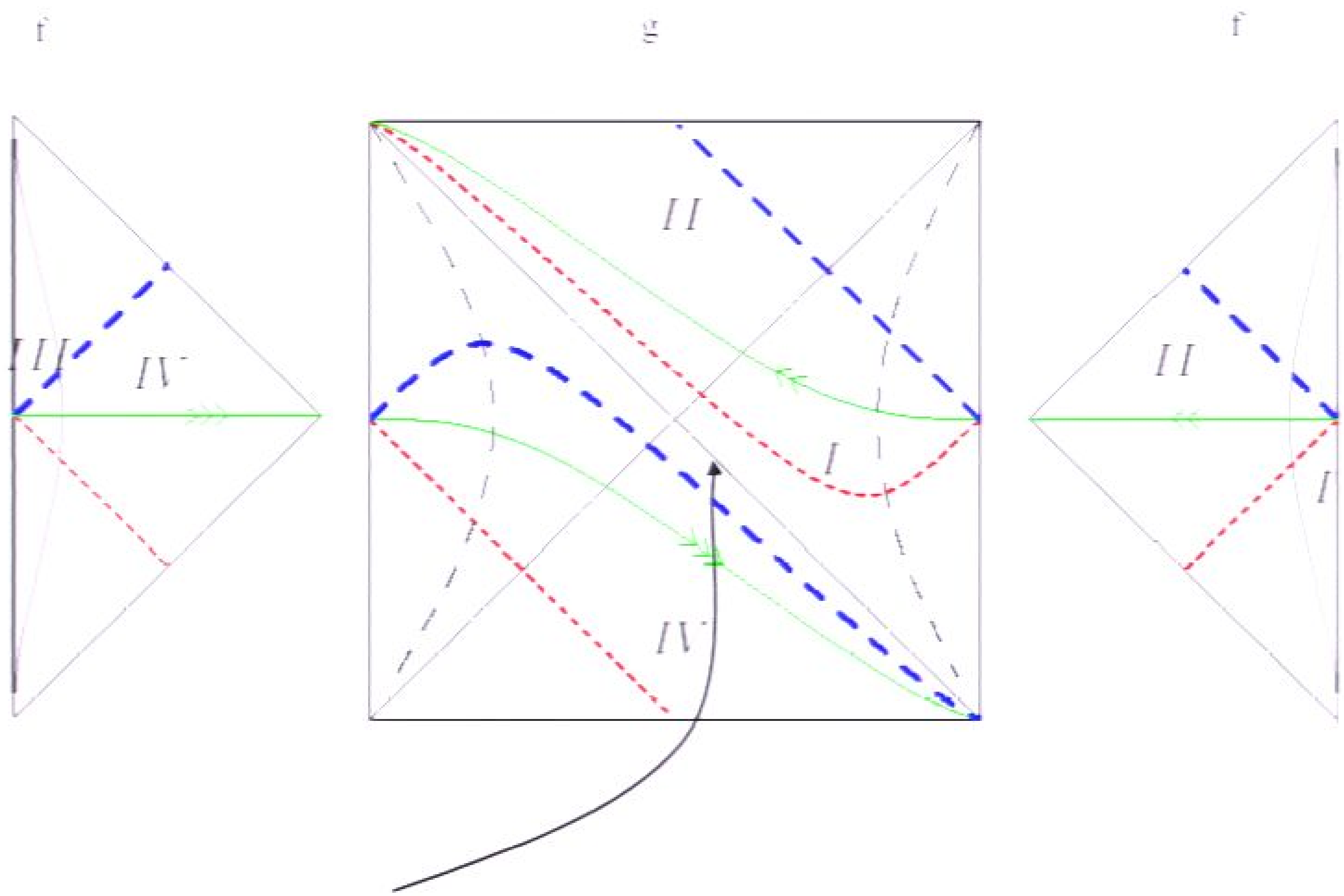


The Two Minkowski metrics are glued together along the time like infinities of $r=r_H$ spheres

… but …

- Not unique
- Not globally hyperbolic
- Not in an obvious way a solution of the e.o.m

