Title: Learning about topological quantum memory

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Abstract: I will introduce Kitaev's suface codes as a block quantum error-correcting code. Recovery procedures will be described in the case of imperfect syndrome measurements. More might be covered if time permits.

Topologi[cal]

Q. Memory

- Overview: State space of QMem.
  - evolution of states
  - general ideas of QEC
  - toric codes: def'n
    - coding space
    - correctability
    - 2 channels (recovery)
  - generalizations
  - Summary

$$\underbrace{010\ldots\ldots1}_{n \text{ bits}}$$

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$$H_2$$

$$\alpha|0\rangle + \beta|1\rangle$$

$$\underbrace{010 \ldots\ldots 1}_{n \text{ bits}}$$

$$\mathcal{H}_2^{\otimes n} \xrightarrow[\mathcal{N}]{\mathcal{I}}$$

$$\alpha|0\rangle + \beta|1\rangle$$

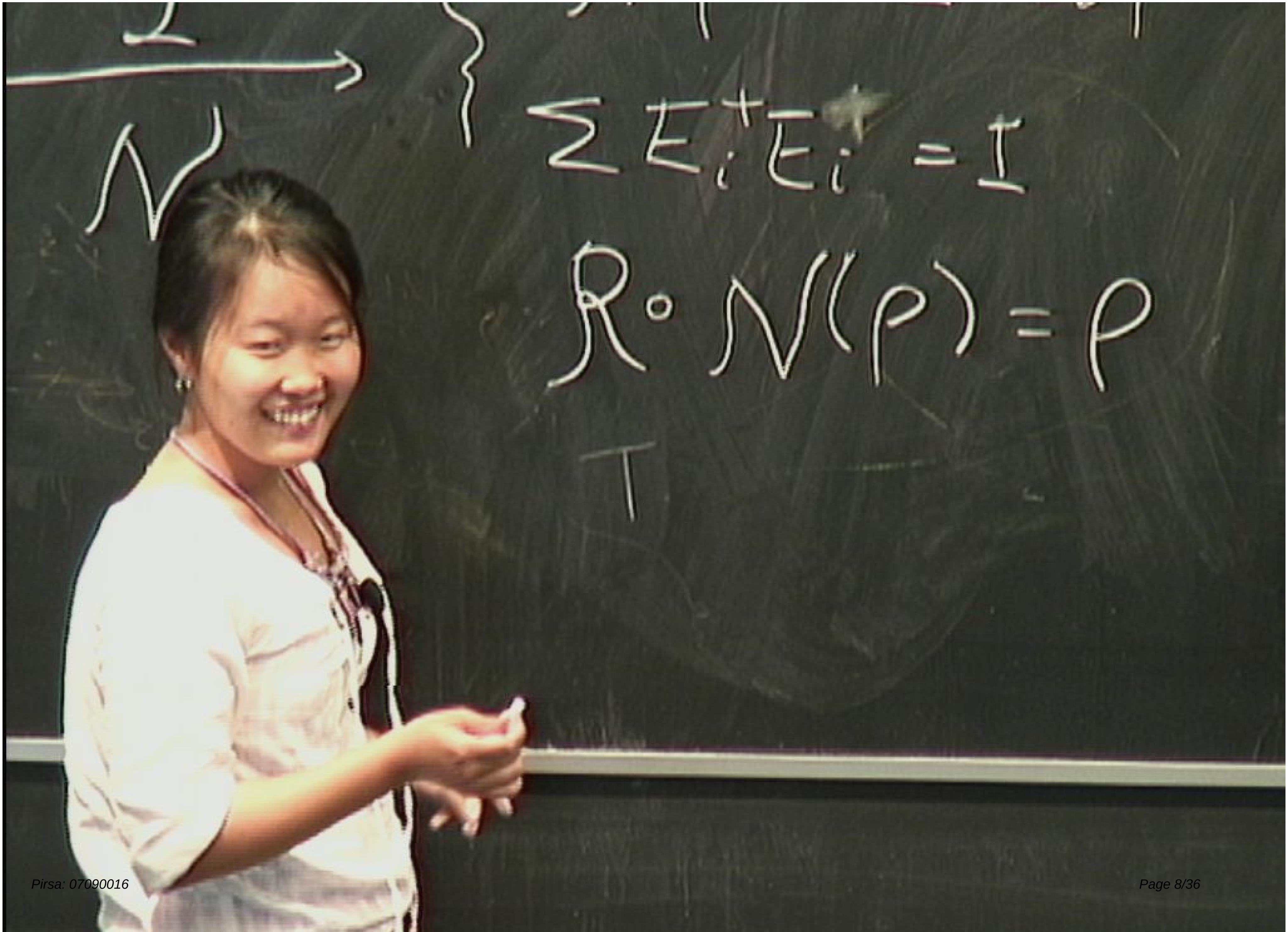
$$\mathcal{N}(\rho) = \sum E_i \rho E_i$$

