Title: Graviton Mass and Cosmological Constant

Date: Sep 11, 2007 12:00 PM

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Abstract:

Plan:

- **Introduction**

  Cosmological Constant Problem

- **Modified GR**

  $D \geq 6$ Brane Induced Gravity

- **Self-acceleration?**

  Developments, Problems & Perspectives in $D=5$

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- **Introduction**

  Cosmological Constant Problem

- **Modified GR**

  $D \geq 6$ Brane Induced Gravity

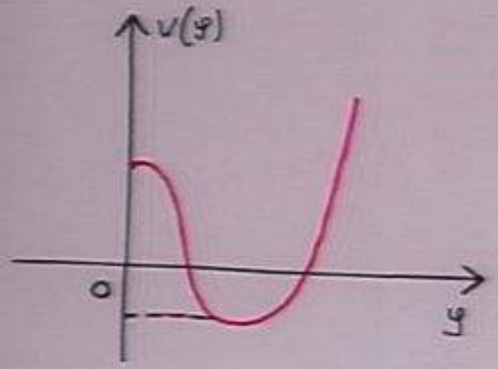
- **Self-acceleration?**

  Developments, Problems & Perspectives in $D=5$

Particle Physics

- Everything that contributes to the vacuum energy acts like a cosmological constant

$$\langle T_{\mu}^{\mu\nu} \rangle = \varepsilon g_{\mu}$$



$\Lambda \sim G_N \varepsilon$

Symmetry breaking: GUT $\sim 10^{16}$ GeV; EW $\sim 10^{3}$ GeV

QCD vacuum ~ 1 GeV

Adjust to zero (?)

Quantum nature: zero-point energy

(Zeldovich, 1967)

$$\mathcal{E} = \int^{\mu} \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{2} \sqrt{\vec{k}^2 + m^2} \sim \mu^4$$

Supersymmetry: particle $\longleftrightarrow$ super particle

$$\Rightarrow \mathcal{E} = \mathcal{E}_p - \mathcal{E}_{sp} = 0$$

However, supersymmetry is broken

at $\mu \gtrsim$ TeV

Particle - Sparticle degeneracy is lifted

Vacuum energy is not protected !

- Combining Particle Physics & General Relativity

$$\mathcal{E} \gtrsim (\text{TeV})^4$$

Einstein's gravity:

$$R \sim G_N \mathcal{E} \gtrsim (10^{-12}\,\text{GeV})^2$$

$$R_{exp} \sim H_0^2 \sim (10^{-42}\,\text{GeV})^2$$

To get $R = R_{exp}$ one needs

$$\mathcal{E} \sim (10^{-15}\,\text{TeV})^4$$

Why is the vacuum energy so small?

S. Weinberg (83) no-go theorem for dynam. adjustment !

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Naturalness in QFT.

Wilson: Higgs (scalar) mass, Cosmological constant, power sensitive to UV physics.

Fine tuning ?

Standard Model ~ 20 param.
(neutrino mass ambiguity)

All the discovered ones – technically Natural

The only unnatural one – Higgs mass yet to be discovered!

How could massive graviton help?

(naive arguments)

$$\frac{1}{r} \longrightarrow \frac{e^{-m_g r}}{r}$$

Scales $\ll 1/m_g$ ≈ massless

Scales $\gtrsim 1/m_g$ = de-gravitation (gravity shuts off?)

$m_g \sim H_0 \sim 10^{-42}$ GeV

Curvature via massive graviton Condensation

GR: $\langle R \rangle = 0$ if no sources

Massive GR: $\langle R \rangle \sim m_g^2 \sim H_0^2$

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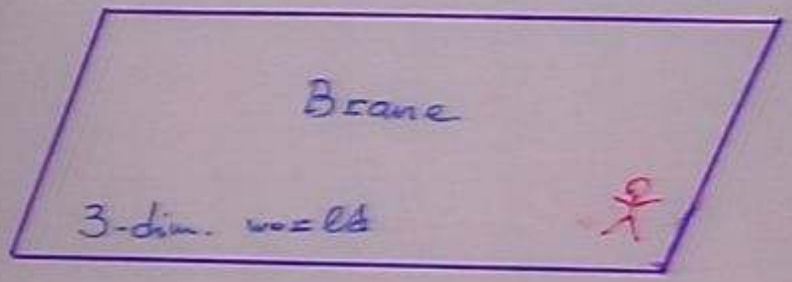
- Suggestive example: $(4+N)$ dim