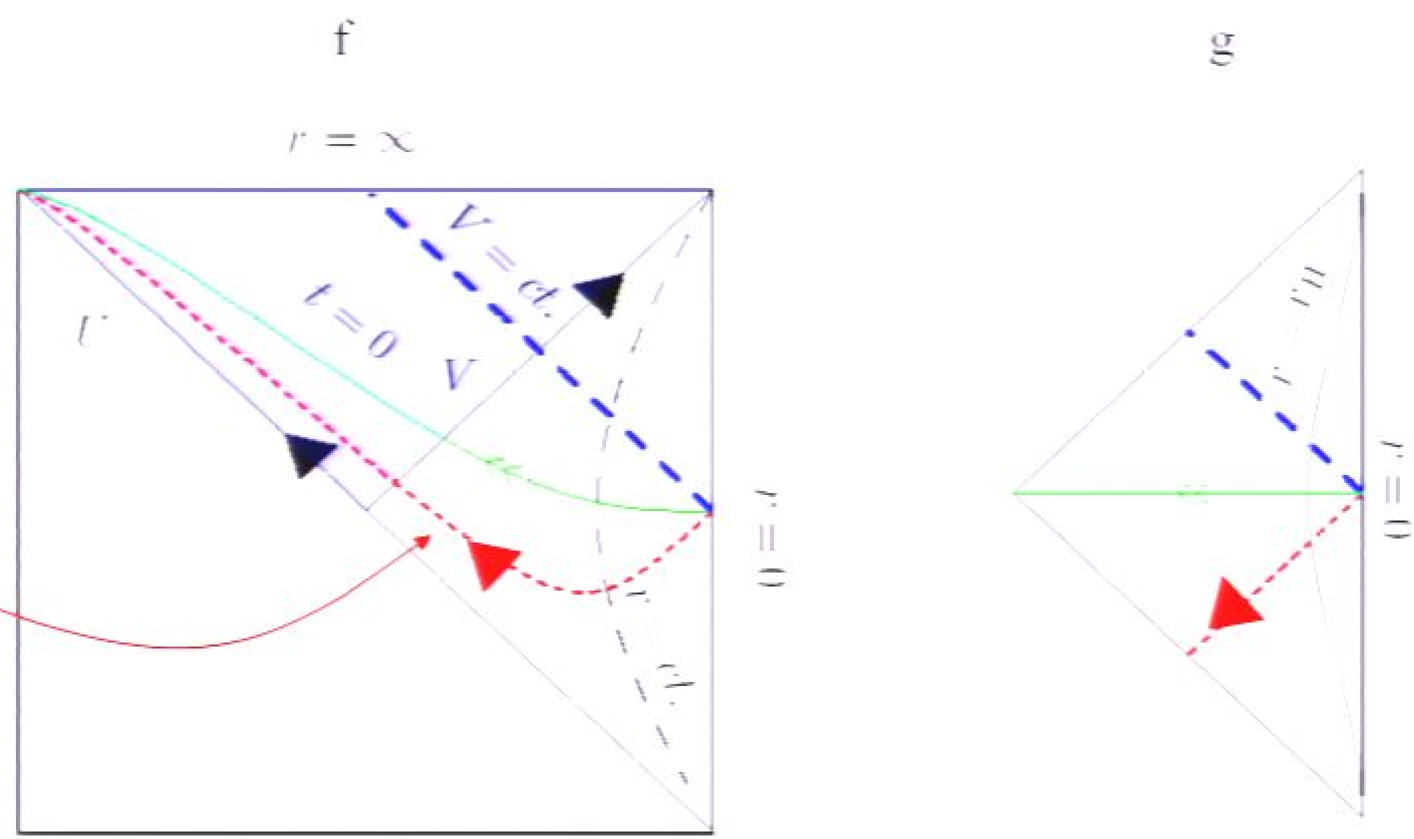
A possible solution to this extension puzzle



A Cauchy Horizon type of instability on the dS horizon?