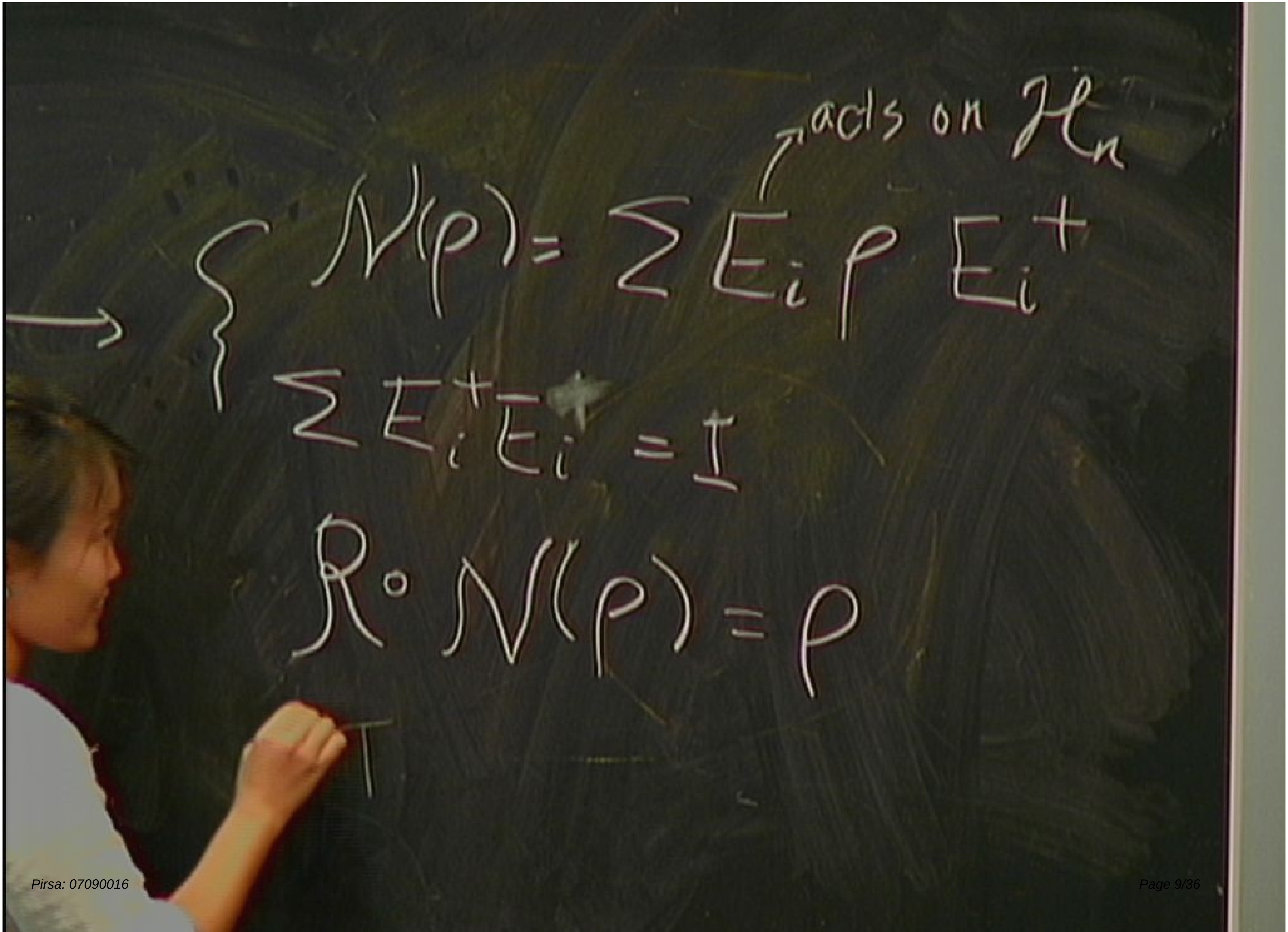
acts on $\mathcal{H}_n$

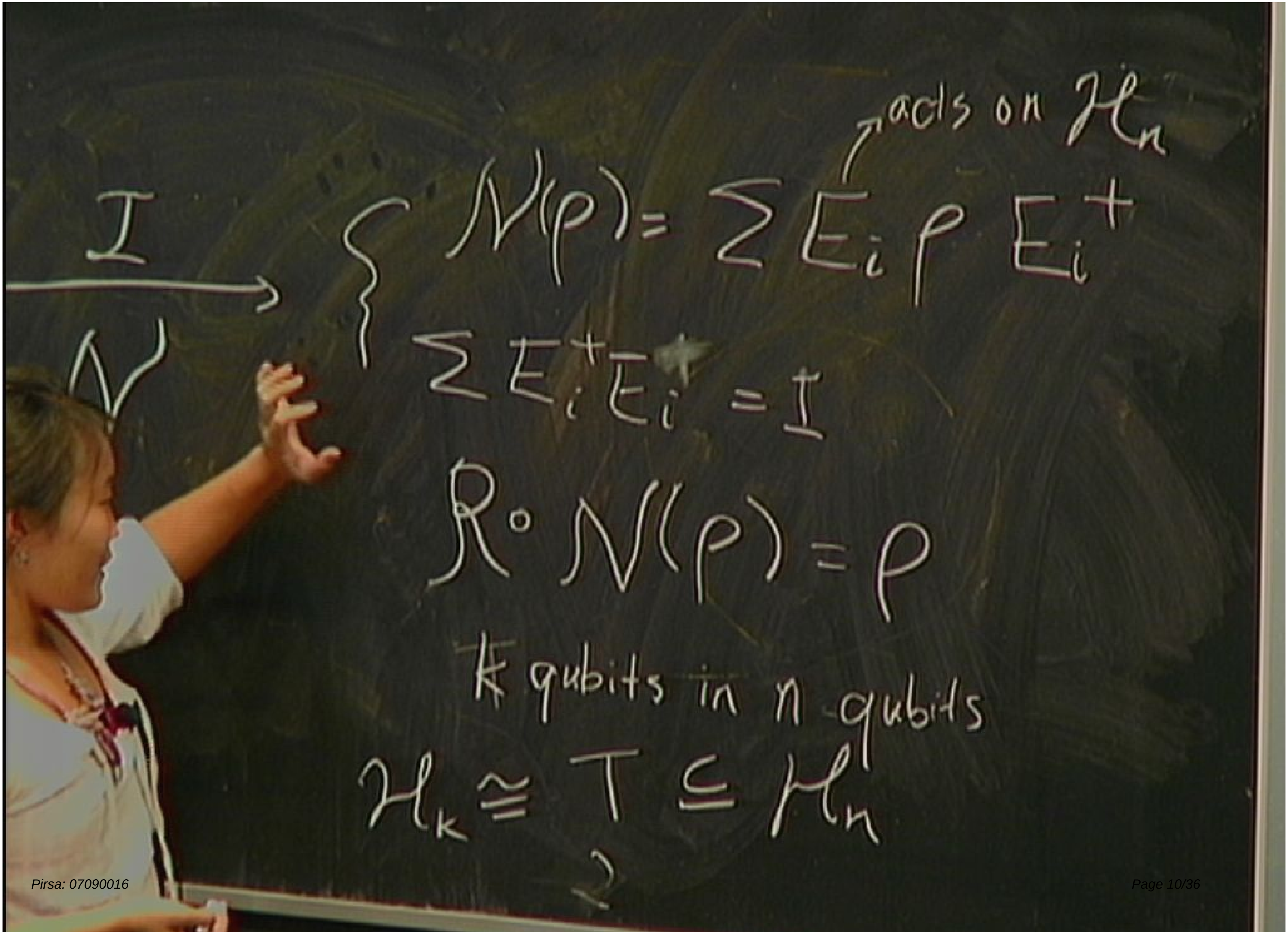
acts on $\mathcal{H}_n$

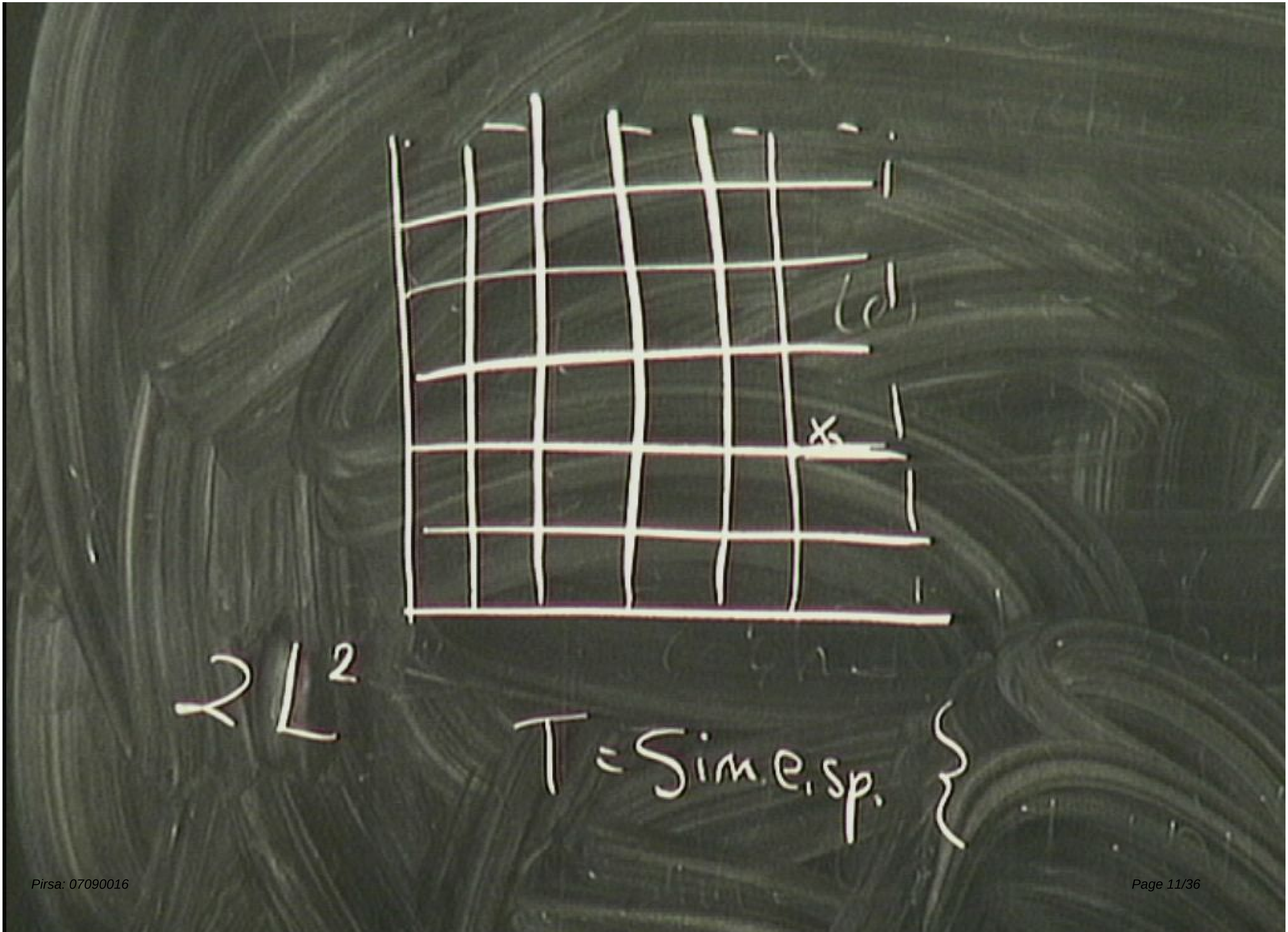
$$\mathcal{N}(\rho) = \sum E_i \, \rho \, E_i^{+}$$

