Title: Degravitating The Vacuum

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Abstract:

Degravitating the Vacuum

Justin Khoury
(Perimeter Institute)

Based on: G. Dvali, S. Hofmann & JK, hep-th/0703027
C. de Rham, G. Dvali, S. Hofmann, JK, O. Pujolas, M. Redi & A. Tolley, to appear

Earlier work: Dvali, Gabadadze & Shifman, hep-th/0202174
Arkani-Hamed, Dimopoulos, Dvali & Gabadadze, hep-th/0209227

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Avoid long receptions with Russian colleagues...

E&M analogue: de-electrification

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$\Longrightarrow$ E-field grows unbounded.

2 possible solns: - give A_μ a mass: $m^2 A_\mu A^\mu$

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Restore gauge inv. with Goldstone-Stuckelberg field:

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with $\tilde{A}_\mu \to \tilde{A}_\mu + \partial_\mu \omega\,; \quad \phi \to \phi - m\omega$

Thus A_μ is gauge-invariant (physical observable)

In terms of helicity-1 and helicity-0 fields:

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Hence, adding mass <=> filtering

Adding mass solves "C.C. problem" in E&M:

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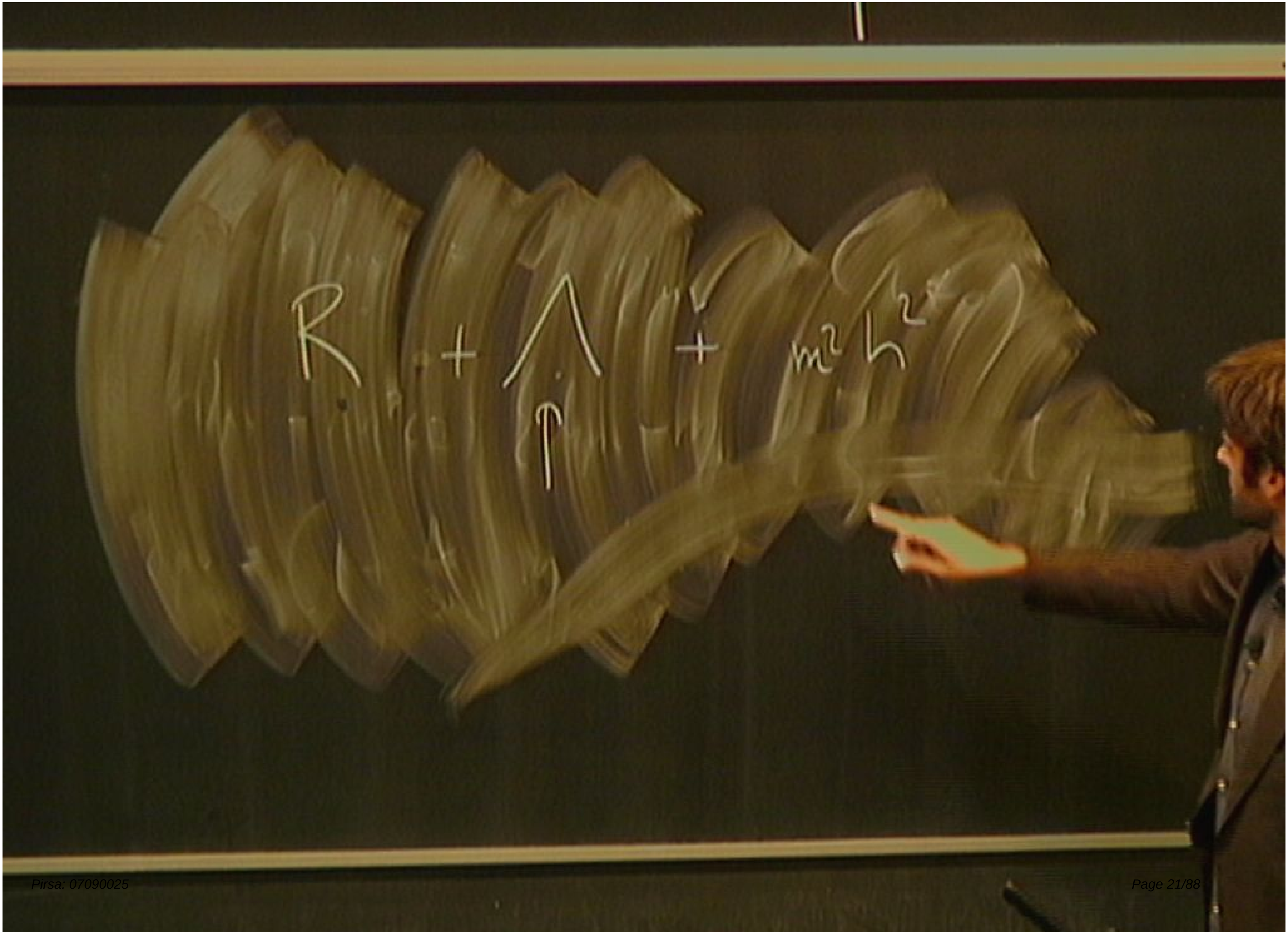
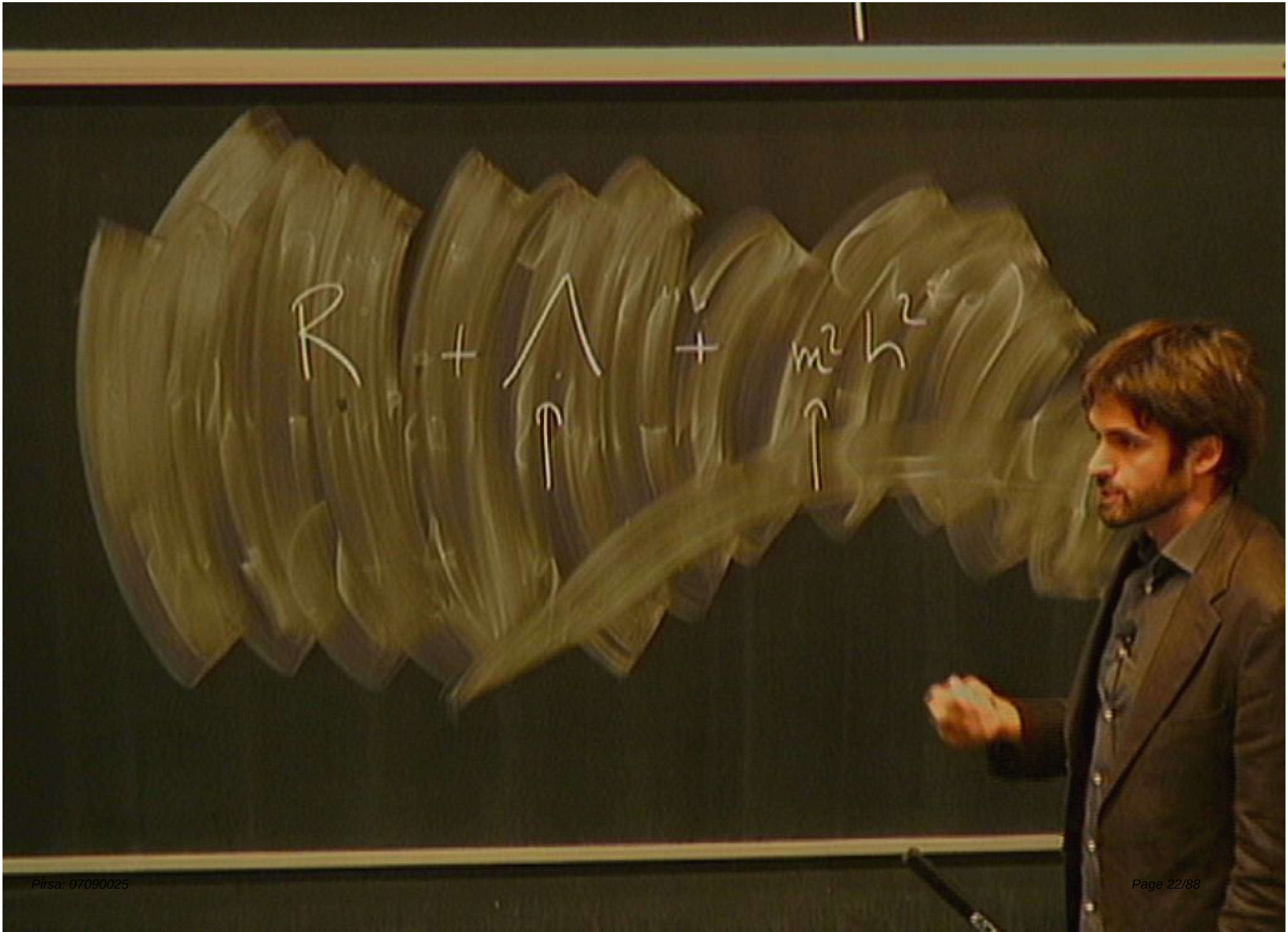
With mass term, vacuum becomes superconducting, and uniform sources get *screened*.

