Title: Quantum Field Theory in Curved Spacetime

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Abstract: Quantum field theory in curved spacetime (QFTCS) is the theory of quantum fields propagating in a classical curved spacetime, as described by general relativity. QFTCS has been applied to describe such important and interesting phenomena as particle creation by black holes and perturbations in the early universe associated with inflation. However, by the mid-1970\'s, it became clear from phenomena such as the Unruh effect that \'particles\' cannot be a fundamental notion in QFTCS. By the mid-1980\'s it was understood how to give a mathematically rigorous formulation of the theory of a free quantum field in curved spacetime. During the past decade, major progress has been made in providing a completely mathematically satisfactory formulation of renormalization in interacting QFTCS, thereby overcoming the difficulties caused by the absence of Poincare symmetry as well as the lack of a preferred vacuum state and a fundamental notion of \'particles\'. This talk will describe these developments and some of the insights that have thereby been attained.

Quantum Field Theory in Curved Spacetime

Robert M. Wald

- R. Brunetti, K. Fredenhagen and R. Verch, Commun. Math. Phys. **237**, 31 (2003).
- S. Hollands and R.M. Wald, Commun. Math. Phys. **223**, 289-326 (2001); Commun. Math. Phys. **231**, 309-345 (2002); Commun. Math. Phys. **237**, 123-160 (2003); Rev. Math. Phys. **17**, 227 (2005).

Some References

Free fields

R.M. Wald: *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*, University of Chicago Press (Chicago, 1994).

Interacting fields

- R. Brunetti, K. Fredenhagen and M. Köhler, Commun. Math. Phys. **180**, 633-652 (1996).
- R. Brunetti and K. Fredenhagen, Commun. Math. Phys. **208**, 623-661 (2000).

- R. Brunetti, K. Fredenhagen and R. Verch, Commun. Math. Phys. **237**, 31 (2003).
- S. Hollands and R.M. Wald, Commun. Math. Phys. **223**, 289-326 (2001); Commun. Math. Phys. **231**, 309-345 (2002); Commun. Math. Phys. **237**, 123-160 (2003); Rev. Math. Phys. **17**, 227 (2005).

Quantum Field Theory in Curved Spacetime

Quantum field theory in curved spacetime (QFTCS) is a theory wherein matter is treated fully in accord with the principles of quantum field theory, but gravity is treated classically in accord with general relativity. It is not expected to be an exact theory of nature, but it should provide a good approximate description in circumstances where the quantum effects of gravity itself do not play a dominant role. Despite its classical treatment of gravity, QFTCS has provided us with some of the deepest insights we presently have into the nature of quantum gravity.

This talk will describe some of the key developments in the theory of quantum fields in curved spacetime and the present status of the theory. I will not treat "back reaction" effects—i.e., effects of the quantum field on the spacetime geometry as described by the semiclassical Einstein equation—but rather will focus on the effects of a curved spacetime geometry on quantum fields.

The Free Klein-Gordon Field in Flat Spacetime

$$\partial^a \partial_a \phi - m^2 \phi = 0$$

(To avoid technical awkwardness, put in cubic box of side L with periodic boundary conditions.)

In terms of the Fourier modes

$$\phi_{\vec{k}} \equiv L^{-3/2} \int e^{-i\vec{k}\cdot\vec{x}} \phi(t, \vec{x}) \, d^3x$$

have

$$H = \sum_{\vec{k}} \frac{1}{2} \left(|\dot{\phi}_{\vec{k}}|^2 + \omega_{\vec{k}}^2 |\phi_{\vec{k}}|^2 \right)$$

where

$$\omega_{\vec{k}}^2 = |\vec{k}|^2 + m^2$$

So, a free Klein-Gordon field, ϕ, is just an infinite collection of decoupled harmonic oscillators. The Heisenberg field operator $\phi(t, \vec{x})$ is given by

$$\phi(t, \vec{x}) = L^{-3/2} \sum_{\vec{k}} \frac{1}{2\omega_{\vec{k}}} \left(e^{i\vec{k}\cdot\vec{x} - i\omega_{\vec{k}} t} a_{\vec{k}} + e^{-i\vec{k}\cdot\vec{x} + i\omega_{\vec{k}} t} a^{\dagger}_{\vec{k}} \right) .$$

However, this formula does <u>not</u> make sense to define ϕ at a point $(t, \vec{x})$. However it does make sense as an "operator valued distribution", i.e., for any (smooth, compactly supported) "test function", f

$$\phi(f) = \int f(t, \vec{x}) \phi(t, \vec{x}) d^4 x$$

is well defined.

The Free Klein-Gordon Field in Flat Spacetime:

Interpretation of states

Interpret the ground state, $|0\rangle$, of all oscillators comprising the KG field as representing the "vacuum". Interpret a state of the form $(a^\dagger)^n|0\rangle$ as one where a total of n "particles" are present.