One possible interest of this weird causal structure …

Past directed in Minkowski, is no longer in de Sitter



An r=0 observer can see the blue area below thanks to signals transmitted in the Minkowski metric

NB: Similar situations have recently been discussed for BH and their hairs (Babichev, Mukhanov and Vikman; Dubovsky, Sibiryakov; Elling, Foster, Jacobson, Wall; Dubosvky, Tinyakov, Zaldarriaga)

An other pathology if $\beta \neq 1$?

$$g_{\mu\nu}dx^\mu dx^\nu = (1-q)dt^2 - (1-q)^{-1}dr^2 - r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)$$

$$f_{\mu\nu}dx^\mu dx^\nu = \frac{2}{3\beta}(1-p)dt^2 - 2Ddtdr - Adr^2 - \frac{2}{3}r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)$$

With $D^2 = \left(\frac{2}{3\beta}\right)^2 (1-q)^{-2}(p-q)(p+\beta-1-\beta q)$

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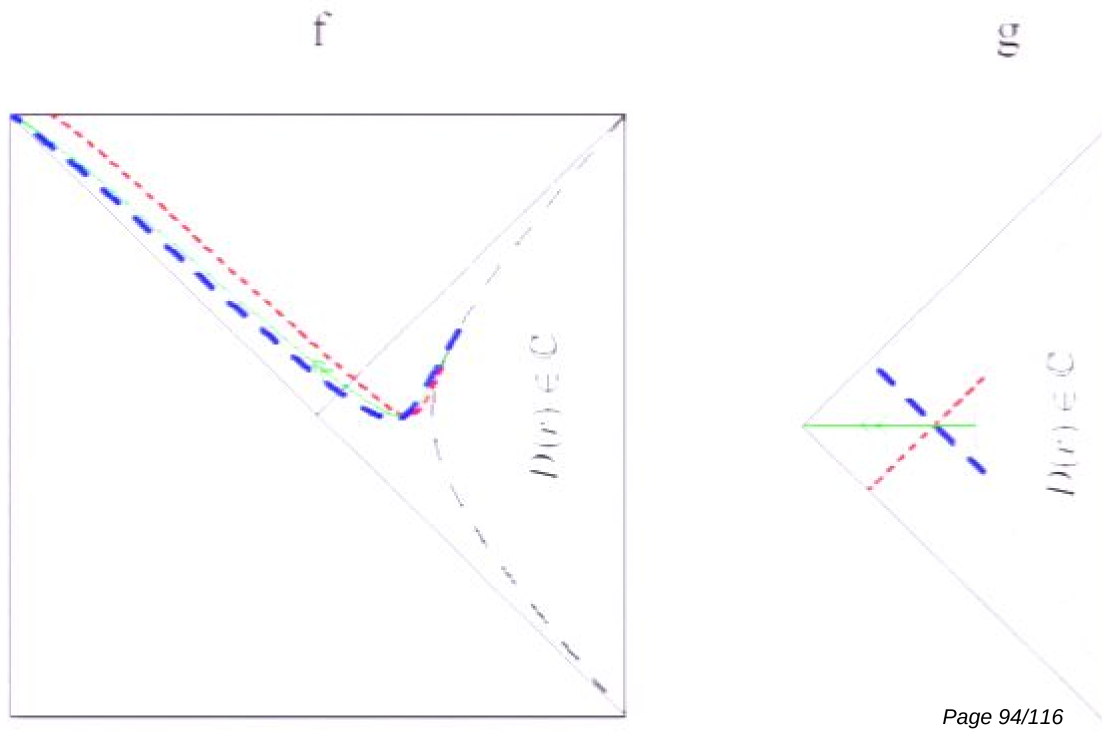
For generic β, *D* can become complex at some radius r….

How to extend the solution beyond ?