Degravitation of c.c. is analogous to screening of uniform charge density in the Higgs vacuum in Maxwell theory


R + Λ + m² h²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$R + \Lambda + m^2 h^2$

Degravitation <=> Massive or Resonance Graviton

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The physical metric $h_{\mu\nu}$ is gauge-inv. => observable

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$\Longrightarrow$ Extra degrees of freedom:

2 helicity-2 + 2 helicity-1 + 1 helicity-0

Strong coupling & r$_*$-effect

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \frac{1}{6}\eta_{\mu\nu}\chi + \frac{1}{3}\frac{\partial_\mu\partial_\nu}{m^2(\Box)}\chi$$

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helicity-0 (couples to $T^\mu{}_\mu$)

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$$\xrightarrow{m \to 0} \quad \frac{T'_{\mu\nu}T^{\mu\nu} - \frac{1}{3}TT'}{\Box}$$

(vDVZ discontinuity)

Near source, however, non-linear interactions in χ are important

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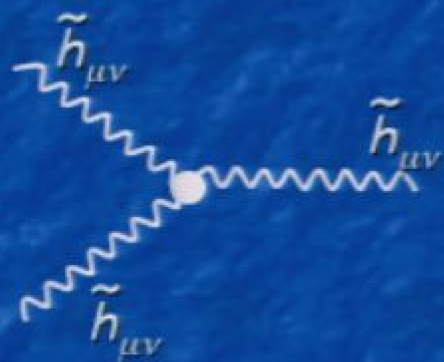
- For $\tilde{h}$ 

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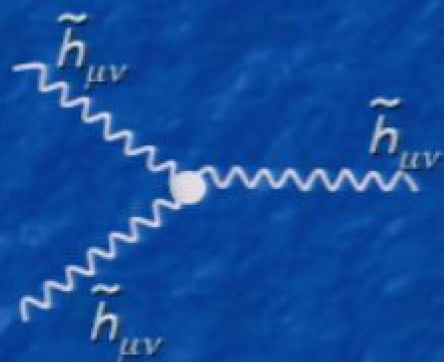
- For $\tilde{h}$ 

