Title: Generating stationary axisymmetric solutions of 5D gravity.

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Abstract: I will discuss a solution generating technique that allows to generate stationary axisymmetric solutions of five-dimensional gravity, starting from static ones. This technique can be used to add angular momentum to static configurations. It can also be used to add KK-monopole charge to asymptotically flat five-dimensional solutions, thus generating geometries that interpolate between five-dimensional and four-dimensional solutions.

Generating stationary axisymmetric solutions of 5D gravity

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PI, September 18, 2007

Based on work with J. Ford, A. Peet and A. Saxena

S.G. and A. Saxena, arXiv:0705.4484 [hep-th];
J. Ford, S.G., A. Peet and A. Saxena, arXiv:0708.3823 [hep-th].

Outline

1. Motivation
2. Review of known results
   - Static solutions
   - Stationary Solutions
   - SL(3,$\mathbb{R}$) invariance
3. Generating angular momentum
4. Generating KK-monopole charge
   - Applications
5. Summary

Motivation

- Finding exact solutions to Einstein equations is hard.
- Most of the advances in our understanding of gravity hinge upon the knowledge of exact solutions.
- In $D > 4$ there is a rich structure of black hole phases, and many new solutions to be discovered. An important testing ground for a theory of quantum gravity will be to explain this phase structure.
- String theory has already been successful in explaining a large part of this structure: microscopic derivation of black hole entropy, Hawking radiation, Gregory-Laflamme instability for black holes in KK spaces.

Why we focus on the case of $D = 5$?

- It is non-trivial. There are black holes with different topologies:
  Myers-Perry Black Hole (topology S^3);
  Black-Ring (topology $S^2 \times S^1$).
- It is close to being fully solved.
  In contrast to the case $D > 5$, where much less is known yet:
  Emparan, Harmark, Niarchos, Obers, Rodriguez.
- For simplicity, we consider vacuum solutions:

$$R_{\mu\nu} = 0$$

Review of known results

Emparan, Reall; Harmark

- Look at solutions which are asymptotically $\mathbb{R}^{4,1}$.
  The rotation group $SO(4)$ has 2 commuting generators
  $\Longrightarrow$ 2 axial symmetries: $\frac{\partial}{\partial\phi}$, $\frac{\partial}{\partial\psi}$.
  We assume that these symmetries are preserved by the full geometry.
  It has been shown (Hollands, Ishibashi, Wald) that b.h. in 5D need only 1 axial symmetry, but no solution with only 1 axial symmetry is known.
- Look at stationary solutions $\Longrightarrow$ $\frac{\partial}{\partial t}$ is Killing.
- In total, 3 commuting Killing vectors: $\frac{\partial}{\partial y^I} = \{\frac{\partial}{\partial t}, \frac{\partial}{\partial\phi}, \frac{\partial}{\partial\psi}\}$, I=0,1,2.
- The 2D subspaces orthogonal to $\frac{\partial}{\partial y^I}$ are integrable (Wald; Emparan,Reall)
  $\Longrightarrow ds^2 = G_{IJ}(x^a)dy^I dy^J + g_{ab}(x^a)dx^a dx^b$, a,b=1,2

- Pick canonical coordinates on the 2D submanifolds.
  Define $r^2 = -\det G_{IJ}$,
  there exists z s.t. $g_{ab}dx^a dx^b = e^{2\nu(r,z)}(dr^2 + \Lambda(r,z)dz^2)$
  $G^{IJ}R_{IJ} = 0 \implies \partial_r \Lambda = 0 \implies \Lambda = \Lambda(z) \implies \Lambda = 1$
  by a redefinition of z.

Canonical form of the metric

$$ds^2 = G_{IJ}dy^I dy^J + e^{2\nu}(dr^2 + dz^2), \quad \det G_{IJ} = -r^2$$

- Define $U = r\partial_r G G^{-1}$, $V = r\partial_z G G^{-1}$

Equations of Motion

$$\partial_r U + \partial_z V = 0 \tag{1}$$

$$\partial_r \nu = -\frac{1}{2r} + \frac{1}{8r}\mathrm{tr}(U^2 - V^2)\,,\ \partial_z \nu = \frac{1}{4r}\mathrm{tr}(UV) \tag{2}$$

Static solutions (Emparan, Reall)

- Eq. (1) simplifies if G_{IJ} is diagonal:

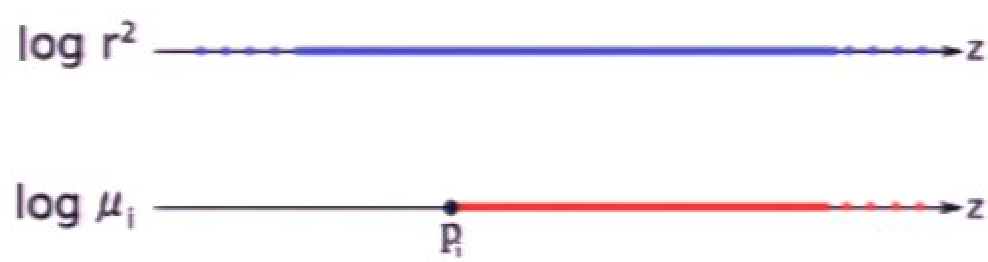
$$G_{IJ} = G_I \delta_{IJ} \Longrightarrow \Box \log G_I = 0, \quad {\scriptstyle I=0,1,2}$$

where $\Box = \partial_r^2 + \partial_z^2 + \frac{\partial_r}{r}$ is the Laplace operator for the metric $ds_3^2 = dr^2 + dz^2 + r^2 d\phi^2$, acting on ϕ-independent functions.

- $\log G_I$ is a harmonic function on $\mathbb{R}^3$, with sources on the $r = 0$ axis. It is a linear combination of:

$$\log r^2 \text{ and } \log \mu_i \text{ with } \mu_i = \sqrt{r^2 + (z - p_i)^2} - (z - p_i)$$

whose sources are

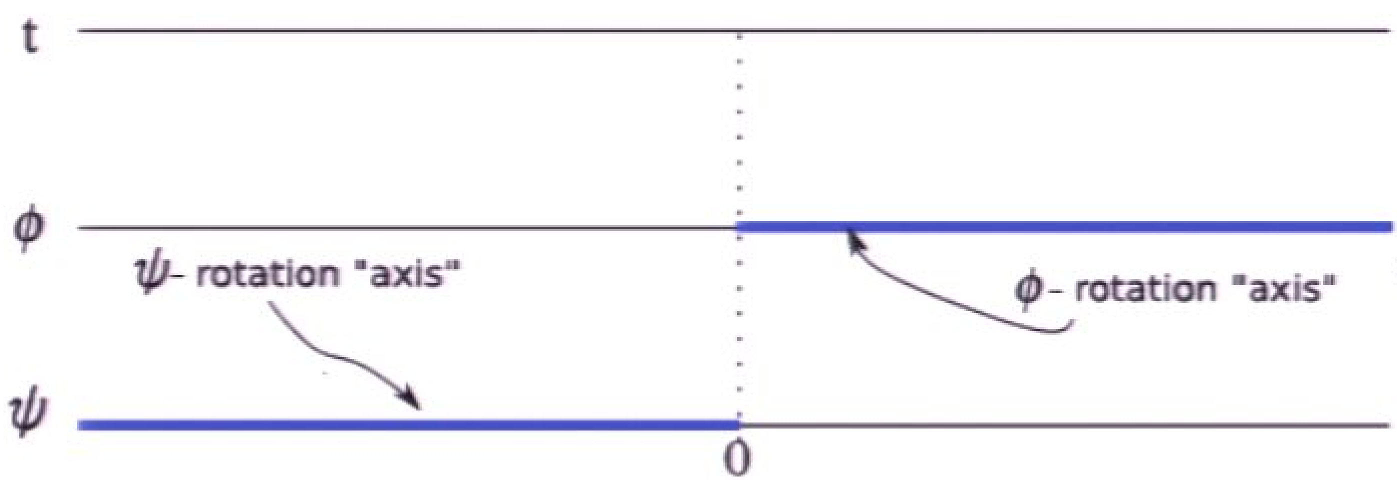


- The condition $\det G = -r^2$ implies that the sources for $\log G_I$ (I=0,1,2) sum up to 1.