An interesting type I solution (and generic β):

$$g_{\mu\nu}dx^\mu dx^\nu = (1-q)dt^2 - (1-q)^{-1}dr^2 - r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)$$
$$f_{\mu\nu}dx^\mu dx^\nu = \frac{2}{3\beta}(1-p)dt^2 - 2Ddtdr - Adr^2 - 2/3r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)$$

With
$$\begin{cases} A = \frac{2}{3\beta}(1-q)^{-2}(p+\beta-q-\beta q) \\ D = \pm\frac{2}{3\beta}(1-q)^{-1}\sqrt{(p-q)(p+\beta-1\beta q)} \\ p = 0 \text{ and } q = 0 \end{cases}$$

$$\Rightarrow \begin{cases} g_{\mu\nu}dx^\mu dx^\nu = dt^2 - dr^2 - r^2\left(d\theta^2 + sin^2\theta d\phi^2\right) \\ f_{\mu\nu}dx^\mu dx^\nu = \frac{2}{3\beta}dt^2 - \frac{2}{3}dr^2 - \frac{2}{3}r^2\left(d\theta^2 + sin^2\theta d\phi^2\right) \end{cases}$$

Both metrics are flat, but this generates a Lorentz violating mass term for one graviton

Spontaneous breaking of Lorentz symmetry

Perturbations around this solution

3. Perturbations

We obtain the mass term for the « gravitons » $h_f^{\mu\nu}$ and $h^g{}_{\mu\nu}$

$$-\frac{M^4}{8}\Big\{n_2(h^g{}_{ij}+h_f{}^{ij})(h^g{}_{ij}+h_f{}^{ij})+n_0(h^g{}_{00}+\beta^{-1}h_f{}^{00})(h^g{}_{00}+\beta^{-1}h_f{}^{00})$$

$$-2n_4(h^g{}_{00}+\beta^{-1}h_f{}^{00})(h^g{}_{ii}+h_f{}^{ii})+n_3(h^g{}_{ii}+h_f{}^{ii})^2\Big\}$$

n_i are potential dependent coefficients

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Case similar with a single graviton were studied

(Rubakov '04, Dubovsky '04)

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Peculiarity of this mass term: components $h_f{}^{0i}$ and $h^g{}_{0i}$ are absent

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Peculiarity of this mass term: components $h_f{}^{0i}$ and $h^g{}_{0i}$ are absent

No propagating scalars and vectors, only tensors are propagating but stable perturbations

(3+1 split of the metric)

$$\begin{cases} h^X{}_{00} &=& 2A^X \\ h^X{}_{0i} &=& B^X_{,i}+V^X_i \\ h^X{}_{ij} &=& 2\psi^X\delta_{ij}-2E^X_{,ij}-2F^X_{(i,j)}-t^X_{ij} \end{cases}$$

Coupling to matter and the vDVZ discontinuity

$$S_{matt} = \frac{1}{4}\int \mathrm{d}^4 x \left(\lambda_g h^g{}_{\mu\nu} T_g^{\mu\nu} + \lambda_f h^f{}_{\mu\nu} T_f^{\mu\nu}\right)$$

We obtain e.g. for the gauge invariant scalar potentials

$$\begin{cases} \Phi^g & \equiv & A^g - \dot{B}^g - \ddot{E}^g \\ \Phi^f & \equiv & A^f - 3\dot{B}^f - 3^2\ddot{E}^f \end{cases}$$

$$\begin{aligned} \Delta\Phi^g = & - \frac{\kappa_g \lambda_g}{4}\left(T_g^{00} + T_g^{ii} - \frac{3}{\Delta}\ddot{T}_g^{00}\right) \\ & - \left(\frac{\kappa_g M^4 n_2}{4\Delta}\right)\frac{n_2 + 3n_3 - 3n_0}{n_2 + n_3 - n_0}\left(\kappa_g \lambda_g T_g^{00} + \kappa_f \lambda_f 3 T_f^{00}\right) \\ \Delta\Phi^f = & - \frac{\kappa_f \lambda_f 3}{4}\left(3T_f^{00} + T_f^{ii} - \frac{3}{\Delta}3^2\ddot{T}_f^{00}\right) \\ & - \left(\frac{\kappa_f 3 M^4 n_2}{4\Delta}\right)\frac{n_2 + 3n_3 - 3n_0}{n_2 + n_3 - n_0}\left(\kappa_g \lambda_g T_g^{00} + \kappa_f \lambda_f 3 T_f^{00}\right) \end{aligned}$$

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- Massless result: no vDVZ discontinuity

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- This interaction leads to correction to the « massless » metric proportional the (squared) graviton mass (analogous to « half massive gravity » of Gabadadze Grisa)

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Question: these corrections are not seen in the known exact solutions…. Linearization instability ? (see e.g. C.D., Gababdaze, Iglesias in the case of DGP gravity)

Conclusions (of Part II)

- Exact, known S^4 of bigravity have weird causal structure, and are geodesically incomplete…

  $\rightarrow$ some new pathologies of these solutions (theories?)