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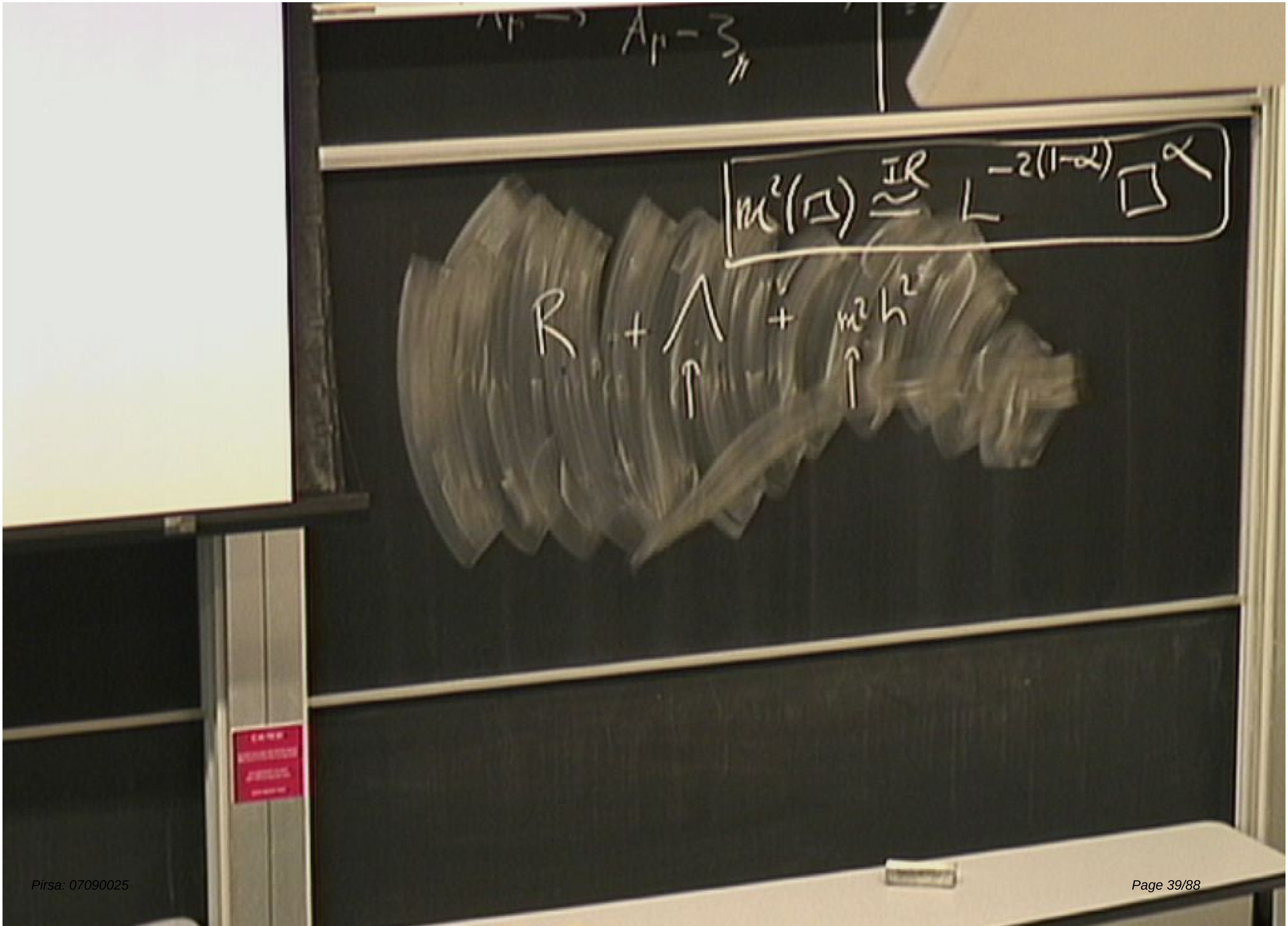
m²(□) ≃(IR) L^(−2(1−α)) □^α
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$$m^2(\Box) \overset{IR}{\simeq} L^{-2(1-\alpha)}\,\Box^{\alpha}$$

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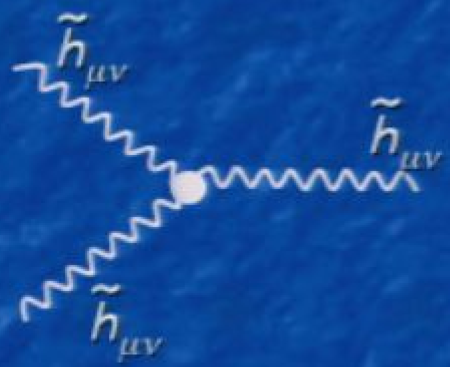
- For $\tilde{h}$ 

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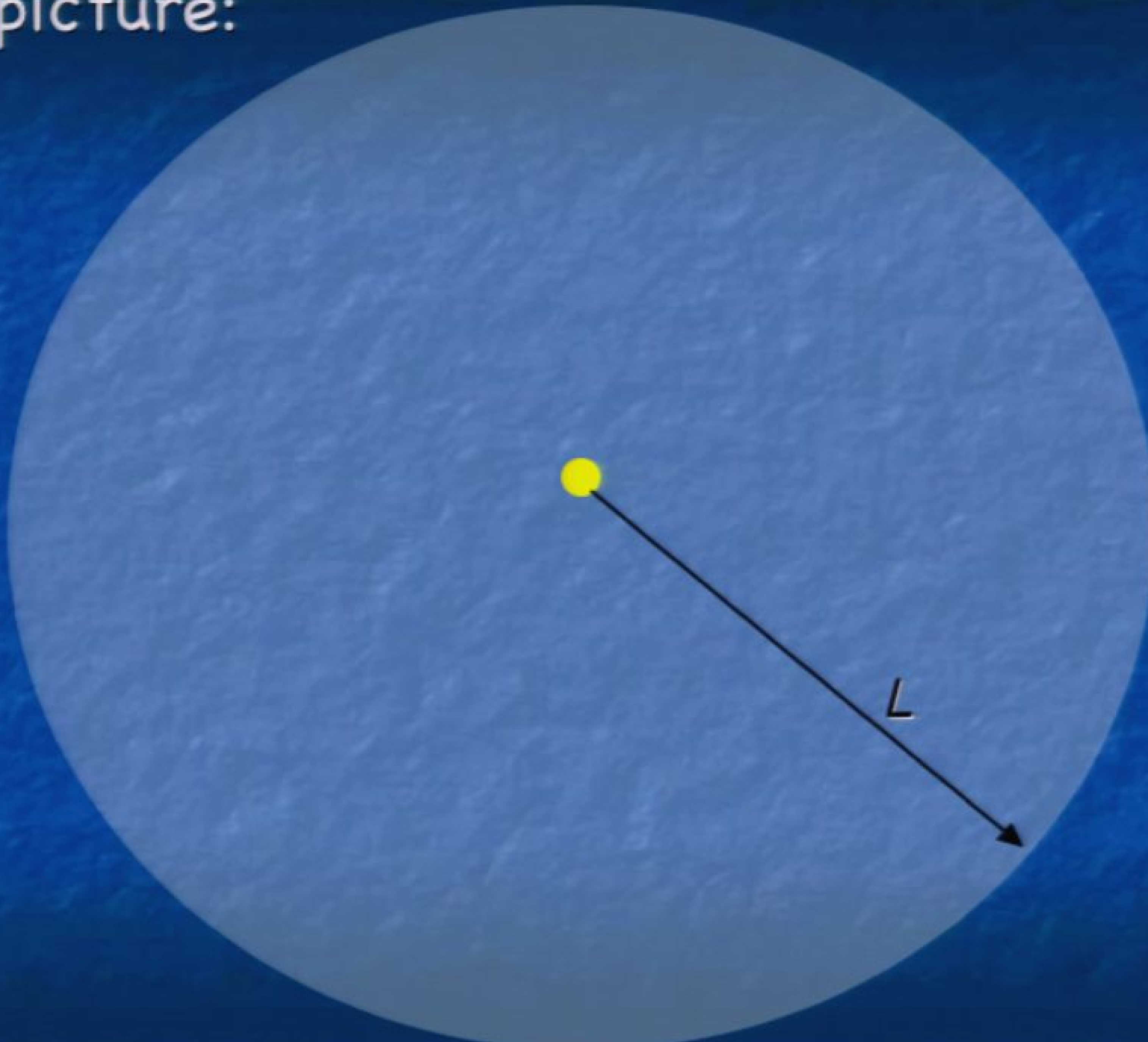