Examples

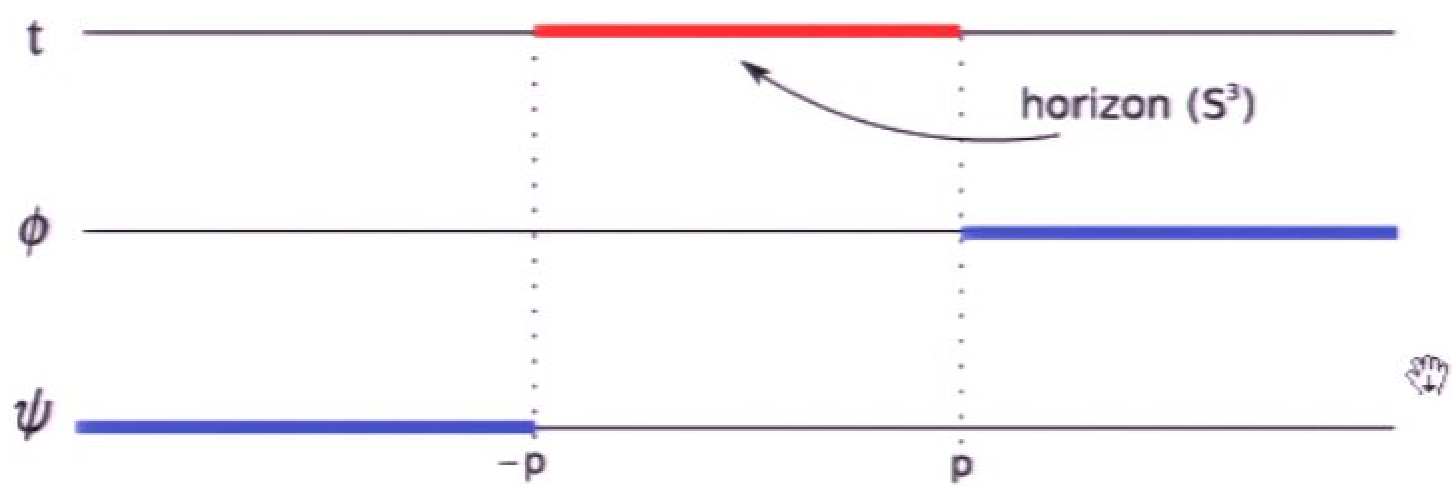
- 5D Minkowski space:

$$\log(-G_0) = 0 \ , \ \log G_1 = \log \mu_0 \ , \ \log G_2 = \log \frac{r^2}{\mu_0}$$



- 5D Schwarzschild Black-Hole:

$$\log(-G_0) = \log\frac{\mu_{-p}}{\mu_p}\ ,\ \log G_1 = \log\mu_p\ ,\ \log G_2 = \log\frac{r^2}{\mu_{-p}}$$



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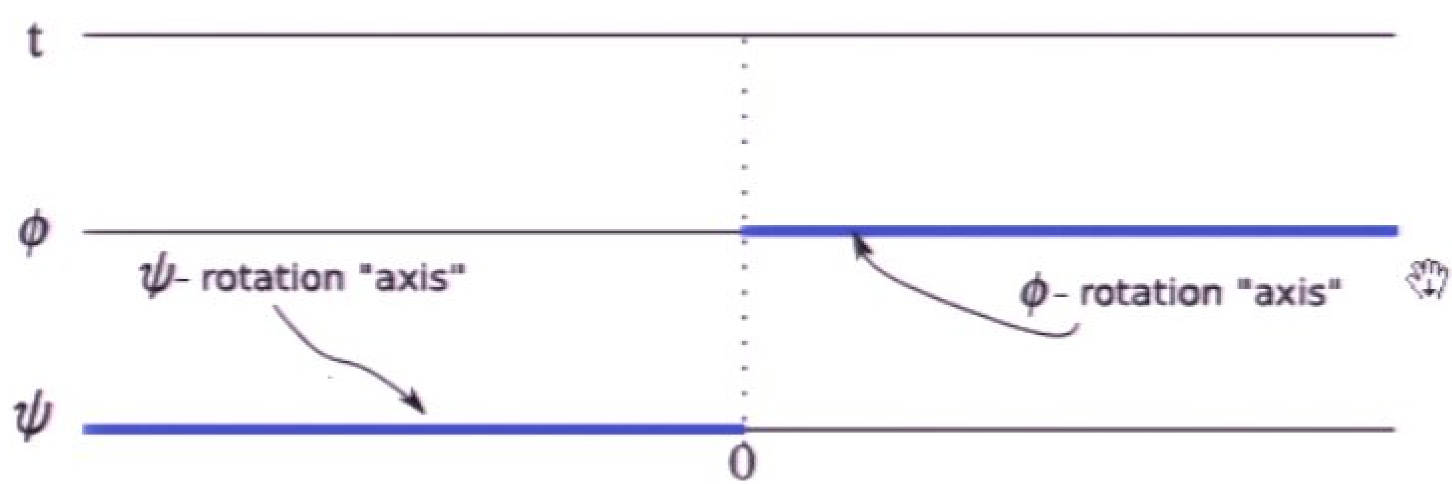