- Is there an exact, non singular solution which shows the Vainsthein recovery of the vDVZ discontinuity ?

- Interesting solution with spontaneous breaking of Lorentz sym.

- Linearization instability (is it true and generic) ?

An interesting type I solution (and generic β):

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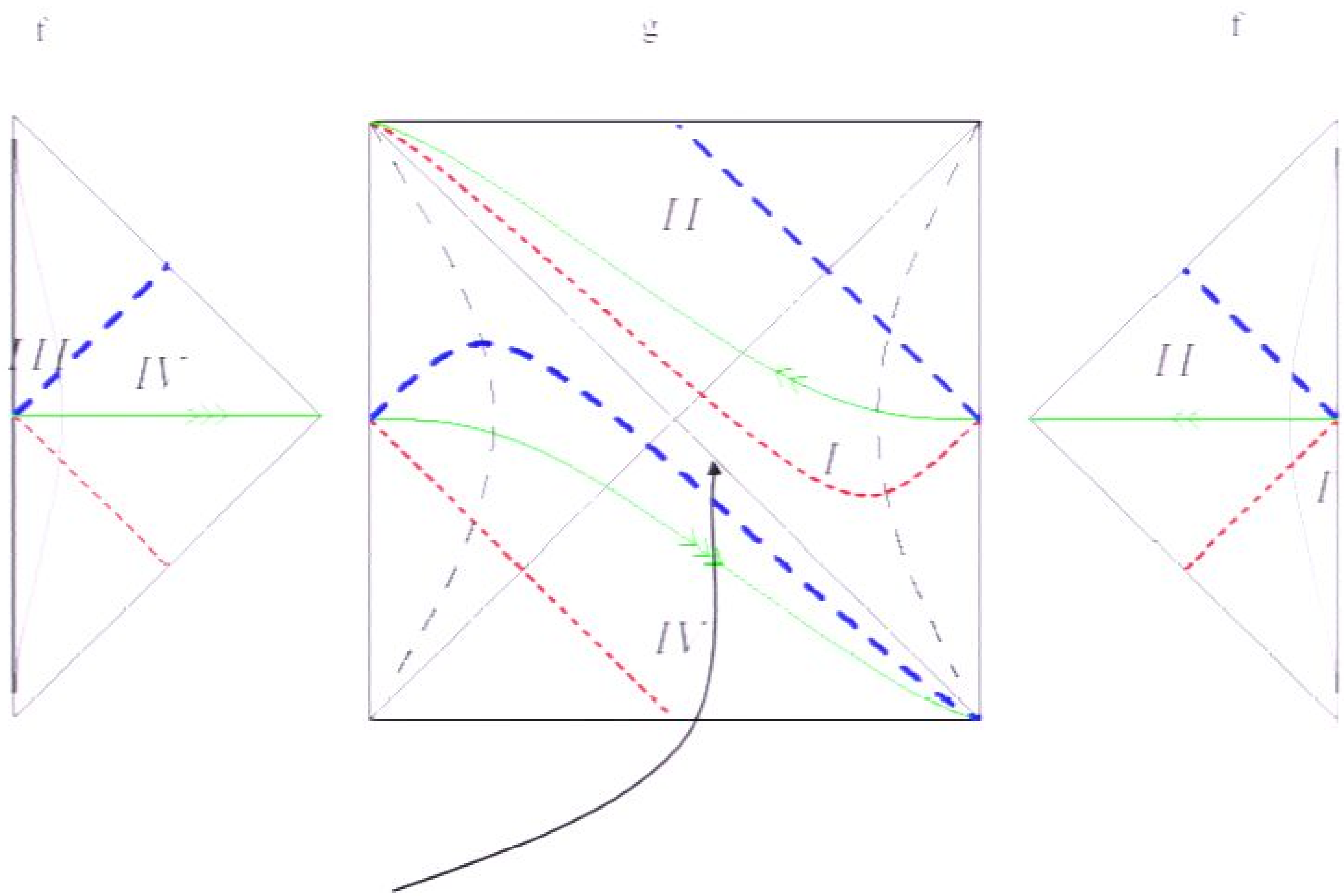
Both metrics are flat, but this generates a Lorentz violating mass term for one graviton