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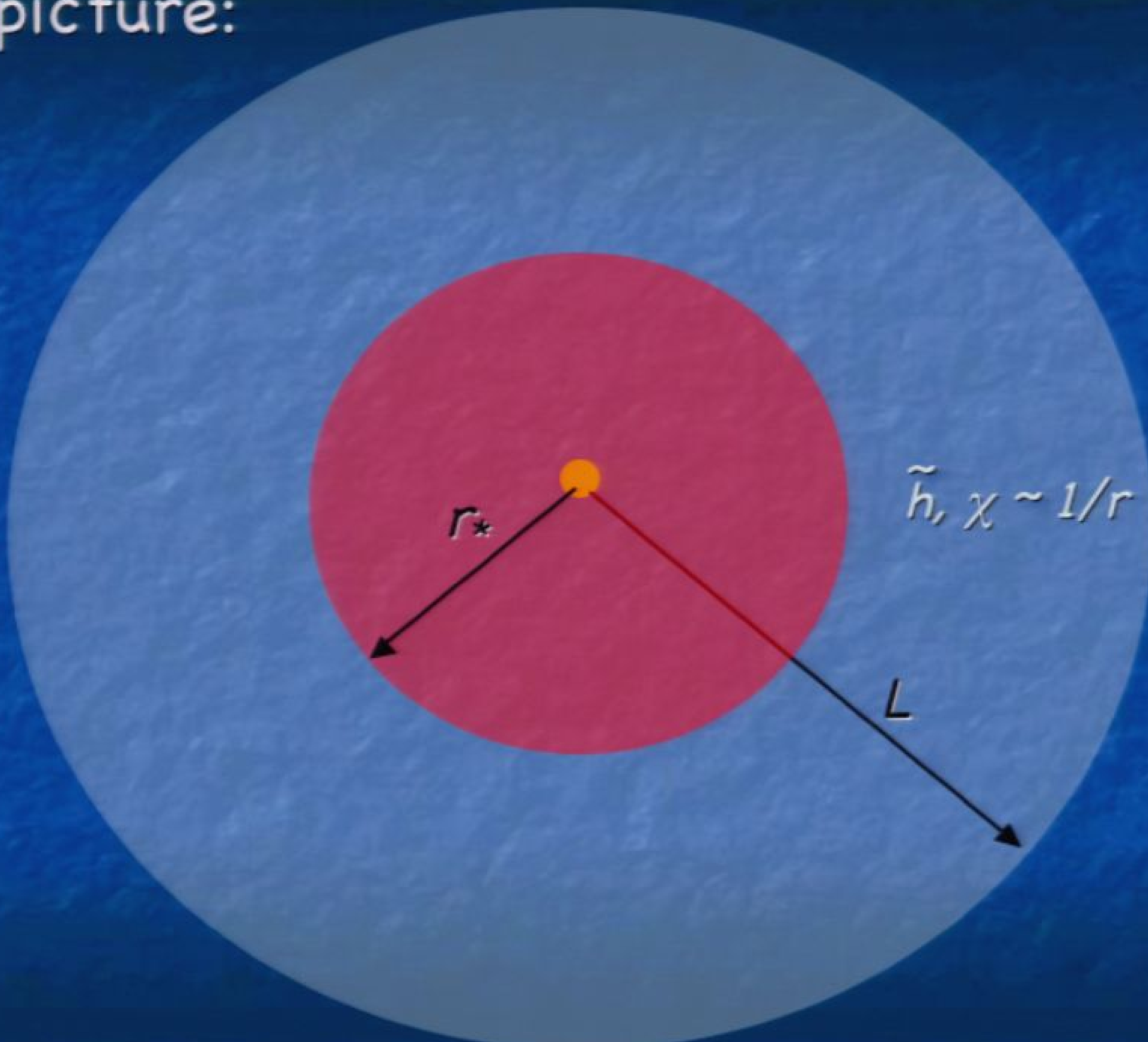

In other words...

- For $r \ll r_*$, χ is non-linear and decouples

$$\chi(r \ll r_\star) \sim \frac{r_{\rm sch}}{r_\star}\left(\frac{r}{r_\star}\right)^{\frac{3}{2}-2\alpha}$$

$\Longrightarrow$ have GR + small $O(r/r_*)$ corrections

Testable in LLR: $\alpha > 1/2$ ruled out; $\alpha < 1/2$ beyond planned accuracy.

Dvali, Gabadadze & Zaldarriaga, hep-ph/0212069

- For $r \gg r_*$, χ is linear and goes like $1/r$

$\Longrightarrow$ have scalar-tensor theory

Implications of r_*-effect for degravitation

- For $H^{-1} \ll L$, then universe is within its own r_*, which means χ is suppressed, and thus

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \frac{1}{6}\eta_{\mu\nu}\chi + \ldots \approx \tilde{h}_{\mu\nu}$$

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- Once $H^{-1} \sim L$, then χ no longer decouples, and presumably *degravitation stops*

(leaves effective c.c. of order L^{-2}?)