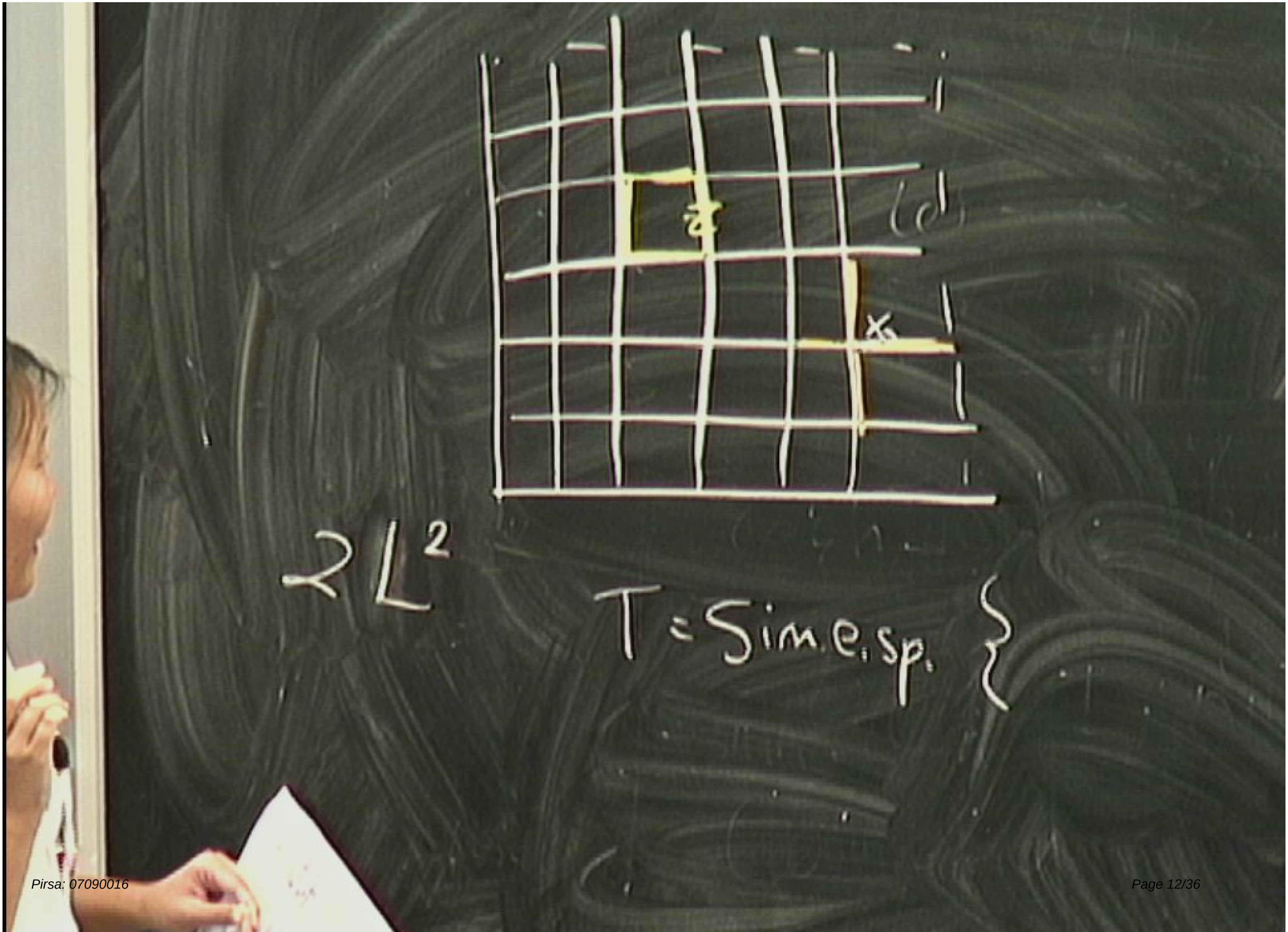
$$\rightarrow \quad \sum E_i E^{+} = I$$

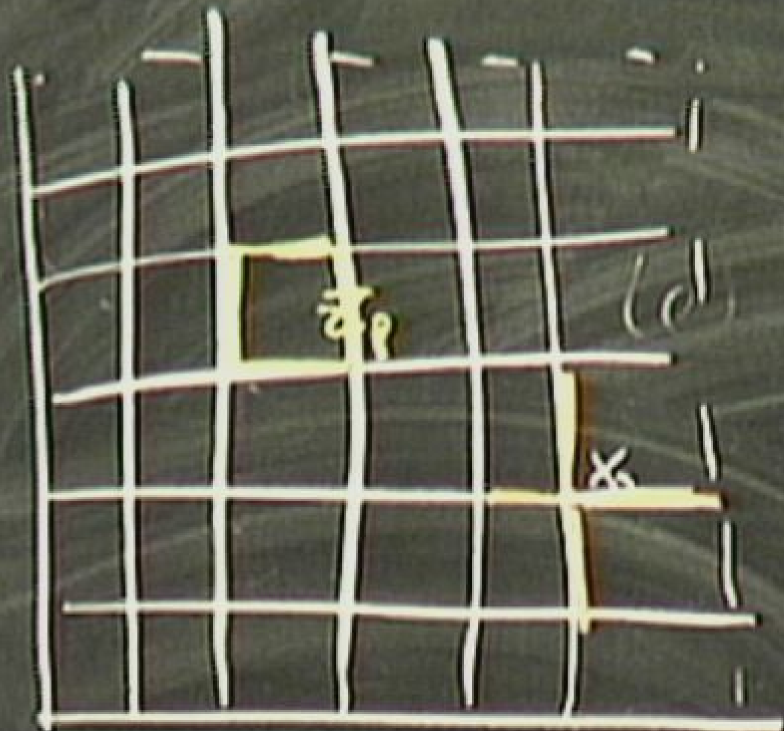

$\mathcal{N}$
$\sum E_i^\dagger E_i = I$
$R \circ \mathcal{N}(\rho) = \rho$
T


acts on $\mathcal{H}_n$
$N(\rho) = \sum E_i \rho E_i^{+}$
$\sum E_i^{+} E_i = \underline{I}$
$R \circ N(\rho) = \rho$
T


acts on $\mathcal{H}_n$
I
N
$N(\rho) = \sum E_i \rho E_i^{\dagger}$
$\sum E_i^{\dagger} E_i = I$
$R \circ N(\rho) = \rho$
k qubits in n qubits
$\mathcal{H}_k \cong T \subseteq \mathcal{H}_n$

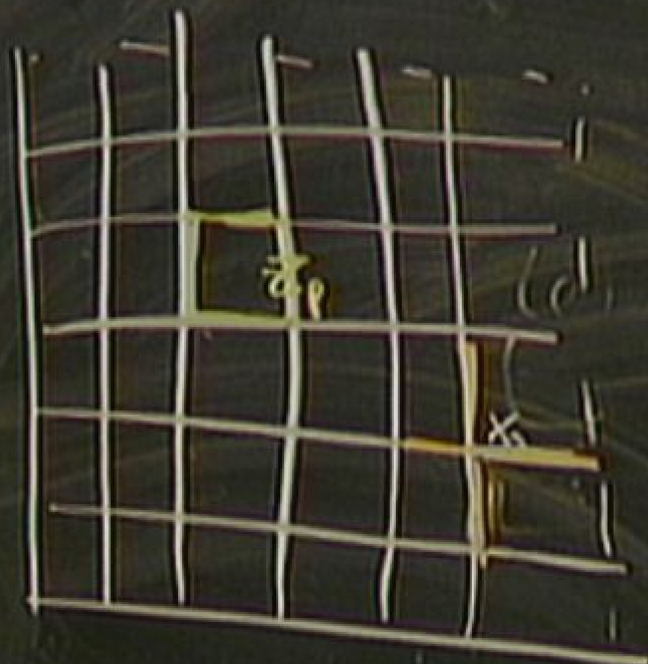

$2L^2$
T = Sim.e.sp.