Tension of the brane $\Rightarrow \mathcal{E}$

- $\mathcal{E}$ could "curve" extra dimensions producing (nearly) flat 4D world !

$$ds^2 = A^2(\rho)\,\underbrace{\bar{g}_{\mu\nu}\,dx^\mu dx^\nu}_{\text{nearly flat}} - B^2(\rho)d\rho^2 - C^2(\rho)\rho^2 d\Omega^2_{N-1}$$

How does this look like from 4D point of view ?

van Dam Veltman Zakharov
(vDVZ) discontinuity

Massive spin-2 =?

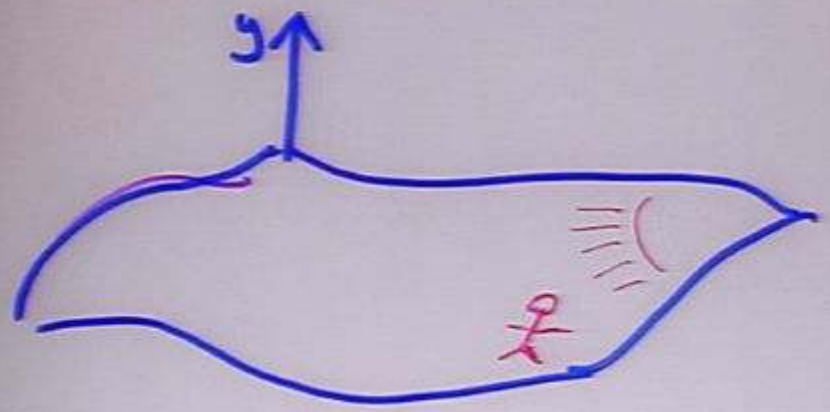
helicity-2; helicity-1;
helicity-0

↑ does couple to T^{μ}_{μ}

"tensor-scalar" gravity

$$S = \frac{M_*^3}{2} \int d^5x \sqrt{g_5} R_5 +$$

$$+ \frac{M_P^2}{2} \int d^4x \sqrt{g_4} R_4 + \dots$$



$$m_c \sim \frac{M_*^3}{M_P^2}$$

(Duals, G.C. Percacci)

$$G_{\mu\nu} + m_c\left(K_{\mu\nu} - g_{\mu\nu}K\right) = T_{\mu\nu}$$

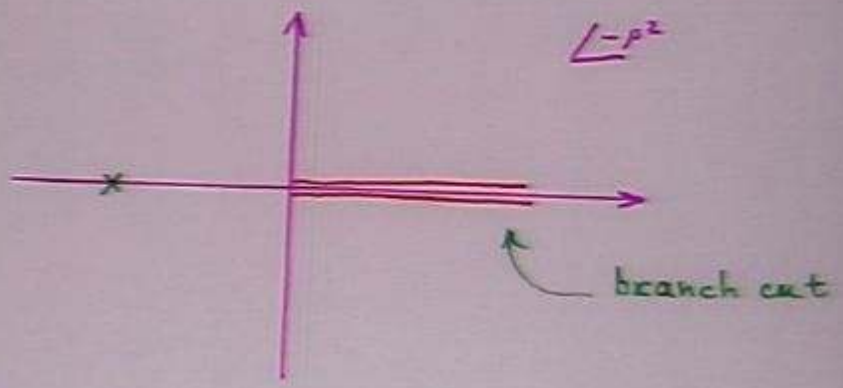
$$\nabla^r K_{\mu\nu} = \nabla_\nu K$$

Bianchi OK.

$$R = K_{\mu\nu}^2 - K^2$$

Perturbative spectrum: Conventional branch

$$\text{Propagator} \sim \frac{1}{p^2 + m_g^2\sqrt{p^2}}\, e^{-\sqrt{p^2}|y|}$$

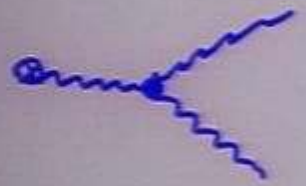


- Residue at $p^2=0$ is zero: no massless graviton
- Pole on the second sheet: resonance graviton

  lifetime $\sim m_g^{-1} \sim H_0^{-1} \sim 13.7$ Gyr
- Non local theory from 4D point of view.