Degravitation in the decoupling limit:

$L \to \infty \, ; \;\; M_{\rm Pl} \to \infty \, ,$ with $\Lambda_{\rm strong} \equiv \left(L^{-4(1-\alpha)} M_{\rm Pl} \right)^{\frac{1}{1+4(1-\alpha)}}$ fixed

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$$(\mathcal{E}\tilde{h})_{\mu\nu} = -\Lambda\eta_{\mu\nu} \qquad \Box\chi + \frac{1}{\Lambda_{\rm strong}^{5-4\alpha}}\Box^{1-2\alpha}(\Box\chi)^2 + \ldots = -4\Lambda$$

Ignoring non-linear terms for a moment, soln is

$$\tilde{h}_{\mu\nu} = -\frac{\Lambda}{12}\eta_{\mu\nu}x_\alpha x^\alpha\,; \quad \chi = \frac{\Lambda}{2}x_\alpha x^\alpha$$

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Summary thus far:

$$m^2(\Box) = L^{2(1-\alpha)}\Box^\alpha$$

- Absence of ghosts: $0 \leq \alpha < 1$
- Non-linear degrav. (in decoupling limit): $\alpha < \frac{1}{2}$

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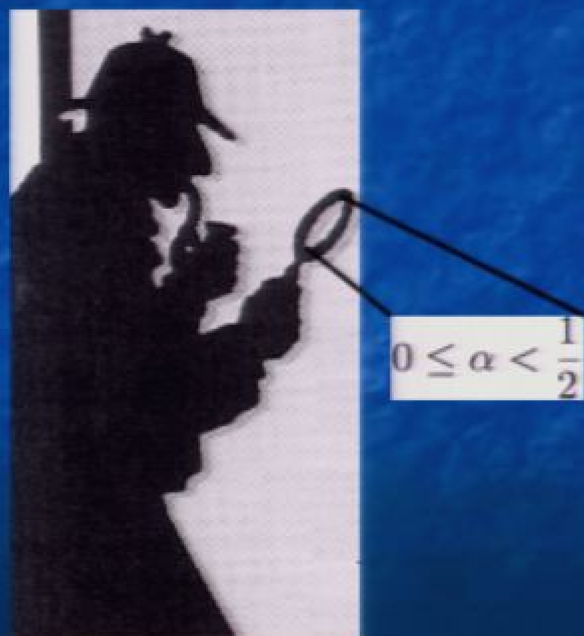

Realizing degravitation in brane-world:

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- Because extra dimension is infinite, no massless graviton. Rather, graviton is a resonance with width $L^{-1} = M_5^{\,3}/M_4^{\,2}$.
- Linearized eom has degravitation form with $\alpha = 1/2$

$$(\mathcal{E}h)_{\mu\nu} + L^{-1}\sqrt{-\Box}(h_{\mu\nu} - \eta_{\mu\nu}h) = T_{\mu\nu}$$

DGP model (cont'd)

- However, at non-linear level, degravitation fails.
- Israel junction condition fixes (local) Friedmann eqn on brane:

$$H^2 = \left(\sqrt{\frac{\rho}{3M_4^2} + \frac{1}{4L^2}} \pm \frac{1}{2L} \right)^2$$

Deffayet, hep-th/0010186

Consistent with earlier conclusion that $\alpha = 1/2$ is borderline.

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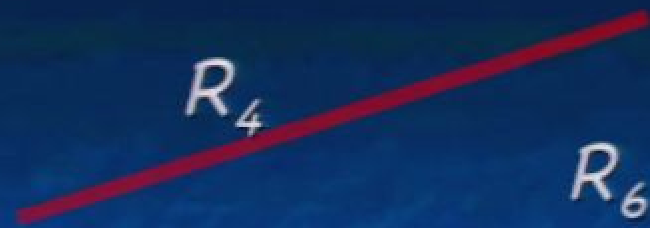
Hence, $$\frac{1}{L^{-2(1-\alpha)}\Box^\alpha} \approx \lim_{\Box \to 0} \int_0 ds\, \frac{s^{N/2-1}}{\Box + s}$$

- Limit is finite for $N>2$ => $\alpha = 0$.
- For $N=2$, get log divergence => $\alpha \approx 0$. Q.E.D.

Codimension-2 DGP:



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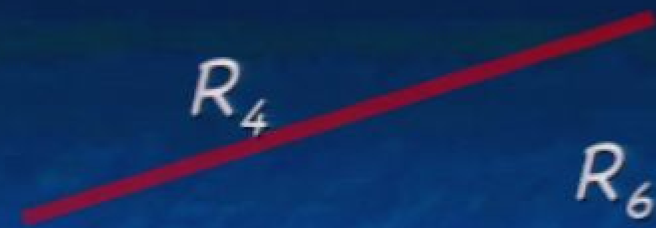

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- Brane-to-brane propagator diverges in thin-brane limit:

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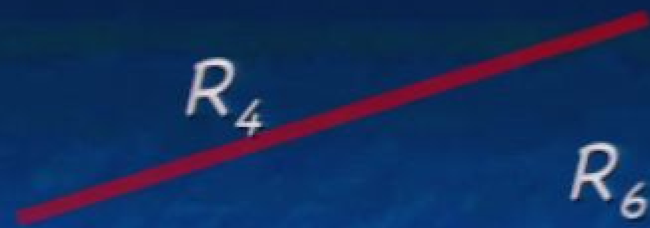

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- Naïve DGP action...

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...leads to a ghost. (More on this later.)