In an interacting theory, the state of the field may be such that the field behaves like a free field at early and late times. In that case, have a particle interpretation at early and late times. The relationship between the early and late time particle descriptions of a state—given by the S-matrix—contains a great deal of the dynamical information about the interacting theory.

The particle interpretation/description of quantum field theory in flat spacetime has been remarkably successful—to the extent that one might easily get the impression that, at a fundamental level, quantum field theory is really a theory of "particles".

Note, however, that the definition and interpretation of "particle states" relies heavily on the presence of a time translation symmetry.

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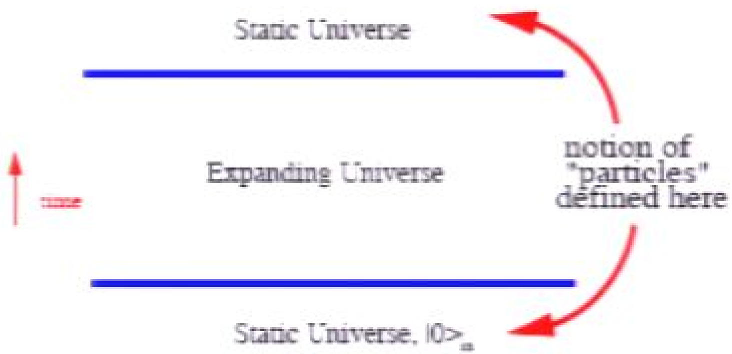
An Early Application of QFTCS:

Particle Creation in an Expanding Universe

In the 1960's, Parker investigated effects of particle creation in an expanding universe. Consider the following (highly artificial!) spacetime:



On account of intermediate expansion, "out" annihilation and creation operators differ from corresponding "in" operators. Consequently, $|0\rangle_{\rm in} \neq |0\rangle_{\rm out}$, i.e., have "spontaneous particle creation from the vacuum."

But, how does one define $|0\rangle_{\mathrm{in}}$ if the universe began at a "big bang" singularity? How does one define "particles" at late times if the universe recollapses?

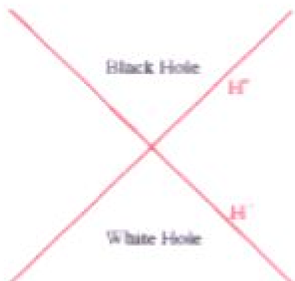
Particle Creation by Rotating Black Holes

In the early 1970's, it was realized that if one sends a classical wave of the appropriate frequency and angular dependence toward a *rotating* black hole, the wave will be amplified by the black hole ("*superradiant scattering*"). The wave extracts some of the rotational energy of the black hole, causing the black hole to spin down.

There is a close physical analogy between superradiant scattering stimulated emission. This suggested that there should also be "spontaneous emission"—i.e., particle creation by a rotating black hole. This was noted by Starobinski and confirmed by Unruh.

The Hawking Effect

The calculation of particle creation by a rotating black hole was done in the idealized spacetime representing the stationary final state of the black hole. This spacetime also contains a white hole, so one has to impose initial conditions on the white hole horizon that express the condition that no particles are emerging.



A natural looking condition was chosen, but it was not clear that this choice was physically correct. In 1974, Hawking realized that this difficulty could be overcome by considering the more physically relevant case of a spacetime describing gravitational collapse to a black hole. When he carried through the calculation, he found that the results were significantly altered: Even for a non-rotating black hole, particle creation occurs at late times and produces an exactly thermal flux of particles at a temperature

$$T = \frac{\kappa}{2\pi} = \frac{\hbar c^3}{8\pi G M}$$

where κ is the surface gravity of the black hole.

Some Implications of the Hawking Effect

1. Black holes are perfect black bodies in the thermodynamic sense at a non-zero temperature!! This ties in beautifully with the mathematical analogy that had previously been discovered between certain laws of black hole physics and the ordinary laws of thermodynamics and leads to the identification of

$$\frac{1}{4}\frac{c^3}{G\hbar}A$$

as representing the physical entropy of a black hole.

2. Particle creation $\rightarrow$ energy flux to infinity and loss of black hole mass $\rightarrow$ "evaporation of the black hole. In this process, semiclassically an initial pure state $\rightarrow$ final

mixed state ("information loss").

These and other ramifications of Hawking's results have provided us with some of the deepest insights we presently have regarding the nature of quantum gravity.

The Unruh Effect

There was a very disturbing aspect of Hawking's calculation: It appeared to show a divergent density of ultra-high-frequency particles present near the horizon of the black hole. What do these "particles" mean? Does their presence destroy the black hole?