Breakdown of Perturbation Theory:

(Deffayet, Dvali, G.G., Vainshtein)



$$\frac{P_\mu P_\nu}{m_c \sqrt{P^2}}$$

- Perturbative expansion in Newton's constant breaks down at

$$r \lesssim r_* \equiv (r_g r_c^2)^{1/3}$$

where $r_c \equiv 1/m_c$ $\quad r_g = 2G_N M$

Sun: $r_* \sim$ kpc

Galaxy: $r_* \sim$ Mpc

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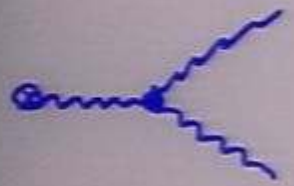

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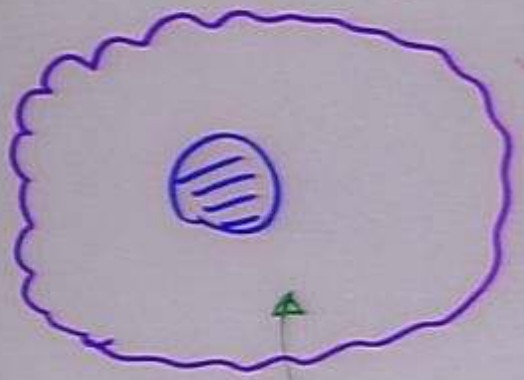
(G.G, Iglesias)

$$R + 3m_c K = -T$$

↳ $R \neq 0$ outside the source



$R \neq 0$ $\quad R(r \sim r_*) = m_c^2$

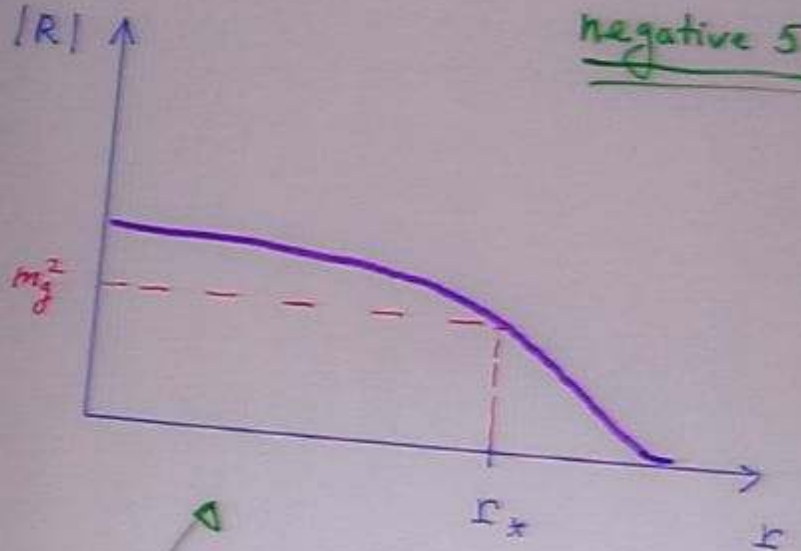
$$\Delta M \sim M_P^2 \int_* R\, d^3x \sim M\,!$$

$$-2\Phi = \frac{r_H}{r} \mp m_g^2 r^2 g_f(r)$$

Exact result (G.G. Izotas)

Self-accelerated branch

negative 5D mass ??