Gabadadze & Shifman, hep-th/0312289
Dubovsky and Rubakov, hep-th/0212222

Cascading DGP:

C. de Rham, G. Dvali, S. Hofmann, JK,
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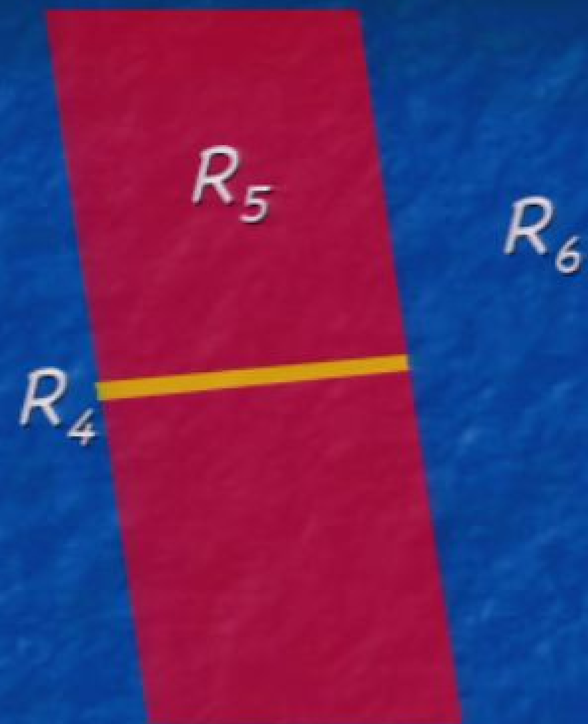

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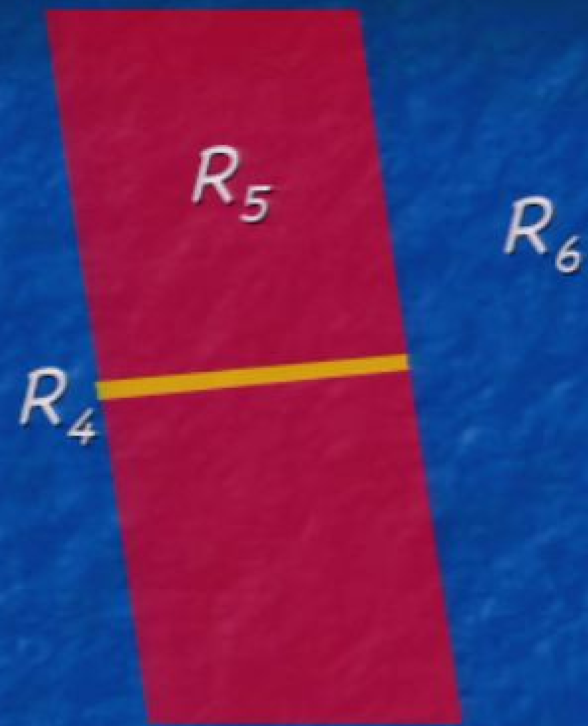

- Brane-to-brane propagator is completely *regular* even in thin-brane limit
- When carefully treated, model does not suffer from ghost-like instabilities

Brane-to-brane propagator is *regular* :

$$G(p) \sim \frac{1}{\sqrt{1 - L_6^2 p^2}} \tanh^{-1}\left(\sqrt{\frac{1 - L_6 p}{1 + L_6 p}}\right)$$

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- Compare with pure cod-2 case: $G(p) \sim \int_0^{\Lambda} \frac{d\omega\, \omega}{p^2 + \omega^2} \sim \log\left(\frac{\Lambda}{p}\right)$

$\Longrightarrow$ finite L_6 plays the role of regulator here!

Ghost stories:

- The naïve action...

$$S = \frac{M_6^4}{2}\int d^6x\sqrt{-g_6}R_6 + \frac{M_5^3}{2}\int d^5x\sqrt{-g_5}R_5 + \frac{M_4^2}{2}\int d^4x\sqrt{-g_4}R_4$$

leads to the exchange amplitude

$$\mathcal{A} \sim \frac{1}{\Box + g(\Box)}\left(T'_{\mu\nu}T^{\mu\nu} - \frac{1}{3}TT'\right) + f(\Box)TT'$$