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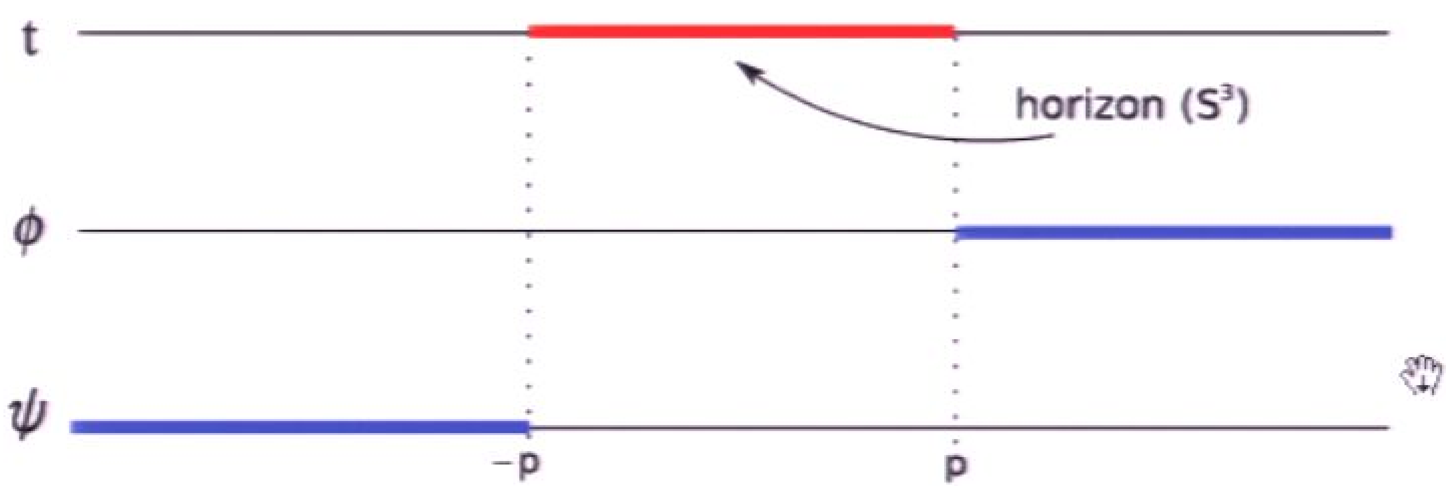
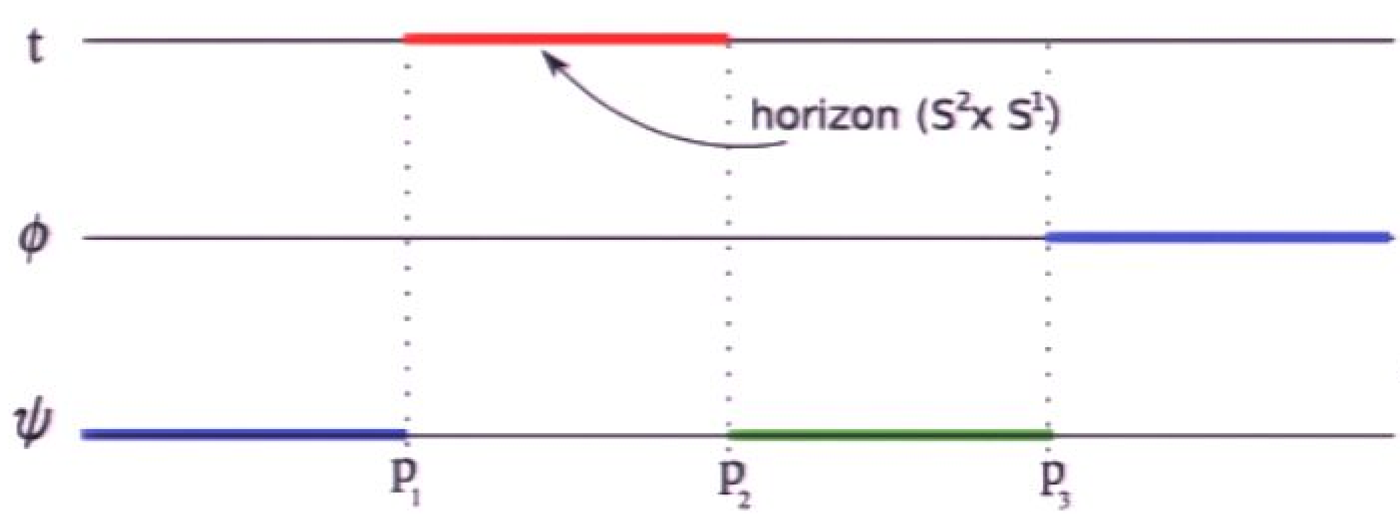


- Static Black-Ring:

$$\log(-G_0) = \log\frac{\mu_{p_1}}{\mu_{p_2}}\ ,\ \log G_1 = \log \mu_{p_3}\ ,\ \log G_2 = \log\frac{r^2}{\mu_{p_1}} + \log\frac{\mu_{p_2}}{\mu_{p_3}}$$



Stationary Solutions

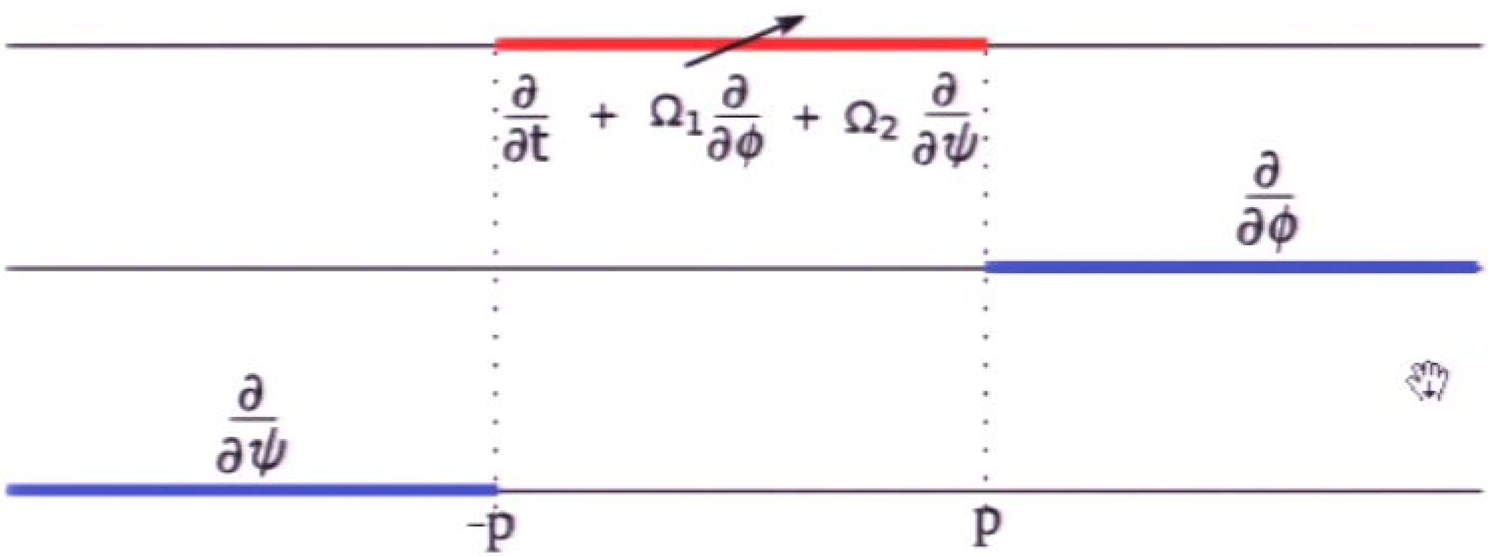
- The concept of rod structure can be generalized to non-static solutions (Harmark).
- $\mathrm{detG} = -r^2$ implies that on the $r = 0$ axis $\ker G \neq \{0\}$.
- For regular solutions, $\dim \ker G = 1$ for *generic* z.
- At isolated points $z = p_i$, one can have $\dim \ker G = 2$.
- Consider $r = 0$ and $p_i < z < p_{i+1}$. There exists v_i s.t. $G v_i = 0$. It can be shown that v_i does not depend on z:

$$v_i = \Omega_i^{(0)} \frac{\partial}{\partial t} + \Omega_i^{(1)} \frac{\partial}{\partial \phi} + \Omega_i^{(2)} \frac{\partial}{\partial \psi}$$

- The rod structure is defined as the collection of data $\{[p_i, p_{i+1}], v_i\}$.