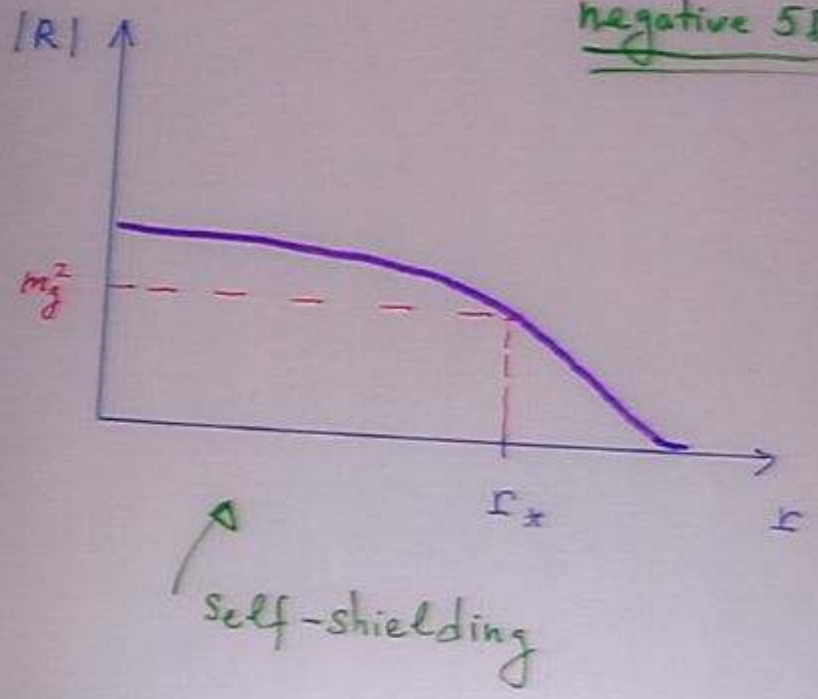
self-shielding

Exact result (G.G. Iglesias)

$$-2\Phi = \frac{r_n}{r} \mp m_g^2 r^2 g_{\pm}(r)$$

Self-accelerated branch

negative 5D mass ??



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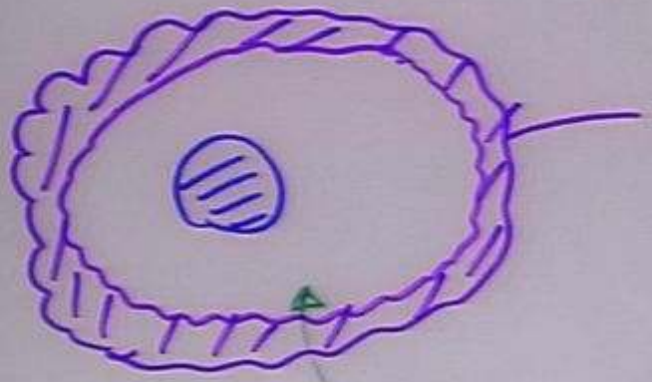


$$\Delta M \sim M_P^2 \int_* R \, d^3x \sim M \, !$$

Additional perihelion precession

$$r \ll r_*$$

$$-2\Phi = \frac{r_s}{r} - \alpha m_c^2 r^2 \left(\frac{r_*}{r}\right)^{3/2-\beta} =$$

$$=$$

$\alpha = \pm 0.84 \qquad \beta = 0.04$

(Dvali, Gruzinov, Zaldarriaga)

(Lue, Stackman)

(G.G., Iglesias)

Domain Walls in GR

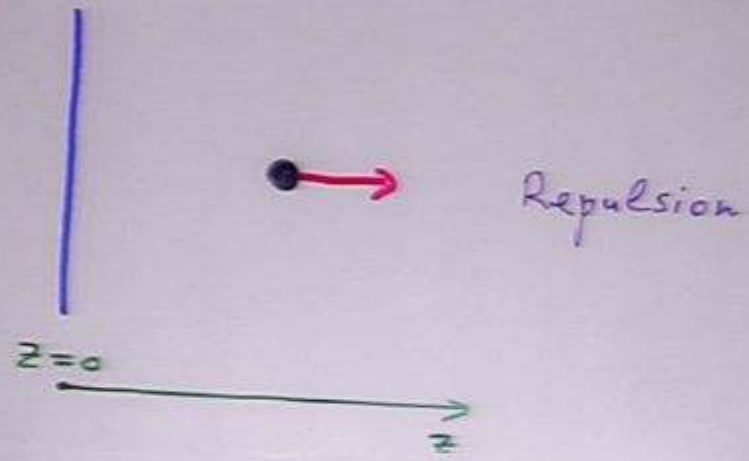
$$G_{\mu\nu} = \sigma\, \delta(z)\, \mathrm{diag}(1, -1, -1, 0)$$