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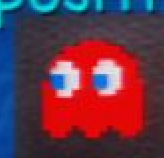
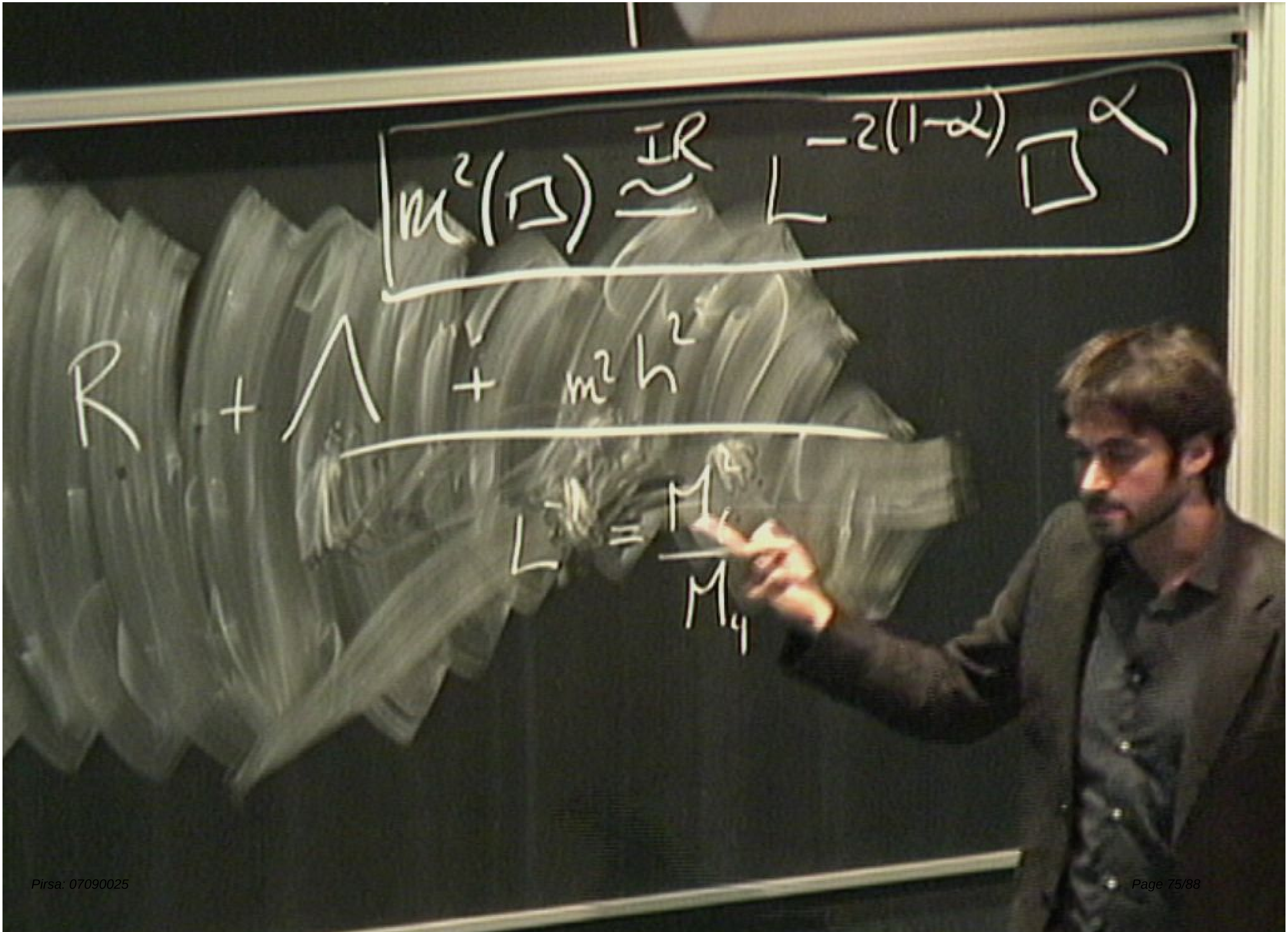
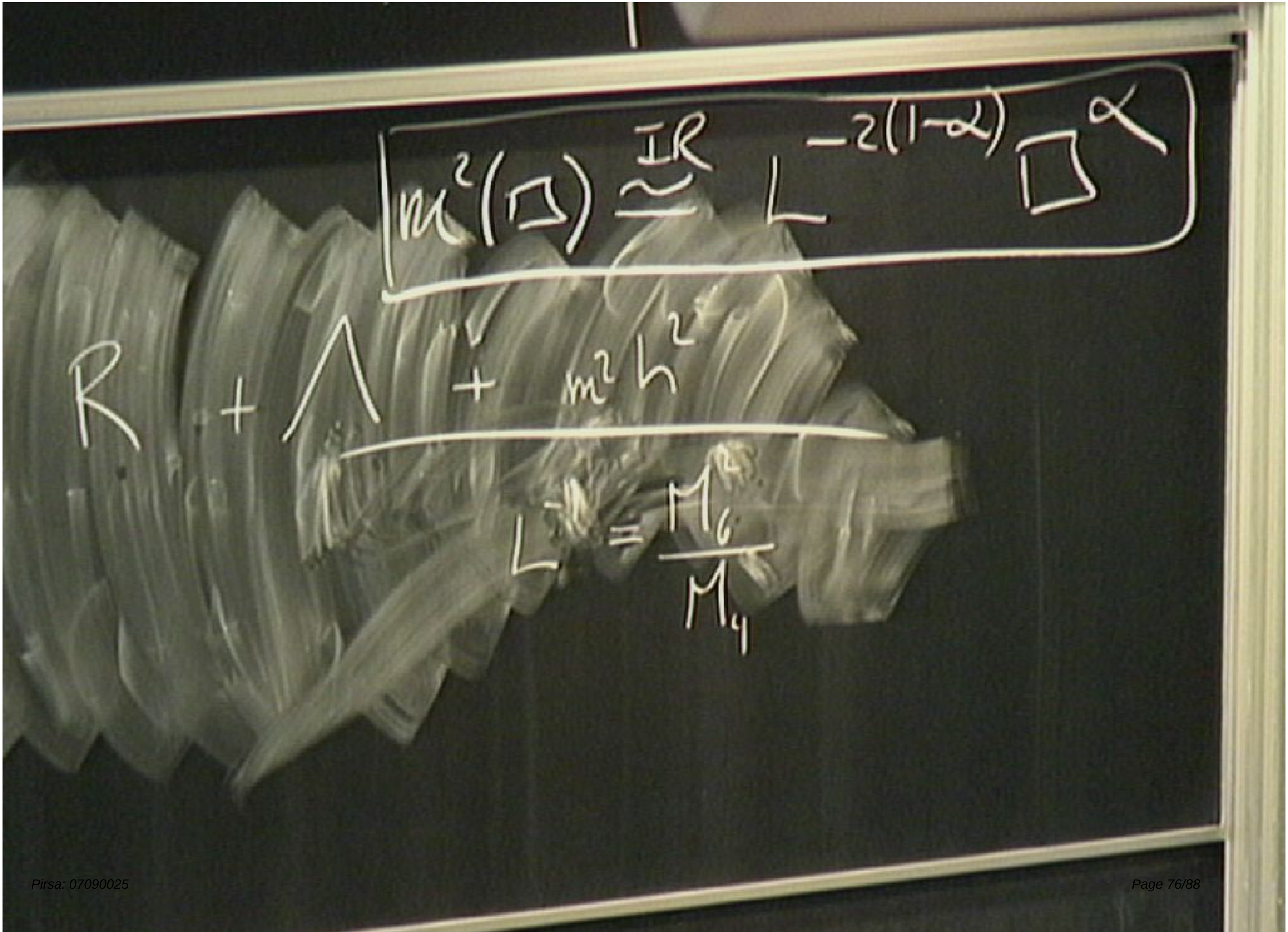
where f goes through zero (i.e. no positive-definite spectral representation) $\Longrightarrow$ 


m²(□) ≃IR L^{-2(1-α)} □^α
R + Λ + m²h²
L⁻¹ = M_5²/M_4


$m^2(\Box) \overset{IR}{\simeq} L^{-2(1-\alpha)} \Box^{\alpha}$
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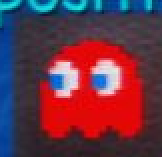
where f goes through zero (i.e. no positive-definite spectral representation) $\Longrightarrow$ 

- Moreover the trace of the metric still diverges on the brane $\Longrightarrow$ must still regulate the cod-2 brane.