2L²
T = Sim.e.sp.



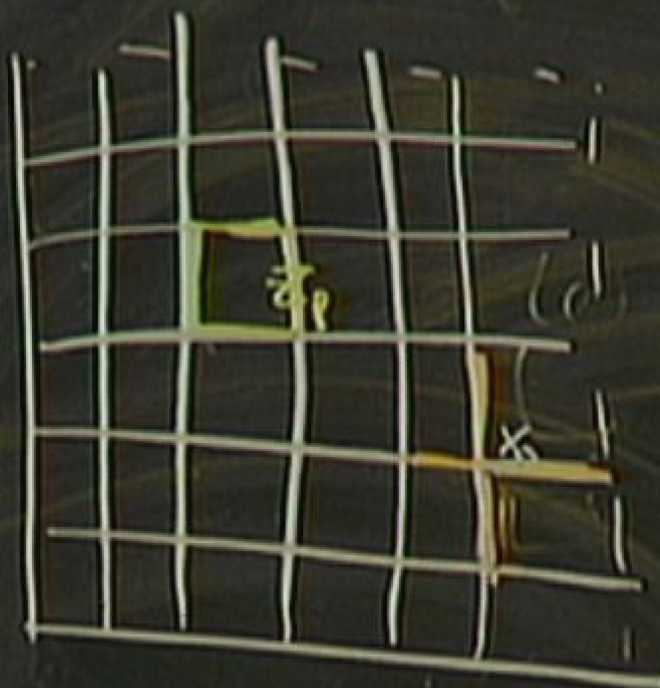
$2L^2$

$$T = \text{Sim.e.sp.}\left\{\{X_s\}_s \cup \{Z_p\}_p\right\}$$



$2L^2$

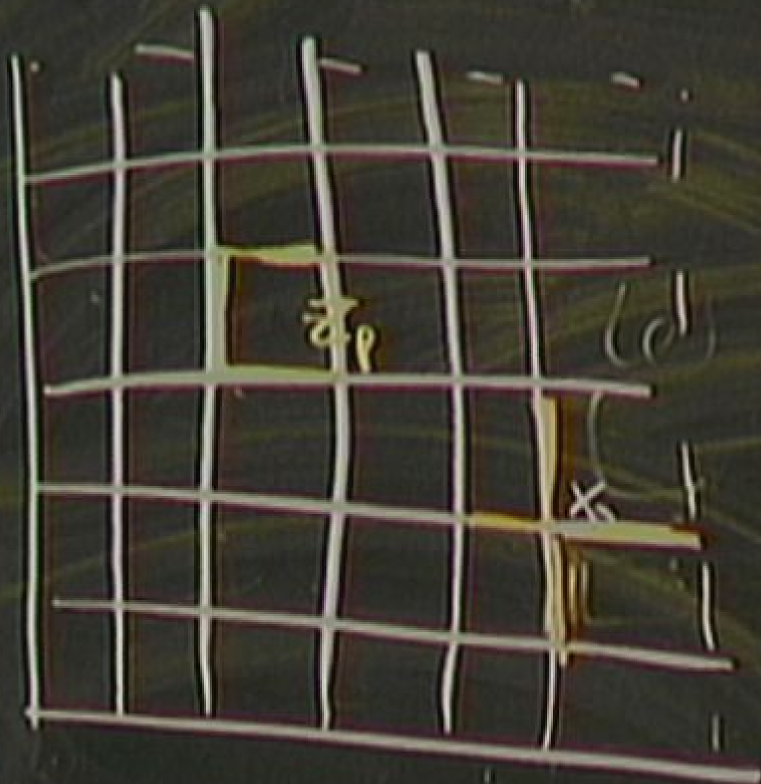
$T = \mathrm{Sim.e.sp.}\{\{X_s\}_s \cup \{Z_p\}_p\}$