σ: tension

$$ds^2 = (1 - H|z|)^2 \left(-dt^2 + e^{2Ht}(dx^2 + dy^2)\right) + dz^2$$

$$H \sim \sigma / M_p^2$$

Vilenkin '84
Ipsec, Sikivie '84



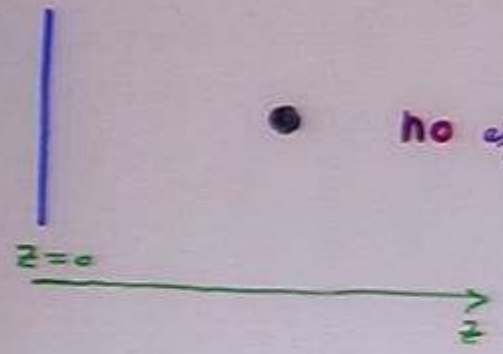
Domain Walls in MG : $\sigma < \pi m_c M_p^2$

(Dvali, G.G., Pujolás, Rahman)

$$G_{\mu\nu} = T^{DW}_{\mu\nu} - m_c(K_{\mu\nu} - g_{\mu\nu}K)$$

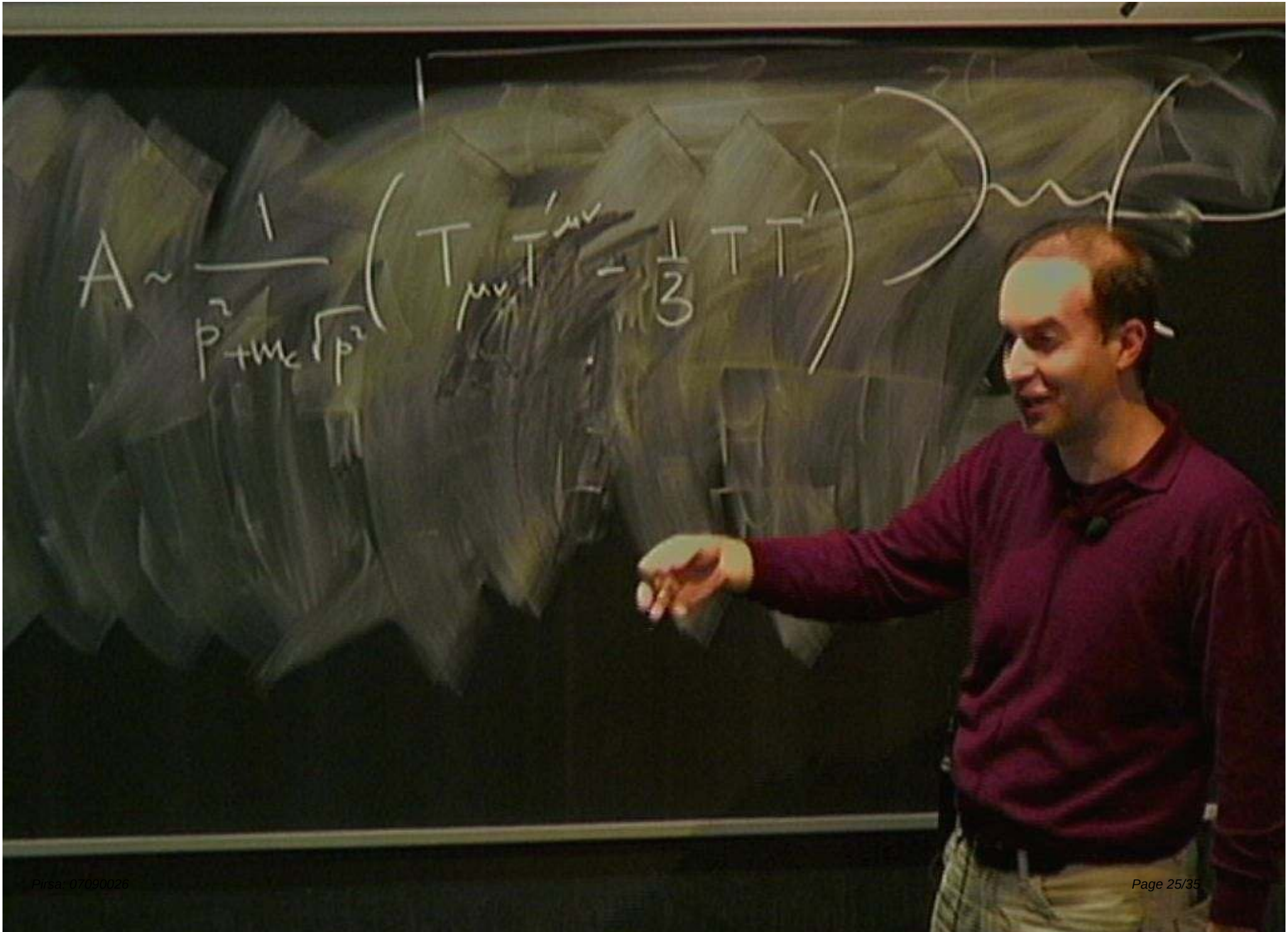
exact cancel.