Example

- 5D Myers-Perry Black-Hole:



Uniqueness Theorem (Hollands, Yazandjiev)

- Two stationary, asymptotically $\mathbb{R}^{4,1}$, vacuum solutions with 2 axial symmetries are identical if and only if their rod structures coincide.
- Problem: How to construct the solution with a given rod structure?
- Solve the coupled equations $\partial_r(r\partial_r GG^{-1}) + \partial_z(r\partial_z GG^{-1}) = 0$.
- This problem has been shown to be integrable (Belinsky, Zakharov; Pomeransky).
- In principle, the most general solution can be generated by an Inverse Scattering transformation.
- In practice, most solutions generated by IS have pathologies (singularities at the rod intersections).
- We propose a different solution, based on the SL(3,$\mathbb{R}$) invariance of 5D GR.

SL(3,$\mathbb{R}$) invariance (Maison)

- Consider a stationary solution in 5D, with a space-like Killing vector $\frac{\partial}{\partial \xi^1}$ (and $\xi^0 \equiv t$):

$$ds^2 = \lambda_{ab}(d\xi^a + \omega^a)(d\xi^b + \omega^b) + \frac{1}{\tau} ds_3^2, \ \tau = -\det \lambda_{ab} \ \ (a,b{=}1,2)$$

- The 1-forms on 3D space, ω^a, can be dualized to scalars, V_a:

$$dV_a = -\tau \lambda_{ab} *_3 d\omega^b$$

- The scalars can be organized into a 3×3 uni-modular symmetric matrix χ:

$$\chi = \begin{pmatrix} \lambda_{ab} - \frac{V_a V_b}{\tau} & \frac{V_a}{\tau} \\ \frac{V_b}{\tau} & -\frac{1}{\tau} \end{pmatrix}$$

Equations of Motion

$$d *_3 (\chi^{-1} d\chi) = 0 \ , \ R^{(3)}_{ij} = \frac{1}{4}\mathrm{tr}(\chi^{-1}\partial_i\chi\chi^{-1}\partial_j\chi)$$

- The equations of motion are invariant under

$$\chi \to \chi' = N\chi N^T \ , \ ds_3^2 \to ds_3^2 \quad \text{with} \quad N \in \mathrm{SL}(3, \mathbb{R})$$

- What is the physical significance of the transformation N?
- In general N changes the behavior at infinity. Need to identify the subgroup that preserves the asymptotic structure.

$\mathbb{R}^{3,1} \times S^1$ asymptotics (Rasheed; Larsen)

- $ds^2 \approx -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + d(x^5)^2$
- $\xi^1 \equiv x^5 \Longrightarrow \chi \approx \eta_4 \equiv \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$
- N preserves the boundary conditions if

$$N\eta_4 N^T = \eta_4 \Leftrightarrow N \in \mathrm{SO}(2,1)$$

SO(2,1) is generated by

- $N_\alpha = \begin{pmatrix} \cosh\alpha & \sinh\alpha & 0 \\ \sinh\alpha & \cosh\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \Longrightarrow$ KK-momentum charge
- $N_\beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cosh\beta & \sinh\beta \\ 0 & \sinh\beta & \cosh\beta \end{pmatrix} \Longrightarrow$ KK-monopole charge
- $N_\gamma = \begin{pmatrix} \cos\gamma & 0 & -\sin\gamma \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\gamma \end{pmatrix} \Longrightarrow$ NUT-charge

$\mathbb{R}^{4,1}$ asymptotics (S.G., Saxena)

- $ds^2 \approx -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\psi^2)$
- ξ^1 could be any linear combination of ϕ and ψ. However, to have a non-trivial subgroup of SL(3,$\mathbb{R}$) that preserves the asymptotic limit, one should take

$$\xi^1 = \ell(\psi + \phi) \quad \text{or} \quad \xi^1 = \ell(\psi - \phi)$$

- Let us make the first choice; we find

$$\chi \approx \eta_5 \equiv \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- η_5 is related to η_4 by an SL(3,$\mathbb{R}$) transformation D:

$$\eta_5 = D^T \eta_4 D \quad \text{with} \quad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- The subgroup of SL(3,$\mathbb{R}$) that preserves the asymptotic form of χ is again isomorphic to SO(2,1):

$$M\eta_5 M^T = \eta_5 \;\Leftrightarrow\; M = D^T N D \text{ with } N \in \mathrm{SO}(2,1)$$

- A more careful analysis shows that $\chi \approx \eta_5$ is sufficient to guarantee that the geometry is asymptotically $\mathbb{R}^{4,1}$.
In particular, no terms of the form $\cos 2\theta dt(d\psi - d\phi)$ are generated.
- In summary, the transformation

$$(\chi, ds_3^2) \xrightarrow{M} (\chi', ds_3^2)$$

connects 5D asymptotically flat solutions of pure gravity.

A technical remark

- To construct the metric from the data (χ', ds_3^2), one needs to solve the duality equations for ω'^a. In most cases, this can be computationally impractical.
- The e.o.m. $d *_3 (\chi^{-1} d\chi) = 0$ allows one to define the matrix of 1-forms κ:

$$\chi^{-1} d\chi = *_3 d\kappa$$

- κ encodes the information on the gauge fields ω^a:

$$\omega^0 = -\kappa^0{}_2 \,,\; \omega^1 = -\kappa^1{}_2$$

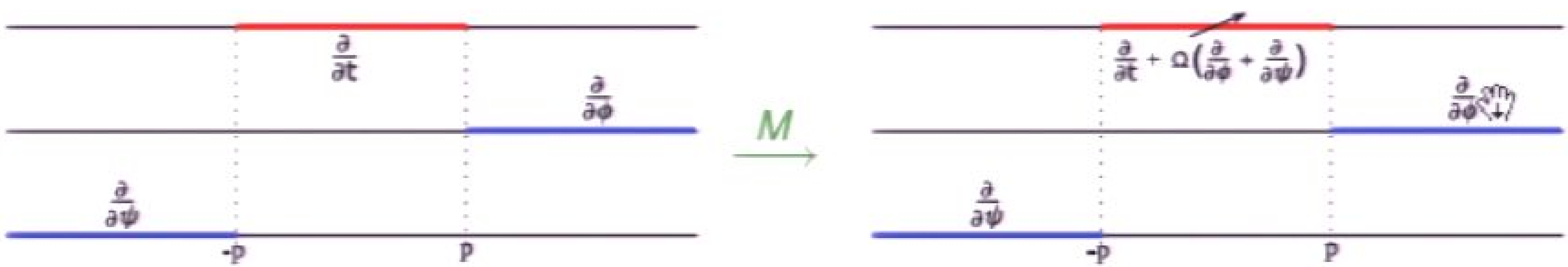
- κ transforms linearly under SL(3,$\mathbb{R}$):

$$\kappa \to \kappa' = (M^T)^{-1} \kappa M^T$$

- No need to solve any differential equation, after the transformation M.

Physical meaning of M

- Naively M has 3 parameters. However ...
- $M_\beta \equiv D^T N_\beta D \Longrightarrow t \to t,\ \xi^1 \to e^{-\beta}\xi^1,\ x^i \to e^\beta x^i.$
- $M_s \equiv D^T N_{\beta(s)} N_{\alpha(s)} N_{\gamma(s)} D \Longrightarrow t \to t + s\,\xi^1,\ \xi^1 \to \xi^1,\ x^i \to x^i.$
- Out of the 3 parameters of SO(2,1), only 1 parameter is physical. Loosely speaking, it corresponds to add angular momentum along $\psi + \phi$.
- Example: action of M on 5D Schwarzschild b.h.