$$2L^2$$

$$T = \mathrm{Sim.e.sp.}\left\{ \{X_s\}_s \cup \{Z_p\}_p \right\}$$

$$k = n - r$$

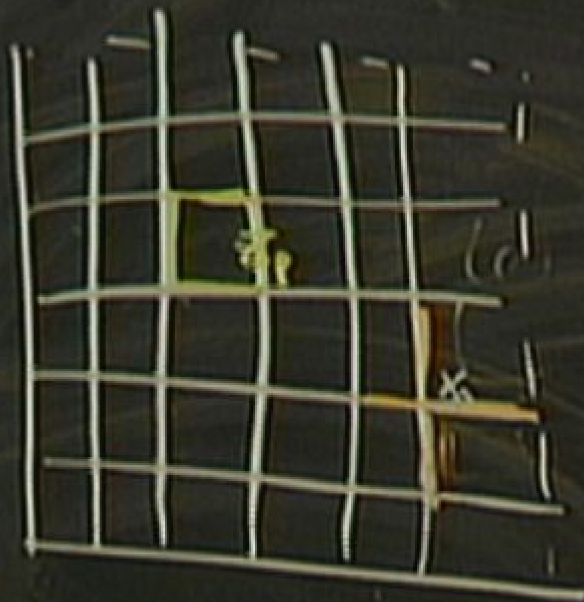


$2L^2$

$$T = \mathrm{Sim.e.sp.}\left\{ \{X_s\}_s \cup \{Z_p\}_p \right\}$$

$$k = n - r$$

$$= \#E - (\#V + \#F)$$

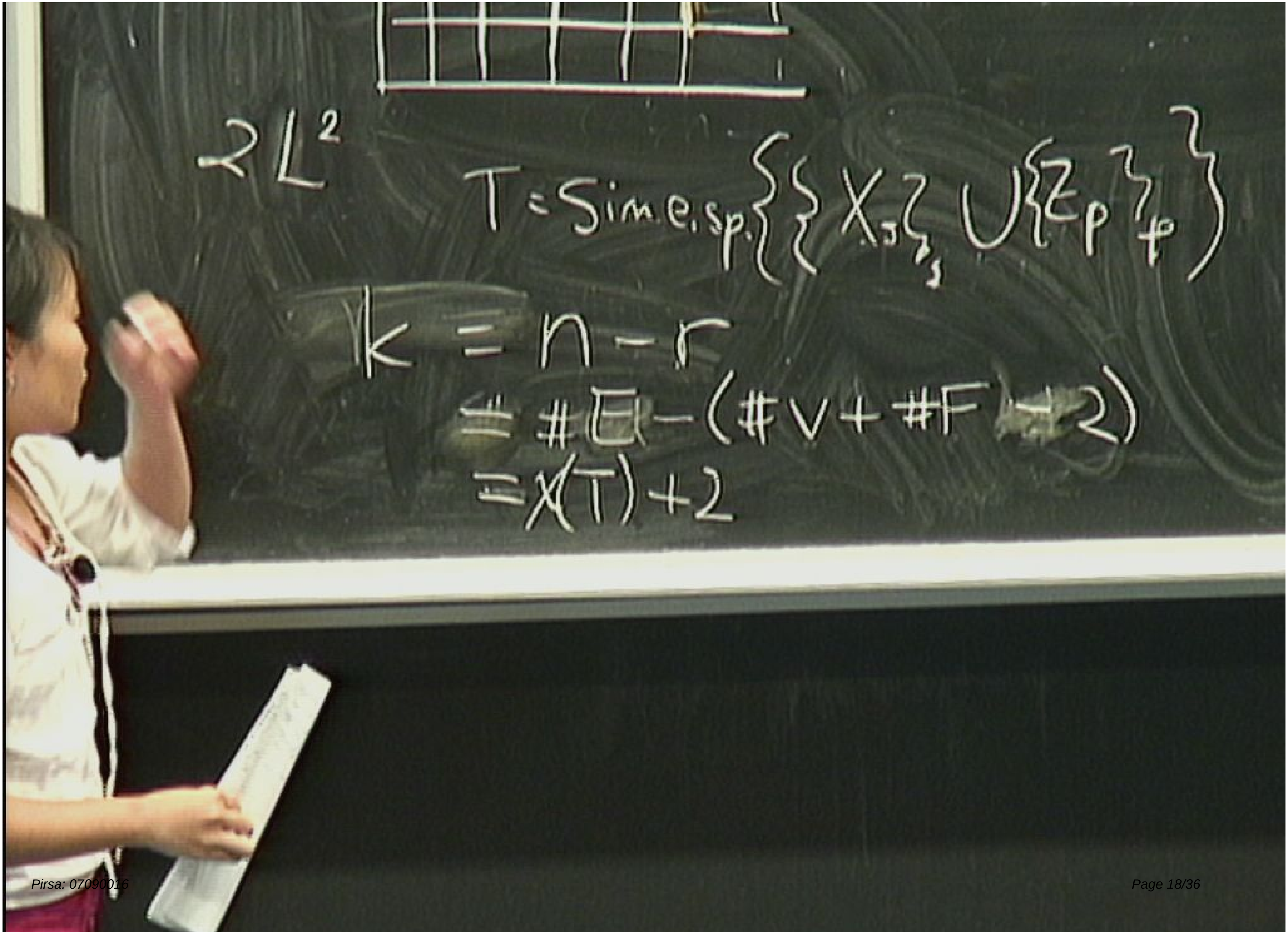


$2L^2$

$T = \mathrm{Simp.sp.}\left(\{\{X_j\}_j \cup \{Z_p\}_p\}\right)$

$k = n - r$

$= \#E - (\#V + \#F - 2)$


2L^2
k = n - r
= #E - (#V + #F - 2)
= χ(T) + 2

$\chi = 2-g$

$k = g$

$\chi = 2-2g$

$k = 2g$

F basis $\mathcal{L}(H_n)$

(

$*$ (

F basis $\mathcal{L}(H_n)$

$\cup|$

F'

$\rho_n = \langle F \rangle$

$*$ (

F basis $\mathcal{L}(H_n)$

$\cup\!|$

F'

$P_n = \langle F \rangle$ $\quad \underbrace{\text{"}M|\psi\rangle = |\psi\rangle}$ $\forall M \in S$

$S \leq P_n$

F basis $\mathcal{L}(H_n)$

$\cup\!|$

F'

$\mathcal{P}_n = \langle F \rangle$ $\underbrace{M|\psi\rangle = |\psi\rangle}$ $\forall M \in S$

$S \leq \mathcal{P}_n \iff$ abelian subgr of $\mathcal{P}_n$

is stabilizer $-I \notin S$

F basis $\mathcal{L}(H_n)$

$\cup\!|$

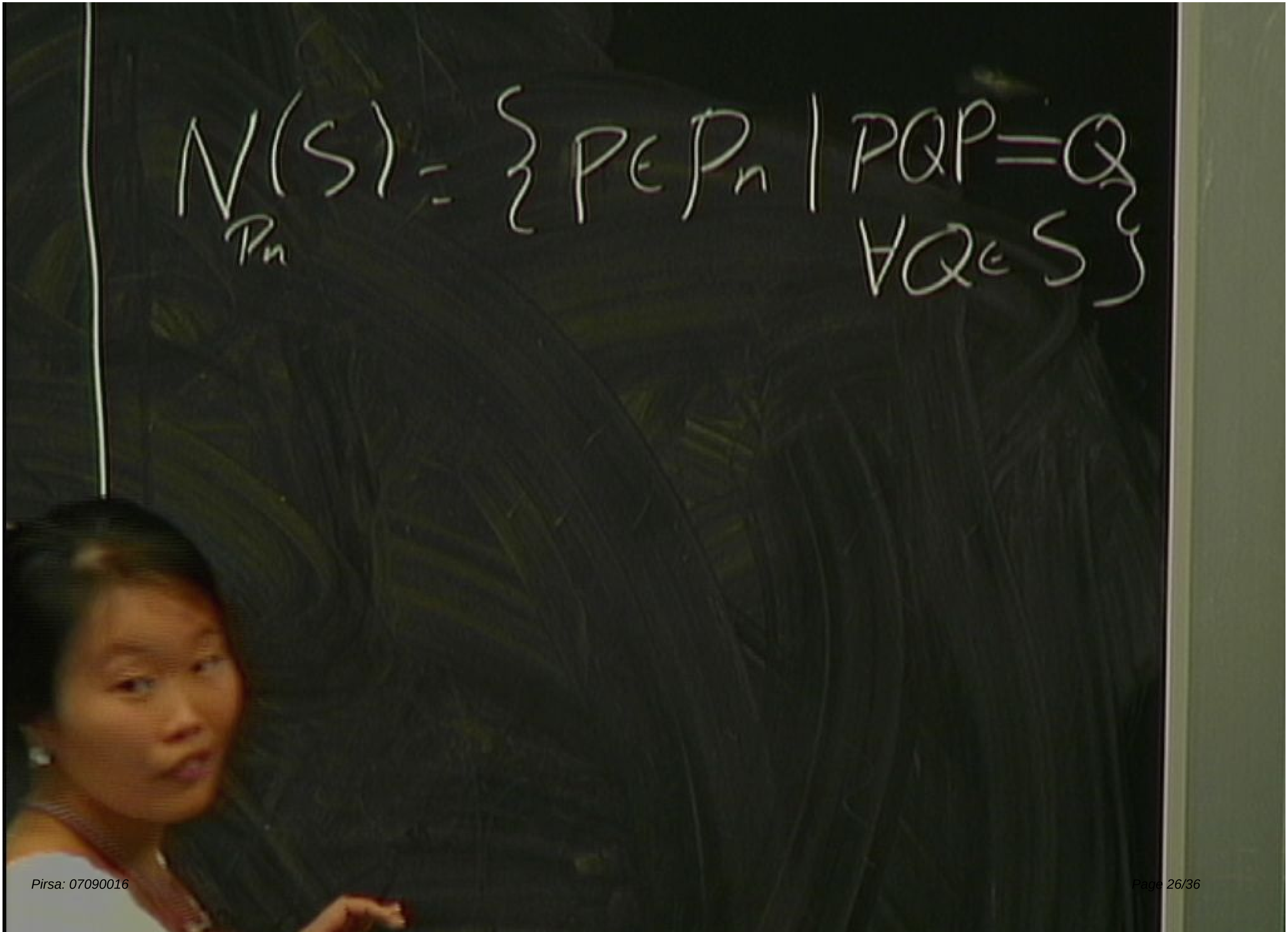
F'