$$ds^2_{4D} = -dt^2 + dx^2 + dy^2 + dz^2$$



$$ds^2_{5D} = -dt^2 + dx^2 + dy^2 + d\rho^2 + \rho^2 d\theta^2$$

$$0 \le \theta < 2\left(\pi - \frac{\sigma}{m_c M_p^2}\right)$$


$A \sim \frac{1}{p^2 + m_c\sqrt{p^2}} \left(T_{\mu\nu} T'^{\mu\nu} - \frac{1}{3} T T' \right)$

Brane Induced Gravity

(Dvali, G.G.)
(G.G., Shifman)

$$\frac{M_P^2}{2}\int d^4x\left(\sqrt{g_4}\,R_4\,a + \sqrt{g_D}\,R_D\,b\right)$$

↳ regularized

$$+\ \frac{M_*^{D-2}}{2}\int d^Dx\,\sqrt{g_D}\,R_D$$

$\equiv$ # of space-time dimensions.

$+b=1$

- $b=0$ ghosts (Dubovsky & Rubakov)
- $b>b_c(D)$ no ghosts (G.G. & Shifman) Softly massive gravity
- $b=b_c$ (this talk)

Strong coupling, no ghosts. $b_c=1/3$ for $D=6$

One graviton exchange

(G.G. Shifman)

$$A \sim \frac{1}{p^2 - m_c^2 f(p^2)} \left[(T_A \cdot T_B) - \frac{[T_A T_B]}{2} \frac{(2b-1)p^2 - m_c^2 f}{(3b-1)p^2 - 2m_c^2 f} \right]$$

↑ 6D for simplicity

Non-linearities determined by: $\frac{P_\mu P_\nu}{(3b-1)p^2 - 2m_c^2 f}$

- $b < 1/3$: ghost; no strong coupling except ($p \sim m_c$)
- $b > 1/3$: no strong coupling. Light bendy?
- $b = 1/3$ strong coupling, like in 5D (similar) !

$$\text{Res } A\Big|_{p^2 = m_c^2 f} \sim (T_A T_B) - \frac{1}{3}[T_A T_B]$$

Massive gravity graph!

$D \geq 6$ Brane Induced Gravity

(Dvali, G.G.)
(G.G., Shifman)

$$S = \frac{M_P^2}{2}\int d^4x\left(\sqrt{g_4}\,R_4\,a + \sqrt{g_D}\,R_D\,b\right)$$

↳ regularized

$$+\ \frac{M_*^{D-2}}{2}\int d^Dx\,\sqrt{g_D}\,R_D$$

$D \equiv$ # of space-time dimensions.

$a + b = 1$

- $b = 0$ ghosts (Dubovsky & Rubakov)
- $b > b_c(D)$ no ghosts
  Softly massive gravity (G.G. & Shifman)
- $b = b_c$ (this talk)
  Strong coupling