To gain insight into this issue, Unruh took an operational definition of "particles": A "particle" is a state of the field that makes a particle detector register. He then found that in Minkowski spacetime, when a quantum field is in it's ordinary vacuum state, a uniformly accelerating observer "sees" an exactly thermal spectrum of particles, at a temperature $T = a/2\pi$.

The "particles" near the horizon of a black hole that appear in the Hawking calculation would similarly be "seen" by a stationary observer just outside the black hole, but not by a freely falling observer.

Lessons

The notion of "particles" is not fundamental in quantum field theory. Quantum field theory is truly the quantum theory of *fields*, not particles.

With the exception of stationary spacetimes (and certain other spacetimes with very special properties), there is no preferred notion of a "vacuum state" in quantum field theory in curved spacetime and, correspondingly, there is no preferred notion of "particles"

However, in general, different choices of vacuum state will give rise to unitarily inequivalent Hilbert space constructions of the theory. So, how does one formulate quantum field theory in a general curved spacetime?

Answer: Use the algebraic approach. Formulate the predictions of the theory in terms of probabilities for measuring local field observables, not "particles".

Free KG Field in Curved Spacetime: Observables

To define a suitable algebra of observables $\mathcal{A}$, start with the free *-algebra, $\mathcal{A}_0$, generated by a unit element I and expressions of the form "$\phi(f)$", where f is an arbitrary test function on M. (An example of an element of $\mathcal{A}_0$ is $c_1\phi(f_1)\phi(f_2) + c_2\phi^*(f_3)\phi(f_4)\phi^*(f_5)$.) Impose the following relations on $\mathcal{A}_0$:

- Linearity of $\phi(f)$ in f
- reality of ϕ: $\phi^*(f) = \phi(\bar{f})$
- The Klein-Gordon equation: $\phi([\nabla^a\nabla_a - m^2]f) = 0$

- The canonical commutation relations:

$$[\phi(f), \phi(g)] = -i\Delta(f, g)I$$

The desired *-algebra, $\mathcal{A}$, is simply $\mathcal{A}_0$ factored by these relations. Note that the observables in $\mathcal{A}$ correspond to the correlation functions of the quantum field ϕ.

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Free KG Field in Curved Spacetime: States

A *state*, ω, is simply a linear map $\omega : \mathcal{A} \to \mathbf{C}$ that satisfies the positivity condition $\omega(A^*A) \geq 0$ for all $A \in \mathcal{A}$. The quantity $\omega(A)$ is interpreted as the expectation value of the observable A in the state ω.

If $\mathcal{H}$ is a Hilbert space which carries a representation, π, of $\mathcal{A}$, and if $\Psi \in \mathcal{H}$ then the map $\omega : \mathcal{A} \to \mathbf{C}$ given by

$$\omega(A) = \langle \Psi | \pi(A) | \Psi \rangle$$

defines a state on $\mathcal{A}$.

Conversely, given a state, ω, on $\mathcal{A}$, we can use it to define

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Conversely, given a state, ω, on $\mathcal{A}$, we can use it to define a (pre-)inner-product on $\mathcal{A}$ by

$$(A_1, A_2) = \omega(A_1^* A_2)\,.$$

By factoring by zero-norm vectors and completing this space to get a Hilbert space $\mathcal{H}$ which carries a natural representation, π, of $\mathcal{A}$. The vector $\Psi \in \mathcal{H}$ corresponding to $I \in \mathcal{A}$ then satisfies $\omega(A) = \langle \Psi | \pi(A) | \Psi \rangle$ for all $A \in \mathcal{A}$.

Thus, every state in the algebraic sense corresponds to a state in the usual Hilbert space sense. However, one may simultaneously consider all states arising in all Hilbert space constructions of the theory without having to make a particular choice of representation at the outset.

Going Beyond the Theory of a Linear Field Based Upon $\mathcal{A}$

Issues:

- Even in the theory of a free Klein-Gordon field, there are many observables of interest, such as T_{ab}, that are not represented in $\mathcal{A}$.
- Perturbative rules for constructing interacting quantum fields require that one be able to define Wick polynomials of the free field as well as time-ordered-products of polynomial expressions in the field.

We need an enlarged algebra of observables.

Difficulties:

- ϕ is a distribution, so, *a priori*, $[\phi(x)]^2$ does not make mathematical sense. Attempt to define $\phi^2(f)$ by

$$\phi^2(f) = \lim_{n \to \infty} \int \phi(x)\phi(y) f(x) F_n(x,y) d^4x d^4y$$

where $F_n(x,y) \to \delta(x,y)$ yield divergent results.