T is

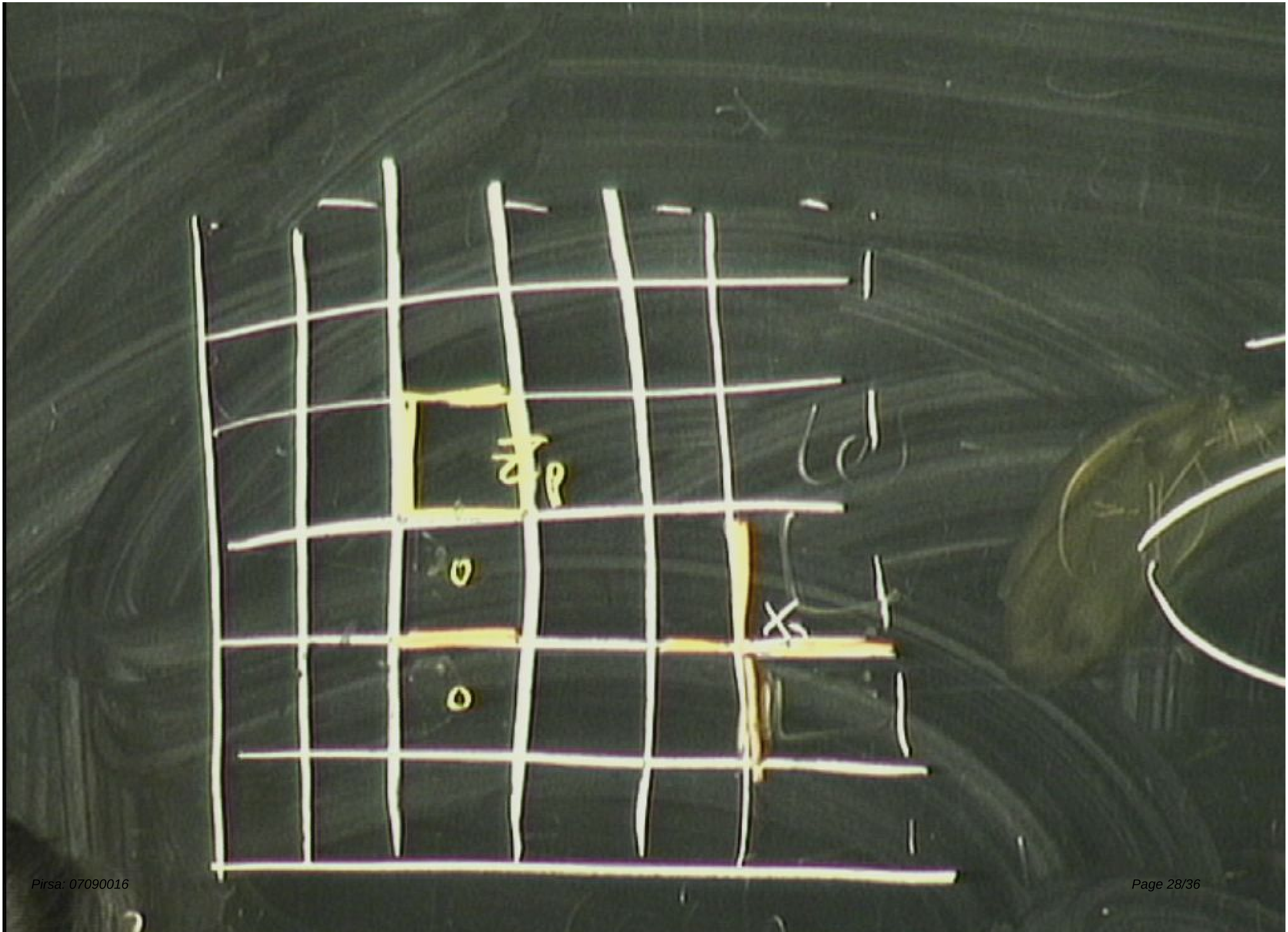
$\mathcal{P}_n = \langle F \rangle$ "$\underbrace{M|\psi\rangle = |\psi\rangle}$ $\forall M \in S$

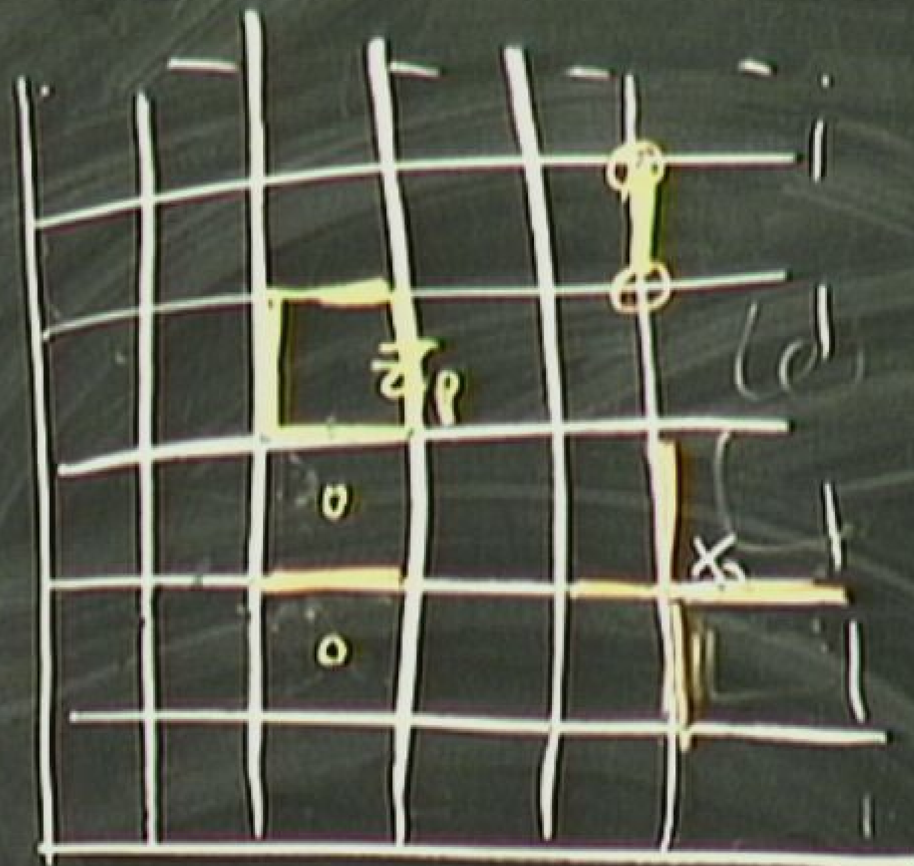
$S \leq \mathcal{P}_n$ is stabilizer $\iff$ abelian subgr of $\mathcal{P}_n$

$-I \notin S$


$N_{\mathcal{P}_n}(S) = \{ P \in \mathcal{P}_n \mid PQP = Q \ \forall Q \in S \}$