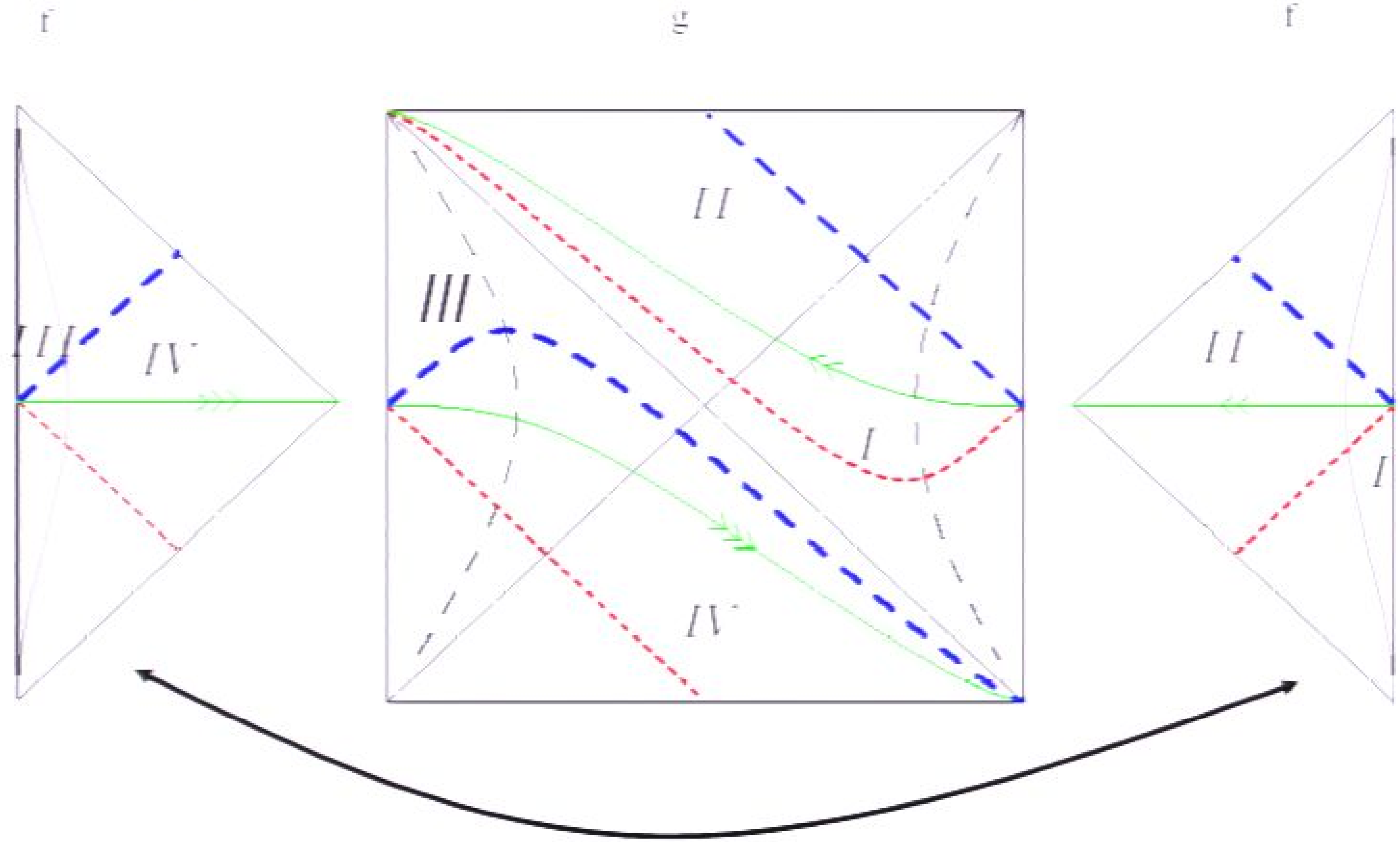
Spontaneous breaking of Lorentz symmetry

A possible solution to this extension puzzle



A Cauchy Horizon type of instability on the dS horizon?