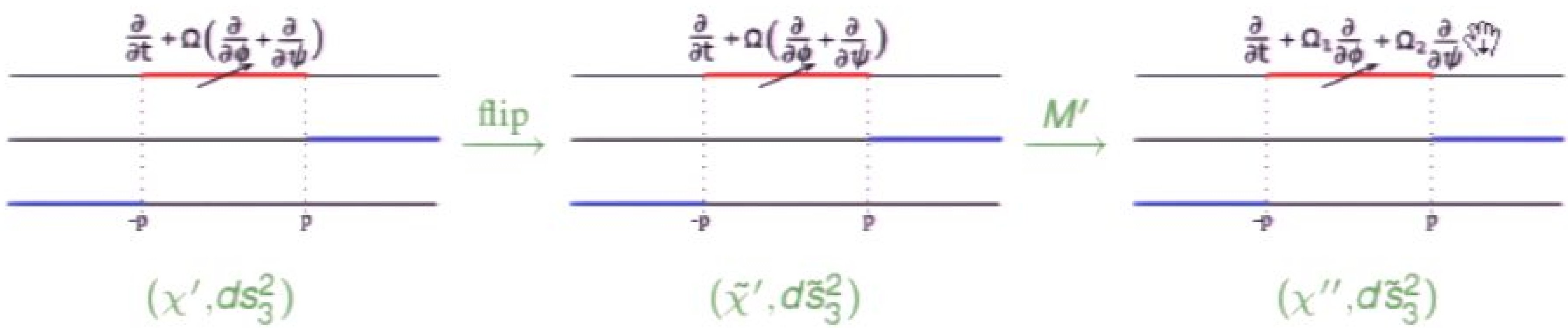
The result is the Myers-Perry b.h. with $\Omega_1 = \Omega_2 = \Omega$.

The “flip”

- Can one generate the full MP solution $(\Omega_1 \neq \Omega_2)$?
- Till now, we have used only 1 axial symmetry $\xi^1 = \ell(\psi + \phi)$. We have another Killing vector, corresponding to $\tilde{\xi}^1 = \tilde{\ell}(\psi - \phi)$.
- Given the data (χ, ds_3^2), we can generate the data $(\tilde{\chi}, d\tilde{s}_3^2)$, corresponding to the *same* metric, by decomposing the metric with respect to $\tilde{\xi}^1$:

$$(\chi, ds_3^2) \xrightarrow{\text{flip}} (\tilde{\chi}, d\tilde{s}_3^2)$$

- Example: MP



General conjecture

- In general we can prove that:
  If we start from a static 5D solution with N finite rods and act with M, we obtain a 5D stationary solution with the same number of rods and the same rod positions. The eigenvectors corresponding to the finite rods (both time-like and space-like) are rotated, by an amount that depends on M and the starting rod configuration.
- One can further act with a sequence of flips and M-transformations.
- Conjecture: after $2N$ steps, one generates the most general solution with N finite rods.
- The "flip" operation requires re-diagonalizing the metric and solving the new duality equations.
  This is computationally impractical, in most cases.

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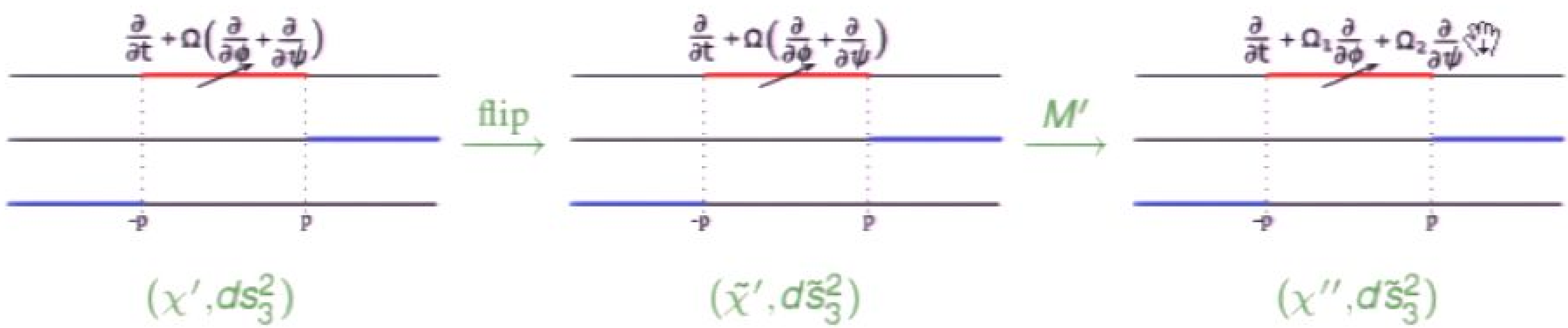

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- One can further act with a sequence of flips and M-transformations.
- Conjecture: after $2N$ steps, one generates the most general solution with N finite rods.
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The 4D-5D connection (Ford, S.G., Peet, Saxena)

- The problem: Start from an asymptotically $\mathbb{R}^{4,1}$ geometry. Construct a solution representing the 5D geometry at the center of a KK-monopole geometry.
- The resulting geometry is locally asymptotic to $\mathbb{R}^{3,1} \times S^1$ (4D).
- It interpolates between a 4D ($R_K \to 0$) and a 5D ($R_K \to \infty$) geometry (R_K is the KK-monopole radius).
- Such geometries are at the basis of a remarkable connection between 4D and 5D partition functions, in cases in which the partition function is independent of R_K (extremal solutions). (Gaiotto, Strominger, Yin)
- In the susy case, there is a systematic way to solve this problem. Add constants to harmonic functions.
- No general solution is known in the non-susy case.

- For axisymmetric vacuum solutions, the SL(3,$\mathbb{R}$) transformation D provides a solution to the problem!
- We have seen that

$$\chi_{5D} \to \eta_5, \; \chi_{4D} \to \eta_4, \; \eta_4 = D\eta_5 D^T$$

- Then

$$\chi_{4D} = D\chi_{5D} D^T$$

gives a metric that interpolates between 4D and 5D.