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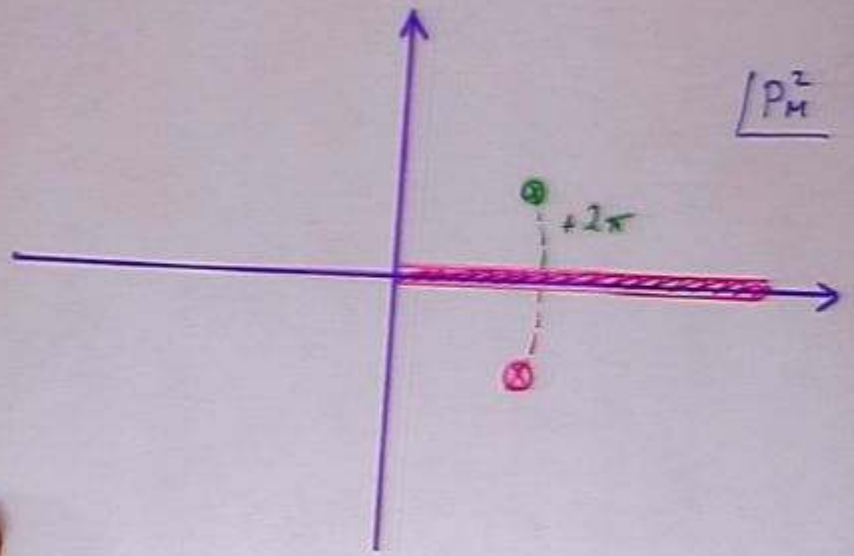
Massive gravity good!

(G.G., M. Shifman)

$$(\text{Mass})^2 \sim \frac{m_c^2}{\ln \frac{\Lambda^2}{m_c^2}}$$

$$\text{Arg}\left(P_M^2\right)_1 \sim -\frac{\pi}{\ln \frac{\Lambda^2}{m_c^2}} \qquad \text{Arg}\left(P_M^2\right)_2 \sim 2\pi + \frac{\pi}{\ln \Lambda^2/m_c^2}$$

$$(\text{Width}) \ll (\text{Mass})$$



$b \neq 0$ important for solution.

$$ds_6^2 = \eta_{\mu\nu} dx^\mu dx^\nu + d\rho^2 + (1-\alpha)^2 \rho^2 d\varphi^2$$

↑ deficit angle (Sundrum)

In our case:

$$\alpha \sim \frac{T}{b\frac{M_p^2}{\Delta^2} + M_*^4}$$

Δ - brane width $\quad \Delta \gtrsim 1/M_p$

T - brane tension $\quad T \ll (M_p)^4$

due to e.g. SUSY $\quad \Rightarrow$

$\Rightarrow \quad \underline{\alpha \ll 1}$

Could solve CCP !

Unlike massive gravity nonlinear diagrams do not blow up as $m_c \to 0$ unless $b = 1/3$.

$$\frac{P_\mu P_\nu}{m_g^2} \Rightarrow \frac{P_\mu P_\nu}{2m_c^2 f(p^2) + p^2(1-3b)}$$

$m_c \to 0$ limit is smooth. $b \neq 1/3$

Softly massive gravity.

$b = 1/3$ Strongly coupled gravity. !

4D Massive Gravity

Pauli-Fierz Action (1935)

$$S_m = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{g}\,R$$

$$+\frac{M_{Pl}^2 m_g^2}{2}\int d^4x\sqrt{g}\left(h_{\mu\nu}^2-(h_2^1)^2\right)$$

$$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$$

- No reparametrization invariance
- No unique nonlinear completion

The ADM formalism:

$g_{\mu\nu} \rightarrow g_{ij} = \gamma_{ij} \; ; \; g_{0j} = N_j \; ; \; N = \frac{1}{\sqrt{-g^{00}}}$

Hamiltonian

$$\boxed{H_{EH} = \sqrt{\gamma}\left(N R^0 + N_j R^j\right)}$$

$$R^0 \equiv -R^3 + \gamma^{-1}\left(\pi_{ij}^2 - \tfrac{1}{2}\pi^2\right)$$

$$R^j \equiv -2\nabla_i\left(\gamma^{-1/2}\pi^{ij}\right)$$

N & N_j are Lagrange multipliers.

$\Rightarrow$ 4 constraints

ADM formalism:

a natural tool for Lorentz non-invariant massive gravity (G.G, Gcisa)

$g_{\mu\nu} \longrightarrow \gamma_{ij} \quad N_j \quad N \qquad j = 1, 2, 3$

$$\mathcal{L} = M_p^2 \Big[\sqrt{\gamma} N \left(R_3 + K_{ij}^2 - K^2 \right) + m_g^2 \sqrt{\gamma} N f(h_{ij}) + m_g^2 \sqrt{\gamma} g(N_j) \Big]$$

- Two massive polarizations + long-range int. no BD problem
- Five massive polarizations + no BD problem