A geometric extension is possible



The Two Minkowski metrics are glued together along the time like infinities of $r=r_H$ spheres

… but …

- Not unique
- Not globally hyperbolic
- Not in an obvious way a solution of the e.o.m

2.1.Type II solutions $(D = 0)$

- The general solution is not known analytically!
- Asymptotic form of the f metric, when one assumes that $g_{\mu}=\eta_{\mu\nu}$ (and is non dynamical) has been found to be of Yukawa type Aragone, Chela-Flores '72
- With the same assumptions, numerical integration of the E.o.m show that the asymptotically flat solutions develop singularities at finite radius Damour, Kogan Papazoglou, '03

So Vainshtein mechanism seems not to work for non linear PF!

$$
\begin{aligned}
S &= \int \mathrm{d}^4x\sqrt{-g}\,\left(\frac{-R_g}{2\kappa_g}+L_g\right)+\int \mathrm{d}^4x\sqrt{-f}\left(\frac{-R_f}{2\kappa_f}+L_f\right)\\
&\quad -\frac{\zeta}{4}(-g)^u(-f)^{\frac{1}{2}-u}(f^{\mu\nu}-g^{\mu\nu})(f^{\sigma\tau}-g^{\sigma\tau})(g_{\mu\sigma}g_{\mu\tau}-g_{\mu\nu}g_{\sigma\tau})
\end{aligned}
$$

However

- Cosmological solutions play a tricky role for the vDVZ discontinuity: No vDVZ discontinuity on AdS (and dS) (Higuchi; Porrati; Kogan, Mouslopoulos, Papazoglou)

- Known solutions of bigravity theory, with « cosmological asymptotics » which are arbitrarily close to « massless » Schwarzschild solutions (Salam, Strathdee '77; Isham Storey '78; Damour, Kogan, Papazoglou '03). See PART 2.

An other pathology of non-linear Pauli-Fierz: at non linear level, it propagates 6 instead of 5 degrees of freedom, the energy of the sixth d.o.f. having no lower bound!

Using the usual ADM decomposition of the metric, the non-linear PF Lagrangian reads (for $\eta_{\mu\nu}$ flat)

$$M_P^2 \int d^4x \left\{ \left(\pi^{ij}\dot{g}_{ij} - NR^0 - N_i R^i\right) - m^2 \left(h_{ij}h_{ij} - 2N_iN_i - h_{ii}h_{jj} + 2h_{ii}\underbrace{\left(1 - N^2 + N_k g^{kl} N_l\right)}\right)\right\}$$

With
$$\begin{cases} N \equiv \left(-g^{00}\right)^{-1/2} \\ N_i \equiv g_{0i} \end{cases}$$

Neither N, nor N_i are Lagrange multipliers

The e.o.m. of N and N_i determine those as functions of the other variables

Boulware, Deser '72

The vDVZ discontinuity is due to the scalar polarization of the graviton being coupled to the trace of the source energy momentum tensor…

While the Vainshtein mecanism is due to this polarization having strong self interaction

C.D.,Gabadadze, Dvali, Vainshtein

This polarization can be described by the following action:

Arkani-Hamed, Georgi, and Schwartz

$$\frac{1}{2}(\nabla\phi)^2 - \frac{1}{M_P}\phi T + \frac{1}{\Lambda^5}\left\{(\nabla^2\phi)^3 + \underbrace{\ \dots\ }_{\text{Other cubic terms omitted}}\right\}$$

With $\Lambda = (m^4 M_P)^{1/5}$