- The time-ordered-product $T(\phi^{k_1}(f_1) \ldots \phi^{k_n}(f_n))$ can be defined by a straightforward "time ordering" of the factors in the case where the supports of $f_1, \ldots, f_n$ have suitable causal properties. However, it is not straightforward to extend this distribution to the "total diagonal", i.e., to the case where the supports of $f_1, \ldots, f_n$ have nonvanishing mutual intersection.

Challenges to Going Beyond the Linear Theory

In Minkowski spacetime, Wick powers like ϕ^2 are defined by a "normal ordering" prescription, which can be interpreted as "subtracting off the (infinite) vacuum expectation values" of the field quantities. But in curved spacetime, there is no preferred vacuum state.

In Minkowski spacetime the renormalization prescriptions used to define time-ordered-products make use of "momentum space methods" (i.e., global Fourier transforms of quantities) and/or "Euclidean methods" (i.e., analytic continuation of expressions defined on Euclidean space rather than Minkowski spacetime). These methods, in turn, require

- Poincare symmetry
- A preferred, Poincare invariant, vacuum state
- The ability to "Euclideanize" via $t \to it$

All of the above features are absent in a general, curved spacetime.

Microlocal Analysis

During the past decade, it has been realized that "microlocal analysis" provides exactly the right mathematical tools for defining nonlinear quantities in QFTCS. In essence, microlocal analysis provides a refined characterization of the singularities of a distribution by examining the decay properties of the Fourier transform of the distribution (after it has been localized near point x). It therefore provides a notion of the singular points and directions (x, k) of a distribution, α, called the the *wavefront set*, denoted $\mathrm{WF}(\alpha)$. This refined characterization of the singularities of distributions can enable one to define operations that normally are ill defined. For example, if α and β are distributions, their product will make sense as a distribution if whenever $(x, k) \in \mathrm{WF}(\alpha)$, we have that $(x, -k) \notin \mathrm{WF}(\beta)$.

Microlocal analysis thereby provides an extremely useful calculus for determining whether proposed regularization/renormalization schemes are well defined.

Local and Covariant Fields

A key, additional idea needed to define field quantities Φ (such as $\phi^2(f)$ or $T(\phi^3(f_1)\phi^4(f_2))$ is that these quantities at point x "be locally and covariantly constructed out of the spacetime geometry" in an arbitrarily small neighborhood of x. More precisely, if we have an isometric embedding of a region, $\mathcal{O}$, of the spacetime (M, g_{ab}) into a region $\mathcal{O}'$, of the spacetime (M', g'_{ab}), this embedding will induce an isomorphism of the local field algebras of $\mathcal{O}$ and $\mathcal{O}'$. We demand that under this isomorphism, the quantity designated as Φ in $\mathcal{O}$ be taken into the quantity with the same designation Φ in $\mathcal{O}'$.

We can isometrically embed all of Minkowski spacetime into itself by a Poincare transformation. The above condition then requires the field definitions in Minkowski spacetime to be Poincare invariant. The above condition contains much of the essential content of Poincare invariance, but is applicable to arbitrary curved spacetimes.

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Axioms for Wick Powers

- ϕ^n should be local and covariant.
- $[\phi^n(x), \phi(y)] = in\Delta(x,y)\phi^{n-1}(x)$
- $(\phi^n(f))^* = \phi^n(\bar{f})$
- For any regular (Hadamard) state ω, $\omega(\phi^n(x))$ is smooth.
- ϕ^n varies smoothly/analytically under a smooth/analytic variation of the metric and coupling parameters.
- Under scaling of the metric, $g_{ab} \rightarrow \lambda^2 g_{ab}$, ϕ^n, we have $\phi^n \rightarrow \lambda^{-n}\phi^n +$ terms polynomial in $\ln\lambda$.

The axioms for time-ordered products are similar, except that the 4th condition is replaced by a much more complicated "microlocal spectrum condition" and there are additional "unitarity" and "causal factorization" conditions.

Sketch of Uniqueness Argument

Consider ϕ^2. The commutation condition

$$[\phi^2(x), \phi(y)] = 2i\Delta(x, y)\phi(x)$$

uniquely determines $\phi^2(x)$ up to a multiple of the identity, i.e., up to $C(x)\mathbb{1}$. Local covariance condition implies that $C(x)$ depends only on the spacetime geometry near x; smoothness/analyticity and scaling implies $C = \alpha R$, so ϕ^2 is unique up to addition of a multiple of the scalar curvature.

By induction, each higher power of ϕ gives rise to a new "multiple of the identity" ambiguity, given by curvature terms of the appropriate dimension.

A similar—but much more complicated—argument for time-ordered products yields uniqueness up to certain specified ambiguities. In the computation of an S-matrix for an interaction Lagrangian, these ambiguities correspond precisely to the addition of "counterterms" (including curvature couplings) of the appropriate dimension to the Lagrangian.