Examples

- $\mathbb{R}^{4,1} \xrightarrow{D}$ Gross-Perry monopole

$$ds^2_{5D} = -dt^2 + V^{-1}(d\xi^1 + \ell\cos\theta d\phi)^2 + V^{-1}ds^2_3, \quad V = \frac{\ell}{r}$$

$$\chi_{5D} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -V \end{pmatrix} \xrightarrow{D} \chi_{4D} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1-\frac{V}{2} & -\frac{V}{2} \\ 0 & -\frac{V}{2} & -(1+\frac{V}{2}) \end{pmatrix}$$

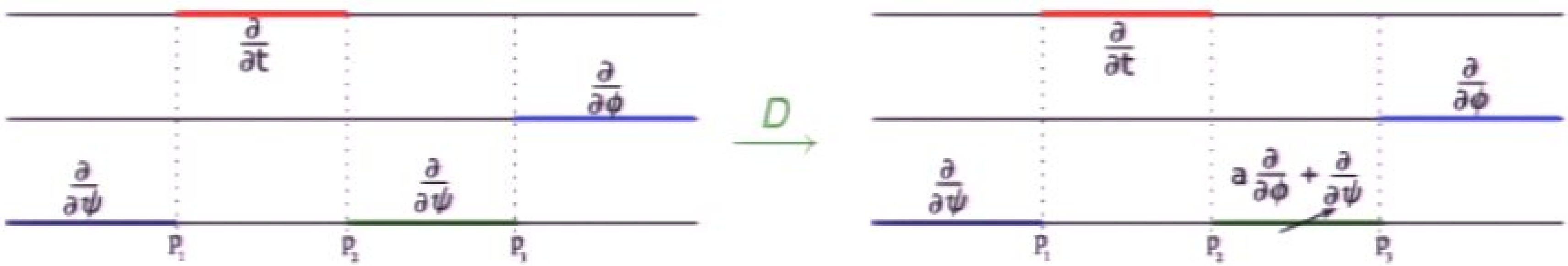
$$ds^2_{4D} = -dt^2 + \tilde{V}^{-1}(d\xi^1 + \frac{\ell}{2}\cos\theta d\phi)^2 + \tilde{V}^{-1}ds^2_3, \quad \tilde{V} = 1 + \frac{\ell}{2r}$$

- The rod structures of ds^2_{5D} and ds^2_{4D} are identical (up to a change of coordinates).

- MP black hole $\xrightarrow{D}$ Rasheed-Larsen geometry
- The Rasheed-Larsen geometry is a a solution of KK gravity carrying KK-monopole charge, KK-momentum charge, and 4D angular momentum.
- It was first obtained by Rasheed, by applying an SO(2,1) transformation to Kerr $\times S^1$.
- It has the property that for R_K much greater than all other scales of the geometry it goes over to the 5D MP black hole.
- Our construction makes this last property obvious.

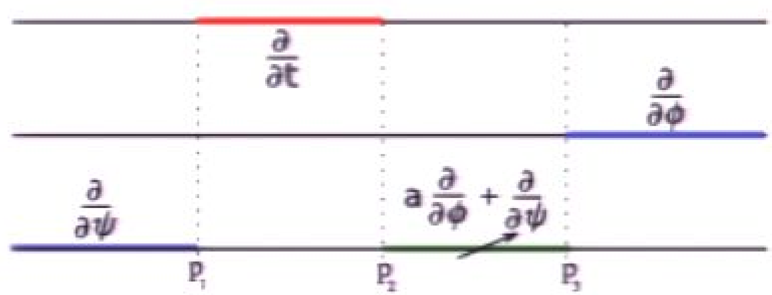
BR in Taub-NUT

- Construct a Black Ring in Taub-NUT:



Problem: the final solution is not a Black Ring!

- Solution: Start from a 5D geometry with rod structure



and tune a in such a way that the finite space-like rod points along $\frac{\partial}{\partial\psi}$ after the D action.

- The 5D geometry with this rod structure could be generated by a sequence of SL(3,$\mathbb{R}$) transformations. In this case we found it more practical to use Inverse Scattering techniques.
- Technical remark: we have derived identities that allow to one compute χ and κ for any solution generated by IS and reduce the problem to purely algebraic manipulations.
- As a result of this construction, we generate a new solution, representing a static Black Ring in Taub-NUT.
- This solution suffers from the same conical defect singularities as the static Black Ring in flat space.
- The regular solution representing a rotating Black Ring in Taub-NUT can be generated in an analogous way (work in progress with Camps, Emparan, Figueras, Saxena).

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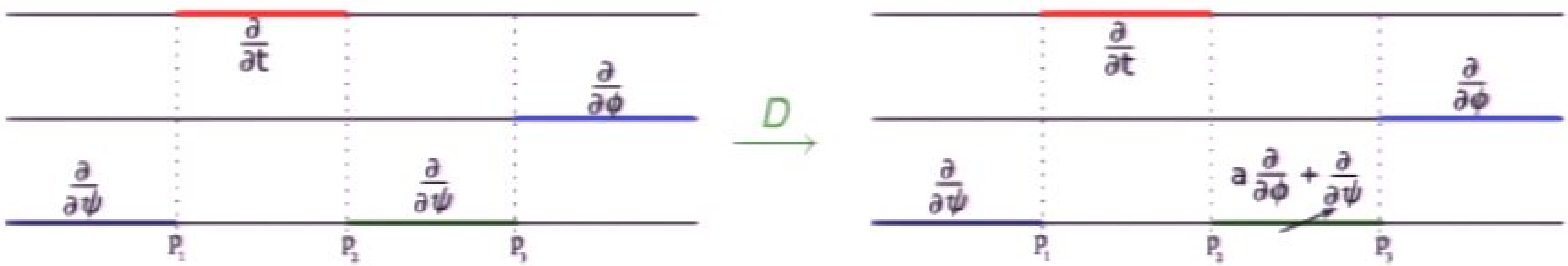

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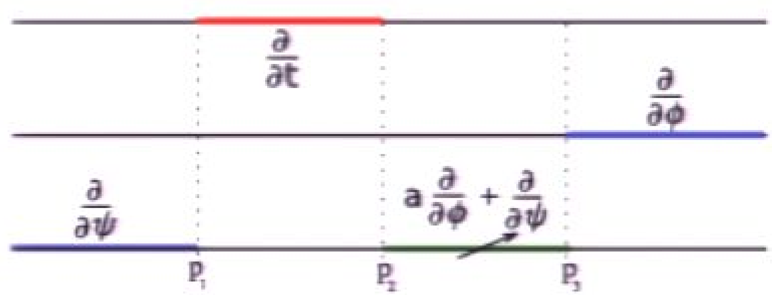

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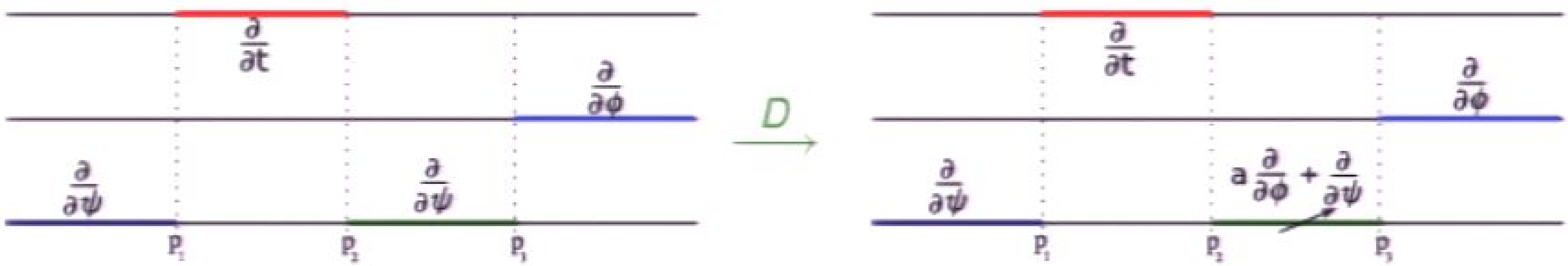

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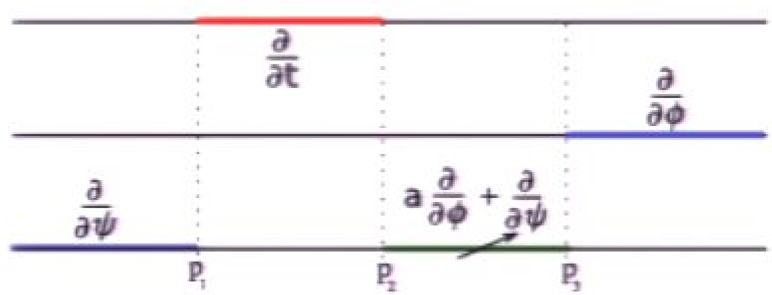

and tune *a* in such a way that the finite space-like rod points along $\frac{\partial}{\partial\psi}$ after the *D* action.