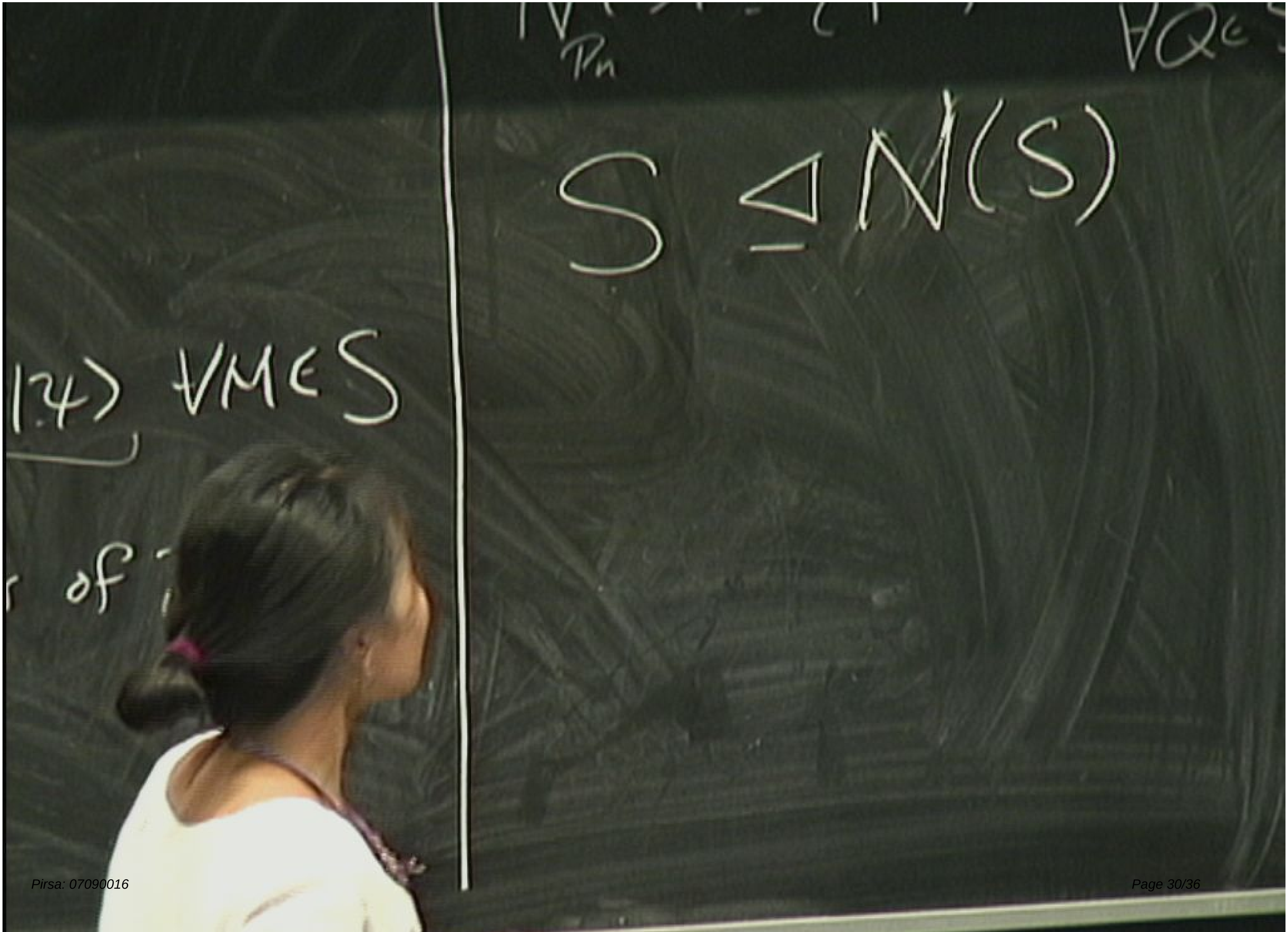
$$N_{P_n}(S) = \{ P \in P_n \mid PQP^{-1} = Q \;\; \forall Q \in S \}$$





$2L^2$

$T = \mathrm{Sim.e.sp.}\{\{X_s\}_s$


$|\psi\rangle$ $\forall M \in S$
$S \trianglelefteq N(S)$
of

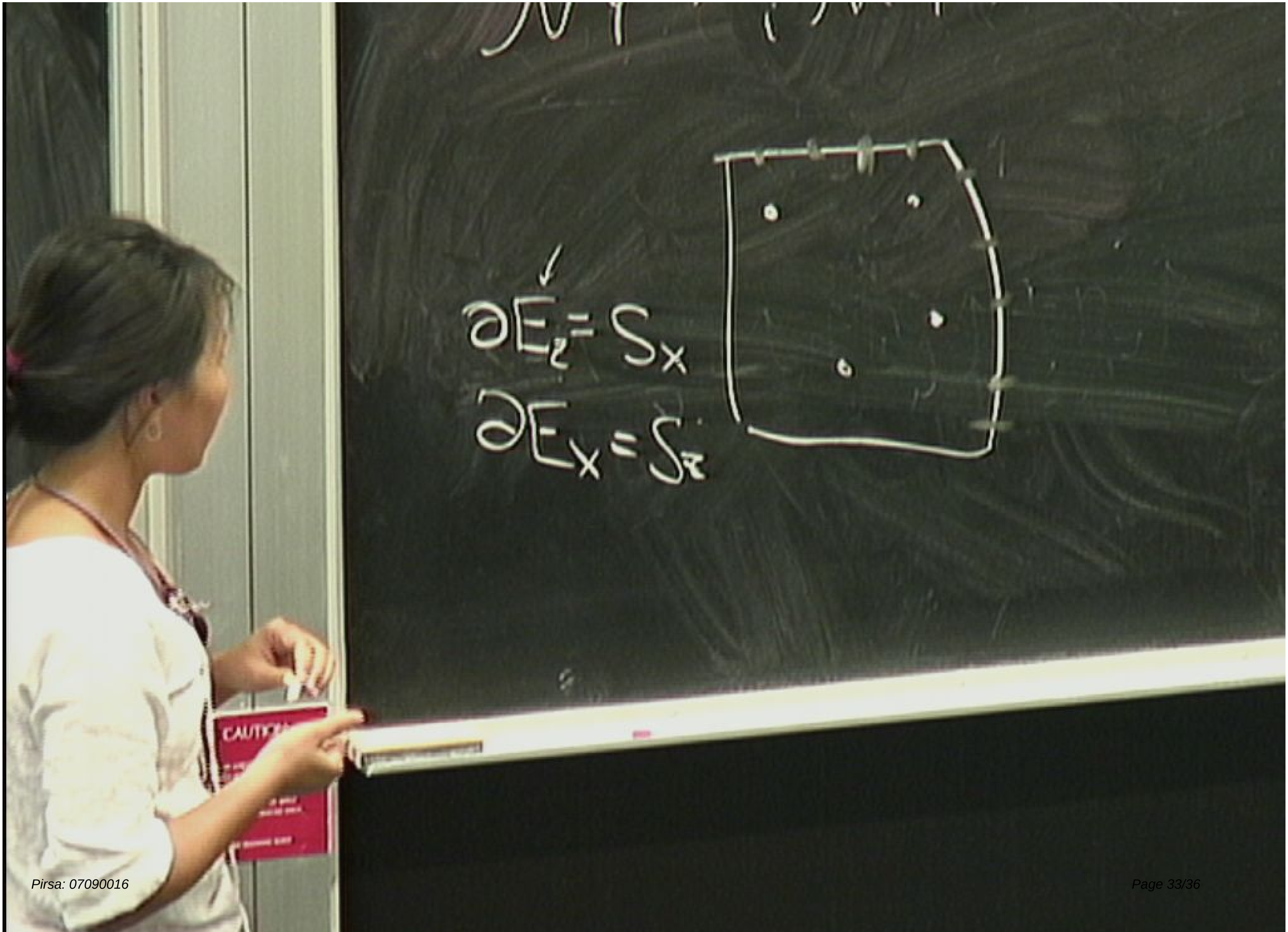
$$N_{\mathcal{P}_n}(S) = \{ P \in \mathcal{P}_n \mid PQP^{-1} = Q \;\; \forall Q \in S \}$$