Existence of Wick Powers

Quantities quadratic in ϕ (such as ϕ^2 or T_{ab}) can be defined by a "point-splitting" prescription as follows. Define

$$:\phi(x)\phi(y):_H = \phi(x)\phi(y) - H(x, y)\mathbb{1}$$

where $H(x, y)$ is a locally and covariantly constructed "Hadamard parametrix". Can then define $\phi^2(f)$ to be $:\phi(x)\phi(y):_H$ smeared with $f(x)\delta(x, y)$. Can define $\phi^n(f)$ by a suitable generalization of this prescription.

Note that the definition of Wick powers involves the subtraction of a locally and covariantly constructed quantity $H(x, y)$, not the subtraction of a vacuum expectation value.

Results

(in collaboration with S. Hollands)

- There exists a well defined prescription for defining all Wick polynomials that is local and covariant and satisfies a list of additional reasonable properties. This prescription is unique up to certain "local curvature ambiguities", e.g., the prescription for ϕ^2 is unique up to

$$\phi^2 \to \phi^2 + (c_1 R + c_2 m^2) I \, .$$

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For a massless field in Minkowski spacetime, all of the ambiguities disappear, and the prescription agrees with normal ordering. However, on a general curved spacetime, the prescription for defining ϕ^2 and other Wick polynomials does not agree with normal ordering with respect to any choice of vacuum state.

- There exists a prescription for defining all time-ordered-products that is local and covariant and satisfies a list of additional reasonable properties. This prescription is unique up to "renormalization ambiguities" of the type expected from Minkowski spacetime analyses, but with additional local curvature ambiguities.
- Theories that are renormalizable in Minkowski spacetime remain renormalizable in curved spacetime. Renormalization group flow can be

defined in terms of the behavior of the quantum field theory under scaling of the spacetime metric, $g_{ab} \to \lambda^2 g_{ab}$.

- Additional renormalization conditions can be imposed so that, in perturbation theory, for an arbitrary interaction, (i) the interacting field satisfies the classical interacting equation of motion and (ii) the stress-energy tensor of the interacting field is conserved.

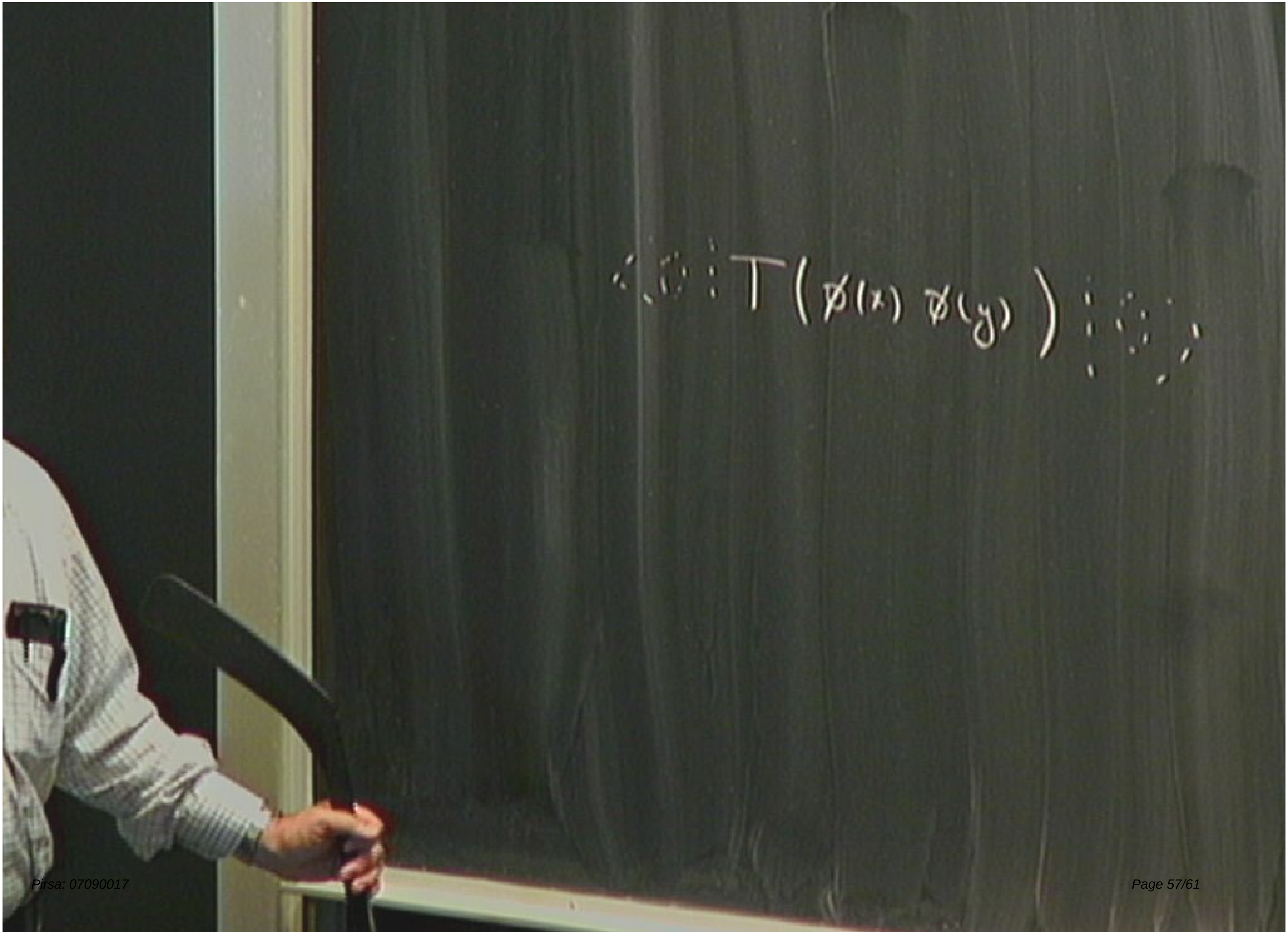
All of the above results have been obtained without any appeal to a notion of "vacuum" or "particles".