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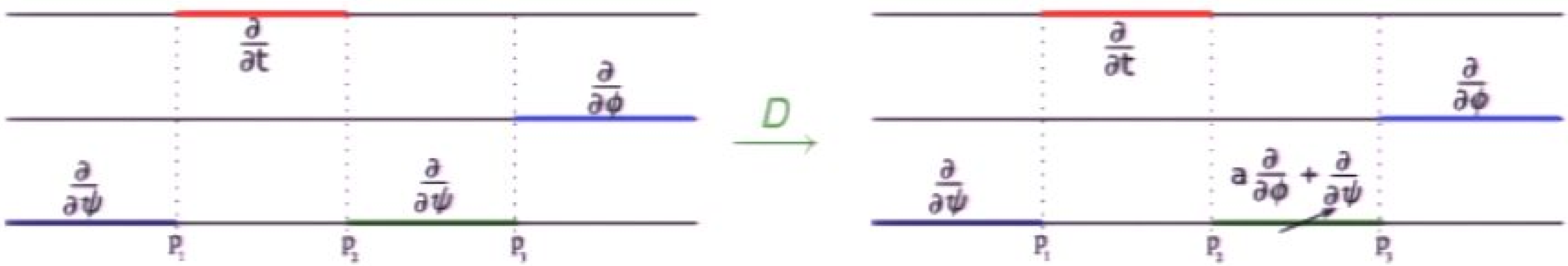

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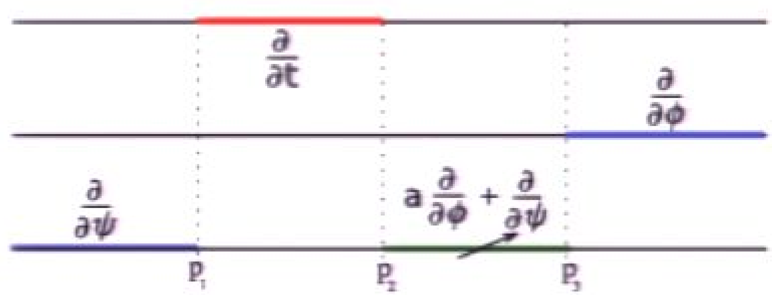


and tune a in such a way that the finite space-like rod points along $\frac{\partial}{\partial \psi}$ after the D action.

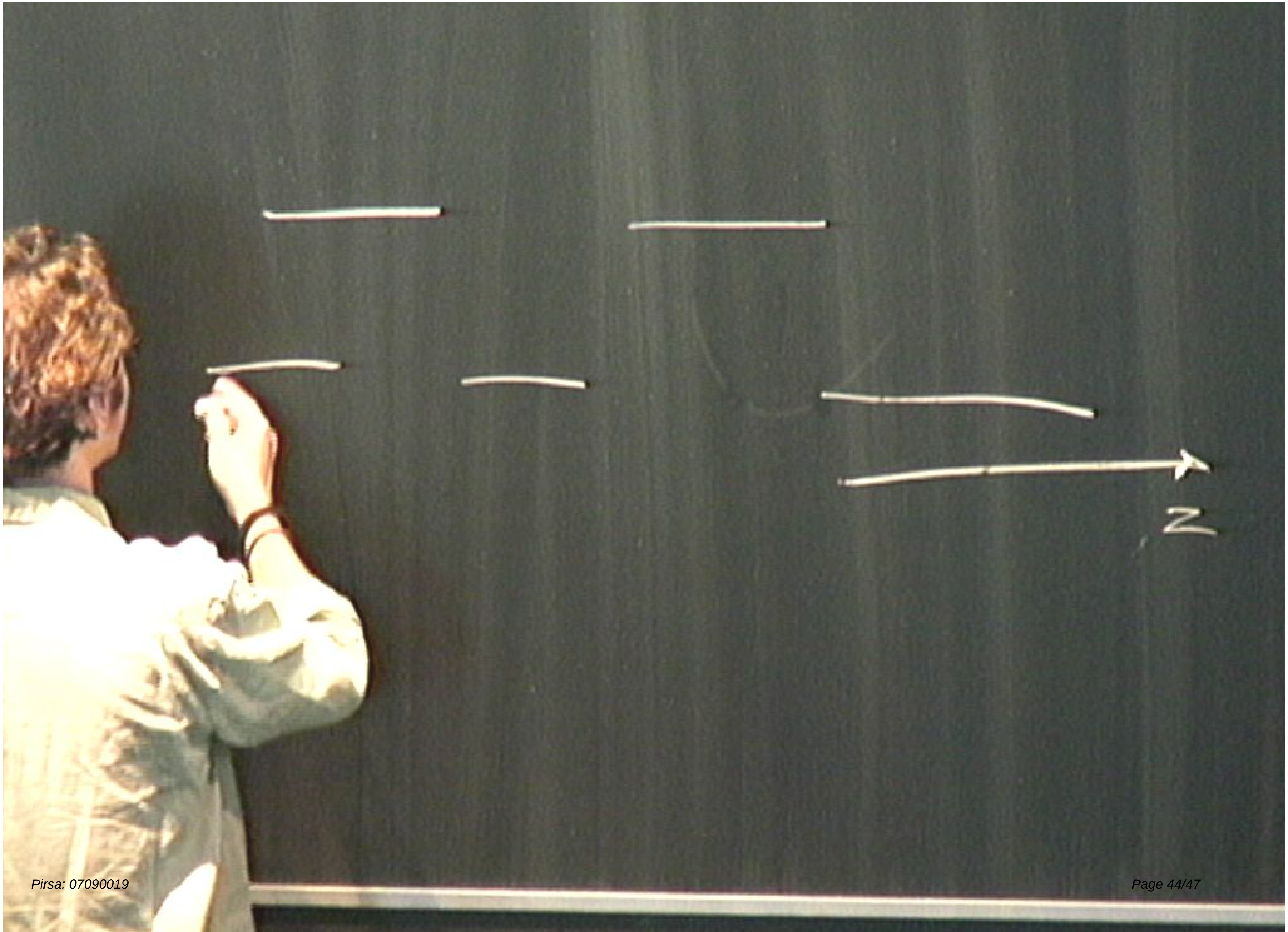
- The 5D geometry with this rod structure could be generated by a sequence of SL(3,$\mathbb{R}$) transformations. In this case we found it more practical to use Inverse Scattering techniques.
- Technical remark: we have derived identities that allow to one compute χ and κ for any solution generated by IS and reduce the problem to purely algebraic manipulations.
- As a result of this construction, we generate a new solution, representing a static Black Ring in Taub-NUT.
- This solution suffers from the same conical defect singularities as the static Black Ring in flat space.
- The regular solution representing a rotating Black Ring in Taub-NUT can be generated in an analogous way (work in progress with Camps, Emparan, Figueras, Saxena).