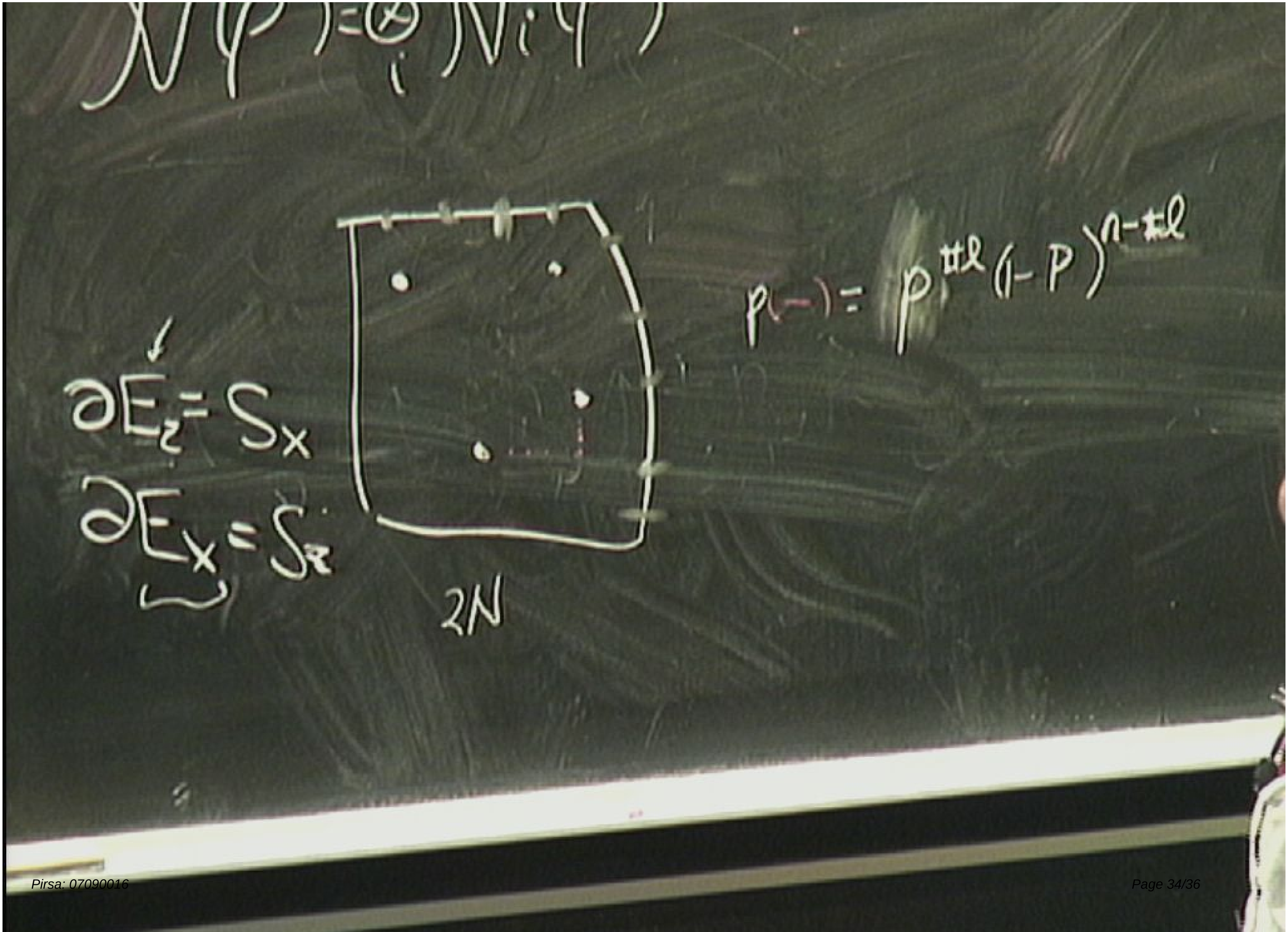
$$S \trianglelefteq N(S)$$

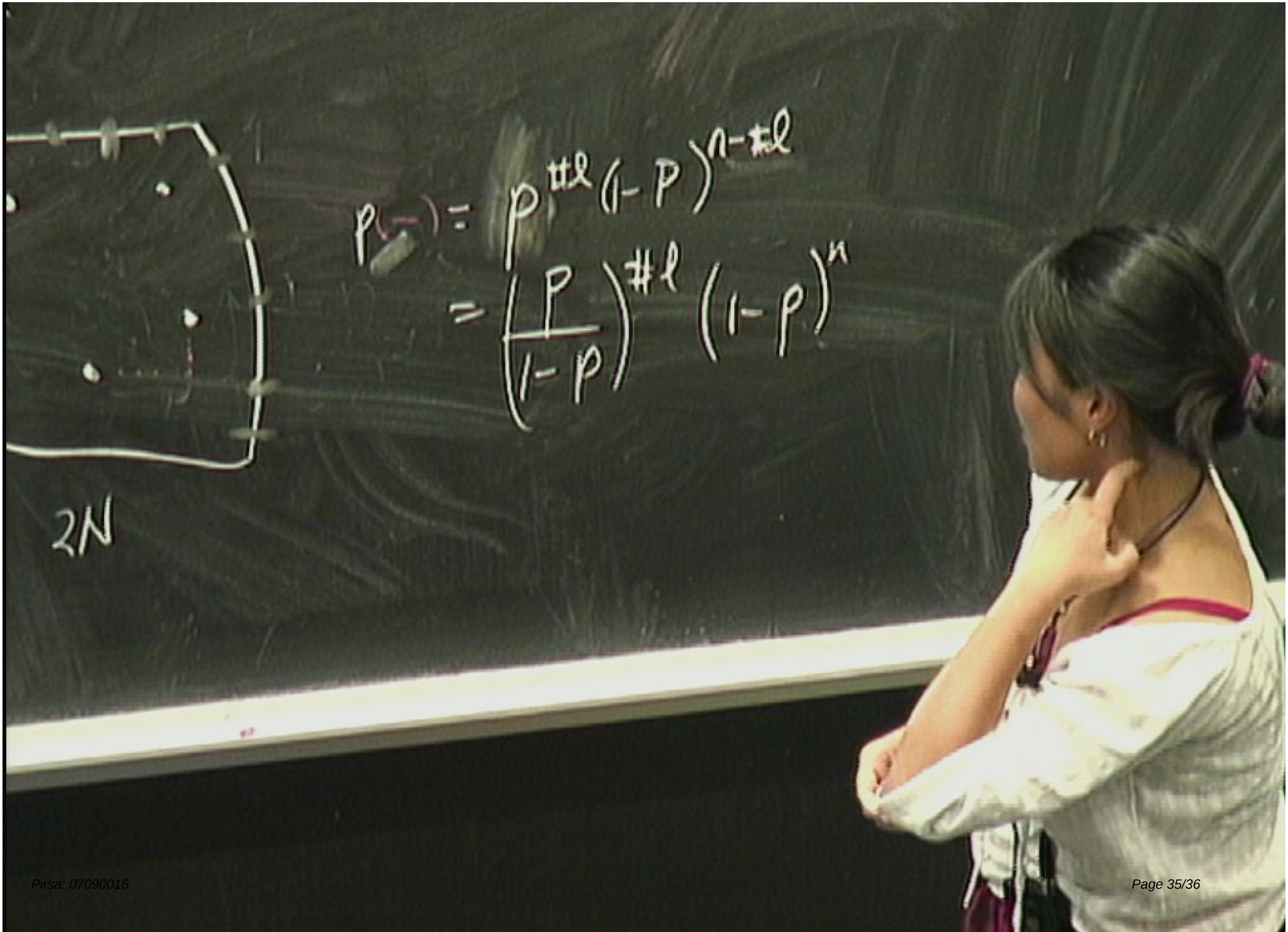
$N \setminus S \rightarrow$ cannot correct

$$\mathcal{N}_i(\rho) = (1-2p)\rho + pX\rho X + pZ\rho Z$$

$$\mathcal{N}(\rho) = \bigotimes_i \mathcal{N}_i(\rho)$$

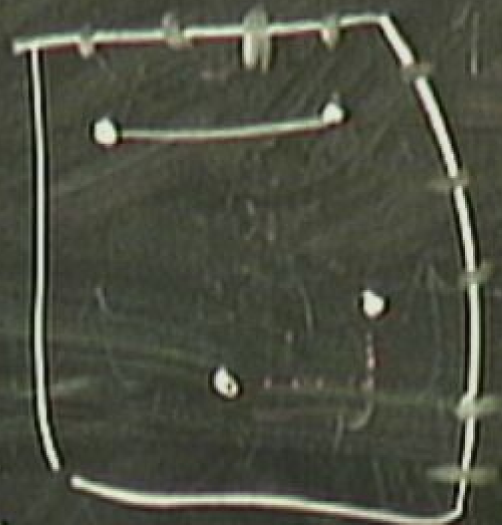

$\partial E_z = S_x$
$\partial E_x = S_z$


$\mathcal{N}(\rho)=\bigotimes_i \mathcal{N}_i(\rho)$
$p(-)= p^{\#\ell}(1-p)^{n-\#\ell}$
$\partial E_z = S_x$
$\partial E_x = S_z$
2N


$P(\text{—}) = p^{\#l}(1-p)^{n-\#l}$
$= \left(\frac{p}{1-p}\right)^{\#l}(1-p)^{n}$
2N

$\partial E_z = S_x$

$\partial E_x = S_z$



$2N$

$$p(\cdot) = p^{\#\ell}(1-p)^{n-\#\ell} = \left(\frac{p}{1-p}\right)^{\#\ell}(1-p)^{n}$$