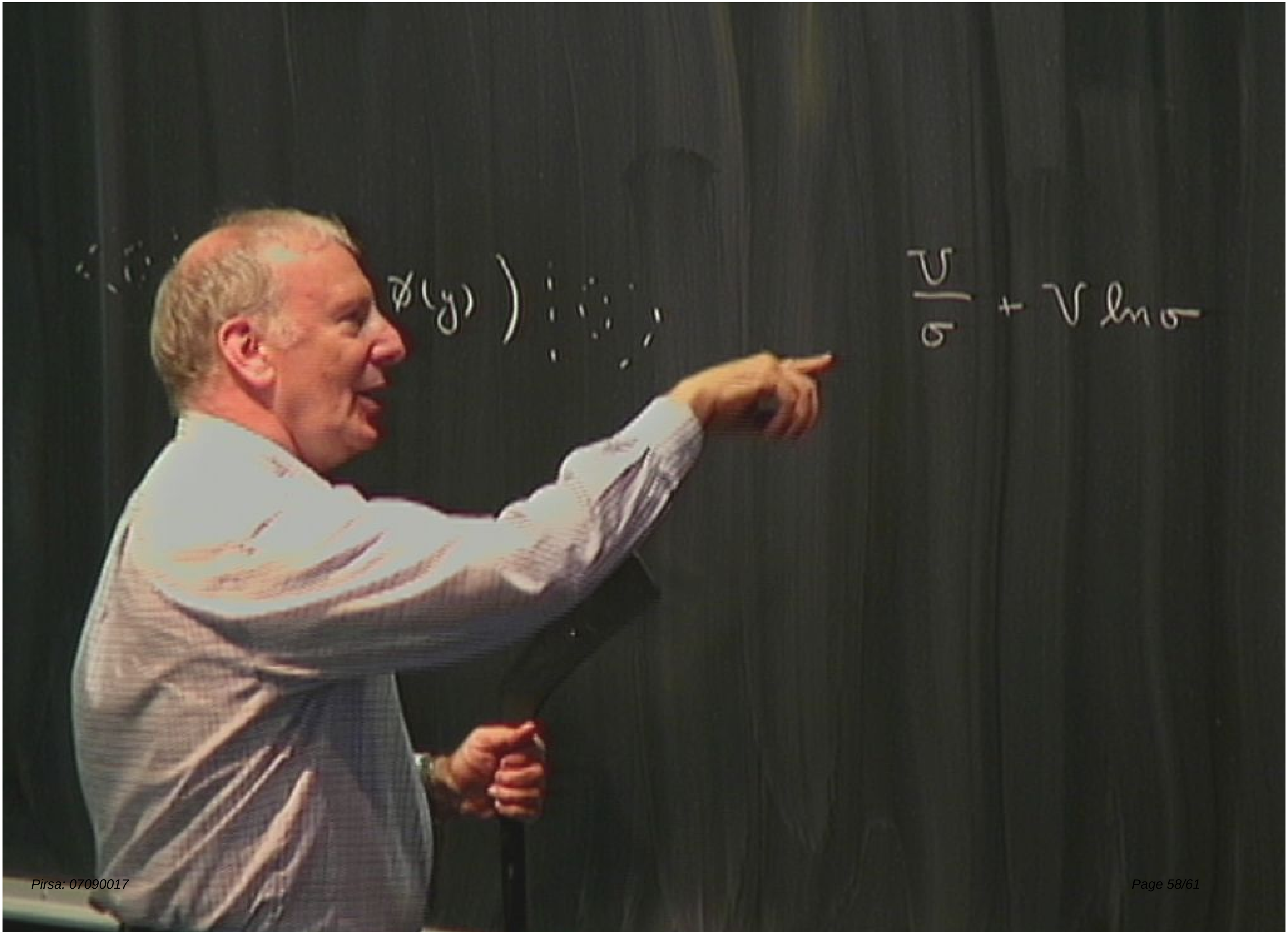
Conclusions

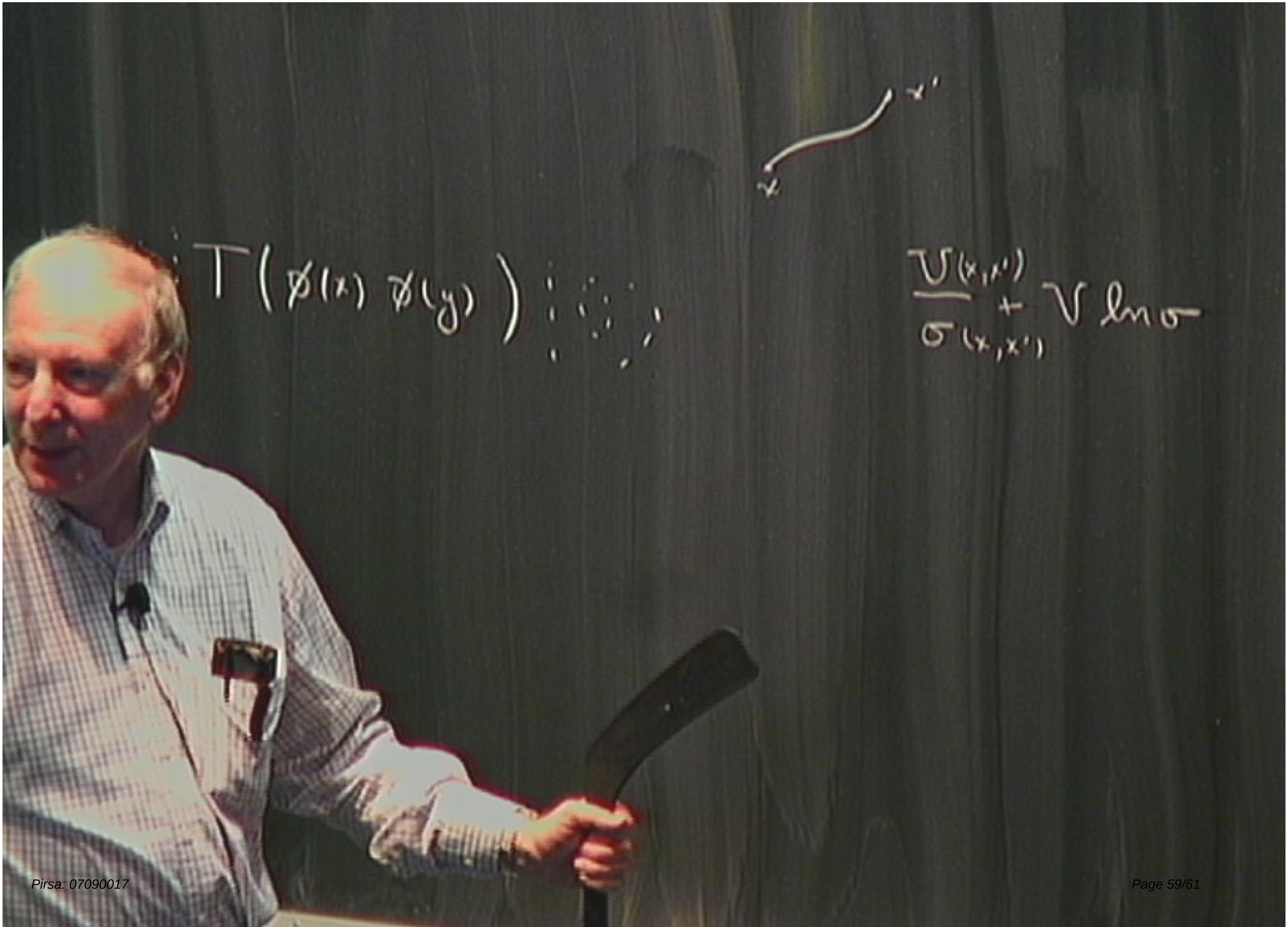
QFTCS has provided us with deep insights into the nature of quantum phenomena in strong gravitational fields and into the nature of QFT itself.