Curing the ghost

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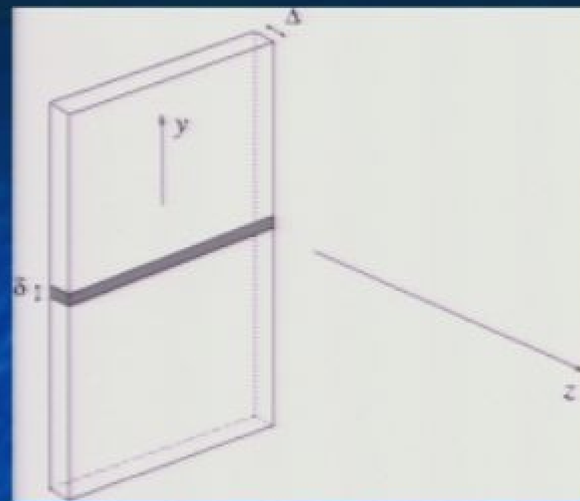

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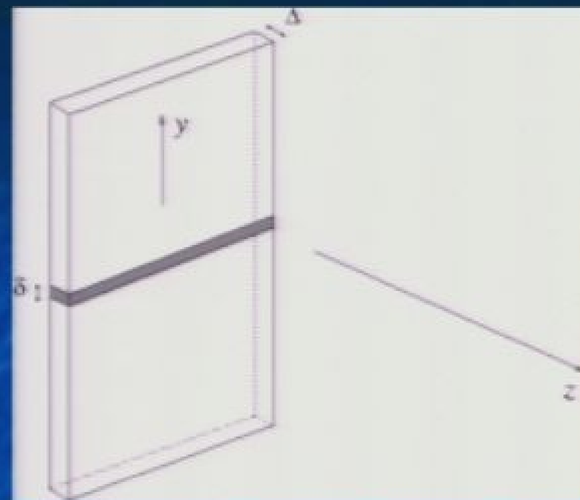

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Kolanovic, Porrati & Rombouts, hep-th/0304148

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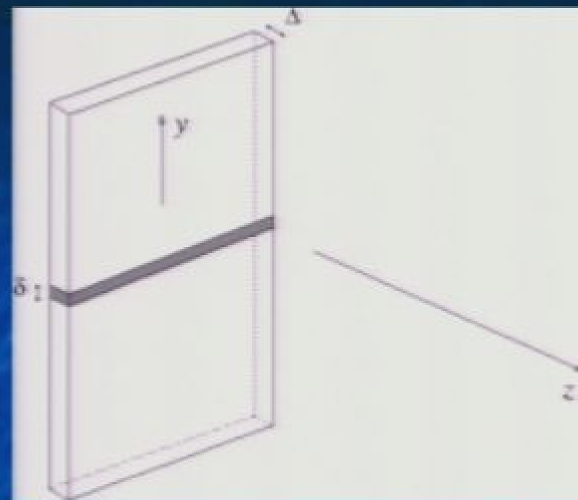

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$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + e^{2\psi}dy^2 + e^{2\phi}dz^2$$

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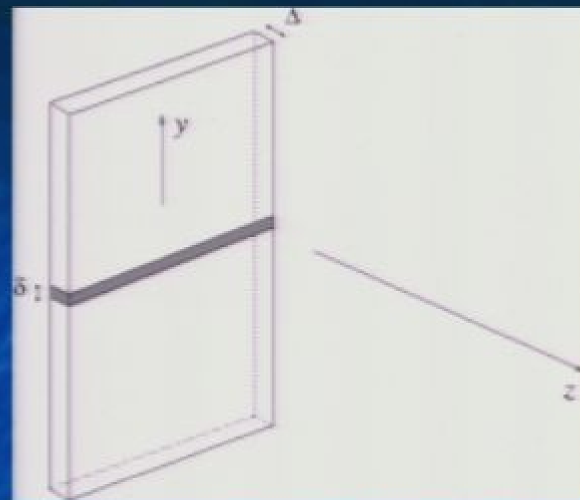

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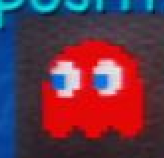
  where f goes through zero (i.e. no positive-definite spectral representation) $\Longrightarrow$ 

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6. Furthermore, trace of metric is now regular, hence thin-brane limit is well-defined.

Work in progress...

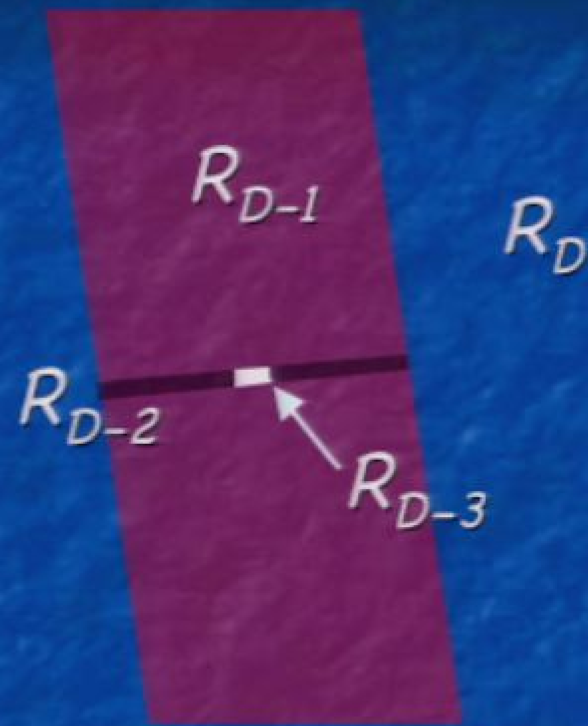
- To get correct tensor structure, scalars ϕ and ψ must get *strongly coupled* (r_*–effect).

$$\Longrightarrow \quad \text{Expect } 1/4 \to 1/3 \to 1/2$$

- Study the cosmology of cascading DGP:

  Is degravitation realized?

Cascading from arbitrary codimension:



...all the way down to R_4 on cod-N brane

- Propagator is *regular* & has tensor structure of graviton in *D* dimns (=> ghost-free).
- For codimension N>2, get behavior of massive gravity (α = 0) at long wavelengths.