4D non-susy microstates S.G., Ross, Saxena

- Jejjala, Madden, Ross, Titchener have constructed regular, horizon-less geometries dual to some non-BPS microstates of the 5D black hole with D1-D5-P charges.
- The corresponding CFT states carry both left and right moving excitations.
- The geometries can be constructed by applying a chain of dualities to a certain vacuum "seed" solution (5D MP).
- By applying D to the vacuum seed, and applying the same chain of dualities, we have constructed new non-BPS geometries dual to microstates of the 4D black hole with D1-D5-P-KK charges.

Summary

- We have presented a method to generate stationary axisymmetric solutions of 5D gravity.
- The method is based on the $SL(3,\mathbb{R})$ invariance of the system.
- It is conjectured that the method generates all the 5D asymptotically flat solutions in this class.
- We have identified an $SL(3,\mathbb{R})$ transformation D that connects 5D and 4D solutions of pure gravity.
- We have applied the D transformation to generate a new Black Ring in Taub-NUT.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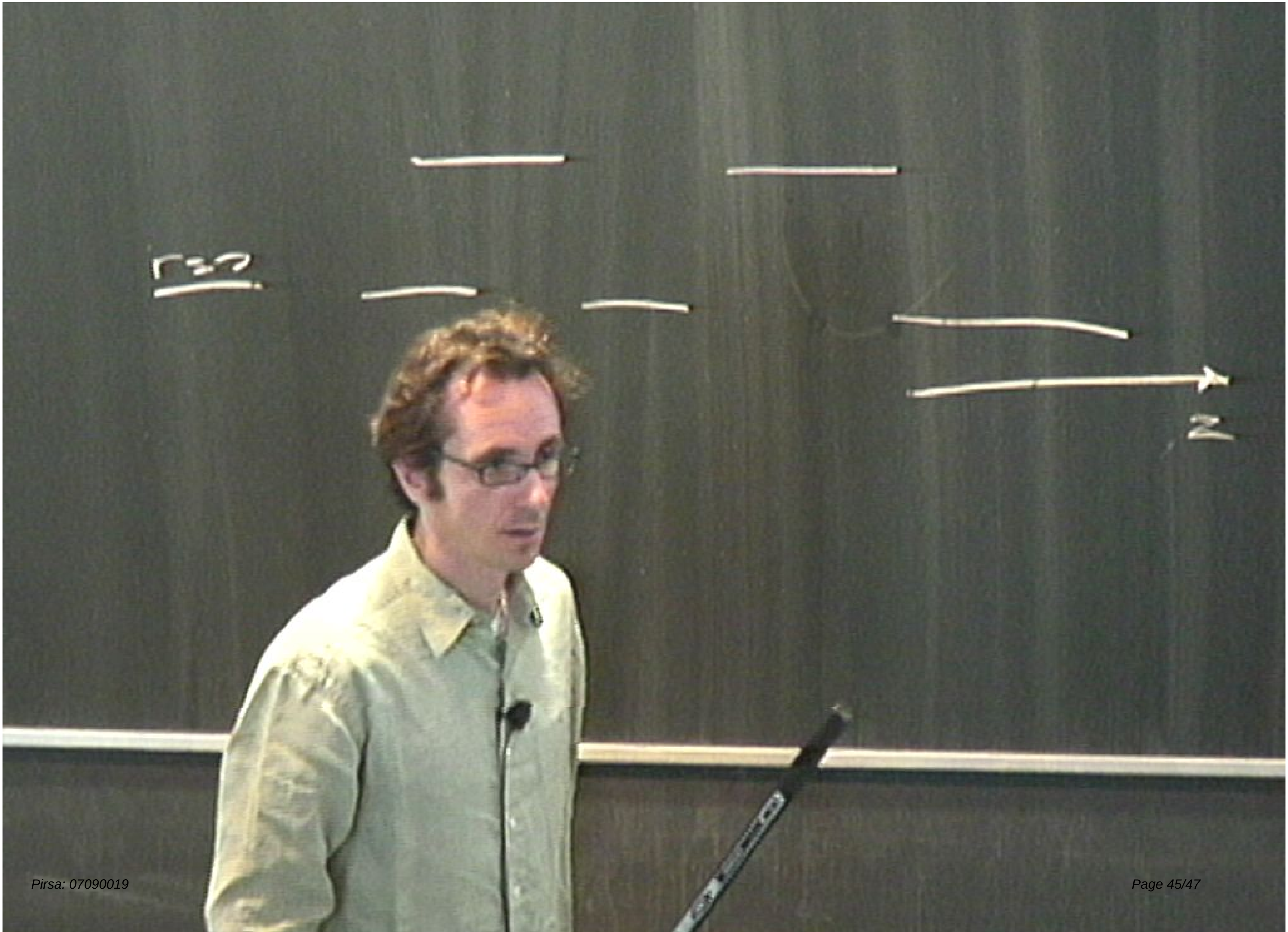
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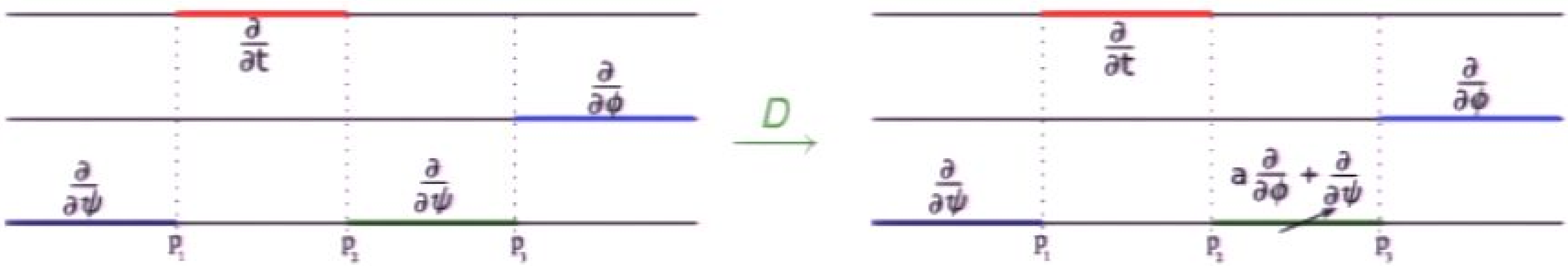
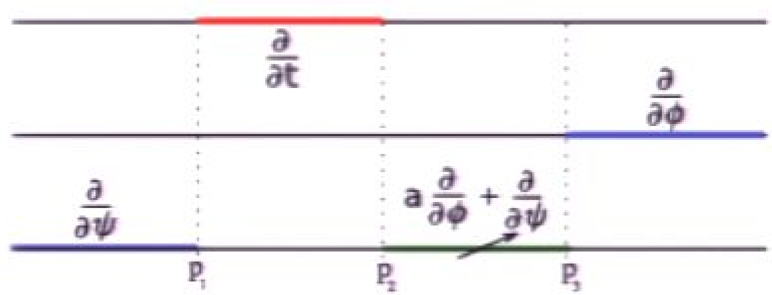
BR in Taub-NUT

- Construct a Black Ring in Taub-NUT:



Problem: the final solution is not a Black Ring!

- Solution: Start from a 5D geometry with rod structure



and tune *a* in such a way that the finite space-like rod points along $\frac{\partial}{\partial \psi}$ after the *D* action.