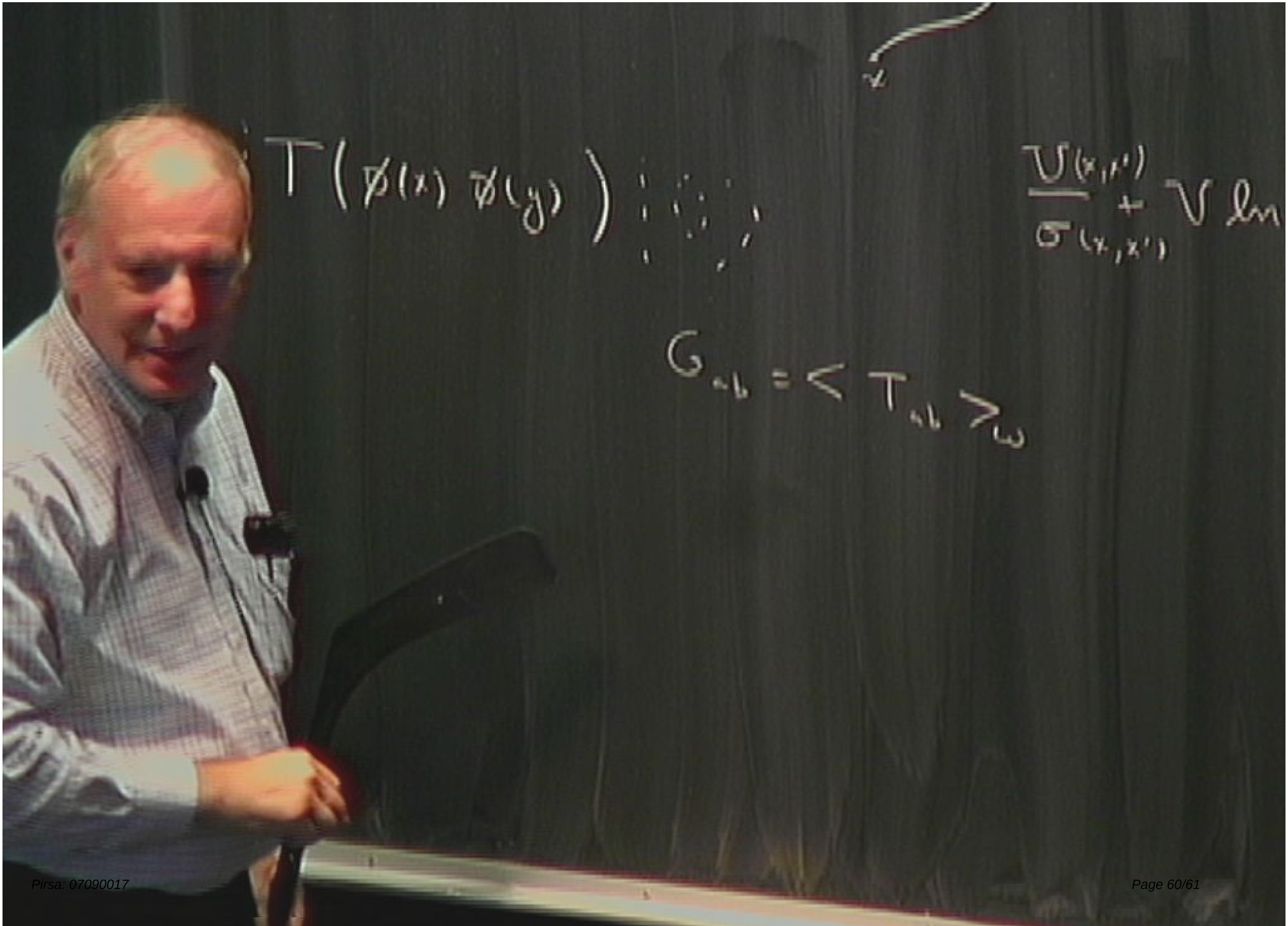
The notion of "particles" can now be seen to properly join the ranks of such notions as "simultaneity" and "gravitational force"—notions that are extremely useful for informal discussions in our everyday lives, but, at a fundamental level, are ambiguous or meaningless.

Despite the lack of a fundamental notion of "particles", QFTCS makes mathematical and physical sense as a theory of locally observable quantum fields.


$\langle 0 | T(\phi(x)\ \phi(y)) | 0 \rangle$


$\phi(y)$)
$\frac{V}{\sigma} + V \ln \sigma$


T(ϕ(x) ϕ(y))
x'
x
σ(x,x')
V ln σ


T(ϕ(x) ϕ(y))
U(x,x')/σ(x,x') + V ln
G_ab = < T_ab >_ω

$$\sigma(x,x') \quad + \nabla \ln \sigma$$

$$G_{ab} = \langle T_{ab} \rangle_\omega \qquad \nabla^a T_{ab} = 0$$